Computer Algebra
Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/339-7.3.4

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**537**]. This is test number [339].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (537)	0.00 (0)
Mathematica	99.44 (534)	0.56 (3)
Maple	94.60 (508)	5.40 (29)
Maxima	50.09 (269)	49.91 (268)
Fricas	48.60 (261)	51.40 (276)
Mupad	32.96 (177)	67.04 (360)
Giac	32.96 (177)	67.04 (360)
Reduce	32.96 (177)	67.04 (360)
Sympy	26.82 (144)	73.18 (393)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

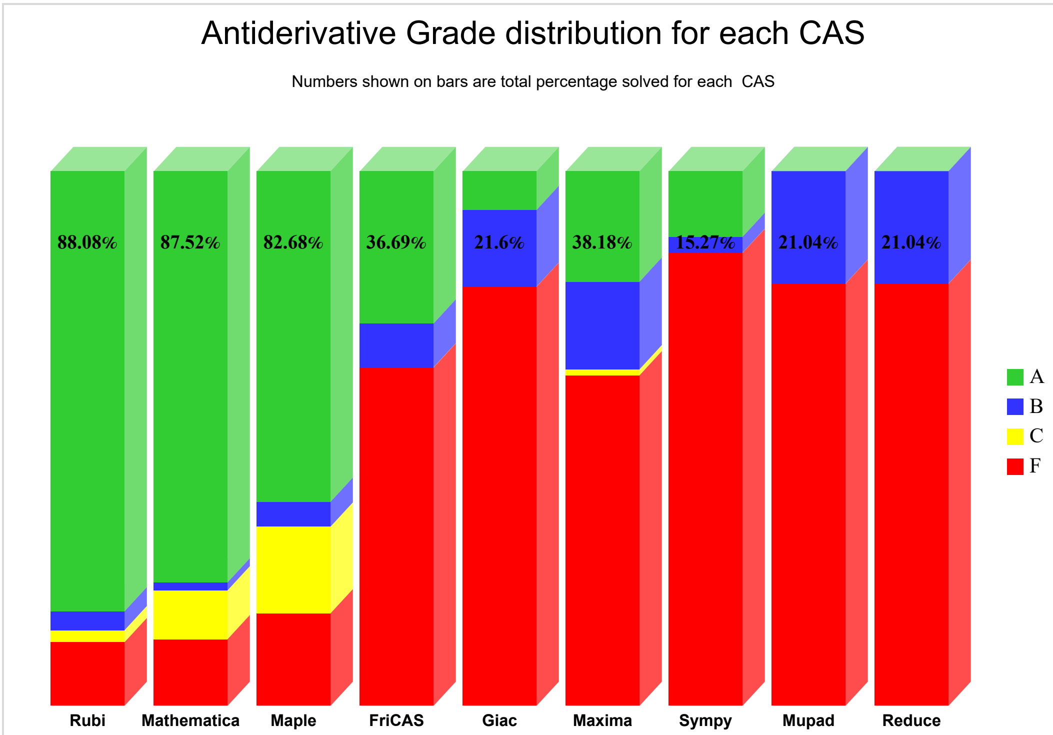
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

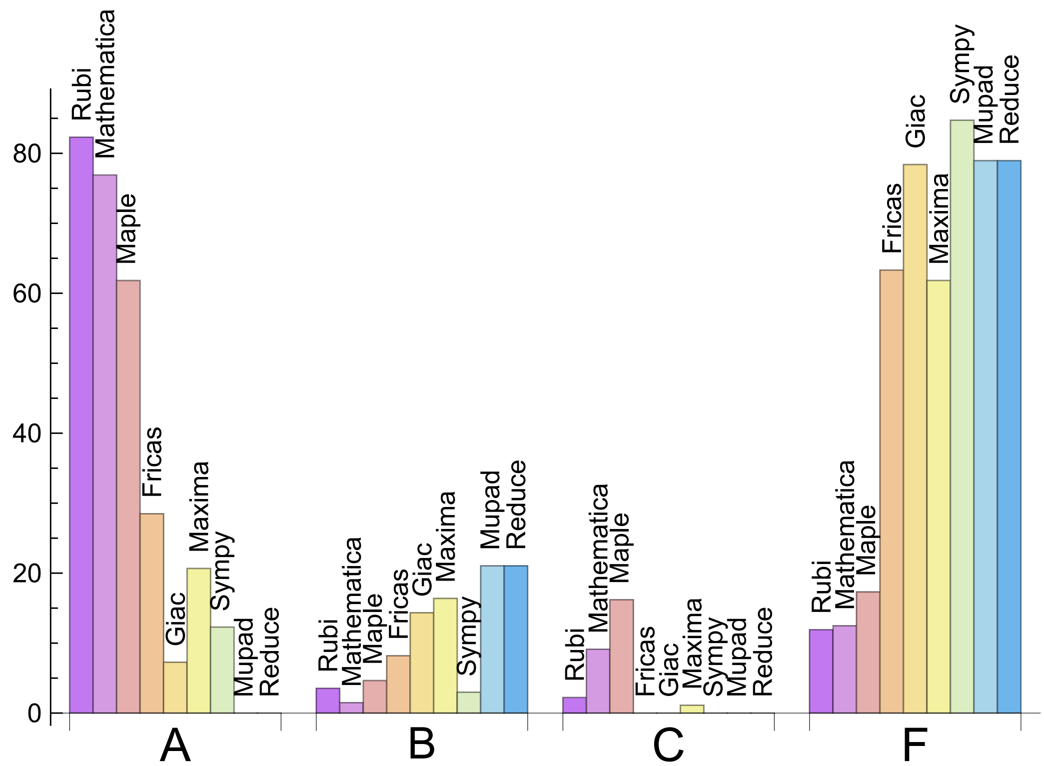
System	% A grade	% B grade	% C grade	% F grade
Rubi	82.309	3.538	2.235	11.918
Mathematica	76.909	1.490	9.125	12.477
Maple	61.825	4.655	16.201	17.318
Fricas	28.492	8.194	0.000	63.315
Maxima	20.670	16.387	1.117	61.825
Sympy	12.291	2.980	0.000	84.730
Giac	7.263	14.339	0.000	78.399
Mupad	0.000	21.043	0.000	78.957
Reduce	0.000	21.043	0.000	78.957

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	29	100.00	0.00	0.00
Maxima	268	98.88	0.00	1.12
Fricas	276	98.91	0.00	1.09
Giac	360	86.67	0.00	13.33
Mupad	360	0.00	100.00	0.00
Reduce	360	100.00	0.00	0.00
Sympy	393	98.22	1.78	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Maxima	0.09
Reduce	0.17
Giac	0.23
Mathematica	0.74
Rubi	0.88
Sympy	3.56
Mupad	3.94
Maple	5.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	93.83	1.33	49.00	1.14
Reduce	102.77	1.50	73.00	1.32
Fricas	122.03	1.60	91.00	1.18
Mupad	145.72	1.52	67.00	1.14
Mathematica	156.43	1.04	101.50	0.98
Rubi	163.23	1.13	135.00	1.00
Maxima	197.73	2.03	134.00	1.49
Giac	211.31	2.24	122.00	1.49
Maple	406.23	2.43	137.00	1.06

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

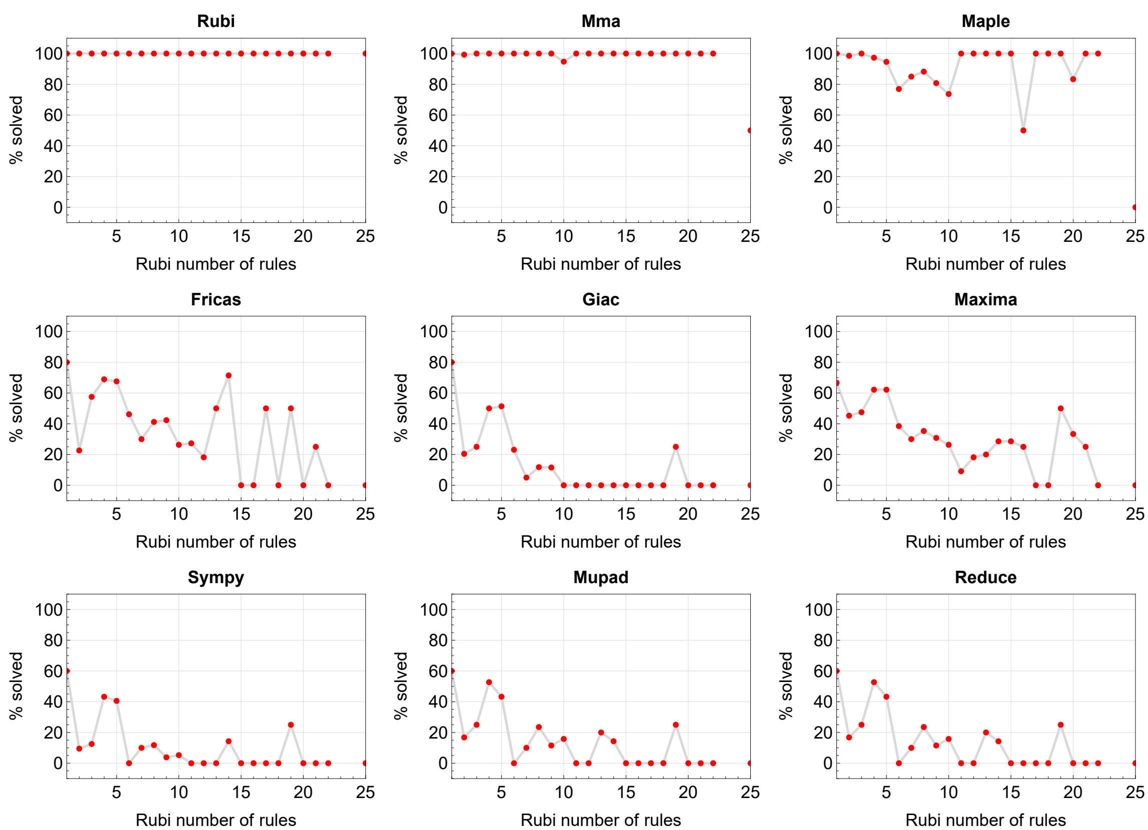


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

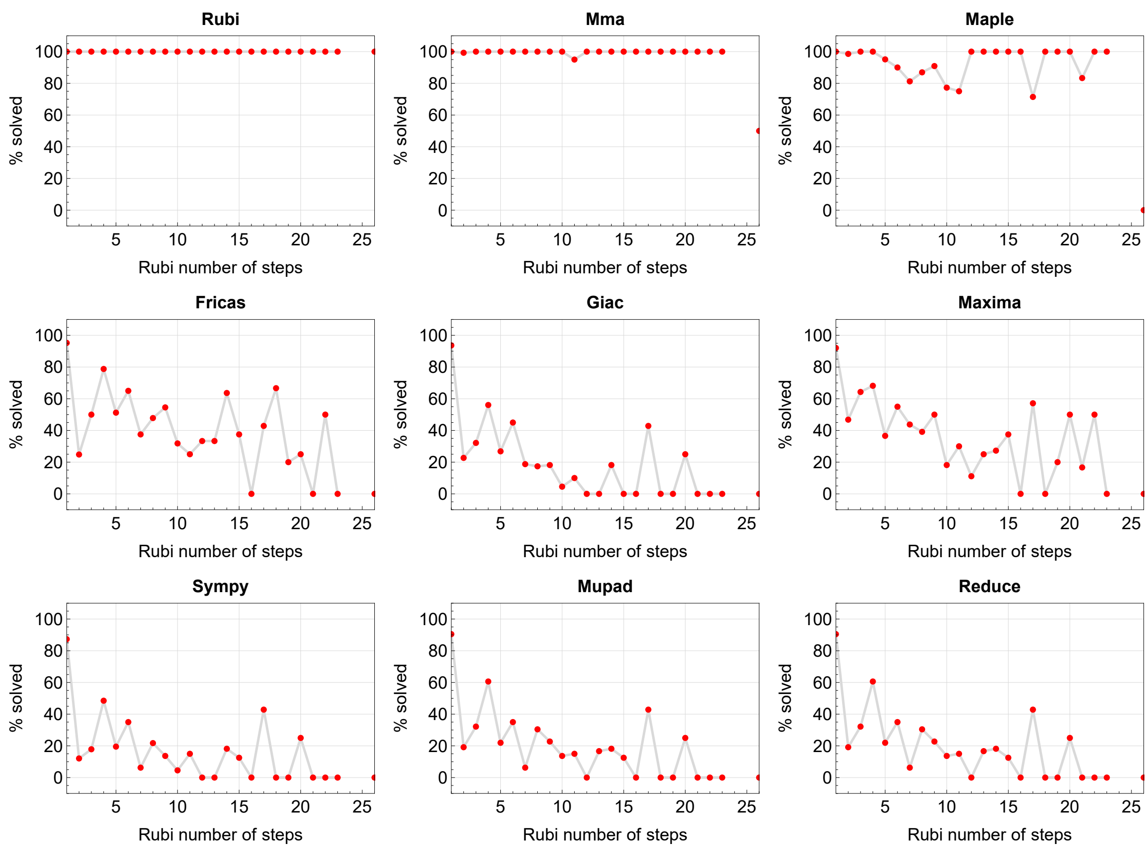


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

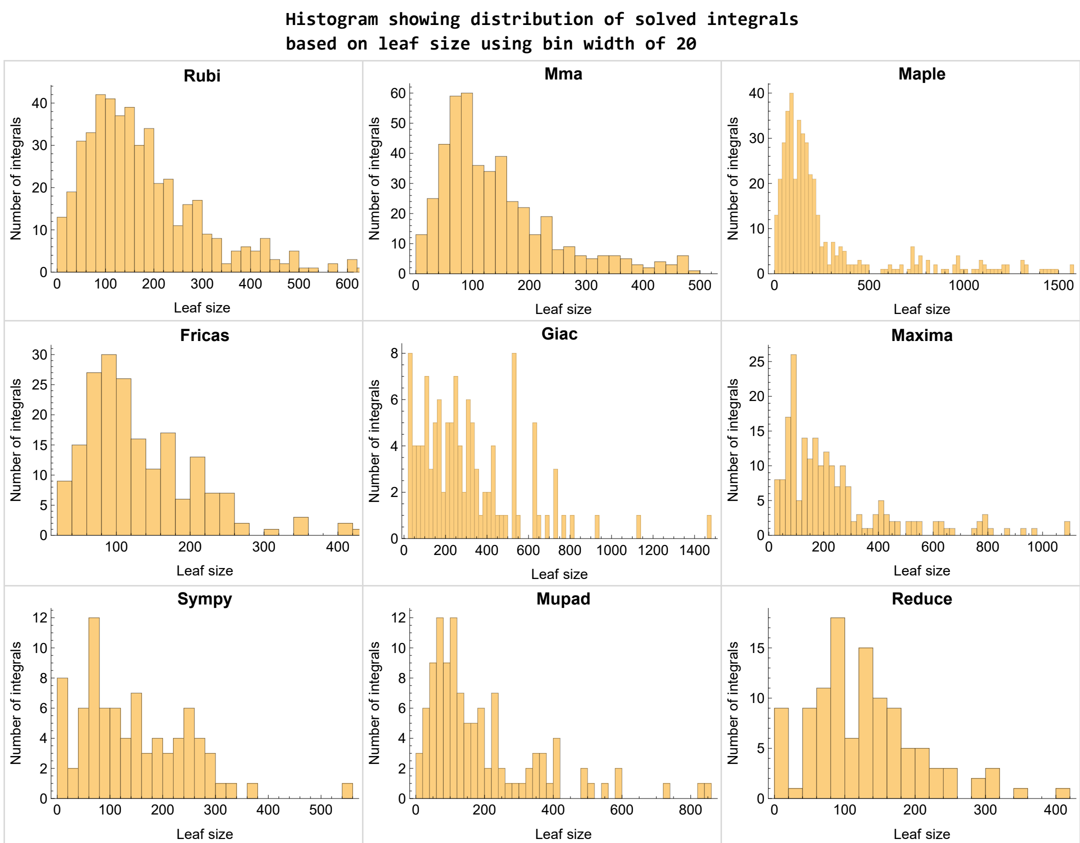


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

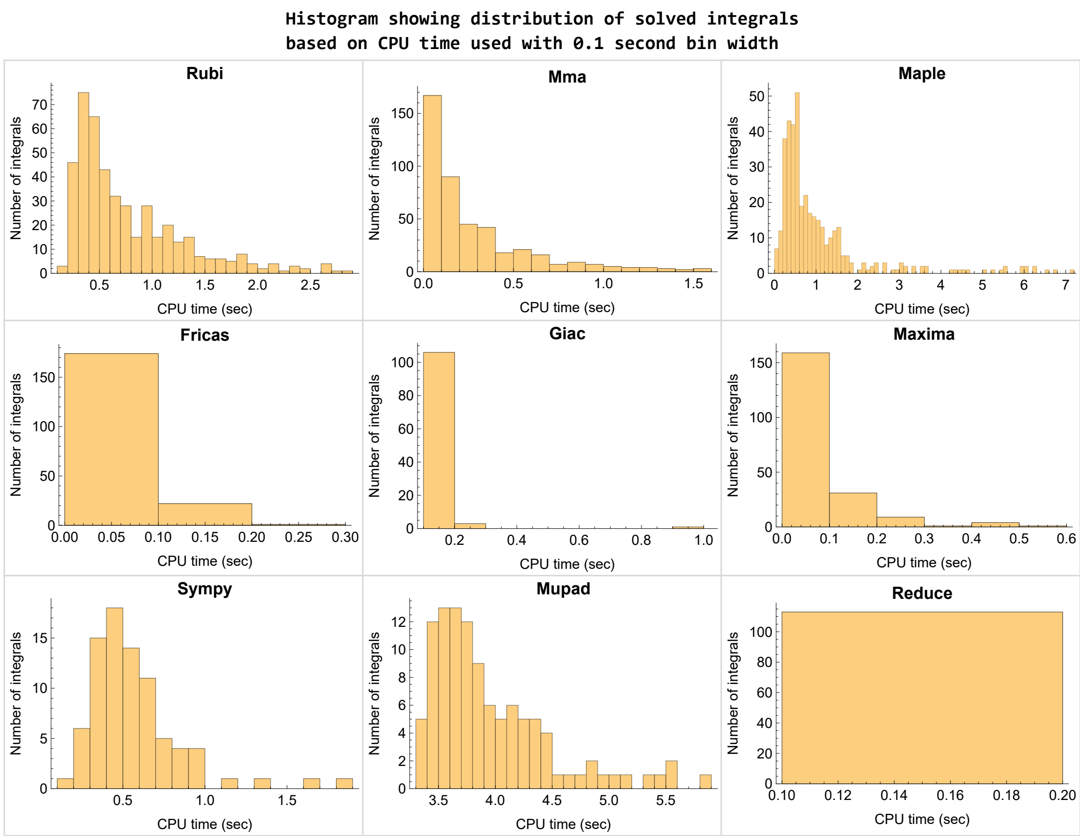


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

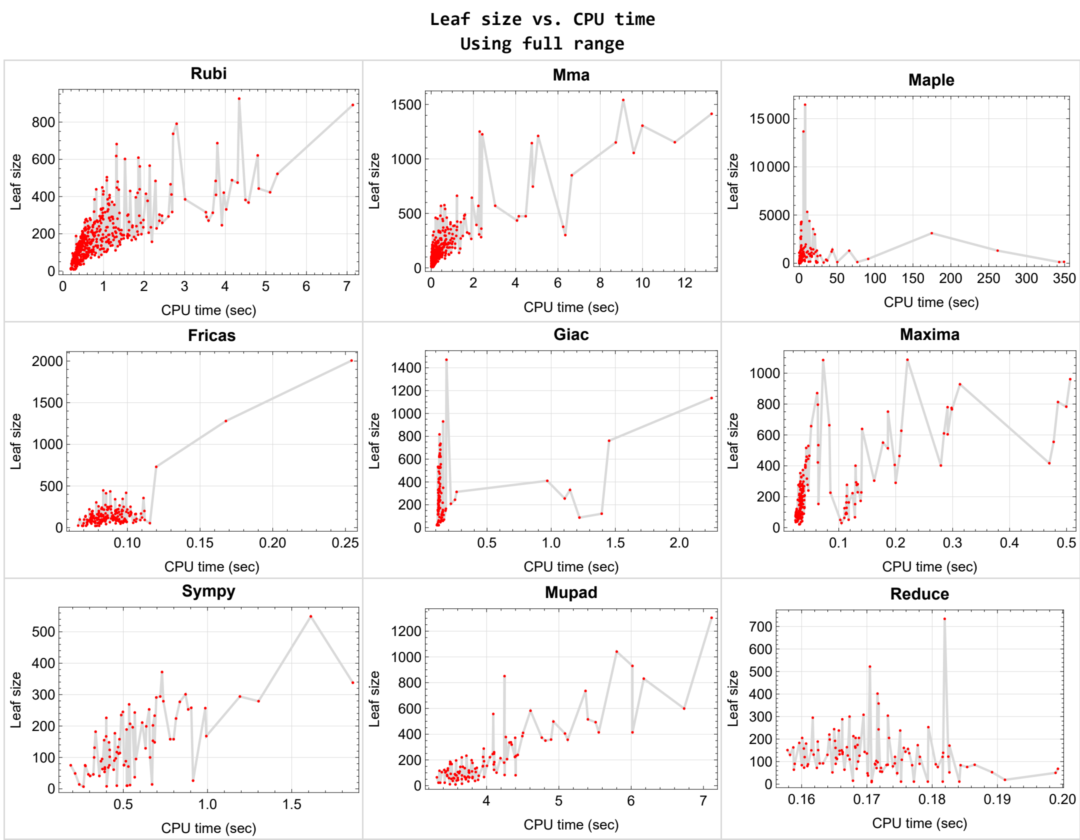


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 249, 251, 252, 254, 255, 257, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 479, 480, 485, 486, 491, 492, 507, 508, 534}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {511, 527, 529, 531}

Mathematica {150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 386, 394, 401, 402, 409, 426, 435, 437, 446, 454, 456, 458, 502, 503, 504, 505, 532, 533, 535, 536}

Maple {72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 135, 136, 137, 138, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 235, 236, 238, 239, 240, 242, 243, 245, 246, 266, 270, 271, 272, 273, 277, 312, 313, 316, 318, 497, 526, 533}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

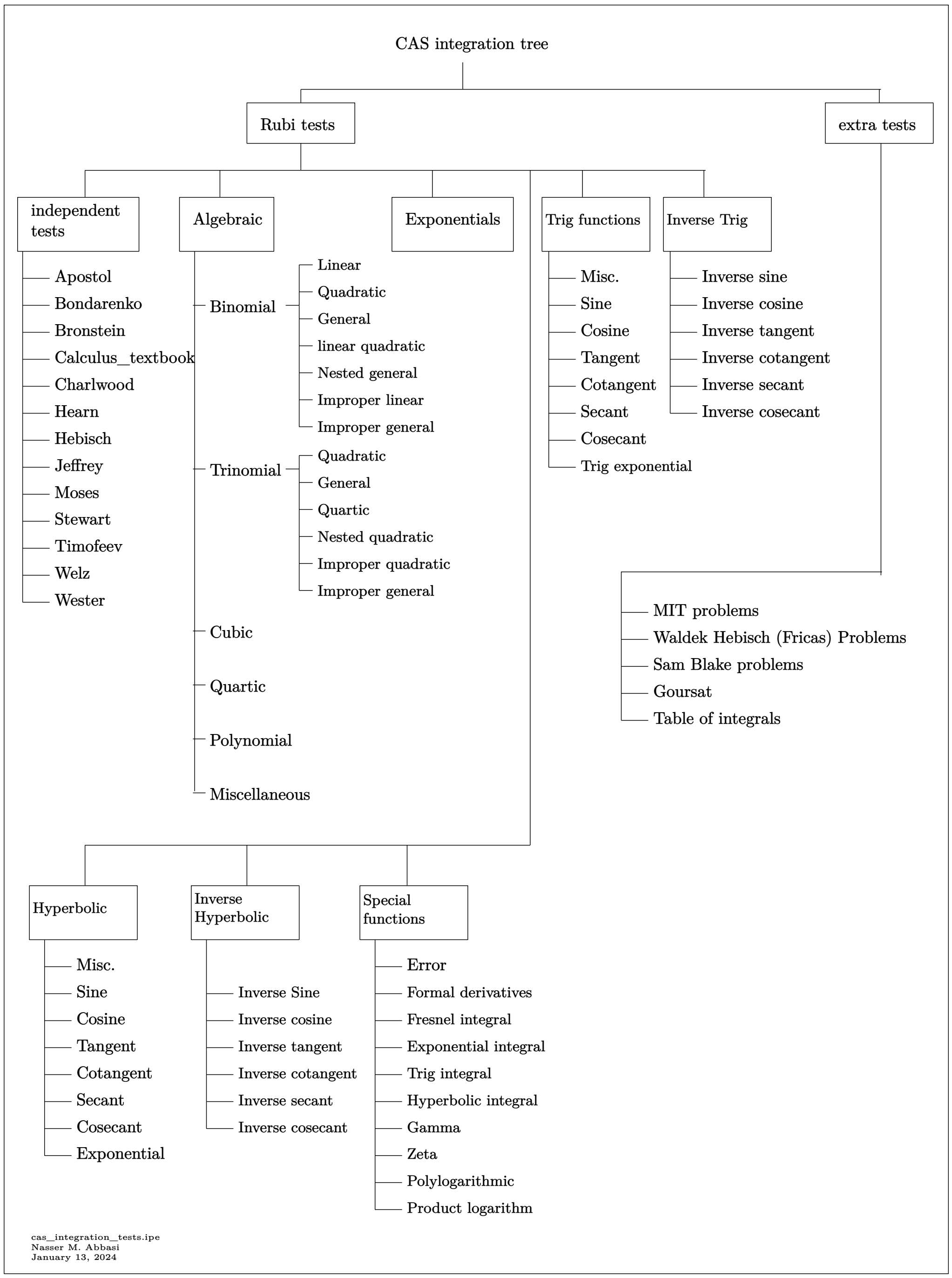
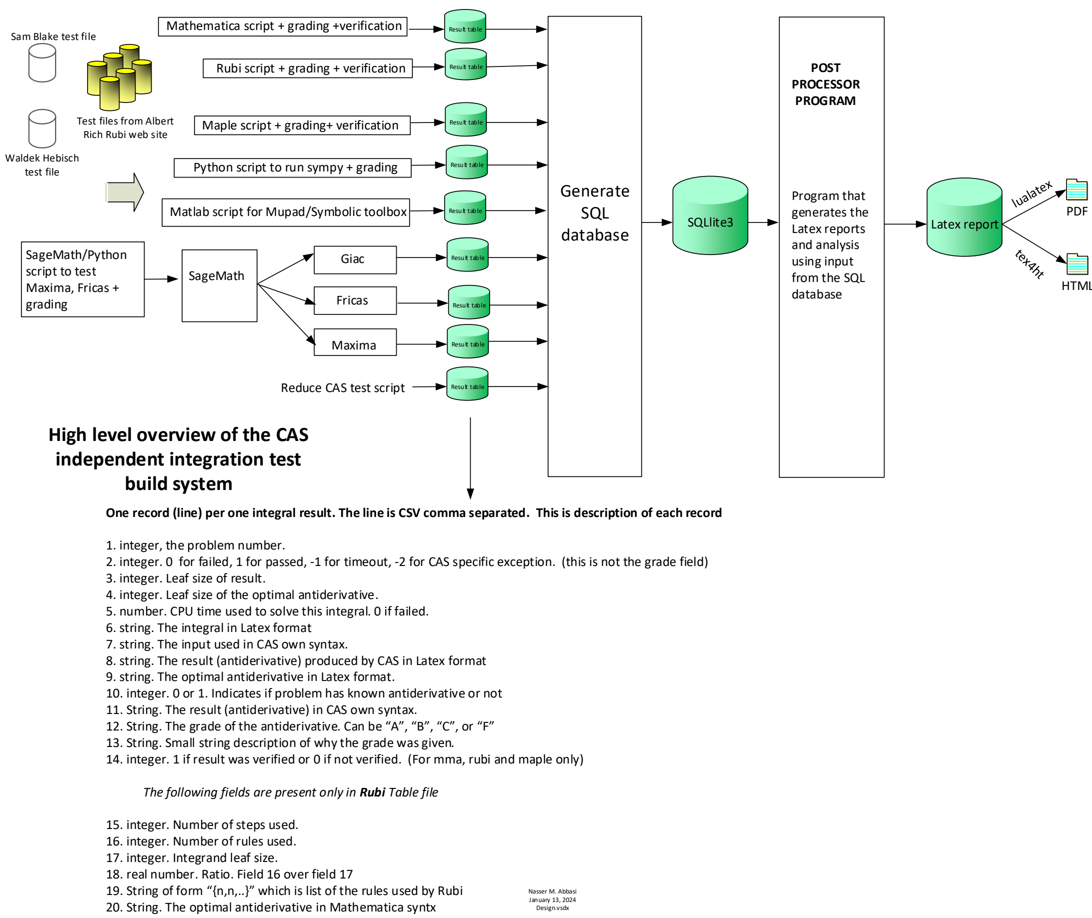


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 253, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 283, 284, 285, 288, 290, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 333, 334, 339, 340, 342, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 401, 404, 405, 406, 407, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 437, 440, 441, 442, 443, 444, 446, 450, 451, 452, 453, 454, 455, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537 }

B grade { 172, 173, 174, 289, 331, 332, 336, 337, 338, 343, 344, 436, 438, 439, 447, 448, 449, 456, 458 }

C grade { 280, 377, 379, 384, 385, 386, 400, 402, 408, 409, 410, 445 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 111, 112, 113, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 250, 253, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 506, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 528, 530, 537 }

B grade { 4, 120, 121, 383, 405, 504, 527, 529 }

C grade { 72, 73, 80, 81, 82, 88, 89, 90, 91, 99, 100, 101, 102, 108, 109, 110, 116, 117, 132, 133, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 212, 240, 246, 270, 272, 278, 312, 319, 502, 505, 532, 533, 535, 536 }

F normal fail { 518, 526, 531 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 107, 114, 115, 118, 124, 125, 126, 127, 134, 147, 148, 149, 150, 151, 152, 153, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 181, 182, 184, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 214, 215, 216, 224, 225, 227, 228, 229, 230, 232, 237, 241, 244, 248, 250, 253, 256, 258, 259, 260, 261, 262, 264, 265, 267, 268, 269, 274, 275, 276, 278, 279, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 317, 319, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 371, 372, 373, 375, 377, 378, 379, 384, 385, 386, 389, 390, 391, 392, 396, 398, 399, 400, 402, 406, 407, 408, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 434, 435, 436, 437, 439, 441, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 476, 477, 478, 481, 482, 483, 484, 487, 493, 498, 499, 500, 501, 502, 505, 506, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 537 }

B grade { 47, 67, 98, 133, 139, 140, 231, 233, 247, 263, 306, 393, 394, 401, 409, 488, 489, 490, 494, 495, 496, 503, 504, 532, 536 }

C grade { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 135, 136, 137, 138, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 235, 236, 238, 239, 240, 242, 243, 245, 246, 266, 270, 271, 272, 273, 277, 312, 313, 316, 318, 364, 366, 368, 388, 427, 429, 448, 450, 497, 526, 533 }

F normal fail { 280, 320, 348, 374, 376, 380, 381, 382, 383, 397, 404, 405, 438, 440, 442, 443, 444, 471, 475, 509, 510, 511, 512, 527, 528, 529, 530, 531, 535 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 119, 124, 125, 126, 137, 138, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 228, 230, 232, 237, 244, 253, 256, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 344, 345, 346, 347, 364, 366, 368, 371, 388, 390, 391, 393, 398, 399, 406, 407, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 468, 469, 470, 472, 473, 474, 476, 477, 478, 498, 499, 500, 501, 506, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 537 }

B grade { 248, 250, 258, 283, 284, 285, 288, 289, 290, 293, 294, 295, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 509, 510, 511, 512 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 306, 312, 313, 318, 319, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 505, 513, 514, 518, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { 280, 320, 348 }

Maxima

A grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 14, 18, 19, 20, 21, 24, 25, 30, 31, 35, 36, 42, 54, 62, 66, 69, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 182, 184, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 214, 215, 216, 224, 225, 227, 259, 261, 302, 304, 307, 310, 364, 366, 368, 371, 388, 390, 391, 393, 398, 399, 406, 407, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 469, 470, 498, 499, 500, 501, 503, 505, 506, 515, 516, 517, 524, 526 }

B grade { 4, 13, 17, 22, 23, 28, 29, 32, 33, 34, 37, 40, 41, 61, 67, 71, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 107, 114, 115, 118, 124, 125, 126, 166, 181, 228, 229, 230, 231, 232, 233, 235, 237, 239, 244, 250, 253, 256, 260, 262, 263, 264, 265, 267, 268, 269, 271, 274, 275, 276, 303, 305, 306, 308, 309, 311, 313, 314, 315, 316, 317, 345, 346, 347, 468, 472, 473, 474, 504, 509, 510, 511, 512, 519, 520 }

C grade { 502, 521, 522, 523, 525, 537 }

F normal fail { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 68, 70, 72, 73, 74, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 236, 238, 240, 241, 242, 243, 245, 246, 247, 248, 258, 266, 270, 272, 273, 277, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 312, 318, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 513, 514, 527, 528, 529, 530, 531, 532, 533, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { 278, 319, 518 }

Giac

A grade { 54, 61, 62, 107, 114, 115, 124, 125, 230, 237, 244, 253, 256, 258, 268, 269, 275, 276, 368, 390, 391, 462, 463, 464, 468, 469, 470, 509, 510, 511, 512, 515, 516, 517, 521, 523, 524, 525, 537 }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 66, 76, 78, 118, 126, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 248, 250, 261, 262, 302, 304, 308, 310, 314, 316, 371, 393, 498, 499, 500, 501, 506 }

C grade { }

F normal fail { 5, 6, 14, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 307, 309, 311, 312, 313, 315, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 372, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 385, 386, 389, 392, 394, 397, 398, 399, 400, 401, 402, 405, 406, 407, 408, 409, 410, 414, 419, 424, 426, 428, 438, 440, 447, 449, 467, 472, 473, 474, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 505, 513, 514, 518, 519, 520, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { 364, 366, 373, 380, 388, 396, 404, 413, 415, 418, 420, 423, 425, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 441, 442, 443, 444, 445, 446, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 465, 466, 471, 475, 522 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 126, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 228, 230, 232, 237, 244, 248, 250, 253, 256, 258, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 345, 346, 347, 498, 499, 500, 501, 506, 519, 520, 521, 522, 523, 524, 525, 537 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 306, 312, 313, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 505, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 41, 42, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 228, 230, 232, 237, 244, 248, 250, 253, 256, 261, 498, 499, 500, 501, 506, 521, 522, 523, 524, 525 }

B grade { 4, 13, 17, 23, 28, 34, 40, 54, 61, 62, 66, 258, 264, 302, 304, 307 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 505, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 526, 527, 528, 529, 530, 531, 537 }

F(-1) timedout fail { 416, 421, 504, 532, 533, 535, 536 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 126, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 224, 228, 230, 232, 237, 244, 248, 250, 253, 256, 258, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 345, 346, 347, 498, 499, 500, 501, 506, 519, 520, 521, 522, 523, 524, 525, 537 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 306, 312, 313, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 505, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	99	97	92	121	114	124	491	103	103
N.S.	1	0.92	0.90	0.85	1.12	1.06	1.15	4.55	0.95	0.95
time (sec)	N/A	0.307	0.042	0.232	0.026	0.085	0.419	0.124	0.171	3.429

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	89	87	84	110	102	112	394	92	92
N.S.	1	0.93	0.91	0.88	1.15	1.06	1.17	4.10	0.96	0.96
time (sec)	N/A	0.317	0.040	0.222	0.031	0.092	0.473	0.124	0.163	3.504

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	81	79	76	99	93	100	305	84	83
N.S.	1	0.96	0.94	0.90	1.18	1.11	1.19	3.63	1.00	0.99
time (sec)	N/A	0.289	0.033	0.162	0.031	0.081	0.327	0.120	0.160	3.453

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	46	95	59	85	77	75	211	64	65
N.S.	1	1.05	2.16	1.34	1.93	1.75	1.70	4.80	1.45	1.48
time (sec)	N/A	0.231	0.013	0.155	0.028	0.076	0.188	0.119	0.159	3.312

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	0	0	0	0	49	0
N.S.	1	1.00	0.95	1.22	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.264	0.023	0.173	0.000	0.000	0.000	0.000	0.168	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	87	0	0	0	0	66	0
N.S.	1	1.00	1.01	1.24	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.289	0.053	0.227	0.000	0.000	0.000	0.000	0.174	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	52	76	76	89	89	95	192	79	75
N.S.	1	0.93	1.36	1.36	1.59	1.59	1.70	3.43	1.41	1.34
time (sec)	N/A	0.262	0.050	0.214	0.025	0.090	0.374	0.124	0.174	3.455

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	87	86	86	99	101	117	306	87	110
N.S.	1	0.89	0.88	0.88	1.01	1.03	1.19	3.12	0.89	1.12
time (sec)	N/A	0.309	0.052	0.214	0.029	0.094	0.462	0.120	0.171	3.437

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	95	94	94	114	110	129	401	97	120
N.S.	1	0.86	0.85	0.85	1.04	1.00	1.17	3.65	0.88	1.09
time (sec)	N/A	0.322	0.052	0.218	0.026	0.089	0.632	0.122	0.172	3.462

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	131	125	124	210	162	196	620	136	146
N.S.	1	0.83	0.80	0.79	1.34	1.03	1.25	3.95	0.87	0.93
time (sec)	N/A	0.450	0.055	0.414	0.031	0.105	0.556	0.127	0.170	3.636

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	123	115	116	184	146	177	525	127	134
N.S.	1	0.86	0.80	0.81	1.29	1.02	1.24	3.67	0.89	0.94
time (sec)	N/A	0.416	0.050	0.287	0.034	0.086	0.450	0.135	0.167	3.465

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	113	107	108	179	137	167	425	117	122
N.S.	1	0.88	0.83	0.84	1.39	1.06	1.29	3.29	0.91	0.95
time (sec)	N/A	0.403	0.045	0.320	0.031	0.090	0.398	0.124	0.165	3.414

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	58	92	80	147	114	131	330	100	105
N.S.	1	0.82	1.30	1.13	2.07	1.61	1.85	4.65	1.41	1.48
time (sec)	N/A	0.246	0.051	0.179	0.030	0.084	0.329	0.122	0.169	3.380

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	103	103	173	0	0	0	82	0
N.S.	1	1.00	0.90	0.90	1.52	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.330	0.067	0.236	0.138	0.000	0.000	0.000	0.162	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	80	73	89	0	0	0	410	66	0
N.S.	1	1.31	1.20	1.46	0.00	0.00	0.00	6.72	1.08	0.00
time (sec)	N/A	0.331	0.067	0.290	0.000	0.000	0.000	0.970	0.168	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	143	123	0	0	0	0	111	0
N.S.	1	1.00	1.04	0.90	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.372	0.078	0.361	0.000	0.000	0.000	0.000	0.163	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	68	103	110	157	128	158	330	112	116
N.S.	1	0.84	1.27	1.36	1.94	1.58	1.95	4.07	1.38	1.43
time (sec)	N/A	0.293	0.067	0.296	0.034	0.096	0.477	0.119	0.165	3.348

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	122	114	118	178	147	189	431	120	168
N.S.	1	0.83	0.78	0.80	1.21	1.00	1.29	2.93	0.82	1.14
time (sec)	N/A	0.427	0.072	0.289	0.033	0.102	0.520	0.126	0.160	3.517

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	132	122	126	194	156	199	532	130	182
N.S.	1	0.82	0.76	0.78	1.20	0.97	1.24	3.30	0.81	1.13
time (sec)	N/A	0.461	0.071	0.301	0.031	0.102	0.637	0.130	0.158	3.501

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	158	151	152	285	190	243	722	167	177
N.S.	1	0.82	0.79	0.79	1.48	0.99	1.27	3.76	0.87	0.92
time (sec)	N/A	0.476	0.067	0.589	0.032	0.099	0.568	0.131	0.168	3.805

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	150	142	144	265	178	235	621	158	165
N.S.	1	0.84	0.80	0.81	1.49	1.00	1.32	3.49	0.89	0.93
time (sec)	N/A	0.547	0.062	0.510	0.031	0.103	0.488	0.138	0.160	4.098

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	112	133	134	244	165	211	527	148	153
N.S.	1	0.83	0.99	0.99	1.81	1.22	1.56	3.90	1.10	1.13
time (sec)	N/A	0.529	0.052	0.410	0.033	0.092	0.612	0.127	0.166	3.950

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	76	115	101	219	149	182	425	131	136
N.S.	1	0.90	1.37	1.20	2.61	1.77	2.17	5.06	1.56	1.62
time (sec)	N/A	0.447	0.059	0.266	0.031	0.083	0.336	0.130	0.162	3.698

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	148	131	228	0	0	0	115	0
N.S.	1	1.00	0.97	0.86	1.50	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.488	0.079	0.306	0.140	0.000	0.000	0.000	0.173	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	131	229	0	0	0	122	0
N.S.	1	1.00	0.99	0.87	1.53	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.648	0.086	0.416	0.133	0.000	0.000	0.000	0.173	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	165	137	0	0	0	0	133	0
N.S.	1	1.00	1.03	0.86	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.604	0.086	0.495	0.000	0.000	0.000	0.000	0.174	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	175	151	0	0	0	0	142	0
N.S.	1	1.00	0.99	0.86	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.520	0.091	0.531	0.000	0.000	0.000	0.000	0.175	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	76	131	138	228	163	207	431	143	147
N.S.	1	0.82	1.41	1.48	2.45	1.75	2.23	4.63	1.54	1.58
time (sec)	N/A	0.307	0.080	0.391	0.035	0.096	0.540	0.129	0.170	3.671

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	108	140	146	250	175	233	533	151	233
N.S.	1	0.79	1.02	1.07	1.82	1.28	1.70	3.89	1.10	1.70
time (sec)	N/A	0.345	0.076	0.407	0.033	0.093	0.686	0.118	0.158	3.721

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	161	149	154	273	188	257	634	161	220
N.S.	1	0.82	0.76	0.79	1.39	0.96	1.31	3.23	0.82	1.12
time (sec)	N/A	0.474	0.081	0.410	0.036	0.094	0.987	0.129	0.165	3.657

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	186	177	178	373	222	294	817	198	337
N.S.	1	0.83	0.79	0.79	1.67	0.99	1.31	3.65	0.88	1.50
time (sec)	N/A	0.510	0.071	0.842	0.037	0.094	0.720	0.129	0.164	4.336

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	147	168	170	339	208	279	723	189	196
N.S.	1	0.86	0.98	0.99	1.98	1.22	1.63	4.23	1.11	1.15
time (sec)	N/A	0.413	0.069	0.722	0.038	0.096	0.738	0.134	0.162	3.578

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	126	159	164	326	198	269	621	180	185
N.S.	1	0.82	1.04	1.07	2.13	1.29	1.76	4.06	1.18	1.21
time (sec)	N/A	0.344	0.062	0.639	0.034	0.092	0.535	0.130	0.161	3.496

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	90	146	121	283	177	226	526	163	168
N.S.	1	0.84	1.36	1.13	2.64	1.65	2.11	4.92	1.52	1.57
time (sec)	N/A	0.272	0.064	0.388	0.036	0.089	0.399	0.120	0.159	3.495

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	179	159	276	0	0	0	146	0
N.S.	1	1.00	0.97	0.86	1.49	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.437	0.101	0.426	0.132	0.000	0.000	0.000	0.166	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	194	159	281	0	0	0	155	0
N.S.	1	1.00	1.09	0.89	1.58	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.440	0.102	0.576	0.133	0.000	0.000	0.000	0.160	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	143	147	293	0	0	0	129	0
N.S.	1	1.00	0.92	0.94	1.88	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.424	0.098	0.717	0.130	0.000	0.000	0.000	0.158	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	197	165	0	0	0	0	164	0
N.S.	1	1.00	1.04	0.87	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.468	0.105	0.780	0.000	0.000	0.000	0.000	0.177	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	179	0	0	0	0	173	0
N.S.	1	1.00	0.99	0.86	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.487	0.108	0.797	0.000	0.000	0.000	0.000	0.163	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	88	157	166	299	191	253	532	174	179
N.S.	1	0.81	1.44	1.52	2.74	1.75	2.32	4.88	1.60	1.64
time (sec)	N/A	0.313	0.095	0.567	0.033	0.102	0.654	0.134	0.169	4.242

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	118	166	174	329	208	291	634	183	248
N.S.	1	0.78	1.10	1.15	2.18	1.38	1.93	4.20	1.21	1.64
time (sec)	N/A	0.348	0.095	0.572	0.034	0.092	0.696	0.133	0.160	4.102

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	192	175	182	353	218	301	735	192	260
N.S.	1	0.84	0.76	0.79	1.54	0.95	1.31	3.21	0.84	1.14
time (sec)	N/A	0.540	0.101	0.585	0.032	0.100	0.870	0.137	0.165	4.104

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	195	129	162	0	0	0	0	64	0
N.S.	1	1.10	0.73	0.92	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.853	0.357	0.315	0.000	0.000	0.000	0.000	0.171	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	138	97	132	0	0	0	0	54	0
N.S.	1	0.95	0.67	0.91	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	1.219	0.237	0.238	0.000	0.000	0.000	0.000	0.201	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	75	106	0	0	0	0	41	0
N.S.	1	0.97	0.80	1.13	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.514	0.170	0.194	0.000	0.000	0.000	0.000	0.187	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	78	0	0	0	0	33	0
N.S.	1	1.00	1.02	1.53	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.314	0.144	0.161	0.000	0.000	0.000	0.000	0.182	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	115	0	0	0	0	52	0
N.S.	1	1.00	1.20	2.50	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.291	0.165	0.227	0.000	0.000	0.000	0.000	0.174	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	89	93	156	0	0	0	0	49	0
N.S.	1	0.96	1.00	1.68	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.566	0.206	0.259	0.000	0.000	0.000	0.000	0.181	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	126	133	188	0	0	0	0	67	0
N.S.	1	0.86	0.91	1.29	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.855	0.298	0.335	0.000	0.000	0.000	0.000	0.170	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	181	172	218	0	0	0	0	182	0
N.S.	1	0.98	0.93	1.18	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	1.275	0.362	0.376	0.000	0.000	0.000	0.000	0.165	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	142	163	0	0	0	0	121	0
N.S.	1	1.00	0.78	0.90	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.465	0.540	0.349	0.000	0.000	0.000	0.000	0.165	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	137	0	0	0	0	109	0
N.S.	1	1.00	0.81	0.92	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.427	0.438	0.289	0.000	0.000	0.000	0.000	0.164	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	99	123	0	0	0	0	95	0
N.S.	1	1.00	0.93	1.16	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.368	0.306	0.280	0.000	0.000	0.000	0.000	0.167	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	55	64	41	96	49	95	63	73	45
N.S.	1	0.96	1.12	0.72	1.68	0.86	1.67	1.11	1.28	0.79
time (sec)	N/A	0.249	0.091	0.211	0.030	0.082	0.576	0.123	0.160	3.882

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	159	0	0	0	0	196	0
N.S.	1	1.00	0.81	1.28	0.00	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	0.417	0.315	0.304	0.000	0.000	0.000	0.000	0.176	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	184	0	0	0	0	243	0
N.S.	1	1.00	0.82	1.08	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.558	0.574	0.364	0.000	0.000	0.000	0.000	0.176	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	189	218	0	0	0	0	321	0
N.S.	1	1.00	0.89	1.03	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.835	0.755	0.493	0.000	0.000	0.000	0.000	0.161	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	189	197	0	0	0	0	209	0
N.S.	1	1.00	0.83	0.87	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.843	0.585	0.509	0.000	0.000	0.000	0.000	0.160	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	167	170	0	0	0	0	201	0
N.S.	1	1.00	0.86	0.88	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.751	0.518	0.416	0.000	0.000	0.000	0.000	0.166	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	145	157	0	0	0	0	190	0
N.S.	1	1.00	0.97	1.05	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.490	0.393	0.414	0.000	0.000	0.000	0.000	0.173	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	99	68	152	84	277	114	126	81
N.S.	1	0.97	1.29	0.88	1.97	1.09	3.60	1.48	1.64	1.05
time (sec)	N/A	0.339	0.086	0.342	0.037	0.082	0.835	0.118	0.169	4.401

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	74	86	72	134	75	224	118	130	123
N.S.	1	0.96	1.12	0.94	1.74	0.97	2.91	1.53	1.69	1.60
time (sec)	N/A	0.266	0.088	0.306	0.036	0.083	0.813	0.117	0.164	4.103

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	147	190	0	0	0	0	343	0
N.S.	1	1.00	0.91	1.18	0.00	0.00	0.00	0.00	2.13	0.00
time (sec)	N/A	0.465	0.382	0.415	0.000	0.000	0.000	0.000	0.186	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	186	222	0	0	0	0	434	0
N.S.	1	1.00	0.85	1.02	0.00	0.00	0.00	0.00	1.99	0.00
time (sec)	N/A	0.545	0.801	0.490	0.000	0.000	0.000	0.000	0.166	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	220	256	0	0	0	0	458	0
N.S.	1	1.00	0.82	0.96	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.603	0.987	0.721	0.000	0.000	0.000	0.000	0.170	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	75	75	132	91	294	161	156	139
N.S.	1	1.02	0.94	0.94	1.65	1.14	3.68	2.01	1.95	1.74
time (sec)	N/A	0.272	0.099	0.356	0.037	0.090	1.192	0.124	0.176	3.865

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	88	120	0	0	0	36	0
N.S.	1	1.00	0.95	2.15	2.93	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.299	0.140	0.231	0.032	0.000	0.000	0.000	0.167	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	271	306	0	0	0	0	296	0
N.S.	1	1.00	1.00	1.13	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.913	0.472	0.533	0.000	0.000	0.000	0.000	0.174	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	234	278	402	0	0	0	259	0
N.S.	1	1.00	0.99	1.18	1.70	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	1.098	0.343	0.456	0.279	0.000	0.000	0.000	0.167	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	201	249	0	0	0	0	224	0
N.S.	1	1.00	1.03	1.27	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.985	0.389	0.397	0.000	0.000	0.000	0.000	0.181	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	122	156	211	290	0	0	0	178	0
N.S.	1	1.09	1.39	1.88	2.59	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.554	0.274	0.330	0.200	0.000	0.000	0.000	0.185	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	228	1827	0	0	0	0	90	0
N.S.	1	1.00	1.19	9.57	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.788	0.312	2.622	0.000	0.000	0.000	0.000	0.180	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	249	1857	0	0	0	0	133	0
N.S.	1	1.00	1.24	9.24	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.739	0.333	2.806	0.000	0.000	0.000	0.000	0.190	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	206	276	0	0	0	0	235	0
N.S.	1	1.00	1.36	1.83	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.598	0.205	0.625	0.000	0.000	0.000	0.000	0.188	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	246	308	417	0	0	0	235	0
N.S.	1	1.00	1.19	1.50	2.02	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.747	0.315	0.625	0.470	0.000	0.000	0.000	0.173	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	329	374	766	0	0	1135	376	0
N.S.	1	1.00	0.92	1.05	2.15	0.00	0.00	3.19	1.06	0.00
time (sec)	N/A	1.265	0.650	0.750	0.299	0.000	0.000	2.251	0.173	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	297	345	604	0	0	0	339	0
N.S.	1	1.00	0.95	1.11	1.94	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	1.393	0.641	0.674	0.291	0.000	0.000	0.000	0.173	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	263	318	610	0	0	761	303	0
N.S.	1	1.00	0.94	1.14	2.18	0.00	0.00	2.72	1.08	0.00
time (sec)	N/A	1.232	0.708	0.598	0.285	0.000	0.000	1.452	0.182	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	170	227	223	464	0	0	0	251	0
N.S.	1	0.97	1.30	1.27	2.65	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.651	0.608	0.469	0.207	0.000	0.000	0.000	0.177	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	324	895	0	0	0	0	189	0
N.S.	1	1.00	1.17	3.22	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.995	0.433	2.454	0.000	0.000	0.000	0.000	0.177	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	341	2519	0	0	0	0	151	0
N.S.	1	1.00	1.20	8.90	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.933	0.351	2.183	0.000	0.000	0.000	0.000	0.181	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	370	952	0	0	0	0	278	0
N.S.	1	1.00	1.18	3.04	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.981	0.573	4.342	0.000	0.000	0.000	0.000	0.188	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	227	270	350	555	0	0	0	301	0
N.S.	1	0.93	1.11	1.43	2.27	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.528	0.424	0.812	0.477	0.000	0.000	0.000	0.187	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	385	436	928	0	0	0	454	0
N.S.	1	1.00	0.93	1.05	2.24	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	2.045	1.051	1.032	0.313	0.000	0.000	0.000	0.176	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	356	408	775	0	0	0	418	0
N.S.	1	1.00	0.94	1.08	2.06	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	2.086	0.802	0.917	0.298	0.000	0.000	0.000	0.175	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	325	377	780	0	0	0	382	0
N.S.	1	1.00	1.14	1.32	2.73	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.971	1.701	0.893	0.291	0.000	0.000	0.000	0.178	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	197	293	276	627	0	0	0	333	0
N.S.	1	0.96	1.42	1.34	3.04	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.474	1.497	0.654	0.210	0.000	0.000	0.000	0.171	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	448	959	0	0	0	0	278	0
N.S.	1	1.00	1.26	2.70	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	1.140	0.540	3.385	0.000	0.000	0.000	0.000	0.176	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	479	1012	0	0	0	0	286	0
N.S.	1	1.00	1.33	2.80	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	1.112	0.437	4.543	0.000	0.000	0.000	0.000	0.173	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	461	1086	0	0	0	0	322	0
N.S.	1	1.00	1.20	2.82	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	1.100	0.739	4.485	0.000	0.000	0.000	0.000	0.183	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	569	1197	0	0	0	0	357	0
N.S.	1	1.00	1.44	3.02	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	1.796	0.465	5.590	0.000	0.000	0.000	0.000	0.175	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	256	343	412	813	0	0	0	379	0
N.S.	1	0.94	1.27	1.52	3.00	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.780	0.550	1.103	0.485	0.000	0.000	0.000	0.168	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	330	372	440	783	0	0	0	415	0
N.S.	1	0.94	1.06	1.25	2.22	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.945	0.755	1.135	0.499	0.000	0.000	0.000	0.170	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	449	402	468	961	0	0	0	451	0
N.S.	1	0.94	0.84	0.98	2.01	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.969	0.911	1.165	0.506	0.000	0.000	0.000	0.165	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	329	423	347	967	0	0	0	0	100	0
N.S.	1	1.29	1.05	2.94	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	5.103	0.684	5.903	0.000	0.000	0.000	0.000	0.172	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	247	271	260	905	0	0	0	0	88	0
N.S.	1	1.10	1.05	3.66	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	3.587	0.470	3.576	0.000	0.000	0.000	0.000	0.177	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	183	140	2602	0	0	0	0	71	0
N.S.	1	1.06	0.81	15.13	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.751	0.546	1.826	0.000	0.000	0.000	0.000	0.164	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	102	287	0	0	0	0	58	0
N.S.	1	1.01	1.21	3.42	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.621	0.454	0.487	0.000	0.000	0.000	0.000	0.164	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	84	132	1148	0	0	0	0	96	0
N.S.	1	1.09	1.71	14.91	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.621	0.691	1.006	0.000	0.000	0.000	0.000	0.160	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	162	166	225	4135	0	0	0	0	82	0
N.S.	1	1.02	1.39	25.52	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	1.694	0.580	2.601	0.000	0.000	0.000	0.000	0.165	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	246	317	1492	0	0	0	0	104	0
N.S.	1	0.98	1.27	5.97	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	3.916	0.854	5.062	0.000	0.000	0.000	0.000	0.177	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	334	368	388	1718	0	0	0	0	392	0
N.S.	1	1.10	1.16	5.14	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	4.572	1.105	7.853	0.000	0.000	0.000	0.000	0.175	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	425	1050	0	0	0	0	215	0
N.S.	1	1.00	1.08	2.66	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.148	1.236	5.906	0.000	0.000	0.000	0.000	0.172	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	354	983	0	0	0	0	204	0
N.S.	1	1.00	1.07	2.97	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.938	0.928	4.275	0.000	0.000	0.000	0.000	0.169	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	295	2674	0	0	0	0	190	0
N.S.	1	1.00	1.13	10.28	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.769	0.683	1.627	0.000	0.000	0.000	0.000	0.187	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	233	729	0	0	0	0	170	0
N.S.	1	1.00	1.24	3.88	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.602	0.636	1.072	0.000	0.000	0.000	0.000	0.179	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	124	96	277	101	0	119	171	97
N.S.	1	1.07	1.16	0.90	2.59	0.94	0.00	1.11	1.60	0.91
time (sec)	N/A	0.365	0.323	0.369	0.045	0.085	0.000	0.121	0.183	3.620

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	254	1239	0	0	0	0	414	0
N.S.	1	1.00	0.86	4.20	0.00	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.967	0.912	1.197	0.000	0.000	0.000	0.000	0.183	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	347	4256	0	0	0	0	627	0
N.S.	1	1.00	0.94	11.47	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	1.211	1.209	2.429	0.000	0.000	0.000	0.000	0.186	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	452	1572	0	0	0	0	739	0
N.S.	1	1.00	0.94	3.28	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	1.348	1.557	6.027	0.000	0.000	0.000	0.000	0.181	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	420	1069	0	0	0	0	359	0
N.S.	1	1.00	1.03	2.62	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	1.149	1.401	5.477	0.000	0.000	0.000	0.000	0.173	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	418	2771	0	0	0	0	349	0
N.S.	1	1.00	1.24	8.22	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.987	0.917	2.484	0.000	0.000	0.000	0.000	0.178	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	310	815	0	0	0	0	336	0
N.S.	1	1.00	1.17	3.08	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.794	1.011	1.525	0.000	0.000	0.000	0.000	0.177	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	150	170	429	164	0	226	300	405
N.S.	1	1.00	0.96	1.08	2.73	1.04	0.00	1.44	1.91	2.58
time (sec)	N/A	0.441	0.332	0.514	0.043	0.106	0.000	0.122	0.167	5.084

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	158	183	177	399	156	0	232	288	373
N.S.	1	1.01	1.17	1.13	2.54	0.99	0.00	1.48	1.83	2.38
time (sec)	N/A	0.422	0.315	0.458	0.042	0.095	0.000	0.126	0.166	4.764

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	376	1321	0	0	0	0	714	0
N.S.	1	1.00	1.04	3.65	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	1.128	1.225	1.707	0.000	0.000	0.000	0.000	0.170	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	479	4301	0	0	0	0	1033	0
N.S.	1	1.00	1.07	9.60	0.00	0.00	0.00	0.00	2.31	0.00
time (sec)	N/A	1.328	1.566	3.033	0.000	0.000	0.000	0.000	0.177	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	174	168	252	445	203	0	333	402	498
N.S.	1	0.99	0.95	1.43	2.53	1.15	0.00	1.89	2.28	2.83
time (sec)	N/A	0.473	0.197	0.507	0.046	0.097	0.000	0.128	0.172	4.924

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	59	647	0	85	0	0	36	0
N.S.	1	1.15	0.88	9.66	0.00	1.27	0.00	0.00	0.54	0.00
time (sec)	N/A	0.513	0.322	3.024	0.000	0.083	0.000	0.000	0.170	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	306	297	644	713	0	0	0	0	601	0
N.S.	1	0.97	2.10	2.33	0.00	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	0.979	1.926	2.187	0.000	0.000	0.000	0.000	0.171	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	240	236	488	610	0	0	0	0	467	0
N.S.	1	0.98	2.03	2.54	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	0.737	1.599	1.178	0.000	0.000	0.000	0.000	0.171	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	191	181	334	3401	0	0	0	0	304	0
N.S.	1	0.95	1.75	17.81	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.566	0.416	2.622	0.000	0.000	0.000	0.000	0.174	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	152	1215	0	0	0	0	80	0
N.S.	1	1.02	1.37	10.95	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.712	0.434	1.537	0.000	0.000	0.000	0.000	0.175	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	142	198	182	529	160	0	172	304	582
N.S.	1	1.02	1.42	1.31	3.81	1.15	0.00	1.24	2.19	4.19
time (sec)	N/A	0.446	0.182	1.184	0.047	0.079	0.000	0.134	0.173	4.607

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	210	215	327	796	250	0	362	522	930
N.S.	1	1.01	1.03	1.57	3.83	1.20	0.00	1.74	2.51	4.47
time (sec)	N/A	0.624	0.230	1.400	0.063	0.097	0.000	0.132	0.170	6.018

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	271	279	472	1085	345	0	555	734	1304
N.S.	1	0.99	1.01	1.72	3.95	1.25	0.00	2.02	2.67	4.74
time (sec)	N/A	0.840	0.222	1.574	0.073	0.097	0.000	0.139	0.182	7.113

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	331	172	352	0	0	0	0	23	0
N.S.	1	1.07	0.56	1.14	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	4.023	0.326	6.730	0.000	0.000	0.000	0.000	0.184	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	205	212	126	736	0	0	0	0	21	0
N.S.	1	1.03	0.61	3.59	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.835	0.325	3.333	0.000	0.000	0.000	0.000	0.181	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	104	101	82	593	0	0	0	0	20	0
N.S.	1	0.97	0.79	5.70	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.622	0.170	1.083	0.000	0.000	0.000	0.000	0.180	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	93	100	86	1094	0	0	0	0	38	0
N.S.	1	1.08	0.92	11.76	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.632	0.654	1.454	0.000	0.000	0.000	0.000	0.173	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	93	100	86	1101	0	0	0	0	38	0
N.S.	1	1.08	0.92	11.84	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.640	0.005	0.316	0.000	0.000	0.000	0.000	0.174	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	191	198	154	1339	0	0	0	0	24	0
N.S.	1	1.04	0.81	7.01	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.597	0.272	3.657	0.000	0.000	0.000	0.000	0.165	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	291	222	602	0	0	0	0	24	0
N.S.	1	0.95	0.73	1.97	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	3.538	0.503	8.987	0.000	0.000	0.000	0.000	0.164	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	421	233	442	0	0	0	0	24	0
N.S.	1	1.10	0.61	1.15	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	3.962	0.306	3.569	0.000	0.000	0.000	0.000	0.179	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	261	274	172	381	0	0	0	0	22	0
N.S.	1	1.05	0.66	1.46	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.984	0.267	1.241	0.000	0.000	0.000	0.000	0.171	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	131	136	112	228	0	0	0	0	21	0
N.S.	1	1.04	0.85	1.74	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.809	0.154	0.754	0.000	0.000	0.000	0.000	0.165	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	136	102	754	0	155	0	0	36	0
N.S.	1	1.15	0.86	6.39	0.00	1.31	0.00	0.00	0.31	0.00
time (sec)	N/A	0.822	0.393	1.154	0.000	0.080	0.000	0.000	0.167	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	136	102	761	0	155	0	0	36	0
N.S.	1	1.15	0.86	6.45	0.00	1.31	0.00	0.00	0.31	0.00
time (sec)	N/A	0.858	0.005	0.520	0.000	0.082	0.000	0.000	0.166	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	259	172	573	0	0	0	0	27	0
N.S.	1	1.08	0.72	2.40	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.447	0.353	1.457	0.000	0.000	0.000	0.000	0.176	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	382	250	779	0	0	0	0	27	0
N.S.	1	1.01	0.66	2.05	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	4.496	0.668	4.694	0.000	0.000	0.000	0.000	0.171	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	17	18	22	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.06	1.12	1.38	1.12
time (sec)	N/A	0.220	2.077	0.092	0.095	0.082	0.552	0.322	0.178	3.545

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	17	17	41	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	1.13	1.13	2.73	1.13
time (sec)	N/A	0.203	0.116	0.091	0.088	0.082	0.594	0.304	0.178	3.503

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	21	20	20	37	20
N.S.	1	1.00	1.11	1.00	1.11	1.17	1.11	1.11	2.06	1.11
time (sec)	N/A	0.242	0.223	0.361	0.093	0.068	0.787	0.307	0.175	3.537

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	68	18	20	18	26	18
N.S.	1	1.00	1.12	1.00	4.25	1.12	1.25	1.12	1.62	1.12
time (sec)	N/A	0.220	1.804	0.088	0.086	0.071	0.631	0.136	0.164	3.591

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	58	17	20	17	51	17
N.S.	1	1.00	1.13	1.00	3.87	1.13	1.33	1.13	3.40	1.13
time (sec)	N/A	0.198	0.907	0.139	0.074	0.075	0.705	0.124	0.162	3.599

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	68	21	24	20	45	20
N.S.	1	1.00	1.11	1.00	3.78	1.17	1.33	1.11	2.50	1.11
time (sec)	N/A	0.232	1.082	0.362	0.090	0.067	0.951	0.131	0.165	3.579

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	474	323	0	0	0	0	68	0
N.S.	1	1.00	1.72	1.17	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.557	4.475	0.903	0.000	0.000	0.000	0.000	0.170	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	394	261	0	0	0	0	55	0
N.S.	1	1.00	1.84	1.22	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.477	2.143	0.343	0.000	0.000	0.000	0.000	0.181	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	315	214	0	0	0	0	39	0
N.S.	1	1.00	2.02	1.37	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.402	1.799	0.283	0.000	0.000	0.000	0.000	0.176	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	257	145	0	0	0	0	30	0
N.S.	1	1.00	2.25	1.27	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.464	0.164	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	294	203	0	0	0	0	119	0
N.S.	1	1.00	1.99	1.37	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.412	1.188	0.305	0.000	0.000	0.000	0.000	0.173	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	360	267	0	0	0	0	116	0
N.S.	1	1.00	1.80	1.34	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.494	2.375	0.362	0.000	0.000	0.000	0.000	0.190	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	435	322	0	0	0	0	189	0
N.S.	1	1.00	1.67	1.23	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.536	4.061	0.460	0.000	0.000	0.000	0.000	0.171	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	1414	1573	0	0	0	0	89	0
N.S.	1	1.00	3.67	4.09	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.778	13.270	14.746	0.000	0.000	0.000	0.000	0.174	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	1153	13674	0	0	0	0	69	0
N.S.	1	1.00	4.13	49.01	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.573	11.529	5.592	0.000	0.000	0.000	0.000	0.169	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	1055	1087	0	0	0	0	55	0
N.S.	1	1.00	5.61	5.78	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.323	9.584	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	1151	1679	0	0	0	0	230	0
N.S.	1	1.00	3.61	5.26	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.767	8.735	2.328	0.000	0.000	0.000	0.000	0.184	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	1305	16456	0	0	0	0	220	0
N.S.	1	1.00	3.17	39.94	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.991	9.997	7.725	0.000	0.000	0.000	0.000	0.183	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	850	1466	0	0	0	0	97	0
N.S.	1	1.00	3.09	5.33	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.681	6.654	2.323	0.000	0.000	0.000	0.000	0.178	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	25	15	20	27	20
N.S.	1	1.00	1.11	1.00	1.11	1.39	0.83	1.11	1.50	1.11
time (sec)	N/A	0.223	0.018	0.772	0.084	0.075	1.438	0.108	0.183	3.740

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	119	72	69	73	76	71	335	72	61
N.S.	1	1.65	1.00	0.96	1.01	1.06	0.99	4.65	1.00	0.85
time (sec)	N/A	0.410	0.025	0.422	0.027	0.075	0.530	0.123	0.172	3.716

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	99	79	57	72	61	54	227	56	51
N.S.	1	1.57	1.25	0.90	1.14	0.97	0.86	3.60	0.89	0.81
time (sec)	N/A	0.368	0.029	0.364	0.031	0.072	0.389	0.129	0.172	3.657

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	99	62	61	65	68	63	268	64	53
N.S.	1	1.60	1.00	0.98	1.05	1.10	1.02	4.32	1.03	0.85
time (sec)	N/A	0.364	0.021	0.318	0.029	0.078	0.391	0.131	0.167	3.511

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	69	47	37	52	46	160	48	44
N.S.	1	1.08	1.72	1.18	0.92	1.30	1.15	4.00	1.20	1.10
time (sec)	N/A	0.229	0.020	0.293	0.024	0.081	0.323	0.120	0.170	3.605

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	47	44	47	53	49	203	52	40
N.S.	1	1.03	0.73	0.69	0.73	0.83	0.77	3.17	0.81	0.62
time (sec)	N/A	0.276	0.016	0.143	0.024	0.081	0.214	0.120	0.165	3.586

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	60	69	89	0	0	0	33	0
N.S.	1	1.19	1.25	1.44	1.85	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.336	0.021	0.165	0.031	0.000	0.000	0.000	0.165	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	63	38	41	36	51	41	145	50	37
N.S.	1	1.66	1.00	1.08	0.95	1.34	1.08	3.82	1.32	0.97
time (sec)	N/A	0.347	0.008	0.194	0.025	0.081	0.303	0.122	0.199	3.840

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	68	78	81	0	0	330	45	0
N.S.	1	0.96	1.21	1.39	1.45	0.00	0.00	5.89	0.80	0.00
time (sec)	N/A	0.343	0.033	0.214	0.033	0.000	0.000	1.147	0.184	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	85	58	59	53	64	63	204	68	49
N.S.	1	1.47	1.00	1.02	0.91	1.10	1.09	3.52	1.17	0.84
time (sec)	N/A	0.342	0.021	0.230	0.029	0.081	0.376	0.122	0.168	3.712

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	71	48	61	52	46	160	48	61
N.S.	1	1.05	1.69	1.14	1.45	1.24	1.10	3.81	1.14	1.45
time (sec)	N/A	0.252	0.022	0.198	0.028	0.070	0.293	0.123	0.175	3.707

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	109	71	68	62	73	75	281	76	59
N.S.	1	1.54	1.00	0.96	0.87	1.03	1.06	3.96	1.07	0.83
time (sec)	N/A	0.370	0.022	0.253	0.028	0.075	0.504	0.125	0.181	3.803

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	411	113	199	190	0	0	0	117	0
N.S.	1	2.54	0.70	1.23	1.17	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	2.674	0.669	0.741	0.033	0.000	0.000	0.000	0.174	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	297	88	105	146	109	114	522	108	101
N.S.	1	2.56	0.76	0.91	1.26	0.94	0.98	4.50	0.93	0.87
time (sec)	N/A	2.440	0.044	0.368	0.033	0.079	0.480	0.126	0.175	3.759

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	297	95	179	173	0	0	0	96	0
N.S.	1	2.15	0.69	1.30	1.25	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	1.880	0.189	0.431	0.036	0.000	0.000	0.000	0.179	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	66	84	74	91	88	305	88	77
N.S.	1	1.04	0.69	0.88	0.78	0.96	0.93	3.21	0.93	0.81
time (sec)	N/A	0.401	0.035	0.216	0.032	0.085	0.367	0.121	0.161	3.891

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	71	154	144	0	0	0	74	0
N.S.	1	1.07	0.62	1.34	1.25	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.643	0.099	0.302	0.032	0.000	0.000	0.000	0.165	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	146	194	154	663	0	0	0	0	61	0
N.S.	1	1.33	1.05	4.54	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.449	0.195	5.256	0.000	0.000	0.000	0.000	0.164	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	136	102	156	152	0	0	0	51	0
N.S.	1	1.46	1.10	1.68	1.63	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.177	0.105	0.257	0.032	0.000	0.000	0.000	0.170	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	193	191	736	0	0	0	0	95	0
N.S.	1	1.12	1.11	4.28	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.501	0.221	6.056	0.000	0.000	0.000	0.000	0.167	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	162	93	198	188	0	0	0	84	0
N.S.	1	1.40	0.80	1.71	1.62	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.106	0.207	0.347	0.035	0.000	0.000	0.000	0.170	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	116	82	113	164	108	102	282	110	246
N.S.	1	1.30	0.92	1.27	1.84	1.21	1.15	3.17	1.24	2.76
time (sec)	N/A	0.478	0.038	0.296	0.033	0.077	0.420	0.123	0.167	4.085

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	244	114	219	228	0	0	0	103	0
N.S.	1	1.71	0.80	1.53	1.59	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.714	0.339	0.343	0.035	0.000	0.000	0.000	0.166	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	175	134	772	0	0	0	0	104	0
N.S.	1	1.11	0.85	4.92	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.955	0.230	2.945	0.000	0.000	0.000	0.000	0.166	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	232	210	277	0	0	0	19	0
N.S.	1	1.00	1.20	1.09	1.44	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.485	0.301	0.379	0.114	0.000	0.000	0.000	0.166	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	21	21	24	20	28	21
N.S.	1	1.00	1.11	1.00	1.17	1.17	1.33	1.11	1.56	1.17
time (sec)	N/A	0.222	0.570	0.141	0.084	0.080	0.675	0.455	0.170	3.527

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	20	19	24	19	72	20
N.S.	1	1.00	1.12	1.00	1.18	1.12	1.41	1.12	4.24	1.18
time (sec)	N/A	0.204	0.244	0.121	0.082	0.069	0.566	0.428	0.163	3.525

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	23	22	24	22	28	23
N.S.	1	1.00	1.10	1.00	1.15	1.10	1.20	1.10	1.40	1.15
time (sec)	N/A	0.237	0.794	0.142	0.100	0.071	0.921	0.425	0.173	3.545

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	86	21	27	20	28	21
N.S.	1	1.00	1.11	1.00	4.78	1.17	1.50	1.11	1.56	1.17
time (sec)	N/A	0.211	0.722	0.138	0.106	0.068	0.800	0.162	0.182	3.575

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	77	19	27	19	26	20
N.S.	1	1.00	1.12	1.00	4.53	1.12	1.59	1.12	1.53	1.18
time (sec)	N/A	0.189	0.939	0.122	0.087	0.076	0.538	0.154	0.176	3.715

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	94	22	27	22	28	23
N.S.	1	1.00	1.10	1.00	4.70	1.10	1.35	1.10	1.40	1.15
time (sec)	N/A	0.211	0.822	0.350	0.120	0.069	1.004	0.165	0.182	3.603

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	155	19	27	19	26	20
N.S.	1	1.00	1.12	1.00	9.12	1.12	1.59	1.12	1.53	1.18
time (sec)	N/A	0.195	0.788	0.123	0.109	0.071	0.636	0.164	0.174	3.672

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	89	89	92	100	383	92	106
N.S.	1	1.00	1.00	0.93	0.93	0.96	1.04	3.99	0.96	1.10
time (sec)	N/A	0.399	0.036	0.638	0.028	0.083	0.659	0.134	0.163	3.850

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	103	77	88	77	76	240	76	101
N.S.	1	1.00	1.18	0.89	1.01	0.89	0.87	2.76	0.87	1.16
time (sec)	N/A	0.339	0.035	0.608	0.029	0.078	0.505	0.135	0.185	4.137

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	81	84	90	319	84	71
N.S.	1	1.00	1.00	0.94	0.94	0.98	1.05	3.71	0.98	0.83
time (sec)	N/A	0.392	0.031	0.450	0.024	0.097	0.447	0.133	0.184	3.744

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	93	68	46	68	68	176	68	64
N.S.	1	1.06	1.86	1.36	0.92	1.36	1.36	3.52	1.36	1.28
time (sec)	N/A	0.266	0.032	0.332	0.028	0.081	0.391	0.121	0.199	3.761

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	71	64	66	72	75	255	72	60
N.S.	1	1.07	0.68	0.62	0.63	0.69	0.72	2.45	0.69	0.58
time (sec)	N/A	0.425	0.027	0.223	0.026	0.075	0.273	0.126	0.162	3.601

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	82	89	106	0	0	0	53	0
N.S.	1	1.00	1.17	1.27	1.51	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.320	0.032	0.281	0.026	0.000	0.000	0.000	0.170	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	64	57	66	68	249	71	57
N.S.	1	1.00	1.00	1.00	0.89	1.03	1.06	3.89	1.11	0.89
time (sec)	N/A	0.320	0.017	0.292	0.030	0.082	0.411	0.125	0.171	3.552

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	73	82	0	0	0	52	0
N.S.	1	1.00	0.98	1.18	1.32	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.294	0.053	0.346	0.025	0.000	0.000	0.000	0.165	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	67	66	72	75	274	80	59
N.S.	1	1.00	1.00	0.99	0.97	1.06	1.10	4.03	1.18	0.87
time (sec)	N/A	0.322	0.020	0.322	0.029	0.082	0.418	0.131	0.161	3.480

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	89	96	112	0	0	0	66	0
N.S.	1	1.00	1.16	1.25	1.45	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.313	0.043	0.396	0.031	0.000	0.000	0.000	0.164	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	80	71	81	88	265	88	70
N.S.	1	1.00	1.00	0.96	0.86	0.98	1.06	3.19	1.06	0.84
time (sec)	N/A	0.347	0.032	0.343	0.025	0.073	0.515	0.126	0.181	3.695

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	233	214	0	0	0	151	0
N.S.	1	1.00	0.68	1.15	1.06	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	1.218	1.322	1.164	0.034	0.000	0.000	0.000	0.183	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	140	170	133	153	683	142	221
N.S.	1	1.00	0.69	0.90	1.09	0.85	0.98	4.38	0.91	1.42
time (sec)	N/A	1.067	0.037	0.579	0.032	0.081	0.676	0.136	0.166	4.046

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	121	213	198	0	0	0	131	0
N.S.	1	1.00	0.68	1.20	1.11	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	1.000	0.849	0.710	0.034	0.000	0.000	0.000	0.177	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	144	82	119	93	116	133	473	122	111
N.S.	1	1.04	0.59	0.86	0.67	0.84	0.96	3.43	0.88	0.80
time (sec)	N/A	0.557	0.032	0.355	0.030	0.080	0.469	0.130	0.182	3.798

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	192	99	188	175	0	0	0	109	0
N.S.	1	1.12	0.58	1.10	1.02	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.877	0.509	0.460	0.035	0.000	0.000	0.000	0.173	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	200	733	0	0	0	0	95	0
N.S.	1	1.00	1.08	3.94	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.771	0.060	7.375	0.000	0.000	0.000	0.000	0.181	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	182	203	200	0	0	0	116	0
N.S.	1	1.00	1.17	1.30	1.28	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.675	0.276	0.365	0.036	0.000	0.000	0.000	0.179	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	200	779	0	0	0	0	77	0
N.S.	1	1.00	1.23	4.81	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.726	0.255	6.240	0.000	0.000	0.000	0.000	0.178	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	153	208	203	0	0	0	99	0
N.S.	1	1.00	0.92	1.25	1.22	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.717	0.067	0.539	0.035	0.000	0.000	0.000	0.175	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	238	1124	0	0	0	0	130	0
N.S.	1	1.00	1.11	5.25	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.901	0.242	9.855	0.000	0.000	0.000	0.000	0.194	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	118	233	239	0	0	0	118	0
N.S.	1	1.00	0.75	1.48	1.52	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.829	0.530	0.565	0.036	0.000	0.000	0.000	0.177	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	114	99	149	188	132	148	440	144	335
N.S.	1	1.01	0.88	1.32	1.66	1.17	1.31	3.89	1.27	2.96
time (sec)	N/A	0.507	0.044	0.441	0.033	0.075	0.686	0.134	0.176	4.316

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	140	253	254	0	0	0	138	0
N.S.	1	1.00	0.77	1.38	1.39	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	1.066	0.947	0.575	0.037	0.000	0.000	0.000	0.164	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	124	169	204	148	168	651	164	357
N.S.	1	1.00	0.73	0.99	1.20	0.87	0.99	3.83	0.96	2.10
time (sec)	N/A	1.130	0.047	0.456	0.035	0.090	0.993	0.132	0.167	5.122

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	248	298	183	858	0	0	0	0	152	0
N.S.	1	1.20	0.74	3.46	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.589	0.412	6.514	0.000	0.000	0.000	0.000	0.169	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	21	26	20	21	44	21
N.S.	1	1.00	1.10	1.00	1.05	1.30	1.00	1.05	2.20	1.05
time (sec)	N/A	0.222	0.693	0.201	0.091	0.077	1.094	0.928	0.184	3.505

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	20	26	19	20	102	20
N.S.	1	1.00	1.11	1.00	1.05	1.37	1.00	1.05	5.37	1.05
time (sec)	N/A	0.206	0.367	0.188	0.091	0.072	1.092	0.877	0.177	3.282

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	29	20	23	44	23
N.S.	1	1.00	1.09	1.00	1.05	1.32	0.91	1.05	2.00	1.05
time (sec)	N/A	0.236	0.938	0.388	0.110	0.069	2.062	0.893	0.174	3.812

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	102	26	22	21	44	21
N.S.	1	1.00	1.10	1.00	5.10	1.30	1.10	1.05	2.20	1.05
time (sec)	N/A	0.235	0.599	0.199	0.116	0.071	1.255	0.193	0.176	3.787

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	90	26	20	20	42	20
N.S.	1	1.00	1.11	1.00	4.74	1.37	1.05	1.05	2.21	1.05
time (sec)	N/A	0.203	0.753	0.179	0.092	0.074	1.129	0.169	0.175	3.824

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	108	29	22	23	44	23
N.S.	1	1.00	1.09	1.00	4.91	1.32	1.00	1.05	2.00	1.05
time (sec)	N/A	0.239	0.785	0.387	0.129	0.069	2.542	0.195	0.177	3.508

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	156	79	84	82	88	97	303	92	80
N.S.	1	1.08	0.55	0.58	0.57	0.61	0.67	2.10	0.64	0.56
time (sec)	N/A	0.522	0.033	0.349	0.025	0.076	0.455	0.144	0.174	3.583

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	271	124	222	199	0	0	0	144	0
N.S.	1	1.19	0.55	0.98	0.88	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.164	0.826	1.079	0.034	0.000	0.000	0.000	0.168	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	338	466	231	978	0	0	0	0	200	0
N.S.	1	1.38	0.68	2.89	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	2.651	0.816	19.283	0.000	0.000	0.000	0.000	0.177	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	101	60	131	120	0	0	0	23	0
N.S.	1	1.16	0.69	1.51	1.38	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.639	0.145	0.292	0.033	0.000	0.000	0.000	0.189	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	42	37	85	56	41	0	38	82
N.S.	1	1.05	1.00	0.88	2.02	1.33	0.98	0.00	0.90	1.95
time (sec)	N/A	0.397	0.076	0.256	0.036	0.078	0.358	0.000	0.170	3.570

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	44	90	125	0	0	0	21	0
N.S.	1	1.09	0.81	1.67	2.31	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.391	0.082	0.282	0.033	0.000	0.000	0.000	0.174	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	65	22	10	22	11	23
N.S.	1	1.00	1.00	0.92	5.00	1.69	0.77	1.69	0.85	1.77
time (sec)	N/A	0.200	0.005	0.197	0.034	0.070	0.462	0.109	0.177	3.334

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	109	132	0	0	0	22	0
N.S.	1	1.00	0.96	2.42	2.93	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.378	0.108	0.259	0.035	0.000	0.000	0.000	0.168	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	41	46	82	63	37	0	49	80
N.S.	1	1.07	1.00	1.12	2.00	1.54	0.90	0.00	1.20	1.95
time (sec)	N/A	0.395	0.101	0.306	0.026	0.080	0.568	0.000	0.175	3.659

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	78	60	170	162	0	0	0	24	0
N.S.	1	0.93	0.71	2.02	1.93	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.609	0.246	0.381	0.035	0.000	0.000	0.000	0.167	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	135	160	112	750	0	0	0	0	25	0
N.S.	1	1.19	0.83	5.56	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.387	0.154	14.624	0.000	0.000	0.000	0.000	0.166	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	90	59	5330	200	0	0	0	25	0
N.S.	1	1.20	0.79	71.07	2.67	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.719	0.179	10.532	0.035	0.000	0.000	0.000	0.168	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	78	89	68	638	0	0	0	0	23	0
N.S.	1	1.14	0.87	8.18	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.592	0.098	9.964	0.000	0.000	0.000	0.000	0.175	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	127	22	10	22	11	68
N.S.	1	1.00	1.00	0.92	9.77	1.69	0.77	1.69	0.85	5.23
time (sec)	N/A	0.192	0.006	1.216	0.040	0.069	0.521	0.113	0.182	3.806

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	78	60	1108	0	0	0	0	24	0
N.S.	1	1.18	0.91	16.79	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.569	0.312	16.303	0.000	0.000	0.000	0.000	0.177	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	61	4380	237	0	0	0	47	0
N.S.	1	1.08	0.92	66.36	3.59	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.691	0.245	13.467	0.038	0.000	0.000	0.000	0.178	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	138	142	133	1316	0	0	0	0	26	0
N.S.	1	1.03	0.96	9.54	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.408	0.333	23.335	0.000	0.000	0.000	0.000	0.182	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	235	142	217	0	0	0	0	25	0
N.S.	1	1.15	0.69	1.06	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.980	0.247	22.132	0.000	0.000	0.000	0.000	0.178	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	103	120	78	736	0	0	0	0	25	0
N.S.	1	1.17	0.76	7.15	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.002	0.231	16.364	0.000	0.000	0.000	0.000	0.181	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	108	116	87	670	0	0	0	0	23	0
N.S.	1	1.07	0.81	6.20	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.748	0.102	16.602	0.000	0.000	0.000	0.000	0.182	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	209	22	10	22	11	90
N.S.	1	1.00	1.00	0.92	16.08	1.69	0.77	1.69	0.85	6.92
time (sec)	N/A	0.198	0.005	1.553	0.040	0.069	0.523	0.113	0.175	3.561

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	91	104	83	1165	0	0	0	0	24	0
N.S.	1	1.14	0.91	12.80	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.755	0.378	21.546	0.000	0.000	0.000	0.000	0.184	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	93	810	0	0	0	0	49	0
N.S.	1	1.16	1.03	9.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.020	0.231	16.441	0.000	0.000	0.000	0.000	0.186	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	198	165	369	0	0	0	0	26	0
N.S.	1	0.99	0.82	1.84	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.934	0.359	35.397	0.000	0.000	0.000	0.000	0.189	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	25	14	25	14	11
N.S.	1	1.00	1.00	0.80	0.00	1.67	0.93	1.67	0.93	0.73
time (sec)	N/A	0.202	0.007	0.299	0.000	0.066	0.671	0.116	0.171	3.492

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	23	22	20	22	26	23
N.S.	1	1.00	1.10	1.00	1.15	1.10	1.00	1.10	1.30	1.15
time (sec)	N/A	0.228	0.967	0.972	0.090	0.070	0.781	0.443	0.163	3.625

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	21	20	7	21	9	9
N.S.	1	1.00	1.00	1.11	2.33	2.22	0.78	2.33	1.00	1.00
time (sec)	N/A	0.200	0.079	0.211	0.031	0.078	0.264	0.123	0.171	3.570

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	25	23	22	24	25	25
N.S.	1	1.00	1.09	1.00	1.14	1.05	1.00	1.09	1.14	1.14
time (sec)	N/A	0.253	0.239	1.344	0.098	0.084	1.144	0.417	0.178	3.473

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	52	22	24	22	30	23
N.S.	1	1.00	1.10	1.00	2.60	1.10	1.20	1.10	1.50	1.15
time (sec)	N/A	0.287	0.145	0.965	0.082	0.073	0.878	0.125	0.192	3.455

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	22	8	22	11	23
N.S.	1	1.00	1.00	1.09	2.09	2.00	0.73	2.00	1.00	2.09
time (sec)	N/A	0.200	0.006	0.217	0.031	0.072	0.401	0.114	0.184	3.358

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	59	23	26	24	29	25
N.S.	1	1.00	1.09	1.00	2.68	1.05	1.18	1.09	1.32	1.14
time (sec)	N/A	0.317	0.177	0.945	0.093	0.072	1.342	0.125	0.181	3.459

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	113	22	24	22	30	23
N.S.	1	1.00	1.10	1.00	5.65	1.10	1.20	1.10	1.50	1.15
time (sec)	N/A	0.273	0.451	0.938	0.085	0.075	1.008	0.139	0.184	3.509

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	42	22	12	22	11	23
N.S.	1	1.00	1.00	0.92	3.23	1.69	0.92	1.69	0.85	1.77
time (sec)	N/A	0.237	0.006	0.239	0.036	0.081	0.540	0.114	0.179	3.420

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	134	23	26	24	29	25
N.S.	1	1.00	1.09	1.00	6.09	1.05	1.18	1.09	1.32	1.14
time (sec)	N/A	0.330	0.599	0.607	0.097	0.071	1.683	0.141	0.189	3.471

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	84	26	30	19	33
N.S.	1	1.00	1.00	1.06	0.00	4.94	1.53	1.76	1.12	1.94
time (sec)	N/A	0.213	0.010	1.073	0.000	0.082	0.914	0.112	0.191	3.748

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	129	64	159	177	0	0	0	29	0
N.S.	1	1.18	0.59	1.46	1.62	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.754	0.125	0.512	0.034	0.000	0.000	0.000	0.195	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	55	126	65	0	0	52	110
N.S.	1	1.00	0.79	0.96	2.21	1.14	0.00	0.00	0.91	1.93
time (sec)	N/A	0.321	0.123	0.595	0.032	0.075	0.000	0.000	0.183	3.628

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	66	37	62	48	61	154	86	37
N.S.	1	1.09	1.20	0.67	1.13	0.87	1.11	2.80	1.56	0.67
time (sec)	N/A	0.242	0.093	0.375	0.032	0.075	0.443	0.116	0.187	3.532

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	54	122	64	0	255	53	106
N.S.	1	1.00	0.81	1.00	2.26	1.19	0.00	4.72	0.98	1.96
time (sec)	N/A	0.223	0.101	0.574	0.032	0.072	0.000	1.105	0.189	3.602

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	109	63	190	206	0	0	0	26	0
N.S.	1	1.20	0.69	2.09	2.26	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.681	0.138	0.463	0.040	0.000	0.000	0.000	0.170	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	102	77	127	150	118	253	0	134	132
N.S.	1	1.24	0.94	1.55	1.83	1.44	3.09	0.00	1.63	1.61
time (sec)	N/A	0.708	0.149	0.529	0.032	0.078	0.885	0.000	0.166	3.843

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	191	83	213	233	0	0	0	28	0
N.S.	1	1.55	0.67	1.73	1.89	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.634	0.254	0.660	0.036	0.000	0.000	0.000	0.163	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	184	103	735	0	0	0	0	31	0
N.S.	1	1.14	0.64	4.57	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.320	0.141	24.043	0.000	0.000	0.000	0.000	0.166	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	105	93	72	273	96	0	0	121	231
N.S.	1	1.12	0.99	0.77	2.90	1.02	0.00	0.00	1.29	2.46
time (sec)	N/A	0.391	0.164	22.652	0.036	0.083	0.000	0.000	0.182	4.406

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	85	43	52	146	66	0	140	55	198
N.S.	1	1.04	0.52	0.63	1.78	0.80	0.00	1.71	0.67	2.41
time (sec)	N/A	0.351	0.052	0.502	0.031	0.075	0.000	0.119	0.172	3.793

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	100	93	72	268	95	0	88	121	213
N.S.	1	1.14	1.06	0.82	3.05	1.08	0.00	1.00	1.38	2.42
time (sec)	N/A	0.340	0.125	23.138	0.037	0.079	0.000	1.220	0.178	4.192

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	136	167	106	1203	0	0	0	0	28	0
N.S.	1	1.23	0.78	8.85	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.248	0.233	42.651	0.000	0.000	0.000	0.000	0.184	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	142	175	97	3005	406	0	0	0	207	0
N.S.	1	1.23	0.68	21.16	2.86	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	1.497	0.298	20.664	0.041	0.000	0.000	0.000	0.177	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	205	313	146	1415	0	0	0	0	30	0
N.S.	1	1.53	0.71	6.90	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	3.692	0.686	43.704	0.000	0.000	0.000	0.000	0.177	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	227	259	139	806	0	0	0	0	31	0
N.S.	1	1.14	0.61	3.55	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.919	0.139	27.717	0.000	0.000	0.000	0.000	0.177	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	132	72	87	465	114	0	0	85	410
N.S.	1	1.09	0.60	0.72	3.84	0.94	0.00	0.00	0.70	3.39
time (sec)	N/A	0.488	0.068	22.721	0.048	0.075	0.000	0.000	0.173	4.504

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	133	91	72	298	97	0	192	121	239
N.S.	1	1.12	0.76	0.61	2.50	0.82	0.00	1.61	1.02	2.01
time (sec)	N/A	0.488	0.057	32.194	0.038	0.070	0.000	0.122	0.174	4.431

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	71	86	459	113	0	122	86	378
N.S.	1	1.10	0.62	0.75	3.99	0.98	0.00	1.06	0.75	3.29
time (sec)	N/A	0.454	0.048	24.033	0.044	0.076	0.000	1.394	0.173	4.266

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	193	241	135	1300	0	0	0	0	28	0
N.S.	1	1.25	0.70	6.74	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.811	0.276	65.895	0.000	0.000	0.000	0.000	0.180	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	235	144	271	0	0	0	0	175	0
N.S.	1	1.23	0.75	1.42	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	2.166	0.259	36.934	0.000	0.000	0.000	0.000	0.175	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	443	215	447	0	0	0	0	30	0
N.S.	1	1.47	0.71	1.48	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	4.824	0.472	90.923	0.000	0.000	0.000	0.000	0.159	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	87	0	0	0	0	0	27	0
N.S.	1	1.08	0.84	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.637	0.182	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	31	22	23	36	23
N.S.	1	1.00	1.09	1.00	1.05	1.41	1.00	1.05	1.64	1.05
time (sec)	N/A	0.241	3.279	1.761	0.102	0.091	0.791	0.542	0.161	3.522

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	31	22	23	36	23
N.S.	1	1.00	1.09	1.00	1.05	1.41	1.00	1.05	1.64	1.05
time (sec)	N/A	0.254	1.083	2.177	0.105	0.080	0.763	0.504	0.165	3.501

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	27	22	0	58	0	0	46	0
N.S.	1	0.93	1.00	0.81	0.00	2.15	0.00	0.00	1.70	0.00
time (sec)	N/A	0.343	0.152	1.009	0.000	0.072	0.000	0.000	0.173	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	38	0	0	34	0
N.S.	1	1.00	1.00	0.93	0.00	2.71	0.00	0.00	2.43	0.00
time (sec)	N/A	0.321	0.117	0.968	0.000	0.072	0.000	0.000	0.169	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	27	22	0	54	0	0	32	0
N.S.	1	0.93	1.00	0.81	0.00	2.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.295	0.108	1.000	0.000	0.080	0.000	0.000	0.174	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	28	22	23	34	23
N.S.	1	1.00	1.09	1.00	1.05	1.27	1.00	1.05	1.55	1.05
time (sec)	N/A	0.242	0.711	1.684	0.115	0.078	1.089	0.421	0.169	3.528

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	117	31	24	23	42	23
N.S.	1	1.00	1.09	1.00	5.32	1.41	1.09	1.05	1.91	1.05
time (sec)	N/A	1.406	2.303	1.731	0.132	0.067	0.968	0.150	0.165	3.606

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	36	0	111	0	0	111	0
N.S.	1	1.00	0.95	0.95	0.00	2.92	0.00	0.00	2.92	0.00
time (sec)	N/A	0.512	0.175	1.271	0.000	0.093	0.000	0.000	0.170	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	75	32	28	0	106	0	0	40	0
N.S.	1	2.08	0.89	0.78	0.00	2.94	0.00	0.00	1.11	0.00
time (sec)	N/A	0.885	0.070	1.471	0.000	0.085	0.000	0.000	0.166	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	102	0	0	112	0
N.S.	1	1.00	0.86	1.03	0.00	2.91	0.00	0.00	3.20	0.00
time (sec)	N/A	0.443	0.117	1.319	0.000	0.084	0.000	0.000	0.164	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	123	28	24	23	40	23
N.S.	1	1.00	1.09	1.00	5.59	1.27	1.09	1.05	1.82	1.05
time (sec)	N/A	1.382	2.960	1.792	0.128	0.074	1.314	0.143	0.164	3.544

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	209	31	24	23	42	23
N.S.	1	1.00	1.09	1.00	9.50	1.41	1.09	1.05	1.91	1.05
time (sec)	N/A	0.952	5.993	1.743	0.149	0.074	1.014	0.176	0.170	3.741

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	108	47	51	0	131	0	0	128	0
N.S.	1	1.69	0.73	0.80	0.00	2.05	0.00	0.00	2.00	0.00
time (sec)	N/A	1.086	0.093	1.487	0.000	0.071	0.000	0.000	0.166	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	43	0	135	0	0	221	0
N.S.	1	1.00	0.92	0.60	0.00	1.88	0.00	0.00	3.07	0.00
time (sec)	N/A	0.522	0.059	1.503	0.000	0.075	0.000	0.000	0.177	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	103	58	51	0	122	0	0	129	0
N.S.	1	1.78	1.00	0.88	0.00	2.10	0.00	0.00	2.22	0.00
time (sec)	N/A	1.068	0.061	1.399	0.000	0.077	0.000	0.000	0.169	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	217	28	24	23	40	23
N.S.	1	1.00	1.09	1.00	9.86	1.27	1.09	1.05	1.82	1.05
time (sec)	N/A	1.044	2.685	1.549	0.150	0.072	1.429	0.173	0.166	3.604

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	103	73	68	0	151	0	0	238	0
N.S.	1	1.06	0.75	0.70	0.00	1.56	0.00	0.00	2.45	0.00
time (sec)	N/A	0.690	0.152	1.451	0.000	0.083	0.000	0.000	0.172	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	171	84	83	0	171	0	0	252	0
N.S.	1	1.42	0.70	0.69	0.00	1.42	0.00	0.00	2.10	0.00
time (sec)	N/A	1.439	0.073	1.512	0.000	0.085	0.000	0.000	0.177	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	168	101	98	0	200	0	0	252	0
N.S.	1	1.09	0.66	0.64	0.00	1.30	0.00	0.00	1.64	0.00
time (sec)	N/A	0.933	0.138	1.615	0.000	0.076	0.000	0.000	0.184	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	236	112	113	0	220	0	0	252	0
N.S.	1	1.33	0.63	0.64	0.00	1.24	0.00	0.00	1.42	0.00
time (sec)	N/A	1.710	0.075	1.683	0.000	0.074	0.000	0.000	0.174	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	233	128	128	0	249	0	0	252	0
N.S.	1	1.10	0.61	0.61	0.00	1.18	0.00	0.00	1.19	0.00
time (sec)	N/A	1.252	0.142	1.714	0.000	0.088	0.000	0.000	0.169	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	91	98	60	99	71	158	239	139	83
N.S.	1	1.18	1.27	0.78	1.29	0.92	2.05	3.10	1.81	1.08
time (sec)	N/A	0.284	0.102	0.440	0.025	0.082	0.778	0.128	0.182	4.249

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	105	61	78	179	95	0	0	90	150
N.S.	1	1.05	0.61	0.78	1.79	0.95	0.00	0.00	0.90	1.50
time (sec)	N/A	0.350	0.108	0.480	0.033	0.072	0.000	0.000	0.165	3.947

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	88	60	82	71	158	239	151	105
N.S.	1	1.09	1.17	0.80	1.09	0.95	2.11	3.19	2.01	1.40
time (sec)	N/A	0.257	0.084	0.405	0.030	0.077	0.799	0.120	0.163	3.754

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	65	90	182	97	0	0	90	154
N.S.	1	1.05	0.69	0.96	1.94	1.03	0.00	0.00	0.96	1.64
time (sec)	N/A	0.330	0.086	0.445	0.035	0.067	0.000	0.000	0.159	4.005

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	195	81	234	268	0	0	0	38	0
N.S.	1	1.51	0.63	1.81	2.08	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.139	0.206	0.484	0.039	0.000	0.000	0.000	0.164	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	205	94	198	204	161	549	0	213	183
N.S.	1	1.67	0.76	1.61	1.66	1.31	4.46	0.00	1.73	1.49
time (sec)	N/A	1.309	0.171	0.544	0.039	0.074	1.613	0.000	0.168	4.106

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	144	71	90	226	99	0	250	90	372
N.S.	1	1.13	0.56	0.71	1.78	0.78	0.00	1.97	0.71	2.93
time (sec)	N/A	0.682	0.126	0.707	0.036	0.084	0.000	0.125	0.178	4.395

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	316	121	120	388	136	0	0	205	350
N.S.	1	1.94	0.74	0.74	2.38	0.83	0.00	0.00	1.26	2.15
time (sec)	N/A	1.039	0.146	50.193	0.041	0.075	0.000	0.000	0.173	4.814

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	132	71	90	206	99	0	251	90	319
N.S.	1	1.06	0.57	0.72	1.65	0.79	0.00	2.01	0.72	2.55
time (sec)	N/A	0.460	0.098	0.526	0.033	0.080	0.000	0.123	0.171	4.347

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	206	127	120	392	137	0	0	205	358
N.S.	1	1.36	0.84	0.79	2.60	0.91	0.00	0.00	1.36	2.37
time (sec)	N/A	0.615	0.120	76.608	0.041	0.083	0.000	0.000	0.169	4.892

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	196	303	129	1305	0	0	0	0	40	0
N.S.	1	1.55	0.66	6.66	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.378	0.457	261.635	0.000	0.000	0.000	0.000	0.181	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	209	385	127	3116	534	0	0	0	357	0
N.S.	1	1.84	0.61	14.91	2.56	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	3.010	0.459	174.773	0.064	0.000	0.000	0.000	0.174	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	264	135	120	437	140	0	341	205	414
N.S.	1	1.38	0.70	0.62	2.28	0.73	0.00	1.78	1.07	2.16
time (sec)	N/A	0.857	0.145	342.845	0.043	0.077	0.000	0.126	0.168	6.018

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	420	107	142	657	161	0	0	150	831
N.S.	1	1.95	0.50	0.66	3.06	0.75	0.00	0.00	0.70	3.87
time (sec)	N/A	1.840	0.146	349.190	0.051	0.076	0.000	0.000	0.166	6.175

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	239	148	848	422	140	0	342	205	414
N.S.	1	1.27	0.79	4.51	2.24	0.74	0.00	1.82	1.09	2.20
time (sec)	N/A	0.733	0.115	1.484	0.063	0.071	0.000	0.128	0.160	5.552

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	283	111	150	663	166	0	0	150	736
N.S.	1	1.39	0.55	0.74	3.27	0.82	0.00	0.00	0.74	3.63
time (sec)	N/A	1.140	0.301	12.401	0.083	0.076	0.000	0.000	0.161	5.369

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	277	484	189	1446	0	0	0	0	40	0
N.S.	1	1.75	0.68	5.22	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	3.762	0.399	15.924	0.000	0.000	0.000	0.000	0.173	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	522	218	351	0	0	0	0	307	0
N.S.	1	1.86	0.78	1.25	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	5.285	0.528	7.113	0.000	0.000	0.000	0.000	0.176	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	151	152	0	0	0	0	0	37	0
N.S.	1	0.90	0.90	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.453	0.415	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	25	40	49	24	52	25
N.S.	1	1.00	1.09	1.00	1.14	1.82	2.23	1.09	2.36	1.14
time (sec)	N/A	0.248	7.119	1.241	0.125	0.075	1.551	0.600	0.166	3.854

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	25	40	49	24	52	25
N.S.	1	1.00	1.09	1.00	1.14	1.82	2.23	1.09	2.36	1.14
time (sec)	N/A	0.245	5.523	1.955	0.120	0.078	1.532	0.544	0.169	3.834

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	31	31	0	118	0	0	113	0
N.S.	1	0.88	0.76	0.76	0.00	2.88	0.00	0.00	2.76	0.00
time (sec)	N/A	0.352	0.163	0.888	0.000	0.086	0.000	0.000	0.169	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	24	24	0	102	0	0	52	0
N.S.	1	0.93	0.83	0.83	0.00	3.52	0.00	0.00	1.79	0.00
time (sec)	N/A	0.333	0.136	0.976	0.000	0.075	0.000	0.000	0.174	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	22	24	0	88	0	0	52	0
N.S.	1	0.93	0.81	0.89	0.00	3.26	0.00	0.00	1.93	0.00
time (sec)	N/A	0.332	0.134	0.950	0.000	0.074	0.000	0.000	0.173	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	24	24	0	102	0	0	50	0
N.S.	1	0.93	0.83	0.83	0.00	3.52	0.00	0.00	1.72	0.00
time (sec)	N/A	0.314	0.129	0.661	0.000	0.085	0.000	0.000	0.163	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	33	31	0	118	0	0	48	0
N.S.	1	0.88	0.80	0.76	0.00	2.88	0.00	0.00	1.17	0.00
time (sec)	N/A	0.314	0.090	0.793	0.000	0.087	0.000	0.000	0.162	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	25	39	49	24	49	25
N.S.	1	1.00	1.09	1.00	1.14	1.77	2.23	1.09	2.23	1.14
time (sec)	N/A	0.239	0.924	1.947	0.115	0.078	2.113	0.446	0.167	3.596

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	151	40	56	24	60	25
N.S.	1	1.00	1.09	1.00	6.86	1.82	2.55	1.09	2.73	1.14
time (sec)	N/A	3.229	7.504	0.129	0.137	0.081	2.194	0.165	0.192	3.954

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	49	62	0	232	0	0	395	0
N.S.	1	1.04	0.92	1.17	0.00	4.38	0.00	0.00	7.45	0.00
time (sec)	N/A	0.491	0.187	1.512	0.000	0.082	0.000	0.000	0.178	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	172	80	54	0	231	0	0	60	0
N.S.	1	3.13	1.45	0.98	0.00	4.20	0.00	0.00	1.09	0.00
time (sec)	N/A	1.793	0.140	1.526	0.000	0.087	0.000	0.000	0.163	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	97	56	38	0	164	0	0	392	0
N.S.	1	2.37	1.37	0.93	0.00	4.00	0.00	0.00	9.56	0.00
time (sec)	N/A	0.808	0.204	1.418	0.000	0.082	0.000	0.000	0.167	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	87	75	54	0	225	0	0	58	0
N.S.	1	1.64	1.42	1.02	0.00	4.25	0.00	0.00	1.09	0.00
time (sec)	N/A	0.948	0.126	1.314	0.000	0.082	0.000	0.000	0.167	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	43	60	0	222	0	0	208	0
N.S.	1	1.06	0.88	1.22	0.00	4.53	0.00	0.00	4.24	0.00
time (sec)	N/A	0.434	0.133	1.523	0.000	0.083	0.000	0.000	0.176	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	153	39	56	24	57	25
N.S.	1	1.00	1.09	1.00	6.95	1.77	2.55	1.09	2.59	1.14
time (sec)	N/A	2.462	3.235	1.812	0.142	0.078	2.818	0.162	0.176	3.894

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	206	60	90	0	256	0	0	441	0
N.S.	1	2.06	0.60	0.90	0.00	2.56	0.00	0.00	4.41	0.00
time (sec)	N/A	2.112	0.207	1.367	0.000	0.089	0.000	0.000	0.180	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	270	66	82	0	267	0	0	479	0
N.S.	1	2.52	0.62	0.77	0.00	2.50	0.00	0.00	4.48	0.00
time (sec)	N/A	2.377	0.219	1.300	0.000	0.095	0.000	0.000	0.184	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	229	56	51	0	193	0	0	453	0
N.S.	1	2.66	0.65	0.59	0.00	2.24	0.00	0.00	5.27	0.00
time (sec)	N/A	2.307	0.175	1.378	0.000	0.076	0.000	0.000	0.179	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	188	96	82	0	256	0	0	622	0
N.S.	1	1.88	0.96	0.82	0.00	2.56	0.00	0.00	6.22	0.00
time (sec)	N/A	1.584	0.185	1.287	0.000	0.080	0.000	0.000	0.181	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	116	86	88	0	241	0	0	239	0
N.S.	1	1.68	1.25	1.28	0.00	3.49	0.00	0.00	3.46	0.00
time (sec)	N/A	1.103	0.312	0.645	0.000	0.082	0.000	0.000	0.170	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	255	39	56	24	57	25
N.S.	1	1.00	1.09	1.00	11.59	1.77	2.55	1.09	2.59	1.14
time (sec)	N/A	2.996	3.845	1.003	0.159	0.083	2.963	0.195	0.177	3.661

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	219	108	122	0	272	0	0	444	0
N.S.	1	1.75	0.86	0.98	0.00	2.18	0.00	0.00	3.55	0.00
time (sec)	N/A	1.627	0.204	0.848	0.000	0.085	0.000	0.000	0.174	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	409	132	152	0	303	0	0	468	0
N.S.	1	2.41	0.78	0.89	0.00	1.78	0.00	0.00	2.75	0.00
time (sec)	N/A	3.771	0.149	2.029	0.000	0.079	0.000	0.000	0.178	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	621	166	182	0	341	0	0	468	0
N.S.	1	2.42	0.65	0.71	0.00	1.33	0.00	0.00	1.82	0.00
time (sec)	N/A	4.799	0.381	1.829	0.000	0.092	0.000	0.000	0.177	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	144	81	124	240	131	0	0	132	206
N.S.	1	1.07	0.60	0.93	1.79	0.98	0.00	0.00	0.99	1.54
time (sec)	N/A	0.453	0.176	0.694	0.047	0.085	0.000	0.000	0.166	4.282

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	334	157	168	516	179	0	0	295	493
N.S.	1	1.56	0.73	0.79	2.41	0.84	0.00	0.00	1.38	2.30
time (sec)	N/A	0.940	0.223	14.471	0.043	0.080	0.000	0.000	0.162	5.508

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	484	143	212	871	216	0	0	220	1041
N.S.	1	1.66	0.49	0.73	2.99	0.74	0.00	0.00	0.76	3.58
time (sec)	N/A	2.281	0.091	7.473	0.062	0.093	0.000	0.000	0.164	5.800

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	226	257	0	0	0	0	0	43	0
N.S.	1	0.90	1.02	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.556	0.483	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	47	22	23	60	23
N.S.	1	1.00	1.09	1.00	1.05	2.14	1.00	1.05	2.73	1.05
time (sec)	N/A	0.228	7.554	0.344	0.118	0.081	1.508	0.552	0.176	4.062

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	47	22	23	60	23
N.S.	1	1.00	1.09	1.00	1.05	2.14	1.00	1.05	2.73	1.05
time (sec)	N/A	0.242	16.259	1.172	0.132	0.082	1.484	0.586	0.175	4.397

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	40	40	0	220	0	0	200	0
N.S.	1	0.85	0.73	0.73	0.00	4.00	0.00	0.00	3.64	0.00
time (sec)	N/A	0.367	0.116	0.914	0.000	0.089	0.000	0.000	0.182	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	38	33	33	0	200	0	0	60	0
N.S.	1	0.88	0.77	0.77	0.00	4.65	0.00	0.00	1.40	0.00
time (sec)	N/A	0.357	0.124	0.885	0.000	0.092	0.000	0.000	0.169	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	40	40	0	216	0	0	60	0
N.S.	1	0.85	0.73	0.73	0.00	3.93	0.00	0.00	1.09	0.00
time (sec)	N/A	0.367	0.099	0.879	0.000	0.084	0.000	0.000	0.170	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	24	24	0	136	0	0	60	0
N.S.	1	0.93	0.83	0.83	0.00	4.69	0.00	0.00	2.07	0.00
time (sec)	N/A	0.346	0.107	0.742	0.000	0.082	0.000	0.000	0.168	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	55	40	0	216	0	0	60	0
N.S.	1	0.85	1.00	0.73	0.00	3.93	0.00	0.00	1.09	0.00
time (sec)	N/A	0.364	0.207	0.871	0.000	0.086	0.000	0.000	0.166	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	38	43	33	0	200	0	0	58	0
N.S.	1	0.88	1.00	0.77	0.00	4.65	0.00	0.00	1.35	0.00
time (sec)	N/A	0.332	0.212	0.725	0.000	0.080	0.000	0.000	0.166	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	40	40	0	216	0	0	56	0
N.S.	1	0.85	0.73	0.73	0.00	3.93	0.00	0.00	1.02	0.00
time (sec)	N/A	0.342	0.187	0.910	0.000	0.086	0.000	0.000	0.180	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	44	22	23	58	23
N.S.	1	1.00	1.09	1.00	1.05	2.00	1.00	1.05	2.64	1.05
time (sec)	N/A	0.232	1.011	0.229	0.130	0.080	2.190	0.423	0.164	3.584

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	23	46	24	23	60	23
N.S.	1	1.00	1.09	1.00	1.05	2.09	1.09	1.05	2.73	1.05
time (sec)	N/A	0.240	2.222	0.416	0.116	0.085	2.099	0.457	0.165	3.572

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	120	56	78	0	418	0	0	68	0
N.S.	1	1.79	0.84	1.16	0.00	6.24	0.00	0.00	1.01	0.00
time (sec)	N/A	0.960	0.137	1.565	0.000	0.099	0.000	0.000	0.168	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	63	56	86	0	413	0	0	316	0
N.S.	1	0.95	0.85	1.30	0.00	6.26	0.00	0.00	4.79	0.00
time (sec)	N/A	0.442	0.252	1.710	0.000	0.085	0.000	0.000	0.163	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	227	73	121	0	447	0	0	954	0
N.S.	1	1.99	0.64	1.06	0.00	3.92	0.00	0.00	8.37	0.00
time (sec)	N/A	1.364	0.236	1.726	0.000	0.083	0.000	0.000	0.169	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	149	83	131	0	435	0	0	365	0
N.S.	1	1.67	0.93	1.47	0.00	4.89	0.00	0.00	4.10	0.00
time (sec)	N/A	1.138	0.132	0.853	0.000	0.088	0.000	0.000	0.166	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	218	79	118	163	91	0	0	23	0
N.S.	1	1.57	0.57	0.85	1.17	0.65	0.00	0.00	0.17	0.00
time (sec)	N/A	0.717	0.068	1.759	0.113	0.098	0.000	0.000	0.166	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	234	160	173	0	0	0	0	23	0
N.S.	1	1.19	0.81	0.88	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.652	0.364	0.773	0.000	0.000	0.000	0.000	0.177	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	109	60	97	88	72	0	0	23	0
N.S.	1	1.25	0.69	1.11	1.01	0.83	0.00	0.00	0.26	0.00
time (sec)	N/A	0.423	0.050	0.754	0.112	0.106	0.000	0.000	0.166	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	150	125	152	0	0	0	0	23	0
N.S.	1	1.03	0.86	1.04	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.385	0.197	0.740	0.000	0.000	0.000	0.000	0.168	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	79	30	58	0	47	21	0
N.S.	1	1.00	0.91	2.47	0.94	1.81	0.00	1.47	0.66	0.00
time (sec)	N/A	0.228	0.025	0.635	0.105	0.097	0.000	0.137	0.175	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	76	113	0	0	0	0	20	0
N.S.	1	1.00	0.80	1.19	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.238	0.081	0.707	0.000	0.000	0.000	0.000	0.167	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	99	0	0	0	0	23	0
N.S.	1	1.00	0.76	1.32	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.270	0.099	0.791	0.000	0.000	0.000	0.000	0.179	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	70	51	58	0	111	23	0
N.S.	1	1.00	1.14	1.67	1.21	1.38	0.00	2.64	0.55	0.00
time (sec)	N/A	0.277	0.039	0.913	0.117	0.104	0.000	0.146	0.172	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	130	126	139	0	0	0	0	23	0
N.S.	1	0.95	0.92	1.01	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.426	0.522	0.911	0.000	0.000	0.000	0.000	0.180	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	321	160	173	0	0	0	0	25	0
N.S.	1	1.57	0.78	0.84	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.098	0.322	0.503	0.000	0.000	0.000	0.000	0.175	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	158	188	0	0	0	0	0	25	0
N.S.	1	0.98	1.17	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.098	0.571	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	104	149	0	0	0	0	23	0
N.S.	1	1.05	0.87	1.24	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.372	0.190	0.477	0.000	0.000	0.000	0.000	0.179	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	90	119	0	0	0	0	0	22	0
N.S.	1	0.87	1.16	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.472	0.103	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	80	100	158	0	0	0	0	25	0
N.S.	1	1.18	1.47	2.32	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.506	0.103	0.461	0.000	0.000	0.000	0.000	0.168	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	104	89	129	0	0	0	0	25	0
N.S.	1	0.99	0.85	1.23	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.434	0.394	0.477	0.000	0.000	0.000	0.000	0.167	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	157	188	229	0	0	0	0	25	0
N.S.	1	1.03	1.24	1.51	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.124	0.883	0.508	0.000	0.000	0.000	0.000	0.172	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	317	215	0	0	0	0	0	25	0
N.S.	1	1.45	0.98	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.691	0.713	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	305	303	570	0	0	0	0	0	25	0
N.S.	1	0.99	1.87	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.609	3.030	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	117	157	0	0	0	0	0	23	0
N.S.	1	0.91	1.23	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.628	0.165	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	138	451	0	0	0	0	0	22	0
N.S.	1	0.90	2.95	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.598	0.302	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	122	146	215	0	0	0	0	25	0
N.S.	1	1.20	1.43	2.11	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.645	0.135	0.537	0.000	0.000	0.000	0.000	0.170	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	107	131	188	0	0	0	0	25	0
N.S.	1	1.09	1.34	1.92	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.710	0.307	0.550	0.000	0.000	0.000	0.000	0.187	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	264	301	384	0	0	0	0	25	0
N.S.	1	0.99	1.13	1.44	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.629	6.348	0.567	0.000	0.000	0.000	0.000	0.172	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	41	24	22	46	22
N.S.	1	1.00	1.09	0.91	1.00	1.86	1.09	1.00	2.09	1.00
time (sec)	N/A	0.251	2.345	2.191	0.222	0.085	46.297	0.201	0.195	3.935

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	85	76	123	96	94	0	0	46	0
N.S.	1	1.15	1.03	1.66	1.30	1.27	0.00	0.00	0.62	0.00
time (sec)	N/A	0.421	0.046	1.129	0.113	0.094	0.000	0.000	0.167	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	143	121	190	0	0	0	0	46	0
N.S.	1	1.04	0.88	1.39	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.400	0.241	1.010	0.000	0.000	0.000	0.000	0.181	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	27	38	39	51	0	61	44	0
N.S.	1	1.00	0.63	0.88	0.91	1.19	0.00	1.42	1.02	0.00
time (sec)	N/A	0.243	0.031	0.645	0.027	0.092	0.000	0.143	0.178	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	27	38	36	47	0	59	43	0
N.S.	1	1.00	0.68	0.95	0.90	1.18	0.00	1.48	1.08	0.00
time (sec)	N/A	0.211	0.024	0.737	0.034	0.095	0.000	0.156	0.176	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	122	97	157	0	0	0	0	44	0
N.S.	1	1.09	0.87	1.40	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.569	0.180	1.230	0.000	0.000	0.000	0.000	0.183	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	89	182	84	107	0	155	46	0
N.S.	1	1.05	1.09	2.22	1.02	1.30	0.00	1.89	0.56	0.00
time (sec)	N/A	0.553	0.087	1.283	0.032	0.098	0.000	0.156	0.179	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	256	182	347	0	0	0	0	46	0
N.S.	1	1.43	1.02	1.94	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.237	1.133	1.406	0.000	0.000	0.000	0.000	0.189	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	26	24	48	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	1.08	1.00	2.00	1.00
time (sec)	N/A	0.277	0.476	0.819	0.296	0.090	43.210	0.156	0.202	3.938

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	206	165	219	0	0	0	0	48	0
N.S.	1	1.11	0.89	1.18	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.796	0.305	0.505	0.000	0.000	0.000	0.000	0.168	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	159	193	0	0	0	0	0	48	0
N.S.	1	0.93	1.13	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.967	0.278	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	34	45	62	69	0	0	46	0
N.S.	1	1.03	0.50	0.66	0.91	1.01	0.00	0.00	0.68	0.00
time (sec)	N/A	0.358	0.044	0.431	0.032	0.099	0.000	0.000	0.180	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	38	49	57	69	0	0	45	0
N.S.	1	1.00	0.60	0.78	0.90	1.10	0.00	0.00	0.71	0.00
time (sec)	N/A	0.241	0.033	0.437	0.034	0.100	0.000	0.000	0.186	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	155	159	232	0	0	0	0	46	0
N.S.	1	1.22	1.25	1.83	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.177	0.216	0.509	0.000	0.000	0.000	0.000	0.184	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	215	336	0	0	0	0	48	0
N.S.	1	1.00	1.26	1.96	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.831	0.781	0.506	0.000	0.000	0.000	0.000	0.188	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	316	266	313	0	0	0	0	48	0
N.S.	1	1.43	1.20	1.42	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	3.522	1.898	0.543	0.000	0.000	0.000	0.000	0.193	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	26	24	48	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	1.08	1.00	2.00	1.00
time (sec)	N/A	0.271	0.483	0.731	0.370	0.094	97.119	0.174	0.244	4.048

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	249	0	0	0	0	0	48	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.394	0.348	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	234	541	0	0	0	0	0	48	0
N.S.	1	0.95	2.20	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.302	0.661	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	93	45	56	88	91	0	0	46	0
N.S.	1	0.99	0.48	0.60	0.94	0.97	0.00	0.00	0.49	0.00
time (sec)	N/A	0.386	0.049	0.524	0.034	0.092	0.000	0.000	0.178	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	90	45	56	86	87	0	0	45	0
N.S.	1	1.02	0.51	0.64	0.98	0.99	0.00	0.00	0.51	0.00
time (sec)	N/A	0.339	0.037	0.510	0.034	0.099	0.000	0.000	0.185	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	220	230	305	0	0	0	0	46	0
N.S.	1	1.19	1.24	1.65	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.441	0.268	0.615	0.000	0.000	0.000	0.000	0.176	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	201	270	488	0	0	0	0	48	0
N.S.	1	1.07	1.44	2.61	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.420	1.399	0.697	0.000	0.000	0.000	0.000	0.182	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	488	377	482	0	0	0	0	48	0
N.S.	1	1.36	1.05	1.34	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	4.165	6.246	0.651	0.000	0.000	0.000	0.000	0.180	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	24	24	50	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	1.00	1.00	2.08	1.00
time (sec)	N/A	0.277	0.428	0.735	0.128	0.087	65.500	0.169	0.165	4.228

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	24	24	50	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	1.00	1.00	2.08	1.00
time (sec)	N/A	0.279	2.385	0.174	0.122	0.083	3.112	0.149	0.165	4.048

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	48	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	5.33	0.00
time (sec)	N/A	0.297	0.091	0.319	0.000	0.000	0.000	0.000	0.181	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	46	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	5.11	0.00
time (sec)	N/A	0.261	0.045	0.329	0.000	0.000	0.000	0.000	0.168	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	40	24	0	47	24
N.S.	1	1.00	1.08	0.92	1.00	1.67	1.00	0.00	1.96	1.00
time (sec)	N/A	0.284	1.437	0.942	0.126	0.077	5.437	0.000	0.174	3.764

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	0	24	54	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	0.00	1.00	2.25	1.00
time (sec)	N/A	0.276	0.540	0.736	0.129	0.083	0.000	0.182	0.184	4.256

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	26	24	54	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	1.08	1.00	2.25	1.00
time (sec)	N/A	0.823	2.851	0.184	0.156	0.084	4.173	0.166	0.189	4.129

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	65	0	0	0	0	52	0
N.S.	1	1.00	0.94	1.81	0.00	0.00	0.00	0.00	1.44	0.00
time (sec)	N/A	0.438	0.068	0.543	0.000	0.000	0.000	0.000	0.180	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0	50	0
N.S.	1	1.00	0.91	1.77	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.422	0.098	0.550	0.000	0.000	0.000	0.000	0.181	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	40	26	0	51	24
N.S.	1	1.00	1.08	0.92	1.00	1.67	1.08	0.00	2.12	1.00
time (sec)	N/A	1.133	5.809	0.197	0.126	0.082	7.019	0.000	0.177	3.782

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	0	24	54	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	0.00	1.00	2.25	1.00
time (sec)	N/A	0.291	0.738	0.763	0.131	0.092	0.000	0.190	0.177	3.935

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	43	26	24	54	24
N.S.	1	1.00	1.08	0.92	1.00	1.79	1.08	1.00	2.25	1.00
time (sec)	N/A	1.035	5.164	0.194	0.150	0.078	5.745	0.173	0.180	3.821

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	43	87	0	0	0	0	52	0
N.S.	1	1.04	0.63	1.28	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.573	0.125	0.533	0.000	0.000	0.000	0.000	0.192	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	44	86	0	0	0	0	50	0
N.S.	1	1.06	0.68	1.32	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.549	0.073	0.537	0.000	0.000	0.000	0.000	0.170	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	40	26	0	51	24
N.S.	1	1.00	1.08	0.92	1.00	1.67	1.08	0.00	2.12	1.00
time (sec)	N/A	1.354	11.499	0.205	0.149	0.085	9.965	0.000	0.168	3.646

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	331	178	195	0	0	0	0	21	0
N.S.	1	1.36	0.73	0.80	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.960	0.511	1.354	0.000	0.000	0.000	0.000	0.164	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	210	79	120	128	91	0	0	21	0
N.S.	1	1.54	0.58	0.88	0.94	0.67	0.00	0.00	0.15	0.00
time (sec)	N/A	0.698	0.048	1.060	0.114	0.109	0.000	0.000	0.160	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	226	160	175	0	0	0	0	21	0
N.S.	1	1.16	0.82	0.90	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.630	0.360	1.053	0.000	0.000	0.000	0.000	0.168	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	49	99	50	72	0	0	19	0
N.S.	1	1.08	0.83	1.68	0.85	1.22	0.00	0.00	0.32	0.00
time (sec)	N/A	0.243	0.033	0.954	0.103	0.094	0.000	0.000	0.168	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	117	152	0	0	0	0	18	0
N.S.	1	1.01	0.82	1.06	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.335	0.222	0.686	0.000	0.000	0.000	0.000	0.166	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	111	0	0	0	0	21	0
N.S.	1	1.00	0.91	1.11	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.395	0.122	0.974	0.000	0.000	0.000	0.000	0.166	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	142	133	186	0	0	0	0	21	0
N.S.	1	1.09	1.02	1.43	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.577	0.354	0.924	0.000	0.000	0.000	0.000	0.174	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	130	126	139	0	0	0	0	21	0
N.S.	1	0.96	0.93	1.02	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.543	0.521	0.831	0.000	0.000	0.000	0.000	0.173	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	96	90	75	0	0	21	0
N.S.	1	1.00	1.13	1.37	1.29	1.07	0.00	0.00	0.30	0.00
time (sec)	N/A	0.278	0.052	1.396	0.114	0.096	0.000	0.000	0.170	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	295	222	164	0	0	0	0	21	0
N.S.	1	1.54	1.16	0.86	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.905	1.136	1.385	0.000	0.000	0.000	0.000	0.179	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	331	104	116	204	93	0	0	21	0
N.S.	1	2.21	0.69	0.77	1.36	0.62	0.00	0.00	0.14	0.00
time (sec)	N/A	1.116	0.086	1.414	0.114	0.112	0.000	0.000	0.172	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	437	307	183	0	0	0	0	21	0
N.S.	1	1.80	1.26	0.75	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.376	2.271	1.396	0.000	0.000	0.000	0.000	0.166	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	892	268	0	0	0	0	0	23	0
N.S.	1	2.65	0.80	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	7.142	1.080	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	926	175	211	0	0	0	0	23	0
N.S.	1	3.30	0.62	0.75	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	4.342	0.477	0.582	0.000	0.000	0.000	0.000	0.172	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	254	475	228	0	0	0	0	0	23	0
N.S.	1	1.87	0.90	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	4.302	0.897	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	179	135	175	0	0	0	0	21	0
N.S.	1	1.02	0.77	1.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.494	0.326	0.525	0.000	0.000	0.000	0.000	0.163	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	150	187	0	0	0	0	0	20	0
N.S.	1	0.95	1.18	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.701	0.530	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	212	203	0	0	0	0	0	23	0
N.S.	1	1.22	1.17	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.292	0.239	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	193	223	0	0	0	0	0	23	0
N.S.	1	0.98	1.13	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.474	0.568	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	188	229	0	0	0	0	23	0
N.S.	1	1.04	1.25	1.52	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.193	1.001	0.500	0.000	0.000	0.000	0.000	0.180	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	163	177	171	0	0	0	0	23	0
N.S.	1	0.96	1.05	1.01	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.731	1.376	0.575	0.000	0.000	0.000	0.000	0.168	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	791	272	215	0	0	0	0	48	0
N.S.	1	2.71	0.93	0.74	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.801	0.980	1.064	0.000	0.000	0.000	0.000	0.195	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	566	79	140	163	106	0	0	48	0
N.S.	1	3.04	0.42	0.75	0.88	0.57	0.00	0.00	0.26	0.00
time (sec)	N/A	2.137	0.063	0.964	0.118	0.106	0.000	0.000	0.191	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	562	224	195	0	0	0	0	48	0
N.S.	1	2.31	0.92	0.80	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.894	0.687	0.952	0.000	0.000	0.000	0.000	0.177	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	88	61	120	67	90	0	0	46	0
N.S.	1	1.09	0.75	1.48	0.83	1.11	0.00	0.00	0.57	0.00
time (sec)	N/A	0.265	0.048	0.897	0.128	0.105	0.000	0.000	0.191	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	193	176	173	0	0	0	0	45	0
N.S.	1	1.02	0.93	0.92	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.481	0.460	0.634	0.000	0.000	0.000	0.000	0.184	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	169	143	130	0	0	0	0	46	0
N.S.	1	1.17	0.99	0.90	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.747	0.203	0.821	0.000	0.000	0.000	0.000	0.190	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	291	181	203	0	0	0	0	45	0
N.S.	1	1.63	1.01	1.13	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.221	0.527	1.425	0.000	0.000	0.000	0.000	0.184	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	235	158	143	0	0	0	0	48	0
N.S.	1	1.40	0.94	0.85	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.074	0.707	0.845	0.000	0.000	0.000	0.000	0.201	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	217	240	218	0	0	0	0	48	0
N.S.	1	1.15	1.27	1.15	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.055	1.016	0.856	0.000	0.000	0.000	0.000	0.183	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	430	282	162	0	0	0	0	48	0
N.S.	1	2.25	1.48	0.85	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.656	2.362	0.904	0.000	0.000	0.000	0.000	0.209	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	104	116	126	93	0	0	48	0
N.S.	1	1.05	1.11	1.23	1.34	0.99	0.00	0.00	0.51	0.00
time (sec)	N/A	0.306	0.064	1.195	0.110	0.099	0.000	0.000	0.186	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	737	474	184	0	0	0	0	48	0
N.S.	1	3.03	1.95	0.76	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.714	4.148	1.119	0.000	0.000	0.000	0.000	0.197	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	242	224	193	0	0	0	0	70	0
N.S.	1	1.04	0.96	0.83	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.620	0.839	1.015	0.000	0.000	0.000	0.000	0.192	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	193	176	173	0	0	0	0	45	0
N.S.	1	1.02	0.93	0.92	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.506	0.040	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	117	152	0	0	0	0	18	0
N.S.	1	1.01	0.82	1.06	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.352	0.043	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	59	74	73	0	90	58	0
N.S.	1	1.00	0.55	0.66	0.83	0.82	0.00	1.01	0.65	0.00
time (sec)	N/A	0.323	0.037	1.676	0.024	0.098	0.000	0.149	0.180	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	138	65	79	108	99	0	114	81	0
N.S.	1	1.04	0.49	0.59	0.81	0.74	0.00	0.86	0.61	0.00
time (sec)	N/A	0.489	0.044	1.549	0.032	0.082	0.000	0.155	0.181	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	187	81	99	140	121	0	138	96	0
N.S.	1	1.06	0.46	0.56	0.79	0.68	0.00	0.78	0.54	0.00
time (sec)	N/A	0.619	0.051	1.559	0.034	0.110	0.000	0.155	0.180	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	228	206	345	0	0	0	0	49	0
N.S.	1	0.78	0.71	1.19	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.625	0.617	1.237	0.000	0.000	0.000	0.000	0.172	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	176	119	319	0	0	0	0	21	0
N.S.	1	0.75	0.51	1.36	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.470	0.271	0.585	0.000	0.000	0.000	0.000	0.170	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	125	109	302	0	0	0	0	25	0
N.S.	1	0.69	0.60	1.66	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.336	0.147	0.549	0.000	0.000	0.000	0.000	0.165	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	74	90	54	0	70	50	0
N.S.	1	1.00	0.90	1.54	1.88	1.12	0.00	1.46	1.04	0.00
time (sec)	N/A	0.228	0.057	0.531	0.112	0.115	0.000	0.164	0.166	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	64	121	90	84	0	111	66	0
N.S.	1	1.03	0.61	1.15	0.86	0.80	0.00	1.06	0.63	0.00
time (sec)	N/A	0.355	0.068	0.899	0.039	0.093	0.000	0.161	0.181	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	168	80	137	132	112	0	149	88	0
N.S.	1	1.07	0.51	0.87	0.84	0.71	0.00	0.95	0.56	0.00
time (sec)	N/A	0.523	0.079	0.788	0.040	0.090	0.000	0.176	0.169	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	150	187	0	0	0	0	0	20	0
N.S.	1	0.95	1.18	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.687	0.061	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	161	70	84	304	105	0	0	60	0
N.S.	1	1.16	0.50	0.60	2.19	0.76	0.00	0.00	0.43	0.00
time (sec)	N/A	0.417	0.055	0.447	0.162	0.091	0.000	0.000	0.164	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	283	94	118	514	139	0	0	83	0
N.S.	1	1.36	0.45	0.57	2.47	0.67	0.00	0.00	0.40	0.00
time (sec)	N/A	0.665	0.063	0.464	0.186	0.102	0.000	0.000	0.190	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	429	120	152	751	169	0	0	98	0
N.S.	1	1.55	0.43	0.55	2.71	0.61	0.00	0.00	0.35	0.00
time (sec)	N/A	0.999	0.078	0.482	0.186	0.109	0.000	0.000	0.173	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	291	569	0	0	0	0	0	20	0
N.S.	1	0.96	1.88	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.197	2.235	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	240	87	105	0	134	0	0	60	0
N.S.	1	1.26	0.46	0.55	0.00	0.70	0.00	0.00	0.31	0.00
time (sec)	N/A	0.890	0.065	0.490	0.000	0.093	0.000	0.000	0.174	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	439	119	153	0	176	0	0	83	0
N.S.	1	1.52	0.41	0.53	0.00	0.61	0.00	0.00	0.29	0.00
time (sec)	N/A	1.890	0.076	0.542	0.000	0.101	0.000	0.000	0.179	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	687	151	201	0	214	0	0	98	0
N.S.	1	1.78	0.39	0.52	0.00	0.56	0.00	0.00	0.25	0.00
time (sec)	N/A	3.803	0.091	0.550	0.000	0.096	0.000	0.000	0.178	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	20	21	20	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	0.95	1.00
time (sec)	N/A	0.207	1.133	0.137	0.087	0.072	0.519	0.148	0.181	3.375

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	33	22	21	22	21
N.S.	1	1.00	1.10	0.90	1.00	1.57	1.05	1.00	1.05	1.00
time (sec)	N/A	0.215	0.310	0.295	0.085	0.075	0.690	0.134	0.176	3.430

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	46	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	5.11	0.00
time (sec)	N/A	0.265	0.034	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	24	21	0	0	0	0	66	0
N.S.	1	0.93	0.89	0.78	0.00	0.00	0.00	0.00	2.44	0.00
time (sec)	N/A	0.324	0.053	0.491	0.000	0.000	0.000	0.000	0.168	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	31	30	0	0	0	0	92	0
N.S.	1	0.88	0.76	0.73	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.347	0.061	0.489	0.000	0.000	0.000	0.000	0.180	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	42	39	0	0	0	0	112	0
N.S.	1	0.85	0.76	0.71	0.00	0.00	0.00	0.00	2.04	0.00
time (sec)	N/A	0.360	0.065	0.497	0.000	0.000	0.000	0.000	0.168	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	22	21	20	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.05	1.00	0.95	1.00
time (sec)	N/A	0.207	1.266	0.148	0.096	0.075	0.768	0.145	0.174	3.509

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	33	24	21	22	21
N.S.	1	1.00	1.10	0.90	1.00	1.57	1.14	1.00	1.05	1.00
time (sec)	N/A	0.218	0.927	0.115	0.092	0.085	1.001	0.151	0.192	3.629

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0	50	0
N.S.	1	1.00	0.91	1.77	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.414	0.092	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	0	0	0	72	0
N.S.	1	1.00	0.88	2.31	0.00	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.461	0.099	1.006	0.000	0.000	0.000	0.000	0.176	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	63	56	176	0	0	0	0	100	0
N.S.	1	0.95	0.85	2.67	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	0.494	0.117	1.178	0.000	0.000	0.000	0.000	0.193	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	74	64	232	0	0	0	0	122	0
N.S.	1	0.92	0.80	2.90	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	0.509	0.141	1.146	0.000	0.000	0.000	0.000	0.174	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	22	21	20	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.05	1.00	0.95	1.00
time (sec)	N/A	0.213	1.383	0.143	0.091	0.075	1.082	0.149	0.161	3.747

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	33	24	21	22	21
N.S.	1	1.00	1.10	0.90	1.00	1.57	1.14	1.00	1.05	1.00
time (sec)	N/A	0.219	0.879	0.609	0.106	0.072	1.416	0.159	0.164	3.740

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	44	86	0	0	0	0	50	0
N.S.	1	1.06	0.68	1.32	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.562	0.066	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	111	56	180	0	0	0	0	72	0
N.S.	1	1.41	0.71	2.28	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	1.296	0.155	1.217	0.000	0.000	0.000	0.000	0.178	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	133	79	272	0	0	0	0	100	0
N.S.	1	1.43	0.85	2.92	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	1.350	0.186	1.047	0.000	0.000	0.000	0.000	0.192	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	155	99	364	0	0	0	0	122	0
N.S.	1	1.45	0.93	3.40	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	1.398	0.174	1.462	0.000	0.000	0.000	0.000	0.169	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	193	1435	0	0	0	0	150	0
N.S.	1	1.00	1.58	11.76	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.599	0.602	18.464	0.000	0.000	0.000	0.000	0.164	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	247	213	245	276	248	372	1471	358	288
N.S.	1	1.01	0.87	1.00	1.13	1.01	1.52	6.00	1.46	1.18
time (sec)	N/A	0.744	0.055	0.545	0.029	0.086	0.730	0.184	0.172	3.960

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	171	150	167	198	178	245	930	253	190
N.S.	1	1.01	0.89	0.99	1.17	1.05	1.45	5.50	1.50	1.12
time (sec)	N/A	0.578	0.041	0.341	0.036	0.092	0.498	0.157	0.179	3.964

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	112	98	105	131	119	155	527	162	118
N.S.	1	1.02	0.89	0.95	1.19	1.08	1.41	4.79	1.47	1.07
time (sec)	N/A	0.404	0.029	0.527	0.034	0.092	0.386	0.137	0.168	3.754

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	69	57	65	65	73	266	84	60
N.S.	1	1.04	1.21	1.00	1.14	1.14	1.28	4.67	1.47	1.05
time (sec)	N/A	0.280	0.018	0.141	0.030	0.088	0.276	0.125	0.177	3.541

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	439	662	364	406	0	0	0	94	0
N.S.	1	1.02	1.54	0.85	0.95	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.820	1.222	3.101	0.198	0.000	0.000	0.000	0.164	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	618	746	1951	550	0	0	0	1167	0
N.S.	1	1.05	1.26	3.31	0.93	0.00	0.00	0.00	1.98	0.00
time (sec)	N/A	1.313	4.812	8.625	0.178	0.000	0.000	0.000	0.180	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	657	682	1541	4047	1087	0	0	0	0	0
N.S.	1	1.04	2.35	6.16	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.323	9.091	2.289	0.221	0.000	0.000	0.000	0.224	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	181	576	160	180	0	0	0	77	0
N.S.	1	1.06	3.37	0.94	1.05	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.537	0.628	3.117	0.028	0.000	0.000	0.000	0.174	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	23	22	14	48	15	15
N.S.	1	1.00	1.00	0.82	1.35	1.29	0.82	2.82	0.88	0.88
time (sec)	N/A	0.219	0.114	0.432	0.032	0.079	0.239	0.118	0.168	3.659

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	15	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	0.94	1.00
time (sec)	N/A	0.201	4.546	1.218	0.427	0.090	0.565	0.133	0.195	3.436

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	17	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.06	1.00
time (sec)	N/A	0.208	2.676	0.609	0.429	0.104	0.485	0.158	0.177	3.508

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	153	356	0	71	33	0
N.S.	1	1.00	1.92	0.00	2.47	5.74	0.00	1.15	0.53	0.00
time (sec)	N/A	0.299	0.074	0.000	0.064	0.111	0.000	0.145	0.188	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	130	226	0	223	730	0	135	52	0
N.S.	1	1.02	1.77	0.00	1.74	5.70	0.00	1.05	0.41	0.00
time (sec)	N/A	0.352	0.187	0.000	0.124	0.120	0.000	0.151	0.210	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	194	329	0	401	1280	0	218	71	0
N.S.	1	0.97	1.64	0.00	2.00	6.40	0.00	1.09	0.36	0.00
time (sec)	N/A	1.120	0.359	0.000	0.129	0.168	0.000	0.154	0.234	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	281	431	0	639	2006	0	349	90	0
N.S.	1	0.99	1.52	0.00	2.26	7.09	0.00	1.23	0.32	0.00
time (sec)	N/A	1.344	0.558	0.000	0.141	0.254	0.000	0.158	0.239	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	139	97	229	0	0	0	0	16	0
N.S.	1	0.75	0.52	1.23	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.414	0.271	1.653	0.000	0.000	0.000	0.000	0.179	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	99	90	210	0	0	0	0	20	0
N.S.	1	0.69	0.62	1.46	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.304	0.134	0.594	0.000	0.000	0.000	0.000	0.171	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	47	63	42	0	54	39	0
N.S.	1	1.00	0.81	1.27	1.70	1.14	0.00	1.46	1.05	0.00
time (sec)	N/A	0.222	0.045	0.786	0.110	0.085	0.000	0.139	0.172	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	45	85	67	62	0	86	49	0
N.S.	1	1.04	0.54	1.02	0.81	0.75	0.00	1.04	0.59	0.00
time (sec)	N/A	0.318	0.053	0.978	0.036	0.095	0.000	0.142	0.169	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	135	55	95	99	82	0	118	65	0
N.S.	1	1.09	0.44	0.77	0.80	0.66	0.00	0.95	0.52	0.00
time (sec)	N/A	0.456	0.059	0.866	0.036	0.092	0.000	0.147	0.176	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	0	434	0	0	0	0	78	0
N.S.	1	1.00	0.00	1.68	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.941	0.000	1.566	0.000	0.000	0.000	0.000	0.179	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	137	95	135	245	118	0	0	238	223
N.S.	1	1.28	0.89	1.26	2.29	1.10	0.00	0.00	2.22	2.08
time (sec)	N/A	0.944	0.159	0.892	0.037	0.086	0.000	0.000	0.175	4.391

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	137	96	135	245	118	0	0	238	223
N.S.	1	1.28	0.90	1.26	2.29	1.10	0.00	0.00	2.22	2.08
time (sec)	N/A	0.945	0.081	0.668	0.037	0.087	0.000	0.000	0.166	3.951

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	286	236	296	317	251	338	313	308	599
N.S.	1	1.09	0.90	1.13	1.21	0.95	1.29	1.19	1.17	2.28
time (sec)	N/A	1.139	0.086	6.273	0.044	0.102	1.863	0.263	0.169	6.733

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	222	192	249	271	196	279	0	244	851
N.S.	1	0.99	0.85	1.11	1.20	0.87	1.24	0.00	1.08	3.78
time (sec)	N/A	0.567	0.081	3.657	0.034	0.096	1.303	0.000	0.165	4.248

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	223	183	229	252	200	258	244	243	515
N.S.	1	1.08	0.89	1.11	1.22	0.97	1.25	1.18	1.18	2.50
time (sec)	N/A	0.924	0.071	3.060	0.038	0.112	0.903	0.250	0.172	5.401

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	174	171	139	202	209	173	557
N.S.	1	1.00	0.92	1.24	1.22	0.99	1.44	1.49	1.24	3.98
time (sec)	N/A	0.418	0.060	2.141	0.031	0.083	0.676	0.218	0.178	4.092

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	116	144	141	178	132	148	165	161	385
N.S.	1	1.12	1.38	1.36	1.71	1.27	1.42	1.59	1.55	3.70
time (sec)	N/A	0.917	0.023	1.859	0.039	0.091	0.415	0.179	0.176	4.494

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	216	190	0	1227	152	0	0	0	174	0
N.S.	1	0.88	0.00	5.68	0.70	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	1.234	0.000	11.179	0.129	0.000	0.000	0.000	0.180	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	85	332	0	0	0	0	0	155	0
N.S.	1	0.81	3.16	0.00	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	0.785	0.124	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	146	152	0	0	0	0	0	112	0
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.401	0.103	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	178	460	0	0	0	0	0	164	0
N.S.	1	0.90	2.34	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.596	0.210	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	234	299	0	0	0	0	0	134	0
N.S.	1	0.96	1.23	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.554	0.086	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	293	0	0	0	0	0	0	324	0
N.S.	1	1.14	0.00	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	2.600	0.000	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	504	1145	970	0	0	0	0	254	0
N.S.	1	0.98	2.24	1.89	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.083	4.763	14.128	0.000	0.000	0.000	0.000	0.183	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	599	609	1251	3550	0	0	0	0	367	0
N.S.	1	1.02	2.09	5.93	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.859	2.295	18.317	0.000	0.000	0.000	0.000	0.180	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	77	37	22	26	148	26
N.S.	1	1.00	1.08	1.00	3.21	1.54	0.92	1.08	6.17	1.08
time (sec)	N/A	0.831	0.141	0.782	0.185	0.091	116.602	0.128	0.209	4.942

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	613	602	1226	0	0	0	0	0	115	0
N.S.	1	0.98	2.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.528	2.419	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	487	1211	961	0	0	0	0	1230	0
N.S.	1	0.95	2.37	1.88	0.00	0.00	0.00	0.00	2.41	0.00
time (sec)	N/A	1.081	5.062	12.714	0.000	0.000	0.000	0.000	0.226	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	102	70	73	226	74	0	93	137	67
N.S.	1	1.31	0.90	0.94	2.90	0.95	0.00	1.19	1.76	0.86
time (sec)	N/A	0.580	0.108	0.789	0.085	0.085	0.000	0.129	0.176	3.964

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [438] had the largest ratio of [1.04167000000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	0.92	18	0.222
2	A	4	4	0.93	18	0.222
3	A	4	4	0.96	16	0.250
4	A	5	5	1.05	15	0.333
5	A	2	2	1.00	18	0.111
6	A	2	2	1.00	18	0.111
7	A	4	4	0.93	18	0.222
8	A	4	4	0.89	18	0.222
9	A	4	4	0.86	18	0.222
10	A	4	4	0.83	20	0.200
11	A	4	4	0.86	20	0.200
12	A	4	4	0.88	18	0.222
13	A	5	5	0.82	17	0.294
14	A	2	2	1.00	20	0.100
15	A	2	2	1.31	20	0.100
16	A	2	2	1.00	20	0.100
17	A	4	4	0.84	20	0.200
18	A	4	4	0.83	20	0.200
19	A	4	4	0.82	20	0.200
20	A	4	4	0.82	20	0.200
21	A	4	4	0.84	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	4	0.83	18	0.222
23	A	5	5	0.90	17	0.294
24	A	2	2	1.00	20	0.100
25	A	2	2	1.00	20	0.100
26	A	2	2	1.00	20	0.100
27	A	2	2	1.00	20	0.100
28	A	4	4	0.82	20	0.200
29	A	4	4	0.79	20	0.200
30	A	4	4	0.82	20	0.200
31	A	4	4	0.83	20	0.200
32	A	4	4	0.86	20	0.200
33	A	4	4	0.82	18	0.222
34	A	5	5	0.84	17	0.294
35	A	2	2	1.00	20	0.100
36	A	2	2	1.00	20	0.100
37	A	2	2	1.00	20	0.100
38	A	2	2	1.00	20	0.100
39	A	2	2	1.00	20	0.100
40	A	4	4	0.81	20	0.200
41	A	4	4	0.78	20	0.200
42	A	4	4	0.84	20	0.200
43	A	16	15	1.10	20	0.750
44	A	11	10	0.95	20	0.500
45	A	7	6	0.97	18	0.333
46	A	4	3	1.00	17	0.176
47	A	2	2	1.00	20	0.100
48	A	10	9	0.96	20	0.450
49	A	14	13	0.86	20	0.650
50	A	19	18	0.98	20	0.900
51	A	2	2	1.00	20	0.100
52	A	2	2	1.00	20	0.100
53	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	5	0.96	17	0.294
55	A	2	2	1.00	20	0.100
56	A	2	2	1.00	20	0.100
57	A	2	2	1.00	20	0.100
58	A	2	2	1.00	20	0.100
59	A	2	2	1.00	20	0.100
60	A	2	2	1.00	20	0.100
61	A	4	4	0.97	18	0.222
62	A	5	5	0.96	17	0.294
63	A	2	2	1.00	20	0.100
64	A	2	2	1.00	20	0.100
65	A	2	2	1.00	20	0.100
66	A	4	4	1.02	16	0.250
67	A	3	3	1.00	17	0.176
68	A	2	2	1.00	20	0.100
69	A	2	2	1.00	20	0.100
70	A	2	2	1.00	18	0.111
71	A	2	2	1.09	17	0.118
72	A	2	2	1.00	20	0.100
73	A	2	2	1.00	20	0.100
74	A	2	2	1.00	20	0.100
75	A	2	2	1.00	20	0.100
76	A	2	2	1.00	22	0.091
77	A	2	2	1.00	22	0.091
78	A	2	2	1.00	20	0.100
79	A	2	2	0.97	19	0.105
80	A	2	2	1.00	22	0.091
81	A	2	2	1.00	22	0.091
82	A	2	2	1.00	22	0.091
83	A	2	2	0.93	22	0.091
84	A	2	2	1.00	22	0.091
85	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	20	0.100
87	A	2	2	0.96	19	0.105
88	A	2	2	1.00	22	0.091
89	A	2	2	1.00	22	0.091
90	A	2	2	1.00	22	0.091
91	A	2	2	1.00	22	0.091
92	A	2	2	0.94	22	0.091
93	A	2	2	0.94	22	0.091
94	A	2	2	0.94	22	0.091
95	A	21	20	1.29	22	0.909
96	A	16	15	1.10	22	0.682
97	A	11	10	1.06	20	0.500
98	A	3	3	1.01	19	0.158
99	A	3	3	1.09	22	0.136
100	A	9	9	1.02	22	0.409
101	A	19	18	0.98	22	0.818
102	A	23	22	1.10	22	1.000
103	A	2	2	1.00	22	0.091
104	A	2	2	1.00	22	0.091
105	A	2	2	1.00	22	0.091
106	A	2	2	1.00	20	0.100
107	A	2	2	1.07	19	0.105
108	A	2	2	1.00	22	0.091
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	2	2	1.00	22	0.091
112	A	2	2	1.00	22	0.091
113	A	2	2	1.00	22	0.091
114	A	2	2	1.00	20	0.100
115	A	2	2	1.01	19	0.105
116	A	2	2	1.00	22	0.091
117	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	0.99	18	0.111
119	A	4	4	1.15	20	0.200
120	A	2	2	0.97	18	0.111
121	A	2	2	0.98	18	0.111
122	A	2	2	0.95	16	0.125
123	A	4	4	1.02	18	0.222
124	A	2	2	1.02	18	0.111
125	A	2	2	1.01	18	0.111
126	A	2	2	0.99	18	0.111
127	A	18	17	1.07	18	0.944
128	A	10	10	1.03	16	0.625
129	A	4	4	0.97	15	0.267
130	A	4	4	1.08	18	0.222
131	A	5	5	1.08	19	0.263
132	A	9	9	1.04	18	0.500
133	A	15	15	0.95	18	0.833
134	A	15	15	1.10	19	0.789
135	A	10	10	1.05	17	0.588
136	A	5	5	1.04	16	0.312
137	A	5	5	1.15	19	0.263
138	A	6	6	1.15	20	0.300
139	A	12	12	1.08	19	0.632
140	A	17	17	1.01	19	0.895
141	N/A	1	0	1.00	16	0.000
142	N/A	1	0	1.00	15	0.000
143	N/A	1	0	1.00	18	0.000
144	N/A	1	0	1.00	16	0.000
145	N/A	1	0	1.00	15	0.000
146	N/A	1	0	1.00	18	0.000
147	A	2	2	1.00	19	0.105
148	A	2	2	1.00	19	0.105
149	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	5	4	1.00	16	0.250
151	A	2	2	1.00	19	0.105
152	A	2	2	1.00	19	0.105
153	A	2	2	1.00	19	0.105
154	A	2	2	1.00	21	0.095
155	A	2	2	1.00	19	0.105
156	A	1	1	1.00	18	0.056
157	A	2	2	1.00	21	0.095
158	A	2	2	1.00	21	0.095
159	A	2	2	1.00	17	0.118
160	N/A	1	0	1.00	18	0.000
161	A	6	5	1.65	18	0.278
162	A	4	4	1.57	18	0.222
163	A	6	5	1.60	18	0.278
164	A	2	2	1.08	16	0.125
165	A	3	3	1.03	15	0.200
166	A	5	5	1.19	18	0.278
167	A	9	8	1.66	18	0.444
168	A	5	5	0.96	18	0.278
169	A	9	8	1.47	18	0.444
170	A	3	3	1.05	18	0.167
171	A	6	5	1.54	18	0.278
172	B	21	20	2.54	20	1.000
173	B	20	19	2.56	20	0.950
174	B	17	16	2.15	20	0.800
175	A	4	4	1.04	18	0.222
176	A	8	7	1.07	17	0.412
177	A	10	10	1.33	20	0.500
178	A	11	10	1.46	20	0.500
179	A	14	13	1.12	20	0.650
180	A	9	9	1.40	20	0.450
181	A	10	9	1.30	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	15	15	1.71	20	0.750
183	A	7	7	1.11	17	0.412
184	A	2	2	1.00	19	0.105
185	N/A	1	0	1.00	18	0.000
186	N/A	1	0	1.00	17	0.000
187	N/A	1	0	1.00	20	0.000
188	N/A	1	0	1.00	18	0.000
189	N/A	1	0	1.00	17	0.000
190	N/A	1	0	1.00	20	0.000
191	N/A	1	0	1.00	17	0.000
192	A	2	2	1.00	20	0.100
193	A	2	2	1.00	20	0.100
194	A	2	2	1.00	20	0.100
195	A	3	3	1.06	18	0.167
196	A	4	4	1.07	17	0.235
197	A	2	2	1.00	20	0.100
198	A	2	2	1.00	20	0.100
199	A	2	2	1.00	20	0.100
200	A	2	2	1.00	20	0.100
201	A	2	2	1.00	20	0.100
202	A	2	2	1.00	20	0.100
203	A	2	2	1.00	22	0.091
204	A	2	2	1.00	22	0.091
205	A	2	2	1.00	22	0.091
206	A	5	5	1.04	20	0.250
207	A	10	9	1.12	19	0.474
208	A	2	2	1.00	22	0.091
209	A	2	2	1.00	22	0.091
210	A	2	2	1.00	22	0.091
211	A	2	2	1.00	22	0.091
212	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
213	A	2	2	1.00	22	0.091
214	A	3	3	1.01	22	0.136
215	A	2	2	1.00	22	0.091
216	A	2	2	1.00	22	0.091
217	A	11	11	1.20	19	0.579
218	N/A	1	0	1.00	20	0.000
219	N/A	1	0	1.00	19	0.000
220	N/A	1	0	1.00	22	0.000
221	N/A	1	0	1.00	20	0.000
222	N/A	1	0	1.00	19	0.000
223	N/A	1	0	1.00	22	0.000
224	A	5	5	1.08	17	0.294
225	A	13	12	1.19	19	0.632
226	A	16	16	1.38	19	0.842
227	A	9	8	1.16	20	0.400
228	A	4	4	1.05	20	0.200
229	A	5	4	1.09	18	0.222
230	A	1	1	1.00	17	0.059
231	A	3	3	1.00	20	0.150
232	A	8	7	1.07	20	0.350
233	A	7	7	0.93	20	0.350
234	A	10	10	1.19	22	0.455
235	A	8	7	1.20	22	0.318
236	A	4	4	1.14	20	0.200
237	A	1	1	1.00	19	0.053
238	A	4	4	1.18	22	0.182
239	A	6	6	1.08	22	0.273
240	A	14	13	1.03	22	0.591
241	A	13	12	1.15	22	0.545
242	A	7	7	1.17	22	0.318
243	A	5	5	1.07	20	0.250
244	A	1	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	5	5	1.14	22	0.227
246	A	7	7	1.16	22	0.318
247	A	11	11	0.99	22	0.500
248	A	1	1	1.00	21	0.048
249	N/A	1	0	1.00	20	0.000
250	A	1	1	1.00	19	0.053
251	N/A	1	0	1.00	22	0.000
252	N/A	2	0	1.00	20	0.000
253	A	1	1	1.00	19	0.053
254	N/A	2	0	1.00	22	0.000
255	N/A	2	0	1.00	20	0.000
256	A	1	1	1.00	19	0.053
257	N/A	2	0	1.00	22	0.000
258	A	1	1	1.00	19	0.053
259	A	9	8	1.18	20	0.400
260	A	2	2	1.00	20	0.100
261	A	3	3	1.09	18	0.167
262	A	2	2	1.00	17	0.118
263	A	7	7	1.20	20	0.350
264	A	11	10	1.24	20	0.500
265	A	15	15	1.55	20	0.750
266	A	8	8	1.14	22	0.364
267	A	4	4	1.12	22	0.182
268	A	3	3	1.04	20	0.150
269	A	4	4	1.14	19	0.211
270	A	8	8	1.23	22	0.364
271	A	11	11	1.23	22	0.500
272	A	21	20	1.53	22	0.909
273	A	11	11	1.14	22	0.500
274	A	4	4	1.09	22	0.182
275	A	5	5	1.12	20	0.250
276	A	4	4	1.10	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
277	A	11	11	1.25	22	0.500
278	A	12	12	1.23	22	0.545
279	A	20	20	1.47	22	0.909
280	C	11	10	1.08	21	0.476
281	N/A	1	0	1.00	22	0.000
282	N/A	1	0	1.00	22	0.000
283	A	6	5	0.93	22	0.227
284	A	7	6	1.00	20	0.300
285	A	5	4	0.93	19	0.211
286	N/A	1	0	1.00	22	0.000
287	N/A	14	0	1.00	22	0.000
288	A	8	7	1.00	22	0.318
289	B	11	10	2.08	20	0.500
290	A	8	7	1.00	19	0.368
291	N/A	14	0	1.00	22	0.000
292	N/A	11	0	1.00	22	0.000
293	A	12	11	1.69	22	0.500
294	A	8	7	1.00	20	0.350
295	A	12	11	1.78	19	0.579
296	N/A	11	0	1.00	22	0.000
297	A	9	8	1.06	19	0.421
298	A	13	12	1.42	19	0.632
299	A	10	9	1.09	19	0.474
300	A	14	13	1.33	19	0.684
301	A	11	10	1.10	19	0.526
302	A	4	4	1.18	20	0.200
303	A	3	3	1.05	20	0.150
304	A	4	4	1.09	18	0.222
305	A	3	3	1.05	17	0.176
306	A	12	12	1.51	20	0.600
307	A	15	14	1.67	20	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	4	4	1.13	22	0.182
309	A	10	10	1.94	22	0.455
310	A	4	4	1.06	20	0.200
311	A	8	8	1.36	19	0.421
312	A	13	13	1.55	22	0.591
313	A	20	20	1.84	22	0.909
314	A	9	9	1.38	22	0.409
315	A	10	10	1.95	22	0.455
316	A	9	9	1.27	20	0.450
317	A	8	8	1.39	19	0.421
318	A	21	21	1.75	22	0.955
319	A	21	21	1.86	22	0.955
320	A	5	4	0.90	21	0.190
321	N/A	1	0	1.00	22	0.000
322	N/A	1	0	1.00	22	0.000
323	A	5	4	0.88	22	0.182
324	A	4	3	0.93	22	0.136
325	A	4	3	0.93	22	0.136
326	A	4	3	0.93	20	0.150
327	A	5	4	0.88	19	0.211
328	N/A	1	0	1.00	22	0.000
329	N/A	17	0	1.00	22	0.000
330	A	5	4	1.04	22	0.182
331	B	14	13	3.13	22	0.591
332	B	10	9	2.37	22	0.409
333	A	9	8	1.64	20	0.400
334	A	5	4	1.06	19	0.211
335	N/A	17	0	1.00	22	0.000
336	B	15	14	2.06	22	0.636
337	B	14	13	2.52	22	0.591
338	B	15	14	2.66	22	0.636
339	A	12	11	1.88	20	0.550

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	10	9	1.68	19	0.474
341	N/A	17	0	1.00	22	0.000
342	A	13	12	1.75	19	0.632
343	B	18	17	2.41	19	0.895
344	B	18	17	2.42	19	0.895
345	A	4	4	1.07	17	0.235
346	A	13	13	1.56	19	0.684
347	A	13	13	1.66	19	0.684
348	A	5	4	0.90	21	0.190
349	N/A	1	0	1.00	22	0.000
350	N/A	1	0	1.00	22	0.000
351	A	6	5	0.85	22	0.227
352	A	4	3	0.88	22	0.136
353	A	4	3	0.85	22	0.136
354	A	4	3	0.93	22	0.136
355	A	4	3	0.85	22	0.136
356	A	4	3	0.88	20	0.150
357	A	5	4	0.85	19	0.211
358	N/A	1	0	1.00	22	0.000
359	N/A	1	0	1.00	22	0.000
360	A	9	8	1.79	20	0.400
361	A	5	4	0.95	19	0.211
362	A	8	7	1.99	20	0.350
363	A	10	9	1.67	19	0.474
364	A	9	9	1.57	22	0.409
365	A	8	7	1.19	22	0.318
366	A	5	5	1.25	22	0.227
367	A	3	3	1.03	22	0.136
368	A	2	2	1.00	20	0.100
369	A	1	1	1.00	19	0.053
370	A	1	1	1.00	22	0.045
371	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
372	A	3	3	0.95	22	0.136
373	A	6	6	1.57	24	0.250
374	A	10	9	0.98	24	0.375
375	A	2	2	1.05	22	0.091
376	A	7	6	0.87	21	0.286
377	C	8	7	1.18	24	0.292
378	A	2	2	0.99	24	0.083
379	C	13	12	1.03	24	0.500
380	A	17	16	1.45	24	0.667
381	A	11	10	0.99	24	0.417
382	A	8	7	0.91	22	0.318
383	A	8	7	0.90	21	0.333
384	C	9	8	1.20	24	0.333
385	C	9	8	1.09	24	0.333
386	C	12	11	0.99	24	0.458
387	N/A	1	0	1.00	22	0.000
388	A	4	4	1.15	22	0.182
389	A	2	2	1.04	22	0.091
390	A	2	2	1.00	20	0.100
391	A	1	1	1.00	19	0.053
392	A	4	4	1.09	22	0.182
393	A	7	6	1.05	22	0.273
394	A	8	8	1.43	22	0.364
395	N/A	1	0	1.00	24	0.000
396	A	4	4	1.11	24	0.167
397	A	10	9	0.93	24	0.375
398	A	2	2	1.03	22	0.091
399	A	2	2	1.00	21	0.095
400	C	11	10	1.22	24	0.417
401	A	5	5	1.00	24	0.208
402	C	23	22	1.43	24	0.917
403	N/A	1	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	A	11	10	1.00	24	0.417
405	A	11	10	0.95	24	0.417
406	A	3	3	0.99	22	0.136
407	A	2	2	1.02	21	0.095
408	C	13	12	1.19	24	0.500
409	C	12	11	1.07	24	0.458
410	C	22	21	1.36	24	0.875
411	N/A	1	0	1.00	24	0.000
412	N/A	1	0	1.00	24	0.000
413	A	5	4	1.00	22	0.182
414	A	4	3	1.00	21	0.143
415	N/A	1	0	1.00	24	0.000
416	N/A	1	0	1.00	24	0.000
417	N/A	8	0	1.00	24	0.000
418	A	5	4	1.00	22	0.182
419	A	6	5	1.00	21	0.238
420	N/A	7	0	1.00	24	0.000
421	N/A	1	0	1.00	24	0.000
422	N/A	8	0	1.00	24	0.000
423	A	7	6	1.04	22	0.273
424	A	6	5	1.06	21	0.238
425	N/A	9	0	1.00	24	0.000
426	A	12	11	1.36	22	0.500
427	A	9	9	1.54	22	0.409
428	A	8	7	1.16	22	0.318
429	A	3	3	1.08	20	0.150
430	A	2	2	1.01	19	0.105
431	A	3	3	1.00	22	0.136
432	A	7	6	1.09	22	0.273
433	A	5	5	0.96	22	0.227
434	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	A	9	9	1.54	22	0.409
436	B	22	21	2.21	22	0.955
437	A	14	14	1.80	22	0.636
438	B	26	25	2.65	24	1.042
439	B	16	15	3.30	24	0.625
440	A	21	20	1.87	24	0.833
441	A	3	3	1.02	22	0.136
442	A	9	8	0.95	21	0.381
443	A	11	10	1.22	24	0.417
444	A	10	9	0.98	24	0.375
445	C	20	19	1.04	24	0.792
446	A	6	6	0.96	24	0.250
447	B	19	18	2.71	22	0.818
448	B	19	19	3.04	22	0.864
449	B	15	14	2.31	22	0.636
450	A	4	4	1.09	20	0.200
451	A	3	3	1.02	19	0.158
452	A	7	7	1.17	22	0.318
453	A	10	9	1.63	22	0.409
454	A	8	8	1.40	22	0.364
455	A	13	12	1.15	22	0.545
456	B	13	13	2.25	22	0.591
457	A	7	6	1.05	22	0.273
458	B	19	19	3.03	22	0.864
459	A	4	4	1.04	19	0.211
460	A	3	3	1.02	19	0.158
461	A	2	2	1.01	19	0.105
462	A	2	2	1.00	19	0.105
463	A	3	3	1.04	19	0.158
464	A	4	4	1.06	19	0.211
465	A	4	4	0.78	20	0.200
466	A	3	3	0.75	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
467	A	2	2	0.69	20	0.100
468	A	1	1	1.00	20	0.050
469	A	2	2	1.03	20	0.100
470	A	3	3	1.07	20	0.150
471	A	9	8	0.95	21	0.381
472	A	5	5	1.16	21	0.238
473	A	9	9	1.36	21	0.429
474	A	14	14	1.55	21	0.667
475	A	10	9	0.96	21	0.429
476	A	5	5	1.26	21	0.238
477	A	9	9	1.52	21	0.429
478	A	14	14	1.78	21	0.667
479	N/A	1	0	1.00	21	0.000
480	N/A	1	0	1.00	21	0.000
481	A	4	3	1.00	21	0.143
482	A	5	4	0.93	21	0.190
483	A	5	4	0.88	21	0.190
484	A	5	4	0.85	21	0.190
485	N/A	1	0	1.00	21	0.000
486	N/A	1	0	1.00	21	0.000
487	A	6	5	1.00	21	0.238
488	A	5	4	1.00	21	0.190
489	A	5	4	0.95	21	0.190
490	A	5	4	0.92	21	0.190
491	N/A	1	0	1.00	21	0.000
492	N/A	1	0	1.00	21	0.000
493	A	6	5	1.06	21	0.238
494	A	10	9	1.41	21	0.429
495	A	10	9	1.43	21	0.429
496	A	10	9	1.45	21	0.429
497	A	2	2	1.00	28	0.071
498	A	6	5	1.01	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	6	5	1.01	14	0.357
500	A	6	5	1.02	14	0.357
501	A	6	5	1.04	12	0.417
502	A	3	3	1.02	14	0.214
503	A	4	4	1.05	14	0.286
504	A	4	4	1.04	14	0.286
505	A	3	3	1.06	14	0.214
506	A	1	1	1.00	20	0.050
507	N/A	1	0	1.00	16	0.000
508	N/A	1	0	1.00	16	0.000
509	A	6	5	1.00	16	0.312
510	A	7	6	1.02	16	0.375
511	A	7	6	0.97	16	0.375
512	A	6	5	0.99	16	0.312
513	A	3	3	0.75	15	0.200
514	A	2	2	0.69	15	0.133
515	A	1	1	1.00	15	0.067
516	A	2	2	1.04	15	0.133
517	A	3	3	1.09	15	0.200
518	A	2	2	1.00	15	0.133
519	A	2	2	1.28	27	0.074
520	A	3	3	1.28	31	0.097
521	A	2	2	1.09	27	0.074
522	A	2	2	0.99	27	0.074
523	A	2	2	1.08	27	0.074
524	A	2	2	1.00	25	0.080
525	A	8	7	1.12	24	0.292
526	A	11	10	0.88	27	0.370
527	A	8	7	0.81	27	0.259
528	A	2	2	0.93	27	0.074
529	A	17	16	0.90	27	0.593
530	A	2	2	0.96	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
531	A	26	25	1.14	27	0.926
532	A	2	2	0.98	22	0.091
533	A	13	12	1.02	21	0.571
534	N/A	8	0	1.00	24	0.000
535	A	10	9	0.98	24	0.375
536	A	2	2	0.95	24	0.083
537	A	4	3	1.31	20	0.150

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(d+cdx)(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))\,dx = \frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4 + \frac{1}{4}dx^4(a+b\text{arctanh}(cx)) + \frac{1}{5}cdx^5(a+b\text{arctanh}(cx)) + \frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(1+cx)}{40c^4}$$

output
```
1/4*b*d*x/c^3+1/10*b*d*x^2/c^2+1/12*b*d*x^3/c+1/20*b*d*x^4+1/4*d*x^4*(a+b*
arctanh(c*x))+1/5*c*d*x^5*(a+b*arctanh(c*x))+9/40*b*d*ln(-c*x+1)/c^4-1/40*
b*d*ln(c*x+1)/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d(30bcx+12bc^2x^2+10bc^3x^3+30ac^4x^4+6bc^4x^4+24ac^5x^5+6bc^4x^4(5+4cx)\text{arctanh}(cx)+27b\log(1}{120c^4}$$

input
```
Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*(30*b*c*x + 12*b*c^2*x^2 + 10*b*c^3*x^3 + 30*a*c^4*x^4 + 6*b*c^4*x^4 +
24*a*c^5*x^5 + 6*b*c^4*x^4*(5 + 4*c*x)*ArcTanh[c*x] + 27*b*Log[1 - c*x] -
3*b*Log[1 + c*x]))/(120*c^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6498, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int \frac{dx^4(4cx+5)}{20(1-c^2x^2)}dx+\frac{1}{5}cdx^5(a+b\text{arctanh}(cx))+\frac{1}{4}dx^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{20}bcd\int \frac{x^4(4cx+5)}{1-c^2x^2}dx+\frac{1}{5}cdx^5(a+b\text{arctanh}(cx))+\frac{1}{4}dx^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{ 523}$$

$$-\frac{1}{20}bcd\int\left(-\frac{4x^3}{c}-\frac{5x^2}{c^2}-\frac{4x}{c^3}+\frac{4cx+5}{c^4(1-c^2x^2)}-\frac{5}{c^4}\right)dx+\frac{1}{5}cdx^5(a+b\text{arctanh}(cx))+\frac{1}{4}dx^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}cdx^5(a+b\text{arctanh}(cx))+\frac{1}{4}dx^4(a+b\text{arctanh}(cx))-\frac{1}{20}bcd\left(\frac{5\text{arctanh}(cx)}{c^5}-\frac{5x}{c^4}-\frac{2x^2}{c^3}-\frac{5x^3}{3c^2}-\frac{2\log(1-c^2x^2)}{c^5}-\frac{x^4}{c}\right)$$

input
```
Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcTanh[c*x]))/4 + (c*d*x^5*(a + b*ArcTanh[c*x]))/5 - (b*c*d
*((-5*x)/c^4 - (2*x^2)/c^3 - (5*x^3)/(3*c^2) - x^4/c + (5*ArcTanh[c*x])/c^
5 - (2*Log[1 - c^2*x^2])/c^5))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

method	result
parts	$ad\left(\frac{1}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{db\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^4x^4}{20}+\frac{x^3c^3}{12}+\frac{c^2x^2}{10}+\frac{cx}{4}+\frac{9\ln(cx-1)}{40}-\frac{\ln(cx+1)}{40}\right)}{c^4}$
derivativedivides	$\frac{ad\left(\frac{1}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^4x^4}{20}+\frac{x^3c^3}{12}+\frac{c^2x^2}{10}+\frac{cx}{4}+\frac{9\ln(cx-1)}{40}-\frac{\ln(cx+1)}{40}\right)}{c^4}$
default	$\frac{ad\left(\frac{1}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^4x^4}{20}+\frac{x^3c^3}{12}+\frac{c^2x^2}{10}+\frac{cx}{4}+\frac{9\ln(cx-1)}{40}-\frac{\ln(cx+1)}{40}\right)}{c^4}$
parallelrisch	$\frac{12b\,c^5d\,\operatorname{arctanh}(cx)x^5+12c^5d\,x^5a+15db\,\operatorname{arctanh}(cx)x^4c^4+15a\,c^4d\,x^4+3d\,x^4c^4b+5b\,c^3d\,x^3+6b\,c^2d\,x^2+15bcdx+12\ln}{60c^4}$
risch	$\frac{db\,x^4(4cx+5)\ln(cx+1)}{40}-\frac{dcb\,x^5\ln(-cx+1)}{10}+\frac{dca\,x^5}{5}-\frac{db\,x^4\ln(-cx+1)}{8}+\frac{da\,x^4}{4}+\frac{bd\,x^4}{20}+\frac{bd\,x^3}{12c}+\frac{bd\,x^2}{10c^2}$

input
```
int(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/5*c*x^5+1/4*x^4)+d*b/c^4*(1/5*arctanh(c*x)*c^5*x^5+1/4*arctanh(c*x)
*c^4*x^4+1/20*c^4*x^4+1/12*x^3*c^3+1/10*c^2*x^2+1/4*c*x+9/40*ln(c*x-1)-1/4
0*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int x^3(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{24\,ac^5dx^5+6\,(5\,a+b)c^4dx^4+10\,bc^3dx^3+12\,bc^2dx^2+30\,bcdx-3\,bd\log\left(cx+1\right)+27\,bd\log\left(cx-1\right)+}{120\,c^4}$$

input
```
integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*d*x^5 + 6*(5*a + b)*c^4*d*x^4 + 10*b*c^3*d*x^3 + 12*b*c^2*
d*x^2 + 30*b*c*d*x - 3*b*d*log(c*x + 1) + 27*b*d*log(c*x - 1) + 3*(4*b*c^5
*d*x^5 + 5*b*c^4*d*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15

$$\int x^3(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\begin{cases} \frac{acdx^5}{5}+\frac{adx^4}{4}+\frac{bcdx^5\operatorname{atanh}(cx)}{5}+\frac{bdx^4\operatorname{atanh}(cx)}{4}+\frac{bdx^4}{20}+\frac{bdx^3}{12c}+\frac{bdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{bd\log\left(x-\frac{1}{c}\right)}{5c^4}-\frac{bd\operatorname{atanh}(cx)}{20c^4} & \text{for } c \neq 0 \\ \frac{adx^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c*d*x**5/5 + a*d*x**4/4 + b*c*d*x**5*atanh(c*x)/5 + b*d*x**4*
atanh(c*x)/4 + b*d*x**4/20 + b*d*x**3/(12*c) + b*d*x**2/(10*c**2) + b*d*x/
(4*c**3) + b*d*log(x - 1/c)/(5*c**4) - b*d*atanh(c*x)/(20*c**4), Ne(c, 0))
, (a*d*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int x^3(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{1}{5}acdx^5+\frac{1}{4}adx^4+\frac{1}{20}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log(c^2x^2-1)}{c^6}\right)\right)bcd$$
$$+\frac{1}{24}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bd$$

input
```
integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x
^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d + 1/24*(6*x^4*arctanh(c*x) + c*(2
*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(92) = 184$.

Time = 0.12 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.55

$$\int x^3(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{1}{15}c\left(\frac{3\left(\frac{10(cx+1)^4bd}{(cx-1)^4}-\frac{5(cx+1)^3bd}{(cx-1)^3}+\frac{15(cx+1)^2bd}{(cx-1)^2}-\frac{5(cx+1)bd}{cx-1}+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^5}{(cx-1)^5}-\frac{5(cx+1)^4c^5}{(cx-1)^4}+\frac{10(cx+1)^3c^5}{(cx-1)^3}-\frac{10(cx+1)^2c^5}{(cx-1)^2}+\frac{5(cx+1)c^5}{cx-1}-c^5}+\frac{\frac{60(cx+1)^4ad}{(cx-1)^4}-\frac{30(cx+1)^3ad}{(cx-1)^3}+\cdots}{\cdots}\right.$$

input
```
integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/15*c*(3*(10*(c*x + 1)^4*b*d/(c*x - 1)^4 - 5*(c*x + 1)^3*b*d/(c*x - 1)^3
+ 15*(c*x + 1)^2*b*d/(c*x - 1)^2 - 5*(c*x + 1)*b*d/(c*x - 1) + b*d)*log(-(
c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^5/(c*x - 1)^5 - 5*(c*x + 1)^4*c^5/(c*x
- 1)^4 + 10*(c*x + 1)^3*c^5/(c*x - 1)^3 - 10*(c*x + 1)^2*c^5/(c*x - 1)^2 +
 5*(c*x + 1)*c^5/(c*x - 1) - c^5) + (60*(c*x + 1)^4*a*d/(c*x - 1)^4 - 30*(
c*x + 1)^3*a*d/(c*x - 1)^3 + 90*(c*x + 1)^2*a*d/(c*x - 1)^2 - 30*(c*x + 1)
*a*d/(c*x - 1) + 6*a*d + 27*(c*x + 1)^4*b*d/(c*x - 1)^4 - 69*(c*x + 1)^3*b
*d/(c*x - 1)^3 + 79*(c*x + 1)^2*b*d/(c*x - 1)^2 - 47*(c*x + 1)*b*d/(c*x -
1) + 10*b*d)/((c*x + 1)^5*c^5/(c*x - 1)^5 - 5*(c*x + 1)^4*c^5/(c*x - 1)^4
+ 10*(c*x + 1)^3*c^5/(c*x - 1)^3 - 10*(c*x + 1)^2*c^5/(c*x - 1)^2 + 5*(c*x
 + 1)*c^5/(c*x - 1) - c^5) - 3*b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 3*b
*d*log(-(c*x + 1)/(c*x - 1))/c^5)
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int x^3(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{\frac{b\,c\,d\,x}{4}-\frac{d\,(15\,b\,\mathrm{atanh}(c\,x)-6\,b\,\ln(c^2\,x^2-1))}{60}+\frac{b\,c^2\,d\,x^2}{10}+\frac{b\,c^3\,d\,x^3}{12}}{c^4}$$
$$+\frac{d\,(15\,a\,x^4+3\,b\,x^4+15\,b\,x^4\,\mathrm{atanh}(c\,x))}{60}+\frac{c\,d\,(12\,a\,x^5+12\,b\,x^5\,\mathrm{atanh}(c\,x))}{60}$$

input
```
int(x^3*(a + b*atanh(c*x))*(d + c*d*x),x)
```

output
```
((b*c*d*x)/4 - (d*(15*b*atanh(c*x) - 6*b*log(c^2*x^2 - 1)))/60 + (b*c^2*d*
x^2)/10 + (b*c^3*d*x^3)/12)/c^4 + (d*(15*a*x^4 + 3*b*x^4 + 15*b*x^4*atanh(
c*x)))/60 + (c*d*(12*a*x^5 + 12*b*x^5*atanh(c*x)))/60
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d(12atanh(cx)\,b\,c^5x^5+15atanh(cx)\,b\,c^4x^4-3atanh(cx)\,b+12\log(c^2x-c)\,b+12a\,c^5x^5+15a\,c^4x^4+3b}{60c^4}$$

input
```
int(x^3*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
(d*(12*atanh(c*x)*b*c**5*x**5 + 15*atanh(c*x)*b*c**4*x**4 - 3*atanh(c*x)*b
 + 12*log(c**2*x - c)*b + 12*a*c**5*x**5 + 15*a*c**4*x**4 + 3*b*c**4*x**4
+ 5*b*c**3*x**3 + 6*b*c**2*x**2 + 15*b*c*x))/(60*c**4)
```

3.2 $\int x^2(d+cdx)(a+b\,\mathbf{arctanh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))\,dx=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3(a+b\text{arctanh}(cx))+\frac{1}{4}cdx^4(a+b\text{arctanh}(cx))+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(1+cx)}{24c^3}$$

output
```
1/4*b*d*x/c^2+1/6*b*d*x^2/c+1/12*b*d*x^3+1/3*d*x^3*(a+b*arctanh(c*x))+1/4*
c*d*x^4*(a+b*arctanh(c*x))+7/24*b*d*ln(-c*x+1)/c^3+1/24*b*d*ln(c*x+1)/c^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d(6bcx+4bc^2x^2+8ac^3x^3+2bc^3x^3+6ac^4x^4+2bc^3x^3(4+3cx)\text{arctanh}(cx)+7b\log(1-cx)+b\log(1+}{24c^3}$$

input
```
Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*(6*b*c*x + 4*b*c^2*x^2 + 8*a*c^3*x^3 + 2*b*c^3*x^3 + 6*a*c^4*x^4 + 2*b*
c^3*x^3*(4 + 3*c*x)*ArcTanh[c*x] + 7*b*Log[1 - c*x] + b*Log[1 + c*x]))/(24
*c^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6498, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{dx^3(3cx+4)}{12(1-c^2x^2)}dx+\frac{1}{4}cdx^4(a+b\text{arctanh}(cx))+\frac{1}{3}dx^3(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bcd\int \frac{x^3(3cx+4)}{1-c^2x^2}dx+\frac{1}{4}cdx^4(a+b\text{arctanh}(cx))+\frac{1}{3}dx^3(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{523}$$

$$-\frac{1}{12}bcd\int \left(-\frac{3x^2}{c}-\frac{4x}{c^2}+\frac{4cx+3}{c^3(1-c^2x^2)}-\frac{3}{c^3}\right)dx+\frac{1}{4}cdx^4(a+b\text{arctanh}(cx))+\frac{1}{3}dx^3(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}cdx^4(a+b\text{arctanh}(cx))+\frac{1}{3}dx^3(a+b\text{arctanh}(cx))-\frac{1}{12}bcd\left(\frac{3\text{arctanh}(cx)}{c^4}-\frac{3x}{c^3}-\frac{2x^2}{c^2}-\frac{2\log(1-c^2x^2)}{c^4}-\frac{x^3}{c}\right)$$

input
```
Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcTanh[c*x]))/3 + (c*d*x^4*(a + b*ArcTanh[c*x]))/4 - (b*c*d
*((-3*x)/c^3 - (2*x^2)/c^2 - x^3/c + (3*ArcTanh[c*x])/c^4 - (2*Log[1 - c^2
*x^2])/c^4))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result
parts	$ad\left(\frac{1}{4}c\,x^4+\frac{1}{3}x^3\right)+\frac{db\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{x^3c^3}{12}+\frac{c^2x^2}{6}+\frac{cx}{4}+\frac{7\ln(cx-1)}{24}+\frac{\ln(cx+1)}{24}\right)}{c^3}$
derivativedivides	$\frac{ad\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+db\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{x^3c^3}{12}+\frac{c^2x^2}{6}+\frac{cx}{4}+\frac{7\ln(cx-1)}{24}+\frac{\ln(cx+1)}{24}\right)}{c^3}$
default	$\frac{ad\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+db\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{x^3c^3}{12}+\frac{c^2x^2}{6}+\frac{cx}{4}+\frac{7\ln(cx-1)}{24}+\frac{\ln(cx+1)}{24}\right)}{c^3}$
parallelrisch	$\frac{3db\operatorname{arctanh}(cx)x^4c^4+3a\,c^4d\,x^4+4db\operatorname{arctanh}(cx)x^3c^3+4a\,c^3d\,x^3+b\,c^3d\,x^3+2b\,c^2d\,x^2+3bcdx+4\ln(cx-1)bd+\operatorname{arctanh}}{12c^3}$
risch	$\frac{db\,x^3(3cx+4)\ln(cx+1)}{24}-\frac{dcb\,x^4\ln(-cx+1)}{8}+\frac{dca\,x^4}{4}-\frac{db\,x^3\ln(-cx+1)}{6}+\frac{da\,x^3}{3}+\frac{bd\,x^3}{12}+\frac{bd\,x^2}{6c}+\frac{bdx}{4c^2}$

input
```
int(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/4*c*x^4+1/3*x^3)+d*b/c^3*(1/4*arctanh(c*x)*c^4*x^4+1/3*arctanh(c*x)
*c^3*x^3+1/12*x^3*c^3+1/6*c^2*x^2+1/4*c*x+7/24*ln(c*x-1)+1/24*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int x^2(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{6\,ac^4dx^4+2\,(4\,a+b)c^3dx^3+4\,bc^2dx^2+6\,bcdx+bd\log\left(cx+1\right)+7\,bd\log\left(cx-1\right)+\left(3\,bc^4dx^4+4\,bc^3d\right.}{24\,c^3}$$

input
```
integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/24*(6*a*c^4*d*x^4 + 2*(4*a + b)*c^3*d*x^3 + 4*b*c^2*d*x^2 + 6*b*c*d*x +
b*d*log(c*x + 1) + 7*b*d*log(c*x - 1) + (3*b*c^4*d*x^4 + 4*b*c^3*d*x^3)*lo
g(-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\begin{cases}\frac{acdx^4}{4}+\frac{adx^3}{3}+\frac{bcdx^4\operatorname{atanh}(cx)}{4}+\frac{bdx^3\operatorname{atanh}(cx)}{3}+\frac{bdx^3}{12}+\frac{bdx^2}{6c}+\frac{bdx}{4c^2}+\frac{bd\log\left(x-\frac{1}{c}\right)}{3c^3}+\frac{bd\operatorname{atanh}(cx)}{12c^3} & \text{for } c\neq 0\\ \frac{adx^3}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c*d*x**4/4 + a*d*x**3/3 + b*c*d*x**4*atanh(c*x)/4 + b*d*x**3*
atanh(c*x)/3 + b*d*x**3/12 + b*d*x**2/(6*c) + b*d*x/(4*c**2) + b*d*log(x -
 1/c)/(3*c**3) + b*d*atanh(c*x)/(12*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{1}{4}acdx^4+\frac{1}{3}adx^3$$
$$+\frac{1}{24}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bcd$$
$$+\frac{1}{6}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bd$$

input
```
integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*c*d*x^4 + 1/3*a*d*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3
*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d + 1/6*(2*x^3*arc
tanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(82) = 164$.

Time = 0.12 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.10

$$\int x^2(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{1}{3}c\left(\frac{\left(\frac{6(cx+1)^3bd}{(cx-1)^3}-\frac{3(cx+1)^2bd}{(cx-1)^2}+\frac{4(cx+1)bd}{cx-1}-bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^4}{(cx-1)^4}-\frac{4(cx+1)^3c^4}{(cx-1)^3}+\frac{6(cx+1)^2c^4}{(cx-1)^2}-\frac{4(cx+1)c^4}{cx-1}+c^4}+\frac{\frac{12(cx+1)^3ad}{(cx-1)^3}-\frac{6(cx+1)^2ad}{(cx-1)^2}+\frac{8(cx+1)ad}{cx-1}-2ad}{\frac{(cx+1)^4c^4}{(cx-1)^4}-\frac{4(cx+1)^3c^4}{(cx-1)^3}+\cdots}\right.$$

input
```
integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/3*c*((6*(c*x + 1)^3*b*d/(c*x - 1)^3 - 3*(c*x + 1)^2*b*d/(c*x - 1)^2 + 4*
(c*x + 1)*b*d/(c*x - 1) - b*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^4/
(c*x - 1)^4 - 4*(c*x + 1)^3*c^4/(c*x - 1)^3 + 6*(c*x + 1)^2*c^4/(c*x - 1)^
2 - 4*(c*x + 1)*c^4/(c*x - 1) + c^4) + (12*(c*x + 1)^3*a*d/(c*x - 1)^3 - 6
*(c*x + 1)^2*a*d/(c*x - 1)^2 + 8*(c*x + 1)*a*d/(c*x - 1) - 2*a*d + 5*(c*x
+ 1)^3*b*d/(c*x - 1)^3 - 10*(c*x + 1)^2*b*d/(c*x - 1)^2 + 7*(c*x + 1)*b*d/
(c*x - 1) - 2*b*d)/((c*x + 1)^4*c^4/(c*x - 1)^4 - 4*(c*x + 1)^3*c^4/(c*x -
 1)^3 + 6*(c*x + 1)^2*c^4/(c*x - 1)^2 - 4*(c*x + 1)*c^4/(c*x - 1) + c^4) -
 b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 + b*d*log(-(c*x + 1)/(c*x - 1))/c^4
)
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^2(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx=\frac{\frac{b\,c\,d\,x}{4}-\frac{d\,\left(3\,b\,\mathrm{atanh}\left(c\,x\right)-2\,b\,\ln\left(c^2\,x^2-1\right)\right)}{12}+\frac{b\,c^2\,d\,x^2}{6}}{c^3}+\frac{d\,\left(4\,a\,x^3+b\,x^3+4\,b\,x^3\,\mathrm{atanh}\left(c\,x\right)\right)}{12}+\frac{c\,d\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atanh}\left(c\,x\right)\right)}{12}$$

input
```
int(x^2*(a + b*atanh(c*x))*(d + c*d*x),x)
```

output
```
((b*c*d*x)/4 - (d*(3*b*atanh(c*x) - 2*b*log(c^2*x^2 - 1)))/12 + (b*c^2*d*x
^2)/6)/c^3 + (d*(4*a*x^3 + b*x^3 + 4*b*x^3*atanh(c*x)))/12 + (c*d*(3*a*x^4
 + 3*b*x^4*atanh(c*x)))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d(3atanh(cx)\,b\,c^4x^4+4atanh(cx)\,b\,c^3x^3+atanh(cx)\,b+4\log(c^2x-c)\,b+3a\,c^4x^4+4a\,c^3x^3+b\,c^3x^3+}{12c^3}$$

input
```
int(x^2*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
(d*(3*atanh(c*x)*b*c**4*x**4 + 4*atanh(c*x)*b*c**3*x**3 + atanh(c*x)*b + 4
*log(c**2*x - c)*b + 3*a*c**4*x**4 + 4*a*c**3*x**3 + b*c**3*x**3 + 2*b*c**
2*x**2 + 3*b*c*x))/(12*c**3)
```

3.3 $\int x(d+cdx)(a+b\,\mathbf{arctanh}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int x(d+cdx)(a+b\text{arctanh}(cx))\,dx=\frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))+\frac{1}{3}cdx^3(a+b\text{arctanh}(cx))+\frac{5bd\log(1-cx)}{12c^2}-\frac{bd\log(1+cx)}{12c^2}$$

output
```
1/2*b*d*x/c+1/6*b*d*x^2+1/2*d*x^2*(a+b*arctanh(c*x))+1/3*c*d*x^3*(a+b*arct
anh(c*x))+5/12*b*d*ln(-c*x+1)/c^2-1/12*b*d*ln(c*x+1)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int x(d+cdx)(a+b\text{arctanh}(cx))\,dx=\frac{d(6bcx+6ac^2x^2+2bc^2x^2+4ac^3x^3+2bc^2x^2(3+2cx)\text{arctanh}(cx)+5b\log(1-cx)-b\log(1+cx))}{12c^2}$$

input
```
Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*(6*b*c*x + 6*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 2*b*c^2*x^2*(3 + 2
*c*x)*ArcTanh[c*x] + 5*b*Log[1 - c*x] - b*Log[1 + c*x]))/(12*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6498, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{dx^2(2cx+3)}{6(1-c^2x^2)}dx+\frac{1}{3}cdx^3(a+b\text{arctanh}(cx))+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}bcd\int \frac{x^2(2cx+3)}{1-c^2x^2}dx+\frac{1}{3}cdx^3(a+b\text{arctanh}(cx))+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{523}$$

$$-\frac{1}{6}bcd\int \left(-\frac{2x}{c}+\frac{2cx+3}{c^2(1-c^2x^2)}-\frac{3}{c^2}\right)dx+\frac{1}{3}cdx^3(a+b\text{arctanh}(cx))+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}cdx^3(a+b\text{arctanh}(cx))+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))-\frac{1}{6}bcd\left(\frac{3\text{arctanh}(cx)}{c^3}-\frac{3x}{c^2}-\frac{\log(1-c^2x^2)}{c^3}-\frac{x^2}{c}\right)$$

input
```
Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*x^2*(a + b*ArcTanh[c*x]))/2 + (c*d*x^3*(a + b*ArcTanh[c*x]))/3 - (b*c*d
*((-3*x)/c^2 - x^2/c + (3*ArcTanh[c*x])/c^3 - Log[1 - c^2*x^2]/c^3))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
parts	$ad\left(\frac{1}{3}c\,x^3+\frac{1}{2}x^2\right)+\frac{db\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{c^2x^2}{6}+\frac{cx}{2}+\frac{5\ln(cx-1)}{12}-\frac{\ln(cx+1)}{12}\right)}{c^2}$
derivativedivides	$\frac{ad\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+db\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{c^2x^2}{6}+\frac{cx}{2}+\frac{5\ln(cx-1)}{12}-\frac{\ln(cx+1)}{12}\right)}{c^2}$
default	$\frac{ad\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+db\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{c^2x^2}{6}+\frac{cx}{2}+\frac{5\ln(cx-1)}{12}-\frac{\ln(cx+1)}{12}\right)}{c^2}$
parallelrisch	$\frac{2db\operatorname{arctanh}(cx)x^3c^3+2a\,c^3d\,x^3+3x^2\operatorname{arctanh}(cx)b\,c^2d+3a\,c^2d\,x^2+b\,c^2d\,x^2+3bcdx+2\ln(cx-1)bd-\operatorname{arctanh}(cx)bd}{6c^2}$
risch	$\frac{db\,x^2(2cx+3)\ln(cx+1)}{12}-\frac{dcb\,x^3\ln(-cx+1)}{6}+\frac{acd\,x^3}{3}-\frac{db\,x^2\ln(-cx+1)}{4}+\frac{da\,x^2}{2}+\frac{bd\,x^2}{6}+\frac{bdx}{2c}-\frac{bd\ln(c}{12c}$

input
```
int(x*(c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/3*c*x^3+1/2*x^2)+d*b/c^2*(1/3*arctanh(c*x)*c^3*x^3+1/2*arctanh(c*x)
*c^2*x^2+1/6*c^2*x^2+1/2*c*x+5/12*ln(c*x-1)-1/12*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int x(d+cdx)(a+b\text{arctanh}(cx))\,dx = \frac{4\,ac^3dx^3+2\,(3\,a+b)c^2dx^2+6\,bcdx-bd\log\left(cx+1\right)+5\,bd\log\left(cx-1\right)+\left(2\,bc^3dx^3+3\,bc^2dx^2\right)\log\left(-\right.}{12\,c^2}$$

input
```
integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/12*(4*a*c^3*d*x^3 + 2*(3*a + b)*c^2*d*x^2 + 6*b*c*d*x - b*d*log(c*x + 1)
 + 5*b*d*log(c*x - 1) + (2*b*c^3*d*x^3 + 3*b*c^2*d*x^2)*log(-(c*x + 1)/(c*
x - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int x(d+cdx)(a+b\text{arctanh}(cx))\,dx = \begin{cases} \frac{acdx^3}{3}+\frac{adx^2}{2}+\frac{bcdx^3\operatorname{atanh}\left(cx\right)}{3}+\frac{bdx^2\operatorname{atanh}\left(cx\right)}{2}+\frac{bdx^2}{6}+\frac{bdx}{2c}+\frac{bd\log\left(x-\frac{1}{c}\right)}{3c^2}-\frac{bd\operatorname{atanh}\left(cx\right)}{6c^2} & \text{for}\ c\neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c*d*x**3/3 + a*d*x**2/2 + b*c*d*x**3*atanh(c*x)/3 + b*d*x**2*
atanh(c*x)/2 + b*d*x**2/6 + b*d*x/(2*c) + b*d*log(x - 1/c)/(3*c**2) - b*d*
atanh(c*x)/(6*c**2), Ne(c, 0)), (a*d*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int x(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{1}{3}\,acdx^3+\frac{1}{6}\left(2\,x^3\,\text{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bcd+\frac{1}{2}\,adx^2$$
$$+\frac{1}{4}\left(2\,x^2\,\text{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+1\right)}{c^3}+\frac{\log\left(cx-1\right)}{c^3}\right)\right)bd$$

input
```
integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*c*d*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^
4))*b*c*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x +
 1)/c^3 + log(c*x - 1)/c^3))*b*d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(72) = 144.

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.63

$$\int x(d+cdx)(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{1}{3}\,c\left(\frac{\left(\frac{6\,(cx+1)^2bd}{(cx-1)^2}-\frac{3\,(cx+1)bd}{cx-1}+bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^3}{(cx-1)^3}-\frac{3\,(cx+1)^2c^3}{(cx-1)^2}+\frac{3\,(cx+1)c^3}{cx-1}-c^3}+\frac{\frac{12\,(cx+1)^2ad}{(cx-1)^2}-\frac{6\,(cx+1)ad}{cx-1}+2\,ad+\frac{5\,(cx+1)^2bd}{(cx-1)^2}-\frac{8\,(cx+1)bd}{cx-1}+}{\frac{(cx+1)^3c^3}{(cx-1)^3}-\frac{3\,(cx+1)^2c^3}{(cx-1)^2}+\frac{3\,(cx+1)c^3}{cx-1}-c^3}\right.$$

input
```
integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/3*c*((6*(c*x + 1)^2*b*d/(c*x - 1)^2 - 3*(c*x + 1)*b*d/(c*x - 1) + b*d)*l
og(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^3/(c*x - 1)^3 - 3*(c*x + 1)^2*c^3/
(c*x - 1)^2 + 3*(c*x + 1)*c^3/(c*x - 1) - c^3) + (12*(c*x + 1)^2*a*d/(c*x
- 1)^2 - 6*(c*x + 1)*a*d/(c*x - 1) + 2*a*d + 5*(c*x + 1)^2*b*d/(c*x - 1)^2
 - 8*(c*x + 1)*b*d/(c*x - 1) + 3*b*d)/((c*x + 1)^3*c^3/(c*x - 1)^3 - 3*(c*
x + 1)^2*c^3/(c*x - 1)^2 + 3*(c*x + 1)*c^3/(c*x - 1) - c^3) - b*d*log(-(c*
x + 1)/(c*x - 1) + 1)/c^3 + b*d*log(-(c*x + 1)/(c*x - 1))/c^3)
```

Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int x(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx=\frac{d\,(3\,a\,x^2+b\,x^2+3\,b\,x^2\,\mathrm{atanh}(c\,x))}{6}-\frac{\frac{d\,(3\,b\,\mathrm{atanh}(c\,x)-b\,\ln(c^2\,x^2-1))}{6}-\frac{b\,c\,d\,x}{2}}{c^2}+\frac{c\,d\,(2\,a\,x^3+2\,b\,x^3\,\mathrm{atanh}(c\,x))}{6}$$

input
```
int(x*(a + b*atanh(c*x))*(d + c*d*x),x)
```

output
```
(d*(3*a*x^2 + b*x^2 + 3*b*x^2*atanh(c*x)))/6 - ((d*(3*b*atanh(c*x) - b*log
(c^2*x^2 - 1)))/6 - (b*c*d*x)/2)/c^2 + (c*d*(2*a*x^3 + 2*b*x^3*atanh(c*x))
)/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int x(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx=\frac{d(2atanh(cx)\,b\,c^3x^3+3atanh(cx)\,b\,c^2x^2-atanh(cx)\,b+2\log(c^2x-c)\,b+2a\,c^3x^3+3a\,c^2x^2+b\,c^2x^2+$$
$$6c^2$$

input
```
int(x*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
(d*(2*atanh(c*x)*b*c**3*x**3 + 3*atanh(c*x)*b*c**2*x**2 - atanh(c*x)*b + 2
*log(c**2*x - c)*b + 2*a*c**3*x**3 + 3*a*c**2*x**2 + b*c**2*x**2 + 3*b*c*x
))/(6*c**2)
```

3.4 $\int (d+cdx)(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int (d+cdx)(a+b\text{arctanh}(cx))\,dx=\frac{bdx}{2}+\frac{d(1+cx)^2(a+b\text{arctanh}(cx))}{2c}+\frac{bd\log(1-cx)}{c}$$

output
```
1/2*b*d*x+1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))/c+b*d*ln(-c*x+1)/c
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.

Time = 0.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\begin{aligned}\int (d+cdx)(a+b\text{arctanh}(cx))\,dx=&\,adx+\frac{bdx}{2}+\frac{1}{2}acdx^2+bdx\text{arctanh}(cx)\\&+\frac{1}{2}bcdx^2\text{arctanh}(cx)+\frac{bd\log(1-cx)}{4c}\\&-\frac{bd\log(1+cx)}{4c}+\frac{bd\log(1-c^2x^2)}{2c}\end{aligned}$$

input
```
Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
a*d*x + (b*d*x)/2 + (a*c*d*x^2)/2 + b*d*x*ArcTanh[c*x] + (b*c*d*x^2*ArcTan
h[c*x])/2 + (b*d*Log[1 - c*x])/(4*c) - (b*d*Log[1 + c*x])/(4*c) + (b*d*Log
[1 - c^2*x^2])/(2*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6478, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx + d)(a + b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2c} - \frac{b\int \frac{d^2(cx+1)^2}{1-c^2x^2}dx}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2c} - \frac{1}{2}bd\int \frac{(cx+1)^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2c} - \frac{1}{2}bd\int \frac{cx+1}{1-cx}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2c} - \frac{1}{2}bd\int \left(-1 - \frac{2}{cx-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2c} - \frac{1}{2}bd\left(-\frac{2\log(1-cx)}{c} - x\right)$$

input
```
Int[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

output
```
(d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*c) - (b*d*(-x - (2*Log[1 - c*x])/c
))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

method	result
parts	$ad\left(\frac{1}{2}c\,x^{2}+x\right)+\frac{db\left(\frac{\operatorname{arctanh}(cx)c^{2}x^{2}}{2}+\operatorname{arctanh}(cx)cx+\frac{cx}{2}+\frac{3\ln(cx-1)}{4}+\frac{\ln(cx+1)}{4}\right)}{c}$
derivativedivides	$\frac{ad\left(\frac{1}{2}c^{2}x^{2}+cx\right)+db\left(\frac{\operatorname{arctanh}(cx)c^{2}x^{2}}{2}+\operatorname{arctanh}(cx)cx+\frac{cx}{2}+\frac{3\ln(cx-1)}{4}+\frac{\ln(cx+1)}{4}\right)}{c}$
default	$\frac{ad\left(\frac{1}{2}c^{2}x^{2}+cx\right)+db\left(\frac{\operatorname{arctanh}(cx)c^{2}x^{2}}{2}+\operatorname{arctanh}(cx)cx+\frac{cx}{2}+\frac{3\ln(cx-1)}{4}+\frac{\ln(cx+1)}{4}\right)}{c}$
parallelrisch	$\frac{x^{2}\operatorname{arctanh}(cx)b\,c^{2}d+a\,c^{2}d\,x^{2}+2bcdx\,\operatorname{arctanh}(cx)+2adxc+bcdx+2\ln(cx-1)bd+\operatorname{arctanh}(cx)bd}{2c}$
risch	$\frac{dbx(cx+2)\ln(cx+1)}{4}-\frac{dcb\,x^{2}\ln(-cx+1)}{4}+\frac{dca\,x^{2}}{2}-\frac{bdx\ln(-cx+1)}{2}+adx+\frac{bdx}{2}+\frac{\ln(cx+1)bd}{4c}+\frac{3\ln(-$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/2*c*x^2+x)+d*b/c*(1/2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x+1/2*c*x
+3/4*ln(c*x-1)+1/4*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int(d+cdx)(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{2\,ac^{2}dx^{2}+2\,(2\,a+b)cdx+bd\log\left(cx+1\right)+3\,bd\log\left(cx-1\right)+\left(bc^{2}dx^{2}+2\,bcdx\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*a*c^2*d*x^2 + 2*(2*a + b)*c*d*x + b*d*log(c*x + 1) + 3*b*d*log(c*x
- 1) + (b*c^2*d*x^2 + 2*b*c*d*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int (d + cdx)(a + b\text{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{acdx^2}{2} + adx + \frac{bcdx^2 \operatorname{atanh}(cx)}{2} + bdx \operatorname{atanh}(cx) + \frac{bdx}{2} + \frac{bd \log\left(x - \frac{1}{c}\right)}{c} + \frac{bd \operatorname{atanh}(cx)}{2c} & \text{for } c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c*d*x**2/2 + a*d*x + b*c*d*x**2*atanh(c*x)/2 + b*d*x*atanh(c*
x) + b*d*x/2 + b*d*log(x - 1/c)/c + b*d*atanh(c*x)/(2*c), Ne(c, 0)), (a*d*
x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int (d + cdx)(a + b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{2}acdx^2 + \frac{1}{4}\left(2x^2 \operatorname{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)bcd$$
$$+ adx + \frac{(2cx \operatorname{artanh}(cx) + \log(-c^2x^2 + 1))bd}{2c}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*c*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
log(c*x - 1)/c^3))*b*c*d + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.80

$$\int (d + cdx)(a + b\text{arctanh}(cx))\,dx = -c\left(\frac{bd\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2} - \frac{\left(\frac{2\,(cx+1)bd}{cx-1} - bd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^2}{(cx-1)^2} - \frac{2\,(cx+1)c^2}{cx-1} + c^2} - \frac{bd\log\left(-\frac{cx+1}{cx-1}\right)}{c^2} - \frac{\frac{4\,(cx+1)ad}{cx-1} - 2\,ad + \frac{(cx+1)bd}{cx-1}}{\frac{(cx+1)^2c^2}{(cx-1)^2} - \frac{2\,(cx+1)c^2}{cx-1} +}\right.$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-c*(b*d*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - (2*(c*x + 1)*b*d/(c*x - 1) - b
*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^2/(c*x - 1)^2 - 2*(c*x + 1)*c
^2/(c*x - 1) + c^2) - b*d*log(-(c*x + 1)/(c*x - 1))/c^2 - (4*(c*x + 1)*a*d
/(c*x - 1) - 2*a*d + (c*x + 1)*b*d/(c*x - 1) - b*d)/((c*x + 1)^2*c^2/(c*x
- 1)^2 - 2*(c*x + 1)*c^2/(c*x - 1) + c^2))
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int (d + cdx)(a + b\text{arctanh}(cx))\,dx = \frac{d\,(2\,a\,x + b\,x + 2\,b\,x\,\text{atanh}(c\,x))}{2} + \frac{c\,d\,(a\,x^2 + b\,x^2\,\text{atanh}(c\,x))}{2} - \frac{d\,(b\,\text{atanh}(c\,x) - b\,\ln\left(c^2\,x^2 - 1\right))}{2\,c}$$

input
```
int((a + b*atanh(c*x))*(d + c*d*x),x)
```

output
```
(d*(2*a*x + b*x + 2*b*x*atanh(c*x)))/2 + (c*d*(a*x^2 + b*x^2*atanh(c*x)))/
2 - (d*(b*atanh(c*x) - b*log(c^2*x^2 - 1)))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int (d + cdx)(a + b\text{arctanh}(cx))\,dx$$
$$= \frac{d(atanh(cx)\,b\,c^2x^2 + 2atanh(cx)\,bcx + atanh(cx)\,b + 2\log(c^2x - c)\,b + a\,c^2x^2 + 2acx + bcx)}{2c}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x)),x)
```

output
```
(d*(atanh(c*x)*b*c**2*x**2 + 2*atanh(c*x)*b*c*x + atanh(c*x)*b + 2*log(c**
2*x - c)*b + a*c**2*x**2 + 2*a*c*x + b*c*x))/(2*c)
```

3.5 $\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x}\,dx = acdx+bcdx\text{arctanh}(cx) + ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right) - \frac{1}{2}bd\,\text{PolyLog}(2,-cx)+\frac{1}{2}bd\,\text{PolyLog}(2,cx)$$

output
```
a*c*d*x+b*c*d*x*arctanh(c*x)+a*d*ln(x)+1/2*b*d*ln(-c^2*x^2+1)-1/2*b*d*poly
log(2,-c*x)+1/2*b*d*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x}\,dx = acdx+bcdx\text{arctanh}(cx) + ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right) + \frac{1}{2}bd(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx))$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
a*c*d*x + b*c*d*x*ArcTanh[c*x] + a*d*Log[x] + (b*d*Log[1 - c^2*x^2])/2 + (
b*d*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))}{x}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(cd(a+b\text{arctanh}(cx))+\frac{d(a+b\text{arctanh}(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$acdx+ad\log(x)+bcdx\text{arctanh}(cx)+\frac{1}{2}bd\log\left(1-c^2x^2\right)-\frac{1}{2}bd\,\text{PolyLog}(2,-cx)+\frac{1}{2}bd\,\text{PolyLog}(2,cx)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
a*c*d*x + b*c*d*x*ArcTanh[c*x] + a*d*Log[x] + (b*d*Log[1 - c^2*x^2])/2 - (
b*d*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
parts	$ad(cx+\ln(x))+db\Big(\operatorname{arctanh}(cx)\ln(cx)+\operatorname{arctanh}(cx)\,cx+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}-\frac{\text{dilo}$
derivativedivides	$ad(cx+\ln(cx))+db\Big(\operatorname{arctanh}(cx)\ln(cx)+\operatorname{arctanh}(cx)\,cx+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}-\frac{\text{di}$
default	$ad(cx+\ln(cx))+db\Big(\operatorname{arctanh}(cx)\ln(cx)+\operatorname{arctanh}(cx)\,cx+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}-\frac{\text{di}$
risch	$-\frac{\ln(-cx+1)bcdx}{2}+adxc+\frac{\ln(-cx+1)bd}{2}+\ln(-cx)\,ad+\frac{\text{dilog}(-cx+1)bd}{2}-ad-db+\frac{\ln(cx+1)b}{2}$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*d*(c*x+ln(x))+d*b*(arctanh(c*x)*ln(c*x)+arctanh(c*x)*c*x+1/2*ln(c*x-1)+1
/2*ln(c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x}\,dx=\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x}\,dx = d\left(\int ac\,dx + \int \frac{a}{x}\,dx + \int bc\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))/x,x)
```

output
```
d*(Integral(a*c, x) + Integral(a/x, x) + Integral(b*c*atanh(c*x), x) + Int
egral(b*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x}\,dx = \int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

output
```
a*c*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d + 1/2*b*d*integ
rate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*d*log(x)
```

Giac [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x}\,dx = \int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,(d+c\,d\,x)}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x}\,dx = d\left(atanh(cx)\,bcx + atanh(cx)\,b + \left(\int \frac{atanh(cx)}{x}dx\right)b + \log\left(c^2x-c\right)b + \log(x)\,a + acx\right)$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))/x,x)
```

output
```
d*(atanh(c*x)*b*c*x + atanh(c*x)*b + int(atanh(c*x)/x,x)*b + log(c**2*x -
c)*b + log(x)*a + a*c*x)
```

3.6 $\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 70

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx = -\frac{d(a+b\text{arctanh}(cx))}{x}+acd\log(x) + bcd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right) - \frac{1}{2}bcd\,\text{PolyLog}(2,-cx)+\frac{1}{2}bcd\,\text{PolyLog}(2,cx)$$

output
```
-d*(a+b*arctanh(c*x))/x+a*c*d*ln(x)+b*c*d*ln(x)-1/2*b*c*d*ln(-c^2*x^2+1)-1
/2*b*c*d*polylog(2,-c*x)+1/2*b*c*d*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx = -\frac{ad}{x}+acd\log(x) + bcd\left(-\frac{\text{arctanh}(cx)}{cx}+\log(cx)-\frac{1}{2}\log\left(1-c^2x^2\right)\right) + \frac{1}{2}bcd(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx))$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*c*d*Log[x] + b*c*d*(-(ArcTanh[c*x]/(c*x)) + Log[c*x] - Log[
1 - c^2*x^2]/2) + (b*c*d*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))}{x^2}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{d(a+b\text{arctanh}(cx))}{x^2}+\frac{cd(a+b\text{arctanh}(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\text{arctanh}(cx))}{x}+acd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)-\frac{1}{2}bcd\,\text{PolyLog}(2,-cx)+\frac{1}{2}bcd\,\text{PolyLog}(2,cx)+bcd\log(x)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcTanh[c*x]))/x) + a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
 - c^2*x^2])/2 - (b*c*d*PolyLog[2, -(c*x)])/2 + (b*c*d*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

method	result
parts	$ad\left(c\ln\left(x\right)-\frac{1}{x}\right)+dbc\left(\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{\ln(cx-1)}{2}+\ln\left(cx\right)-\frac{\ln(cx+1)}{2}\right.$
derivativedivides	$c\left(ad\left(\ln\left(cx\right)-\frac{1}{cx}\right)+db\left(\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{\ln(cx-1)}{2}+\ln\left(cx\right)-\frac{\ln(cx+}{2}\right.\right.$
default	$c\left(ad\left(\ln\left(cx\right)-\frac{1}{cx}\right)+db\left(\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{\ln(cx-1)}{2}+\ln\left(cx\right)-\frac{\ln(cx+}{2}\right.\right.$
risch	$\frac{cdb\operatorname{dilog}(-cx+1)}{2}+\frac{cdb\ln(-cx)}{2}-\frac{\ln(-cx+1)bcd}{2}+\frac{db\ln(-cx+1)}{2x}+cda\ln\left(-cx\right)-\frac{ad}{x}-\frac{bcd\operatorname{dilog}(cx+}{2}$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*d*(c*ln(x)-1/x)+d*b*c*(arctanh(c*x)*ln(c*x)-arctanh(c*x)/c/x-1/2*ln(c*x-
1)+ln(c*x)-1/2*ln(c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*
x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx=d\left(\int \frac{a}{x^2}\,dx+\int \frac{ac}{x}\,dx+\int \frac{b\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{bc\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))/x**2,x)
```

output
```
d*(Integral(a/x**2, x) + Integral(a*c/x, x) + Integral(b*atanh(c*x)/x**2,
x) + Integral(b*c*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/2*b*c*d*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c*d*log(x) -
1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - a*d/x
```

Giac [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\,(d+c\,d\,x)}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x^2,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$=\frac{d\left(-atanh(cx)\,bcx-atanh(cx)\,b+\left(\int \frac{atanh(cx)}{x}dx\right)bcx-\log(c^2x-c)\,bcx+\log(x)\,acx+\log(x)\,bcx-a\right)}{x}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))/x^2,x)
```

output
```
(d*( - atanh(c*x)*b*c*x - atanh(c*x)*b + int(atanh(c*x)/x,x)*b*c*x - log(c
**2*x - c)*b*c*x + log(x)*a*c*x + log(x)*b*c*x - a))/x
```

3.7 $\int \frac{(d+cdx)(a+b\,\text{arctanh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{(d+cdx)(a+b\,\text{arctanh}(cx))}{x^3}\,dx = -\frac{bcd}{2x} - \frac{d(1+cx)^2(a+b\,\text{arctanh}(cx))}{2x^2} + bc^2d\log(x) - bc^2d\log(1-cx)$$

output
```
-1/2*b*c*d/x-1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))/x^2+b*c^2*d*ln(x)-b*c^2*d*
ln(-c*x+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{(d+cdx)(a+b\,\text{arctanh}(cx))}{x^3}\,dx = -\frac{d(2a+4acx+2bcx+2(b+2bcx)\text{arctanh}(cx)-4bc^2x^2\log(x)+3bc^2x^2\log(1-cx)+bc^2x^2\log(1+c}{4x^2}$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
-1/4*(d*(2*a + 4*a*c*x + 2*b*c*x + 2*(b + 2*b*c*x)*ArcTanh[c*x] - 4*b*c^2*
x^2*Log[x] + 3*b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x]))/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6498, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))}{x^3}dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int -\frac{d(cx+1)}{2x^2(1-cx)}dx-\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}bcd\int \frac{cx+1}{x^2(1-cx)}dx-\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}bcd\int \left(-\frac{2c^2}{cx-1}+\frac{2c}{x}+\frac{1}{x^2}\right)dx-\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}bcd\left(2c\log(x)-2c\log(1-cx)-\frac{1}{x}\right)-\frac{d(cx+1)^2(a+b\text{arctanh}(cx))}{2x^2}$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
-1/2*(d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/x^2 + (b*c*d*(-x^(-1) + 2*c*Log[
x] - 2*c*Log[1 - c*x]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

method	result
parts	$ad\left(-\frac{c}{x}-\frac{1}{2x^2}\right)+db\,c^2\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{3\ln(cx-1)}{4}-\frac{1}{2cx}+\ln(cx)-\frac{\ln(cx+1)}{4}\right)$
derivativedivides	$c^2\left(ad\left(-\frac{1}{2c^2x^2}-\frac{1}{cx}\right)+db\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{3\ln(cx-1)}{4}-\frac{1}{2cx}+\ln(cx)-\frac{\ln(cx+1}{4}\right.\right.$
default	$c^2\left(ad\left(-\frac{1}{2c^2x^2}-\frac{1}{cx}\right)+db\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{3\ln(cx-1)}{4}-\frac{1}{2cx}+\ln(cx)-\frac{\ln(cx+1}{4}\right.\right.$
parallelrisch	$\frac{2b\,c^2d\ln(x)x^2-2\ln(cx-1)x^2b\,c^2d-x^2\operatorname{arctanh}(cx)b\,c^2d-a\,c^2d\,x^2-2bcdx\operatorname{arctanh}(cx)-2adxc-bcdx-\operatorname{arctanh}(cx)bd-}{2x^2}$
risch	$-\frac{db(2cx+1)\ln(cx+1)}{4x^2}+\frac{d\left(4b\,c^2\ln(-x)x^2-3b\,c^2x^2\ln(-cx+1)-b\,c^2\ln(cx+1)x^2+2bcx\ln(-cx+1)-4acx-2bcx+b\right.}{4x^2}$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*d*(-c/x-1/2/x^2)+d*b*c^2*(-1/2*arctanh(c*x)/c^2/x^2-arctanh(c*x)/c/x-3/4
*ln(c*x-1)-1/2/c/x+ln(c*x)-1/4*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x^3}\,dx = -\frac{bc^2dx^2\log(cx+1)+3\,bc^2dx^2\log(cx-1)-4\,bc^2dx^2\log(x)+2\,(2\,a+b)cdx+2\,ad+(2\,bcdx+bd)\,\mathrm{lo}}{4\,x^2}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*c^2*d*x^2*log(c*x + 1) + 3*b*c^2*d*x^2*log(c*x - 1) - 4*b*c^2*d*x^
2*log(x) + 2*(2*a + b)*c*d*x + 2*a*d + (2*b*c*d*x + b*d)*log(-(c*x + 1)/(c
*x - 1)))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x^3}\,dx = \begin{cases} -\frac{acd}{x}-\frac{ad}{2x^2}+bc^2d\log(x)-bc^2d\log\left(x-\frac{1}{c}\right)-\frac{bc^2d\operatorname{atanh}(cx)}{2}-\frac{bcd\operatorname{atanh}(cx)}{x}-\frac{bcd}{2x}-\frac{bd\operatorname{atanh}(cx)}{2x^2} & \text{for } c\neq 0 \\ -\frac{ad}{2x^2} & \text{otherwis} \end{cases}$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))/x**3,x)
```

output
```
Piecewise((-a*c*d/x - a*d/(2*x**2) + b*c**2*d*log(x) - b*c**2*d*log(x - 1/
c) - b*c**2*d*atanh(c*x)/2 - b*c*d*atanh(c*x)/x - b*c*d/(2*x) - b*d*atanh(
c*x)/(2*x**2), Ne(c, 0)), (-a*d/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$= -\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\left(cx\right)}{x}\right)bcd$$
$$+\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\left(cx\right)}{x^2}\right)bd-\frac{acd}{x}-\frac{ad}{2\,x^2}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d + 1/4*((c*
log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d - a*c*d/x
 - 1/2*a*d/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.43

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$= \left(bcd\log\left(-\frac{cx+1}{cx-1}-1\right)-bcd\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{2\,(cx+1)bcd}{cx-1}+bcd\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1}+\frac{\frac{4\,(cx+1)acd}{cx-1}+2\,acd-}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(}{c}}\right.$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

output
```
(b*c*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c*d*log(-(c*x + 1)/(c*x - 1)) + (
2*(c*x + 1)*b*c*d/(c*x - 1) + b*c*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^
2/(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1) + (4*(c*x + 1)*a*c*d/(c*x - 1)
+ 2*a*c*d + (c*x + 1)*b*c*d/(c*x - 1) + b*c*d)/((c*x + 1)^2/(c*x - 1)^2 +
2*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^3}\,dx = \frac{d\left(b\,c^2\,\text{atanh}(c\,x)-b\,c^2\,\ln\left(c^2\,x^2-1\right)+2\,b\,c^2\,\ln\left(x\right)\right)}{2} - \frac{\frac{d\,(a+b\,\text{atanh}(c\,x))}{2}+\frac{d\,x\,(2\,a\,c+b\,c+2\,b\,c\,\text{atanh}(c\,x))}{2}}{x^2}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x^3,x)
```

output
```
(d*(b*c^2*atanh(c*x) - b*c^2*log(c^2*x^2 - 1) + 2*b*c^2*log(x)))/2 - ((d*(
a + b*atanh(c*x)))/2 + (d*x*(2*a*c + b*c + 2*b*c*atanh(c*x)))/2)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^3}\,dx = \frac{d(-atanh(cx)\,b\,c^2x^2-2atanh(cx)\,bcx-atanh(cx)\,b-2\log(c^2x-c)\,b\,c^2x^2+2\log(x)\,b\,c^2x^2-2acx-a}{2x^2}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))/x^3,x)
```

output
```
(d*( - atanh(c*x)*b*c**2*x**2 - 2*atanh(c*x)*b*c*x - atanh(c*x)*b - 2*log(
c**2*x - c)*b*c**2*x**2 + 2*log(x)*b*c**2*x**2 - 2*a*c*x - a - b*c*x))/(2*
x**2)
```

3.8 $\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx = -\frac{bcd}{6x^2}-\frac{bc^2d}{2x}-\frac{d(a+b\text{arctanh}(cx))}{3x^3} -\frac{cd(a+b\text{arctanh}(cx))}{2x^2}+\frac{1}{3}bc^3d\log(x) -\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(1+cx)$$

output
```
-1/6*b*c*d/x^2-1/2*b*c^2*d/x-1/3*d*(a+b*arctanh(c*x))/x^3-1/2*c*d*(a+b*arc
tanh(c*x))/x^2+1/3*b*c^3*d*ln(x)-5/12*b*c^3*d*ln(-c*x+1)+1/12*b*c^3*d*ln(c
*x+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx = -\frac{d(4a+6acx+2bcx+6bc^2x^2+2b(2+3cx)\text{arctanh}(cx)-4bc^3x^3\log(x)+5bc^3x^3\log(1-cx)-bc^3x^3}{12x^3}$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
-1/12*(d*(4*a + 6*a*c*x + 2*b*c*x + 6*b*c^2*x^2 + 2*b*(2 + 3*c*x)*ArcTanh[
c*x] - 4*b*c^3*x^3*Log[x] + 5*b*c^3*x^3*Log[1 - c*x] - b*c^3*x^3*Log[1 + c
*x]))/x^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6498, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))}{x^4}dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int -\frac{d(3cx+2)}{6x^3(1-c^2x^2)}dx-\frac{d(a+b\text{arctanh}(cx))}{3x^3}-\frac{cd(a+b\text{arctanh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}bcd\int \frac{3cx+2}{x^3(1-c^2x^2)}dx-\frac{d(a+b\text{arctanh}(cx))}{3x^3}-\frac{cd(a+b\text{arctanh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 523}$$

$$\frac{1}{6}bcd\int \left(-\frac{5c^3}{2(cx-1)}+\frac{c^3}{2(cx+1)}+\frac{2c^2}{x}+\frac{3c}{x^2}+\frac{2}{x^3}\right)dx-\frac{d(a+b\text{arctanh}(cx))}{3x^3}-\frac{cd(a+b\text{arctanh}(cx))}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d(a+b\text{arctanh}(cx))}{3x^3}-\frac{cd(a+b\text{arctanh}(cx))}{2x^2}+\frac{1}{6}bcd\left(2c^2\log(x)-\frac{5}{2}c^2\log(1-cx)+\frac{1}{2}c^2\log(cx+1)-\frac{3c}{x}-\frac{1}{x^2}\right)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcTanh[c*x]))/x^3 - (c*d*(a + b*ArcTanh[c*x]))/(2*x^2) + (
b*c*d*(-x^(-2) - (3*c)/x + 2*c^2*Log[x] - (5*c^2*Log[1 - c*x])/2 + (c^2*Lo
g[1 + c*x])/2))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

method	result
parts	$ad\left(-\frac{c}{2x^2}-\frac{1}{3x^3}\right)+db\,c^3\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{5\ln(cx-1)}{12}-\frac{1}{6c^2x^2}-\frac{1}{2cx}+\frac{\ln(cx)}{3}+\ln\right.$
derivativedivides	$c^3\left(ad\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^2x^2}\right)+db\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{5\ln(cx-1)}{12}-\frac{1}{6c^2x^2}-\frac{1}{2cx}+\frac{\ln(cx)}{3}\right.\right.$
default	$c^3\left(ad\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^2x^2}\right)+db\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{5\ln(cx-1)}{12}-\frac{1}{6c^2x^2}-\frac{1}{2cx}+\frac{\ln(cx)}{3}\right.\right.$
parallelrisch	$\frac{2b\,c^3d\ln(x)x^3-2\ln(cx-1)x^3b\,c^3d+db\operatorname{arctanh}(cx)x^3c^3-3a\,c^3d\,x^3-b\,c^3d\,x^3-3b\,c^2d\,x^2-3bcdx\operatorname{arctanh}(cx)-3adxc-b}{6x^3}$
risch	$-\frac{db(3cx+2)\ln(cx+1)}{12x^3}-\frac{d(5b\,x^3\ln(-cx+1)c^3-4b\,c^3\ln(-x)x^3-b\,c^3\ln(cx+1)x^3+6b\,c^2x^2-3bcx\ln(-cx+1)+6acx+}{12x^3}$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*d*(-1/2*c/x^2-1/3/x^3)+d*b*c^3*(-1/3*arctanh(c*x)/c^3/x^3-1/2*arctanh(c*
x)/c^2/x^2-5/12*ln(c*x-1)-1/6/c^2/x^2-1/2/c/x+1/3*ln(c*x)+1/12*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x^4}\,dx$$
$$=\frac{bc^3dx^3\log(cx+1)-5\,bc^3dx^3\log(cx-1)+4\,bc^3dx^3\log(x)-6\,bc^2dx^2-2\,(3\,a+b)cdx-4\,ad-(3\,bcd}{12\,x^3}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/12*(b*c^3*d*x^3*log(c*x + 1) - 5*b*c^3*d*x^3*log(c*x - 1) + 4*b*c^3*d*x^
3*log(x) - 6*b*c^2*d*x^2 - 2*(3*a + b)*c*d*x - 4*a*d - (3*b*c*d*x + 2*b*d)
*log(-(c*x + 1)/(c*x - 1)))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$=\begin{cases} -\frac{acd}{2x^2}-\frac{ad}{3x^3}+\frac{bc^3d\log(x)}{3}-\frac{bc^3d\log\left(x-\frac{1}{c}\right)}{3}+\frac{bc^3d\,\text{atanh}(cx)}{6}-\frac{bc^2d}{2x}-\frac{bcd\,\text{atanh}(cx)}{2x^2}-\frac{bcd}{6x^2}-\frac{bd\,\text{atanh}(cx)}{3x^3} & \text{for } c\neq 0 \\ -\frac{ad}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))/x**4,x)
```

output
```
Piecewise((-a*c*d/(2*x**2) - a*d/(3*x**3) + b*c**3*d*log(x)/3 - b*c**3*d*l
og(x - 1/c)/3 + b*c**3*d*atanh(c*x)/6 - b*c**2*d/(2*x) - b*c*d*atanh(c*x)/
(2*x**2) - b*c*d/(6*x**2) - b*d*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d/(3*x
**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$=\frac{1}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}(cx)}{x^2}\right)bcd$$
$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}(cx)}{x^3}\right)bd-\frac{acd}{2\,x^2}-\frac{ad}{3\,x^3}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
 - 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x
^3)*b*d - 1/2*a*c*d/x^2 - 1/3*a*d/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(84) = 168$.

Time = 0.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.12

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$=\frac{1}{3}\left(bc^2d\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^2d\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{6\,(cx+1)^2bc^2d}{(cx-1)^2}+\frac{3\,(cx+1)bc^2d}{cx-1}+bc^2d\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3\,(cx+1)^2}{(cx-1)^2}+\frac{3\,(cx+1)}{cx-1}+1}+\right.$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

output
```
1/3*(b*c^2*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^2*d*log(-(c*x + 1)/(c*x -
 1)) + (6*(c*x + 1)^2*b*c^2*d/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*d/(c*x - 1)
+ b*c^2*d)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1
)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^2*a*c^2*d/(c*
x - 1)^2 + 6*(c*x + 1)*a*c^2*d/(c*x - 1) + 2*a*c^2*d + 5*(c*x + 1)^2*b*c^2
*d/(c*x - 1)^2 + 8*(c*x + 1)*b*c^2*d/(c*x - 1) + 3*b*c^2*d)/((c*x + 1)^3/(
c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^4}\,dx=\frac{b\,c^3\,d\,\ln(x)}{3}-\frac{a\,c\,d}{2\,x^2}-\frac{b\,c\,d}{6\,x^2}-\frac{b\,d\,\text{atanh}(c\,x)}{3\,x^3}-\frac{b\,c^3\,d\,\ln(c^2\,x^2-1)}{6}-\frac{b\,c^2\,d}{2\,x}-\frac{a\,d}{3\,x^3}-\frac{b\,c^4\,d\,\text{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{2\,\sqrt{-c^2}}-\frac{b\,c\,d\,\text{atanh}(c\,x)}{2\,x^2}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x^4,x)
```

output
```
(b*c^3*d*log(x))/3 - (a*c*d)/(2*x^2) - (b*c*d)/(6*x^2) - (b*d*atanh(c*x))/
(3*x^3) - (b*c^3*d*log(c^2*x^2 - 1))/6 - (b*c^2*d)/(2*x) - (a*d)/(3*x^3) -
 (b*c^4*d*atan((c^2*x)/(-c^2)^(1/2)))/(2*(-c^2)^(1/2)) - (b*c*d*atanh(c*x)
)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x^4}\,dx$$
$$= \frac{d(atanh(cx)\,b\,c^3x^3 - 3atanh(cx)\,bcx - 2atanh(cx)\,b - 2\log(c^2x-c)\,b\,c^3x^3 + 2\log(x)\,b\,c^3x^3 - 3acx - 2a}{6x^3}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))/x^4,x)
```

output
```
(d*(atanh(c*x)*b*c**3*x**3 - 3*atanh(c*x)*b*c*x - 2*atanh(c*x)*b - 2*log(c
**2*x - c)*b*c**3*x**3 + 2*log(x)*b*c**3*x**3 - 3*a*c*x - 2*a - 3*b*c**2*x
**2 - b*c*x))/(6*x**3)
```

3.9 $\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 110

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^5}\,dx = -\frac{bcd}{12x^3}-\frac{bc^2d}{6x^2}-\frac{bc^3d}{4x}-\frac{d(a+b\text{arctanh}(cx))}{4x^4}-\frac{cd(a+b\text{arctanh}(cx))}{3x^3}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4d\log(1+cx)$$

output
```
-1/12*b*c*d/x^3-1/6*b*c^2*d/x^2-1/4*b*c^3*d/x-1/4*d*(a+b*arctanh(c*x))/x^4
-1/3*c*d*(a+b*arctanh(c*x))/x^3+1/3*b*c^4*d*ln(x)-7/24*b*c^4*d*ln(-c*x+1)-
1/24*b*c^4*d*ln(c*x+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^5}\,dx = -\frac{d(6a+8acx+2bcx+4bc^2x^2+6bc^3x^3+2b(3+4cx)\text{arctanh}(cx)-8bc^4x^4\log(x)+7bc^4x^4\log(1-cx}{24x^4}$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/24*(d*(6*a + 8*a*c*x + 2*b*c*x + 4*b*c^2*x^2 + 6*b*c^3*x^3 + 2*b*(3 + 4
*c*x)*ArcTanh[c*x] - 8*b*c^4*x^4*Log[x] + 7*b*c^4*x^4*Log[1 - c*x] + b*c^4
*x^4*Log[1 + c*x]))/x^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6498, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))}{x^5}dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d(4cx+3)}{12x^4(1-c^2x^2)}dx-\frac{d(a+b\text{arctanh}(cx))}{4x^4}-\frac{cd(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bcd\int \frac{4cx+3}{x^4(1-c^2x^2)}dx-\frac{d(a+b\text{arctanh}(cx))}{4x^4}-\frac{cd(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{12}bcd\int \left(-\frac{7c^4}{2(cx-1)}-\frac{c^4}{2(cx+1)}+\frac{4c^3}{x}+\frac{3c^2}{x^2}+\frac{4c}{x^3}+\frac{3}{x^4}\right)dx-\frac{d(a+b\text{arctanh}(cx))}{4x^4}-\frac{cd(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\text{arctanh}(cx))}{4x^4}-\frac{cd(a+b\text{arctanh}(cx))}{3x^3}+\frac{1}{12}bcd\left(4c^3\log(x)-\frac{7}{2}c^3\log(1-cx)-\frac{1}{2}c^3\log(cx+1)-\frac{3c^2}{x}-\frac{2c}{x^2}-\frac{1}{x^3}\right)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTanh[c*x]))/x^4 - (c*d*(a + b*ArcTanh[c*x]))/(3*x^3) + (
b*c*d*(-x^(-3) - (2*c)/x^2 - (3*c^2)/x + 4*c^3*Log[x] - (7*c^3*Log[1 - c*x
])/2 - (c^3*Log[1 + c*x])/2))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

method	result
parts	$ad\left(-\frac{1}{4x^4}-\frac{c}{3x^3}\right)+db\,c^4\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{7\ln(cx-1)}{24}-\frac{1}{12c^3x^3}-\frac{1}{6c^2x^2}-\frac{1}{4cx}+\right.$
derivativedivides	$c^4\left(ad\left(-\frac{1}{3c^3x^3}-\frac{1}{4c^4x^4}\right)+db\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{7\ln(cx-1)}{24}-\frac{1}{12c^3x^3}-\frac{1}{6c^2x^2}-\frac{1}{4cx}\right.\right.$
default	$c^4\left(ad\left(-\frac{1}{3c^3x^3}-\frac{1}{4c^4x^4}\right)+db\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{7\ln(cx-1)}{24}-\frac{1}{12c^3x^3}-\frac{1}{6c^2x^2}-\frac{1}{4cx}\right.\right.$
parallelrisch	$\frac{4b\,c^4d\ln(x)x^4-4\ln(cx-1)x^4b\,c^4d-db\operatorname{arctanh}(cx)x^4c^4-2d\,x^4c^4b-3b\,c^3d\,x^3-2b\,c^2d\,x^2-4bcdx\operatorname{arctanh}(cx)-4adxc-}{12x^4}$
risch	$-\frac{db(4cx+3)\ln(cx+1)}{24x^4}+\frac{d\left(8b\,c^4\ln(-x)x^4-7b\,x^4\ln(-cx+1)c^4-b\,c^4\ln(cx+1)x^4-6b\,c^3x^3-4b\,c^2x^2+4bcx\ln(-cx+1\right.}{24x^4}$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*d*(-1/4/x^4-1/3*c/x^3)+d*b*c^4*(-1/3*arctanh(c*x)/c^3/x^3-1/4*arctanh(c*
x)/c^4/x^4-7/24*ln(c*x-1)-1/12/c^3/x^3-1/6/c^2/x^2-1/4/c/x+1/3*ln(c*x)-1/2
4*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int\frac{(d+cdx)(a+b\operatorname{arctanh}(cx))}{x^5}\,dx=-\frac{bc^4dx^4\log(cx+1)+7\,bc^4dx^4\log(cx-1)-8\,bc^4dx^4\log(x)+6\,bc^3dx^3+4\,bc^2dx^2+2\,(4\,a+b)cdx+}{24\,x^4}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

output
```
-1/24*(b*c^4*d*x^4*log(c*x + 1) + 7*b*c^4*d*x^4*log(c*x - 1) - 8*b*c^4*d*x
^4*log(x) + 6*b*c^3*d*x^3 + 4*b*c^2*d*x^2 + 2*(4*a + b)*c*d*x + 6*a*d + (4
*b*c*d*x + 3*b*d)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{(d + cdx)(a + b\text{arctanh}(cx))}{x^5}\,dx$$
$$= \begin{cases} -\frac{acd}{3x^3} - \frac{ad}{4x^4} + \frac{bc^4d\log(x)}{3} - \frac{bc^4d\log\left(x-\frac{1}{c}\right)}{3} - \frac{bc^4d\,\text{atanh}(cx)}{12} - \frac{bc^3d}{4x} - \frac{bc^2d}{6x^2} - \frac{bcd\,\text{atanh}(cx)}{3x^3} - \frac{bcd}{12x^3} - \frac{bd\,\text{atanh}(cx)}{4x^4} & \text{f} \\ -\frac{ad}{4x^4} & \text{o} \end{cases}$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))/x**5,x)
```

output
```
Piecewise((-a*c*d/(3*x**3) - a*d/(4*x**4) + b*c**4*d*log(x)/3 - b*c**4*d*l
og(x - 1/c)/3 - b*c**4*d*atanh(c*x)/12 - b*c**3*d/(4*x) - b*c**2*d/(6*x**2
) - b*c*d*atanh(c*x)/(3*x**3) - b*c*d/(12*x**3) - b*d*atanh(c*x)/(4*x**4),
 Ne(c, 0)), (-a*d/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

$$\int \frac{(d + cdx)(a + b\text{arctanh}(cx))}{x^5}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right) - c^2\log\left(x^2\right) + \frac{1}{x^2}\right)c + \frac{2\,\text{artanh}(cx)}{x^3}\right)bcd$$
$$+ \frac{1}{24}\left(\left(3\,c^3\log(cx+1) - 3\,c^3\log(cx-1) - \frac{2\,(3\,c^2x^2+1)}{x^3}\right)c - \frac{6\,\text{artanh}(cx)}{x^4}\right)bd$$
$$- \frac{acd}{3\,x^3} - \frac{ad}{4\,x^4}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*c*d + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d - 1/3*a*c*d/x^3 - 1/4*a*d/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(94) = 188$.

Time = 0.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.65

$$\int \frac{(d + cdx)(a + b\operatorname{arctanh}(cx))}{x^5}\,dx$$
$$= \frac{1}{3}\left(bc^3d\log\left(-\frac{cx+1}{cx-1} - 1\right) - bc^3d\log\left(-\frac{cx+1}{cx-1}\right) + \frac{\left(\frac{6\,(cx+1)^3bc^3d}{(cx-1)^3} + \frac{3\,(cx+1)^2bc^3d}{(cx-1)^2} + \frac{4\,(cx+1)bc^3d}{cx-1} + bc^3d\right) \mathrm{l}}{\frac{(cx+1)^4}{(cx-1)^4} + \frac{4\,(cx+1)^3}{(cx-1)^3} + \frac{6\,(cx+1)^2}{(cx-1)^2} + \frac{4\,(cx+1)}{cx-1} +}\right.$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

output
```
1/3*(b*c^3*d*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^3*d*log(-(c*x + 1)/(c*x -
 1)) + (6*(c*x + 1)^3*b*c^3*d/(c*x - 1)^3 + 3*(c*x + 1)^2*b*c^3*d/(c*x - 1
)^2 + 4*(c*x + 1)*b*c^3*d/(c*x - 1) + b*c^3*d)*log(-(c*x + 1)/(c*x - 1))/(
(c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x -
 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x + 1)^3*a*c^3*d/(c*x - 1)^3 +
 6*(c*x + 1)^2*a*c^3*d/(c*x - 1)^2 + 8*(c*x + 1)*a*c^3*d/(c*x - 1) + 2*a*c
^3*d + 5*(c*x + 1)^3*b*c^3*d/(c*x - 1)^3 + 10*(c*x + 1)^2*b*c^3*d/(c*x - 1
)^2 + 7*(c*x + 1)*b*c^3*d/(c*x - 1) + 2*b*c^3*d)/((c*x + 1)^4/(c*x - 1)^4
+ 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x
 - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{(d + cdx)(a + b\operatorname{arctanh}(cx))}{x^5}\,dx = \frac{b\,c^4\,d\,\ln(x)}{3} - \frac{a\,c\,d}{3\,x^3} - \frac{b\,c\,d}{12\,x^3} - \frac{b\,d\,\mathrm{atanh}(c\,x)}{4\,x^4} - \frac{b\,c^4\,d\,\ln(c^2\,x^2 - 1)}{6} - \frac{b\,c^2\,d}{6\,x^2} - \frac{b\,c^3\,d}{4\,x} - \frac{a\,d}{4\,x^4} - \frac{b\,c^5\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{4\sqrt{-c^2}} - \frac{b\,c\,d\,\mathrm{atanh}(c\,x)}{3\,x^3}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x))/x^5,x)
```

output
```
(b*c^4*d*log(x))/3 - (a*c*d)/(3*x^3) - (b*c*d)/(12*x^3) - (b*d*atanh(c*x))
/(4*x^4) - (b*c^4*d*log(c^2*x^2 - 1))/6 - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4
*x) - (a*d)/(4*x^4) - (b*c^5*d*atan((c^2*x)/(-c^2)^(1/2)))/(4*(-c^2)^(1/2)
) - (b*c*d*atanh(c*x))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$= \frac{d(-atanh(cx)\,b\,c^4x^4 - 4atanh(cx)\,bcx - 3atanh(cx)\,b - 4\log(c^2x-c)\,b\,c^4x^4 + 4\log(x)\,b\,c^4x^4 - 4acx -}{12x^4}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))/x^5,x)
```

output
```
(d*( - atanh(c*x)*b*c**4*x**4 - 4*atanh(c*x)*b*c*x - 3*atanh(c*x)*b - 4*lo
g(c**2*x - c)*b*c**4*x**4 + 4*log(x)*b*c**4*x**4 - 4*a*c*x - 3*a - 3*b*c**
3*x**3 - 2*b*c**2*x**2 - b*c*x))/(12*x**4)
```

3.10 $\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\begin{aligned}\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))\,dx &= \frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4\\&+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))\\&+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))\\&+\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))\\&+\frac{49bd^2\log(1-cx)}{120c^4}-\frac{bd^2\log(1+cx)}{120c^4}\end{aligned}$$

output
```
5/12*b*d^2*x/c^3+1/5*b*d^2*x^2/c^2+5/36*b*d^2*x^3/c+1/10*b*d^2*x^4+1/30*b*
c*d^2*x^5+1/4*d^2*x^4*(a+b*arctanh(c*x))+2/5*c*d^2*x^5*(a+b*arctanh(c*x))+
1/6*c^2*d^2*x^6*(a+b*arctanh(c*x))+49/120*b*d^2*ln(-c*x+1)/c^4-1/120*b*d^2
*ln(c*x+1)/c^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^2(150bcx+72bc^2x^2+50bc^3x^3+90ac^4x^4+36bc^4x^4+144ac^5x^5+12bc^5x^5+60ac^6x^6+6bc^4x^4(15+24}{360c^4}$$

input
```
Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*(150*b*c*x + 72*b*c^2*x^2 + 50*b*c^3*x^3 + 90*a*c^4*x^4 + 36*b*c^4*x^
4 + 144*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^4*x^4*(15 + 24*c*x
 + 10*c^2*x^2)*ArcTanh[c*x] + 147*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(360
*c^4)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)^2(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{d^2x^4(10c^2x^2+24cx+15)}{60(1-c^2x^2)}dx+\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{60}bcd^2\int \frac{x^4(10c^2x^2+24cx+15)}{1-c^2x^2}dx+\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{60}bcd^2\int\left(-10x^4-\frac{24x^3}{c}-\frac{25x^2}{c^2}-\frac{24x}{c^3}+\frac{24cx+25}{c^4(1-c^2x^2)}-\frac{25}{c^4}\right)dx+\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))-\frac{1}{60}bcd^2\left(\frac{25\text{arctanh}(cx)}{c^5}-\frac{25x}{c^4}-\frac{12x^2}{c^3}-\frac{25x^3}{3c^2}-\frac{12\log(1-c^2x^2)}{c^5}-\frac{6x^4}{c}-2x^5\right)$$

input
```
Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcTanh[c*x]))/4 + (2*c*d^2*x^5*(a + b*ArcTanh[c*x]))/5 +
(c^2*d^2*x^6*(a + b*ArcTanh[c*x]))/6 - (b*c*d^2*((-25*x)/c^4 - (12*x^2)/c^
3 - (25*x^3)/(3*c^2) - (6*x^4)/c - 2*x^5 + (25*ArcTanh[c*x])/c^5 - (12*Log
[1 - c^2*x^2])/c^5))/60
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

method	result
parts	$d^2a\left(\frac{1}{6}c^2x^6+\frac{2}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{d^2b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{2\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^5x^5}{30}+\frac{c^4x^4}{10}+\frac{5x^3c}{36}\right.}{c^4}$
derivativedivides	$\frac{d^2a\left(\frac{1}{6}c^6x^6+\frac{2}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{2\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^5x^5}{30}+\frac{c^4x^4}{10}+\frac{5x^3c^3}{36}+\frac{c^2x^2}{5}+\right.}{c^4}$
default	$\frac{d^2a\left(\frac{1}{6}c^6x^6+\frac{2}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{2\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^5x^5}{30}+\frac{c^4x^4}{10}+\frac{5x^3c^3}{36}+\frac{c^2x^2}{5}+\right.}{c^4}$
parallelrisch	$\frac{30b\,c^6d^2\operatorname{arctanh}(cx)x^6+30c^6d^2x^6a+72b\,c^5d^2\operatorname{arctanh}(cx)x^5+72a\,c^5d^2x^5+6c^5d^2x^5b+45d^2b\operatorname{arctanh}(cx)x^4c^4+45a\,c}{180c^4}$
risch	$\frac{d^2b\,x^4(10c^2x^2+24cx+15)\ln(cx+1)}{120}-\frac{d^2c^2b\,x^6\ln(-cx+1)}{12}+\frac{d^2c^2a\,x^6}{6}-\frac{d^2cb\,x^5\ln(-cx+1)}{5}+\frac{2d^2ca\,x^5}{5}+bc$

input
```
int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/6*c^2*x^6+2/5*c*x^5+1/4*x^4)+d^2*b/c^4*(1/6*arctanh(c*x)*c^6*x^6+
2/5*arctanh(c*x)*c^5*x^5+1/4*arctanh(c*x)*c^4*x^4+1/30*c^5*x^5+1/10*c^4*x^
4+5/36*x^3*c^3+1/5*c^2*x^2+5/12*c*x+49/120*ln(c*x-1)-1/120*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int x^3(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{60\,ac^6d^2x^6+12\,(12\,a+b)c^5d^2x^5+18\,(5\,a+2\,b)c^4d^2x^4+50\,bc^3d^2x^3+72\,bc^2d^2x^2+150\,bcd^2x-3\,bd^2\log}{360\,c^4}$$

input
```
integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/360*(60*a*c^6*d^2*x^6 + 12*(12*a + b)*c^5*d^2*x^5 + 18*(5*a + 2*b)*c^4*d
^2*x^4 + 50*b*c^3*d^2*x^3 + 72*b*c^2*d^2*x^2 + 150*b*c*d^2*x - 3*b*d^2*log
(c*x + 1) + 147*b*d^2*log(c*x - 1) + 3*(10*b*c^6*d^2*x^6 + 24*b*c^5*d^2*x^
5 + 15*b*c^4*d^2*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.25

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$

$$= \begin{cases} \frac{ac^2d^2x^6}{6} + \frac{2acd^2x^5}{5} + \frac{ad^2x^4}{4} + \frac{bc^2d^2x^6\,\text{atanh}\,(cx)}{6} + \frac{2bcd^2x^5\,\text{atanh}\,(cx)}{5} + \frac{bcd^2x^5}{30} + \frac{bd^2x^4\,\text{atanh}\,(cx)}{4} + \frac{bd^2x^4}{10} + \frac{5bd^2x^3}{36c} + \frac{b \\ \frac{ad^2x^4}{4} \end{cases}$$

input
```
integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**2*d**2*x**6/6 + 2*a*c*d**2*x**5/5 + a*d**2*x**4/4 + b*c**2
*d**2*x**6*atanh(c*x)/6 + 2*b*c*d**2*x**5*atanh(c*x)/5 + b*c*d**2*x**5/30
+ b*d**2*x**4*atanh(c*x)/4 + b*d**2*x**4/10 + 5*b*d**2*x**3/(36*c) + b*d**
2*x**2/(5*c**2) + 5*b*d**2*x/(12*c**3) + 2*b*d**2*log(x - 1/c)/(5*c**4) -
b*d**2*atanh(c*x)/(60*c**4), Ne(c, 0)), (a*d**2*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.34

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))\,dx = \frac{1}{6}\,ac^2d^2x^6 + \frac{2}{5}\,acd^2x^5 + \frac{1}{4}\,ad^2x^4$$
$$+ \frac{1}{180}\left(30\,x^6\,\text{artanh}\,(cx) + c\left(\frac{2\,(3\,c^4x^5 + 5\,c^2x^3 + 15\,x)}{c^6} - \frac{15\,\log\,(cx+1)}{c^7} + \frac{15\,\log\,(cx-1)}{c^7}\right)\right)bc^2d^2$$
$$+ \frac{1}{10}\left(4\,x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\,(c^2x^2-1)}{c^6}\right)\right)bcd^2$$
$$+ \frac{1}{24}\left(6\,x^4\,\text{artanh}\,(cx) + c\left(\frac{2\,(c^2x^3 + 3\,x)}{c^4} - \frac{3\,\log\,(cx+1)}{c^5} + \frac{3\,\log\,(cx-1)}{c^5}\right)\right)bd^2$$

input
```
integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^2*d^2*x^6 + 2/5*a*c*d^2*x^5 + 1/4*a*d^2*x^4 + 1/180*(30*x^6*arctan
h(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 1
5*log(c*x - 1)/c^7))*b*c^2*d^2 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 +
2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d^2 + 1/24*(6*x^4*arctanh(c*x) +
 c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(137) = 274$.

Time = 0.13 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.95

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/45*c*(6*(30*(c*x + 1)^5*b*d^2/(c*x - 1)^5 - 30*(c*x + 1)^4*b*d^2/(c*x -
1)^4 + 70*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 45*(c*x + 1)^2*b*d^2/(c*x - 1)^2
 + 18*(c*x + 1)*b*d^2/(c*x - 1) - 3*b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x
 + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c
^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x
- 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) + (360*(c*x + 1)^5*a*d^2/(c*x -
1)^5 - 360*(c*x + 1)^4*a*d^2/(c*x - 1)^4 + 840*(c*x + 1)^3*a*d^2/(c*x - 1)
^3 - 540*(c*x + 1)^2*a*d^2/(c*x - 1)^2 + 216*(c*x + 1)*a*d^2/(c*x - 1) - 3
6*a*d^2 + 162*(c*x + 1)^5*b*d^2/(c*x - 1)^5 - 531*(c*x + 1)^4*b*d^2/(c*x -
 1)^4 + 818*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 696*(c*x + 1)^2*b*d^2/(c*x - 1
)^2 + 300*(c*x + 1)*b*d^2/(c*x - 1) - 53*b*d^2)/((c*x + 1)^6*c^5/(c*x - 1)
^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1)^4 - 20*(
c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*(c*x + 1)*
c^5/(c*x - 1) + c^5) - 18*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 18*b*d
^2*log(-(c*x + 1)/(c*x - 1))/c^5)
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int x^3(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{\frac{b\,c^2\,d^2\,x^2}{5}-\frac{d^2\,\left(75\,b\,\mathrm{atanh}(c\,x)-36\,b\,\ln\left(c^2\,x^2-1\right)\right)}{180}+\frac{5\,b\,c^3\,d^2\,x^3}{36}+\frac{5\,b\,c\,d^2\,x}{12}}{c^4}+\frac{d^2\,\left(45\,a\,x^4+18\,b\,x^4+45\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{180}+\frac{c^2\,d^2\,\left(30\,a\,x^6+30\,b\,x^6\,\mathrm{atanh}(c\,x)\right)}{180}+\frac{c\,d^2\,\left(72\,a\,x^5+6\,b\,x^5+72\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{180}$$

input
```
int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

output
```
((b*c^2*d^2*x^2)/5 - (d^2*(75*b*atanh(c*x) - 36*b*log(c^2*x^2 - 1)))/180 +
 (5*b*c^3*d^2*x^3)/36 + (5*b*c*d^2*x)/12)/c^4 + (d^2*(45*a*x^4 + 18*b*x^4
+ 45*b*x^4*atanh(c*x)))/180 + (c^2*d^2*(30*a*x^6 + 30*b*x^6*atanh(c*x)))/1
80 + (c*d^2*(72*a*x^5 + 6*b*x^5 + 72*b*x^5*atanh(c*x)))/180
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int x^3(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{d^2(30atanh(cx)\,b\,c^6x^6+72atanh(cx)\,b\,c^5x^5+45atanh(cx)\,b\,c^4x^4-3atanh(cx)\,b+72\log(c^2x-c)\,b+3}{180c^4}$$

input
```
int(x^3*(c*d*x+d)^2*(a+b*atanh(c*x)),x)
```

output
```
(d**2*(30*atanh(c*x)*b*c**6*x**6 + 72*atanh(c*x)*b*c**5*x**5 + 45*atanh(c*
x)*b*c**4*x**4 - 3*atanh(c*x)*b + 72*log(c**2*x - c)*b + 30*a*c**6*x**6 +
72*a*c**5*x**5 + 45*a*c**4*x**4 + 6*b*c**5*x**5 + 18*b*c**4*x**4 + 25*b*c*
*3*x**3 + 36*b*c**2*x**2 + 75*b*c*x))/(180*c**4)
```

3.11 $\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\begin{aligned}\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))\,dx =& \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4\\&+\frac{1}{3}d^2x^3(a+b\text{arctanh}(cx))\\&+\frac{1}{2}cd^2x^4(a+b\text{arctanh}(cx))\\&+\frac{1}{5}c^2d^2x^5(a+b\text{arctanh}(cx))\\&+\frac{31bd^2\log(1-cx)}{60c^3}+\frac{bd^2\log(1+cx)}{60c^3}\end{aligned}$$

output
```
1/2*b*d^2*x/c^2+4/15*b*d^2*x^2/c+1/6*b*d^2*x^3+1/20*b*c*d^2*x^4+1/3*d^2*x^
3*(a+b*arctanh(c*x))+1/2*c*d^2*x^4*(a+b*arctanh(c*x))+1/5*c^2*d^2*x^5*(a+b
*arctanh(c*x))+31/60*b*d^2*ln(-c*x+1)/c^3+1/60*b*d^2*ln(c*x+1)/c^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int x^2(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{d^2(30bcx+16bc^2x^2+20ac^3x^3+10bc^3x^3+30ac^4x^4+3bc^4x^4+12ac^5x^5+2bc^3x^3(10+15cx+6c^2x^2)\operatorname{ar}}{60c^3}$$

input
```
Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*(30*b*c*x + 16*b*c^2*x^2 + 20*a*c^3*x^3 + 10*b*c^3*x^3 + 30*a*c^4*x^4
 + 3*b*c^4*x^4 + 12*a*c^5*x^5 + 2*b*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2)*ArcT
anh[c*x] + 31*b*Log[1 - c*x] + b*Log[1 + c*x]))/(60*c^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)^2(a+b\operatorname{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{d^2x^3(6c^2x^2+15cx+10)}{30(1-c^2x^2)}dx+\frac{1}{5}c^2d^2x^5(a+b\operatorname{arctanh}(cx))+\frac{1}{2}cd^2x^4(a+b\operatorname{arctanh}(cx))+\frac{1}{3}d^2x^3(a+b\operatorname{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{30}bcd^2\int \frac{x^3(6c^2x^2+15cx+10)}{1-c^2x^2}dx+\frac{1}{5}c^2d^2x^5(a+b\operatorname{arctanh}(cx))+\frac{1}{2}cd^2x^4(a+b\operatorname{arctanh}(cx))+\frac{1}{3}d^2x^3(a+b\operatorname{arctanh}(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{30}bcd^2\int\left(-6x^3-\frac{15x^2}{c}-\frac{16x}{c^2}+\frac{16cx+15}{c^3(1-c^2x^2)}-\frac{15}{c^3}\right)dx+\frac{1}{5}c^2d^2x^5(a+b\text{arctanh}(cx))+\frac{1}{2}cd^2x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^2d^2x^5(a+b\text{arctanh}(cx))+\frac{1}{2}cd^2x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arctanh}(cx))-\frac{1}{30}bcd^2\left(\frac{15\text{arctanh}(cx)}{c^4}-\frac{15x}{c^3}-\frac{8x^2}{c^2}-\frac{8\log(1-c^2x^2)}{c^4}-\frac{5x^3}{c}-\frac{3x^4}{2}\right)$$

input
```
Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c*d^2*x^4*(a + b*ArcTanh[c*x]))/2 + (c
^2*d^2*x^5*(a + b*ArcTanh[c*x]))/5 - (b*c*d^2*((-15*x)/c^3 - (8*x^2)/c^2 -
 (5*x^3)/c - (3*x^4)/2 + (15*ArcTanh[c*x])/c^4 - (8*Log[1 - c^2*x^2])/c^4)
)/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

method	result
parts	$d^2a\left(\frac{1}{5}c^2x^5+\frac{1}{2}c\,x^4+\frac{1}{3}x^3\right)+\frac{d^2b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{2}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{c^4x^4}{20}+\frac{x^3c^3}{6}+\frac{4c^2x^2}{15}\cdots\right.}{c^3}$
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{1}{2}c^4x^4+\frac{1}{3}x^3c^3\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{2}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{c^4x^4}{20}+\frac{x^3c^3}{6}+\frac{4c^2x^2}{15}+\frac{cx}{2}+\frac{31\cdots}{}\right.}{c^3}$
default	$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{1}{2}c^4x^4+\frac{1}{3}x^3c^3\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{2}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{c^4x^4}{20}+\frac{x^3c^3}{6}+\frac{4c^2x^2}{15}+\frac{cx}{2}+\frac{31\cdots}{}\right.}{c^3}$
parallelrisch	$\frac{12b\,c^5d^2\operatorname{arctanh}(cx)x^5+12a\,c^5d^2x^5+30d^2b\operatorname{arctanh}(cx)x^4c^4+30a\,c^4d^2x^4+3b\,c^4d^2x^4+20d^2b\operatorname{arctanh}(cx)x^3c^3+20a\,c\cdots}{60c^3}$
risch	$\frac{d^2b\,x^3(6c^2x^2+15cx+10)\ln(cx+1)}{60}-\frac{d^2c^2b\,x^5\ln(-cx+1)}{10}+\frac{d^2c^2a\,x^5}{5}-\frac{d^2cb\,x^4\ln(-cx+1)}{4}+\frac{d^2ca\,x^4}{2}+\frac{bc\,d^2\cdots}{20\cdots}$

input

```
int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output

```
d^2*a*(1/5*c^2*x^5+1/2*c*x^4+1/3*x^3)+d^2*b/c^3*(1/5*arctanh(c*x)*c^5*x^5+
1/2*arctanh(c*x)*c^4*x^4+1/3*arctanh(c*x)*c^3*x^3+1/20*c^4*x^4+1/6*x^3*c^3
+4/15*c^2*x^2+1/2*c*x+31/60*ln(c*x-1)+1/60*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int x^2(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{12\,ac^5d^2x^5+3\,(10\,a+b)c^4d^2x^4+10\,(2\,a+b)c^3d^2x^3+16\,bc^2d^2x^2+30\,bcd^2x+bd^2\log{(cx+1)}+31\,bd^2\cdots}{60\,c^3}$$

input

```
integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output

```
1/60*(12*a*c^5*d^2*x^5 + 3*(10*a + b)*c^4*d^2*x^4 + 10*(2*a + b)*c^3*d^2*x
^3 + 16*b*c^2*d^2*x^2 + 30*b*c*d^2*x + b*d^2*log(c*x + 1) + 31*b*d^2*log(c
*x - 1) + (6*b*c^5*d^2*x^5 + 15*b*c^4*d^2*x^4 + 10*b*c^3*d^2*x^3)*log(-(c*
x + 1)/(c*x - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.24

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$
$$=\begin{cases} \frac{ac^2d^2x^5}{5}+\frac{acd^2x^4}{2}+\frac{ad^2x^3}{3}+\frac{bc^2d^2x^5\,\text{atanh}\,(cx)}{5}+\frac{bcd^2x^4\,\text{atanh}\,(cx)}{2}+\frac{bcd^2x^4}{20}+\frac{bd^2x^3\,\text{atanh}\,(cx)}{3}+\frac{bd^2x^3}{6}+\frac{4bd^2x^2}{15c}+\frac{bd^2}{2c^2} \\ \frac{ad^2x^3}{3} \end{cases}$$

input
```
integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**2*d**2*x**5/5 + a*c*d**2*x**4/2 + a*d**2*x**3/3 + b*c**2*d
**2*x**5*atanh(c*x)/5 + b*c*d**2*x**4*atanh(c*x)/2 + b*c*d**2*x**4/20 + b*
d**2*x**3*atanh(c*x)/3 + b*d**2*x**3/6 + 4*b*d**2*x**2/(15*c) + b*d**2*x/(
2*c**2) + 8*b*d**2*log(x - 1/c)/(15*c**3) + b*d**2*atanh(c*x)/(30*c**3), N
e(c, 0)), (a*d**2*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.29

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{1}{5}\,ac^2d^2x^5+\frac{1}{2}\,acd^2x^4$$
$$+\frac{1}{20}\left(4\,x^5\,\text{artanh}\,(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\,(c^2x^2-1)}{c^6}\right)\right)bc^2d^2+\frac{1}{3}\,ad^2x^3$$
$$+\frac{1}{12}\left(6\,x^4\,\text{artanh}\,(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\,(cx+1)}{c^5}+\frac{3\,\log\,(cx-1)}{c^5}\right)\right)bcd^2$$
$$+\frac{1}{6}\left(2\,x^3\,\text{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\,(c^2x^2-1)}{c^4}\right)\right)bd^2$$

input
```
integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^2*d^2*x^5 + 1/2*a*c*d^2*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x
^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/1
2*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*
log(c*x - 1)/c^5))*b*c*d^2 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^
2*x^2 - 1)/c^4))*b*d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(125) = 250$.

Time = 0.13 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.67

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{4}{15}c\left(\frac{\left(\frac{15\,(cx+1)^4bd^2}{(cx-1)^4}-\frac{15\,(cx+1)^3bd^2}{(cx-1)^3}+\frac{20\,(cx+1)^2bd^2}{(cx-1)^2}-\frac{10\,(cx+1)bd^2}{cx-1}+2\,bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^4}{(cx-1)^5}-\frac{5\,(cx+1)^4c^4}{(cx-1)^4}+\frac{10\,(cx+1)^3c^4}{(cx-1)^3}-\frac{10\,(cx+1)^2c^4}{(cx-1)^2}+\frac{5\,(cx+1)c^4}{cx-1}-c^4}+\frac{\frac{30\,(cx+1)^4ad^2}{(cx-1)^4}-\frac{30\,(cx\cdots}{(cx\cdots}\right.$$

input
```
integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
4/15*c*((15*(c*x + 1)^4*b*d^2/(c*x - 1)^4 - 15*(c*x + 1)^3*b*d^2/(c*x - 1)
^3 + 20*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 10*(c*x + 1)*b*d^2/(c*x - 1) + 2*b
*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^4/(c*x - 1)^5 - 5*(c*x + 1)
^4*c^4/(c*x - 1)^4 + 10*(c*x + 1)^3*c^4/(c*x - 1)^3 - 10*(c*x + 1)^2*c^4/(
c*x - 1)^2 + 5*(c*x + 1)*c^4/(c*x - 1) - c^4) + (30*(c*x + 1)^4*a*d^2/(c*x
 - 1)^4 - 30*(c*x + 1)^3*a*d^2/(c*x - 1)^3 + 40*(c*x + 1)^2*a*d^2/(c*x - 1
)^2 - 20*(c*x + 1)*a*d^2/(c*x - 1) + 4*a*d^2 + 13*(c*x + 1)^4*b*d^2/(c*x -
 1)^4 - 36*(c*x + 1)^3*b*d^2/(c*x - 1)^3 + 41*(c*x + 1)^2*b*d^2/(c*x - 1)^
2 - 23*(c*x + 1)*b*d^2/(c*x - 1) + 5*b*d^2)/((c*x + 1)^5*c^4/(c*x - 1)^5 -
 5*(c*x + 1)^4*c^4/(c*x - 1)^4 + 10*(c*x + 1)^3*c^4/(c*x - 1)^3 - 10*(c*x
+ 1)^2*c^4/(c*x - 1)^2 + 5*(c*x + 1)*c^4/(c*x - 1) - c^4) - 2*b*d^2*log(-(
c*x + 1)/(c*x - 1) + 1)/c^4 + 2*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^4)
```

Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx = \frac{\frac{4\,b\,c^2\,d^2\,x^2}{15}-\frac{d^2\,\left(30\,b\,\mathrm{atanh}(c\,x)-16\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60}+\frac{b\,c\,d^2\,x}{2}}{c^3} + \frac{d^2\,\left(20\,a\,x^3+10\,b\,x^3+20\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{60} + \frac{c^2\,d^2\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{60} + \frac{c\,d^2\,\left(30\,a\,x^4+3\,b\,x^4+30\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{60}$$

input
```
int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

output
```
((4*b*c^2*d^2*x^2)/15 - (d^2*(30*b*atanh(c*x) - 16*b*log(c^2*x^2 - 1)))/60
 + (b*c*d^2*x)/2)/c^3 + (d^2*(20*a*x^3 + 10*b*x^3 + 20*b*x^3*atanh(c*x)))/
60 + (c^2*d^2*(12*a*x^5 + 12*b*x^5*atanh(c*x)))/60 + (c*d^2*(30*a*x^4 + 3*
b*x^4 + 30*b*x^4*atanh(c*x)))/60
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int x^2(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^2(12atanh(cx)\,b\,c^5x^5+30atanh(cx)\,b\,c^4x^4+20atanh(cx)\,b\,c^3x^3+2atanh(cx)\,b+32\log(c^2x-c)\,b+12}{60c^3}$$

input
```
int(x^2*(c*d*x+d)^2*(a+b*atanh(c*x)),x)
```

output
```
(d**2*(12*atanh(c*x)*b*c**5*x**5 + 30*atanh(c*x)*b*c**4*x**4 + 20*atanh(c*
x)*b*c**3*x**3 + 2*atanh(c*x)*b + 32*log(c**2*x - c)*b + 12*a*c**5*x**5 +
30*a*c**4*x**4 + 20*a*c**3*x**3 + 3*b*c**4*x**4 + 10*b*c**3*x**3 + 16*b*c*
*2*x**2 + 30*b*c*x))/(60*c**3)
```

3.12 $\int x(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\begin{aligned}\int x(d+cdx)^2(a+b\text{arctanh}(cx))\,dx &= \frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3\\&\quad+\frac{1}{2}d^2x^2(a+b\text{arctanh}(cx))\\&\quad+\frac{2}{3}cd^2x^3(a+b\text{arctanh}(cx))\\&\quad+\frac{1}{4}c^2d^2x^4(a+b\text{arctanh}(cx))\\&\quad+\frac{17bd^2\log(1-cx)}{24c^2}-\frac{bd^2\log(1+cx)}{24c^2}\end{aligned}$$

output

```
3/4*b*d^2*x/c+1/3*b*d^2*x^2+1/12*b*c*d^2*x^3+1/2*d^2*x^2*(a+b*arctanh(c*x)
)+2/3*c*d^2*x^3*(a+b*arctanh(c*x))+1/4*c^2*d^2*x^4*(a+b*arctanh(c*x))+17/2
4*b*d^2*ln(-c*x+1)/c^2-1/24*b*d^2*ln(c*x+1)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^2(18bcx+12ac^2x^2+8bc^2x^2+16ac^3x^3+2bc^3x^3+6ac^4x^4+2bc^2x^2(6+8cx+3c^2x^2)\operatorname{arctanh}(cx)+17}{24c^2}$$

input
```
Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*(18*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 16*a*c^3*x^3 + 2*b*c^3*x^3 +
 6*a*c^4*x^4 + 2*b*c^2*x^2*(6 + 8*c*x + 3*c^2*x^2)*ArcTanh[c*x] + 17*b*Log
[1 - c*x] - b*Log[1 + c*x]))/(24*c^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)^2(a+b\operatorname{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{d^2x^2(3c^2x^2+8cx+6)}{12(1-c^2x^2)}dx+\frac{1}{4}c^2d^2x^4(a+b\operatorname{arctanh}(cx))+\frac{2}{3}cd^2x^3(a+b\operatorname{arctanh}(cx))+\frac{1}{2}d^2x^2(a+b\operatorname{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bcd^2\int \frac{x^2(3c^2x^2+8cx+6)}{1-c^2x^2}dx+\frac{1}{4}c^2d^2x^4(a+b\operatorname{arctanh}(cx))+\frac{2}{3}cd^2x^3(a+b\operatorname{arctanh}(cx))+\frac{1}{2}d^2x^2(a+b\operatorname{arctanh}(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{12}bcd^2\int\left(-3x^2-\frac{8x}{c}+\frac{8cx+9}{c^2(1-c^2x^2)}-\frac{9}{c^2}\right)dx+\frac{1}{4}c^2d^2x^4(a+b\text{arctanh}(cx))+\frac{2}{3}cd^2x^3(a+b\text{arctanh}(cx))+\frac{1}{2}d^2x^2(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}c^2d^2x^4(a+b\text{arctanh}(cx))+\frac{2}{3}cd^2x^3(a+b\text{arctanh}(cx))+\frac{1}{2}d^2x^2(a+b\text{arctanh}(cx))-\frac{1}{12}bcd^2\left(\frac{9\text{arctanh}(cx)}{c^3}-\frac{9x}{c^2}-\frac{4\log(1-c^2x^2)}{c^3}-\frac{4x^2}{c}-x^3\right)$$

input
```
Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*x^2*(a + b*ArcTanh[c*x]))/2 + (2*c*d^2*x^3*(a + b*ArcTanh[c*x]))/3 +
(c^2*d^2*x^4*(a + b*ArcTanh[c*x]))/4 - (b*c*d^2*((-9*x)/c^2 - (4*x^2)/c -
x^3 + (9*ArcTanh[c*x])/c^3 - (4*Log[1 - c^2*x^2])/c^3))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

method	result
parts	$d^2a\left(\frac{1}{4}c^2x^4+\frac{2}{3}c\,x^3+\frac{1}{2}x^2\right)+\frac{d^2b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{2\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{x^3c^3}{12}+\frac{c^2x^2}{3}+\frac{3cx}{4}+\right.}{c^2}$
derivativedivides	$\frac{d^2a\left(\frac{1}{4}c^4x^4+\frac{2}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{2\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{x^3c^3}{12}+\frac{c^2x^2}{3}+\frac{3cx}{4}+\frac{17\ln(cx-}{24}\right.}{c^2}$
default	$\frac{d^2a\left(\frac{1}{4}c^4x^4+\frac{2}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{2\operatorname{arctanh}(cx)c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{x^3c^3}{12}+\frac{c^2x^2}{3}+\frac{3cx}{4}+\frac{17\ln(cx-}{24}\right.}{c^2}$
parallelrisch	$\frac{3d^2b\operatorname{arctanh}(cx)x^4c^4+3a\,c^4d^2x^4+8d^2b\operatorname{arctanh}(cx)x^3c^3+8a\,c^3d^2x^3+b\,c^3d^2x^3+6x^2\operatorname{arctanh}(cx)b\,c^2d^2+6a\,c^2d^2x^2+}{12c^2}$
risch	$\frac{d^2b\,x^2\left(3c^2x^2+8cx+6\right)\ln(cx+1)}{24}-\frac{d^2c^2b\,x^4\ln(-cx+1)}{8}+\frac{d^2c^2a\,x^4}{4}-\frac{d^2cb\,x^3\ln(-cx+1)}{3}+\frac{2d^2ca\,x^3}{3}+\frac{bc\,d^2x}{12}$

input
```
int(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/4*c^2*x^4+2/3*c*x^3+1/2*x^2)+d^2*b/c^2*(1/4*arctanh(c*x)*c^4*x^4+
2/3*arctanh(c*x)*c^3*x^3+1/2*arctanh(c*x)*c^2*x^2+1/12*x^3*c^3+1/3*c^2*x^2
+3/4*c*x+17/24*ln(c*x-1)-1/24*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.06

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{6\,ac^4d^2x^4+2\,(8\,a+b)c^3d^2x^3+4\,(3\,a+2\,b)c^2d^2x^2+18\,bcd^2x-bd^2\log{(cx+1)}+17\,bd^2\log{(cx-1)}+}{24\,c^2}$$

input
```
integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/24*(6*a*c^4*d^2*x^4 + 2*(8*a + b)*c^3*d^2*x^3 + 4*(3*a + 2*b)*c^2*d^2*x^
2 + 18*b*c*d^2*x - b*d^2*log(c*x + 1) + 17*b*d^2*log(c*x - 1) + (3*b*c^4*d
^2*x^4 + 8*b*c^3*d^2*x^3 + 6*b*c^2*d^2*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.29

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^2d^2x^4}{4}+\frac{2acd^2x^3}{3}+\frac{ad^2x^2}{2}+\frac{bc^2d^2x^4\operatorname{atanh}(cx)}{4}+\frac{2bcd^2x^3\operatorname{atanh}(cx)}{3}+\frac{bcd^2x^3}{12}+\frac{bd^2x^2\operatorname{atanh}(cx)}{2}+\frac{bd^2x^2}{3}+\frac{3bd^2x}{4c}+\frac{2b}{} \\ \frac{ad^2x^2}{2}\end{cases}$$

input
```
integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**2*d**2*x**4/4 + 2*a*c*d**2*x**3/3 + a*d**2*x**2/2 + b*c**2
*d**2*x**4*atanh(c*x)/4 + 2*b*c*d**2*x**3*atanh(c*x)/3 + b*c*d**2*x**3/12
+ b*d**2*x**2*atanh(c*x)/2 + b*d**2*x**2/3 + 3*b*d**2*x/(4*c) + 2*b*d**2*l
og(x - 1/c)/(3*c**2) - b*d**2*atanh(c*x)/(12*c**2), Ne(c, 0)), (a*d**2*x**
2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.39

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{1}{4}ac^2d^2x^4+\frac{2}{3}acd^2x^3$$
$$+\frac{1}{24}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2(c^2x^3+3x)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)bc^2d^2$$
$$+\frac{1}{3}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bcd^2+\frac{1}{2}ad^2x^2$$
$$+\frac{1}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd^2$$

input
```
integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*c^2*d^2*x^4 + 2/3*a*c*d^2*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2
*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^2*d^2 + 1/
3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c*d^2 + 1/2*
a*d^2*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(
c*x - 1)/c^3))*b*d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(113) = 226$.

Time = 0.12 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.29

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx =$$

$$-\frac{1}{3}c\left(\frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3}-\frac{2\left(\frac{6\,(cx+1)^3bd^2}{(cx-1)^3}-\frac{6\,(cx+1)^2bd^2}{(cx-1)^2}+\frac{4\,(cx+1)bd^2}{cx-1}-bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^3}{(cx-1)^4}-\frac{4\,(cx+1)^3c^3}{(cx-1)^3}+\frac{6\,(cx+1)^2c^3}{(cx-1)^2}-\frac{4\,(cx+1)c^3}{cx-1}+c^3}-\frac{2\,bd^2\log\left(\right.}{c^3}$$

input
```
integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-1/3*c*(2*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 2*(6*(c*x + 1)^3*b*d^2
/(c*x - 1)^3 - 6*(c*x + 1)^2*b*d^2/(c*x - 1)^2 + 4*(c*x + 1)*b*d^2/(c*x -
1) - b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^3/(c*x - 1)^4 - 4*(c*
x + 1)^3*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*c^3/(c*x - 1)^2 - 4*(c*x + 1)*c^3
/(c*x - 1) + c^3) - 2*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^3 - (24*(c*x + 1)^
3*a*d^2/(c*x - 1)^3 - 24*(c*x + 1)^2*a*d^2/(c*x - 1)^2 + 16*(c*x + 1)*a*d^
2/(c*x - 1) - 4*a*d^2 + 10*(c*x + 1)^3*b*d^2/(c*x - 1)^3 - 23*(c*x + 1)^2*
b*d^2/(c*x - 1)^2 + 18*(c*x + 1)*b*d^2/(c*x - 1) - 5*b*d^2)/((c*x + 1)^4*c
^3/(c*x - 1)^4 - 4*(c*x + 1)^3*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*c^3/(c*x -
1)^2 - 4*(c*x + 1)*c^3/(c*x - 1) + c^3))
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^2\,(6\,a\,x^2+4\,b\,x^2+6\,b\,x^2\,\mathrm{atanh}(c\,x))}{12} - \frac{\frac{d^2\,(9\,b\,\mathrm{atanh}(c\,x)-4\,b\,\ln(c^2\,x^2-1))}{12}-\frac{3\,b\,c\,d^2\,x}{4}}{c^2} + \frac{c^2\,d^2\,(3\,a\,x^4+3\,b\,x^4\,\mathrm{atanh}(c\,x))}{12} + \frac{c\,d^2\,(8\,a\,x^3+b\,x^3+8\,b\,x^3\,\mathrm{atanh}(c\,x))}{12}$$

input
```
int(x*(a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

output
```
(d^2*(6*a*x^2 + 4*b*x^2 + 6*b*x^2*atanh(c*x)))/12 - ((d^2*(9*b*atanh(c*x)
- 4*b*log(c^2*x^2 - 1)))/12 - (3*b*c*d^2*x)/4)/c^2 + (c^2*d^2*(3*a*x^4 + 3
*b*x^4*atanh(c*x)))/12 + (c*d^2*(8*a*x^3 + b*x^3 + 8*b*x^3*atanh(c*x)))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^2(3atanh(cx)\,b\,c^4x^4+8atanh(cx)\,b\,c^3x^3+6atanh(cx)\,b\,c^2x^2-atanh(cx)\,b+8\log(c^2x-c)\,b+3a\,c^4x^4}{12c^2}$$

input
```
int(x*(c*d*x+d)^2*(a+b*atanh(c*x)),x)
```

output
```
(d**2*(3*atanh(c*x)*b*c**4*x**4 + 8*atanh(c*x)*b*c**3*x**3 + 6*atanh(c*x)*
b*c**2*x**2 - atanh(c*x)*b + 8*log(c**2*x - c)*b + 3*a*c**4*x**4 + 8*a*c**
3*x**3 + 6*a*c**2*x**2 + b*c**3*x**3 + 4*b*c**2*x**2 + 9*b*c*x))/(12*c**2)
```

3.13 $\int (d+cdx)^2(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))\,dx = \frac{2}{3}bd^2x + \frac{bd^2(1+cx)^2}{6c} + \frac{d^2(1+cx)^3(a+b\text{arctanh}(cx))}{3c} + \frac{4bd^2\log(1-cx)}{3c}$$

output
```
2/3*b*d^2*x+1/6*b*d^2*(c*x+1)^2/c+1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))/c+4
/3*b*d^2*ln(-c*x+1)/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))\,dx = \frac{d^2(6acx+6bcx+6ac^2x^2+bc^2x^2+2ac^3x^3+2bcx(3+3cx+c^2x^2)\,\text{arctanh}(cx)+6b\log(1-cx)+b\log}{6c}$$

input
```
Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*(6*a*c*x + 6*b*c*x + 6*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + 2*b*c*x*
(3 + 3*c*x + c^2*x^2)*ArcTanh[c*x] + 6*b*Log[1 - c*x] + b*Log[1 - c^2*x^2]
))/(6*c)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6478, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^2(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3c}-\frac{b\int \frac{d^3(cx+1)^3}{1-c^2x^2}dx}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3c}-\frac{1}{3}bd^2\int \frac{(cx+1)^3}{1-c^2x^2}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3c}-\frac{1}{3}bd^2\int \frac{(cx+1)^2}{1-cx}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3c}-\frac{1}{3}bd^2\int \left(-cx+\frac{4}{1-cx}-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3c}-\frac{1}{3}bd^2\left(-\frac{cx^2}{2}-\frac{4\log(1-cx)}{c}-3x\right)$$

input
```
Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x]))/(3*c) - (b*d^2*(-3*x - (c*x^2)/2 -
(4*Log[1 - c*x])/c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{d^2 a(cx+1)^3}{3}+d^2 b\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\operatorname{arctanh}(cx)c^2x^2+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{3}+\frac{c^2x^2}{6}+cx+\frac{4\ln(cx-1)}{3}\right)}{c}$
default	$\frac{\frac{d^2 a(cx+1)^3}{3}+d^2 b\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\operatorname{arctanh}(cx)c^2x^2+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{3}+\frac{c^2x^2}{6}+cx+\frac{4\ln(cx-1)}{3}\right)}{c}$
parts	$\frac{d^2 a(cx+1)^3}{3c}+\frac{d^2 b\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\operatorname{arctanh}(cx)c^2x^2+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{3}+\frac{c^2x^2}{6}+cx+\frac{4\ln(cx-1)}{3}\right)}{c}$
parallelrisch	$\frac{2d^2b\operatorname{arctanh}(cx)x^3c^3+2a c^3d^2x^3+6x^2\operatorname{arctanh}(cx)b c^2d^2+6a c^2d^2x^2+b c^2d^2x^2+6bc d^2x\operatorname{arctanh}(cx)+6d^2acx+6bc d}{6c}$
risch	$\frac{d^2(cx+1)^3b\ln(cx+1)}{6c}-\frac{d^2c^2b x^3\ln(-cx+1)}{6}+\frac{d^2c^2a x^3}{3}-\frac{d^2cb x^2\ln(-cx+1)}{2}+d^2ca x^2+\frac{d^2cb x^2}{6}-\frac{b d^2}{}$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*d^2*a*(c*x+1)^3+d^2*b*(1/3*arctanh(c*x)*c^3*x^3+arctanh(c*x)*c^2*
x^2+arctanh(c*x)*c*x+1/3*arctanh(c*x)+1/6*c^2*x^2+c*x+4/3*ln(c*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int (d+cdx)^2(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{2\,ac^3d^2x^3+(6\,a+b)c^2d^2x^2+6\,(a+b)cd^2x+bd^2\log(cx+1)+7\,bd^2\log(cx-1)+(bc^3d^2x^3+3\,bc^2d^2x}{6\,c}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*d^2*x^3 + (6*a + b)*c^2*d^2*x^2 + 6*(a + b)*c*d^2*x + b*d^2*l
og(c*x + 1) + 7*b*d^2*log(c*x - 1) + (b*c^3*d^2*x^3 + 3*b*c^2*d^2*x^2 + 3*
b*c*d^2*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(63) = 126$.

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int (d + cdx)^2(a + b\text{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^2d^2x^3}{3} + acd^2x^2 + ad^2x + \frac{bc^2d^2x^3\,\text{atanh}\,(cx)}{3} + bcd^2x^2\,\text{atanh}\,(cx) + \frac{bcd^2x^2}{6} + bd^2x\,\text{atanh}\,(cx) + bd^2x + \frac{4bd^2}{} \\ ad^2x \end{cases}$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**2*d**2*x**3/3 + a*c*d**2*x**2 + a*d**2*x + b*c**2*d**2*x**
3*atanh(c*x)/3 + b*c*d**2*x**2*atanh(c*x) + b*c*d**2*x**2/6 + b*d**2*x*ata
nh(c*x) + b*d**2*x + 4*b*d**2*log(x - 1/c)/(3*c) + b*d**2*atanh(c*x)/(3*c)
, Ne(c, 0)), (a*d**2*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(63) = 126$.

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.07

$$\int (d + cdx)^2(a + b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{3}\,ac^2d^2x^3 + \frac{1}{6}\left(2\,x^3\,\text{artanh}\,(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\,(c^2x^2-1)}{c^4}\right)\right)bc^2d^2 + acd^2x^2$$
$$+ \frac{1}{2}\left(2\,x^2\,\text{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log\,(cx+1)}{c^3} + \frac{\log\,(cx-1)}{c^3}\right)\right)bcd^2$$
$$+ ad^2x + \frac{(2\,cx\,\text{artanh}\,(cx) + \log\,(-c^2x^2+1))bd^2}{2\,c}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*c^2*d^2*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1
)/c^4))*b*c^2*d^2 + a*c*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - l
og(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctan
h(c*x) + log(-c^2*x^2 + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(63) = 126$.

Time = 0.12 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.65

$$\int (d + cdx)^2(a + b\operatorname{arctanh}(cx))\,dx =$$

$$-\frac{2}{3}\left(\frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2} - \frac{2\,bd^2\log\left(-\frac{cx+1}{cx-1}\right)}{c^2} - \frac{2\left(\frac{3\,(cx+1)^2bd^2}{(cx-1)^2} - \frac{3\,(cx+1)bd^2}{cx-1} + bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^2}{(cx-1)^3} - \frac{3\,(cx+1)^2c^2}{(cx-1)^2} + \frac{3\,(cx+1)c^2}{cx-1} - c^2} - \frac{12\,(\right.$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-2/3*(2*b*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 2*b*d^2*log(-(c*x + 1)/(
c*x - 1))/c^2 - 2*(3*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 3*(c*x + 1)*b*d^2/(c*
x - 1) + b*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3
*(c*x + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2) - (12*(c*x
 + 1)^2*a*d^2/(c*x - 1)^2 - 12*(c*x + 1)*a*d^2/(c*x - 1) + 4*a*d^2 + 4*(c*
x + 1)^2*b*d^2/(c*x - 1)^2 - 7*(c*x + 1)*b*d^2/(c*x - 1) + 3*b*d^2)/((c*x
+ 1)^3*c^2/(c*x - 1)^3 - 3*(c*x + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(
c*x - 1) - c^2))*c
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))\,dx = \frac{d^2\,(6\,a\,x+6\,b\,x+6\,b\,x\,\text{atanh}(c\,x))}{6} + \frac{c^2\,d^2\,(2\,a\,x^3+2\,b\,x^3\,\text{atanh}(c\,x))}{6} - \frac{d^2\,(6\,b\,\text{atanh}(c\,x)-4\,b\,\ln(c^2\,x^2-1))}{6\,c} + \frac{c\,d^2\,(6\,a\,x^2+b\,x^2+6\,b\,x^2\,\text{atanh}(c\,x))}{6}$$

input
```
int((a + b*atanh(c*x))*(d + c*d*x)^2,x)
```

output
```
(d^2*(6*a*x + 6*b*x + 6*b*x*atanh(c*x)))/6 + (c^2*d^2*(2*a*x^3 + 2*b*x^3*a
tanh(c*x)))/6 - (d^2*(6*b*atanh(c*x) - 4*b*log(c^2*x^2 - 1)))/(6*c) + (c*d
^2*(6*a*x^2 + b*x^2 + 6*b*x^2*atanh(c*x)))/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))\,dx = \frac{d^2(2atanh(cx)\,b\,c^3x^3+6atanh(cx)\,b\,c^2x^2+6atanh(cx)\,bcx+2atanh(cx)\,b+8\log(c^2x-c)\,b+2a\,c^3x^3+}{6c}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x)),x)
```

output
```
(d**2*(2*atanh(c*x)*b*c**3*x**3 + 6*atanh(c*x)*b*c**2*x**2 + 6*atanh(c*x)*
b*c*x + 2*atanh(c*x)*b + 8*log(c**2*x - c)*b + 2*a*c**3*x**3 + 6*a*c**2*x*
*2 + 6*a*c*x + b*c**2*x**2 + 6*b*c*x))/(6*c)
```

3.14 $\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x}\,dx = 2acd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\text{arctanh}(cx) + 2bcd^2x\text{arctanh}(cx)+\frac{1}{2}c^2d^2x^2(a+b\text{arctanh}(cx)) + ad^2\log(x)+bd^2\log\left(1-c^2x^2\right) - \frac{1}{2}bd^2\operatorname{PolyLog}(2,-cx)+\frac{1}{2}bd^2\operatorname{PolyLog}(2,cx)$$

output
```
2*a*c*d^2*x+1/2*b*c*d^2*x-1/2*b*d^2*arctanh(c*x)+2*b*c*d^2*x*arctanh(c*x)+
1/2*c^2*d^2*x^2*(a+b*arctanh(c*x))+a*d^2*ln(x)+b*d^2*ln(-c^2*x^2+1)-1/2*b*
d^2*polylog(2,-c*x)+1/2*b*d^2*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x}\,dx = \frac{1}{4}d^2\big(8acx+2bcx+2ac^2x^2+8bcx\text{arctanh}(cx) + 2bc^2x^2\text{arctanh}(cx)+4a\log(x)+b\log(1-cx) - b\log(1+cx)+4b\log\left(1-c^2x^2\right) - 2b\operatorname{PolyLog}(2,-cx)+2b\operatorname{PolyLog}(2,cx)\big)$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
(d^2*(8*a*c*x + 2*b*c*x + 2*a*c^2*x^2 + 8*b*c*x*ArcTanh[c*x] + 2*b*c^2*x^2
*ArcTanh[c*x] + 4*a*Log[x] + b*Log[1 - c*x] - b*Log[1 + c*x] + 4*b*Log[1 -
 c^2*x^2] - 2*b*PolyLog[2, -(c*x)] + 2*b*PolyLog[2, c*x]))/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))}{x}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^2d^2x(a+b\text{arctanh}(cx))+2cd^2(a+b\text{arctanh}(cx))+\frac{d^2(a+b\text{arctanh}(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^2d^2x^2(a+b\text{arctanh}(cx))+2acd^2x+ad^2\log(x)-\frac{1}{2}bd^2\text{arctanh}(cx)+2bcd^2x\text{arctanh}(cx)+bd^2\log\left(1-c^2x^2\right)-\frac{1}{2}bd^2\text{PolyLog}(2,-cx)+\frac{1}{2}bd^2\text{PolyLog}(2,cx)+\frac{1}{2}bcd^2x$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
2*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTanh[c*x])/2 + 2*b*c*d^2*x*ArcTanh
[c*x] + (c^2*d^2*x^2*(a + b*ArcTanh[c*x]))/2 + a*d^2*Log[x] + b*d^2*Log[1
- c^2*x^2] - (b*d^2*PolyLog[2, -(c*x)])/2 + (b*d^2*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result
parts	$d^2a\left(\frac{c^2x^2}{2}+2cx+\ln\left(x\right)\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+2\operatorname{arctanh}\left(cx\right)cx+\operatorname{arctanh}\left(cx\right)\ln\left(c\right.\right.$
derivativedivides	$d^2a\left(\frac{c^2x^2}{2}+2cx+\ln\left(cx\right)\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+2\operatorname{arctanh}\left(cx\right)cx+\operatorname{arctanh}\left(cx\right)\ln\left(\right.\right.$
default	$d^2a\left(\frac{c^2x^2}{2}+2cx+\ln\left(cx\right)\right)+d^2b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+2\operatorname{arctanh}\left(cx\right)cx+\operatorname{arctanh}\left(cx\right)\ln\left(\right.\right.$
risch	$\frac{a\,c^2d^2x^2}{2}+2d^2acx+d^2a\ln\left(-cx\right)-\frac{5d^2a}{2}-\frac{d^2b\ln(-cx+1)c^2x^2}{4}-d^2b\ln\left(-cx+1\right)cx+\frac{5d^2b\ldots}{}$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/2*c^2*x^2+2*c*x+ln(x))+d^2*b*(1/2*arctanh(c*x)*c^2*x^2+2*arctanh(
c*x)*c*x+arctanh(c*x)*ln(c*x)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*
ln(c*x+1)+1/2*c*x+5/4*ln(c*x-1)+3/4*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x}\,dx=\int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2
*x + b*d^2)*arctanh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x}\,dx=d^2\left(\int 2ac\,dx+\int \frac{a}{x}\,dx+\int ac^2x\,dx+\int 2bc\,\text{atanh}\,(cx)\,dx+\int \frac{b\,\text{atanh}\,(cx)}{x}\,dx+\int bc^2x\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x,x)
```

output
```
d**2*(Integral(2*a*c, x) + Integral(a/x, x) + Integral(a*c**2*x, x) + Inte
gral(2*b*c*atanh(c*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(b*c**2*
x*atanh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.52

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x}\,dx = \frac{1}{4}\,bc^2d^2x^2\log\left(cx+1\right) - \frac{1}{4}\,bc^2d^2x^2\log\left(-cx+1\right) + \frac{1}{2}\,ac^2d^2x^2 + 2\,acd^2x + \frac{1}{2}\,bcd^2x + \left(2\,cx\,\operatorname{artanh}\left(cx\right) + \log\left(-c^2x^2+1\right)\right)bd^2 - \frac{1}{2}\,(\log\left(cx\right)\log\left(-cx+1\right) + \operatorname{Li}_2(-cx+1))bd^2 + \frac{1}{2}\,(\log\left(cx+1\right)\log\left(-cx\right) + \operatorname{Li}_2(cx+1))bd^2 - \frac{1}{4}\,bd^2\log\left(cx+1\right) + \frac{1}{4}\,bd^2\log\left(cx-1\right) + ad^2\log\left(x\right)$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*b*c^2*d^2*x^2*log(c*x + 1) - 1/4*b*c^2*d^2*x^2*log(-c*x + 1) + 1/2*a*c
^2*d^2*x^2 + 2*a*c*d^2*x + 1/2*b*c*d^2*x + (2*c*x*arctanh(c*x) + log(-c^2*
x^2 + 1))*b*d^2 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^2 + 1
/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^2 - 1/4*b*d^2*log(c*x + 1
) + 1/4*b*d^2*log(c*x - 1) + a*d^2*log(x)
```

Giac [F]

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x}\,dx = \int \frac{(cdx+d)^2(b\operatorname{artanh}\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+c\,d\,x\right)^2}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x}\,dx$$
$$=\frac{d^2\left(atanh(cx)\,b\,c^2x^2+4atanh(cx)\,bcx+3atanh(cx)\,b+2\left(\int \frac{atanh(cx)}{x}dx\right)b+4\log(c^2x-c)\,b+2\log(x)\,a\right.}{2}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))/x,x)
```

output
```
(d**2*(atanh(c*x)*b*c**2*x**2 + 4*atanh(c*x)*b*c*x + 3*atanh(c*x)*b + 2*in
t(atanh(c*x)/x,x)*b + 4*log(c**2*x - c)*b + 2*log(x)*a + a*c**2*x**2 + 4*a
*c*x + b*c*x))/2
```

3.15 $\int \frac{(d+cdx)^2(a+b\,\mathbf{arctanh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx = \frac{d^2(-1+c^2x^2)\,(a+b\text{arctanh}(cx))}{x} + (2a+b)cd^2\log(x) - bcd^2\,\text{PolyLog}(2,-cx) + bcd^2\,\text{PolyLog}(2,cx)$$

output
```
d^2*(c^2*x^2-1)*(a+b*arctanh(c*x))/x+(2*a+b)*c*d^2*ln(x)-b*c*d^2*polylog(2
,-c*x)+b*c*d^2*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx = \frac{d^2(-a+ac^2x^2-b\text{arctanh}(cx)+bc^2x^2\text{arctanh}(cx)+2acx\log(x)+bcx\log(cx)-bcx\,\text{PolyLog}(2,-cx)}{x}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
(d^2*(-a + a*c^2*x^2 - b*ArcTanh[c*x] + b*c^2*x^2*ArcTanh[c*x] + 2*a*c*x*L
og[x] + b*c*x*Log[c*x] - b*c*x*PolyLog[2, -(c*x)] + b*c*x*PolyLog[2, c*x])
)/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^2d^2(a+b\text{arctanh}(cx))+\frac{d^2(a+b\text{arctanh}(cx))}{x^2}+\frac{2cd^2(a+b\text{arctanh}(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\text{arctanh}(cx))}{x}+ac^2d^2x+2acd^2\log(x)+bc^2d^2x\text{arctanh}(cx)-bcd^2\operatorname{PolyLog}(2,-cx)+bcd^2\operatorname{PolyLog}(2,cx)+bcd^2\log(x)$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
a*c^2*d^2*x + b*c^2*d^2*x*ArcTanh[c*x] - (d^2*(a + b*ArcTanh[c*x]))/x + 2*
a*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, -(c*x)] + b*c*d^2*Pol
yLog[2, c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

method	result
parts	$d^2a\left(c^2x+2c\ln\left(x\right)-\frac{1}{x}\right)+d^2bc\left(\operatorname{arctanh}\left(cx\right)cx+2\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)-\frac{\operatorname{arctanh}(cx)}{cx}\right.$
derivativedivides	$c\left(d^2a\left(cx+2\ln\left(cx\right)-\frac{1}{cx}\right)+d^2b\left(\operatorname{arctanh}\left(cx\right)cx+2\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)-\frac{\operatorname{arctanh}(cx}{cx}\right.\right.$
default	$c\left(d^2a\left(cx+2\ln\left(cx\right)-\frac{1}{cx}\right)+d^2b\left(\operatorname{arctanh}\left(cx\right)cx+2\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)-\frac{\operatorname{arctanh}(cx}{cx}\right.\right.$
risch	$xa\,c^2d^2-c\,d^2a+2c\,d^2a\ln\left(-cx\right)-\frac{d^2a}{x}-\frac{c^2d^2b\ln(-cx+1)x}{2}-bc\,d^2+c\,d^2b\operatorname{dilog}\left(-cx+1\right.$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(c^2*x+2*c*ln(x)-1/x)+d^2*b*c*(arctanh(c*x)*c*x+2*arctanh(c*x)*ln(c*
x)-arctanh(c*x)/c/x+ln(c*x)-dilog(c*x)-dilog(c*x+1)-ln(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int\frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx=\int\frac{(cdx+d)^2(b\operatorname{artanh}\left(cx\right)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2
*x + b*d^2)*arctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx = d^2\left(\int ac^2\,dx + \int \frac{a}{x^2}\,dx + \int \frac{2ac}{x}\,dx + \int bc^2\,\text{atanh}\,(cx)\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{2bc\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**2,x)
```

output
```
d**2*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(2*a*c/x, x) + I
ntegral(b*c**2*atanh(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(
2*b*c*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

output
```
a*c^2*d^2*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^2 + b*c*d
^2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 2*a*c*d^2*log(x) - 1/2
*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^2 - a*d^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(59) = 118$.

Time = 0.97 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.72

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx$$

$$=\frac{1}{6}\left(\frac{6\,ad^2}{\frac{(cx+1)c^2}{cx-1}+c^2}+\frac{5\,bd^2\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}+\frac{3\,bd^2\log\left(-\frac{cx+1}{cx-1}-1\right)}{c^2}+\left(\frac{3\,bd^2}{\frac{(cx+1)c^2}{cx-1}+c^2}-\frac{\frac{3\,(cx+1)^2bd^2}{(cx-1)^2}-1}{\frac{(cx+1)^3c^2}{(cx-1)^3}-\frac{3\,(cx+1}{(cx-1}}\right.\right.$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

output
```
1/6*(6*a*d^2/((c*x + 1)*c^2/(c*x - 1) + c^2) + 5*b*d^2*log(-(c*x + 1)/(c*x
 - 1) + 1)/c^2 + 3*b*d^2*log(-(c*x + 1)/(c*x - 1) - 1)/c^2 + (3*b*d^2/((c*
x + 1)*c^2/(c*x - 1) + c^2) - (3*(c*x + 1)^2*b*d^2/(c*x - 1)^2 - 12*(c*x +
 1)*b*d^2/(c*x - 1) + 5*b*d^2)/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c*x + 1)^
2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2))*log(-(c*x + 1)/(c*x
- 1)) - 8*b*d^2*log(-(c*x + 1)/(c*x - 1))/c^2 - 2*(3*(c*x + 1)^2*a*d^2/(c*
x - 1)^2 - 12*(c*x + 1)*a*d^2/(c*x - 1) + 5*a*d^2 - (c*x + 1)^2*b*d^2/(c*x
 - 1)^2 + (c*x + 1)*b*d^2/(c*x - 1))/((c*x + 1)^3*c^2/(c*x - 1)^3 - 3*(c*x
 + 1)^2*c^2/(c*x - 1)^2 + 3*(c*x + 1)*c^2/(c*x - 1) - c^2))*c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^2}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^2,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$=\frac{d^2\left(atanh(cx)\,b\,c^2x^2-atanh(cx)\,b+2\left(\int \frac{atanh(cx)}{x}dx\right)bcx+2\log(x)\,acx+\log(x)\,bcx+a\,c^2x^2-a\right)}{x}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))/x^2,x)
```

output
```
(d**2*(atanh(c*x)*b*c**2*x**2 - atanh(c*x)*b + 2*int(atanh(c*x)/x,x)*b*c*x
 + 2*log(x)*a*c*x + log(x)*b*c*x + a*c**2*x**2 - a))/x
```

3.16 $\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\begin{aligned}\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx = &-\frac{bcd^2}{2x}+\frac{1}{2}bc^2d^2\text{arctanh}(cx)\\ &-\frac{d^2(a+b\text{arctanh}(cx))}{2x^2}\\ &-\frac{2cd^2(a+b\text{arctanh}(cx))}{x}+ac^2d^2\log(x)\\ &+2bc^2d^2\log(x)-bc^2d^2\log\left(1-c^2x^2\right)\\ &-\frac{1}{2}bc^2d^2\operatorname{PolyLog}(2,-cx)\\ &+\frac{1}{2}bc^2d^2\operatorname{PolyLog}(2,cx)\end{aligned}$$

output
```
-1/2*b*c*d^2/x+1/2*b*c^2*d^2*arctanh(c*x)-1/2*d^2*(a+b*arctanh(c*x))/x^2-2
*c*d^2*(a+b*arctanh(c*x))/x+a*c^2*d^2*ln(x)+2*b*c^2*d^2*ln(x)-b*c^2*d^2*ln
(-c^2*x^2+1)-1/2*b*c^2*d^2*polylog(2,-c*x)+1/2*b*c^2*d^2*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$=\frac{d^2(-2a-8acx-2bcx-2b\text{arctanh}(cx)-8bcx\text{arctanh}(cx)+4ac^2x^2\log(x)+8bc^2x^2\log(cx)-bc^2x^2\log}{4}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
(d^2*(-2*a - 8*a*c*x - 2*b*c*x - 2*b*ArcTanh[c*x] - 8*b*c*x*ArcTanh[c*x] +
 4*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^
2*x^2*Log[1 + c*x] - 4*b*c^2*x^2*Log[1 - c^2*x^2] - 2*b*c^2*x^2*PolyLog[2,
 -(c*x)] + 2*b*c^2*x^2*PolyLog[2, c*x]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(\frac{c^2d^2(a+b\text{arctanh}(cx))}{x}+\frac{d^2(a+b\text{arctanh}(cx))}{x^3}+\frac{2cd^2(a+b\text{arctanh}(cx))}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{d^2(a+b\text{arctanh}(cx))}{2x^2}-\frac{2cd^2(a+b\text{arctanh}(cx))}{x}+ac^2d^2\log(x)+\frac{1}{2}bc^2d^2\text{arctanh}(cx)-\frac{1}{2}bc^2d^2\operatorname{PolyLog}(2,-cx)+\frac{1}{2}bc^2d^2\operatorname{PolyLog}(2,cx)-bc^2d^2\log\left(1-c^2x^2\right)+2bc^2d^2\log(x)-\frac{bcd^2}{2x}$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^2)/x + (b*c^2*d^2*ArcTanh[c*x])/2 - (d^2*(a + b*ArcTanh[c*x]))
/(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x]))/x + a*c^2*d^2*Log[x] + 2*b*c^2*d
^2*Log[x] - b*c^2*d^2*Log[1 - c^2*x^2] - (b*c^2*d^2*PolyLog[2, -(c*x)])/2
+ (b*c^2*d^2*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result
parts	$d^2 a\left(c^2\ln(x)-\frac{2c}{x}-\frac{1}{2x^2}\right)+d^2 b\,c^2\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln(cx)-\frac{2\operatorname{arctanh}(cx)}{cx}-\right.$
derivativedivides	$c^2\left(d^2 a\left(-\frac{1}{2c^2x^2}+\ln(cx)-\frac{2}{cx}\right)+d^2 b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln(cx)-\frac{2\operatorname{arctanh}(cx)}{cx}\right.\right.$
default	$c^2\left(d^2 a\left(-\frac{1}{2c^2x^2}+\ln(cx)-\frac{2}{cx}\right)+d^2 b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln(cx)-\frac{2\operatorname{arctanh}(cx)}{cx}\right.\right.$
risch	$-\frac{d^2a}{2x^2}+c^2d^2a\ln(-cx)-\frac{2c\,d^2a}{x}+\frac{5c^2d^2b\ln(-cx)}{4}-\frac{bc\,d^2}{2x}-\frac{5b\,c^2d^2\ln(-cx+1)}{4}+\frac{d^2b\ln(-cx+1)}{4x^2}+$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(c^2*ln(x)-2*c/x-1/2/x^2)+d^2*b*c^2*(-1/2*arctanh(c*x)/c^2/x^2+arcta
nh(c*x)*ln(c*x)-2*arctanh(c*x)/c/x-5/4*ln(c*x-1)-1/2/c/x+2*ln(c*x)-3/4*ln(
c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx=\int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2
*x + b*d^2)*arctanh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx=d^2\left(\int \frac{a}{x^3}\,dx+\int \frac{2ac}{x^2}\,dx+\int \frac{ac^2}{x}\,dx+\int \frac{b\,\text{atanh}\,(cx)}{x^3}\,dx+\int \frac{2bc\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{bc^2\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**3,x)
```

output
```
d**2*(Integral(a/x**3, x) + Integral(2*a*c/x**2, x) + Integral(a*c**2/x, x
) + Integral(b*atanh(c*x)/x**3, x) + Integral(2*b*c*atanh(c*x)/x**2, x) +
Integral(b*c**2*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*b*c^2*d^2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^2*d^2*l
og(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^2 + 1/4
*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^2 -
2*a*c*d^2/x - 1/2*a*d^2/x^2
```

Giac [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,(d+c\,d\,x)^2}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^3,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$=\frac{d^2\left(-3atanh(cx)\,b\,c^2x^2-4atanh(cx)\,bcx-atanh(cx)\,b+2\left(\int \frac{atanh(cx)}{x}dx\right)b\,c^2x^2-4\log(c^2x-c)\,b\,c^2x^2\right.}{2x^2}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))/x^3,x)
```

output
```
(d**2*( - 3*atanh(c*x)*b*c**2*x**2 - 4*atanh(c*x)*b*c*x - atanh(c*x)*b + 2
*int(atanh(c*x)/x,x)*b*c**2*x**2 - 4*log(c**2*x - c)*b*c**2*x**2 + 2*log(x
)*a*c**2*x**2 + 4*log(x)*b*c**2*x**2 - 4*a*c*x - a - b*c*x))/(2*x**2)
```

3.17 $\int \frac{(d+cdx)^2(a+b\,\mathrm{arctanh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{(d+cdx)^2(a+b\mathrm{arctanh}(cx))}{x^4}\,dx = -\frac{bcd^2}{6x^2} - \frac{bc^2d^2}{x} - \frac{d^2(1+cx)^3(a+b\mathrm{arctanh}(cx))}{3x^3} + \frac{4}{3}bc^3d^2\log(x) - \frac{4}{3}bc^3d^2\log(1-cx)$$

output
```
-1/6*b*c*d^2/x^2-b*c^2*d^2/x-1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x))/x^3+4/3*
b*c^3*d^2*ln(x)-4/3*b*c^3*d^2*ln(-c*x+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

$$\int \frac{(d+cdx)^2(a+b\mathrm{arctanh}(cx))}{x^4}\,dx = -\frac{d^2(2a+6acx+bcx+6ac^2x^2+6bc^2x^2+2b(1+3cx+3c^2x^2)\mathrm{arctanh}(cx)-8bc^3x^3\log(x)+7bc^3x^3\mathrm{lo}}{6x^3}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
-1/6*(d^2*(2*a + 6*a*c*x + b*c*x + 6*a*c^2*x^2 + 6*b*c^2*x^2 + 2*b*(1 + 3*
c*x + 3*c^2*x^2)*ArcTanh[c*x] - 8*b*c^3*x^3*Log[x] + 7*b*c^3*x^3*Log[1 - c
*x] + b*c^3*x^3*Log[1 + c*x]))/x^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d^2(cx+1)^2}{3x^3(1-cx)}dx-\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^2\int \frac{(cx+1)^2}{x^3(1-cx)}dx-\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{3}bcd^2\int \left(-\frac{4c^3}{cx-1}+\frac{4c^2}{x}+\frac{3c}{x^2}+\frac{1}{x^3}\right)dx-\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}bcd^2\left(4c^2\log(x)-4c^2\log(1-cx)-\frac{3c}{x}-\frac{1}{2x^2}\right)-\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))}{3x^3}$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x]))/x^3 + (b*c*d^2*(-1/2*1/x^2 - (
3*c)/x + 4*c^2*Log[x] - 4*c^2*Log[1 - c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

method	result
parts	$d^2 a\left(-\frac{c^2}{x}-\frac{c}{x^2}-\frac{1}{3x^3}\right)+d^2 b\,c^3\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{7\ln(cx-1)}{6}-\frac{1}{6c^2x^2}\right.$
derivativedivides	$c^3\left(d^2 a\left(-\frac{1}{3c^3x^3}-\frac{1}{c^2x^2}-\frac{1}{cx}\right)+d^2 b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{7\ln(cx-1)}{6}-\right.\right.$
default	$c^3\left(d^2 a\left(-\frac{1}{3c^3x^3}-\frac{1}{c^2x^2}-\frac{1}{cx}\right)+d^2 b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}-\frac{7\ln(cx-1)}{6}-\right.\right.$
risch	$-\frac{d^2 b(3c^2x^2+3cx+1)\ln(cx+1)}{6x^3}-\frac{d^2\left(7b\,x^3\ln(-cx+1)c^3+b\,c^3\ln(cx+1)x^3-8b\,c^3\ln(-x)x^3-3b\,c^2x^2\ln(-cx+1)+6a\right.}{6x^3}$
parallelrisch	$\frac{8b\,c^3d^2\ln(x)x^3-8\ln(cx-1)x^3b\,c^3d^2-2d^2b\operatorname{arctanh}(cx)x^3c^3-6a\,c^3d^2x^3-b\,c^3d^2x^3-6x^2\operatorname{arctanh}(cx)b\,c^2d^2-6a\,c^2d^2x^2}{6x^3}$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-c^2/x-c/x^2-1/3/x^3)+d^2*b*c^3*(-1/3*arctanh(c*x)/c^3/x^3-arctanh(
c*x)/c^2/x^2-arctanh(c*x)/c/x-7/6*ln(c*x-1)-1/6/c^2/x^2-1/c/x+4/3*ln(c*x)-
1/6*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.58

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^4}\,dx = -\frac{bc^3d^2x^3\log(cx+1)+7\,bc^3d^2x^3\log(cx-1)-8\,bc^3d^2x^3\log(x)+6\,(a+b)c^2d^2x^2+(6\,a+b)cd^2x+2}{6\,x^3}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*c^3*d^2*x^3*log(c*x + 1) + 7*b*c^3*d^2*x^3*log(c*x - 1) - 8*b*c^3*
d^2*x^3*log(x) + 6*(a + b)*c^2*d^2*x^2 + (6*a + b)*c*d^2*x + 2*a*d^2 + (3*
b*c^2*d^2*x^2 + 3*b*c*d^2*x + b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.

Time = 0.48 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.95

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^4}\,dx = \begin{cases} -\frac{ac^2d^2}{x}-\frac{acd^2}{x^2}-\frac{ad^2}{3x^3}+\frac{4bc^3d^2\log(x)}{3}-\frac{4bc^3d^2\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^3d^2\,\text{atanh}(cx)}{3}-\frac{bc^2d^2\,\text{atanh}(cx)}{x}-\frac{bc^2d^2}{x}-\frac{bcd^2\,\text{atanh}(cx)}{x^2} \\ -\frac{ad^2}{3x^3} \end{cases}$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**4,x)
```

output
```
Piecewise((-a*c**2*d**2/x - a*c*d**2/x**2 - a*d**2/(3*x**3) + 4*b*c**3*d**
2*log(x)/3 - 4*b*c**3*d**2*log(x - 1/c)/3 - b*c**3*d**2*atanh(c*x)/3 - b*c
**2*d**2*atanh(c*x)/x - b*c**2*d**2/x - b*c*d**2*atanh(c*x)/x**2 - b*c*d**
2/(6*x**2) - b*d**2*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d**2/(3*x**3), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(73) = 146$.

Time = 0.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.94

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$= -\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\left(cx\right)}{x}\right)bc^2d^2$$
$$+\frac{1}{2}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\left(cx\right)}{x^2}\right)bcd^2$$
$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}\left(cx\right)}{x^3}\right)bd^2$$
$$-\frac{ac^2d^2}{x}-\frac{acd^2}{x^2}-\frac{ad^2}{3\,x^3}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^2 + 1/2*
((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^2 -
 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*d^2 - a*c^2*d^2/x - a*c*d^2/x^2 - 1/3*a*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.07

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$= \frac{2}{3}\left(2\,bc^2d^2\log\left(-\frac{cx+1}{cx-1}-1\right) - 2\,bc^2d^2\log\left(-\frac{cx+1}{cx-1}\right) + \frac{2\left(\frac{3\,(cx+1)^2bc^2d^2}{(cx-1)^2} + \frac{3\,(cx+1)bc^2d^2}{cx-1} + bc^2d^2\right)\log\left(-\right.}{\frac{(cx+1)^3}{(cx-1)^3} + \frac{3\,(cx+1)^2}{(cx-1)^2} + \frac{3\,(cx+1)}{cx-1} + 1}\right.$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

output
```
2/3*(2*b*c^2*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^2*d^2*log(-(c*x + 1
)/(c*x - 1)) + 2*(3*(c*x + 1)^2*b*c^2*d^2/(c*x - 1)^2 + 3*(c*x + 1)*b*c^2*
d^2/(c*x - 1) + b*c^2*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3/(c*x - 1
)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1) + (12*(c*x +
1)^2*a*c^2*d^2/(c*x - 1)^2 + 12*(c*x + 1)*a*c^2*d^2/(c*x - 1) + 4*a*c^2*d^
2 + 4*(c*x + 1)^2*b*c^2*d^2/(c*x - 1)^2 + 7*(c*x + 1)*b*c^2*d^2/(c*x - 1)
+ 3*b*c^2*d^2)/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c
*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$= \frac{d^2\left(6\,b\,c^3\,\text{atanh}(c\,x) - 4\,b\,c^3\,\ln\left(c^2\,x^2-1\right) + 8\,b\,c^3\,\ln(x)\right)}{6} - \frac{\frac{d^2\,(2\,a+2\,b\,\text{atanh}(c\,x))}{6} + \frac{d^2\,x\,(6\,a\,c+b\,c+6\,b\,c\,\text{atanh}(c\,x))}{6} + \frac{d^2\,x^2\,\left(6\,a\,c^2+6\,b\,c^2+6\,b\,c^2\,\text{atanh}(c\,x)\right)}{6}}{x^3}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^4,x)
```

output
```
(d^2*(6*b*c^3*atanh(c*x) - 4*b*c^3*log(c^2*x^2 - 1) + 8*b*c^3*log(x)))/6 -
 ((d^2*(2*a + 2*b*atanh(c*x)))/6 + (d^2*x*(6*a*c + b*c + 6*b*c*atanh(c*x))
)/6 + (d^2*x^2*(6*a*c^2 + 6*b*c^2 + 6*b*c^2*atanh(c*x)))/6)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$=\frac{d^2(-2atanh(cx)\,b\,c^3x^3-6atanh(cx)\,b\,c^2x^2-6atanh(cx)\,bcx-2atanh(cx)\,b-8\log(c^2x-c)\,b\,c^3x^3+8\,\mathrm{l}}{6x^3}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))/x^4,x)
```

output
```
(d**2*( - 2*atanh(c*x)*b*c**3*x**3 - 6*atanh(c*x)*b*c**2*x**2 - 6*atanh(c*
x)*b*c*x - 2*atanh(c*x)*b - 8*log(c**2*x - c)*b*c**3*x**3 + 8*log(x)*b*c**
3*x**3 - 6*a*c**2*x**2 - 6*a*c*x - 2*a - 6*b*c**2*x**2 - b*c*x))/(6*x**3)
```

3.18 $\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 147

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx = -\frac{bcd^2}{12x^3}-\frac{bc^2d^2}{3x^2}-\frac{3bc^3d^2}{4x}-\frac{d^2(a+b\text{arctanh}(cx))}{4x^4}-\frac{2cd^2(a+b\text{arctanh}(cx))}{3x^3}-\frac{c^2d^2(a+b\text{arctanh}(cx))}{2x^2}+\frac{2}{3}bc^4d^2\log(x)-\frac{17}{24}bc^4d^2\log(1-cx)+\frac{1}{24}bc^4d^2\log(1+cx)$$

output
```
-1/12*b*c*d^2/x^3-1/3*b*c^2*d^2/x^2-3/4*b*c^3*d^2/x-1/4*d^2*(a+b*arctanh(c
*x))/x^4-2/3*c*d^2*(a+b*arctanh(c*x))/x^3-1/2*c^2*d^2*(a+b*arctanh(c*x))/x
^2+2/3*b*c^4*d^2*ln(x)-17/24*b*c^4*d^2*ln(-c*x+1)+1/24*b*c^4*d^2*ln(c*x+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx = -\frac{d^2(6a+16acx+2bcx+12ac^2x^2+8bc^2x^2+18bc^3x^3+2b(3+8cx+6c^2x^2)\,\text{arctanh}(cx)-16bc^4x^4\log}{24x^4}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/24*(d^2*(6*a + 16*a*c*x + 2*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 18*b*c
^3*x^3 + 2*b*(3 + 8*c*x + 6*c^2*x^2)*ArcTanh[c*x] - 16*b*c^4*x^4*Log[x] +
17*b*c^4*x^4*Log[1 - c*x] - b*c^4*x^4*Log[1 + c*x]))/x^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))}{x^5}\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d^2\left(6c^2x^2+8cx+3\right)}{12x^4\left(1-c^2x^2\right)}dx-\frac{c^2d^2(a+b\text{arctanh}(cx))}{2x^2}-\frac{d^2(a+b\text{arctanh}(cx))}{4x^4}-\frac{2cd^2(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bcd^2\int \frac{6c^2x^2+8cx+3}{x^4\left(1-c^2x^2\right)}dx-\frac{c^2d^2(a+b\text{arctanh}(cx))}{2x^2}-\frac{d^2(a+b\text{arctanh}(cx))}{4x^4}-\frac{2cd^2(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{12}bcd^2\int\left(-\frac{17c^4}{2(cx-1)}+\frac{c^4}{2(cx+1)}+\frac{8c^3}{x}+\frac{9c^2}{x^2}+\frac{8c}{x^3}+\frac{3}{x^4}\right)dx-\frac{c^2d^2(a+b\text{arctanh}(cx))}{2x^2}-\frac{d^2(a+b\text{arctanh}(cx))}{4x^4}-\frac{2cd^2(a+b\text{arctanh}(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2d^2(a+b\text{arctanh}(cx))}{2x^2}-\frac{d^2(a+b\text{arctanh}(cx))}{4x^4}-\frac{2cd^2(a+b\text{arctanh}(cx))}{3x^3}+\frac{1}{12}bcd^2\left(8c^3\log(x)-\frac{17}{2}c^3\log(1-cx)+\frac{1}{2}c^3\log(cx+1)-\frac{9c^2}{x}-\frac{4c}{x^2}-\frac{1}{x^3}\right)$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/4*(d^2*(a + b*ArcTanh[c*x]))/x^4 - (2*c*d^2*(a + b*ArcTanh[c*x]))/(3*x^
3) - (c^2*d^2*(a + b*ArcTanh[c*x]))/(2*x^2) + (b*c*d^2*(-x^(-3) - (4*c)/x^
2 - (9*c^2)/x + 8*c^3*Log[x] - (17*c^3*Log[1 - c*x])/2 + (c^3*Log[1 + c*x]
)/2))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

method	result
parts	$d^2 a\left(-\frac{1}{4x^4}-\frac{c^2}{2x^2}-\frac{2c}{3x^3}\right)+d^2 b\, c^4\left(-\frac{2\,\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{17\ln(cx-1)}{24}-\right.$
derivativedivides	$c^4\left(d^2 a\left(-\frac{2}{3c^3x^3}-\frac{1}{2c^2x^2}-\frac{1}{4c^4x^4}\right)+d^2 b\left(-\frac{2\,\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{17\ln(cx-}{24}\right.\right.$
default	$c^4\left(d^2 a\left(-\frac{2}{3c^3x^3}-\frac{1}{2c^2x^2}-\frac{1}{4c^4x^4}\right)+d^2 b\left(-\frac{2\,\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{17\ln(cx-}{24}\right.\right.$
risch	$-\frac{d^2 b(6c^2x^2+8cx+3)\ln(cx+1)}{24x^4}+\frac{d^2\left(16b\,c^4\ln(-x)x^4-17b\,x^4\ln(-cx+1)c^4+b\,c^4\ln(cx+1)x^4-18b\,c^3x^3+6b\,c^2x^2\ln(\right.}{24}$
parallelrisch	$\frac{8b\,c^4d^2\ln(x)x^4-8\ln(cx-1)x^4b\,c^4d^2+d^2b\,\operatorname{arctanh}(cx)x^4c^4-6a\,c^4d^2x^4-4b\,c^4d^2x^4-9b\,c^3d^2x^3-6x^2\operatorname{arctanh}(cx)b\,c^2d^2}{12x^4}$

input

```
int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output

```
d^2*a*(-1/4/x^4-1/2*c^2/x^2-2/3*c/x^3)+d^2*b*c^4*(-2/3*arctanh(c*x)/c^3/x^
3-1/2*arctanh(c*x)/c^2/x^2-1/4*arctanh(c*x)/c^4/x^4-17/24*ln(c*x-1)-1/12/c
^3/x^3-1/3/c^2/x^2-3/4/c/x+2/3*ln(c*x)+1/24*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{bc^4d^2x^4\log(cx+1)-17\,bc^4d^2x^4\log(cx-1)+16\,bc^4d^2x^4\log(x)-18\,bc^3d^2x^3-4\,(3\,a+2\,b)c^2d^2x^2-2}{24\,x^4}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(b*c^4*d^2*x^4*log(c*x + 1) - 17*b*c^4*d^2*x^4*log(c*x - 1) + 16*b*c^
4*d^2*x^4*log(x) - 18*b*c^3*d^2*x^3 - 4*(3*a + 2*b)*c^2*d^2*x^2 - 2*(8*a +
 b)*c*d^2*x - 6*a*d^2 - (6*b*c^2*d^2*x^2 + 8*b*c*d^2*x + 3*b*d^2)*log(-(c*
x + 1)/(c*x - 1)))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.29

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$= \begin{cases} -\frac{ac^2d^2}{2x^2} - \frac{2acd^2}{3x^3} - \frac{ad^2}{4x^4} + \frac{2bc^4d^2\log(x)}{3} - \frac{2bc^4d^2\log\left(x-\frac{1}{c}\right)}{3} + \frac{bc^4d^2\,\text{atanh}(cx)}{12} - \frac{3bc^3d^2}{4x} - \frac{bc^2d^2\,\text{atanh}(cx)}{2x^2} - \frac{bc^2d^2}{3x^2} - \frac{2bc}{} \\ -\frac{ad^2}{4x^4} \end{cases}$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**5,x)
```

output
```
Piecewise((-a*c**2*d**2/(2*x**2) - 2*a*c*d**2/(3*x**3) - a*d**2/(4*x**4) +
 2*b*c**4*d**2*log(x)/3 - 2*b*c**4*d**2*log(x - 1/c)/3 + b*c**4*d**2*atanh
(c*x)/12 - 3*b*c**3*d**2/(4*x) - b*c**2*d**2*atanh(c*x)/(2*x**2) - b*c**2*
d**2/(3*x**2) - 2*b*c*d**2*atanh(c*x)/(3*x**3) - b*c*d**2/(12*x**3) - b*d*
*2*atanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**2/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\left(cx\right)}{x^2}\right)bc^2d^2$$
$$-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}\left(cx\right)}{x^3}\right)bcd^2$$
$$+\frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\text{artanh}\left(cx\right)}{x^4}\right)bd^2$$
$$-\frac{ac^2d^2}{2\,x^2}-\frac{2\,acd^2}{3\,x^3}-\frac{ad^2}{4\,x^4}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2
*d^2 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*
x)/x^3)*b*c*d^2 + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^
2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^2 - 1/2*a*c^2*d^2/x^2 - 2/3*a*
c*d^2/x^3 - 1/4*a*d^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(129) = 258.

Time = 0.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.93

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{1}{3}\left(2\,bc^3d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^3d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{2\left(\frac{6\,(cx+1)^3bc^3d^2}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3d^2}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d^2}{cx-1}\right.}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+4}\right.$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

output
```
1/3*(2*b*c^3*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^3*d^2*log(-(c*x + 1
)/(c*x - 1)) + 2*(6*(c*x + 1)^3*b*c^3*d^2/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^
3*d^2/(c*x - 1)^2 + 4*(c*x + 1)*b*c^3*d^2/(c*x - 1) + b*c^3*d^2)*log(-(c*x
 + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*
(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (24*(c*x + 1)^3*a*c
^3*d^2/(c*x - 1)^3 + 24*(c*x + 1)^2*a*c^3*d^2/(c*x - 1)^2 + 16*(c*x + 1)*a
*c^3*d^2/(c*x - 1) + 4*a*c^3*d^2 + 10*(c*x + 1)^3*b*c^3*d^2/(c*x - 1)^3 +
23*(c*x + 1)^2*b*c^3*d^2/(c*x - 1)^2 + 18*(c*x + 1)*b*c^3*d^2/(c*x - 1) +
5*b*c^3*d^2)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x
 + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x^5}\,dx = \frac{2\,b\,c^4\,d^2\,\ln(x)}{3} - \frac{b\,c^4\,d^2\,\ln(c^2\,x^2-1)}{3} - \frac{a\,c^2\,d^2}{2\,x^2} - \frac{b\,c^2\,d^2}{3\,x^2} - \frac{3\,b\,c^3\,d^2}{4\,x} - \frac{a\,d^2}{4\,x^4} - \frac{2\,a\,c\,d^2}{3\,x^3} - \frac{b\,c\,d^2}{12\,x^3} - \frac{b\,d^2\,\mathrm{atanh}(c\,x)}{4\,x^4} - \frac{3\,b\,c^5\,d^2\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{4\,\sqrt{-c^2}} - \frac{2\,b\,c\,d^2\,\mathrm{atanh}(c\,x)}{3\,x^3} - \frac{b\,c^2\,d^2\,\mathrm{atanh}(c\,x)}{2\,x^2}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^5,x)
```

output
```
(2*b*c^4*d^2*log(x))/3 - (b*c^4*d^2*log(c^2*x^2 - 1))/3 - (a*c^2*d^2)/(2*x
^2) - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (a*d^2)/(4*x^4) - (2*a*c
*d^2)/(3*x^3) - (b*c*d^2)/(12*x^3) - (b*d^2*atanh(c*x))/(4*x^4) - (3*b*c^5
*d^2*atan((c^2*x)/(-c^2)^(1/2)))/(4*(-c^2)^(1/2)) - (2*b*c*d^2*atanh(c*x))
/(3*x^3) - (b*c^2*d^2*atanh(c*x))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$= \frac{d^2(atanh(cx)\,b\,c^4x^4 - 6atanh(cx)\,b\,c^2x^2 - 8atanh(cx)\,bcx - 3atanh(cx)\,b - 8\log(c^2x - c)\,b\,c^4x^4 + 8\log}{12x^4}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))/x^5,x)
```

output
```
(d**2*(atanh(c*x)*b*c**4*x**4 - 6*atanh(c*x)*b*c**2*x**2 - 8*atanh(c*x)*b*
c*x - 3*atanh(c*x)*b - 8*log(c**2*x - c)*b*c**4*x**4 + 8*log(x)*b*c**4*x**
4 - 6*a*c**2*x**2 - 8*a*c*x - 3*a - 9*b*c**3*x**3 - 4*b*c**2*x**2 - b*c*x)
)/(12*x**4)
```

3.19 $\int \frac{(d+cdx)^2(a+b\,\mathrm{arctanh}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{(d+cdx)^2(a+b\mathrm{arctanh}(cx))}{x^6}\,dx = -\frac{bcd^2}{20x^4}-\frac{bc^2d^2}{6x^3}-\frac{4bc^3d^2}{15x^2}-\frac{bc^4d^2}{2x} -\frac{d^2(a+b\mathrm{arctanh}(cx))}{5x^5}-\frac{cd^2(a+b\mathrm{arctanh}(cx))}{2x^4} -\frac{c^2d^2(a+b\mathrm{arctanh}(cx))}{3x^3}+\frac{8}{15}bc^5d^2\log(x) -\frac{31}{60}bc^5d^2\log(1-cx)-\frac{1}{60}bc^5d^2\log(1+cx)$$

output
```
-1/20*b*c*d^2/x^4-1/6*b*c^2*d^2/x^3-4/15*b*c^3*d^2/x^2-1/2*b*c^4*d^2/x-1/5
*d^2*(a+b*arctanh(c*x))/x^5-1/2*c*d^2*(a+b*arctanh(c*x))/x^4-1/3*c^2*d^2*(
a+b*arctanh(c*x))/x^3+8/15*b*c^5*d^2*ln(x)-31/60*b*c^5*d^2*ln(-c*x+1)-1/60
*b*c^5*d^2*ln(c*x+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^6}\,dx = -\frac{d^2(12a+30acx+3bcx+20ac^2x^2+10bc^2x^2+16bc^3x^3+30bc^4x^4+2b(6+15cx+10c^2x^2)\,\text{arctanh}(c}{60x^5}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

output
```
-1/60*(d^2*(12*a + 30*a*c*x + 3*b*c*x + 20*a*c^2*x^2 + 10*b*c^2*x^2 + 16*b
*c^3*x^3 + 30*b*c^4*x^4 + 2*b*(6 + 15*c*x + 10*c^2*x^2)*ArcTanh[c*x] - 32*
b*c^5*x^5*Log[x] + 31*b*c^5*x^5*Log[1 - c*x] + b*c^5*x^5*Log[1 + c*x]))/x^
5
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d^2\left(10c^2x^2+15cx+6\right)}{30x^5\left(1-c^2x^2\right)}dx-\frac{c^2d^2(a+b\text{arctanh}(cx))}{3x^3}-\frac{d^2(a+b\text{arctanh}(cx))}{5x^5}-\frac{cd^2(a+b\text{arctanh}(cx))}{2x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}bcd^2\int \frac{10c^2x^2+15cx+6}{x^5\left(1-c^2x^2\right)}dx-\frac{c^2d^2(a+b\text{arctanh}(cx))}{3x^3}-\frac{d^2(a+b\text{arctanh}(cx))}{5x^5}-\frac{cd^2(a+b\text{arctanh}(cx))}{2x^4}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{30}bcd^2\int\left(-\frac{31c^5}{2(cx-1)}-\frac{c^5}{2(cx+1)}+\frac{16c^4}{x}+\frac{15c^3}{x^2}+\frac{16c^2}{x^3}+\frac{15c}{x^4}+\frac{6}{x^5}\right)dx-\frac{c^2d^2(a+b\text{arctanh}(cx))}{3x^3}-\frac{d^2(a+b\text{arctanh}(cx))}{5x^5}-\frac{cd^2(a+b\text{arctanh}(cx))}{2x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2d^2(a+b\text{arctanh}(cx))}{3x^3}-\frac{d^2(a+b\text{arctanh}(cx))}{5x^5}-\frac{cd^2(a+b\text{arctanh}(cx))}{2x^4}+\frac{1}{30}bcd^2\left(16c^4\log(x)-\frac{31}{2}c^4\log(1-cx)-\frac{1}{2}c^4\log(cx+1)-\frac{15c^3}{x}-\frac{8c^2}{x^2}-\frac{5c}{x^3}-\frac{3}{2x^4}\right)$$

input

```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

output

```
-1/5*(d^2*(a + b*ArcTanh[c*x]))/x^5 - (c*d^2*(a + b*ArcTanh[c*x]))/(2*x^4)
 - (c^2*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) + (b*c*d^2*(-3/(2*x^4) - (5*c)/x
^3 - (8*c^2)/x^2 - (15*c^3)/x + 16*c^4*Log[x] - (31*c^4*Log[1 - c*x])/2 -
(c^4*Log[1 + c*x])/2))/30
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

method	result
parts	$d^2a\left(-\frac{c}{2x^4}-\frac{1}{5x^5}-\frac{c^2}{3x^3}\right)+d^2b\,c^5\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^4x^4}-\frac{\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{31\ln(cx-1)}{60}-\frac{}{2}\right.$
derivativedivides	$c^5\left(d^2a\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^4x^4}-\frac{1}{5c^5x^5}\right)+d^2b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^4x^4}-\frac{\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{31\ln(cx-1}{60}\right.\right.$
default	$c^5\left(d^2a\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^4x^4}-\frac{1}{5c^5x^5}\right)+d^2b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^4x^4}-\frac{\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{31\ln(cx-1}{60}\right.\right.$
risch	$-\frac{d^2b(10c^2x^2+15cx+6)\ln(cx+1)}{60x^5}+\frac{d^2(32b\,c^5\ln(-x)x^5-31b\,c^5x^5\ln(-cx+1)-b\,c^5\ln(cx+1)x^5-30b\,c^4x^4-16b\,c^3x^3}{}$
parallelrisch	$\frac{32b\,c^5d^2\ln(x)x^5-32\ln(cx-1)x^5b\,c^5d^2-2b\,c^5d^2\operatorname{arctanh}(cx)x^5-16c^5d^2x^5b-30b\,c^4d^2x^4-16b\,c^3d^2x^3-20x^2\operatorname{arctanh}(c}{60x^5}$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-1/2*c/x^4-1/5/x^5-1/3*c^2/x^3)+d^2*b*c^5*(-1/3*arctanh(c*x)/c^3/x^
3-1/2*arctanh(c*x)/c^4/x^4-1/5*arctanh(c*x)/c^5/x^5-31/60*ln(c*x-1)-1/20/c
^4/x^4-1/6/c^3/x^3-4/15/c^2/x^2-1/2/c/x+8/15*ln(c*x)-1/60*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97

$$\int\frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x^6}\,dx=$$
$$-\frac{bc^5d^2x^5\log(cx+1)+31\,bc^5d^2x^5\log(cx-1)-32\,bc^5d^2x^5\log(x)+30\,bc^4d^2x^4+16\,bc^3d^2x^3+10\,(2\,a}{60\,x^5}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(b*c^5*d^2*x^5*log(c*x + 1) + 31*b*c^5*d^2*x^5*log(c*x - 1) - 32*b*c
^5*d^2*x^5*log(x) + 30*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a + b)*c^2
*d^2*x^2 + 3*(10*a + b)*c*d^2*x + 12*a*d^2 + (10*b*c^2*d^2*x^2 + 15*b*c*d^
2*x + 6*b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \begin{cases} -\frac{ac^2d^2}{3x^3} - \frac{acd^2}{2x^4} - \frac{ad^2}{5x^5} + \frac{8bc^5d^2\log(x)}{15} - \frac{8bc^5d^2\log\left(x-\frac{1}{c}\right)}{15} - \frac{bc^5d^2\,\text{atanh}\,(cx)}{30} - \frac{bc^4d^2}{2x} - \frac{4bc^3d^2}{15x^2} - \frac{bc^2d^2\,\text{atanh}\,(cx)}{3x^3} - \frac{bc^2d}{6x^3} \\ -\frac{ad^2}{5x^5} \end{cases}$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**6,x)
```

output
```
Piecewise((-a*c**2*d**2/(3*x**3) - a*c*d**2/(2*x**4) - a*d**2/(5*x**5) + 8
*b*c**5*d**2*log(x)/15 - 8*b*c**5*d**2*log(x - 1/c)/15 - b*c**5*d**2*atanh
(c*x)/30 - b*c**4*d**2/(2*x) - 4*b*c**3*d**2/(15*x**2) - b*c**2*d**2*atanh
(c*x)/(3*x**3) - b*c**2*d**2/(6*x**3) - b*c*d**2*atanh(c*x)/(2*x**4) - b*c
*d**2/(20*x**4) - b*d**2*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**2/(5*x**5)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}\,(cx)}{x^3}\right)bc^2d^2 + \frac{1}{12}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\,(3\,c^2x^2+1)}{x^3}\right)c-\frac{6\,\text{artanh}\,(cx)}{x^4}\right)bcd^2 - \frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\text{artanh}\,(cx)}{x^5}\right)bd^2 - \frac{ac^2d^2}{3\,x^3}-\frac{acd^2}{2\,x^4}-\frac{ad^2}{5\,x^5}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*c^2*d^2 + 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^
2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 - 1
) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^2 -
1/3*a*c^2*d^2/x^3 - 1/2*a*c*d^2/x^4 - 1/5*a*d^2/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(141) = 282.

Time = 0.13 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.30

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{4}{15}\left(2\,bc^4d^2\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,bc^4d^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{15\,(cx+1)^4bc^4d^2}{(cx-1)^4}+\frac{15\,(cx+1)^3bc^4d^2}{(cx-1)^3}+\frac{20\,(cx+1)^2bc}{(cx-1)^2}\right.}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}}\right.$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

output
```
4/15*(2*b*c^4*d^2*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b*c^4*d^2*log(-(c*x +
1)/(c*x - 1)) + (15*(c*x + 1)^4*b*c^4*d^2/(c*x - 1)^4 + 15*(c*x + 1)^3*b*c
^4*d^2/(c*x - 1)^3 + 20*(c*x + 1)^2*b*c^4*d^2/(c*x - 1)^2 + 10*(c*x + 1)*b
*c^4*d^2/(c*x - 1) + 2*b*c^4*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(
c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(
c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (30*(c*x + 1)^4*a*c^
4*d^2/(c*x - 1)^4 + 30*(c*x + 1)^3*a*c^4*d^2/(c*x - 1)^3 + 40*(c*x + 1)^2*
a*c^4*d^2/(c*x - 1)^2 + 20*(c*x + 1)*a*c^4*d^2/(c*x - 1) + 4*a*c^4*d^2 + 1
3*(c*x + 1)^4*b*c^4*d^2/(c*x - 1)^4 + 36*(c*x + 1)^3*b*c^4*d^2/(c*x - 1)^3
 + 41*(c*x + 1)^2*b*c^4*d^2/(c*x - 1)^2 + 23*(c*x + 1)*b*c^4*d^2/(c*x - 1)
 + 5*b*c^4*d^2)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*
(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x -
1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.13

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))}{x^6}\,dx = -\frac{12\,a\,d^2+12\,b\,d^2\operatorname{atanh}(c\,x)+20\,a\,c^2\,d^2\,x^2+10\,b\,c^2\,d^2\,x^2+16\,b\,c^3\,d^2\,x^3+30\,b\,c^4\,d^2\,x^4+30\,a\,c\,d^2\,x+3}{}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^2)/x^6,x)
```

output
```
-(12*a*d^2 + 12*b*d^2*atanh(c*x) + 20*a*c^2*d^2*x^2 + 10*b*c^2*d^2*x^2 + 1
6*b*c^3*d^2*x^3 + 30*b*c^4*d^2*x^4 + 30*a*c*d^2*x + 3*b*c*d^2*x - 32*b*c^5
*d^2*x^5*log(x) + 20*b*c^2*d^2*x^2*atanh(c*x) + 16*b*c^5*d^2*x^5*log(c^2*x
^2 - 1) + 30*b*c*d^2*x*atanh(c*x) - 30*b*c^4*d^2*x^5*atan((c^2*x)/(-c^2)^(
1/2))*(-c^2)^(1/2))/(60*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{d^2(-2atanh(cx)\,b\,c^5x^5-20atanh(cx)\,b\,c^2x^2-30atanh(cx)\,bcx-12atanh(cx)\,b-32\log(c^2x-c)\,b\,c^5x^5}{60x^5}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))/x^6,x)
```

output
```
(d**2*( - 2*atanh(c*x)*b*c**5*x**5 - 20*atanh(c*x)*b*c**2*x**2 - 30*atanh(
c*x)*b*c*x - 12*atanh(c*x)*b - 32*log(c**2*x - c)*b*c**5*x**5 + 32*log(x)*
b*c**5*x**5 - 20*a*c**2*x**2 - 30*a*c*x - 12*a - 30*b*c**4*x**4 - 16*b*c**
3*x**3 - 10*b*c**2*x**2 - 3*b*c*x))/(60*x**5)
```

3.20 $\int x^3(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 192

$$\begin{aligned}\int x^3(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))\,dx &= \frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5 \\ &+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x^4(a+b\,\mathrm{arctanh}(cx)) \\ &+\frac{3}{5}cd^3x^5(a+b\,\mathrm{arctanh}(cx)) \\ &+\frac{1}{2}c^2d^3x^6(a+b\,\mathrm{arctanh}(cx)) \\ &+\frac{1}{7}c^3d^3x^7(a+b\,\mathrm{arctanh}(cx)) \\ &+\frac{209bd^3\log(1-cx)}{280c^4}-\frac{bd^3\log(1+cx)}{280c^4}\end{aligned}$$

output
```
3/4*b*d^3*x/c^3+13/35*b*d^3*x^2/c^2+1/4*b*d^3*x^3/c+13/70*b*d^3*x^4+1/10*b
*c*d^3*x^5+1/42*b*c^2*d^3*x^6+1/4*d^3*x^4*(a+b*arctanh(c*x))+3/5*c*d^3*x^5
*(a+b*arctanh(c*x))+1/2*c^2*d^3*x^6*(a+b*arctanh(c*x))+1/7*c^3*d^3*x^7*(a+
b*arctanh(c*x))+209/280*b*d^3*ln(-c*x+1)/c^4-1/280*b*d^3*ln(c*x+1)/c^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{d^3(630bcx+312bc^2x^2+210bc^3x^3+210ac^4x^4+156bc^4x^4+504ac^5x^5+84bc^5x^5+420ac^6x^6+20bc^6x^6+}{840c}$$

input
```
Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*(630*b*c*x + 312*b*c^2*x^2 + 210*b*c^3*x^3 + 210*a*c^4*x^4 + 156*b*c^
4*x^4 + 504*a*c^5*x^5 + 84*b*c^5*x^5 + 420*a*c^6*x^6 + 20*b*c^6*x^6 + 120*
a*c^7*x^7 + 6*b*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*
x] + 627*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(840*c^4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)^3(a+b\text{arctanh}(cx))\,dx$$
$$\Big\downarrow \text{6498}$$
$$-bc\int \frac{d^3x^4(20c^3x^3+70c^2x^2+84cx+35)}{140(1-c^2x^2)}dx+\frac{1}{7}c^3d^3x^7(a+b\text{arctanh}(cx))+\frac{1}{2}c^2d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}cd^3x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^3x^4(a+b\text{arctanh}(cx))$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{140}bcd^3\int \frac{x^4(20c^3x^3+70c^2x^2+84cx+35)}{1-c^2x^2}dx+\frac{1}{7}c^3d^3x^7(a+b\text{arctanh}(cx))+\frac{1}{2}c^2d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}cd^3x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^3x^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{140}bcd^3\int\left(-20cx^5-70x^4-\frac{104x^3}{c}-\frac{105x^2}{c^2}-\frac{104x}{c^3}+\frac{104cx+105}{c^4(1-c^2x^2)}-\frac{105}{c^4}\right)dx+\frac{1}{7}c^3d^3x^7(a+b\text{arctanh}(cx))+\frac{1}{2}c^2d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}cd^3x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^3x^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}c^3d^3x^7(a+b\text{arctanh}(cx))+\frac{1}{2}c^2d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}cd^3x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^3x^4(a+b\text{arctanh}(cx))-\frac{1}{140}bcd^3\left(\frac{105\text{arctanh}(cx)}{c^5}-\frac{105x}{c^4}-\frac{52x^2}{c^3}-\frac{35x^3}{c^2}-\frac{52\log(1-c^2x^2)}{c^5}-\frac{10cx^6}{3}-\frac{26x^4}{c}-14x^5\right)$$

input
```
Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*x^4*(a + b*ArcTanh[c*x]))/4 + (3*c*d^3*x^5*(a + b*ArcTanh[c*x]))/5 +
(c^2*d^3*x^6*(a + b*ArcTanh[c*x]))/2 + (c^3*d^3*x^7*(a + b*ArcTanh[c*x]))/
7 - (b*c*d^3*((-105*x)/c^4 - (52*x^2)/c^3 - (35*x^3)/c^2 - (26*x^4)/c - 14
*x^5 - (10*c*x^6)/3 + (105*ArcTanh[c*x])/c^5 - (52*Log[1 - c^2*x^2])/c^5))
/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.79

method	result
parts	$d^3 a\left(\frac{1}{7}c^3x^7+\frac{1}{2}c^2x^6+\frac{3}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{d^3 b\left(\frac{\operatorname{arctanh}(cx)c^7x^7}{7}+\frac{\operatorname{arctanh}(cx)c^6x^6}{2}+\frac{3\operatorname{arctanh}(cx)c^5x^5}{5}+\operatorname{arctanh}\right.}{}$
derivativedivides	$\frac{d^3 a\left(\frac{1}{7}c^7x^7+\frac{1}{2}c^6x^6+\frac{3}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^3 b\left(\frac{\operatorname{arctanh}(cx)c^7x^7}{7}+\frac{\operatorname{arctanh}(cx)c^6x^6}{2}+\frac{3\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^6}{4}\right.}{c^4}$
default	$\frac{d^3 a\left(\frac{1}{7}c^7x^7+\frac{1}{2}c^6x^6+\frac{3}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^3 b\left(\frac{\operatorname{arctanh}(cx)c^7x^7}{7}+\frac{\operatorname{arctanh}(cx)c^6x^6}{2}+\frac{3\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{c^6}{4}\right.}{c^4}$
parallelrisch	$60b\,c^7d^3\operatorname{arctanh}(cx)x^7+60c^7d^3x^7a+210b\,c^6d^3\operatorname{arctanh}(cx)x^6+210a\,c^6d^3x^6+10c^6d^3x^6b+252b\,c^5d^3\operatorname{arctanh}(cx)x^5+2$
risch	$\frac{d^3 b\,x^4\left(20x^3c^3+70c^2x^2+84cx+35\right)\ln(cx+1)}{280}-\frac{d^3c^3b\,x^7\ln(-cx+1)}{14}+\frac{d^3c^3a\,x^7}{7}-\frac{d^3c^2b\,x^6\ln(-cx+1)}{4}+\frac{d^3c^2a}{2}$

input
```
int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/7*c^3*x^7+1/2*c^2*x^6+3/5*c*x^5+1/4*x^4)+d^3*b/c^4*(1/7*arctanh(c
*x)*c^7*x^7+1/2*arctanh(c*x)*c^6*x^6+3/5*arctanh(c*x)*c^5*x^5+1/4*arctanh(
c*x)*c^4*x^4+1/42*c^6*x^6+1/10*c^5*x^5+13/70*c^4*x^4+1/4*x^3*c^3+13/35*c^2
*x^2+3/4*c*x+209/280*ln(c*x-1)-1/280*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.99

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{120\,ac^7d^3x^7 + 20\,(21\,a+b)c^6d^3x^6 + 84\,(6\,a+b)c^5d^3x^5 + 6\,(35\,a+26\,b)c^4d^3x^4 + 210\,bc^3d^3x^3 + 312\,bc^2d^3}{}$$

input
```
integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/840*(120*a*c^7*d^3*x^7 + 20*(21*a + b)*c^6*d^3*x^6 + 84*(6*a + b)*c^5*d^
3*x^5 + 6*(35*a + 26*b)*c^4*d^3*x^4 + 210*b*c^3*d^3*x^3 + 312*b*c^2*d^3*x^
2 + 630*b*c*d^3*x - 3*b*d^3*log(c*x + 1) + 627*b*d^3*log(c*x - 1) + 3*(20*
b*c^7*d^3*x^7 + 70*b*c^6*d^3*x^6 + 84*b*c^5*d^3*x^5 + 35*b*c^4*d^3*x^4)*lo
g(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.27

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^3d^3x^7}{7} + \frac{ac^2d^3x^6}{2} + \frac{3acd^3x^5}{5} + \frac{ad^3x^4}{4} + \frac{bc^3d^3x^7\,\text{atanh}\,(cx)}{7} + \frac{bc^2d^3x^6\,\text{atanh}\,(cx)}{2} + \frac{bc^2d^3x^6}{42} + \frac{3bcd^3x^5\,\text{atanh}\,(cx)}{5} + \frac{bcd^3x^5}{10} \\ \frac{ad^3x^4}{4} \end{cases}$$

input
```
integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**3*d**3*x**7/7 + a*c**2*d**3*x**6/2 + 3*a*c*d**3*x**5/5 + a
*d**3*x**4/4 + b*c**3*d**3*x**7*atanh(c*x)/7 + b*c**2*d**3*x**6*atanh(c*x)
/2 + b*c**2*d**3*x**6/42 + 3*b*c*d**3*x**5*atanh(c*x)/5 + b*c*d**3*x**5/10
 + b*d**3*x**4*atanh(c*x)/4 + 13*b*d**3*x**4/70 + b*d**3*x**3/(4*c) + 13*b
*d**3*x**2/(35*c**2) + 3*b*d**3*x/(4*c**3) + 26*b*d**3*log(x - 1/c)/(35*c*
*4) - b*d**3*atanh(c*x)/(140*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.48

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{1}{7}\,ac^3d^3x^7 + \frac{1}{2}\,ac^2d^3x^6 + \frac{3}{5}\,acd^3x^5 + \frac{1}{84}\left(12\,x^7\,\text{artanh}\,(cx) + c\left(\frac{2\,c^4x^6 + 3\,c^2x^4 + 6\,x^2}{c^6} + \frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)bc^3d^3 + \frac{1}{4}\,ad^3x^4 + \frac{1}{60}\left(30\,x^6\,\text{artanh}\,(cx) + c\left(\frac{2\left(3\,c^4x^5 + 5\,c^2x^3 + 15\,x\right)}{c^6} - \frac{15\,\log\left(cx+1\right)}{c^7} + \frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)bc^2d^3 + \frac{3}{20}\left(4\,x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bcd^3 + \frac{1}{24}\left(6\,x^4\,\text{artanh}\,(cx) + c\left(\frac{2\left(c^2x^3 + 3\,x\right)}{c^4} - \frac{3\,\log\left(cx+1\right)}{c^5} + \frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bd^3$$

input
```
integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^3*d^3*x^7 + 1/2*a*c^2*d^3*x^6 + 3/5*a*c*d^3*x^5 + 1/84*(12*x^7*arc
tanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^
8))*b*c^3*d^3 + 1/4*a*d^3*x^4 + 1/60*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^
5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*
c^2*d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*
x^2 - 1)/c^6))*b*c*d^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c
^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(168) = 336.

Time = 0.13 (sec) , antiderivative size = 722, normalized size of antiderivative = 3.76

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/105*c*(6*(140*(c*x + 1)^6*b*d^3/(c*x - 1)^6 - 210*(c*x + 1)^5*b*d^3/(c*x
 - 1)^5 + 490*(c*x + 1)^4*b*d^3/(c*x - 1)^4 - 455*(c*x + 1)^3*b*d^3/(c*x -
 1)^3 + 273*(c*x + 1)^2*b*d^3/(c*x - 1)^2 - 91*(c*x + 1)*b*d^3/(c*x - 1) +
 13*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^7*c^5/(c*x - 1)^7 - 7*(c*x
 + 1)^6*c^5/(c*x - 1)^6 + 21*(c*x + 1)^5*c^5/(c*x - 1)^5 - 35*(c*x + 1)^4*
c^5/(c*x - 1)^4 + 35*(c*x + 1)^3*c^5/(c*x - 1)^3 - 21*(c*x + 1)^2*c^5/(c*x
 - 1)^2 + 7*(c*x + 1)*c^5/(c*x - 1) - c^5) + (1680*(c*x + 1)^6*a*d^3/(c*x
- 1)^6 - 2520*(c*x + 1)^5*a*d^3/(c*x - 1)^5 + 5880*(c*x + 1)^4*a*d^3/(c*x
- 1)^4 - 5460*(c*x + 1)^3*a*d^3/(c*x - 1)^3 + 3276*(c*x + 1)^2*a*d^3/(c*x
- 1)^2 - 1092*(c*x + 1)*a*d^3/(c*x - 1) + 156*a*d^3 + 762*(c*x + 1)^6*b*d^
3/(c*x - 1)^6 - 3063*(c*x + 1)^5*b*d^3/(c*x - 1)^5 + 5959*(c*x + 1)^4*b*d^
3/(c*x - 1)^4 - 6694*(c*x + 1)^3*b*d^3/(c*x - 1)^3 + 4344*(c*x + 1)^2*b*d^
3/(c*x - 1)^2 - 1539*(c*x + 1)*b*d^3/(c*x - 1) + 231*b*d^3)/((c*x + 1)^7*c
^5/(c*x - 1)^7 - 7*(c*x + 1)^6*c^5/(c*x - 1)^6 + 21*(c*x + 1)^5*c^5/(c*x -
 1)^5 - 35*(c*x + 1)^4*c^5/(c*x - 1)^4 + 35*(c*x + 1)^3*c^5/(c*x - 1)^3 -
21*(c*x + 1)^2*c^5/(c*x - 1)^2 + 7*(c*x + 1)*c^5/(c*x - 1) - c^5) - 78*b*d
^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 78*b*d^3*log(-(c*x + 1)/(c*x - 1))/
c^5)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.92

$$\int x^3(d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx$$
$$= \frac{\frac{13\,b\,c^2\,d^3\,x^2}{35}-\frac{d^3\,\left(315\,b\,\mathrm{atanh}(c\,x)-156\,b\,\ln\left(c^2\,x^2-1\right)\right)}{420}+\frac{b\,c^3\,d^3\,x^3}{4}+\frac{3\,b\,c\,d^3\,x}{4}}{c^4} + \frac{d^3\,\left(105\,a\,x^4+78\,b\,x^4+105\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{420} + \frac{c^3\,d^3\,\left(60\,a\,x^7+60\,b\,x^7\,\mathrm{atanh}(c\,x)\right)}{420} + \frac{c\,d^3\,\left(252\,a\,x^5+42\,b\,x^5+252\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{420} + \frac{c^2\,d^3\,\left(210\,a\,x^6+10\,b\,x^6+210\,b\,x^6\,\mathrm{atanh}(c\,x)\right)}{420}$$

input
```
int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

output
```
((13*b*c^2*d^3*x^2)/35 - (d^3*(315*b*atanh(c*x) - 156*b*log(c^2*x^2 - 1)))
/420 + (b*c^3*d^3*x^3)/4 + (3*b*c*d^3*x)/4)/c^4 + (d^3*(105*a*x^4 + 78*b*x
^4 + 105*b*x^4*atanh(c*x)))/420 + (c^3*d^3*(60*a*x^7 + 60*b*x^7*atanh(c*x)
))/420 + (c*d^3*(252*a*x^5 + 42*b*x^5 + 252*b*x^5*atanh(c*x)))/420 + (c^2*
d^3*(210*a*x^6 + 10*b*x^6 + 210*b*x^6*atanh(c*x)))/420
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{d^3(60atanh(cx)\,b\,c^7x^7+210atanh(cx)\,b\,c^6x^6+252atanh(cx)\,b\,c^5x^5+105atanh(cx)\,b\,c^4x^4-3atanh(cx)}{}$$

input
```
int(x^3*(c*d*x+d)^3*(a+b*atanh(c*x)),x)
```

output
```
(d**3*(60*atanh(c*x)*b*c**7*x**7 + 210*atanh(c*x)*b*c**6*x**6 + 252*atanh(
c*x)*b*c**5*x**5 + 105*atanh(c*x)*b*c**4*x**4 - 3*atanh(c*x)*b + 312*log(c
**2*x - c)*b + 60*a*c**7*x**7 + 210*a*c**6*x**6 + 252*a*c**5*x**5 + 105*a*
c**4*x**4 + 10*b*c**6*x**6 + 42*b*c**5*x**5 + 78*b*c**4*x**4 + 105*b*c**3*
x**3 + 156*b*c**2*x**2 + 315*b*c*x))/(420*c**4)
```

3.21 $\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 178

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))+\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))+\frac{37bd^3\log(1-cx)}{40c^3}+\frac{bd^3\log(1+cx)}{120c^3}$$

output
```
11/12*b*d^3*x/c^2+7/15*b*d^3*x^2/c+11/36*b*d^3*x^3+3/20*b*c*d^3*x^4+1/30*b
*c^2*d^3*x^5+1/3*d^3*x^3*(a+b*arctanh(c*x))+3/4*c*d^3*x^4*(a+b*arctanh(c*x
))+3/5*c^2*d^3*x^5*(a+b*arctanh(c*x))+1/6*c^3*d^3*x^6*(a+b*arctanh(c*x))+3
7/40*b*d^3*ln(-c*x+1)/c^3+1/120*b*d^3*ln(c*x+1)/c^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^3(330bcx+168bc^2x^2+120ac^3x^3+110bc^3x^3+270ac^4x^4+54bc^4x^4+216ac^5x^5+12bc^5x^5+60ac^6x^6+}{360c^3}$$

input
```
Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*(330*b*c*x + 168*b*c^2*x^2 + 120*a*c^3*x^3 + 110*b*c^3*x^3 + 270*a*c^
4*x^4 + 54*b*c^4*x^4 + 216*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c
^3*x^3*(20 + 45*c*x + 36*c^2*x^2 + 10*c^3*x^3)*ArcTanh[c*x] + 333*b*Log[1
- c*x] + 3*b*Log[1 + c*x]))/(360*c^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)^3(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{d^3x^3(10c^3x^3+36c^2x^2+45cx+20)}{60(1-c^2x^2)}dx+\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{60}bcd^3\int \frac{x^3(10c^3x^3+36c^2x^2+45cx+20)}{1-c^2x^2}dx+\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))$$

$\Big\downarrow$ 2333

$$-\frac{1}{60}bcd^3\int\left(-10cx^4-36x^3-\frac{55x^2}{c}-\frac{56x}{c^2}+\frac{56cx+55}{c^3\left(1-c^2x^2\right)}-\frac{55}{c^3}\right)dx+\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))$$

$\Big\downarrow$ 2009

$$\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))-\frac{1}{60}bcd^3\left(\frac{55\text{arctanh}(cx)}{c^4}-\frac{55x}{c^3}-\frac{28x^2}{c^2}-\frac{28\log\left(1-c^2x^2\right)}{c^4}-2cx^5-\frac{55x^3}{3c}-9x^4\right)$$

input
```
Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*x^3*(a + b*ArcTanh[c*x]))/3 + (3*c*d^3*x^4*(a + b*ArcTanh[c*x]))/4 +
(3*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (c^3*d^3*x^6*(a + b*ArcTanh[c*x])
)/6 - (b*c*d^3*((-55*x)/c^3 - (28*x^2)/c^2 - (55*x^3)/(3*c) - 9*x^4 - 2*c*
x^5 + (55*ArcTanh[c*x])/c^4 - (28*Log[1 - c^2*x^2])/c^4))/60
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

method	result
parts	$d^3 a\left(\frac{1}{6}c^3x^6+\frac{3}{5}c^2x^5+\frac{3}{4}c\,x^4+\frac{1}{3}x^3\right)+\frac{d^3 b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{3\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arcta}\right.}{}$
derivativedivides	$\frac{d^3 a\left(\frac{1}{6}c^6x^6+\frac{3}{5}c^5x^5+\frac{3}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+d^3 b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{3\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\right.}{c^3}$
default	$\frac{d^3 a\left(\frac{1}{6}c^6x^6+\frac{3}{5}c^5x^5+\frac{3}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+d^3 b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{3\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^3x^3}{3}+\right.}{c^3}$
parallelrisch	$\frac{30b\,c^6d^3\operatorname{arctanh}(cx)x^6+30a\,c^6d^3x^6+108b\,c^5d^3\operatorname{arctanh}(cx)x^5+108a\,c^5d^3x^5+6b\,c^5d^3x^5+135d^3b\operatorname{arctanh}(cx)x^4c^4+1}{}$
risch	$\frac{d^3 b\,x^3\left(10x^3c^3+36c^2x^2+45cx+20\right)\ln(cx+1)}{120}-\frac{d^3c^3b\,x^6\ln(-cx+1)}{12}+\frac{d^3c^3a\,x^6}{6}-\frac{3d^3c^2b\,x^5\ln(-cx+1)}{10}+\frac{3d^3c}{}$

input
```
int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/6*c^3*x^6+3/5*c^2*x^5+3/4*c*x^4+1/3*x^3)+d^3*b/c^3*(1/6*arctanh(c
*x)*c^6*x^6+3/5*arctanh(c*x)*c^5*x^5+3/4*arctanh(c*x)*c^4*x^4+1/3*arctanh(
c*x)*c^3*x^3+1/30*c^5*x^5+3/20*c^4*x^4+11/36*x^3*c^3+7/15*c^2*x^2+11/12*c*
x+37/40*ln(c*x-1)+1/120*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00

$$\int x^2(d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx$$
$$= \frac{60\,ac^6d^3x^6 + 12\,(18\,a+b)c^5d^3x^5 + 54\,(5\,a+b)c^4d^3x^4 + 10\,(12\,a+11\,b)c^3d^3x^3 + 168\,bc^2d^3x^2 + 330\,bcd^3x}{}$$

input
```
integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/360*(60*a*c^6*d^3*x^6 + 12*(18*a + b)*c^5*d^3*x^5 + 54*(5*a + b)*c^4*d^3
*x^4 + 10*(12*a + 11*b)*c^3*d^3*x^3 + 168*b*c^2*d^3*x^2 + 330*b*c*d^3*x +
3*b*d^3*log(c*x + 1) + 333*b*d^3*log(c*x - 1) + 3*(10*b*c^6*d^3*x^6 + 36*b
*c^5*d^3*x^5 + 45*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3)*log(-(c*x + 1)/(c*x -
1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.32

$$\int x^2(d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^3d^3x^6}{6} + \frac{3ac^2d^3x^5}{5} + \frac{3acd^3x^4}{4} + \frac{ad^3x^3}{3} + \frac{bc^3d^3x^6\operatorname{atanh}(cx)}{6} + \frac{3bc^2d^3x^5\operatorname{atanh}(cx)}{5} + \frac{bc^2d^3x^5}{30} + \frac{3bcd^3x^4\operatorname{atanh}(cx)}{4} + \frac{3bcd}{2} \\ \frac{ad^3x^3}{3} \end{cases}$$

input
```
integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**3*d**3*x**6/6 + 3*a*c**2*d**3*x**5/5 + 3*a*c*d**3*x**4/4 +
 a*d**3*x**3/3 + b*c**3*d**3*x**6*atanh(c*x)/6 + 3*b*c**2*d**3*x**5*atanh(
c*x)/5 + b*c**2*d**3*x**5/30 + 3*b*c*d**3*x**4*atanh(c*x)/4 + 3*b*c*d**3*x
**4/20 + b*d**3*x**3*atanh(c*x)/3 + 11*b*d**3*x**3/36 + 7*b*d**3*x**2/(15*
c) + 11*b*d**3*x/(12*c**2) + 14*b*d**3*log(x - 1/c)/(15*c**3) + b*d**3*ata
nh(c*x)/(60*c**3), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.49

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{1}{6}\,ac^3d^3x^6 + \frac{3}{5}\,ac^2d^3x^5 + \frac{3}{4}\,acd^3x^4 + \frac{1}{180}\left(30\,x^6\,\text{artanh}\,(cx) + c\left(\frac{2\,(3\,c^4x^5 + 5\,c^2x^3 + 15\,x)}{c^6} - \frac{15\,\log\,(cx+1)}{c^7} + \frac{15\,\log\,(cx-1)}{c^7}\right)\right)bc^3d^3 + \frac{3}{20}\left(4\,x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\,(c^2x^2-1)}{c^6}\right)\right)bc^2d^3 + \frac{1}{3}\,ad^3x^3 + \frac{1}{8}\left(6\,x^4\,\text{artanh}\,(cx) + c\left(\frac{2\,(c^2x^3 + 3\,x)}{c^4} - \frac{3\,\log\,(cx+1)}{c^5} + \frac{3\,\log\,(cx-1)}{c^5}\right)\right)bcd^3 + \frac{1}{6}\left(2\,x^3\,\text{artanh}\,(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\,(c^2x^2-1)}{c^4}\right)\right)bd^3$$

input
```
integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^3*d^3*x^6 + 3/5*a*c^2*d^3*x^5 + 3/4*a*c*d^3*x^4 + 1/180*(30*x^6*ar
ctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7
 + 15*log(c*x - 1)/c^7))*b*c^3*d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^
4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/8*
(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*lo
g(c*x - 1)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*
x^2 - 1)/c^4))*b*d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(156) = 312$.

Time = 0.14 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.49

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-1/45*c*(42*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 - 6*(60*(c*x + 1)^5*b*
d^3/(c*x - 1)^5 - 90*(c*x + 1)^4*b*d^3/(c*x - 1)^4 + 140*(c*x + 1)^3*b*d^3
/(c*x - 1)^3 - 105*(c*x + 1)^2*b*d^3/(c*x - 1)^2 + 42*(c*x + 1)*b*d^3/(c*x
 - 1) - 7*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^4/(c*x - 1)^6 -
6*(c*x + 1)^5*c^4/(c*x - 1)^5 + 15*(c*x + 1)^4*c^4/(c*x - 1)^4 - 20*(c*x +
 1)^3*c^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^4/(c*x - 1)^2 - 6*(c*x + 1)*c^4/(
c*x - 1) + c^4) - 42*b*d^3*log(-(c*x + 1)/(c*x - 1))/c^4 - (720*(c*x + 1)^
5*a*d^3/(c*x - 1)^5 - 1080*(c*x + 1)^4*a*d^3/(c*x - 1)^4 + 1680*(c*x + 1)^
3*a*d^3/(c*x - 1)^3 - 1260*(c*x + 1)^2*a*d^3/(c*x - 1)^2 + 504*(c*x + 1)*a
*d^3/(c*x - 1) - 84*a*d^3 + 318*(c*x + 1)^5*b*d^3/(c*x - 1)^5 - 1119*(c*x
+ 1)^4*b*d^3/(c*x - 1)^4 + 1742*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 1464*(c*x
+ 1)^2*b*d^3/(c*x - 1)^2 + 636*(c*x + 1)*b*d^3/(c*x - 1) - 113*b*d^3)/((c*
x + 1)^6*c^4/(c*x - 1)^6 - 6*(c*x + 1)^5*c^4/(c*x - 1)^5 + 15*(c*x + 1)^4*
c^4/(c*x - 1)^4 - 20*(c*x + 1)^3*c^4/(c*x - 1)^3 + 15*(c*x + 1)^2*c^4/(c*x
 - 1)^2 - 6*(c*x + 1)*c^4/(c*x - 1) + c^4))
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{\frac{7\,b\,c^2\,d^3\,x^2}{15}-\frac{d^3\,\left(165\,b\,\mathrm{atanh}(c\,x)-84\,b\,\ln\left(c^2\,x^2-1\right)\right)}{180}+\frac{11\,b\,c\,d^3\,x}{12}}{c^3}$$
$$+\frac{d^3\,\left(60\,a\,x^3+55\,b\,x^3+60\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{180}+\frac{c^3\,d^3\,\left(30\,a\,x^6+30\,b\,x^6\,\mathrm{atanh}(c\,x)\right)}{180}$$
$$+\frac{c\,d^3\,\left(135\,a\,x^4+27\,b\,x^4+135\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{180}$$
$$+\frac{c^2\,d^3\,\left(108\,a\,x^5+6\,b\,x^5+108\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{180}$$

input
```
int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

output
```
((7*b*c^2*d^3*x^2)/15 - (d^3*(165*b*atanh(c*x) - 84*b*log(c^2*x^2 - 1)))/1
80 + (11*b*c*d^3*x)/12)/c^3 + (d^3*(60*a*x^3 + 55*b*x^3 + 60*b*x^3*atanh(c
*x)))/180 + (c^3*d^3*(30*a*x^6 + 30*b*x^6*atanh(c*x)))/180 + (c*d^3*(135*a
*x^4 + 27*b*x^4 + 135*b*x^4*atanh(c*x)))/180 + (c^2*d^3*(108*a*x^5 + 6*b*x
^5 + 108*b*x^5*atanh(c*x)))/180
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^3(30atanh(cx)\,b\,c^6x^6+108atanh(cx)\,b\,c^5x^5+135atanh(cx)\,b\,c^4x^4+60atanh(cx)\,b\,c^3x^3+3atanh(cx)\,b}{}$$

input
```
int(x^2*(c*d*x+d)^3*(a+b*atanh(c*x)),x)
```

output
```
(d**3*(30*atanh(c*x)*b*c**6*x**6 + 108*atanh(c*x)*b*c**5*x**5 + 135*atanh(
c*x)*b*c**4*x**4 + 60*atanh(c*x)*b*c**3*x**3 + 3*atanh(c*x)*b + 168*log(c*
*2*x - c)*b + 30*a*c**6*x**6 + 108*a*c**5*x**5 + 135*a*c**4*x**4 + 60*a*c*
*3*x**3 + 6*b*c**5*x**5 + 27*b*c**4*x**4 + 55*b*c**3*x**3 + 84*b*c**2*x**2
 + 165*b*c*x))/(180*c**3)
```

3.22 $\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 135

$$\begin{aligned}\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx &= \frac{3bd^3x}{5c}+\frac{3bd^3(1+cx)^2}{20c^2}+\frac{bd^3(1+cx)^3}{20c^2}\\&\quad+\frac{bd^3(1+cx)^4}{20c^2}-\frac{d^3(1+cx)^4(a+b\text{arctanh}(cx))}{4c^2}\\&\quad+\frac{d^3(1+cx)^5(a+b\text{arctanh}(cx))}{5c^2}\\&\quad+\frac{6bd^3\log(1-cx)}{5c^2}\end{aligned}$$

output
```
3/5*b*d^3*x/c+3/20*b*d^3*(c*x+1)^2/c^2+1/20*b*d^3*(c*x+1)^3/c^2+1/20*b*d^3
*(c*x+1)^4/c^2-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))/c^2+1/5*d^3*(c*x+1)^5*
(a+b*arctanh(c*x))/c^2+6/5*b*d^3*ln(-c*x+1)/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^3(50bcx+20ac^2x^2+24bc^2x^2+40ac^3x^3+10bc^3x^3+30ac^4x^4+2bc^4x^4+8ac^5x^5+2bc^2x^2(10+20cx+}{40c^2}$$

input
```
Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*(50*b*c*x + 20*a*c^2*x^2 + 24*b*c^2*x^2 + 40*a*c^3*x^3 + 10*b*c^3*x^3
 + 30*a*c^4*x^4 + 2*b*c^4*x^4 + 8*a*c^5*x^5 + 2*b*c^2*x^2*(10 + 20*c*x + 1
5*c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x] + 49*b*Log[1 - c*x] - b*Log[1 + c*x]))
/(40*c^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6498, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)^3(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d^3(1-4cx)(cx+1)^3}{20c^2(1-cx)}dx+\frac{d^3(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2}-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^3\int \frac{(1-4cx)(cx+1)^3}{1-cx}dx}{20c}+\frac{d^3(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2}-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c^2}$$

$$\Big\downarrow \text{86}$$

$$\frac{bd^3 \int \left(4(cx+1)^3 + 3(cx+1)^2 + 6(cx+1) + \frac{24}{cx-1} + 12\right) dx}{20c} + \frac{d^3(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2} - \frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2} - \frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c^2} + \frac{bd^3\left(\frac{(cx+1)^4}{c} + \frac{(cx+1)^3}{c} + \frac{3(cx+1)^2}{c} + \frac{24\log(1-cx)}{c} + 12x\right)}{20c}$$

input
```
Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
-1/4*(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/c^2 + (d^3*(1 + c*x)^5*(a + b*
ArcTanh[c*x]))/(5*c^2) + (b*d^3*(12*x + (3*(1 + c*x)^2)/c + (1 + c*x)^3/c
+ (1 + c*x)^4/c + (24*Log[1 - c*x])/c))/(20*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

method	result
parts	$d^3a\left(\frac{1}{5}c^3x^5+\frac{3}{4}c^2x^4+c\,x^3+\frac{1}{2}x^2\right)+\frac{d^3b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arctanh}(cx)c^3x^3+\frac{\operatorname{arctanh}(}{}\right.}{c^2}$
derivativedivides	$\frac{d^3a\left(\frac{1}{5}c^5x^5+\frac{3}{4}c^4x^4+x^3c^3+\frac{1}{2}c^2x^2\right)+d^3b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arctanh}(cx)c^3x^3+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{c^4}{2}\right.}{c^2}$
default	$\frac{d^3a\left(\frac{1}{5}c^5x^5+\frac{3}{4}c^4x^4+x^3c^3+\frac{1}{2}c^2x^2\right)+d^3b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arctanh}(cx)c^3x^3+\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\frac{c^4}{2}\right.}{c^2}$
parallelrisch	$\frac{4b\,c^5d^3\operatorname{arctanh}(cx)x^5+4a\,c^5d^3x^5+15d^3b\operatorname{arctanh}(cx)x^4c^4+15a\,c^4d^3x^4+b\,c^4d^3x^4+20d^3b\operatorname{arctanh}(cx)x^3c^3+20a\,c^3d^3}{20c^2}$
risch	$\frac{d^3b\,x^2\left(4x^3c^3+15c^2x^2+20cx+10\right)\ln(cx+1)}{40}-\frac{d^3c^3b\,x^5\ln(-cx+1)}{10}+\frac{d^3c^3a\,x^5}{5}-\frac{3d^3c^2b\,x^4\ln(-cx+1)}{8}+\frac{3d^3c^2}{4}$

input
```
int(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/5*c^3*x^5+3/4*c^2*x^4+c*x^3+1/2*x^2)+d^3*b/c^2*(1/5*arctanh(c*x)*
c^5*x^5+3/4*arctanh(c*x)*c^4*x^4+arctanh(c*x)*c^3*x^3+1/2*arctanh(c*x)*c^2
*x^2+1/20*c^4*x^4+1/4*x^3*c^3+3/5*c^2*x^2+5/4*c*x+49/40*ln(c*x-1)-1/40*ln(
c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.22

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{8\,ac^5d^3x^5+2\,(15\,a+b)c^4d^3x^4+10\,(4\,a+b)c^3d^3x^3+4\,(5\,a+6\,b)c^2d^3x^2+50\,bcd^3x-bd^3\log{(cx+1)}+\cdots}{40\,c^2}$$

input
```
integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/40*(8*a*c^5*d^3*x^5 + 2*(15*a + b)*c^4*d^3*x^4 + 10*(4*a + b)*c^3*d^3*x^
3 + 4*(5*a + 6*b)*c^2*d^3*x^2 + 50*b*c*d^3*x - b*d^3*log(c*x + 1) + 49*b*d
^3*log(c*x - 1) + (4*b*c^5*d^3*x^5 + 15*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3 +
 10*b*c^2*d^3*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.56

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \begin{cases} \frac{ac^3d^3x^5}{5}+\frac{3ac^2d^3x^4}{4}+acd^3x^3+\frac{ad^3x^2}{2}+\frac{bc^3d^3x^5\operatorname{atanh}{(cx)}}{5}+\frac{3bc^2d^3x^4\operatorname{atanh}{(cx)}}{4}+\frac{bc^2d^3x^4}{20}+bcd^3x^3\operatorname{atanh}{(cx)}+\cdots \\ \frac{ad^3x^2}{2} \end{cases}$$

input
```
integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**3*d**3*x**5/5 + 3*a*c**2*d**3*x**4/4 + a*c*d**3*x**3 + a*d
**3*x**2/2 + b*c**3*d**3*x**5*atanh(c*x)/5 + 3*b*c**2*d**3*x**4*atanh(c*x)
/4 + b*c**2*d**3*x**4/20 + b*c*d**3*x**3*atanh(c*x) + b*c*d**3*x**3/4 + b*
d**3*x**2*atanh(c*x)/2 + 3*b*d**3*x**2/5 + 5*b*d**3*x/(4*c) + 6*b*d**3*log
(x - 1/c)/(5*c**2) - b*d**3*atanh(c*x)/(20*c**2), Ne(c, 0)), (a*d**3*x**2/
2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(121) = 242$.

Time = 0.03 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.81

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{5}ac^3d^3x^5 + \frac{3}{4}ac^2d^3x^4 + \frac{1}{20}\left(4\,x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4+2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^3d^3 + acd^3x^3 + \frac{1}{8}\left(6\,x^4\,\text{artanh}\,(cx) + c\left(\frac{2\,(c^2x^3+3\,x)}{c^4} - \frac{3\,\log\left(cx+1\right)}{c^5} + \frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bc^2d^3 + \frac{1}{2}\left(2\,x^3\,\text{artanh}\,(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bcd^3 + \frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2\,x^2\,\text{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log\left(cx+1\right)}{c^3} + \frac{\log\left(cx-1\right)}{c^3}\right)\right)bd^3$$

input
```
integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^3*d^3*x^5 + 3/4*a*c^2*d^3*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2
*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^3 + a*c*d^3*x^3 + 1/8
*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*l
og(c*x - 1)/c^5))*b*c^2*d^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c
^2*x^2 - 1)/c^4))*b*c*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(121) = 242$.

Time = 0.13 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.90

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx =$$
$$-\frac{1}{5}\left(\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^3} - \frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^3} - \frac{2\left(\frac{20\,(cx+1)^4bd^3}{(cx-1)^4} - \frac{30\,(cx+1)^3bd^3}{(cx-1)^3} + \frac{30\,(cx+1)^2bd^3}{(cx-1)^2} - \frac{15\,(cx+1)}{cx-1}\right.}{\frac{(cx+1)^5c^3}{(cx-1)^5} - \frac{5\,(cx+1)^4c^3}{(cx-1)^4} + \frac{10\,(cx+1)^3c^3}{(cx-1)^3} - \frac{10\,(cx+1)^2c}{(cx-1)^2}}\right.$$

input
```
integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-1/5*(6*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 6*b*d^3*log(-(c*x + 1)/(
c*x - 1))/c^3 - 2*(20*(c*x + 1)^4*b*d^3/(c*x - 1)^4 - 30*(c*x + 1)^3*b*d^3
/(c*x - 1)^3 + 30*(c*x + 1)^2*b*d^3/(c*x - 1)^2 - 15*(c*x + 1)*b*d^3/(c*x
- 1) + 3*b*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^3/(c*x - 1)^5 - 5
*(c*x + 1)^4*c^3/(c*x - 1)^4 + 10*(c*x + 1)^3*c^3/(c*x - 1)^3 - 10*(c*x +
1)^2*c^3/(c*x - 1)^2 + 5*(c*x + 1)*c^3/(c*x - 1) - c^3) - (80*(c*x + 1)^4*
a*d^3/(c*x - 1)^4 - 120*(c*x + 1)^3*a*d^3/(c*x - 1)^3 + 120*(c*x + 1)^2*a*
d^3/(c*x - 1)^2 - 60*(c*x + 1)*a*d^3/(c*x - 1) + 12*a*d^3 + 34*(c*x + 1)^4
*b*d^3/(c*x - 1)^4 - 103*(c*x + 1)^3*b*d^3/(c*x - 1)^3 + 123*(c*x + 1)^2*b
*d^3/(c*x - 1)^2 - 69*(c*x + 1)*b*d^3/(c*x - 1) + 15*b*d^3)/((c*x + 1)^5*c
^3/(c*x - 1)^5 - 5*(c*x + 1)^4*c^3/(c*x - 1)^4 + 10*(c*x + 1)^3*c^3/(c*x -
 1)^3 - 10*(c*x + 1)^2*c^3/(c*x - 1)^2 + 5*(c*x + 1)*c^3/(c*x - 1) - c^3))
*c
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{d^3\,\left(10\,a\,x^2+12\,b\,x^2+10\,b\,x^2\,\mathrm{atanh}(c\,x)\right)}{20} - \frac{\frac{d^3\,\left(25\,b\,\mathrm{atanh}(c\,x)-12\,b\,\ln\left(c^2\,x^2-1\right)\right)}{20}-\frac{5\,b\,c\,d^3\,x}{4}}{c^2} + \frac{c^3\,d^3\,\left(4\,a\,x^5+4\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{20} + \frac{c\,d^3\,\left(20\,a\,x^3+5\,b\,x^3+20\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{20} + \frac{c^2\,d^3\,\left(15\,a\,x^4+b\,x^4+15\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{20}$$

input
```
int(x*(a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

output
```
(d^3*(10*a*x^2 + 12*b*x^2 + 10*b*x^2*atanh(c*x)))/20 - ((d^3*(25*b*atanh(c
*x) - 12*b*log(c^2*x^2 - 1)))/20 - (5*b*c*d^3*x)/4)/c^2 + (c^3*d^3*(4*a*x^
5 + 4*b*x^5*atanh(c*x)))/20 + (c*d^3*(20*a*x^3 + 5*b*x^3 + 20*b*x^3*atanh(
c*x)))/20 + (c^2*d^3*(15*a*x^4 + b*x^4 + 15*b*x^4*atanh(c*x)))/20
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))\,dx = \frac{d^3(4atanh(cx)\,b\,c^5x^5+15atanh(cx)\,b\,c^4x^4+20atanh(cx)\,b\,c^3x^3+10atanh(cx)\,b\,c^2x^2-atanh(cx)\,b+24}{20c^2}$$

input
```
int(x*(c*d*x+d)^3*(a+b*atanh(c*x)),x)
```

output
```
(d**3*(4*atanh(c*x)*b*c**5*x**5 + 15*atanh(c*x)*b*c**4*x**4 + 20*atanh(c*x
)*b*c**3*x**3 + 10*atanh(c*x)*b*c**2*x**2 - atanh(c*x)*b + 24*log(c**2*x -
 c)*b + 4*a*c**5*x**5 + 15*a*c**4*x**4 + 20*a*c**3*x**3 + 10*a*c**2*x**2 +
 b*c**4*x**4 + 5*b*c**3*x**3 + 12*b*c**2*x**2 + 25*b*c*x))/(20*c**2)
```

3.23 $\int (d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int (d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx = bd^3x+\frac{bd^3(1+cx)^2}{4c}+\frac{bd^3(1+cx)^3}{12c}+\frac{d^3(1+cx)^4(a+b\operatorname{arctanh}(cx))}{4c}+\frac{2bd^3\log(1-cx)}{c}$$

output
```
b*d^3*x+1/4*b*d^3*(c*x+1)^2/c+1/12*b*d^3*(c*x+1)^3/c+1/4*d^3*(c*x+1)^4*(a+
b*arctanh(c*x))/c+2*b*d^3*ln(-c*x+1)/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

$$\int (d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^3(24acx+42bcx+36ac^2x^2+12bc^2x^2+24ac^3x^3+2bc^3x^3+6ac^4x^4+6bcx(4+6cx+4c^2x^2+c^3x^3)\operatorname{ar}}{24c}$$

input
```
Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*(24*a*c*x + 42*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c^3*x^3 + 2
*b*c^3*x^3 + 6*a*c^4*x^4 + 6*b*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3)*ArcTa
nh[c*x] + 45*b*Log[1 - c*x] + 3*b*Log[1 + c*x]))/(24*c)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6478, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^3(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c}-\frac{b\int \frac{d^4(cx+1)^4}{1-c^2x^2}dx}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c}-\frac{1}{4}bd^3\int \frac{(cx+1)^4}{1-c^2x^2}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c}-\frac{1}{4}bd^3\int \frac{(cx+1)^3}{1-cx}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c}-\frac{1}{4}bd^3\int \left(-(cx+1)^2-2(cx+1)+\frac{8}{1-cx}-4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4c}-\frac{1}{4}bd^3\left(-\frac{(cx+1)^3}{3c}-\frac{(cx+1)^2}{c}-\frac{8\log(1-cx)}{c}-4x\right)$$

input
```
Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/(4*c) - (b*d^3*(-4*x - (1 + c*x)^2/
c - (1 + c*x)^3/(3*c) - (8*Log[1 - c*x])/c))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{d^3a(cx+1)^4}{4}+d^3b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arctanh}(cx)c^3x^3+\frac{3\operatorname{arctanh}(cx)c^2x^2}{2}+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{4}+\frac{x^3c^3}{12}+\frac{c^2x^2}{2}+\right.}{c}$
default	$\frac{\frac{d^3a(cx+1)^4}{4}+d^3b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arctanh}(cx)c^3x^3+\frac{3\operatorname{arctanh}(cx)c^2x^2}{2}+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{4}+\frac{x^3c^3}{12}+\frac{c^2x^2}{2}+\right.}{c}$
parts	$\frac{d^3a(cx+1)^4}{4c}+\frac{d^3b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\operatorname{arctanh}(cx)c^3x^3+\frac{3\operatorname{arctanh}(cx)c^2x^2}{2}+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{4}+\frac{x^3c^3}{12}+\frac{c^2x^2}{2}\right.}{c}$
parallelrisch	$\frac{3d^3b\operatorname{arctanh}(cx)x^4c^4+3a\,c^4d^3x^4+12d^3b\operatorname{arctanh}(cx)x^3c^3+12a\,c^3d^3x^3+b\,c^3d^3x^3+18x^2\operatorname{arctanh}(cx)b\,c^2d^3+18a\,c^2d^3}{12c}$
risch	$\frac{d^3(cx+1)^4b\ln(cx+1)}{8c}-\frac{d^3c^3b\,x^4\ln(-cx+1)}{8}+\frac{d^3c^3a\,x^4}{4}-\frac{d^3c^2b\,x^3\ln(-cx+1)}{2}+d^3c^2a\,x^3+\frac{d^3c^2b\,x^3}{12}-$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/4*d^3*a*(c*x+1)^4+d^3*b*(1/4*arctanh(c*x)*c^4*x^4+arctanh(c*x)*c^3*
x^3+3/2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x+1/4*arctanh(c*x)+1/12*x^3*c^
3+1/2*c^2*x^2+7/4*c*x+2*ln(c*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int(d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{6\,ac^4d^3x^4+2\,(12\,a+b)c^3d^3x^3+12\,(3\,a+b)c^2d^3x^2+6\,(4\,a+7\,b)cd^3x+3\,bd^3\log{(cx+1)}+45\,bd^3\log{(}}{24\,c}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/24*(6*a*c^4*d^3*x^4 + 2*(12*a + b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^
2 + 6*(4*a + 7*b)*c*d^3*x + 3*b*d^3*log(c*x + 1) + 45*b*d^3*log(c*x - 1) +
 3*(b*c^4*d^3*x^4 + 4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c*d^3*x)*log(-
(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(73) = 146$.

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.17

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^3d^3x^4}{4} + ac^2d^3x^3 + \frac{3acd^3x^2}{2} + ad^3x + \frac{bc^3d^3x^4\,\text{atanh}\,(cx)}{4} + bc^2d^3x^3\,\text{atanh}\,(cx) + \frac{bc^2d^3x^3}{12} + \frac{3bcd^3x^2\,\text{atanh}\,(cx)}{2} + \\ ad^3x \end{cases}$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**3*d**3*x**4/4 + a*c**2*d**3*x**3 + 3*a*c*d**3*x**2/2 + a*d
**3*x + b*c**3*d**3*x**4*atanh(c*x)/4 + b*c**2*d**3*x**3*atanh(c*x) + b*c*
*2*d**3*x**3/12 + 3*b*c*d**3*x**2*atanh(c*x)/2 + b*c*d**3*x**2/2 + b*d**3*
x*atanh(c*x) + 7*b*d**3*x/4 + 2*b*d**3*log(x - 1/c)/c + b*d**3*atanh(c*x)/
(4*c), Ne(c, 0)), (a*d**3*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(78) = 156$.

Time = 0.03 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.61

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{4}\,ac^3d^3x^4 + ac^2d^3x^3 + \frac{1}{24}\left(6\,x^4\,\text{artanh}\,(cx) + c\left(\frac{2\,(c^2x^3+3\,x)}{c^4} - \frac{3\,\log\,(cx+1)}{c^5} + \frac{3\,\log\,(cx-1)}{c^5}\right)\right)bc^3d^3 + \frac{1}{2}\left(2\,x^3\,\text{artanh}\,(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\,(c^2x^2-1)}{c^4}\right)\right)bc^2d^3 + \frac{3}{2}\,acd^3x^2 + \frac{3}{4}\left(2\,x^2\,\text{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log\,(cx+1)}{c^3} + \frac{\log\,(cx-1)}{c^3}\right)\right)bcd^3 + ad^3x + \frac{(2\,cx\,\text{artanh}\,(cx) + \log\,(-c^2x^2+1))bd^3}{2\,c}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*c^3*d^3*x^4 + a*c^2*d^3*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x
^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^3*d^3 + 1/2*
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + 3/2*
a*c*d^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*b*d^3/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(78) = 156$.

Time = 0.13 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.06

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))\,dx =$$

$$-\frac{1}{3}\left(\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2}-\frac{6\,bd^3\log\left(-\frac{cx+1}{cx-1}\right)}{c^2}-\frac{6\left(\frac{4\,(cx+1)^3bd^3}{(cx-1)^3}-\frac{6\,(cx+1)^2bd^3}{(cx-1)^2}+\frac{4\,(cx+1)bd^3}{cx-1}-bd^3\right)\log\left(-\right.}{\frac{(cx+1)^4c^2}{(cx-1)^4}-\frac{4\,(cx+1)^3c^2}{(cx-1)^3}+\frac{6\,(cx+1)^2c^2}{(cx-1)^2}-\frac{4\,(cx+1)c^2}{cx-1}+c}\right.$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-1/3*(6*b*d^3*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 6*b*d^3*log(-(c*x + 1)/(
c*x - 1))/c^2 - 6*(4*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 6*(c*x + 1)^2*b*d^3/(
c*x - 1)^2 + 4*(c*x + 1)*b*d^3/(c*x - 1) - b*d^3)*log(-(c*x + 1)/(c*x - 1)
)/((c*x + 1)^4*c^2/(c*x - 1)^4 - 4*(c*x + 1)^3*c^2/(c*x - 1)^3 + 6*(c*x +
1)^2*c^2/(c*x - 1)^2 - 4*(c*x + 1)*c^2/(c*x - 1) + c^2) - (48*(c*x + 1)^3*
a*d^3/(c*x - 1)^3 - 72*(c*x + 1)^2*a*d^3/(c*x - 1)^2 + 48*(c*x + 1)*a*d^3/
(c*x - 1) - 12*a*d^3 + 18*(c*x + 1)^3*b*d^3/(c*x - 1)^3 - 45*(c*x + 1)^2*b
*d^3/(c*x - 1)^2 + 38*(c*x + 1)*b*d^3/(c*x - 1) - 11*b*d^3)/((c*x + 1)^4*c
^2/(c*x - 1)^4 - 4*(c*x + 1)^3*c^2/(c*x - 1)^3 + 6*(c*x + 1)^2*c^2/(c*x -
1)^2 - 4*(c*x + 1)*c^2/(c*x - 1) + c^2))*c
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int (d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^3\,(12\,a\,x+21\,b\,x+12\,b\,x\,\operatorname{atanh}(c\,x))}{12} + \frac{c^3\,d^3\,(3\,a\,x^4+3\,b\,x^4\,\operatorname{atanh}(c\,x))}{12} - \frac{d^3\,(21\,b\,\operatorname{atanh}(c\,x)-12\,b\,\ln\left(c^2\,x^2-1\right))}{12\,c} + \frac{c\,d^3\,(18\,a\,x^2+6\,b\,x^2+18\,b\,x^2\,\operatorname{atanh}(c\,x))}{12} + \frac{c^2\,d^3\,(12\,a\,x^3+b\,x^3+12\,b\,x^3\,\operatorname{atanh}(c\,x))}{12}$$

input
```
int((a + b*atanh(c*x))*(d + c*d*x)^3,x)
```

output
```
(d^3*(12*a*x + 21*b*x + 12*b*x*atanh(c*x)))/12 + (c^3*d^3*(3*a*x^4 + 3*b*x
^4*atanh(c*x)))/12 - (d^3*(21*b*atanh(c*x) - 12*b*log(c^2*x^2 - 1)))/(12*c
) + (c*d^3*(18*a*x^2 + 6*b*x^2 + 18*b*x^2*atanh(c*x)))/12 + (c^2*d^3*(12*a
*x^3 + b*x^3 + 12*b*x^3*atanh(c*x)))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int (d+cdx)^3(a+b\operatorname{arctanh}(cx))\,dx = \frac{d^3(3atanh(cx)\,b\,c^4x^4+12atanh(cx)\,b\,c^3x^3+18atanh(cx)\,b\,c^2x^2+12atanh(cx)\,bcx+3atanh(cx)\,b+24}{12c}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x)),x)
```

output
```
(d**3*(3*atanh(c*x)*b*c**4*x**4 + 12*atanh(c*x)*b*c**3*x**3 + 18*atanh(c*x
)*b*c**2*x**2 + 12*atanh(c*x)*b*c*x + 3*atanh(c*x)*b + 24*log(c**2*x - c)*
b + 3*a*c**4*x**4 + 12*a*c**3*x**3 + 18*a*c**2*x**2 + 12*a*c*x + b*c**3*x*
*3 + 6*b*c**2*x**2 + 21*b*c*x))/(12*c)
```

3.24 $\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x} dx = 3acd^3x + \frac{3}{2}bcd^3x + \frac{1}{6}bc^2d^3x^2 - \frac{3}{2}bd^3\text{arctanh}(cx) + 3bcd^3x\text{arctanh}(cx) + \frac{3}{2}c^2d^3x^2(a+b\text{arctanh}(cx)) + \frac{1}{3}c^3d^3x^3(a+b\text{arctanh}(cx)) + ad^3\log(x) + \frac{5}{3}bd^3\log\left(1-c^2x^2\right) - \frac{1}{2}bd^3\,\text{PolyLog}(2,-cx) + \frac{1}{2}bd^3\,\text{PolyLog}(2,cx)$$

output
```
3*a*c*d^3*x+3/2*b*c*d^3*x+1/6*b*c^2*d^3*x^2-3/2*b*d^3*arctanh(c*x)+3*b*c*d
^3*x*arctanh(c*x)+3/2*c^2*d^3*x^2*(a+b*arctanh(c*x))+1/3*c^3*d^3*x^3*(a+b*
arctanh(c*x))+a*d^3*ln(x)+5/3*b*d^3*ln(-c^2*x^2+1)-1/2*b*d^3*polylog(2,-c*
x)+1/2*b*d^3*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x}\,dx = \frac{1}{12}d^3\big(36acx+18bcx+18ac^2x^2+2bc^2x^2 + 4ac^3x^3+36bcx\text{arctanh}(cx) + 18bc^2x^2\text{arctanh}(cx)+4bc^3x^3\text{arctanh}(cx) + 12a\log(x)+9b\log(1-cx)-9b\log(1+cx) + 18b\log\left(1-c^2x^2\right)+2b\log\left(-1+c^2x^2\right) - 6b\,\text{PolyLog}(2,-cx)+6b\,\text{PolyLog}(2,cx)\big)$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
(d^3*(36*a*c*x + 18*b*c*x + 18*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 36*
b*c*x*ArcTanh[c*x] + 18*b*c^2*x^2*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x]
+ 12*a*Log[x] + 9*b*Log[1 - c*x] - 9*b*Log[1 + c*x] + 18*b*Log[1 - c^2*x^2
] + 2*b*Log[-1 + c^2*x^2] - 6*b*PolyLog[2, -(c*x)] + 6*b*PolyLog[2, c*x]))
/12
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))}{x}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^3d^3x^2(a+b\text{arctanh}(cx))+3c^2d^3x(a+b\text{arctanh}(cx))+3cd^3(a+b\text{arctanh}(cx))+\frac{d^3(a+b\text{arctanh}(cx))}{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^3d^3x^3(a+b\text{arctanh}(cx))+\frac{3}{2}c^2d^3x^2(a+b\text{arctanh}(cx))+3acd^3x+ad^3\log(x)-\frac{3}{2}bd^3\text{arctanh}(cx)+3bcd^3x\text{arctanh}(cx)+\frac{1}{6}bc^2d^3x^2+\frac{5}{3}bd^3\log\left(1-c^2x^2\right)-\frac{1}{2}bd^3\,\text{PolyLog}(2,-cx)+\frac{1}{2}bd^3\,\text{PolyLog}(2,cx)+\frac{3}{2}bcd^3x$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
3*a*c*d^3*x + (3*b*c*d^3*x)/2 + (b*c^2*d^3*x^2)/6 - (3*b*d^3*ArcTanh[c*x])
/2 + 3*b*c*d^3*x*ArcTanh[c*x] + (3*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/2 + (
c^3*d^3*x^3*(a + b*ArcTanh[c*x]))/3 + a*d^3*Log[x] + (5*b*d^3*Log[1 - c^2*
x^2])/3 - (b*d^3*PolyLog[2, -(c*x)])/2 + (b*d^3*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

method	result
parts	$d^3a\left(\frac{x^3c^3}{3}+\frac{3c^2x^2}{2}+3cx+\ln\left(x\right)\right)+d^3b\left(\frac{\text{arctanh}(cx)c^3x^3}{3}+\frac{3\,\text{arctanh}(cx)c^2x^2}{2}+3\,\text{arctanh}\left(c\right.\right.$
derivativedivides	$d^3a\left(\frac{x^3c^3}{3}+\frac{3c^2x^2}{2}+3cx+\ln\left(cx\right)\right)+d^3b\left(\frac{\text{arctanh}(cx)c^3x^3}{3}+\frac{3\,\text{arctanh}(cx)c^2x^2}{2}+3\,\text{arctanh}\left(\right.\right.$
default	$d^3a\left(\frac{x^3c^3}{3}+\frac{3c^2x^2}{2}+3cx+\ln\left(cx\right)\right)+d^3b\left(\frac{\text{arctanh}(cx)c^3x^3}{3}+\frac{3\,\text{arctanh}(cx)c^2x^2}{2}+3\,\text{arctanh}\left(\right.\right.$
risch	$-\frac{d^3b\ln(-cx+1)c^3x^3}{6}-\frac{3d^3b\ln(-cx+1)c^2x^2}{4}-\frac{3d^3b\ln(-cx+1)cx}{2}+\frac{29d^3b\ln(-cx+1)}{12}+\frac{b\,c^2d^3x^2}{6}+\frac{3bc\,d^3x}{2}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/3*x^3*c^3+3/2*c^2*x^2+3*c*x+ln(x))+d^3*b*(1/3*arctanh(c*x)*c^3*x^
3+3/2*arctanh(c*x)*c^2*x^2+3*arctanh(c*x)*c*x+arctanh(c*x)*ln(c*x)-1/2*dil
og(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1)+1/6*c^2*x^2+3/2*c*x+29/12*l
n(c*x-1)+11/12*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\mathrm{arctanh}(cx))}{x}\,dx = \int \frac{(cdx+d)^3(b\,\mathrm{artanh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d
^3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\mathrm{arctanh}(cx))}{x}\,dx = d^3\left(\int 3ac\,dx + \int \frac{a}{x}\,dx + \int 3ac^2x\,dx + \int ac^3x^2\,dx + \int 3bc\,\mathrm{atanh}\,(cx)\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x}\,dx + \int 3bc^2x\,\mathrm{atanh}\,(cx)\,dx + \int bc^3x^2\,\mathrm{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x,x)
```

output
```
d**3*(Integral(3*a*c, x) + Integral(a/x, x) + Integral(3*a*c**2*x, x) + In
tegral(a*c**3*x**2, x) + Integral(3*b*c*atanh(c*x), x) + Integral(b*atanh(
c*x)/x, x) + Integral(3*b*c**2*x*atanh(c*x), x) + Integral(b*c**3*x**2*ata
nh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.50

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x}\,dx = \frac{1}{3}\,ac^3d^3x^3 + \frac{3}{2}\,ac^2d^3x^2 + \frac{1}{6}\,bc^2d^3x^2 + 3\,acd^3x + \frac{3}{2}\,bcd^3x + \frac{3}{2}\left(2\,cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)bd^3 - \frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right) + \text{Li}_2(-cx+1)\right)bd^3 + \frac{1}{2}\left(\log\left(cx+1\right)\log\left(-cx\right) + \text{Li}_2(cx+1)\right)bd^3 - \frac{7}{12}\,bd^3\log\left(cx+1\right) + \frac{11}{12}\,bd^3\log\left(cx-1\right) + ad^3\log\left(x\right) + \frac{1}{12}\left(2\,bc^3d^3x^3 + 9\,bc^2d^3x^2\right)\log\left(cx+1\right) - \frac{1}{12}\left(2\,bc^3d^3x^3 + 9\,bc^2d^3x^2\right)\log\left(-cx+1\right)$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

output
```
1/3*a*c^3*d^3*x^3 + 3/2*a*c^2*d^3*x^2 + 1/6*b*c^2*d^3*x^2 + 3*a*c*d^3*x +
3/2*b*c*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d^3 - 1/2*(
log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^3 + 1/2*(log(c*x + 1)*log(-c
*x) + dilog(c*x + 1))*b*d^3 - 7/12*b*d^3*log(c*x + 1) + 11/12*b*d^3*log(c*
x - 1) + a*d^3*log(x) + 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(c*x +
 1) - 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(-c*x + 1)
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^3}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x}\,dx$$
$$=\frac{d^3\left(2atanh(cx)\,b\,c^3x^3+9atanh(cx)\,b\,c^2x^2+18atanh(cx)\,bcx+11atanh(cx)\,b+6\left(\int\frac{atanh(cx)}{x}dx\right)b+20\right.}{6}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x,x)
```

output
```
(d**3*(2*atanh(c*x)*b*c**3*x**3 + 9*atanh(c*x)*b*c**2*x**2 + 18*atanh(c*x)
*b*c*x + 11*atanh(c*x)*b + 6*int(atanh(c*x)/x,x)*b + 20*log(c**2*x - c)*b
+ 6*log(x)*a + 2*a*c**3*x**3 + 9*a*c**2*x**2 + 18*a*c*x + b*c**2*x**2 + 9*
b*c*x))/6
```

3.25 $\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\begin{aligned}\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx &= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\text{arctanh}(cx)\\ &\quad+3bc^2d^3x\text{arctanh}(cx)-\frac{d^3(a+b\text{arctanh}(cx))}{x}\\ &\quad+\frac{1}{2}c^3d^3x^2(a+b\text{arctanh}(cx))+3acd^3\log(x)\\ &\quad+bcd^3\log(x)+bcd^3\log\left(1-c^2x^2\right)\\ &\quad-\frac{3}{2}bcd^3\operatorname{PolyLog}(2,-cx)+\frac{3}{2}bcd^3\operatorname{PolyLog}(2,cx)\end{aligned}$$

output
```
3*a*c^2*d^3*x+1/2*b*c^2*d^3*x-1/2*b*c*d^3*arctanh(c*x)+3*b*c^2*d^3*x*arcta
nh(c*x)-d^3*(a+b*arctanh(c*x))/x+1/2*c^3*d^3*x^2*(a+b*arctanh(c*x))+3*a*c*
d^3*ln(x)+b*c*d^3*ln(x)+b*c*d^3*ln(-c^2*x^2+1)-3/2*b*c*d^3*polylog(2,-c*x)
+3/2*b*c*d^3*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$= \frac{d^3(-4a+12ac^2x^2+2bc^2x^2+2ac^3x^3-4b\text{arctanh}(cx)+12bc^2x^2\text{arctanh}(cx)+2bc^3x^3\text{arctanh}(cx)+12}{}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
(d^3*(-4*a + 12*a*c^2*x^2 + 2*b*c^2*x^2 + 2*a*c^3*x^3 - 4*b*ArcTanh[c*x] +
 12*b*c^2*x^2*ArcTanh[c*x] + 2*b*c^3*x^3*ArcTanh[c*x] + 12*a*c*x*Log[x] +
4*b*c*x*Log[c*x] + b*c*x*Log[1 - c*x] - b*c*x*Log[1 + c*x] + 4*b*c*x*Log[1
 - c^2*x^2] - 6*b*c*x*PolyLog[2, -(c*x)] + 6*b*c*x*PolyLog[2, c*x]))/(4*x)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$\Big\downarrow \ 6502$$
$$\int \left(c^3d^3x(a+b\text{arctanh}(cx))+3c^2d^3(a+b\text{arctanh}(cx))+\frac{d^3(a+b\text{arctanh}(cx))}{x^2}+\frac{3cd^3(a+b\text{arctanh}(cx))}{x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}c^3d^3x^2(a+b\text{arctanh}(cx))-\frac{d^3(a+b\text{arctanh}(cx))}{x}+3ac^2d^3x+3acd^3\log(x)+ 3bc^2d^3x\text{arctanh}(cx)-\frac{1}{2}bcd^3\text{arctanh}(cx)+bcd^3\log\left(1-c^2x^2\right)+\frac{1}{2}bc^2d^3x- \frac{3}{2}bcd^3\,\text{PolyLog}(2,-cx)+\frac{3}{2}bcd^3\,\text{PolyLog}(2,cx)+bcd^3\log(x)$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
3*a*c^2*d^3*x + (b*c^2*d^3*x)/2 - (b*c*d^3*ArcTanh[c*x])/2 + 3*b*c^2*d^3*x
*ArcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/x + (c^3*d^3*x^2*(a + b*ArcTanh
[c*x]))/2 + 3*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 - c^2*x^2] -
 (3*b*c*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c*d^3*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

method	result
parts	$d^3a\left(\frac{c^3x^2}{2}+3c^2x+3c\ln\left(x\right)-\frac{1}{x}\right)+d^3bc\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+3\ \operatorname{arctanh}\left(cx\right)cx+3\ \operatorname{arcta}\right.$
derivativedivides	$c\left(d^3a\left(\frac{c^2x^2}{2}+3cx+3\ln\left(cx\right)-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+3\ \operatorname{arctanh}\left(cx\right)cx+3\ \operatorname{arcta}\right.\right.$
default	$c\left(d^3a\left(\frac{c^2x^2}{2}+3cx+3\ln\left(cx\right)-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+3\ \operatorname{arctanh}\left(cx\right)cx+3\ \operatorname{arcta}\right.\right.$
risch	$-\frac{c^3d^3b\ln(-cx+1)x^2}{4}-\frac{3c^2d^3b\ln(-cx+1)x}{2}+\frac{5c\,d^3b\ln(-cx+1)}{4}+\frac{b\,c^2d^3x}{2}-3bc\,d^3+\frac{3c\,d^3b\,\operatorname{dilog}(-cx+1)}{2}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/2*c^3*x^2+3*c^2*x+3*c*ln(x)-1/x)+d^3*b*c*(1/2*arctanh(c*x)*c^2*x^
2+3*arctanh(c*x)*c*x+3*arctanh(c*x)*ln(c*x)-arctanh(c*x)/c/x-3/2*dilog(c*x
)-3/2*dilog(c*x+1)-3/2*ln(c*x)*ln(c*x+1)+1/2*c*x+5/4*ln(c*x-1)+ln(c*x)+3/4
*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d
^3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx = d^3\left(\int 3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int \frac{3ac}{x}\,dx + \int ac^3x\,dx + \int 3bc^2\,\text{atanh}\,(cx)\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{3bc\,\text{atanh}\,(cx)}{x}\,dx + \int bc^3x\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**2,x)
```

output
```
d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c/x, x) +
 Integral(a*c**3*x, x) + Integral(3*b*c**2*atanh(c*x), x) + Integral(b*ata
nh(c*x)/x**2, x) + Integral(3*b*c*atanh(c*x)/x, x) + Integral(b*c**3*x*ata
nh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$\begin{aligned}&= \frac{1}{4}\,bc^3d^3x^2\log(cx+1) - \frac{1}{4}\,bc^3d^3x^2\log(-cx+1) + \frac{1}{2}\,ac^3d^3x^2 \\ &\quad + 3\,ac^2d^3x + \frac{1}{2}\,bc^2d^3x + \frac{3}{2}\left(2\,cx\,\text{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)bcd^3 \\ &\quad - \frac{3}{2}\left(\log(cx)\log(-cx+1) + \text{Li}_2(-cx+1)\right)bcd^3 \\ &\quad + \frac{3}{2}\left(\log(cx+1)\log(-cx) + \text{Li}_2(cx+1)\right)bcd^3 \\ &\quad - \frac{1}{4}\,bcd^3\log(cx+1) + \frac{1}{4}\,bcd^3\log(cx-1) + 3\,acd^3\log(x) \\ &\quad - \frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\text{artanh}(cx)}{x}\right)bd^3 - \frac{ad^3}{x}\end{aligned}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/4*b*c^3*d^3*x^2*log(c*x + 1) - 1/4*b*c^3*d^3*x^2*log(-c*x + 1) + 1/2*a*c
^3*d^3*x^2 + 3*a*c^2*d^3*x + 1/2*b*c^2*d^3*x + 3/2*(2*c*x*arctanh(c*x) + l
og(-c^2*x^2 + 1))*b*c*d^3 - 3/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))
*b*c*d^3 + 3/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c*d^3 - 1/4*b*c
*d^3*log(c*x + 1) + 1/4*b*c*d^3*log(c*x - 1) + 3*a*c*d^3*log(x) - 1/2*(c*(
log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^3 - a*d^3/x
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^3}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^2,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^2}\,dx = \frac{d^3\left(atanh(cx)\,b\,c^3x^3+6atanh(cx)\,b\,c^2x^2+3atanh(cx)\,bcx-2atanh(cx)\,b+6\left(\int\frac{atanh(cx)}{x}dx\right)bcx+4\log\cdots\right)}{2x}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x^2,x)
```

output
```
(d**3*(atanh(c*x)*b*c**3*x**3 + 6*atanh(c*x)*b*c**2*x**2 + 3*atanh(c*x)*b*
c*x - 2*atanh(c*x)*b + 6*int(atanh(c*x)/x,x)*b*c*x + 4*log(c**2*x - c)*b*c
*x + 6*log(x)*a*c*x + 2*log(x)*b*c*x + a*c**3*x**3 + 6*a*c**2*x**2 - 2*a +
 b*c**2*x**2))/(2*x)
```

3.26 $\int \frac{(d+cdx)^3(a+b\,\mathbf{arctanh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 160

$$\begin{aligned}\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx &= -\frac{bcd^3}{2x}+ac^3d^3x+\frac{1}{2}bc^2d^3\text{arctanh}(cx)\\ &\quad+bc^3d^3x\text{arctanh}(cx)-\frac{d^3(a+b\text{arctanh}(cx))}{2x^2}\\ &\quad-\frac{3cd^3(a+b\text{arctanh}(cx))}{x}+3ac^2d^3\log(x)\\ &\quad+3bc^2d^3\log(x)-bc^2d^3\log\left(1-c^2x^2\right)\\ &\quad-\frac{3}{2}bc^2d^3\operatorname{PolyLog}(2,-cx)\\ &\quad+\frac{3}{2}bc^2d^3\operatorname{PolyLog}(2,cx)\end{aligned}$$

output
```
-1/2*b*c*d^3/x+a*c^3*d^3*x+1/2*b*c^2*d^3*arctanh(c*x)+b*c^3*d^3*x*arctanh(
c*x)-1/2*d^3*(a+b*arctanh(c*x))/x^2-3*c*d^3*(a+b*arctanh(c*x))/x+3*a*c^2*d
^3*ln(x)+3*b*c^2*d^3*ln(x)-b*c^2*d^3*ln(-c^2*x^2+1)-3/2*b*c^2*d^3*polylog(
2,-c*x)+3/2*b*c^2*d^3*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))}{x^3}\,dx$$
$$= \frac{d^3(-2a-12acx-2bcx+4ac^3x^3-2b\operatorname{arctanh}(cx)-12bcx\operatorname{arctanh}(cx)+4bc^3x^3\operatorname{arctanh}(cx)+12ac^2x^2}{}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
(d^3*(-2*a - 12*a*c*x - 2*b*c*x + 4*a*c^3*x^3 - 2*b*ArcTanh[c*x] - 12*b*c*
x*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] + 12*a*c^2*x^2*Log[x] + 12*b*c^2
*x^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x] - 4*b*c^2*
x^2*Log[1 - c^2*x^2] - 6*b*c^2*x^2*PolyLog[2, -(c*x)] + 6*b*c^2*x^2*PolyLo
g[2, c*x]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\operatorname{arctanh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(c^3d^3(a+b\operatorname{arctanh}(cx))+\frac{3c^2d^3(a+b\operatorname{arctanh}(cx))}{x}+\frac{d^3(a+b\operatorname{arctanh}(cx))}{x^3}+\frac{3cd^3(a+b\operatorname{arctanh}(cx))}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\text{arctanh}(cx))}{2x^2}-\frac{3cd^3(a+b\text{arctanh}(cx))}{x}+ac^3d^3x+3ac^2d^3\log(x)+bc^3d^3x\text{arctanh}(cx)+\frac{1}{2}bc^2d^3\text{arctanh}(cx)-\frac{3}{2}bc^2d^3\operatorname{PolyLog}(2,-cx)+\frac{3}{2}bc^2d^3\operatorname{PolyLog}(2,cx)-bc^2d^3\log\left(1-c^2x^2\right)+3bc^2d^3\log(x)-\frac{bcd^3}{2x}$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^3)/x + a*c^3*d^3*x + (b*c^2*d^3*ArcTanh[c*x])/2 + b*c^3*d^3*x*
ArcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/(2*x^2) - (3*c*d^3*(a + b*ArcTan
h[c*x]))/x + 3*a*c^2*d^3*Log[x] + 3*b*c^2*d^3*Log[x] - b*c^2*d^3*Log[1 - c
^2*x^2] - (3*b*c^2*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c^2*d^3*PolyLog[2, c*x
])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

method	result
parts	$d^3a\left(x\,c^3+3c^2\ln\left(x\right)-\frac{3c}{x}-\frac{1}{2x^2}\right)+d^3b\,c^2\Big(\text{arctanh}\left(cx\right)cx-\frac{\text{arctanh}(cx)}{2c^2x^2}+3\ \text{arctanh}\left(c\right.$
derivativedivides	$c^2\Big(d^3a\left(cx-\frac{1}{2c^2x^2}+3\ln\left(cx\right)-\frac{3}{cx}\right)+d^3b\Big(\text{arctanh}\left(cx\right)cx-\frac{\text{arctanh}(cx)}{2c^2x^2}+3\ \text{arctanh}\left(c\right.$
default	$c^2\Big(d^3a\left(cx-\frac{1}{2c^2x^2}+3\ln\left(cx\right)-\frac{3}{cx}\right)+d^3b\Big(\text{arctanh}\left(cx\right)cx-\frac{\text{arctanh}(cx)}{2c^2x^2}+3\ \text{arctanh}\left(c\right.$
risch	$-\frac{c^3d^3b\ln(-cx+1)x}{2}-\frac{5c^2d^3b\ln(-cx+1)}{4}-b\,c^2d^3+\frac{7c^2d^3b\ln(-cx)}{4}-\frac{bc\,d^3}{2x}+\frac{d^3b\ln(-cx+1)}{4x^2}+\frac{3c^2d^3b\,d}{}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(x*c^3+3*c^2*ln(x)-3*c/x-1/2/x^2)+d^3*b*c^2*(arctanh(c*x)*c*x-1/2*ar
ctanh(c*x)/c^2/x^2+3*arctanh(c*x)*ln(c*x)-3*arctanh(c*x)/c/x-3/2*dilog(c*x
)-3/2*dilog(c*x+1)-3/2*ln(c*x)*ln(c*x+1)-5/4*ln(c*x-1)-1/2/c/x+3*ln(c*x)-3
/4*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx=\int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d
^3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx=d^3\left(\int ac^3\,dx+\int \frac{a}{x^3}\,dx+\int \frac{3ac}{x^2}\,dx+\int \frac{3ac^2}{x}\,dx+\int bc^3\operatorname{atanh}(cx)\,dx+\int \frac{b\operatorname{atanh}(cx)}{x^3}\,dx+\int \frac{3bc\operatorname{atanh}(cx)}{x^2}\,dx+\int \frac{3bc^2\operatorname{atanh}(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**3,x)
```

output
```
d**3*(Integral(a*c**3, x) + Integral(a/x**3, x) + Integral(3*a*c/x**2, x)
+ Integral(3*a*c**2/x, x) + Integral(b*c**3*atanh(c*x), x) + Integral(b*at
anh(c*x)/x**3, x) + Integral(3*b*c*atanh(c*x)/x**2, x) + Integral(3*b*c**2
*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

output
```
a*c^3*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^2*d^3 + 3/2
*b*c^2*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 3*a*c^2*d^3*lo
g(x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^3 +
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^3
 - 3*a*c*d^3/x - 1/2*a*d^3/x^2
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^3}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^3,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$=\frac{d^3\left(2atanh(cx)\,b\,c^3x^3-3atanh(cx)\,b\,c^2x^2-6atanh(cx)\,bcx-atanh(cx)\,b+6\left(\int\frac{atanh(cx)}{x}dx\right)b\,c^2x^2-4\right.}{2x^2}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x^3,x)
```

output
```
(d**3*(2*atanh(c*x)*b*c**3*x**3 - 3*atanh(c*x)*b*c**2*x**2 - 6*atanh(c*x)*
b*c*x - atanh(c*x)*b + 6*int(atanh(c*x)/x,x)*b*c**2*x**2 - 4*log(c**2*x -
c)*b*c**2*x**2 + 6*log(x)*a*c**2*x**2 + 6*log(x)*b*c**2*x**2 + 2*a*c**3*x*
*3 - 6*a*c*x - a - b*c*x))/(2*x**2)
```

3.27 $\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 176

$$\begin{aligned}\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4} dx = &-\frac{bcd^3}{6x^2}-\frac{3bc^2d^3}{2x}+\frac{3}{2}bc^3d^3\text{arctanh}(cx)\\ &-\frac{d^3(a+b\text{arctanh}(cx))}{3x^3}\\ &-\frac{3cd^3(a+b\text{arctanh}(cx))}{2x^2}\\ &-\frac{3c^2d^3(a+b\text{arctanh}(cx))}{x}+ac^3d^3\log(x)\\ &+\frac{10}{3}bc^3d^3\log(x)-\frac{5}{3}bc^3d^3\log\left(1-c^2x^2\right)\\ &-\frac{1}{2}bc^3d^3\,\text{PolyLog}(2,-cx)\\ &+\frac{1}{2}bc^3d^3\,\text{PolyLog}(2,cx)\end{aligned}$$

output
```
-1/6*b*c*d^3/x^2-3/2*b*c^2*d^3/x+3/2*b*c^3*d^3*arctanh(c*x)-1/3*d^3*(a+b*a
rctanh(c*x))/x^3-3/2*c*d^3*(a+b*arctanh(c*x))/x^2-3*c^2*d^3*(a+b*arctanh(c
*x))/x+a*c^3*d^3*ln(x)+10/3*b*c^3*d^3*ln(x)-5/3*b*c^3*d^3*ln(-c^2*x^2+1)-1
/2*b*c^3*d^3*polylog(2,-c*x)+1/2*b*c^3*d^3*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$= \frac{d^3(-4a-18acx-2bcx-36ac^2x^2-18bc^2x^2-4b\text{arctanh}(cx)-18bcx\text{arctanh}(cx)-36bc^2x^2\text{arctanh}(c$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
(d^3*(-4*a - 18*a*c*x - 2*b*c*x - 36*a*c^2*x^2 - 18*b*c^2*x^2 - 4*b*ArcTan
h[c*x] - 18*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 12*a*c^3*x^3*
Log[x] + 40*b*c^3*x^3*Log[c*x] - 9*b*c^3*x^3*Log[1 - c*x] + 9*b*c^3*x^3*Lo
g[1 + c*x] - 20*b*c^3*x^3*Log[1 - c^2*x^2] - 6*b*c^3*x^3*PolyLog[2, -(c*x)
] + 6*b*c^3*x^3*PolyLog[2, c*x]))/(12*x^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{ 6502}$$
$$\int \left(\frac{c^3d^3(a+b\text{arctanh}(cx))}{x}+\frac{3c^2d^3(a+b\text{arctanh}(cx))}{x^2}+\frac{d^3(a+b\text{arctanh}(cx))}{x^4}+\frac{3cd^3(a+b\text{arctanh}(cx))}{x^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{3c^2d^3(a+b\operatorname{arctanh}(cx))}{x}-\frac{d^3(a+b\operatorname{arctanh}(cx))}{3x^3}-\frac{3cd^3(a+b\operatorname{arctanh}(cx))}{2x^2}+ac^3d^3\log(x)+\frac{3}{2}bc^3d^3\operatorname{arctanh}(cx)-\frac{1}{2}bc^3d^3\operatorname{PolyLog}(2,-cx)+\frac{1}{2}bc^3d^3\operatorname{PolyLog}(2,cx)+\frac{10}{3}bc^3d^3\log(x)-\frac{3bc^2d^3}{2x}-\frac{5}{3}bc^3d^3\log\left(1-c^2x^2\right)-\frac{bcd^3}{6x^2}$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
-1/6*(b*c*d^3)/x^2 - (3*b*c^2*d^3)/(2*x) + (3*b*c^3*d^3*ArcTanh[c*x])/2 -
(d^3*(a + b*ArcTanh[c*x]))/(3*x^3) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(2*x^2
) - (3*c^2*d^3*(a + b*ArcTanh[c*x]))/x + a*c^3*d^3*Log[x] + (10*b*c^3*d^3*
Log[x])/3 - (5*b*c^3*d^3*Log[1 - c^2*x^2])/3 - (b*c^3*d^3*PolyLog[2, -(c*x
)])/2 + (b*c^3*d^3*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.86

method	result
parts	$d^3a\left(c^3\ln(x)-\frac{3c^2}{x}-\frac{3c}{2x^2}-\frac{1}{3x^3}\right)+d^3b\,c^3\Big(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{3\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln$
derivativedivides	$c^3\Big(d^3a\left(-\frac{1}{3c^3x^3}-\frac{3}{2c^2x^2}+\ln(cx)-\frac{3}{cx}\right)+d^3b\Big(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{3\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)$
default	$c^3\Big(d^3a\left(-\frac{1}{3c^3x^3}-\frac{3}{2c^2x^2}+\ln(cx)-\frac{3}{cx}\right)+d^3b\Big(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{3\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)$
risch	$\frac{29c^3d^3b\ln(-cx)}{12}-\frac{3b\,c^2d^3}{2x}-\frac{29\ln(-cx+1)b\,c^3d^3}{12}+\frac{3c\,d^3b\ln(-cx+1)}{4x^2}+\frac{c^3d^3b\operatorname{dilog}(-cx+1)}{2}-\frac{b\,c\,d^3}{6x^2}+\frac{d^3b}{}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(c^3*ln(x)-3*c^2/x-3/2*c/x^2-1/3/x^3)+d^3*b*c^3*(-1/3*arctanh(c*x)/c
^3/x^3-3/2*arctanh(c*x)/c^2/x^2+arctanh(c*x)*ln(c*x)-3*arctanh(c*x)/c/x-29
/12*ln(c*x-1)-1/6/c^2/x^2-3/2/c/x+10/3*ln(c*x)-11/12*ln(c*x+1)-1/2*dilog(c
*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d
^3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx=d^3\left(\int \frac{a}{x^4}\,dx+\int \frac{3ac}{x^3}\,dx+\int \frac{3ac^2}{x^2}\,dx+\int \frac{ac^3}{x}\,dx+\int \frac{b\,\text{atanh}\,(cx)}{x^4}\,dx+\int \frac{3bc\,\text{atanh}\,(cx)}{x^3}\,dx+\int \frac{3bc^2\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{bc^3\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**4,x)
```

output
```
d**3*(Integral(a/x**4, x) + Integral(3*a*c/x**3, x) + Integral(3*a*c**2/x*
*2, x) + Integral(a*c**3/x, x) + Integral(b*atanh(c*x)/x**4, x) + Integral
(3*b*c*atanh(c*x)/x**3, x) + Integral(3*b*c**2*atanh(c*x)/x**2, x) + Integ
ral(b*c**3*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/2*b*c^3*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^3*d^3*l
og(x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^3
 + 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*
c*d^3 - 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c
*x)/x^3)*b*d^3 - 3*a*c^2*d^3/x - 3/2*a*c*d^3/x^2 - 1/3*a*d^3/x^3
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,(d+c\,d\,x)^3}{x^4}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^4,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^4, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$= \frac{d^3\left(-11atanh(cx)\,b\,c^3x^3-18atanh(cx)\,b\,c^2x^2-9atanh(cx)\,bcx-2atanh(cx)\,b+6\left(\int \frac{atanh(cx)}{x}dx\right)b\,c^3x^3\right.}{6x^3}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x^4,x)
```

output
```
(d**3*( - 11*atanh(c*x)*b*c**3*x**3 - 18*atanh(c*x)*b*c**2*x**2 - 9*atanh(
c*x)*b*c*x - 2*atanh(c*x)*b + 6*int(atanh(c*x)/x,x)*b*c**3*x**3 - 20*log(c
**2*x - c)*b*c**3*x**3 + 6*log(x)*a*c**3*x**3 + 20*log(x)*b*c**3*x**3 - 18
*a*c**2*x**2 - 9*a*c*x - 2*a - 9*b*c**2*x**2 - b*c*x))/(6*x**3)
```

3.28 $\int \frac{(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))}{x^5}\,dx = -\frac{bcd^3}{12x^3}-\frac{bc^2d^3}{2x^2}-\frac{7bc^3d^3}{4x}-\frac{d^3(1+cx)^4(a+b\,\mathrm{arctanh}(cx))}{4x^4}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)$$

output
```
-1/12*b*c*d^3/x^3-1/2*b*c^2*d^3/x^2-7/4*b*c^3*d^3/x-1/4*d^3*(c*x+1)^4*(a+b
*arctanh(c*x))/x^4+2*b*c^4*d^3*ln(x)-2*b*c^4*d^3*ln(-c*x+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int \frac{(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))}{x^5}\,dx = -\frac{d^3(6a+24acx+2bcx+36ac^2x^2+12bc^2x^2+24ac^3x^3+42bc^3x^3+6b(1+4cx+6c^2x^2+4c^3x^3)\,\mathrm{arcta}}{24x^4}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/24*(d^3*(6*a + 24*a*c*x + 2*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*
c^3*x^3 + 42*b*c^3*x^3 + 6*b*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3)*ArcTanh[c
*x] - 48*b*c^4*x^4*Log[x] + 45*b*c^4*x^4*Log[1 - c*x] + 3*b*c^4*x^4*Log[1
+ c*x]))/x^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))}{x^5}dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int -\frac{d^3(cx+1)^3}{4x^4(1-cx)}dx-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}bcd^3\int \frac{(cx+1)^3}{x^4(1-cx)}dx-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{4}bcd^3\int \left(-\frac{8c^4}{cx-1}+\frac{8c^3}{x}+\frac{7c^2}{x^2}+\frac{4c}{x^3}+\frac{1}{x^4}\right)dx-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4x^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}bcd^3\left(8c^3\log(x)-8c^3\log(1-cx)-\frac{7c^2}{x}-\frac{2c}{x^2}-\frac{1}{3x^3}\right)-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{4x^4}$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/4*(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/x^4 + (b*c*d^3*(-1/3*1/x^3 - (
2*c)/x^2 - (7*c^2)/x + 8*c^3*Log[x] - 8*c^3*Log[1 - c*x]))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.48

method	result
parts	$d^{3} a \left(-\frac{1}{4 x^{4}}-\frac{c^{3}}{x}-\frac{3 c^{2}}{2 x^{2}}-\frac{c}{x^{3}}\right)+d^{3} b\, c^{4} \bigl(-\frac{\operatorname{arctanh}\left(c x \right)}{c^{3} x^{3}}-\frac{3 \,\operatorname{arctanh}\left(c x \right)}{2 c^{2} x^{2}}-\frac{\operatorname{arctanh}\left(c x \right)}{c x}-\frac{\operatorname{arctanh}\left(c x \right)}{4 c^{4} x^{4}}$
derivativedivides	$c^{4} \bigl(d^{3} a \left(-\frac{1}{c^{3} x^{3}}-\frac{3}{2 c^{2} x^{2}}-\frac{1}{c x}-\frac{1}{4 c^{4} x^{4}}\right)+d^{3} b \bigl(-\frac{\operatorname{arctanh}\left(c x \right)}{c^{3} x^{3}}-\frac{3 \,\operatorname{arctanh}\left(c x \right)}{2 c^{2} x^{2}}-\frac{\operatorname{arctanh}\left(c x \right)}{c x}-$ arc
default	$c^{4} \bigl(d^{3} a \left(-\frac{1}{c^{3} x^{3}}-\frac{3}{2 c^{2} x^{2}}-\frac{1}{c x}-\frac{1}{4 c^{4} x^{4}}\right)+d^{3} b \bigl(-\frac{\operatorname{arctanh}\left(c x \right)}{c^{3} x^{3}}-\frac{3 \,\operatorname{arctanh}\left(c x \right)}{2 c^{2} x^{2}}-\frac{\operatorname{arctanh}\left(c x \right)}{c x}-$ arc
risch	$-\frac{d^{3} b \left(4 x^{3} c^{3}+6 c^{2} x^{2}+4 c x +1\right) \ln \left(c x +1\right)}{8 x^{4}}+$ $d^{3} \bigl(48 b\, c^{4} \ln \left(-x \right) x^{4}-3 b\, c^{4} \ln \left(c x +1\right) x^{4}-45 b\, x^{4} \ln \left(-c x +1\right) c^{4}+12 b\, x^{3} \ln($
parallelrisch	$24 b\, c^{4} d^{3} \ln \left(x \right) x^{4}-24 \ln \left(c x -1\right) x^{4} b\, c^{4} d^{3}-3 d^{3} b \,\operatorname{arctanh}\left(c x \right) x^{4} c^{4}-18 a\, c^{4} d^{3} x^{4}-6 b\, c^{4} d^{3} x^{4}-12 d^{3} b \,\operatorname{arctanh}\left(c x \right) x^{3} c^{3}-12 a$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/4/x^4-c^3/x-3/2*c^2/x^2-c/x^3)+d^3*b*c^4*(-arctanh(c*x)/c^3/x^3-
3/2*arctanh(c*x)/c^2/x^2-arctanh(c*x)/c/x-1/4*arctanh(c*x)/c^4/x^4-15/8*ln
(c*x-1)-1/12/c^3/x^3-1/2/c^2/x^2-7/4/c/x+2*ln(c*x)-1/8*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.75

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^5}\,dx = -\frac{3\,bc^4d^3x^4\log(cx+1)+45\,bc^4d^3x^4\log(cx-1)-48\,bc^4d^3x^4\log(x)+6\,(4\,a+7\,b)c^3d^3x^3+12\,(3\,a+b}{24\,x^4}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

output
```
-1/24*(3*b*c^4*d^3*x^4*log(c*x + 1) + 45*b*c^4*d^3*x^4*log(c*x - 1) - 48*b
*c^4*d^3*x^4*log(x) + 6*(4*a + 7*b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2
 + 2*(12*a + b)*c*d^3*x + 6*a*d^3 + 3*(4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 +
 4*b*c*d^3*x + b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(92) = 184.

Time = 0.54 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.23

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^5}\,dx = \begin{cases} -\frac{ac^3d^3}{x}-\frac{3ac^2d^3}{2x^2}-\frac{acd^3}{x^3}-\frac{ad^3}{4x^4}+2bc^4d^3\log(x)-2bc^4d^3\log\left(x-\frac{1}{c}\right)-\frac{bc^4d^3\,\text{atanh}(cx)}{4}-\frac{bc^3d^3\,\text{atanh}(cx)}{x}-\frac{7bc^3}{4x} \\ -\frac{ad^3}{4x^4} \end{cases}$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**5,x)
```

output
```
Piecewise((-a*c**3*d**3/x - 3*a*c**2*d**3/(2*x**2) - a*c*d**3/x**3 - a*d**
3/(4*x**4) + 2*b*c**4*d**3*log(x) - 2*b*c**4*d**3*log(x - 1/c) - b*c**4*d*
*3*atanh(c*x)/4 - b*c**3*d**3*atanh(c*x)/x - 7*b*c**3*d**3/(4*x) - 3*b*c**
2*d**3*atanh(c*x)/(2*x**2) - b*c**2*d**3/(2*x**2) - b*c*d**3*atanh(c*x)/x*
*3 - b*c*d**3/(12*x**3) - b*d**3*atanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**3/
(4*x**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(85) = 170$.

Time = 0.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.45

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$= -\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\,(cx)}{x}\right)bc^3d^3 + \frac{3}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\,(cx)}{x^2}\right)bc^2d^3 - \frac{1}{2}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}\,(cx)}{x^3}\right)bcd^3 - \frac{ac^3d^3}{x} + \frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\text{artanh}\,(cx)}{x^4}\right)bd^3 - \frac{3\,ac^2d^3}{2\,x^2} - \frac{acd^3}{x^3} - \frac{ad^3}{4\,x^4}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^3 + 3/4*
((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d^3
 - 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x
^3)*b*c*d^3 - a*c^3*d^3/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1)
 - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^3 - 3/2*a*c^2*d^3/x^
2 - a*c*d^3/x^3 - 1/4*a*d^3/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(85) = 170$.

Time = 0.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.63

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{1}{3}\left(6\,bc^3d^3\log\left(-\frac{cx+1}{cx-1}-1\right)-6\,bc^3d^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{6\left(\frac{4\,(cx+1)^3bc^3d^3}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3d^3}{(cx-1)^2}+\frac{4\,(cx+1)bc^3d^3}{cx-1}\right.}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+4}\right.$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

output
```
1/3*(6*b*c^3*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 6*b*c^3*d^3*log(-(c*x + 1
)/(c*x - 1)) + 6*(4*(c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 + 6*(c*x + 1)^2*b*c^
3*d^3/(c*x - 1)^2 + 4*(c*x + 1)*b*c^3*d^3/(c*x - 1) + b*c^3*d^3)*log(-(c*x
 + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*
(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (48*(c*x + 1)^3*a*c
^3*d^3/(c*x - 1)^3 + 72*(c*x + 1)^2*a*c^3*d^3/(c*x - 1)^2 + 48*(c*x + 1)*a
*c^3*d^3/(c*x - 1) + 12*a*c^3*d^3 + 18*(c*x + 1)^3*b*c^3*d^3/(c*x - 1)^3 +
 45*(c*x + 1)^2*b*c^3*d^3/(c*x - 1)^2 + 38*(c*x + 1)*b*c^3*d^3/(c*x - 1) +
 11*b*c^3*d^3)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c
*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.58

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{d^3\,(21\,b\,c^4\,\text{atanh}(c\,x)-12\,b\,c^4\,\ln(c^2\,x^2-1)+24\,b\,c^4\,\ln(x))}{12}$$
$$-\frac{\frac{d^3\,(3\,a+3\,b\,\text{atanh}(c\,x))}{12}+\frac{d^3\,x\,(12\,a\,c+b\,c+12\,b\,c\,\text{atanh}(c\,x))}{12}+\frac{d^3\,x^2\,(18\,a\,c^2+6\,b\,c^2+18\,b\,c^2\,\text{atanh}(c\,x))}{12}+\frac{d^3\,x^3\,(12\,a\,c^3+21\,b\,c^3+12\,b\,c}{12}}{x^4}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^5,x)
```

output
```
(d^3*(21*b*c^4*atanh(c*x) - 12*b*c^4*log(c^2*x^2 - 1) + 24*b*c^4*log(x)))/
12 - ((d^3*(3*a + 3*b*atanh(c*x)))/12 + (d^3*x*(12*a*c + b*c + 12*b*c*atan
h(c*x)))/12 + (d^3*x^2*(18*a*c^2 + 6*b*c^2 + 18*b*c^2*atanh(c*x)))/12 + (d
^3*x^3*(12*a*c^3 + 21*b*c^3 + 12*b*c^3*atanh(c*x)))/12)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{d^3(-3atanh(cx)\,b\,c^4x^4-12atanh(cx)\,b\,c^3x^3-18atanh(cx)\,b\,c^2x^2-12atanh(cx)\,bcx-3atanh(cx)\,b-2\cdots}{12\cdots}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x^5,x)
```

output
```
(d**3*( - 3*atanh(c*x)*b*c**4*x**4 - 12*atanh(c*x)*b*c**3*x**3 - 18*atanh(
c*x)*b*c**2*x**2 - 12*atanh(c*x)*b*c*x - 3*atanh(c*x)*b - 24*log(c**2*x -
c)*b*c**4*x**4 + 24*log(x)*b*c**4*x**4 - 12*a*c**3*x**3 - 18*a*c**2*x**2 -
 12*a*c*x - 3*a - 21*b*c**3*x**3 - 6*b*c**2*x**2 - b*c*x))/(12*x**4)
```

3.29 $\int \frac{(d+cdx)^3(a+b\textbf{arctanh}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\begin{aligned}\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^6}\,dx =& -\frac{bcd^3}{20x^4}-\frac{bc^2d^3}{4x^3}-\frac{3bc^3d^3}{5x^2}-\frac{5bc^4d^3}{4x}\\ &-\frac{d^3(1+cx)^4(a+b\text{arctanh}(cx))}{5x^5}\\ &+\frac{cd^3(1+cx)^4(a+b\text{arctanh}(cx))}{20x^4}\\ &+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\log(1-cx)\end{aligned}$$

output
```
-1/20*b*c*d^3/x^4-1/4*b*c^2*d^3/x^3-3/5*b*c^3*d^3/x^2-5/4*b*c^4*d^3/x-1/5*
d^3*(c*x+1)^4*(a+b*arctanh(c*x))/x^5+1/20*c*d^3*(c*x+1)^4*(a+b*arctanh(c*x
))/x^4+6/5*b*c^5*d^3*ln(x)-6/5*b*c^5*d^3*ln(-c*x+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^6}\,dx = -\frac{d^3(8a+30acx+2bcx+40ac^2x^2+10bc^2x^2+20ac^3x^3+24bc^3x^3+50bc^4x^4+2b(4+15cx+20c^2x^2+}{40x^5}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

output
```
-1/40*(d^3*(8*a + 30*a*c*x + 2*b*c*x + 40*a*c^2*x^2 + 10*b*c^2*x^2 + 20*a*
c^3*x^3 + 24*b*c^3*x^3 + 50*b*c^4*x^4 + 2*b*(4 + 15*c*x + 20*c^2*x^2 + 10*
c^3*x^3)*ArcTanh[c*x] - 48*b*c^5*x^5*Log[x] + 49*b*c^5*x^5*Log[1 - c*x] -
b*c^5*x^5*Log[1 + c*x]))/x^5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))}{x^6}\,dx$$

$$\Big\downarrow \ 6498$$

$$-bc\int -\frac{d^3(4-cx)(cx+1)^3}{20x^5(1-cx)}dx-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{5x^5}+\frac{cd^3(cx+1)^4(a+b\text{arctanh}(cx))}{20x^4}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{20}bcd^3\int \frac{(4-cx)(cx+1)^3}{x^5(1-cx)}dx-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{5x^5}+\frac{cd^3(cx+1)^4(a+b\text{arctanh}(cx))}{20x^4}$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{20}bcd^3\int\left(-\frac{24c^5}{cx-1}+\frac{24c^4}{x}+\frac{25c^3}{x^2}+\frac{24c^2}{x^3}+\frac{15c}{x^4}+\frac{4}{x^5}\right)dx-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{5x^5}+\frac{cd^3(cx+1)^4(a+b\text{arctanh}(cx))}{20x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))}{5x^5}+\frac{cd^3(cx+1)^4(a+b\text{arctanh}(cx))}{20x^4}+\frac{1}{20}bcd^3\left(24c^4\log(x)-24c^4\log(1-cx)-\frac{25c^3}{x}-\frac{12c^2}{x^2}-\frac{5c}{x^3}-\frac{1}{x^4}\right)$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

output
```
-1/5*(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/x^5 + (c*d^3*(1 + c*x)^4*(a +
b*ArcTanh[c*x]))/(20*x^4) + (b*c*d^3*(-x^(-4) - (5*c)/x^3 - (12*c^2)/x^2 -
 (25*c^3)/x + 24*c^4*Log[x] - 24*c^4*Log[1 - c*x]))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

method	result
parts	$d^3 a\left(-\frac{3c}{4x^4}-\frac{1}{5x^5}-\frac{c^3}{2x^2}-\frac{c^2}{x^3}\right)+d^3 b\,c^5\left(-\frac{\operatorname{arctanh}(cx)}{c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{3\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{\operatorname{arctanh}(}{5c^5x^5}\right.$
derivativedivides	$c^5\left(d^3a\left(-\frac{1}{c^3x^3}-\frac{1}{2c^2x^2}-\frac{3}{4c^4x^4}-\frac{1}{5c^5x^5}\right)+d^3b\left(-\frac{\operatorname{arctanh}(cx)}{c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{3\operatorname{arctanh}(cx)}{4c^4x^4}-\right.\right.$
default	$c^5\left(d^3a\left(-\frac{1}{c^3x^3}-\frac{1}{2c^2x^2}-\frac{3}{4c^4x^4}-\frac{1}{5c^5x^5}\right)+d^3b\left(-\frac{\operatorname{arctanh}(cx)}{c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{3\operatorname{arctanh}(cx)}{4c^4x^4}-\right.\right.$
risch	$-\frac{d^3b(10x^3c^3+20c^2x^2+15cx+4)\ln(cx+1)}{40x^5}+\frac{d^3(b\,c^5\ln(cx+1)x^5+48b\,c^5\ln(-x)x^5-49b\,c^5x^5\ln(-cx+1)-50b\,c^4x^4-}{}$
parallelrisch	$\frac{24b\,c^5d^3\ln(x)x^5-24\ln(cx-1)x^5b\,c^5d^3+b\,c^5d^3\operatorname{arctanh}(cx)x^5-10a\,c^5d^3x^5-12b\,c^5d^3x^5-25b\,c^4d^3x^4-10d^3b\operatorname{arctanh}(}{}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-3/4*c/x^4-1/5/x^5-1/2*c^3/x^2-c^2/x^3)+d^3*b*c^5*(-arctanh(c*x)/c^
3/x^3-1/2*arctanh(c*x)/c^2/x^2-3/4*arctanh(c*x)/c^4/x^4-1/5*arctanh(c*x)/c
^5/x^5-49/40*ln(c*x-1)-1/20/c^4/x^4-1/4/c^3/x^3-3/5/c^2/x^2-5/4/c/x+6/5*ln
(c*x)+1/40*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int\frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))}{x^6}\,dx$$
$$=\frac{bc^5d^3x^5\log(cx+1)-49\,bc^5d^3x^5\log(cx-1)+48\,bc^5d^3x^5\log(x)-50\,bc^4d^3x^4-4\,(5\,a+6\,b)c^3d^3x^3-1}{40}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/40*(b*c^5*d^3*x^5*log(c*x + 1) - 49*b*c^5*d^3*x^5*log(c*x - 1) + 48*b*c^
5*d^3*x^5*log(x) - 50*b*c^4*d^3*x^4 - 4*(5*a + 6*b)*c^3*d^3*x^3 - 10*(4*a
+ b)*c^2*d^3*x^2 - 2*(15*a + b)*c*d^3*x - 8*a*d^3 - (10*b*c^3*d^3*x^3 + 20
*b*c^2*d^3*x^2 + 15*b*c*d^3*x + 4*b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.70

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \begin{cases} -\frac{ac^3d^3}{2x^2} - \frac{ac^2d^3}{x^3} - \frac{3acd^3}{4x^4} - \frac{ad^3}{5x^5} + \frac{6bc^5d^3\log(x)}{5} - \frac{6bc^5d^3\log\left(x-\frac{1}{c}\right)}{5} + \frac{bc^5d^3\,\text{atanh}(cx)}{20} - \frac{5bc^4d^3}{4x} - \frac{bc^3d^3\,\text{atanh}(cx)}{2x^2} - \\ -\frac{ad^3}{5x^5} \end{cases}$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**6,x)
```

output
```
Piecewise((-a*c**3*d**3/(2*x**2) - a*c**2*d**3/x**3 - 3*a*c*d**3/(4*x**4)
- a*d**3/(5*x**5) + 6*b*c**5*d**3*log(x)/5 - 6*b*c**5*d**3*log(x - 1/c)/5
+ b*c**5*d**3*atanh(c*x)/20 - 5*b*c**4*d**3/(4*x) - b*c**3*d**3*atanh(c*x)
/(2*x**2) - 3*b*c**3*d**3/(5*x**2) - b*c**2*d**3*atanh(c*x)/x**3 - b*c**2*
d**3/(4*x**3) - 3*b*c*d**3*atanh(c*x)/(4*x**4) - b*c*d**3/(20*x**4) - b*d*
*3*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**3/(5*x**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(121) = 242$.

Time = 0.03 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.82

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{1}{4}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\text{artanh}(cx)}{x^2}\right)bc^3d^3$$
$$- \frac{1}{2}\left(\left(c^2\log(c^2x^2-1) - c^2\log(x^2) + \frac{1}{x^2}\right)c + \frac{2\,\text{artanh}(cx)}{x^3}\right)bc^2d^3$$
$$+ \frac{1}{8}\left(\left(3\,c^3\log(cx+1) - 3\,c^3\log(cx-1) - \frac{2\,(3\,c^2x^2+1)}{x^3}\right)c - \frac{6\,\text{artanh}(cx)}{x^4}\right)bcd^3$$
$$- \frac{1}{20}\left(\left(2\,c^4\log(c^2x^2-1) - 2\,c^4\log(x^2) + \frac{2\,c^2x^2+1}{x^4}\right)c + \frac{4\,\text{artanh}(cx)}{x^5}\right)bd^3$$
$$- \frac{ac^3d^3}{2\,x^2} - \frac{ac^2d^3}{x^3} - \frac{3\,acd^3}{4\,x^4} - \frac{ad^3}{5\,x^5}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3
*d^3 - 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*
x)/x^3)*b*c^2*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c
^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^3 - 1/20*((2*c^4*log(c^2*x^
2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d
^3 - 1/2*a*c^3*d^3/x^2 - a*c^2*d^3/x^3 - 3/4*a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(121) = 242.

Time = 0.12 (sec) , antiderivative size = 533, normalized size of antiderivative = 3.89

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{1}{5}\left(6\,bc^4d^3\log\left(-\frac{cx+1}{cx-1} - 1\right) - 6\,bc^4d^3\log\left(-\frac{cx+1}{cx-1}\right) + \frac{2\left(\frac{20\,(cx+1)^4bc^4d^3}{(cx-1)^4} + \frac{30\,(cx+1)^3bc^4d^3}{(cx-1)^3} + \frac{30\,(cx+1)^2bc}{(cx-1)^2}\right.}{\frac{(cx+1)^5}{(cx-1)^5} + \frac{5\,(cx+1)^4}{(cx-1)^4} + \frac{10\,(cx+1)^3}{(cx-1)^3}}\right.$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

output
```
1/5*(6*b*c^4*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 6*b*c^4*d^3*log(-(c*x + 1
)/(c*x - 1)) + 2*(20*(c*x + 1)^4*b*c^4*d^3/(c*x - 1)^4 + 30*(c*x + 1)^3*b*
c^4*d^3/(c*x - 1)^3 + 30*(c*x + 1)^2*b*c^4*d^3/(c*x - 1)^2 + 15*(c*x + 1)*
b*c^4*d^3/(c*x - 1) + 3*b*c^4*d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/
(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*
(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (80*(c*x + 1)^4*a*c
^4*d^3/(c*x - 1)^4 + 120*(c*x + 1)^3*a*c^4*d^3/(c*x - 1)^3 + 120*(c*x + 1)
^2*a*c^4*d^3/(c*x - 1)^2 + 60*(c*x + 1)*a*c^4*d^3/(c*x - 1) + 12*a*c^4*d^3
 + 34*(c*x + 1)^4*b*c^4*d^3/(c*x - 1)^4 + 103*(c*x + 1)^3*b*c^4*d^3/(c*x -
 1)^3 + 123*(c*x + 1)^2*b*c^4*d^3/(c*x - 1)^2 + 69*(c*x + 1)*b*c^4*d^3/(c*
x - 1) + 15*b*c^4*d^3)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^
4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/
(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.70

$$\int \frac{(d + cdx)^3(a + b\text{arctanh}(cx))}{x^6} \, dx =$$

$$-\frac{4\,a\,d^3 + 4\,b\,d^3\,\text{atanh}(c\,x) + 20\,a\,c^2\,d^3\,x^2 + 10\,a\,c^3\,d^3\,x^3 + 10\,a\,c^5\,d^3\,x^5 + 5\,b\,c^2\,d^3\,x^2 + 12\,b\,c^3\,d^3\,x^3 + 25}{}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^6,x)
```

output
```
-(4*a*d^3 + 4*b*d^3*atanh(c*x) + 20*a*c^2*d^3*x^2 + 10*a*c^3*d^3*x^3 + 10*
a*c^5*d^3*x^5 + 5*b*c^2*d^3*x^2 + 12*b*c^3*d^3*x^3 + 25*b*c^4*d^3*x^4 + 12
*b*c^5*d^3*x^5 + 15*a*c*d^3*x + b*c*d^3*x - 24*b*c^5*d^3*x^5*log(x) + 20*b
*c^2*d^3*x^2*atanh(c*x) + 10*b*c^3*d^3*x^3*atanh(c*x) + 12*b*c^5*d^3*x^5*l
og(c^2*x^2 - 1) + 15*b*c*d^3*x*atanh(c*x) - 25*b*c^4*d^3*x^5*atan((c^2*x)/
(-c^2)^(1/2))*(-c^2)^(1/2))/(20*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.10

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{d^3(atanh(cx)\,b\,c^5x^5 - 10atanh(cx)\,b\,c^3x^3 - 20atanh(cx)\,b\,c^2x^2 - 15atanh(cx)\,bcx - 4atanh(cx)\,b - 24\,\mathrm{lo}}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x^6,x)
```

output
```
(d**3*(atanh(c*x)*b*c**5*x**5 - 10*atanh(c*x)*b*c**3*x**3 - 20*atanh(c*x)*
b*c**2*x**2 - 15*atanh(c*x)*b*c*x - 4*atanh(c*x)*b - 24*log(c**2*x - c)*b*
c**5*x**5 + 24*log(x)*b*c**5*x**5 - 10*a*c**3*x**3 - 20*a*c**2*x**2 - 15*a
*c*x - 4*a - 25*b*c**4*x**4 - 12*b*c**3*x**3 - 5*b*c**2*x**2 - b*c*x))/(20
*x**5)
```

3.30 $\int \frac{(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\begin{aligned}\int \frac{(d+cdx)^3(a+b\mathrm{arctanh}(cx))}{x^7}\,dx = &-\frac{bcd^3}{30x^5}-\frac{3bc^2d^3}{20x^4}-\frac{11bc^3d^3}{36x^3}-\frac{7bc^4d^3}{15x^2}\\ &-\frac{11bc^5d^3}{12x}-\frac{d^3(a+b\mathrm{arctanh}(cx))}{6x^6}\\ &-\frac{3cd^3(a+b\mathrm{arctanh}(cx))}{5x^5}\\ &-\frac{3c^2d^3(a+b\mathrm{arctanh}(cx))}{4x^4}\\ &-\frac{c^3d^3(a+b\mathrm{arctanh}(cx))}{3x^3}+\frac{14}{15}bc^6d^3\log(x)\\ &-\frac{37}{40}bc^6d^3\log(1-cx)-\frac{1}{120}bc^6d^3\log(1+cx)\end{aligned}$$

output
```
-1/30*b*c*d^3/x^5-3/20*b*c^2*d^3/x^4-11/36*b*c^3*d^3/x^3-7/15*b*c^4*d^3/x^
2-11/12*b*c^5*d^3/x-1/6*d^3*(a+b*arctanh(c*x))/x^6-3/5*c*d^3*(a+b*arctanh(
c*x))/x^5-3/4*c^2*d^3*(a+b*arctanh(c*x))/x^4-1/3*c^3*d^3*(a+b*arctanh(c*x)
)/x^3+14/15*b*c^6*d^3*ln(x)-37/40*b*c^6*d^3*ln(-c*x+1)-1/120*b*c^6*d^3*ln(
c*x+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^7}\,dx = -\frac{d^3(60a+216acx+12bcx+270ac^2x^2+54bc^2x^2+120ac^3x^3+110bc^3x^3+168bc^4x^4+330bc^5x^5+6b(\cdots}{3\cdots}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

output
```
-1/360*(d^3*(60*a + 216*a*c*x + 12*b*c*x + 270*a*c^2*x^2 + 54*b*c^2*x^2 +
120*a*c^3*x^3 + 110*b*c^3*x^3 + 168*b*c^4*x^4 + 330*b*c^5*x^5 + 6*b*(10 +
36*c*x + 45*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x] - 336*b*c^6*x^6*Log[x] + 33
3*b*c^6*x^6*Log[1 - c*x] + 3*b*c^6*x^6*Log[1 + c*x]))/x^6
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))}{x^7}dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d^3\left(20c^3x^3+45c^2x^2+36cx+10\right)}{60x^6\left(1-c^2x^2\right)}dx-\frac{c^3d^3(a+b\text{arctanh}(cx))}{3x^3}-\frac{3c^2d^3(a+b\text{arctanh}(cx))}{4x^4}-\frac{d^3(a+b\text{arctanh}(cx))}{6x^6}-\frac{3cd^3(a+b\text{arctanh}(cx))}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{60}bcd^3\int \frac{20c^3x^3+45c^2x^2+36cx+10}{x^6\left(1-c^2x^2\right)}dx-\frac{c^3d^3(a+b\text{arctanh}(cx))}{3x^3}-\frac{3c^2d^3(a+b\text{arctanh}(cx))}{4x^4}-\frac{d^3(a+b\text{arctanh}(cx))}{6x^6}-\frac{3cd^3(a+b\text{arctanh}(cx))}{5x^5}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{60}bcd^3\int\left(-\frac{111c^6}{2(cx-1)}-\frac{c^6}{2(cx+1)}+\frac{56c^5}{x}+\frac{55c^4}{x^2}+\frac{56c^3}{x^3}+\frac{55c^2}{x^4}+\frac{36c}{x^5}+\frac{10}{x^6}\right)dx-\frac{c^3d^3(a+b\text{arctanh}(cx))}{3x^3}-\frac{3c^2d^3(a+b\text{arctanh}(cx))}{4x^4}-\frac{d^3(a+b\text{arctanh}(cx))}{6x^6}-\frac{3cd^3(a+b\text{arctanh}(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3d^3(a+b\text{arctanh}(cx))}{3x^3}-\frac{3c^2d^3(a+b\text{arctanh}(cx))}{4x^4}-\frac{d^3(a+b\text{arctanh}(cx))}{6x^6}-\frac{3cd^3(a+b\text{arctanh}(cx))}{5x^5}+\frac{1}{60}bcd^3\left(56c^5\log(x)-\frac{111}{2}c^5\log(1-cx)-\frac{1}{2}c^5\log(cx+1)-\frac{55c^4}{x}-\frac{28c^3}{x^2}-\frac{55c^2}{3x^3}-\frac{9c}{x^4}-\frac{2}{x^5}\right)$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

output
```
-1/6*(d^3*(a + b*ArcTanh[c*x]))/x^6 - (3*c*d^3*(a + b*ArcTanh[c*x]))/(5*x^
5) - (3*c^2*d^3*(a + b*ArcTanh[c*x]))/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*
x]))/(3*x^3) + (b*c*d^3*(-2/x^5 - (9*c)/x^4 - (55*c^2)/(3*x^3) - (28*c^3)/
x^2 - (55*c^4)/x + 56*c^5*Log[x] - (111*c^5*Log[1 - c*x])/2 - (c^5*Log[1 +
 c*x])/2))/60
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.79

method	result
parts	$d^3a\left(-\frac{1}{6x^6}-\frac{3c^2}{4x^4}-\frac{3c}{5x^5}-\frac{c^3}{3x^3}\right)+d^3b\,c^6\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{3\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{3\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{\operatorname{arctan}}{6c^6}\right.$
derivativedivides	$c^6\left(d^3a\left(-\frac{1}{3c^3x^3}-\frac{3}{4c^4x^4}-\frac{3}{5c^5x^5}-\frac{1}{6c^6x^6}\right)+d^3b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{3\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{3\operatorname{arctanh}(cx)}{5c^5x^5}\right.\right.$
default	$c^6\left(d^3a\left(-\frac{1}{3c^3x^3}-\frac{3}{4c^4x^4}-\frac{3}{5c^5x^5}-\frac{1}{6c^6x^6}\right)+d^3b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{3\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{3\operatorname{arctanh}(cx)}{5c^5x^5}\right.\right.$
risch	$-\frac{d^3b(20x^3c^3+45c^2x^2+36cx+10)\ln(cx+1)}{120x^6}+\frac{d^3(336b\,c^6\ln(-x)x^6-333b\,c^6x^6\ln(-cx+1)-3b\,c^6\ln(cx+1)x^6-330b}{}$
parallelrisch	$\frac{168b\,c^6d^3\ln(x)x^6-168\ln(cx-1)x^6b\,c^6d^3-3b\,c^6d^3\operatorname{arctanh}(cx)x^6-84c^6d^3x^6b-165b\,c^5d^3x^5-84b\,c^4d^3x^4-60d^3b\operatorname{arcta}}{}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/6/x^6-3/4*c^2/x^4-3/5*c/x^5-1/3*c^3/x^3)+d^3*b*c^6*(-1/3*arctanh
(c*x)/c^3/x^3-3/4*arctanh(c*x)/c^4/x^4-3/5*arctanh(c*x)/c^5/x^5-1/6*arctan
h(c*x)/c^6/x^6-37/40*ln(c*x-1)-1/30/c^5/x^5-3/20/c^4/x^4-11/36/c^3/x^3-7/1
5/c^2/x^2-11/12/c/x+14/15*ln(c*x)-1/120*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int\frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))}{x^7}\,dx=$$

$$-\frac{3\,bc^6d^3x^6\log(cx+1)+333\,bc^6d^3x^6\log(cx-1)-336\,bc^6d^3x^6\log(x)+330\,bc^5d^3x^5+168\,bc^4d^3x^4+1}{}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

output
```
-1/360*(3*b*c^6*d^3*x^6*log(c*x + 1) + 333*b*c^6*d^3*x^6*log(c*x - 1) - 33
6*b*c^6*d^3*x^6*log(x) + 330*b*c^5*d^3*x^5 + 168*b*c^4*d^3*x^4 + 10*(12*a
+ 11*b)*c^3*d^3*x^3 + 54*(5*a + b)*c^2*d^3*x^2 + 12*(18*a + b)*c*d^3*x + 6
0*a*d^3 + 3*(20*b*c^3*d^3*x^3 + 45*b*c^2*d^3*x^2 + 36*b*c*d^3*x + 10*b*d^3
)*log(-(c*x + 1)/(c*x - 1)))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.31

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^7}\,dx$$
$$= \begin{cases} -\frac{ac^3d^3}{3x^3} - \frac{3ac^2d^3}{4x^4} - \frac{3acd^3}{5x^5} - \frac{ad^3}{6x^6} + \frac{14bc^6d^3\log(x)}{15} - \frac{14bc^6d^3\log\left(x-\frac{1}{c}\right)}{15} - \frac{bc^6d^3\,\text{atanh}(cx)}{60} - \frac{11bc^5d^3}{12x} - \frac{7bc^4d^3}{15x^2} - \frac{bc^3d^3\,\text{at}}{3x} \\ -\frac{ad^3}{6x^6} \end{cases}$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**7,x)
```

output
```
Piecewise((-a*c**3*d**3/(3*x**3) - 3*a*c**2*d**3/(4*x**4) - 3*a*c*d**3/(5*
x**5) - a*d**3/(6*x**6) + 14*b*c**6*d**3*log(x)/15 - 14*b*c**6*d**3*log(x
- 1/c)/15 - b*c**6*d**3*atanh(c*x)/60 - 11*b*c**5*d**3/(12*x) - 7*b*c**4*d
**3/(15*x**2) - b*c**3*d**3*atanh(c*x)/(3*x**3) - 11*b*c**3*d**3/(36*x**3)
 - 3*b*c**2*d**3*atanh(c*x)/(4*x**4) - 3*b*c**2*d**3/(20*x**4) - 3*b*c*d**
3*atanh(c*x)/(5*x**5) - b*c*d**3/(30*x**5) - b*d**3*atanh(c*x)/(6*x**6), N
e(c, 0)), (-a*d**3/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.39

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^7}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}\,(cx)}{x^3}\right)bc^3d^3$$
$$+\frac{1}{8}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\,(3\,c^2x^2+1)}{x^3}\right)c-\frac{6\,\text{artanh}\,(cx)}{x^4}\right)bc^2d^3$$
$$-\frac{3}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\text{artanh}\,(cx)}{x^5}\right)bcd^3$$
$$+\frac{1}{180}\left(\left(15\,c^5\log\left(cx+1\right)-15\,c^5\log\left(cx-1\right)-\frac{2\,(15\,c^4x^4+5\,c^2x^2+3)}{x^5}\right)c-\frac{30\,\text{artanh}\,(cx)}{x^6}\right)bd^3$$
$$-\frac{ac^3d^3}{3\,x^3}-\frac{3\,ac^2d^3}{4\,x^4}-\frac{3\,acd^3}{5\,x^5}-\frac{ad^3}{6\,x^6}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*c^3*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2
 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^3 - 3/20*((2*c^4*log(c^2*x^2 -
1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^3
 + 1/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c
^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^3 - 1/3*a*c^3*d^3/x^3 - 3/4*
a*c^2*d^3/x^4 - 3/5*a*c*d^3/x^5 - 1/6*a*d^3/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(172) = 344.

Time = 0.13 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.23

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

output
```
1/45*(42*b*c^5*d^3*log(-(c*x + 1)/(c*x - 1) - 1) - 42*b*c^5*d^3*log(-(c*x
+ 1)/(c*x - 1)) + 6*(60*(c*x + 1)^5*b*c^5*d^3/(c*x - 1)^5 + 90*(c*x + 1)^4
*b*c^5*d^3/(c*x - 1)^4 + 140*(c*x + 1)^3*b*c^5*d^3/(c*x - 1)^3 + 105*(c*x
+ 1)^2*b*c^5*d^3/(c*x - 1)^2 + 42*(c*x + 1)*b*c^5*d^3/(c*x - 1) + 7*b*c^5*
d^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c
*x - 1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(
c*x + 1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1) + (720*(c*x + 1)^5*a*c
^5*d^3/(c*x - 1)^5 + 1080*(c*x + 1)^4*a*c^5*d^3/(c*x - 1)^4 + 1680*(c*x +
1)^3*a*c^5*d^3/(c*x - 1)^3 + 1260*(c*x + 1)^2*a*c^5*d^3/(c*x - 1)^2 + 504*
(c*x + 1)*a*c^5*d^3/(c*x - 1) + 84*a*c^5*d^3 + 318*(c*x + 1)^5*b*c^5*d^3/(
c*x - 1)^5 + 1119*(c*x + 1)^4*b*c^5*d^3/(c*x - 1)^4 + 1742*(c*x + 1)^3*b*c
^5*d^3/(c*x - 1)^3 + 1464*(c*x + 1)^2*b*c^5*d^3/(c*x - 1)^2 + 636*(c*x + 1
)*b*c^5*d^3/(c*x - 1) + 113*b*c^5*d^3)/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x +
 1)^5/(c*x - 1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^
3 + 15*(c*x + 1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.12

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))}{x^7}\,dx = \frac{14\,b\,c^6\,d^3\,\ln(x)}{15} - \frac{7\,b\,c^6\,d^3\,\ln(c^2\,x^2-1)}{15} - \frac{3\,a\,c^2\,d^3}{4\,x^4} - \frac{a\,c^3\,d^3}{3\,x^3} - \frac{3\,b\,c^2\,d^3}{20\,x^4} - \frac{11\,b\,c^3\,d^3}{36\,x^3} - \frac{7\,b\,c^4\,d^3}{15\,x^2} - \frac{11\,b\,c^5\,d^3}{12\,x} - \frac{a\,d^3}{6\,x^6} - \frac{3\,a\,c\,d^3}{5\,x^5} - \frac{b\,c\,d^3}{30\,x^5} - \frac{b\,d^3\,\mathrm{atanh}(c\,x)}{6\,x^6} - \frac{11\,b\,c^7\,d^3\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{12\,\sqrt{-c^2}} - \frac{3\,b\,c\,d^3\,\mathrm{atanh}(c\,x)}{5\,x^5} - \frac{3\,b\,c^2\,d^3\,\mathrm{atanh}(c\,x)}{4\,x^4} - \frac{b\,c^3\,d^3\,\mathrm{atanh}(c\,x)}{3\,x^3}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^3)/x^7,x)
```

output
```
(14*b*c^6*d^3*log(x))/15 - (7*b*c^6*d^3*log(c^2*x^2 - 1))/15 - (3*a*c^2*d^
3)/(4*x^4) - (a*c^3*d^3)/(3*x^3) - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)
/(36*x^3) - (7*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (a*d^3)/(6*x^
6) - (3*a*c*d^3)/(5*x^5) - (b*c*d^3)/(30*x^5) - (b*d^3*atanh(c*x))/(6*x^6)
 - (11*b*c^7*d^3*atan((c^2*x)/(-c^2)^(1/2)))/(12*(-c^2)^(1/2)) - (3*b*c*d^
3*atanh(c*x))/(5*x^5) - (3*b*c^2*d^3*atanh(c*x))/(4*x^4) - (b*c^3*d^3*atan
h(c*x))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))}{x^7}\,dx$$
$$= \frac{d^3(-3atanh(cx)\,b\,c^6x^6 - 60atanh(cx)\,b\,c^3x^3 - 135atanh(cx)\,b\,c^2x^2 - 108atanh(cx)\,bcx - 30atanh(cx)\,b}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))/x^7,x)
```

output
```
(d**3*( - 3*atanh(c*x)*b*c**6*x**6 - 60*atanh(c*x)*b*c**3*x**3 - 135*atanh
(c*x)*b*c**2*x**2 - 108*atanh(c*x)*b*c*x - 30*atanh(c*x)*b - 168*log(c**2*
x - c)*b*c**6*x**6 + 168*log(x)*b*c**6*x**6 - 60*a*c**3*x**3 - 135*a*c**2*
x**2 - 108*a*c*x - 30*a - 165*b*c**5*x**5 - 84*b*c**4*x**4 - 55*b*c**3*x**
3 - 27*b*c**2*x**2 - 6*b*c*x))/(180*x**6)
```

3.31 $\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx &= \frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c} \\ &+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6 \\ &+\frac{1}{56}bc^3d^4x^7+\frac{1}{4}d^4x^4(a+b\text{arctanh}(cx)) \\ &+\frac{4}{5}cd^4x^5(a+b\text{arctanh}(cx)) \\ &+c^2d^4x^6(a+b\text{arctanh}(cx)) \\ &+\frac{4}{7}c^3d^4x^7(a+b\text{arctanh}(cx)) \\ &+\frac{1}{8}c^4d^4x^8(a+b\text{arctanh}(cx)) \\ &+\frac{769bd^4\log(1-cx)}{560c^4}-\frac{bd^4\log(1+cx)}{560c^4}\end{aligned}$$

output

```
11/8*b*d^4*x/c^3+24/35*b*d^4*x^2/c^2+11/24*b*d^4*x^3/c+12/35*b*d^4*x^4+9/4
0*b*c*d^4*x^5+2/21*b*c^2*d^4*x^6+1/56*b*c^3*d^4*x^7+1/4*d^4*x^4*(a+b*arcta
nh(c*x))+4/5*c*d^4*x^5*(a+b*arctanh(c*x))+c^2*d^4*x^6*(a+b*arctanh(c*x))+4
/7*c^3*d^4*x^7*(a+b*arctanh(c*x))+1/8*c^4*d^4*x^8*(a+b*arctanh(c*x))+769/5
60*b*d^4*ln(-c*x+1)/c^4-1/560*b*d^4*ln(c*x+1)/c^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.79

$$\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^4(2310bcx+1152bc^2x^2+770bc^3x^3+420ac^4x^4+576bc^4x^4+1344ac^5x^5+378bc^5x^5+1680ac^6x^6+160}{}$$

input
```
Integrate[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^4*(2310*b*c*x + 1152*b*c^2*x^2 + 770*b*c^3*x^3 + 420*a*c^4*x^4 + 576*b*
c^4*x^4 + 1344*a*c^5*x^5 + 378*b*c^5*x^5 + 1680*a*c^6*x^6 + 160*b*c^6*x^6
+ 960*a*c^7*x^7 + 30*b*c^7*x^7 + 210*a*c^8*x^8 + 6*b*c^4*x^4*(70 + 224*c*x
 + 280*c^2*x^2 + 160*c^3*x^3 + 35*c^4*x^4)*ArcTanh[c*x] + 2307*b*Log[1 - c
*x] - 3*b*Log[1 + c*x]))/(1680*c^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)^4(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{d^4x^4(35c^4x^4+160c^3x^3+280c^2x^2+224cx+70)}{280(1-c^2x^2)}dx+\frac{1}{8}c^4d^4x^8(a+b\text{arctanh}(cx))+\frac{4}{7}c^3d^4x^7(a+b\text{arctanh}(cx))+c^2d^4x^6(a+b\text{arctanh}(cx))+\frac{4}{5}cd^4x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^4x^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{280}bcd^4\int\frac{x^4(35c^4x^4+160c^3x^3+280c^2x^2+224cx+70)}{1-c^2x^2}dx+\frac{1}{8}c^4d^4x^8(a+\text{barctanh}(cx))+\frac{4}{7}c^3d^4x^7(a+\text{barctanh}(cx))+c^2d^4x^6(a+\text{barctanh}(cx))+\frac{4}{5}cd^4x^5(a+\text{barctanh}(cx))+\frac{1}{4}d^4x^4(a+\text{barctanh}(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{280}bcd^4\int\left(-35c^2x^6-160cx^5-315x^4-\frac{384x^3}{c}-\frac{385x^2}{c^2}-\frac{384x}{c^3}+\frac{384cx+385}{c^4(1-c^2x^2)}-\frac{385}{c^4}\right)dx+\frac{1}{8}c^4d^4x^8(a+\text{barctanh}(cx))+\frac{4}{7}c^3d^4x^7(a+\text{barctanh}(cx))+c^2d^4x^6(a+\text{barctanh}(cx))+\frac{4}{5}cd^4x^5(a+\text{barctanh}(cx))+\frac{1}{4}d^4x^4(a+\text{barctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}c^4d^4x^8(a+\text{barctanh}(cx))+\frac{4}{7}c^3d^4x^7(a+\text{barctanh}(cx))+c^2d^4x^6(a+\text{barctanh}(cx))+\frac{4}{5}cd^4x^5(a+\text{barctanh}(cx))+\frac{1}{4}d^4x^4(a+\text{barctanh}(cx))-\frac{1}{280}bcd^4\left(\frac{385\text{arctanh}(cx)}{c^5}-\frac{385x}{c^4}-\frac{192x^2}{c^3}-5c^2x^7-\frac{385x^3}{3c^2}-\frac{192\log(1-c^2x^2)}{c^5}-\frac{80cx^6}{3}-\frac{96x^4}{c}-63x^5\right)$$

input

```
Int[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output

```
(d^4*x^4*(a + b*ArcTanh[c*x]))/4 + (4*c*d^4*x^5*(a + b*ArcTanh[c*x]))/5 +
c^2*d^4*x^6*(a + b*ArcTanh[c*x]) + (4*c^3*d^4*x^7*(a + b*ArcTanh[c*x]))/7
+ (c^4*d^4*x^8*(a + b*ArcTanh[c*x]))/8 - (b*c*d^4*((-385*x)/c^4 - (192*x^2
)/c^3 - (385*x^3)/(3*c^2) - (96*x^4)/c - 63*x^5 - (80*c*x^6)/3 - 5*c^2*x^7
 + (385*ArcTanh[c*x])/c^5 - (192*Log[1 - c^2*x^2])/c^5))/280
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

method	result
parts	$d^4a\left(\frac{1}{8}c^4x^8+\frac{4}{7}c^3x^7+c^2x^6+\frac{4}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{d^4b\left(\frac{\operatorname{arctanh}(cx)c^8x^8}{8}+\frac{4\operatorname{arctanh}(cx)c^7x^7}{7}+\operatorname{arctanh}(cx)c^6\right.}{}$
derivativedivides	$\frac{d^4a\left(\frac{1}{8}c^8x^8+\frac{4}{7}c^7x^7+c^6x^6+\frac{4}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^8x^8}{8}+\frac{4\operatorname{arctanh}(cx)c^7x^7}{7}+\operatorname{arctanh}(cx)c^6x^6+\frac{4\operatorname{arctanh}(cx}{5}\right.}{c^4}$
default	$\frac{d^4a\left(\frac{1}{8}c^8x^8+\frac{4}{7}c^7x^7+c^6x^6+\frac{4}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^8x^8}{8}+\frac{4\operatorname{arctanh}(cx)c^7x^7}{7}+\operatorname{arctanh}(cx)c^6x^6+\frac{4\operatorname{arctanh}(cx}{5}\right.}{c^4}$
parallelrisch	$\frac{105b\,c^8d^4\operatorname{arctanh}(cx)x^8+105c^8d^4x^8a+480b\,c^7d^4\operatorname{arctanh}(cx)x^7+480a\,c^7d^4x^7+15c^7d^4x^7b+840b\,c^6d^4\operatorname{arctanh}(cx)x^6}{}$
risch	$\frac{d^4b\,x^4\left(35c^4x^4+160x^3c^3+280c^2x^2+224cx+70\right)\ln(cx+1)}{560}-\frac{d^4c^4b\,x^8\ln(-cx+1)}{16}+\frac{d^4c^4a\,x^8}{8}-\frac{2d^4c^3b\,x^7\ln(-cx}{7}$

input
```
int(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/8*c^4*x^8+4/7*c^3*x^7+c^2*x^6+4/5*c*x^5+1/4*x^4)+d^4*b/c^4*(1/8*a
rctanh(c*x)*c^8*x^8+4/7*arctanh(c*x)*c^7*x^7+arctanh(c*x)*c^6*x^6+4/5*arct
anh(c*x)*c^5*x^5+1/4*arctanh(c*x)*c^4*x^4+1/56*c^7*x^7+2/21*c^6*x^6+9/40*c
^5*x^5+12/35*c^4*x^4+11/24*x^3*c^3+24/35*c^2*x^2+11/8*c*x+769/560*ln(c*x-1
)-1/560*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99

$$\int x^3(d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx$$
$$= \frac{210\,ac^8d^4x^8+30\,(32\,a+b)c^7d^4x^7+80\,(21\,a+2\,b)c^6d^4x^6+42\,(32\,a+9\,b)c^5d^4x^5+12\,(35\,a+48\,b)c^4d^4x}{}$$

input
```
integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(210*a*c^8*d^4*x^8 + 30*(32*a + b)*c^7*d^4*x^7 + 80*(21*a + 2*b)*c^
6*d^4*x^6 + 42*(32*a + 9*b)*c^5*d^4*x^5 + 12*(35*a + 48*b)*c^4*d^4*x^4 + 7
70*b*c^3*d^4*x^3 + 1152*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 3*b*d^4*log(c*x +
 1) + 2307*b*d^4*log(c*x - 1) + 3*(35*b*c^8*d^4*x^8 + 160*b*c^7*d^4*x^7 +
280*b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 + 70*b*c^4*d^4*x^4)*log(-(c*x + 1)/(
c*x - 1)))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.31

$$\int x^3(d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^4d^4x^8}{8}+\frac{4ac^3d^4x^7}{7}+ac^2d^4x^6+\frac{4acd^4x^5}{5}+\frac{ad^4x^4}{4}+\frac{bc^4d^4x^8\operatorname{atanh}(cx)}{8}+\frac{4bc^3d^4x^7\operatorname{atanh}(cx)}{7}+\frac{bc^3d^4x^7}{56}+bc^2d^4x^6\operatorname{at} \\ \frac{ad^4x^4}{4} \end{cases}$$

input
```
integrate(x**3*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**4*x**8/8 + 4*a*c**3*d**4*x**7/7 + a*c**2*d**4*x**6 +
4*a*c*d**4*x**5/5 + a*d**4*x**4/4 + b*c**4*d**4*x**8*atanh(c*x)/8 + 4*b*c*
*3*d**4*x**7*atanh(c*x)/7 + b*c**3*d**4*x**7/56 + b*c**2*d**4*x**6*atanh(c
*x) + 2*b*c**2*d**4*x**6/21 + 4*b*c*d**4*x**5*atanh(c*x)/5 + 9*b*c*d**4*x*
*5/40 + b*d**4*x**4*atanh(c*x)/4 + 12*b*d**4*x**4/35 + 11*b*d**4*x**3/(24*
c) + 24*b*d**4*x**2/(35*c**2) + 11*b*d**4*x/(8*c**3) + 48*b*d**4*log(x - 1
/c)/(35*c**4) - b*d**4*atanh(c*x)/(280*c**4), Ne(c, 0)), (a*d**4*x**4/4, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.67

$$\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \frac{1}{8}\,ac^4d^4x^8 + \frac{4}{7}\,ac^3d^4x^7 + ac^2d^4x^6 + \frac{4}{5}\,acd^4x^5$$
$$+ \frac{1}{1680}\left(210\,x^8\,\text{artanh}\,(cx) + c\left(\frac{2\,(15\,c^6x^7 + 21\,c^4x^5 + 35\,c^2x^3 + 105\,x)}{c^8} - \frac{105\,\log\,(cx+1)}{c^9} + \frac{105\,\log\,(}{c^9}\right.\right.$$
$$+ \frac{1}{21}\left(12\,x^7\,\text{artanh}\,(cx) + c\left(\frac{2\,c^4x^6 + 3\,c^2x^4 + 6\,x^2}{c^6} + \frac{6\,\log\,(c^2x^2-1)}{c^8}\right)\right)bc^3d^4$$
$$+ \frac{1}{4}\,ad^4x^4$$
$$+ \frac{1}{30}\left(30\,x^6\,\text{artanh}\,(cx) + c\left(\frac{2\,(3\,c^4x^5 + 5\,c^2x^3 + 15\,x)}{c^6} - \frac{15\,\log\,(cx+1)}{c^7} + \frac{15\,\log\,(cx-1)}{c^7}\right)\right)bc^2d^4$$
$$+ \frac{1}{5}\left(4\,x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\,(c^2x^2-1)}{c^6}\right)\right)bcd^4$$
$$+ \frac{1}{24}\left(6\,x^4\,\text{artanh}\,(cx) + c\left(\frac{2\,(c^2x^3 + 3\,x)}{c^4} - \frac{3\,\log\,(cx+1)}{c^5} + \frac{3\,\log\,(cx-1)}{c^5}\right)\right)bd^4$$

input
```
integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^4*d^4*x^8 + 4/7*a*c^3*d^4*x^7 + a*c^2*d^4*x^6 + 4/5*a*c*d^4*x^5 +
1/1680*(210*x^8*arctanh(c*x) + c*(2*(15*c^6*x^7 + 21*c^4*x^5 + 35*c^2*x^3
+ 105*x)/c^8 - 105*log(c*x + 1)/c^9 + 105*log(c*x - 1)/c^9))*b*c^4*d^4 + 1
/21*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(
c^2*x^2 - 1)/c^8))*b*c^3*d^4 + 1/4*a*d^4*x^4 + 1/30*(30*x^6*arctanh(c*x) +
 c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*
x - 1)/c^7))*b*c^2*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^
4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^
2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(198) = 396$.

Time = 0.13 (sec) , antiderivative size = 817, normalized size of antiderivative = 3.65

$$\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output

```
-4/105*c*(36*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 - 12*(35*(c*x + 1)^7*
b*d^4/(c*x - 1)^7 - 70*(c*x + 1)^6*b*d^4/(c*x - 1)^6 + 175*(c*x + 1)^5*b*d
^4/(c*x - 1)^5 - 210*(c*x + 1)^4*b*d^4/(c*x - 1)^4 + 168*(c*x + 1)^3*b*d^4
/(c*x - 1)^3 - 84*(c*x + 1)^2*b*d^4/(c*x - 1)^2 + 24*(c*x + 1)*b*d^4/(c*x
- 1) - 3*b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^8*c^5/(c*x - 1)^8 - 8
*(c*x + 1)^7*c^5/(c*x - 1)^7 + 28*(c*x + 1)^6*c^5/(c*x - 1)^6 - 56*(c*x +
1)^5*c^5/(c*x - 1)^5 + 70*(c*x + 1)^4*c^5/(c*x - 1)^4 - 56*(c*x + 1)^3*c^5
/(c*x - 1)^3 + 28*(c*x + 1)^2*c^5/(c*x - 1)^2 - 8*(c*x + 1)*c^5/(c*x - 1)
+ c^5) - 36*b*d^4*log(-(c*x + 1)/(c*x - 1))/c^5 - (840*(c*x + 1)^7*a*d^4/(
c*x - 1)^7 - 1680*(c*x + 1)^6*a*d^4/(c*x - 1)^6 + 4200*(c*x + 1)^5*a*d^4/(
c*x - 1)^5 - 5040*(c*x + 1)^4*a*d^4/(c*x - 1)^4 + 4032*(c*x + 1)^3*a*d^4/(
c*x - 1)^3 - 2016*(c*x + 1)^2*a*d^4/(c*x - 1)^2 + 576*(c*x + 1)*a*d^4/(c*x
 - 1) - 72*a*d^4 + 384*(c*x + 1)^7*b*d^4/(c*x - 1)^7 - 1830*(c*x + 1)^6*b*
d^4/(c*x - 1)^6 + 4304*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 6031*(c*x + 1)^4*b*
d^4/(c*x - 1)^4 + 5228*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 2782*(c*x + 1)^2*b*
d^4/(c*x - 1)^2 + 836*(c*x + 1)*b*d^4/(c*x - 1) - 109*b*d^4)/((c*x + 1)^8*
c^5/(c*x - 1)^8 - 8*(c*x + 1)^7*c^5/(c*x - 1)^7 + 28*(c*x + 1)^6*c^5/(c*x
- 1)^6 - 56*(c*x + 1)^5*c^5/(c*x - 1)^5 + 70*(c*x + 1)^4*c^5/(c*x - 1)^4 -
 56*(c*x + 1)^3*c^5/(c*x - 1)^3 + 28*(c*x + 1)^2*c^5/(c*x - 1)^2 - 8*(c*x
+ 1)*c^5/(c*x - 1) + c^5))
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.50

$$\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \frac{a\,d^4\,x^4}{4}+\frac{12\,b\,d^4\,x^4}{35}+a\,c^2\,d^4\,x^6+\frac{4\,a\,c^3\,d^4\,x^7}{7}+\frac{a\,c^4\,d^4\,x^8}{8}+\frac{11\,b\,d^4\,x^3}{24\,c}+\frac{24\,b\,d^4\,x^2}{35\,c^2}+\frac{2\,b\,c^2\,d^4\,x^6}{21}+\frac{b\,c^3\,d^4\,x^7}{56}+\frac{769\,b\,d^4\,\ln\left(c\,x-1\right)}{560\,c^4}-\frac{b\,d^4\,\ln\left(c\,x+1\right)}{560\,c^4}+\frac{b\,d^4\,x^4\,\ln\left(c\,x+1\right)}{8}-\frac{b\,d^4\,x^4\,\ln\left(1-c\,x\right)}{8}+\frac{4\,a\,c\,d^4\,x^5}{5}+\frac{11\,b\,d^4\,x}{8\,c^3}+\frac{9\,b\,c\,d^4\,x^5}{40}+\frac{b\,c^2\,d^4\,x^6\,\ln\left(c\,x+1\right)}{2}-\frac{b\,c^2\,d^4\,x^6\,\ln\left(1-c\,x\right)}{2}+\frac{2\,b\,c^3\,d^4\,x^7\,\ln\left(c\,x+1\right)}{7}-\frac{2\,b\,c^3\,d^4\,x^7\,\ln\left(1-c\,x\right)}{7}+\frac{b\,c^4\,d^4\,x^8\,\ln\left(c\,x+1\right)}{16}-\frac{b\,c^4\,d^4\,x^8\,\ln\left(1-c\,x\right)}{16}+\frac{2\,b\,c\,d^4\,x^5\,\ln\left(c\,x+1\right)}{5}-\frac{2\,b\,c\,d^4\,x^5\,\ln\left(1-c\,x\right)}{5}$$

input

```
int(x^3*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

output

```
(a*d^4*x^4)/4 + (12*b*d^4*x^4)/35 + a*c^2*d^4*x^6 + (4*a*c^3*d^4*x^7)/7 +
(a*c^4*d^4*x^8)/8 + (11*b*d^4*x^3)/(24*c) + (24*b*d^4*x^2)/(35*c^2) + (2*b
*c^2*d^4*x^6)/21 + (b*c^3*d^4*x^7)/56 + (769*b*d^4*log(c*x - 1))/(560*c^4)
 - (b*d^4*log(c*x + 1))/(560*c^4) + (b*d^4*x^4*log(c*x + 1))/8 - (b*d^4*x^
4*log(1 - c*x))/8 + (4*a*c*d^4*x^5)/5 + (11*b*d^4*x)/(8*c^3) + (9*b*c*d^4*
x^5)/40 + (b*c^2*d^4*x^6*log(c*x + 1))/2 - (b*c^2*d^4*x^6*log(1 - c*x))/2
+ (2*b*c^3*d^4*x^7*log(c*x + 1))/7 - (2*b*c^3*d^4*x^7*log(1 - c*x))/7 + (b
*c^4*d^4*x^8*log(c*x + 1))/16 - (b*c^4*d^4*x^8*log(1 - c*x))/16 + (2*b*c*d
^4*x^5*log(c*x + 1))/5 - (2*b*c*d^4*x^5*log(1 - c*x))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.88

$$\int x^3(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^4(105atanh(cx)\,b\,c^8x^8+480atanh(cx)\,b\,c^7x^7+840atanh(cx)\,b\,c^6x^6+672atanh(cx)\,b\,c^5x^5+210atanh(}{}$$

input
```
int(x^3*(c*d*x+d)^4*(a+b*atanh(c*x)),x)
```

output
```
(d**4*(105*atanh(c*x)*b*c**8*x**8 + 480*atanh(c*x)*b*c**7*x**7 + 840*atanh
(c*x)*b*c**6*x**6 + 672*atanh(c*x)*b*c**5*x**5 + 210*atanh(c*x)*b*c**4*x**
4 - 3*atanh(c*x)*b + 1152*log(c**2*x - c)*b + 105*a*c**8*x**8 + 480*a*c**7
*x**7 + 840*a*c**6*x**6 + 672*a*c**5*x**5 + 210*a*c**4*x**4 + 15*b*c**7*x*
*7 + 80*b*c**6*x**6 + 189*b*c**5*x**5 + 288*b*c**4*x**4 + 385*b*c**3*x**3
+ 576*b*c**2*x**2 + 1155*b*c*x))/(840*c**4)
```

3.32 $\int x^2(d+cdx)^4(a+b\,\mathrm{arctanh}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\begin{aligned}\int x^2(d+cdx)^4(a+b\,\mathrm{arctanh}(cx))\,dx &= \frac{5bd^4x}{3c^2}+\frac{88bd^4x^2}{105c}+\frac{5}{9}bd^4x^3\\&+\frac{47}{140}bcd^4x^4+\frac{2}{15}bc^2d^4x^5+\frac{1}{42}bc^3d^4x^6\\&+\frac{d^4(1+cx)^5(a+b\,\mathrm{arctanh}(cx))}{5c^3}\\&-\frac{d^4(1+cx)^6(a+b\,\mathrm{arctanh}(cx))}{3c^3}\\&+\frac{d^4(1+cx)^7(a+b\,\mathrm{arctanh}(cx))}{7c^3}\\&+\frac{176bd^4\log(1-cx)}{105c^3}\end{aligned}$$

output
```
5/3*b*d^4*x/c^2+88/105*b*d^4*x^2/c+5/9*b*d^4*x^3+47/140*b*c*d^4*x^4+2/15*b
*c^2*d^4*x^5+1/42*b*c^3*d^4*x^6+1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/c^3-1
/3*d^4*(c*x+1)^6*(a+b*arctanh(c*x))/c^3+1/7*d^4*(c*x+1)^7*(a+b*arctanh(c*x
))/c^3+176/105*b*d^4*ln(-c*x+1)/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98

$$\int x^2(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{d^4(2100bcx+1056bc^2x^2+420ac^3x^3+700bc^3x^3+1260ac^4x^4+423bc^4x^4+1512ac^5x^5+168bc^5x^5+840$$

input
```
Integrate[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^4*(2100*b*c*x + 1056*b*c^2*x^2 + 420*a*c^3*x^3 + 700*b*c^3*x^3 + 1260*a
*c^4*x^4 + 423*b*c^4*x^4 + 1512*a*c^5*x^5 + 168*b*c^5*x^5 + 840*a*c^6*x^6
+ 30*b*c^6*x^6 + 180*a*c^7*x^7 + 12*b*c^3*x^3*(35 + 105*c*x + 126*c^2*x^2
+ 70*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] + 2106*b*Log[1 - c*x] + 6*b*Log[1
+ c*x]))/(1260*c^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)^4(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int \frac{d^4(cx+1)^4\left(15c^2x^2-5cx+1\right)}{105c^3(1-cx)}dx+\frac{d^4(cx+1)^7(a+b\text{arctanh}(cx))}{7c^3}-\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{3c^3}+\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd^4\int \frac{(cx+1)^4(15c^2x^2-5cx+1)}{1-cx}dx}{105c^2}+\frac{d^4(cx+1)^7(a+b\text{arctanh}(cx))}{7c^3}-\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{3c^3}+\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^3}$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bd^4\int \left(-15c^5x^5-70c^4x^4-141c^3x^3-175c^2x^2-176cx-\frac{176}{cx-1}-175\right)dx}{105c^2}+\frac{d^4(cx+1)^7(a+b\text{arctanh}(cx))}{7c^3}-\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{3c^3}+\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^4(cx+1)^7(a+b\text{arctanh}(cx))}{7c^3}-\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{3c^3}+\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^3}-\frac{bd^4\left(-\frac{5}{2}c^5x^6-14c^4x^5-\frac{141c^3x^4}{4}-\frac{175c^2x^3}{3}-88cx^2-\frac{176\log(1-cx)}{c}-175x\right)}{105c^2}$$

input
```
Int[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^3) - (d^4*(1 + c*x)^6*(a + b*A
rcTanh[c*x]))/(3*c^3) + (d^4*(1 + c*x)^7*(a + b*ArcTanh[c*x]))/(7*c^3) - (
b*d^4*(-175*x - 88*c*x^2 - (175*c^2*x^3)/3 - (141*c^3*x^4)/4 - 14*c^4*x^5
- (5*c^5*x^6)/2 - (176*Log[1 - c*x])/c))/(105*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

method	result
parts	$d^4a\left(\frac{1}{7}c^4x^7+\frac{2}{3}c^3x^6+\frac{6}{5}c^2x^5+c\,x^4+\frac{1}{3}x^3\right)+\frac{d^4b\left(\frac{\operatorname{arctanh}(cx)c^7x^7}{7}+\frac{2\operatorname{arctanh}(cx)c^6x^6}{3}+\frac{6\operatorname{arctanh}(cx)c^5}{5}\right.}{}$
derivativedivides	$\frac{d^4a\left(\frac{1}{7}c^7x^7+\frac{2}{3}c^6x^6+\frac{6}{5}c^5x^5+c^4x^4+\frac{1}{3}x^3c^3\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^7x^7}{7}+\frac{2\operatorname{arctanh}(cx)c^6x^6}{3}+\frac{6\operatorname{arctanh}(cx)c^5x^5}{5}+\operatorname{arctanh}(cx\right.}{c^3}$
default	$\frac{d^4a\left(\frac{1}{7}c^7x^7+\frac{2}{3}c^6x^6+\frac{6}{5}c^5x^5+c^4x^4+\frac{1}{3}x^3c^3\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^7x^7}{7}+\frac{2\operatorname{arctanh}(cx)c^6x^6}{3}+\frac{6\operatorname{arctanh}(cx)c^5x^5}{5}+\operatorname{arctanh}(cx\right.}{c^3}$
parallelrisch	$\frac{180b\,c^7d^4\operatorname{arctanh}(cx)x^7+180a\,c^7d^4x^7+840b\,c^6d^4\operatorname{arctanh}(cx)x^6+840a\,c^6d^4x^6+30b\,c^6d^4x^6+1512b\,c^5d^4\operatorname{arctanh}(cx)x}{}$
risch	$\frac{d^4b\,x^3\left(15c^4x^4+70x^3c^3+126c^2x^2+105cx+35\right)\ln(cx+1)}{210}-\frac{d^4c^4b\,x^7\ln(-cx+1)}{14}+\frac{d^4c^4a\,x^7}{7}-\frac{d^4c^3b\,x^6\ln(-cx+1}{3}$

input
```
int(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/7*c^4*x^7+2/3*c^3*x^6+6/5*c^2*x^5+c*x^4+1/3*x^3)+d^4*b/c^3*(1/7*a
rctanh(c*x)*c^7*x^7+2/3*arctanh(c*x)*c^6*x^6+6/5*arctanh(c*x)*c^5*x^5+arct
anh(c*x)*c^4*x^4+1/3*arctanh(c*x)*c^3*x^3+1/42*c^6*x^6+2/15*c^5*x^5+47/140
*c^4*x^4+5/9*x^3*c^3+88/105*c^2*x^2+5/3*c*x+117/70*ln(c*x-1)+1/210*ln(c*x+
1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.22

$$\int x^2(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$= \frac{180\,ac^7d^4x^7+30\,(28\,a+b)c^6d^4x^6+168\,(9\,a+b)c^5d^4x^5+9\,(140\,a+47\,b)c^4d^4x^4+140\,(3\,a+5\,b)c^3d^4x^3}{}$$

input
```
integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/1260*(180*a*c^7*d^4*x^7 + 30*(28*a + b)*c^6*d^4*x^6 + 168*(9*a + b)*c^5*
d^4*x^5 + 9*(140*a + 47*b)*c^4*d^4*x^4 + 140*(3*a + 5*b)*c^3*d^4*x^3 + 105
6*b*c^2*d^4*x^2 + 2100*b*c*d^4*x + 6*b*d^4*log(c*x + 1) + 2106*b*d^4*log(c
*x - 1) + 6*(15*b*c^7*d^4*x^7 + 70*b*c^6*d^4*x^6 + 126*b*c^5*d^4*x^5 + 105
*b*c^4*d^4*x^4 + 35*b*c^3*d^4*x^3)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.63

$$\int x^2(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$= \begin{cases} \frac{ac^4d^4x^7}{7}+\frac{2ac^3d^4x^6}{3}+\frac{6ac^2d^4x^5}{5}+acd^4x^4+\frac{ad^4x^3}{3}+\frac{bc^4d^4x^7\operatorname{atanh}(cx)}{7}+\frac{2bc^3d^4x^6\operatorname{atanh}(cx)}{3}+\frac{bc^3d^4x^6}{42}+\frac{6bc^2d^4x^5\operatorname{atan}}{5} \\ \frac{ad^4x^3}{3} \end{cases}$$

input
```
integrate(x**2*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**4*x**7/7 + 2*a*c**3*d**4*x**6/3 + 6*a*c**2*d**4*x**5/
5 + a*c*d**4*x**4 + a*d**4*x**3/3 + b*c**4*d**4*x**7*atanh(c*x)/7 + 2*b*c*
*3*d**4*x**6*atanh(c*x)/3 + b*c**3*d**4*x**6/42 + 6*b*c**2*d**4*x**5*atanh
(c*x)/5 + 2*b*c**2*d**4*x**5/15 + b*c*d**4*x**4*atanh(c*x) + 47*b*c*d**4*x
**4/140 + b*d**4*x**3*atanh(c*x)/3 + 5*b*d**4*x**3/9 + 88*b*d**4*x**2/(105
*c) + 5*b*d**4*x/(3*c**2) + 176*b*d**4*log(x - 1/c)/(105*c**3) + b*d**4*at
anh(c*x)/(105*c**3), Ne(c, 0)), (a*d**4*x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(151) = 302$.

Time = 0.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.98

$$\int x^2(d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx = \frac{1}{7}\,ac^4d^4x^7 + \frac{2}{3}\,ac^3d^4x^6 + \frac{6}{5}\,ac^2d^4x^5 + \frac{1}{84}\left(12\,x^7\operatorname{artanh}(cx) + c\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6} + \frac{6\,\log(c^2x^2-1)}{c^8}\right)\right)bc^4d^4 + acd^4x^4 + \frac{1}{45}\left(30\,x^6\operatorname{artanh}(cx) + c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6} - \frac{15\,\log(cx+1)}{c^7} + \frac{15\,\log(cx-1)}{c^7}\right)\right)bc^3d^4 + \frac{3}{10}\left(4\,x^5\operatorname{artanh}(cx) + c\left(\frac{c^2x^4+2\,x^2}{c^4} + \frac{2\,\log(c^2x^2-1)}{c^6}\right)\right)bc^2d^4 + \frac{1}{3}\,ad^4x^3 + \frac{1}{6}\left(6\,x^4\operatorname{artanh}(cx) + c\left(\frac{2\,(c^2x^3+3\,x)}{c^4} - \frac{3\,\log(cx+1)}{c^5} + \frac{3\,\log(cx-1)}{c^5}\right)\right)bcd^4 + \frac{1}{6}\left(2\,x^3\operatorname{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log(c^2x^2-1)}{c^4}\right)\right)bd^4$$

input
```
integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^4*d^4*x^7 + 2/3*a*c^3*d^4*x^6 + 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7*a
rctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/
c^8))*b*c^4*d^4 + a*c*d^4*x^4 + 1/45*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^
5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*
c^3*d^4 + 3/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*
x^2 - 1)/c^6))*b*c^2*d^4 + 1/3*a*d^4*x^3 + 1/6*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d^4 +
1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(151) = 302$.

Time = 0.13 (sec) , antiderivative size = 723, normalized size of antiderivative = 4.23

$$\int x^2(d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx = \text{Too large to display}$$

input

```
integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output

```
-4/315*(132*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^4 - 132*b*d^4*log(-(c*x
+ 1)/(c*x - 1))/c^4 - 12*(105*(c*x + 1)^6*b*d^4/(c*x - 1)^6 - 210*(c*x + 1
)^5*b*d^4/(c*x - 1)^5 + 385*(c*x + 1)^4*b*d^4/(c*x - 1)^4 - 385*(c*x + 1)^
3*b*d^4/(c*x - 1)^3 + 231*(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 77*(c*x + 1)*b*d
^4/(c*x - 1) + 11*b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^7*c^4/(c*x -
 1)^7 - 7*(c*x + 1)^6*c^4/(c*x - 1)^6 + 21*(c*x + 1)^5*c^4/(c*x - 1)^5 - 3
5*(c*x + 1)^4*c^4/(c*x - 1)^4 + 35*(c*x + 1)^3*c^4/(c*x - 1)^3 - 21*(c*x +
 1)^2*c^4/(c*x - 1)^2 + 7*(c*x + 1)*c^4/(c*x - 1) - c^4) - (2520*(c*x + 1)
^6*a*d^4/(c*x - 1)^6 - 5040*(c*x + 1)^5*a*d^4/(c*x - 1)^5 + 9240*(c*x + 1)
^4*a*d^4/(c*x - 1)^4 - 9240*(c*x + 1)^3*a*d^4/(c*x - 1)^3 + 5544*(c*x + 1)
^2*a*d^4/(c*x - 1)^2 - 1848*(c*x + 1)*a*d^4/(c*x - 1) + 264*a*d^4 + 1128*(
c*x + 1)^6*b*d^4/(c*x - 1)^6 - 4812*(c*x + 1)^5*b*d^4/(c*x - 1)^5 + 9476*(
c*x + 1)^4*b*d^4/(c*x - 1)^4 - 10631*(c*x + 1)^3*b*d^4/(c*x - 1)^3 + 6933*
(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 2465*(c*x + 1)*b*d^4/(c*x - 1) + 371*b*d^4
)/((c*x + 1)^7*c^4/(c*x - 1)^7 - 7*(c*x + 1)^6*c^4/(c*x - 1)^6 + 21*(c*x +
 1)^5*c^4/(c*x - 1)^5 - 35*(c*x + 1)^4*c^4/(c*x - 1)^4 + 35*(c*x + 1)^3*c^
4/(c*x - 1)^3 - 21*(c*x + 1)^2*c^4/(c*x - 1)^2 + 7*(c*x + 1)*c^4/(c*x - 1)
 - c^4))*c
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

$$\int x^2(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{\frac{88\,b\,c^2\,d^4\,x^2}{105}-\frac{d^4\,(2100\,b\,\text{atanh}(c\,x)-1056\,b\,\ln(c^2\,x^2-1))}{1260}+\frac{5\,b\,c\,d^4\,x}{3}}{c^3}$$
$$+\frac{d^4\,(420\,a\,x^3+700\,b\,x^3+420\,b\,x^3\,\text{atanh}(c\,x))}{1260}$$
$$+\frac{c^4\,d^4\,(180\,a\,x^7+180\,b\,x^7\,\text{atanh}(c\,x))}{1260}$$
$$+\frac{c\,d^4\,(1260\,a\,x^4+423\,b\,x^4+1260\,b\,x^4\,\text{atanh}(c\,x))}{1260}$$
$$+\frac{c^3\,d^4\,(840\,a\,x^6+30\,b\,x^6+840\,b\,x^6\,\text{atanh}(c\,x))}{1260}$$
$$+\frac{c^2\,d^4\,(1512\,a\,x^5+168\,b\,x^5+1512\,b\,x^5\,\text{atanh}(c\,x))}{1260}$$

input
```
int(x^2*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

output
```
((88*b*c^2*d^4*x^2)/105 - (d^4*(2100*b*atanh(c*x) - 1056*b*log(c^2*x^2 - 1
)))/1260 + (5*b*c*d^4*x)/3)/c^3 + (d^4*(420*a*x^3 + 700*b*x^3 + 420*b*x^3*
atanh(c*x)))/1260 + (c^4*d^4*(180*a*x^7 + 180*b*x^7*atanh(c*x)))/1260 + (c
*d^4*(1260*a*x^4 + 423*b*x^4 + 1260*b*x^4*atanh(c*x)))/1260 + (c^3*d^4*(84
0*a*x^6 + 30*b*x^6 + 840*b*x^6*atanh(c*x)))/1260 + (c^2*d^4*(1512*a*x^5 +
168*b*x^5 + 1512*b*x^5*atanh(c*x)))/1260
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

$$\int x^2(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^4(180atanh(cx)\,b\,c^7x^7+840atanh(cx)\,b\,c^6x^6+1512atanh(cx)\,b\,c^5x^5+1260atanh(cx)\,b\,c^4x^4+420atan}{}$$

input
```
int(x^2*(c*d*x+d)^4*(a+b*atanh(c*x)),x)
```

output
```
(d**4*(180*atanh(c*x)*b*c**7*x**7 + 840*atanh(c*x)*b*c**6*x**6 + 1512*atan
h(c*x)*b*c**5*x**5 + 1260*atanh(c*x)*b*c**4*x**4 + 420*atanh(c*x)*b*c**3*x
**3 + 12*atanh(c*x)*b + 2112*log(c**2*x - c)*b + 180*a*c**7*x**7 + 840*a*c
**6*x**6 + 1512*a*c**5*x**5 + 1260*a*c**4*x**4 + 420*a*c**3*x**3 + 30*b*c*
*6*x**6 + 168*b*c**5*x**5 + 423*b*c**4*x**4 + 700*b*c**3*x**3 + 1056*b*c**
2*x**2 + 2100*b*c*x))/(1260*c**3)
```

3.33 $\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\begin{aligned}\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx &= \frac{16bd^4x}{15c}+\frac{4bd^4(1+cx)^2}{15c^2}+\frac{4bd^4(1+cx)^3}{45c^2}\\ &\quad+\frac{bd^4(1+cx)^4}{30c^2}+\frac{bd^4(1+cx)^5}{30c^2}\\ &\quad-\frac{d^4(1+cx)^5(a+b\text{arctanh}(cx))}{5c^2}\\ &\quad+\frac{d^4(1+cx)^6(a+b\text{arctanh}(cx))}{6c^2}\\ &\quad+\frac{32bd^4\log(1-cx)}{15c^2}\end{aligned}$$

output
```
16/15*b*d^4*x/c+4/15*b*d^4*(c*x+1)^2/c^2+4/45*b*d^4*(c*x+1)^3/c^2+1/30*b*d
^4*(c*x+1)^4/c^2+1/30*b*d^4*(c*x+1)^5/c^2-1/5*d^4*(c*x+1)^5*(a+b*arctanh(c
*x))/c^2+1/6*d^4*(c*x+1)^6*(a+b*arctanh(c*x))/c^2+32/15*b*d^4*ln(-c*x+1)/c
^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04

$$\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{d^4(390bcx+90ac^2x^2+192bc^2x^2+240ac^3x^3+100bc^3x^3+270ac^4x^4+36bc^4x^4+144ac^5x^5+6bc^5x^5+3}{18}$$

input
```
Integrate[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^4*(390*b*c*x + 90*a*c^2*x^2 + 192*b*c^2*x^2 + 240*a*c^3*x^3 + 100*b*c^3
*x^3 + 270*a*c^4*x^4 + 36*b*c^4*x^4 + 144*a*c^5*x^5 + 6*b*c^5*x^5 + 30*a*c
^6*x^6 + 6*b*c^2*x^2*(15 + 40*c*x + 45*c^2*x^2 + 24*c^3*x^3 + 5*c^4*x^4)*A
rcTanh[c*x] + 387*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(180*c^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6498, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)^4(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int -\frac{d^4(1-5cx)(cx+1)^4}{30c^2(1-cx)}dx+\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{6c^2}-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bd^4\int \frac{(1-5cx)(cx+1)^4}{1-cx}dx}{30c}+\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{6c^2}-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{bd^4 \int \left(5(cx+1)^4 + 4(cx+1)^3 + 8(cx+1)^2 + 16(cx+1) + \frac{64}{cx-1} + 32\right) dx}{30c} + \frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{6c^2} - \frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{d^4(cx+1)^6(a+b\text{arctanh}(cx))}{6c^2} - \frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c^2} + \frac{bd^4\left(\frac{(cx+1)^5}{c} + \frac{(cx+1)^4}{c} + \frac{8(cx+1)^3}{3c} + \frac{8(cx+1)^2}{c} + \frac{64\log(1-cx)}{c} + 32x\right)}{30c}$$

input
```
Int[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
-1/5*(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/c^2 + (d^4*(1 + c*x)^6*(a + b*
ArcTanh[c*x]))/(6*c^2) + (b*d^4*(32*x + (8*(1 + c*x)^2)/c + (8*(1 + c*x)^3
)/(3*c) + (1 + c*x)^4/c + (1 + c*x)^5/c + (64*Log[1 - c*x])/c))/(30*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

method	result
parts	$d^4a\left(\frac{1}{6}c^4x^6+\frac{4}{5}c^3x^5+\frac{3}{2}c^2x^4+\frac{4}{3}c\,x^3+\frac{1}{2}x^2\right)+\frac{d^4b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{4\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)}{2}\right.}{}$
derivativedivides	$\frac{d^4a\left(\frac{1}{6}c^6x^6+\frac{4}{5}c^5x^5+\frac{3}{2}c^4x^4+\frac{4}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{4\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{2}+\frac{4\operatorname{arctanh}(}{3}\right.}{c^2}$
default	$\frac{d^4a\left(\frac{1}{6}c^6x^6+\frac{4}{5}c^5x^5+\frac{3}{2}c^4x^4+\frac{4}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^6x^6}{6}+\frac{4\operatorname{arctanh}(cx)c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)c^4x^4}{2}+\frac{4\operatorname{arctanh}(}{3}\right.}{c^2}$
parallelrisch	$15b\,c^6d^4\operatorname{arctanh}(cx)x^6+15a\,c^6d^4x^6+72b\,c^5d^4\operatorname{arctanh}(cx)x^5+72a\,c^5d^4x^5+3b\,c^5d^4x^5+135d^4b\operatorname{arctanh}(cx)x^4c^4+135$
risch	$\frac{d^4b\,x^2\left(5c^4x^4+24x^3c^3+45c^2x^2+40cx+15\right)\ln(cx+1)}{60}-\frac{d^4c^4b\,x^6\ln(-cx+1)}{12}+\frac{d^4c^4a\,x^6}{6}-\frac{2d^4c^3b\,x^5\ln(-cx+1)}{5}$

input
```
int(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/6*c^4*x^6+4/5*c^3*x^5+3/2*c^2*x^4+4/3*c*x^3+1/2*x^2)+d^4*b/c^2*(1
/6*arctanh(c*x)*c^6*x^6+4/5*arctanh(c*x)*c^5*x^5+3/2*arctanh(c*x)*c^4*x^4+
4/3*arctanh(c*x)*c^3*x^3+1/2*arctanh(c*x)*c^2*x^2+1/30*c^5*x^5+1/5*c^4*x^4
+5/9*x^3*c^3+16/15*c^2*x^2+13/6*c*x+43/20*ln(c*x-1)-1/60*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

$$\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{30\,ac^6d^4x^6+6\,(24\,a+b)c^5d^4x^5+18\,(15\,a+2\,b)c^4d^4x^4+20\,(12\,a+5\,b)c^3d^4x^3+6\,(15\,a+32\,b)c^2d^4x^2+\cdots}{}$$

input
```
integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*d^4*x^6 + 6*(24*a + b)*c^5*d^4*x^5 + 18*(15*a + 2*b)*c^4*d
^4*x^4 + 20*(12*a + 5*b)*c^3*d^4*x^3 + 6*(15*a + 32*b)*c^2*d^4*x^2 + 390*b
*c*d^4*x - 3*b*d^4*log(c*x + 1) + 387*b*d^4*log(c*x - 1) + 3*(5*b*c^6*d^4*
x^6 + 24*b*c^5*d^4*x^5 + 45*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 15*b*c^2*d^
4*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.76

$$\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^4x^6}{6}+\frac{4ac^3d^4x^5}{5}+\frac{3ac^2d^4x^4}{2}+\frac{4acd^4x^3}{3}+\frac{ad^4x^2}{2}+\frac{bc^4d^4x^6\operatorname{atanh}(cx)}{6}+\frac{4bc^3d^4x^5\operatorname{atanh}(cx)}{5}+\frac{bc^3d^4x^5}{30}+\frac{3bc^2d^4x^4\operatorname{atan}\cdots}{2} \\ \frac{ad^4x^2}{2}\end{cases}$$

input
```
integrate(x*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**4*x**6/6 + 4*a*c**3*d**4*x**5/5 + 3*a*c**2*d**4*x**4/
2 + 4*a*c*d**4*x**3/3 + a*d**4*x**2/2 + b*c**4*d**4*x**6*atanh(c*x)/6 + 4*
b*c**3*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**5/30 + 3*b*c**2*d**4*x**4*a
tanh(c*x)/2 + b*c**2*d**4*x**4/5 + 4*b*c*d**4*x**3*atanh(c*x)/3 + 5*b*c*d*
*4*x**3/9 + b*d**4*x**2*atanh(c*x)/2 + 16*b*d**4*x**2/15 + 13*b*d**4*x/(6*
c) + 32*b*d**4*log(x - 1/c)/(15*c**2) - b*d**4*atanh(c*x)/(30*c**2), Ne(c,
 0)), (a*d**4*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(137) = 274$.

Time = 0.03 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.13

$$\begin{aligned}\int x(d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx &= \frac{1}{6}\,ac^4d^4x^6 + \frac{4}{5}\,ac^3d^4x^5 + \frac{3}{2}\,ac^2d^4x^4 \\ &+ \frac{1}{180}\left(30\,x^6\operatorname{artanh}(cx) + c\left(\frac{2\,(3\,c^4x^5 + 5\,c^2x^3 + 15\,x)}{c^6} - \frac{15\,\log(cx+1)}{c^7} + \frac{15\,\log(cx-1)}{c^7}\right)\right)bc^4d^4 \\ &+ \frac{1}{5}\left(4\,x^5\operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2-1)}{c^6}\right)\right)bc^3d^4 + \frac{4}{3}\,acd^4x^3 \\ &+ \frac{1}{4}\left(6\,x^4\operatorname{artanh}(cx) + c\left(\frac{2\,(c^2x^3 + 3\,x)}{c^4} - \frac{3\,\log(cx+1)}{c^5} + \frac{3\,\log(cx-1)}{c^5}\right)\right)bc^2d^4 \\ &+ \frac{2}{3}\left(2\,x^3\operatorname{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log(c^2x^2-1)}{c^4}\right)\right)bcd^4 + \frac{1}{2}\,ad^4x^2 \\ &+ \frac{1}{4}\left(2\,x^2\operatorname{artanh}(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)bd^4\end{aligned}$$

input
```
integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^4*d^4*x^6 + 4/5*a*c^3*d^4*x^5 + 3/2*a*c^2*d^4*x^4 + 1/180*(30*x^6*
arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c
^7 + 15*log(c*x - 1)/c^7))*b*c^4*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x
^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^4 + 4/3*a*c*d^4*x^3 + 1
/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3
*log(c*x - 1)/c^5))*b*c^2*d^4 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log
(c^2*x^2 - 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*
(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(137) = 274$.

Time = 0.13 (sec) , antiderivative size = 621, normalized size of antiderivative = 4.06

$$\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-8/45*(12*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 - 12*b*d^4*log(-(c*x + 1
)/(c*x - 1))/c^3 - 6*(15*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 30*(c*x + 1)^4*b*
d^4/(c*x - 1)^4 + 40*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 30*(c*x + 1)^2*b*d^4/
(c*x - 1)^2 + 12*(c*x + 1)*b*d^4/(c*x - 1) - 2*b*d^4)*log(-(c*x + 1)/(c*x
 - 1))/((c*x + 1)^6*c^3/(c*x - 1)^6 - 6*(c*x + 1)^5*c^3/(c*x - 1)^5 + 15*(c
*x + 1)^4*c^3/(c*x - 1)^4 - 20*(c*x + 1)^3*c^3/(c*x - 1)^3 + 15*(c*x + 1)^
2*c^3/(c*x - 1)^2 - 6*(c*x + 1)*c^3/(c*x - 1) + c^3) - (180*(c*x + 1)^5*a*
d^4/(c*x - 1)^5 - 360*(c*x + 1)^4*a*d^4/(c*x - 1)^4 + 480*(c*x + 1)^3*a*d^
4/(c*x - 1)^3 - 360*(c*x + 1)^2*a*d^4/(c*x - 1)^2 + 144*(c*x + 1)*a*d^4/(c
*x - 1) - 24*a*d^4 + 78*(c*x + 1)^5*b*d^4/(c*x - 1)^5 - 294*(c*x + 1)^4*b*
d^4/(c*x - 1)^4 + 472*(c*x + 1)^3*b*d^4/(c*x - 1)^3 - 399*(c*x + 1)^2*b*d^
4/(c*x - 1)^2 + 174*(c*x + 1)*b*d^4/(c*x - 1) - 31*b*d^4)/((c*x + 1)^6*c^3
/(c*x - 1)^6 - 6*(c*x + 1)^5*c^3/(c*x - 1)^5 + 15*(c*x + 1)^4*c^3/(c*x - 1
)^4 - 20*(c*x + 1)^3*c^3/(c*x - 1)^3 + 15*(c*x + 1)^2*c^3/(c*x - 1)^2 - 6*
(c*x + 1)*c^3/(c*x - 1) + c^3))*c
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \frac{d^4\,(45\,a\,x^2+96\,b\,x^2+45\,b\,x^2\,\mathrm{atanh}(c\,x))}{90} - \frac{\frac{d^4\,(195\,b\,\mathrm{atanh}(c\,x)-96\,b\,\ln(c^2\,x^2-1))}{90}-\frac{13\,b\,c\,d^4\,x}{6}}{c^2} + \frac{c^4\,d^4\,(15\,a\,x^6+15\,b\,x^6\,\mathrm{atanh}(c\,x))}{90} + \frac{c\,d^4\,(120\,a\,x^3+50\,b\,x^3+120\,b\,x^3\,\mathrm{atanh}(c\,x))}{90} + \frac{c^3\,d^4\,(72\,a\,x^5+3\,b\,x^5+72\,b\,x^5\,\mathrm{atanh}(c\,x))}{90} + \frac{c^2\,d^4\,(135\,a\,x^4+18\,b\,x^4+135\,b\,x^4\,\mathrm{atanh}(c\,x))}{90}$$

input
```
int(x*(a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

output
```
(d^4*(45*a*x^2 + 96*b*x^2 + 45*b*x^2*atanh(c*x)))/90 - ((d^4*(195*b*atanh(
c*x) - 96*b*log(c^2*x^2 - 1)))/90 - (13*b*c*d^4*x)/6)/c^2 + (c^4*d^4*(15*a
*x^6 + 15*b*x^6*atanh(c*x)))/90 + (c*d^4*(120*a*x^3 + 50*b*x^3 + 120*b*x^3
*atanh(c*x)))/90 + (c^3*d^4*(72*a*x^5 + 3*b*x^5 + 72*b*x^5*atanh(c*x)))/90
 + (c^2*d^4*(135*a*x^4 + 18*b*x^4 + 135*b*x^4*atanh(c*x)))/90
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

$$\int x(d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \frac{d^4(15atanh(cx)\,b\,c^6x^6+72atanh(cx)\,b\,c^5x^5+135atanh(cx)\,b\,c^4x^4+120atanh(cx)\,b\,c^3x^3+45atanh(cx)$$

input
```
int(x*(c*d*x+d)^4*(a+b*atanh(c*x)),x)
```

output
```
(d**4*(15*atanh(c*x)*b*c**6*x**6 + 72*atanh(c*x)*b*c**5*x**5 + 135*atanh(c
*x)*b*c**4*x**4 + 120*atanh(c*x)*b*c**3*x**3 + 45*atanh(c*x)*b*c**2*x**2 -
 3*atanh(c*x)*b + 192*log(c**2*x - c)*b + 15*a*c**6*x**6 + 72*a*c**5*x**5
+ 135*a*c**4*x**4 + 120*a*c**3*x**3 + 45*a*c**2*x**2 + 3*b*c**5*x**5 + 18*
b*c**4*x**4 + 50*b*c**3*x**3 + 96*b*c**2*x**2 + 195*b*c*x))/(90*c**2)
```

3.34 $\int (d+cdx)^4(a+b\text{arctanh}(cx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \frac{8}{5}bd^4x+\frac{2bd^4(1+cx)^2}{5c}+\frac{2bd^4(1+cx)^3}{15c}+\frac{bd^4(1+cx)^4}{20c}+\frac{d^4(1+cx)^5(a+b\text{arctanh}(cx))}{5c}+\frac{16bd^4\log(1-cx)}{5c}$$

output
```
8/5*b*d^4*x+2/5*b*d^4*(c*x+1)^2/c+2/15*b*d^4*(c*x+1)^3/c+1/20*b*d^4*(c*x+1
)^4/c+1/5*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/c+16/5*b*d^4*ln(-c*x+1)/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int (d+cdx)^4(a+b\text{arctanh}(cx))\,dx = \frac{d^4(60acx+180bcx+120ac^2x^2+66bc^2x^2+120ac^3x^3+20bc^3x^3+60ac^4x^4+3bc^4x^4+12ac^5x^5+12bcx}{60c}$$

input
```
Integrate[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^4*(60*a*c*x + 180*b*c*x + 120*a*c^2*x^2 + 66*b*c^2*x^2 + 120*a*c^3*x^3
+ 20*b*c^3*x^3 + 60*a*c^4*x^4 + 3*b*c^4*x^4 + 12*a*c^5*x^5 + 12*b*c*x*(5 +
 10*c*x + 10*c^2*x^2 + 5*c^3*x^3 + c^4*x^4)*ArcTanh[c*x] + 180*b*Log[1 - c
*x] + 6*b*Log[1 - c^2*x^2]))/(60*c)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6478, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^4(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c}-\frac{b\int \frac{d^5(cx+1)^5}{1-c^2x^2}dx}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c}-\frac{1}{5}bd^4\int \frac{(cx+1)^5}{1-c^2x^2}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c}-\frac{1}{5}bd^4\int \frac{(cx+1)^4}{1-cx}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c}-\frac{1}{5}bd^4\int \left(-(cx+1)^3-2(cx+1)^2-4(cx+1)+\frac{16}{1-cx}-8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5c}-\frac{1}{5}bd^4\left(-\frac{(cx+1)^4}{4c}-\frac{2(cx+1)^3}{3c}-\frac{2(cx+1)^2}{c}-\frac{16\log(1-cx)}{c}-8x\right)$$

input
```
Int[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c) - (b*d^4*(-8*x - (2*(1 + c*x)
^2)/c - (2*(1 + c*x)^3)/(3*c) - (1 + c*x)^4/(4*c) - (16*Log[1 - c*x])/c))/
5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{d^4a(cx+1)^5}{5}+d^4b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\operatorname{arctanh}(cx)c^4x^4+2\operatorname{arctanh}(cx)c^3x^3+2\operatorname{arctanh}(cx)c^2x^2+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctan}}{5}\right.}{c}$
default	$\frac{\frac{d^4a(cx+1)^5}{5}+d^4b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\operatorname{arctanh}(cx)c^4x^4+2\operatorname{arctanh}(cx)c^3x^3+2\operatorname{arctanh}(cx)c^2x^2+\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctan}}{5}\right.}{c}$
parts	$\frac{d^4a(cx+1)^5}{5c}+\frac{d^4b\left(\frac{\operatorname{arctanh}(cx)c^5x^5}{5}+\operatorname{arctanh}(cx)c^4x^4+2\operatorname{arctanh}(cx)c^3x^3+2\operatorname{arctanh}(cx)c^2x^2+\operatorname{arctanh}(cx)cx+\operatorname{ar}\right.}{c}$
parallelrisch	$12bc^5d^4\operatorname{arctanh}(cx)x^5+12ac^5d^4x^5+60d^4b\operatorname{arctanh}(cx)x^4c^4+60ac^4d^4x^4+3bc^4d^4x^4+120d^4b\operatorname{arctanh}(cx)x^3c^3+120$
risch	$\frac{d^4(cx+1)^5b\ln(cx+1)}{10c}-\frac{d^4c^4bx^5\ln(-cx+1)}{10}+\frac{d^4c^4ax^5}{5}-\frac{d^4c^3bx^4\ln(-cx+1)}{2}+d^4c^3ax^4+\frac{d^4c^3bx^4}{20}-$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/5*d^4*a*(c*x+1)^5+d^4*b*(1/5*arctanh(c*x)*c^5*x^5+arctanh(c*x)*c^4*
x^4+2*arctanh(c*x)*c^3*x^3+2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x+1/5*arc
tanh(c*x)+1/20*c^4*x^4+1/3*x^3*c^3+11/10*c^2*x^2+3*c*x+16/5*ln(c*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

$$\int (d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx$$
$$=\frac{12\,ac^5d^4x^5+3\,(20\,a+b)c^4d^4x^4+20\,(6\,a+b)c^3d^4x^3+6\,(20\,a+11\,b)c^2d^4x^2+60\,(a+3\,b)cd^4x+6\,bd^4\,\mathrm{lo}}{}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*d^4*x^5 + 3*(20*a + b)*c^4*d^4*x^4 + 20*(6*a + b)*c^3*d^4*x
^3 + 6*(20*a + 11*b)*c^2*d^4*x^2 + 60*(a + 3*b)*c*d^4*x + 6*b*d^4*log(c*x
+ 1) + 186*b*d^4*log(c*x - 1) + 6*(b*c^5*d^4*x^5 + 5*b*c^4*d^4*x^4 + 10*b*
c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(97) = 194$.

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.11

$$\int (d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$=\begin{cases} \frac{ac^4d^4x^5}{5}+ac^3d^4x^4+2ac^2d^4x^3+2acd^4x^2+ad^4x+\frac{bc^4d^4x^5\operatorname{atanh}(cx)}{5}+bc^3d^4x^4\operatorname{atanh}(cx)+\frac{bc^3d^4x^4}{20}+2bc^2 \\ ad^4x \end{cases}$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*c**4*d**4*x**5/5 + a*c**3*d**4*x**4 + 2*a*c**2*d**4*x**3 + 2*
a*c*d**4*x**2 + a*d**4*x + b*c**4*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**
4*atanh(c*x) + b*c**3*d**4*x**4/20 + 2*b*c**2*d**4*x**3*atanh(c*x) + b*c**
2*d**4*x**3/3 + 2*b*c*d**4*x**2*atanh(c*x) + 11*b*c*d**4*x**2/10 + b*d**4*
x*atanh(c*x) + 3*b*d**4*x + 16*b*d**4*log(x - 1/c)/(5*c) + b*d**4*atanh(c*
x)/(5*c), Ne(c, 0)), (a*d**4*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(95) = 190$.

Time = 0.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.64

$$\int (d+cdx)^4(a+b\text{arctanh}(cx))\,dx$$
$$\begin{aligned}
&=\frac{1}{5}\,ac^4d^4x^5+ac^3d^4x^4\\
&\quad+\frac{1}{20}\left(4\,x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log(c^2x^2-1)}{c^6}\right)\right)bc^4d^4+2\,ac^2d^4x^3\\
&\quad+\frac{1}{6}\left(6\,x^4\operatorname{artanh}(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log(cx+1)}{c^5}+\frac{3\,\log(cx-1)}{c^5}\right)\right)bc^3d^4\\
&\quad+\left(2\,x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log(c^2x^2-1)}{c^4}\right)\right)bc^2d^4+2\,acd^4x^2\\
&\quad+\left(2\,x^2\operatorname{artanh}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bcd^4\\
&\quad+ad^4x+\frac{(2\,cx\operatorname{artanh}(cx)+\log(-c^2x^2+1))bd^4}{2\,c}
\end{aligned}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^4*x^5 + a*c^3*d^4*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4
 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^4*d^4 + 2*a*c^2*d^4*x^3 + 1/6
*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*l
og(c*x - 1)/c^5))*b*c^3*d^4 + (2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c^2*d^4 + 2*a*c*d^4*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2
 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^4 + a*d^4*x + 1/2*(2*c*x*ar
ctanh(c*x) + log(-c^2*x^2 + 1))*b*d^4/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(95) = 190$.

Time = 0.12 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.92

$$\int (d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx = -\frac{4}{15}\left(\frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^2} - \frac{12\,bd^4\log\left(-\frac{cx+1}{cx-1}\right)}{c^2} - \frac{12\left(\frac{5\,(cx+1)^4bd^4}{(cx-1)^4} - \frac{10\,(cx+1)^3bd^4}{(cx-1)^3} + \frac{10\,(cx+1)^2bd^4}{(cx-1)^2} - \frac{5\,(c\cdots}{c\cdots}\right.}{\frac{(cx+1)^5c^2}{(cx-1)^5} - \frac{5\,(cx+1)^4c^2}{(cx-1)^4} + \frac{10\,(cx+1)^3c^2}{(cx-1)^3} - \frac{10\,(cx+\cdots}{(cx-1\cdots}}\right.$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
-4/15*(12*b*d^4*log(-(c*x + 1)/(c*x - 1) + 1)/c^2 - 12*b*d^4*log(-(c*x + 1
)/(c*x - 1))/c^2 - 12*(5*(c*x + 1)^4*b*d^4/(c*x - 1)^4 - 10*(c*x + 1)^3*b*
d^4/(c*x - 1)^3 + 10*(c*x + 1)^2*b*d^4/(c*x - 1)^2 - 5*(c*x + 1)*b*d^4/(c*
x - 1) + b*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^2/(c*x - 1)^5 - 5
*(c*x + 1)^4*c^2/(c*x - 1)^4 + 10*(c*x + 1)^3*c^2/(c*x - 1)^3 - 10*(c*x +
1)^2*c^2/(c*x - 1)^2 + 5*(c*x + 1)*c^2/(c*x - 1) - c^2) - (120*(c*x + 1)^4
*a*d^4/(c*x - 1)^4 - 240*(c*x + 1)^3*a*d^4/(c*x - 1)^3 + 240*(c*x + 1)^2*a
*d^4/(c*x - 1)^2 - 120*(c*x + 1)*a*d^4/(c*x - 1) + 24*a*d^4 + 48*(c*x + 1)
^4*b*d^4/(c*x - 1)^4 - 156*(c*x + 1)^3*b*d^4/(c*x - 1)^3 + 196*(c*x + 1)^2
*b*d^4/(c*x - 1)^2 - 113*(c*x + 1)*b*d^4/(c*x - 1) + 25*b*d^4)/((c*x + 1)^
5*c^2/(c*x - 1)^5 - 5*(c*x + 1)^4*c^2/(c*x - 1)^4 + 10*(c*x + 1)^3*c^2/(c*
x - 1)^3 - 10*(c*x + 1)^2*c^2/(c*x - 1)^2 + 5*(c*x + 1)*c^2/(c*x - 1) - c^
2))*c
```

Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.57

$$\int (d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx=\frac{d^4\,\left(60\,a\,x+180\,b\,x+60\,b\,x\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c^4\,d^4\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atanh}(c\,x)\right)}{60}-\frac{d^4\,\left(180\,b\,\mathrm{atanh}(c\,x)-96\,b\,\ln\left(c^2\,x^2-1\right)\right)}{60\,c}+\frac{c\,d^4\,\left(120\,a\,x^2+66\,b\,x^2+120\,b\,x^2\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c^3\,d^4\,\left(60\,a\,x^4+3\,b\,x^4+60\,b\,x^4\,\mathrm{atanh}(c\,x)\right)}{60}+\frac{c^2\,d^4\,\left(120\,a\,x^3+20\,b\,x^3+120\,b\,x^3\,\mathrm{atanh}(c\,x)\right)}{60}$$

input
```
int((a + b*atanh(c*x))*(d + c*d*x)^4,x)
```

output
```
(d^4*(60*a*x + 180*b*x + 60*b*x*atanh(c*x)))/60 + (c^4*d^4*(12*a*x^5 + 12*
b*x^5*atanh(c*x)))/60 - (d^4*(180*b*atanh(c*x) - 96*b*log(c^2*x^2 - 1)))/(
60*c) + (c*d^4*(120*a*x^2 + 66*b*x^2 + 120*b*x^2*atanh(c*x)))/60 + (c^3*d^
4*(60*a*x^4 + 3*b*x^4 + 60*b*x^4*atanh(c*x)))/60 + (c^2*d^4*(120*a*x^3 + 2
0*b*x^3 + 120*b*x^3*atanh(c*x)))/60
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int (d+cdx)^4(a+b\operatorname{arctanh}(cx))\,dx=\frac{d^4(12atanh(cx)\,b\,c^5x^5+60atanh(cx)\,b\,c^4x^4+120atanh(cx)\,b\,c^3x^3+120atanh(cx)\,b\,c^2x^2+60atanh(cx)}{}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x)),x)
```

output
```
(d**4*(12*atanh(c*x)*b*c**5*x**5 + 60*atanh(c*x)*b*c**4*x**4 + 120*atanh(c
*x)*b*c**3*x**3 + 120*atanh(c*x)*b*c**2*x**2 + 60*atanh(c*x)*b*c*x + 12*at
anh(c*x)*b + 192*log(c**2*x - c)*b + 12*a*c**5*x**5 + 60*a*c**4*x**4 + 120
*a*c**3*x**3 + 120*a*c**2*x**2 + 60*a*c*x + 3*b*c**4*x**4 + 20*b*c**3*x**3
 + 66*b*c**2*x**2 + 180*b*c*x))/(60*c)
```

3.35 $\int \frac{(d+cdx)^4(a+b\,\mathbf{arctanh}(cx))}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\begin{aligned}\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx &= 4acd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}bc^2d^4x^2+\frac{1}{12}bc^3d^4x^3\\&\quad-\frac{13}{4}bd^4\text{arctanh}(cx)+4bcd^4x\text{arctanh}(cx)\\&\quad+3c^2d^4x^2(a+b\text{arctanh}(cx))\\&\quad+\frac{4}{3}c^3d^4x^3(a+b\text{arctanh}(cx))\\&\quad+\frac{1}{4}c^4d^4x^4(a+b\text{arctanh}(cx))\\&\quad+ad^4\log(x)+\frac{8}{3}bd^4\log\left(1-c^2x^2\right)\\&\quad-\frac{1}{2}bd^4\,\text{PolyLog}(2,-cx)+\frac{1}{2}bd^4\,\text{PolyLog}(2,cx)\end{aligned}$$

output

```
4*a*c*d^4*x+13/4*b*c*d^4*x+2/3*b*c^2*d^4*x^2+1/12*b*c^3*d^4*x^3-13/4*b*d^4
*arctanh(c*x)+4*b*c*d^4*x*arctanh(c*x)+3*c^2*d^4*x^2*(a+b*arctanh(c*x))+4/
3*c^3*d^4*x^3*(a+b*arctanh(c*x))+1/4*c^4*d^4*x^4*(a+b*arctanh(c*x))+a*d^4*
ln(x)+8/3*b*d^4*ln(-c^2*x^2+1)-1/2*b*d^4*polylog(2,-c*x)+1/2*b*d^4*polylog
(2,c*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx = \frac{1}{24}d^4\big(96acx+78bcx+72ac^2x^2+16bc^2x^2 + 32ac^3x^3+2bc^3x^3+6ac^4x^4+96bcx\text{arctanh}(cx) + 72bc^2x^2\text{arctanh}(cx)+32bc^3x^3\text{arctanh}(cx) + 6bc^4x^4\text{arctanh}(cx)+24a\log(x) + 39b\log(1-cx)-39b\log(1+cx) + 48b\log\left(1-c^2x^2\right)+16b\log\left(-1+c^2x^2\right) - 12b\,\text{PolyLog}(2,-cx)+12b\,\text{PolyLog}(2,cx)\big)$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
(d^4*(96*a*c*x + 78*b*c*x + 72*a*c^2*x^2 + 16*b*c^2*x^2 + 32*a*c^3*x^3 + 2
*b*c^3*x^3 + 6*a*c^4*x^4 + 96*b*c*x*ArcTanh[c*x] + 72*b*c^2*x^2*ArcTanh[c*
x] + 32*b*c^3*x^3*ArcTanh[c*x] + 6*b*c^4*x^4*ArcTanh[c*x] + 24*a*Log[x] +
39*b*Log[1 - c*x] - 39*b*Log[1 + c*x] + 48*b*Log[1 - c^2*x^2] + 16*b*Log[-
1 + c^2*x^2] - 12*b*PolyLog[2, -(c*x)] + 12*b*PolyLog[2, c*x]))/24
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \Big(c^4d^4x^3(a+b\text{arctanh}(cx))+4c^3d^4x^2(a+b\text{arctanh}(cx))+6c^2d^4x(a+b\text{arctanh}(cx))+4cd^4(a+b\text{arctanh}(c$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}c^4d^4x^4(a+b\text{arctanh}(cx))+\frac{4}{3}c^3d^4x^3(a+b\text{arctanh}(cx))+3c^2d^4x^2(a+b\text{arctanh}(cx))+4acd^4x+ad^4\log(x)-\frac{13}{4}bd^4\text{arctanh}(cx)+4bcd^4x\text{arctanh}(cx)+\frac{1}{12}bc^3d^4x^3+\frac{2}{3}bc^2d^4x^2+\frac{8}{3}bd^4\log\left(1-c^2x^2\right)-\frac{1}{2}bd^4\,\text{PolyLog}(2,-cx)+\frac{1}{2}bd^4\,\text{PolyLog}(2,cx)+\frac{13}{4}bcd^4x$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

output
```
4*a*c*d^4*x + (13*b*c*d^4*x)/4 + (2*b*c^2*d^4*x^2)/3 + (b*c^3*d^4*x^3)/12
- (13*b*d^4*ArcTanh[c*x])/4 + 4*b*c*d^4*x*ArcTanh[c*x] + 3*c^2*d^4*x^2*(a
+ b*ArcTanh[c*x]) + (4*c^3*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + (c^4*d^4*x^4*
(a + b*ArcTanh[c*x]))/4 + a*d^4*Log[x] + (8*b*d^4*Log[1 - c^2*x^2])/3 - (b
*d^4*PolyLog[2, -(c*x)])/2 + (b*d^4*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

method	result
parts	$d^4a\left(\frac{c^4x^4}{4}+\frac{4x^3c^3}{3}+3c^2x^2+4cx+\ln\left(x\right)\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{4\operatorname{arctanh}(cx)c^3x^3}{3}+3\operatorname{ar}\right.$
derivativedivides	$d^4a\left(\frac{c^4x^4}{4}+\frac{4x^3c^3}{3}+3c^2x^2+4cx+\ln\left(cx\right)\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{4\operatorname{arctanh}(cx)c^3x^3}{3}+3\operatorname{a}\right.$
default	$d^4a\left(\frac{c^4x^4}{4}+\frac{4x^3c^3}{3}+3c^2x^2+4cx+\ln\left(cx\right)\right)+d^4b\left(\frac{\operatorname{arctanh}(cx)c^4x^4}{4}+\frac{4\operatorname{arctanh}(cx)c^3x^3}{3}+3\operatorname{a}\right.$
risch	$-\frac{103d^4a}{12}+\frac{a\,c^4d^4x^4}{4}+\frac{25d^4b\ln(cx+1)}{24}-\frac{d^4b\operatorname{dilog}(cx+1)}{2}+d^4a\ln\left(-cx\right)+\frac{103d^4b\ln(-cx+1)}{24}+\frac{d^4b\,\mathrm{d}}{}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/4*c^4*x^4+4/3*x^3*c^3+3*c^2*x^2+4*c*x+ln(x))+d^4*b*(1/4*arctanh(c
*x)*c^4*x^4+4/3*arctanh(c*x)*c^3*x^3+3*arctanh(c*x)*c^2*x^2+4*arctanh(c*x)
*c*x+arctanh(c*x)*ln(c*x)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c
*x+1)+1/12*x^3*c^3+2/3*c^2*x^2+13/4*c*x+103/24*ln(c*x-1)+25/24*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x}\,dx=\int \frac{(cdx+d)^4(b\operatorname{artanh}\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x
+ a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x
 + b*d^4)*arctanh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx = d^4\left(\int 4ac\,dx + \int \frac{a}{x}\,dx + \int 6ac^2x\,dx + \int 4ac^3x^2\,dx + \int ac^4x^3\,dx + \int 4bc\,\text{atanh}(cx)\,dx + \int \frac{b\,\text{atanh}(cx)}{x}\,dx + \int 6bc^2x\,\text{atanh}(cx)\,dx + \int 4bc^3x^2\,\text{atanh}(cx)\,dx + \int bc^4x^3\,\text{atanh}(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x,x)
```

output
```
d**4*(Integral(4*a*c, x) + Integral(a/x, x) + Integral(6*a*c**2*x, x) + In
tegral(4*a*c**3*x**2, x) + Integral(a*c**4*x**3, x) + Integral(4*b*c*atanh
(c*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(6*b*c**2*x*atanh(c*x),
x) + Integral(4*b*c**3*x**2*atanh(c*x), x) + Integral(b*c**4*x**3*atanh(c*
x), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.49

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx$$
$$= \frac{1}{4}\,ac^4d^4x^4 + \frac{4}{3}\,ac^3d^4x^3 + \frac{1}{12}\,bc^3d^4x^3 + 3\,ac^2d^4x^2 + \frac{2}{3}\,bc^2d^4x^2$$
$$+ 4\,acd^4x + \frac{13}{4}\,bcd^4x + 2\left(2\,cx\,\text{artanh}\left(cx\right) + \log\left(-c^2x^2+1\right)\right)bd^4$$
$$- \frac{1}{2}\,(\log\left(cx\right)\log\left(-cx+1\right) + \text{Li}_2(-cx+1))bd^4$$
$$+ \frac{1}{2}\,(\log\left(cx+1\right)\log\left(-cx\right) + \text{Li}_2(cx+1))bd^4$$
$$- \frac{23}{24}\,bd^4\log\left(cx+1\right) + \frac{55}{24}\,bd^4\log\left(cx-1\right) + ad^4\log\left(x\right)$$
$$+ \frac{1}{24}\left(3\,bc^4d^4x^4 + 16\,bc^3d^4x^3 + 36\,bc^2d^4x^2\right)\log\left(cx+1\right)$$
$$- \frac{1}{24}\left(3\,bc^4d^4x^4 + 16\,bc^3d^4x^3 + 36\,bc^2d^4x^2\right)\log\left(-cx+1\right)$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*c^4*d^4*x^4 + 4/3*a*c^3*d^4*x^3 + 1/12*b*c^3*d^4*x^3 + 3*a*c^2*d^4*x
^2 + 2/3*b*c^2*d^4*x^2 + 4*a*c*d^4*x + 13/4*b*c*d^4*x + 2*(2*c*x*arctanh(c
*x) + log(-c^2*x^2 + 1))*b*d^4 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x
+ 1))*b*d^4 + 1/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^4 - 23/24*
b*d^4*log(c*x + 1) + 55/24*b*d^4*log(c*x - 1) + a*d^4*log(x) + 1/24*(3*b*c
^4*d^4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(c*x + 1) - 1/24*(3*b
*c^4*d^4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(-c*x + 1)
```

Giac [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx = \int \frac{(cdx+d)^4(b\,\text{artanh}\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x}\,dx$$
$$=\frac{d^4\Big(3atanh(cx)\,b\,c^4x^4+16atanh(cx)\,b\,c^3x^3+36atanh(cx)\,b\,c^2x^2+48atanh(cx)\,bcx+25atanh(cx)\,b+1}{}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x,x)
```

output
```
(d**4*(3*atanh(c*x)*b*c**4*x**4 + 16*atanh(c*x)*b*c**3*x**3 + 36*atanh(c*x
)*b*c**2*x**2 + 48*atanh(c*x)*b*c*x + 25*atanh(c*x)*b + 12*int(atanh(c*x)/
x,x)*b + 64*log(c**2*x - c)*b + 12*log(x)*a + 3*a*c**4*x**4 + 16*a*c**3*x*
*3 + 36*a*c**2*x**2 + 48*a*c*x + b*c**3*x**3 + 8*b*c**2*x**2 + 39*b*c*x))/
12
```

3.36 $\int \frac{(d+cdx)^4(a+b\,\mathbf{arctanh}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 178

$$\begin{aligned}\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx &= 6ac^2d^4x+2bc^2d^4x+\frac{1}{6}bc^3d^4x^2-2bcd^4\text{arctanh}(cx)\\ &\quad+6bc^2d^4x\text{arctanh}(cx)-\frac{d^4(a+b\text{arctanh}(cx))}{x}\\ &\quad+2c^3d^4x^2(a+b\text{arctanh}(cx))\\ &\quad+\frac{1}{3}c^4d^4x^3(a+b\text{arctanh}(cx))+4acd^4\log(x)\\ &\quad+bcd^4\log(x)+\frac{8}{3}bcd^4\log\left(1-c^2x^2\right)\\ &\quad-2bcd^4\,\text{PolyLog}(2,-cx)+2bcd^4\,\text{PolyLog}(2,cx)\end{aligned}$$

output
```
6*a*c^2*d^4*x+2*b*c^2*d^4*x+1/6*b*c^3*d^4*x^2-2*b*c*d^4*arctanh(c*x)+6*b*c
^2*d^4*x*arctanh(c*x)-d^4*(a+b*arctanh(c*x))/x+2*c^3*d^4*x^2*(a+b*arctanh(
c*x))+1/3*c^4*d^4*x^3*(a+b*arctanh(c*x))+4*a*c*d^4*ln(x)+b*c*d^4*ln(x)+8/3
*b*c*d^4*ln(-c^2*x^2+1)-2*b*c*d^4*polylog(2,-c*x)+2*b*c*d^4*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.09

$$\int \frac{(d + cdx)^4(a + b\text{arctanh}(cx))}{x^2}\,dx$$
$$= \frac{d^4(-6a + 36ac^2x^2 + 12bc^2x^2 + 12ac^3x^3 + bc^3x^3 + 2ac^4x^4 - 6b\text{arctanh}(cx) + 36bc^2x^2\text{arctanh}(cx) + 12bc$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
(d^4*(-6*a + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 12*a*c^3*x^3 + b*c^3*x^3 + 2*a*
c^4*x^4 - 6*b*ArcTanh[c*x] + 36*b*c^2*x^2*ArcTanh[c*x] + 12*b*c^3*x^3*ArcT
anh[c*x] + 2*b*c^4*x^4*ArcTanh[c*x] + 24*a*c*x*Log[x] + 6*b*c*x*Log[c*x] +
 6*b*c*x*Log[1 - c*x] - 6*b*c*x*Log[1 + c*x] + 15*b*c*x*Log[1 - c^2*x^2] +
 b*c*x*Log[-1 + c^2*x^2] - 12*b*c*x*PolyLog[2, -(c*x)] + 12*b*c*x*PolyLog[
2, c*x]))/(6*x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx + d)^4(a + b\text{arctanh}(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(c^4d^4x^2(a + b\text{arctanh}(cx)) + 4c^3d^4x(a + b\text{arctanh}(cx)) + 6c^2d^4(a + b\text{arctanh}(cx)) + \frac{d^4(a + b\text{arctanh}(cx))}{x^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^4d^4x^3(a+b\text{arctanh}(cx))+2c^3d^4x^2(a+b\text{arctanh}(cx))-\frac{d^4(a+b\text{arctanh}(cx))}{x}+6ac^2d^4x+4acd^4\log(x)+6bc^2d^4x\text{arctanh}(cx)-2bcd^4\text{arctanh}(cx)+\frac{1}{6}bc^3d^4x^2+\frac{8}{3}bcd^4\log(1-c^2x^2)+2bc^2d^4x-2bcd^4\,\text{PolyLog}(2,-cx)+2bcd^4\,\text{PolyLog}(2,cx)+bcd^4\log(x)$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

output
```
6*a*c^2*d^4*x + 2*b*c^2*d^4*x + (b*c^3*d^4*x^2)/6 - 2*b*c*d^4*ArcTanh[c*x]
 + 6*b*c^2*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/x + 2*c^3*d^4*x
^2*(a + b*ArcTanh[c*x]) + (c^4*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + 4*a*c*d^4
*Log[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 - c^2*x^2])/3 - 2*b*c*d^4*Poly
Log[2, -(c*x)] + 2*b*c*d^4*PolyLog[2, c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

method	result
parts	$d^4a\left(\frac{x^3c^4}{3}+2c^3x^2+6c^2x+4c\ln(x)-\frac{1}{x}\right)+d^4bc\left(\frac{\text{arctanh}(cx)c^3x^3}{3}+2\ \text{arctanh}(cx)\,c^2x^2\right.$
derivativedivides	$c\left(d^4a\left(\frac{x^3c^3}{3}+2c^2x^2+6cx+4\ln(cx)-\frac{1}{cx}\right)+d^4b\left(\frac{\text{arctanh}(cx)c^3x^3}{3}+2\ \text{arctanh}(cx)\,c^2x^2\right.\right.$
default	$c\left(d^4a\left(\frac{x^3c^3}{3}+2c^2x^2+6cx+4\ln(cx)-\frac{1}{cx}\right)+d^4b\left(\frac{\text{arctanh}(cx)c^3x^3}{3}+2\ \text{arctanh}(cx)\,c^2x^2\right.\right.$
risch	$-\frac{119bc\,d^4}{18}-\frac{25c\,d^4a}{3}+\frac{b\,c^4d^4\ln(cx+1)x^3}{6}+b\,c^3d^4\ln(cx+1)\,x^2+3b\,c^2d^4\ln(cx+1)\,x+\frac{5bc\,d}{}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/3*x^3*c^4+2*c^3*x^2+6*c^2*x+4*c*ln(x)-1/x)+d^4*b*c*(1/3*arctanh(c
*x)*c^3*x^3+2*arctanh(c*x)*c^2*x^2+6*arctanh(c*x)*c*x+4*arctanh(c*x)*ln(c*
x)-arctanh(c*x)/c/x-2*dilog(c*x)-2*dilog(c*x+1)-2*ln(c*x)*ln(c*x+1)+1/6*c^
2*x^2+2*c*x+11/3*ln(c*x-1)+ln(c*x)+5/3*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx = \int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x
+ a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x
 + b*d^4)*arctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx = d^4\left(\int 6ac^2\,dx + \int \frac{a}{x^2}\,dx + \int \frac{4ac}{x}\,dx + \int 4ac^3x\,dx + \int ac^4x^2\,dx + \int 6bc^2\,\text{atanh}\,(cx)\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{4bc\,\text{atanh}\,(cx)}{x}\,dx + \int 4bc^3x\,\text{atanh}\,(cx)\,dx + \int bc^4x^2\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**2,x)
```

output
```
d**4*(Integral(6*a*c**2, x) + Integral(a/x**2, x) + Integral(4*a*c/x, x) +
 Integral(4*a*c**3*x, x) + Integral(a*c**4*x**2, x) + Integral(6*b*c**2*at
anh(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(4*b*c*atanh(c*x)/
x, x) + Integral(4*b*c**3*x*atanh(c*x), x) + Integral(b*c**4*x**2*atanh(c*
x), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.58

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$= \frac{1}{3}\,ac^4d^4x^3 + 2\,ac^3d^4x^2 + \frac{1}{6}\,bc^3d^4x^2 + 6\,ac^2d^4x$$
$$+ 2\,bc^2d^4x + 3\left(2\,cx\,\text{artanh}\left(cx\right) + \log\left(-c^2x^2+1\right)\right)bcd^4$$
$$- 2\left(\log\left(cx\right)\log\left(-cx+1\right) + \text{Li}_2(-cx+1)\right)bcd^4$$
$$+ 2\left(\log\left(cx+1\right)\log\left(-cx\right) + \text{Li}_2(cx+1)\right)bcd^4$$
$$- \frac{5}{6}\,bcd^4\log\left(cx+1\right) + \frac{7}{6}\,bcd^4\log\left(cx-1\right) + 4\,acd^4\log\left(x\right)$$
$$- \frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\text{artanh}\left(cx\right)}{x}\right)bd^4 - \frac{ad^4}{x}$$
$$+ \frac{1}{6}\left(bc^4d^4x^3 + 6\,bc^3d^4x^2\right)\log\left(cx+1\right) - \frac{1}{6}\left(bc^4d^4x^3 + 6\,bc^3d^4x^2\right)\log\left(-cx+1\right)$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*c^4*d^4*x^3 + 2*a*c^3*d^4*x^2 + 1/6*b*c^3*d^4*x^2 + 6*a*c^2*d^4*x +
2*b*c^2*d^4*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^4 - 2*(lo
g(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c*d^4 + 2*(log(c*x + 1)*log(-c*x
) + dilog(c*x + 1))*b*c*d^4 - 5/6*b*c*d^4*log(c*x + 1) + 7/6*b*c*d^4*log(c
*x - 1) + 4*a*c*d^4*log(x) - 1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arct
anh(c*x)/x)*b*d^4 - a*d^4/x + 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(c*
x + 1) - 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(-c*x + 1)
```

Giac [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^2,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^2}\,dx$$
$$=\frac{d^4\Big(2atanh(cx)\,b\,c^4x^4+12atanh(cx)\,b\,c^3x^3+36atanh(cx)\,b\,c^2x^2+20atanh(cx)\,bcx-6atanh(cx)\,b+24$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^2,x)
```

output

```
(d**4*(2*atanh(c*x)*b*c**4*x**4 + 12*atanh(c*x)*b*c**3*x**3 + 36*atanh(c*x
)*b*c**2*x**2 + 20*atanh(c*x)*b*c*x - 6*atanh(c*x)*b + 24*int(atanh(c*x)/x
,x)*b*c*x + 32*log(c**2*x - c)*b*c*x + 24*log(x)*a*c*x + 6*log(x)*b*c*x +
2*a*c**4*x**4 + 12*a*c**3*x**3 + 36*a*c**2*x**2 - 6*a + b*c**3*x**3 + 12*b
*c**2*x**2))/(6*x)
```

3.37 $\int \frac{(d+cdx)^4(a+b\,\mathrm{arctanh}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{(d+cdx)^4(a+b\,\mathrm{arctanh}(cx))}{x^3}\,dx = -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\,\mathrm{arctanh}(cx)-\frac{d^4(a+b\,\mathrm{arctanh}(cx))}{2x^2}-\frac{4cd^4(a+b\,\mathrm{arctanh}(cx))}{x}+\frac{1}{2}c^4d^4x^2(a+b\,\mathrm{arctanh}(cx))+6ac^2d^4\log(x)+4bc^2d^4\log(x)-3bc^2d^4\,\mathrm{PolyLog}(2,-cx)+3bc^2d^4\,\mathrm{PolyLog}(2,cx)$$

output
```
-1/2*b*c*d^4/x+4*a*c^3*d^4*x+1/2*b*c^3*d^4*x+4*b*c^3*d^4*x*arctanh(c*x)-1/
2*d^4*(a+b*arctanh(c*x))/x^2-4*c*d^4*(a+b*arctanh(c*x))/x+1/2*c^4*d^4*x^2*
(a+b*arctanh(c*x))+6*a*c^2*d^4*ln(x)+4*b*c^2*d^4*ln(x)-3*b*c^2*d^4*polylog
(2,-c*x)+3*b*c^2*d^4*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$= \frac{d^4(-a-8acx-bcx+8ac^3x^3+bc^3x^3+ac^4x^4-b\text{arctanh}(cx)-8bcx\text{arctanh}(cx)+8bc^3x^3\text{arctanh}(cx)}{}$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
(d^4*(-a - 8*a*c*x - b*c*x + 8*a*c^3*x^3 + b*c^3*x^3 + a*c^4*x^4 - b*ArcTa
nh[c*x] - 8*b*c*x*ArcTanh[c*x] + 8*b*c^3*x^3*ArcTanh[c*x] + b*c^4*x^4*ArcT
anh[c*x] + 12*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - 6*b*c^2*x^2*PolyLo
g[2, -(c*x)] + 6*b*c^2*x^2*PolyLog[2, c*x]))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$\Big\downarrow \; 6502$$
$$\int \left(c^4d^4x(a+b\text{arctanh}(cx))+4c^3d^4(a+b\text{arctanh}(cx))+\frac{6c^2d^4(a+b\text{arctanh}(cx))}{x}+\frac{d^4(a+b\text{arctanh}(cx))}{x^3}+\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{1}{2}c^4d^4x^2(a+b\text{arctanh}(cx))-\frac{d^4(a+b\text{arctanh}(cx))}{2x^2}-\frac{4cd^4(a+b\text{arctanh}(cx))}{x}+4ac^3d^4x+6ac^2d^4\log(x)+4bc^3d^4x\text{arctanh}(cx)+\frac{1}{2}bc^3d^4x-3bc^2d^4\text{PolyLog}(2,-cx)+3bc^2d^4\text{PolyLog}(2,cx)+4bc^2d^4\log(x)-\frac{bcd^4}{2x}$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^4)/x + 4*a*c^3*d^4*x + (b*c^3*d^4*x)/2 + 4*b*c^3*d^4*x*ArcTanh
[c*x] - (d^4*(a + b*ArcTanh[c*x]))/(2*x^2) - (4*c*d^4*(a + b*ArcTanh[c*x])
)/x + (c^4*d^4*x^2*(a + b*ArcTanh[c*x]))/2 + 6*a*c^2*d^4*Log[x] + 4*b*c^2*
d^4*Log[x] - 3*b*c^2*d^4*PolyLog[2, -(c*x)] + 3*b*c^2*d^4*PolyLog[2, c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

method	result
parts	$d^4a\left(\frac{c^4x^2}{2}+4x\,c^3+6c^2\ln\left(x\right)-\frac{4c}{x}-\frac{1}{2x^2}\right)+d^4b\,c^2\left(\frac{\text{arctanh}(cx)c^2x^2}{2}+4\,\text{arctanh}\left(cx\right)cx\right.$
derivativedivides	$c^2\left(d^4a\left(\frac{c^2x^2}{2}+4cx-\frac{1}{2c^2x^2}+6\ln\left(cx\right)-\frac{4}{cx}\right)+d^4b\left(\frac{\text{arctanh}(cx)c^2x^2}{2}+4\,\text{arctanh}\left(cx\right)cx\right.\right.$
default	$c^2\left(d^4a\left(\frac{c^2x^2}{2}+4cx-\frac{1}{2c^2x^2}+6\ln\left(cx\right)-\frac{4}{cx}\right)+d^4b\left(\frac{\text{arctanh}(cx)c^2x^2}{2}+4\,\text{arctanh}\left(cx\right)cx\right.\right.$
risch	$-\frac{c^4d^4b\ln(-cx+1)x^2}{4}-2c^3d^4b\ln\left(-cx+1\right)x+\frac{2c\,d^4b\ln(-cx+1)}{x}+4d^4a\,c^3x-4b\,c^2d^4-\frac{9c^2d^4}{2}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/2*c^4*x^2+4*x*c^3+6*c^2*ln(x)-4*c/x-1/2/x^2)+d^4*b*c^2*(1/2*arcta
nh(c*x)*c^2*x^2+4*arctanh(c*x)*c*x-1/2*arctanh(c*x)/c^2/x^2+6*arctanh(c*x)
*ln(c*x)-4*arctanh(c*x)/c/x-3*dilog(c*x)-3*dilog(c*x+1)-3*ln(c*x)*ln(c*x+1
)+1/2*c*x+4*ln(c*x)-1/2/c/x)
```

Fricas [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^3}\,dx=\int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x
+ a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x
 + b*d^4)*arctanh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^3}\,dx=d^4\left(\int 4ac^3\,dx+\int \frac{a}{x^3}\,dx+\int \frac{4ac}{x^2}\,dx+\int \frac{6ac^2}{x}\,dx+\int ac^4x\,dx+\int 4bc^3\,\text{atanh}\,(cx)\,dx+\int \frac{b\,\text{atanh}\,(cx)}{x^3}\,dx+\int \frac{4bc\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{6bc^2\,\text{atanh}\,(cx)}{x}\,dx+\int bc^4x\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**3,x)
```

output
```
d**4*(Integral(4*a*c**3, x) + Integral(a/x**3, x) + Integral(4*a*c/x**2, x
) + Integral(6*a*c**2/x, x) + Integral(a*c**4*x, x) + Integral(4*b*c**3*at
anh(c*x), x) + Integral(b*atanh(c*x)/x**3, x) + Integral(4*b*c*atanh(c*x)/
x**2, x) + Integral(6*b*c**2*atanh(c*x)/x, x) + Integral(b*c**4*x*atanh(c*
x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(146) = 292$.

Time = 0.13 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.88

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^3}\,dx$$
$$= \frac{1}{4}\,bc^4d^4x^2\log(cx+1) - \frac{1}{4}\,bc^4d^4x^2\log(-cx+1) + \frac{1}{2}\,ac^4d^4x^2 + 4\,ac^3d^4x + \frac{1}{2}\,bc^3d^4x + 2\,\big(2\,cx\,\text{artanh}\,(cx) + \log\big(-c^2x^2+1\big)\big)bc^2d^4 - 3\,(\log(cx)\log(-cx+1) + \text{Li}_2(-cx+1))bc^2d^4 + 3\,(\log(cx+1)\log(-cx) + \text{Li}_2(cx+1))bc^2d^4 - \frac{1}{4}\,bc^2d^4\log(cx+1) + \frac{1}{4}\,bc^2d^4\log(cx-1) + 6\,ac^2d^4\log(x) - 2\left(c\big(\log\big(c^2x^2-1\big) - \log\big(x^2\big)\big) + \frac{2\,\text{artanh}\,(cx)}{x}\right)bcd^4 + \frac{1}{4}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\text{artanh}\,(cx)}{x^2}\right)bd^4 - \frac{4\,acd^4}{x} - \frac{ad^4}{2\,x^2}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*b*c^4*d^4*x^2*log(c*x + 1) - 1/4*b*c^4*d^4*x^2*log(-c*x + 1) + 1/2*a*c
^4*d^4*x^2 + 4*a*c^3*d^4*x + 1/2*b*c^3*d^4*x + 2*(2*c*x*arctanh(c*x) + log
(-c^2*x^2 + 1))*b*c^2*d^4 - 3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b
*c^2*d^4 + 3*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c^2*d^4 - 1/4*b*c
^2*d^4*log(c*x + 1) + 1/4*b*c^2*d^4*log(c*x - 1) + 6*a*c^2*d^4*log(x) - 2*
(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^4 + 1/4*((c*log
(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^4 - 4*a*c*d^
4/x - 1/2*a*d^4/x^2
```

Giac [F]

$$\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^3}\,dx = \int \frac{(cdx+d)^4(b\operatorname{artanh}(cx)+a)}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{atanh}(c\,x))\,(d+c\,d\,x)^4}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^3,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^3}\,dx = \frac{d^4\left(atanh(cx)\,b\,c^4x^4+8atanh(cx)\,b\,c^3x^3-8atanh(cx)\,bcx-atanh(cx)\,b+12\left(\int \frac{atanh(cx)}{x}dx\right)b\,c^2x^2+12\right.}{2x^2}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^3,x)
```

output
```
(d**4*(atanh(c*x)*b*c**4*x**4 + 8*atanh(c*x)*b*c**3*x**3 - 8*atanh(c*x)*b*
c*x - atanh(c*x)*b + 12*int(atanh(c*x)/x,x)*b*c**2*x**2 + 12*log(x)*a*c**2
*x**2 + 8*log(x)*b*c**2*x**2 + a*c**4*x**4 + 8*a*c**3*x**3 - 8*a*c*x - a +
 b*c**3*x**3 - b*c*x))/(2*x**2)
```

3.38 $\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\begin{aligned}\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^4}\,dx = &-\frac{bcd^4}{6x^2}-\frac{2bc^2d^4}{x}+ac^4d^4x+2bc^3d^4\operatorname{arctanh}(cx)\\&+bc^4d^4x\operatorname{arctanh}(cx)-\frac{d^4(a+b\operatorname{arctanh}(cx))}{3x^3}\\&-\frac{2cd^4(a+b\operatorname{arctanh}(cx))}{x^2}\\&-\frac{6c^2d^4(a+b\operatorname{arctanh}(cx))}{x}+4ac^3d^4\log(x)\\&+\frac{19}{3}bc^3d^4\log(x)-\frac{8}{3}bc^3d^4\log\left(1-c^2x^2\right)\\&-2bc^3d^4\operatorname{PolyLog}(2,-cx)+2bc^3d^4\operatorname{PolyLog}(2,cx)\end{aligned}$$

output
```
-1/6*b*c*d^4/x^2-2*b*c^2*d^4/x+a*c^4*d^4*x+2*b*c^3*d^4*arctanh(c*x)+b*c^4*
d^4*x*arctanh(c*x)-1/3*d^4*(a+b*arctanh(c*x))/x^3-2*c*d^4*(a+b*arctanh(c*x
))/x^2-6*c^2*d^4*(a+b*arctanh(c*x))/x+4*a*c^3*d^4*ln(x)+19/3*b*c^3*d^4*ln(
x)-8/3*b*c^3*d^4*ln(-c^2*x^2+1)-2*b*c^3*d^4*polylog(2,-c*x)+2*b*c^3*d^4*po
lylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$= \frac{d^4(-2a-12acx-bcx-36ac^2x^2-12bc^2x^2+6ac^4x^4-2b\text{arctanh}(cx)-12bcx\text{arctanh}(cx)-36bc^2x^2\text{ar}}{}$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
(d^4*(-2*a - 12*a*c*x - b*c*x - 36*a*c^2*x^2 - 12*b*c^2*x^2 + 6*a*c^4*x^4
- 2*b*ArcTanh[c*x] - 12*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 6
*b*c^4*x^4*ArcTanh[c*x] + 24*a*c^3*x^3*Log[x] + 38*b*c^3*x^3*Log[c*x] - 6*
b*c^3*x^3*Log[1 - c*x] + 6*b*c^3*x^3*Log[1 + c*x] - 16*b*c^3*x^3*Log[1 - c
^2*x^2] - 12*b*c^3*x^3*PolyLog[2, -(c*x)] + 12*b*c^3*x^3*PolyLog[2, c*x]))
/(6*x^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{6502, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(c^4d^4(a+b\text{arctanh}(cx))+\frac{4c^3d^4(a+b\text{arctanh}(cx))}{x}+\frac{6c^2d^4(a+b\text{arctanh}(cx))}{x^2}+\frac{d^4(a+b\text{arctanh}(cx))}{x^4}+\frac{4}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2d^4(a+b\text{arctanh}(cx))}{x}-\frac{d^4(a+b\text{arctanh}(cx))}{3x^3}-\frac{2cd^4(a+b\text{arctanh}(cx))}{x^2}+ac^4d^4x+4ac^3d^4\log(x)+bc^4d^4x\text{arctanh}(cx)+2bc^3d^4\text{arctanh}(cx)-2bc^3d^4\,\text{PolyLog}(2,-cx)+2bc^3d^4\,\text{PolyLog}(2,cx)+\frac{19}{3}bc^3d^4\log(x)-\frac{2bc^2d^4}{x}-\frac{8}{3}bc^3d^4\log\left(1-c^2x^2\right)-\frac{bcd^4}{6x^2}$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

output
```
-1/6*(b*c*d^4)/x^2 - (2*b*c^2*d^4)/x + a*c^4*d^4*x + 2*b*c^3*d^4*ArcTanh[c
*x] + b*c^4*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/(3*x^3) - (2*c
*d^4*(a + b*ArcTanh[c*x]))/x^2 - (6*c^2*d^4*(a + b*ArcTanh[c*x]))/x + 4*a*
c^3*d^4*Log[x] + (19*b*c^3*d^4*Log[x])/3 - (8*b*c^3*d^4*Log[1 - c^2*x^2])/
3 - 2*b*c^3*d^4*PolyLog[2, -(c*x)] + 2*b*c^3*d^4*PolyLog[2, c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

method	result
parts	$d^4a\left(c^4x+4c^3\ln(x)-\frac{6c^2}{x}-\frac{2c}{x^2}-\frac{1}{3x^3}\right)+d^4b\,c^3\Big(\text{arctanh}(cx)\,cx-\frac{\text{arctanh}(cx)}{3c^3x^3}-\frac{2\,\text{arctan}}{c^2x}$
derivativedivides	$c^3\Big(d^4a\big(cx-\frac{1}{3c^3x^3}-\frac{2}{c^2x^2}+4\ln(cx)-\frac{6}{cx}\big)+d^4b\Big(\text{arctanh}(cx)\,cx-\frac{\text{arctanh}(cx)}{3c^3x^3}-\frac{2\,\text{arcta}}{c^2}$
default	$c^3\Big(d^4a\big(cx-\frac{1}{3c^3x^3}-\frac{2}{c^2x^2}+4\ln(cx)-\frac{6}{cx}\big)+d^4b\Big(\text{arctanh}(cx)\,cx-\frac{\text{arctanh}(cx)}{3c^3x^3}-\frac{2\,\text{arcta}}{c^2}$
risch	$-\frac{bc\,d^4\ln(cx+1)}{x^2}-\frac{3b\,c^2d^4\ln(cx+1)}{x}-b\,c^3d^4-c^3d^4a-\frac{c^4d^4b\ln(-cx+1)x}{2}+\frac{c\,d^4b\ln(-cx+1)}{x^2}+\frac{3c^2d^4b}{}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(c^4*x+4*c^3*ln(x)-6*c^2/x-2*c/x^2-1/3/x^3)+d^4*b*c^3*(arctanh(c*x)*
c*x-1/3*arctanh(c*x)/c^3/x^3-2*arctanh(c*x)/c^2/x^2+4*arctanh(c*x)*ln(c*x)
-6*arctanh(c*x)/c/x-2*dilog(c*x)-2*dilog(c*x+1)-2*ln(c*x)*ln(c*x+1)-11/3*l
n(c*x-1)-1/6/c^2/x^2-2/c/x+19/3*ln(c*x)-5/3*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx=\int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x
+ a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x
 + b*d^4)*arctanh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx=d^4\left(\int ac^4\,dx+\int \frac{a}{x^4}\,dx+\int \frac{4ac}{x^3}\,dx+\int \frac{6ac^2}{x^2}\,dx+\int \frac{4ac^3}{x}\,dx+\int bc^4\,\text{atanh}\,(cx)\,dx+\int \frac{b\,\text{atanh}\,(cx)}{x^4}\,dx+\int \frac{4bc\,\text{atanh}\,(cx)}{x^3}\,dx+\int \frac{6bc^2\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{4bc^3\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**4,x)
```

output
```
d**4*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(4*a*c/x**3, x)
+ Integral(6*a*c**2/x**2, x) + Integral(4*a*c**3/x, x) + Integral(b*c**4*a
tanh(c*x), x) + Integral(b*atanh(c*x)/x**4, x) + Integral(4*b*c*atanh(c*x)
/x**3, x) + Integral(6*b*c**2*atanh(c*x)/x**2, x) + Integral(4*b*c**3*atan
h(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx = \int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

output
```
a*c^4*d^4*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^3*d^4 + 2*b
*c^3*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 4*a*c^3*d^4*log(
x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^4 + ((
c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^4 - 1
/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*
b*d^4 - 6*a*c^2*d^4/x - 2*a*c*d^4/x^2 - 1/3*a*d^4/x^3
```

Giac [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx = \int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^4}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^4,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^4, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^4}\,dx = \frac{d^4\Big(6atanh(cx)\,b\,c^4x^4 - 20atanh(cx)\,b\,c^3x^3 - 36atanh(cx)\,b\,c^2x^2 - 12atanh(cx)\,bcx - 2atanh(cx)\,b + 24}{}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^4,x)
```

output
```
(d**4*(6*atanh(c*x)*b*c**4*x**4 - 20*atanh(c*x)*b*c**3*x**3 - 36*atanh(c*x
)*b*c**2*x**2 - 12*atanh(c*x)*b*c*x - 2*atanh(c*x)*b + 24*int(atanh(c*x)/x
,x)*b*c**3*x**3 - 32*log(c**2*x - c)*b*c**3*x**3 + 24*log(x)*a*c**3*x**3 +
 38*log(x)*b*c**3*x**3 + 6*a*c**4*x**4 - 36*a*c**2*x**2 - 12*a*c*x - 2*a -
 12*b*c**2*x**2 - b*c*x))/(6*x**3)
```

3.39 $\int \frac{(d+cdx)^4(a+b\,\text{arctanh}(cx))}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\begin{aligned}\int \frac{(d+cdx)^4(a+b\,\text{arctanh}(cx))}{x^5}\,dx =& -\frac{bcd^4}{12x^3}-\frac{2bc^2d^4}{3x^2}-\frac{13bc^3d^4}{4x} \\ &+\frac{13}{4}bc^4d^4\text{arctanh}(cx)-\frac{d^4(a+b\,\text{arctanh}(cx))}{4x^4} \\ &-\frac{4cd^4(a+b\,\text{arctanh}(cx))}{3x^3} \\ &-\frac{3c^2d^4(a+b\,\text{arctanh}(cx))}{x^2} \\ &-\frac{4c^3d^4(a+b\,\text{arctanh}(cx))}{x}+ac^4d^4\log(x) \\ &+\frac{16}{3}bc^4d^4\log(x)-\frac{8}{3}bc^4d^4\log\left(1-c^2x^2\right) \\ &-\frac{1}{2}bc^4d^4\,\text{PolyLog}(2,-cx) \\ &+\frac{1}{2}bc^4d^4\,\text{PolyLog}(2,cx)\end{aligned}$$

output
```
-1/12*b*c*d^4/x^3-2/3*b*c^2*d^4/x^2-13/4*b*c^3*d^4/x+13/4*b*c^4*d^4*arctan
h(c*x)-1/4*d^4*(a+b*arctanh(c*x))/x^4-4/3*c*d^4*(a+b*arctanh(c*x))/x^3-3*c
^2*d^4*(a+b*arctanh(c*x))/x^2-4*c^3*d^4*(a+b*arctanh(c*x))/x+a*c^4*d^4*ln(
x)+16/3*b*c^4*d^4*ln(x)-8/3*b*c^4*d^4*ln(-c^2*x^2+1)-1/2*b*c^4*d^4*polylog
(2,-c*x)+1/2*b*c^4*d^4*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$= \frac{d^4(-6a-32acx-2bcx-72ac^2x^2-16bc^2x^2-96ac^3x^3-78bc^3x^3-6b\text{arctanh}(cx)-32bcx\text{arctanh}(cx}{}$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
(d^4*(-6*a - 32*a*c*x - 2*b*c*x - 72*a*c^2*x^2 - 16*b*c^2*x^2 - 96*a*c^3*x
^3 - 78*b*c^3*x^3 - 6*b*ArcTanh[c*x] - 32*b*c*x*ArcTanh[c*x] - 72*b*c^2*x^
2*ArcTanh[c*x] - 96*b*c^3*x^3*ArcTanh[c*x] + 24*a*c^4*x^4*Log[x] + 128*b*c
^4*x^4*Log[c*x] - 39*b*c^4*x^4*Log[1 - c*x] + 39*b*c^4*x^4*Log[1 + c*x] -
64*b*c^4*x^4*Log[1 - c^2*x^2] - 12*b*c^4*x^4*PolyLog[2, -(c*x)] + 12*b*c^4
*x^4*PolyLog[2, c*x]))/(24*x^4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$\Big\downarrow \text{ 6502}$$
$$\int \left(\frac{c^4d^4(a+b\text{arctanh}(cx))}{x}+\frac{4c^3d^4(a+b\text{arctanh}(cx))}{x^2}+\frac{6c^2d^4(a+b\text{arctanh}(cx))}{x^3}+\frac{d^4(a+b\text{arctanh}(cx))}{x^5}+\right.$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{4c^3d^4(a+b\operatorname{arctanh}(cx))}{x}-\frac{3c^2d^4(a+b\operatorname{arctanh}(cx))}{x^2}-\frac{d^4(a+b\operatorname{arctanh}(cx))}{4x^4}-\frac{4cd^4(a+b\operatorname{arctanh}(cx))}{3x^3}+ac^4d^4\log(x)+\frac{13}{4}bc^4d^4\operatorname{arctanh}(cx)-\frac{1}{2}bc^4d^4\operatorname{PolyLog}(2,-cx)+\frac{1}{2}bc^4d^4\operatorname{PolyLog}(2,cx)+\frac{16}{3}bc^4d^4\log(x)-\frac{13bc^3d^4}{4x}-\frac{2bc^2d^4}{3x^2}-\frac{8}{3}bc^4d^4\log\left(1-c^2x^2\right)-\frac{bcd^4}{12x^3}$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^4)/x^3 - (2*b*c^2*d^4)/(3*x^2) - (13*b*c^3*d^4)/(4*x) + (13*b
*c^4*d^4*ArcTanh[c*x])/4 - (d^4*(a + b*ArcTanh[c*x]))/(4*x^4) - (4*c*d^4*(
a + b*ArcTanh[c*x]))/(3*x^3) - (3*c^2*d^4*(a + b*ArcTanh[c*x]))/x^2 - (4*c
^3*d^4*(a + b*ArcTanh[c*x]))/x + a*c^4*d^4*Log[x] + (16*b*c^4*d^4*Log[x])/
3 - (8*b*c^4*d^4*Log[1 - c^2*x^2])/3 - (b*c^4*d^4*PolyLog[2, -(c*x)])/2 +
(b*c^4*d^4*PolyLog[2, c*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

method	result
parts	$d^4a\left(c^4\ln(x)-\frac{1}{4x^4}-\frac{4c^3}{x}-\frac{3c^2}{x^2}-\frac{4c}{3x^3}\right)+d^4b\,c^4\left(-\frac{4\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{3\,\mathrm{arctanh}(cx)}{c^2x^2}+\mathrm{arctanh}\right.$
derivativedivides	$c^4\left(d^4a\left(-\frac{4}{3c^3x^3}-\frac{3}{c^2x^2}+\ln(cx)-\frac{4}{cx}-\frac{1}{4c^4x^4}\right)+d^4b\left(-\frac{4\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{3\,\mathrm{arctanh}(cx)}{c^2x^2}+\mathrm{arct}\right.\right.$
default	$c^4\left(d^4a\left(-\frac{4}{3c^3x^3}-\frac{3}{c^2x^2}+\ln(cx)-\frac{4}{cx}-\frac{1}{4c^4x^4}\right)+d^4b\left(-\frac{4\,\mathrm{arctanh}(cx)}{3c^3x^3}-\frac{3\,\mathrm{arctanh}(cx)}{c^2x^2}+\mathrm{arct}\right.\right.$
risch	$\frac{3c^2d^4b\ln(-cx+1)}{2x^2}-\frac{103\ln(-cx+1)b\,c^4d^4}{24}-\frac{25\ln(cx+1)b\,c^4d^4}{24}+\frac{2c\,d^4b\ln(-cx+1)}{3x^3}+\frac{2c^3d^4b\ln(-cx+1)}{x}-3$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(c^4*ln(x)-1/4/x^4-4*c^3/x-3*c^2/x^2-4/3*c/x^3)+d^4*b*c^4*(-4/3*arct
anh(c*x)/c^3/x^3-3*arctanh(c*x)/c^2/x^2+arctanh(c*x)*ln(c*x)-4*arctanh(c*x
)/c/x-1/4*arctanh(c*x)/c^4/x^4-103/24*ln(c*x-1)-1/12/c^3/x^3-2/3/c^2/x^2-1
3/4/c/x+16/3*ln(c*x)-25/24*ln(c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*l
n(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^4(a+b\mathrm{arctanh}(cx))}{x^5}\,dx=\int \frac{(cdx+d)^4(b\,\mathrm{artanh}\,(cx)+a)}{x^5}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x
+ a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x
 + b*d^4)*arctanh(c*x))/x^5, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^5}\,dx = d^4\left(\int \frac{a}{x^5}\,dx + \int \frac{4ac}{x^4}\,dx + \int \frac{6ac^2}{x^3}\,dx + \int \frac{4ac^3}{x^2}\,dx + \int \frac{ac^4}{x}\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^5}\,dx + \int \frac{4bc\,\text{atanh}\,(cx)}{x^4}\,dx + \int \frac{6bc^2\,\text{atanh}\,(cx)}{x^3}\,dx + \int \frac{4bc^3\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{bc^4\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**5,x)
```

output
```
d**4*(Integral(a/x**5, x) + Integral(4*a*c/x**4, x) + Integral(6*a*c**2/x*
*3, x) + Integral(4*a*c**3/x**2, x) + Integral(a*c**4/x, x) + Integral(b*a
tanh(c*x)/x**5, x) + Integral(4*b*c*atanh(c*x)/x**4, x) + Integral(6*b*c**
2*atanh(c*x)/x**3, x) + Integral(4*b*c**3*atanh(c*x)/x**2, x) + Integral(b
*c**4*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^5}\,dx = \int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^5}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*b*c^4*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^4*d^4*l
og(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^4 +
 3/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^
2*d^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c
*x)/x^3)*b*c*d^4 - 4*a*c^3*d^4/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c
*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^4 - 3*a*c^2*d
^4/x^2 - 4/3*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

Giac [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^5}\,dx=\int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^5}\,dx$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^5}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+c\,d\,x\right)^4}{x^5}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^5,x)
```

output
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^5, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^5}\,dx$$
$$=\frac{d^4\Big(-25atanh(cx)\,b\,c^4x^4-48atanh(cx)\,b\,c^3x^3-36atanh(cx)\,b\,c^2x^2-16atanh(cx)\,bcx-3atanh(cx)\,b+}{}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^5,x)
```

output

```
(d**4*( - 25*atanh(c*x)*b*c**4*x**4 - 48*atanh(c*x)*b*c**3*x**3 - 36*atanh
(c*x)*b*c**2*x**2 - 16*atanh(c*x)*b*c*x - 3*atanh(c*x)*b + 12*int(atanh(c*
x)/x,x)*b*c**4*x**4 - 64*log(c**2*x - c)*b*c**4*x**4 + 12*log(x)*a*c**4*x*
*4 + 64*log(x)*b*c**4*x**4 - 48*a*c**3*x**3 - 36*a*c**2*x**2 - 16*a*c*x -
3*a - 39*b*c**3*x**3 - 8*b*c**2*x**2 - b*c*x))/(12*x**4)
```

3.40 $\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^6}\,dx = -\frac{bcd^4}{20x^4}-\frac{bc^2d^4}{3x^3}-\frac{11bc^3d^4}{10x^2}-\frac{3bc^4d^4}{x} -\frac{d^4(1+cx)^5(a+b\text{arctanh}(cx))}{5x^5} +\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)$$

output
```
-1/20*b*c*d^4/x^4-1/3*b*c^2*d^4/x^3-11/10*b*c^3*d^4/x^2-3*b*c^4*d^4/x-1/5*
d^4*(c*x+1)^5*(a+b*arctanh(c*x))/x^5+16/5*b*c^5*d^4*ln(x)-16/5*b*c^5*d^4*l
n(-c*x+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.44

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^6}\,dx = -\frac{d^4(12a+60acx+3bcx+120ac^2x^2+20bc^2x^2+120ac^3x^3+66bc^3x^3+60ac^4x^4+180bc^4x^4+12b(1+$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

output
```
-1/60*(d^4*(12*a + 60*a*c*x + 3*b*c*x + 120*a*c^2*x^2 + 20*b*c^2*x^2 + 120
*a*c^3*x^3 + 66*b*c^3*x^3 + 60*a*c^4*x^4 + 180*b*c^4*x^4 + 12*b*(1 + 5*c*x
 + 10*c^2*x^2 + 10*c^3*x^3 + 5*c^4*x^4)*ArcTanh[c*x] - 192*b*c^5*x^5*Log[x
] + 186*b*c^5*x^5*Log[1 - c*x] + 6*b*c^5*x^5*Log[1 + c*x]))/x^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6498, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{6498}$$

$$-bc\int -\frac{d^4(cx+1)^4}{5x^5(1-cx)}dx-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}bcd^4\int \frac{(cx+1)^4}{x^5(1-cx)}dx-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5x^5}$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{5}bcd^4\int \left(-\frac{16c^5}{cx-1}+\frac{16c^4}{x}+\frac{15c^3}{x^2}+\frac{11c^2}{x^3}+\frac{5c}{x^4}+\frac{1}{x^5}\right)dx-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}bcd^4\left(16c^4\log(x)-16c^4\log(1-cx)-\frac{15c^3}{x}-\frac{11c^2}{2x^2}-\frac{5c}{3x^3}-\frac{1}{4x^4}\right)-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{5x^5}$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

output
```
-1/5*(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/x^5 + (b*c*d^4*(-1/4*1/x^4 - (
5*c)/(3*x^3) - (11*c^2)/(2*x^2) - (15*c^3)/x + 16*c^4*Log[x] - 16*c^4*Log[
1 - c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

method	result
parts	$d^4a\left(-\frac{c}{x^4}-\frac{c^4}{x}-\frac{1}{5x^5}-\frac{2c^3}{x^2}-\frac{2c^2}{x^3}\right)+d^4b\,c^5\left(-\frac{2\,\mathrm{arctanh}(cx)}{c^3x^3}-\frac{2\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{\mathrm{arctanh}(cx)}{cx}-\right.$
derivativedivides	$c^5\left(d^4a\left(-\frac{2}{c^3x^3}-\frac{2}{c^2x^2}-\frac{1}{cx}-\frac{1}{c^4x^4}-\frac{1}{5c^5x^5}\right)+d^4b\left(-\frac{2\,\mathrm{arctanh}(cx)}{c^3x^3}-\frac{2\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{\mathrm{arctanh}(c}{cx}\right.\right.$
default	$c^5\left(d^4a\left(-\frac{2}{c^3x^3}-\frac{2}{c^2x^2}-\frac{1}{cx}-\frac{1}{c^4x^4}-\frac{1}{5c^5x^5}\right)+d^4b\left(-\frac{2\,\mathrm{arctanh}(cx)}{c^3x^3}-\frac{2\,\mathrm{arctanh}(cx)}{c^2x^2}-\frac{\mathrm{arctanh}(c}{cx}\right.\right.$
risch	$-\frac{d^4b(5c^4x^4+10x^3c^3+10c^2x^2+5cx+1)\ln(cx+1)}{10x^5}-\frac{d^4(6b\,c^5\ln(cx+1)x^5+186b\,c^5x^5\ln(-cx+1)-192b\,c^5\ln(-x)x^5-}{}$
parallelrisch	$\frac{192b\,c^5d^4\ln(x)x^5-192\ln(cx-1)x^5b\,c^5d^4-12b\,c^5d^4\,\mathrm{arctanh}(cx)x^5-120a\,c^5d^4x^5-66b\,c^5d^4x^5-60d^4b\,\mathrm{arctanh}(cx)x^4c^4}{}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-c/x^4-c^4/x-1/5/x^5-2*c^3/x^2-2*c^2/x^3)+d^4*b*c^5*(-2*arctanh(c*x
)/c^3/x^3-2*arctanh(c*x)/c^2/x^2-arctanh(c*x)/c/x-arctanh(c*x)/c^4/x^4-1/5
*arctanh(c*x)/c^5/x^5-31/10*ln(c*x-1)-1/20/c^4/x^4-1/3/c^3/x^3-11/10/c^2/x
^2-3/c/x+16/5*ln(c*x)-1/10*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.75

$$\int\frac{(d+cdx)^4(a+b\mathrm{arctanh}(cx))}{x^6}\,dx=$$
$$-\frac{6\,bc^5d^4x^5\log(cx+1)+186\,bc^5d^4x^5\log(cx-1)-192\,bc^5d^4x^5\log(x)+60\,(a+3\,b)c^4d^4x^4+6\,(20\,a+}{}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(6*b*c^5*d^4*x^5*log(c*x + 1) + 186*b*c^5*d^4*x^5*log(c*x - 1) - 192
*b*c^5*d^4*x^5*log(x) + 60*(a + 3*b)*c^4*d^4*x^4 + 6*(20*a + 11*b)*c^3*d^4
*x^3 + 20*(6*a + b)*c^2*d^4*x^2 + 3*(20*a + b)*c*d^4*x + 12*a*d^4 + 6*(5*b
*c^4*d^4*x^4 + 10*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x + b*d^4)*
log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(109) = 218$.

Time = 0.65 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \begin{cases} -\frac{ac^4d^4}{x} - \frac{2ac^3d^4}{x^2} - \frac{2ac^2d^4}{x^3} - \frac{acd^4}{x^4} - \frac{ad^4}{5x^5} + \frac{16bc^5d^4\log(x)}{5} - \frac{16bc^5d^4\log\left(x-\frac{1}{c}\right)}{5} - \frac{bc^5d^4\operatorname{atanh}(cx)}{5} - \frac{bc^4d^4\operatorname{atanh}(cx)}{x} - \\ -\frac{ad^4}{5x^5} \end{cases}$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**6,x)
```

output
```
Piecewise((-a*c**4*d**4/x - 2*a*c**3*d**4/x**2 - 2*a*c**2*d**4/x**3 - a*c*
d**4/x**4 - a*d**4/(5*x**5) + 16*b*c**5*d**4*log(x)/5 - 16*b*c**5*d**4*log
(x - 1/c)/5 - b*c**5*d**4*atanh(c*x)/5 - b*c**4*d**4*atanh(c*x)/x - 3*b*c*
*4*d**4/x - 2*b*c**3*d**4*atanh(c*x)/x**2 - 11*b*c**3*d**4/(10*x**2) - 2*b
*c**2*d**4*atanh(c*x)/x**3 - b*c**2*d**4/(3*x**3) - b*c*d**4*atanh(c*x)/x*
*4 - b*c*d**4/(20*x**4) - b*d**4*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**4/
(5*x**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(97) = 194$.

Time = 0.03 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.74

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= -\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\,(cx)}{x}\right)bc^4d^4$$
$$+\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\,(cx)}{x^2}\right)bc^3d^4$$
$$-\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}\,(cx)}{x^3}\right)bc^2d^4-\frac{ac^4d^4}{x}$$
$$+\frac{1}{6}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\text{artanh}\,(cx)}{x^4}\right)bcd^4$$
$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\text{artanh}\,(cx)}{x^5}\right)bd^4$$
$$-\frac{2\,ac^3d^4}{x^2}-\frac{2\,ac^2d^4}{x^3}-\frac{acd^4}{x^4}-\frac{ad^4}{5\,x^5}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^4*d^4 + ((c*
log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*d^4 - (
(c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*c^
2*d^4 - a*c^4*d^4/x + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3
*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^4 - 1/20*((2*c^4*log(c^2*
x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b
*d^4 - 2*a*c^3*d^4/x^2 - 2*a*c^2*d^4/x^3 - a*c*d^4/x^4 - 1/5*a*d^4/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(97) = 194.

Time = 0.13 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.88

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{4}{15}\left(12\,bc^4d^4\log\left(-\frac{cx+1}{cx-1}-1\right)-12\,bc^4d^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{12\left(\frac{5\,(cx+1)^4bc^4d^4}{(cx-1)^4}+\frac{10\,(cx+1)^3bc^4d^4}{(cx-1)^3}+\frac{10\,(cx+\cdots}{(cx\cdots}\right.}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+\cdots}{(cx-1\cdots}}\right.$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

output
```
4/15*(12*b*c^4*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 12*b*c^4*d^4*log(-(c*x
+ 1)/(c*x - 1)) + 12*(5*(c*x + 1)^4*b*c^4*d^4/(c*x - 1)^4 + 10*(c*x + 1)^3
*b*c^4*d^4/(c*x - 1)^3 + 10*(c*x + 1)^2*b*c^4*d^4/(c*x - 1)^2 + 5*(c*x + 1
)*b*c^4*d^4/(c*x - 1) + b*c^4*d^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/
(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*
(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1) + (120*(c*x + 1)^4*a*
c^4*d^4/(c*x - 1)^4 + 240*(c*x + 1)^3*a*c^4*d^4/(c*x - 1)^3 + 240*(c*x + 1
)^2*a*c^4*d^4/(c*x - 1)^2 + 120*(c*x + 1)*a*c^4*d^4/(c*x - 1) + 24*a*c^4*d
^4 + 48*(c*x + 1)^4*b*c^4*d^4/(c*x - 1)^4 + 156*(c*x + 1)^3*b*c^4*d^4/(c*x
 - 1)^3 + 196*(c*x + 1)^2*b*c^4*d^4/(c*x - 1)^2 + 113*(c*x + 1)*b*c^4*d^4/
(c*x - 1) + 25*b*c^4*d^4)/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x -
1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x +
1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

$$\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^6}\,dx = \frac{d^4\left(180\,b\,c^5\operatorname{atanh}(c\,x)-96\,b\,c^5\,\ln\left(c^2\,x^2-1\right)+192\,b\,c^5\,\ln\left(x\right)\right)}{60} - \frac{\frac{d^4\,\left(12\,a+12\,b\operatorname{atanh}(c\,x)\right)}{60}+\frac{d^4\,x\,\left(60\,a\,c+3\,b\,c+60\,b\,c\operatorname{atanh}(c\,x)\right)}{60}+\frac{d^4\,x^2\,\left(120\,a\,c^2+20\,b\,c^2+120\,b\,c^2\operatorname{atanh}(c\,x)\right)}{60}+\frac{d^4\,x^4\,\left(60\,a\,c^4+180\,b\,c\dots\right.}{6\dots}}{x^5}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^6,x)
```

output
```
(d^4*(180*b*c^5*atanh(c*x) - 96*b*c^5*log(c^2*x^2 - 1) + 192*b*c^5*log(x))
)/60 - ((d^4*(12*a + 12*b*atanh(c*x)))/60 + (d^4*x*(60*a*c + 3*b*c + 60*b*
c*atanh(c*x)))/60 + (d^4*x^2*(120*a*c^2 + 20*b*c^2 + 120*b*c^2*atanh(c*x))
)/60 + (d^4*x^4*(60*a*c^4 + 180*b*c^4 + 60*b*c^4*atanh(c*x)))/60 + (d^4*x^
3*(120*a*c^3 + 66*b*c^3 + 120*b*c^3*atanh(c*x)))/60)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.60

$$\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^6}\,dx$$
$$= \frac{d^4(-12atanh(cx)\,b\,c^5x^5 - 60atanh(cx)\,b\,c^4x^4 - 120atanh(cx)\,b\,c^3x^3 - 120atanh(cx)\,b\,c^2x^2 - 60atanh(c}{}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^6,x)
```

output
```
(d**4*( - 12*atanh(c*x)*b*c**5*x**5 - 60*atanh(c*x)*b*c**4*x**4 - 120*atan
h(c*x)*b*c**3*x**3 - 120*atanh(c*x)*b*c**2*x**2 - 60*atanh(c*x)*b*c*x - 12
*atanh(c*x)*b - 192*log(c**2*x - c)*b*c**5*x**5 + 192*log(x)*b*c**5*x**5 -
 60*a*c**4*x**4 - 120*a*c**3*x**3 - 120*a*c**2*x**2 - 60*a*c*x - 12*a - 18
0*b*c**4*x**4 - 66*b*c**3*x**3 - 20*b*c**2*x**2 - 3*b*c*x))/(60*x**5)
```

3.41 $\int \frac{(d+cdx)^4(a+b\,\mathbf{arctanh}(cx))}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\begin{aligned}\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx =& -\frac{bcd^4}{30x^5}-\frac{bc^2d^4}{5x^4}-\frac{5bc^3d^4}{9x^3}-\frac{16bc^4d^4}{15x^2}\\ &-\frac{13bc^5d^4}{6x}-\frac{d^4(1+cx)^5(a+b\text{arctanh}(cx))}{6x^6}\\ &+\frac{cd^4(1+cx)^5(a+b\text{arctanh}(cx))}{30x^5}\\ &+\frac{32}{15}bc^6d^4\log(x)-\frac{32}{15}bc^6d^4\log(1-cx)\end{aligned}$$

output
```
-1/30*b*c*d^4/x^5-1/5*b*c^2*d^4/x^4-5/9*b*c^3*d^4/x^3-16/15*b*c^4*d^4/x^2-
13/6*b*c^5*d^4/x-1/6*d^4*(c*x+1)^5*(a+b*arctanh(c*x))/x^6+1/30*c*d^4*(c*x+
1)^5*(a+b*arctanh(c*x))/x^5+32/15*b*c^6*d^4*ln(x)-32/15*b*c^6*d^4*ln(-c*x+
1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx = -\frac{d^4(30a+144acx+6bcx+270ac^2x^2+36bc^2x^2+240ac^3x^3+100bc^3x^3+90ac^4x^4+192bc^4x^4+390bc}{}$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

output
```
-1/180*(d^4*(30*a + 144*a*c*x + 6*b*c*x + 270*a*c^2*x^2 + 36*b*c^2*x^2 + 2
40*a*c^3*x^3 + 100*b*c^3*x^3 + 90*a*c^4*x^4 + 192*b*c^4*x^4 + 390*b*c^5*x^
5 + 6*b*(5 + 24*c*x + 45*c^2*x^2 + 40*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] -
 384*b*c^6*x^6*Log[x] + 387*b*c^6*x^6*Log[1 - c*x] - 3*b*c^6*x^6*Log[1 + c
*x]))/x^6
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x^7}\,dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int -\frac{d^4(5-cx)(cx+1)^4}{30x^6(1-cx)}dx-\frac{d^4(cx+1)^5(a+b\text{arctanh}(cx))}{6x^6}+\frac{cd^4(cx+1)^5(a+b\text{arctanh}(cx))}{30x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{30}bcd^4\int\frac{(5-cx)(cx+1)^4}{x^6(1-cx)}dx-\frac{d^4(cx+1)^5(a+b\operatorname{arctanh}(cx))}{6x^6}+\frac{cd^4(cx+1)^5(a+b\operatorname{arctanh}(cx))}{30x^5}$$

$$\Big\downarrow \textbf{165}$$

$$\frac{1}{30}bcd^4\int\left(-\frac{64c^6}{cx-1}+\frac{64c^5}{x}+\frac{65c^4}{x^2}+\frac{64c^3}{x^3}+\frac{50c^2}{x^4}+\frac{24c}{x^5}+\frac{5}{x^6}\right)dx-\frac{d^4(cx+1)^5(a+b\operatorname{arctanh}(cx))}{6x^6}+\frac{cd^4(cx+1)^5(a+b\operatorname{arctanh}(cx))}{30x^5}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{d^4(cx+1)^5(a+b\operatorname{arctanh}(cx))}{6x^6}+\frac{cd^4(cx+1)^5(a+b\operatorname{arctanh}(cx))}{30x^5}+\frac{1}{30}bcd^4\left(64c^5\log(x)-64c^5\log(1-cx)-\frac{65c^4}{x}-\frac{32c^3}{x^2}-\frac{50c^2}{3x^3}-\frac{6c}{x^4}-\frac{1}{x^5}\right)$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

output
```
-1/6*(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/x^6 + (c*d^4*(1 + c*x)^5*(a +
b*ArcTanh[c*x]))/(30*x^5) + (b*c*d^4*(-x^(-5) - (6*c)/x^4 - (50*c^2)/(3*x^
3) - (32*c^3)/x^2 - (65*c^4)/x + 64*c^5*Log[x] - 64*c^5*Log[1 - c*x]))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.15

method	result
parts	$d^4a\left(-\frac{1}{6x^6}-\frac{3c^2}{2x^4}-\frac{4c}{5x^5}-\frac{c^4}{2x^2}-\frac{4c^3}{3x^3}\right)+d^4b\,c^6\left(-\frac{4\,\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{3\,\operatorname{arctanh}(cx)}{2c^4x^4}\right.$
derivativedivides	$c^6\left(d^4a\left(-\frac{4}{3c^3x^3}-\frac{1}{2c^2x^2}-\frac{3}{2c^4x^4}-\frac{4}{5c^5x^5}-\frac{1}{6c^6x^6}\right)+d^4b\left(-\frac{4\,\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{3\,\operatorname{arc}}{2}\right.\right.$
default	$c^6\left(d^4a\left(-\frac{4}{3c^3x^3}-\frac{1}{2c^2x^2}-\frac{3}{2c^4x^4}-\frac{4}{5c^5x^5}-\frac{1}{6c^6x^6}\right)+d^4b\left(-\frac{4\,\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{3\,\operatorname{arc}}{2}\right.\right.$
risch	$-\frac{d^4b\left(15c^4x^4+40x^3c^3+45c^2x^2+24cx+5\right)\ln(cx+1)}{60x^6}+\frac{d^4\left(3b\,c^6\ln(cx+1)x^6+384b\,c^6\ln(-x)x^6-387b\,c^6x^6\ln(-cx+1\right.}{}$
parallelrisch	$\frac{192b\,c^6d^4\ln(x)x^6-192\ln(cx-1)x^6b\,c^6d^4+3b\,c^6d^4\operatorname{arctanh}(cx)x^6-45a\,c^6d^4x^6-96b\,c^6d^4x^6-195b\,c^5d^4x^5-45d^4b\,\operatorname{arcta}}{}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-1/6/x^6-3/2*c^2/x^4-4/5*c/x^5-1/2*c^4/x^2-4/3*c^3/x^3)+d^4*b*c^6*(
-4/3*arctanh(c*x)/c^3/x^3-1/2*arctanh(c*x)/c^2/x^2-3/2*arctanh(c*x)/c^4/x^
4-4/5*arctanh(c*x)/c^5/x^5-1/6*arctanh(c*x)/c^6/x^6-43/20*ln(c*x-1)-1/30/c
^5/x^5-1/5/c^4/x^4-5/9/c^3/x^3-16/15/c^2/x^2-13/6/c/x+32/15*ln(c*x)+1/60*l
n(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.38

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx$$
$$= \frac{3\,bc^6d^4x^6\log(cx+1) - 387\,bc^6d^4x^6\log(cx-1) + 384\,bc^6d^4x^6\log(x) - 390\,bc^5d^4x^5 - 6\,(15\,a+32\,b)c^4d}{}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/180*(3*b*c^6*d^4*x^6*log(c*x + 1) - 387*b*c^6*d^4*x^6*log(c*x - 1) + 384
*b*c^6*d^4*x^6*log(x) - 390*b*c^5*d^4*x^5 - 6*(15*a + 32*b)*c^4*d^4*x^4 -
20*(12*a + 5*b)*c^3*d^4*x^3 - 18*(15*a + 2*b)*c^2*d^4*x^2 - 6*(24*a + b)*c
*d^4*x - 30*a*d^4 - 3*(15*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 45*b*c^2*d^4*
x^2 + 24*b*c*d^4*x + 5*b*d^4)*log(-(c*x + 1)/(c*x - 1)))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.93

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx$$
$$= \begin{cases} -\frac{ac^4d^4}{2x^2} - \frac{4ac^3d^4}{3x^3} - \frac{3ac^2d^4}{2x^4} - \frac{4acd^4}{5x^5} - \frac{ad^4}{6x^6} + \frac{32bc^6d^4\log(x)}{15} - \frac{32bc^6d^4\log\left(x-\frac{1}{c}\right)}{15} + \frac{bc^6d^4\,\text{atanh}(cx)}{30} - \frac{13bc^5d^4}{6x} - \frac{bc^4d^4\,\text{at}}{2} \\ -\frac{ad^4}{6x^6} \end{cases}$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**7,x)
```

output
```
Piecewise((-a*c**4*d**4/(2*x**2) - 4*a*c**3*d**4/(3*x**3) - 3*a*c**2*d**4/
(2*x**4) - 4*a*c*d**4/(5*x**5) - a*d**4/(6*x**6) + 32*b*c**6*d**4*log(x)/1
5 - 32*b*c**6*d**4*log(x - 1/c)/15 + b*c**6*d**4*atanh(c*x)/30 - 13*b*c**5
*d**4/(6*x) - b*c**4*d**4*atanh(c*x)/(2*x**2) - 16*b*c**4*d**4/(15*x**2) -
 4*b*c**3*d**4*atanh(c*x)/(3*x**3) - 5*b*c**3*d**4/(9*x**3) - 3*b*c**2*d**
4*atanh(c*x)/(2*x**4) - b*c**2*d**4/(5*x**4) - 4*b*c*d**4*atanh(c*x)/(5*x*
*5) - b*c*d**4/(30*x**5) - b*d**4*atanh(c*x)/(6*x**6), Ne(c, 0)), (-a*d**4
/(6*x**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(133) = 266$.

Time = 0.03 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.18

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx$$
$$= \frac{1}{4}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\text{artanh}(cx)}{x^2}\right)bc^4d^4$$
$$- \frac{2}{3}\left(\left(c^2\log(c^2x^2-1) - c^2\log(x^2) + \frac{1}{x^2}\right)c + \frac{2\,\text{artanh}(cx)}{x^3}\right)bc^3d^4$$
$$+ \frac{1}{4}\left(\left(3\,c^3\log(cx+1) - 3\,c^3\log(cx-1) - \frac{2\,(3\,c^2x^2+1)}{x^3}\right)c - \frac{6\,\text{artanh}(cx)}{x^4}\right)bc^2d^4$$
$$- \frac{1}{5}\left(\left(2\,c^4\log(c^2x^2-1) - 2\,c^4\log(x^2) + \frac{2\,c^2x^2+1}{x^4}\right)c + \frac{4\,\text{artanh}(cx)}{x^5}\right)bcd^4$$
$$- \frac{ac^4d^4}{2\,x^2}$$
$$+ \frac{1}{180}\left(\left(15\,c^5\log(cx+1) - 15\,c^5\log(cx-1) - \frac{2\,(15\,c^4x^4+5\,c^2x^2+3)}{x^5}\right)c - \frac{30\,\text{artanh}(cx)}{x^6}\right)bd^4$$
$$- \frac{4\,ac^3d^4}{3\,x^3} - \frac{3\,ac^2d^4}{2\,x^4} - \frac{4\,acd^4}{5\,x^5} - \frac{ad^4}{6\,x^6}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^4
*d^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*
x)/x^3)*b*c^3*d^4 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c
^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^4 - 1/5*((2*c^4*log(c^2*x
^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*
c*d^4 - 1/2*a*c^4*d^4/x^2 + 1/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x -
 1) - 2*(15*c^4*x^4 + 5*c^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^4 -
 4/3*a*c^3*d^4/x^3 - 3/2*a*c^2*d^4/x^4 - 4/5*a*c*d^4/x^5 - 1/6*a*d^4/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(133) = 266$.

Time = 0.13 (sec) , antiderivative size = 634, normalized size of antiderivative = 4.20

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

output
```
8/45*(12*b*c^5*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 12*b*c^5*d^4*log(-(c*x
+ 1)/(c*x - 1)) + 6*(15*(c*x + 1)^5*b*c^5*d^4/(c*x - 1)^5 + 30*(c*x + 1)^4
*b*c^5*d^4/(c*x - 1)^4 + 40*(c*x + 1)^3*b*c^5*d^4/(c*x - 1)^3 + 30*(c*x +
1)^2*b*c^5*d^4/(c*x - 1)^2 + 12*(c*x + 1)*b*c^5*d^4/(c*x - 1) + 2*b*c^5*d^
4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x
 - 1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*
x + 1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1) + (180*(c*x + 1)^5*a*c^5
*d^4/(c*x - 1)^5 + 360*(c*x + 1)^4*a*c^5*d^4/(c*x - 1)^4 + 480*(c*x + 1)^3
*a*c^5*d^4/(c*x - 1)^3 + 360*(c*x + 1)^2*a*c^5*d^4/(c*x - 1)^2 + 144*(c*x
+ 1)*a*c^5*d^4/(c*x - 1) + 24*a*c^5*d^4 + 78*(c*x + 1)^5*b*c^5*d^4/(c*x -
1)^5 + 294*(c*x + 1)^4*b*c^5*d^4/(c*x - 1)^4 + 472*(c*x + 1)^3*b*c^5*d^4/(
c*x - 1)^3 + 399*(c*x + 1)^2*b*c^5*d^4/(c*x - 1)^2 + 174*(c*x + 1)*b*c^5*d
^4/(c*x - 1) + 31*b*c^5*d^4)/((c*x + 1)^6/(c*x - 1)^6 + 6*(c*x + 1)^5/(c*x
 - 1)^5 + 15*(c*x + 1)^4/(c*x - 1)^4 + 20*(c*x + 1)^3/(c*x - 1)^3 + 15*(c*
x + 1)^2/(c*x - 1)^2 + 6*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.64

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx = \frac{32\,b\,c^6\,d^4\,\ln(x)}{15} - \frac{16\,b\,c^6\,d^4\,\ln(c^2\,x^2-1)}{15} - \frac{3\,a\,c^2\,d^4}{2\,x^4} - \frac{4\,a\,c^3\,d^4}{3\,x^3} - \frac{a\,c^4\,d^4}{2\,x^2} - \frac{b\,c^2\,d^4}{5\,x^4} - \frac{5\,b\,c^3\,d^4}{9\,x^3} - \frac{16\,b\,c^4\,d^4}{15\,x^2} - \frac{13\,b\,c^5\,d^4}{6\,x} - \frac{a\,d^4}{6\,x^6} - \frac{4\,a\,c\,d^4}{5\,x^5} - \frac{b\,c\,d^4}{30\,x^5} - \frac{b\,d^4\,\text{atanh}(c\,x)}{6\,x^6} - \frac{13\,b\,c^7\,d^4\,\text{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{6\,\sqrt{-c^2}} - \frac{4\,b\,c\,d^4\,\text{atanh}(c\,x)}{5\,x^5} - \frac{3\,b\,c^2\,d^4\,\text{atanh}(c\,x)}{2\,x^4} - \frac{4\,b\,c^3\,d^4\,\text{atanh}(c\,x)}{3\,x^3} - \frac{b\,c^4\,d^4\,\text{atanh}(c\,x)}{2\,x^2}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^7,x)
```

output
```
(32*b*c^6*d^4*log(x))/15 - (16*b*c^6*d^4*log(c^2*x^2 - 1))/15 - (3*a*c^2*d
^4)/(2*x^4) - (4*a*c^3*d^4)/(3*x^3) - (a*c^4*d^4)/(2*x^2) - (b*c^2*d^4)/(5
*x^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(
6*x) - (a*d^4)/(6*x^6) - (4*a*c*d^4)/(5*x^5) - (b*c*d^4)/(30*x^5) - (b*d^4
*atanh(c*x))/(6*x^6) - (13*b*c^7*d^4*atan((c^2*x)/(-c^2)^(1/2)))/(6*(-c^2)
^(1/2)) - (4*b*c*d^4*atanh(c*x))/(5*x^5) - (3*b*c^2*d^4*atanh(c*x))/(2*x^4
) - (4*b*c^3*d^4*atanh(c*x))/(3*x^3) - (b*c^4*d^4*atanh(c*x))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^7}\,dx = \frac{d^4(3atanh(cx)\,b\,c^6x^6 - 45atanh(cx)\,b\,c^4x^4 - 120atanh(cx)\,b\,c^3x^3 - 135atanh(cx)\,b\,c^2x^2 - 72atanh(cx)\,b}{}$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^7,x)
```

output

```
(d**4*(3*atanh(c*x)*b*c**6*x**6 - 45*atanh(c*x)*b*c**4*x**4 - 120*atanh(c*
x)*b*c**3*x**3 - 135*atanh(c*x)*b*c**2*x**2 - 72*atanh(c*x)*b*c*x - 15*ata
nh(c*x)*b - 192*log(c**2*x - c)*b*c**6*x**6 + 192*log(x)*b*c**6*x**6 - 45*
a*c**4*x**4 - 120*a*c**3*x**3 - 135*a*c**2*x**2 - 72*a*c*x - 15*a - 195*b*
c**5*x**5 - 96*b*c**4*x**4 - 50*b*c**3*x**3 - 18*b*c**2*x**2 - 3*b*c*x))/(
90*x**6)
```

3.42 $\int \frac{(d+cdx)^4(a+b\textbf{arctanh}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 229

$$\begin{aligned}\int \frac{(d+cdx)^4(a+b\operatorname{arctanh}(cx))}{x^8}\,dx = &-\frac{bcd^4}{42x^6}-\frac{2bc^2d^4}{15x^5}-\frac{47bc^3d^4}{140x^4}-\frac{5bc^4d^4}{9x^3}\\ &-\frac{88bc^5d^4}{105x^2}-\frac{5bc^6d^4}{3x}-\frac{d^4(a+b\operatorname{arctanh}(cx))}{7x^7}\\ &-\frac{2cd^4(a+b\operatorname{arctanh}(cx))}{3x^6}\\ &-\frac{6c^2d^4(a+b\operatorname{arctanh}(cx))}{5x^5}\\ &-\frac{c^3d^4(a+b\operatorname{arctanh}(cx))}{x^4}\\ &-\frac{c^4d^4(a+b\operatorname{arctanh}(cx))}{3x^3}+\frac{176}{105}bc^7d^4\log(x)\\ &-\frac{117}{70}bc^7d^4\log(1-cx)-\frac{1}{210}bc^7d^4\log(1+cx)\end{aligned}$$

output
```
-1/42*b*c*d^4/x^6-2/15*b*c^2*d^4/x^5-47/140*b*c^3*d^4/x^4-5/9*b*c^4*d^4/x^
3-88/105*b*c^5*d^4/x^2-5/3*b*c^6*d^4/x-1/7*d^4*(a+b*arctanh(c*x))/x^7-2/3*
c*d^4*(a+b*arctanh(c*x))/x^6-6/5*c^2*d^4*(a+b*arctanh(c*x))/x^5-c^3*d^4*(a
+b*arctanh(c*x))/x^4-1/3*c^4*d^4*(a+b*arctanh(c*x))/x^3+176/105*b*c^7*d^4*
ln(x)-117/70*b*c^7*d^4*ln(-c*x+1)-1/210*b*c^7*d^4*ln(c*x+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^8}\,dx =$$
$$-\frac{d^4(180a+840acx+30bcx+1512ac^2x^2+168bc^2x^2+1260ac^3x^3+423bc^3x^3+420ac^4x^4+700bc^4x^4 -$$

input
```
Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

output
```
-1/1260*(d^4*(180*a + 840*a*c*x + 30*b*c*x + 1512*a*c^2*x^2 + 168*b*c^2*x^
2 + 1260*a*c^3*x^3 + 423*b*c^3*x^3 + 420*a*c^4*x^4 + 700*b*c^4*x^4 + 1056*
b*c^5*x^5 + 2100*b*c^6*x^6 + 12*b*(15 + 70*c*x + 126*c^2*x^2 + 105*c^3*x^3
 + 35*c^4*x^4)*ArcTanh[c*x] - 2112*b*c^7*x^7*Log[x] + 2106*b*c^7*x^7*Log[1
 - c*x] + 6*b*c^7*x^7*Log[1 + c*x]))/x^7
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6498, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^4(a+b\text{arctanh}(cx))}{x^8}\,dx$$

$$\Big\downarrow \text{ 6498}$$

$$-bc\int -\frac{d^4\left(35c^4x^4+105c^3x^3+126c^2x^2+70cx+15\right)}{105x^7\left(1-c^2x^2\right)}dx-\frac{c^4d^4(a+b\text{arctanh}(cx))}{3x^3}-\frac{c^3d^4(a+b\text{arctanh}(cx))}{x^4}-\frac{6c^2d^4(a+b\text{arctanh}(cx))}{5x^5}-\frac{d^4(a+b\text{arctanh}(cx))}{7x^7}-\frac{2cd^4(a+b\text{arctanh}(cx))}{3x^6}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{105}bcd^4\int \frac{35c^4x^4+105c^3x^3+126c^2x^2+70cx+15}{x^7\left(1-c^2x^2\right)}dx-\frac{c^4d^4(a+b\text{arctanh}(cx))}{3x^3}-\frac{c^3d^4(a+b\text{arctanh}(cx))}{x^4}-\frac{6c^2d^4(a+b\text{arctanh}(cx))}{5x^5}-\frac{d^4(a+b\text{arctanh}(cx))}{7x^7}-\frac{2cd^4(a+b\text{arctanh}(cx))}{3x^6}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{105}bcd^4\int \left(-\frac{351c^7}{2(cx-1)}-\frac{c^7}{2(cx+1)}+\frac{176c^6}{x}+\frac{175c^5}{x^2}+\frac{176c^4}{x^3}+\frac{175c^3}{x^4}+\frac{141c^2}{x^5}+\frac{70c}{x^6}+\frac{15}{x^7}\right)dx-\frac{c^4d^4(a+b\text{arctanh}(cx))}{3x^3}-\frac{c^3d^4(a+b\text{arctanh}(cx))}{x^4}-\frac{6c^2d^4(a+b\text{arctanh}(cx))}{5x^5}-\frac{d^4(a+b\text{arctanh}(cx))}{7x^7}-\frac{2cd^4(a+b\text{arctanh}(cx))}{3x^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4d^4(a+b\text{arctanh}(cx))}{3x^3}-\frac{c^3d^4(a+b\text{arctanh}(cx))}{x^4}-\frac{6c^2d^4(a+b\text{arctanh}(cx))}{5x^5}-\frac{d^4(a+b\text{arctanh}(cx))}{7x^7}-\frac{2cd^4(a+b\text{arctanh}(cx))}{3x^6}+\frac{1}{105}bcd^4\left(176c^6\log(x)-\frac{351}{2}c^6\log(1-cx)-\frac{1}{2}c^6\log(cx+1)-\frac{175c^5}{x}-\frac{88c^4}{x^2}-\frac{175c^3}{3x^3}-\frac{141c^2}{4x^4}-\frac{14c}{x^5}-\frac{5}{2x^6}\right)$$

input
```
Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

output
```
-1/7*(d^4*(a + b*ArcTanh[c*x]))/x^7 - (2*c*d^4*(a + b*ArcTanh[c*x]))/(3*x^
6) - (6*c^2*d^4*(a + b*ArcTanh[c*x]))/(5*x^5) - (c^3*d^4*(a + b*ArcTanh[c*
x]))/x^4 - (c^4*d^4*(a + b*ArcTanh[c*x]))/(3*x^3) + (b*c*d^4*(-5/(2*x^6) -
 (14*c)/x^5 - (141*c^2)/(4*x^4) - (175*c^3)/(3*x^3) - (88*c^4)/x^2 - (175*
c^5)/x + 176*c^6*Log[x] - (351*c^6*Log[1 - c*x])/2 - (c^6*Log[1 + c*x])/2)
)/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

method	result
parts	$d^4a\left(-\frac{2c}{3x^6}-\frac{c^3}{x^4}-\frac{1}{7x^7}-\frac{6c^2}{5x^5}-\frac{c^4}{3x^3}\right)+d^4b\,c^7\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{7c^7x^7}-\frac{\operatorname{arctanh}(cx)}{c^4x^4}-\frac{6\ a}{}\right.$
derivativedivides	$c^7\left(d^4a\left(-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}-\frac{1}{c^4x^4}-\frac{6}{5c^5x^5}-\frac{2}{3c^6x^6}\right)+d^4b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{7c^7x^7}-\frac{\operatorname{arctanh}}{c^4x^4}\right.\right.$
default	$c^7\left(d^4a\left(-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}-\frac{1}{c^4x^4}-\frac{6}{5c^5x^5}-\frac{2}{3c^6x^6}\right)+d^4b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{7c^7x^7}-\frac{\operatorname{arctanh}}{c^4x^4}\right.\right.$
risch	$-\frac{d^4b(35c^4x^4+105x^3c^3+126c^2x^2+70cx+15)\ln(cx+1)}{210x^7}-\frac{d^4(6b\,c^7\ln(cx+1)x^7-2112b\,c^7\ln(-x)x^7+2106b\,c^7x^7\ln(-}{}$
parallelrisch	$\frac{2112b\,c^7d^4\ln(x)x^7-2112\ln(cx-1)x^7b\,c^7d^4-12b\,c^7d^4\operatorname{arctanh}(cx)x^7-1056c^7d^4x^7b-2100b\,c^6d^4x^6-1056b\,c^5d^4x^5-42}{}$

input
```
int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-2/3*c/x^6-c^3/x^4-1/7/x^7-6/5*c^2/x^5-1/3*c^4/x^3)+d^4*b*c^7*(-1/3
*arctanh(c*x)/c^3/x^3-1/7*arctanh(c*x)/c^7/x^7-arctanh(c*x)/c^4/x^4-6/5*ar
ctanh(c*x)/c^5/x^5-2/3*arctanh(c*x)/c^6/x^6-117/70*ln(c*x-1)-1/42/c^6/x^6-
2/15/c^5/x^5-47/140/c^4/x^4-5/9/c^3/x^3-88/105/c^2/x^2-5/3/c/x+176/105*ln(
c*x)-1/210*ln(c*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.95

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^8}\,dx =$$
$$-\frac{6\,bc^7d^4x^7\log(cx+1)+2106\,bc^7d^4x^7\log(cx-1)-2112\,bc^7d^4x^7\log(x)+2100\,bc^6d^4x^6+1056\,bc^5d^4x}{}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/1260*(6*b*c^7*d^4*x^7*log(c*x + 1) + 2106*b*c^7*d^4*x^7*log(c*x - 1) -
2112*b*c^7*d^4*x^7*log(x) + 2100*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^5 + 140*
(3*a + 5*b)*c^4*d^4*x^4 + 9*(140*a + 47*b)*c^3*d^4*x^3 + 168*(9*a + b)*c^2
*d^4*x^2 + 30*(28*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*b*c^4*d^4*x^4 + 105*b
*c^3*d^4*x^3 + 126*b*c^2*d^4*x^2 + 70*b*c*d^4*x + 15*b*d^4)*log(-(c*x + 1)
/(c*x - 1)))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.31

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^8}\,dx$$
$$= \begin{cases} -\frac{ac^4d^4}{3x^3} - \frac{ac^3d^4}{x^4} - \frac{6ac^2d^4}{5x^5} - \frac{2acd^4}{3x^6} - \frac{ad^4}{7x^7} + \frac{176bc^7d^4\log(x)}{105} - \frac{176bc^7d^4\log\left(x-\frac{1}{c}\right)}{105} - \frac{bc^7d^4\,\text{atanh}(cx)}{105} - \frac{5bc^6d^4}{3x} - \frac{88bc^5d^4}{105x^2} \\ -\frac{ad^4}{7x^7} \end{cases}$$

input
```
integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**8,x)
```

output
```
Piecewise((-a*c**4*d**4/(3*x**3) - a*c**3*d**4/x**4 - 6*a*c**2*d**4/(5*x**
5) - 2*a*c*d**4/(3*x**6) - a*d**4/(7*x**7) + 176*b*c**7*d**4*log(x)/105 -
176*b*c**7*d**4*log(x - 1/c)/105 - b*c**7*d**4*atanh(c*x)/105 - 5*b*c**6*d
**4/(3*x) - 88*b*c**5*d**4/(105*x**2) - b*c**4*d**4*atanh(c*x)/(3*x**3) -
5*b*c**4*d**4/(9*x**3) - b*c**3*d**4*atanh(c*x)/x**4 - 47*b*c**3*d**4/(140
*x**4) - 6*b*c**2*d**4*atanh(c*x)/(5*x**5) - 2*b*c**2*d**4/(15*x**5) - 2*b
*c*d**4*atanh(c*x)/(3*x**6) - b*c*d**4/(42*x**6) - b*d**4*atanh(c*x)/(7*x*
*7), Ne(c, 0)), (-a*d**4/(7*x**7), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.54

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^8}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\ \text{artanh}\left(cx\right)}{x^3}\right)bc^4d^4$$
$$+\frac{1}{6}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\ \text{artanh}\left(cx\right)}{x^4}\right)bc^3d^4$$
$$-\frac{3}{10}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\ \text{artanh}\left(cx\right)}{x^5}\right)bc^2d^4$$
$$+\frac{1}{45}\left(\left(15\,c^5\log\left(cx+1\right)-15\,c^5\log\left(cx-1\right)-\frac{2\left(15\,c^4x^4+5\,c^2x^2+3\right)}{x^5}\right)c-\frac{30\ \text{artanh}\left(cx\right)}{x^6}\right)bcd^4$$
$$-\frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2-1\right)-6\,c^6\log\left(x^2\right)+\frac{6\,c^4x^4+3\,c^2x^2+2}{x^6}\right)c+\frac{12\ \text{artanh}\left(cx\right)}{x^7}\right)bd^4$$
$$-\frac{ac^4d^4}{3\,x^3}-\frac{ac^3d^4}{x^4}-\frac{6\,ac^2d^4}{5\,x^5}-\frac{2\,acd^4}{3\,x^6}-\frac{ad^4}{7\,x^7}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*c^4*d^4 + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2
 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^3*d^4 - 3/10*((2*c^4*log(c^2*x^2 -
1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c^2*d
^4 + 1/45*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*
c^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*c*d^4 - 1/84*((6*c^6*log(c^2*
x^2 - 1) - 6*c^6*log(x^2) + (6*c^4*x^4 + 3*c^2*x^2 + 2)/x^6)*c + 12*arctan
h(c*x)/x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 - a*c^3*d^4/x^4 - 6/5*a*c^2*d^4/x^5
- 2/3*a*c*d^4/x^6 - 1/7*a*d^4/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(203) = 406$.

Time = 0.14 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.21

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="giac")
```

output
```
4/315*(132*b*c^6*d^4*log(-(c*x + 1)/(c*x - 1) - 1) - 132*b*c^6*d^4*log(-(c
*x + 1)/(c*x - 1)) + 12*(105*(c*x + 1)^6*b*c^6*d^4/(c*x - 1)^6 + 210*(c*x
+ 1)^5*b*c^6*d^4/(c*x - 1)^5 + 385*(c*x + 1)^4*b*c^6*d^4/(c*x - 1)^4 + 385
*(c*x + 1)^3*b*c^6*d^4/(c*x - 1)^3 + 231*(c*x + 1)^2*b*c^6*d^4/(c*x - 1)^2
 + 77*(c*x + 1)*b*c^6*d^4/(c*x - 1) + 11*b*c^6*d^4)*log(-(c*x + 1)/(c*x -
1))/((c*x + 1)^7/(c*x - 1)^7 + 7*(c*x + 1)^6/(c*x - 1)^6 + 21*(c*x + 1)^5/
(c*x - 1)^5 + 35*(c*x + 1)^4/(c*x - 1)^4 + 35*(c*x + 1)^3/(c*x - 1)^3 + 21
*(c*x + 1)^2/(c*x - 1)^2 + 7*(c*x + 1)/(c*x - 1) + 1) + (2520*(c*x + 1)^6*
a*c^6*d^4/(c*x - 1)^6 + 5040*(c*x + 1)^5*a*c^6*d^4/(c*x - 1)^5 + 9240*(c*x
 + 1)^4*a*c^6*d^4/(c*x - 1)^4 + 9240*(c*x + 1)^3*a*c^6*d^4/(c*x - 1)^3 + 5
544*(c*x + 1)^2*a*c^6*d^4/(c*x - 1)^2 + 1848*(c*x + 1)*a*c^6*d^4/(c*x - 1)
 + 264*a*c^6*d^4 + 1128*(c*x + 1)^6*b*c^6*d^4/(c*x - 1)^6 + 4812*(c*x + 1)
^5*b*c^6*d^4/(c*x - 1)^5 + 9476*(c*x + 1)^4*b*c^6*d^4/(c*x - 1)^4 + 10631*
(c*x + 1)^3*b*c^6*d^4/(c*x - 1)^3 + 6933*(c*x + 1)^2*b*c^6*d^4/(c*x - 1)^2
 + 2465*(c*x + 1)*b*c^6*d^4/(c*x - 1) + 371*b*c^6*d^4)/((c*x + 1)^7/(c*x -
 1)^7 + 7*(c*x + 1)^6/(c*x - 1)^6 + 21*(c*x + 1)^5/(c*x - 1)^5 + 35*(c*x +
 1)^4/(c*x - 1)^4 + 35*(c*x + 1)^3/(c*x - 1)^3 + 21*(c*x + 1)^2/(c*x - 1)^
2 + 7*(c*x + 1)/(c*x - 1) + 1))*c
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

$$\int \frac{(d+cdx)^4(a+b\mathrm{arctanh}(cx))}{x^8}\,dx = \frac{176\,b\,c^7\,d^4\,\ln(x)}{105} - \frac{88\,b\,c^7\,d^4\,\ln(c^2\,x^2-1)}{105} - \frac{6\,a\,c^2\,d^4}{5\,x^5} - \frac{a\,c^3\,d^4}{x^4} - \frac{a\,c^4\,d^4}{3\,x^3} - \frac{2\,b\,c^2\,d^4}{15\,x^5} - \frac{47\,b\,c^3\,d^4}{140\,x^4} - \frac{5\,b\,c^4\,d^4}{9\,x^3} - \frac{88\,b\,c^5\,d^4}{105\,x^2} - \frac{5\,b\,c^6\,d^4}{3\,x} - \frac{a\,d^4}{7\,x^7} - \frac{2\,a\,c\,d^4}{3\,x^6} - \frac{b\,c\,d^4}{42\,x^6} - \frac{b\,d^4\,\mathrm{atanh}(c\,x)}{7\,x^7} - \frac{5\,b\,c^8\,d^4\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)}{3\,\sqrt{-c^2}} - \frac{2\,b\,c\,d^4\,\mathrm{atanh}(c\,x)}{3\,x^6} - \frac{6\,b\,c^2\,d^4\,\mathrm{atanh}(c\,x)}{5\,x^5} - \frac{b\,c^3\,d^4\,\mathrm{atanh}(c\,x)}{x^4} - \frac{b\,c^4\,d^4\,\mathrm{atanh}(c\,x)}{3\,x^3}$$

input
```
int(((a + b*atanh(c*x))*(d + c*d*x)^4)/x^8,x)
```

output
```
(176*b*c^7*d^4*log(x))/105 - (88*b*c^7*d^4*log(c^2*x^2 - 1))/105 - (6*a*c^
2*d^4)/(5*x^5) - (a*c^3*d^4)/x^4 - (a*c^4*d^4)/(3*x^3) - (2*b*c^2*d^4)/(15
*x^5) - (47*b*c^3*d^4)/(140*x^4) - (5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/
(105*x^2) - (5*b*c^6*d^4)/(3*x) - (a*d^4)/(7*x^7) - (2*a*c*d^4)/(3*x^6) -
(b*c*d^4)/(42*x^6) - (b*d^4*atanh(c*x))/(7*x^7) - (5*b*c^8*d^4*atan((c^2*x
)/(-c^2)^(1/2)))/(3*(-c^2)^(1/2)) - (2*b*c*d^4*atanh(c*x))/(3*x^6) - (6*b*
c^2*d^4*atanh(c*x))/(5*x^5) - (b*c^3*d^4*atanh(c*x))/x^4 - (b*c^4*d^4*atan
h(c*x))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84

$$\int \frac{(d+cdx)^4(a+b\text{arctanh}(cx))}{x^8}\,dx$$
$$= \frac{d^4(-12atanh(cx)\,b\,c^7x^7 - 420atanh(cx)\,b\,c^4x^4 - 1260atanh(cx)\,b\,c^3x^3 - 1512atanh(cx)\,b\,c^2x^2 - 840ata$$

input
```
int((c*d*x+d)^4*(a+b*atanh(c*x))/x^8,x)
```

output
```
(d**4*( - 12*atanh(c*x)*b*c**7*x**7 - 420*atanh(c*x)*b*c**4*x**4 - 1260*at
anh(c*x)*b*c**3*x**3 - 1512*atanh(c*x)*b*c**2*x**2 - 840*atanh(c*x)*b*c*x
- 180*atanh(c*x)*b - 2112*log(c**2*x - c)*b*c**7*x**7 + 2112*log(x)*b*c**7
*x**7 - 420*a*c**4*x**4 - 1260*a*c**3*x**3 - 1512*a*c**2*x**2 - 840*a*c*x
- 180*a - 2100*b*c**6*x**6 - 1056*b*c**5*x**5 - 700*b*c**4*x**4 - 423*b*c*
*3*x**3 - 168*b*c**2*x**2 - 30*b*c*x))/(1260*x**7)
```

3.43 $\int \frac{x^3(a+b\text{arctanh}(cx))}{d+cdx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+cdx}\,dx = \frac{ax}{c^3d}-\frac{bx}{2c^3d}+\frac{bx^2}{6c^2d}+\frac{b\text{arctanh}(cx)}{2c^4d}+\frac{bx\text{arctanh}(cx)}{c^3d}-\frac{x^2(a+b\text{arctanh}(cx))}{2c^2d}+\frac{x^3(a+b\text{arctanh}(cx))}{3cd}+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^4d}+\frac{2b\log(1-c^2x^2)}{3c^4d}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^4d}$$

output
```
a*x/c^3/d-1/2*b*x/c^3/d+1/6*b*x^2/c^2/d+1/2*b*arctanh(c*x)/c^4/d+b*x*arcta
nh(c*x)/c^3/d-1/2*x^2*(a+b*arctanh(c*x))/c^2/d+1/3*x^3*(a+b*arctanh(c*x))/
c/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^4/d+2/3*b*ln(-c^2*x^2+1)/c^4/d-1/2*
b*polylog(2,1-2/(c*x+1))/c^4/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.73

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+cdx}\,dx$$
$$= \frac{-b+6acx-3bcx-3ac^2x^2+bc^2x^2+2ac^3x^3+b\text{arctanh}(cx)\left(3+6cx-3c^2x^2+2c^3x^3+6\log\left(1+e^{-2\ldots}\right.\right.}{6c^4d}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

output
```
(-b + 6*a*c*x - 3*b*c*x - 3*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + b*ArcTan
h[c*x]*(3 + 6*c*x - 3*c^2*x^2 + 2*c^3*x^3 + 6*Log[1 + E^(-2*ArcTanh[c*x])]
) - 6*a*Log[1 + c*x] + 4*b*Log[1 - c^2*x^2] - 3*b*PolyLog[2, -E^(-2*ArcTan
h[c*x])])/(6*c^4*d)
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6492, 27, 6452, 243, 49, 2009, 6492, 6452, 262, 219, 6492, 2009, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{cdx+d}\,dx$$

$$\Big\downarrow \; 6492$$

$$\frac{\int x^2(a+b\text{arctanh}(cx))dx}{cd}-\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{d(cx+1)}dx}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x^2(a+b\text{arctanh}(cx))dx}{cd}-\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \; 6452$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{3}bc\int\frac{x^3}{1-c^2x^2}dx}{cd}-\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int\frac{x^2}{1-c^2x^2}dx^2}{cd}-\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{49}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^2-1)}\right)dx^2}{cd}-\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{6492}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\int x(a+b\text{arctanh}(cx))dx}{c}-\frac{\int\frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\int\frac{x^2}{1-c^2x^2}dx}{c}-\frac{\int\frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}\right)}{c}-\frac{\int\frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c}-\frac{\int\frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \; 6492$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c}-\frac{\frac{\int(a+b\text{arctanh}(cx))dx}{c}-\frac{\int\frac{a+b\text{arctanh}(cx)}{cx+1}dx}{c}}{c}}{cd}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c}-\frac{\int\frac{a+b\text{arctanh}(cx)}{cx+1}dx}{c}}{c}}{cd}$$

$$\Big\downarrow \; 6470$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c}-\frac{b\int\frac{\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{c}}{c}}{cd}$$

$$\Big\downarrow \; 2849$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c}-\frac{\frac{b\int\frac{\log\left(\frac{2}{cx+1}\right)}{1-\frac{2}{cx+1}}d\frac{1}{cx+1}}{c}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{c}}{cd}$$

$$\Big\downarrow \; 2752$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{c}}{cd}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

output
```
((x^3*(a + b*ArcTanh[c*x]))/3 - (b*c*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/
6)/(c*d) - (((x^2*(a + b*ArcTanh[c*x]))/2 - (b*c*(-(x/c^2) + ArcTanh[c*x]/
c^3))/2)/c - ((a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c - (-
(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/c) + (b*PolyLog[2, 1 - 2/(1 + c*x
)])/(2*c))/c)/c)/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{a\left(\frac{x^3c^3}{3}-\frac{c^2x^2}{2}+cx-\ln(cx+1)\right)}{d}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}-\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\operatorname{arctanh}(cx)cx-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-(\ln\cdots\right.}{\cdots}}{c^4}$
default	$\frac{\frac{a\left(\frac{x^3c^3}{3}-\frac{c^2x^2}{2}+cx-\ln(cx+1)\right)}{d}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}-\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\operatorname{arctanh}(cx)cx-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-(\ln\cdots\right.}{\cdots}}{c^4}$
parts	$\frac{a\left(\frac{\frac{1}{3}c^2x^3-\frac{1}{2}cx^2+x}{c^3}-\frac{\ln(cx+1)}{c^4}\right)}{d}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^3x^3}{3}-\frac{\operatorname{arctanh}(cx)c^2x^2}{2}+\operatorname{arctanh}(cx)cx-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(c\cdots}{}\right.}{\cdots}$
risch	$-\frac{b\ln(cx+1)^2}{4c^4d}+\frac{b\left(\frac{1}{3}c^2x^3-\frac{1}{2}cx^2+x\right)\ln(cx+1)}{2c^3d}-\frac{b\ln(-cx+1)x^3}{6dc}+\frac{\ln(-cx+1)bx^2}{4dc^2}-\frac{b\ln(-cx+1)x}{2dc^3}+\frac{5b\ln(-\cdots}{12\cdots}$

input
```
int(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d*(1/3*x^3*c^3-1/2*c^2*x^2+c*x-ln(c*x+1))+b/d*(1/3*arctanh(c*x)*c
^3*x^3-1/2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x-arctanh(c*x)*ln(c*x+1)+1/
4*ln(c*x+1)^2-1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/2*dilog(1
/2*c*x+1/2)+1/6*(c*x+1)^2-5/6*c*x-5/6+11/12*ln(c*x+1)+5/12*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))}{d+cdx}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x^3}{cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arctanh(c*x) + a*x^3)/(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + cdx}\,dx = \frac{\int \frac{ax^3}{cx+1}\,dx + \int \frac{bx^3\,\text{atanh}\,(cx)}{cx+1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(Integral(a*x**3/(c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(c*x + 1), x))
/d
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + cdx}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^3}{cdx + d}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/72*(2*c^4*(2*(c^2*x^3 + 3*x)/(c^7*d) - 3*log(c*x + 1)/(c^8*d) + 3*log(c*
x - 1)/(c^8*d)) + 216*c^4*integrate(1/6*x^4*log(c*x + 1)/(c^5*d*x^2 - c^3*
d), x) - 3*c^3*(x^2/(c^5*d) + log(c^2*x^2 - 1)/(c^7*d)) - 216*c^3*integrat
e(1/6*x^3*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x) + 9*c^2*(2*x/(c^5*d) - log(
c*x + 1)/(c^6*d) + log(c*x - 1)/(c^6*d)) - 216*c*integrate(1/6*x*log(c*x +
 1)/(c^5*d*x^2 - c^3*d), x) - 6*(2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*log(c*x
 + 1))*log(-c*x + 1)/(c^4*d) + 18*log(6*c^5*d*x^2 - 6*c^3*d)/(c^4*d) - 216
*integrate(1/6*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x))*b + 1/6*a*((2*c^2*x^3
 - 3*c*x^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d))
```

Giac [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+cdx}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)x^3}{cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+cdx}\,dx = \int \frac{x^3\,(a+b\,\text{atanh}(c\,x))}{d+c\,d\,x}\,dx$$

input
```
int((x^3*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

output
```
int((x^3*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+cdx}\,dx = \frac{6\left(\int \frac{atanh(cx)x^3}{cx+1}dx\right)b\,c^4-6\log(cx+1)\,a+2a\,c^3x^3-3a\,c^2x^2+6acx}{6c^4d}$$

input
```
int(x^3*(a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(6*int((atanh(c*x)*x**3)/(c*x + 1),x)*b*c**4 - 6*log(c*x + 1)*a + 2*a*c**3
*x**3 - 3*a*c**2*x**2 + 6*a*c*x)/(6*c**4*d)
```

3.44 $\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx = -\frac{ax}{c^2d}+\frac{bx}{2c^2d}-\frac{b\text{arctanh}(cx)}{2c^3d}-\frac{bx\text{arctanh}(cx)}{c^2d}+\frac{x^2(a+b\text{arctanh}(cx))}{2cd}-\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^3d}-\frac{b\log(1-c^2x^2)}{2c^3d}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^3d}$$

output
```
-a*x/c^2/d+1/2*b*x/c^2/d-1/2*b*arctanh(c*x)/c^3/d-b*x*arctanh(c*x)/c^2/d+1
/2*x^2*(a+b*arctanh(c*x))/c/d-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^3/d-1/2*b
*ln(-c^2*x^2+1)/c^3/d+1/2*b*polylog(2,1-2/(c*x+1))/c^3/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx = \frac{-2acx+bcx+ac^2x^2+b\text{arctanh}(cx)\left(-1-2cx+c^2x^2-2\log\left(1+e^{-2\text{arctanh}(cx)}\right)\right)+2a\log(1+cx)-b}{2c^3d}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

output
```
(-2*a*c*x + b*c*x + a*c^2*x^2 + b*ArcTanh[c*x]*(-1 - 2*c*x + c^2*x^2 - 2*L
og[1 + E^(-2*ArcTanh[c*x])]) + 2*a*Log[1 + c*x] - b*Log[1 - c^2*x^2] + b*P
olyLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^3*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6492, 27, 6452, 262, 219, 6492, 2009, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{cdx+d}dx$$

$$\Big\downarrow \text{6492}$$

$$\frac{\int x(a+b\text{arctanh}(cx))dx}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))}{d(cx+1)}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x(a+b\text{arctanh}(cx))dx}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\int \frac{x^2}{1-c^2x^2}dx}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\int \frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}\right)}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{6492}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{cd}-\frac{\frac{\int (a+b\text{arctanh}(cx))dx}{c}-\frac{\int \frac{a+b\text{arctanh}(cx)}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{cd}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c}-\frac{\int \frac{a+b\text{arctanh}(cx)}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{cd}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c}-\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{c}}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{cd}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c}-\frac{\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-\frac{2}{cx+1}}d\frac{1}{cx+1}}{c}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{c}}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{cd}-\frac{\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c}-\frac{\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{c}}{cd}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

output
```
((x^2*(a + b*ArcTanh[c*x]))/2 - (b*c*(-(x/c^2) + ArcTanh[c*x]/c^3))/2)/(c*
d) - ((a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c - (-(((a + b
*ArcTanh[c*x])*Log[2/(1 + c*x)])/c) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c
))/c)/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{a\left(\frac{c^2x^2}{2}-cx+\ln(cx+1)\right)}{d}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-\operatorname{arctanh}(cx)cx+\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln(-}{2}\right.}{d}}{c^3}$
default	$\frac{\frac{a\left(\frac{c^2x^2}{2}-cx+\ln(cx+1)\right)}{d}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-\operatorname{arctanh}(cx)cx+\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln(-}{2}\right.}{d}}{c^3}$
parts	$\frac{a\left(\frac{\frac{1}{2}cx^2-x}{c^2}+\frac{\ln(cx+1)}{c^3}\right)}{d}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-\operatorname{arctanh}(cx)cx+\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}\right.\right.}{}\right.}{d\,c^3}$
risch	$\frac{b\ln(cx+1)^2}{4c^3d}+\frac{b\left(\frac{1}{2}cx^2-x\right)\ln(cx+1)}{2c^2d}-\frac{b\ln(-cx+1)x^2}{4dc}+\frac{\ln(-cx+1)bx}{2d\,c^2}-\frac{b\ln(-cx+1)}{4d\,c^3}+\frac{bx}{2c^2d}+\frac{b}{8c^3d}+\frac{b}{}$

input
```
int(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d*(1/2*c^2*x^2-c*x+ln(c*x+1))+b/d*(1/2*arctanh(c*x)*c^2*x^2-arcta
nh(c*x)*c*x+arctanh(c*x)*ln(c*x+1)-1/4*ln(c*x+1)^2+1/2*(ln(c*x+1)-ln(1/2*c
*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2)+1/2*c*x+1/2-3/4*ln(c*x+1)
-1/4*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x^2}{cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctanh(c*x) + a*x^2)/(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx=\frac{\int \frac{ax^2}{cx+1}\,dx+\int \frac{bx^2\,\text{atanh}\,(cx)}{cx+1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(Integral(a*x**2/(c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(c*x + 1), x))
/d
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x^2}{cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/8*(c^3*(x^2/(c^4*d) + log(c^2*x^2 - 1)/(c^6*d)) + 8*c^3*integrate(1/2*x^
3*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x) - c^2*(2*x/(c^4*d) - log(c*x + 1)/(
c^5*d) + log(c*x - 1)/(c^5*d)) - 8*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^4
*d*x^2 - c^2*d), x) + 8*c*integrate(1/2*x*log(c*x + 1)/(c^4*d*x^2 - c^2*d)
, x) - 2*(c^2*x^2 - 2*c*x + 2*log(c*x + 1))*log(-c*x + 1)/(c^3*d) - 2*log(
2*c^4*d*x^2 - 2*c^2*d)/(c^3*d) + 8*integrate(1/2*log(c*x + 1)/(c^4*d*x^2 -
 c^2*d), x))*b + 1/2*a*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d))
```

Giac [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x^2}{cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+cdx}\,dx=\int \frac{x^2\,(a+b\,\text{atanh}(c\,x))}{d+c\,d\,x}\,dx$$

input
```
int((x^2*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

output
```
int((x^2*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{d + cdx}\, dx = \frac{2\left(\int \frac{atanh(cx)x^2}{cx+1}dx\right) b\, c^3 + 2\log(cx + 1)\, a + a\, c^2x^2 - 2acx}{2c^3d}$$

input
```
int(x^2*(a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(2*int((atanh(c*x)*x**2)/(c*x + 1),x)*b*c**3 + 2*log(c*x + 1)*a + a*c**2*x
**2 - 2*a*c*x)/(2*c**3*d)
```

3.45 $\int \frac{x(a+b\text{arctanh}(cx))}{d+cdx} dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{x(a+b\text{arctanh}(cx))}{d+cdx} dx = \frac{ax}{cd} + \frac{bx\text{arctanh}(cx)}{cd} + \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^2d} + \frac{b\log(1-c^2x^2)}{2c^2d} - \frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2d}$$

output
```
a*x/c/d+b*x*arctanh(c*x)/c/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^2/d+1/2*b*
ln(-c^2*x^2+1)/c^2/d-1/2*b*polylog(2,1-2/(c*x+1))/c^2/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{x(a+b\text{arctanh}(cx))}{d+cdx} dx$$
$$= \frac{2acx + 2b\text{arctanh}(cx)\left(cx + \log\left(1+e^{-2\text{arctanh}(cx)}\right)\right) - 2a\log(1+cx) + b\log(1-c^2x^2) - b\,\text{PolyLog}(2,}{2c^2d}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

output
```
(2*a*c*x + 2*b*ArcTanh[c*x]*(c*x + Log[1 + E^(-2*ArcTanh[c*x])]) - 2*a*Log
[1 + c*x] + b*Log[1 - c^2*x^2] - b*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^
2*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6492, 27, 2009, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arctanh}(cx))}{cdx+d}dx$$

$$\Big\downarrow \text{6492}$$

$$\frac{\int (a+b\text{arctanh}(cx))dx}{cd}-\frac{\int \frac{a+b\text{arctanh}(cx)}{d(cx+1)}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\text{arctanh}(cx))dx}{cd}-\frac{\int \frac{a+b\text{arctanh}(cx)}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{cd}-\frac{\int \frac{a+b\text{arctanh}(cx)}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{6470}$$

$$\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{cd}-\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{cd}-\frac{\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-\frac{2}{cx+1}}d\frac{1}{cx+1}}{c}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{ax + bx\text{arctanh}(cx) + \frac{b\log(1-c^2x^2)}{2c}}{cd} - \frac{\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c} - \frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c}}{cd}$$

input
```
Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

output
```
(a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/(c*d) - (-(((a + b*A
rcTanh[c*x])*Log[2/(1 + c*x)])/c) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c))
/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{a(cx-\ln(cx+1))}{d}+\frac{b\left(-\operatorname{arctanh}(cx)\ln(cx+1)+\operatorname{arctanh}(cx)cx+\frac{\ln(cx+1)^2}{4}-\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2}\right.}{d}}{c^2}$
default	$\frac{\frac{a(cx-\ln(cx+1))}{d}+\frac{b\left(-\operatorname{arctanh}(cx)\ln(cx+1)+\operatorname{arctanh}(cx)cx+\frac{\ln(cx+1)^2}{4}-\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2}\right.}{d}}{c^2}$
parts	$\frac{a\left(\frac{x}{c}-\frac{\ln(cx+1)}{c^2}\right)}{d}+\frac{b\left(-\operatorname{arctanh}(cx)\ln(cx+1)+\operatorname{arctanh}(cx)cx+\frac{\ln(cx+1)^2}{4}-\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}+\right.}{d\,c^2}$
risch	$-\frac{b\ln(cx+1)^2}{4c^2d}+\frac{bx\ln(cx+1)}{2cd}+\frac{b\ln(cx+1)}{2c^2d}-\frac{\ln(-cx+1)bx}{2dc}-\frac{\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b}{2d\,c^2}+\frac{\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+}{2d\,c^2}$

input
```
int(x*(a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a/d*(c*x-ln(c*x+1))+b/d*(-arctanh(c*x)*ln(c*x+1)+arctanh(c*x)*c*x+1
/4*ln(c*x+1)^2-1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/2*dilog(
1/2*c*x+1/2)+1/2*ln((c*x-1)*(c*x+1))))
```

Fricas [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+cdx}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x}{cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*x*arctanh(c*x) + a*x)/(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{x(a + b\operatorname{arctanh}(cx))}{d + cdx}\,dx = \frac{\int \frac{ax}{cx+1}\,dx + \int \frac{bx\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

input
```
integrate(x*(a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(Integral(a*x/(c*x + 1), x) + Integral(b*x*atanh(c*x)/(c*x + 1), x))/d
```

Maxima [F]

$$\int \frac{x(a + b\operatorname{arctanh}(cx))}{d + cdx}\,dx = \int \frac{(b\operatorname{artanh}(cx) + a)x}{cdx + d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/4*(c^2*(2*x/(c^3*d) - log(c*x + 1)/(c^4*d) + log(c*x - 1)/(c^4*d)) + 2*c
^2*integrate(x^2*log(c*x + 1)/(c^3*d*x^2 - c*d), x) - 4*c*integrate(x*log(
c*x + 1)/(c^3*d*x^2 - c*d), x) - 2*(c*x - log(c*x + 1))*log(-c*x + 1)/(c^2
*d) + log(c^3*d*x^2 - c*d)/(c^2*d) - 2*integrate(log(c*x + 1)/(c^3*d*x^2 -
 c*d), x))*b + a*(x/(c*d) - log(c*x + 1)/(c^2*d))
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+cdx}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x}{cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+cdx}\,dx=\int \frac{x\,(a+b\operatorname{atanh}(c\,x))}{d+c\,d\,x}\,dx$$

input
```
int((x*(a + b*atanh(c*x)))/(d + c*d*x),x)
```

output
```
int((x*(a + b*atanh(c*x)))/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+cdx}\,dx=\frac{\left(\int \frac{atanh(cx)x}{cx+1}dx\right)b\,c^2-\log(cx+1)\,a+acx}{c^2d}$$

input
```
int(x*(a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(int((atanh(c*x)*x)/(c*x + 1),x)*b*c**2 - log(c*x + 1)*a + a*c*x)/(c**2*d)
```

3.46 $\int \frac{a+b\text{arctanh}(cx)}{d+cdx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{a+b\text{arctanh}(cx)}{d+cdx}\,dx=-\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2cd}$$

output
```
-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c/d+1/2*b*polylog(2,1-2/(c*x+1))/c/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arctanh}(cx)}{d+cdx}\,dx =\frac{-2b\text{arctanh}(cx)\log\left(1+e^{-2\text{arctanh}(cx)}\right)+2a\log(1+cx)+b\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)}{2cd}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

output
```
(-2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a*Log[1 + c*x] + b*Pol
yLog[2, -E^(-2*ArcTanh[c*x])])/(2*c*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{cdx+d}dx$$

$$\Big\downarrow \text{6470}$$

$$\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx}{d}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-\frac{2}{cx+1}}d\frac{1}{cx+1}}{cd}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{cd}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

output
```
-(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c*d)) + (b*PolyLog[2, 1 - 2/(1
+ c*x)])/(2*c*d)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$\frac{\frac{a\ln(cx+1)}{d}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2}\right)}{d}}{c}$	78
default	$\frac{\frac{a\ln(cx+1)}{d}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2}\right)}{d}}{c}$	78
parts	$\frac{a\ln(cx+1)}{dc}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{2}\right)}{dc}$	80
risch	$\frac{b\ln(cx+1)^2}{4cd}-\frac{\ln(-cx+1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b}{2cd}+\frac{\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b}{2cd}+\frac{a\ln(-cx-1)}{cd}+\frac{\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)b}{2cd}$	96

input
```
int((a+b*arctanh(c*x))/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d*ln(c*x+1)+b/d*(arctanh(c*x)*ln(c*x+1)-1/4*ln(c*x+1)^2+1/2*(ln(c*x
+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{arctanh}(cx)}{d+cdx}\,dx=\int \frac{b\,\text{artanh}\,(cx)+a}{cdx+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{d+cdx}\,dx=\frac{\int \frac{a}{cx+1}\,dx+\int \frac{b\,\text{atanh}\,(cx)}{cx+1}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(Integral(a/(c*x + 1), x) + Integral(b*atanh(c*x)/(c*x + 1), x))/d
```

Maxima [F]

$$\int \frac{a+b\text{arctanh}(cx)}{d+cdx}\,dx=\int \frac{b\,\text{artanh}\,(cx)+a}{cdx+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*c*integrate(x*log(c*x + 1)/(c^2*d*x^2 - d), x) - log(c*x + 1)*log(-
c*x + 1)/(c*d))*b + a*log(c*d*x + d)/(c*d)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{d + cdx}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{cdx + d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx)}{d + cdx}\,dx = \int \frac{a + b\,\text{atanh}(c\,x)}{d + c\,d\,x}\,dx$$

input
```
int((a + b*atanh(c*x))/(d + c*d*x),x)
```

output
```
int((a + b*atanh(c*x))/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{a + b\text{arctanh}(cx)}{d + cdx}\,dx = \frac{\left(\int \frac{atanh(cx)}{cx+1}dx\right)bc + \log(cx+1)\,a}{cd}$$

input
```
int((a+b*atanh(c*x))/(c*d*x+d),x)
```

output
```
(int(atanh(c*x)/(c*x + 1),x)*b*c + log(c*x + 1)*a)/(c*d)
```

3.47 $\int \frac{a+b\,\text{arctanh}(cx)}{x(d+cdx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)}\,dx=\frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{2d}$$

output
```
(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d-1/2*b*polylog(2,-1+2/(c*x+1))/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)}\,dx=\frac{2b\text{arctanh}(cx)\log\left(1-e^{-2\text{arctanh}(cx)}\right)+2a\log(x)-2a\log(1+cx)-b\,\text{PolyLog}\left(2,e^{-2\text{arctanh}(cx)}\right)}{2d}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

output
```
(2*b*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + 2*a*Log[x] - 2*a*Log[1 +
c*x] - b*PolyLog[2, E^(-2*ArcTanh[c*x])])/(2*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x(cdx+d)}dx$$

$$\Big\downarrow \text{ 6494}$$

$$\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d}-\frac{bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx}{d}$$

$$\Big\downarrow \text{ 2897}$$

$$\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d}-\frac{b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

output
```
((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*PolyLog[2, -1 + 2/(1 +
c*x)])/(2*d)
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

method	result
parts	$\frac{a(-\ln(cx+1)+\ln(x))}{d}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx)-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right.}{2}\right.}{d}$
derivativedivides	$\frac{a(\ln(cx)-\ln(cx+1))}{d}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx)-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}\right.}{d}$
default	$\frac{a(\ln(cx)-\ln(cx+1))}{d}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx)-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2}\right.}{d}$
risch	$-\frac{\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b}{2d}+\frac{\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)b}{2d}-\frac{a\ln(-cx-1)}{d}+\frac{a\ln(-cx)}{d}+\frac{\operatorname{dilog}(-cx+1)b}{2d}-\frac{\operatorname{dilog}(-}{}$

input
```
int((a+b*arctanh(c*x))/x/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*(-ln(c*x+1)+ln(x))+b/d*(arctanh(c*x)*ln(c*x)-arctanh(c*x)*ln(c*x+1)+1/
4*ln(c*x+1)^2-1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/2*dilog(1
/2*c*x+1/2)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c*d*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)}\,dx = \frac{\int \frac{a}{cx^2+x}\,dx + \int \frac{b\,\text{atanh}\,(cx)}{cx^2+x}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))/x/(c*d*x+d),x)
```

output
```
(Integral(a/(c*x**2 + x), x) + Integral(b*atanh(c*x)/(c*x**2 + x), x))/d
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="maxima")
```

output
```
-a*(log(c*x + 1)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x
+ 1))/(c*d*x^2 + d*x), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)}\,dx=\int \frac{a+b\operatorname{atanh}(c\,x)}{x\,(d+c\,d\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)}\,dx=\frac{-atanh(cx)^2\,b-2\left(\int \frac{atanh(cx)}{c^2x^3-x}dx\right)b-2\log(cx+1)\,a+2\log(x)\,a}{2d}$$

input
```
int((a+b*atanh(c*x))/x/(c*d*x+d),x)
```

output
```
( - atanh(c*x)**2*b - 2*int(atanh(c*x)/(c**2*x**3 - x),x)*b - 2*log(c*x +
1)*a + 2*log(x)*a)/(2*d)
```

3.48 $\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)}\,dx = -\frac{a+b\text{arctanh}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{bc\log\left(1-c^2x^2\right)}{2d} - \frac{c(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d} + \frac{bc\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{2d}$$

output
```
-(a+b*arctanh(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(-c^2*x^2+1)/d-c*(a+b*arctan
h(c*x))*ln(2-2/(c*x+1))/d+1/2*b*c*polylog(2,-1+2/(c*x+1))/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)}\,dx = \frac{-2\Big(a+b\text{arctanh}(cx)\left(1+cx\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right)+acx\log(x)-acx\log(1+cx)-bcx\log\Big(\frac{cx}{\sqrt{1-c^2x^2}}}{2dx}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

output
```
(-2*(a + b*ArcTanh[c*x]*(1 + c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + a*c*x*Log
[x] - a*c*x*Log[1 + c*x] - b*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + b*c*x*Pol
yLog[2, E^(-2*ArcTanh[c*x])])/(2*d*x)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6496, 27, 6452, 243, 47, 14, 16, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(cdx+d)}dx$$

$$\Big\downarrow \text{6496}$$

$$\frac{\int \frac{a+b\text{arctanh}(cx)}{x^2}dx}{d}-c\int \frac{a+b\text{arctanh}(cx)}{dx(cx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arctanh}(cx)}{x^2}dx}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \ 14$$

$$\frac{\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \ 6494$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)}{d}$$

$$\Big\downarrow \ 2897$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}}{d}-\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

output
```
(-((a + b*ArcTanh[c*x])/x) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2)/d - (c
*((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*
x)])/2))/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

method	result
parts	$\frac{a(c\ln(cx+1)-\frac{1}{x}-c\ln(x))}{d}+$ bc(− arctanh(cx)/(cx) − arctanh(cx) ln(cx) + arctanh(cx) ln(cx+1) − ln(cx−1)/2 + ln(cx) − ln(cx…
derivativedivides	$c\Big($ $\frac{a(-\frac{1}{cx}-\ln(cx)+\ln(cx+1))}{d}+$ b(− arctanh(cx)/(cx) − arctanh(cx) ln(cx) + arctanh(cx) ln(cx+1) − ln(cx−1)/2 + ln(cx) − l…
default	$c\Big($ $\frac{a(-\frac{1}{cx}-\ln(cx)+\ln(cx+1))}{d}+$ b(− arctanh(cx)/(cx) − arctanh(cx) ln(cx) + arctanh(cx) ln(cx+1) − ln(cx−1)/2 + ln(cx) − l…
risch	$-\frac{cb\operatorname{dilog}(-cx+1)}{2d}+\frac{cb\ln(\frac{cx}{2}+\frac{1}{2})\ln(-\frac{cx}{2}+\frac{1}{2})}{2d}-\frac{cb\ln(\frac{cx}{2}+\frac{1}{2})\ln(-cx+1)}{2d}+\frac{cb\operatorname{dilog}(-\frac{cx}{2}+\frac{1}{2})}{2d}+\frac{cb\ln(-cx)}{2d}$ …

input
```
int((a+b*arctanh(c*x))/x^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*(c*ln(c*x+1)-1/x-c*ln(x))+b/d*c*(-arctanh(c*x)/c/x-arctanh(c*x)*ln(c*x
)+arctanh(c*x)*ln(c*x+1)-1/2*ln(c*x-1)+ln(c*x)-1/2*ln(c*x+1)+1/2*dilog(c*x
)+1/2*dilog(c*x+1)+1/2*ln(c*x)*ln(c*x+1)-1/4*ln(c*x+1)^2+1/2*(ln(c*x+1)-ln
(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2))
```

Fricas [F]

$$\int\frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)}\,dx=\int\frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c*d*x^3 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)}\,dx = \frac{\int \frac{a}{cx^3+x^2}\,dx + \int \frac{b\operatorname{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))/x**2/(c*d*x+d),x)
```

output
```
(Integral(a/(c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c*x**3 + x**2), x
))/d
```

Maxima [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)}\,dx = \int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="maxima")
```

output
```
a*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/2*b*integrate((log(c*x + 1
) - log(-c*x + 1))/(c*d*x^3 + d*x^2), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)}\,dx = \int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)}\,dx=\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^2\,\left(d+c\,d\,x\right)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^2*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x^2*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)}\,dx=\frac{\left(\int \frac{atanh(cx)}{c\,x^3+x^2}dx\right)bx+\log(cx+1)\,acx-\log(x)\,acx-a}{dx}$$

input
```
int((a+b*atanh(c*x))/x^2/(c*d*x+d),x)
```

output
```
(int(atanh(c*x)/(c*x**3 + x**2),x)*b*x + log(c*x + 1)*a*c*x - log(x)*a*c*x
 - a)/(d*x)
```

3.49 $\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)}\,dx &= -\frac{bc}{2dx}+\frac{bc^2\text{arctanh}(cx)}{2d}-\frac{a+b\text{arctanh}(cx)}{2dx^2}\\ &\quad+\frac{c(a+b\text{arctanh}(cx))}{dx}-\frac{bc^2\log(x)}{d}+\frac{bc^2\log\left(1-c^2x^2\right)}{2d}\\ &\quad+\frac{c^2(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}\\ &\quad-\frac{bc^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

output
```
-1/2*b*c/d/x+1/2*b*c^2*arctanh(c*x)/d-1/2*(a+b*arctanh(c*x))/d/x^2+c*(a+b*
arctanh(c*x))/d/x-b*c^2*ln(x)/d+1/2*b*c^2*ln(-c^2*x^2+1)/d+c^2*(a+b*arctan
h(c*x))*ln(2-2/(c*x+1))/d-1/2*b*c^2*polylog(2,-1+2/(c*x+1))/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.91

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)}\,dx =$$
$$-\frac{a-2acx+bcx-b\text{arctanh}(cx)\left(-1+2cx+c^2x^2+2c^2x^2\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right)-2ac^2x^2\log(x)+2}{2dx^2}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

output
```
-1/2*(a - 2*a*c*x + b*c*x - b*ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x
^2*Log[1 - E^(-2*ArcTanh[c*x])]) - 2*a*c^2*x^2*Log[x] + 2*a*c^2*x^2*Log[1
+ c*x] + 2*b*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + b*c^2*x^2*PolyLog[2, E
^(-2*ArcTanh[c*x])])/(d*x^2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6496, 27, 6452, 264, 219, 6496, 6452, 243, 47, 14, 16, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(cdx+d)}\,dx$$

$$\Big\downarrow \ 6496$$

$$\frac{\int \frac{a+b\text{arctanh}(cx)}{x^3}dx}{d}-c\int \frac{a+b\text{arctanh}(cx)}{dx^2(cx+1)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a+b\text{arctanh}(cx)}{x^3}dx}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx}{d}$$

$$\Big\downarrow \ 6452$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6496}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(\int \frac{a+b\text{arctanh}(cx)}{x^2}dx-c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)}{d}$$

$$\Big\downarrow \text{6494}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-bc\int\frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2\right.\right.\right.}{d}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}}{d}-\frac{c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTanh[c*x])/x^2 + (b*c*(-x^(-1) + c*ArcTanh[c*x]))/2)/d - (
c*(-((a + b*ArcTanh[c*x])/x) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2 - c*(
(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)
])/2)))/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.29

method	result
parts	$\frac{a\left(-c^2\ln(cx+1)-\frac{1}{2x^2}+c^2\ln(x)+\frac{c}{x}\right)}{d}+\frac{b\,c^2\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{cx}-\operatorname{arctanh}(cx)\ln(cx+1)\right.}{}$
derivativedivides	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}+\ln(cx)+\frac{1}{cx}-\ln(cx+1)\right)}{d}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{cx}-\operatorname{arctanh}(cx)\ln(cx+1\right.}{}\right.$
default	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}+\ln(cx)+\frac{1}{cx}-\ln(cx+1)\right)}{d}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{cx}-\operatorname{arctanh}(cx)\ln(cx+1\right.}{}\right.$
risch	$-\frac{c^2b\ln(-cx)}{4d}-\frac{bc}{2dx}+\frac{c^2b\ln(-cx+1)}{4d}+\frac{b\ln(-cx+1)}{4d\,x^2}+\frac{c^2b\,\operatorname{dilog}(-cx+1)}{2d}-\frac{c^2b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d}+$

input
```
int((a+b*arctanh(c*x))/x^3/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*(-c^2*ln(c*x+1)-1/2/x^2+c^2*ln(x)+c/x)+b/d*c^2*(-1/2*arctanh(c*x)/c^2/
x^2+arctanh(c*x)*ln(c*x)+arctanh(c*x)/c/x-arctanh(c*x)*ln(c*x+1)-1/2*dilog
(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1)+1/4*ln(c*x+1)^2-1/2*(ln(c*x+1
)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/2*dilog(1/2*c*x+1/2)+1/4*ln(c*x-1)-1
/2/c/x-ln(c*x)+3/4*ln(c*x+1))
```

Fricas [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c*d*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)}\,dx = \frac{\int \frac{a}{cx^4+x^3}\,dx + \int \frac{b\,\text{atanh}\,(cx)}{cx^4+x^3}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))/x**3/(c*d*x+d),x)
```

output
```
(Integral(a/(c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c*x**4 + x**3), x
))/d
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="maxima")
```

output
```
-1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a + 1/2
*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^4 + d*x^3), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(d+cdx)}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(d+cdx)}\,dx=\int \frac{a+b\operatorname{atanh}(c\,x)}{x^3\,(d+c\,d\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^3*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x^3*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(d+cdx)}\,dx=\frac{2\left(\int \frac{atanh(cx)}{c\,x^4+x^3}dx\right)b\,x^2-2\log(cx+1)\,a\,c^2x^2+2\log(x)\,a\,c^2x^2+2acx-a}{2d\,x^2}$$

input
```
int((a+b*atanh(c*x))/x^3/(c*d*x+d),x)
```

output
```
(2*int(atanh(c*x)/(c*x**4 + x**3),x)*b*x**2 - 2*log(c*x + 1)*a*c**2*x**2 +
 2*log(x)*a*c**2*x**2 + 2*a*c*x - a)/(2*d*x**2)
```

3.50 $\int \frac{a+b\text{arctanh}(cx)}{x^4(d+cdx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx)}{x^4(d+cdx)}\,dx = &-\frac{bc}{6dx^2}+\frac{bc^2}{2dx}-\frac{bc^3\text{arctanh}(cx)}{2d}-\frac{a+b\text{arctanh}(cx)}{3dx^3}\\&+\frac{c(a+b\text{arctanh}(cx))}{2dx^2}-\frac{c^2(a+b\text{arctanh}(cx))}{dx}+\frac{4bc^3\log(x)}{3d}\\&-\frac{2bc^3\log(1-c^2x^2)}{3d}-\frac{c^3(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}\\&+\frac{bc^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

output
```
-1/6*b*c/d/x^2+1/2*b*c^2/d/x-1/2*b*c^3*arctanh(c*x)/d-1/3*(a+b*arctanh(c*x
))/d/x^3+1/2*c*(a+b*arctanh(c*x))/d/x^2-c^2*(a+b*arctanh(c*x))/d/x+4/3*b*c
^3*ln(x)/d-2/3*b*c^3*ln(-c^2*x^2+1)/d-c^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1
))/d+1/2*b*c^3*polylog(2,-1+2/(c*x+1))/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arctanh}(cx)}{x^4(d+cdx)}\,dx$$
$$= \frac{-2a+3acx-bcx-6ac^2x^2+3bc^2x^2+bc^3x^3-b\text{arctanh}(cx)\left(2-3cx+6c^2x^2+3c^3x^3+6c^3x^3\log\left(1-\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

output
```
(-2*a + 3*a*c*x - b*c*x - 6*a*c^2*x^2 + 3*b*c^2*x^2 + b*c^3*x^3 - b*ArcTan
h[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTan
h[c*x])]) - 6*a*c^3*x^3*Log[x] + 6*a*c^3*x^3*Log[1 + c*x] + 8*b*c^3*x^3*Lo
g[(c*x)/Sqrt[1 - c^2*x^2]] + 3*b*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])])/
(6*d*x^3)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6496, 27, 6452, 243, 54, 2009, 6496, 6452, 264, 219, 6496, 6452, 243, 47, 14, 16, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^4(cdx+d)}dx$$

$$\Big\downarrow \text{6496}$$

$$\frac{\int \frac{a+b\text{arctanh}(cx)}{x^4}dx}{d}-c\int \frac{a+b\text{arctanh}(cx)}{dx^3(cx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arctanh}(cx)}{x^4}dx}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{3}bc\int \frac{1}{x^3(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{6}bc\int \frac{1}{x^4(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{\frac{1}{6}bc\int \left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\int \frac{a+b\text{arctanh}(cx)}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6496}$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(\int \frac{a+b\text{arctanh}(cx)}{x^3}dx-c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\int \frac{a+b\text{arctanh}(cx)}{x^2(cx+1)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \ 6496$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\left(\int\frac{a+b\text{arctanh}(cx)}{x^2}dx-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \ 6452$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\left(-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+bc\int\frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\left(-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{1}{2}bc\int\frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\right.\right.}{d}$$

$$\Big\downarrow \ 47$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\left(-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\int\frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(\text{carc}\right.\right.}{d}$$

$$\Big\downarrow \ 14$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\left(-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(\text{carc}\right.\right.}{d}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-\frac{c\left(-c\left(-c\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(\text{carct}\right.\right.}{d}$$

$$\Big\downarrow \ 6494$$

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-$$
$$\frac{c\left(-c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-bc\int\frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\left(1\right.\right.\right.}{d}$$

$\Big\downarrow$ 2897

$$\frac{\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}}{d}-$$
$$\frac{c\left(-c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\text{l}\right.\right.}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

output
```
(-1/3*(a + b*ArcTanh[c*x])/x^3 + (b*c*(-x^(-2) + c^2*Log[x^2] - c^2*Log[1
- c^2*x^2]))/6)/d - (c*(-1/2*(a + b*ArcTanh[c*x])/x^2 + (b*c*(-x^(-1) + c*
ArcTanh[c*x]))/2 - c*(-((a + b*ArcTanh[c*x])/x) + (b*c*(Log[x^2] - Log[1 -
 c^2*x^2]))/2 - c*((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[
2, -1 + 2/(1 + c*x)])/2))))/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

method	result
parts	$\frac{a\left(c^3\ln(cx+1)-\frac{1}{3x^3}-\frac{c^2}{x}+\frac{c}{2x^2}-c^3\ln(x)\right)}{d}+\frac{bc^3\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{cx}+\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\operatorname{arctanh}(cx)\ln(cx)+\operatorname{arct}\right.}{}$
derivativedivides	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}+\frac{1}{2c^2x^2}-\ln(cx)+\ln(cx+1)\right)}{d}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{cx}+\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\operatorname{arctanh}(cx)\ln(cx)+\right.}{}\right.$
default	$c^3\left(\frac{a\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}+\frac{1}{2c^2x^2}-\ln(cx)+\ln(cx+1)\right)}{d}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)}{cx}+\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\operatorname{arctanh}(cx)\ln(cx)+\right.}{}\right.$
risch	$-\frac{a}{3d\,x^3}+\frac{ca}{2d\,x^2}-\frac{c^2a}{dx}+\frac{5c^3b\ln(-cx)}{12d}-\frac{5c^3b\ln(-cx+1)}{12d}-\frac{c^3b\operatorname{dilog}(-cx+1)}{2d}+\frac{b\ln(-cx+1)}{6d\,x^3}+\frac{c^3b\operatorname{dilog}(}{2}$

input
```
int((a+b*arctanh(c*x))/x^4/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*(c^3*ln(c*x+1)-1/3/x^3-c^2/x+1/2*c/x^2-c^3*ln(x))+b/d*c^3*(-1/3*arctan
h(c*x)/c^3/x^3-arctanh(c*x)/c/x+1/2*arctanh(c*x)/c^2/x^2-arctanh(c*x)*ln(c
*x)+arctanh(c*x)*ln(c*x+1)-5/12*ln(c*x-1)-1/6/c^2/x^2+1/2/c/x+4/3*ln(c*x)-
11/12*ln(c*x+1)+1/2*dilog(c*x)+1/2*dilog(c*x+1)+1/2*ln(c*x)*ln(c*x+1)-1/4*
ln(c*x+1)^2+1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2
*c*x+1/2))
```

Fricas [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x^4(d+cdx)}\,dx=\int \frac{b\,\text{artanh}\,(cx)+a}{(cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c*d*x^5 + d*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x^4(d+cdx)}\,dx=\frac{\int \frac{a}{cx^5+x^4}\,dx+\int \frac{b\,\text{atanh}\,(cx)}{cx^5+x^4}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))/x**4/(c*d*x+d),x)
```

output
```
(Integral(a/(c*x**5 + x**4), x) + Integral(b*atanh(c*x)/(c*x**5 + x**4), x
))/d
```

Maxima [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^4(d+cdx)}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^
3))*a + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^5 + d*x^4),
x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^4(d+cdx)}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^4(d+cdx)}\,dx=\int \frac{a+b\operatorname{atanh}(c\,x)}{x^4\,(d+c\,d\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^4*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x^4*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^4(d + cdx)}\,dx$$
$$= \frac{3atanh(cx)^2\,b\,c^3x^3 - 11atanh(cx)\,b\,c^3x^3 - 6atanh(cx)\,b\,c^2x^2 + 3atanh(cx)\,bcx - 2atanh(cx)\,b + 6\Big(\int \frac{ata}{c^2}}{}$$

input
```
int((a+b*atanh(c*x))/x^4/(c*d*x+d),x)
```

output
```
(3*atanh(c*x)**2*b*c**3*x**3 - 11*atanh(c*x)*b*c**3*x**3 - 6*atanh(c*x)*b*
c**2*x**2 + 3*atanh(c*x)*b*c*x - 2*atanh(c*x)*b + 6*int(atanh(c*x)/(c**2*x
**3 - x),x)*b*c**3*x**3 - 8*log(c**2*x - c)*b*c**3*x**3 + 6*log(c*x + 1)*a
*c**3*x**3 - 6*log(x)*a*c**3*x**3 + 8*log(x)*b*c**3*x**3 - 6*a*c**2*x**2 +
 3*a*c*x - 2*a + 3*b*c**2*x**2 - b*c*x)/(6*d*x**3)
```

3.51 $\int \frac{x^3(a+b\,\mathrm{arctanh}(cx))}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\int \frac{x^3(a+b\,\mathrm{arctanh}(cx))}{(d+cdx)^2}\,dx = -\frac{2ax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(1+cx)}-\frac{b\,\mathrm{arctanh}(cx)}{c^4d^2}-\frac{2bx\,\mathrm{arctanh}(cx)}{c^3d^2}+\frac{x^2(a+b\,\mathrm{arctanh}(cx))}{2c^2d^2}+\frac{a+b\,\mathrm{arctanh}(cx)}{c^4d^2(1+cx)}-\frac{3(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^4d^2}-\frac{b\log(1-c^2x^2)}{c^4d^2}+\frac{3b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^4d^2}$$

output
```
-2*a*x/c^3/d^2+1/2*b*x/c^3/d^2+1/2*b/c^4/d^2/(c*x+1)-b*arctanh(c*x)/c^4/d^
2-2*b*x*arctanh(c*x)/c^3/d^2+1/2*x^2*(a+b*arctanh(c*x))/c^2/d^2+(a+b*arcta
nh(c*x))/c^4/d^2/(c*x+1)-3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^4/d^2-b*ln(-
c^2*x^2+1)/c^4/d^2+3/2*b*polylog(2,1-2/(c*x+1))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx$$
$$= \frac{-8acx+2ac^2x^2+\frac{4a}{1+cx}+12a\log(1+cx)+b\big(2cx+\cosh(2\text{arctanh}(cx))-4\log\left(1-c^2x^2\right)+6\,\text{PolyLog}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

output
```
(-8*a*c*x + 2*a*c^2*x^2 + (4*a)/(1 + c*x) + 12*a*Log[1 + c*x] + b*(2*c*x +
 Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2*ArcTanh[c
*x])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x]] - 6*Lo
g[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]])
)/(4*c^4*d^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(cdx+d)^2}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(\frac{3(a+b\text{arctanh}(cx))}{c^3d^2(cx+1)}-\frac{2(a+b\text{arctanh}(cx))}{c^3d^2}-\frac{a+b\text{arctanh}(cx)}{c^3d^2(cx+1)^2}+\frac{x(a+b\text{arctanh}(cx))}{c^2d^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a+b\text{arctanh}(cx)}{c^4d^2(cx+1)}-\frac{3\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^4d^2}+\frac{x^2(a+b\text{arctanh}(cx))}{2c^2d^2}-\frac{2ax}{c^3d^2}-\frac{b\text{arctanh}(cx)}{c^4d^2}-\frac{2bx\text{arctanh}(cx)}{c^3d^2}+\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{b}{2c^4d^2(cx+1)}+\frac{bx}{2c^3d^2}-\frac{b\log(1-c^2x^2)}{c^4d^2}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

output
```
(-2*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(1 + c*x)) - (b*ArcT
anh[c*x])/(c^4*d^2) - (2*b*x*ArcTanh[c*x])/(c^3*d^2) + (x^2*(a + b*ArcTanh
[c*x]))/(2*c^2*d^2) + (a + b*ArcTanh[c*x])/(c^4*d^2*(1 + c*x)) - (3*(a + b
*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^2) - (b*Log[1 - c^2*x^2])/(c^4*d^2
) + (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{a\left(\frac{c^2x^2}{2}-2cx+\frac{1}{cx+1}+3\ln(cx+1)\right)}{d^2}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-2\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{cx+1}+3\operatorname{arctanh}(cx)\ln(cx+1)-\frac{3\ln(cx+1)^2}{4}+\right.}{}}{c^4}$
default	$\frac{\frac{a\left(\frac{c^2x^2}{2}-2cx+\frac{1}{cx+1}+3\ln(cx+1)\right)}{d^2}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-2\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{cx+1}+3\operatorname{arctanh}(cx)\ln(cx+1)-\frac{3\ln(cx+1)^2}{4}+\right.}{}}{c^4}$
parts	$\frac{a\left(\frac{\frac{1}{2}c\,x^2-2x}{c^3}+\frac{1}{c^4(cx+1)}+\frac{3\ln(cx+1)}{c^4}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-2\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{cx+1}+3\operatorname{arctanh}(cx)\ln(cx+1)\right.}{}$
risch	$\frac{3b\ln(cx+1)^2}{4c^4d^2}+\left(\frac{b\left(\frac{1}{2}c\,x^2-2x\right)}{2c^3d^2}+\frac{b}{2c^4d^2(cx+1)}\right)\ln(cx+1)+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(cx+1)}-\frac{5b\ln(cx+1)}{4c^4d^2}+$

input
```
int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d^2*(1/2*c^2*x^2-2*c*x+1/(c*x+1)+3*ln(c*x+1))+b/d^2*(1/2*arctanh(
c*x)*c^2*x^2-2*arctanh(c*x)*c*x+1/(c*x+1)*arctanh(c*x)+3*arctanh(c*x)*ln(c
*x+1)-3/4*ln(c*x+1)^2+3/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-3/2
*dilog(1/2*c*x+1/2)+1/2*c*x+1/2+1/2/(c*x+1)-3/2*ln(c*x+1)-1/2*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x^3}{(cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arctanh(c*x) + a*x^3)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))}{(d+cdx)^2}\,dx = \frac{\int \frac{ax^3}{c^2x^2+2cx+1}\,dx + \int \frac{bx^3\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

output
```
(Integral(a*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**3*atanh(c*x)/
(c**2*x**2 + 2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))}{(d+cdx)^2}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)x^3}{(cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*(c^4*(2/(c^9*d^2*x + c^8*d^2) + 2*(c*x^2 - 2*x)/(c^7*d^2) + 7*log(c*x
 + 1)/(c^8*d^2) + log(c*x - 1)/(c^8*d^2)) + 16*c^4*integrate(1/2*x^4*log(c
*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c^3*(2/(
c^8*d^2*x + c^7*d^2) - 4*x/(c^6*d^2) + 5*log(c*x + 1)/(c^7*d^2) - log(c*x
- 1)/(c^7*d^2)) - 16*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^2*x^3 + c^5
*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) - 7*c^2*(2/(c^7*d^2*x + c^6*d^2) + 3*l
og(c*x + 1)/(c^6*d^2) + log(c*x - 1)/(c^6*d^2)) + 48*c^2*integrate(1/2*x^2
*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c*
(2/(c^6*d^2*x + c^5*d^2) + log(c*x + 1)/(c^5*d^2) - log(c*x - 1)/(c^5*d^2)
) + 96*c*integrate(1/2*x*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2
*x - c^3*d^2), x) - 4*(c^3*x^3 - 3*c^2*x^2 - 4*c*x + 6*(c*x + 1)*log(c*x +
 1) + 2)*log(-c*x + 1)/(c^5*d^2*x + c^4*d^2) + 4/(c^5*d^2*x + c^4*d^2) - 2
*log(c*x + 1)/(c^4*d^2) + 2*log(c*x - 1)/(c^4*d^2) + 48*integrate(1/2*log(
c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x))*b + 1/2*a*
(2/(c^5*d^2*x + c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)/(c^4*d
^2))
```

Giac [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x^3}{(cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{x^3\,(a+b\,\text{atanh}(c\,x))}{(d+c\,d\,x)^2}\,dx$$

input
```
int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

output
```
int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx = \frac{2\left(\int \frac{atanh(cx)x^3}{c^2x^2+2cx+1}dx\right)b\,c^5x+2\left(\int \frac{atanh(cx)x^3}{c^2x^2+2cx+1}dx\right)b\,c^4+6\log(cx+1)\,acx+6\log(cx+1)\,a+a\,c^3x^3-3a\,c^2x}{2c^4d^2\,(cx+1)}$$

input
```
int(x^3*(a+b*atanh(c*x))/(c*d*x+d)^2,x)
```

output
```
(2*int((atanh(c*x)*x**3)/(c**2*x**2 + 2*c*x + 1),x)*b*c**5*x + 2*int((atan
h(c*x)*x**3)/(c**2*x**2 + 2*c*x + 1),x)*b*c**4 + 6*log(c*x + 1)*a*c*x + 6*
log(c*x + 1)*a + a*c**3*x**3 - 3*a*c**2*x**2 - 6*a*c*x)/(2*c**4*d**2*(c*x
+ 1))
```

3.52 $\int \frac{x^2(a+b\,\mathbf{arctanh}(cx))}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(d+cdx)^2}\,dx = \frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{b\,\mathrm{arctanh}(cx)}{2c^3d^2}+\frac{bx\,\mathrm{arctanh}(cx)}{c^2d^2}-\frac{a+b\,\mathrm{arctanh}(cx)}{c^3d^2(1+cx)}+\frac{2(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\log(1-c^2x^2)}{2c^3d^2}-\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^3d^2}$$

output

```
a*x/c^2/d^2-1/2*b/c^3/d^2/(c*x+1)+1/2*b*arctanh(c*x)/c^3/d^2+b*x*arctanh(c
*x)/c^2/d^2-(a+b*arctanh(c*x))/c^3/d^2/(c*x+1)+2*(a+b*arctanh(c*x))*ln(2/(
c*x+1))/c^3/d^2+1/2*b*ln(-c^2*x^2+1)/c^3/d^2-b*polylog(2,1-2/(c*x+1))/c^3/
d^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx = \frac{4acx-\frac{4a}{1+cx}-8a\log(1+cx)+b\left(-\cosh(2\text{arctanh}(cx))+2\log\left(1-c^2x^2\right)-4\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)\ldots\right.}{\ldots}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

output
```
(4*a*c*x - (4*a)/(1 + c*x) - 8*a*Log[1 + c*x] + b*(-Cosh[2*ArcTanh[c*x]] +
 2*Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[
c*x]] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*Arc
Tanh[c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(-\frac{2(a+b\text{arctanh}(cx))}{c^2d^2(cx+1)}+\frac{a+b\text{arctanh}(cx)}{c^2d^2}+\frac{a+b\text{arctanh}(cx)}{c^2d^2(cx+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\text{arctanh}(cx)}{c^3d^2(cx+1)}+\frac{2\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^3d^2}+\frac{ax}{c^2d^2}+\frac{b\text{arctanh}(cx)}{2c^3d^2}+\frac{bx\text{arctanh}(cx)}{c^2d^2}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b}{2c^3d^2(cx+1)}+\frac{b\log\left(1-c^2x^2\right)}{2c^3d^2}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

output
```
(a*x)/(c^2*d^2) - b/(2*c^3*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c^3*d^2) +
 (b*x*ArcTanh[c*x])/(c^2*d^2) - (a + b*ArcTanh[c*x])/(c^3*d^2*(1 + c*x)) +
 (2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^3*d^2) + (b*Log[1 - c^2*x^2]
)/(2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{a\left(cx-\frac{1}{cx+1}-2\ln(cx+1)\right)}{d^2}+\frac{b\left(\operatorname{arctanh}(cx)cx-\frac{\operatorname{arctanh}(cx)}{cx+1}-2\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{2}-\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{c}{}\right.}{d^2}}{c^3}$
default	$\frac{\frac{a\left(cx-\frac{1}{cx+1}-2\ln(cx+1)\right)}{d^2}+\frac{b\left(\operatorname{arctanh}(cx)cx-\frac{\operatorname{arctanh}(cx)}{cx+1}-2\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{2}-\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{c}{}\right.}{d^2}}{c^3}$
parts	$\frac{a\left(\frac{x}{c^2}-\frac{1}{c^3(cx+1)}-\frac{2\ln(cx+1)}{c^3}\right)}{d^2}+\frac{b\left(\operatorname{arctanh}(cx)cx-\frac{\operatorname{arctanh}(cx)}{cx+1}-2\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{2}-(\ln(cx+1)-\ln\right.}{d^2c^3}$
risch	$-\frac{b\ln(cx+1)^2}{2c^3d^2}+\left(\frac{bx}{2c^2d^2}-\frac{b}{2c^3d^2(cx+1)}\right)\ln\left(cx+1\right)-\frac{b}{2c^3d^2(cx+1)}+\frac{b\ln(cx+1)}{2c^3d^2}+\frac{ax}{c^2d^2}-\frac{a}{d^2c^3}+$

input
```
int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d^2*(c*x-1/(c*x+1)-2*ln(c*x+1))+b/d^2*(arctanh(c*x)*c*x-1/(c*x+1)
*arctanh(c*x)-2*arctanh(c*x)*ln(c*x+1)+1/2*ln(c*x+1)^2-(ln(c*x+1)-ln(1/2*c
*x+1/2))*ln(-1/2*c*x+1/2)+dilog(1/2*c*x+1/2)-1/2/(c*x+1)+3/4*ln(c*x+1)+1/4
*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x^2}{(cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arctanh(c*x) + a*x^2)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\frac{\int \frac{ax^2}{c^2x^2+2cx+1}\,dx+\int \frac{bx^2\,\text{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

output
```
(Integral(a*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**2*atanh(c*x)/
(c**2*x**2 + 2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(d + cdx)^2} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^2}{(cdx + d)^2} dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-1/8*(c^3*(2/(c^7*d^2*x + c^6*d^2) - 4*x/(c^5*d^2) + 5*log(c*x + 1)/(c^6*d
^2) - log(c*x - 1)/(c^6*d^2)) - 4*c^3*integrate(x^3*log(c*x + 1)/(c^5*d^2*
x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) - 2*c^2*(2/(c^6*d^2*x + c^5*d
^2) + 3*log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 12*c^2*integrat
e(x^2*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) +
 16*c*integrate(x*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^
2*d^2), x) + 4*(c^2*x^2 + c*x - 2*(c*x + 1)*log(c*x + 1) - 1)*log(-c*x + 1
)/(c^4*d^2*x + c^3*d^2) + 2/(c^4*d^2*x + c^3*d^2) - log(c*x + 1)/(c^3*d^2)
 + log(c*x - 1)/(c^3*d^2) + 8*integrate(log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^
2*x^2 - c^3*d^2*x - c^2*d^2), x))*b - a*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*
d^2) + 2*log(c*x + 1)/(c^3*d^2))
```

Giac [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(d + cdx)^2} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^2}{(cdx + d)^2} dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{x^2\,\left(a+b\,\text{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^2}\,dx$$

input
```
int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

output
```
int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx$$
$$=\frac{\left(\int \frac{atanh(cx)x^2}{c^2x^2+2cx+1}dx\right)b\,c^4x+\left(\int \frac{atanh(cx)x^2}{c^2x^2+2cx+1}dx\right)b\,c^3-2\log(cx+1)\,acx-2\log(cx+1)\,a+a\,c^2x^2+2acx}{c^3d^2\,(cx+1)}$$

input
```
int(x^2*(a+b*atanh(c*x))/(c*d*x+d)^2,x)
```

output
```
(int((atanh(c*x)*x**2)/(c**2*x**2 + 2*c*x + 1),x)*b*c**4*x + int((atanh(c*
x)*x**2)/(c**2*x**2 + 2*c*x + 1),x)*b*c**3 - 2*log(c*x + 1)*a*c*x - 2*log(
c*x + 1)*a + a*c**2*x**2 + 2*a*c*x)/(c**3*d**2*(c*x + 1))
```

3.53 $\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx = \frac{b}{2c^2d^2(1+cx)} - \frac{b\text{arctanh}(cx)}{2c^2d^2} + \frac{a+b\text{arctanh}(cx)}{c^2d^2(1+cx)} - \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^2d^2} + \frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2d^2}$$

output
```
1/2*b/c^2/d^2/(c*x+1)-1/2*b*arctanh(c*x)/c^2/d^2+(a+b*arctanh(c*x))/c^2/d^
2/(c*x+1)-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^2/d^2+1/2*b*polylog(2,1-2/(c*
x+1))/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx$$
$$= \frac{\frac{4a}{1+cx} + 4a\log(1+cx) + b\Big(\cosh(2\text{arctanh}(cx)) + 2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right) + 2\text{arctanh}(cx)\left(\cosh(2\text{ar}\right.}{4c^2d^2}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

output
```
((4*a)/(1 + c*x) + 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]] + 2*PolyLog[
2, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*Log[1
+ E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4
*c^2*d^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(\frac{a+b\operatorname{arctanh}(cx)}{cd^2(cx+1)}-\frac{a+b\operatorname{arctanh}(cx)}{cd^2(cx+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a+b\operatorname{arctanh}(cx)}{c^2d^2(cx+1)}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{c^2d^2}-\frac{b\operatorname{arctanh}(cx)}{2c^2d^2}+\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b}{2c^2d^2(cx+1)}$$

input
```
Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

output
```
b/(2*c^2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*c^2*d^2) + (a + b*ArcTanh[c*
x])/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^2*d^2
) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{cx+1}+\ln(cx+1)\right)}{d^2}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{cx+1}+\operatorname{arctanh}(cx)\ln(cx+1)+\frac{1}{2cx+2}-\frac{\ln(cx+1)}{4}+\frac{\ln(cx-1)}{4}-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right.}{2}\right.}{d^2}}{c^2}$
default	$\frac{\frac{a\left(\frac{1}{cx+1}+\ln(cx+1)\right)}{d^2}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{cx+1}+\operatorname{arctanh}(cx)\ln(cx+1)+\frac{1}{2cx+2}-\frac{\ln(cx+1)}{4}+\frac{\ln(cx-1)}{4}-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\right.}{2}\right.}{d^2}}{c^2}$
parts	$\frac{a\left(\frac{1}{c^2(cx+1)}+\frac{\ln(cx+1)}{c^2}\right)}{d^2}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{cx+1}+\operatorname{arctanh}(cx)\ln(cx+1)+\frac{1}{2cx+2}-\frac{\ln(cx+1)}{4}+\frac{\ln(cx-1)}{4}-\frac{\ln(cx+1)^2}{4}+\frac{(\ln(cx+1}{}\right.}{d^2c^2}$
risch	$\frac{b\ln(cx+1)^2}{4c^2d^2}+\frac{b\ln(cx+1)}{2c^2d^2(cx+1)}+\frac{b}{2c^2d^2(cx+1)}-\frac{a}{d^2c^2(-cx-1)}+\frac{a\ln(-cx-1)}{d^2c^2}-\frac{b\ln(-cx-1)}{4d^2c^2}-\frac{b\ln(-cx+1)}{4d^2c(-cx-1}$

input
```
int(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a/d^2*(1/(c*x+1)+ln(c*x+1))+b/d^2*(1/(c*x+1)*arctanh(c*x)+arctanh(c
*x)*ln(c*x+1)+1/2/(c*x+1)-1/4*ln(c*x+1)+1/4*ln(c*x-1)-1/4*ln(c*x+1)^2+1/2*
(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2)))
```

Fricas [F]

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x}{(cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x*arctanh(c*x) + a*x)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\frac{\int \frac{ax}{c^2x^2+2cx+1}\,dx+\int \frac{bx\,\text{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

input
```
integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

output
```
(Integral(a*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)x}{(cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*(8*c^2*integrate(x^2*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^2*d^2
*x - c*d^2), x) - c*(2/(c^4*d^2*x + c^3*d^2) + log(c*x + 1)/(c^3*d^2) - lo
g(c*x - 1)/(c^3*d^2)) + 4*c*integrate(x*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^
2*x^2 - c^2*d^2*x - c*d^2), x) - 4*((c*x + 1)*log(c*x + 1) + 1)*log(-c*x +
 1)/(c^3*d^2*x + c^2*d^2) + 2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^
2) + log(c*x - 1)/(c^2*d^2) + 4*integrate(log(c*x + 1)/(c^4*d^2*x^3 + c^3*
d^2*x^2 - c^2*d^2*x - c*d^2), x))*b + a*(1/(c^3*d^2*x + c^2*d^2) + log(c*x
 + 1)/(c^2*d^2))
```

Giac [F]

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)x}{(cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx = \int \frac{x\,(a+b\,\text{atanh}(c\,x))}{(d+c\,d\,x)^2}\,dx$$

input
```
int((x*(a + b*atanh(c*x)))/(d + c*d*x)^2,x)
```

output
```
int((x*(a + b*atanh(c*x)))/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^2}\,dx$$
$$=\frac{\left(\int \frac{atanh(cx)x}{c^2x^2+2cx+1}dx\right) b\,c^3x+\left(\int \frac{atanh(cx)x}{c^2x^2+2cx+1}dx\right) b\,c^2+\log(cx+1)\,acx+\log(cx+1)\,a-acx}{c^2d^2\,(cx+1)}$$

input
```
int(x*(a+b*atanh(c*x))/(c*d*x+d)^2,x)
```

output
```
(int((atanh(c*x)*x)/(c**2*x**2 + 2*c*x + 1),x)*b*c**3*x + int((atanh(c*x)*
x)/(c**2*x**2 + 2*c*x + 1),x)*b*c**2 + log(c*x + 1)*a*c*x + log(c*x + 1)*a
 - a*c*x)/(c**2*d**2*(c*x + 1))
```

3.54 $\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^2} dx = -\frac{b}{2cd^2(1+cx)} + \frac{b\text{arctanh}(cx)}{2cd^2} - \frac{a+b\text{arctanh}(cx)}{cd^2(1+cx)}$$

output
```
-1/2*b/c/d^2/(c*x+1)+1/2*b*arctanh(c*x)/c/d^2-(a+b*arctanh(c*x))/c/d^2/(c*
x+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^2} dx = \frac{-4a-2b-4b\text{arctanh}(cx)-(b+bcx)\log(1-cx)+b\log(1+cx)+bcx\log(1+cx)}{4cd^2(1+cx)}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

output
```
(-4*a - 2*b - 4*b*ArcTanh[c*x] - (b + b*c*x)*Log[1 - c*x] + b*Log[1 + c*x]
 + b*c*x*Log[1 + c*x])/(4*c*d^2*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6478, 27, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{b\int \frac{1}{d(cx+1)(1-c^2x^2)}dx}{d}-\frac{a+b\text{arctanh}(cx)}{cd^2(cx+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{1}{(cx+1)(1-c^2x^2)}dx}{d^2}-\frac{a+b\text{arctanh}(cx)}{cd^2(cx+1)}$$

$$\Big\downarrow \text{456}$$

$$\frac{b\int \frac{1}{(1-cx)(cx+1)^2}dx}{d^2}-\frac{a+b\text{arctanh}(cx)}{cd^2(cx+1)}$$

$$\Big\downarrow \text{54}$$

$$\frac{b\int \left(\frac{1}{2(cx+1)^2}-\frac{1}{2(c^2x^2-1)}\right)dx}{d^2}-\frac{a+b\text{arctanh}(cx)}{cd^2(cx+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{\text{arctanh}(cx)}{2c}-\frac{1}{2c(cx+1)}\right)}{d^2}-\frac{a+b\text{arctanh}(cx)}{cd^2(cx+1)}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

output
```
-((a + b*ArcTanh[c*x])/(c*d^2*(1 + c*x))) + (b*(-1/2*1/(c*(1 + c*x)) + Arc
Tanh[c*x]/(2*c)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result	s
parallelrisch	$\frac{\operatorname{arctanh}(cx)bcx+2acx+bcx-b\operatorname{arctanh}(cx)}{2d^2(cx+1)c}$	4
derivativedivides	$\frac{-\frac{a}{d^2(cx+1)}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{\ln(cx-1)}{4}-\frac{1}{2(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}}{c}$	6
default	$\frac{-\frac{a}{d^2(cx+1)}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{\ln(cx-1)}{4}-\frac{1}{2(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}}{c}$	6
parts	$-\frac{a}{d^2c(cx+1)}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{\ln(cx-1)}{4}-\frac{1}{2(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2c}$	6
risch	$-\frac{b\ln(cx+1)}{2c\,d^2(cx+1)}+\frac{\ln(-cx-1)bcx-\ln(cx-1)bcx+b\ln(-cx-1)-b\ln(cx-1)+2b\ln(-cx+1)-4a-2b}{4d^2(cx+1)c}$	9
orering	$-\frac{(cx+1)(2c^2x^2-3cx+1)(a+b\operatorname{arctanh}(cx))}{2c(cdx+d)^2}-\frac{x(cx-1)(cx+1)^2\left(\frac{bc}{(-c^2x^2+1)(cdx+d)^2}-\frac{2(a+b\operatorname{arctanh}(cx))cd}{(cdx+d)^3}\right)}{2c}$	1

input
```
int((a+b*arctanh(c*x))/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctanh(c*x)*b*c*x+2*a*c*x+b*c*x-b*arctanh(c*x))/d^2/(c*x+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{a+b\operatorname{arctanh}(cx)}{(d+cdx)^2}\,dx=\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)-4\,a-2\,b}{4\,(c^2d^2x+cd^2)}$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1)) - 4*a - 2*b)/(c^2*d^2*x + c*d^2
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int \frac{a + b\text{arctanh}(cx)}{(d + cdx)^2}\,dx = \begin{cases} -\frac{2a}{2c^2d^2x+2cd^2} + \frac{bcx\,\text{atanh}\,(cx)}{2c^2d^2x+2cd^2} - \frac{b\,\text{atanh}\,(cx)}{2c^2d^2x+2cd^2} - \frac{b}{2c^2d^2x+2cd^2} & \text{for } c \neq 0 \\ \frac{ax}{d^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

output
```
Piecewise((-2*a/(2*c**2*d**2*x + 2*c*d**2) + b*c*x*atanh(c*x)/(2*c**2*d**2
*x + 2*c*d**2) - b*atanh(c*x)/(2*c**2*d**2*x + 2*c*d**2) - b/(2*c**2*d**2*
x + 2*c*d**2), Ne(c, 0)), (a*x/d**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int \frac{a + b\text{arctanh}(cx)}{(d + cdx)^2}\,dx = -\frac{1}{4}\left(c\left(\frac{2}{c^3d^2x + c^2d^2} - \frac{\log(cx + 1)}{c^2d^2} + \frac{\log(cx - 1)}{c^2d^2}\right) + \frac{4\,\text{artanh}\,(cx)}{c^2d^2x + cd^2}\right)b - \frac{a}{c^2d^2x + cd^2}$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-1/4*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(
c^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*b - a/(c^2*d^2*x + c*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^2}\,dx=\frac{1}{4}\,c\left(\frac{(cx-1)b\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2d^2}+\frac{(cx-1)(2\,a+b)}{(cx+1)c^2d^2}\right)$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
1/4*c*((c*x - 1)*b*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2) + (c*x -
1)*(2*a + b)/((c*x + 1)*c^2*d^2))
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^2}\,dx=-\frac{b\,\text{atanh}(c\,x)-c\,(2\,a\,x+b\,x+b\,x\,\text{atanh}(c\,x))}{2\,x\,c^2\,d^2+2\,c\,d^2}$$

input
```
int((a + b*atanh(c*x))/(d + c*d*x)^2,x)
```

output
```
-(b*atanh(c*x) - c*(2*a*x + b*x + b*x*atanh(c*x)))/(2*c*d^2 + 2*c^2*d^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^2}\,dx$$
$$=\frac{4atanh(cx)\,bcx+\log(cx-1)\,bcx+\log(cx-1)\,b-\log(cx+1)\,bcx-\log(cx+1)\,b+4acx+2bcx}{4c\,d^2\,(cx+1)}$$

input
```
int((a+b*atanh(c*x))/(c*d*x+d)^2,x)
```

output
```
(4*atanh(c*x)*b*c*x + log(c*x - 1)*b*c*x + log(c*x - 1)*b - log(c*x + 1)*b
*c*x - log(c*x + 1)*b + 4*a*c*x + 2*b*c*x)/(4*c*d**2*(c*x + 1))
```

3.55 $\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^2}\,dx = \frac{b}{2d^2(1+cx)}-\frac{b\text{arctanh}(cx)}{2d^2}+\frac{a+b\text{arctanh}(cx)}{d^2(1+cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^2}-\frac{b\,\text{PolyLog}(2,-cx)}{2d^2}+\frac{b\,\text{PolyLog}(2,cx)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d^2}$$

output
```
1/2*b/d^2/(c*x+1)-1/2*b*arctanh(c*x)/d^2+(a+b*arctanh(c*x))/d^2/(c*x+1)+a*
ln(x)/d^2+(a+b*arctanh(c*x))*ln(2/(c*x+1))/d^2-1/2*b*polylog(2,-c*x)/d^2+1
/2*b*polylog(2,c*x)/d^2-1/2*b*polylog(2,1-2/(c*x+1))/d^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^2}\,dx = \frac{\frac{4a}{1+cx}+4a\log(x)-4a\log(1+cx)+b\big(\cosh(2\text{arctanh}(cx))-2\,\text{PolyLog}\left(2,e^{-2\text{arctanh}(cx)}\right)+2\text{arctanh}(cx)}{4d^2}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

output
```
((4*a)/(1 + c*x) + 4*a*Log[x] - 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]]
 - 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]
] + 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTan
h[c*x]]))/(4*d^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x(cdx+d)^2}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{a+b\text{arctanh}(cx)}{d^2x}-\frac{c(a+b\text{arctanh}(cx))}{d^2(cx+1)}-\frac{c(a+b\text{arctanh}(cx))}{d^2(cx+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a+b\text{arctanh}(cx)}{d^2(cx+1)}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}+\frac{a\log(x)}{d^2}-\frac{b\text{arctanh}(cx)}{2d^2}-\frac{b\text{PolyLog}(2,-cx)}{2d^2}+\frac{b\text{PolyLog}(2,cx)}{2d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{b}{2d^2(cx+1)}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

output
```
b/(2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*d^2) + (a + b*ArcTanh[c*x])/(d^2
*(1 + c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2
 - (b*PolyLog[2, -(c*x)])/(2*d^2) + (b*PolyLog[2, c*x])/(2*d^2) - (b*PolyL
og[2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.28

method	result
parts	$\frac{a\left(\frac{1}{cx+1}-\ln(cx+1)+\ln(x)\right)}{d^2}+\frac{b\big(\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{cx+1}-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-\frac{\big(\ln(cx+1)-\ln(}{}}{}$
derivativedivides	$\frac{a\left(\ln(cx)+\frac{1}{cx+1}-\ln(cx+1)\right)}{d^2}+\frac{b\big(\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{cx+1}-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-\frac{\big(\ln(cx+1)-\ln}{}}{}$
default	$\frac{a\left(\ln(cx)+\frac{1}{cx+1}-\ln(cx+1)\right)}{d^2}+\frac{b\big(\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{cx+1}-\operatorname{arctanh}(cx)\ln(cx+1)+\frac{\ln(cx+1)^2}{4}-\frac{\big(\ln(cx+1)-\ln}{}}{}$
risch	$\frac{a\ln(-cx)}{d^2}-\frac{a}{d^2(-cx-1)}-\frac{a\ln(-cx-1)}{d^2}-\frac{b\ln(-cx-1)}{4d^2}-\frac{b\ln(-cx+1)cx}{4d^2(-cx-1)}+\frac{b\ln(-cx+1)}{4d^2(-cx-1)}+\frac{b\operatorname{dilog}(-cx+1)}{2d^2}$

input
```
int((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*(1/(c*x+1)-ln(c*x+1)+ln(x))+b/d^2*(arctanh(c*x)*ln(c*x)+1/(c*x+1)*ar
ctanh(c*x)-arctanh(c*x)*ln(c*x+1)+1/4*ln(c*x+1)^2-1/2*(ln(c*x+1)-ln(1/2*c*
x+1/2))*ln(-1/2*c*x+1/2)+1/2*dilog(1/2*c*x+1/2)+1/4*ln(c*x-1)+1/2/(c*x+1)-
1/4*ln(c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1))
```

Fricas [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^2} dx = \int \frac{b \text{artanh}(cx) + a}{(cdx+d)^2 x} dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^2} dx = \frac{\int \frac{a}{c^2x^3+2cx^2+x} dx + \int \frac{b \text{atanh}(cx)}{c^2x^3+2cx^2+x} dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))/x/(c*d*x+d)**2,x)
```

output
```
(Integral(a/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b*atanh(c*x)/(c**2*x
**3 + 2*c*x**2 + x), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^2} dx = \int \frac{b \text{artanh}(cx) + a}{(cdx+d)^2 x} dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
a*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/2*b*integrate((l
og(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)^2}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)^2}\,dx=\int \frac{a+b\,\mathrm{atanh}(c\,x)}{x\,(d+c\,d\,x)^2}\,dx$$

input
```
int((a + b*atanh(c*x))/(x*(d + c*d*x)^2),x)
```

output
```
int((a + b*atanh(c*x))/(x*(d + c*d*x)^2), x)
```

Reduce [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)^2}\,dx$$
$$=\frac{-2atanh(cx)^2bcx-2atanh(cx)^2b-4atanh(cx)\,bcx-8\left(\int\frac{atanh(cx)}{c^3x^4+c^2x^3-c\,x^2-x}dx\right)bcx-8\left(\int\frac{atanh(cx)}{c^3x^4+c^2x^3-c\,x^2-}\right.}{}$$

input
```
int((a+b*atanh(c*x))/x/(c*d*x+d)^2,x)
```

output
```
( - 2*atanh(c*x)**2*b*c*x - 2*atanh(c*x)**2*b - 4*atanh(c*x)*b*c*x - 8*int
(atanh(c*x)/(c**3*x**4 + c**2*x**3 - c*x**2 - x),x)*b*c*x - 8*int(atanh(c*
x)/(c**3*x**4 + c**2*x**3 - c*x**2 - x),x)*b - log(c*x - 1)*b*c*x - log(c*
x - 1)*b - 8*log(c*x + 1)*a*c*x - 8*log(c*x + 1)*a + log(c*x + 1)*b*c*x +
log(c*x + 1)*b + 8*log(x)*a*c*x + 8*log(x)*a - 8*a*c*x - 2*b*c*x)/(8*d**2*
(c*x + 1))
```

3.56 $\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2(d+cdx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2(d+cdx)^2}\,dx = -\frac{bc}{2d^2(1+cx)}+\frac{bc\,\mathrm{arctanh}(cx)}{2d^2}-\frac{a+b\,\mathrm{arctanh}(cx)}{d^2x} - \frac{c(a+b\,\mathrm{arctanh}(cx))}{d^2(1+cx)}-\frac{2ac\log(x)}{d^2} + \frac{bc\log(x)}{d^2}-\frac{2c(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{bc\log\left(1-c^2x^2\right)}{2d^2}+\frac{bc\,\mathrm{PolyLog}(2,-cx)}{d^2} - \frac{bc\,\mathrm{PolyLog}(2,cx)}{d^2}+\frac{bc\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^2}$$

output

```
-1/2*b*c/d^2/(c*x+1)+1/2*b*c*arctanh(c*x)/d^2-(a+b*arctanh(c*x))/d^2/x-c*(
a+b*arctanh(c*x))/d^2/(c*x+1)-2*a*c*ln(x)/d^2+b*c*ln(x)/d^2-2*c*(a+b*arcta
nh(c*x))*ln(2/(c*x+1))/d^2-1/2*b*c*ln(-c^2*x^2+1)/d^2+b*c*polylog(2,-c*x)/
d^2-b*c*polylog(2,c*x)/d^2+b*c*polylog(2,1-2/(c*x+1))/d^2
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^2}\,dx$$
$$= \frac{-\frac{4a}{x}-\frac{4ac}{1+cx}-8ac\log(x)+8ac\log(1+cx)+bc\Big(-\cosh(2\text{arctanh}(cx))+4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+4\,\text{PolyLog}\,(}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

output
```
((-4*a)/x - (4*a*c)/(1 + c*x) - 8*a*c*Log[x] + 8*a*c*Log[1 + c*x] + b*c*(-
Cosh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2
*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2
*ArcTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]]))
)/(4*d^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(cdx+d)^2}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(\frac{2c^2(a+b\text{arctanh}(cx))}{d^2(cx+1)}+\frac{c^2(a+b\text{arctanh}(cx))}{d^2(cx+1)^2}+\frac{a+b\text{arctanh}(cx)}{d^2x^2}-\frac{2c(a+b\text{arctanh}(cx))}{d^2x}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{c(a+b\text{arctanh}(cx))}{d^2(cx+1)}-\frac{a+b\text{arctanh}(cx)}{d^2x}-\frac{2c\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{2ac\log(x)}{d^2}+\frac{b\text{carctanh}(cx)}{2d^2}-\frac{bc\log\left(1-c^2x^2\right)}{2d^2}+\frac{bc\,\text{PolyLog}(2,-cx)}{d^2}-\frac{bc\,\text{PolyLog}(2,cx)}{d^2}+\frac{bc\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^2}-\frac{bc}{2d^2(cx+1)}+\frac{bc\log(x)}{d^2}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

output
```
-1/2*(b*c)/(d^2*(1 + c*x)) + (b*c*ArcTanh[c*x])/(2*d^2) - (a + b*ArcTanh[c
*x])/(d^2*x) - (c*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*a*c*Log[x])/d
^2 + (b*c*Log[x])/d^2 - (2*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 -
(b*c*Log[1 - c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, -(c*x)])/d^2 - (b*c*PolyL
og[2, c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + c*x)])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

method	result
parts	$\frac{a\left(-\frac{c}{cx+1}+2c\ln(cx+1)-\frac{1}{x}-2c\ln(x)\right)}{d^2}+\frac{bc\left(-\frac{\operatorname{arctanh}(cx)}{cx}-2\operatorname{arctanh}(cx)\ln(cx)-\frac{\operatorname{arctanh}(cx)}{cx+1}+2\operatorname{arctanh}(cx)\ln(cx-\right.}{}$
derivativedivides	$c\left(\frac{a\left(-\frac{1}{cx}-2\ln(cx)-\frac{1}{cx+1}+2\ln(cx+1)\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx}-2\operatorname{arctanh}(cx)\ln(cx)-\frac{\operatorname{arctanh}(cx)}{cx+1}+2\operatorname{arctanh}(cx)\ln(\right.}{}\right.$
default	$c\left(\frac{a\left(-\frac{1}{cx}-2\ln(cx)-\frac{1}{cx+1}+2\ln(cx+1)\right)}{d^2}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx}-2\operatorname{arctanh}(cx)\ln(cx)-\frac{\operatorname{arctanh}(cx)}{cx+1}+2\operatorname{arctanh}(cx)\ln(\right.}{}\right.$
risch	$-\frac{a}{d^2x}-\frac{2ca\ln(-cx)}{d^2}+\frac{ca}{d^2(-cx-1)}+\frac{2ca\ln(-cx-1)}{d^2}+\frac{bc\ln(-cx-1)}{4d^2}+\frac{c^2b\ln(-cx+1)x}{4d^2(-cx-1)}-\frac{cb\ln(-cx+1)}{4d^2(-cx-1)}$

input
```
int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*(-c/(c*x+1)+2*c*ln(c*x+1)-1/x-2*c*ln(x))+b/d^2*c*(-arctanh(c*x)/c/x-
2*arctanh(c*x)*ln(c*x)-1/(c*x+1)*arctanh(c*x)+2*arctanh(c*x)*ln(c*x+1)-3/4
*ln(c*x-1)+ln(c*x)-1/2/(c*x+1)-1/4*ln(c*x+1)+dilog(c*x)+dilog(c*x+1)+ln(c*
x)*ln(c*x+1)-1/2*ln(c*x+1)^2+(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-
dilog(1/2*c*x+1/2))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)^2}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c^2*d^2*x^4 + 2*c*d^2*x^3 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + cdx)^2} dx = \frac{\int \frac{a}{c^2x^4+2cx^3+x^2} dx + \int \frac{b \operatorname{atanh}(cx)}{c^2x^4+2cx^3+x^2} dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c**
2*x**4 + 2*c*x**3 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + cdx)^2} dx = \int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)^2 x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-a*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d^
2) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^4 + 2*c*d^2
*x^3 + d^2*x^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + cdx)^2} dx = \int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)^2 x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^2}\,dx = \int \frac{a+b\,\text{atanh}(c\,x)}{x^2\,(d+c\,d\,x)^2}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^2),x)
```

output
```
int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^2}\,dx$$
$$= \frac{-atanh(cx)^2\,b\,c^2x^2 - atanh(cx)^2\,bcx - 2atanh(cx)\,b\,c^2x^2 + 2atanh(cx)\,b - 4\left(\int \frac{atanh(cx)}{c^3x^5+c^2x^4-c\,x^3-x^2}dx\right) bc\,x}{}$$

input
```
int((a+b*atanh(c*x))/x^2/(c*d*x+d)^2,x)
```

output
```
( - atanh(c*x)**2*b*c**2*x**2 - atanh(c*x)**2*b*c*x - 2*atanh(c*x)*b*c**2*
x**2 + 2*atanh(c*x)*b - 4*int(atanh(c*x)/(c**3*x**5 + c**2*x**4 - c*x**3 -
 x**2),x)*b*c*x**2 - 4*int(atanh(c*x)/(c**3*x**5 + c**2*x**4 - c*x**3 - x*
*2),x)*b*x + 4*log(c*x + 1)*a*c**2*x**2 + 4*log(c*x + 1)*a*c*x + 2*log(c*x
 + 1)*b*c**2*x**2 + 2*log(c*x + 1)*b*c*x - 4*log(x)*a*c**2*x**2 - 4*log(x)
*a*c*x - 2*log(x)*b*c**2*x**2 - 2*log(x)*b*c*x + 4*a*c**2*x**2 - 2*a)/(2*d
**2*x*(c*x + 1))
```

3.57 $\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 212

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^2}\,dx = -\frac{bc}{2d^2x}+\frac{bc^2}{2d^2(1+cx)}-\frac{a+b\text{arctanh}(cx)}{2d^2x^2}+\frac{2c(a+b\text{arctanh}(cx))}{d^2x}+\frac{c^2(a+b\text{arctanh}(cx))}{d^2(1+cx)}+\frac{3ac^2\log(x)}{d^2}-\frac{2bc^2\log(x)}{d^2}+\frac{3c^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^2}+\frac{bc^2\log(1-c^2x^2)}{d^2}-\frac{3bc^2\,\text{PolyLog}(2,-cx)}{2d^2}+\frac{3bc^2\,\text{PolyLog}(2,cx)}{2d^2}-\frac{3bc^2\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d^2}$$

output
```
-1/2*b*c/d^2/x+1/2*b*c^2/d^2/(c*x+1)-1/2*(a+b*arctanh(c*x))/d^2/x^2+2*c*(a
+b*arctanh(c*x))/d^2/x+c^2*(a+b*arctanh(c*x))/d^2/(c*x+1)+3*a*c^2*ln(x)/d^
2-2*b*c^2*ln(x)/d^2+3*c^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/d^2+b*c^2*ln(-c
^2*x^2+1)/d^2-3/2*b*c^2*polylog(2,-c*x)/d^2+3/2*b*c^2*polylog(2,c*x)/d^2-3
/2*b*c^2*polylog(2,1-2/(c*x+1))/d^2
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)^2}\,dx$$

$$= \frac{-\frac{2a}{x^2} + \frac{8ac}{x} - \frac{2bc}{x} + \frac{4ac^2}{1+cx} + bc^2\cosh(2\text{arctanh}(cx)) + 12ac^2\log(x) - 12ac^2\log(1+cx) - 8bc^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

output
```
((-2*a)/x^2 + (8*a*c)/x - (2*b*c)/x + (4*a*c^2)/(1 + c*x) + b*c^2*Cosh[2*A
rcTanh[c*x]] + 12*a*c^2*Log[x] - 12*a*c^2*Log[1 + c*x] - 8*b*c^2*Log[(c*x)
/Sqrt[1 - c^2*x^2]] - 6*b*c^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - b*c^2*Sinh
[2*ArcTanh[c*x]] + 2*b*ArcTanh[c*x]*(c^2 - x^(-2) + (4*c)/x + c^2*Cosh[2*A
rcTanh[c*x]] + 6*c^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*Sinh[2*ArcTanh[c*x
]]))/(4*d^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(cdx + d)^2}\,dx$$

$$\Big\downarrow \; 6502$$

$$\int \left(-\frac{3c^3(a + b\text{arctanh}(cx))}{d^2(cx+1)} - \frac{c^3(a + b\text{arctanh}(cx))}{d^2(cx+1)^2} + \frac{3c^2(a + b\text{arctanh}(cx))}{d^2x} + \frac{a + b\text{arctanh}(cx)}{d^2x^3} - \frac{2c(a + ba}{d}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2(a+b\operatorname{arctanh}(cx))}{d^2(cx+1)}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{d^2}-\frac{a+b\operatorname{arctanh}(cx)}{2d^2x^2}+\frac{2c(a+b\operatorname{arctanh}(cx))}{d^2x}+\frac{3ac^2\log(x)}{d^2}-\frac{3bc^2\operatorname{PolyLog}(2,-cx)}{2d^2}+\frac{3bc^2\operatorname{PolyLog}(2,cx)}{2d^2}-\frac{3bc^2\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{bc^2\log(1-c^2x^2)}{d^2}+\frac{bc^2}{2d^2(cx+1)}-\frac{2bc^2\log(x)}{d^2}-\frac{bc}{2d^2x}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

output
```
-1/2*(b*c)/(d^2*x) + (b*c^2)/(2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])/(2*d
^2*x^2) + (2*c*(a + b*ArcTanh[c*x]))/(d^2*x) + (c^2*(a + b*ArcTanh[c*x]))/
(d^2*(1 + c*x)) + (3*a*c^2*Log[x])/d^2 - (2*b*c^2*Log[x])/d^2 + (3*c^2*(a
+ b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (b*c^2*Log[1 - c^2*x^2])/d^2 - (
3*b*c^2*PolyLog[2, -(c*x)])/(2*d^2) + (3*b*c^2*PolyLog[2, c*x])/(2*d^2) -
(3*b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03

method	result
derivativedivides	$c^{2}\left(\frac{a\left(-\frac{1}{2c^{2}x^{2}}+\frac{2}{cx}+3\ln\left(cx\right)+\frac{1}{cx+1}-3\ln\left(cx+1\right)\right)}{d^{2}}+\frac{b\left(-\frac{\operatorname{arctanh}\left(cx\right)}{2c^{2}x^{2}}+\frac{2\operatorname{arctanh}\left(cx\right)}{cx}+3\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)+\frac{\operatorname{arctan}}{cx+}\right.}{}\right.$
default	$c^{2}\left(\frac{a\left(-\frac{1}{2c^{2}x^{2}}+\frac{2}{cx}+3\ln\left(cx\right)+\frac{1}{cx+1}-3\ln\left(cx+1\right)\right)}{d^{2}}+\frac{b\left(-\frac{\operatorname{arctanh}\left(cx\right)}{2c^{2}x^{2}}+\frac{2\operatorname{arctanh}\left(cx\right)}{cx}+3\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)+\frac{\operatorname{arctan}}{cx+}\right.}{}\right.$
parts	$\frac{a\left(\frac{c^{2}}{cx+1}-3c^{2}\ln\left(cx+1\right)-\frac{1}{2x^{2}}+\frac{2c}{x}+3c^{2}\ln\left(x\right)\right)}{d^{2}}+\frac{b\,c^{2}\left(-\frac{\operatorname{arctanh}\left(cx\right)}{2c^{2}x^{2}}+\frac{2\operatorname{arctanh}\left(cx\right)}{cx}+3\operatorname{arctanh}\left(cx\right)\ln\left(cx\right)+\frac{\operatorname{arctanh}\left(cx\right.}{cx+1}\right.}{}$
risch	$-\frac{c^{3}b\ln\left(-cx+1\right)x}{4d^{2}\left(-cx-1\right)}+\frac{5c^{2}b\ln\left(cx+1\right)}{4d^{2}}+\frac{c^{2}b\ln\left(-cx+1\right)}{4d^{2}\left(-cx-1\right)}+\frac{3c^{2}b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-cx+1\right)}{2d^{2}}-\frac{3c^{2}b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\right.}{2d^{2}}$

input
```
int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d^2*(-1/2/c^2/x^2+2/c/x+3*ln(c*x)+1/(c*x+1)-3*ln(c*x+1))+b/d^2*(-1/
2*arctanh(c*x)/c^2/x^2+2*arctanh(c*x)/c/x+3*arctanh(c*x)*ln(c*x)+1/(c*x+1)
*arctanh(c*x)-3*arctanh(c*x)*ln(c*x+1)-3/2*dilog(c*x)-3/2*dilog(c*x+1)-3/2
*ln(c*x)*ln(c*x+1)+3/4*ln(c*x+1)^2-3/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2
*c*x+1/2)+3/2*dilog(1/2*c*x+1/2)+ln(c*x-1)-1/2/c/x-2*ln(c*x)+1/2/(c*x+1)+l
n(c*x+1)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(d+cdx)^2}\,dx=\int \frac{b\operatorname{artanh}\left(cx\right)+a}{\left(cdx+d\right)^{2}x^{3}}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)^2} dx = \frac{\int \frac{a}{c^2x^5+2cx^4+x^3} dx + \int \frac{b\,\text{atanh}(cx)}{c^2x^5+2cx^4+x^3} dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**2,x)
```

output
```
(Integral(a/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c**
2*x**5 + 2*c*x**4 + x**3), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)^2} dx = \int \frac{b\,\text{artanh}(cx) + a}{(cdx + d)^2x^3} dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x - 1
)/(c*d^2*x^3 + d^2*x^2)) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/
(c^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + cdx)^2} dx = \int \frac{b\,\text{artanh}(cx) + a}{(cdx + d)^2x^3} dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^2}\,dx=\int \frac{a+b\,\text{atanh}(c\,x)}{x^3\,{(d+c\,d\,x)}^2}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^2),x)
```

output
```
int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^2), x)
```

Reduce [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^2}\,dx$$
$$=\frac{-6atanh(cx)^2\,b\,c^3x^3-6atanh(cx)^2\,b\,c^2x^2-12atanh(cx)\,b\,c^3x^3+12atanh(cx)\,bcx+4atanh(cx)\,b-24\big(}{}$$

input
```
int((a+b*atanh(c*x))/x^3/(c*d*x+d)^2,x)
```

output
```
( - 6*atanh(c*x)**2*b*c**3*x**3 - 6*atanh(c*x)**2*b*c**2*x**2 - 12*atanh(c
*x)*b*c**3*x**3 + 12*atanh(c*x)*b*c*x + 4*atanh(c*x)*b - 24*int(atanh(c*x)
/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*b*c*x**3 - 24*int(atanh(c*x)/(
c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*b*x**2 + log(c*x - 1)*b*c**3*x**
3 + log(c*x - 1)*b*c**2*x**2 - 48*log(c*x + 1)*a*c**3*x**3 - 48*log(c*x +
1)*a*c**2*x**2 + 7*log(c*x + 1)*b*c**3*x**3 + 7*log(c*x + 1)*b*c**2*x**2 +
 48*log(x)*a*c**3*x**3 + 48*log(x)*a*c**2*x**2 - 8*log(x)*b*c**3*x**3 - 8*
log(x)*b*c**2*x**2 - 48*a*c**3*x**3 + 24*a*c*x - 8*a - 6*b*c**3*x**3 + 4*b
*c*x)/(16*d**2*x**2*(c*x + 1))
```

3.58 $\int \frac{x^4(a+b\mathbf{arctanh}(cx))}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\begin{aligned}\int \frac{x^4(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = &-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b}{8c^5d^3(1+cx)^2}+\frac{15b}{8c^5d^3(1+cx)}\\&-\frac{19b\text{arctanh}(cx)}{8c^5d^3}-\frac{3bx\text{arctanh}(cx)}{c^4d^3}\\&+\frac{x^2(a+b\text{arctanh}(cx))}{2c^3d^3}-\frac{a+b\text{arctanh}(cx)}{2c^5d^3(1+cx)^2}\\&+\frac{4(a+b\text{arctanh}(cx))}{c^5d^3(1+cx)}-\frac{6(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^5d^3}\\&-\frac{3b\log(1-c^2x^2)}{2c^5d^3}+\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^5d^3}\end{aligned}$$

output

```
-3*a*x/c^4/d^3+1/2*b*x/c^4/d^3-1/8*b/c^5/d^3/(c*x+1)^2+15/8*b/c^5/d^3/(c*x
+1)-19/8*b*arctanh(c*x)/c^5/d^3-3*b*x*arctanh(c*x)/c^4/d^3+1/2*x^2*(a+b*ar
ctanh(c*x))/c^3/d^3-1/2*(a+b*arctanh(c*x))/c^5/d^3/(c*x+1)^2+4*(a+b*arctan
h(c*x))/c^5/d^3/(c*x+1)-6*(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^5/d^3-3/2*b*l
n(-c^2*x^2+1)/c^5/d^3+3*b*polylog(2,1-2/(c*x+1))/c^5/d^3
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{x^4(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx$$
$$= \frac{-96acx+16ac^2x^2-\frac{16a}{(1+cx)^2}+\frac{128a}{1+cx}+192a\log(1+cx)+b\big(16cx+28\cosh(2\text{arctanh}(cx))-\cosh(4\text{arct}}{}$$

input
```
Integrate[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
(-96*a*c*x + 16*a*c^2*x^2 - (16*a)/(1 + c*x)^2 + (128*a)/(1 + c*x) + 192*a
*Log[1 + c*x] + b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]]
 - 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sinh[2*A
rcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*c^2*
x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*Ar
cTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^5*d
^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arctanh}(cx))}{(cdx+d)^3}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(\frac{6(a+b\text{arctanh}(cx))}{c^4d^3(cx+1)}-\frac{4(a+b\text{arctanh}(cx))}{c^4d^3(cx+1)^2}-\frac{3(a+b\text{arctanh}(cx))}{c^4d^3}+\frac{a+b\text{arctanh}(cx)}{c^4d^3(cx+1)^3}+\frac{x(a+b\text{arctanh}(}{c^3d^3}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{4(a+b\text{arctanh}(cx))}{c^5d^3(cx+1)}-\frac{a+b\text{arctanh}(cx)}{2c^5d^3(cx+1)^2}-\frac{6\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^5d^3}+\frac{x^2(a+b\text{arctanh}(cx))}{2c^3d^3}-\frac{3ax}{c^4d^3}-\frac{19b\text{arctanh}(cx)}{8c^5d^3}-\frac{3bx\text{arctanh}(cx)}{c^4d^3}+\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{15b}{8c^5d^3(cx+1)}-\frac{b}{8c^5d^3(cx+1)^2}+\frac{bx}{2c^4d^3}-\frac{3b\log(1-c^2x^2)}{2c^5d^3}$$

input
```
Int[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
(-3*a*x)/(c^4*d^3) + (b*x)/(2*c^4*d^3) - b/(8*c^5*d^3*(1 + c*x)^2) + (15*b
)/(8*c^5*d^3*(1 + c*x)) - (19*b*ArcTanh[c*x])/(8*c^5*d^3) - (3*b*x*ArcTanh
[c*x])/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*c^3*d^3) - (a + b*ArcTanh
[c*x])/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh[c*x]))/(c^5*d^3*(1 + c*
x)) - (6*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^5*d^3) - (3*b*Log[1 - c
^2*x^2])/(2*c^5*d^3) + (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{a\left(\frac{c^2x^2}{2}-3cx-\frac{1}{2(cx+1)^2}+\frac{4}{cx+1}+6\ln(cx+1)\right)}{d^3}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-3\operatorname{arctanh}(cx)cx-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{4\operatorname{arctanh}(cx)}{cx+1}+6\operatorname{arctanh}(\right.}{}}{}$
default	$\frac{\frac{a\left(\frac{c^2x^2}{2}-3cx-\frac{1}{2(cx+1)^2}+\frac{4}{cx+1}+6\ln(cx+1)\right)}{d^3}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-3\operatorname{arctanh}(cx)cx-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{4\operatorname{arctanh}(cx)}{cx+1}+6\operatorname{arctanh}(\right.}{}}{}$
parts	$\frac{a\left(\frac{\frac{1}{2}cx^2-3x}{c^4}+\frac{4}{c^5(cx+1)}+\frac{6\ln(cx+1)}{c^5}-\frac{1}{2c^5(cx+1)^2}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arctanh}(cx)c^2x^2}{2}-3\operatorname{arctanh}(cx)cx-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{4\operatorname{arctanh}(c}{cx+1}\right.}{}$
risch	$\frac{9b}{8c^5d^3}-\frac{7b\ln(cx+1)}{4c^5d^3}+\frac{b}{8d^3c^5(-cx-1)}+\frac{a\,x^2}{2d^3c^3}-\frac{4a}{d^3c^5(-cx-1)}-\frac{a}{2d^3c^5(-cx-1)^2}-\frac{5b\ln(-cx+1)}{4d^3c^5}+\frac{5a}{2d^3}$

input
```
int(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a/d^3*(1/2*c^2*x^2-3*c*x-1/2/(c*x+1)^2+4/(c*x+1)+6*ln(c*x+1))+b/d^3
*(1/2*arctanh(c*x)*c^2*x^2-3*arctanh(c*x)*c*x-1/2/(c*x+1)^2*arctanh(c*x)+4
/(c*x+1)*arctanh(c*x)+6*arctanh(c*x)*ln(c*x+1)-3/2*ln(c*x+1)^2+3*(ln(c*x+1
)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-3*dilog(1/2*c*x+1/2)+1/2*c*x+1/2-1/8/(
c*x+1)^2+15/8/(c*x+1)-43/16*ln(c*x+1)-5/16*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^4(a+b\operatorname{arctanh}(cx))}{(d+cdx)^3}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x^4}{(cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arctanh(c*x) + a*x^4)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d
^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a + b\text{arctanh}(cx))}{(d + cdx)^3}\,dx = \frac{\int \frac{ax^4}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^4\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x**4*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

output
```
(Integral(a*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x*
*4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a + b\text{arctanh}(cx))}{(d + cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^4}{(cdx + d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*(c^5*(2*(9*c*x + 8)/(c^12*d^3*x^2 + 2*c^11*d^3*x + c^10*d^3) + 4*(c*x
^2 - 4*x)/(c^9*d^3) + 31*log(c*x + 1)/(c^10*d^3) + log(c*x - 1)/(c^10*d^3)
) + 32*c^5*integrate(1/2*x^5*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2
*c^5*d^3*x - c^4*d^3), x) + 3*c^4*(2*(7*c*x + 6)/(c^11*d^3*x^2 + 2*c^10*d^
3*x + c^9*d^3) - 8*x/(c^8*d^3) + 17*log(c*x + 1)/(c^9*d^3) - log(c*x - 1)/
(c^9*d^3)) - 32*c^4*integrate(1/2*x^4*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^
3*x^3 - 2*c^5*d^3*x - c^4*d^3), x) - 15*c^3*(2*(5*c*x + 4)/(c^10*d^3*x^2 +
 2*c^9*d^3*x + c^8*d^3) + 7*log(c*x + 1)/(c^8*d^3) + log(c*x - 1)/(c^8*d^3
)) + 192*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 -
 2*c^5*d^3*x - c^4*d^3), x) + 9*c^2*(2*(3*c*x + 2)/(c^9*d^3*x^2 + 2*c^8*d^
3*x + c^7*d^3) + log(c*x + 1)/(c^7*d^3) - log(c*x - 1)/(c^7*d^3)) + 576*c^
2*integrate(1/2*x^2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*
x - c^4*d^3), x) + 9*c*(2*x/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - log(c*
x + 1)/(c^6*d^3) + log(c*x - 1)/(c^6*d^3)) + 576*c*integrate(1/2*x*log(c*x
 + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3), x) - 8*(c^4*x
^4 - 4*c^3*x^3 - 11*c^2*x^2 + 2*c*x + 12*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1
) + 7)*log(-c*x + 1)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + 14*(c*x + 2)/
(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - 7*log(c*x + 1)/(c^5*d^3) + 7*log(c
*x - 1)/(c^5*d^3) + 192*integrate(1/2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^
3*x^3 - 2*c^5*d^3*x - c^4*d^3), x))*b + 1/2*a*((8*c*x + 7)/(c^7*d^3*x^2...
```

Giac [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)x^4}{(cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^4/(c*d*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{atanh}(c\,x)\right)}{{\left(d+c\,d\,x\right)}^3}\,dx$$

input
```
int((x^4*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

output
```
int((x^4*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{2\left(\int \frac{atanh(cx)x^4}{c^3x^3+3c^2x^2+3cx+1}dx\right)b\,c^7x^2+4\left(\int \frac{atanh(cx)x^4}{c^3x^3+3c^2x^2+3cx+1}dx\right)b\,c^6x+2\left(\int \frac{atanh(cx)x^4}{c^3x^3+3c^2x^2+3cx+1}dx\right)b\,c^5+12\log(c}{2c^5d^3\left(c^2x^2+2cx+\right.}$$

input
```
int(x^4*(a+b*atanh(c*x))/(c*d*x+d)^3,x)
```

output
```
(2*int((atanh(c*x)*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b*c**7*x
**2 + 4*int((atanh(c*x)*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b*c
**6*x + 2*int((atanh(c*x)*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b
*c**5 + 12*log(c*x + 1)*a*c**2*x**2 + 24*log(c*x + 1)*a*c*x + 12*log(c*x +
 1)*a + a*c**4*x**4 - 4*a*c**3*x**3 - 12*a*c**2*x**2 + 6*a)/(2*c**5*d**3*(
c**2*x**2 + 2*c*x + 1))
```

3.59 $\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{ax}{c^3d^3}+\frac{b}{8c^4d^3(1+cx)^2}-\frac{11b}{8c^4d^3(1+cx)}+\frac{11b\text{arctanh}(cx)}{8c^4d^3}+\frac{bx\text{arctanh}(cx)}{c^3d^3}+\frac{a+b\text{arctanh}(cx)}{2c^4d^3(1+cx)^2}-\frac{3(a+b\text{arctanh}(cx))}{c^4d^3(1+cx)}+\frac{3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^4d^3}+\frac{b\log(1-c^2x^2)}{2c^4d^3}-\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^4d^3}$$

output
```
a*x/c^3/d^3+1/8*b/c^4/d^3/(c*x+1)^2-11/8*b/c^4/d^3/(c*x+1)+11/8*b*arctanh(
c*x)/c^4/d^3+b*x*arctanh(c*x)/c^3/d^3+1/2*(a+b*arctanh(c*x))/c^4/d^3/(c*x+
1)^2-3*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)+3*(a+b*arctanh(c*x))*ln(2/(c*x+1
))/c^4/d^3+1/2*b*ln(-c^2*x^2+1)/c^4/d^3-3/2*b*polylog(2,1-2/(c*x+1))/c^4/d
^3
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.86

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{32acx+\frac{16a}{(1+cx)^2}-\frac{96a}{1+cx}-96a\log(1+cx)+b\left(-20\cosh(2\text{arctanh}(cx))+\cosh(4\text{arctanh}(cx))+16\log(1\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
(32*a*c*x + (16*a)/(1 + c*x)^2 - (96*a)/(1 + c*x) - 96*a*Log[1 + c*x] + b*
(-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 - c^2*x^2] - 4
8*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c
*x]*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24*Log[1 + E
^(-2*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Si
nh[4*ArcTanh[c*x]]))/(32*c^4*d^3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(cdx+d)^3}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(-\frac{3(a+b\text{arctanh}(cx))}{c^3d^3(cx+1)}+\frac{3(a+b\text{arctanh}(cx))}{c^3d^3(cx+1)^2}+\frac{a+b\text{arctanh}(cx)}{c^3d^3}-\frac{a+b\text{arctanh}(cx)}{c^3d^3(cx+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(a+b\operatorname{arctanh}(cx))}{c^4d^3(cx+1)}+\frac{a+b\operatorname{arctanh}(cx)}{2c^4d^3(cx+1)^2}+\frac{3\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{c^4d^3}+\frac{ax}{c^3d^3}+\frac{11b\operatorname{arctanh}(cx)}{8c^4d^3}+\frac{bx\operatorname{arctanh}(cx)}{c^3d^3}-\frac{3b\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{11b}{8c^4d^3(cx+1)}+\frac{b}{8c^4d^3(cx+1)^2}+\frac{b\log(1-c^2x^2)}{2c^4d^3}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
(a*x)/(c^3*d^3) + b/(8*c^4*d^3*(1 + c*x)^2) - (11*b)/(8*c^4*d^3*(1 + c*x))
 + (11*b*ArcTanh[c*x])/(8*c^4*d^3) + (b*x*ArcTanh[c*x])/(c^3*d^3) + (a + b
*ArcTanh[c*x])/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x]))/(c^4*d^3
*(1 + c*x)) + (3*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^3) + (b*Log
[1 - c^2*x^2])/(2*c^4*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{a\left(cx+\frac{1}{2(cx+1)^2}-\frac{3}{cx+1}-3\ln(cx+1)\right)}{d^3}+\frac{b\left(\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{cx+1}-3\operatorname{arctanh}(cx)\ln(cx+1)+\frac{3\ln(cx+1)^2}{4}-\right.}{}}{c^4}$
default	$\frac{\frac{a\left(cx+\frac{1}{2(cx+1)^2}-\frac{3}{cx+1}-3\ln(cx+1)\right)}{d^3}+\frac{b\left(\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{cx+1}-3\operatorname{arctanh}(cx)\ln(cx+1)+\frac{3\ln(cx+1)^2}{4}-\right.}{}}{c^4}$
parts	$\frac{a\left(\frac{x}{c^3}-\frac{3}{c^4(cx+1)}-\frac{3\ln(cx+1)}{c^4}+\frac{1}{2c^4(cx+1)^2}\right)}{d^3}+\frac{b\left(\operatorname{arctanh}(cx)cx+\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{cx+1}-3\operatorname{arctanh}(cx)\ln(cx+1\right.}{}$
risch	$\frac{a}{2d^3c^4(-cx-1)^2}+\frac{b\ln(-cx+1)}{2d^3c^4}+\frac{11b\ln(-cx-1)}{16d^3c^4}-\frac{3b\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d^3c^4}-\frac{3a\ln(-cx-1)}{d^3c^4}+\frac{b\ln(-cx+1)x}{8d^3c^3(-cx-1)^2}+$

input
```
int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d^3*(c*x+1/2/(c*x+1)^2-3/(c*x+1)-3*ln(c*x+1))+b/d^3*(arctanh(c*x)
*c*x+1/2/(c*x+1)^2*arctanh(c*x)-3/(c*x+1)*arctanh(c*x)-3*arctanh(c*x)*ln(c
*x+1)+3/4*ln(c*x+1)^2-3/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+3/2
*dilog(1/2*c*x+1/2)+1/8/(c*x+1)^2-11/8/(c*x+1)+19/16*ln(c*x+1)-3/16*ln(c*x
-1)))
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))}{(d+cdx)^3}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x^3}{(cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^3*arctanh(c*x) + a*x^3)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d
^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{\int \frac{ax^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^3\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

output
```
(Integral(a*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)x^3}{(cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*(2*c^4*(2*(7*c*x + 6)/(c^10*d^3*x^2 + 2*c^9*d^3*x + c^8*d^3) - 8*x/(c^7*d^3) + 17*log(c*x + 1)/(c^8*d^3) - log(c*x - 1)/(c^8*d^3)) - 32*c^4*integrate(1/2*x^4*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) - 6*c^3*(2*(5*c*x + 4)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*d^3) + 7*log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c^7*d^3)) + 128*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) + 288*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) + 9*c*(2*x/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - log(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3)) + 288*c*integrate(1/2*x*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) + 8*(2*c^3*x^3 + 4*c^2*x^2 - 4*c*x - 6*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) - 5)*log(-c*x + 1)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) + 10*(c*x + 2)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - 5*log(c*x + 1)/(c^4*d^3) + 5*log(c*x - 1)/(c^4*d^3) + 96*integrate(1/2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x))*b - 1/2*a*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - 2*x/(c^3*d^3) + 6*log(c*x + 1)/(c^4*d^3))
```

Giac [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)x^3}{(cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \int \frac{x^3\,(a+b\,\text{atanh}(c\,x))}{(d+c\,d\,x)^3}\,dx$$

input
```
int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

output
```
int((x^3*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{2\left(\int \frac{atanh(cx)x^3}{c^3x^3+3c^2x^2+3cx+1}dx\right)b\,c^6x^2+4\left(\int \frac{atanh(cx)x^3}{c^3x^3+3c^2x^2+3cx+1}dx\right)b\,c^5x+2\left(\int \frac{atanh(cx)x^3}{c^3x^3+3c^2x^2+3cx+1}dx\right)b\,c^4-6\log(cx}{2c^4d^3\,(c^2x^2+2cx+1)}$$

input
```
int(x^3*(a+b*atanh(c*x))/(c*d*x+d)^3,x)
```

output

```
(2*int((atanh(c*x)*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b*c**6*x
**2 + 4*int((atanh(c*x)*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b*c
**5*x + 2*int((atanh(c*x)*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b
*c**4 - 6*log(c*x + 1)*a*c**2*x**2 - 12*log(c*x + 1)*a*c*x - 6*log(c*x + 1
)*a + 2*a*c**3*x**3 + 6*a*c**2*x**2 - 3*a)/(2*c**4*d**3*(c**2*x**2 + 2*c*x
 + 1))
```

3.60 $\int \frac{x^2(a+b\,\text{arctanh}(cx))}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\begin{aligned}\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = &-\frac{b}{8c^3d^3(1+cx)^2}+\frac{7b}{8c^3d^3(1+cx)}-\frac{7b\text{arctanh}(cx)}{8c^3d^3}\\ &-\frac{a+b\text{arctanh}(cx)}{2c^3d^3(1+cx)^2}+\frac{2(a+b\text{arctanh}(cx))}{c^3d^3(1+cx)}\\ &-\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{c^3d^3}\\ &+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^3d^3}\end{aligned}$$

output
```
-1/8*b/c^3/d^3/(c*x+1)^2+7/8*b/c^3/d^3/(c*x+1)-7/8*b*arctanh(c*x)/c^3/d^3-
1/2*(a+b*arctanh(c*x))/c^3/d^3/(c*x+1)^2+2*(a+b*arctanh(c*x))/c^3/d^3/(c*x
+1)-(a+b*arctanh(c*x))*ln(2/(c*x+1))/c^3/d^3+1/2*b*polylog(2,1-2/(c*x+1))/
c^3/d^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(d + cdx)^3} \, dx$$
$$= \frac{-\frac{16a}{(1+cx)^2} + \frac{64a}{1+cx} + 32a\log(1 + cx) + b\big(12\cosh(2\text{arctanh}(cx)) - \cosh(4\text{arctanh}(cx)) + 16\,\text{PolyLog}\,(2, -}{}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
((-16*a)/(1 + c*x)^2 + (64*a)/(1 + c*x) + 32*a*Log[1 + c*x] + b*(12*Cosh[2
*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])
] - 12*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cos
h[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])]
- 6*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^3*d^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(cdx + d)^3} \, dx$$
$$\Big\downarrow \; 6502$$
$$\int \left(\frac{a + b\text{arctanh}(cx)}{c^2d^3(cx + 1)} - \frac{2(a + b\text{arctanh}(cx))}{c^2d^3(cx + 1)^2} + \frac{a + b\text{arctanh}(cx)}{c^2d^3(cx + 1)^3}\right) dx$$
$$\Big\downarrow \; 2009$$

$$\frac{2(a+b\text{arctanh}(cx))}{c^3d^3(cx+1)}-\frac{a+b\text{arctanh}(cx)}{2c^3d^3(cx+1)^2}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^3d^3}-\frac{7b\text{arctanh}(cx)}{8c^3d^3}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{7b}{8c^3d^3(cx+1)}-\frac{b}{8c^3d^3(cx+1)^2}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
-1/8*b/(c^3*d^3*(1 + c*x)^2) + (7*b)/(8*c^3*d^3*(1 + c*x)) - (7*b*ArcTanh[
c*x])/(8*c^3*d^3) - (a + b*ArcTanh[c*x])/(2*c^3*d^3*(1 + c*x)^2) + (2*(a +
 b*ArcTanh[c*x]))/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c
*x)])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{2(cx+1)^2}+\frac{2}{cx+1}+\ln(cx+1)\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{2\operatorname{arctanh}(cx)}{cx+1}+\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right.\right.}{2}\cdots\right.}{d^3}}{c^3}$
default	$\frac{\frac{a\left(-\frac{1}{2(cx+1)^2}+\frac{2}{cx+1}+\ln(cx+1)\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{2\operatorname{arctanh}(cx)}{cx+1}+\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{\left(\ln(cx+1)-\ln\left(\frac{cx}{2}+\frac{1}{2}\right.\right.}{2}\cdots\right.}{d^3}}{c^3}$
parts	$\frac{a\left(\frac{2}{c^3(cx+1)}+\frac{\ln(cx+1)}{c^3}-\frac{1}{2c^3(cx+1)^2}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{2\operatorname{arctanh}(cx)}{cx+1}+\operatorname{arctanh}(cx)\ln(cx+1)-\frac{\ln(cx+1)^2}{4}+\frac{(\ln(cx+1\cdots}{}\right.}{}$
risch	$\frac{b\ln(cx+1)^2}{4c^3d^3}+\frac{\left(\frac{bx}{c^2}+\frac{3b}{4c^3}\right)\ln(cx+1)}{d^3(cx+1)^2}-\frac{7b\ln(-cx-1)}{16d^3c^3}-\frac{b\ln(-cx+1)x}{2d^3c^2(-cx-1)}+\frac{b\ln(-cx+1)}{2d^3c^3(-cx-1)}+\frac{b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-\cdots}{2d^3c^3}$

input
```
int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d^3*(-1/2/(c*x+1)^2+2/(c*x+1)+ln(c*x+1))+b/d^3*(-1/2/(c*x+1)^2*ar
ctanh(c*x)+2/(c*x+1)*arctanh(c*x)+arctanh(c*x)*ln(c*x+1)-1/4*ln(c*x+1)^2+1
/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2)-1/8
/(c*x+1)^2+7/8/(c*x+1)-7/16*ln(c*x+1)+7/16*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{(d+cdx)^3}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x^2}{(cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arctanh(c*x) + a*x^2)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d
^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{\int \frac{ax^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^2\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

output
```
(Integral(a*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
*2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)x^2}{(cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*(64*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 -
 2*c^3*d^3*x - c^2*d^3), x) - 4*c^2*(2*(3*c*x + 2)/(c^7*d^3*x^2 + 2*c^6*d^
3*x + c^5*d^3) + log(c*x + 1)/(c^5*d^3) - log(c*x - 1)/(c^5*d^3)) + 64*c^2
*integrate(1/2*x^2*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x
 - c^2*d^3), x) + 7*c*(2*x/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - log(c*x
 + 1)/(c^4*d^3) + log(c*x - 1)/(c^4*d^3)) + 96*c*integrate(1/2*x*log(c*x +
 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x) - 8*(4*c*x +
 2*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) + 3)*log(-c*x + 1)/(c^5*d^3*x^2 + 2*
c^4*d^3*x + c^3*d^3) + 6*(c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) -
 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3) + 32*integrate(1/2*lo
g(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x))*b +
1/2*a*((4*c*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)/
(c^3*d^3))
```

Giac [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(d + cdx)^3} \, dx = \int \frac{(b \,\text{artanh}\,(cx) + a)x^2}{(cdx + d)^3} \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(d + cdx)^3} \, dx = \int \frac{x^2 \left(a + b \,\text{atanh}(c\, x)\right)}{\left(d + c\, d\, x\right)^3} \, dx$$

input
```
int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

output
```
int((x^2*(a + b*atanh(c*x)))/(d + c*d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{(d + cdx)^3} \, dx$$
$$= \frac{2\left(\int \frac{atanh(cx)x^2}{c^3x^3+3c^2x^2+3cx+1}dx\right) b\, c^5x^2 + 4\left(\int \frac{atanh(cx)x^2}{c^3x^3+3c^2x^2+3cx+1}dx\right) b\, c^4x + 2\left(\int \frac{atanh(cx)x^2}{c^3x^3+3c^2x^2+3cx+1}dx\right) b\, c^3 + 2\log(ca}{2c^3d^3\left(c^2x^2 + 2cx + 1\right)}$$

input
```
int(x^2*(a+b*atanh(c*x))/(c*d*x+d)^3,x)
```

output
```
(2*int((atanh(c*x)*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b*c**5*x
**2 + 4*int((atanh(c*x)*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b*c
**4*x + 2*int((atanh(c*x)*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*b
*c**3 + 2*log(c*x + 1)*a*c**2*x**2 + 4*log(c*x + 1)*a*c*x + 2*log(c*x + 1)
*a - 2*a*c**2*x**2 + a)/(2*c**3*d**3*(c**2*x**2 + 2*c*x + 1))
```

3.61 $\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{b}{8c^2d^3(1+cx)^2}-\frac{3b}{8c^2d^3(1+cx)} - \frac{b\text{arctanh}(cx)}{8c^2d^3}+\frac{x^2(a+b\text{arctanh}(cx))}{2d^3(1+cx)^2}$$

output
```
1/8*b/c^2/d^3/(c*x+1)^2-3/8*b/c^2/d^3/(c*x+1)-1/8*b*arctanh(c*x)/c^2/d^3+1
/2*x^2*(a+b*arctanh(c*x))/d^3/(c*x+1)^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = -\frac{8a+4b+16acx+6bcx+8(b+2bcx)\text{arctanh}(cx)+3b(1+cx)^2\log(1-cx)-3b\log(1+cx)-6bcx}{16c^2d^3(1+cx)^2}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
-1/16*(8*a + 4*b + 16*a*c*x + 6*b*c*x + 8*(b + 2*b*c*x)*ArcTanh[c*x] + 3*b
*(1 + c*x)^2*Log[1 - c*x] - 3*b*Log[1 + c*x] - 6*b*c*x*Log[1 + c*x] - 3*b*
c^2*x^2*Log[1 + c*x])/(c^2*d^3*(1 + c*x)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6498, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arctanh}(cx))}{(cdx+d)^3}\,dx$$

$$\Big\downarrow \text{6498}$$

$$\frac{x^2(a+b\text{arctanh}(cx))}{2d^3(cx+1)^2}-bc\int \frac{x^2}{2d^3(1-cx)(cx+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2(a+b\text{arctanh}(cx))}{2d^3(cx+1)^2}-\frac{bc\int \frac{x^2}{(1-cx)(cx+1)^3}dx}{2d^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{x^2(a+b\text{arctanh}(cx))}{2d^3(cx+1)^2}-\frac{bc\int \left(-\frac{3}{4c^2(cx+1)^2}+\frac{1}{2c^2(cx+1)^3}-\frac{1}{4c^2(c^2x^2-1)}\right)dx}{2d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(a+b\text{arctanh}(cx))}{2d^3(cx+1)^2}-\frac{bc\left(\frac{\text{arctanh}(cx)}{4c^3}+\frac{3}{4c^3(cx+1)}-\frac{1}{4c^3(cx+1)^2}\right)}{2d^3}$$

input
```
Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

output
```
(x^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c*x)^2) - (b*c*(-1/4*1/(c^3*(1 + c*
x)^2) + 3/(4*c^3*(1 + c*x)) + ArcTanh[c*x]/(4*c^3)))/(2*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6498
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(
a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x
^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && Intege
rQ[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0
]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{3 \operatorname{arctanh}(cx) b c^2 x^2+4 a c^2 x^2+2 b c^2 x^2-2 \operatorname{arctanh}(cx) b c x+b c x-b \operatorname{arctanh}(cx)}{8 d^3 (cx+1)^2 c^2}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{2(cx+1)^2}-\frac{1}{cx+1}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{3\ln(cx-1)}{16}+\frac{1}{8(cx+1)^2}-\frac{3}{8(cx+1)}+\frac{3\ln(cx+1)}{16}\right)}{d^3}}{c^2}$
default	$\frac{\frac{a\left(\frac{1}{2(cx+1)^2}-\frac{1}{cx+1}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{3\ln(cx-1)}{16}+\frac{1}{8(cx+1)^2}-\frac{3}{8(cx+1)}+\frac{3\ln(cx+1)}{16}\right)}{d^3}}{c^2}$
parts	$\frac{a\left(-\frac{1}{c^2(cx+1)}+\frac{1}{2c^2(cx+1)^2}\right)}{d^3}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{3\ln(cx-1)}{16}+\frac{1}{8(cx+1)^2}-\frac{3}{8(cx+1)}+\frac{3\ln(cx+1)}{16}\right)}{d^3 c^2}$
orering	$-\frac{(cx+1)(4x^3c^3-7c^2x^2+cx+2)(a+b \operatorname{arctanh}(cx))}{8c^2(cdx+d)^3}-\frac{(2cx+1)(cx-1)(cx+1)^2\left(\frac{a+b \operatorname{arctanh}(cx)}{(cdx+d)^3}+\frac{xbc}{(-c^2x^2+1)(cdx+d\cdots}\cdots\right)}{8c^2}$
risch	$-\frac{b(2cx+1)\ln(cx+1)}{4c^2d^3(cx+1)^2}-\frac{3\ln(cx-1)b c^2x^2-3\ln(-cx-1)b c^2x^2+6\ln(cx-1)bcx-6\ln(-cx-1)bcx-8bcx\ln(-cx+1)+\cdots}{16c^2d^3(cx+1)^2}$

input
```
int(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(3*arctanh(c*x)*b*c^2*x^2+4*a*c^2*x^2+2*b*c^2*x^2-2*arctanh(c*x)*b*c*x
+b*c*x-b*arctanh(c*x))/d^3/(c*x+1)^2/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = -\frac{2\,(8\,a+3\,b)cx-(3\,bc^2x^2-2\,bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(8*a + 3*b)*c*x - (3*b*c^2*x^2 - 2*b*c*x - b)*log(-(c*x + 1)/(c*x
 - 1)) + 8*a + 4*b)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(71) = 142.

Time = 0.84 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.60

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \begin{cases} -\frac{8acx}{8c^4d^3x^2+16c^3d^3x+8c^2d^3} - \frac{4a}{8c^4d^3x^2+16c^3d^3x+8c^2d^3} + \frac{3bc^2x^2\operatorname{atanh}(cx)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3} - \frac{2bcx\operatorname{atanh}(cx)}{8c^4d^3x^2+16c^3d^3x+8c^2d^3} - \frac{3bcx}{8c^4d^3x^2+16c^3d^3x+8c^2d^3} \\ \frac{ax^2}{2d^3} \end{cases}$$

input
```
integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

output
```
Piecewise((-8*a*c*x/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 4*
a/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) + 3*b*c**2*x**2*atanh(
c*x)/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 2*b*c*x*atanh(c*x
)/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - 3*b*c*x/(8*c**4*d**3
*x**2 + 16*c**3*d**3*x + 8*c**2*d**3) - b*atanh(c*x)/(8*c**4*d**3*x**2 + 1
6*c**3*d**3*x + 8*c**2*d**3) - 2*b/(8*c**4*d**3*x**2 + 16*c**3*d**3*x + 8*
c**2*d**3), Ne(c, 0)), (a*x**2/(2*d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(69) = 138$.

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.97

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = -\frac{1}{16}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3}-\frac{3\,\log\left(cx+1\right)}{c^3d^3}+\frac{3\,\log\left(cx-1\right)}{c^3d^3}\right)+\frac{8\,(2\,cx+1)\,\text{artanh}\left(cx\right)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}\right)b - \frac{(2\,cx+1)a}{2\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/16*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x
+ 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c
^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*b - 1/2*(2*c*x + 1)*a/(c^4*d^3*x^2 +
2*c^3*d^3*x + c^2*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx = \frac{1}{32}\,c\left(\frac{2\,(cx-1)^2\left(\frac{2\,(cx+1)b}{cx-1}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^3d^3}+\frac{(cx-1)^2\left(\frac{8\,(cx+1)a}{cx-1}+4\,a+\frac{4\,(cx+1)b}{cx-1}+b\right)}{(cx+1)^2c^3d^3}\right)$$

input
```
integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
1/32*c*(2*(c*x - 1)^2*(2*(c*x + 1)*b/(c*x - 1) + b)*log(-(c*x + 1)/(c*x -
1))/((c*x + 1)^2*c^3*d^3) + (c*x - 1)^2*(8*(c*x + 1)*a/(c*x - 1) + 4*a + 4
*(c*x + 1)*b/(c*x - 1) + b)/((c*x + 1)^2*c^3*d^3))
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx$$
$$=\frac{c\,(b\,x-2\,b\,x\,\text{atanh}(c\,x))-b\,\text{atanh}(c\,x)+c^2\,(4\,a\,x^2+2\,b\,x^2+3\,b\,x^2\,\text{atanh}(c\,x))}{8\,c^4\,d^3\,x^2+16\,c^3\,d^3\,x+8\,c^2\,d^3}$$

input
```
int((x*(a + b*atanh(c*x)))/(d + c*d*x)^3,x)
```

output
```
(c*(b*x - 2*b*x*atanh(c*x)) - b*atanh(c*x) + c^2*(4*a*x^2 + 2*b*x^2 + 3*b*
x^2*atanh(c*x)))/(8*c^2*d^3 + 16*c^3*d^3*x + 8*c^4*d^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int \frac{x(a+b\text{arctanh}(cx))}{(d+cdx)^3}\,dx$$
$$=\frac{8atanh(cx)\,b\,c^2x^2+\log(cx-1)\,b\,c^2x^2+2\log(cx-1)\,bcx+\log(cx-1)\,b-\log(cx+1)\,b\,c^2x^2-2\log(cx}{16c^2d^3\,(c^2x^2+2cx+1)}$$

input
```
int(x*(a+b*atanh(c*x))/(c*d*x+d)^3,x)
```

output
```
(8*atanh(c*x)*b*c**2*x**2 + log(c*x - 1)*b*c**2*x**2 + 2*log(c*x - 1)*b*c*
x + log(c*x - 1)*b - log(c*x + 1)*b*c**2*x**2 - 2*log(c*x + 1)*b*c*x - log
(c*x + 1)*b + 8*a*c**2*x**2 + 3*b*c**2*x**2 - b)/(16*c**2*d**3*(c**2*x**2
+ 2*c*x + 1))
```

3.62 $\int \frac{a+b\textbf{arctanh}(cx)}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{a+b\operatorname{arctanh}(cx)}{(d+cdx)^3}\,dx = -\frac{b}{8cd^3(1+cx)^2}-\frac{b}{8cd^3(1+cx)}+\frac{b\operatorname{arctanh}(cx)}{8cd^3}-\frac{a+b\operatorname{arctanh}(cx)}{2cd^3(1+cx)^2}$$

output
```
-1/8*b/c/d^3/(c*x+1)^2-1/8*b/c/d^3/(c*x+1)+1/8*b*arctanh(c*x)/c/d^3-1/2*(a
+b*arctanh(c*x))/c/d^3/(c*x+1)^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{a+b\operatorname{arctanh}(cx)}{(d+cdx)^3}\,dx = \frac{-8a-4b-2bcx-8b\operatorname{arctanh}(cx)-b(1+cx)^2\log(1-cx)+b\log(1+cx)+2bcx\log(1+cx)+bc^2x^2\,\mathrm{lo}}{16cd^3(1+cx)^2}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

output
```
(-8*a - 4*b - 2*b*c*x - 8*b*ArcTanh[c*x] - b*(1 + c*x)^2*Log[1 - c*x] + b*
Log[1 + c*x] + 2*b*c*x*Log[1 + c*x] + b*c^2*x^2*Log[1 + c*x])/(16*c*d^3*(1
 + c*x)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6478, 27, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(cdx+d)^3}dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{b\int \frac{1}{d^2(cx+1)^2(1-c^2x^2)}dx}{2d}-\frac{a+b\text{arctanh}(cx)}{2cd^3(cx+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{1}{(cx+1)^2(1-c^2x^2)}dx}{2d^3}-\frac{a+b\text{arctanh}(cx)}{2cd^3(cx+1)^2}$$

$$\Big\downarrow \text{456}$$

$$\frac{b\int \frac{1}{(1-cx)(cx+1)^3}dx}{2d^3}-\frac{a+b\text{arctanh}(cx)}{2cd^3(cx+1)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{b\int \left(\frac{1}{4(cx+1)^2}+\frac{1}{2(cx+1)^3}-\frac{1}{4(c^2x^2-1)}\right)dx}{2d^3}-\frac{a+b\text{arctanh}(cx)}{2cd^3(cx+1)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{\text{arctanh}(cx)}{4c}-\frac{1}{4c(cx+1)}-\frac{1}{4c(cx+1)^2}\right)}{2d^3}-\frac{a+b\text{arctanh}(cx)}{2cd^3(cx+1)^2}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x])/(c*d^3*(1 + c*x)^2) + (b*(-1/4*1/(c*(1 + c*x)^2)
 - 1/(4*c*(1 + c*x)) + ArcTanh[c*x]/(4*c)))/(2*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{a}{2d^3(cx+1)^2}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{\ln(cx-1)}{16}-\frac{1}{8(cx+1)^2}-\frac{1}{8(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}}{c}$
default	$\frac{-\frac{a}{2d^3(cx+1)^2}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{\ln(cx-1)}{16}-\frac{1}{8(cx+1)^2}-\frac{1}{8(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}}{c}$
parallelrisch	$\frac{\operatorname{arctanh}(cx)b\,c^2x^2+4a\,c^2x^2+2b\,c^2x^2+2\operatorname{arctanh}(cx)bcx+8acx+3bcx-3b\operatorname{arctanh}(cx)}{8d^3(cx+1)^2c}$
parts	$-\frac{a}{2d^3c(cx+1)^2}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{\ln(cx-1)}{16}-\frac{1}{8(cx+1)^2}-\frac{1}{8(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3c}$
orering	$-\frac{(cx+1)(6x^3c^3+2c^2x^2-11cx+3)(a+b\operatorname{arctanh}(cx))}{8c(cdx+d)^3}-\frac{x(2cx+3)(cx-1)(cx+1)^2\left(\frac{bc}{(-c^2x^2+1)(cdx+d)^3}-\frac{3(a+b\operatorname{arct}}{(cdx}\right.}{8c}$
risch	$-\frac{b\ln(cx+1)}{4c\,d^3(cx+1)^2}-\frac{\ln(cx-1)b\,c^2x^2-\ln(-cx-1)b\,c^2x^2+2\ln(cx-1)bcx-2\ln(-cx-1)bcx+2bcx+b\ln(cx-1)-b\ln(-ca}{16d^3(cx+1)^2c}$

input
```
int((a+b*arctanh(c*x))/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*a/d^3/(c*x+1)^2+b/d^3*(-1/2/(c*x+1)^2*arctanh(c*x)-1/16*ln(c*x-1
)-1/8/(c*x+1)^2-1/8/(c*x+1)+1/16*ln(c*x+1)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^3}\,dx=-\frac{2\,bcx-(bc^2x^2+2\,bcx-3\,b)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\,(c^3d^3x^2+2\,c^2d^3x+cd^3)}$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*b*c*x - (b*c^2*x^2 + 2*b*c*x - 3*b)*log(-(c*x + 1)/(c*x - 1)) + 8
*a + 4*b)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(65) = 130$.

Time = 0.81 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.91

$$\int \frac{a + b\text{arctanh}(cx)}{(d + cdx)^3} \, dx = \begin{cases} -\frac{4a}{8c^3d^3x^2+16c^2d^3x+8cd^3} + \frac{bc^2x^2 \operatorname{atanh}(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3} + \frac{2bcx \operatorname{atanh}(cx)}{8c^3d^3x^2+16c^2d^3x+8cd^3} - \frac{bcx}{8c^3d^3x^2+16c^2d^3x+8cd^3} - \frac{3b \operatorname{atanh}(cx)}{8c^3d^3x^2+16c^2d^3x+ \\ \frac{ax}{d^3} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

output
```
Piecewise((-4*a/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) + b*c**2*x*
*2*atanh(c*x)/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) + 2*b*c*x*ata
nh(c*x)/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d**3) - b*c*x/(8*c**3*d**
3*x**2 + 16*c**2*d**3*x + 8*c*d**3) - 3*b*atanh(c*x)/(8*c**3*d**3*x**2 + 1
6*c**2*d**3*x + 8*c*d**3) - 2*b/(8*c**3*d**3*x**2 + 16*c**2*d**3*x + 8*c*d
**3), Ne(c, 0)), (a*x/d**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int \frac{a + b\text{arctanh}(cx)}{(d + cdx)^3} \, dx = -\frac{1}{16} \left(c \left(\frac{2(cx+2)}{c^4d^3x^2 + 2c^3d^3x + c^2d^3} - \frac{\log(cx+1)}{c^2d^3} + \frac{\log(cx-1)}{c^2d^3} \right) + \frac{8 \operatorname{artanh}(cx)}{c^3d^3x^2 + 2c^2d^3x + cd^3} \right) b - \frac{a}{2(c^3d^3x^2 + 2c^2d^3x + cd^3)}$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/16*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)
/(c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2
*d^3*x + c*d^3))*b - 1/2*a/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^3}\,dx$$
$$=\frac{1}{32}c\left(\frac{2\,(cx-1)^2\left(\frac{2\,(cx+1)b}{cx-1}-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^2c^2d^3}+\frac{(cx-1)^2\left(\frac{8\,(cx+1)a}{cx-1}-4\,a+\frac{4\,(cx+1)b}{cx-1}-b\right)}{(cx+1)^2c^2d^3}\right)$$

input
```
integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
1/32*c*(2*(c*x - 1)^2*(2*(c*x + 1)*b/(c*x - 1) - b)*log(-(c*x + 1)/(c*x -
1))/((c*x + 1)^2*c^2*d^3) + (c*x - 1)^2*(8*(c*x + 1)*a/(c*x - 1) - 4*a + 4
*(c*x + 1)*b/(c*x - 1) - b)/((c*x + 1)^2*c^2*d^3))
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.60

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^3}\,dx$$
$$=\frac{c^2\left(\frac{a\,x^2}{2}+\frac{b\,x^2}{4}-\frac{b\,x^2\,\ln(c^2\,x^2-1)}{16}+\frac{b\,x^2\,\ln(c\,x+1)}{8}\right)-\frac{b\,\ln(c^2\,x^2-1)}{16}-\frac{b\,\text{atanh}(c\,x)}{2}+\frac{b\,\ln(c\,x+1)}{8}+c\left(a\,x+\frac{3\,b\,x}{8}+\frac{b\,x\,\ln}{}\right.}{c\,d^3\,(c\,x+1)^2}$$

input
```
int((a + b*atanh(c*x))/(d + c*d*x)^3,x)
```

output
```
(c^2*((a*x^2)/2 + (b*x^2)/4 - (b*x^2*log(c^2*x^2 - 1))/16 + (b*x^2*log(c*x
 + 1))/8) - (b*log(c^2*x^2 - 1))/16 - (b*atanh(c*x))/2 + (b*log(c*x + 1))/
8 + c*(a*x + (3*b*x)/8 + (b*x*log(c*x + 1))/4 - (b*x*log(c^2*x^2 - 1))/8))
/(c*d^3*(c*x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int \frac{a+b\text{arctanh}(cx)}{(d+cdx)^3}\,dx$$
$$= \frac{8atanh(cx)\,b\,c^2x^2+16atanh(cx)\,bcx+3\log(cx-1)\,b\,c^2x^2+6\log(cx-1)\,bcx+3\log(cx-1)\,b-3\log(}{16c\,d^3\,(c^2x^2+2cx+1)}$$

input
```
int((a+b*atanh(c*x))/(c*d*x+d)^3,x)
```

output
```
(8*atanh(c*x)*b*c**2*x**2 + 16*atanh(c*x)*b*c*x + 3*log(c*x - 1)*b*c**2*x*
*2 + 6*log(c*x - 1)*b*c*x + 3*log(c*x - 1)*b - 3*log(c*x + 1)*b*c**2*x**2
- 6*log(c*x + 1)*b*c*x - 3*log(c*x + 1)*b - 8*a + b*c**2*x**2 - 3*b)/(16*c
*d**3*(c**2*x**2 + 2*c*x + 1))
```

3.63 $\int \frac{a+b\,\mathbf{arctanh}(cx)}{x(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx)}{x(d+cdx)^3}\,dx &= \frac{b}{8d^3(1+cx)^2}+\frac{5b}{8d^3(1+cx)}-\frac{5b\text{arctanh}(cx)}{8d^3}\\ &\quad+\frac{a+b\text{arctanh}(cx)}{2d^3(1+cx)^2}+\frac{a+b\text{arctanh}(cx)}{d^3(1+cx)}+\frac{a\log(x)}{d^3}\\ &\quad+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3}-\frac{b\,\text{PolyLog}(2,-cx)}{2d^3}\\ &\quad+\frac{b\,\text{PolyLog}(2,cx)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d^3}\end{aligned}$$

output
```
1/8*b/d^3/(c*x+1)^2+5/8*b/d^3/(c*x+1)-5/8*b*arctanh(c*x)/d^3+1/2*(a+b*arct
anh(c*x))/d^3/(c*x+1)^2+(a+b*arctanh(c*x))/d^3/(c*x+1)+a*ln(x)/d^3+(a+b*ar
ctanh(c*x))*ln(2/(c*x+1))/d^3-1/2*b*polylog(2,-c*x)/d^3+1/2*b*polylog(2,c*
x)/d^3-1/2*b*polylog(2,1-2/(c*x+1))/d^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)^3} dx$$
$$= \frac{\frac{16a}{(1+cx)^2} + \frac{32a}{1+cx} + 32a\log(x) - 32a\log(1 + cx) + b\left(12\cosh(2\text{arctanh}(cx)) + \cosh(4\text{arctanh}(cx)) - 16\,\text{Po}\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

output
```
((16*a)/(1 + c*x)^2 + (32*a)/(1 + c*x) + 32*a*Log[x] - 32*a*Log[1 + c*x] +
 b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*PolyLog[2, E^(-2*A
rcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh
[c*x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x])] - 6*Sinh[2*
ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arctanh}(cx)}{x(cdx + d)^3} dx$$
$$\Big\downarrow \text{ 6502}$$
$$\int \left(\frac{a + b\text{arctanh}(cx)}{d^3x} - \frac{c(a + b\text{arctanh}(cx))}{d^3(cx + 1)} - \frac{c(a + b\text{arctanh}(cx))}{d^3(cx + 1)^2} - \frac{c(a + b\text{arctanh}(cx))}{d^3(cx + 1)^3}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{a+b\text{arctanh}(cx)}{d^3(cx+1)}+\frac{a+b\text{arctanh}(cx)}{2d^3(cx+1)^2}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^3}+\frac{a\log(x)}{d^3}-\frac{5b\text{arctanh}(cx)}{8d^3}-\frac{b\,\text{PolyLog}(2,-cx)}{2d^3}+\frac{b\,\text{PolyLog}(2,cx)}{2d^3}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{5b}{8d^3(cx+1)}+\frac{b}{8d^3(cx+1)^2}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

output
```
b/(8*d^3*(1 + c*x)^2) + (5*b)/(8*d^3*(1 + c*x)) - (5*b*ArcTanh[c*x])/(8*d^
3) + (a + b*ArcTanh[c*x])/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])/(d^3*
(1 + c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3
- (b*PolyLog[2, -(c*x)])/(2*d^3) + (b*PolyLog[2, c*x])/(2*d^3) - (b*PolyLo
g[2, 1 - 2/(1 + c*x)])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.18

method	result
parts	$\frac{a\left(\frac{1}{2(cx+1)^2}+\frac{1}{cx+1}-\ln(cx+1)+\ln(x)\right)}{d^3}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{\operatorname{arctanh}(cx)}{cx+1}-\operatorname{arctanh}(cx)\ln(cx+1)-\right.}{}$
derivativedivides	$\frac{a\left(\ln(cx)+\frac{1}{2(cx+1)^2}+\frac{1}{cx+1}-\ln(cx+1)\right)}{d^3}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{\operatorname{arctanh}(cx)}{cx+1}-\operatorname{arctanh}(cx)\ln(cx+1)-\right.}{}$
default	$\frac{a\left(\ln(cx)+\frac{1}{2(cx+1)^2}+\frac{1}{cx+1}-\ln(cx+1)\right)}{d^3}+\frac{b\left(\operatorname{arctanh}(cx)\ln(cx)+\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}+\frac{\operatorname{arctanh}(cx)}{cx+1}-\operatorname{arctanh}(cx)\ln(cx+1)-\right.}{}$
risch	$-\frac{5b\ln(-cx-1)}{16d^3}-\frac{b\ln(-cx+1)cx}{4d^3(-cx-1)}+\frac{b\ln(-cx+1)}{4d^3(-cx-1)}+\frac{b\operatorname{dilog}(-cx+1)}{2d^3}-\frac{b\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d^3}+$

input
```
int((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*(1/2/(c*x+1)^2+1/(c*x+1)-ln(c*x+1)+ln(x))+b/d^3*(arctanh(c*x)*ln(c*x
)+1/2/(c*x+1)^2*arctanh(c*x)+1/(c*x+1)*arctanh(c*x)-arctanh(c*x)*ln(c*x+1)
-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1)+1/4*ln(c*x+1)^2-1/2
*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/2*dilog(1/2*c*x+1/2)+5/16*
ln(c*x-1)+1/8/(c*x+1)^2+5/8/(c*x+1)-5/16*ln(c*x+1))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+cdx)^3}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c^3*d^3*x^4 + 3*c^2*d^3*x^3 + 3*c*d^3*x^2 +
 d^3*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)^3}\,dx = \frac{\int \frac{a}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{b\,\text{atanh}\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

input
```
integrate((a+b*atanh(c*x))/x/(c*d*x+d)**3,x)
```

output
```
(Integral(a/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b*atan
h(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)^3}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)^3x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*a*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 +
2*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^
4 + 3*c^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)^3}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)^3x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)^3} dx = \int \frac{a + b\,\text{atanh}(c\,x)}{x\,(d + c\,d\,x)^3} dx$$

input
```
int((a + b*atanh(c*x))/(x*(d + c*d*x)^3),x)
```

output
```
int((a + b*atanh(c*x))/(x*(d + c*d*x)^3), x)
```

Reduce [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + cdx)^3} dx = \frac{-4atanh(cx)^2 bc^2x^2 - 8atanh(cx)^2 bcx - 4atanh(cx)^2 b + 8atanh(cx) bcx + 16atanh(cx) b - 32\left(\int \frac{}{c^4x^5+}\right.}{}$$

input
```
int((a+b*atanh(c*x))/x/(c*d*x+d)^3,x)
```

output
```
( - 4*atanh(c*x)**2*b*c**2*x**2 - 8*atanh(c*x)**2*b*c*x - 4*atanh(c*x)**2*
b + 8*atanh(c*x)*b*c*x + 16*atanh(c*x)*b - 32*int(atanh(c*x)/(c**4*x**5 +
2*c**3*x**4 - 2*c*x**2 - x),x)*b*c**2*x**2 - 64*int(atanh(c*x)/(c**4*x**5
+ 2*c**3*x**4 - 2*c*x**2 - x),x)*b*c*x - 32*int(atanh(c*x)/(c**4*x**5 + 2*
c**3*x**4 - 2*c*x**2 - x),x)*b + 3*log(c*x - 1)*b*c**2*x**2 + 6*log(c*x -
1)*b*c*x + 3*log(c*x - 1)*b - 32*log(c*x + 1)*a*c**2*x**2 - 64*log(c*x + 1
)*a*c*x - 32*log(c*x + 1)*a - 3*log(c*x + 1)*b*c**2*x**2 - 6*log(c*x + 1)*
b*c*x - 3*log(c*x + 1)*b + 32*log(x)*a*c**2*x**2 + 64*log(x)*a*c*x + 32*lo
g(x)*a - 16*a*c**2*x**2 + 32*a - 3*b*c**2*x**2 + 5*b)/(32*d**3*(c**2*x**2
+ 2*c*x + 1))
```

3.64 $\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 218

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^3}\,dx = & -\frac{bc}{8d^3(1+cx)^2}-\frac{9bc}{8d^3(1+cx)}+\frac{9bc\text{arctanh}(cx)}{8d^3} \\ & -\frac{a+b\text{arctanh}(cx)}{d^3x}-\frac{c(a+b\text{arctanh}(cx))}{2d^3(1+cx)^2} \\ & -\frac{2c(a+b\text{arctanh}(cx))}{d^3(1+cx)}-\frac{3ac\log(x)}{d^3} \\ & +\frac{bc\log(x)}{d^3}-\frac{3c(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3} \\ & -\frac{bc\log(1-c^2x^2)}{2d^3}+\frac{3bc\,\text{PolyLog}(2,-cx)}{2d^3} \\ & -\frac{3bc\,\text{PolyLog}(2,cx)}{2d^3}+\frac{3bc\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d^3}\end{aligned}$$

output
```
-1/8*b*c/d^3/(c*x+1)^2-9/8*b*c/d^3/(c*x+1)+9/8*b*c*arctanh(c*x)/d^3-(a+b*a
rctanh(c*x))/d^3/x-1/2*c*(a+b*arctanh(c*x))/d^3/(c*x+1)^2-2*c*(a+b*arctanh
(c*x))/d^3/(c*x+1)-3*a*c*ln(x)/d^3+b*c*ln(x)/d^3-3*c*(a+b*arctanh(c*x))*ln
(2/(c*x+1))/d^3-1/2*b*c*ln(-c^2*x^2+1)/d^3+3/2*b*c*polylog(2,-c*x)/d^3-3/2
*b*c*polylog(2,c*x)/d^3+3/2*b*c*polylog(2,1-2/(c*x+1))/d^3
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.85

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^3}\,dx$$
$$= \frac{-\frac{32a}{x}-\frac{16ac}{(1+cx)^2}-\frac{64ac}{1+cx}-96ac\log(x)+96ac\log(1+cx)+bc\Big(-20\cosh(2\text{arctanh}(cx))-\cosh(4\text{arctanh}(cx)\Big.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

output
```
((-32*a)/x - (16*a*c)/(1 + c*x)^2 - (64*a*c)/(1 + c*x) - 96*a*c*Log[x] + 9
6*a*c*Log[1 + c*x] + b*c*(-20*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]]
+ 32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 48*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2
0*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-8/(c*x) -
 10*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 24*Log[1 - E^(-2*ArcTanh
[c*x])] + 10*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*d^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(cdx+d)^3}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \Big(\frac{3c^2(a+b\text{arctanh}(cx))}{d^3(cx+1)}+\frac{2c^2(a+b\text{arctanh}(cx))}{d^3(cx+1)^2}+\frac{c^2(a+b\text{arctanh}(cx))}{d^3(cx+1)^3}+\frac{a+b\text{arctanh}(cx)}{d^3x^2}-\frac{3c(a+b\text{arc}}{d^3}$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2c(a+b\text{arctanh}(cx))}{d^3(cx+1)}-\frac{c(a+b\text{arctanh}(cx))}{2d^3(cx+1)^2}-\frac{a+b\text{arctanh}(cx)}{d^3x}-\frac{3c\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^3}-\frac{3ac\log(x)}{d^3}+\frac{9bc\text{arctanh}(cx)}{8d^3}-\frac{bc\log(1-c^2x^2)}{2d^3}+\frac{3bc\,\text{PolyLog}(2,-cx)}{2d^3}-\frac{3bc\,\text{PolyLog}(2,cx)}{2d^3}+\frac{3bc\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}-\frac{9bc}{8d^3(cx+1)}-\frac{bc}{8d^3(cx+1)^2}+\frac{bc\log(x)}{d^3}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

output
```
-1/8*(b*c)/(d^3*(1 + c*x)^2) - (9*b*c)/(8*d^3*(1 + c*x)) + (9*b*c*ArcTanh[
c*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(d^3*x) - (c*(a + b*ArcTanh[c*x]))/(2
*d^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) - (3*a*c*Lo
g[x])/d^3 + (b*c*Log[x])/d^3 - (3*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])
/d^3 - (b*c*Log[1 - c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, -(c*x)])/(2*d^3)
 - (3*b*c*PolyLog[2, c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2/(1 + c*x)])/(
2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02

method	result
parts	$\frac{a\left(-\frac{c}{2(cx+1)^2}-\frac{2c}{cx+1}+3c\ln(cx+1)-\frac{1}{x}-3c\ln(x)\right)}{d^3}+\frac{bc\left(-\frac{\operatorname{arctanh}(cx)}{cx}-3\operatorname{arctanh}(cx)\ln(cx)-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{2\operatorname{arctanh}(}{cx+1}\right.}{}$
derivativedivides	$c\left(\frac{a\left(-\frac{1}{cx}-3\ln(cx)-\frac{1}{2(cx+1)^2}-\frac{2}{cx+1}+3\ln(cx+1)\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx}-3\operatorname{arctanh}(cx)\ln(cx)-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{2\operatorname{arcta}}{cx}\right.}{}\right.$
default	$c\left(\frac{a\left(-\frac{1}{cx}-3\ln(cx)-\frac{1}{2(cx+1)^2}-\frac{2}{cx+1}+3\ln(cx+1)\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{cx}-3\operatorname{arctanh}(cx)\ln(cx)-\frac{\operatorname{arctanh}(cx)}{2(cx+1)^2}-\frac{2\operatorname{arcta}}{cx}\right.}{}\right.$
risch	$\frac{cb}{8d^3(-cx-1)}-\frac{a}{d^3x}-\frac{ca}{2d^3(-cx-1)^2}+\frac{2ca}{d^3(-cx-1)}-\frac{3cb\operatorname{dilog}(-cx+1)}{2d^3}+\frac{3cb\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{2d^3}+\frac{cb\ln(-cx)}{2d^3}$

input
```
int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*(-1/2/(c*x+1)^2*c-2*c/(c*x+1)+3*c*ln(c*x+1)-1/x-3*c*ln(x))+b/d^3*c*(
-arctanh(c*x)/c/x-3*arctanh(c*x)*ln(c*x)-1/2/(c*x+1)^2*arctanh(c*x)-2/(c*x
+1)*arctanh(c*x)+3*arctanh(c*x)*ln(c*x+1)-17/16*ln(c*x-1)+ln(c*x)-1/8/(c*x
+1)^2-9/8/(c*x+1)+1/16*ln(c*x+1)+3/2*dilog(c*x)+3/2*dilog(c*x+1)+3/2*ln(c*
x)*ln(c*x+1)-3/4*ln(c*x+1)^2+3/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1
/2)-3/2*dilog(1/2*c*x+1/2))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+cdx)^3}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 +
 d^3*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + cdx)^3}\,dx = \frac{\int \frac{a}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{b\,\text{atanh}\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

input
```
integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b*a
tanh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + cdx)^3}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)^3 x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*a*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c*
log(c*x + 1)/d^3 + 6*c*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-
c*x + 1))/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + cdx)^3}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(cdx + d)^3 x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^3}\,dx=\int \frac{a+b\,\text{atanh}(c\,x)}{x^2\,{(d+c\,d\,x)}^3}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^3),x)
```

output
```
int((a + b*atanh(c*x))/(x^2*(d + c*d*x)^3), x)
```

Reduce [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+cdx)^3}\,dx$$
$$=\frac{-64a-12atanh(cx)^2\,b\,c^3x^3+192\log(cx+1)\,a\,c^3x^3-32\log(x)\,b\,c^3x^3-384\log(x)\,a\,c^2x^2-12atanh(cx}{}$$

input
```
int((a+b*atanh(c*x))/x^2/(c*d*x+d)^3,x)
```

output
```
( - 12*atanh(c*x)**2*b*c**3*x**3 - 24*atanh(c*x)**2*b*c**2*x**2 - 12*atanh
(c*x)**2*b*c*x - 12*atanh(c*x)*b*c**3*x**3 + 36*atanh(c*x)*b*c*x + 32*atan
h(c*x)*b - 96*int(atanh(c*x)/(c**4*x**6 + 2*c**3*x**5 - 2*c*x**3 - x**2),x
)*b*c**2*x**3 - 192*int(atanh(c*x)/(c**4*x**6 + 2*c**3*x**5 - 2*c*x**3 - x
**2),x)*b*c*x**2 - 96*int(atanh(c*x)/(c**4*x**6 + 2*c**3*x**5 - 2*c*x**3 -
 x**2),x)*b*x + 7*log(c*x - 1)*b*c**3*x**3 + 14*log(c*x - 1)*b*c**2*x**2 +
 7*log(c*x - 1)*b*c*x + 192*log(c*x + 1)*a*c**3*x**3 + 384*log(c*x + 1)*a*
c**2*x**2 + 192*log(c*x + 1)*a*c*x + 25*log(c*x + 1)*b*c**3*x**3 + 50*log(
c*x + 1)*b*c**2*x**2 + 25*log(c*x + 1)*b*c*x - 192*log(x)*a*c**3*x**3 - 38
4*log(x)*a*c**2*x**2 - 192*log(x)*a*c*x - 32*log(x)*b*c**3*x**3 - 64*log(x
)*b*c**2*x**2 - 32*log(x)*b*c*x + 96*a*c**3*x**3 - 192*a*c*x - 64*a + 3*b*
c**3*x**3 - 5*b*c*x)/(64*d**3*x*(c**2*x**2 + 2*c*x + 1))
```

3.65 $\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 268

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^3}\,dx = -\frac{bc}{2d^3x}+\frac{bc^2}{8d^3(1+cx)^2}+\frac{13bc^2}{8d^3(1+cx)}-\frac{9bc^2\text{arctanh}(cx)}{8d^3} -\frac{a+b\text{arctanh}(cx)}{2d^3x^2}+\frac{3c(a+b\text{arctanh}(cx))}{d^3x} +\frac{c^2(a+b\text{arctanh}(cx))}{2d^3(1+cx)^2}+\frac{3c^2(a+b\text{arctanh}(cx))}{d^3(1+cx)}+\frac{6ac^2\log(x)}{d^3} -\frac{3bc^2\log(x)}{d^3}+\frac{6c^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3} +\frac{3bc^2\log(1-c^2x^2)}{2d^3}-\frac{3bc^2\,\text{PolyLog}(2,-cx)}{d^3} +\frac{3bc^2\,\text{PolyLog}(2,cx)}{d^3}-\frac{3bc^2\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^3}$$

output
```
-1/2*b*c/d^3/x+1/8*b*c^2/d^3/(c*x+1)^2+13/8*b*c^2/d^3/(c*x+1)-9/8*b*c^2*ar
ctanh(c*x)/d^3-1/2*(a+b*arctanh(c*x))/d^3/x^2+3*c*(a+b*arctanh(c*x))/d^3/x
+1/2*c^2*(a+b*arctanh(c*x))/d^3/(c*x+1)^2+3*c^2*(a+b*arctanh(c*x))/d^3/(c*
x+1)+6*a*c^2*ln(x)/d^3-3*b*c^2*ln(x)/d^3+6*c^2*(a+b*arctanh(c*x))*ln(2/(c*
x+1))/d^3+3/2*b*c^2*ln(-c^2*x^2+1)/d^3-3*b*c^2*polylog(2,-c*x)/d^3+3*b*c^2
*polylog(2,c*x)/d^3-3*b*c^2*polylog(2,1-2/(c*x+1))/d^3
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(d+cdx)^3}\,dx$$
$$= \frac{-\frac{16a}{x^2}+\frac{96ac}{x}+\frac{16ac^2}{(1+cx)^2}+\frac{96ac^2}{1+cx}+192ac^2\log(x)-192ac^2\log(1+cx)+bc^2\left(-\frac{16}{cx}+28\cosh(2\operatorname{arctanh}(cx))\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

output
```
((-16*a)/x^2 + (96*a*c)/x + (16*a*c^2)/(1 + c*x)^2 + (96*a*c^2)/(1 + c*x)
+ 192*a*c^2*Log[x] - 192*a*c^2*Log[1 + c*x] + b*c^2*(-16/(c*x) + 28*Cosh[2
*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 96*Log[(c*x)/Sqrt[1 - c^2*x^2]] -
96*PolyLog[2, E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c
*x]*(4 - 4/(c^2*x^2) + 24/(c*x) + 14*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh
[c*x]] + 48*Log[1 - E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] - Sinh[
4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(cdx+d)^3}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(-\frac{6c^3(a+b\operatorname{arctanh}(cx))}{d^3(cx+1)}-\frac{3c^3(a+b\operatorname{arctanh}(cx))}{d^3(cx+1)^2}-\frac{c^3(a+b\operatorname{arctanh}(cx))}{d^3(cx+1)^3}+\frac{6c^2(a+b\operatorname{arctanh}(cx))}{d^3x}+\frac{a+\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{3c^2(a+b\operatorname{arctanh}(cx))}{d^3(cx+1)}+\frac{c^2(a+b\operatorname{arctanh}(cx))}{2d^3(cx+1)^2}+\frac{6c^2\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{d^3}-\frac{a+b\operatorname{arctanh}(cx)}{2d^3x^2}+\frac{3c(a+b\operatorname{arctanh}(cx))}{d^3x}+\frac{6ac^2\log(x)}{d^3}-\frac{9bc^2\operatorname{arctanh}(cx)}{8d^3}-\frac{3bc^2\operatorname{PolyLog}(2,-cx)}{d^3}+\frac{3bc^2\operatorname{PolyLog}(2,cx)}{d^3}-\frac{3bc^2\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^3}+\frac{3bc^2\log\left(1-c^2x^2\right)}{2d^3}+\frac{13bc^2}{8d^3(cx+1)}+\frac{bc^2}{8d^3(cx+1)^2}-\frac{3bc^2\log(x)}{d^3}-\frac{bc}{2d^3x}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

output
```
-1/2*(b*c)/(d^3*x) + (b*c^2)/(8*d^3*(1 + c*x)^2) + (13*b*c^2)/(8*d^3*(1 +
c*x)) - (9*b*c^2*ArcTanh[c*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(2*d^3*x^2)
+ (3*c*(a + b*ArcTanh[c*x]))/(d^3*x) + (c^2*(a + b*ArcTanh[c*x]))/(2*d^3*(
1 + c*x)^2) + (3*c^2*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) + (6*a*c^2*Log[
x])/d^3 - (3*b*c^2*Log[x])/d^3 + (6*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*
x)])/d^3 + (3*b*c^2*Log[1 - c^2*x^2])/(2*d^3) - (3*b*c^2*PolyLog[2, -(c*x)
])/d^3 + (3*b*c^2*PolyLog[2, c*x])/d^3 - (3*b*c^2*PolyLog[2, 1 - 2/(1 + c*
x)])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

method	result
derivativedivides	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}+\frac{3}{cx}+6\ln(cx)+\frac{1}{2(cx+1)^2}+\frac{3}{cx+1}-6\ln(cx+1)\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\frac{3\operatorname{arctanh}(cx)}{cx}+6\operatorname{arctanh}(cx)\ln(c\right.}{}\right.$
default	$c^2\left(\frac{a\left(-\frac{1}{2c^2x^2}+\frac{3}{cx}+6\ln(cx)+\frac{1}{2(cx+1)^2}+\frac{3}{cx+1}-6\ln(cx+1)\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\frac{3\operatorname{arctanh}(cx)}{cx}+6\operatorname{arctanh}(cx)\ln(c\right.}{}\right.$
parts	$\frac{a\left(\frac{c^2}{2(cx+1)^2}+\frac{3c^2}{cx+1}-6c^2\ln(cx+1)-\frac{1}{2x^2}+\frac{3c}{x}+6c^2\ln(x)\right)}{d^3}+\frac{b\,c^2\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}+\frac{3\operatorname{arctanh}(cx)}{cx}+6\operatorname{arctanh}(cx)\ln(cx)+\right.}{}$
risch	$-\frac{3c^2b\ln(cx+1)^2}{2d^3}-\frac{7c^2b\ln(cx)}{4d^3}-\frac{b\ln(cx+1)}{4d^3x^2}-\frac{3c^2b\operatorname{dilog}(cx+1)}{d^3}-\frac{5c^2b\ln(-cx)}{4d^3}+\frac{5c^2b\ln(-cx+1)}{4d^3}+\frac{b\ln(}{4}$

input
```
int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d^3*(-1/2/c^2/x^2+3/c/x+6*ln(c*x)+1/2/(c*x+1)^2+3/(c*x+1)-6*ln(c*x+
1))+b/d^3*(-1/2*arctanh(c*x)/c^2/x^2+3*arctanh(c*x)/c/x+6*arctanh(c*x)*ln(
c*x)+1/2/(c*x+1)^2*arctanh(c*x)+3/(c*x+1)*arctanh(c*x)-6*arctanh(c*x)*ln(c
*x+1)-3*dilog(c*x)-3*dilog(c*x+1)-3*ln(c*x)*ln(c*x+1)+3/2*ln(c*x+1)^2-3*(l
n(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+3*dilog(1/2*c*x+1/2)+33/16*ln(c
*x-1)-1/2/c/x-3*ln(c*x)+1/8/(c*x+1)^2+13/8/(c*x+1)+15/16*ln(c*x+1)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3(d+cdx)^3}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(cdx+d)^3x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 +
 d^3*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{arctanh}(cx)}{x^3(d + cdx)^3} \, dx = \frac{\int \frac{a}{c^3x^6+3c^2x^5+3cx^4+x^3} \, dx + \int \frac{b \operatorname{atanh}(cx)}{c^3x^6+3c^2x^5+3cx^4+x^3} \, dx}{d^3}$$

input
```
integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**3,x)
```

output
```
(Integral(a/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x) + Integral(b*a
tanh(c*x)/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\operatorname{arctanh}(cx)}{x^3(d + cdx)^3} \, dx = \int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)^3 x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*a*((12*c^3*x^3 + 18*c^2*x^2 + 4*c*x - 1)/(c^2*d^3*x^4 + 2*c*d^3*x^3 +
d^3*x^2) - 12*c^2*log(c*x + 1)/d^3 + 12*c^2*log(x)/d^3) + 1/2*b*integrate(
(log(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4
+ d^3*x^3), x)
```

Giac [F]

$$\int \frac{a + b\operatorname{arctanh}(cx)}{x^3(d + cdx)^3} \, dx = \int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)^3 x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^3}\,dx=\int \frac{a+b\,\text{atanh}(c\,x)}{x^3\,{\left(d+c\,d\,x\right)}^3}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^3),x)
```

output
```
int((a + b*atanh(c*x))/(x^3*(d + c*d*x)^3), x)
```

Reduce [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+cdx)^3}\,dx$$
$$=\frac{-32a-24atanh(cx)^2\,b\,c^3x^3-768\log(cx+1)\,a\,c^3x^3-96\left(\int \frac{atanh(cx)}{c^4x^7+2c^3x^6-2c\,x^4-x^3}dx\right)b\,c^2x^4-192\left(\int \frac{}{c^4x^7+}\right.}{}$$

input
```
int((a+b*atanh(c*x))/x^3/(c*d*x+d)^3,x)
```

output
```
( - 12*atanh(c*x)**2*b*c**4*x**4 - 24*atanh(c*x)**2*b*c**3*x**3 - 12*atanh
(c*x)**2*b*c**2*x**2 - 12*atanh(c*x)*b*c**4*x**4 + 36*atanh(c*x)*b*c**2*x*
*2 + 32*atanh(c*x)*b*c*x + 16*atanh(c*x)*b - 96*int(atanh(c*x)/(c**4*x**7
+ 2*c**3*x**6 - 2*c*x**4 - x**3),x)*b*c**2*x**4 - 192*int(atanh(c*x)/(c**4
*x**7 + 2*c**3*x**6 - 2*c*x**4 - x**3),x)*b*c*x**3 - 96*int(atanh(c*x)/(c*
*4*x**7 + 2*c**3*x**6 - 2*c*x**4 - x**3),x)*b*x**2 + 9*log(c*x - 1)*b*c**4
*x**4 + 18*log(c*x - 1)*b*c**3*x**3 + 9*log(c*x - 1)*b*c**2*x**2 - 384*log
(c*x + 1)*a*c**4*x**4 - 768*log(c*x + 1)*a*c**3*x**3 - 384*log(c*x + 1)*a*
c**2*x**2 - 9*log(c*x + 1)*b*c**4*x**4 - 18*log(c*x + 1)*b*c**3*x**3 - 9*l
og(c*x + 1)*b*c**2*x**2 + 384*log(x)*a*c**4*x**4 + 768*log(x)*a*c**3*x**3
+ 384*log(x)*a*c**2*x**2 - 192*a*c**4*x**4 + 384*a*c**2*x**2 + 128*a*c*x -
 32*a - 15*b*c**4*x**4 + 33*b*c**2*x**2 + 16*b*c*x)/(64*d**3*x**2*(c**2*x*
*2 + 2*c*x + 1))
```

3.66 $\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4} dx = -\frac{b}{18c(1+cx)^3} - \frac{b}{24c(1+cx)^2} - \frac{b}{24c(1+cx)} + \frac{b\text{arctanh}(cx)}{24c} - \frac{a+b\text{arctanh}(cx)}{3c(1+cx)^3}$$

output
```
-1/18*b/c/(c*x+1)^3-1/24*b/c/(c*x+1)^2-1/24*b/c/(c*x+1)+1/24*b*arctanh(c*x
)/c-1/3*(a+b*arctanh(c*x))/c/(c*x+1)^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4} dx = -\frac{48a+2b(10+9cx+3c^2x^2)+48b\text{arctanh}(cx)+3b(1+cx)^3\log(1-cx)-3b(1+cx)^3\log(1+cx)}{144c(1+cx)^3}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

output
```
-1/144*(48*a + 2*b*(10 + 9*c*x + 3*c^2*x^2) + 48*b*ArcTanh[c*x] + 3*b*(1 +
 c*x)^3*Log[1 - c*x] - 3*b*(1 + c*x)^3*Log[1 + c*x])/(c*(1 + c*x)^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6478, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(cx+1)^4}dx$$

$$\Big\downarrow \text{6478}$$

$$\frac{1}{3}b\int \frac{1}{(cx+1)^3(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{3c(cx+1)^3}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{3}b\int \frac{1}{(1-cx)(cx+1)^4}dx-\frac{a+b\text{arctanh}(cx)}{3c(cx+1)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}b\int \left(\frac{1}{8(cx+1)^2}+\frac{1}{4(cx+1)^3}+\frac{1}{2(cx+1)^4}-\frac{1}{8(c^2x^2-1)}\right)dx-\frac{a+b\text{arctanh}(cx)}{3c(cx+1)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}b\left(\frac{\text{arctanh}(cx)}{8c}-\frac{1}{8c(cx+1)}-\frac{1}{8c(cx+1)^2}-\frac{1}{6c(cx+1)^3}\right)-\frac{a+b\text{arctanh}(cx)}{3c(cx+1)^3}$$

input
```
Int[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x])/(c*(1 + c*x)^3) + (b*(-1/6*1/(c*(1 + c*x)^3) - 1
/(8*c*(1 + c*x)^2) - 1/(8*c*(1 + c*x)) + ArcTanh[c*x]/(8*c)))/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{a}{3(cx+1)^3}+b\left(-\frac{\operatorname{arctanh}(cx)}{3(cx+1)^3}-\frac{\ln(cx-1)}{48}-\frac{1}{18(cx+1)^3}-\frac{1}{24(cx+1)^2}-\frac{1}{24(cx+1)}+\frac{\ln(cx+1)}{48}\right)}{c}$
default	$\frac{-\frac{a}{3(cx+1)^3}+b\left(-\frac{\operatorname{arctanh}(cx)}{3(cx+1)^3}-\frac{\ln(cx-1)}{48}-\frac{1}{18(cx+1)^3}-\frac{1}{24(cx+1)^2}-\frac{1}{24(cx+1)}+\frac{\ln(cx+1)}{48}\right)}{c}$
parts	$-\frac{a}{3(cx+1)^3c}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)}{3(cx+1)^3}-\frac{\ln(cx-1)}{48}-\frac{1}{18(cx+1)^3}-\frac{1}{24(cx+1)^2}-\frac{1}{24(cx+1)}+\frac{\ln(cx+1)}{48}\right)}{c}$
parallelrisch	$-\frac{-72a\,c^2x^2-21bcx-9\operatorname{arctanh}(cx)bcx-9\operatorname{arctanh}(cx)b\,c^2x^2-24a\,c^3x^3-27b\,c^2x^2-72acx+21b\operatorname{arctanh}(cx)-10b\,c^3x}{72(cx+1)^3c}$
orering	$-\frac{(40c^4x^4+65x^3c^3-33c^2x^2-93cx+21)(a+b\operatorname{arctanh}(cx))}{72(cx+1)^3c}-\frac{x(10c^2x^2+27cx+21)(cx-1)(cx+1)^2\left(\frac{bc}{(-c^2x^2+1)(cx-}\right.}{72c}$
risch	$-\frac{b\ln(cx+1)}{6c(cx+1)^3}-\frac{3\ln(cx-1)b\,c^3x^3-3\ln(-cx-1)b\,c^3x^3+9\ln(cx-1)b\,c^2x^2-9\ln(-cx-1)b\,c^2x^2+6b\,c^2x^2+9\ln(cx-1)b}{144(cx+1)^3c}$

input
```
int((a+b*arctanh(c*x))/(c*x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*a/(c*x+1)^3+b*(-1/3/(c*x+1)^3*arctanh(c*x)-1/48*ln(c*x-1)-1/18/(
c*x+1)^3-1/24/(c*x+1)^2-1/24/(c*x+1)+1/48*ln(c*x+1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4}\,dx = -\frac{6\,bc^2x^2+18\,bcx-3\,(bc^3x^3+3\,bc^2x^2+3\,bcx-7\,b)\log\left(-\frac{cx+1}{cx-1}\right)+48\,a+20\,b}{144\,(c^4x^3+3\,c^3x^2+3\,c^2x+c)}$$

input
```
integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="fricas")
```

output
```
-1/144*(6*b*c^2*x^2 + 18*b*c*x - 3*(b*c^3*x^3 + 3*b*c^2*x^2 + 3*b*c*x - 7*
b)*log(-(c*x + 1)/(c*x - 1)) + 48*a + 20*b)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x
 + c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(63) = 126.

Time = 1.19 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.68

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4}\,dx = \begin{cases} -\frac{24a}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{3bc^3x^3\,\text{atanh}\,(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c}+\frac{9bc^2x^2\,\text{atanh}\,(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c}-\frac{3bc^2x^2}{72c^4x^3+216c^3x^2+216c^2x+72} \\ ax \end{cases}$$

input
```
integrate((a+b*atanh(c*x))/(c*x+1)**4,x)
```

output
```
Piecewise((-24*a/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 3*b*
c**3*x**3*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) +
9*b*c**2*x**2*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c
) - 3*b*c**2*x**2/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*b
*c*x*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 9*b*c
*x/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 21*b*atanh(c*x)/(7
2*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 10*b/(72*c**4*x**3 + 21
6*c**3*x**2 + 216*c**2*x + 72*c), Ne(c, 0)), (a*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.65

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4}\,dx = -\frac{1}{144}\left(c\left(\frac{2\,(3\,c^2x^2+9\,cx+10)}{c^5x^3+3\,c^4x^2+3\,c^3x+c^2}-\frac{3\,\log\left(cx+1\right)}{c^2}+\frac{3\,\log\left(cx-1\right)}{c^2}\right)+\frac{48\,\text{artanh}\left(cx\right)}{c^4x^3+3\,c^3x^2+3\,c^2x+c}\right) - \frac{a}{3\,(c^4x^3+3\,c^3x^2+3\,c^2x+c)}$$

input
```
integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="maxima")
```

output
```
-1/144*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2
) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 +
3*c^3*x^2 + 3*c^2*x + c))*b - 1/3*a/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.01

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4}\,dx = \frac{1}{288}\,c\left(\frac{6\,(cx-1)^3\left(\frac{3\,(cx+1)^2b}{(cx-1)^2}-\frac{3\,(cx+1)b}{cx-1}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)^3c^2}+\frac{(cx-1)^3\left(\frac{36\,(cx+1)^2a}{(cx-1)^2}-\frac{36\,(cx+1)a}{cx-1}+12\,a+\frac{18}{(}\right.}{(cx+1)^3c^2}\right.$$

input
```
integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="giac")
```

output
```
1/288*c*(6*(c*x - 1)^3*(3*(c*x + 1)^2*b/(c*x - 1)^2 - 3*(c*x + 1)*b/(c*x -
 1) + b)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (c*x - 1)^3*(36*(c*
x + 1)^2*a/(c*x - 1)^2 - 36*(c*x + 1)*a/(c*x - 1) + 12*a + 18*(c*x + 1)^2*
b/(c*x - 1)^2 - 9*(c*x + 1)*b/(c*x - 1) + 2*b)/((c*x + 1)^3*c^2))
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.74

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4}\,dx = \frac{\frac{b\,c^2\,x^3}{8}-\frac{b\,x}{8}-\frac{b\,\mathrm{atanh}(c\,x)}{3\,c}-\frac{12\,a+5\,b}{36\,c}+\frac{b\,c^3\,x^4}{24}+\frac{c\,x^2\,(24\,a+7\,b)}{72}+\frac{b\,c\,x^2\,\mathrm{atanh}(c\,x)}{3}}{-c^5\,x^5-3\,c^4\,x^4-2\,c^3\,x^3+2\,c^2\,x^2+3\,c\,x+1} - \frac{b\,\ln\left(c^2\,x^2-1\right)}{48\,c}+\frac{b\,\ln\left(c\,x+1\right)}{24\,c}$$

input
```
int((a + b*atanh(c*x))/(c*x + 1)^4,x)
```

output
```
((b*c^2*x^3)/8 - (b*x)/8 - (b*atanh(c*x))/(3*c) - (12*a + 5*b)/(36*c) + (b
*c^3*x^4)/24 + (c*x^2*(24*a + 7*b))/72 + (b*c*x^2*atanh(c*x))/3)/(3*c*x +
2*c^2*x^2 - 2*c^3*x^3 - 3*c^4*x^4 - c^5*x^5 + 1) - (b*log(c^2*x^2 - 1))/(4
8*c) + (b*log(c*x + 1))/(24*c)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int \frac{a+b\text{arctanh}(cx)}{(1+cx)^4}\,dx = \frac{-48atanh(cx)\,b-3\log(cx-1)\,b\,c^3x^3-9\log(cx-1)\,b\,c^2x^2-9\log(cx-1)\,bcx-3\log(cx-1)\,b+3\log}{144c\,(c^3x^3+3}$$

input
```
int((a+b*atanh(c*x))/(c*x+1)^4,x)
```

output
```
( - 48*atanh(c*x)*b - 3*log(c*x - 1)*b*c**3*x**3 - 9*log(c*x - 1)*b*c**2*x
**2 - 9*log(c*x - 1)*b*c*x - 3*log(c*x - 1)*b + 3*log(c*x + 1)*b*c**3*x**3
 + 9*log(c*x + 1)*b*c**2*x**2 + 9*log(c*x + 1)*b*c*x + 3*log(c*x + 1)*b -
48*a + 2*b*c**3*x**3 - 12*b*c*x - 18*b)/(144*c*(c**3*x**3 + 3*c**2*x**2 +
3*c*x + 1))
```

3.67 $\int \frac{\text{arctanh}(ax)}{cx+acx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{\text{arctanh}(ax)}{cx+acx^2}\,dx = \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c}$$

output
```
arctanh(a*x)*ln(2-2/(a*x+1))/c-1/2*polylog(2,-1+2/(a*x+1))/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{\text{arctanh}(ax)}{cx+acx^2}\,dx = \frac{\text{arctanh}(ax)\log\left(1-e^{-2\text{arctanh}(ax)}\right)}{c} - \frac{\text{PolyLog}\left(2,e^{-2\text{arctanh}(ax)}\right)}{2c}$$

input
```
Integrate[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

output
```
(ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])])/c - PolyLog[2, E^(-2*ArcTanh[a
*x])]/(2*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2026, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{acx^2+cx}dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{\text{arctanh}(ax)}{x(acx+c)}dx$$
$$\Big\downarrow \text{6494}$$
$$\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx}{c}$$
$$\Big\downarrow \text{2897}$$
$$\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}$$

input
```
Int[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

output
```
(ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - PolyLog[2, -1 + 2/(1 + a*x)]/(2*c)
```

Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

method	result
risch	$-\frac{\ln(ax+1)^2}{4c}-\frac{\text{dilog}(ax+1)}{2c}-\frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2c}+\frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln(-ax+1)}{2c}+\frac{\text{dilog}(-ax+1)}{2c}-\frac{\text{dilog}(-}{2c}$
derivativedivides	$\frac{\frac{a\ \text{arctanh}(ax)\ln(ax)}{c}-\frac{a\ \text{arctanh}(ax)\ln(ax+1)}{c}-\frac{a\left(-\frac{\ln(ax+1)^2}{4}+\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\text{dilog}\right.}{c}}{a}$
default	$\frac{\frac{a\ \text{arctanh}(ax)\ln(ax)}{c}-\frac{a\ \text{arctanh}(ax)\ln(ax+1)}{c}-\frac{a\left(-\frac{\ln(ax+1)^2}{4}+\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\text{dilog}\right.}{c}}{a}$
parts	$\frac{\text{arctanh}(ax)\ln(x)}{c}-\frac{\text{arctanh}(ax)\ln(ax+1)}{c}-\frac{a\left(\frac{-\frac{\ln(ax+1)^2}{4}+\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}}{a}\right.}{c}$

input
```
int(arctanh(a*x)/(a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c*ln(a*x+1)^2-1/2/c*dilog(a*x+1)-1/2/c*ln(1/2*a*x+1/2)*ln(-1/2*a*x+1/
2)+1/2/c*ln(1/2*a*x+1/2)*ln(-a*x+1)+1/2/c*dilog(-a*x+1)-1/2/c*dilog(-1/2*a
*x+1/2)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{cx + acx^2}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)}{acx^2 + cx}\,dx$$

input
```
integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{cx + acx^2}\,dx = \frac{\int \frac{\operatorname{atanh}\left(ax\right)}{ax^2+x}\,dx}{c}$$

input
```
integrate(atanh(a*x)/(a*c*x**2+c*x),x)
```

output
```
Integral(atanh(a*x)/(a*x**2 + x), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.93

$$\int \frac{\operatorname{arctanh}(ax)}{cx + acx^2}\,dx$$
$$= \frac{1}{4}\,a\left(\frac{\log\left(ax+1\right)^2}{ac} - \frac{2\left(\log\left(ax+1\right)\log\left(-\frac{1}{2}\,ax+\frac{1}{2}\right) + \mathrm{Li}_2\left(\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{ac} - \frac{2\left(\log\left(ax+1\right)\log\left(x\right) + \mathrm{Li}_2(-\right.}{ac}\right.$$
$$- \left(\frac{\log\left(ax+1\right)}{c} - \frac{\log\left(x\right)}{c}\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="maxima")
```

output
```
1/4*a*(log(a*x + 1)^2/(a*c) - 2*(log(a*x + 1)*log(-1/2*a*x + 1/2) + dilog(
1/2*a*x + 1/2))/(a*c) - 2*(log(a*x + 1)*log(x) + dilog(-a*x))/(a*c) + 2*(l
og(-a*x + 1)*log(x) + dilog(a*x))/(a*c)) - (log(a*x + 1)/c - log(x)/c)*arc
tanh(a*x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)}{cx+acx^2}\,dx = \int \frac{\text{artanh}\,(ax)}{acx^2+cx}\,dx$$

input
```
integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)}{cx+acx^2}\,dx = \int \frac{\text{atanh}(a\,x)}{a\,c\,x^2+c\,x}\,dx$$

input
```
int(atanh(a*x)/(c*x + a*c*x^2),x)
```

output
```
int(atanh(a*x)/(c*x + a*c*x^2), x)
```

Reduce [F]

$$\int \frac{\text{arctanh}(ax)}{cx+acx^2}\,dx = \frac{-atanh(ax)^2-2\left(\int \frac{atanh(ax)}{a^2x^3-x}dx\right)}{2c}$$

input
```
int(atanh(a*x)/(a*c*x^2+c*x),x)
```

output
```
( - atanh(a*x)**2 - 2*int(atanh(a*x)/(a**2*x**3 - x),x))/(2*c)
```

3.68 $\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 270

$$\begin{aligned}\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx &= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}\\ &\quad-\frac{3b^2d\text{arctanh}(cx)}{10c^4}+\frac{b^2dx\text{arctanh}(cx)}{2c^3}\\ &\quad+\frac{bdx^2(a+b\text{arctanh}(cx))}{5c^2}\\ &\quad+\frac{bdx^3(a+b\text{arctanh}(cx))}{6c}\\ &\quad+\frac{1}{10}bdx^4(a+b\text{arctanh}(cx))\\ &\quad-\frac{d(a+b\text{arctanh}(cx))^2}{20c^4}\\ &\quad+\frac{1}{4}dx^4(a+b\text{arctanh}(cx))^2\\ &\quad+\frac{1}{5}cdx^5(a+b\text{arctanh}(cx))^2\\ &\quad-\frac{2bd(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{5c^4}\\ &\quad+\frac{b^2d\log(1-c^2x^2)}{3c^4}\\ &\quad-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}\end{aligned}$$

output
```
1/2*a*b*d*x/c^3+3/10*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*b^2*d*x^3/c-3/10*
b^2*d*arctanh(c*x)/c^4+1/2*b^2*d*x*arctanh(c*x)/c^3+1/5*b*d*x^2*(a+b*arcta
nh(c*x))/c^2+1/6*b*d*x^3*(a+b*arctanh(c*x))/c+1/10*b*d*x^4*(a+b*arctanh(c*
x))-1/20*d*(a+b*arctanh(c*x))^2/c^4+1/4*d*x^4*(a+b*arctanh(c*x))^2+1/5*c*d
*x^5*(a+b*arctanh(c*x))^2-2/5*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+1/
3*b^2*d*ln(-c^2*x^2+1)/c^4-1/5*b^2*d*polylog(2,1-2/(-c*x+1))/c^4
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d(-18ab-5b^2+30abcx+18b^2cx+12abc^2x^2+5b^2c^2x^2+10abc^3x^3+2b^2c^3x^3+15a^2c^4x^4+6abc^4x^4+}{}$$

input
```
Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d*(-18*a*b - 5*b^2 + 30*a*b*c*x + 18*b^2*c*x + 12*a*b*c^2*x^2 + 5*b^2*c^2
*x^2 + 10*a*b*c^3*x^3 + 2*b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 1
2*a^2*c^5*x^5 + 3*b^2*(-9 + 5*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 2*b*Ar
cTanh[c*x]*(3*a*c^4*x^4*(5 + 4*c*x) + b*(-9 + 15*c*x + 6*c^2*x^2 + 5*c^3*x
^3 + 3*c^4*x^4) - 12*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x]
 - 15*a*b*Log[1 + c*x] + 20*b^2*Log[1 - c^2*x^2] + 12*a*b*Log[-1 + c^2*x^2
] + 12*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^4)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(cdx^4(a+b\text{arctanh}(cx))^2+dx^3(a+b\text{arctanh}(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\text{arctanh}(cx))^2}{20c^4}-\frac{2bd\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{5c^4}+\frac{bdx^2(a+b\text{arctanh}(cx))}{5c^2}+\frac{1}{5}cdx^5(a+b\text{arctanh}(cx))^2+\frac{1}{4}dx^4(a+b\text{arctanh}(cx))^2+\frac{1}{10}bdx^4(a+b\text{arctanh}(cx))+\frac{bdx^3(a+b\text{arctanh}(cx))}{6c}+\frac{abdx}{2c^3}-\frac{3b^2d\text{arctanh}(cx)}{10c^4}+\frac{b^2dx\text{arctanh}(cx)}{2c^3}-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2d\log\left(1-c^2x^2\right)}{3c^4}+\frac{b^2dx^3}{30c}$$

input
```
Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(a*b*d*x)/(2*c^3) + (3*b^2*d*x)/(10*c^3) + (b^2*d*x^2)/(12*c^2) + (b^2*d*x
^3)/(30*c) - (3*b^2*d*ArcTanh[c*x])/(10*c^4) + (b^2*d*x*ArcTanh[c*x])/(2*c
^3) + (b*d*x^2*(a + b*ArcTanh[c*x]))/(5*c^2) + (b*d*x^3*(a + b*ArcTanh[c*x
]))/(6*c) + (b*d*x^4*(a + b*ArcTanh[c*x]))/10 - (d*(a + b*ArcTanh[c*x])^2)
/(20*c^4) + (d*x^4*(a + b*ArcTanh[c*x])^2)/4 + (c*d*x^5*(a + b*ArcTanh[c*x
])^2)/5 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (b^2*d*L
og[1 - c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.13

method	result
parts	$d\,a^2\left(\frac{1}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^4x^4}{10}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{\operatorname{arctanh}(cx)\ldots}{5}\ldots\right.}{\ldots}$
derivativedivides	$\frac{d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^4x^4}{10}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{\operatorname{arctanh}(cx)c^2x^2}{5}+\ldots\right.}{\ldots}$
default	$\frac{d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^4x^4}{10}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{\operatorname{arctanh}(cx)c^2x^2}{5}+\ldots\right.}{\ldots}$
risch	$\frac{db\,x^4a}{10}-\frac{d\,b^2\ln(-cx+1)x^4}{20}-\frac{dcab\ln(-cx+1)x^5}{5}+\frac{dc\,x^5a^2}{5}-\frac{9d\,b^2\ln(-cx+1)^2}{80c^4}+\frac{d\,b^2\ln(-cx+1)^2x^4}{16}+\frac{d\,b\ldots}{}$

input
```
int(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/5*c*x^5+1/4*x^4)+d*b^2/c^4*(1/5*arctanh(c*x)^2*c^5*x^5+1/4*arctan
h(c*x)^2*c^4*x^4+1/10*arctanh(c*x)*c^4*x^4+1/6*arctanh(c*x)*c^3*x^3+1/5*ar
ctanh(c*x)*c^2*x^2+1/2*arctanh(c*x)*c*x+9/20*arctanh(c*x)*ln(c*x-1)-1/20*a
rctanh(c*x)*ln(c*x+1)+9/80*ln(c*x-1)^2-1/5*dilog(1/2*c*x+1/2)-9/40*ln(c*x-
1)*ln(1/2*c*x+1/2)+1/80*ln(c*x+1)^2-1/40*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1
/2*c*x+1/2)+1/30*x^3*c^3+1/12*c^2*x^2+3/10*c*x+29/60*ln(c*x-1)+11/60*ln(c*
x+1))+2*d*a*b/c^4*(1/5*arctanh(c*x)*c^5*x^5+1/4*arctanh(c*x)*c^4*x^4+1/20*
c^4*x^4+1/12*x^3*c^3+1/10*c^2*x^2+1/4*c*x+9/40*ln(c*x-1)-1/40*ln(c*x+1))
```

Fricas [F]

$$\int x^3(d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx=\int(cdx+d)(b\operatorname{artanh}(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c*d*x^4 + a^2*d*x^3 + (b^2*c*d*x^4 + b^2*d*x^3)*arctanh(c*x)^
2 + 2*(a*b*c*d*x^4 + a*b*d*x^3)*arctanh(c*x), x)
```

Sympy [F]

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx = d\left(\int a^2x^3\,dx+\int a^2cx^4\,dx+\int b^2x^3\operatorname{atanh}^2(cx)\,dx+\int 2abx^3\operatorname{atanh}(cx)\,dx+\int b^2cx^4\operatorname{atanh}^2(cx)\,dx+\int 2abcx^4\operatorname{atanh}(cx)\,dx\right)$$

input
```
integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

output
```
d*(Integral(a**2*x**3, x) + Integral(a**2*c*x**4, x) + Integral(b**2*x**3*
atanh(c*x)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(b**2*c*x
**4*atanh(c*x)**2, x) + Integral(2*a*b*c*x**4*atanh(c*x), x))
```

Maxima [F]

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx = \int (cdx+d)(b\operatorname{artanh}(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctanh(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*(4*x
^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*
c*d - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^10 - 15*log(c*x
+ 1)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x^2)/c^8 + 2*log(
c^2*x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*x + 1)/(c^6*x^2
 - c^4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)/c^9 + 3*log(c
*x - 1)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900*c^2*(2*x/c^6
 - log(c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1/150*x*log(c
*x + 1)/(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (12*c^5*x^5
- 15*c^4*x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 + 1)*log(c*x
 + 1))*log(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 - 10*log(-
c*x + 1) + 2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x + 1) + 1)
 + 2000*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 9000*(c*x
- 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(log(-c*
x + 1)^2 - 2*log(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4)/c^5
 - 540000*integrate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2*c*d + 1/12
*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*l
og(c*x - 1)/c^5))*a*b*d + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1
)/c^5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1)
- 8)*log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - ...
```

Giac [F]

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)(b\,\text{artanh}\,(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int x^3\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)\,dx$$

input
```
int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

output
```
int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

Reduce [F]

$$\int x^3(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d\left(12atanh(cx)^2\,b^2c^5x^5+15atanh(cx)^2\,b^2c^4x^4-12atanh(cx)^2\,b^2cx-15atanh(cx)^2\,b^2+24atanh(cx)\,ab\,c\right.}{}$$

input
```
int(x^3*(c*d*x+d)*(a+b*atanh(c*x))^2,x)
```

output
```
(d*(12*atanh(c*x)**2*b**2*c**5*x**5 + 15*atanh(c*x)**2*b**2*c**4*x**4 - 12
*atanh(c*x)**2*b**2*c*x - 15*atanh(c*x)**2*b**2 + 24*atanh(c*x)*a*b*c**5*x
**5 + 30*atanh(c*x)*a*b*c**4*x**4 - 6*atanh(c*x)*a*b + 6*atanh(c*x)*b**2*c
**4*x**4 + 10*atanh(c*x)*b**2*c**3*x**3 + 12*atanh(c*x)*b**2*c**2*x**2 + 3
0*atanh(c*x)*b**2*c*x + 22*atanh(c*x)*b**2 + 12*int(atanh(c*x)**2,x)*b**2*
c + 24*log(c**2*x - c)*a*b + 40*log(c**2*x - c)*b**2 + 12*a**2*c**5*x**5 +
 15*a**2*c**4*x**4 + 6*a*b*c**4*x**4 + 10*a*b*c**3*x**3 + 12*a*b*c**2*x**2
 + 30*a*b*c*x + 2*b**2*c**3*x**3 + 5*b**2*c**2*x**2 + 18*b**2*c*x))/(60*c*
*4)
```

3.69 $\int x^2(d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 236

$$
\begin{aligned}
\int x^2(d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx &= \frac{abdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{b^2dx^2}{12c}-\frac{b^2d\operatorname{arctanh}(cx)}{3c^3} \\
&\quad+\frac{b^2dx\operatorname{arctanh}(cx)}{2c^2}+\frac{bdx^2(a+b\operatorname{arctanh}(cx))}{3c} \\
&\quad+\frac{1}{6}bdx^3(a+b\operatorname{arctanh}(cx)) \\
&\quad+\frac{d(a+b\operatorname{arctanh}(cx))^2}{12c^3} \\
&\quad+\frac{1}{3}dx^3(a+b\operatorname{arctanh}(cx))^2 \\
&\quad+\frac{1}{4}cdx^4(a+b\operatorname{arctanh}(cx))^2 \\
&\quad-\frac{2bd(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c^3} \\
&\quad+\frac{b^2d\log\left(1-c^2x^2\right)}{3c^3} \\
&\quad-\frac{b^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}
\end{aligned}
$$

output
```
1/2*a*b*d*x/c^2+1/3*b^2*d*x/c^2+1/12*b^2*d*x^2/c-1/3*b^2*d*arctanh(c*x)/c^
3+1/2*b^2*d*x*arctanh(c*x)/c^2+1/3*b*d*x^2*(a+b*arctanh(c*x))/c+1/6*b*d*x^
3*(a+b*arctanh(c*x))+1/12*d*(a+b*arctanh(c*x))^2/c^3+1/3*d*x^3*(a+b*arctan
h(c*x))^2+1/4*c*d*x^4*(a+b*arctanh(c*x))^2-2/3*b*d*(a+b*arctanh(c*x))*ln(2
/(-c*x+1))/c^3+1/3*b^2*d*ln(-c^2*x^2+1)/c^3-1/3*b^2*d*polylog(2,1-2/(-c*x+
1))/c^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.99

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d\left(-b^2+6abcx+4b^2cx+4abc^2x^2+b^2c^2x^2+4a^2c^3x^3+2abc^3x^3+3a^2c^4x^4+b^2\left(-7+4c^3x^3+3c^4x^4\right)\text{ar}\right.}{}$$

input
```
Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d*(-b^2 + 6*a*b*c*x + 4*b^2*c*x + 4*a*b*c^2*x^2 + b^2*c^2*x^2 + 4*a^2*c^3
*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-7 + 4*c^3*x^3 + 3*c^4*x^4)*Ar
cTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^3*x^3*(4 + 3*c*x) + b*(-2 + 3*c*x + 2
*c^2*x^2 + c^3*x^3) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*
x] - 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 - c^2*x^2] + 4*a*b*Log[-1 + c^2*x^2]
 + 4*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(12*c^3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(cdx^3(a+b\text{arctanh}(cx))^2+dx^2(a+b\text{arctanh}(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(a+b\text{arctanh}(cx))^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{3c^3}+\frac{1}{4}cdx^4(a+b\text{arctanh}(cx))^2+\frac{1}{3}dx^3(a+b\text{arctanh}(cx))^2+\frac{1}{6}bdx^3(a+b\text{arctanh}(cx))+\frac{bdx^2(a+b\text{arctanh}(cx))}{3c}+\frac{abdx}{2c^2}-\frac{b^2d\text{arctanh}(cx)}{3c^3}+\frac{b^2dx\text{arctanh}(cx)}{2c^2}-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{b^2dx}{3c^2}+\frac{b^2d\log(1-c^2x^2)}{3c^3}+\frac{b^2dx^2}{12c}$$

input
```
Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(a*b*d*x)/(2*c^2) + (b^2*d*x)/(3*c^2) + (b^2*d*x^2)/(12*c) - (b^2*d*ArcTan
h[c*x])/(3*c^3) + (b^2*d*x*ArcTanh[c*x])/(2*c^2) + (b*d*x^2*(a + b*ArcTanh
[c*x]))/(3*c) + (b*d*x^3*(a + b*ArcTanh[c*x]))/6 + (d*(a + b*ArcTanh[c*x])
^2)/(12*c^3) + (d*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d*x^4*(a + b*ArcTanh[
c*x])^2)/4 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*
d*Log[1 - c^2*x^2])/(3*c^3) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.18

method	result
parts	$d\,a^2\left(\frac{1}{4}c\,x^4+\frac{1}{3}x^3\right)+\frac{d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+\frac{\operatorname{arctanh}(cx)c}{2}\right.}{}$
derivativedivides	$\frac{d\,a^2\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+\frac{\operatorname{arctanh}(cx)cx}{2}+\frac{7}{}\right.}{}$
default	$\frac{d\,a^2\left(\frac{1}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+\frac{\operatorname{arctanh}(cx)cx}{2}+\frac{7}{}\right.}{}$
risch	$\frac{d\,b^2\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{3c^3}+\frac{d\,b^2\ln(-cx-1)}{6c^3}+\frac{db\,x^3a}{6}+\frac{dc\,x^4a^2}{4}+\frac{d\,b^2\ln(-cx+1)^2x^3}{12}-\frac{d\,b^2\ln(-cx+1)x^3}{12}-\frac{7d\,b}{}$

input
```
int(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/4*c*x^4+1/3*x^3)+d*b^2/c^3*(1/4*arctanh(c*x)^2*c^4*x^4+1/3*arctan
h(c*x)^2*c^3*x^3+1/6*arctanh(c*x)*c^3*x^3+1/3*arctanh(c*x)*c^2*x^2+1/2*arc
tanh(c*x)*c*x+7/12*arctanh(c*x)*ln(c*x-1)+1/12*arctanh(c*x)*ln(c*x+1)-1/48
*ln(c*x+1)^2+1/24*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/3*dilog(1
/2*c*x+1/2)+7/48*ln(c*x-1)^2-7/24*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*c^2*x^2+1
/3*c*x+1/2*ln(c*x-1)+1/6*ln(c*x+1))+2*d*a*b/c^3*(1/4*arctanh(c*x)*c^4*x^4+
1/3*arctanh(c*x)*c^3*x^3+1/12*x^3*c^3+1/6*c^2*x^2+1/4*c*x+7/24*ln(c*x-1)+1
/24*ln(c*x+1))
```

Fricas [F]

$$\int x^2(d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx=\int(cdx+d)(b\operatorname{artanh}(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c*d*x^3 + a^2*d*x^2 + (b^2*c*d*x^3 + b^2*d*x^2)*arctanh(c*x)^
2 + 2*(a*b*c*d*x^3 + a*b*d*x^2)*arctanh(c*x), x)
```

Sympy [F]

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))^2\,dx = d\left(\int a^2x^2\,dx + \int a^2cx^3\,dx + \int b^2x^2\,\text{atanh}^2(cx)\,dx + \int 2abx^2\,\text{atanh}(cx)\,dx + \int b^2cx^3\,\text{atanh}^2(cx)\,dx + \int 2abcx^3\,\text{atanh}(cx)\,dx\right)$$

input
```
integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

output
```
d*(Integral(a**2*x**2, x) + Integral(a**2*c*x**3, x) + Integral(b**2*x**2*
atanh(c*x)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(b**2*c*x
**3*atanh(c*x)**2, x) + Integral(2*a*b*c*x**3*atanh(c*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.70

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))^2\,dx = \frac{1}{4}a^2cdx^4 + \frac{1}{3}a^2dx^3 + \frac{1}{12}\left(6x^4\,\text{artanh}(cx) + c\left(\frac{2(c^2x^3+3x)}{c^4} - \frac{3\log(cx+1)}{c^5} + \frac{3\log(cx-1)}{c^5}\right)\right)abcd + \frac{1}{3}\left(2x^3\,\text{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log(c^2x^2-1)}{c^4}\right)\right)abd + \frac{\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right) + \text{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d}{3c^3} + \frac{b^2d\log(cx+1)}{6c^3} + \frac{b^2d\log(cx-1)}{2c^3} + \frac{4b^2c^2dx^2 + 16b^2cdx + (3b^2c^4dx^4 + 4b^2c^3dx^3 + b^2d)\log(cx+1)^2 + (3b^2c^4dx^4 + 4b^2c^3dx^3 - 7b^2d)\log}{}$$

input
```
integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c*d + 1/3*(2*x
^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*d + 1/3*(log(c*x
 + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d/c^3 + 1/6*b^2*d*lo
g(c*x + 1)/c^3 + 1/2*b^2*d*log(c*x - 1)/c^3 + 1/48*(4*b^2*c^2*d*x^2 + 16*b
^2*c*d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x + 1)^2 + (3
*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 - 7*b^2*d)*log(-c*x + 1)^2 + 4*(b^2*c^3*d
*x^3 + 2*b^2*c^2*d*x^2 + 3*b^2*c*d*x)*log(c*x + 1) - 2*(2*b^2*c^3*d*x^3 +
4*b^2*c^2*d*x^2 + 6*b^2*c*d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d
)*log(c*x + 1))*log(-c*x + 1))/c^3
```

Giac [F]

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)(b\,\text{artanh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int x^2\,\left(a+b\,\text{atanh}(c\,x)\right)^2\,\left(d+c\,d\,x\right)\,dx$$

input
```
int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

output
```
int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

Reduce [F]

$$\int x^2(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d\big(3atanh(cx)^2\,b^2c^4x^4+4atanh(cx)^2\,b^2c^3x^3-4atanh(cx)^2\,b^2cx-3atanh(cx)^2\,b^2+6atanh(cx)\,ab\,c^4x^4+}{}$$

input
```
int(x^2*(c*d*x+d)*(a+b*atanh(c*x))^2,x)
```

output
```
(d*(3*atanh(c*x)**2*b**2*c**4*x**4 + 4*atanh(c*x)**2*b**2*c**3*x**3 - 4*at
anh(c*x)**2*b**2*c*x - 3*atanh(c*x)**2*b**2 + 6*atanh(c*x)*a*b*c**4*x**4 +
 8*atanh(c*x)*a*b*c**3*x**3 + 2*atanh(c*x)*a*b + 2*atanh(c*x)*b**2*c**3*x*
*3 + 4*atanh(c*x)*b**2*c**2*x**2 + 6*atanh(c*x)*b**2*c*x + 4*atanh(c*x)*b*
*2 + 4*int(atanh(c*x)**2,x)*b**2*c + 8*log(c**2*x - c)*a*b + 8*log(c**2*x
- c)*b**2 + 3*a**2*c**4*x**4 + 4*a**2*c**3*x**3 + 2*a*b*c**3*x**3 + 4*a*b*
c**2*x**2 + 6*a*b*c*x + b**2*c**2*x**2 + 4*b**2*c*x))/(12*c**3)
```

3.70 $\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 196

$$\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\frac{abdx}{c}+\frac{b^2dx}{3c}-\frac{b^2d\text{arctanh}(cx)}{3c^2}+\frac{b^2dx\text{arctanh}(cx)}{c}+\frac{1}{3}bdx^2(a+b\text{arctanh}(cx))-\frac{d(a+b\text{arctanh}(cx))^2}{6c^2}+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))^2+\frac{1}{3}cdx^3(a+b\text{arctanh}(cx))^2-\frac{2bd(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c^2}+\frac{b^2d\log(1-c^2x^2)}{2c^2}-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}$$

output
```
a*b*d*x/c+1/3*b^2*d*x/c-1/3*b^2*d*arctanh(c*x)/c^2+b^2*d*x*arctanh(c*x)/c+
1/3*b*d*x^2*(a+b*arctanh(c*x))-1/6*d*(a+b*arctanh(c*x))^2/c^2+1/2*d*x^2*(a
+b*arctanh(c*x))^2+1/3*c*d*x^3*(a+b*arctanh(c*x))^2-2/3*b*d*(a+b*arctanh(c
*x))*ln(2/(-c*x+1))/c^2+1/2*b^2*d*ln(-c^2*x^2+1)/c^2-1/3*b^2*d*polylog(2,1
-2/(-c*x+1))/c^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d\left(6abcx+2b^2cx+3a^2c^2x^2+2abc^2x^2+2a^2c^3x^3+b^2(-5+3c^2x^2+2c^3x^3)\,\text{arctanh}(cx)^2+2b\text{arctanh}(cx)\ldots\right.}{}$$

input
```
Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d*(6*a*b*c*x + 2*b^2*c*x + 3*a^2*c^2*x^2 + 2*a*b*c^2*x^2 + 2*a^2*c^3*x^3
+ b^2*(-5 + 3*c^2*x^2 + 2*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^
2*x^2*(3 + 2*c*x) + b*(-1 + 3*c*x + c^2*x^2) - 2*b*Log[1 + E^(-2*ArcTanh[c
*x])]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*b^2*Log[1 - c^2*x^2]
+ 2*a*b*Log[-1 + c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(6*c^
2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)(a+b\text{arctanh}(cx))^2\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(cdx^2(a+b\text{arctanh}(cx))^2+dx(a+b\text{arctanh}(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\operatorname{arctanh}(cx))^2}{6c^2}-\frac{2bd\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{3c^2}+\frac{1}{3}cdx^3(a+b\operatorname{arctanh}(cx))^2+\frac{1}{2}dx^2(a+b\operatorname{arctanh}(cx))^2+\frac{1}{3}bdx^2(a+b\operatorname{arctanh}(cx))+\frac{abdx}{c}-\frac{b^2d\operatorname{arctanh}(cx)}{3c^2}+\frac{b^2dx\operatorname{arctanh}(cx)}{c}-\frac{b^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}+\frac{b^2d\log(1-c^2x^2)}{2c^2}+\frac{b^2dx}{3c}$$

input
```
Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(a*b*d*x)/c + (b^2*d*x)/(3*c) - (b^2*d*ArcTanh[c*x])/(3*c^2) + (b^2*d*x*Ar
cTanh[c*x])/c + (b*d*x^2*(a + b*ArcTanh[c*x]))/3 - (d*(a + b*ArcTanh[c*x])
^2)/(6*c^2) + (d*x^2*(a + b*ArcTanh[c*x])^2)/2 + (c*d*x^3*(a + b*ArcTanh[c
*x])^2)/3 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (b^2*d
*Log[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.27

method	result
parts	$d\,a^2\left(\frac{1}{3}c\,x^3+\frac{1}{2}x^2\right)+\frac{d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)^2c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+\operatorname{arctanh}(cx)cx+\frac{5\operatorname{arctanh}(cx)}{6}\right.}{}$
derivativedivides	$\frac{d\,a^2\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)^2c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+\operatorname{arctanh}(cx)cx+\frac{5\operatorname{arctanh}(cx)\ln(cx-}{6}\right.}{}$
default	$\frac{d\,a^2\left(\frac{1}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d\,b^2\left(\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)^2c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+\operatorname{arctanh}(cx)cx+\frac{5\operatorname{arctanh}(cx)\ln(cx-}{6}\right.}{}$
risch	$\frac{d\,b^2\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{3c^2}+\left(-\frac{d\,b^2x^2(2cx+3)\ln(-cx+1)}{12}-\frac{db\left(-4a\,c^3x^3-6a\,c^2x^2-2b\,c^2x^2-6bcx-5b\ln(-cx+1)\right)}{12c^2}\right)\,\mathrm{l}$

input
```
int(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/3*c*x^3+1/2*x^2)+d*b^2/c^2*(1/3*arctanh(c*x)^2*c^3*x^3+1/2*arctan
h(c*x)^2*c^2*x^2+1/3*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x+5/6*arctanh(c*x
)*ln(c*x-1)-1/6*arctanh(c*x)*ln(c*x+1)+5/24*ln(c*x-1)^2-1/3*dilog(1/2*c*x+
1/2)-5/12*ln(c*x-1)*ln(1/2*c*x+1/2)+1/24*ln(c*x+1)^2-1/12*(ln(c*x+1)-ln(1/
2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/3*c*x+2/3*ln(c*x-1)+1/3*ln(c*x+1))+2*d*a*b/
c^2*(1/3*arctanh(c*x)*c^3*x^3+1/2*arctanh(c*x)*c^2*x^2+1/6*c^2*x^2+1/2*c*x
+5/12*ln(c*x-1)-1/12*ln(c*x+1))
```

Fricas [F]

$$\int x(d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx=\int (cdx+d)(b\operatorname{artanh}(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c*d*x^2 + a^2*d*x + (b^2*c*d*x^2 + b^2*d*x)*arctanh(c*x)^2 +
2*(a*b*c*d*x^2 + a*b*d*x)*arctanh(c*x), x)
```

Sympy [F]

$$\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx = d\left(\int a^2x\,dx + \int a^2cx^2\,dx + \int b^2x\,\text{atanh}^2\,(cx)\,dx + \int 2abx\,\text{atanh}\,(cx)\,dx + \int b^2cx^2\,\text{atanh}^2\,(cx)\,dx + \int 2abcx^2\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate(x*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

output
```
d*(Integral(a**2*x, x) + Integral(a**2*c*x**2, x) + Integral(b**2*x*atanh(
c*x)**2, x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(b**2*c*x**2*atanh
(c*x)**2, x) + Integral(2*a*b*c*x**2*atanh(c*x), x))
```

Maxima [F]

$$\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx = \int (cdx+d)(b\,\text{artanh}\,(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctanh(c*x)^2 + 1/3*(2*x^3*arctanh(c*x) +
 c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d - 1/216*(2*c^4*(2*(c^2*x^3 +
3*x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x - 1)/c^7) - 3*c^3*(x^2/c^4 + log
(c^2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9*x^3*log(c*x + 1)/(c^4*x^2 - c^2
), x) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 + log(c*x - 1)/c^5) - 324*c*inte
grate(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x) - 6*(3*c^3*x^3*log(c*x + 1)^2
 + (2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 + 1)*log(c*x + 1))*log(-c*x
 + 1))/c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 27
*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 54*(c*x - 1)*(log
(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*log(9*c^4*x^2 - 9*c^2)/c^3 -
 324*integrate(1/9*log(c*x + 1)/(c^4*x^2 - c^2), x))*b^2*c*d + 1/2*a^2*d*x
^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1
)/c^3))*a*b*d + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*a
rctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*
x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2*d
```

Giac [F]

$$\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)(b\,\text{artanh}\,(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int x\,\left(a+b\,\mathrm{atanh}(c\,x)\right)^2\,\left(d+c\,d\,x\right)\,dx$$

input
```
int(x*(a + b*atanh(c*x))^2*(d + c*d*x),x)
```

output
```
int(x*(a + b*atanh(c*x))^2*(d + c*d*x), x)
```

Reduce [F]

$$\int x(d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$=\frac{d\left(2atanh(cx)^2b^2c^3x^3+3atanh(cx)^2b^2c^2x^2-2atanh(cx)^2b^2cx-3atanh(cx)^2b^2+4atanh(cx)\,ab\,c^3x^3+\right.}{}$$

input
```
int(x*(c*d*x+d)*(a+b*atanh(c*x))^2,x)
```

output
```
(d*(2*atanh(c*x)**2*b**2*c**3*x**3 + 3*atanh(c*x)**2*b**2*c**2*x**2 - 2*at
anh(c*x)**2*b**2*c*x - 3*atanh(c*x)**2*b**2 + 4*atanh(c*x)*a*b*c**3*x**3 +
 6*atanh(c*x)*a*b*c**2*x**2 - 2*atanh(c*x)*a*b + 2*atanh(c*x)*b**2*c**2*x*
*2 + 6*atanh(c*x)*b**2*c*x + 4*atanh(c*x)*b**2 + 2*int(atanh(c*x)**2,x)*b*
*2*c + 4*log(c**2*x - c)*a*b + 6*log(c**2*x - c)*b**2 + 2*a**2*c**3*x**3 +
 3*a**2*c**2*x**2 + 2*a*b*c**2*x**2 + 6*a*b*c*x + 2*b**2*c*x))/(6*c**2)
```

3.71 $\int (d+cdx)(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 112

$$\int (d+cdx)(a+b\text{arctanh}(cx))^2\,dx = abdx+b^2dx\text{arctanh}(cx) + \frac{d(1+cx)^2(a+b\text{arctanh}(cx))^2}{2c} - \frac{2bd(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c} + \frac{b^2d\log(1-c^2x^2)}{2c} - \frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}$$

output

```
a*b*d*x+b^2*d*x*arctanh(c*x)+1/2*d*(c*x+1)^2*(a+b*arctanh(c*x))^2/c-2*b*d*
(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+1/2*b^2*d*ln(-c^2*x^2+1)/c-b^2*d*polyl
og(2,1-2/(-c*x+1))/c
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int (d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$=\frac{d\left(2a^2cx+2abcx+a^2c^2x^2+b^2(-3+2cx+c^2x^2)\operatorname{arctanh}(cx)^2+2b\operatorname{arctanh}(cx)\left(cx(2a+b+acx)-2b\right.\right.}{}$$

input
```
Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d*(2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-3 + 2*c*x + c^2*x^2)*ArcTa
nh[c*x]^2 + 2*b*ArcTanh[c*x]*(c*x*(2*a + b + a*c*x) - 2*b*Log[1 + E^(-2*Ar
cTanh[c*x])]) + a*b*Log[1 - c*x] - a*b*Log[1 + c*x] + 2*a*b*Log[1 - c^2*x^
2] + b^2*Log[1 - c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)(a+b\operatorname{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6480}$$

$$\frac{d(cx+1)^2(a+b\operatorname{arctanh}(cx))^2}{2c}-\frac{b\int \left(\frac{2d^2(cx+1)(a+b\operatorname{arctanh}(cx))}{1-c^2x^2}-d^2(a+b\operatorname{arctanh}(cx))\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(cx+1)^2(a+b\operatorname{arctanh}(cx))^2}{2c}-\frac{b\left(\frac{2d^2\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{c}-ad^2x-bd^2x\operatorname{arctanh}(cx)-\frac{bd^2\log(1-c^2x^2)}{2c}+\frac{bd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}\right)}{d}$$

input
```
Int[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d*(1 + c*x)^2*(a + b*ArcTanh[c*x])^2)/(2*c) - (b*(-(a*d^2*x) - b*d^2*x*Ar
cTanh[c*x] + (2*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c - (b*d^2*Log[
1 - c^2*x^2])/(2*c) + (b*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/c))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.88

method	result
parts	$d\,a^{2}\left(\frac{1}{2}c\,x^{2}+x\right)+\frac{d\,b^{2}\left(\frac{\operatorname{arctanh}(cx)^{2}c^{2}x^{2}}{2}+\operatorname{arctanh}(cx)^{2}cx+\operatorname{arctanh}(cx)cx+\frac{3\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{\operatorname{arctanh}(cx)}{2}\right.}{}$
derivativedivides	$\frac{d\,a^{2}\left(\frac{1}{2}c^{2}x^{2}+cx\right)+d\,b^{2}\left(\frac{\operatorname{arctanh}(cx)^{2}c^{2}x^{2}}{2}+\operatorname{arctanh}(cx)^{2}cx+\operatorname{arctanh}(cx)cx+\frac{3\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{\operatorname{arctanh}(cx)\ln(cx+}{2}\right.}{}$
default	$\frac{d\,a^{2}\left(\frac{1}{2}c^{2}x^{2}+cx\right)+d\,b^{2}\left(\frac{\operatorname{arctanh}(cx)^{2}c^{2}x^{2}}{2}+\operatorname{arctanh}(cx)^{2}cx+\operatorname{arctanh}(cx)cx+\frac{3\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{\operatorname{arctanh}(cx)\ln(cx+}{2}\right.}{}$
risch	$a^{2}dx+\frac{b^{2}\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)d}{c}-\frac{3\ln(-cx+1)^{2}b^{2}d}{8c}+\frac{\ln(-cx+1)^{2}b^{2}dx}{4}+\frac{b\ln(-cx-1)ad}{2c}-\ln\left(-cx+1\right)ab$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/2*c*x^2+x)+d*b^2/c*(1/2*arctanh(c*x)^2*c^2*x^2+arctanh(c*x)^2*c*x
+arctanh(c*x)*c*x+3/2*arctanh(c*x)*ln(c*x-1)+1/2*arctanh(c*x)*ln(c*x+1)-1/
8*ln(c*x+1)^2+1/4*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-dilog(1/2*c
*x+1/2)+1/2*ln(c*x-1)+1/2*ln(c*x+1)+3/8*ln(c*x-1)^2-3/4*ln(c*x-1)*ln(1/2*c
*x+1/2))+2*d*a*b/c*(1/2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x+1/2*c*x+3/4*
ln(c*x-1)+1/4*ln(c*x+1))
```

Fricas [F]

$$\int (d+cdx)(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)(b\,\text{artanh}\,(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x), x)
```

Sympy [F]

$$\int (d+cdx)(a+b\text{arctanh}(cx))^2\,dx=d\left(\int a^2\,dx+\int b^2\,\text{atanh}^2\,(cx)\,dx+\int 2ab\,\text{atanh}\,(cx)\,dx+\int a^2cx\,dx+\int b^2cx\,\text{atanh}^2\,(cx)\,dx+\int 2abcx\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

output
```
d*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*at
anh(c*x), x) + Integral(a**2*c*x, x) + Integral(b**2*c*x*atanh(c*x)**2, x)
 + Integral(2*a*b*c*x*atanh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(105) = 210$.

Time = 0.20 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.59

$$\int (d+cdx)(a+b\text{arctanh}(cx))^2\,dx$$
$$= \frac{1}{2}a^2cdx^2 + \frac{1}{2}\left(2x^2\,\text{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)abcd$$
$$+ a^2dx + \frac{(2cx\,\text{artanh}(cx) + \log(-c^2x^2+1))abd}{c}$$
$$+ \frac{\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right) + \text{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d}{c}$$
$$+ \frac{b^2d\log(cx+1)}{2c} + \frac{b^2d\log(cx-1)}{2c}$$
$$+ \frac{4b^2cdx\log(cx+1) + (b^2c^2dx^2 + 2b^2cdx + b^2d)\log(cx+1)^2 + (b^2c^2dx^2 + 2b^2cdx - 3b^2d)\log(-cx +}{8c}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*c*d*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3
+ log(c*x - 1)/c^3))*a*b*c*d + a^2*d*x + (2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*a*b*d/c + (log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2)
)*b^2*d/c + 1/2*b^2*d*log(c*x + 1)/c + 1/2*b^2*d*log(c*x - 1)/c + 1/8*(4*b
^2*c*d*x*log(c*x + 1) + (b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1)
^2 + (b^2*c^2*d*x^2 + 2*b^2*c*d*x - 3*b^2*d)*log(-c*x + 1)^2 - 2*(2*b^2*c*
d*x + (b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1))*log(-c*x + 1))/c
```

Giac [F]

$$\int (d+cdx)(a+b\text{arctanh}(cx))^2\,dx = \int (cdx+d)(b\,\text{artanh}(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx = \int (a+b\,\mathrm{atanh}(c\,x))^2\,(d+c\,d\,x)\,dx$$

input
```
int((a + b*atanh(c*x))^2*(d + c*d*x),x)
```

output
```
int((a + b*atanh(c*x))^2*(d + c*d*x), x)
```

Reduce [F]

$$\int (d+cdx)(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$= \frac{d\Big(atanh(cx)^2\,b^2c^2x^2 + 2atanh(cx)^2\,b^2cx - atanh(cx)^2\,b^2 + 2atanh(cx)\,ab\,c^2x^2 + 4atanh(cx)\,abcx + 2ata}{}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))^2,x)
```

output
```
(d*(atanh(c*x)**2*b**2*c**2*x**2 + 2*atanh(c*x)**2*b**2*c*x - atanh(c*x)**
2*b**2 + 2*atanh(c*x)*a*b*c**2*x**2 + 4*atanh(c*x)*a*b*c*x + 2*atanh(c*x)*
a*b + 2*atanh(c*x)*b**2*c*x + 2*atanh(c*x)*b**2 + 4*int((atanh(c*x)*x)/(c*
*2*x**2 - 1),x)*b**2*c**2 + 4*log(c**2*x - c)*a*b + 2*log(c**2*x - c)*b**2
 + a**2*c**2*x**2 + 2*a**2*c*x + 2*a*b*c*x))/(2*c)
```

3.72 $\int \frac{(d+cdx)(a+b\,\mathrm{arctanh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 191

$$\begin{aligned}\int \frac{(d+cdx)(a+b\,\mathrm{arctanh}(cx))^2}{x}\,dx &= d(a+b\,\mathrm{arctanh}(cx))^2 + cdx(a+b\,\mathrm{arctanh}(cx))^2 \\ &\quad + 2d(a+b\,\mathrm{arctanh}(cx))^2\mathrm{arctanh}\left(1-\frac{2}{1-cx}\right) \\ &\quad - 2bd(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) \\ &\quad - b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right) \\ &\quad - bd(a+b\,\mathrm{arctanh}(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right) \\ &\quad + bd(a+b\,\mathrm{arctanh}(cx))\,\mathrm{PolyLog}\left(2,-1 + \frac{2}{1-cx}\right) + \frac{1}{2}b^2d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-cx}\right) \\ &\quad - \frac{1}{2}b^2d\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)\end{aligned}$$

output
```
d*(a+b*arctanh(c*x))^2+c*d*x*(a+b*arctanh(c*x))^2-2*d*(a+b*arctanh(c*x))^2
*arctanh(-1+2/(-c*x+1))-2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-b^2*d*poly
log(2,1-2/(-c*x+1))-b*d*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*d*(a+
b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*d*polylog(3,1-2/(-c*x+1))
-1/2*b^2*d*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.19

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx$$
$$= d\bigg(a^2cx + a^2\log(cx) + ab\big(2cx\text{arctanh}(cx) + \log\left(1-c^2x^2\right)\big) + b^2\big(\text{arctanh}(cx)\left((-1+cx)\text{arctanh}(cx) - 2\log\left(1+e^{-2\text{arctanh}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)\big) + ab(-\text{PolyLog}(2,-cx) + \text{PolyLog}(2,cx)) + b^2\bigg(\frac{i\pi^3}{24} - \frac{2}{3}\text{arctanh}(cx)^3 - \text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right) + \text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right) + \text{arctanh}(cx)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right) + \text{arctanh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right) + \frac{1}{2}\,\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\bigg)\bigg)$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

output
```
d*(a^2*c*x + a^2*Log[c*x] + a*b*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2]) +
b^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])
]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + a*b*(-PolyLog[2, -(c*x)] + PolyLo
g[2, c*x]) + b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[
1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + Ar
cTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2
*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*Arc
Tanh[c*x])]/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))^2}{x}dx$$

$$\Big\downarrow \ 6502$$

$$\int \left(cd(a+b\text{arctanh}(cx))^2+\frac{d(a+b\text{arctanh}(cx))^2}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-bd\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+bd\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+d(a+b\text{arctanh}(cx))^2+cdx(a+b\text{arctanh}(cx))^2+2d\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-2bd\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+b^2(-d)\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}b^2d\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

output
```
d*(a + b*ArcTanh[c*x])^2 + c*d*x*(a + b*ArcTanh[c*x])^2 + 2*d*(a + b*ArcTa
nh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1
- c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 - c*x)] - b*d*(a + b*ArcTanh[c*x])*Pol
yLog[2, 1 - 2/(1 - c*x)] + b*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
 c*x)] + (b^2*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d*PolyLog[3, -1 + 2/
(1 - c*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 1827, normalized size of antiderivative = 9.57

method	result	size
parts	Expression too large to display	1827
derivativedivides	Expression too large to display	1837
default	Expression too large to display	1837

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(c*x+ln(x))+d*b^2*(arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2
*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^2*ln(1+(c
*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(
1/2))-arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+arctanh(c*x)^2*ln(c*x)-1
/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^
(1/2))+arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1)/(
-c^2*x^2+1)^(1/2))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-1/2*I*Pi*csgn(I/
(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/
(c^2*x^2-1)))^2*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c
*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2
))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-arctanh(c*x)*ln(1+(c*x+1)^2/(-c^
2*x^2+1))-arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)*ln(
1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-dilo
g(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/4*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1
-(c*x+1)^2/(c^2*x^2-1)))*(2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-2*ar
ctanh(c*x)^2+polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))+arctanh(c*x)^2*c*x+arctan
h(c*x)^2-1/2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(
-c^2*x^2+1))-arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-2*polylog(...
```

Fricas [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*
c*d*x + a*b*d)*arctanh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx = d\left(\int a^2c\,dx + \int \frac{a^2}{x}\,dx + \int b^2c\,\text{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x}\,dx + \int 2abc\,\text{atanh}\,(cx)\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x,x)
```

output
```
d*(Integral(a**2*c, x) + Integral(a**2/x, x) + Integral(b**2*c*atanh(c*x)*
*2, x) + Integral(b**2*atanh(c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x),
x) + Integral(2*a*b*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*b^2*c*d*x*log(-c*x + 1)^2 + a^2*c*d*x + (2*c*x*arctanh(c*x) + log(-c^2
*x^2 + 1))*a*b*d + a^2*d*log(x) - integrate(-1/4*((b^2*c^2*d*x^2 - b^2*d)*
log(c*x + 1)^2 + 4*(a*b*c*d*x - a*b*d)*log(c*x + 1) - 2*(b^2*c^2*d*x^2 + 2
*a*b*c*d*x - 2*a*b*d + (b^2*c^2*d*x^2 - b^2*d)*log(c*x + 1))*log(-c*x + 1)
)/(c*x^2 - x), x)
```

Giac [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x}\,dx=d\Bigg(2atanh(cx)\,abcx+2atanh(cx)\,ab+\left(\int atanh(cx)^2\,dx\right)b^2c+2\left(\int \frac{atanh(cx)}{x}dx\right)ab+\left(\int \frac{atanh(cx)^2}{x}dx\right)b^2+2\log\left(c^2x-c\right)ab+\log(x)\,a^2+a^2cx\Bigg)$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))^2/x,x)
```

output
```
d*(2*atanh(c*x)*a*b*c*x + 2*atanh(c*x)*a*b + int(atanh(c*x)**2,x)*b**2*c +
 2*int(atanh(c*x)/x,x)*a*b + int(atanh(c*x)**2/x,x)*b**2 + 2*log(c**2*x -
c)*a*b + log(x)*a**2 + a**2*c*x)
```

3.73 $\int \frac{(d+cdx)(a+b\,\text{arctanh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\begin{aligned}\int \frac{(d+cdx)(a+b\,\text{arctanh}(cx))^2}{x^2}\,dx &= cd(a+b\,\text{arctanh}(cx))^2-\frac{d(a+b\,\text{arctanh}(cx))^2}{x}\\ &\quad+2cd(a+b\,\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)\\ &\quad+2bcd(a+b\,\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)\\ &\quad-bcd(a+b\,\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\\ &\quad+bcd(a+b\,\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)-b^2cd\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\\ &\quad+\frac{1}{2}b^2cd\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)\\ &\quad-\frac{1}{2}b^2cd\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)\end{aligned}$$

output
```
c*d*(a+b*arctanh(c*x))^2-d*(a+b*arctanh(c*x))^2/x-2*c*d*(a+b*arctanh(c*x))
^2*arctanh(-1+2/(-c*x+1))+2*b*c*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c*d
*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*c*d*(a+b*arctanh(c*x))*polyl
og(2,-1+2/(-c*x+1))-b^2*c*d*polylog(2,-1+2/(c*x+1))+1/2*b^2*c*d*polylog(3,
1-2/(-c*x+1))-1/2*b^2*c*d*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.24

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^2}\,dx =$$

$$-\frac{d\Big(a^2-a^2cx\log(x)+ab(2\text{arctanh}(cx)+cx(-2\log(cx)+\log(1-c^2x^2)))+b^2\big(\text{arctanh}(cx)\,((1-cx)}{}$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

output
```
-((d*(a^2 - a^2*c*x*Log[x] + a*b*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[
1 - c^2*x^2])) + b^2*(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 -
 E^(-2*ArcTanh[c*x])]) + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + a*b*c*x*(P
olyLog[2, -(c*x)] - PolyLog[2, c*x]) - b^2*c*x*((I/24)*Pi^3 - (2*ArcTanh[c
*x]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Lo
g[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])]
+ ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[
c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/x)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(\frac{d(a+b\text{arctanh}(cx))^2}{x^2}+\frac{cd(a+b\text{arctanh}(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-bcd\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+bcd\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+cd(a+b\text{arctanh}(cx))^2-\frac{d(a+b\text{arctanh}(cx))^2}{x}+2cd\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2+2bcd\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))+b^2(-c)d\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{1}{2}b^2cd\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2cd\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

output
```
c*d*(a + b*ArcTanh[c*x])^2 - (d*(a + b*ArcTanh[c*x])^2)/x + 2*c*d*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 2*b*c*d*(a + b*ArcTanh[c*x])*Log
[2 - 2/(1 + c*x)] - b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]
 + b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d*PolyL
og[2, -1 + 2/(1 + c*x)] + (b^2*c*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*c
*d*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.81 (sec) , antiderivative size = 1857, normalized size of antiderivative = 9.24

method	result	size
parts	Expression too large to display	1857
derivativedivides	Expression too large to display	1860
default	Expression too large to display	1860

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(c*ln(x)-1/x)+d*b^2*c*(arctanh(c*x)^2*ln(c*x)-arctanh(c*x)^2/c/x-arc
tanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+arctanh(c*x)^2*ln(1-(c*x+1)^2/(-c
^2*x^2+1))+arctanh(c*x)*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-1/2*polylog(3,(c
*x+1)^2/(-c^2*x^2+1))-arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*
polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/8*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1
)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*(4*arct
anh(c*x)^2-2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))-2*arctanh(c*x)*ln(1
+(c*x+1)^2/(-c^2*x^2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-polylog(2,(c*x
+1)^2/(-c^2*x^2+1)))-1/8*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(
c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*(2*arctanh(c*x)*ln(1+
(c*x+1)^2/(-c^2*x^2+1))-dilog((c*x+1)^2/(-c^2*x^2+1))+dilog(1+(c*x+1)^2/(-
c^2*x^2+1)))+1/8*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*
x^2-1)))^3*(4*arctanh(c*x)^2-2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))-2
*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+
1))-polylog(2,(c*x+1)^2/(-c^2*x^2+1)))+1/8*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x
^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))*(4*arctanh(c*x)^2-2*arctanh(c*x)*ln(1-(c*x+1
)^2/(-c^2*x^2+1))-2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-polylog(2,-(
c*x+1)^2/(-c^2*x^2+1))-polylog(2,(c*x+1)^2/(-c^2*x^2+1)))-arctanh(c*x)^2+3
/2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+3/4*polylog(2,(c*x+1)^2/(-...
```

Fricas [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*
c*d*x + a*b*d)*arctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^2}\,dx=d\left(\int \frac{a^2}{x^2}\,dx+\int \frac{a^2c}{x}\,dx+\int \frac{b^2\,\text{atanh}^2\,(cx)}{x^2}\,dx+\int \frac{2ab\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{b^2c\,\text{atanh}^2\,(cx)}{x}\,dx+\int \frac{2abc\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**2,x)
```

output
```
d*(Integral(a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(b**2*atanh(c*
x)**2/x**2, x) + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(b**2*c*atan
h(c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^2}\,dx = \int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
a^2*c*d*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*
d - 1/4*b^2*d*log(-c*x + 1)^2/x - a^2*d/x - integrate(-1/4*((b^2*c^2*d*x^2
 - b^2*d)*log(c*x + 1)^2 + 4*(a*b*c^2*d*x^2 - a*b*c*d*x)*log(c*x + 1) - 2*
(2*a*b*c^2*d*x^2 - (2*a*b*c*d + b^2*c*d)*x + (b^2*c^2*d*x^2 - b^2*d)*log(c
*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

Giac [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^2}\,dx = \int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^2,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx$$
$$= \frac{d\left(-atanh(cx)^2 b^2 - 2atanh(cx)\,abcx - 2atanh(cx)\,ab - 2\left(\int \frac{atanh(cx)}{c^2x^3-x}dx\right)b^2cx + 2\left(\int \frac{atanh(cx)}{x}dx\right)abcx \cdots\right.}{x}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))^2/x^2,x)
```

output
```
(d*( - atanh(c*x)**2*b**2 - 2*atanh(c*x)*a*b*c*x - 2*atanh(c*x)*a*b - 2*in
t(atanh(c*x)/(c**2*x**3 - x),x)*b**2*c*x + 2*int(atanh(c*x)/x,x)*a*b*c*x +
 int(atanh(c*x)**2/x,x)*b**2*c*x - 2*log(c**2*x - c)*a*b*c*x + log(x)*a**2
*c*x + 2*log(x)*a*b*c*x - a**2))/x
```

3.74 $\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\begin{aligned}\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^3}\,dx = &-\frac{bcd(a+b\text{arctanh}(cx))}{x} \\ &+\frac{3}{2}c^2d(a+b\text{arctanh}(cx))^2 \\ &-\frac{d(a+b\text{arctanh}(cx))^2}{2x^2}-\frac{cd(a+b\text{arctanh}(cx))^2}{x} \\ &+b^2c^2d\log(x)-\frac{1}{2}b^2c^2d\log\left(1-c^2x^2\right) \\ &+2bc^2d(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) \\ &-b^2c^2d\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
-b*c*d*(a+b*arctanh(c*x))/x+3/2*c^2*d*(a+b*arctanh(c*x))^2-1/2*d*(a+b*arct
anh(c*x))^2/x^2-c*d*(a+b*arctanh(c*x))^2/x+b^2*c^2*d*ln(x)-1/2*b^2*c^2*d*l
n(-c^2*x^2+1)+2*b*c^2*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^2*c^2*d*polyl
og(2,-1+2/(c*x+1))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^3}\,dx =$$

$$-\frac{d\Big(a^2+2a^2cx+2abcx+b^2(1+2cx-3c^2x^2)\,\text{arctanh}(cx)^2+2b\text{arctanh}(cx)\,\big(a+2acx+bcx-2bc^2x^2}{}$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

output
```
-1/2*(d*(a^2 + 2*a^2*c*x + 2*a*b*c*x + b^2*(1 + 2*c*x - 3*c^2*x^2)*ArcTanh
[c*x]^2 + 2*b*ArcTanh[c*x]*(a + 2*a*c*x + b*c*x - 2*b*c^2*x^2*Log[1 - E^(-
2*ArcTanh[c*x])]) - 4*a*b*c^2*x^2*Log[c*x] + a*b*c^2*x^2*Log[1 - c*x] - a*
b*c^2*x^2*Log[1 + c*x] - 2*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*
b*c^2*x^2*Log[1 - c^2*x^2] + 2*b^2*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]
))/x^2
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(\frac{d(a+b\text{arctanh}(cx))^2}{x^3}+\frac{cd(a+b\text{arctanh}(cx))^2}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{2}c^2d(a+b\text{arctanh}(cx))^2+2bc^2d\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{d(a+b\text{arctanh}(cx))^2}{2x^2}-\frac{cd(a+b\text{arctanh}(cx))^2}{x}-\frac{bcd(a+b\text{arctanh}(cx))}{x}-b^2c^2d\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{1}{2}b^2c^2d\log\left(1-c^2x^2\right)+b^2c^2d\log(x)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

output
```
-((b*c*d*(a + b*ArcTanh[c*x]))/x) + (3*c^2*d*(a + b*ArcTanh[c*x])^2)/2 - (
d*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (c*d*(a + b*ArcTanh[c*x])^2)/x + b^2*c
^2*d*Log[x] - (b^2*c^2*d*Log[1 - c^2*x^2])/2 + 2*b*c^2*d*(a + b*ArcTanh[c*
x])*Log[2 - 2/(1 + c*x)] - b^2*c^2*d*PolyLog[2, -1 + 2/(1 + c*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.83

method	result
parts	$d\,a^2\left(-\frac{c}{x}-\frac{1}{2x^2}\right)+d\,b^2c^2\left(-\frac{\text{arctanh}(cx)^2}{2c^2x^2}-\frac{\text{arctanh}(cx)^2}{cx}-\frac{3\,\text{arctanh}(cx)\ln(cx-1)}{2}-\frac{\text{arctanh}(cx)}{cx}+\right.$
derivativedivides	$c^2\left(d\,a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{cx}\right)+d\,b^2\left(-\frac{\text{arctanh}(cx)^2}{2c^2x^2}-\frac{\text{arctanh}(cx)^2}{cx}-\frac{3\,\text{arctanh}(cx)\ln(cx-1)}{2}-\frac{\text{arctanh}(c\right.\right.$
default	$c^2\left(d\,a^2\left(-\frac{1}{2c^2x^2}-\frac{1}{cx}\right)+d\,b^2\left(-\frac{\text{arctanh}(cx)^2}{2c^2x^2}-\frac{\text{arctanh}(cx)^2}{cx}-\frac{3\,\text{arctanh}(cx)\ln(cx-1)}{2}-\frac{\text{arctanh}(c\right.\right.$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(-c/x-1/2/x^2)+d*b^2*c^2*(-1/2*arctanh(c*x)^2/c^2/x^2-arctanh(c*x)^2
/c/x-3/2*arctanh(c*x)*ln(c*x-1)-arctanh(c*x)/c/x+2*arctanh(c*x)*ln(c*x)-1/
2*arctanh(c*x)*ln(c*x+1)-1/2*ln(c*x-1)+ln(c*x)-1/2*ln(c*x+1)-dilog(c*x)-di
log(c*x+1)-ln(c*x)*ln(c*x+1)-3/8*ln(c*x-1)^2+dilog(1/2*c*x+1/2)+3/4*ln(c*x
-1)*ln(1/2*c*x+1/2)+1/8*ln(c*x+1)^2-1/4*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/
2*c*x+1/2))+2*d*a*b*c^2*(-1/2*arctanh(c*x)/c^2/x^2-arctanh(c*x)/c/x-3/4*ln
(c*x-1)-1/2/c/x+ln(c*x)-1/4*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^3}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*
c*d*x + a*b*d)*arctanh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^3}\,dx=d\left(\int \frac{a^2}{x^3}\,dx+\int \frac{a^2c}{x^2}\,dx+\int \frac{b^2\,\text{atanh}^2\,(cx)}{x^3}\,dx+\int \frac{2ab\,\text{atanh}\,(cx)}{x^3}\,dx+\int \frac{b^2c\,\text{atanh}^2\,(cx)}{x^2}\,dx+\int \frac{2abc\,\text{atanh}\,(cx)}{x^2}\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**3,x)
```

output
```
d*(Integral(a**2/x**3, x) + Integral(a**2*c/x**2, x) + Integral(b**2*atanh
(c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(b**2*c*a
tanh(c*x)**2/x**2, x) + Integral(2*a*b*c*atanh(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^3}\,dx=\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c*d - 1/4*b^2*c*
d*(log(-c*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*
x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) + 1/2*((c*log(c*x +
 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*d + 1/8*((2*(log(c
*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x -
1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c*arctanh(c
*x))*b^2*d - a^2*c*d/x - 1/2*b^2*d*arctanh(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

Giac [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^3}\,dx=\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^3,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^3}\,dx$$
$$= \frac{d\Big(atanh(cx)^2\,b^2c^2x^2 - 2atanh(cx)^2\,b^2cx - atanh(cx)^2\,b^2 - 2atanh(cx)\,ab\,c^2x^2 - 4atanh(cx)\,abcx - 2at}{}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))^2/x^3,x)
```

output
```
(d*(atanh(c*x)**2*b**2*c**2*x**2 - 2*atanh(c*x)**2*b**2*c*x - atanh(c*x)**
2*b**2 - 2*atanh(c*x)*a*b*c**2*x**2 - 4*atanh(c*x)*a*b*c*x - 2*atanh(c*x)*
a*b - 4*atanh(c*x)*b**2*c**2*x**2 - 2*atanh(c*x)*b**2*c*x + 2*atanh(c*x)*b
**2 - 4*int(atanh(c*x)/(c**2*x**5 - x**3),x)*b**2*x**2 - 4*log(c**2*x - c)
*a*b*c**2*x**2 - 2*log(c**2*x - c)*b**2*c**2*x**2 + 4*log(x)*a*b*c**2*x**2
 + 2*log(x)*b**2*c**2*x**2 - 2*a**2*c*x - a**2 - 2*a*b*c*x + 2*b**2*c*x))/
(2*x**2)
```

3.75 $\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 206

$$\begin{aligned}\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^4}\,dx = &-\frac{b^2c^2d}{3x}+\frac{1}{3}b^2c^3d\operatorname{arctanh}(cx)\\ &-\frac{bcd(a+b\operatorname{arctanh}(cx))}{3x^2}\\ &-\frac{bc^2d(a+b\operatorname{arctanh}(cx))}{x}\\ &+\frac{5}{6}c^3d(a+b\operatorname{arctanh}(cx))^2\\ &-\frac{d(a+b\operatorname{arctanh}(cx))^2}{3x^3}-\frac{cd(a+b\operatorname{arctanh}(cx))^2}{2x^2}\\ &+b^2c^3d\log(x)-\frac{1}{2}b^2c^3d\log\left(1-c^2x^2\right)\\ &+\frac{2}{3}bc^3d(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)\\ &-\frac{1}{3}b^2c^3d\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d/x+1/3*b^2*c^3*d*arctanh(c*x)-1/3*b*c*d*(a+b*arctanh(c*x))/x
^2-b*c^2*d*(a+b*arctanh(c*x))/x+5/6*c^3*d*(a+b*arctanh(c*x))^2-1/3*d*(a+b*
arctanh(c*x))^2/x^3-1/2*c*d*(a+b*arctanh(c*x))^2/x^2+b^2*c^3*d*ln(x)-1/2*b
^2*c^3*d*ln(-c^2*x^2+1)+2/3*b*c^3*d*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-1/3
*b^2*c^3*d*polylog(2,-1+2/(c*x+1))
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.19

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^4}\,dx =$$

$$-\frac{d\left(2a^2+3a^2cx+2abcx+6abc^2x^2+2b^2c^2x^2+b^2(2+3cx-5c^3x^3)\,\text{arctanh}(cx)^2+2b\text{arctanh}(cx)\,(a(\right.}{}$$

input
```
Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

output
```
-1/6*(d*(2*a^2 + 3*a^2*c*x + 2*a*b*c*x + 6*a*b*c^2*x^2 + 2*b^2*c^2*x^2 + b
^2*(2 + 3*c*x - 5*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*(2 + 3*c*x
) + b*c*x*(1 + 3*c*x - c^2*x^2) - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]
) - 4*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*Log[1 - c*x] - 3*a*b*c^3*x^3*Lo
g[1 + c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^3*x^3*Lo
g[1 - c^2*x^2] + 2*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^3
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{d(a+b\text{arctanh}(cx))^2}{x^4}+\frac{cd(a+b\text{arctanh}(cx))^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{6}c^3d(a+b\operatorname{arctanh}(cx))^2+\frac{2}{3}bc^3d\log\left(2-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))-\frac{bc^2d(a+b\operatorname{arctanh}(cx))}{x}-\frac{d(a+b\operatorname{arctanh}(cx))^2}{3x^3}-\frac{cd(a+b\operatorname{arctanh}(cx))^2}{2x^2}-\frac{bcd(a+b\operatorname{arctanh}(cx))}{3x^2}+\frac{1}{3}b^2c^3d\operatorname{arctanh}(cx)-\frac{1}{3}b^2c^3d\operatorname{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+b^2c^3d\log(x)-\frac{b^2c^2d}{3x}-\frac{1}{2}b^2c^3d\log\left(1-c^2x^2\right)$$

input
```
Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

output
```
-1/3*(b^2*c^2*d)/x + (b^2*c^3*d*ArcTanh[c*x])/3 - (b*c*d*(a + b*ArcTanh[c*
x]))/(3*x^2) - (b*c^2*d*(a + b*ArcTanh[c*x]))/x + (5*c^3*d*(a + b*ArcTanh[
c*x])^2)/6 - (d*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (c*d*(a + b*ArcTanh[c*x]
)^2)/(2*x^2) + b^2*c^3*d*Log[x] - (b^2*c^3*d*Log[1 - c^2*x^2])/2 + (2*b*c^
3*d*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*d*PolyLog[2, -
1 + 2/(1 + c*x)])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.50

method	result
parts	$d\,a^2\left(-\frac{c}{2x^2}-\frac{1}{3x^3}\right)+d\,b^2c^3\left(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{5\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{\operatorname{arctanh}(cx)}{3c^2x^2}\right.$
derivativedivides	$c^3\left(d\,a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{5\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{\operatorname{arctan}}{3c^2}\right.\right.$
default	$c^3\left(d\,a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{5\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{\operatorname{arctan}}{3c^2}\right.\right.$

input
```
int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(-1/2*c/x^2-1/3/x^3)+d*b^2*c^3*(-1/3*arctanh(c*x)^2/c^3/x^3-1/2*arct
anh(c*x)^2/c^2/x^2-5/6*arctanh(c*x)*ln(c*x-1)-1/3*arctanh(c*x)/c^2/x^2-arc
tanh(c*x)/c/x+2/3*arctanh(c*x)*ln(c*x)+1/6*arctanh(c*x)*ln(c*x+1)-1/24*ln(
c*x+1)^2+1/12*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/3*dilog(1/2*c
*x+1/2)-1/3*dilog(c*x)-1/3*dilog(c*x+1)-1/3*ln(c*x)*ln(c*x+1)-5/24*ln(c*x-
1)^2+5/12*ln(c*x-1)*ln(1/2*c*x+1/2)-2/3*ln(c*x-1)-1/3/c/x+ln(c*x)-1/3*ln(c
*x+1))+2*d*a*b*c^3*(-1/3*arctanh(c*x)/c^3/x^3-1/2*arctanh(c*x)/c^2/x^2-5/1
2*ln(c*x-1)-1/6/c^2/x^2-1/2/c/x+1/3*ln(c*x)+1/12*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)(a+b\operatorname{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*
c*d*x + a*b*d)*arctanh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^4}\,dx = d\left(\int \frac{a^2}{x^4}\,dx + \int \frac{a^2c}{x^3}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^4}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^4}\,dx + \int \frac{b^2c\operatorname{atanh}^2(cx)}{x^3}\,dx + \int \frac{2abc\operatorname{atanh}(cx)}{x^3}\,dx\right)$$

input
```
integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**4,x)
```

output
```
d*(Integral(a**2/x**4, x) + Integral(a**2*c/x**3, x) + Integral(b**2*atanh
(c*x)**2/x**4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(b**2*c*a
tanh(c*x)**2/x**3, x) + Integral(2*a*b*c*atanh(c*x)/x**3, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(187) = 374.

Time = 0.47 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.02

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$
$$= -\frac{1}{3}\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2c^3d - \frac{1}{3}(\log(cx)\log(-cx+1)+\text{Li}_2(-cx+1))b^2c^3d + \frac{1}{3}(\log(cx+1)\log(-cx)+\text{Li}_2(cx+1))b^2c^3d - \frac{1}{3}b^2c^3d\log(cx+1) - \frac{2}{3}b^2c^3d\log(cx-1) + b^2c^3d\log(x) + \frac{1}{2}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}(cx)}{x^2}\right)abcd - \frac{1}{3}\left(\left(c^2\log(c^2x^2-1)-c^2\log(x^2)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}(cx)}{x^3}\right)abd - \frac{a^2cd}{2x^2} - \frac{a^2d}{3x^3} - \frac{8b^2c^2dx^2-(b^2c^3dx^3-3b^2cdx-2b^2d)\log(cx+1)^2-(5b^2c^3dx^3-3b^2cdx-2b^2d)\log(-cx+1)^2+}{}$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d -
 1/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d + 1/3*(log(c*x +
 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d - 1/3*b^2*c^3*d*log(c*x + 1) - 2
/3*b^2*c^3*d*log(c*x - 1) + b^2*c^3*d*log(x) + 1/2*((c*log(c*x + 1) - c*lo
g(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d - 1/3*((c^2*log(c^2*x^2
- 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d - 1/2*a^2*c*d/x
^2 - 1/3*a^2*d/x^3 - 1/24*(8*b^2*c^2*d*x^2 - (b^2*c^3*d*x^3 - 3*b^2*c*d*x
- 2*b^2*d)*log(c*x + 1)^2 - (5*b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(
-c*x + 1)^2 + 4*(3*b^2*c^2*d*x^2 + b^2*c*d*x)*log(c*x + 1) - 2*(6*b^2*c^2*
d*x^2 + 2*b^2*c*d*x - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(c*x + 1)
)*log(-c*x + 1))/x^3
```

Giac [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)}{x^4}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^4,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x))/x^4, x)
```

Reduce [F]

$$\int \frac{(d+cdx)(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$
$$= \frac{d\Big(3atanh(cx)^2 b^2c^3x^3 - 3atanh(cx)^2 b^2cx - 2atanh(cx)^2 b^2 + 2atanh(cx)\,ab\,c^3x^3 - 6atanh(cx)\,abcx - 4}{}$$

input
```
int((c*d*x+d)*(a+b*atanh(c*x))^2/x^4,x)
```

output
```
(d*(3*atanh(c*x)**2*b**2*c**3*x**3 - 3*atanh(c*x)**2*b**2*c*x - 2*atanh(c*
x)**2*b**2 + 2*atanh(c*x)*a*b*c**3*x**3 - 6*atanh(c*x)*a*b*c*x - 4*atanh(c
*x)*a*b - 6*atanh(c*x)*b**2*c**3*x**3 - 6*atanh(c*x)*b**2*c**2*x**2 - 4*in
t(atanh(c*x)/(c**2*x**5 - x**3),x)*b**2*c*x**3 - 4*log(c**2*x - c)*a*b*c**
3*x**3 - 6*log(c**2*x - c)*b**2*c**3*x**3 + 4*log(x)*a*b*c**3*x**3 + 6*log
(x)*b**2*c**3*x**3 - 3*a**2*c*x - 2*a**2 - 6*a*b*c**2*x**2 - 2*a*b*c*x))/(
6*x**3)
```

3.76 $\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 356

$$\begin{aligned}
&\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx\\
&=\frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\text{arctanh}(cx)}{5c^4}\\
&\quad+\frac{5b^2d^2x\text{arctanh}(cx)}{6c^3}+\frac{2bd^2x^2(a+b\text{arctanh}(cx))}{5c^2}+\frac{5bd^2x^3(a+b\text{arctanh}(cx))}{18c}\\
&\quad+\frac{1}{5}bd^2x^4(a+b\text{arctanh}(cx))+\frac{1}{15}bcd^2x^5(a+b\text{arctanh}(cx))\\
&\quad-\frac{d^2(a+b\text{arctanh}(cx))^2}{60c^4}+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))^2+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))^2\\
&\quad+\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))^2-\frac{4bd^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{5c^4}\\
&\quad+\frac{53b^2d^2\log(1-c^2x^2)}{90c^4}-\frac{2b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}
\end{aligned}$$

output
```
5/6*a*b*d^2*x/c^3+3/5*b^2*d^2*x/c^3+31/180*b^2*d^2*x^2/c^2+1/15*b^2*d^2*x^
3/c+1/60*b^2*d^2*x^4-3/5*b^2*d^2*arctanh(c*x)/c^4+5/6*b^2*d^2*x*arctanh(c*
x)/c^3+2/5*b*d^2*x^2*(a+b*arctanh(c*x))/c^2+5/18*b*d^2*x^3*(a+b*arctanh(c*
x))/c+1/5*b*d^2*x^4*(a+b*arctanh(c*x))+1/15*b*c*d^2*x^5*(a+b*arctanh(c*x))
-1/60*d^2*(a+b*arctanh(c*x))^2/c^4+1/4*d^2*x^4*(a+b*arctanh(c*x))^2+2/5*c*
d^2*x^5*(a+b*arctanh(c*x))^2+1/6*c^2*d^2*x^6*(a+b*arctanh(c*x))^2-4/5*b*d^
2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+53/90*b^2*d^2*ln(-c^2*x^2+1)/c^4-2
/5*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^4
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.92

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^2\left(-108ab-34b^2+150abcx+108b^2cx+72abc^2x^2+31b^2c^2x^2+50abc^3x^3+12b^2c^3x^3+45a^2c^4x^4+36\right.}{}$$

input
```
Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^2*(-108*a*b - 34*b^2 + 150*a*b*c*x + 108*b^2*c*x + 72*a*b*c^2*x^2 + 31*
b^2*c^2*x^2 + 50*a*b*c^3*x^3 + 12*b^2*c^3*x^3 + 45*a^2*c^4*x^4 + 36*a*b*c^
4*x^4 + 3*b^2*c^4*x^4 + 72*a^2*c^5*x^5 + 12*a*b*c^5*x^5 + 30*a^2*c^6*x^6 +
 3*b^2*(-49 + 15*c^4*x^4 + 24*c^5*x^5 + 10*c^6*x^6)*ArcTanh[c*x]^2 + 2*b*A
rcTanh[c*x]*(3*a*c^4*x^4*(15 + 24*c*x + 10*c^2*x^2) + b*(-54 + 75*c*x + 36
*c^2*x^2 + 25*c^3*x^3 + 18*c^4*x^4 + 6*c^5*x^5) - 72*b*Log[1 + E^(-2*ArcTa
nh[c*x])]) + 75*a*b*Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 106*b^2*Log[1 - c
^2*x^2] + 72*a*b*Log[-1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x]
)]))/(180*c^4)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)^2(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^2d^2x^5(a+b\text{arctanh}(cx))^2+2cd^2x^4(a+b\text{arctanh}(cx))^2+d^2x^3(a+b\text{arctanh}(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\text{arctanh}(cx))^2}{60c^4}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{5c^4}+\frac{1}{6}c^2d^2x^6(a+b\text{arctanh}(cx))^2+\frac{2bd^2x^2(a+b\text{arctanh}(cx))}{5c^2}+\frac{2}{5}cd^2x^5(a+b\text{arctanh}(cx))^2+\frac{1}{15}bcd^2x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^2x^4(a+b\text{arctanh}(cx))^2+\frac{1}{5}bd^2x^4(a+b\text{arctanh}(cx))+\frac{5bd^2x^3(a+b\text{arctanh}(cx))}{18c}+\frac{5abd^2x}{6c^3}-\frac{3b^2d^2\text{arctanh}(cx)}{5c^4}+\frac{5b^2d^2x\text{arctanh}(cx)}{6c^3}-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{3b^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{53b^2d^2\log(1-c^2x^2)}{90c^4}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4$$

input
```
Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(5*a*b*d^2*x)/(6*c^3) + (3*b^2*d^2*x)/(5*c^3) + (31*b^2*d^2*x^2)/(180*c^2)
 + (b^2*d^2*x^3)/(15*c) + (b^2*d^2*x^4)/60 - (3*b^2*d^2*ArcTanh[c*x])/(5*c
^4) + (5*b^2*d^2*x*ArcTanh[c*x])/(6*c^3) + (2*b*d^2*x^2*(a + b*ArcTanh[c*x
]))/(5*c^2) + (5*b*d^2*x^3*(a + b*ArcTanh[c*x]))/(18*c) + (b*d^2*x^4*(a +
b*ArcTanh[c*x]))/5 + (b*c*d^2*x^5*(a + b*ArcTanh[c*x]))/15 - (d^2*(a + b*A
rcTanh[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 + (2*c*d^2*x
^5*(a + b*ArcTanh[c*x])^2)/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x])^2)/6 - (4
*b*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (53*b^2*d^2*Log[1
- c^2*x^2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.05

method	result
parts	$d^2a^2\left(\frac{1}{6}c^2x^6+\frac{2}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^6x^6}{6}+\frac{2\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^5}{15}\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{6}c^6x^6+\frac{2}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^6x^6}{6}+\frac{2\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^5x^5}{15}+\operatorname{arcta}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{6}c^6x^6+\frac{2}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^6x^6}{6}+\frac{2\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)c^5x^5}{15}+\operatorname{arcta}\right.}{}$
risch	$-\frac{d^2b\ln(-cx-1)a}{60c^4}-\frac{2d^2cab\ln(-cx+1)x^5}{5}-\frac{d^2c^2ab\ln(-cx+1)x^6}{6}-\frac{16d^2ba}{9c^4}+\frac{2d^2b^2\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{5c^4}+\frac{d^2c^2b}{}$

input
```
int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/6*c^2*x^6+2/5*c*x^5+1/4*x^4)+d^2*b^2/c^4*(1/6*arctanh(c*x)^2*c^
6*x^6+2/5*arctanh(c*x)^2*c^5*x^5+1/4*arctanh(c*x)^2*c^4*x^4+1/15*arctanh(c
*x)*c^5*x^5+1/5*arctanh(c*x)*c^4*x^4+5/18*arctanh(c*x)*c^3*x^3+2/5*arctanh
(c*x)*c^2*x^2+5/6*arctanh(c*x)*c*x+49/60*arctanh(c*x)*ln(c*x-1)-1/60*arcta
nh(c*x)*ln(c*x+1)+49/240*ln(c*x-1)^2-2/5*dilog(1/2*c*x+1/2)-49/120*ln(c*x-
1)*ln(1/2*c*x+1/2)+1/240*ln(c*x+1)^2-1/120*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(
-1/2*c*x+1/2)+1/60*c^4*x^4+1/15*x^3*c^3+31/180*c^2*x^2+3/5*c*x+8/9*ln(c*x-
1)+13/45*ln(c*x+1))+2*d^2*a*b/c^4*(1/6*arctanh(c*x)*c^6*x^6+2/5*arctanh(c*
x)*c^5*x^5+1/4*arctanh(c*x)*c^4*x^4+1/30*c^5*x^5+1/10*c^4*x^4+5/36*x^3*c^3
+1/5*c^2*x^2+5/12*c*x+49/120*ln(c*x-1)-1/120*ln(c*x+1))
```

Fricas [F]

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^2*d^2*x^5 + 2*a^2*c*d^2*x^4 + a^2*d^2*x^3 + (b^2*c^2*d^2*x^
5 + 2*b^2*c*d^2*x^4 + b^2*d^2*x^3)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^5 + 2
*a*b*c*d^2*x^4 + a*b*d^2*x^3)*arctanh(c*x), x)
```

Sympy [F]

$$\begin{aligned}&\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx\\&=d^2\bigg(\int a^2x^3\,dx+\int 2a^2cx^4\,dx+\int a^2c^2x^5\,dx+\int b^2x^3\,\text{atanh}^2\,(cx)\,dx\\&\quad+\int 2abx^3\,\text{atanh}\,(cx)\,dx+\int 2b^2cx^4\,\text{atanh}^2\,(cx)\,dx+\int b^2c^2x^5\,\text{atanh}^2\,(cx)\,dx\\&\quad+\int 4abcx^4\,\text{atanh}\,(cx)\,dx+\int 2abc^2x^5\,\text{atanh}\,(cx)\,dx\bigg)\end{aligned}$$

input
```
integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

output
```
d**2*(Integral(a**2*x**3, x) + Integral(2*a**2*c*x**4, x) + Integral(a**2*
c**2*x**5, x) + Integral(b**2*x**3*atanh(c*x)**2, x) + Integral(2*a*b*x**3
*atanh(c*x), x) + Integral(2*b**2*c*x**4*atanh(c*x)**2, x) + Integral(b**2
*c**2*x**5*atanh(c*x)**2, x) + Integral(4*a*b*c*x**4*atanh(c*x), x) + Inte
gral(2*a*b*c**2*x**5*atanh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(317) = 634$.

Time = 0.30 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.15

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*c^2*d^2*x^6 + 2/5*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctanh(c*x)^2 +
 1/4*a^2*d^2*x^4 + 1/90*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3
 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^2 + 1
/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6
))*a*b*c*d^2 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log
(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*d^2 + 1/48*(4*c*(2*(c^2*x^3 + 3*x
)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2
 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)
^2 + 16*log(c*x - 1))/c^4)*b^2*d^2 + 2/5*(log(c*x + 1)*log(-1/2*c*x + 1/2)
 + dilog(1/2*c*x + 1/2))*b^2*d^2/c^4 - 2/45*b^2*d^2*log(c*x + 1)/c^4 + 5/9
*b^2*d^2*log(c*x - 1)/c^4 + 1/360*(6*b^2*c^4*d^2*x^4 + 24*b^2*c^3*d^2*x^3
+ 32*b^2*c^2*d^2*x^2 + 216*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5
*d^2*x^5 + 7*b^2*d^2)*log(c*x + 1)^2 + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d
^2*x^5 - 17*b^2*d^2)*log(-c*x + 1)^2 + 4*(3*b^2*c^5*d^2*x^5 + 9*b^2*c^4*d^
2*x^4 + 5*b^2*c^3*d^2*x^3 + 18*b^2*c^2*d^2*x^2 + 15*b^2*c*d^2*x)*log(c*x +
 1) - 2*(6*b^2*c^5*d^2*x^5 + 18*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 36*
b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x
^5 + 7*b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/c^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(317) = 634$.

Time = 2.25 (sec) , antiderivative size = 1135, normalized size of antiderivative = 3.19

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
1/63*(84*((c*x + 1)^5*b^2*d^2/(c*x - 1)^5 + (c*x + 1)^4*b^2*d^2/(c*x - 1)^
4 + (c*x + 1)^3*b^2*d^2/(c*x - 1)^3)*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1
)^8*c^7/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^7/(
c*x - 1)^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x - 1)
^4 - 56*(c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 - 8*(
c*x + 1)*c^7/(c*x - 1) + c^7) + 2*(168*(c*x + 1)^5*a*b*d^2/(c*x - 1)^5 + 1
68*(c*x + 1)^4*a*b*d^2/(c*x - 1)^4 + 168*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 +
 28*(c*x + 1)^5*b^2*d^2/(c*x - 1)^5 - 35*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 +
 28*(c*x + 1)^3*b^2*d^2/(c*x - 1)^3 - 28*(c*x + 1)^2*b^2*d^2/(c*x - 1)^2 +
 8*(c*x + 1)*b^2*d^2/(c*x - 1) - b^2*d^2)*log(-(c*x + 1)/(c*x - 1))/((c*x
+ 1)^8*c^7/(c*x - 1)^8 - 8*(c*x + 1)^7*c^7/(c*x - 1)^7 + 28*(c*x + 1)^6*c^
7/(c*x - 1)^6 - 56*(c*x + 1)^5*c^7/(c*x - 1)^5 + 70*(c*x + 1)^4*c^7/(c*x -
 1)^4 - 56*(c*x + 1)^3*c^7/(c*x - 1)^3 + 28*(c*x + 1)^2*c^7/(c*x - 1)^2 -
8*(c*x + 1)*c^7/(c*x - 1) + c^7) + (336*(c*x + 1)^5*a^2*d^2/(c*x - 1)^5 +
336*(c*x + 1)^4*a^2*d^2/(c*x - 1)^4 + 336*(c*x + 1)^3*a^2*d^2/(c*x - 1)^3
+ 112*(c*x + 1)^5*a*b*d^2/(c*x - 1)^5 - 140*(c*x + 1)^4*a*b*d^2/(c*x - 1)^
4 + 112*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 - 112*(c*x + 1)^2*a*b*d^2/(c*x - 1
)^2 + 32*(c*x + 1)*a*b*d^2/(c*x - 1) - 4*a*b*d^2 - 2*(c*x + 1)^7*b^2*d^2/(
c*x - 1)^7 + 15*(c*x + 1)^6*b^2*d^2/(c*x - 1)^6 - 30*(c*x + 1)^5*b^2*d^2/(
c*x - 1)^5 + 34*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 - 30*(c*x + 1)^3*b^2*d^...
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int x^3\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

input
```
int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

output
```
int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

Reduce [F]

$$\int x^3(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^2\left(144\log(c^2x-c)\,ab+72a^2c^5x^5+108b^2cx-6atanh(cx)\,ab+31b^2c^2x^2+30atanh(cx)^2\,b^2c^6x^6+12ata\right.}{}$$

input
```
int(x^3*(c*d*x+d)^2*(a+b*atanh(c*x))^2,x)
```

output
```
(d**2*(30*atanh(c*x)**2*b**2*c**6*x**6 + 72*atanh(c*x)**2*b**2*c**5*x**5 +
 45*atanh(c*x)**2*b**2*c**4*x**4 - 72*atanh(c*x)**2*b**2*c*x - 75*atanh(c*
x)**2*b**2 + 60*atanh(c*x)*a*b*c**6*x**6 + 144*atanh(c*x)*a*b*c**5*x**5 +
90*atanh(c*x)*a*b*c**4*x**4 - 6*atanh(c*x)*a*b + 12*atanh(c*x)*b**2*c**5*x
**5 + 36*atanh(c*x)*b**2*c**4*x**4 + 50*atanh(c*x)*b**2*c**3*x**3 + 72*ata
nh(c*x)*b**2*c**2*x**2 + 150*atanh(c*x)*b**2*c*x + 104*atanh(c*x)*b**2 + 7
2*int(atanh(c*x)**2,x)*b**2*c + 144*log(c**2*x - c)*a*b + 212*log(c**2*x -
 c)*b**2 + 30*a**2*c**6*x**6 + 72*a**2*c**5*x**5 + 45*a**2*c**4*x**4 + 12*
a*b*c**5*x**5 + 36*a*b*c**4*x**4 + 50*a*b*c**3*x**3 + 72*a*b*c**2*x**2 + 1
50*a*b*c*x + 3*b**2*c**4*x**4 + 12*b**2*c**3*x**3 + 31*b**2*c**2*x**2 + 10
8*b**2*c*x))/(180*c**4)
```

3.77 $\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 312

$$\begin{aligned}\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx &= \frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\text{arctanh}(cx)}{30c^3}\\ &+\frac{b^2d^2x\text{arctanh}(cx)}{c^2}+\frac{8bd^2x^2(a+b\text{arctanh}(cx))}{15c}+\frac{1}{3}bd^2x^3(a+b\text{arctanh}(cx))\\ &+\frac{1}{10}bcd^2x^4(a+b\text{arctanh}(cx))+\frac{d^2(a+b\text{arctanh}(cx))^2}{30c^3}\\ &+\frac{1}{3}d^2x^3(a+b\text{arctanh}(cx))^2+\frac{1}{2}cd^2x^4(a+b\text{arctanh}(cx))^2\\ &+\frac{1}{5}c^2d^2x^5(a+b\text{arctanh}(cx))^2-\frac{16bd^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{15c^3}\\ &+\frac{2b^2d^2\log(1-c^2x^2)}{3c^3}-\frac{8b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}\end{aligned}$$

output
```
a*b*d^2*x/c^2+19/30*b^2*d^2*x/c^2+1/6*b^2*d^2*x^2/c+1/30*b^2*d^2*x^3-19/30
*b^2*d^2*arctanh(c*x)/c^3+b^2*d^2*x*arctanh(c*x)/c^2+8/15*b*d^2*x^2*(a+b*a
rctanh(c*x))/c+1/3*b*d^2*x^3*(a+b*arctanh(c*x))+1/10*b*c*d^2*x^4*(a+b*arct
anh(c*x))+1/30*d^2*(a+b*arctanh(c*x))^2/c^3+1/3*d^2*x^3*(a+b*arctanh(c*x))
^2+1/2*c*d^2*x^4*(a+b*arctanh(c*x))^2+1/5*c^2*d^2*x^5*(a+b*arctanh(c*x))^2
-16/15*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^3+2/3*b^2*d^2*ln(-c^2*x^2
+1)/c^3-8/15*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^3
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.95

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^2\left(-9ab-5b^2+30abcx+19b^2cx+16abc^2x^2+5b^2c^2x^2+10a^2c^3x^3+10abc^3x^3+b^2c^3x^3+15a^2c^4x^4+\right.}{}$$

input
```
Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^2*(-9*a*b - 5*b^2 + 30*a*b*c*x + 19*b^2*c*x + 16*a*b*c^2*x^2 + 5*b^2*c^
2*x^2 + 10*a^2*c^3*x^3 + 10*a*b*c^3*x^3 + b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 3
*a*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-31 + 10*c^3*x^3 + 15*c^4*x^4 + 6*c^5*
x^5)*ArcTanh[c*x]^2 + b*ArcTanh[c*x]*(2*a*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2
) + b*(-19 + 30*c*x + 16*c^2*x^2 + 10*c^3*x^3 + 3*c^4*x^4) - 32*b*Log[1 +
E^(-2*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15*a*b*Log[1 + c*x] + 20*b^2
*Log[1 - c^2*x^2] + 16*a*b*Log[-1 + c^2*x^2] + 16*b^2*PolyLog[2, -E^(-2*Ar
cTanh[c*x])]))/(30*c^3)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(c^2d^2x^4(a+b\text{arctanh}(cx))^2+2cd^2x^3(a+b\text{arctanh}(cx))^2+d^2x^2(a+b\text{arctanh}(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{d^2(a+b\text{arctanh}(cx))^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{15c^3}+\frac{1}{5}c^2d^2x^5(a+b\text{arctanh}(cx))^2+\frac{1}{2}cd^2x^4(a+b\text{arctanh}(cx))^2+\frac{1}{10}bcd^2x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^2x^3(a+b\text{arctanh}(cx))^2+\frac{1}{3}bd^2x^3(a+b\text{arctanh}(cx))+\frac{8bd^2x^2(a+b\text{arctanh}(cx))}{15c}+\frac{abd^2x}{c^2}-\frac{19b^2d^2\text{arctanh}(cx)}{30c^3}+\frac{b^2d^2x\text{arctanh}(cx)}{c^2}-\frac{8b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{19b^2d^2x}{30c^2}+\frac{2b^2d^2\log(1-c^2x^2)}{3c^3}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3$$

input
```
Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + (b^2*d^2*x^2)/(6*c) + (b^2*d^2
*x^3)/30 - (19*b^2*d^2*ArcTanh[c*x])/(30*c^3) + (b^2*d^2*x*ArcTanh[c*x])/c
^2 + (8*b*d^2*x^2*(a + b*ArcTanh[c*x]))/(15*c) + (b*d^2*x^3*(a + b*ArcTanh
[c*x]))/3 + (b*c*d^2*x^4*(a + b*ArcTanh[c*x]))/10 + (d^2*(a + b*ArcTanh[c*
x])^2)/(30*c^3) + (d^2*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d^2*x^4*(a + b*A
rcTanh[c*x])^2)/2 + (c^2*d^2*x^5*(a + b*ArcTanh[c*x])^2)/5 - (16*b*d^2*(a
+ b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(15*c^3) + (2*b^2*d^2*Log[1 - c^2*x^2]
)/(3*c^3) - (8*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.11

method	result
parts	$d^2a^2\left(\frac{1}{5}c^2x^5+\frac{1}{2}c\,x^4+\frac{1}{3}x^3\right)+\frac{d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{2}+\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^4x}{10}\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{5}c^5x^5+\frac{1}{2}c^4x^4+\frac{1}{3}x^3c^3\right)+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{2}+\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^4x^4}{10}+\operatorname{arctan}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{5}c^5x^5+\frac{1}{2}c^4x^4+\frac{1}{3}x^3c^3\right)+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4x^4}{2}+\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)c^4x^4}{10}+\operatorname{arctan}\right.}{}$
risch	$-\frac{d^2c^2ab\ln(-cx+1)x^5}{5}-\frac{d^2cab\ln(-cx+1)x^4}{2}-\frac{d^2b^2\ln(-cx+1)x^3}{6}-\frac{31d^2b^2\ln(-cx+1)^2}{120c^3}+\frac{d^2b^2\ln(-cx+1)^2}{12}$

input
```
int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/5*c^2*x^5+1/2*c*x^4+1/3*x^3)+d^2*b^2/c^3*(1/5*arctanh(c*x)^2*c^
5*x^5+1/2*arctanh(c*x)^2*c^4*x^4+1/3*arctanh(c*x)^2*c^3*x^3+1/10*arctanh(c
*x)*c^4*x^4+1/3*arctanh(c*x)*c^3*x^3+8/15*arctanh(c*x)*c^2*x^2+arctanh(c*x
)*c*x+31/30*arctanh(c*x)*ln(c*x-1)+1/30*arctanh(c*x)*ln(c*x+1)-1/120*ln(c*
x+1)^2+1/60*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-8/15*dilog(1/2*c*
x+1/2)+1/30*x^3*c^3+1/6*c^2*x^2+19/30*c*x+59/60*ln(c*x-1)+7/20*ln(c*x+1)+3
1/120*ln(c*x-1)^2-31/60*ln(c*x-1)*ln(1/2*c*x+1/2))+2*d^2*a*b/c^3*(1/5*arct
anh(c*x)*c^5*x^5+1/2*arctanh(c*x)*c^4*x^4+1/3*arctanh(c*x)*c^3*x^3+1/20*c^
4*x^4+1/6*x^3*c^3+4/15*c^2*x^2+1/2*c*x+31/60*ln(c*x-1)+1/60*ln(c*x+1))
```

Fricas [F]

$$\int x^2(d+cdx)^2(a+b\operatorname{arctanh}(cx))^2\,dx=\int (cdx+d)^2(b\operatorname{artanh}(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^2*d^2*x^4 + 2*a^2*c*d^2*x^3 + a^2*d^2*x^2 + (b^2*c^2*d^2*x^
4 + 2*b^2*c*d^2*x^3 + b^2*d^2*x^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^4 + 2
*a*b*c*d^2*x^3 + a*b*d^2*x^2)*arctanh(c*x), x)
```

Sympy [F]

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$= d^2\left(\int a^2x^2\,dx + \int 2a^2cx^3\,dx + \int a^2c^2x^4\,dx + \int b^2x^2\,\text{atanh}^2(cx)\,dx + \int 2abx^2\,\text{atanh}(cx)\,dx + \int 2b^2cx^3\,\text{atanh}^2(cx)\,dx + \int b^2c^2x^4\,\text{atanh}^2(cx)\,dx + \int 4abcx^3\,\text{atanh}(cx)\,dx + \int 2abc^2x^4\,\text{atanh}(cx)\,dx\right)$$

input
```
integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

output
```
d**2*(Integral(a**2*x**2, x) + Integral(2*a**2*c*x**3, x) + Integral(a**2*
c**2*x**4, x) + Integral(b**2*x**2*atanh(c*x)**2, x) + Integral(2*a*b*x**2
*atanh(c*x), x) + Integral(2*b**2*c*x**3*atanh(c*x)**2, x) + Integral(b**2
*c**2*x**4*atanh(c*x)**2, x) + Integral(4*a*b*c*x**3*atanh(c*x), x) + Inte
gral(2*a*b*c**2*x**4*atanh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(281) = 562$.

Time = 0.29 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.94

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = \frac{1}{5}a^2c^2d^2x^5 + \frac{1}{2}a^2cd^2x^4 + \frac{1}{10}\left(4x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)abc^2d^2 + \frac{1}{3}a^2d^2x^3 + \frac{1}{6}\left(6x^4\,\text{artanh}\,(cx) + c\left(\frac{2\left(c^2x^3+3x\right)}{c^4} - \frac{3\log\left(cx+1\right)}{c^5} + \frac{3\log\left(cx-1\right)}{c^5}\right)\right)abcd^2 + \frac{1}{3}\left(2x^3\,\text{artanh}\,(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)abd^2 + \frac{8\left(\log\left(cx+1\right)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right) + \text{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d^2}{15c^3} + \frac{7b^2d^2\log\left(cx+1\right)}{20c^3} + \frac{59b^2d^2\log\left(cx-1\right)}{60c^3} + \frac{4b^2c^3d^2x^3 + 20b^2c^2d^2x^2 + 76b^2cd^2x + \left(6b^2c^5d^2x^5 + 15b^2c^4d^2x^4 + 10b^2c^3d^2x^3 + b^2d^2\right)\log\left(cx+1\right)^2 +}{}$$

input
```
integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*c^2*d^2*x^5 + 1/2*a^2*c*d^2*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((c
^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2*x
^3 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c
^5 + 3*log(c*x - 1)/c^5))*a*b*c*d^2 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2
 + log(c^2*x^2 - 1)/c^4))*a*b*d^2 + 8/15*(log(c*x + 1)*log(-1/2*c*x + 1/2)
 + dilog(1/2*c*x + 1/2))*b^2*d^2/c^3 + 7/20*b^2*d^2*log(c*x + 1)/c^3 + 59/
60*b^2*d^2*log(c*x - 1)/c^3 + 1/120*(4*b^2*c^3*d^2*x^3 + 20*b^2*c^2*d^2*x^
2 + 76*b^2*c*d^2*x + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*
d^2*x^3 + b^2*d^2)*log(c*x + 1)^2 + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^
4 + 10*b^2*c^3*d^2*x^3 - 31*b^2*d^2)*log(-c*x + 1)^2 + 2*(3*b^2*c^4*d^2*x^
4 + 10*b^2*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x)*log(c*x + 1)
 - 2*(3*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2
*c*d^2*x + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 +
b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/c^3
```

Giac [F]

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int x^2\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

input
```
int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

output
```
int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

Reduce [F]

$$\int x^2(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^2\left(32\log(c^2x-c)\,ab+6a^2c^5x^5+19b^2cx+2atanh(cx)\,ab+5b^2c^2x^2+10atanh(cx)\,b^2c^3x^3+30atanh(c$$

input
```
int(x^2*(c*d*x+d)^2*(a+b*atanh(c*x))^2,x)
```

output
```
(d**2*(6*atanh(c*x)**2*b**2*c**5*x**5 + 15*atanh(c*x)**2*b**2*c**4*x**4 +
10*atanh(c*x)**2*b**2*c**3*x**3 - 16*atanh(c*x)**2*b**2*c*x - 15*atanh(c*x
)**2*b**2 + 12*atanh(c*x)*a*b*c**5*x**5 + 30*atanh(c*x)*a*b*c**4*x**4 + 20
*atanh(c*x)*a*b*c**3*x**3 + 2*atanh(c*x)*a*b + 3*atanh(c*x)*b**2*c**4*x**4
 + 10*atanh(c*x)*b**2*c**3*x**3 + 16*atanh(c*x)*b**2*c**2*x**2 + 30*atanh(
c*x)*b**2*c*x + 21*atanh(c*x)*b**2 + 16*int(atanh(c*x)**2,x)*b**2*c + 32*l
og(c**2*x - c)*a*b + 40*log(c**2*x - c)*b**2 + 6*a**2*c**5*x**5 + 15*a**2*
c**4*x**4 + 10*a**2*c**3*x**3 + 3*a*b*c**4*x**4 + 10*a*b*c**3*x**3 + 16*a*
b*c**2*x**2 + 30*a*b*c*x + b**2*c**3*x**3 + 5*b**2*c**2*x**2 + 19*b**2*c*x
))/(30*c**3)
```

3.78 $\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 280

$$\begin{aligned}\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx =& \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{1}{12}b^2d^2x^2\\&-\frac{2b^2d^2\text{arctanh}(cx)}{3c^2}+\frac{3b^2d^2x\text{arctanh}(cx)}{2c}\\&+\frac{2}{3}bd^2x^2(a+b\text{arctanh}(cx))\\&+\frac{1}{6}bcd^2x^3(a+b\text{arctanh}(cx))\\&-\frac{d^2(a+b\text{arctanh}(cx))^2}{12c^2}\\&+\frac{1}{2}d^2x^2(a+b\text{arctanh}(cx))^2\\&+\frac{2}{3}cd^2x^3(a+b\text{arctanh}(cx))^2\\&+\frac{1}{4}c^2d^2x^4(a+b\text{arctanh}(cx))^2\\&-\frac{4bd^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c^2}\\&+\frac{5b^2d^2\log(1-c^2x^2)}{6c^2}\\&-\frac{2b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}\end{aligned}$$

output
```
3/2*a*b*d^2*x/c+2/3*b^2*d^2*x/c+1/12*b^2*d^2*x^2-2/3*b^2*d^2*arctanh(c*x)/
c^2+3/2*b^2*d^2*x*arctanh(c*x)/c+2/3*b*d^2*x^2*(a+b*arctanh(c*x))+1/6*b*c*
d^2*x^3*(a+b*arctanh(c*x))-1/12*d^2*(a+b*arctanh(c*x))^2/c^2+1/2*d^2*x^2*(
a+b*arctanh(c*x))^2+2/3*c*d^2*x^3*(a+b*arctanh(c*x))^2+1/4*c^2*d^2*x^4*(a+
b*arctanh(c*x))^2-4/3*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2+5/6*b^2*
d^2*ln(-c^2*x^2+1)/c^2-2/3*b^2*d^2*polylog(2,1-2/(-c*x+1))/c^2
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$= \frac{d^2\left(-b^2+18abcx+8b^2cx+6a^2c^2x^2+8abc^2x^2+b^2c^2x^2+8a^2c^3x^3+2abc^3x^3+3a^2c^4x^4+b^2(-17+6c^2 \ldots\right.}{}$$

input
```
Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^2*(-b^2 + 18*a*b*c*x + 8*b^2*c*x + 6*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + b^2*
c^2*x^2 + 8*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-17 + 6*c^2
*x^2 + 8*c^3*x^3 + 3*c^4*x^4)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*x^2
*(6 + 8*c*x + 3*c^2*x^2) + b*(-4 + 9*c*x + 4*c^2*x^2 + c^3*x^3) - 8*b*Log[
1 + E^(-2*ArcTanh[c*x])]) + 9*a*b*Log[1 - c*x] - 9*a*b*Log[1 + c*x] + 10*b
^2*Log[1 - c^2*x^2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^(-2*Ar
cTanh[c*x])]))/(12*c^2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)^2(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(c^2d^2x^3(a+b\text{arctanh}(cx))^2+2cd^2x^2(a+b\text{arctanh}(cx))^2+d^2x(a+b\text{arctanh}(cx))^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}c^2d^2x^4(a+b\text{arctanh}(cx))^2-\frac{d^2(a+b\text{arctanh}(cx))^2}{12c^2}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{3c^2}+\frac{2}{3}cd^2x^3(a+b\text{arctanh}(cx))^2+\frac{1}{6}bcd^2x^3(a+b\text{arctanh}(cx))+\frac{1}{2}d^2x^2(a+b\text{arctanh}(cx))^2+\frac{2}{3}bd^2x^2(a+b\text{arctanh}(cx))+\frac{3abd^2x}{2c}-\frac{2b^2d^2\text{arctanh}(cx)}{3c^2}+\frac{3b^2d^2x\text{arctanh}(cx)}{2c}-\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}+\frac{5b^2d^2\log(1-c^2x^2)}{6c^2}+\frac{2b^2d^2x}{3c}+\frac{1}{12}b^2d^2x^2$$

input
```
Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(3*a*b*d^2*x)/(2*c) + (2*b^2*d^2*x)/(3*c) + (b^2*d^2*x^2)/12 - (2*b^2*d^2*
ArcTanh[c*x])/(3*c^2) + (3*b^2*d^2*x*ArcTanh[c*x])/(2*c) + (2*b*d^2*x^2*(a
 + b*ArcTanh[c*x]))/3 + (b*c*d^2*x^3*(a + b*ArcTanh[c*x]))/6 - (d^2*(a + b
*ArcTanh[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTanh[c*x])^2)/2 + (2*c*d^2
*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c^2*d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 -
(4*b*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (5*b^2*d^2*Log[1
 - c^2*x^2])/(6*c^2) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.14

method	result
parts	$d^2a^2\left(\frac{1}{4}c^2x^4+\frac{2}{3}c\,x^3+\frac{1}{2}x^2\right)+\frac{d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{2\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)^2c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)c^3}{6}\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{4}c^4x^4+\frac{2}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{2\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)^2c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{2\operatorname{arc}}{}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{4}c^4x^4+\frac{2}{3}x^3c^3+\frac{1}{2}c^2x^2\right)+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{2\operatorname{arctanh}(cx)^2c^3x^3}{3}+\frac{\operatorname{arctanh}(cx)^2c^2x^2}{2}+\frac{\operatorname{arctanh}(cx)c^3x^3}{6}+\frac{2\operatorname{arc}}{}\right.}{}$
risch	$\frac{2d^2b^2\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{3c^2}-\frac{2d^2b^2\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{3c^2}-\frac{d^2b\ln(-cx-1)a}{12c^2}+\frac{2d^2b^2\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{3c^2}+2d^2$

input
```
int(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/4*c^2*x^4+2/3*c*x^3+1/2*x^2)+d^2*b^2/c^2*(1/4*arctanh(c*x)^2*c^
4*x^4+2/3*arctanh(c*x)^2*c^3*x^3+1/2*arctanh(c*x)^2*c^2*x^2+1/6*arctanh(c*
x)*c^3*x^3+2/3*arctanh(c*x)*c^2*x^2+3/2*arctanh(c*x)*c*x+17/12*arctanh(c*x
)*ln(c*x-1)-1/12*arctanh(c*x)*ln(c*x+1)+17/48*ln(c*x-1)^2-2/3*dilog(1/2*c*
x+1/2)-17/24*ln(c*x-1)*ln(1/2*c*x+1/2)+1/48*ln(c*x+1)^2-1/24*(ln(c*x+1)-ln
(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/12*c^2*x^2+2/3*c*x+7/6*ln(c*x-1)+1/2*ln(
c*x+1))+2*d^2*a*b/c^2*(1/4*arctanh(c*x)*c^4*x^4+2/3*arctanh(c*x)*c^3*x^3+1
/2*arctanh(c*x)*c^2*x^2+1/12*x^3*c^3+1/3*c^2*x^2+3/4*c*x+17/24*ln(c*x-1)-1
/24*ln(c*x+1))
```

Fricas [F]

$$\int x(d+cdx)^2(a+b\operatorname{arctanh}(cx))^2\,dx=\int(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^2*d^2*x^3 + 2*a^2*c*d^2*x^2 + a^2*d^2*x + (b^2*c^2*d^2*x^3
+ 2*b^2*c*d^2*x^2 + b^2*d^2*x)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^3 + 2*a*b
*c*d^2*x^2 + a*b*d^2*x)*arctanh(c*x), x)
```

Sympy [F]

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = d^2\left(\int a^2x\,dx + \int 2a^2cx^2\,dx + \int a^2c^2x^3\,dx + \int b^2x\,\text{atanh}^2\,(cx)\,dx + \int 2abx\,\text{atanh}\,(cx)\,dx + \int 2b^2cx^2\,\text{atanh}^2\,(cx)\,dx + \int b^2c^2x^3\,\text{atanh}^2\,(cx)\,dx + \int 4abcx^2\,\text{atanh}\,(cx)\,dx + \int 2abc^2x^3\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

output
```
d**2*(Integral(a**2*x, x) + Integral(2*a**2*c*x**2, x) + Integral(a**2*c**
2*x**3, x) + Integral(b**2*x*atanh(c*x)**2, x) + Integral(2*a*b*x*atanh(c*
x), x) + Integral(2*b**2*c*x**2*atanh(c*x)**2, x) + Integral(b**2*c**2*x**
3*atanh(c*x)**2, x) + Integral(4*a*b*c*x**2*atanh(c*x), x) + Integral(2*a*
b*c**2*x**3*atanh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(249) = 498$.

Time = 0.28 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.18

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*c^2*d^2*x^4 + 2/3*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctanh(c*x)^2 +
 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5
+ 3*log(c*x - 1)/c^5))*a*b*c^2*d^2 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2
+ log(c^2*x^2 - 1)/c^4))*a*b*c*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arctanh(
c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*d^2 + 1/8*(4
*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(
c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x -
 1))/c^2)*b^2*d^2 + 2/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x
+ 1/2))*b^2*d^2/c^2 + 2/3*b^2*d^2*log(c*x - 1)/c^2 + 1/48*(4*b^2*c^2*d^2*x
^2 + 32*b^2*c*d^2*x + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*
log(c*x + 1)^2 + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 - 11*b^2*d^2)*log(
-c*x + 1)^2 + 4*(b^2*c^3*d^2*x^3 + 4*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x)*log(
c*x + 1) - 2*(2*b^2*c^3*d^2*x^3 + 8*b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (3*b
^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x + 1))*log(-c*x + 1
))/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(249) = 498$.

Time = 1.45 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.72

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
2/45*(30*(c*x + 1)^3*b^2*d^2*log(-(c*x + 1)/(c*x - 1))^2/((((c*x + 1)^6*c^5
/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1
)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*
(c*x + 1)*c^5/(c*x - 1) + c^5)*(c*x - 1)^3) + 2*(60*(c*x + 1)^3*a*b*d^2/(c
*x - 1)^3 + 10*(c*x + 1)^3*b^2*d^2/(c*x - 1)^3 - 15*(c*x + 1)^2*b^2*d^2/(c
*x - 1)^2 + 6*(c*x + 1)*b^2*d^2/(c*x - 1) - b^2*d^2)*log(-(c*x + 1)/(c*x -
 1))/((c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x - 1)^5 + 15*(c*
x + 1)^4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 + 15*(c*x + 1)^2
*c^5/(c*x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) + (120*(c*x + 1)^3*a^2
*d^2/(c*x - 1)^3 + 40*(c*x + 1)^3*a*b*d^2/(c*x - 1)^3 - 60*(c*x + 1)^2*a*b
*d^2/(c*x - 1)^2 + 24*(c*x + 1)*a*b*d^2/(c*x - 1) - 4*a*b*d^2 - 2*(c*x + 1
)^5*b^2*d^2/(c*x - 1)^5 + 11*(c*x + 1)^4*b^2*d^2/(c*x - 1)^4 - 18*(c*x + 1
)^3*b^2*d^2/(c*x - 1)^3 + 11*(c*x + 1)^2*b^2*d^2/(c*x - 1)^2 - 2*(c*x + 1)
*b^2*d^2/(c*x - 1))/((c*x + 1)^6*c^5/(c*x - 1)^6 - 6*(c*x + 1)^5*c^5/(c*x
- 1)^5 + 15*(c*x + 1)^4*c^5/(c*x - 1)^4 - 20*(c*x + 1)^3*c^5/(c*x - 1)^3 +
 15*(c*x + 1)^2*c^5/(c*x - 1)^2 - 6*(c*x + 1)*c^5/(c*x - 1) + c^5) - 2*b^2
*d^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 2*b^2*d^2*log(-(c*x + 1)/(c*x - 1
))/c^5)*c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int x\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

input
```
int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

output
```
int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

Reduce [F]

$$\int x(d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$= \frac{d^2\left(3atanh(cx)^2 b^2c^4x^4+8atanh(cx)^2 b^2c^3x^3+6atanh(cx)^2 b^2c^2x^2-8atanh(cx)^2 b^2cx-9atanh(cx)^2 b^2\right.}{}$$

input
```
int(x*(c*d*x+d)^2*(a+b*atanh(c*x))^2,x)
```

output
```
(d**2*(3*atanh(c*x)**2*b**2*c**4*x**4 + 8*atanh(c*x)**2*b**2*c**3*x**3 + 6
*atanh(c*x)**2*b**2*c**2*x**2 - 8*atanh(c*x)**2*b**2*c*x - 9*atanh(c*x)**2
*b**2 + 6*atanh(c*x)*a*b*c**4*x**4 + 16*atanh(c*x)*a*b*c**3*x**3 + 12*atan
h(c*x)*a*b*c**2*x**2 - 2*atanh(c*x)*a*b + 2*atanh(c*x)*b**2*c**3*x**3 + 8*
atanh(c*x)*b**2*c**2*x**2 + 18*atanh(c*x)*b**2*c*x + 12*atanh(c*x)*b**2 +
8*int(atanh(c*x)**2,x)*b**2*c + 16*log(c**2*x - c)*a*b + 20*log(c**2*x - c
)*b**2 + 3*a**2*c**4*x**4 + 8*a**2*c**3*x**3 + 6*a**2*c**2*x**2 + 2*a*b*c*
*3*x**3 + 8*a*b*c**2*x**2 + 18*a*b*c*x + b**2*c**2*x**2 + 8*b**2*c*x))/(12
*c**2)
```

3.79 $\int (d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 175

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx = 2abd^2x+\frac{1}{3}b^2d^2x-\frac{b^2d^2\text{arctanh}(cx)}{3c}+2b^2d^2x\text{arctanh}(cx)+\frac{1}{3}bcd^2x^2(a+b\text{arctanh}(cx))+\frac{d^2(1+cx)^3(a+b\text{arctanh}(cx))^2}{3c}-\frac{8bd^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c}+\frac{b^2d^2\log(1-c^2x^2)}{c}-\frac{4b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c}$$

output
```
2*a*b*d^2*x+1/3*b^2*d^2*x-1/3*b^2*d^2*arctanh(c*x)/c+2*b^2*d^2*x*arctanh(c
*x)+1/3*b*c*d^2*x^2*(a+b*arctanh(c*x))+1/3*d^2*(c*x+1)^3*(a+b*arctanh(c*x)
)^2/c-8/3*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c+b^2*d^2*ln(-c^2*x^2+1)
/c-4/3*b^2*d^2*polylog(2,1-2/(-c*x+1))/c
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int (d+cdx)^2(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$= \frac{d^2\left(3a^2cx+6abcx+b^2cx+3a^2c^2x^2+abc^2x^2+a^2c^3x^3+b^2(-7+3cx+3c^2x^2+c^3x^3)\operatorname{arctanh}(cx)^2+b\right.}{}$$

input
```
Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^2*(3*a^2*c*x + 6*a*b*c*x + b^2*c*x + 3*a^2*c^2*x^2 + a*b*c^2*x^2 + a^2*
c^3*x^3 + b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTan
h[c*x]*(2*a*c*x*(3 + 3*c*x + c^2*x^2) + b*(-1 + 6*c*x + c^2*x^2) - 8*b*Log
[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*a
*b*Log[1 - c^2*x^2] + 3*b^2*Log[1 - c^2*x^2] + a*b*Log[-1 + c^2*x^2] + 4*b
^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^2(a+b\operatorname{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6480}$$

$$\frac{d^2(cx+1)^3(a+b\operatorname{arctanh}(cx))^2}{3c} - \frac{2b\int \left(-cx(a+b\operatorname{arctanh}(cx))d^3+\frac{4(cx+1)(a+b\operatorname{arctanh}(cx))d^3}{1-c^2x^2}-3(a+b\operatorname{arctanh}(cx))d^3\right)dx}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))^2}{3c} - \frac{2b\left(-\frac{1}{2}cd^3x^2(a+b\text{arctanh}(cx))+\frac{4d^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-3ad^3x-3bd^3x\text{arctanh}(cx)+\frac{bd^3\text{arctanh}(cx)}{2c}-\right.}{3d}$$

input
```
Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x])^2)/(3*c) - (2*b*(-3*a*d^3*x - (b*d^3
*x)/2 + (b*d^3*ArcTanh[c*x])/(2*c) - 3*b*d^3*x*ArcTanh[c*x] - (c*d^3*x^2*(
a + b*ArcTanh[c*x]))/2 + (4*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c -
 (3*b*d^3*Log[1 - c^2*x^2])/(2*c) + (2*b*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/
c))/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{d^2a^2(cx+1)^3}{3}+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\operatorname{arctanh}(cx)^2c^2x^2+\operatorname{arctanh}(cx)^2cx+\frac{\operatorname{arctanh}(cx)^2}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+2\operatorname{arctanh}\right.}{}$
default	$\frac{\frac{d^2a^2(cx+1)^3}{3}+d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\operatorname{arctanh}(cx)^2c^2x^2+\operatorname{arctanh}(cx)^2cx+\frac{\operatorname{arctanh}(cx)^2}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+2\operatorname{arctanh}\right.}{}$
parts	$\frac{d^2a^2(cx+1)^3}{3c}+\frac{d^2b^2\left(\frac{\operatorname{arctanh}(cx)^2c^3x^3}{3}+\operatorname{arctanh}(cx)^2c^2x^2+\operatorname{arctanh}(cx)^2cx+\frac{\operatorname{arctanh}(cx)^2}{3}+\frac{\operatorname{arctanh}(cx)c^2x^2}{3}+2\operatorname{arct}\right.}{}$
risch	$\frac{5d^2b^2\ln(-cx-1)}{6c}+\frac{4b^2\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)d^2}{3c}-\frac{4b^2\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)d^2}{3c}+\frac{7\ln(-cx+1)ab\,d^2}{3c}-\ln(-$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*d^2*a^2*(c*x+1)^3+d^2*b^2*(1/3*arctanh(c*x)^2*c^3*x^3+arctanh(c*x
)^2*c^2*x^2+arctanh(c*x)^2*c*x+1/3*arctanh(c*x)^2+1/3*arctanh(c*x)*c^2*x^2
+2*arctanh(c*x)*c*x+8/3*arctanh(c*x)*ln(c*x-1)+1/3*c*x-1/3+7/6*ln(c*x-1)+5
/6*ln(c*x+1)+2/3*ln(c*x-1)^2-4/3*dilog(1/2*c*x+1/2)-4/3*ln(c*x-1)*ln(1/2*c
*x+1/2))+2*d^2*a*b*(1/3*arctanh(c*x)*c^3*x^3+arctanh(c*x)*c^2*x^2+arctanh(
c*x)*c*x+1/3*arctanh(c*x)+1/6*c^2*x^2+c*x+4/3*ln(c*x-1)))
```

Fricas [F]

$$\int(d+cdx)^2(a+b\operatorname{arctanh}(cx))^2\,dx=\int(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
 + a*b*d^2)*arctanh(c*x), x)
```

Sympy [F]

$$\int (d+cdx)^2(a+b\operatorname{arctanh}(cx))^2\,dx = d^2\left(\int a^2\,dx + \int b^2\operatorname{atanh}^2(cx)\,dx + \int 2ab\operatorname{atanh}(cx)\,dx + \int 2a^2cx\,dx + \int a^2c^2x^2\,dx + \int 2b^2cx\operatorname{atanh}^2(cx)\,dx + \int b^2c^2x^2\operatorname{atanh}^2(cx)\,dx + \int 4abcx\operatorname{atanh}(cx)\,dx + \int 2abc^2x^2\operatorname{atanh}(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

output
```
d**2*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b
*atanh(c*x), x) + Integral(2*a**2*c*x, x) + Integral(a**2*c**2*x**2, x) +
Integral(2*b**2*c*x*atanh(c*x)**2, x) + Integral(b**2*c**2*x**2*atanh(c*x)
**2, x) + Integral(4*a*b*c*x*atanh(c*x), x) + Integral(2*a*b*c**2*x**2*ata
nh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(160) = 320$.

Time = 0.21 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.65

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$= \frac{1}{3}a^2c^2d^2x^3 + \frac{1}{3}\left(2x^3\,\text{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log(c^2x^2-1)}{c^4}\right)\right)abc^2d^2 + a^2cd^2x^2$$
$$+ \left(2x^2\,\text{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)abcd^2$$
$$+ a^2d^2x + \frac{(2cx\,\text{artanh}(cx) + \log(-c^2x^2+1))abd^2}{c}$$
$$+ \frac{4\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right) + \text{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2d^2}{3c}$$
$$+ \frac{5b^2d^2\log(cx+1)}{6c} + \frac{7b^2d^2\log(cx-1)}{6c}$$
$$+ \frac{4b^2cd^2x + (b^2c^3d^2x^3 + 3b^2c^2d^2x^2 + 3b^2cd^2x + b^2d^2)\log(cx+1)^2 + (b^2c^3d^2x^3 + 3b^2c^2d^2x^2 + 3b^2cd^2x}{}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*d^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
 1)/c^4))*a*b*c^2*d^2 + a^2*c*d^2*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 -
 log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*c*d^2 + a^2*d^2*x + (2*c*x*arct
anh(c*x) + log(-c^2*x^2 + 1))*a*b*d^2/c + 4/3*(log(c*x + 1)*log(-1/2*c*x +
 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^2/c + 5/6*b^2*d^2*log(c*x + 1)/c + 7/6
*b^2*d^2*log(c*x - 1)/c + 1/12*(4*b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c
^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 + (b^2*c^3*d^2*x^3 +
3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x - 7*b^2*d^2)*log(-c*x + 1)^2 + 2*(b^2*c^
2*d^2*x^2 + 6*b^2*c*d^2*x)*log(c*x + 1) - 2*(b^2*c^2*d^2*x^2 + 6*b^2*c*d^2
*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c
*x + 1))*log(-c*x + 1))/c
```

Giac [F]

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx=\int (a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2\,dx$$

input
```
int((a + b*atanh(c*x))^2*(d + c*d*x)^2,x)
```

output
```
int((a + b*atanh(c*x))^2*(d + c*d*x)^2, x)
```

Reduce [F]

$$\int (d+cdx)^2(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^2\left(atanh(cx)^2\,b^2c^3x^3+3atanh(cx)^2\,b^2c^2x^2+3atanh(cx)^2\,b^2cx-3atanh(cx)^2\,b^2+2atanh(cx)\,ab\,c^3x^3+\right.}{}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))^2,x)
```

output
```
(d**2*(atanh(c*x)**2*b**2*c**3*x**3 + 3*atanh(c*x)**2*b**2*c**2*x**2 + 3*a
tanh(c*x)**2*b**2*c*x - 3*atanh(c*x)**2*b**2 + 2*atanh(c*x)*a*b*c**3*x**3
+ 6*atanh(c*x)*a*b*c**2*x**2 + 6*atanh(c*x)*a*b*c*x + 2*atanh(c*x)*a*b + a
tanh(c*x)*b**2*c**2*x**2 + 6*atanh(c*x)*b**2*c*x + 5*atanh(c*x)*b**2 + 8*i
nt((atanh(c*x)*x)/(c**2*x**2 - 1),x)*b**2*c**2 + 8*log(c**2*x - c)*a*b + 6
*log(c**2*x - c)*b**2 + a**2*c**3*x**3 + 3*a**2*c**2*x**2 + 3*a**2*c*x + a
*b*c**2*x**2 + 6*a*b*c*x + b**2*c*x))/(3*c)
```

3.80 $\int \frac{(d+cdx)^2(a+b\,\mathbf{arctanh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 278

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx = abcd^2x + b^2cd^2x\text{arctanh}(cx) + \frac{3}{2}d^2(a+b\text{arctanh}(cx))^2 + 2cd^2x(a+b\text{arctanh}(cx))^2 + \frac{1}{2}c^2d^2x^2(a+b\text{arctanh}(cx))^2 + 2d^2(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right) - 4bd^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) + \frac{1}{2}b^2d^2\log\left(1-c^2x^2\right) - 2b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right) - bd^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right) + bd^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) + \frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right) - \frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)$$

output
```
a*b*c*d^2*x+b^2*c*d^2*x*arctanh(c*x)+3/2*d^2*(a+b*arctanh(c*x))^2+2*c*d^2*
x*(a+b*arctanh(c*x))^2+1/2*c^2*d^2*x^2*(a+b*arctanh(c*x))^2-2*d^2*(a+b*arc
tanh(c*x))^2*arctanh(-1+2/(-c*x+1))-4*b*d^2*(a+b*arctanh(c*x))*ln(2/(-c*x+
1))+1/2*b^2*d^2*ln(-c^2*x^2+1)-2*b^2*d^2*polylog(2,1-2/(-c*x+1))-b*d^2*(a+
b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+b*d^2*(a+b*arctanh(c*x))*polylog(2
,-1+2/(-c*x+1))+1/2*b^2*d^2*polylog(3,1-2/(-c*x+1))-1/2*b^2*d^2*polylog(3,
-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.17

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx$$
$$= \frac{1}{2}d^2\Bigg(4a^2cx+a^2c^2x^2+2a^2\log(cx) + ab\big(2cx+2c^2x^2\text{arctanh}(cx)+\log(1-cx)-\log(1+cx)\big) + 4ab\big(2cx\text{arctanh}(cx)+\log\left(1-c^2x^2\right)\big) + b^2\big(2cx\text{arctanh}(cx)+\left(-1+c^2x^2\right)\text{arctanh}(cx)^2+\log\left(1-c^2x^2\right)\big) + 4b^2\big(\text{arctanh}(cx)\left((-1+cx)\text{arctanh}(cx)-2\log\left(1+e^{-2\text{arctanh}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)\big) + 2ab(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx)) + 2b^2\bigg(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}(cx)^3-\text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right) + \text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right)+\text{arctanh}(cx)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right) + \text{arctanh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right)+\frac{1}{2}\,\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\bigg)\Bigg)$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

output
```
(d^2*(4*a^2*c*x + a^2*c^2*x^2 + 2*a^2*Log[c*x] + a*b*(2*c*x + 2*c^2*x^2*Ar
cTanh[c*x] + Log[1 - c*x] - Log[1 + c*x]) + 4*a*b*(2*c*x*ArcTanh[c*x] + Lo
g[1 - c^2*x^2]) + b^2*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2
+ Log[1 - c^2*x^2]) + 4*b^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log
[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 2*a*b*(-P
olyLog[2, -(c*x)] + PolyLog[2, c*x]) + 2*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x
]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[
1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] +
ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*
x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/2
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))^2}{x}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^2d^2x(a+b\text{arctanh}(cx))^2+2cd^2(a+b\text{arctanh}(cx))^2+\frac{d^2(a+b\text{arctanh}(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^2d^2x^2(a+b\text{arctanh}(cx))^2-bd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+bd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+2cd^2x(a+b\text{arctanh}(cx))^2+\frac{3}{2}d^2(a+b\text{arctanh}(cx))^2+2d^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-4bd^2\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+abcd^2x+b^2cd^2x\text{arctanh}(cx)+\frac{1}{2}b^2d^2\log\left(1-c^2x^2\right)-2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}b^2d^2\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d^2\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

output
```
a*b*c*d^2*x + b^2*c*d^2*x*ArcTanh[c*x] + (3*d^2*(a + b*ArcTanh[c*x])^2)/2
+ 2*c*d^2*x*(a + b*ArcTanh[c*x])^2 + (c^2*d^2*x^2*(a + b*ArcTanh[c*x])^2)/
2 + 2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 4*b*d^2*(a + b
*ArcTanh[c*x])*Log[2/(1 - c*x)] + (b^2*d^2*Log[1 - c^2*x^2])/2 - 2*b^2*d^2
*PolyLog[2, 1 - 2/(1 - c*x)] - b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2
/(1 - c*x)] + b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b
^2*d^2*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^2*PolyLog[3, -1 + 2/(1 - c*
x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.45 (sec) , antiderivative size = 895, normalized size of antiderivative = 3.22

method	result	size
parts	Expression too large to display	895
derivativedivides	Expression too large to display	897
default	Expression too large to display	897

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/2*c^2*x^2+2*c*x+ln(x))+d^2*b^2*(1/2*arctanh(c*x)^2*c^2*x^2+2*ar
ctanh(c*x)^2*c*x+arctanh(c*x)^2*ln(c*x)-arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*
x^2+1)-1)+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*p
olylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2
))+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*polylog(
2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-ar
ctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(-c
^2*x^2+1))+1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(
c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1))
)*arctanh(c*x)^2-4*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-4*dilog(1-I*(c*x+
1)/(-c^2*x^2+1)^(1/2))+3/2*arctanh(c*x)^2+(c*x+1)*arctanh(c*x)-1/2*I*Pi*cs
gn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x
+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-ln(1+(c*x+1)^2/(-c^2*x^2+1))-4*arctan
h(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-4*arctanh(c*x)*ln(1-I*(c*x+1)/(-
c^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x
+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+1/2*I*Pi*
csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x
)^2)+2*d^2*a*b*(1/2*arctanh(c*x)*c^2*x^2+2*arctanh(c*x)*c*x+arctanh(c*x)*l
n(c*x)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1)+1/2*c*x+5/4*l
n(c*x-1)+3/4*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2
*b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*
x + a*b*d^2)*arctanh(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx = d^2\left(\int 2a^2c\,dx + \int \frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int 2b^2c\,\text{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x}\,dx + \int 4abc\,\text{atanh}\,(cx)\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x}\,dx + \int b^2c^2x\,\text{atanh}^2\,(cx)\,dx + \int 2abc^2x\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x,x)
```

output
```
d**2*(Integral(2*a**2*c, x) + Integral(a**2/x, x) + Integral(a**2*c**2*x,
x) + Integral(2*b**2*c*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x,
x) + Integral(4*a*b*c*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x, x) + I
ntegral(b**2*c**2*x*atanh(c*x)**2, x) + Integral(2*a*b*c**2*x*atanh(c*x),
x))
```

Maxima [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/2*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + 2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
 + 1))*a*b*d^2 + a^2*d^2*log(x) + 1/8*(b^2*c^2*d^2*x^2 + 4*b^2*c*d^2*x)*lo
g(-c*x + 1)^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c
*d^2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*d^2*x^2 +
a*b*c*d^2*x - a*b*d^2)*log(c*x + 1) - (4*a*b*c*d^2*x - 4*a*b*d^2 + (4*a*b*
c^3*d^2 + b^2*c^3*d^2)*x^3 - 4*(a*b*c^2*d^2 - b^2*c^2*d^2)*x^2 + 2*(b^2*c^
3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1))*log(-c*
x + 1))/(c*x^2 - x), x)
```

Giac [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x}\,dx$$
$$= \frac{d^2\left(atanh(cx)^2 b^2c^2x^2 - atanh(cx)^2 b^2 + 2atanh(cx)\,ab\,c^2x^2 + 8atanh(cx)\,abcx + 6atanh(cx)\,ab + 2atan\right.}{}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))^2/x,x)
```

output
```
(d**2*(atanh(c*x)**2*b**2*c**2*x**2 - atanh(c*x)**2*b**2 + 2*atanh(c*x)*a*
b*c**2*x**2 + 8*atanh(c*x)*a*b*c*x + 6*atanh(c*x)*a*b + 2*atanh(c*x)*b**2*
c*x + 2*atanh(c*x)*b**2 + 4*int(atanh(c*x)**2,x)*b**2*c + 4*int(atanh(c*x)
/x,x)*a*b + 2*int(atanh(c*x)**2/x,x)*b**2 + 8*log(c**2*x - c)*a*b + 2*log(
c**2*x - c)*b**2 + 2*log(x)*a**2 + a**2*c**2*x**2 + 4*a**2*c*x + 2*a*b*c*x
))/2
```

3.81 $\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 283

$$\begin{aligned}\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx &= 2cd^2(a+b\text{arctanh}(cx))^2-\frac{d^2(a+b\text{arctanh}(cx))^2}{x} \\ &\quad+c^2d^2x(a+b\text{arctanh}(cx))^2 \\ &\quad+4cd^2(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right) \\ &\quad-2bcd^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) \\ &\quad+2bcd^2(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) \\ &\quad-b^2cd^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-2bcd^2(a \\ &\qquad+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right) \\ &\quad+2bcd^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1\right. \\ &\quad\left.+\frac{2}{1-cx}\right)-b^2cd^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right) \\ &\quad+b^2cd^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right) \\ &\quad-b^2cd^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)\end{aligned}$$

output
```
2*c*d^2*(a+b*arctanh(c*x))^2-d^2*(a+b*arctanh(c*x))^2/x+c^2*d^2*x*(a+b*arc
tanh(c*x))^2-4*c*d^2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-2*b*c*d^2
*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+2*b*c*d^2*(a+b*arctanh(c*x))*ln(2-2/(c*
x+1))-b^2*c*d^2*polylog(2,1-2/(-c*x+1))-2*b*c*d^2*(a+b*arctanh(c*x))*polyl
og(2,1-2/(-c*x+1))+2*b*c*d^2*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-b
^2*c*d^2*polylog(2,-1+2/(c*x+1))+b^2*c*d^2*polylog(3,1-2/(-c*x+1))-b^2*c*d
^2*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.20

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx$$
$$= \frac{d^2\left(-12a^2+ib^2c\pi^3x+12a^2c^2x^2-24ab\text{arctanh}(cx)+24abc^2x^2\text{arctanh}(cx)-12b^2\text{arctanh}(cx)^2+12b^2c\right.}{}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

output
```
(d^2*(-12*a^2 + I*b^2*c*Pi^3*x + 12*a^2*c^2*x^2 - 24*a*b*ArcTanh[c*x] + 24
*a*b*c^2*x^2*ArcTanh[c*x] - 12*b^2*ArcTanh[c*x]^2 + 12*b^2*c^2*x^2*ArcTanh
[c*x]^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 24*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-
2*ArcTanh[c*x])] - 24*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] -
24*b^2*c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTan
h[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 24*a*b*c*x*Log[
c*x] + 12*b^2*c*x*(1 + 2*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] -
12*b^2*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]*PolyL
og[2, E^(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x)] + 24*a*b*c*x*Pol
yLog[2, c*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c*x*Po
lyLog[3, E^(2*ArcTanh[c*x])]))/(12*x)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^2d^2(a+b\text{arctanh}(cx))^2+\frac{d^2(a+b\text{arctanh}(cx))^2}{x^2}+\frac{2cd^2(a+b\text{arctanh}(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^2d^2x(a+b\text{arctanh}(cx))^2-2bcd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+2bcd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+2cd^2(a+b\text{arctanh}(cx))^2-\frac{d^2(a+b\text{arctanh}(cx))^2}{x}+4cd^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-2bcd^2\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+2bcd^2\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-b^2cd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-b^2cd^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+b^2cd^2\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-b^2cd^2\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

output
```
2*c*d^2*(a + b*ArcTanh[c*x])^2 - (d^2*(a + b*ArcTanh[c*x])^2)/x + c^2*d^2*
x*(a + b*ArcTanh[c*x])^2 + 4*c*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1
 - c*x)] - 2*b*c*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] + 2*b*c*d^2*(a
+ b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*d^2*PolyLog[2, 1 - 2/(1 - c
*x)] - 2*b*c*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 2*b*c*
d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^2*PolyLog[
2, -1 + 2/(1 + c*x)] + b^2*c*d^2*PolyLog[3, 1 - 2/(1 - c*x)] - b^2*c*d^2*P
olyLog[3, -1 + 2/(1 - c*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.18 (sec) , antiderivative size = 2519, normalized size of antiderivative = 8.90

method	result	size
parts	Expression too large to display	2519
derivativedivides	Expression too large to display	2521
default	Expression too large to display	2521

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(c^2*x+2*c*ln(x)-1/x)+d^2*b^2*c*(2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c
^2*x^2+1)^(1/2))+4*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*ar
ctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+4*arctanh(c*x)*polylog(2,-(c
*x+1)/(-c^2*x^2+1)^(1/2))-2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+1/
2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)
)*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*
x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))+arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-dilog((c*x+1)/
(-c^2*x^2+1)^(1/2))+dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2)))+2*arctanh(c*x)^2*
ln(c*x)+2*arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*ln(
1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))+
arctanh(c*x)^2*c*x-polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+polylog(3,-(c*x+1)^2
/(-c^2*x^2+1))-2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+2*polylog
(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-4*
polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-4*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1
/2))-arctanh(c*x)^2/c/x-1/4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*
(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2
/(c^2*x^2-1)))*(2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-4*arctanh(
c*x)^2+2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))+2*arctanh(c*x)*ln(1-...
```

Fricas [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2
*b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*
x + a*b*d^2)*arctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx = d^2\left(\int a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int \frac{2a^2c}{x}\,dx + \int b^2c^2\,\text{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x^2}\,dx + \int 2abc^2\,\text{atanh}\,(cx)\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{2b^2c\,\text{atanh}^2\,(cx)}{x}\,dx + \int \frac{4abc\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**2,x)
```

output
```
d**2*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(2*a**2*c/
x, x) + Integral(b**2*c**2*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2
/x**2, x) + Integral(2*a*b*c**2*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)
/x**2, x) + Integral(2*b**2*c*atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh
(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
a^2*c^2*d^2*x - 1/2*b^2*c^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1), x) +
 1/4*b^2*c^2*d^2*integrate(log(c*x + 1)^2/(c^2*x^2), x) + (2*c*x*arctanh(c
*x) + log(-c^2*x^2 + 1))*a*b*c*d^2 + 1/2*(c*x - (c*x - 1)*log(-c*x + 1) -
1)*b^2*c*d^2 + 1/4*b^2*c*d^2*gamma(3, -log(c*x + 1)) + 1/2*b^2*c*d^2*integ
rate(log(c*x + 1)^2/x, x) - b^2*c*d^2*integrate(log(c*x + 1)*log(-c*x + 1)
/x, x) + 2*a*b*c*d^2*integrate(log(c*x + 1)/x, x) - 2*a*b*c*d^2*integrate(
log(-c*x + 1)/x, x) - 1/2*b^2*c*d^2*integrate(log(-c*x + 1)/x, x) + 2*a^2*
c*d^2*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d^
2 - 1/2*b^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x^2, x) - a^2*d^2/x +
 1/4*(b^2*c^2*d^2*x^2 - b^2*d^2)*log(-c*x + 1)^2/x
```

Giac [F]

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(a+b\operatorname{atanh}(c\,x))^2\,(d+c\,d\,x)^2}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^2,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^2}\,dx$$
$$=\frac{d^2\left(-atanh(cx)^2b^2+2atanh(cx)\,ab\,c^2x^2-2atanh(cx)\,ab+\left(\int atanh(cx)^2\,dx\right)b^2c^2x-2\left(\int\frac{atanh(cx)}{c^2x^3-x}dx\right)\right.}{x}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))^2/x^2,x)
```

output
```
(d**2*( - atanh(c*x)**2*b**2 + 2*atanh(c*x)*a*b*c**2*x**2 - 2*atanh(c*x)*a
*b + int(atanh(c*x)**2,x)*b**2*c**2*x - 2*int(atanh(c*x)/(c**2*x**3 - x),x
)*b**2*c*x + 4*int(atanh(c*x)/x,x)*a*b*c*x + 2*int(atanh(c*x)**2/x,x)*b**2
*c*x + 2*log(x)*a**2*c*x + 2*log(x)*a*b*c*x + a**2*c**2*x**2 - a**2))/x
```

3.82 $\int \frac{(d+cdx)^2(a+b\,\mathbf{arctanh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx = -\frac{bcd^2(a+b\text{arctanh}(cx))}{x} + \frac{5}{2}c^2d^2(a+b\text{arctanh}(cx))^2 - \frac{d^2(a+b\text{arctanh}(cx))^2}{2x^2} - \frac{2cd^2(a+b\text{arctanh}(cx))^2}{x} + 2c^2d^2(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right) + b^2c^2d^2\log(x) - \frac{1}{2}b^2c^2d^2\log\left(1-c^2x^2\right) + 4bc^2d^2(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) - bc^2d^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right) + bc^2d^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) - 2b^2c^2d^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right) + \frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right) - \frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)$$

output
```
-b*c*d^2*(a+b*arctanh(c*x))/x+5/2*c^2*d^2*(a+b*arctanh(c*x))^2-1/2*d^2*(a+
b*arctanh(c*x))^2/x^2-2*c*d^2*(a+b*arctanh(c*x))^2/x-2*c^2*d^2*(a+b*arctan
h(c*x))^2*arctanh(-1+2/(-c*x+1))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(-c^2
*x^2+1)+4*b*c^2*d^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c^2*d^2*(a+b*arct
anh(c*x))*polylog(2,1-2/(-c*x+1))+b*c^2*d^2*(a+b*arctanh(c*x))*polylog(2,-
1+2/(-c*x+1))-2*b^2*c^2*d^2*polylog(2,-1+2/(c*x+1))+1/2*b^2*c^2*d^2*polylo
g(3,1-2/(-c*x+1))-1/2*b^2*c^2*d^2*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.18

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx =$$
$$-\frac{d^2\Big(a^2+4a^2cx-2a^2c^2x^2\log(x)+ab(2\text{arctanh}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))+b^2\Big(2$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

output
```
-1/2*(d^2*(a^2 + 4*a^2*c*x - 2*a^2*c^2*x^2*Log[x] + a*b*(2*ArcTanh[c*x] +
c*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])) + b^2*(2*c*x*ArcTanh[c*x] +
 (1 - c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) +
4*a*b*c*x*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])) + 4*b^2*
c*x*(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E^(-2*ArcTanh[c*
x])]) + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + 2*a*b*c^2*x^2*(PolyLog[2, -
(c*x)] - PolyLog[2, c*x]) - 2*b^2*c^2*x^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3
)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 -
 E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Arc
Tanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])
]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/x^2
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx$$
$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{c^2d^2(a+b\text{arctanh}(cx))^2}{x}+\frac{d^2(a+b\text{arctanh}(cx))^2}{x^3}+\frac{2cd^2(a+b\text{arctanh}(cx))^2}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-bc^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+bc^2d^2\operatorname{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+\frac{5}{2}c^2d^2(a+b\text{arctanh}(cx))^2+2c^2d^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2+4bc^2d^2\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{d^2(a+b\text{arctanh}(cx))^2}{2x^2}-\frac{2cd^2(a+b\text{arctanh}(cx))^2}{x}-\frac{bcd^2(a+b\text{arctanh}(cx))}{x}-2b^2c^2d^2\operatorname{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{1}{2}b^2c^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2c^2d^2\operatorname{PolyLog}\left(3,\frac{2}{1-cx}-1\right)-\frac{1}{2}b^2c^2d^2\log\left(1-c^2x^2\right)+b^2c^2d^2\log(x)$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

output
```
-((b*c*d^2*(a + b*ArcTanh[c*x]))/x) + (5*c^2*d^2*(a + b*ArcTanh[c*x])^2)/2
 - (d^2*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x])^2)
/x + 2*c^2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + b^2*c^2*d
^2*Log[x] - (b^2*c^2*d^2*Log[1 - c^2*x^2])/2 + 4*b*c^2*d^2*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)] - b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 -
 2/(1 - c*x)] + b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)
] - 2*b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 + c*x)] + (b^2*c^2*d^2*PolyLog[3, 1
 - 2/(1 - c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.34 (sec) , antiderivative size = 952, normalized size of antiderivative = 3.04

method	result	size
parts	Expression too large to display	952
derivativedivides	Expression too large to display	953
default	Expression too large to display	953

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(c^2*ln(x)-2*c/x-1/2/x^2)+d^2*b^2*c^2*(-1/2*arctanh(c*x)^2/c^2/x^2
+arctanh(c*x)^2*ln(c*x)-2*arctanh(c*x)^2/c/x-arctanh(c*x)^2*ln((c*x+1)^2/(
-c^2*x^2+1)-1)+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c
*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(c*x+1)/(-c^2*x^2+1)
^(1/2))+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*pol
ylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2
))-arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^
2/(-c^2*x^2+1))+1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1
)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^
2-1)))*arctanh(c*x)^2-1/2*(c*x-(-c^2*x^2+1)^(1/2)+1)/c/x*arctanh(c*x)-1/2*
arctanh(c*x)*(c*x+(-c^2*x^2+1)^(1/2)+1)/c/x+4*arctanh(c*x)*ln(1+(c*x+1)/(-
c^2*x^2+1)^(1/2))-3/2*arctanh(c*x)^2-4*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+4
*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+1/2*
I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctan
h(c*x)^2-1/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c
^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+ln(1+(c*x+1)/(-c^
2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1
)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2)+2*d^2*a*b*
c^2*(-1/2*arctanh(c*x)/c^2/x^2+arctanh(c*x)*ln(c*x)-2*arctanh(c*x)/c/x-5/4
*ln(c*x-1)-1/2/c/x+2*ln(c*x)-3/4*ln(c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x...
```

Fricas [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2
*b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*
x + a*b*d^2)*arctanh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx = d^2\left(\int \frac{a^2}{x^3}\,dx + \int \frac{2a^2c}{x^2}\,dx + \int \frac{a^2c^2}{x}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x^3}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x^3}\,dx + \int \frac{2b^2c\,\text{atanh}^2\,(cx)}{x^2}\,dx + \int \frac{b^2c^2\,\text{atanh}^2\,(cx)}{x}\,dx + \int \frac{4abc\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{2abc^2\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**3,x)
```

output
```
d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c/x**2, x) + Integral(a**2*
c**2/x, x) + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*atanh(c
*x)/x**3, x) + Integral(2*b**2*c*atanh(c*x)**2/x**2, x) + Integral(b**2*c*
*2*atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x)/x**2, x) + Integral(2
*a*b*c**2*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
a^2*c^2*d^2*log(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x
)*a*b*c*d^2 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c
*x)/x^2)*a*b*d^2 - 2*a^2*c*d^2/x - 1/2*a^2*d^2/x^2 - 1/8*(4*b^2*c*d^2*x +
b^2*d^2)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*
d^2*x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b
*c^2*d^2*x^2)*log(c*x + 1) - (4*a*b*c^3*d^2*x^3 - b^2*c*d^2*x - 4*(a*b*c^2
*d^2 + b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2
*x - b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)
```

Giac [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^3,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^3}\,dx$$
$$= \frac{d^2\Big(atanh(cx)^2 b^2c^2x^2 - 4atanh(cx)^2 b^2cx - atanh(cx)^2 b^2 - 6atanh(cx)\,ab\,c^2x^2 - 8atanh(cx)\,abcx - 2a}{}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))^2/x^3,x)
```

output
```
(d**2*(atanh(c*x)**2*b**2*c**2*x**2 - 4*atanh(c*x)**2*b**2*c*x - atanh(c*x
)**2*b**2 - 6*atanh(c*x)*a*b*c**2*x**2 - 8*atanh(c*x)*a*b*c*x - 2*atanh(c*
x)*a*b - 2*atanh(c*x)*b**2*c**2*x**2 - 2*atanh(c*x)*b**2*c*x - 8*int(atanh
(c*x)/(c**2*x**3 - x),x)*b**2*c**2*x**2 + 4*int(atanh(c*x)/x,x)*a*b*c**2*x
**2 + 2*int(atanh(c*x)**2/x,x)*b**2*c**2*x**2 - 8*log(c**2*x - c)*a*b*c**2
*x**2 - 2*log(c**2*x - c)*b**2*c**2*x**2 + 2*log(x)*a**2*c**2*x**2 + 8*log
(x)*a*b*c**2*x**2 + 2*log(x)*b**2*c**2*x**2 - 4*a**2*c*x - a**2 - 2*a*b*c*
x))/(2*x**2)
```

3.83 $\int \frac{(d+cdx)^2(a+b\mathbf{arctanh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 244

$$\begin{aligned}\int \frac{(d+cdx)^2(a+b\,\mathrm{arctanh}(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d^2}{3x}+\frac{1}{3}b^2c^3d^2\mathrm{arctanh}(cx)\\ &\quad-\frac{bcd^2(a+b\,\mathrm{arctanh}(cx))}{3x^2}\\ &\quad-\frac{2bc^2d^2(a+b\,\mathrm{arctanh}(cx))}{x}\\ &\quad-\frac{d^2(1+cx)^3(a+b\,\mathrm{arctanh}(cx))^2}{3x^3}\\ &\quad+\frac{8}{3}abc^3d^2\log(x)+2b^2c^3d^2\log(x)\\ &\quad+\frac{8}{3}bc^3d^2(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)\\ &\quad-b^2c^3d^2\log\left(1-c^2x^2\right)-\frac{4}{3}b^2c^3d^2\,\mathrm{PolyLog}(2,-cx)\\ &\quad+\frac{4}{3}b^2c^3d^2\,\mathrm{PolyLog}(2,cx)\\ &\quad+\frac{4}{3}b^2c^3d^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^2/x+1/3*b^2*c^3*d^2*arctanh(c*x)-1/3*b*c*d^2*(a+b*arctanh(c
*x))/x^2-2*b*c^2*d^2*(a+b*arctanh(c*x))/x-1/3*d^2*(c*x+1)^3*(a+b*arctanh(c
*x))^2/x^3+8/3*a*b*c^3*d^2*ln(x)+2*b^2*c^3*d^2*ln(x)+8/3*b*c^3*d^2*(a+b*ar
ctanh(c*x))*ln(2/(-c*x+1))-b^2*c^3*d^2*ln(-c^2*x^2+1)-4/3*b^2*c^3*d^2*poly
log(2,-c*x)+4/3*b^2*c^3*d^2*polylog(2,c*x)+4/3*b^2*c^3*d^2*polylog(2,1-2/(
-c*x+1))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.11

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx =$$

$$-\frac{d^2\left(a^2+3a^2cx+abcx+3a^2c^2x^2+6abc^2x^2+b^2c^2x^2+b^2(1+3cx+3c^2x^2-7c^3x^3)\,\text{arctanh}(cx)^2+ba\right.}{}$$

input
```
Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^2*(a^2 + 3*a^2*c*x + a*b*c*x + 3*a^2*c^2*x^2 + 6*a*b*c^2*x^2 + b^2
*c^2*x^2 + b^2*(1 + 3*c*x + 3*c^2*x^2 - 7*c^3*x^3)*ArcTanh[c*x]^2 + b*ArcT
anh[c*x]*(b*c*x*(1 + 6*c*x - c^2*x^2) + a*(2 + 6*c*x + 6*c^2*x^2) - 8*b*c^
3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x
^3*Log[1 - c*x] - 3*a*b*c^3*x^3*Log[1 + c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqr
t[1 - c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 - c^2*x^2] + 4*b^2*c^3*x^3*PolyLog[2
, E^(-2*ArcTanh[c*x])]))/x^3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6500, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$

$\Big\downarrow$ 6500

$$-2bc\int\left(-\frac{4d^2(a+b\text{arctanh}(cx))c^3}{3(1-cx)}-\frac{4d^2(a+b\text{arctanh}(cx))c^2}{3x}-\frac{d^2(a+b\text{arctanh}(cx))c}{x^2}-\frac{d^2(a+b\text{arctanh}(cx}{3x^3}\right.$$
$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2009

$$-2bc\left(-\frac{4}{3}c^2d^2\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+\frac{d^2(a+b\text{arctanh}(cx))}{6x^2}+\frac{cd^2(a+b\text{arctanh}(cx))}{x}-\frac{4}{3}ac^2d^2\log\right.$$
$$\frac{d^2(cx+1)^3(a+b\text{arctanh}(cx))^2}{3x^3}$$

input
```
Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x])^2)/x^3 - 2*b*c*((b*c*d^2)/(6*x)
 - (b*c^2*d^2*ArcTanh[c*x])/6 + (d^2*(a + b*ArcTanh[c*x]))/(6*x^2) + (c*d^
2*(a + b*ArcTanh[c*x]))/x - (4*a*c^2*d^2*Log[x])/3 - b*c^2*d^2*Log[x] - (4
*c^2*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/3 + (b*c^2*d^2*Log[1 - c^2
*x^2])/2 + (2*b*c^2*d^2*PolyLog[2, -(c*x)])/3 - (2*b*c^2*d^2*PolyLog[2, c*
x])/3 - (2*b*c^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6500
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Si
mp[(a + b*ArcTanh[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b
*ArcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && N
eQ[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.43

method	result
parts	$d^2a^2\left(-\frac{c^2}{x}-\frac{c}{x^2}-\frac{1}{3x^3}\right)+d^2b^2c^3\left(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)^2}{c^2x^2}-\frac{\operatorname{arctanh}(cx)^2}{cx}-\frac{7\operatorname{arctanh}(cx)\ \mathrm{l}}{3}\right.$
derivativedivides	$c^3\left(d^2a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{c^2x^2}-\frac{1}{cx}\right)+d^2b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)^2}{c^2x^2}-\frac{\operatorname{arctanh}(cx)^2}{cx}-\frac{7\operatorname{arctanh}}{}\right.\right.$
default	$c^3\left(d^2a^2\left(-\frac{1}{3c^3x^3}-\frac{1}{c^2x^2}-\frac{1}{cx}\right)+d^2b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3}-\frac{\operatorname{arctanh}(cx)^2}{c^2x^2}-\frac{\operatorname{arctanh}(cx)^2}{cx}-\frac{7\operatorname{arctanh}}{}\right.\right.$

input
```
int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-c^2/x-c/x^2-1/3/x^3)+d^2*b^2*c^3*(-1/3*arctanh(c*x)^2/c^3/x^3-ar
ctanh(c*x)^2/c^2/x^2-arctanh(c*x)^2/c/x-7/3*arctanh(c*x)*ln(c*x-1)-1/3*arc
tanh(c*x)/c^2/x^2-2*arctanh(c*x)/c/x+8/3*arctanh(c*x)*ln(c*x)-1/3*arctanh(
c*x)*ln(c*x+1)-4/3*dilog(c*x)-4/3*dilog(c*x+1)-4/3*ln(c*x)*ln(c*x+1)-7/12*
ln(c*x-1)^2+4/3*dilog(1/2*c*x+1/2)+7/6*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*ln(c
*x+1)^2-1/6*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-7/6*ln(c*x-1)-1/3
/c/x+2*ln(c*x)-5/6*ln(c*x+1))+2*d^2*a*b*c^3*(-1/3*arctanh(c*x)/c^3/x^3-arc
tanh(c*x)/c^2/x^2-arctanh(c*x)/c/x-7/6*ln(c*x-1)-1/6/c^2/x^2-1/c/x+4/3*ln(
c*x)-1/6*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2
*b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*
x + a*b*d^2)*arctanh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx = d^2\left(\int \frac{a^2}{x^4}\,dx + \int \frac{2a^2c}{x^3}\,dx + \int \frac{a^2c^2}{x^2}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x^4}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x^4}\,dx + \int \frac{2b^2c\,\text{atanh}^2\,(cx)}{x^3}\,dx + \int \frac{b^2c^2\,\text{atanh}^2\,(cx)}{x^2}\,dx + \int \frac{4abc\,\text{atanh}\,(cx)}{x^3}\,dx + \int \frac{2abc^2\,\text{atanh}\,(cx)}{x^2}\,dx\right)$$

input
```
integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**4,x)
```

output
```
d**2*(Integral(a**2/x**4, x) + Integral(2*a**2*c/x**3, x) + Integral(a**2*
c**2/x**2, x) + Integral(b**2*atanh(c*x)**2/x**4, x) + Integral(2*a*b*atan
h(c*x)/x**4, x) + Integral(2*b**2*c*atanh(c*x)**2/x**3, x) + Integral(b**2
*c**2*atanh(c*x)**2/x**2, x) + Integral(4*a*b*c*atanh(c*x)/x**3, x) + Inte
gral(2*a*b*c**2*atanh(c*x)/x**2, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(221) = 442$.

Time = 0.48 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.27

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$
$$= -\frac{4}{3}\left(\log(cx+1)\log\left(-\frac{1}{2}cx+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}cx+\frac{1}{2}\right)\right)b^2c^3d^2$$
$$-\frac{4}{3}\left(\log(cx)\log(-cx+1)+\text{Li}_2(-cx+1)\right)b^2c^3d^2$$
$$+\frac{4}{3}\left(\log(cx+1)\log(-cx)+\text{Li}_2(cx+1)\right)b^2c^3d^2$$
$$-\frac{5}{6}b^2c^3d^2\log(cx+1)-\frac{7}{6}b^2c^3d^2\log(cx-1)+2b^2c^3d^2\log(x)$$
$$-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}(cx)}{x}\right)abc^2d^2$$
$$+\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}(cx)}{x^2}\right)abcd^2$$
$$-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}(cx)}{x^3}\right)abd^2$$
$$-\frac{a^2c^2d^2}{x}-\frac{a^2cd^2}{x^2}-\frac{a^2d^2}{3x^3}$$
$$-\frac{4b^2c^2d^2x^2+(b^2c^3d^2x^3+3b^2c^2d^2x^2+3b^2cd^2x+b^2d^2)\log(cx+1)^2-(7b^2c^3d^2x^3-3b^2c^2d^2x^2-3b^2c}{}$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d^2
 - 4/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d^2 + 4/3*(log(c
*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d^2 - 5/6*b^2*c^3*d^2*log(c*x
+ 1) - 7/6*b^2*c^3*d^2*log(c*x - 1) + 2*b^2*c^3*d^2*log(x) - (c*(log(c^2*x
^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c^2*d^2 + ((c*log(c*x + 1) - c
*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d^2 - 1/3*((c^2*log(c^2
*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d^2 - a^2*c^
2*d^2/x - a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/12*(4*b^2*c^2*d^2*x^2 + (b^2
*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2
 - (7*b^2*c^3*d^2*x^3 - 3*b^2*c^2*d^2*x^2 - 3*b^2*c*d^2*x - b^2*d^2)*log(-
c*x + 1)^2 + 2*(6*b^2*c^2*d^2*x^2 + b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*c
^2*d^2*x^2 + b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*
d^2*x + b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/x^3
```

Giac [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(cdx+d)^2(b\,\text{artanh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^2}{x^4}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^4,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^2(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$
$$= \frac{d^2\Big(3atanh(cx)^2\,b^2c^3x^3 - 3atanh(cx)^2\,b^2c^2x^2 - 3atanh(cx)^2\,b^2cx - atanh(cx)^2\,b^2 - 2atanh(cx)\,ab\,c^3x^3 -}{}$$

input
```
int((c*d*x+d)^2*(a+b*atanh(c*x))^2/x^4,x)
```

output
```
(d**2*(3*atanh(c*x)**2*b**2*c**3*x**3 - 3*atanh(c*x)**2*b**2*c**2*x**2 - 3
*atanh(c*x)**2*b**2*c*x - atanh(c*x)**2*b**2 - 2*atanh(c*x)*a*b*c**3*x**3
- 6*atanh(c*x)*a*b*c**2*x**2 - 6*atanh(c*x)*a*b*c*x - 2*atanh(c*x)*a*b - 9
*atanh(c*x)*b**2*c**3*x**3 - 6*atanh(c*x)*b**2*c**2*x**2 + 3*atanh(c*x)*b*
*2*c*x - 8*int(atanh(c*x)/(c**2*x**5 - x**3),x)*b**2*c*x**3 - 8*log(c**2*x
 - c)*a*b*c**3*x**3 - 6*log(c**2*x - c)*b**2*c**3*x**3 + 8*log(x)*a*b*c**3
*x**3 + 6*log(x)*b**2*c**3*x**3 - 3*a**2*c**2*x**2 - 3*a**2*c*x - a**2 - 6
*a*b*c**2*x**2 - a*b*c*x + 3*b**2*c**2*x**2))/(3*x**3)
```

3.84 $\int x^3(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 415

$$\begin{aligned}\int x^3(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))^2\,dx &= \frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5\\ &\quad-\frac{122b^2d^3\,\mathrm{arctanh}(cx)}{105c^4}+\frac{3b^2d^3x\,\mathrm{arctanh}(cx)}{2c^3}+\frac{26bd^3x^2(a+b\,\mathrm{arctanh}(cx))}{35c^2}\\ &\quad+\frac{bd^3x^3(a+b\,\mathrm{arctanh}(cx))}{2c}+\frac{13}{35}bd^3x^4(a+b\,\mathrm{arctanh}(cx))+\frac{1}{5}bcd^3x^5(a+b\,\mathrm{arctanh}(cx))\\ &\quad+\frac{1}{21}bc^2d^3x^6(a+b\,\mathrm{arctanh}(cx))-\frac{d^3(a+b\,\mathrm{arctanh}(cx))^2}{140c^4}+\frac{1}{4}d^3x^4(a+b\,\mathrm{arctanh}(cx))^2\\ &\quad+\frac{3}{5}cd^3x^5(a+b\,\mathrm{arctanh}(cx))^2+\frac{1}{2}c^2d^3x^6(a+b\,\mathrm{arctanh}(cx))^2\\ &\quad+\frac{1}{7}c^3d^3x^7(a+b\,\mathrm{arctanh}(cx))^2-\frac{52bd^3(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{35c^4}\\ &\quad+\frac{11b^2d^3\log(1-c^2x^2)}{10c^4}-\frac{26b^2d^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{35c^4}\end{aligned}$$

output
```
3/2*a*b*d^3*x/c^3+122/105*b^2*d^3*x/c^3+7/20*b^2*d^3*x^2/c^2+44/315*b^2*d^
3*x^3/c+1/20*b^2*d^3*x^4+1/105*b^2*c*d^3*x^5-122/105*b^2*d^3*arctanh(c*x)/
c^4+3/2*b^2*d^3*x*arctanh(c*x)/c^3+26/35*b*d^3*x^2*(a+b*arctanh(c*x))/c^2+
1/2*b*d^3*x^3*(a+b*arctanh(c*x))/c+13/35*b*d^3*x^4*(a+b*arctanh(c*x))+1/5*
b*c*d^3*x^5*(a+b*arctanh(c*x))+1/21*b*c^2*d^3*x^6*(a+b*arctanh(c*x))-1/140
*d^3*(a+b*arctanh(c*x))^2/c^4+1/4*d^3*x^4*(a+b*arctanh(c*x))^2+3/5*c*d^3*x
^5*(a+b*arctanh(c*x))^2+1/2*c^2*d^3*x^6*(a+b*arctanh(c*x))^2+1/7*c^3*d^3*x
^7*(a+b*arctanh(c*x))^2-52/35*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4+
11/10*b^2*d^3*ln(-c^2*x^2+1)/c^4-26/35*b^2*d^3*polylog(2,1-2/(-c*x+1))/c^4
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.93

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(-1464ab-504b^2+1890abcx+1464b^2cx+936abc^2x^2+441b^2c^2x^2+630abc^3x^3+176b^2c^3x^3+315a\right.}{}$$

input
```
Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^3*(-1464*a*b - 504*b^2 + 1890*a*b*c*x + 1464*b^2*c*x + 936*a*b*c^2*x^2
+ 441*b^2*c^2*x^2 + 630*a*b*c^3*x^3 + 176*b^2*c^3*x^3 + 315*a^2*c^4*x^4 +
468*a*b*c^4*x^4 + 63*b^2*c^4*x^4 + 756*a^2*c^5*x^5 + 252*a*b*c^5*x^5 + 12*
b^2*c^5*x^5 + 630*a^2*c^6*x^6 + 60*a*b*c^6*x^6 + 180*a^2*c^7*x^7 + 9*b^2*(
-209 + 35*c^4*x^4 + 84*c^5*x^5 + 70*c^6*x^6 + 20*c^7*x^7)*ArcTanh[c*x]^2 +
 6*b*ArcTanh[c*x]*(3*a*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3) + b
*(-244 + 315*c*x + 156*c^2*x^2 + 105*c^3*x^3 + 78*c^4*x^4 + 42*c^5*x^5 + 1
0*c^6*x^6) - 312*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 945*a*b*Log[1 - c*x] -
945*a*b*Log[1 + c*x] + 1386*b^2*Log[1 - c^2*x^2] + 936*a*b*Log[-1 + c^2*x^
2] + 936*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(1260*c^4)
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(cdx+d)^3(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^3d^3x^6(a+b\text{arctanh}(cx))^2+3c^2d^3x^5(a+b\text{arctanh}(cx))^2+3cd^3x^4(a+b\text{arctanh}(cx))^2+d^3x^3(a+b\text{arctanh}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\text{arctanh}(cx))^2}{140c^4}-\frac{52bd^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{35c^4}+\frac{1}{7}c^3d^3x^7(a+b\text{arctanh}(cx))^2+\frac{1}{2}c^2d^3x^6(a+b\text{arctanh}(cx))^2+\frac{1}{21}bc^2d^3x^6(a+b\text{arctanh}(cx))+\frac{26bd^3x^2(a+b\text{arctanh}(cx))}{35c^2}+\frac{3}{5}cd^3x^5(a+b\text{arctanh}(cx))^2+\frac{1}{5}bcd^3x^5(a+b\text{arctanh}(cx))+\frac{1}{4}d^3x^4(a+b\text{arctanh}(cx))^2+\frac{13}{35}bd^3x^4(a+b\text{arctanh}(cx))+\frac{bd^3x^3(a+b\text{arctanh}(cx))}{2c}+\frac{3abd^3x}{2c^3}-\frac{122b^2d^3\text{arctanh}(cx)}{105c^4}+\frac{3b^2d^3x\text{arctanh}(cx)}{2c^3}-\frac{26b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{35c^4}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{11b^2d^3\log\left(1-c^2x^2\right)}{10c^4}+\frac{1}{105}b^2cd^3x^5+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4$$

input
```
Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(3*a*b*d^3*x)/(2*c^3) + (122*b^2*d^3*x)/(105*c^3) + (7*b^2*d^3*x^2)/(20*c^
2) + (44*b^2*d^3*x^3)/(315*c) + (b^2*d^3*x^4)/20 + (b^2*c*d^3*x^5)/105 - (
122*b^2*d^3*ArcTanh[c*x])/(105*c^4) + (3*b^2*d^3*x*ArcTanh[c*x])/(2*c^3) +
 (26*b*d^3*x^2*(a + b*ArcTanh[c*x]))/(35*c^2) + (b*d^3*x^3*(a + b*ArcTanh[
c*x]))/(2*c) + (13*b*d^3*x^4*(a + b*ArcTanh[c*x]))/35 + (b*c*d^3*x^5*(a +
b*ArcTanh[c*x]))/5 + (b*c^2*d^3*x^6*(a + b*ArcTanh[c*x]))/21 - (d^3*(a + b
*ArcTanh[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c*d^
3*x^5*(a + b*ArcTanh[c*x])^2)/5 + (c^2*d^3*x^6*(a + b*ArcTanh[c*x])^2)/2 +
 (c^3*d^3*x^7*(a + b*ArcTanh[c*x])^2)/7 - (52*b*d^3*(a + b*ArcTanh[c*x])*L
og[2/(1 - c*x)])/(35*c^4) + (11*b^2*d^3*Log[1 - c^2*x^2])/(10*c^4) - (26*b
^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(35*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.05

method	result
parts	$d^3a^2\left(\frac{1}{7}c^3x^7+\frac{1}{2}c^2x^6+\frac{3}{5}c\,x^5+\frac{1}{4}x^4\right)+\frac{d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^7x^7}{7}+\frac{\operatorname{arctanh}(cx)^2c^6x^6}{2}+\frac{3\operatorname{arctanh}(cx)^2c^5x^5}{5}+\right.}{}$
derivativedivides	$\frac{d^3a^2\left(\frac{1}{7}c^7x^7+\frac{1}{2}c^6x^6+\frac{3}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^7x^7}{7}+\frac{\operatorname{arctanh}(cx)^2c^6x^6}{2}+\frac{3\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4}{4}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{7}c^7x^7+\frac{1}{2}c^6x^6+\frac{3}{5}c^5x^5+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^7x^7}{7}+\frac{\operatorname{arctanh}(cx)^2c^6x^6}{2}+\frac{3\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{\operatorname{arctanh}(cx)^2c^4}{4}\right.}{}$
risch	$\frac{3ab\,d^3x}{2c^3}+\frac{26d^3b^2\operatorname{dilog}\left(-\frac{cx}{2}+\frac{1}{2}\right)}{35c^4}-\frac{13d^3b^2\ln(-cx+1)x^4}{70}+\frac{13d^3b\,x^4a}{35}-\frac{353d^3ba}{105c^4}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+$

input
```
int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/7*c^3*x^7+1/2*c^2*x^6+3/5*c*x^5+1/4*x^4)+d^3*b^2/c^4*(1/7*arcta
nh(c*x)^2*c^7*x^7+1/2*arctanh(c*x)^2*c^6*x^6+3/5*arctanh(c*x)^2*c^5*x^5+1/
4*arctanh(c*x)^2*c^4*x^4+1/21*arctanh(c*x)*c^6*x^6+1/5*arctanh(c*x)*c^5*x^
5+13/35*arctanh(c*x)*c^4*x^4+1/2*arctanh(c*x)*c^3*x^3+26/35*arctanh(c*x)*c
^2*x^2+3/2*arctanh(c*x)*c*x+209/140*arctanh(c*x)*ln(c*x-1)-1/140*arctanh(c
*x)*ln(c*x+1)+209/560*ln(c*x-1)^2-26/35*dilog(1/2*c*x+1/2)-209/280*ln(c*x-
1)*ln(1/2*c*x+1/2)+1/560*ln(c*x+1)^2-1/280*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(
-1/2*c*x+1/2)+1/105*c^5*x^5+1/20*c^4*x^4+44/315*x^3*c^3+7/20*c^2*x^2+122/1
05*c*x+353/210*ln(c*x-1)+109/210*ln(c*x+1))+2*d^3*a*b/c^4*(1/7*arctanh(c*x
)*c^7*x^7+1/2*arctanh(c*x)*c^6*x^6+3/5*arctanh(c*x)*c^5*x^5+1/4*arctanh(c*
x)*c^4*x^4+1/42*c^6*x^6+1/10*c^5*x^5+13/70*c^4*x^4+1/4*x^3*c^3+13/35*c^2*x
^2+3/4*c*x+209/280*ln(c*x-1)-1/280*ln(c*x+1))
```

Fricas [F]

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^3*d^3*x^6 + 3*a^2*c^2*d^3*x^5 + 3*a^2*c*d^3*x^4 + a^2*d^3*x
^3 + (b^2*c^3*d^3*x^6 + 3*b^2*c^2*d^3*x^5 + 3*b^2*c*d^3*x^4 + b^2*d^3*x^3)
*arctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^6 + 3*a*b*c^2*d^3*x^5 + 3*a*b*c*d^3*x^4
 + a*b*d^3*x^3)*arctanh(c*x), x)
```

Sympy [F]

$$\int x^3(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$= d^3\left(\int a^2x^3\,dx + \int 3a^2cx^4\,dx + \int 3a^2c^2x^5\,dx + \int a^2c^3x^6\,dx + \int b^2x^3\operatorname{atanh}^2(cx)\,dx + \int 2abx^3\operatorname{atanh}(cx)\,dx + \int 3b^2cx^4\operatorname{atanh}^2(cx)\,dx + \int 3b^2c^2x^5\operatorname{atanh}^2(cx)\,dx + \int b^2c^3x^6\operatorname{atanh}^2(cx)\,dx + \int 6abcx^4\operatorname{atanh}(cx)\,dx + \int 6abc^2x^5\operatorname{atanh}(cx)\,dx + \int 2abc^3x^6\operatorname{atanh}(cx)\,dx\right)$$

input
```
integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

output
```
d**3*(Integral(a**2*x**3, x) + Integral(3*a**2*c*x**4, x) + Integral(3*a**
2*c**2*x**5, x) + Integral(a**2*c**3*x**6, x) + Integral(b**2*x**3*atanh(c
*x)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(3*b**2*c*x**4*a
tanh(c*x)**2, x) + Integral(3*b**2*c**2*x**5*atanh(c*x)**2, x) + Integral(
b**2*c**3*x**6*atanh(c*x)**2, x) + Integral(6*a*b*c*x**4*atanh(c*x), x) +
Integral(6*a*b*c**2*x**5*atanh(c*x), x) + Integral(2*a*b*c**3*x**6*atanh(c
*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(370) = 740$.

Time = 0.31 (sec) , antiderivative size = 928, normalized size of antiderivative = 2.24

$$\int x^3(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*a^2*c^3*d^3*x^7 + 1/2*a^2*c^2*d^3*x^6 + 3/5*a^2*c*d^3*x^5 + 1/4*b^2*d^
3*x^4*arctanh(c*x)^2 + 1/42*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x
^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*x^4 +
 1/30*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*
log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^3 + 3/10*(4*x^5*arctanh
(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*d^3 + 1/
12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3
*log(c*x - 1)/c^5))*a*b*d^3 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x
 + 1)/c^5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x -
 1) - 8)*log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x -
 1))/c^4)*b^2*d^3 + 26/35*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*
x + 1/2))*b^2*d^3/c^4 + 13/70*b^2*d^3*log(c*x + 1)/c^4 + 283/210*b^2*d^3*l
og(c*x - 1)/c^4 + 1/2520*(24*b^2*c^5*d^3*x^5 + 126*b^2*c^4*d^3*x^4 + 352*b
^2*c^3*d^3*x^3 + 672*b^2*c^2*d^3*x^2 + 2928*b^2*c*d^3*x + 9*(10*b^2*c^7*d^
3*x^7 + 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1)
^2 + 9*(10*b^2*c^7*d^3*x^7 + 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 - 87*
b^2*d^3)*log(-c*x + 1)^2 + 12*(5*b^2*c^6*d^3*x^6 + 21*b^2*c^5*d^3*x^5 + 39
*b^2*c^4*d^3*x^4 + 35*b^2*c^3*d^3*x^3 + 78*b^2*c^2*d^3*x^2 + 105*b^2*c*d^3
*x)*log(c*x + 1) - 6*(10*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 78*b^2*c^4
*d^3*x^4 + 70*b^2*c^3*d^3*x^3 + 156*b^2*c^2*d^3*x^2 + 210*b^2*c*d^3*x +...
```

Giac [F]

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int x^3\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3\,dx$$

input
```
int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

output
```
int(x^3*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

Reduce [F]

$$\int x^3(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(180atanh(cx)^2\,b^2c^7x^7+60atanh(cx)\,b^2c^6x^6+1872\log(c^2x-c)\,ab+756a^2c^5x^5+1464b^2cx-18atan\right.}{}$$

input
```
int(x^3*(c*d*x+d)^3*(a+b*atanh(c*x))^2,x)
```

output
```
(d**3*(180*atanh(c*x)**2*b**2*c**7*x**7 + 630*atanh(c*x)**2*b**2*c**6*x**6
 + 756*atanh(c*x)**2*b**2*c**5*x**5 + 315*atanh(c*x)**2*b**2*c**4*x**4 - 9
36*atanh(c*x)**2*b**2*c*x - 945*atanh(c*x)**2*b**2 + 360*atanh(c*x)*a*b*c*
*7*x**7 + 1260*atanh(c*x)*a*b*c**6*x**6 + 1512*atanh(c*x)*a*b*c**5*x**5 +
630*atanh(c*x)*a*b*c**4*x**4 - 18*atanh(c*x)*a*b + 60*atanh(c*x)*b**2*c**6
*x**6 + 252*atanh(c*x)*b**2*c**5*x**5 + 468*atanh(c*x)*b**2*c**4*x**4 + 63
0*atanh(c*x)*b**2*c**3*x**3 + 936*atanh(c*x)*b**2*c**2*x**2 + 1890*atanh(c
*x)*b**2*c*x + 1308*atanh(c*x)*b**2 + 936*int(atanh(c*x)**2,x)*b**2*c + 18
72*log(c**2*x - c)*a*b + 2772*log(c**2*x - c)*b**2 + 180*a**2*c**7*x**7 +
630*a**2*c**6*x**6 + 756*a**2*c**5*x**5 + 315*a**2*c**4*x**4 + 60*a*b*c**6
*x**6 + 252*a*b*c**5*x**5 + 468*a*b*c**4*x**4 + 630*a*b*c**3*x**3 + 936*a*
b*c**2*x**2 + 1890*a*b*c*x + 12*b**2*c**5*x**5 + 63*b**2*c**4*x**4 + 176*b
**2*c**3*x**3 + 441*b**2*c**2*x**2 + 1464*b**2*c*x))/(1260*c**4)
```

3.85 $\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 377

$$\begin{aligned}
&\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx \\
&= \frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2d^3\text{arctanh}(cx)}{30c^3} \\
&+\frac{11b^2d^3x\text{arctanh}(cx)}{6c^2}+\frac{14bd^3x^2(a+b\text{arctanh}(cx))}{15c}+\frac{11}{18}bd^3x^3(a+b\text{arctanh}(cx)) \\
&+\frac{3}{10}bcd^3x^4(a+b\text{arctanh}(cx))+\frac{1}{15}bc^2d^3x^5(a+b\text{arctanh}(cx)) \\
&+\frac{d^3(a+b\text{arctanh}(cx))^2}{60c^3}+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))^2 \\
&+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))^2+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))^2 \\
&+\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))^2-\frac{28bd^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{15c^3} \\
&+\frac{113b^2d^3\log(1-c^2x^2)}{90c^3}-\frac{14b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}
\end{aligned}$$

output
```
11/6*a*b*d^3*x/c^2+37/30*b^2*d^3*x/c^2+61/180*b^2*d^3*x^2/c+1/10*b^2*d^3*x
^3+1/60*b^2*c*d^3*x^4-37/30*b^2*d^3*arctanh(c*x)/c^3+11/6*b^2*d^3*x*arctan
h(c*x)/c^2+14/15*b*d^3*x^2*(a+b*arctanh(c*x))/c+11/18*b*d^3*x^3*(a+b*arcta
nh(c*x))+3/10*b*c*d^3*x^4*(a+b*arctanh(c*x))+1/15*b*c^2*d^3*x^5*(a+b*arcta
nh(c*x))+1/60*d^3*(a+b*arctanh(c*x))^2/c^3+1/3*d^3*x^3*(a+b*arctanh(c*x))^
2+3/4*c*d^3*x^4*(a+b*arctanh(c*x))^2+3/5*c^2*d^3*x^5*(a+b*arctanh(c*x))^2+
1/6*c^3*d^3*x^6*(a+b*arctanh(c*x))^2-28/15*b*d^3*(a+b*arctanh(c*x))*ln(2/(
-c*x+1))/c^3+113/90*b^2*d^3*ln(-c^2*x^2+1)/c^3-14/15*b^2*d^3*polylog(2,1-2
/(-c*x+1))/c^3
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.94

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(-162ab-64b^2+330abcx+222b^2cx+168abc^2x^2+61b^2c^2x^2+60a^2c^3x^3+110abc^3x^3+18b^2c^3x^3+\right.}{}$$

input
```
Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^3*(-162*a*b - 64*b^2 + 330*a*b*c*x + 222*b^2*c*x + 168*a*b*c^2*x^2 + 61
*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3 + 135*a^2
*c^4*x^4 + 54*a*b*c^4*x^4 + 3*b^2*c^4*x^4 + 108*a^2*c^5*x^5 + 12*a*b*c^5*x
^5 + 30*a^2*c^6*x^6 + 3*b^2*(-111 + 20*c^3*x^3 + 45*c^4*x^4 + 36*c^5*x^5 +
 10*c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*c^3*x^3*(20 + 45*c*x +
 36*c^2*x^2 + 10*c^3*x^3) + b*(-111 + 165*c*x + 84*c^2*x^2 + 55*c^3*x^3 +
27*c^4*x^4 + 6*c^5*x^5) - 168*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 165*a*b*Lo
g[1 - c*x] - 165*a*b*Log[1 + c*x] + 226*b^2*Log[1 - c^2*x^2] + 168*a*b*Log
[-1 + c^2*x^2] + 168*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(180*c^3)
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)^3(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^3d^3x^5(a+b\text{arctanh}(cx))^2+3c^2d^3x^4(a+b\text{arctanh}(cx))^2+3cd^3x^3(a+b\text{arctanh}(cx))^2+d^3x^2(a+b\text{arctanh}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}c^3d^3x^6(a+b\text{arctanh}(cx))^2+\frac{d^3(a+b\text{arctanh}(cx))^2}{60c^3}-\frac{28bd^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{15c^3}+\frac{3}{5}c^2d^3x^5(a+b\text{arctanh}(cx))^2+\frac{1}{15}bc^2d^3x^5(a+b\text{arctanh}(cx))+\frac{3}{4}cd^3x^4(a+b\text{arctanh}(cx))^2+\frac{3}{10}bcd^3x^4(a+b\text{arctanh}(cx))+\frac{1}{3}d^3x^3(a+b\text{arctanh}(cx))^2+\frac{11}{18}bd^3x^3(a+b\text{arctanh}(cx))+\frac{14bd^3x^2(a+b\text{arctanh}(cx))}{15c}+\frac{11abd^3x}{6c^2}-\frac{37b^2d^3\text{arctanh}(cx)}{30c^3}+\frac{11b^2d^3x\text{arctanh}(cx)}{6c^2}-\frac{14b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{37b^2d^3x}{30c^2}+\frac{113b^2d^3\log\left(1-c^2x^2\right)}{90c^3}+\frac{1}{60}b^2cd^3x^4+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3$$

input
```
Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(11*a*b*d^3*x)/(6*c^2) + (37*b^2*d^3*x)/(30*c^2) + (61*b^2*d^3*x^2)/(180*c
) + (b^2*d^3*x^3)/10 + (b^2*c*d^3*x^4)/60 - (37*b^2*d^3*ArcTanh[c*x])/(30*
c^3) + (11*b^2*d^3*x*ArcTanh[c*x])/(6*c^2) + (14*b*d^3*x^2*(a + b*ArcTanh[
c*x]))/(15*c) + (11*b*d^3*x^3*(a + b*ArcTanh[c*x]))/18 + (3*b*c*d^3*x^4*(a
 + b*ArcTanh[c*x]))/10 + (b*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/15 + (d^3*(a
 + b*ArcTanh[c*x])^2)/(60*c^3) + (d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + (3*c
*d^3*x^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x])^2
)/5 + (c^3*d^3*x^6*(a + b*ArcTanh[c*x])^2)/6 - (28*b*d^3*(a + b*ArcTanh[c*
x])*Log[2/(1 - c*x)])/(15*c^3) + (113*b^2*d^3*Log[1 - c^2*x^2])/(90*c^3) -
 (14*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.08

method	result
parts	$d^3a^2\left(\frac{1}{6}c^3x^6+\frac{3}{5}c^2x^5+\frac{3}{4}c\,x^4+\frac{1}{3}x^3\right)+\frac{d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^6x^6}{6}+\frac{3\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)^2c^4x^4}{4}+\right.}{}$
derivativedivides	$\frac{d^3a^2\left(\frac{1}{6}c^6x^6+\frac{3}{5}c^5x^5+\frac{3}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^6x^6}{6}+\frac{3\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)^2c}{3}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{6}c^6x^6+\frac{3}{5}c^5x^5+\frac{3}{4}c^4x^4+\frac{1}{3}x^3c^3\right)+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^6x^6}{6}+\frac{3\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)^2c^4x^4}{4}+\frac{\operatorname{arctanh}(cx)^2c}{3}\right.}{}$
risch	$\frac{11ab\,d^3x}{6c^2}-\frac{d^3ab\ln(-cx+1)x^3}{3}+\frac{d^3c^3b^2\ln(-cx+1)^2x^6}{24}+\frac{3d^3c^2b^2\ln(-cx+1)^2x^5}{20}+\frac{3d^3c\,b^2\ln(-cx+1)^2x^4}{16}+$

input
```
int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/6*c^3*x^6+3/5*c^2*x^5+3/4*c*x^4+1/3*x^3)+d^3*b^2/c^3*(1/6*arcta
nh(c*x)^2*c^6*x^6+3/5*arctanh(c*x)^2*c^5*x^5+3/4*arctanh(c*x)^2*c^4*x^4+1/
3*arctanh(c*x)^2*c^3*x^3+1/15*arctanh(c*x)*c^5*x^5+3/10*arctanh(c*x)*c^4*x
^4+11/18*arctanh(c*x)*c^3*x^3+14/15*arctanh(c*x)*c^2*x^2+11/6*arctanh(c*x)
*c*x+37/20*arctanh(c*x)*ln(c*x-1)+1/60*arctanh(c*x)*ln(c*x+1)+37/80*ln(c*x
-1)^2-14/15*dilog(1/2*c*x+1/2)-37/40*ln(c*x-1)*ln(1/2*c*x+1/2)-1/240*ln(c*
x+1)^2+1/120*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/60*c^4*x^4+1/1
0*x^3*c^3+61/180*c^2*x^2+37/30*c*x+337/180*ln(c*x-1)+23/36*ln(c*x+1))+2*d^
3*a*b/c^3*(1/6*arctanh(c*x)*c^6*x^6+3/5*arctanh(c*x)*c^5*x^5+3/4*arctanh(c
*x)*c^4*x^4+1/3*arctanh(c*x)*c^3*x^3+1/30*c^5*x^5+3/20*c^4*x^4+11/36*x^3*c
^3+7/15*c^2*x^2+11/12*c*x+37/40*ln(c*x-1)+1/120*ln(c*x+1))
```

Fricas [F]

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^3*d^3*x^5 + 3*a^2*c^2*d^3*x^4 + 3*a^2*c*d^3*x^3 + a^2*d^3*x
^2 + (b^2*c^3*d^3*x^5 + 3*b^2*c^2*d^3*x^4 + 3*b^2*c*d^3*x^3 + b^2*d^3*x^2)
*arctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^5 + 3*a*b*c^2*d^3*x^4 + 3*a*b*c*d^3*x^3
 + a*b*d^3*x^2)*arctanh(c*x), x)
```

Sympy [F]

$$\begin{aligned}&\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx\\&=d^3\bigg(\int a^2x^2\,dx+\int 3a^2cx^3\,dx+\int 3a^2c^2x^4\,dx+\int a^2c^3x^5\,dx\\&\quad+\int b^2x^2\,\text{atanh}^2\,(cx)\,dx+\int 2abx^2\,\text{atanh}\,(cx)\,dx+\int 3b^2cx^3\,\text{atanh}^2\,(cx)\,dx\\&\quad+\int 3b^2c^2x^4\,\text{atanh}^2\,(cx)\,dx+\int b^2c^3x^5\,\text{atanh}^2\,(cx)\,dx+\int 6abcx^3\,\text{atanh}\,(cx)\,dx\\&\quad+\int 6abc^2x^4\,\text{atanh}\,(cx)\,dx+\int 2abc^3x^5\,\text{atanh}\,(cx)\,dx\bigg)\end{aligned}$$

input
```
integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

output
```
d**3*(Integral(a**2*x**2, x) + Integral(3*a**2*c*x**3, x) + Integral(3*a**
2*c**2*x**4, x) + Integral(a**2*c**3*x**5, x) + Integral(b**2*x**2*atanh(c
*x)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(3*b**2*c*x**3*a
tanh(c*x)**2, x) + Integral(3*b**2*c**2*x**4*atanh(c*x)**2, x) + Integral(
b**2*c**3*x**5*atanh(c*x)**2, x) + Integral(6*a*b*c*x**3*atanh(c*x), x) +
Integral(6*a*b*c**2*x**4*atanh(c*x), x) + Integral(2*a*b*c**3*x**5*atanh(c
*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(336) = 672$.

Time = 0.30 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.06

$$\int x^2(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*c^3*d^3*x^6 + 3/5*a^2*c^2*d^3*x^5 + 3/4*a^2*c*d^3*x^4 + 1/90*(30*x
^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1
)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^3*d^3 + 3/10*(4*x^5*arctanh(c*x) + c*(
(c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3
*x^3 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)
/c^5 + 3*log(c*x - 1)/c^5))*a*b*c*d^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c
^2 + log(c^2*x^2 - 1)/c^4))*a*b*d^3 + 14/15*(log(c*x + 1)*log(-1/2*c*x + 1
/2) + dilog(1/2*c*x + 1/2))*b^2*d^3/c^3 + 23/36*b^2*d^3*log(c*x + 1)/c^3 +
 337/180*b^2*d^3*log(c*x - 1)/c^3 + 1/720*(12*b^2*c^4*d^3*x^4 + 72*b^2*c^3
*d^3*x^3 + 244*b^2*c^2*d^3*x^2 + 888*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 +
 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*l
og(c*x + 1)^2 + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^
3*x^4 + 20*b^2*c^3*d^3*x^3 - 111*b^2*d^3)*log(-c*x + 1)^2 + 4*(6*b^2*c^5*d
^3*x^5 + 27*b^2*c^4*d^3*x^4 + 55*b^2*c^3*d^3*x^3 + 84*b^2*c^2*d^3*x^2 + 16
5*b^2*c*d^3*x)*log(c*x + 1) - 2*(12*b^2*c^5*d^3*x^5 + 54*b^2*c^4*d^3*x^4 +
 110*b^2*c^3*d^3*x^3 + 168*b^2*c^2*d^3*x^2 + 330*b^2*c*d^3*x + 3*(10*b^2*c
^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3
+ b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c^3
```

Giac [F]

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int x^2\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3\,dx$$

input
```
int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

output
```
int(x^2*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

Reduce [F]

$$\int x^2(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(336\log(c^2x-c)\,ab+108a^2c^5x^5+222b^2cx+6atanh(cx)\,ab+61b^2c^2x^2+30atanh(cx)^2\,b^2c^6x^6+12a\right.}{}$$

input
```
int(x^2*(c*d*x+d)^3*(a+b*atanh(c*x))^2,x)
```

output
```
(d**3*(30*atanh(c*x)**2*b**2*c**6*x**6 + 108*atanh(c*x)**2*b**2*c**5*x**5
+ 135*atanh(c*x)**2*b**2*c**4*x**4 + 60*atanh(c*x)**2*b**2*c**3*x**3 - 168
*atanh(c*x)**2*b**2*c*x - 165*atanh(c*x)**2*b**2 + 60*atanh(c*x)*a*b*c**6*
x**6 + 216*atanh(c*x)*a*b*c**5*x**5 + 270*atanh(c*x)*a*b*c**4*x**4 + 120*a
tanh(c*x)*a*b*c**3*x**3 + 6*atanh(c*x)*a*b + 12*atanh(c*x)*b**2*c**5*x**5
+ 54*atanh(c*x)*b**2*c**4*x**4 + 110*atanh(c*x)*b**2*c**3*x**3 + 168*atanh
(c*x)*b**2*c**2*x**2 + 330*atanh(c*x)*b**2*c*x + 230*atanh(c*x)*b**2 + 168
*int(atanh(c*x)**2,x)*b**2*c + 336*log(c**2*x - c)*a*b + 452*log(c**2*x -
c)*b**2 + 30*a**2*c**6*x**6 + 108*a**2*c**5*x**5 + 135*a**2*c**4*x**4 + 60
*a**2*c**3*x**3 + 12*a*b*c**5*x**5 + 54*a*b*c**4*x**4 + 110*a*b*c**3*x**3
+ 168*a*b*c**2*x**2 + 330*a*b*c*x + 3*b**2*c**4*x**4 + 18*b**2*c**3*x**3 +
 61*b**2*c**2*x**2 + 222*b**2*c*x))/(180*c**3)
```

3.86 $\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 286

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx = \frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{4}b^2d^3x^2+\frac{1}{30}b^2cd^3x^3-\frac{13b^2d^3\text{arctanh}(cx)}{10c^2}+\frac{5b^2d^3x\text{arctanh}(cx)}{2c}+\frac{6}{5}bd^3x^2(a+b\text{arctanh}(cx))+\frac{1}{2}bcd^3x^3(a+b\text{arctanh}(cx))+\frac{1}{10}bc^2d^3x^4(a+b\text{arctanh}(cx))-\frac{d^3(1+cx)^4(a+b\text{arctanh}(cx))^2}{4c^2}+\frac{d^3(1+cx)^5(a+b\text{arctanh}(cx))^2}{5c^2}-\frac{12bd^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{5c^2}+\frac{3b^2d^3\log(1-c^2x^2)}{2c^2}-\frac{6b^2d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^2}$$

output
```
5/2*a*b*d^3*x/c+13/10*b^2*d^3*x/c+1/4*b^2*d^3*x^2+1/30*b^2*c*d^3*x^3-13/10
*b^2*d^3*arctanh(c*x)/c^2+5/2*b^2*d^3*x*arctanh(c*x)/c+6/5*b*d^3*x^2*(a+b*
arctanh(c*x))+1/2*b*c*d^3*x^3*(a+b*arctanh(c*x))+1/10*b*c^2*d^3*x^4*(a+b*a
rctanh(c*x))-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/c^2+1/5*d^3*(c*x+1)^5*
(a+b*arctanh(c*x))^2/c^2-12/5*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^2+
3/2*b^2*d^3*ln(-c^2*x^2+1)/c^2-6/5*b^2*d^3*polylog(2,1-2/(-c*x+1))/c^2
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.14

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(-18ab-15b^2+150abcx+78b^2cx+30a^2c^2x^2+72abc^2x^2+15b^2c^2x^2+60a^2c^3x^3+30abc^3x^3+2b^2c^3\right.}{}$$

input
```
Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^3*(-18*a*b - 15*b^2 + 150*a*b*c*x + 78*b^2*c*x + 30*a^2*c^2*x^2 + 72*a*
b*c^2*x^2 + 15*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 30*a*b*c^3*x^3 + 2*b^2*c^3*x
^3 + 45*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*a^2*c^5*x^5 + 3*b^2*(-49 + 10*c^2
*x^2 + 20*c^3*x^3 + 15*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c
*x]*(a*c^2*x^2*(10 + 20*c*x + 15*c^2*x^2 + 4*c^3*x^3) + b*(-13 + 25*c*x +
12*c^2*x^2 + 5*c^3*x^3 + c^4*x^4) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 7
5*a*b*Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 90*b^2*Log[1 - c^2*x^2] + 72*a*
b*Log[-1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int\left(\frac{(cdx+d)^4(a+b\text{arctanh}(cx))^2}{cd}-\frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{c}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}bc^2d^3x^4(a+b\text{arctanh}(cx))+\frac{d^3(cx+1)^5(a+b\text{arctanh}(cx))^2}{5c^2}-\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{4c^2}-\frac{12bd^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{5c^2}+\frac{1}{2}bcd^3x^3(a+b\text{arctanh}(cx))+\frac{6}{5}bd^3x^2(a+b\text{arctanh}(cx))+\frac{5abd^3x}{2c}-\frac{13b^2d^3\text{arctanh}(cx)}{10c^2}+\frac{5b^2d^3x\text{arctanh}(cx)}{2c}-\frac{6b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^2}+\frac{3b^2d^3\log(1-c^2x^2)}{2c^2}+\frac{1}{30}b^2cd^3x^3+\frac{13b^2d^3x}{10c}+\frac{1}{4}b^2d^3x^2$$

input

```
Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output

```
(5*a*b*d^3*x)/(2*c) + (13*b^2*d^3*x)/(10*c) + (b^2*d^3*x^2)/4 + (b^2*c*d^3
*x^3)/30 - (13*b^2*d^3*ArcTanh[c*x])/(10*c^2) + (5*b^2*d^3*x*ArcTanh[c*x])
/(2*c) + (6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/5 + (b*c*d^3*x^3*(a + b*ArcTan
h[c*x]))/2 + (b*c^2*d^3*x^4*(a + b*ArcTanh[c*x]))/10 - (d^3*(1 + c*x)^4*(a
 + b*ArcTanh[c*x])^2)/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x])^2)/(
5*c^2) - (12*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^2) + (3*b^2
*d^3*Log[1 - c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(
5*c^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.32

method	result
parts	$d^3a^2\left(\frac{1}{5}c^3x^5+\frac{3}{4}c^2x^4+c\,x^3+\frac{1}{2}x^2\right)+\frac{d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)^2c^4x^4}{4}+\operatorname{arctanh}(cx)^2c^3x^3+\right.}{}$
derivativedivides	$\frac{d^3a^2\left(\frac{1}{5}c^5x^5+\frac{3}{4}c^4x^4+x^3c^3+\frac{1}{2}c^2x^2\right)+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)^2c^4x^4}{4}+\operatorname{arctanh}(cx)^2c^3x^3+\frac{\operatorname{arctanh}(cx)^2c^2}{2}\right.}{}$
default	$\frac{d^3a^2\left(\frac{1}{5}c^5x^5+\frac{3}{4}c^4x^4+x^3c^3+\frac{1}{2}c^2x^2\right)+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^5x^5}{5}+\frac{3\operatorname{arctanh}(cx)^2c^4x^4}{4}+\operatorname{arctanh}(cx)^2c^3x^3+\frac{\operatorname{arctanh}(cx)^2c^2}{2}\right.}{}$
risch	$\frac{5ab\,d^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{b^2c\,d^3x^3}{30}+\left(-\frac{d^3b^2x^2\left(4x^3c^3+15c^2x^2+20cx+10\right)\ln(-cx+1)}{40}-\frac{d^3b\left(-8a\,c^5x^5-30a\,c^4x^4\right.}{}\right.$

input
```
int(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/5*c^3*x^5+3/4*c^2*x^4+c*x^3+1/2*x^2)+d^3*b^2/c^2*(1/5*arctanh(c
*x)^2*c^5*x^5+3/4*arctanh(c*x)^2*c^4*x^4+arctanh(c*x)^2*c^3*x^3+1/2*arctan
h(c*x)^2*c^2*x^2+1/10*arctanh(c*x)*c^4*x^4+1/2*arctanh(c*x)*c^3*x^3+6/5*ar
ctanh(c*x)*c^2*x^2+5/2*arctanh(c*x)*c*x+49/20*arctanh(c*x)*ln(c*x-1)-1/20*
arctanh(c*x)*ln(c*x+1)+49/80*ln(c*x-1)^2-6/5*dilog(1/2*c*x+1/2)-49/40*ln(c
*x-1)*ln(1/2*c*x+1/2)+1/80*ln(c*x+1)^2-1/40*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln
(-1/2*c*x+1/2)+1/30*x^3*c^3+1/4*c^2*x^2+13/10*c*x+43/20*ln(c*x-1)+17/20*ln
(c*x+1))+2*d^3*a*b/c^2*(1/5*arctanh(c*x)*c^5*x^5+3/4*arctanh(c*x)*c^4*x^4+
arctanh(c*x)*c^3*x^3+1/2*arctanh(c*x)*c^2*x^2+1/20*c^4*x^4+1/4*x^3*c^3+3/5
*c^2*x^2+5/4*c*x+49/40*ln(c*x-1)-1/40*ln(c*x+1))
```

Fricas [F]

$$\int x(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\operatorname{artanh}(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^3*d^3*x^4 + 3*a^2*c^2*d^3*x^3 + 3*a^2*c*d^3*x^2 + a^2*d^3*x
 + (b^2*c^3*d^3*x^4 + 3*b^2*c^2*d^3*x^3 + 3*b^2*c*d^3*x^2 + b^2*d^3*x)*arc
tanh(c*x)^2 + 2*(a*b*c^3*d^3*x^4 + 3*a*b*c^2*d^3*x^3 + 3*a*b*c*d^3*x^2 + a
*b*d^3*x)*arctanh(c*x), x)
```

Sympy [F]

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx = d^3\left(\int a^2x\,dx+\int 3a^2cx^2\,dx+\int 3a^2c^2x^3\,dx+\int a^2c^3x^4\,dx+\int b^2x\,\text{atanh}^2\,(cx)\,dx+\int 2abx\,\text{atanh}\,(cx)\,dx+\int 3b^2cx^2\,\text{atanh}^2\,(cx)\,dx+\int 3b^2c^2x^3\,\text{atanh}^2\,(cx)\,dx+\int b^2c^3x^4\,\text{atanh}^2\,(cx)\,dx+\int 6abcx^2\,\text{atanh}\,(cx)\,dx+\int 6abc^2x^3\,\text{atanh}\,(cx)\,dx+\int 2abc^3x^4\,\text{atanh}\,(cx)\,dx\right)$$

input
```
integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

output
```
d**3*(Integral(a**2*x, x) + Integral(3*a**2*c*x**2, x) + Integral(3*a**2*c
**2*x**3, x) + Integral(a**2*c**3*x**4, x) + Integral(b**2*x*atanh(c*x)**2
, x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(3*b**2*c*x**2*atanh(c*x)
**2, x) + Integral(3*b**2*c**2*x**3*atanh(c*x)**2, x) + Integral(b**2*c**3
*x**4*atanh(c*x)**2, x) + Integral(6*a*b*c*x**2*atanh(c*x), x) + Integral(
6*a*b*c**2*x**3*atanh(c*x), x) + Integral(2*a*b*c**3*x**4*atanh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(255) = 510$.

Time = 0.29 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.73

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*c^3*d^3*x^5 + 3/4*a^2*c^2*d^3*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*(
(c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^3*d^3 + a^2*c*d^3*x
^3 + 1/2*b^2*d^3*x^2*arctanh(c*x)^2 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*
x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^2*d^3 + (
2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d^3 + 1/2*a
^2*d^3*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*a*b*d^3 + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x -
 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^
2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2*d^3 + 6/5*(log(c*x + 1)*log(
-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^3/c^2 + 7/20*b^2*d^3*log(c*x
 + 1)/c^2 + 33/20*b^2*d^3*log(c*x - 1)/c^2 + 1/240*(8*b^2*c^3*d^3*x^3 + 60
*b^2*c^2*d^3*x^2 + 312*b^2*c*d^3*x + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3
*x^4 + 20*b^2*c^3*d^3*x^3 + 9*b^2*d^3)*log(c*x + 1)^2 + 3*(4*b^2*c^5*d^3*x
^5 + 15*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 - 39*b^2*d^3)*log(-c*x + 1)^2
 + 12*(b^2*c^4*d^3*x^4 + 5*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 15*b^2*c
*d^3*x)*log(c*x + 1) - 6*(2*b^2*c^4*d^3*x^4 + 10*b^2*c^3*d^3*x^3 + 24*b^2*
c^2*d^3*x^2 + 30*b^2*c*d^3*x + (4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 2
0*b^2*c^3*d^3*x^3 + 9*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c^2
```

Giac [F]

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int x\,(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3\,dx$$

input
```
int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

output
```
int(x*(a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

Reduce [F]

$$\int x(d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(144\log(c^2x-c)\,ab+12a^2c^5x^5+78b^2cx-6atanh(cx)\,ab+15b^2c^2x^2+30atanh(cx)\,b^2c^3x^3+150ata\right.}{}$$

input
```
int(x*(c*d*x+d)^3*(a+b*atanh(c*x))^2,x)
```

output
```
(d**3*(12*atanh(c*x)**2*b**2*c**5*x**5 + 45*atanh(c*x)**2*b**2*c**4*x**4 +
 60*atanh(c*x)**2*b**2*c**3*x**3 + 30*atanh(c*x)**2*b**2*c**2*x**2 - 72*at
anh(c*x)**2*b**2*c*x - 75*atanh(c*x)**2*b**2 + 24*atanh(c*x)*a*b*c**5*x**5
 + 90*atanh(c*x)*a*b*c**4*x**4 + 120*atanh(c*x)*a*b*c**3*x**3 + 60*atanh(c
*x)*a*b*c**2*x**2 - 6*atanh(c*x)*a*b + 6*atanh(c*x)*b**2*c**4*x**4 + 30*at
anh(c*x)*b**2*c**3*x**3 + 72*atanh(c*x)*b**2*c**2*x**2 + 150*atanh(c*x)*b*
*2*c*x + 102*atanh(c*x)*b**2 + 72*int(atanh(c*x)**2,x)*b**2*c + 144*log(c*
*2*x - c)*a*b + 180*log(c**2*x - c)*b**2 + 12*a**2*c**5*x**5 + 45*a**2*c**
4*x**4 + 60*a**2*c**3*x**3 + 30*a**2*c**2*x**2 + 6*a*b*c**4*x**4 + 30*a*b*
c**3*x**3 + 72*a*b*c**2*x**2 + 150*a*b*c*x + 2*b**2*c**3*x**3 + 15*b**2*c*
*2*x**2 + 78*b**2*c*x))/(60*c**2)
```

3.87 $\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 206

$$\begin{aligned}\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx &= \frac{7}{2}abd^3x+b^2d^3x+\frac{1}{12}b^2cd^3x^2-\frac{b^2d^3\text{arctanh}(cx)}{c}\\ &\quad+\frac{7}{2}b^2d^3x\text{arctanh}(cx)+bcd^3x^2(a+b\text{arctanh}(cx))\\ &\quad+\frac{1}{6}bc^2d^3x^3(a+b\text{arctanh}(cx))\\ &\quad+\frac{d^3(1+cx)^4(a+b\text{arctanh}(cx))^2}{4c}\\ &\quad-\frac{4bd^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c}\\ &\quad+\frac{11b^2d^3\log(1-c^2x^2)}{6c}\\ &\quad-\frac{2b^2d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}\end{aligned}$$

output
```
7/2*a*b*d^3*x+b^2*d^3*x+1/12*b^2*c*d^3*x^2-b^2*d^3*arctanh(c*x)/c+7/2*b^2*
d^3*x*arctanh(c*x)+b*c*d^3*x^2*(a+b*arctanh(c*x))+1/6*b*c^2*d^3*x^3*(a+b*a
rctanh(c*x))+1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/c-4*b*d^3*(a+b*arctanh
(c*x))*ln(2/(-c*x+1))/c+11/6*b^2*d^3*ln(-c^2*x^2+1)/c-2*b^2*d^3*polylog(2,
1-2/(-c*x+1))/c
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.42

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{d^3\left(-b^2+12a^2cx+42abcx+12b^2cx+18a^2c^2x^2+12abc^2x^2+b^2c^2x^2+12a^2c^3x^3+2abc^3x^3+3a^2c^4x^4+\right.}{}$$

input
```
Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^3*(-b^2 + 12*a^2*c*x + 42*a*b*c*x + 12*b^2*c*x + 18*a^2*c^2*x^2 + 12*a*
b*c^2*x^2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 +
 3*b^2*(-15 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 + c^4*x^4)*ArcTanh[c*x]^2 + 2*
b*ArcTanh[c*x]*(3*a*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3) + b*(-6 + 21*c*x
 + 6*c^2*x^2 + c^3*x^3) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 21*a*b*Log[
1 - c*x] - 21*a*b*Log[1 + c*x] + 12*a*b*Log[1 - c^2*x^2] + 22*b^2*Log[1 -
c^2*x^2] + 12*a*b*Log[-1 + c^2*x^2] + 24*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x
])]))/(12*c)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$\Big\downarrow \; 6480$$
$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{4c}-$$
$$\frac{b\int\left(-c^2x^2(a+b\text{arctanh}(cx))d^4-4cx(a+b\text{arctanh}(cx))d^4+\frac{8(cx+1)(a+b\text{arctanh}(cx))d^4}{1-c^2x^2}-7(a+b\text{arctanh}(cx))d\right.}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{4c} - \frac{b\left(-\frac{1}{3}c^2d^4x^3(a+b\text{arctanh}(cx)) - 2cd^4x^2(a+b\text{arctanh}(cx)) + \frac{8d^4\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c} - 7ad^4x - 7bd^4x\text{arc}\right.}{2d}$$

input
```
Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)/(4*c) - (b*(-7*a*d^4*x - 2*b*d^4*
x - (b*c*d^4*x^2)/6 + (2*b*d^4*ArcTanh[c*x])/c - 7*b*d^4*x*ArcTanh[c*x] -
2*c*d^4*x^2*(a + b*ArcTanh[c*x]) - (c^2*d^4*x^3*(a + b*ArcTanh[c*x]))/3 +
(8*d^4*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c - (11*b*d^4*Log[1 - c^2*x^
2])/(3*c) + (4*b*d^4*PolyLog[2, 1 - 2/(1 - c*x)])/c))/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{d^3a^2(cx+1)^4}{4}+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\operatorname{arctanh}(cx)^2c^3x^3+\frac{3\operatorname{arctanh}(cx)^2c^2x^2}{2}+\operatorname{arctanh}(cx)^2cx+\frac{\operatorname{arctanh}(cx)^2}{4}+\operatorname{arctanh}\ldots\right.}{}$
default	$\frac{\frac{d^3a^2(cx+1)^4}{4}+d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\operatorname{arctanh}(cx)^2c^3x^3+\frac{3\operatorname{arctanh}(cx)^2c^2x^2}{2}+\operatorname{arctanh}(cx)^2cx+\frac{\operatorname{arctanh}(cx)^2}{4}+\operatorname{arctanh}\ldots\right.}{}$
parts	$\frac{d^3a^2(cx+1)^4}{4c}+\frac{d^3b^2\left(\frac{\operatorname{arctanh}(cx)^2c^4x^4}{4}+\operatorname{arctanh}(cx)^2c^3x^3+\frac{3\operatorname{arctanh}(cx)^2c^2x^2}{2}+\operatorname{arctanh}(cx)^2cx+\frac{\operatorname{arctanh}(cx)^2}{4}+\operatorname{arc}\ldots\right.}{}$
risch	$\frac{2b^2d^3\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)}{c}-\frac{2b^2d^3\ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{c}+\frac{b^2c\,d^3x^2}{12}+\frac{d^3c^3a^2x^4}{4}+d^3c^2a^2x^3+\frac{3d^3c\,a^2\ldots}{2}$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/4*d^3*a^2*(c*x+1)^4+d^3*b^2*(1/4*arctanh(c*x)^2*c^4*x^4+arctanh(c*x
)^2*c^3*x^3+3/2*arctanh(c*x)^2*c^2*x^2+arctanh(c*x)^2*c*x+1/4*arctanh(c*x)
^2+1/6*arctanh(c*x)*c^3*x^3+arctanh(c*x)*c^2*x^2+7/2*arctanh(c*x)*c*x+4*ar
ctanh(c*x)*ln(c*x-1)+1/12*(c*x-1)^2+7/6*c*x-7/6+7/3*ln(c*x-1)+4/3*ln(c*x+1
)+ln(c*x-1)^2-2*dilog(1/2*c*x+1/2)-2*ln(c*x-1)*ln(1/2*c*x+1/2))+2*d^3*a*b*
(1/4*arctanh(c*x)*c^4*x^4+arctanh(c*x)*c^3*x^3+3/2*arctanh(c*x)*c^2*x^2+ar
ctanh(c*x)*c*x+1/4*arctanh(c*x)+1/12*x^3*c^3+1/2*c^2*x^2+7/4*c*x+2*ln(c*x-
1)))
```

Fricas [F]

$$\int (d+cdx)^3(a+b\operatorname{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\operatorname{artanh}(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x
)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*ar
ctanh(c*x), x)
```

Sympy [F]

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx = d^3\left(\int a^2\,dx + \int b^2\operatorname{atanh}^2(cx)\,dx + \int 2ab\operatorname{atanh}(cx)\,dx + \int 3a^2cx\,dx + \int 3a^2c^2x^2\,dx + \int a^2c^3x^3\,dx + \int 3b^2cx\operatorname{atanh}^2(cx)\,dx + \int 3b^2c^2x^2\operatorname{atanh}^2(cx)\,dx + \int b^2c^3x^3\operatorname{atanh}^2(cx)\,dx + \int 6abcx\operatorname{atanh}(cx)\,dx + \int 6abc^2x^2\operatorname{atanh}(cx)\,dx + \int 2abc^3x^3\operatorname{atanh}(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

output
```
d**3*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b
*atanh(c*x), x) + Integral(3*a**2*c*x, x) + Integral(3*a**2*c**2*x**2, x)
+ Integral(a**2*c**3*x**3, x) + Integral(3*b**2*c*x*atanh(c*x)**2, x) + In
tegral(3*b**2*c**2*x**2*atanh(c*x)**2, x) + Integral(b**2*c**3*x**3*atanh(
c*x)**2, x) + Integral(6*a*b*c*x*atanh(c*x), x) + Integral(6*a*b*c**2*x**2
*atanh(c*x), x) + Integral(2*a*b*c**3*x**3*atanh(c*x), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(191) = 382$.

Time = 0.21 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.04

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*c^3*d^3*x^4 + a^2*c^2*d^3*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c
^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^3*d^3
+ (2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c^2*d^3 +
3/2*a^2*c*d^3*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^
3 + log(c*x - 1)/c^3))*a*b*c*d^3 + a^2*d^3*x + (2*c*x*arctanh(c*x) + log(-
c^2*x^2 + 1))*a*b*d^3/c + 2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*
c*x + 1/2))*b^2*d^3/c + 4/3*b^2*d^3*log(c*x + 1)/c + 7/3*b^2*d^3*log(c*x -
 1)/c + 1/48*(4*b^2*c^2*d^3*x^2 + 48*b^2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*
b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x + b^2*d^3)*log(c*x + 1
)^2 + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c
*d^3*x - 15*b^2*d^3)*log(-c*x + 1)^2 + 4*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*
x^2 + 21*b^2*c*d^3*x)*log(c*x + 1) - 2*(2*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3
*x^2 + 42*b^2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2
*d^3*x^2 + 4*b^2*c*d^3*x + b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c
```

Giac [F]

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx=\int (a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3\,dx$$

input
```
int((a + b*atanh(c*x))^2*(d + c*d*x)^3,x)
```

output
```
int((a + b*atanh(c*x))^2*(d + c*d*x)^3, x)
```

Reduce [F]

$$\int (d+cdx)^3(a+b\text{arctanh}(cx))^2\,dx$$
$$= \frac{d^3\Big(48\log(c^2x-c)\,ab+12b^2cx+6atanh(cx)\,ab+b^2c^2x^2+2atanh(cx)\,b^2c^3x^3+42atanh(cx)\,b^2cx+2ab}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2,x)
```

output
```
(d**3*(3*atanh(c*x)**2*b**2*c**4*x**4 + 12*atanh(c*x)**2*b**2*c**3*x**3 +
18*atanh(c*x)**2*b**2*c**2*x**2 + 12*atanh(c*x)**2*b**2*c*x - 21*atanh(c*x
)**2*b**2 + 6*atanh(c*x)*a*b*c**4*x**4 + 24*atanh(c*x)*a*b*c**3*x**3 + 36*
atanh(c*x)*a*b*c**2*x**2 + 24*atanh(c*x)*a*b*c*x + 6*atanh(c*x)*a*b + 2*at
anh(c*x)*b**2*c**3*x**3 + 12*atanh(c*x)*b**2*c**2*x**2 + 42*atanh(c*x)*b**
2*c*x + 32*atanh(c*x)*b**2 + 48*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*b**2
*c**2 + 48*log(c**2*x - c)*a*b + 44*log(c**2*x - c)*b**2 + 3*a**2*c**4*x**
4 + 12*a**2*c**3*x**3 + 18*a**2*c**2*x**2 + 12*a**2*c*x + 2*a*b*c**3*x**3
+ 12*a*b*c**2*x**2 + 42*a*b*c*x + b**2*c**2*x**2 + 12*b**2*c*x))/(12*c)
```

3.88 $\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 355

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx = 3abcd^3x+\frac{1}{3}b^2cd^3x-\frac{1}{3}b^2d^3\text{arctanh}(cx)+3b^2cd^3x\text{arctanh}(cx)+\frac{1}{3}bc^2d^3x^2(a+b\text{arctanh}(cx))+\frac{11}{6}d^3(a+b\text{arctanh}(cx))^2+3cd^3x(a+b\text{arctanh}(cx))^2+\frac{3}{2}c^2d^3x^2(a+b\text{arctanh}(cx))^2+\frac{1}{3}c^3d^3x^3(a+b\text{arctanh}(cx))^2+2d^3(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)-\frac{20}{3}bd^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)+\frac{3}{2}b^2d^3\log\left(1-c^2x^2\right)-\frac{10}{3}b^2d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-bd^3(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+bd^3(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)+\frac{1}{2}b^2d^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d^3\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)$$

output
```
3*a*b*c*d^3*x+1/3*b^2*c*d^3*x-1/3*b^2*d^3*arctanh(c*x)+3*b^2*c*d^3*x*arcta
nh(c*x)+1/3*b*c^2*d^3*x^2*(a+b*arctanh(c*x))+11/6*d^3*(a+b*arctanh(c*x))^2
+3*c*d^3*x*(a+b*arctanh(c*x))^2+3/2*c^2*d^3*x^2*(a+b*arctanh(c*x))^2+1/3*c
^3*d^3*x^3*(a+b*arctanh(c*x))^2-2*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-
c*x+1))-20/3*b*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+3/2*b^2*d^3*ln(-c^2*x
^2+1)-10/3*b^2*d^3*polylog(2,1-2/(-c*x+1))-b*d^3*(a+b*arctanh(c*x))*polylo
g(2,1-2/(-c*x+1))+b*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^
2*d^3*polylog(3,1-2/(-c*x+1))-1/2*b^2*d^3*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.26

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx = \frac{1}{24}d^3\left(ib^2\pi^3+72a^2cx+72abcx+8b^2cx+36a^2c^2x^2+8abc^2x^2+8a^2c^3x^3-8b^2\text{arctanh}(cx)+144abcx\text{arctanh}(cx)+72b^2cx\text{arctanh}(cx)+72abc^2x^2\text{arctanh}(cx)+8b^2c^2x^2\text{arctanh}(cx)+16abc^3x^3\text{arctanh}(cx)-116b^2\text{arctanh}(cx)^2+72b^2cx\text{arctanh}(cx)^2+36b^2c^2x^2\text{arctanh}(cx)^2+8b^2c^3x^3\text{arctanh}(cx)^2-16b^2\text{arctanh}(cx)^3-160b^2\text{arctanh}(cx)\log\left(1+e^{-2\text{arctanh}(cx)}\right)-24b^2\text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right)+24b^2\text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right)+24a^2\log(cx)+36ab\log(1-cx)-36ab\log(1+cx)+72ab\log\left(1-c^2x^2\right)+36b^2\log\left(1-c^2x^2\right)+8ab\log\left(-1+c^2x^2\right)+8b^2(10+3\text{arctanh}(cx))\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)+24b^2\text{arctanh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right)-24ab\,\text{PolyLog}(2,-cx)+24ab\,\text{PolyLog}(2,cx)+12b^2\,\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right)-12b^2\,\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\right)$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

output
```
(d^3*(I*b^2*Pi^3 + 72*a^2*c*x + 72*a*b*c*x + 8*b^2*c*x + 36*a^2*c^2*x^2 +
8*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - 8*b^2*ArcTanh[c*x] + 144*a*b*c*x*ArcTanh[c
*x] + 72*b^2*c*x*ArcTanh[c*x] + 72*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^
2*ArcTanh[c*x] + 16*a*b*c^3*x^3*ArcTanh[c*x] - 116*b^2*ArcTanh[c*x]^2 + 72
*b^2*c*x*ArcTanh[c*x]^2 + 36*b^2*c^2*x^2*ArcTanh[c*x]^2 + 8*b^2*c^3*x^3*Ar
cTanh[c*x]^2 - 16*b^2*ArcTanh[c*x]^3 - 160*b^2*ArcTanh[c*x]*Log[1 + E^(-2*
ArcTanh[c*x])] - 24*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b
^2*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*Log[c*x] + 36*a*b*L
og[1 - c*x] - 36*a*b*Log[1 + c*x] + 72*a*b*Log[1 - c^2*x^2] + 36*b^2*Log[1
 - c^2*x^2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*(10 + 3*ArcTanh[c*x])*PolyLo
g[2, -E^(-2*ArcTanh[c*x])] + 24*b^2*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c
*x])] - 24*a*b*PolyLog[2, -(c*x)] + 24*a*b*PolyLog[2, c*x] + 12*b^2*PolyLo
g[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*PolyLog[3, E^(2*ArcTanh[c*x])]))/24
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^3d^3x^2(a+b\text{arctanh}(cx))^2+3c^2d^3x(a+b\text{arctanh}(cx))^2+3cd^3(a+b\text{arctanh}(cx))^2+\frac{d^3(a+b\text{arctanh}(cx))^2}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^3d^3x^3(a+b\operatorname{arctanh}(cx))^2+\frac{3}{2}c^2d^3x^2(a+b\operatorname{arctanh}(cx))^2+\frac{1}{3}bc^2d^3x^2(a+b\operatorname{arctanh}(cx))-bd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))+bd^3\operatorname{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\operatorname{arctanh}(cx))+3cd^3x(a+b\operatorname{arctanh}(cx))^2+\frac{11}{6}d^3(a+b\operatorname{arctanh}(cx))^2+2d^3\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))^2-\frac{20}{3}bd^3\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))+3abcd^3x-\frac{1}{3}b^2d^3\operatorname{arctanh}(cx)+3b^2cd^3x\operatorname{arctanh}(cx)+\frac{3}{2}b^2d^3\log\left(1-c^2x^2\right)-\frac{10}{3}b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,\frac{2}{1-cx}-1\right)+\frac{1}{3}b^2cd^3x$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

output
```
3*a*b*c*d^3*x + (b^2*c*d^3*x)/3 - (b^2*d^3*ArcTanh[c*x])/3 + 3*b^2*c*d^3*x
*ArcTanh[c*x] + (b*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/3 + (11*d^3*(a + b*Ar
cTanh[c*x])^2)/6 + 3*c*d^3*x*(a + b*ArcTanh[c*x])^2 + (3*c^2*d^3*x^2*(a +
b*ArcTanh[c*x])^2)/2 + (c^3*d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + 2*d^3*(a +
 b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - (20*b*d^3*(a + b*ArcTanh[c*x
])*Log[2/(1 - c*x)])/3 + (3*b^2*d^3*Log[1 - c^2*x^2])/2 - (10*b^2*d^3*Poly
Log[2, 1 - 2/(1 - c*x)])/3 - b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)] + b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2
*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)
])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.38 (sec) , antiderivative size = 959, normalized size of antiderivative = 2.70

method	result	size
parts	Expression too large to display	959
derivativedivides	Expression too large to display	961
default	Expression too large to display	961

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/3*x^3*c^3+3/2*c^2*x^2+3*c*x+ln(x))+d^3*b^2*(arctanh(c*x)^2*ln(1
-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)
^(1/2))+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*pol
ylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+
1)-1)+1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x
^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*arc
tanh(c*x)^2+arctanh(c*x)^2*ln(c*x)-1/3-20/3*arctanh(c*x)*ln(1+I*(c*x+1)/(-
c^2*x^2+1)^(1/2))-20/3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/
3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*dilog(1-I*(c*x+1)/(-c^2*x^2+1
)^(1/2))+1/3*c*x+1/3*arctanh(c*x)^2*c^3*x^3+3/2*arctanh(c*x)^2*c^2*x^2+3*a
rctanh(c*x)^2*c*x+11/6*arctanh(c*x)^2+1/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1
))+11/3*(c*x+1)*arctanh(c*x)-3*ln(1+(c*x+1)^2/(-c^2*x^2+1))-arctanh(c*x)*p
olylog(2,-(c*x+1)^2/(-c^2*x^2+1))-2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))
-2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+1/3*(c*x-3)*(c*x+1)*arctanh(c*x)-
1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-1/2*I*Pi*csgn(I*(-(c*x+1)^2
/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1
)))^2*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)
^2/(c^2*x^2-1)))^3*arctanh(c*x)^2)+2*d^3*a*b*(1/3*arctanh(c*x)*c^3*x^3+3/2
*arctanh(c*x)*c^2*x^2+3*arctanh(c*x)*c*x+arctanh(c*x)*ln(c*x)-1/2*dilog...
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x, x)
```

Sympy [F]

$$\begin{aligned}\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx=d^3\bigg(&\int 3a^2c\,dx+\int \frac{a^2}{x}\,dx+\int 3a^2c^2x\,dx\\&+\int a^2c^3x^2\,dx+\int 3b^2c\,\text{atanh}^2\,(cx)\,dx\\&+\int \frac{b^2\,\text{atanh}^2\,(cx)}{x}\,dx+\int 6abc\,\text{atanh}\,(cx)\,dx\\&+\int \frac{2ab\,\text{atanh}\,(cx)}{x}\,dx\\&+\int 3b^2c^2x\,\text{atanh}^2\,(cx)\,dx\\&+\int b^2c^3x^2\,\text{atanh}^2\,(cx)\,dx\\&+\int 6abc^2x\,\text{atanh}\,(cx)\,dx\\&+\int 2abc^3x^2\,\text{atanh}\,(cx)\,dx\bigg)\end{aligned}$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x,x)
```

output
```
d**3*(Integral(3*a**2*c, x) + Integral(a**2/x, x) + Integral(3*a**2*c**2*x
, x) + Integral(a**2*c**3*x**2, x) + Integral(3*b**2*c*atanh(c*x)**2, x) +
 Integral(b**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x), x) + Int
egral(2*a*b*atanh(c*x)/x, x) + Integral(3*b**2*c**2*x*atanh(c*x)**2, x) +
Integral(b**2*c**3*x**2*atanh(c*x)**2, x) + Integral(6*a*b*c**2*x*atanh(c*
x), x) + Integral(2*a*b*c**3*x**2*atanh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/3*a^2*c^3*d^3*x^3 + 3/2*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + 3*(2*c*x*arcta
nh(c*x) + log(-c^2*x^2 + 1))*a*b*d^3 + a^2*d^3*log(x) + 1/24*(2*b^2*c^3*d^
3*x^3 + 9*b^2*c^2*d^3*x^2 + 18*b^2*c*d^3*x)*log(-c*x + 1)^2 - integrate(-1
/12*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log
(c*x + 1)^2 + 12*(a*b*c^4*d^3*x^4 + 2*a*b*c^3*d^3*x^3 - 3*a*b*c^2*d^3*x^2
+ a*b*c*d^3*x - a*b*d^3)*log(c*x + 1) - (12*a*b*c*d^3*x - 12*a*b*d^3 + 2*(
6*a*b*c^4*d^3 + b^2*c^4*d^3)*x^4 + 3*(8*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 -
 18*(2*a*b*c^2*d^3 - b^2*c^2*d^3)*x^2 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3
*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^2 - x),
x)
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x}\,dx = \frac{d^3\left(2atanh(cx)^2\,b^2c^3x^3+9atanh(cx)^2\,b^2c^2x^2-2atanh(cx)^2\,b^2cx-9atanh(cx)^2\,b^2+4atanh(cx)\,ab\,c^3x^3\right.}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x,x)
```

output
```
(d**3*(2*atanh(c*x)**2*b**2*c**3*x**3 + 9*atanh(c*x)**2*b**2*c**2*x**2 - 2
*atanh(c*x)**2*b**2*c*x - 9*atanh(c*x)**2*b**2 + 4*atanh(c*x)*a*b*c**3*x**
3 + 18*atanh(c*x)*a*b*c**2*x**2 + 36*atanh(c*x)*a*b*c*x + 22*atanh(c*x)*a*
b + 2*atanh(c*x)*b**2*c**2*x**2 + 18*atanh(c*x)*b**2*c*x + 16*atanh(c*x)*b
**2 + 20*int(atanh(c*x)**2,x)*b**2*c + 12*int(atanh(c*x)/x,x)*a*b + 6*int(
atanh(c*x)**2/x,x)*b**2 + 40*log(c**2*x - c)*a*b + 18*log(c**2*x - c)*b**2
 + 6*log(x)*a**2 + 2*a**2*c**3*x**3 + 9*a**2*c**2*x**2 + 18*a**2*c*x + 2*a
*b*c**2*x**2 + 18*a*b*c*x + 2*b**2*c*x))/6
```

3.89 $\int \frac{(d+cdx)^3(a+b\,\text{arctanh}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 361

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx = abc^2d^3x + b^2c^2d^3x\operatorname{arctanh}(cx) + \frac{7}{2}cd^3(a+b\operatorname{arctanh}(cx))^2 - \frac{d^3(a+b\operatorname{arctanh}(cx))^2}{x} + 3c^2d^3x(a+b\operatorname{arctanh}(cx))^2 + \frac{1}{2}c^3d^3x^2(a+b\operatorname{arctanh}(cx))^2 + 6cd^3(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right) - 6bcd^3(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) + \frac{1}{2}b^2cd^3\log\left(1-c^2x^2\right) + 2bcd^3(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) - 3b^2cd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) - 3bcd^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) + 3bcd^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) - b^2cd^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right) + \frac{3}{2}b^2cd^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right) - \frac{3}{2}b^2cd^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)$$

output
```
a*b*c^2*d^3*x+b^2*c^2*d^3*x*arctanh(c*x)+7/2*c*d^3*(a+b*arctanh(c*x))^2-d^
3*(a+b*arctanh(c*x))^2/x+3*c^2*d^3*x*(a+b*arctanh(c*x))^2+1/2*c^3*d^3*x^2*
(a+b*arctanh(c*x))^2-6*c*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-6
*b*c*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))+1/2*b^2*c*d^3*ln(-c^2*x^2+1)+2*
b*c*d^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-3*b^2*c*d^3*polylog(2,1-2/(-c*x
+1))-3*b*c*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))+3*b*c*d^3*(a+b*a
rctanh(c*x))*polylog(2,-1+2/(-c*x+1))-b^2*c*d^3*polylog(2,-1+2/(c*x+1))+3/
2*b^2*c*d^3*polylog(3,1-2/(-c*x+1))-3/2*b^2*c*d^3*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.33

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^2}\,dx$$
$$= \frac{d^3\left(-8a^2+ib^2c\pi^3x+24a^2c^2x^2+8abc^2x^2+4a^2c^3x^3-16ab\text{arctanh}(cx)+48abc^2x^2\text{arctanh}(cx)+8b^2c^2\right.}{}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

output
```
(d^3*(-8*a^2 + I*b^2*c*Pi^3*x + 24*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + 4*a^2*c^3
*x^3 - 16*a*b*ArcTanh[c*x] + 48*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*A
rcTanh[c*x] + 8*a*b*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 20*b^2*c
*x*ArcTanh[c*x]^2 + 24*b^2*c^2*x^2*ArcTanh[c*x]^2 + 4*b^2*c^3*x^3*ArcTanh[
c*x]^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 16*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2
*ArcTanh[c*x])] - 48*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 2
4*b^2*c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh
[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c
*x] + 4*a*b*c*x*Log[1 - c*x] - 4*a*b*c*x*Log[1 + c*x] + 16*a*b*c*x*Log[1 -
 c^2*x^2] + 4*b^2*c*x*Log[1 - c^2*x^2] + 24*b^2*c*x*(1 + ArcTanh[c*x])*Pol
yLog[2, -E^(-2*ArcTanh[c*x])] - 8*b^2*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]
+ 24*b^2*c*x*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c*x*Poly
Log[2, -(c*x)] + 24*a*b*c*x*PolyLog[2, c*x] + 12*b^2*c*x*PolyLog[3, -E^(-2
*ArcTanh[c*x])] - 12*b^2*c*x*PolyLog[3, E^(2*ArcTanh[c*x])]))/(8*x)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^3d^3x(a+b\text{arctanh}(cx))^2+3c^2d^3(a+b\text{arctanh}(cx))^2+\frac{d^3(a+b\text{arctanh}(cx))^2}{x^2}+\frac{3cd^3(a+b\text{arctanh}(cx))^2}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^3d^3x^2(a+b\text{arctanh}(cx))^2+3c^2d^3x(a+b\text{arctanh}(cx))^2-3bcd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+3bcd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+\frac{7}{2}cd^3(a+b\text{arctanh}(cx))^2-\frac{d^3(a+b\text{arctanh}(cx))^2}{x}+6cd^3\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-6bcd^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+2bcd^3\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))+abc^2d^3x+b^2c^2d^3x\text{arctanh}(cx)+\frac{1}{2}b^2cd^3\log\left(1-c^2x^2\right)-3b^2cd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-b^2cd^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{3}{2}b^2cd^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{3}{2}b^2cd^3\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

output
```
a*b*c^2*d^3*x + b^2*c^2*d^3*x*ArcTanh[c*x] + (7*c*d^3*(a + b*ArcTanh[c*x])
^2)/2 - (d^3*(a + b*ArcTanh[c*x])^2)/x + 3*c^2*d^3*x*(a + b*ArcTanh[c*x])^
2 + (c^3*d^3*x^2*(a + b*ArcTanh[c*x])^2)/2 + 6*c*d^3*(a + b*ArcTanh[c*x])^
2*ArcTanh[1 - 2/(1 - c*x)] - 6*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x
)] + (b^2*c*d^3*Log[1 - c^2*x^2])/2 + 2*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - 3*b^2*c*d^3*PolyLog[2, 1 - 2/(1 - c*x)] - 3*b*c*d^3*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c*d^3*(a + b*ArcTanh[c*
x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^3*PolyLog[2, -1 + 2/(1 + c*x)]
+ (3*b^2*c*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (3*b^2*c*d^3*PolyLog[3, -1
 + 2/(1 - c*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.54 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.80

method	result	size
parts	Expression too large to display	1012
derivativedivides	Expression too large to display	1014
default	Expression too large to display	1014

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/2*c^3*x^2+3*c^2*x+3*c*ln(x)-1/x)+d^3*b^2*c*(3*arctanh(c*x)^2*ln
(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+
1)^(1/2))+3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*arctanh(c*x)
*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^
2*x^2+1)-1)+3/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/
(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)
))*arctanh(c*x)^2+3*arctanh(c*x)^2*ln(c*x)+2*arctanh(c*x)*ln(1+(c*x+1)/(-c
^2*x^2+1)^(1/2))-6*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6*arcta
nh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6*dilog(1+I*(c*x+1)/(-c^2*x^2+1
)^(1/2))-6*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*arctanh(c*x)^2*c^2*x^
2+3*arctanh(c*x)^2*c*x-2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+2*dilog(1+(c*x+
1)/(-c^2*x^2+1)^(1/2))+3/2*arctanh(c*x)^2+3/2*polylog(3,-(c*x+1)^2/(-c^2*x
^2+1))+(c*x+1)*arctanh(c*x)-ln(1+(c*x+1)^2/(-c^2*x^2+1))-3*arctanh(c*x)*po
lylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1
))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(
c*x)^2-3/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*
x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-6*polylog(3,-(c*x+1)
/(-c^2*x^2+1)^(1/2))-6*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^
2/c/x+3/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1))
)^3*arctanh(c*x)^2)+2*d^3*b*a*c*(1/2*arctanh(c*x)*c^2*x^2+3*arctanh(c*x...
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx = d^3\left(\int 3a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int \frac{3a^2c}{x}\,dx + \int a^2c^3x\,dx + \int 3b^2c^2\operatorname{atanh}^2(cx)\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^2}\,dx + \int 6abc^2\operatorname{atanh}(cx)\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^2}\,dx + \int \frac{3b^2c\operatorname{atanh}^2(cx)}{x}\,dx + \int b^2c^3x\operatorname{atanh}^2(cx)\,dx + \int \frac{6abc\operatorname{atanh}(cx)}{x}\,dx + \int 2abc^3x\operatorname{atanh}(cx)\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**2,x)
```

output
```
d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*
c/x, x) + Integral(a**2*c**3*x, x) + Integral(3*b**2*c**2*atanh(c*x)**2, x
) + Integral(b**2*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*atanh(c*x),
 x) + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(3*b**2*c*atanh(c*x)**2
/x, x) + Integral(b**2*c**3*x*atanh(c*x)**2, x) + Integral(6*a*b*c*atanh(c
*x)/x, x) + Integral(2*a*b*c**3*x*atanh(c*x), x))
```

Maxima [F]

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx = \int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
1/2*a^2*c^3*d^3*x^2 + 3*a^2*c^2*d^3*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x
^2 + 1))*a*b*c*d^3 + 3*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2))
 + 2*arctanh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/8*(b^2*c^3*d^3*x^3 + 6*b^2*c^
2*d^3*x^2 - 2*b^2*d^3)*log(-c*x + 1)^2/x - integrate(-1/4*((b^2*c^4*d^3*x^
4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 4*(a*b*c
^4*d^3*x^4 - a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 - 3*a*b*c*d^3*x)*log(c*x
+ 1) - (12*a*b*c^2*d^3*x^2 + (4*a*b*c^4*d^3 + b^2*c^4*d^3)*x^4 - 2*(2*a*b*
c^3*d^3 - 3*b^2*c^3*d^3)*x^3 - 2*(6*a*b*c*d^3 + b^2*c*d^3)*x + 2*(b^2*c^4*
d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-
c*x + 1))/(c*x^3 - x^2), x)
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^2,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^2}\,dx$$
$$= \frac{d^3\Big(atanh(cx)^2\,b^2c^3x^3 - atanh(cx)^2\,b^2cx - 2atanh(cx)^2\,b^2 + 2atanh(cx)\,ab\,c^3x^3 + 12atanh(cx)\,ab\,c^2x^2 +}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x^2,x)
```

output
```
(d**3*(atanh(c*x)**2*b**2*c**3*x**3 - atanh(c*x)**2*b**2*c*x - 2*atanh(c*x
)**2*b**2 + 2*atanh(c*x)*a*b*c**3*x**3 + 12*atanh(c*x)*a*b*c**2*x**2 + 6*a
tanh(c*x)*a*b*c*x - 4*atanh(c*x)*a*b + 2*atanh(c*x)*b**2*c**2*x**2 + 2*ata
nh(c*x)*b**2*c*x + 6*int(atanh(c*x)**2,x)*b**2*c**2*x - 4*int(atanh(c*x)/(
c**2*x**3 - x),x)*b**2*c*x + 12*int(atanh(c*x)/x,x)*a*b*c*x + 6*int(atanh(
c*x)**2/x,x)*b**2*c*x + 8*log(c**2*x - c)*a*b*c*x + 2*log(c**2*x - c)*b**2
*c*x + 6*log(x)*a**2*c*x + 4*log(x)*a*b*c*x + a**2*c**3*x**3 + 6*a**2*c**2
*x**2 - 2*a**2 + 2*a*b*c**2*x**2))/(2*x)
```

3.90 $\int \frac{(d+cdx)^3(a+b\,\mathbf{arctanh}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 385

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^3}\,dx = -\frac{bcd^3(a+b\operatorname{arctanh}(cx))}{x} + \frac{9}{2}c^2d^3(a+b\operatorname{arctanh}(cx))^2 - \frac{d^3(a+b\operatorname{arctanh}(cx))^2}{2x^2} - \frac{3cd^3(a+b\operatorname{arctanh}(cx))^2}{x} + c^3d^3x(a+b\operatorname{arctanh}(cx))^2 + 6c^2d^3(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right) + b^2c^2d^3\log(x) - 2bc^2d^3(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) - \frac{1}{2}b^2c^2d^3\log\left(1-c^2x^2\right) + 6bc^2d^3(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) - b^2c^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) - 3bc^2d^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) + 3bc^2d^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) - 3b^2c^2d^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right) + \frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right) - \frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)$$

output
```
-b*c*d^3*(a+b*arctanh(c*x))/x+9/2*c^2*d^3*(a+b*arctanh(c*x))^2-1/2*d^3*(a+
b*arctanh(c*x))^2/x^2-3*c*d^3*(a+b*arctanh(c*x))^2/x+c^3*d^3*x*(a+b*arctan
h(c*x))^2-6*c^2*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))+b^2*c^2*d^
3*ln(x)-2*b*c^2*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-1/2*b^2*c^2*d^3*ln(-
c^2*x^2+1)+6*b*c^2*d^3*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^2*c^2*d^3*poly
log(2,1-2/(-c*x+1))-3*b*c^2*d^3*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))
+3*b*c^2*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-3*b^2*c^2*d^3*pol
ylog(2,-1+2/(c*x+1))+3/2*b^2*c^2*d^3*polylog(3,1-2/(-c*x+1))-3/2*b^2*c^2*d
^3*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.20

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \frac{1}{2}d^3\Bigg(-\frac{a^2}{x^2}-\frac{6a^2c}{x}+2a^2c^3x+6a^2c^2\log(x) - \frac{ab(2\text{arctanh}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))}{x^2} + \frac{b^2\left(-2cx\text{arctanh}(cx)+(-1+c^2x^2)\,\text{arctanh}(cx)^2+2c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)\right)}{x^2} + 2abc^2\left(2cx\text{arctanh}(cx)+\log\left(1-c^2x^2\right)\right) - \frac{6abc(2\text{arctanh}(cx)+cx(-2\log(cx)+\log(1-c^2x^2)))}{x} + 2b^2c^2\left(\text{arctanh}(cx)\left((-1+cx)\text{arctanh}(cx)-2\log\left(1+e^{-2\text{arctanh}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)\right) + \frac{6b^2c\left(\text{arctanh}(cx)\left((-1+cx)\text{arctanh}(cx)+2cx\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right)-cx\,\text{PolyLog}\left(2,e^{-2\text{arctanh}(cx)}\right.\right.}{x} - 6abc^2(\text{PolyLog}(2,-cx)-\text{PolyLog}(2,cx)) + 6b^2c^2\Bigg(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}(cx)^3 - \text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right)+\text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right) + \text{arctanh}(cx)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)+\text{arctanh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\Bigg)\Bigg)$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

output
```
(d^3*(-(a^2/x^2) - (6*a^2*c)/x + 2*a^2*c^3*x + 6*a^2*c^2*Log[x] - (a*b*(2*
ArcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])))/x^2 + (b^2*
(-2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + 2*c^2*x^2*Log[(c*x)
/Sqrt[1 - c^2*x^2]]))/x^2 + 2*a*b*c^2*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^
2]) - (6*a*b*c*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])))/x
+ 2*b^2*c^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTa
nh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + (6*b^2*c*(ArcTanh[c*x]*((
-1 + c*x)*ArcTanh[c*x] + 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*PolyLog
[2, E^(-2*ArcTanh[c*x])]))/x - 6*a*b*c^2*(PolyLog[2, -(c*x)] - PolyLog[2,
c*x]) + 6*b^2*c^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log
[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + A
rcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(
2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*Ar
cTanh[c*x])]/2)))/2
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(c^3d^3(a+b\text{arctanh}(cx))^2+\frac{3c^2d^3(a+b\text{arctanh}(cx))^2}{x}+\frac{d^3(a+b\text{arctanh}(cx))^2}{x^3}+\frac{3cd^3(a+b\text{arctanh}(cx))^2}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$c^3d^3x(a+b\text{arctanh}(cx))^2-3bc^2d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+3bc^2d^3\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))+\frac{9}{2}c^2d^3(a+b\text{arctanh}(cx))^2+6c^2d^3\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-2bc^2d^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+6bc^2d^3\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{d^3(a+b\text{arctanh}(cx))^2}{2x^2}-\frac{3cd^3(a+b\text{arctanh}(cx))^2}{x}-\frac{bcd^3(a+b\text{arctanh}(cx))}{x}-b^2c^2d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-3b^2c^2d^3\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{3}{2}b^2c^2d^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{3}{2}b^2c^2d^3\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)-\frac{1}{2}b^2c^2d^3\log\left(1-c^2x^2\right)+b^2c^2d^3\log(x)$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

output
```
-((b*c*d^3*(a + b*ArcTanh[c*x]))/x) + (9*c^2*d^3*(a + b*ArcTanh[c*x])^2)/2
 - (d^3*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)
/x + c^3*d^3*x*(a + b*ArcTanh[c*x])^2 + 6*c^2*d^3*(a + b*ArcTanh[c*x])^2*A
rcTanh[1 - 2/(1 - c*x)] + b^2*c^2*d^3*Log[x] - 2*b*c^2*d^3*(a + b*ArcTanh[
c*x])*Log[2/(1 - c*x)] - (b^2*c^2*d^3*Log[1 - c^2*x^2])/2 + 6*b*c^2*d^3*(a
 + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c^2*d^3*PolyLog[2, 1 - 2/(1
- c*x)] - 3*b*c^2*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3
*b*c^2*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 3*b^2*c^2*d
^3*PolyLog[2, -1 + 2/(1 + c*x)] + (3*b^2*c^2*d^3*PolyLog[3, 1 - 2/(1 - c*x
)])/2 - (3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.48 (sec) , antiderivative size = 1086, normalized size of antiderivative = 2.82

method	result	size
derivativedivides	Expression too large to display	1086
default	Expression too large to display	1086
parts	Expression too large to display	1086

input

```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output

```
c^2*(d^3*a^2*(c*x-1/2/c^2/x^2+3*ln(c*x)-3/c/x)+d^3*b^2*(3*arctanh(c*x)^2*l
n(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2
+1)^(1/2))+3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*arctanh(c*x
)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3*arctanh(c*x)^2*ln((c*x+1)^2/(-c
^2*x^2+1)-1)+3/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2
/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1
)))*arctanh(c*x)^2+3*arctanh(c*x)^2*ln(c*x)+6*arctanh(c*x)*ln(1+(c*x+1)/(-
c^2*x^2+1)^(1/2))-2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*arct
anh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*dilog(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+ln((c*x+1)/(-c^2*x^2+1)^
(1/2)-1)-1/2*arctanh(c*x)^2/c^2/x^2+arctanh(c*x)^2*c*x-6*dilog((c*x+1)/(-c
^2*x^2+1)^(1/2))+6*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*arctanh(c*x)^2+
3/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-3*arctanh(c*x)*polylog(2,-(c*x+1)^2
/(-c^2*x^2+1))-3/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1
)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-3/2*I*Pi*cs
gn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+
1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-6*pol
ylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2)
)-3*arctanh(c*x)^2/c/x-1/2*(c*x-(-c^2*x^2+1)^(1/2)+1)/c/x*arctanh(c*x)-1/2
*arctanh(c*x)*(c*x+(-c^2*x^2+1)^(1/2)+1)/c/x+3/2*I*Pi*csgn(I*(-(c*x+1)^...
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx = d^3\left(\int a^2c^3\,dx + \int \frac{a^2}{x^3}\,dx + \int \frac{3a^2c}{x^2}\,dx + \int \frac{3a^2c^2}{x}\,dx + \int b^2c^3\,\text{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x^3}\,dx + \int 2abc^3\,\text{atanh}\,(cx)\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x^3}\,dx + \int \frac{3b^2c\,\text{atanh}^2\,(cx)}{x^2}\,dx + \int \frac{3b^2c^2\,\text{atanh}^2\,(cx)}{x}\,dx + \int \frac{6abc\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{6abc^2\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**3,x)
```

output
```
d**3*(Integral(a**2*c**3, x) + Integral(a**2/x**3, x) + Integral(3*a**2*c/
x**2, x) + Integral(3*a**2*c**2/x, x) + Integral(b**2*c**3*atanh(c*x)**2,
x) + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*c**3*atanh(c*x)
, x) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(3*b**2*c*atanh(c*x)**
2/x**2, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*a
tanh(c*x)/x**2, x) + Integral(6*a*b*c**2*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
a^2*c^3*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b*c^2*d^3 + 3*a
^2*c^2*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c*d^3 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*
x)/x^2)*a*b*d^3 - 3*a^2*c*d^3/x - 1/2*a^2*d^3/x^2 + 1/8*(2*b^2*c^3*d^3*x^3
 - 6*b^2*c*d^3*x - b^2*d^3)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^4
*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 1
2*(a*b*c^3*d^3*x^3 - a*b*c^2*d^3*x^2)*log(c*x + 1) - (2*b^2*c^4*d^3*x^4 +
12*a*b*c^3*d^3*x^3 - b^2*c*d^3*x - 6*(2*a*b*c^2*d^3 + b^2*c^2*d^3)*x^2 + 2
*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x +
 1))*log(-c*x + 1))/(c*x^4 - x^3), x)
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^3,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^3}\,dx$$
$$= \frac{d^3\left(atanh(cx)^2\,b^2c^2x^2 - 6atanh(cx)^2\,b^2cx - atanh(cx)^2\,b^2 + 4atanh(cx)\,ab\,c^3x^3 - 6atanh(cx)\,ab\,c^2x^2 - \right.}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x^3,x)
```

output
```
(d**3*(atanh(c*x)**2*b**2*c**2*x**2 - 6*atanh(c*x)**2*b**2*c*x - atanh(c*x
)**2*b**2 + 4*atanh(c*x)*a*b*c**3*x**3 - 6*atanh(c*x)*a*b*c**2*x**2 - 12*a
tanh(c*x)*a*b*c*x - 2*atanh(c*x)*a*b - 2*atanh(c*x)*b**2*c**2*x**2 - 2*ata
nh(c*x)*b**2*c*x + 2*int(atanh(c*x)**2,x)*b**2*c**3*x**2 - 12*int(atanh(c*
x)/(c**2*x**3 - x),x)*b**2*c**2*x**2 + 12*int(atanh(c*x)/x,x)*a*b*c**2*x**
2 + 6*int(atanh(c*x)**2/x,x)*b**2*c**2*x**2 - 8*log(c**2*x - c)*a*b*c**2*x
**2 - 2*log(c**2*x - c)*b**2*c**2*x**2 + 6*log(x)*a**2*c**2*x**2 + 12*log(
x)*a*b*c**2*x**2 + 2*log(x)*b**2*c**2*x**2 + 2*a**2*c**3*x**3 - 6*a**2*c*x
 - a**2 - 2*a*b*c*x))/(2*x**2)
```

3.91 $\int \frac{(d+cdx)^3(a+b\,\text{arctanh}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 396

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^4}\,dx = -\frac{b^2c^2d^3}{3x}+\frac{1}{3}b^2c^3d^3\operatorname{arctanh}(cx) - \frac{bcd^3(a+b\operatorname{arctanh}(cx))}{3x^2} - \frac{3bc^2d^3(a+b\operatorname{arctanh}(cx))}{x} + \frac{29}{6}c^3d^3(a+b\operatorname{arctanh}(cx))^2 - \frac{d^3(a+b\operatorname{arctanh}(cx))^2}{3x^3} - \frac{3cd^3(a+b\operatorname{arctanh}(cx))^2}{2x^2} - \frac{3c^2d^3(a+b\operatorname{arctanh}(cx))^2}{x} + 2c^3d^3(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right) + 3b^2c^3d^3\log(x) - \frac{3}{2}b^2c^3d^3\log\left(1-c^2x^2\right) + \frac{20}{3}bc^3d^3(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) - bc^3d^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) + bc^3d^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) - \frac{10}{3}b^2c^3d^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right) + \frac{1}{2}b^2c^3d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right) - \frac{1}{2}b^2c^3d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)$$

output
```
-1/3*b^2*c^2*d^3/x+1/3*b^2*c^3*d^3*arctanh(c*x)-1/3*b*c*d^3*(a+b*arctanh(c
*x))/x^2-3*b*c^2*d^3*(a+b*arctanh(c*x))/x+29/6*c^3*d^3*(a+b*arctanh(c*x))^
2-1/3*d^3*(a+b*arctanh(c*x))^2/x^3-3/2*c*d^3*(a+b*arctanh(c*x))^2/x^2-3*c^
2*d^3*(a+b*arctanh(c*x))^2/x-2*c^3*d^3*(a+b*arctanh(c*x))^2*arctanh(-1+2/(
-c*x+1))+3*b^2*c^3*d^3*ln(x)-3/2*b^2*c^3*d^3*ln(-c^2*x^2+1)+20/3*b*c^3*d^3
*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b*c^3*d^3*(a+b*arctanh(c*x))*polylog(2
,1-2/(-c*x+1))+b*c^3*d^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))-10/3*
b^2*c^3*d^3*polylog(2,-1+2/(c*x+1))+1/2*b^2*c^3*d^3*polylog(3,1-2/(-c*x+1)
)-1/2*b^2*c^3*d^3*polylog(3,-1+2/(-c*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.44

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$

$$= \frac{d^3\left(-8a^2-36a^2cx-8abcx-72a^2c^2x^2-72abc^2x^2-8b^2c^2x^2+ib^2c^3\pi^3x^3-16ab\text{arctanh}(cx)-72abcx\right.}{}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

output
```
(d^3*(-8*a^2 - 36*a^2*c*x - 8*a*b*c*x - 72*a^2*c^2*x^2 - 72*a*b*c^2*x^2 -
8*b^2*c^2*x^2 + I*b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTanh[c*x] - 72*a*b*c*x*ArcT
anh[c*x] - 8*b^2*c*x*ArcTanh[c*x] - 144*a*b*c^2*x^2*ArcTanh[c*x] - 72*b^2*
c^2*x^2*ArcTanh[c*x] + 8*b^2*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 -
 36*b^2*c*x*ArcTanh[c*x]^2 - 72*b^2*c^2*x^2*ArcTanh[c*x]^2 + 116*b^2*c^3*x
^3*ArcTanh[c*x]^2 - 16*b^2*c^3*x^3*ArcTanh[c*x]^3 + 160*b^2*c^3*x^3*ArcTan
h[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1
+ E^(-2*ArcTanh[c*x])] + 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTan
h[c*x])] + 24*a^2*c^3*x^3*Log[x] + 160*a*b*c^3*x^3*Log[c*x] - 36*a*b*c^3*x
^3*Log[1 - c*x] + 36*a*b*c^3*x^3*Log[1 + c*x] + 72*b^2*c^3*x^3*Log[(c*x)/S
qrt[1 - c^2*x^2]] - 80*a*b*c^3*x^3*Log[1 - c^2*x^2] + 24*b^2*c^3*x^3*ArcTa
nh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 80*b^2*c^3*x^3*PolyLog[2, E^(-2
*ArcTanh[c*x])] + 24*b^2*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x]
)] - 24*a*b*c^3*x^3*PolyLog[2, -(c*x)] + 24*a*b*c^3*x^3*PolyLog[2, c*x] +
12*b^2*c^3*x^3*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c^3*x^3*PolyLog[3
, E^(2*ArcTanh[c*x])]))/(24*x^3)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{c^3d^3(a+b\text{arctanh}(cx))^2}{x}+\frac{3c^2d^3(a+b\text{arctanh}(cx))^2}{x^2}+\frac{d^3(a+b\text{arctanh}(cx))^2}{x^4}+\frac{3cd^3(a+b\text{arctanh}(cx))^2}{x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-bc^3d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))+bc^3d^3\operatorname{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\operatorname{arctanh}(cx))+\frac{29}{6}c^3d^3(a+b\operatorname{arctanh}(cx))^2+2c^3d^3\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))^2+\frac{20}{3}bc^3d^3\log\left(2-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))-\frac{3c^2d^3(a+b\operatorname{arctanh}(cx))^2}{x}-\frac{3bc^2d^3(a+b\operatorname{arctanh}(cx))}{x}-\frac{d^3(a+b\operatorname{arctanh}(cx))^2}{3x^3}-\frac{3cd^3(a+b\operatorname{arctanh}(cx))^2}{2x^2}-\frac{bcd^3(a+b\operatorname{arctanh}(cx))}{3x^2}+\frac{1}{3}b^2c^3d^3\operatorname{arctanh}(cx)-\frac{10}{3}b^2c^3d^3\operatorname{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{1}{2}b^2c^3d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)-\frac{1}{2}b^2c^3d^3\operatorname{PolyLog}\left(3,\frac{2}{1-cx}-1\right)+3b^2c^3d^3\log(x)-\frac{b^2c^2d^3}{3x}-\frac{3}{2}b^2c^3d^3\log\left(1-c^2x^2\right)$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

output
```
-1/3*(b^2*c^2*d^3)/x + (b^2*c^3*d^3*ArcTanh[c*x])/3 - (b*c*d^3*(a + b*ArcT
anh[c*x]))/(3*x^2) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/x + (29*c^3*d^3*(a
 + b*ArcTanh[c*x])^2)/6 - (d^3*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (3*c*d^3*
(a + b*ArcTanh[c*x])^2)/(2*x^2) - (3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/x + 2
*c^3*d^3*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 3*b^2*c^3*d^3*L
og[x] - (3*b^2*c^3*d^3*Log[1 - c^2*x^2])/2 + (20*b*c^3*d^3*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)])/3 - b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 - c*x)] + b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c
*x)] - (10*b^2*c^3*d^3*PolyLog[2, -1 + 2/(1 + c*x)])/3 + (b^2*c^3*d^3*Poly
Log[3, 1 - 2/(1 - c*x)])/2 - (b^2*c^3*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.59 (sec) , antiderivative size = 1197, normalized size of antiderivative = 3.02

method	result	size
derivativedivides	Expression too large to display	1197
default	Expression too large to display	1197
parts	Expression too large to display	1255

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(1/2*I*d^3*b^2*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*
x^2-1)))^3*arctanh(c*x)^2+1/2*I*d^3*b^2*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)
))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1
-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2+d^3*b^2*arctanh(c*x)^2*ln(c*x)+d^3
*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*d^3*b^2*polylog(3
,-(c*x+1)^2/(-c^2*x^2+1))-8/3*b^2*d^3*arctanh(c*x)-11/6*d^3*b^2*arctanh(c*
x)^2+d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*arc
tanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*arctanh(c*x)*pol
ylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1
)^2/(-c^2*x^2+1))-1/3*d^3*b^2/(c*x-(-c^2*x^2+1)^(1/2)+1)*(-c^2*x^2+1)^(1/2
)+1/3*d^3*b^2/(c*x+(-c^2*x^2+1)^(1/2)+1)*(-c^2*x^2+1)^(1/2)+20/3*d^3*b^2*a
rctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-d^3*b^2*arctanh(c*x)^2*ln((c*
x+1)^2/(-c^2*x^2+1)-1)+2*d^3*b*a*(-1/3*arctanh(c*x)/c^3/x^3-3/2*arctanh(c*
x)/c^2/x^2+arctanh(c*x)*ln(c*x)-3*arctanh(c*x)/c/x-29/12*ln(c*x-1)-1/6/c^2
/x^2-3/2/c/x+10/3*ln(c*x)-11/12*ln(c*x+1)-1/2*dilog(c*x)-1/2*dilog(c*x+1)-
1/2*ln(c*x)*ln(c*x+1))-1/2*I*d^3*b^2*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))
*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*
x)^2-1/2*I*d^3*b^2*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2
/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-1/3*d^3*b^2*ar
ctanh(c*x)^2/c^3/x^3-3/2*d^3*b^2*arctanh(c*x)^2/c^2/x^2-3*d^3*b^2*arcta...
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx=d^3\left(\int \frac{a^2}{x^4}\,dx+\int \frac{3a^2c}{x^3}\,dx+\int \frac{3a^2c^2}{x^2}\,dx+\int \frac{a^2c^3}{x}\,dx+\int \frac{b^2\,\text{atanh}^2\,(cx)}{x^4}\,dx+\int \frac{2ab\,\text{atanh}\,(cx)}{x^4}\,dx+\int \frac{3b^2c\,\text{atanh}^2\,(cx)}{x^3}\,dx+\int \frac{3b^2c^2\,\text{atanh}^2\,(cx)}{x^2}\,dx+\int \frac{b^2c^3\,\text{atanh}^2\,(cx)}{x}\,dx+\int \frac{6abc\,\text{atanh}\,(cx)}{x^3}\,dx+\int \frac{6abc^2\,\text{atanh}\,(cx)}{x^2}\,dx+\int \frac{2abc^3\,\text{atanh}\,(cx)}{x}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**4,x)
```

output
```
d**3*(Integral(a**2/x**4, x) + Integral(3*a**2*c/x**3, x) + Integral(3*a**
2*c**2/x**2, x) + Integral(a**2*c**3/x, x) + Integral(b**2*atanh(c*x)**2/x
**4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**3, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**2, x) + Integral(b**
2*c**3*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x)/x**3, x) + Integr
al(6*a*b*c**2*atanh(c*x)/x**2, x) + Integral(2*a*b*c**3*atanh(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
a^2*c^3*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x
)*a*b*c^2*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh
(c*x)/x^2)*a*b*c*d^3 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*
c + 2*arctanh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 3/2*a^2*c*d^3/x^2 - 1/
3*a^2*d^3/x^3 - 1/24*(18*b^2*c^2*d^3*x^2 + 9*b^2*c*d^3*x + 2*b^2*d^3)*log(
-c*x + 1)^2/x^3 - integrate(-1/12*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3
- 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b*c^4*d^3*x^4 - a*b*c^3*
d^3*x^3)*log(c*x + 1) - (12*a*b*c^4*d^3*x^4 - 9*b^2*c^2*d^3*x^2 - 2*b^2*c*
d^3*x - 6*(2*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 + 6*(b^2*c^4*d^3*x^4 + 2*b^2
*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^
5 - x^4), x)
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^4}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^4,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^4, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^4}\,dx = \frac{d^3\Big(6\log(x)\,a^2c^3x^3-4atanh(cx)\,ab-2b^2c^2x^2-16atanh(cx)\,b^2c^3x^3-2atanh(cx)\,b^2cx-2abcx-9atan$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x^4,x)
```

output
```
(d**3*(9*atanh(c*x)**2*b**2*c**3*x**3 - 18*atanh(c*x)**2*b**2*c**2*x**2 -
9*atanh(c*x)**2*b**2*c*x - 2*atanh(c*x)**2*b**2 - 22*atanh(c*x)*a*b*c**3*x
**3 - 36*atanh(c*x)*a*b*c**2*x**2 - 18*atanh(c*x)*a*b*c*x - 4*atanh(c*x)*a
*b - 16*atanh(c*x)*b**2*c**3*x**3 - 18*atanh(c*x)*b**2*c**2*x**2 - 2*atanh
(c*x)*b**2*c*x - 40*int(atanh(c*x)/(c**2*x**3 - x),x)*b**2*c**3*x**3 + 12*
int(atanh(c*x)/x,x)*a*b*c**3*x**3 + 6*int(atanh(c*x)**2/x,x)*b**2*c**3*x**
3 - 40*log(c**2*x - c)*a*b*c**3*x**3 - 18*log(c**2*x - c)*b**2*c**3*x**3 +
 6*log(x)*a**2*c**3*x**3 + 40*log(x)*a*b*c**3*x**3 + 18*log(x)*b**2*c**3*x
**3 - 18*a**2*c**2*x**2 - 9*a**2*c*x - 2*a**2 - 18*a*b*c**2*x**2 - 2*a*b*c
*x - 2*b**2*c**2*x**2))/(6*x**3)
```

3.92 $\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}
&\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx \\
&= -\frac{b^2c^2d^3}{12x^2}-\frac{b^2c^3d^3}{x}+b^2c^4d^3\text{arctanh}(cx)-\frac{bcd^3(a+b\text{arctanh}(cx))}{6x^3} \\
&\quad -\frac{bc^2d^3(a+b\text{arctanh}(cx))}{x^2}-\frac{7bc^3d^3(a+b\text{arctanh}(cx))}{2x} \\
&\quad -\frac{d^3(1+cx)^4(a+b\text{arctanh}(cx))^2}{4x^4}+4abc^4d^3\log(x) \\
&\quad +\frac{11}{3}b^2c^4d^3\log(x)+4bc^4d^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) \\
&\quad -\frac{11}{6}b^2c^4d^3\log\left(1-c^2x^2\right)-2b^2c^4d^3\,\text{PolyLog}(2,-cx) \\
&\quad +2b^2c^4d^3\,\text{PolyLog}(2,cx)+2b^2c^4d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)
\end{aligned}$$

output
```
-1/12*b^2*c^2*d^3/x^2-b^2*c^3*d^3/x+b^2*c^4*d^3*arctanh(c*x)-1/6*b*c*d^3*(
a+b*arctanh(c*x))/x^3-b*c^2*d^3*(a+b*arctanh(c*x))/x^2-7/2*b*c^3*d^3*(a+b*
arctanh(c*x))/x-1/4*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/x^4+4*a*b*c^4*d^3*l
n(x)+11/3*b^2*c^4*d^3*ln(x)+4*b*c^4*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-
11/6*b^2*c^4*d^3*ln(-c^2*x^2+1)-2*b^2*c^4*d^3*polylog(2,-c*x)+2*b^2*c^4*d^
3*polylog(2,c*x)+2*b^2*c^4*d^3*polylog(2,1-2/(-c*x+1))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.27

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^5}\,dx = -\frac{d^3\left(3a^2+12a^2cx+2abcx+18a^2c^2x^2+12abc^2x^2+b^2c^2x^2+12a^2c^3x^3+42abc^3x^3+12b^2c^3x^3-b^2c^4x\right.}{}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

output
```
-1/12*(d^3*(3*a^2 + 12*a^2*c*x + 2*a*b*c*x + 18*a^2*c^2*x^2 + 12*a*b*c^2*x
^2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 42*a*b*c^3*x^3 + 12*b^2*c^3*x^3 - b^2*
c^4*x^4 + 3*b^2*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 - 15*c^4*x^4)*ArcTanh[c
*x]^2 + 2*b*ArcTanh[c*x]*(b*c*x*(1 + 6*c*x + 21*c^2*x^2 - 6*c^3*x^3) + 3*a
*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3) - 24*b*c^4*x^4*Log[1 - E^(-2*ArcTanh[
c*x])]) - 48*a*b*c^4*x^4*Log[c*x] + 21*a*b*c^4*x^4*Log[1 - c*x] - 21*a*b*c
^4*x^4*Log[1 + c*x] - 44*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 24*a*b
*c^4*x^4*Log[1 - c^2*x^2] + 24*b^2*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])]
))/x^4
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6500, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\operatorname{arctanh}(cx))^2}{x^5}\,dx$$

$$\Big\downarrow \; 6500$$

$$-2bc\int\left(-\frac{2d^3(a+b\text{arctanh}(cx))c^4}{1-cx}-\frac{2d^3(a+b\text{arctanh}(cx))c^3}{x}-\frac{7d^3(a+b\text{arctanh}(cx))c^2}{4x^2}-\frac{d^3(a+b\text{arctanh}(\right.$$
$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{4x^4}$$

$\downarrow$ 2009

$$-2bc\left(-2c^3d^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+\frac{7c^2d^3(a+b\text{arctanh}(cx))}{4x}+\frac{d^3(a+b\text{arctanh}(cx))}{12x^3}+\frac{cd^3(a+b\right.$$
$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{4x^4}$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

output
```
-1/4*(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)/x^4 - 2*b*c*((b*c*d^3)/(24*x
^2) + (b*c^2*d^3)/(2*x) - (b*c^3*d^3*ArcTanh[c*x])/2 + (d^3*(a + b*ArcTanh
[c*x]))/(12*x^3) + (c*d^3*(a + b*ArcTanh[c*x]))/(2*x^2) + (7*c^2*d^3*(a +
b*ArcTanh[c*x]))/(4*x) - 2*a*c^3*d^3*Log[x] - (11*b*c^3*d^3*Log[x])/6 - 2*
c^3*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] + (11*b*c^3*d^3*Log[1 - c^2*
x^2])/12 + b*c^3*d^3*PolyLog[2, -(c*x)] - b*c^3*d^3*PolyLog[2, c*x] - b*c^
3*d^3*PolyLog[2, 1 - 2/(1 - c*x)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6500
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Si
mp[(a + b*ArcTanh[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b
*ArcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && N
eQ[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.52

method	result
parts	$d^3a^2\left(-\frac{1}{4x^4}-\frac{c^3}{x}-\frac{3c^2}{2x^2}-\frac{c}{x^3}\right)+d^3b^2c^4\left(-\frac{\operatorname{arctanh}(cx)^2}{c^3x^3}-\frac{3\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)^2}{cx}-\right.$
derivativedivides	$c^4\left(d^3a^2\left(-\frac{1}{c^3x^3}-\frac{3}{2c^2x^2}-\frac{1}{cx}-\frac{1}{4c^4x^4}\right)+d^3b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{c^3x^3}-\frac{3\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)^2}{cx}\right.\right.$
default	$c^4\left(d^3a^2\left(-\frac{1}{c^3x^3}-\frac{3}{2c^2x^2}-\frac{1}{cx}-\frac{1}{4c^4x^4}\right)+d^3b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{c^3x^3}-\frac{3\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)^2}{cx}\right.\right.$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/4/x^4-c^3/x-3/2*c^2/x^2-c/x^3)+d^3*b^2*c^4*(-arctanh(c*x)^2/c^
3/x^3-3/2*arctanh(c*x)^2/c^2/x^2-arctanh(c*x)^2/c/x-1/4*arctanh(c*x)^2/c^4
/x^4-15/4*arctanh(c*x)*ln(c*x-1)-1/6*arctanh(c*x)/c^3/x^3-arctanh(c*x)/c^2
/x^2-7/2*arctanh(c*x)/c/x+4*arctanh(c*x)*ln(c*x)-1/4*arctanh(c*x)*ln(c*x+1
)-2*dilog(c*x)-2*dilog(c*x+1)-2*ln(c*x)*ln(c*x+1)-15/16*ln(c*x-1)^2+2*dilo
g(1/2*c*x+1/2)+15/8*ln(c*x-1)*ln(1/2*c*x+1/2)+1/16*ln(c*x+1)^2-1/8*(ln(c*x
+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-7/3*ln(c*x-1)-1/12/c^2/x^2-1/c/x+11/
3*ln(c*x)-4/3*ln(c*x+1))+2*d^3*b*a*c^4*(-arctanh(c*x)/c^3/x^3-3/2*arctanh(
c*x)/c^2/x^2-arctanh(c*x)/c/x-1/4*arctanh(c*x)/c^4/x^4-15/8*ln(c*x-1)-1/12
/c^3/x^3-1/2/c^2/x^2-7/4/c/x+2*ln(c*x)-1/8*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^5}\,dx=\int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x^5, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx = d^3\left(\int \frac{a^2}{x^5}\,dx + \int \frac{3a^2c}{x^4}\,dx + \int \frac{3a^2c^2}{x^3}\,dx + \int \frac{a^2c^3}{x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^5}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^5}\,dx + \int \frac{3b^2c\operatorname{atanh}^2(cx)}{x^4}\,dx + \int \frac{3b^2c^2\operatorname{atanh}^2(cx)}{x^3}\,dx + \int \frac{b^2c^3\operatorname{atanh}^2(cx)}{x^2}\,dx + \int \frac{6abc\operatorname{atanh}(cx)}{x^4}\,dx + \int \frac{6abc^2\operatorname{atanh}(cx)}{x^3}\,dx + \int \frac{2abc^3\operatorname{atanh}(cx)}{x^2}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**5,x)
```

output
```
d**3*(Integral(a**2/x**5, x) + Integral(3*a**2*c/x**4, x) + Integral(3*a**
2*c**2/x**3, x) + Integral(a**2*c**3/x**2, x) + Integral(b**2*atanh(c*x)**
2/x**5, x) + Integral(2*a*b*atanh(c*x)/x**5, x) + Integral(3*b**2*c*atanh(
c*x)**2/x**4, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**3, x) + Integral(
b**2*c**3*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c*atanh(c*x)/x**4, x) +
Integral(6*a*b*c**2*atanh(c*x)/x**3, x) + Integral(2*a*b*c**3*atanh(c*x)/x
**2, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(254) = 508$.

Time = 0.48 (sec) , antiderivative size = 813, normalized size of antiderivative = 3.00

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
-2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^4*d^3 -
 2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^4*d^3 + 2*(log(c*x + 1
)*log(-c*x) + dilog(c*x + 1))*b^2*c^4*d^3 - b^2*c^4*d^3*log(c*x + 1) - 2*b
^2*c^4*d^3*log(c*x - 1) + 3*b^2*c^4*d^3*log(x) - (c*(log(c^2*x^2 - 1) - lo
g(x^2)) + 2*arctanh(c*x)/x)*a*b*c^3*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x
 - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^2*d^3 - ((c^2*log(c^2*x^2 - 1)
- c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c*d^3 - a^2*c^3*d^3/x
+ 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*
c - 6*arctanh(c*x)/x^4)*a*b*d^3 + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*
x + 1)^2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x
^2*log(c*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x
 + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2*d^
3 - 3/2*a^2*c^2*d^3/x^2 - a^2*c*d^3/x^3 - 1/4*b^2*d^3*arctanh(c*x)^2/x^4 -
 1/4*a^2*d^3/x^4 - 1/8*(8*b^2*c^3*d^3*x^2 + (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d
^3*x^2 + 3*b^2*c^2*d^3*x + 2*b^2*c*d^3)*log(c*x + 1)^2 - (7*b^2*c^4*d^3*x^
3 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c^2*d^3*x - 2*b^2*c*d^3)*log(-c*x + 1)^2 + 4
*(3*b^2*c^3*d^3*x^2 + b^2*c^2*d^3*x)*log(c*x + 1) - 2*(6*b^2*c^3*d^3*x^2 +
 2*b^2*c^2*d^3*x + (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x
+ 2*b^2*c*d^3)*log(c*x + 1))*log(-c*x + 1))/x^3
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx = \int \frac{\left(a+b\,\text{atanh}(c\,x)\right)^2\,\left(d+c\,d\,x\right)^3}{x^5}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^5,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^5, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^5}\,dx$$
$$= \frac{d^3\left(-48\left(\int \frac{atanh(cx)}{c^2x^3-x}dx\right)b^2c^4x^4 - 6atanh(cx)\,ab - b^2c^2x^2 - 42atanh(cx)\,b^2c^3x^3 - 2atanh(cx)\,b^2cx - 42ab\right.}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x^5,x)
```

output
```
(d**3*(21*atanh(c*x)**2*b**2*c**4*x**4 - 12*atanh(c*x)**2*b**2*c**3*x**3 -
 18*atanh(c*x)**2*b**2*c**2*x**2 - 12*atanh(c*x)**2*b**2*c*x - 3*atanh(c*x
)**2*b**2 - 6*atanh(c*x)*a*b*c**4*x**4 - 24*atanh(c*x)*a*b*c**3*x**3 - 36*
atanh(c*x)*a*b*c**2*x**2 - 24*atanh(c*x)*a*b*c*x - 6*atanh(c*x)*a*b - 32*a
tanh(c*x)*b**2*c**4*x**4 - 42*atanh(c*x)*b**2*c**3*x**3 - 12*atanh(c*x)*b*
*2*c**2*x**2 - 2*atanh(c*x)*b**2*c*x - 48*int(atanh(c*x)/(c**2*x**3 - x),x
)*b**2*c**4*x**4 - 48*log(c**2*x - c)*a*b*c**4*x**4 - 44*log(c**2*x - c)*b
**2*c**4*x**4 + 48*log(x)*a*b*c**4*x**4 + 44*log(x)*b**2*c**4*x**4 - 12*a*
*2*c**3*x**3 - 18*a**2*c**2*x**2 - 12*a**2*c*x - 3*a**2 - 42*a*b*c**3*x**3
 - 12*a*b*c**2*x**2 - 2*a*b*c*x - 12*b**2*c**3*x**3 - b**2*c**2*x**2))/(12
*x**4)
```

3.93 $\int \frac{(d+cdx)^3(a+b\,\mathbf{arctanh}(cx))^2}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 352

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx = -\frac{b^2c^2d^3}{30x^3}-\frac{b^2c^3d^3}{4x^2}-\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\text{arctanh}(cx)-\frac{bcd^3(a+b\text{arctanh}(cx))}{10x^4}-\frac{bc^2d^3(a+b\text{arctanh}(cx))}{2x^3}-\frac{6bc^3d^3(a+b\text{arctanh}(cx))}{5x^2}-\frac{5bc^4d^3(a+b\text{arctanh}(cx))}{2x}-\frac{d^3(1+cx)^4(a+b\text{arctanh}(cx))^2}{5x^5}+\frac{cd^3(1+cx)^4(a+b\text{arctanh}(cx))^2}{20x^4}+\frac{12}{5}abc^5d^3\log(x)+3b^2c^5d^3\log(x)+\frac{12}{5}bc^5d^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)-\frac{3}{2}b^2c^5d^3\log\left(1-c^2x^2\right)-\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,-cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)$$

output
```
-1/30*b^2*c^2*d^3/x^3-1/4*b^2*c^3*d^3/x^2-13/10*b^2*c^4*d^3/x+13/10*b^2*c^
5*d^3*arctanh(c*x)-1/10*b*c*d^3*(a+b*arctanh(c*x))/x^4-1/2*b*c^2*d^3*(a+b*
arctanh(c*x))/x^3-6/5*b*c^3*d^3*(a+b*arctanh(c*x))/x^2-5/2*b*c^4*d^3*(a+b*
arctanh(c*x))/x-1/5*d^3*(c*x+1)^4*(a+b*arctanh(c*x))^2/x^5+1/20*c*d^3*(c*x
+1)^4*(a+b*arctanh(c*x))^2/x^4+12/5*a*b*c^5*d^3*ln(x)+3*b^2*c^5*d^3*ln(x)+
12/5*b*c^5*d^3*(a+b*arctanh(c*x))*ln(2/(-c*x+1))-3/2*b^2*c^5*d^3*ln(-c^2*x
^2+1)-6/5*b^2*c^5*d^3*polylog(2,-c*x)+6/5*b^2*c^5*d^3*polylog(2,c*x)+6/5*b
^2*c^5*d^3*polylog(2,1-2/(-c*x+1))
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.06

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx =$$

$$-\frac{d^3\left(12a^2+45a^2cx+6abcx+60a^2c^2x^2+30abc^2x^2+2b^2c^2x^2+30a^2c^3x^3+72abc^3x^3+15b^2c^3x^3+150\right.}{}$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

output
```
-1/60*(d^3*(12*a^2 + 45*a^2*c*x + 6*a*b*c*x + 60*a^2*c^2*x^2 + 30*a*b*c^2*
x^2 + 2*b^2*c^2*x^2 + 30*a^2*c^3*x^3 + 72*a*b*c^3*x^3 + 15*b^2*c^3*x^3 + 1
50*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - 15*b^2*c^5*x^5 + 3*b^2*(4 + 15*c*x + 20*
c^2*x^2 + 10*c^3*x^3 - 49*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*(a*(4
 + 15*c*x + 20*c^2*x^2 + 10*c^3*x^3) + b*c*x*(1 + 5*c*x + 12*c^2*x^2 + 25*
c^3*x^3 - 13*c^4*x^4) - 24*b*c^5*x^5*Log[1 - E^(-2*ArcTanh[c*x])]) - 144*a
*b*c^5*x^5*Log[c*x] + 75*a*b*c^5*x^5*Log[1 - c*x] - 75*a*b*c^5*x^5*Log[1 +
 c*x] - 180*b^2*c^5*x^5*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 72*a*b*c^5*x^5*Log[
1 - c^2*x^2] + 72*b^2*c^5*x^5*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^5
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6500, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx$$

$$\Big\downarrow \; 6500$$

$$-2bc\int \left(-\frac{6d^3(a+b\text{arctanh}(cx))c^5}{5(1-cx)}-\frac{6d^3(a+b\text{arctanh}(cx))c^4}{5x}-\frac{5d^3(a+b\text{arctanh}(cx))c^3}{4x^2}-\frac{6d^3(a+b\text{arctanh}\right.$$
$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{5x^5}+\frac{cd^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{20x^4}$$

$\Big\downarrow$ 2009

$$-2bc\left(-\frac{6}{5}c^4d^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))+\frac{5c^3d^3(a+b\text{arctanh}(cx))}{4x}+\frac{3c^2d^3(a+b\text{arctanh}(cx))}{5x^2}+\frac{d^3(a\right.$$
$$\frac{d^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{5x^5}+\frac{cd^3(cx+1)^4(a+b\text{arctanh}(cx))^2}{20x^4}$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

output
```
-1/5*(d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)/x^5 + (c*d^3*(1 + c*x)^4*(a
+ b*ArcTanh[c*x])^2)/(20*x^4) - 2*b*c*((b*c*d^3)/(60*x^3) + (b*c^2*d^3)/(8
*x^2) + (13*b*c^3*d^3)/(20*x) - (13*b*c^4*d^3*ArcTanh[c*x])/20 + (d^3*(a +
 b*ArcTanh[c*x]))/(20*x^4) + (c*d^3*(a + b*ArcTanh[c*x]))/(4*x^3) + (3*c^2
*d^3*(a + b*ArcTanh[c*x]))/(5*x^2) + (5*c^3*d^3*(a + b*ArcTanh[c*x]))/(4*x
) - (6*a*c^4*d^3*Log[x])/5 - (3*b*c^4*d^3*Log[x])/2 - (6*c^4*d^3*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)])/5 + (3*b*c^4*d^3*Log[1 - c^2*x^2])/4 + (3*b*
c^4*d^3*PolyLog[2, -(c*x)])/5 - (3*b*c^4*d^3*PolyLog[2, c*x])/5 - (3*b*c^4
*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6500
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Si
mp[(a + b*ArcTanh[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b
*ArcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && N
eQ[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.25

method	result
parts	$d^3a^2\left(-\frac{3c}{4x^4}-\frac{1}{5x^5}-\frac{c^3}{2x^2}-\frac{c^2}{x^3}\right)+d^3b^2c^5\left(-\frac{5\,\mathrm{arctanh}(cx)}{2cx}-\frac{6\,\mathrm{arctanh}(cx)}{5c^2x^2}-\frac{\mathrm{arctanh}(cx)}{2c^3x^3}-\frac{43\ln}{}\right.$
derivativedivides	$c^5\left(d^3a^2\left(-\frac{1}{c^3x^3}-\frac{1}{2c^2x^2}-\frac{3}{4c^4x^4}-\frac{1}{5c^5x^5}\right)+d^3b^2\left(-\frac{5\,\mathrm{arctanh}(cx)}{2cx}-\frac{6\,\mathrm{arctanh}(cx)}{5c^2x^2}-\frac{\mathrm{arctanh}(cx)}{2c^3x^3}\right.\right.$
default	$c^5\left(d^3a^2\left(-\frac{1}{c^3x^3}-\frac{1}{2c^2x^2}-\frac{3}{4c^4x^4}-\frac{1}{5c^5x^5}\right)+d^3b^2\left(-\frac{5\,\mathrm{arctanh}(cx)}{2cx}-\frac{6\,\mathrm{arctanh}(cx)}{5c^2x^2}-\frac{\mathrm{arctanh}(cx)}{2c^3x^3}\right.\right.$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-3/4*c/x^4-1/5/x^5-1/2*c^3/x^2-c^2/x^3)+d^3*b^2*c^5*(-5/2*arctanh
(c*x)/c/x-6/5*arctanh(c*x)/c^2/x^2-1/2*arctanh(c*x)/c^3/x^3-43/20*ln(c*x-1
)-17/20*ln(c*x+1)-13/10/c/x+12/5*arctanh(c*x)*ln(c*x)-6/5*ln(c*x)*ln(c*x+1
)-1/30/c^3/x^3-1/4/c^2/x^2-3/4*arctanh(c*x)^2/c^4/x^4-arctanh(c*x)^2/c^3/x
^3-1/5*arctanh(c*x)^2/c^5/x^5+1/20*arctanh(c*x)*ln(c*x+1)-49/20*arctanh(c*
x)*ln(c*x-1)-1/2*arctanh(c*x)^2/c^2/x^2-1/10*arctanh(c*x)/c^4/x^4+6/5*dilo
g(1/2*c*x+1/2)-49/80*ln(c*x-1)^2-1/80*ln(c*x+1)^2+3*ln(c*x)+1/40*(ln(c*x+1
)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+49/40*ln(c*x-1)*ln(1/2*c*x+1/2)-6/5*di
log(c*x)-6/5*dilog(c*x+1))+2*d^3*b*a*c^5*(-arctanh(c*x)/c^3/x^3-1/2*arctan
h(c*x)/c^2/x^2-3/4*arctanh(c*x)/c^4/x^4-1/5*arctanh(c*x)/c^5/x^5-49/40*ln(
c*x-1)-1/20/c^4/x^4-1/4/c^3/x^3-3/5/c^2/x^2-5/4/c/x+6/5*ln(c*x)+1/40*ln(c*
x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\,\mathrm{arctanh}(cx))^2}{x^6}\,dx=\int \frac{(cdx+d)^3(b\,\mathrm{artanh}\,(cx)+a)^2}{x^6}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x^6, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^6}\,dx = d^3\left(\int \frac{a^2}{x^6}\,dx + \int \frac{3a^2c}{x^5}\,dx + \int \frac{3a^2c^2}{x^4}\,dx + \int \frac{a^2c^3}{x^3}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^6}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^6}\,dx + \int \frac{3b^2c\operatorname{atanh}^2(cx)}{x^5}\,dx + \int \frac{3b^2c^2\operatorname{atanh}^2(cx)}{x^4}\,dx + \int \frac{b^2c^3\operatorname{atanh}^2(cx)}{x^3}\,dx + \int \frac{6abc\operatorname{atanh}(cx)}{x^5}\,dx + \int \frac{6abc^2\operatorname{atanh}(cx)}{x^4}\,dx + \int \frac{2abc^3\operatorname{atanh}(cx)}{x^3}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**6,x)
```

output
```
d**3*(Integral(a**2/x**6, x) + Integral(3*a**2*c/x**5, x) + Integral(3*a**
2*c**2/x**4, x) + Integral(a**2*c**3/x**3, x) + Integral(b**2*atanh(c*x)**
2/x**6, x) + Integral(2*a*b*atanh(c*x)/x**6, x) + Integral(3*b**2*c*atanh(
c*x)**2/x**5, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**4, x) + Integral(
b**2*c**3*atanh(c*x)**2/x**3, x) + Integral(6*a*b*c*atanh(c*x)/x**5, x) +
Integral(6*a*b*c**2*atanh(c*x)/x**4, x) + Integral(2*a*b*c**3*atanh(c*x)/x
**3, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(315) = 630$.

Time = 0.50 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.22

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="maxima")
```

output
```
-6/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^5*d^3
 - 6/5*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^5*d^3 + 6/5*(log(c
*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^5*d^3 - 17/20*b^2*c^5*d^3*log(c*
x + 1) - 43/20*b^2*c^5*d^3*log(c*x - 1) + 3*b^2*c^5*d^3*log(x) + 1/2*((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^3*d^3 -
((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b
*c^2*d^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
1)/x^3)*c - 6*arctanh(c*x)/x^4)*a*b*c*d^3 - 1/10*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*d^3 -
1/2*a^2*c^3*d^3/x^2 - a^2*c^2*d^3/x^3 - 3/4*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^
5 - 1/240*(312*b^2*c^4*d^3*x^4 + 60*b^2*c^3*d^3*x^3 + 8*b^2*c^2*d^3*x^2 -
3*(b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b^2*c*d^
3*x - 4*b^2*d^3)*log(c*x + 1)^2 - 3*(49*b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x
^3 - 20*b^2*c^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*d^3)*log(-c*x + 1)^2 + 12
*(25*b^2*c^4*d^3*x^4 + 12*b^2*c^3*d^3*x^3 + 5*b^2*c^2*d^3*x^2 + b^2*c*d^3*
x)*log(c*x + 1) - 6*(50*b^2*c^4*d^3*x^4 + 24*b^2*c^3*d^3*x^3 + 10*b^2*c^2*
d^3*x^2 + 2*b^2*c*d^3*x - (b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c
^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/x^5
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx = \int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^6}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^6}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^6,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^6, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^6}\,dx$$
$$=\frac{d^3\Big(-24atanh(cx)\,ab-2b^2c^2x^2-102atanh(cx)\,b^2c^5x^5-72atanh(cx)\,b^2c^3x^3-6atanh(cx)\,b^2cx-72ab\,c$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x^6,x)
```

output
```
(d**3*(75*atanh(c*x)**2*b**2*c**5*x**5 - 30*atanh(c*x)**2*b**2*c**3*x**3 -
 60*atanh(c*x)**2*b**2*c**2*x**2 - 45*atanh(c*x)**2*b**2*c*x - 12*atanh(c*
x)**2*b**2 + 6*atanh(c*x)*a*b*c**5*x**5 - 60*atanh(c*x)*a*b*c**3*x**3 - 12
0*atanh(c*x)*a*b*c**2*x**2 - 90*atanh(c*x)*a*b*c*x - 24*atanh(c*x)*a*b - 1
02*atanh(c*x)*b**2*c**5*x**5 - 150*atanh(c*x)*b**2*c**4*x**4 - 72*atanh(c*
x)*b**2*c**3*x**3 - 30*atanh(c*x)*b**2*c**2*x**2 - 6*atanh(c*x)*b**2*c*x -
 144*int(atanh(c*x)/(c**2*x**3 - x),x)*b**2*c**5*x**5 - 144*log(c**2*x - c
)*a*b*c**5*x**5 - 180*log(c**2*x - c)*b**2*c**5*x**5 + 144*log(x)*a*b*c**5
*x**5 + 180*log(x)*b**2*c**5*x**5 - 30*a**2*c**3*x**3 - 60*a**2*c**2*x**2
- 45*a**2*c*x - 12*a**2 - 150*a*b*c**4*x**4 - 72*a*b*c**3*x**3 - 30*a*b*c*
*2*x**2 - 6*a*b*c*x - 78*b**2*c**4*x**4 - 15*b**2*c**3*x**3 - 2*b**2*c**2*
x**2))/(60*x**5)
```

3.94 $\int \frac{(d+cdx)^3(a+b\,\mathbf{arctanh}(cx))^2}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 479

$$\int \frac{(d+cdx)^3(a+b\operatorname{arctanh}(cx))^2}{x^7}\,dx = -\frac{b^2c^2d^3}{60x^4} - \frac{b^2c^3d^3}{10x^3} - \frac{61b^2c^4d^3}{180x^2} - \frac{37b^2c^5d^3}{30x} + \frac{37}{30}b^2c^6d^3\operatorname{arctanh}(cx) - \frac{bcd^3(a+b\operatorname{arctanh}(cx))}{15x^5} - \frac{3bc^2d^3(a+b\operatorname{arctanh}(cx))}{10x^4} - \frac{11bc^3d^3(a+b\operatorname{arctanh}(cx))}{18x^3} - \frac{14bc^4d^3(a+b\operatorname{arctanh}(cx))}{15x^2} - \frac{11bc^5d^3(a+b\operatorname{arctanh}(cx))}{6x} - \frac{d^3(a+b\operatorname{arctanh}(cx))^2}{6x^6} - \frac{3cd^3(a+b\operatorname{arctanh}(cx))^2}{5x^5} - \frac{3c^2d^3(a+b\operatorname{arctanh}(cx))^2}{4x^4} - \frac{c^3d^3(a+b\operatorname{arctanh}(cx))^2}{3x^3} + \frac{28}{15}abc^6d^3\log(x) + \frac{113}{45}b^2c^6d^3\log(x) + \frac{37}{20}bc^6d^3(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right) + \frac{1}{60}bc^6d^3(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1+cx}\right) - \frac{113}{90}b^2c^6d^3\log\left(1-c^2x^2\right) - \frac{14}{15}b^2c^6d^3\operatorname{PolyLog}(2,-cx) + \frac{14}{15}b^2c^6d^3\operatorname{PolyLog}(2,cx) + \frac{37}{40}b^2c^6d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) - \frac{1}{120}b^2c^6d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)$$

output
```
-1/60*b^2*c^2*d^3/x^4-1/10*b^2*c^3*d^3/x^3-61/180*b^2*c^4*d^3/x^2-37/30*b^
2*c^5*d^3/x+37/30*b^2*c^6*d^3*arctanh(c*x)-1/15*b*c*d^3*(a+b*arctanh(c*x))
/x^5-3/10*b*c^2*d^3*(a+b*arctanh(c*x))/x^4-11/18*b*c^3*d^3*(a+b*arctanh(c*
x))/x^3-14/15*b*c^4*d^3*(a+b*arctanh(c*x))/x^2-11/6*b*c^5*d^3*(a+b*arctanh
(c*x))/x-1/6*d^3*(a+b*arctanh(c*x))^2/x^6-3/5*c*d^3*(a+b*arctanh(c*x))^2/x
^5-3/4*c^2*d^3*(a+b*arctanh(c*x))^2/x^4-1/3*c^3*d^3*(a+b*arctanh(c*x))^2/x
^3+28/15*a*b*c^6*d^3*ln(x)+113/45*b^2*c^6*d^3*ln(x)+37/20*b*c^6*d^3*(a+b*a
rctanh(c*x))*ln(2/(-c*x+1))+1/60*b*c^6*d^3*(a+b*arctanh(c*x))*ln(2/(c*x+1)
)-113/90*b^2*c^6*d^3*ln(-c^2*x^2+1)-14/15*b^2*c^6*d^3*polylog(2,-c*x)+14/1
5*b^2*c^6*d^3*polylog(2,c*x)+37/40*b^2*c^6*d^3*polylog(2,1-2/(-c*x+1))-1/1
20*b^2*c^6*d^3*polylog(2,1-2/(c*x+1))
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.84

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx =$$

$$-\frac{d^3\Big(30a^2+108a^2cx+12abcx+135a^2c^2x^2+54abc^2x^2+3b^2c^2x^2+60a^2c^3x^3+110abc^3x^3+18b^2c^3x^3+$$

input
```
Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

output
```
-1/180*(d^3*(30*a^2 + 108*a^2*c*x + 12*a*b*c*x + 135*a^2*c^2*x^2 + 54*a*b*
c^2*x^2 + 3*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^
3 + 168*a*b*c^4*x^4 + 61*b^2*c^4*x^4 + 330*a*b*c^5*x^5 + 222*b^2*c^5*x^5 -
 64*b^2*c^6*x^6 + 3*b^2*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x^3 - 111*c^6*x
^6)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*(10 + 36*c*x + 45*c^2*x^2 + 20*
c^3*x^3) + b*c*x*(6 + 27*c*x + 55*c^2*x^2 + 84*c^3*x^3 + 165*c^4*x^4 - 111
*c^5*x^5) - 168*b*c^6*x^6*Log[1 - E^(-2*ArcTanh[c*x])]) - 336*a*b*c^6*x^6*
Log[c*x] + 165*a*b*c^6*x^6*Log[1 - c*x] - 165*a*b*c^6*x^6*Log[1 + c*x] - 4
52*b^2*c^6*x^6*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 168*a*b*c^6*x^6*Log[1 - c^2*
x^2] + 168*b^2*c^6*x^6*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^6
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6500, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^3(a+b\text{arctanh}(cx))^2}{x^7}dx$$

$$\Big\downarrow \text{6500}$$

$$-2bc\int \left(-\frac{37d^3(a+b\text{arctanh}(cx))c^6}{40(1-cx)}+\frac{d^3(a+b\text{arctanh}(cx))c^6}{120(cx+1)}-\frac{14d^3(a+b\text{arctanh}(cx))c^5}{15x}-\frac{11d^3(a+b\text{arcta}}{12x^2}\right.$$
$$\frac{c^3d^3(a+b\text{arctanh}(cx))^2}{3x^3}-\frac{3c^2d^3(a+b\text{arctanh}(cx))^2}{4x^4}-\frac{d^3(a+b\text{arctanh}(cx))^2}{6x^6}-$$
$$\frac{3cd^3(a+b\text{arctanh}(cx))^2}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3d^3(a+b\text{arctanh}(cx))^2}{3x^3}-\frac{3c^2d^3(a+b\text{arctanh}(cx))^2}{4x^4}-$$
$$2bc\left(-\frac{37}{40}c^5d^3\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))-\frac{1}{120}c^5d^3\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))+\frac{11c^4d^3(a+b\text{arct}}{12x}\right.$$
$$\frac{d^3(a+b\text{arctanh}(cx))^2}{6x^6}-\frac{3cd^3(a+b\text{arctanh}(cx))^2}{5x^5}$$

input
```
Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

output
```
-1/6*(d^3*(a + b*ArcTanh[c*x])^2)/x^6 - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/(
5*x^5) - (3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/(4*x^4) - (c^3*d^3*(a + b*ArcT
anh[c*x])^2)/(3*x^3) - 2*b*c*((b*c*d^3)/(120*x^4) + (b*c^2*d^3)/(20*x^3) +
 (61*b*c^3*d^3)/(360*x^2) + (37*b*c^4*d^3)/(60*x) - (37*b*c^5*d^3*ArcTanh[
c*x])/60 + (d^3*(a + b*ArcTanh[c*x]))/(30*x^5) + (3*c*d^3*(a + b*ArcTanh[c
*x]))/(20*x^4) + (11*c^2*d^3*(a + b*ArcTanh[c*x]))/(36*x^3) + (7*c^3*d^3*(
a + b*ArcTanh[c*x]))/(15*x^2) + (11*c^4*d^3*(a + b*ArcTanh[c*x]))/(12*x) -
 (14*a*c^5*d^3*Log[x])/15 - (113*b*c^5*d^3*Log[x])/90 - (37*c^5*d^3*(a + b
*ArcTanh[c*x])*Log[2/(1 - c*x)])/40 - (c^5*d^3*(a + b*ArcTanh[c*x])*Log[2/
(1 + c*x)])/120 + (113*b*c^5*d^3*Log[1 - c^2*x^2])/180 + (7*b*c^5*d^3*Poly
Log[2, -(c*x)])/15 - (7*b*c^5*d^3*PolyLog[2, c*x])/15 - (37*b*c^5*d^3*Poly
Log[2, 1 - 2/(1 - c*x)])/80 + (b*c^5*d^3*PolyLog[2, 1 - 2/(1 + c*x)])/240)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6500
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Si
mp[(a + b*ArcTanh[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b
*ArcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && N
eQ[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.98

method	result
parts	$d^3a^2\left(-\frac{1}{6x^6}-\frac{3c^2}{4x^4}-\frac{3c}{5x^5}-\frac{c^3}{3x^3}\right)+d^3b^2c^6\left(-\frac{11\,\mathrm{arctanh}(cx)}{6cx}-\frac{14\,\mathrm{arctanh}(cx)}{15c^2x^2}-\frac{11\,\mathrm{arctanh}(cx)}{18c^3x^3}-\right.$
derivativedivides	$c^6\left(d^3a^2\left(-\frac{1}{3c^3x^3}-\frac{3}{4c^4x^4}-\frac{3}{5c^5x^5}-\frac{1}{6c^6x^6}\right)+d^3b^2\left(-\frac{11\,\mathrm{arctanh}(cx)}{6cx}-\frac{14\,\mathrm{arctanh}(cx)}{15c^2x^2}-\frac{11\,\mathrm{arcta}}{18c^3}\right.\right.$
default	$c^6\left(d^3a^2\left(-\frac{1}{3c^3x^3}-\frac{3}{4c^4x^4}-\frac{3}{5c^5x^5}-\frac{1}{6c^6x^6}\right)+d^3b^2\left(-\frac{11\,\mathrm{arctanh}(cx)}{6cx}-\frac{14\,\mathrm{arctanh}(cx)}{15c^2x^2}-\frac{11\,\mathrm{arcta}}{18c^3}\right.\right.$

input
```
int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/6/x^6-3/4*c^2/x^4-3/5*c/x^5-1/3*c^3/x^3)+d^3*b^2*c^6*(-11/6*ar
ctanh(c*x)/c/x-14/15*arctanh(c*x)/c^2/x^2-11/18*arctanh(c*x)/c^3/x^3-337/1
80*ln(c*x-1)-23/36*ln(c*x+1)-1/60/c^4/x^4-37/30/c/x+28/15*arctanh(c*x)*ln(
c*x)-14/15*ln(c*x)*ln(c*x+1)-1/10/c^3/x^3-61/180/c^2/x^2-3/4*arctanh(c*x)^
2/c^4/x^4-1/6*arctanh(c*x)^2/c^6/x^6-1/3*arctanh(c*x)^2/c^3/x^3-3/5*arctan
h(c*x)^2/c^5/x^5-1/60*arctanh(c*x)*ln(c*x+1)-37/20*arctanh(c*x)*ln(c*x-1)-
1/15*arctanh(c*x)/c^5/x^5-3/10*arctanh(c*x)/c^4/x^4+14/15*dilog(1/2*c*x+1/
2)-37/80*ln(c*x-1)^2+1/240*ln(c*x+1)^2+113/45*ln(c*x)-1/120*(ln(c*x+1)-ln(
1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+37/40*ln(c*x-1)*ln(1/2*c*x+1/2)-14/15*dilog
(c*x)-14/15*dilog(c*x+1))+2*d^3*b*a*c^6*(-1/3*arctanh(c*x)/c^3/x^3-3/4*arc
tanh(c*x)/c^4/x^4-3/5*arctanh(c*x)/c^5/x^5-1/6*arctanh(c*x)/c^6/x^6-37/40*
ln(c*x-1)-1/30/c^5/x^5-3/20/c^4/x^4-11/36/c^3/x^3-7/15/c^2/x^2-11/12/c/x+1
4/15*ln(c*x)-1/120*ln(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx=\int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)^2}{x^7}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="fricas")
```

output
```
integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 +
(b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*
x)^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*a
rctanh(c*x))/x^7, x)
```

Sympy [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx = d^3\left(\int \frac{a^2}{x^7}\,dx + \int \frac{3a^2c}{x^6}\,dx + \int \frac{3a^2c^2}{x^5}\,dx + \int \frac{a^2c^3}{x^4}\,dx + \int \frac{b^2\,\text{atanh}^2(cx)}{x^7}\,dx + \int \frac{2ab\,\text{atanh}(cx)}{x^7}\,dx + \int \frac{3b^2c\,\text{atanh}^2(cx)}{x^6}\,dx + \int \frac{3b^2c^2\,\text{atanh}^2(cx)}{x^5}\,dx + \int \frac{b^2c^3\,\text{atanh}^2(cx)}{x^4}\,dx + \int \frac{6abc\,\text{atanh}(cx)}{x^6}\,dx + \int \frac{6abc^2\,\text{atanh}(cx)}{x^5}\,dx + \int \frac{2abc^3\,\text{atanh}(cx)}{x^4}\,dx\right)$$

input
```
integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**7,x)
```

output
```
d**3*(Integral(a**2/x**7, x) + Integral(3*a**2*c/x**6, x) + Integral(3*a**
2*c**2/x**5, x) + Integral(a**2*c**3/x**4, x) + Integral(b**2*atanh(c*x)**
2/x**7, x) + Integral(2*a*b*atanh(c*x)/x**7, x) + Integral(3*b**2*c*atanh(
c*x)**2/x**6, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**5, x) + Integral(
b**2*c**3*atanh(c*x)**2/x**4, x) + Integral(6*a*b*c*atanh(c*x)/x**6, x) +
Integral(6*a*b*c**2*atanh(c*x)/x**5, x) + Integral(2*a*b*c**3*atanh(c*x)/x
**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(427) = 854$.

Time = 0.51 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.01

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="maxima")
```

output
```
-14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^6*d
^3 - 14/15*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^6*d^3 + 14/15*
(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^6*d^3 - 23/60*b^2*c^6*d^3*
log(c*x + 1) - 97/60*b^2*c^6*d^3*log(c*x - 1) + 2*b^2*c^6*d^3*log(x) - 1/3
*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*
b*c^3*d^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*a*b*c^2*d^3 - 3/10*((2*c^4*log(c^2*x^2 -
1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*c*d
^3 + 1/90*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*
c^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*a*b*d^3 + 1/360*((184*c^4*log(x
) - (15*c^4*x^4*log(c*x + 1)^2 + 15*c^4*x^4*log(c*x - 1)^2 + 92*c^4*x^4*lo
g(c*x - 1) + 32*c^2*x^2 - 2*(15*c^4*x^4*log(c*x - 1) - 46*c^4*x^4)*log(c*x
 + 1) + 6)/x^4)*c^2 + 4*(15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15
*c^4*x^4 + 5*c^2*x^2 + 3)/x^5)*c*arctanh(c*x))*b^2*d^3 - 1/3*a^2*c^3*d^3/x
^3 - 3/4*a^2*c^2*d^3/x^4 - 3/5*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctanh(c*x)^2/
x^6 - 1/6*a^2*d^3/x^6 - 1/240*(296*b^2*c^5*d^3*x^4 + 60*b^2*c^4*d^3*x^3 +
24*b^2*c^3*d^3*x^2 + (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2
*d^3*x + 36*b^2*c*d^3)*log(c*x + 1)^2 - (101*b^2*c^6*d^3*x^5 - 20*b^2*c^3*
d^3*x^2 - 45*b^2*c^2*d^3*x - 36*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(45*b^2*c^5
*d^3*x^4 + 28*b^2*c^4*d^3*x^3 + 15*b^2*c^3*d^3*x^2 + 9*b^2*c^2*d^3*x)*l...
```

Giac [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx=\int \frac{(cdx+d)^3(b\,\text{artanh}\,(cx)+a)^2}{x^7}\,dx$$

input
```
integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="giac")
```

output
```
integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^2\,(d+c\,d\,x)^3}{x^7}\,dx$$

input
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^7,x)
```

output
```
int(((a + b*atanh(c*x))^2*(d + c*d*x)^3)/x^7, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^3(a+b\text{arctanh}(cx))^2}{x^7}\,dx$$
$$= \frac{d^3\Big(-230atanh(cx)\,b^2c^6x^6 - 452\log(c^2x-c)\,b^2c^6x^6 + 452\log(x)\,b^2c^6x^6 - 60atanh(cx)\,ab - 3b^2c^2x^2 - 2}{}$$

input
```
int((c*d*x+d)^3*(a+b*atanh(c*x))^2/x^7,x)
```

output
```
(d**3*(165*atanh(c*x)**2*b**2*c**6*x**6 - 60*atanh(c*x)**2*b**2*c**3*x**3
- 135*atanh(c*x)**2*b**2*c**2*x**2 - 108*atanh(c*x)**2*b**2*c*x - 30*atanh
(c*x)**2*b**2 - 6*atanh(c*x)*a*b*c**6*x**6 - 120*atanh(c*x)*a*b*c**3*x**3
- 270*atanh(c*x)*a*b*c**2*x**2 - 216*atanh(c*x)*a*b*c*x - 60*atanh(c*x)*a*
b - 230*atanh(c*x)*b**2*c**6*x**6 - 330*atanh(c*x)*b**2*c**5*x**5 - 168*at
anh(c*x)*b**2*c**4*x**4 - 110*atanh(c*x)*b**2*c**3*x**3 - 54*atanh(c*x)*b*
*2*c**2*x**2 - 12*atanh(c*x)*b**2*c*x - 336*int(atanh(c*x)/(c**2*x**3 - x)
,x)*b**2*c**6*x**6 - 336*log(c**2*x - c)*a*b*c**6*x**6 - 452*log(c**2*x -
c)*b**2*c**6*x**6 + 336*log(x)*a*b*c**6*x**6 + 452*log(x)*b**2*c**6*x**6 -
 60*a**2*c**3*x**3 - 135*a**2*c**2*x**2 - 108*a**2*c*x - 30*a**2 - 330*a*b
*c**5*x**5 - 168*a*b*c**4*x**4 - 110*a*b*c**3*x**3 - 54*a*b*c**2*x**2 - 12
*a*b*c*x - 222*b**2*c**5*x**5 - 61*b**2*c**4*x**4 - 18*b**2*c**3*x**3 - 3*
b**2*c**2*x**2))/(180*x**6)
```

3.95 $\int \frac{x^3(a+b\,\mathrm{arctanh}(cx))^2}{d+cdx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 329

$$\begin{aligned}\int \frac{x^3(a+b\,\mathrm{arctanh}(cx))^2}{d+cdx}\,dx = &-\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\mathrm{arctanh}(cx)}{3c^4d}\\&-\frac{b^2x\,\mathrm{arctanh}(cx)}{c^3d}+\frac{bx^2(a+b\,\mathrm{arctanh}(cx))}{3c^2d}\\&+\frac{11(a+b\,\mathrm{arctanh}(cx))^2}{6c^4d}+\frac{x(a+b\,\mathrm{arctanh}(cx))^2}{c^3d}\\&-\frac{x^2(a+b\,\mathrm{arctanh}(cx))^2}{2c^2d}+\frac{x^3(a+b\,\mathrm{arctanh}(cx))^2}{3cd}\\&-\frac{8b(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c^4d}\\&+\frac{(a+b\,\mathrm{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^4d}\\&-\frac{b^2\log(1-c^2x^2)}{2c^4d}-\frac{4b^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^4d}\\&-\frac{b(a+b\,\mathrm{arctanh}(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^4d}\\&-\frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^4d}\end{aligned}$$

output
```
-a*b*x/c^3/d+1/3*b^2*x/c^3/d-1/3*b^2*arctanh(c*x)/c^4/d-b^2*x*arctanh(c*x)
/c^3/d+1/3*b*x^2*(a+b*arctanh(c*x))/c^2/d+11/6*(a+b*arctanh(c*x))^2/c^4/d+
x*(a+b*arctanh(c*x))^2/c^3/d-1/2*x^2*(a+b*arctanh(c*x))^2/c^2/d+1/3*x^3*(a
+b*arctanh(c*x))^2/c/d-8/3*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4/d+(a+b*
arctanh(c*x))^2*ln(2/(c*x+1))/c^4/d-1/2*b^2*ln(-c^2*x^2+1)/c^4/d-4/3*b^2*p
olylog(2,1-2/(-c*x+1))/c^4/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c
^4/d-1/2*b^2*polylog(3,1-2/(c*x+1))/c^4/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \frac{a^2x}{c^3d}-\frac{a^2x^2}{2c^2d}+\frac{a^2x^3}{3cd}-\frac{a^2\log(1+cx)}{c^4d}$$
$$+\frac{ab\Big(-3cx+8cx\text{arctanh}(cx)+(1-c^2x^2)\left(-1+3\text{arctanh}(cx)-2cx\text{arctanh}(cx)\right)+6\text{arctanh}(cx)\log\big(\cdots}{3c^4d}$$
$$+\frac{b^2\Big(2cx-6cx\text{arctanh}(cx)-2(1-c^2x^2)\,\text{arctanh}(cx)-8\text{arctanh}(cx)^2+8cx\text{arctanh}(cx)^2+3(1-c^2x\cdots}{\cdots}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

output
```
(a^2*x)/(c^3*d) - (a^2*x^2)/(2*c^2*d) + (a^2*x^3)/(3*c*d) - (a^2*Log[1 + c
*x])/(c^4*d) + (a*b*(-3*c*x + 8*c*x*ArcTanh[c*x] + (1 - c^2*x^2)*(-1 + 3*A
rcTanh[c*x] - 2*c*x*ArcTanh[c*x]) + 6*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] - 8*Log[1/Sqrt[1 - c^2*x^2]] - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/
(3*c^4*d) + (b^2*(2*c*x - 6*c*x*ArcTanh[c*x] - 2*(1 - c^2*x^2)*ArcTanh[c*x
] - 8*ArcTanh[c*x]^2 + 8*c*x*ArcTanh[c*x]^2 + 3*(1 - c^2*x^2)*ArcTanh[c*x]
^2 - 2*c*x*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 16*ArcTanh[c*x]*Log[1 + E^(-2*Ar
cTanh[c*x])] + 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*Log[1/Sqr
t[1 - c^2*x^2]] + (8 - 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] -
3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(6*c^4*d)
```

Rubi [A] (verified)

Time = 5.10 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.29, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {6492, 27, 6452, 6492, 6452, 6492, 6436, 6470, 6542, 2009, 6452, 262, 219, 6510, 6546, 6470, 2849, 2752, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{cdx+d}dx$$

$$\Big\downarrow \text{6492}$$

$$\frac{\int x^2(a+b\text{arctanh}(cx))^2dx}{cd}-\frac{\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d(cx+1)}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^2(a+b\text{arctanh}(cx))^2dx}{cd}-\frac{\int \frac{x^2(a+b\text{arctanh}(cx))^2}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\int \frac{x^2(a+b\text{arctanh}(cx))^2}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{6492}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{\int x(a+b\text{arctanh}(cx))^2dx}{c}-\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{cx+1}dx}{c}}{cd}$$

$$\Big\downarrow \text{6492}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{\frac{\int(a+b\text{arctanh}(cx))^2dx}{c}-\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{cx+1}dx}{c}}{c}}{cd}$$

$\Big\downarrow$ 6436

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{cx+1}dx}{c}}{c}}{cd}$$

$\Big\downarrow$ 6470

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{\cdots}{cx\cdots}\right)}{1-c^2x^2}\cdots}{c}\cdots}{cd}$$

$\Big\downarrow$ 6542

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))dx}{c^2}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))dx}{c^2}\right)}{c}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{2b\int\cdots}{\cdots}\cdots}{\cdots}}{cd}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))dx}{c^2}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}\right)}{c}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(c\cdots}{1-c^2x^2}\cdots}{c}\cdots}{\cdots}}{cd}$$

$\Big\downarrow$ 6452

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\int\frac{x^2}{1-c^2x^2}dx}{c^2}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(c}{1-c^2x^2}}{c}}{cd}$$

$\downarrow$ 262

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}\right)}{c^2}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(c}{1-c^2x^2}}{c}}{cd}$$

$\downarrow$ 219

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(c}{1-c^2x^2}}{c}}{cd}$$

$\downarrow$ 6510

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)}{cd}-\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx)}{1-c^2x^2}}{c}}{cd}$$

$\downarrow$ 6546

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right.}{cd}$$

$$\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}}{c}\right.}{c}}{cd}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-}{c^2}\right.}{cd}$$

$$\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arct}}{c}}{}\right.}{}}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx}{}\right.}{cd}$$

$$\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-c}}{c}}{}\right.}{}}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(c\cdots}{\cdots}\right.}{cd}$$

$$\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arc}\cdots}{c}}{\cdots}\right.}{\cdots}}{cd}$$

$\big\downarrow$ 6618

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(c\cdots}{\cdots}\right.}{cd}$$

$$\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arc}\cdots}{c}}{\cdots}\right.}{\cdots}}{\cdots}$$

$\big\downarrow$ 7164

$$\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(c\cdots}{\cdots}\right.}{cd}$$

$$\frac{\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{c}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arc}\cdots}{c}}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

output
```
((x^3*(a + b*ArcTanh[c*x])^2)/3 - (2*b*c*(-(((x^2*(a + b*ArcTanh[c*x]))/2
- (b*c*(-(x/c^2) + ArcTanh[c*x]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*x])
^2/(b*c^2) + (((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1
- 2/(1 - c*x)])/(2*c))/c)/c^2))/3)/(c*d) - (((x^2*(a + b*ArcTanh[c*x])^2)/
2 - b*c*((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (a*x + b*x*ArcTanh[c*x] + (b*L
og[1 - c^2*x^2])/(2*c))/c^2))/c - ((x*(a + b*ArcTanh[c*x])^2 - 2*b*c*(-1/2
*(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])
/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c))/c))/c - (-(((a + b*ArcTanh[c*x
])^2*Log[2/(1 + c*x)])/c) + 2*b*(((a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1
 + c*x)])/(2*c) + (b*PolyLog[3, 1 - 2/(1 + c*x)])/(4*c)))/c)/c)/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.90 (sec) , antiderivative size = 967, normalized size of antiderivative = 2.94

method	result	size
derivativedivides	Expression too large to display	967
default	Expression too large to display	967
parts	Expression too large to display	973

input
```
int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2/d*(1/3*x^3*c^3-1/2*c^2*x^2+c*x-ln(c*x+1))+b^2/d*(-1/2*I*Pi*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x
^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-1/3-arctanh(c*x)^
2*ln(c*x+1)-8/3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-8/3*arctan
h(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-8/3*dilog(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-8/3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/3*c*x+1/3*arctanh(c*
x)^2*c^3*x^3-2/3*arctanh(c*x)^3-1/2*arctanh(c*x)^2*c^2*x^2+arctanh(c*x)^2*
c*x+11/6*arctanh(c*x)^2-1/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+1/2*I*Pi*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^
2/(c^2*x^2-1)))*arctanh(c*x)^2-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-1/3*(
c*x+1)*arctanh(c*x)+ln(1+(c*x+1)^2/(-c^2*x^2+1))+arctanh(c*x)*polylog(2,-(
c*x+1)^2/(-c^2*x^2+1))+1/3*(c*x-3)*(c*x+1)*arctanh(c*x)+1/2*I*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+I*Pi*csgn(I
*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^
2+2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+ln(2)*arctanh(c*x)^2)+2*
b*a/d*(1/3*arctanh(c*x)*c^3*x^3-1/2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x-
arctanh(c*x)*ln(c*x+1)+1/4*ln(c*x+1)^2-1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*...
```

Fricas [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2x^3}{cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c*d*
x + d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \frac{\int \frac{a^2x^3}{cx+1}\,dx + \int \frac{b^2x^3\,\text{atanh}^2\,(cx)}{cx+1}\,dx + \int \frac{2abx^3\,\text{atanh}\,(cx)}{cx+1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

output
```
(Integral(a**2*x**3/(c*x + 1), x) + Integral(b**2*x**3*atanh(c*x)**2/(c*x
+ 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c*x + 1), x))/d
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/6*a^2*((2*c^2*x^3 - 3*c*x^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d)) + 1
/24*(2*b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 6*b^2*c*x - 6*b^2*log(c*x + 1))*log(-
c*x + 1)^2/(c^4*d) - integrate(-1/12*(3*(b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*
x + 1)^2 + 12*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (3*b^2*c^2*x^2 +
2*(6*a*b*c^4 + b^2*c^4)*x^4 + 6*b^2*c*x - (12*a*b*c^3 + b^2*c^3)*x^3 + 6*(
b^2*c^4*x^4 - b^2*c^3*x^3 - b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c
^5*d*x^2 - c^3*d), x)
```

Giac [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{x^3\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+c\,d\,x}\,dx$$

input
```
int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

output
```
int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx$$
$$=\frac{12\left(\int \frac{atanh(cx)x^3}{cx+1}dx\right)ab\,c^4+6\left(\int \frac{atanh(cx)^2x^3}{cx+1}dx\right)b^2c^4-6\log(cx+1)\,a^2+2a^2c^3x^3-3a^2c^2x^2+6a^2cx}{6c^4d}$$

input
```
int(x^3*(a+b*atanh(c*x))^2/(c*d*x+d),x)
```

output
```
(12*int((atanh(c*x)*x**3)/(c*x + 1),x)*a*b*c**4 + 6*int((atanh(c*x)**2*x**
3)/(c*x + 1),x)*b**2*c**4 - 6*log(c*x + 1)*a**2 + 2*a**2*c**3*x**3 - 3*a**
2*c**2*x**2 + 6*a**2*c*x)/(6*c**4*d)
```

3.96 $\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\begin{aligned}\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = {} & \frac{abx}{c^2d}+\frac{b^2x\text{arctanh}(cx)}{c^2d}-\frac{3(a+b\text{arctanh}(cx))^2}{2c^3d} \\ & -\frac{x(a+b\text{arctanh}(cx))^2}{c^2d}+\frac{x^2(a+b\text{arctanh}(cx))^2}{2cd} \\ & +\frac{2b(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^3d} \\ & -\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^3d} \\ & +\frac{b^2\log(1-c^2x^2)}{2c^3d}+\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d} \\ & +\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^3d} \\ & +\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^3d}\end{aligned}$$

output
```
a*b*x/c^2/d+b^2*x*arctanh(c*x)/c^2/d-3/2*(a+b*arctanh(c*x))^2/c^3/d-x*(a+b
*arctanh(c*x))^2/c^2/d+1/2*x^2*(a+b*arctanh(c*x))^2/c/d+2*b*(a+b*arctanh(c
*x))*ln(2/(-c*x+1))/c^3/d-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^3/d+1/2*b^2
*ln(-c^2*x^2+1)/c^3/d+b^2*polylog(2,1-2/(-c*x+1))/c^3/d+b*(a+b*arctanh(c*x
))*polylog(2,1-2/(c*x+1))/c^3/d+1/2*b^2*polylog(3,1-2/(c*x+1))/c^3/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx$$
$$= \frac{-2a^2cx+2abcx+a^2c^2x^2-2ab\text{arctanh}(cx)-4abcx\text{arctanh}(cx)+2b^2cx\text{arctanh}(cx)+2abc^2x^2\text{arctanh}}{}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

output
```
(-2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 - 2*a*b*ArcTanh[c*x] - 4*a*b*c*x*Arc
Tanh[c*x] + 2*b^2*c*x*ArcTanh[c*x] + 2*a*b*c^2*x^2*ArcTanh[c*x] + b^2*ArcT
anh[c*x]^2 - 2*b^2*c*x*ArcTanh[c*x]^2 + b^2*c^2*x^2*ArcTanh[c*x]^2 - 4*a*b
*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 4*b^2*ArcTanh[c*x]*Log[1 + E^
(-2*ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 2
*a^2*Log[1 + c*x] - 2*a*b*Log[1 - c^2*x^2] + b^2*Log[1 - c^2*x^2] + 2*b*(a
 - b + b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -
E^(-2*ArcTanh[c*x])])/(2*c^3*d)
```

Rubi [A] (verified)

Time = 3.59 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6492, 27, 6452, 6492, 6436, 6470, 6542, 2009, 6510, 6546, 6470, 2849, 2752, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{cdx+d}\,dx$$

$$\Big\downarrow \text{6492}$$

$$\frac{\int x(a+b\text{arctanh}(cx))^2dx}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{d(cx+1)}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x(a+b\text{arctanh}(cx))^2dx}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{cx+1}dx}{cd}$$

$\Big\downarrow$ 6452

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{cx+1}dx}{cd}$$

$\Big\downarrow$ 6492

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{\int (a+b\text{arctanh}(cx))^2dx}{c}-\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{cx+1}dx}{c}}{cd}$$

$\Big\downarrow$ 6436

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{cx+1}dx}{c}}{cd}$$

$\Big\downarrow$ 6470

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{c}}{cd}$$

$\Big\downarrow$ 6542

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int (a+b\text{arctanh}(cx))dx}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{c}}{cd}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{c}}{cd}$$

$\downarrow$ 6510

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c}-\frac{2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{c}}{cd}$$

$\downarrow$ 6546

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c}-\frac{2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctan}\cdots}{c}}{c}}{cd}$$

$\downarrow$ 6470

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c}-\frac{2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}d\cdots}{c}}{cd}$$

$\downarrow$ 2849

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c}-\frac{2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+\cdots}\right)}{1-c^2x^2}\cdots}{\cdots}}{cd}$$

$\downarrow$ 2752

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c}-2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+\ldots}\right)}{1-c^2x^2}}{cd}$$

$\Big\downarrow$ 6618

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c}-2b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arc}\ldots}{2c}\right.}{cd}$$

$\Big\downarrow$ 7164

$$\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)}{cd}-\frac{\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c}-2b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arc}\ldots}{2c}\right.}{cd}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

output
```
((x^2*(a + b*ArcTanh[c*x])^2)/2 - b*c*((a + b*ArcTanh[c*x])^2/(2*b*c^3) -
(a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2))/(c*d) - ((x*(a
 + b*ArcTanh[c*x])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a +
 b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*
c))/c))/c - (-(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/c) + 2*b*(((a + b
*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, 1 - 2/(1
 + c*x)])/(4*c)))/c)/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.58 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.66

method	result	size
derivativedivides	Expression too large to display	905
default	Expression too large to display	905
parts	Expression too large to display	913

input
```
int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^2/d*(1/2*c^2*x^2-c*x+ln(c*x+1))+b^2/d*(1/2*arctanh(c*x)^2*c^2*x^2
-arctanh(c*x)^2*c*x+arctanh(c*x)^2*ln(c*x+1)-arctanh(c*x)*polylog(2,-(c*x+
1)^2/(-c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*arctanh(c*x)^2
*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3*arctanh(c*x)^3-ln(2)*arctanh(c*x)^2+2*
dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/
2))-3/2*arctanh(c*x)^2-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*
x)^2-1/2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)
))*arctanh(c*x)^2+(c*x+1)*arctanh(c*x)-ln(1+(c*x+1)^2/(-c^2*x^2+1))+2*arct
anh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*ln(1-I*(c*x+1)/
(-c^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^
2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-I*Pi*csgn(I*
(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2
+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c
*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2)+2/d*a*b*(1/2*arctanh(c*x)*
c^2*x^2-arctanh(c*x)*c*x+arctanh(c*x)*ln(c*x+1)-1/4*ln(c*x+1)^2+1/2*(ln(c*
x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2)+1/2*c*x+1/2-
3/4*ln(c*x+1)-1/4*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2x^2}{cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c*d*
x + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\frac{\int \frac{a^2x^2}{cx+1}\,dx+\int \frac{b^2x^2\operatorname{atanh}^2(cx)}{cx+1}\,dx+\int \frac{2abx^2\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

output
```
(Integral(a**2*x**2/(c*x + 1), x) + Integral(b**2*x**2*atanh(c*x)**2/(c*x
+ 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c*x + 1), x))/d
```

Maxima [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2x^2}{cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/2*a^2*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d)) + 1/8*(b^2*c^2*x^
2 - 2*b^2*c*x + 2*b^2*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d) - integrate(-1
/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*
x^2)*log(c*x + 1) + (2*b^2*c*x - (4*a*b*c^3 + b^2*c^3)*x^3 + (4*a*b*c^2 +
b^2*c^2)*x^2 - 2*(b^2*c^3*x^3 - b^2*c^2*x^2 + b^2*c*x + b^2)*log(c*x + 1))
*log(-c*x + 1))/(c^4*d*x^2 - c^2*d), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^2}{cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{x^2\,\left(a+b\,\text{atanh}(c\,x)\right)^2}{d+c\,d\,x}\,dx$$

input
```
int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

output
```
int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx$$
$$=\frac{4\left(\int\frac{atanh(cx)x^2}{cx+1}dx\right)ab\,c^3+2\left(\int\frac{atanh(cx)^2x^2}{cx+1}dx\right)b^2c^3+2\log(cx+1)\,a^2+a^2c^2x^2-2a^2cx}{2c^3d}$$

input
```
int(x^2*(a+b*atanh(c*x))^2/(c*d*x+d),x)
```

output
```
(4*int((atanh(c*x)*x**2)/(c*x + 1),x)*a*b*c**3 + 2*int((atanh(c*x)**2*x**2
)/(c*x + 1),x)*b**2*c**3 + 2*log(c*x + 1)*a**2 + a**2*c**2*x**2 - 2*a**2*c
*x)/(2*c**3*d)
```

3.97 $\int \frac{x(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 172

$$\begin{aligned}\int \frac{x(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx &= \frac{(a+b\text{arctanh}(cx))^2}{c^2d}+\frac{x(a+b\text{arctanh}(cx))^2}{cd}\\ &\quad-\frac{2b(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^2d}\\ &\quad+\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^2d}\\ &\quad-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^2d}\\ &\quad-\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^2d}\\ &\quad-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^2d}\end{aligned}$$

output
```
(a+b*arctanh(c*x))^2/c^2/d+x*(a+b*arctanh(c*x))^2/c/d-2*b*(a+b*arctanh(c*x
))*ln(2/(-c*x+1))/c^2/d+(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^2/d-b^2*polyl
og(2,1-2/(-c*x+1))/c^2/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^2/d
-1/2*b^2*polylog(3,1-2/(c*x+1))/c^2/d
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \frac{2b^2\text{arctanh}(cx)^2\left(-1+cx+\log\left(1+e^{-2\text{arctanh}(cx)}\right)\right)+4b\text{arctanh}(cx)\left(acx+(a-b)\log\left(1+e^{-2\text{arctanh}(c}\right.\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

output
```
(2*b^2*ArcTanh[c*x]^2*(-1 + c*x + Log[1 + E^(-2*ArcTanh[c*x])]) + 4*b*ArcT
anh[c*x]*(a*c*x + (a - b)*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*a*(a*c*x - a*L
og[1 + c*x] + b*Log[1 - c^2*x^2]) - 2*b*(a - b + b*ArcTanh[c*x])*PolyLog[2
, -E^(-2*ArcTanh[c*x])] - b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2*c^2*d)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6492, 27, 6436, 6470, 6546, 6470, 2849, 2752, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{cdx+d}\,dx$$

$$\Big\downarrow \text{ 6492}$$

$$\frac{\int (a+b\text{arctanh}(cx))^2dx}{cd}-\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{d(cx+1)}dx}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (a+b\text{arctanh}(cx))^2dx}{cd}-\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{cx+1}dx}{cd}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{cd}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{cd}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int \frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{cd}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{cd}-\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{cd}-\frac{2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

$$\Big\downarrow \text{6618}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{cd}-\frac{2b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

$$\Big\downarrow \text{7164}$$

$$\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{cd}-\frac{2b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{4c}\right)-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c}}{cd}$$

input
```
Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

output
```
(x*(a + b*ArcTanh[c*x])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + (
((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)]
)/(2*c))/c))/(c*d) - (-(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/c) + 2*b
*(((a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (b*PolyLog[3,
 1 - 2/(1 + c*x)])/(4*c)))/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 2602, normalized size of antiderivative = 15.13

method	result	size
derivativedivides	Expression too large to display	2602
default	Expression too large to display	2602
parts	Expression too large to display	2609

input
```
int(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a^2/d*(c*x-ln(c*x+1))+b^2/d*(-1/4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1
)^2/(c^2*x^2-1)))*(2*arctanh(c*x)^2-2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^
2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))-arctanh(c*x)^2*ln(c*x+1)-arctanh
(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1
)^(1/2))-arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*ln(2)*polylog
(2,-(c*x+1)^2/(-c^2*x^2+1))+ln(2)*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+ln
(2)*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(
1/2))-dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1
-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+
arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1)/(-c^2*x^
2+1)^(1/2))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-2/3*arctanh(c*x)^3+arct
anh(c*x)^2*c*x+arctanh(c*x)^2-1/2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/2*p
olylog(3,-(c*x+1)^2/(-c^2*x^2+1))+arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*
x^2+1))+ln(2)*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+ln(2)*arctan
h(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-ln(2)*arctanh(c*x)*ln(1+(c*x+1)^
2/(-c^2*x^2+1))+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^
2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*x)*ln(1+I*(c*x+1)/(-
c^2*x^2+1)^(1/2))+arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog...
```

Fricas [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x}{cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c*d*x + d)
, x)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx = \frac{\int \frac{a^2x}{cx+1}\,dx + \int \frac{b^2x\,\operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2abx\,\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

input
```
integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

output
```
(Integral(a**2*x/(c*x + 1), x) + Integral(b**2*x*atanh(c*x)**2/(c*x + 1),
x) + Integral(2*a*b*x*atanh(c*x)/(c*x + 1), x))/d
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2x}{cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

output
```
a^2*(x/(c*d) - log(c*x + 1)/(c^2*d)) + 1/4*(b^2*c*x - b^2*log(c*x + 1))*lo
g(-c*x + 1)^2/(c^2*d) - integrate(-1/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x +
1)^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(c*x + 1) - 2*((2*a*b*c^2 + b^2*c^2)*x
^2 - (2*a*b*c - b^2*c)*x + (b^2*c^2*x^2 - 2*b^2*c*x - b^2)*log(c*x + 1))*l
og(-c*x + 1))/(c^3*d*x^2 - c*d), x)
```

Giac [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2x}{cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{x\,(a+b\,\text{atanh}(c\,x))^2}{d+c\,d\,x}\,dx$$

input
```
int((x*(a + b*atanh(c*x))^2)/(d + c*d*x),x)
```

output
```
int((x*(a + b*atanh(c*x))^2)/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \frac{2\left(\int \frac{atanh(cx)x}{cx+1}dx\right)ab\,c^2+\left(\int \frac{atanh(cx)^2x}{cx+1}dx\right)b^2c^2-\log(cx+1)\,a^2+a^2cx}{c^2d}$$

input
```
int(x*(a+b*atanh(c*x))^2/(c*d*x+d),x)
```

output
```
(2*int((atanh(c*x)*x)/(c*x + 1),x)*a*b*c**2 + int((atanh(c*x)**2*x)/(c*x +
 1),x)*b**2*c**2 - log(c*x + 1)*a**2 + a**2*c*x)/(c**2*d)
```

3.98 $\int \frac{(a+b\textbf{arctanh}(cx))^2}{d+cdx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = -\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{cd} + \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2cd}$$

output
```
-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c/d+b*(a+b*arctanh(c*x))*polylog(2,1-2
/(c*x+1))/c/d+1/2*b^2*polylog(3,1-2/(c*x+1))/c/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \frac{-4ab\text{arctanh}(cx)\log\left(1+e^{-2\text{arctanh}(cx)}\right) - 2b^2\text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right) + 2a^2\log(1+cx) + 2b}{2cd}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

output
```
(-4*a*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*L
og[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*Log[1 + c*x] + 2*b*(a + b*ArcTanh[c*x]
)*PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])
/(2*c*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6470, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{cdx+d}dx$$

$$\Big\downarrow \text{6470}$$

$$\frac{2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx}{d}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{cd}$$

$$\Big\downarrow \text{6618}$$

$$\frac{2b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)}{d}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{cd}$$

$$\Big\downarrow \text{7164}$$

$$\frac{2b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{4c}\right)}{d}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{cd}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

output
```
-(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c*d)) + (2*b*(((a + b*ArcTanh
[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, 1 - 2/(1 + c*x)]
)/(4*c)))/d
```

Defintions of rubi rules used

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(82) = 164$.

Time = 0.49 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.42

method	result
risch	$\frac{\ln(-cx+1)^2\ln\left(\frac{cx}{2}+\frac{1}{2}\right)b^2}{4dc} - \frac{\ln(-cx+1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)ab}{dc} + \frac{\ln(-cx+1)\operatorname{polylog}\left(2,-\frac{cx}{2}+\frac{1}{2}\right)b^2}{2dc} + \frac{\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\ldots\right.}{dc}$ …
derivativedivides	$\frac{a^2\ln(cx+1)}{d} + \frac{b^2\left(\operatorname{arctanh}(cx)^2\ln(cx+1)-2\operatorname{arctanh}(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)+\frac{2\operatorname{arctanh}(cx)^3}{3}-\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)^2\operatorname{csgn}\left(\frac{i(cx\ldots}{c^2x\ldots}\right.\right.}{\ldots}\right.}{\ldots}$ …
default	$\frac{a^2\ln(cx+1)}{d} + \frac{b^2\left(\operatorname{arctanh}(cx)^2\ln(cx+1)-2\operatorname{arctanh}(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)+\frac{2\operatorname{arctanh}(cx)^3}{3}-\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)^2\operatorname{csgn}\left(\frac{i(cx\ldots}{c^2x\ldots}\right.\right.}{\ldots}\right.}{\ldots}$ …
parts	$\frac{a^2\ln(cx+1)}{dc} + \frac{b^2\left(\operatorname{arctanh}(cx)^2\ln(cx+1)-2\operatorname{arctanh}(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)+\frac{2\operatorname{arctanh}(cx)^3}{3}-\frac{\left(i\pi\operatorname{csgn}\left(\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)^2\ldots\right.}{\ldots}\right.}{\ldots}$ …

input
```
int((a+b*arctanh(c*x))^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/c*ln(-c*x+1)^2*ln(1/2*c*x+1/2)*b^2-1/d/c*ln(-c*x+1)*ln(1/2*c*x+1/2)*
a*b+1/2/d/c*ln(-c*x+1)*polylog(2,-1/2*c*x+1/2)*b^2+1/d/c*ln(-1/2*c*x+1/2)*
ln(1/2*c*x+1/2)*a*b+1/d/c*dilog(-1/2*c*x+1/2)*a*b-1/2/d/c*polylog(3,-1/2*c
*x+1/2)*b^2+1/d/c*a^2*ln(-c*x-1)+1/12*b^2/d/c*ln(c*x+1)^3+1/2*b/d*a*ln(c*x
+1)^2/c-1/4*b^2/d/c*ln(-c*x+1)*ln(c*x+1)^2+1/4*b^2/d/c*ln(c*x+1)^2*ln(-1/2
*c*x+1/2)+1/2*b^2/d/c*ln(c*x+1)*polylog(2,1/2*c*x+1/2)-1/2*b^2/d/c*polylog
(3,1/2*c*x+1/2)
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2}{cdx+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \frac{\int \frac{a^2}{cx+1}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{cx+1}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{cx+1}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))**2/(c*d*x+d),x)
```

output
```
(Integral(a**2/(c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c*x + 1), x) +
 Integral(2*a*b*atanh(c*x)/(c*x + 1), x))/d
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+cdx}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2}{cdx+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/(c*d) + a^2*log(c*d*x + d)/(c*d) - in
tegrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x +
 1) - 4*(b^2*c*x*log(c*x + 1) + a*b*c*x - a*b)*log(-c*x + 1))/(c^2*d*x^2 -
 d), x)
```

Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(b\,\operatorname{artanh}(cx)+a)^2}{cdx+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\int \frac{(a+b\,\operatorname{atanh}(c\,x))^2}{d+c\,d\,x}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(d + c*d*x),x)
```

output
```
int((a + b*atanh(c*x))^2/(d + c*d*x), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+cdx}\,dx=\frac{2\left(\int \frac{atanh(cx)}{cx+1}dx\right)abc+\left(\int \frac{atanh(cx)^2}{cx+1}dx\right)b^2c+\log(cx+1)\,a^2}{cd}$$

input
```
int((a+b*atanh(c*x))^2/(c*d*x+d),x)
```

output
```
(2*int(atanh(c*x)/(c*x + 1),x)*a*b*c + int(atanh(c*x)**2/(c*x + 1),x)*b**2
*c + log(c*x + 1)*a**2)/(c*d)
```

3.99 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x(d+cdx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)}\,dx = \frac{(a+b\text{arctanh}(cx))^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d} - \frac{b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)}{2d}$$

output
```
(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-b*(a+b*arctanh(c*x))*polylog(2,-1+2
/(c*x+1))/d-1/2*b^2*polylog(3,-1+2/(c*x+1))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)}\,dx$$

$$= \frac{a^2\log(cx)-a^2\log(1+cx)+ab\left(2\text{arctanh}(cx)\log\left(1-e^{-2\text{arctanh}(cx)}\right)-\text{PolyLog}\left(2,e^{-2\text{arctanh}(cx)}\right)\right)+b^2\left(\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

output
```
(a^2*Log[c*x] - a^2*Log[1 + c*x] + a*b*(2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTa
nh[c*x])] - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^2*((I/24)*Pi^3 - (2*ArcTa
nh[c*x]^3)/3 + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*P
olyLog[2, E^(2*ArcTanh[c*x])] - PolyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(cdx+d)}dx$$

$$\Big\downarrow \text{ 6494}$$

$$\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d}-\frac{2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx}{d}$$

$$\Big\downarrow \text{ 6618}$$

$$\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d}-\frac{2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{1-c^2x^2}dx\right)}{d}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d}-\frac{2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}+\frac{b\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{4c}\right)}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

output
```
((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (2*b*c*(((a + b*ArcTanh[
c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 + c*x)
])/(4*c)))/d
```

Defintions of rubi rules used

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.01 (sec) , antiderivative size = 1148, normalized size of antiderivative = 14.91

method	result	size
parts	Expression too large to display	1148
derivativedivides	Expression too large to display	1150
default	Expression too large to display	1150

input
```
int((a+b*arctanh(c*x))^2/x/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(-ln(c*x+1)+ln(x))+b^2/d*(arctanh(c*x)^2*ln(c*x)-arctanh(c*x)^2*ln(c
*x+1)+2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2*ln((c
*x+1)^2/(-c^2*x^2+1)-1)-2/3*arctanh(c*x)^3+1/2*arctanh(c*x)^2*(I*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^3+2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^2-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^
2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c
*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^
2-1)))^3+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*cs
gn(I/(1-(c*x+1)^2/(c^2*x^2-1)))-I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csg
n(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+I*Pi*csgn(I/(1
-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+
1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(-(c*x+1)^2/(c^
2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3-I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-
1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+2*ln
(2))+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*polylo
g(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+ar
ctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*polylog(2,-(c
*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2)))+2*b...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^2 + d*x),
x)
```

Sympy [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x(d + cdx)}\,dx = \frac{\int \frac{a^2}{cx^2+x}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{cx^2+x}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx^2+x}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))**2/x/(c*d*x+d),x)
```

output
```
(Integral(a**2/(c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2/(c*x**2 + x)
, x) + Integral(2*a*b*atanh(c*x)/(c*x**2 + x), x))/d
```

Maxima [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x(d + cdx)}\,dx = \int \frac{(b\operatorname{artanh}(cx) + a)^2}{(cdx + d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="maxima")
```

output
```
-1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/d - a^2*(log(c*x + 1)/d - log(x)/d)
+ integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*
x + 1) - 2*(2*a*b*c*x - 2*a*b - (b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x
 + 1))/(c^2*d*x^3 - d*x), x)
```

Giac [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x(d + cdx)}\,dx = \int \frac{(b\operatorname{artanh}(cx) + a)^2}{(cdx + d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)}\,dx=\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x\,\left(d+c\,d\,x\right)} \,d x$$

input
```
int((a + b*atanh(c*x))^2/(x*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))^2/(x*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)}\,dx = \frac{-atanh(cx)^3 b^2-3atanh(cx)^2 ab-6\left(\int \frac{atanh(cx)}{c^2x^3-x}dx\right)ab-3\left(\int \frac{atanh(cx)^2}{c^2x^3-x}dx\right)b^2-3\log(cx+1)\,a^2+3\log}{3d}$$

input
```
int((a+b*atanh(c*x))^2/x/(c*d*x+d),x)
```

output
```
( - atanh(c*x)**3*b**2 - 3*atanh(c*x)**2*a*b - 6*int(atanh(c*x)/(c**2*x**3
 - x),x)*a*b - 3*int(atanh(c*x)**2/(c**2*x**3 - x),x)*b**2 - 3*log(c*x + 1
)*a**2 + 3*log(x)*a**2)/(3*d)
```

3.100 $\int \frac{(a+b\textbf{arctanh}(cx))^2}{x^2(d+cdx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)}\,dx &= \frac{c(a+b\text{arctanh}(cx))^2}{d}-\frac{(a+b\text{arctanh}(cx))^2}{dx}\\ &+\frac{2bc(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d}\\ &-\frac{c(a+b\text{arctanh}(cx))^2\log\left(2-\frac{2}{1+cx}\right)}{d}\\ &-\frac{b^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d}\\ &+\frac{bc(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d}\\ &+\frac{b^2c\,\text{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

output
```
c*(a+b*arctanh(c*x))^2/d-(a+b*arctanh(c*x))^2/d/x+2*b*c*(a+b*arctanh(c*x))
*ln(2-2/(c*x+1))/d-c*(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-b^2*c*polylog(
2,-1+2/(c*x+1))/d+b*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))/d+1/2*b^2
*c*polylog(3,-1+2/(c*x+1))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx$$

$$= \frac{-\frac{a^2}{x} - a^2c\log(x) + a^2c\log(1+cx) + \frac{ab\left(-2\operatorname{arctanh}(cx)\left(1+cx\log\left(1-e^{-2\operatorname{arctanh}(cx)}\right)\right)+2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+cx\operatorname{PolyLog}(\ldots}{x}}{\ldots}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

output
```
(-(a^2/x) - a^2*c*Log[x] + a^2*c*Log[1 + c*x] + (a*b*(-2*ArcTanh[c*x]*(1 +
 c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]] +
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x + b^2*c*((-1/24*I)*Pi^3 + ArcTanh[
c*x]^2 - ArcTanh[c*x]^2/(c*x) + (2*ArcTanh[c*x]^3)/3 + 2*ArcTanh[c*x]*Log[
1 - E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - Po
lyLog[2, E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])
] + PolyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6496, 27, 6452, 6494, 6550, 6494, 2897, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(cdx+d)}\,dx$$

$$\Big\downarrow \text{ 6496}$$

$$\frac{\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2}dx}{d} - c\int \frac{(a+b\operatorname{arctanh}(cx))^2}{dx(cx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}dx}{d}-\frac{c\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-\frac{c\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6494}$$

$$\frac{2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)}{d}$$

$$\Big\downarrow \text{6550}$$

$$\frac{2bc\left(\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)}{d}$$

$$\Big\downarrow \text{6494}$$

$$\frac{2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)}{d}$$

$$\Big\downarrow \text{2897}$$

$$\frac{2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)}{d}$$

$$\Big\downarrow \text{6618}$$

$$\frac{2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-$$
$$\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{1-c^2x^2}dx\right)\right)}{d}$$

$$\Big\downarrow \; 7164$$

$$\frac{2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}}{d}-$$
$$\frac{c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{4c}\right)\right)}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

output
```
(-((a + b*ArcTanh[c*x])^2/x) + 2*b*c*((a + b*ArcTanh[c*x])^2/(2*b) + (a +
b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)])/2)
)/d - (c*((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - 2*b*c*(((a + b*Arc
Tanh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 +
 c*x)])/(4*c))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.60 (sec) , antiderivative size = 4135, normalized size of antiderivative = 25.52

method	result	size
parts	Expression too large to display	4135
derivativedivides	Expression too large to display	4170
default	Expression too large to display	4170

input
```
int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(c*ln(c*x+1)-1/x-c*ln(x))+b^2/d*c*(-arctanh(c*x)^2*ln(1-(c*x+1)/(-c^
2*x^2+1)^(1/2))-2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-arcta
nh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*arctanh(c*x)*polylog(2,-(c*x+
1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-arctanh
(c*x)^2*ln(c*x)+ln(2)*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-ln(2)*dilog(1+(c*x
+1)/(-c^2*x^2+1)^(1/2))+ln(2)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+ln(2)*
polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^2*ln(c*x+1)+arctanh(c*
x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+
1)^(1/2))+2/3*arctanh(c*x)^3-dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+(c*
x+1)/(-c^2*x^2+1)^(1/2))+ln(2)*arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2
))-arctanh(c*x)^2+polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+polylog(2,-(c*x+1)
/(-c^2*x^2+1)^(1/2))+2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*polylog(3,
(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2/c/x+1/2*I*Pi*csgn(I*(c*x+1)^2/(
c^2*x^2-1))^3*(arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)*
ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2+polylog(2,(c*x+1)/(-c^2*x^
2+1)^(1/2))+polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2)))+1/2*I*Pi*csgn(I*(-(c*x
+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*(arctanh(c*x)*ln(1-(c*x+
1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-arcta
nh(c*x)^2+polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+polylog(2,-(c*x+1)/(-c^2*x
^2+1)^(1/2)))+1/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x...
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2}{(cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^3 + d*x^2)
, x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx=\frac{\int \frac{a^2}{cx^3+x^2}\,dx+\int \frac{b^2\operatorname{atanh}^2(cx)}{cx^3+x^2}\,dx+\int \frac{2ab\operatorname{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d),x)
```

output
```
(Integral(a**2/(c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)**2/(c*x**3 +
 x**2), x) + Integral(2*a*b*atanh(c*x)/(c*x**3 + x**2), x))/d
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2}{(cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="maxima")
```

output
```
a^2*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/4*(b^2*c*x*log(c*x + 1)
- b^2)*log(-c*x + 1)^2/(d*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1
)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(b^2*c^2*x^2 + 2*a*b - (2*a*b*c -
 b^2*c)*x - (b^2*c^3*x^3 + b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*log(
-c*x + 1))/(c^2*d*x^4 - d*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2}{(cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx=\int \frac{(a+b\operatorname{atanh}(c\,x))^2}{x^2\,(d+c\,d\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)}\,dx$$
$$=\frac{2\left(\int \frac{atanh(cx)}{c\,x^3+x^2}dx\right)abx+\left(\int \frac{atanh(cx)^2}{c\,x^3+x^2}dx\right)b^2x+\log(cx+1)\,a^2cx-\log(x)\,a^2cx-a^2}{dx}$$

input
```
int((a+b*atanh(c*x))^2/x^2/(c*d*x+d),x)
```

output
```
(2*int(atanh(c*x)/(c*x**3 + x**2),x)*a*b*x + int(atanh(c*x)**2/(c*x**3 + x
**2),x)*b**2*x + log(c*x + 1)*a**2*c*x - log(x)*a**2*c*x - a**2)/(d*x)
```

3.101 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x^3(d+cdx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 250

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^3(d+cdx)}\,dx = & -\frac{bc(a+b\operatorname{arctanh}(cx))}{dx}-\frac{c^2(a+b\operatorname{arctanh}(cx))^2}{2d} \\
& -\frac{(a+b\operatorname{arctanh}(cx))^2}{2dx^2}+\frac{c(a+b\operatorname{arctanh}(cx))^2}{dx} \\
& +\frac{b^2c^2\log(x)}{d}-\frac{b^2c^2\log\left(1-c^2x^2\right)}{2d} \\
& -\frac{2bc^2(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d} \\
& +\frac{c^2(a+b\operatorname{arctanh}(cx))^2\log\left(2-\frac{2}{1+cx}\right)}{d} \\
& +\frac{b^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d} \\
& -\frac{bc^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d} \\
& -\frac{b^2c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)}{2d}
\end{aligned}
$$

output
```
-b*c*(a+b*arctanh(c*x))/d/x-1/2*c^2*(a+b*arctanh(c*x))^2/d-1/2*(a+b*arctan
h(c*x))^2/d/x^2+c*(a+b*arctanh(c*x))^2/d/x+b^2*c^2*ln(x)/d-1/2*b^2*c^2*ln(
-c^2*x^2+1)/d-2*b*c^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d+c^2*(a+b*arctan
h(c*x))^2*ln(2-2/(c*x+1))/d+b^2*c^2*polylog(2,-1+2/(c*x+1))/d-b*c^2*(a+b*a
rctanh(c*x))*polylog(2,-1+2/(c*x+1))/d-1/2*b^2*c^2*polylog(3,-1+2/(c*x+1))
/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)}\,dx$$

$$= \frac{-\frac{a^2}{x^2}+\frac{2a^2c}{x}+2a^2c^2\log(x)-2a^2c^2\log(1+cx)+\frac{2ab\left(\text{arctanh}(cx)\left(-1+2cx+c^2x^2+2c^2x^2\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right)-cx\left(\right.}{x^2}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

output
```
(-(a^2/x^2) + (2*a^2*c)/x + 2*a^2*c^2*Log[x] - 2*a^2*c^2*Log[1 + c*x] + (2
*a*b*(ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*ArcTanh
[c*x])]) - c*x*(1 + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - c^2*x^2*PolyLog[
2, E^(-2*ArcTanh[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTanh[c*x]/(c*x
) - ArcTanh[c*x]^2/2 - ArcTanh[c*x]^2/(2*c^2*x^2) + ArcTanh[c*x]^2/(c*x) -
 (2*ArcTanh[c*x]^3)/3 - 2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + ArcT
anh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + Log[(c*x)/Sqrt[1 - c^2*x^2]] + Po
lyLog[2, E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])
] - PolyLog[3, E^(2*ArcTanh[c*x])]/2))/(2*d)
```

Rubi [A] (verified)

Time = 3.92 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {6496, 27, 6452, 6496, 6452, 6494, 6544, 6452, 243, 47, 14, 16, 6510, 6550, 6494, 2897, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(cdx+d)}dx$$

$$\Big\downarrow \text{6496}$$

$$\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}dx}{d}-c\int \frac{(a+b\text{arctanh}(cx))^2}{dx^2(cx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}dx}{d}-\frac{c\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6496}$$

$$\frac{bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}dx-c\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)}{d}$$

$$\Big\downarrow \text{6494}$$

$$\frac{bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx - \frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d} - \frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx - c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2 - 2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right) - \frac{(a}{}\right.}{d}$$

$\big\downarrow$ 6544

$$\frac{bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx + \int \frac{a+b\text{arctanh}(cx)}{x^2}dx\right) - \frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d} - \frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx - c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2 - 2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right) - \frac{(a}{}\right.}{d}$$

$\big\downarrow$ 6452

$$\frac{bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx + bc\int \frac{1}{x(1-c^2x^2)}dx - \frac{a+b\text{arctanh}(cx)}{x}\right) - \frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d} - \frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx - c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2 - 2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right) - \frac{(a}{}\right.}{d}$$

$\big\downarrow$ 243

$$\frac{bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx + \frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2 - \frac{a+b\text{arctanh}(cx)}{x}\right) - \frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d} - \frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx - c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2 - 2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right) - \frac{(a}{}\right.}{d}$$

$\big\downarrow$ 47

$$\frac{bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx + \frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2 + \int \frac{1}{x^2}dx^2\right) - \frac{a+b\text{arctanh}(cx)}{x}\right) - \frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d} - \frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx - c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2 - 2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right) - \frac{(a}{}\right.}{d}$$

$\big\downarrow$ 14

$$\frac{bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{(a}{}\right.}{d}$$

$$\Big\downarrow \ 16$$

$$\frac{bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{(a}{}\right.}{d}$$

$$\Big\downarrow \ 6510$$

$$\frac{bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{(a}{}\right.}{d}$$

$$\Big\downarrow \ 6550$$

$$\frac{bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)+2bc\left(\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\right.\right.}{d}$$

$$\Big\downarrow \ 6494$$

$$\frac{bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)-c\left(\log\left(2-\frac{2}{cx+1}\right)(a-\right.\right.}{d}$$

$$\Big\downarrow \ 2897$$

$$\frac{bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int\frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)+2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\text{lo}\right.\right.}{d}$$

$$\Big\downarrow \text{6618}$$

$$\frac{bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{1-c^2x^2}dx\right.\right.\right.}{}$$

$$\Big\downarrow \text{7164}$$

$$\frac{bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}}{d}-\frac{c\left(2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-c\left(\log\left(2-\frac{2}{cx+1}\right)\right.\right.}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTanh[c*x])^2/x^2 + b*c*(-((a + b*ArcTanh[c*x])/x) + (c*(a
+ b*ArcTanh[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2))/d - (
c*(-((a + b*ArcTanh[c*x])^2/x) + 2*b*c*((a + b*ArcTanh[c*x])^2/(2*b) + (a
+ b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)])/
2) - c*((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - 2*b*c*(((a + b*ArcTa
nh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 + c
*x)])/(4*c)))))/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.06 (sec) , antiderivative size = 1492, normalized size of antiderivative = 5.97

method	result	size
derivativedivides	Expression too large to display	1492
default	Expression too large to display	1492
parts	Expression too large to display	1494

input
```
int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/d*(-1/2/c^2/x^2+ln(c*x)+1/c/x-ln(c*x+1))+b^2/d*(arctanh(c*x)^2*ln
(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+
1)^(1/2))+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*p
olylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^
2+1)-1)-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x
)^2+1/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2
-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*arcta
nh(c*x)^2+arctanh(c*x)^2*ln(c*x)-arctanh(c*x)^2*ln(c*x+1)-2*arctanh(c*x)*l
n(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)-1/2*arcta
nh(c*x)^2/c^2/x^2-2/3*arctanh(c*x)^3+2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-2
*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*
x^2-1)))*arctanh(c*x)^2-1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+ln(1+(c*x+1)
/(-c^2*x^2+1)^(1/2))-2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,
(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^2/c/x+1/2*I*Pi*csgn(I*(c*x+1)^2/(
c^2*x^2-1))^3*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x
+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2+1/2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^
(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+I*Pi*csgn(I*(c*x+...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^4 + d*x^3)
, x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)}\,dx = \frac{\int \frac{a^2}{cx^4+x^3}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{cx^4+x^3}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{cx^4+x^3}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d),x)
```

output
```
(Integral(a**2/(c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)**2/(c*x**4 +
 x**3), x) + Integral(2*a*b*atanh(c*x)/(c*x**4 + x**3), x))/d
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="maxima")
```

output
```
-1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a^2 - 1
/8*(2*b^2*c^2*x^2*log(c*x + 1) - 2*b^2*c*x + b^2)*log(-c*x + 1)^2/(d*x^2)
+ integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*
x + 1) - (2*b^2*c^3*x^3 + b^2*c^2*x^2 - 4*a*b + (4*a*b*c - b^2*c)*x - 2*(b
^2*c^4*x^4 + b^2*c^3*x^3 - b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^
2*d*x^5 - d*x^3), x)
```

Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^3(d + cdx)} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{(cdx + d)x^3} dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^3(d + cdx)} dx = \int \frac{(a + b\,\text{atanh}(c\,x))^2}{x^3\,(d + c\,d\,x)} dx$$

input
```
int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)}\,dx = \frac{4\left(\int \frac{atanh(cx)}{c\,x^4+x^3}dx\right)ab\,x^2+2\left(\int \frac{atanh(cx)^2}{c\,x^4+x^3}dx\right)b^2x^2-2\log(cx+1)\,a^2c^2x^2+2\log(x)\,a^2c^2x^2+2a^2cx-a^2}{2d\,x^2}$$

input
```
int((a+b*atanh(c*x))^2/x^3/(c*d*x+d),x)
```

output
```
(4*int(atanh(c*x)/(c*x**4 + x**3),x)*a*b*x**2 + 2*int(atanh(c*x)**2/(c*x**
4 + x**3),x)*b**2*x**2 - 2*log(c*x + 1)*a**2*c**2*x**2 + 2*log(x)*a**2*c**
2*x**2 + 2*a**2*c*x - a**2)/(2*d*x**2)
```

3.102 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x^4(d+cdx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4(d+cdx)}\,dx =& -\frac{b^2c^2}{3dx}+\frac{b^2c^3\operatorname{arctanh}(cx)}{3d}-\frac{bc(a+b\operatorname{arctanh}(cx))}{3dx^2}\\&+\frac{bc^2(a+b\operatorname{arctanh}(cx))}{dx}+\frac{5c^3(a+b\operatorname{arctanh}(cx))^2}{6d}\\&-\frac{(a+b\operatorname{arctanh}(cx))^2}{3dx^3}+\frac{c(a+b\operatorname{arctanh}(cx))^2}{2dx^2}\\&-\frac{c^2(a+b\operatorname{arctanh}(cx))^2}{dx}-\frac{b^2c^3\log(x)}{d}+\frac{b^2c^3\log\left(1-c^2x^2\right)}{2d}\\&+\frac{8bc^3(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{3d}\\&-\frac{c^3(a+b\operatorname{arctanh}(cx))^2\log\left(2-\frac{2}{1+cx}\right)}{d}\\&-\frac{4b^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{3d}\\&+\frac{bc^3(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d}\\&+\frac{b^2c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

output
```
-1/3*b^2*c^2/d/x+1/3*b^2*c^3*arctanh(c*x)/d-1/3*b*c*(a+b*arctanh(c*x))/d/x
^2+b*c^2*(a+b*arctanh(c*x))/d/x+5/6*c^3*(a+b*arctanh(c*x))^2/d-1/3*(a+b*ar
ctanh(c*x))^2/d/x^3+1/2*c*(a+b*arctanh(c*x))^2/d/x^2-c^2*(a+b*arctanh(c*x)
)^2/d/x-b^2*c^3*ln(x)/d+1/2*b^2*c^3*ln(-c^2*x^2+1)/d+8/3*b*c^3*(a+b*arctan
h(c*x))*ln(2-2/(c*x+1))/d-c^3*(a+b*arctanh(c*x))^2*ln(2-2/(c*x+1))/d-4/3*b
^2*c^3*polylog(2,-1+2/(c*x+1))/d+b*c^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(
c*x+1))/d+1/2*b^2*c^3*polylog(3,-1+2/(c*x+1))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4(d+cdx)}\,dx$$

$$= \frac{-\frac{8a^2}{x^3}+\frac{12a^2c}{x^2}-\frac{24a^2c^2}{x}-24a^2c^3\log(x)+24a^2c^3\log(1+cx)-\frac{8ab\left(\text{arctanh}(cx)\left(2-3cx+6c^2x^2+3c^3x^3+6c^3x^3\log\left(1-\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

output
```
((-8*a^2)/x^3 + (12*a^2*c)/x^2 - (24*a^2*c^2)/x - 24*a^2*c^3*Log[x] + 24*a
^2*c^3*Log[1 + c*x] - (8*a*b*(ArcTanh[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*
x^3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*(-1 + 3*c*x + c^2*x^2
+ 8*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - 3*c^3*x^3*PolyLog[2, E^(-2*Arc
Tanh[c*x])]))/x^3 + b^2*c^3*((-I)*Pi^3 - 8/(c*x) + 8*ArcTanh[c*x] - (8*Arc
Tanh[c*x])/(c^2*x^2) + (24*ArcTanh[c*x])/(c*x) + 20*ArcTanh[c*x]^2 - (8*Ar
cTanh[c*x]^2)/(c^3*x^3) + (12*ArcTanh[c*x]^2)/(c^2*x^2) - (24*ArcTanh[c*x]
^2)/(c*x) + 16*ArcTanh[c*x]^3 + 64*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x]
)] - 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*Log[(c*x)/Sqrt[1 -
 c^2*x^2]] - 32*PolyLog[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[
2, E^(2*ArcTanh[c*x])] + 12*PolyLog[3, E^(2*ArcTanh[c*x])]))/(24*d)
```

Rubi [A] (verified)

Time = 4.57 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.10, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6496, 27, 6452, 6496, 6452, 6496, 6452, 6494, 6544, 6452, 243, 47, 14, 16, 264, 219, 6510, 6550, 6494, 2897, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4(cdx+d)}dx$$

$$\Big\downarrow \text{6496}$$

$$\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{x^4}dx}{d}-c\int \frac{(a+b\text{arctanh}(cx))^2}{dx^3(cx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{x^4}dx}{d}-\frac{c\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(cx+1)}dx}{d}$$

$$\Big\downarrow \text{6496}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}dx-c\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(cx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-c\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(cx+1)}dx-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{6496}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-c\left(\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}dx-c\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)-\frac{(a+b\text{arcta}}{2x^{}}\right.}{d}$$

$$\Big\downarrow \text{6494}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int \frac{(a+b\text{arcta}}{}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6544}$$

$$\frac{\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\int \frac{a+b\text{arctanh}(cx)}{x^3}dx\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\int \frac{a+b\text{arctanh}(cx)}{x^2}dx\right)-c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arcta}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx\cdots}\right.\right.\right.\right.}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-\frac{c\left(bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)-c\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\right.\right.\right.\right.}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int\frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-$$
$$c\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\int\frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-c\left(2bc\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\right.\right.$$

$\downarrow$ 14

$$\frac{\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int\frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-$$
$$c\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-c\left(2bc\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\right.\right.$$

$\downarrow$ 16

$$\frac{\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int\frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-$$
$$c\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-c\left(2bc\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\right.\right.$$

$\downarrow$ 264

$$\frac{\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-$$
$$c\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-c\left(2bc\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\right.\right.$$

$\downarrow$ 219

$$\frac{\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-$$
$$c\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-c\left(2bc\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\right.\right.$$

$\downarrow$ 6510

$$\frac{\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}}{d}-$$

$$c\left(-c\left(2bc\int\frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int\frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)\right.\right.$$

$$\Big\downarrow \; 6550$$

$$\frac{\frac{2}{3}bc\left(c^2\left(\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\text{arctanh}(cx)}{3x^3}}{d}$$

$$c\left(-c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int\frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)+2bc\left(\int\frac{a+b\text{arctanh}(cx)}{x(cx+1)}\right.\right.\right.$$

$$\Big\downarrow \; 6494$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-bc\int\frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}b\right.}{d}$$

$$c\left(-c\left(2bc\left(-bc\int\frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)-c\left(\log\left(2-\frac{2}{cx+1}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2897$$

$$\frac{\frac{2}{3}bc\left(c^2\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\right.}{d}$$

$$c\left(-c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\int\frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)+2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}\right.\right.\right.$$

$$\Big\downarrow \; 6618$$

$$\frac{\frac{2}{3}bc\left(c^2\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\right.}{d}$$

$$c\left(-c\left(-c\left(\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right.}{1-c^2x^2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{\frac{2}{3}bc\left(c^2\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\ldots\right.}{d}$$

$$\frac{c\left(bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-c\left(2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(\ldots\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

output
```
(-1/3*(a + b*ArcTanh[c*x])^2/x^3 + (2*b*c*(-1/2*(a + b*ArcTanh[c*x])/x^2 +
 (b*c*(-x^(-1) + c*ArcTanh[c*x]))/2 + c^2*((a + b*ArcTanh[c*x])^2/(2*b) +
(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)
])/2)))/3)/d - (c*(-1/2*(a + b*ArcTanh[c*x])^2/x^2 + b*c*(-((a + b*ArcTanh
[c*x])/x) + (c*(a + b*ArcTanh[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 - c^
2*x^2]))/2) - c*(-((a + b*ArcTanh[c*x])^2/x) + 2*b*c*((a + b*ArcTanh[c*x])
^2/(2*b) + (a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 +
2/(1 + c*x)])/2) - c*((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - 2*b*c*
(((a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/(2*c) + (b*PolyLog[3,
 -1 + 2/(1 + c*x)])/(4*c))))))/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.85 (sec) , antiderivative size = 1718, normalized size of antiderivative = 5.14

method	result	size
derivativedivides	Expression too large to display	1718
default	Expression too large to display	1718
parts	Expression too large to display	1720

input
```
int((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x,method=_RETURNVERBOSE)
```

output
```
c^3*(a^2/d*(-1/3/c^3/x^3-1/c/x+1/2/c^2/x^2-ln(c*x)+ln(c*x+1))+b^2/d*(-arct
anh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*arctanh(c*x)*polylog(2,(c*x+
1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*a
rctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^2*ln((c*x+
1)^2/(-c^2*x^2+1)-1)+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1))
)*arctanh(c*x)^2-arctanh(c*x)^2*ln(c*x)+arctanh(c*x)^2*ln(c*x+1)+8/3*arcta
nh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)-
1/3*arctanh(c*x)^2/c^3/x^3+1/2*arctanh(c*x)^2/c^2/x^2+2/3*arctanh(c*x)^3-1
/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-1/2*I*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2-8/3*dilog((
c*x+1)/(-c^2*x^2+1)^(1/2))+8/3*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-11/6*ar
ctanh(c*x)^2-ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*polylog(3,-(c*x+1)/(-c^2*x
^2+1)^(1/2))+2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^2/c/x+1/
6*((-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-(-c^2*x^2+1)^(1/2)+c*x-1)*arctanh(c*x)
/c^2/x^2+1/6*(-(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2+(-c^2*x^2+1)^(1/2)+c*x-1)*
arctanh(c*x)/c^2/x^2-1/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)
^2/(c^2*x^2-1)))^3*arctanh(c*x)^2-2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)
^(1/2))-ln(2)*arctanh(c*x)^2+1/3*(c*x-(-c^2*x^2+1)^(1/2)+1)/c/x*arctanh(c*
x)+1/3*arctanh(c*x)*(c*x+(-c^2*x^2+1)^(1/2)+1)/c/x+1/2*I*Pi*csgn(I*(c*x...
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4(d+cdx)}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2}{(cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^5 + d*x^4)
, x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4(d+cdx)}\,dx = \frac{\int \frac{a^2}{cx^5+x^4}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{cx^5+x^4}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx^5+x^4}\,dx}{d}$$

input
```
integrate((a+b*atanh(c*x))**2/x**4/(c*d*x+d),x)
```

output
```
(Integral(a**2/(c*x**5 + x**4), x) + Integral(b**2*atanh(c*x)**2/(c*x**5 +
 x**4), x) + Integral(2*a*b*atanh(c*x)/(c*x**5 + x**4), x))/d
```

Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4(d+cdx)}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2}{(cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="maxima")
```

output
```
1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^
3))*a^2 + 1/24*(6*b^2*c^3*x^3*log(c*x + 1) - 6*b^2*c^2*x^2 + 3*b^2*c*x - 2
*b^2)*log(-c*x + 1)^2/(d*x^3) - integrate(-1/12*(3*(b^2*c*x - b^2)*log(c*x
 + 1)^2 + 12*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 3*b^2*c^3*x^3
 - b^2*c^2*x^2 + 12*a*b - 2*(6*a*b*c - b^2*c)*x - 6*(b^2*c^5*x^5 + b^2*c^4
*x^4 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x^6 - d*x^4), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4(d+cdx)}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4(d+cdx)}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2}{x^4\,(d+c\,d\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x^4*(d + c*d*x)),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^4*(d + c*d*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4(d+cdx)}\,dx$$
$$= \frac{6\left(\int \frac{atanh(cx)^2}{c^2x^3-x}dx\right) b^2c^3x^3 - 6\log(x)\,a^2c^3x^3 + 2atanh(cx)^3\,b^2c^3x^3 + 6\log(cx+1)\,a^2c^3x^3 - 4atanh(cx)\,ab}{}$$

input
```
int((a+b*atanh(c*x))^2/x^4/(c*d*x+d),x)
```

output
```
(2*atanh(c*x)**3*b**2*c**3*x**3 + 6*atanh(c*x)**2*a*b*c**3*x**3 - 3*atanh(
c*x)**2*b**2*c**3*x**3 - 6*atanh(c*x)**2*b**2*c**2*x**2 + 3*atanh(c*x)**2*
b**2*c*x - 2*atanh(c*x)**2*b**2 - 16*atanh(c*x)*a*b*c**3*x**3 - 12*atanh(c
*x)*a*b*c**2*x**2 - 4*atanh(c*x)*a*b + 6*atanh(c*x)*b**2*c**2*x**2 + 6*ata
nh(c*x)*b**2*c*x + 12*int(atanh(c*x)/(c**2*x**5 - x**3),x)*a*b*c*x**3 - 16
*int(atanh(c*x)/(c**2*x**5 - x**3),x)*b**2*c*x**3 + 6*int(atanh(c*x)**2/(c
**2*x**3 - x),x)*b**2*c**3*x**3 - 16*log(c**2*x - c)*a*b*c**3*x**3 + 6*log
(c**2*x - c)*b**2*c**3*x**3 + 6*log(c*x + 1)*a**2*c**3*x**3 - 6*log(x)*a**
2*c**3*x**3 + 16*log(x)*a*b*c**3*x**3 - 6*log(x)*b**2*c**3*x**3 - 6*a**2*c
**2*x**2 + 3*a**2*c*x - 2*a**2 - 2*a*b*c*x + 6*b**2*c**2*x**2)/(6*d*x**3)
```

3.103 $\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 394

$$\begin{aligned}\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx &= -\frac{2abx}{c^4d^2}+\frac{b^2x}{3c^4d^2}-\frac{b^2}{2c^5d^2(1+cx)}+\frac{b^2\text{arctanh}(cx)}{6c^5d^2}\\
&-\frac{2b^2x\text{arctanh}(cx)}{c^4d^2}+\frac{bx^2(a+b\text{arctanh}(cx))}{3c^3d^2}\\
&-\frac{b(a+b\text{arctanh}(cx))}{c^5d^2(1+cx)}+\frac{29(a+b\text{arctanh}(cx))^2}{6c^5d^2}\\
&+\frac{3x(a+b\text{arctanh}(cx))^2}{c^4d^2}-\frac{x^2(a+b\text{arctanh}(cx))^2}{c^3d^2}\\
&+\frac{x^3(a+b\text{arctanh}(cx))^2}{3c^2d^2}-\frac{(a+b\text{arctanh}(cx))^2}{c^5d^2(1+cx)}\\
&-\frac{20b(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c^5d^2}\\
&+\frac{4(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^5d^2}\\
&-\frac{b^2\log(1-c^2x^2)}{c^5d^2}-\frac{10b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^5d^2}\\
&-\frac{4b(a+b\text{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^5d^2}\\
&-\frac{2b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{c^5d^2}\end{aligned}$$

output
```
-2*a*b*x/c^4/d^2+1/3*b^2*x/c^4/d^2-1/2*b^2/c^5/d^2/(c*x+1)+1/6*b^2*arctanh
(c*x)/c^5/d^2-2*b^2*x*arctanh(c*x)/c^4/d^2+1/3*b*x^2*(a+b*arctanh(c*x))/c^
3/d^2-b*(a+b*arctanh(c*x))/c^5/d^2/(c*x+1)+29/6*(a+b*arctanh(c*x))^2/c^5/d
^2+3*x*(a+b*arctanh(c*x))^2/c^4/d^2-x^2*(a+b*arctanh(c*x))^2/c^3/d^2+1/3*x
^3*(a+b*arctanh(c*x))^2/c^2/d^2-(a+b*arctanh(c*x))^2/c^5/d^2/(c*x+1)-20/3*
b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^5/d^2+4*(a+b*arctanh(c*x))^2*ln(2/(c
*x+1))/c^5/d^2-b^2*ln(-c^2*x^2+1)/c^5/d^2-10/3*b^2*polylog(2,1-2/(-c*x+1))
/c^5/d^2-4*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^5/d^2-2*b^2*polyl
og(3,1-2/(c*x+1))/c^5/d^2
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.08

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$$
$$= \frac{36a^2cx - 12a^2c^2x^2 + 4a^2c^3x^3 - \frac{12a^2}{1+cx} - 48a^2\log(1+cx) + b^2\big(4cx - 4\text{arctanh}(cx) - 24cx\text{arctanh}(cx) +}{}$$

input
```
Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
(36*a^2*c*x - 12*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(1 + c*x) - 48*a^2
*Log[1 + c*x] + b^2*(4*c*x - 4*ArcTanh[c*x] - 24*c*x*ArcTanh[c*x] + 4*c^2*
x^2*ArcTanh[c*x] - 28*ArcTanh[c*x]^2 + 36*c*x*ArcTanh[c*x]^2 - 12*c^2*x^2*
ArcTanh[c*x]^2 + 4*c^3*x^3*ArcTanh[c*x]^2 - 3*Cosh[2*ArcTanh[c*x]] - 6*Arc
Tanh[c*x]*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 8
0*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 48*ArcTanh[c*x]^2*Log[1 + E^
(-2*ArcTanh[c*x])] - 12*Log[1 - c^2*x^2] - 8*(-5 + 6*ArcTanh[c*x])*PolyLog
[2, -E^(-2*ArcTanh[c*x])] - 24*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2
*ArcTanh[c*x]] + 6*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Si
nh[2*ArcTanh[c*x]]) + 2*a*b*(-2 - 12*c*x + 2*c^2*x^2 - 3*Cosh[2*ArcTanh[c*
x]] + 20*Log[1 - c^2*x^2] - 24*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2
*ArcTanh[c*x]] + 2*ArcTanh[c*x]*(6 + 18*c*x - 6*c^2*x^2 + 2*c^3*x^3 - 3*Co
sh[2*ArcTanh[c*x]] + 24*Log[1 + E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*
x]])))/(12*c^5*d^2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(cdx+d)^2}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(-\frac{4(a+b\text{arctanh}(cx))^2}{c^4d^2(cx+1)}+\frac{3(a+b\text{arctanh}(cx))^2}{c^4d^2}+\frac{(a+b\text{arctanh}(cx))^2}{c^4d^2(cx+1)^2}-\frac{2x(a+b\text{arctanh}(cx))^2}{c^3d^2}+\frac{x^2(a+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^5d^2}-\frac{(a+b\text{arctanh}(cx))^2}{c^5d^2(cx+1)}+\frac{29(a+b\text{arctanh}(cx))^2}{6c^5d^2}-\frac{b(a+b\text{arctanh}(cx))}{c^5d^2(cx+1)}-\frac{20b\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{3c^5d^2}+\frac{4\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c^5d^2}+\frac{3x(a+b\text{arctanh}(cx))^2}{c^4d^2}-\frac{x^2(a+b\text{arctanh}(cx))^2}{c^3d^2}+\frac{bx^2(a+b\text{arctanh}(cx))}{3c^3d^2}+\frac{x^3(a+b\text{arctanh}(cx))^2}{3c^2d^2}-\frac{2abx}{c^4d^2}+\frac{b^2\text{arctanh}(cx)}{6c^5d^2}-\frac{2b^2x\text{arctanh}(cx)}{c^4d^2}-\frac{10b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^5d^2}-\frac{2b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^2}-\frac{b^2}{2c^5d^2(cx+1)}+\frac{b^2x}{3c^4d^2}-\frac{b^2\log(1-c^2x^2)}{c^5d^2}$$

input
```
Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
(-2*a*b*x)/(c^4*d^2) + (b^2*x)/(3*c^4*d^2) - b^2/(2*c^5*d^2*(1 + c*x)) + (
b^2*ArcTanh[c*x])/(6*c^5*d^2) - (2*b^2*x*ArcTanh[c*x])/(c^4*d^2) + (b*x^2*
(a + b*ArcTanh[c*x]))/(3*c^3*d^2) - (b*(a + b*ArcTanh[c*x]))/(c^5*d^2*(1 +
 c*x)) + (29*(a + b*ArcTanh[c*x])^2)/(6*c^5*d^2) + (3*x*(a + b*ArcTanh[c*x
])^2)/(c^4*d^2) - (x^2*(a + b*ArcTanh[c*x])^2)/(c^3*d^2) + (x^3*(a + b*Arc
Tanh[c*x])^2)/(3*c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^5*d^2*(1 + c*x)) - (
20*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^5*d^2) + (4*(a + b*ArcTan
h[c*x])^2*Log[2/(1 + c*x)])/(c^5*d^2) - (b^2*Log[1 - c^2*x^2])/(c^5*d^2) -
 (10*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^5*d^2) - (4*b*(a + b*ArcTanh[c*
x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^2) - (2*b^2*PolyLog[3, 1 - 2/(1 +
c*x)])/(c^5*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.91 (sec) , antiderivative size = 1050, normalized size of antiderivative = 2.66

method	result	size
derivativedivides	Expression too large to display	1050
default	Expression too large to display	1050
parts	Expression too large to display	1060

input
```
int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a^2/d^2*(1/3*x^3*c^3-c^2*x^2+3*c*x-1/(c*x+1)-4*ln(c*x+1))+b^2/d^2*(
-2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x
+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-1/3-4
*arctanh(c*x)^2*ln(c*x+1)-20/3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1
/2))-20/3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*dilog(1+I*(
c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/3*
c*x+1/3*arctanh(c*x)^2*c^3*x^3-8/3*arctanh(c*x)^3-arctanh(c*x)^2*c^2*x^2+3
*arctanh(c*x)^2*c*x+1/4/(c*x+1)*(c*x-1)+29/6*arctanh(c*x)^2-2*polylog(3,-(
c*x+1)^2/(-c^2*x^2+1))-4/3*(c*x+1)*arctanh(c*x)+2*ln(1+(c*x+1)^2/(-c^2*x^2
+1))+4*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+2*I*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2+2*I*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/3*(c*x-3)*(c*x+1)*arctanh(c*x)+
2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
arctanh(c*x)^2+2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1
)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-2*I*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^
2*arctanh(c*x)^2+4*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^
2/(c^2*x^2-1))^2*arctanh(c*x)^2+8*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(
1/2))+4*ln(2)*arctanh(c*x)^2+1/2*arctanh(c*x)*(c*x-1)/(c*x+1)-1/(c*x+1)*ar
ctanh(c*x)^2)+2*b*a/d^2*(1/3*arctanh(c*x)*c^3*x^3-arctanh(c*x)*c^2*x^2+...
```

Fricas [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^4}{(cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^2*
d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{\int \frac{a^2x^4}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^4\,\text{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^4\,\text{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

input
```
integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2*x**4/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**4*atanh
(c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**4*atanh(c*x)/(c**
2*x**2 + 2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}(cx)+a)^2x^4}{(cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-1/3*a^2*(3/(c^6*d^2*x + c^5*d^2) - (c^2*x^3 - 3*c*x^2 + 9*x)/(c^4*d^2) +
12*log(c*x + 1)/(c^5*d^2)) + 1/12*(b^2*c^4*x^4 - 2*b^2*c^3*x^3 + 6*b^2*c^2
*x^2 + 9*b^2*c*x - 3*b^2 - 12*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^
2/(c^6*d^2*x + c^5*d^2) - integrate(-1/12*(3*(b^2*c^5*x^5 - b^2*c^4*x^4)*l
og(c*x + 1)^2 + 12*(a*b*c^5*x^5 - a*b*c^4*x^4)*log(c*x + 1) - 2*(4*b^2*c^3
*x^3 + 15*b^2*c^2*x^2 + (6*a*b*c^5 + b^2*c^5)*x^5 - (6*a*b*c^4 + b^2*c^4)*
x^4 + 6*b^2*c*x - 3*b^2 + 3*(b^2*c^5*x^5 - b^2*c^4*x^4 - 4*b^2*c^2*x^2 - 8
*b^2*c*x - 4*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^7*d^2*x^3 + c^6*d^2*x^2
- c^5*d^2*x - c^4*d^2), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^4}{(cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{x^4\,(a+b\,\text{atanh}(c\,x))^2}{(d+c\,d\,x)^2}\,dx$$

input
```
int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

output
```
int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{6\left(\int \frac{atanh(cx)x^4}{c^2x^2+2cx+1}dx\right)ab\,c^6x+6\left(\int \frac{atanh(cx)x^4}{c^2x^2+2cx+1}dx\right)ab\,c^5+3\left(\int \frac{atanh(cx)^2x^4}{c^2x^2+2cx+1}dx\right)b^2c^6x+3\left(\int \frac{atanh(cx)^2x^4}{c^2x^2+2cx+1}dx\right)b^2}{3c^5d^2\,(cx+1)}$$

input
```
int(x^4*(a+b*atanh(c*x))^2/(c*d*x+d)^2,x)
```

output
```
(6*int((atanh(c*x)*x**4)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**6*x + 6*int((at
anh(c*x)*x**4)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**5 + 3*int((atanh(c*x)**2*
x**4)/(c**2*x**2 + 2*c*x + 1),x)*b**2*c**6*x + 3*int((atanh(c*x)**2*x**4)/
(c**2*x**2 + 2*c*x + 1),x)*b**2*c**5 - 12*log(c*x + 1)*a**2*c*x - 12*log(c
*x + 1)*a**2 + a**2*c**4*x**4 - 2*a**2*c**3*x**3 + 6*a**2*c**2*x**2 + 12*a
**2*c*x)/(3*c**5*d**2*(c*x + 1))
```

3.104 $\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 331

$$\begin{aligned}\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = &\frac{abx}{c^3d^2}+\frac{b^2}{2c^4d^2(1+cx)}-\frac{b^2\text{arctanh}(cx)}{2c^4d^2}+\frac{b^2x\text{arctanh}(cx)}{c^3d^2}\\&+\frac{b(a+b\text{arctanh}(cx))}{c^4d^2(1+cx)}-\frac{3(a+b\text{arctanh}(cx))^2}{c^4d^2}\\&-\frac{2x(a+b\text{arctanh}(cx))^2}{c^3d^2}+\frac{x^2(a+b\text{arctanh}(cx))^2}{2c^2d^2}\\&+\frac{(a+b\text{arctanh}(cx))^2}{c^4d^2(1+cx)}+\frac{4b(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^4d^2}\\&-\frac{3(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^4d^2}\\&+\frac{b^2\log(1-c^2x^2)}{2c^4d^2}+\frac{2b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^2}\\&+\frac{3b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^4d^2}\\&+\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^4d^2}\end{aligned}$$

output
```
a*b*x/c^3/d^2+1/2*b^2/c^4/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/c^4/d^2+b^2*x*a
rctanh(c*x)/c^3/d^2+b*(a+b*arctanh(c*x))/c^4/d^2/(c*x+1)-3*(a+b*arctanh(c*
x))^2/c^4/d^2-2*x*(a+b*arctanh(c*x))^2/c^3/d^2+1/2*x^2*(a+b*arctanh(c*x))^
2/c^2/d^2+(a+b*arctanh(c*x))^2/c^4/d^2/(c*x+1)+4*b*(a+b*arctanh(c*x))*ln(2
/(-c*x+1))/c^4/d^2-3*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^4/d^2+1/2*b^2*ln
(-c^2*x^2+1)/c^4/d^2+2*b^2*polylog(2,1-2/(-c*x+1))/c^4/d^2+3*b*(a+b*arctan
h(c*x))*polylog(2,1-2/(c*x+1))/c^4/d^2+3/2*b^2*polylog(3,1-2/(c*x+1))/c^4/
d^2
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$$
$$= \frac{-8a^2cx+2a^2c^2x^2+\frac{4a^2}{1+cx}+12a^2\log(1+cx)+2ab\big(2cx+\cosh(2\text{arctanh}(cx))-4\log\left(1-c^2x^2\right)+6\,\text{Po}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
(-8*a^2*c*x + 2*a^2*c^2*x^2 + (4*a^2)/(1 + c*x) + 12*a^2*Log[1 + c*x] + 2*
a*b*(2*c*x + Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(
-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[
c*x]] - 6*Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*Ar
cTanh[c*x]]) + b^2*(4*c*x*ArcTanh[c*x] + 6*ArcTanh[c*x]^2 - 8*c*x*ArcTanh[
c*x]^2 + 2*c^2*x^2*ArcTanh[c*x]^2 + Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*
Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 16*ArcTanh[
c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 12*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTan
h[c*x])] + 2*Log[1 - c^2*x^2] + 4*(-2 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*
ArcTanh[c*x])] + 6*PolyLog[3, -E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]
 - 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c
*x]]))/(4*c^4*d^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{3(a+b\text{arctanh}(cx))^2}{c^3d^2(cx+1)}-\frac{2(a+b\text{arctanh}(cx))^2}{c^3d^2}-\frac{(a+b\text{arctanh}(cx))^2}{c^3d^2(cx+1)^2}+\frac{x(a+b\text{arctanh}(cx))^2}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^4d^2}+\frac{b(a+b\text{arctanh}(cx))}{c^4d^2(cx+1)}+\frac{(a+b\text{arctanh}(cx))^2}{c^4d^2(cx+1)}-\frac{3(a+b\text{arctanh}(cx))^2}{c^4d^2}+\frac{4b\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c^4d^2}-\frac{3\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c^4d^2}-\frac{2x(a+b\text{arctanh}(cx))^2}{c^3d^2}+\frac{x^2(a+b\text{arctanh}(cx))^2}{2c^2d^2}+\frac{abx}{c^3d^2}-\frac{b^2\text{arctanh}(cx)}{2c^4d^2}+\frac{b^2x\text{arctanh}(cx)}{c^3d^2}+\frac{2b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^2}+\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{b^2}{2c^4d^2(cx+1)}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^4d^2}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
(a*b*x)/(c^3*d^2) + b^2/(2*c^4*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^4*
d^2) + (b^2*x*ArcTanh[c*x])/(c^3*d^2) + (b*(a + b*ArcTanh[c*x]))/(c^4*d^2*
(1 + c*x)) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^2) - (2*x*(a + b*ArcTanh[c*
x])^2)/(c^3*d^2) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^2*d^2) + (a + b*ArcTa
nh[c*x])^2/(c^4*d^2*(1 + c*x)) + (4*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)
])/(c^4*d^2) - (3*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^2) + (b^
2*Log[1 - c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^4
*d^2) + (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^4*d^2) +
 (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.28 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.97

method	result	size
derivativedivides	Expression too large to display	983
default	Expression too large to display	983
parts	Expression too large to display	994

input
```
int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2/d^2*(1/2*c^2*x^2-2*c*x+1/(c*x+1)+3*ln(c*x+1))+b^2/d^2*(3/2*I*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(
c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2+3*arctanh(c*
x)^2*ln(c*x+1)+4*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+4*arctanh
(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+4*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^
(1/2))+4*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)^3+1/2*arctan
h(c*x)^2*c^2*x^2-2*arctanh(c*x)^2*c*x-1/4/(c*x+1)*(c*x-1)-3*arctanh(c*x)^2
+3/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+(c*x+1)*arctanh(c*x)-ln(1+(c*x+1)^
2/(-c^2*x^2+1))-3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/2*I*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2-
3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-6*arctanh(c*x)^2*l
n((c*x+1)/(-c^2*x^2+1)^(1/2))-3*ln(2)*arctanh(c*x)^2-1/2*arctanh(c*x)*(c*x
-1)/(c*x+1)+1/(c*x+1)*arctanh(c*x)^2+3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2
-3*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2
*arctanh(c*x)^2-3/2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+
1)^2/(c^2*x^2-1))*arctanh(c*x)^2-3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-
(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2
)+2/d^2*a*b*(1/2*arctanh(c*x)*c^2*x^2-2*arctanh(c*x)*c*x+1/(c*x+1)*arctanh
(c*x)+3*arctanh(c*x)*ln(c*x+1)-3/4*ln(c*x+1)^2+3/2*(ln(c*x+1)-ln(1/2*c*...
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{(cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^2*
d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{\int \frac{a^2x^3}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^3\,\text{atanh}^2\,(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^3\,\text{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**3*atanh
(c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c**
2*x**2 + 2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{(cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(2/(c^5*d^2*x + c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1
)/(c^4*d^2)) + 1/8*(b^2*c^3*x^3 - 3*b^2*c^2*x^2 - 4*b^2*c*x + 2*b^2 + 6*(b
^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^5*d^2*x + c^4*d^2) - integr
ate(-1/4*((b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*
b*c^3*x^3)*log(c*x + 1) + (7*b^2*c^2*x^2 - (4*a*b*c^4 + b^2*c^4)*x^4 + 2*b
^2*c*x + 2*(2*a*b*c^3 + b^2*c^3)*x^3 - 2*b^2 - 2*(b^2*c^4*x^4 - b^2*c^3*x^
3 + 3*b^2*c^2*x^2 + 6*b^2*c*x + 3*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^6*d
^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x)
```

Giac [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{(cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx=\int \frac{x^3\,(a+b\,\text{atanh}(c\,x))^2}{(d+c\,d\,x)^2}\,dx$$

input
```
int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

output
```
int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$$
$$=\frac{4\left(\int \frac{atanh(cx)x^3}{c^2x^2+2cx+1}dx\right)ab\,c^5x+4\left(\int \frac{atanh(cx)x^3}{c^2x^2+2cx+1}dx\right)ab\,c^4+2\left(\int \frac{atanh(cx)^2x^3}{c^2x^2+2cx+1}dx\right)b^2c^5x+2\left(\int \frac{atanh(cx)^2x^3}{c^2x^2+2cx+1}dx\right)b^2 \cdots}{2c^4d^2\,(cx+1)}$$

input
```
int(x^3*(a+b*atanh(c*x))^2/(c*d*x+d)^2,x)
```

output
```
(4*int((atanh(c*x)*x**3)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**5*x + 4*int((at
anh(c*x)*x**3)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**4 + 2*int((atanh(c*x)**2*
x**3)/(c**2*x**2 + 2*c*x + 1),x)*b**2*c**5*x + 2*int((atanh(c*x)**2*x**3)/
(c**2*x**2 + 2*c*x + 1),x)*b**2*c**4 + 6*log(c*x + 1)*a**2*c*x + 6*log(c*x
 + 1)*a**2 + a**2*c**3*x**3 - 3*a**2*c**2*x**2 - 6*a**2*c*x)/(2*c**4*d**2*
(c*x + 1))
```

3.105 $\int \frac{x^2(a+b\textbf{arctanh}(cx))^2}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 260

$$
\begin{aligned}
\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = & -\frac{b^2}{2c^3d^2(1+cx)}+\frac{b^2\text{arctanh}(cx)}{2c^3d^2}-\frac{b(a+b\text{arctanh}(cx))}{c^3d^2(1+cx)} \\
& +\frac{3(a+b\text{arctanh}(cx))^2}{2c^3d^2}+\frac{x(a+b\text{arctanh}(cx))^2}{c^2d^2} \\
& -\frac{(a+b\text{arctanh}(cx))^2}{c^3d^2(1+cx)}-\frac{2b(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^3d^2} \\
& +\frac{2(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^3d^2} \\
& -\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d^2} \\
& -\frac{2b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^3d^2} \\
& -\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{c^3d^2}
\end{aligned}
$$

output
```
-1/2*b^2/c^3/d^2/(c*x+1)+1/2*b^2*arctanh(c*x)/c^3/d^2-b*(a+b*arctanh(c*x))
/c^3/d^2/(c*x+1)+3/2*(a+b*arctanh(c*x))^2/c^3/d^2+x*(a+b*arctanh(c*x))^2/c
^2/d^2-(a+b*arctanh(c*x))^2/c^3/d^2/(c*x+1)-2*b*(a+b*arctanh(c*x))*ln(2/(-
c*x+1))/c^3/d^2+2*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^3/d^2-b^2*polylog(2
,1-2/(-c*x+1))/c^3/d^2-2*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^3/d
^2-b^2*polylog(3,1-2/(c*x+1))/c^3/d^2
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.13

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$$
$$= \frac{4a^2cx - \frac{4a^2}{1+cx} - 8a^2\log(1+cx) + b^2\left(-4\text{arctanh}(cx)^2 + 4cx\text{arctanh}(cx)^2 - \cosh(2\text{arctanh}(cx)) - 2\text{arcta}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
(4*a^2*c*x - (4*a^2)/(1 + c*x) - 8*a^2*Log[1 + c*x] + b^2*(-4*ArcTanh[c*x]
^2 + 4*c*x*ArcTanh[c*x]^2 - Cosh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Cosh[2*A
rcTanh[c*x]] - 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 8*ArcTanh[c*x]*Log[
1 + E^(-2*ArcTanh[c*x])] + 8*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] +
 (4 - 8*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 4*PolyLog[3, -E^(
-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c
*x]] + 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 2*a*b*(-Cosh[2*ArcTanh[c*x
]] + 2*Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcT
anh[c*x]] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2
*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(-\frac{2(a+b\text{arctanh}(cx))^2}{c^2d^2(cx+1)}+\frac{(a+b\text{arctanh}(cx))^2}{c^2d^2}+\frac{(a+b\text{arctanh}(cx))^2}{c^2d^2(cx+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^3d^2}-\frac{b(a+b\text{arctanh}(cx))}{c^3d^2(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{c^3d^2(cx+1)}+\frac{3(a+b\text{arctanh}(cx))^2}{2c^3d^2}-\frac{2b\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c^3d^2}+\frac{2\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c^3d^2}+\frac{x(a+b\text{arctanh}(cx))^2}{c^2d^2}+\frac{b^2\text{arctanh}(cx)}{2c^3d^2}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d^2}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b^2}{2c^3d^2(cx+1)}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
-1/2*b^2/(c^3*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c^3*d^2) - (b*(a + b*
ArcTanh[c*x]))/(c^3*d^2*(1 + c*x)) + (3*(a + b*ArcTanh[c*x])^2)/(2*c^3*d^2
) + (x*(a + b*ArcTanh[c*x])^2)/(c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^3*d^2
*(1 + c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d^2) + (2*(
a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d^2) - (b^2*PolyLog[2, 1 - 2/
(1 - c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*
x)])/(c^3*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 2674, normalized size of antiderivative = 10.28

method	result	size
derivativedivides	Expression too large to display	2674
default	Expression too large to display	2674
parts	Expression too large to display	2684

input
```
int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^2/d^2*(c*x-1/(c*x+1)-2*ln(c*x+1))+b^2/d^2*(1/2*I*Pi*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*c
sgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+
1))-2*arctanh(c*x)^2+polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))-2*arctanh(c*x)^2*
ln(c*x+1)-3/2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-1/2*arctanh(c*x)*l
n(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^
2+1)^(1/2))-ln(2)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+2*ln(2)*dilog(1+I*(c*
x+1)/(-c^2*x^2+1)^(1/2))+2*ln(2)*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2
*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^
(1/2))-1/2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*(2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-2*arctanh(c*x)^2+pol
ylog(2,-(c*x+1)^2/(-c^2*x^2+1)))+1/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*(2*arctanh(c*x)*ln
(1+(c*x+1)^2/(-c^2*x^2+1))-2*arctanh(c*x)^2+polylog(2,-(c*x+1)^2/(-c^2*x^2
+1)))-I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)^2*(2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-2*arctanh(c*x)^2+polylog(
2,-(c*x+1)^2/(-c^2*x^2+1)))+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2
/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*x)*ln(1+I*(c
*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+
dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1...
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{\left(b\,\text{artanh}\left(cx\right)+a\right)^2x^2}{\left(cdx+d\right)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^2*
d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{\int \frac{a^2x^2}{c^2x^2+2cx+1}\,dx+\int \frac{b^2x^2\,\text{atanh}^2\,(cx)}{c^2x^2+2cx+1}\,dx+\int \frac{2abx^2\,\text{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**2*atanh
(c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c**
2*x**2 + 2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^2}{(cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-a^2*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*log(c*x + 1)/(c^3*d^2)) +
1/4*(b^2*c^2*x^2 + b^2*c*x - b^2 - 2*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*
x + 1)^2/(c^4*d^2*x + c^3*d^2) - integrate(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^
2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2)*log(c*x + 1) - 2*((2*a*b
*c^3 + b^2*c^3)*x^3 - 2*(a*b*c^2 - b^2*c^2)*x^2 - b^2 + (b^2*c^3*x^3 - 3*b
^2*c^2*x^2 - 4*b^2*c*x - 2*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d^2*x^3
+ c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^2}{(cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \int \frac{x^2\,(a+b\,\text{atanh}(c\,x))^2}{(d+c\,d\,x)^2}\,dx$$

input
```
int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

output
```
int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{2\left(\int \frac{atanh(cx)x^2}{c^2x^2+2cx+1}dx\right)ab\,c^4x+2\left(\int \frac{atanh(cx)x^2}{c^2x^2+2cx+1}dx\right)ab\,c^3+\left(\int \frac{atanh(cx)^2x^2}{c^2x^2+2cx+1}dx\right)b^2c^4x+\left(\int \frac{atanh(cx)^2x^2}{c^2x^2+2cx+1}dx\right)b^2c^3}{c^3d^2\,(cx+1)}$$

input
```
int(x^2*(a+b*atanh(c*x))^2/(c*d*x+d)^2,x)
```

output
```
(2*int((atanh(c*x)*x**2)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**4*x + 2*int((at
anh(c*x)*x**2)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**3 + int((atanh(c*x)**2*x*
*2)/(c**2*x**2 + 2*c*x + 1),x)*b**2*c**4*x + int((atanh(c*x)**2*x**2)/(c**
2*x**2 + 2*c*x + 1),x)*b**2*c**3 - 2*log(c*x + 1)*a**2*c*x - 2*log(c*x + 1
)*a**2 + a**2*c**2*x**2 + 2*a**2*c*x)/(c**3*d**2*(c*x + 1))
```

3.106 $\int \frac{x(a+b\textbf{arctanh}(cx))^2}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{b^2}{2c^2d^2(1+cx)} - \frac{b^2\text{arctanh}(cx)}{2c^2d^2} + \frac{b(a+b\text{arctanh}(cx))}{c^2d^2(1+cx)} - \frac{(a+b\text{arctanh}(cx))^2}{2c^2d^2} + \frac{(a+b\text{arctanh}(cx))^2}{c^2d^2(1+cx)} - \frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2} + \frac{b(a+b\text{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^2d^2} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^2d^2}$$

output
```
1/2*b^2/c^2/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/c^2/d^2+b*(a+b*arctanh(c*x))/
c^2/d^2/(c*x+1)-1/2*(a+b*arctanh(c*x))^2/c^2/d^2+(a+b*arctanh(c*x))^2/c^2/
d^2/(c*x+1)-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^2/d^2+b*(a+b*arctanh(c*x)
)*polylog(2,1-2/(c*x+1))/c^2/d^2+1/2*b^2*polylog(3,1-2/(c*x+1))/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.24

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$$
$$=\frac{\frac{4a^2}{1+cx}+4a^2\log(1+cx)+2ab\left(\cosh(2\text{arctanh}(cx))+2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)+2\text{arctanh}(cx)\left(\cosh\right.\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
((4*a^2)/(1 + c*x) + 4*a^2*Log[1 + c*x] + 2*a*b*(Cosh[2*ArcTanh[c*x]] + 2*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] -
2*Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*
x]]) + b^2*(Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 2
*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcT
anh[c*x])] + 4*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*PolyLog[3
, -E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Sinh[2*Arc
Tanh[c*x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c^2*d^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(cdx+d)^2}\,dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(\frac{(a+b\text{arctanh}(cx))^2}{cd^2(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{cd^2(cx+1)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{b \operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{c^2d^2}+\frac{b(a+b\operatorname{arctanh}(cx))}{c^2d^2(cx+1)}+\frac{(a+b\operatorname{arctanh}(cx))^2}{c^2d^2(cx+1)}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2c^2d^2}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))^2}{c^2d^2}-\frac{b^2\operatorname{arctanh}(cx)}{2c^2d^2}+\frac{b^2 \operatorname{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b^2}{2c^2d^2(cx+1)}$$

input
```
Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

output
```
b^2/(2*c^2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^2*d^2) + (b*(a + b*Arc
Tanh[c*x]))/(c^2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*c^2*d^2) + (a
+ b*ArcTanh[c*x])^2/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(1
 + c*x)])/(c^2*d^2) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])
/(c^2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.88

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{cx+1}+\ln(cx+1)\right)}{d^2}+\frac{b^2\left(\frac{\operatorname{arctanh}(cx)^2}{cx+1}+\operatorname{arctanh}(cx)^2\ln(cx+1)-2\operatorname{arctanh}(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)+\frac{2\operatorname{arctanh}(cx)^3}{3}-\frac{\operatorname{arctanh}(cx}{2(cx-}\right.}{}$
default	$\frac{a^2\left(\frac{1}{cx+1}+\ln(cx+1)\right)}{d^2}+\frac{b^2\left(\frac{\operatorname{arctanh}(cx)^2}{cx+1}+\operatorname{arctanh}(cx)^2\ln(cx+1)-2\operatorname{arctanh}(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)+\frac{2\operatorname{arctanh}(cx)^3}{3}-\frac{\operatorname{arctanh}(cx}{2(cx-}\right.}{}$
parts	$\frac{a^2\left(\frac{1}{c^2(cx+1)}+\frac{\ln(cx+1)}{c^2}\right)}{d^2}+\frac{b^2\left(\frac{\operatorname{arctanh}(cx)^2}{cx+1}+\operatorname{arctanh}(cx)^2\ln(cx+1)-2\operatorname{arctanh}(cx)^2\ln\left(\frac{cx+1}{\sqrt{-c^2x^2+1}}\right)+\frac{2\operatorname{arctanh}(cx}{3}\right.}{}$

input
```
int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a^2/d^2*(1/(c*x+1)+ln(c*x+1))+b^2/d^2*(1/(c*x+1)*arctanh(c*x)^2+arc
tanh(c*x)^2*ln(c*x+1)-2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3*
arctanh(c*x)^3-1/2*arctanh(c*x)*(c*x-1)/(c*x+1)-1/4/(c*x+1)*(c*x-1)-1/2*(1
+I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+2
*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+I
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*cs
gn(I/(1-(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+
1)^2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+2*ln(2))*arctanh(c*x)^2-arctan
h(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(-c^2*x
^2+1)))+2/d^2*a*b*(1/(c*x+1)*arctanh(c*x)+arctanh(c*x)*ln(c*x+1)+1/2/(c*x+
1)-1/4*ln(c*x+1)+1/4*ln(c*x-1)-1/4*ln(c*x+1)^2+1/2*(ln(c*x+1)-ln(1/2*c*x+1
/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1/2)))
```

Fricas [F]

$$\int \frac{x(a + b\text{arctanh}(cx))^2}{(d + cdx)^2} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)^2 x}{(cdx + d)^2} \, dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c^2*d^2*x^
2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{x(a + b\text{arctanh}(cx))^2}{(d + cdx)^2} \, dx = \frac{\int \frac{a^2 x}{c^2 x^2 + 2cx + 1} \, dx + \int \frac{b^2 x \operatorname{atanh}^2(cx)}{c^2 x^2 + 2cx + 1} \, dx + \int \frac{2abx \operatorname{atanh}(cx)}{c^2 x^2 + 2cx + 1} \, dx}{d^2}$$

input
```
integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x*atanh(c*x)*
*2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x*atanh(c*x)/(c**2*x**2 +
2*c*x + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x(a + b\text{arctanh}(cx))^2}{(d + cdx)^2} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)^2 x}{(cdx + d)^2} \, dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
a^2*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(c^2*d^2)) + 1/4*(b^2 + (b^2*c
*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d^2*x + c^2*d^2) - integrate(
-1/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*l
og(c*x + 1) - 2*(2*a*b*c^2*x^2 + b^2 - (2*a*b*c - b^2*c)*x + (2*b^2*c^2*x^
2 + b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d^2*x^3 + c^3*d^2*x^2
 - c^2*d^2*x - c*d^2), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x}{(cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx=\int \frac{x\,(a+b\,\text{atanh}(c\,x))^2}{(d+c\,d\,x)^2}\,dx$$

input
```
int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^2,x)
```

output
```
int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\,\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$$
$$=\frac{2\left(\int \frac{atanh(cx)x}{c^2x^2+2cx+1}dx\right)ab\,c^3x+2\left(\int \frac{atanh(cx)x}{c^2x^2+2cx+1}dx\right)ab\,c^2+\left(\int \frac{atanh(cx)^2x}{c^2x^2+2cx+1}dx\right)b^2c^3x+\left(\int \frac{atanh(cx)^2x}{c^2x^2+2cx+1}dx\right)b^2c^2+}{c^2d^2\,(cx+1)}$$

input
```
int(x*(a+b*atanh(c*x))^2/(c*d*x+d)^2,x)
```

output
```
(2*int((atanh(c*x)*x)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**3*x + 2*int((atanh
(c*x)*x)/(c**2*x**2 + 2*c*x + 1),x)*a*b*c**2 + int((atanh(c*x)**2*x)/(c**2
*x**2 + 2*c*x + 1),x)*b**2*c**3*x + int((atanh(c*x)**2*x)/(c**2*x**2 + 2*c
*x + 1),x)*b**2*c**2 + log(c*x + 1)*a**2*c*x + log(c*x + 1)*a**2 - a**2*c*
x)/(c**2*d**2*(c*x + 1))
```

3.107 $\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = -\frac{b^2}{2cd^2(1+cx)}+\frac{b^2\text{arctanh}(cx)}{2cd^2}-\frac{b(a+b\text{arctanh}(cx))}{cd^2(1+cx)}+\frac{(a+b\text{arctanh}(cx))^2}{2cd^2}-\frac{(a+b\text{arctanh}(cx))^2}{cd^2(1+cx)}$$

output
```
-1/2*b^2/c/d^2/(c*x+1)+1/2*b^2*arctanh(c*x)/c/d^2-b*(a+b*arctanh(c*x))/c/d
^2/(c*x+1)+1/2*(a+b*arctanh(c*x))^2/c/d^2-(a+b*arctanh(c*x))^2/c/d^2/(c*x+
1)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{-4a^2-4ab-2b^2-4b(2a+b)\text{arctanh}(cx)+2b^2(-1+cx)\text{arctanh}(cx)^2-b(2a+b)(1+cx)\log(1-cx}{4cd^2(1+cx)}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

output
```
(-4*a^2 - 4*a*b - 2*b^2 - 4*b*(2*a + b)*ArcTanh[c*x] + 2*b^2*(-1 + c*x)*Ar
cTanh[c*x]^2 - b*(2*a + b)*(1 + c*x)*Log[1 - c*x] + 2*a*b*Log[1 + c*x] + b
^2*Log[1 + c*x] + 2*a*b*c*x*Log[1 + c*x] + b^2*c*x*Log[1 + c*x])/(4*c*d^2*
(1 + c*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{6480, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(cdx+d)^2} \, dx$$

$$\Big\downarrow \text{ 6480}$$

$$\frac{2b\int \left(\frac{a+b\text{arctanh}(cx)}{2d(1-c^2x^2)}+\frac{a+b\text{arctanh}(cx)}{2d(cx+1)^2}\right)dx}{d}-\frac{(a+b\text{arctanh}(cx))^2}{cd^2(cx+1)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2b\left(\frac{(a+b\text{arctanh}(cx))^2}{4bcd}-\frac{a+b\text{arctanh}(cx)}{2cd(cx+1)}+\frac{b\text{arctanh}(cx)}{4cd}-\frac{b}{4cd(cx+1)}\right)}{d}-\frac{(a+b\text{arctanh}(cx))^2}{cd^2(cx+1)}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

output
```
-((a + b*ArcTanh[c*x])^2/(c*d^2*(1 + c*x))) + (2*b*(-1/4*b/(c*d*(1 + c*x))
 + (b*ArcTanh[c*x])/(4*c*d) - (a + b*ArcTanh[c*x])/(2*c*d*(1 + c*x)) + (a
+ b*ArcTanh[c*x])^2/(4*b*c*d)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{b^2 cx \operatorname{arctanh}(cx)^2+2\operatorname{arctanh}(cx)abcx+c b^2\operatorname{arctanh}(cx)x+2a^2cx+2abcx+b^2cx-b^2\operatorname{arctanh}(cx)^2-2\operatorname{arctanh}(cx)ab-}{2d^2(cx+1)c}$
derivativedivides	$-\frac{a^2}{d^2(cx+1)}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)}{cx+1}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}+\frac{\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(cx-1)^2}{8}\right.}{d^2}$
default	$-\frac{a^2}{d^2(cx+1)}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)}{cx+1}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}+\frac{\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(cx-1)^2}{8}\right.}{d^2}$
parts	$-\frac{a^2}{d^2(cx+1)c}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)}{cx+1}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}+\frac{\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}-\ln(\right.}{d^2c}$
risch	$\frac{b^2(cx-1)\ln(cx+1)^2}{8d^2(cx+1)c}-\frac{b(bcx\ln(-cx+1)-b\ln(-cx+1)+4a+2b)\ln(cx+1)}{4d^2(cx+1)c}-\frac{-b^2cx\ln(-cx+1)^2+4abc\ln(cx-1)x+}{}$
orering	$-\frac{(4x^3c^3-3c^2x^2-4cx+3)(a+b\operatorname{arctanh}(cx))^2}{2c(cdx+d)^2}-\frac{(cx+1)^2(cx-1)(7cx-5)\left(\frac{2(a+b\operatorname{arctanh}(cx))bc}{(cdx+d)^2(-c^2x^2+1)}-\frac{2(a+b\operatorname{arctanh}(cx))^2}{(cdx+d)^3}\right.}{4c^2}$

input
```
int((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*c*x*arctanh(c*x)^2+2*arctanh(c*x)*a*b*c*x+c*b^2*arctanh(c*x)*x+2*
a^2*c*x+2*a*b*c*x+b^2*c*x-b^2*arctanh(c*x)^2-2*arctanh(c*x)*a*b-b^2*arctan
h(c*x))/d^2/(c*x+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(d + cdx)^2} dx = \frac{(b^2cx - b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 8a^2 - 8ab - 4b^2 + 2((2ab + b^2)cx - 2ab - b^2)\log\left(-\frac{cx+1}{cx-1}\right)}{8(c^2d^2x + cd^2)}$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*((b^2*c*x - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 8*a^2 - 8*a*b - 4*b^2 +
 2*((2*a*b + b^2)*c*x - 2*a*b - b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^2*d^2*x
 + c*d^2)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(d + cdx)^2} dx = \frac{\int \frac{a^2}{c^2x^2+2cx+1} dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{c^2x^2+2cx+1} dx + \int \frac{2ab \operatorname{atanh}(cx)}{c^2x^2+2cx+1} dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(
c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**2 + 2*c*x
+ 1), x))/d**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(101) = 202$.

Time = 0.05 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.59

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(d + cdx)^2}\,dx$$
$$= -\frac{1}{2}\left(c\left(\frac{2}{c^3d^2x + c^2d^2} - \frac{\log(cx+1)}{c^2d^2} + \frac{\log(cx-1)}{c^2d^2}\right) + \frac{4\,\text{artanh}(cx)}{c^2d^2x + cd^2}\right)ab$$
$$-\frac{1}{8}\left(4\,c\left(\frac{2}{c^3d^2x + c^2d^2} - \frac{\log(cx+1)}{c^2d^2} + \frac{\log(cx-1)}{c^2d^2}\right)\text{artanh}(cx) + \frac{((cx+1)\log(cx+1)^2 + (cx+1}{}\right.$$
$$-\frac{b^2\,\text{artanh}(cx)^2}{c^2d^2x + cd^2} - \frac{a^2}{c^2d^2x + cd^2}$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(
c^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*a*b - 1/8*(4*c*(2/(c^3*d^2
*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c^2*d^2))*arctanh(c
*x) + ((c*x + 1)*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x
 + 1)*log(c*x - 1) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(
c^4*d^2*x + c^3*d^2))*b^2 - b^2*arctanh(c*x)^2/(c^2*d^2*x + c*d^2) - a^2/(
c^2*d^2*x + c*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(d + cdx)^2}\,dx$$
$$= \frac{1}{8}\,c\left(\frac{(cx-1)b^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2d^2} + \frac{2\,(2\,ab + b^2)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2d^2} + \frac{2\,(2\,a^2 + 2\,ab + b^2)(cx-1)}{(cx+1)c^2d^2}\right)$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
1/8*c*((c*x - 1)*b^2*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2*d^2) + 2*(
2*a*b + b^2)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2*d^2) + 2*(
2*a^2 + 2*a*b + b^2)*(c*x - 1)/((c*x + 1)*c^2*d^2))
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{b^2\,\text{atanh}(c\,x)^2+b^2\,\text{atanh}(c\,x)+2\,a\,b\,\text{atanh}(c\,x)}{2\,c\,d^2} - \frac{2\,a^2+4\,a\,b\,\text{atanh}(c\,x)+2\,a\,b+2\,b^2\,\text{atanh}(c\,x)^2+2\,b^2\,\text{atanh}(c\,x)+b^2}{2\,x\,c^2\,d^2+2\,c\,d^2}$$

input
```
int((a + b*atanh(c*x))^2/(d + c*d*x)^2,x)
```

output
```
(b^2*atanh(c*x)^2 + b^2*atanh(c*x) + 2*a*b*atanh(c*x))/(2*c*d^2) - (2*b^2*
atanh(c*x)^2 + 2*a*b + 2*b^2*atanh(c*x) + 2*a^2 + b^2 + 4*a*b*atanh(c*x))/
(2*c*d^2 + 2*c^2*d^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^2}\,dx = \frac{2atanh(cx)^2\,b^2cx-2atanh(cx)^2\,b^2+8atanh(cx)\,abcx+4atanh(cx)\,b^2cx+2\log(cx-1)\,abcx+2\log(c}{}$$

input
```
int((a+b*atanh(c*x))^2/(c*d*x+d)^2,x)
```

output
```
(2*atanh(c*x)**2*b**2*c*x - 2*atanh(c*x)**2*b**2 + 8*atanh(c*x)*a*b*c*x +
4*atanh(c*x)*b**2*c*x + 2*log(c*x - 1)*a*b*c*x + 2*log(c*x - 1)*a*b + log(
c*x - 1)*b**2*c*x + log(c*x - 1)*b**2 - 2*log(c*x + 1)*a*b*c*x - 2*log(c*x
 + 1)*a*b - log(c*x + 1)*b**2*c*x - log(c*x + 1)*b**2 + 4*a**2*c*x + 4*a*b
*c*x + 2*b**2*c*x)/(4*c*d**2*(c*x + 1))
```

3.108 $\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx &= \frac{b^2}{2d^2(1+cx)}-\frac{b^2\text{arctanh}(cx)}{2d^2}+\frac{b(a+b\text{arctanh}(cx))}{d^2(1+cx)}\\&\quad-\frac{(a+b\text{arctanh}(cx))^2}{2d^2}+\frac{(a+b\text{arctanh}(cx))^2}{d^2(1+cx)}\\&\quad+\frac{2(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)}{d^2}\\&\quad+\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d^2}\\&\quad-\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d^2}\\&\quad+\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d^2}\\&\quad-\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^2}\\&\quad+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^2}-\frac{b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d^2}\\&\quad-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d^2}\end{aligned}$$

output
```
1/2*b^2/d^2/(c*x+1)-1/2*b^2*arctanh(c*x)/d^2+b*(a+b*arctanh(c*x))/d^2/(c*x
+1)-1/2*(a+b*arctanh(c*x))^2/d^2+(a+b*arctanh(c*x))^2/d^2/(c*x+1)-2*(a+b*a
rctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^2+(a+b*arctanh(c*x))^2*ln(2/(c*x+1
))/d^2-b*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2+b*(a+b*arctanh(c*x
))*polylog(2,-1+2/(-c*x+1))/d^2-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1)
)/d^2+1/2*b^2*polylog(3,1-2/(-c*x+1))/d^2-1/2*b^2*polylog(3,-1+2/(-c*x+1))
/d^2-1/2*b^2*polylog(3,1-2/(c*x+1))/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx$$
$$= \frac{\frac{24a^2}{1+cx}+24a^2\log(cx)-24a^2\log(1+cx)+12ab\left(\cosh(2\text{arctanh}(cx))-2\,\text{PolyLog}\left(2,e^{-2\text{arctanh}(cx)}\right)+2\text{arc}\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

output
```
((24*a^2)/(1 + c*x) + 24*a^2*Log[c*x] - 24*a^2*Log[1 + c*x] + 12*a*b*(Cosh
[2*ArcTanh[c*x]] - 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cos
h[2*ArcTanh[c*x]] + 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]])
 - Sinh[2*ArcTanh[c*x]]) + b^2*(I*Pi^3 - 16*ArcTanh[c*x]^3 + 6*Cosh[2*ArcT
anh[c*x]] + 12*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 12*ArcTanh[c*x]^2*Cosh[
2*ArcTanh[c*x]] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*ArcTa
nh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])]
 - 6*Sinh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 12*ArcT
anh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(24*d^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{(a+b\text{arctanh}(cx))^2}{d^2x}-\frac{c(a+b\text{arctanh}(cx))^2}{d^2(cx+1)}-\frac{c(a+b\text{arctanh}(cx))^2}{d^2(cx+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{d^2}+\frac{b\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}+\frac{b(a+b\text{arctanh}(cx))}{d^2(cx+1)}+\frac{(a+b\text{arctanh}(cx))^2}{d^2(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{2d^2}+\frac{2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{d^2}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d^2}-\frac{b^2\text{arctanh}(cx)}{2d^2}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^2}-\frac{b^2\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d^2}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{b^2}{2d^2(cx+1)}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

output
```
b^2/(2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*d^2) + (b*(a + b*ArcTanh[c*x
]))/(d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*d^2) + (a + b*ArcTanh[c*x]
)^2/(d^2*(1 + c*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/
d^2 + ((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b*(a + b*ArcTanh[c*
x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2 + (b*(a + b*ArcTanh[c*x])*PolyLog[2,
-1 + 2/(1 - c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x
)])/d^2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) - (b^2*PolyLog[3, -1 +
 2/(1 - c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 1239, normalized size of antiderivative = 4.20

method	result	size
parts	Expression too large to display	1239
derivativedivides	Expression too large to display	1241
default	Expression too large to display	1241

input
```
int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^2*(1/(c*x+1)-ln(c*x+1)+ln(x))+b^2/d^2*(arctanh(c*x)^2*ln(c*x)+1/(c*x
+1)*arctanh(c*x)^2-arctanh(c*x)^2*ln(c*x+1)+2*arctanh(c*x)^2*ln((c*x+1)/(-
c^2*x^2+1)^(1/2))-2/3*arctanh(c*x)^3+1/2*arctanh(c*x)^2*(I*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^3+2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^2-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^
2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2
/(c^2*x^2-1))+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))
^3+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I/(
1-(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-
(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(
c^2*x^2-1)))-I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^
2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-
1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+I*Pi
*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+2*ln(2)-1)
-1/2*arctanh(c*x)*(c*x-1)/(c*x+1)-1/4/(c*x+1)*(c*x-1)-arctanh(c*x)^2*ln((c
*x+1)^2/(-c^2*x^2+1)-1)+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*
arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(c*x+1)/(-c
^2*x^2+1)^(1/2))+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arct...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^3 + 2*
c*d^2*x^2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx = \frac{\int \frac{a^2}{c^2x^3+2cx^2+x}\,dx + \int \frac{b^2\,\text{atanh}^2(cx)}{c^2x^3+2cx^2+x}\,dx + \int \frac{2ab\,\text{atanh}(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b**2*atanh(c*x)**
2/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**3 +
2*c*x**2 + x), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}(cx)+a)^2}{(cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
a^2*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/4*(b^2 - (b^2*
c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x + d^2) + integrate(1/4*(
(b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - 2*(b^2*c
^2*x^2 - 2*a*b + (2*a*b*c + b^2*c)*x - (b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2)
*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^4 + c^2*d^2*x^3 - c*d^2*x^2 - d^2
*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2}{x\,(d+c\,d\,x)^2}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^2),x)
```

output
```
int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^2}\,dx$$
$$=\frac{-48\left(\int \frac{atanh(cx)}{c^3x^4+c^2x^3-c\,x^2-x}dx\right)ab-4atanh(cx)^3\,b^2-24\log(cx+1)\,a^2-6b^2cx-12atanh(cx)^2\,abcx-6\log}{}$$

input
```
int((a+b*atanh(c*x))^2/x/(c*d*x+d)^2,x)
```

output
```
( - 4*atanh(c*x)**3*b**2*c*x - 4*atanh(c*x)**3*b**2 - 12*atanh(c*x)**2*a*b
*c*x - 12*atanh(c*x)**2*a*b - 6*atanh(c*x)**2*b**2*c*x + 6*atanh(c*x)**2*b
**2 - 24*atanh(c*x)*a*b*c*x - 12*atanh(c*x)*b**2*c*x - 48*int(atanh(c*x)/(
c**3*x**4 + c**2*x**3 - c*x**2 - x),x)*a*b*c*x - 48*int(atanh(c*x)/(c**3*x
**4 + c**2*x**3 - c*x**2 - x),x)*a*b - 24*int(atanh(c*x)**2/(c**3*x**4 + c
**2*x**3 - c*x**2 - x),x)*b**2*c*x - 24*int(atanh(c*x)**2/(c**3*x**4 + c**
2*x**3 - c*x**2 - x),x)*b**2 - 6*log(c*x - 1)*a*b*c*x - 6*log(c*x - 1)*a*b
 - 3*log(c*x - 1)*b**2*c*x - 3*log(c*x - 1)*b**2 - 24*log(c*x + 1)*a**2*c*
x - 24*log(c*x + 1)*a**2 + 6*log(c*x + 1)*a*b*c*x + 6*log(c*x + 1)*a*b + 3
*log(c*x + 1)*b**2*c*x + 3*log(c*x + 1)*b**2 + 24*log(x)*a**2*c*x + 24*log
(x)*a**2 - 24*a**2*c*x - 12*a*b*c*x - 6*b**2*c*x)/(24*d**2*(c*x + 1))
```

3.109 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 371

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx = -\frac{b^2c}{2d^2(1+cx)}+\frac{b^2c\operatorname{arctanh}(cx)}{2d^2} - \frac{bc(a+b\operatorname{arctanh}(cx))}{d^2(1+cx)}+\frac{3c(a+b\operatorname{arctanh}(cx))^2}{2d^2} - \frac{(a+b\operatorname{arctanh}(cx))^2}{d^2x}-\frac{c(a+b\operatorname{arctanh}(cx))^2}{d^2(1+cx)} - \frac{4c(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)}{d^2} - \frac{2c(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{2bc(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d^2} + \frac{2bc(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d^2} - \frac{2bc(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d^2} + \frac{2bc(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^2} - \frac{b^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d^2}-\frac{b^2c\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{d^2} + \frac{b^2c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{d^2}+\frac{b^2c\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{d^2}$$

output
```
-1/2*b^2*c/d^2/(c*x+1)+1/2*b^2*c*arctanh(c*x)/d^2-b*c*(a+b*arctanh(c*x))/d
^2/(c*x+1)+3/2*c*(a+b*arctanh(c*x))^2/d^2-(a+b*arctanh(c*x))^2/d^2/x-c*(a+
b*arctanh(c*x))^2/d^2/(c*x+1)+4*c*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+
1))/d^2-2*c*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x)
)*ln(2-2/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2
-2*b*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2+2*b*c*(a+b*arctanh(
c*x))*polylog(2,1-2/(c*x+1))/d^2-b^2*c*polylog(2,-1+2/(c*x+1))/d^2-b^2*c*p
olylog(3,1-2/(-c*x+1))/d^2+b^2*c*polylog(3,-1+2/(-c*x+1))/d^2+b^2*c*polylo
g(3,1-2/(c*x+1))/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx$$
$$= \frac{-\frac{12a^2}{x}-\frac{12a^2c}{1+cx}-24a^2c\log(x)+24a^2c\log(1+cx)+b^2c\Big(-i\pi^3+12\text{arctanh}(cx)^2-\frac{12\text{arctanh}(cx)^2}{cx}+16\text{ar}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

output
```
((-12*a^2)/x - (12*a^2*c)/(1 + c*x) - 24*a^2*c*Log[x] + 24*a^2*c*Log[1 + c
*x] + b^2*c*((-I)*Pi^3 + 12*ArcTanh[c*x]^2 - (12*ArcTanh[c*x]^2)/(c*x) + 1
6*ArcTanh[c*x]^3 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]*Cosh[2*ArcTanh[
c*x]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 24*ArcTanh[c*x]*Log[1 - E^
(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 12*Po
lyLog[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*
x])] + 12*PolyLog[3, E^(2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]] + 6*ArcT
anh[c*x]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 6
*a*b*c*(-Cosh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog
[2, E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) -
 2*Cosh[2*ArcTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTan
h[c*x]])))/(12*d^2)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{2c^2(a+b\text{arctanh}(cx))^2}{d^2(cx+1)}+\frac{c^2(a+b\text{arctanh}(cx))^2}{d^2(cx+1)^2}+\frac{(a+b\text{arctanh}(cx))^2}{d^2x^2}-\frac{2c(a+b\text{arctanh}(cx))^2}{d^2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bc\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{2bc\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))}{d^2}+\frac{2bc\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{bc(a+b\text{arctanh}(cx))}{d^2(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{d^2x}-\frac{c(a+b\text{arctanh}(cx))^2}{d^2(cx+1)}+\frac{3c(a+b\text{arctanh}(cx))^2}{2d^2}-\frac{4c\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{d^2}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{2c\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d^2}+\frac{b^2c\text{arctanh}(cx)}{2d^2}-\frac{b^2c\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d^2}-\frac{b^2c\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{d^2}+\frac{b^2c\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{d^2}+\frac{b^2c\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{d^2}-\frac{b^2c}{2d^2(cx+1)}$$

input

```
Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

output

```
-1/2*(b^2*c)/(d^2*(1 + c*x)) + (b^2*c*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*
ArcTanh[c*x]))/(d^2*(1 + c*x)) + (3*c*(a + b*ArcTanh[c*x])^2)/(2*d^2) - (a
 + b*ArcTanh[c*x])^2/(d^2*x) - (c*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c*x))
- (4*c*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (2*c*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2
- 2/(1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*
x)])/d^2 - (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 +
 (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*Pol
yLog[2, -1 + 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/d^2 +
 (b^2*c*PolyLog[3, -1 + 2/(1 - c*x)])/d^2 + (b^2*c*PolyLog[3, 1 - 2/(1 + c
*x)])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.43 (sec) , antiderivative size = 4256, normalized size of antiderivative = 11.47

method	result	size
parts	Expression too large to display	4256
derivativedivides	Expression too large to display	4258
default	Expression too large to display	4258

input
```
int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^2*(-c/(c*x+1)+2*c*ln(c*x+1)-1/x-2*c*ln(x))+b^2/d^2*c*(-2*arctanh(c*x
)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-4*arctanh(c*x)*polylog(2,(c*x+1)/(-c^
2*x^2+1)^(1/2))-2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-4*arctan
h(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)^2*ln((c*x+1)^
2/(-c^2*x^2+1)-1)+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(arct
anh(c*x)^2-arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)*ln(1
+(c*x+1)/(-c^2*x^2+1)^(1/2))-polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-polylog
(2,-(c*x+1)/(-c^2*x^2+1)^(1/2)))-2*arctanh(c*x)^2*ln(c*x)+2*ln(2)*dilog((c
*x+1)/(-c^2*x^2+1)^(1/2))-2*ln(2)*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*ln
(2)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*ln(2)*polylog(2,-(c*x+1)/(-c^2
*x^2+1)^(1/2))+2*arctanh(c*x)^2*ln(c*x+1)+1/2*arctanh(c*x)*ln(1-(c*x+1)/(-
c^2*x^2+1)^(1/2))+2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+4/3*arct
anh(c*x)^3+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*x
)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-dil
og((c*x+1)/(-c^2*x^2+1)^(1/2)))-I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csg
n(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+
1)^2/(c^2*x^2-1)))*(arctanh(c*x)^2-arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^
(1/2))-arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-polylog(2,(c*x+1)/...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^4 + 2*
c*d^2*x^3 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx = \frac{\int \frac{a^2}{c^2x^4+2cx^3+x^2}\,dx + \int \frac{b^2\,\text{atanh}^2(cx)}{c^2x^4+2cx^3+x^2}\,dx + \int \frac{2ab\,\text{atanh}(cx)}{c^2x^4+2cx^3+x^2}\,dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b**2*atanh(c*x
)**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(2*a*b*atanh(c*x)/(c**2*x
**4 + 2*c*x**3 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}(cx)+a)^2}{(cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-a^2*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/
d^2) - 1/4*(2*b^2*c*x + b^2 - 2*(b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(
-c*x + 1)^2/(c*d^2*x^2 + d^2*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x
+ 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(2*b^2*c^3*x^3 + 3*b^2*c^2*x^2
 + 2*a*b - (2*a*b*c - b^2*c)*x - (2*b^2*c^4*x^4 + 4*b^2*c^3*x^3 + 2*b^2*c^
2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^5 + c^2*d^2
*x^4 - c*d^2*x^3 - d^2*x^2), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^2}{x^2\,(d+c\,d\,x)^2}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^2),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*atanh(c*x))^2/x^2/(c*d*x+d)^2,x)
```

output
```
( - 4*atanh(c*x)**3*b**2*c**2*x**2 - 4*atanh(c*x)**3*b**2*c*x - 12*atanh(c
*x)**2*a*b*c**2*x**2 - 12*atanh(c*x)**2*a*b*c*x - 18*atanh(c*x)**2*b**2*c*
*2*x**2 - 6*atanh(c*x)**2*b**2*c*x + 12*atanh(c*x)**2*b**2 - 24*atanh(c*x)
*a*b*c**2*x**2 + 24*atanh(c*x)*a*b - 36*atanh(c*x)*b**2*c**2*x**2 + 24*ata
nh(c*x)*b**2 - 48*int(atanh(c*x)/(c**3*x**5 + c**2*x**4 - c*x**3 - x**2),x
)*a*b*c*x**2 - 48*int(atanh(c*x)/(c**3*x**5 + c**2*x**4 - c*x**3 - x**2),x
)*a*b*x - 24*int(atanh(c*x)/(c**3*x**5 + c**2*x**4 - c*x**3 - x**2),x)*b**
2*c*x**2 - 24*int(atanh(c*x)/(c**3*x**5 + c**2*x**4 - c*x**3 - x**2),x)*b*
*2*x - 24*int(atanh(c*x)**2/(c**3*x**5 + c**2*x**4 - c*x**3 - x**2),x)*b**
2*c*x**2 - 24*int(atanh(c*x)**2/(c**3*x**5 + c**2*x**4 - c*x**3 - x**2),x)
*b**2*x - 3*log(c*x - 1)*b**2*c**2*x**2 - 3*log(c*x - 1)*b**2*c*x + 24*log
(c*x + 1)*a**2*c**2*x**2 + 24*log(c*x + 1)*a**2*c*x + 24*log(c*x + 1)*a*b*
c**2*x**2 + 24*log(c*x + 1)*a*b*c*x + 27*log(c*x + 1)*b**2*c**2*x**2 + 27*
log(c*x + 1)*b**2*c*x - 24*log(x)*a**2*c**2*x**2 - 24*log(x)*a**2*c*x - 24
*log(x)*a*b*c**2*x**2 - 24*log(x)*a*b*c*x - 24*log(x)*b**2*c**2*x**2 - 24*
log(x)*b**2*c*x + 24*a**2*c**2*x**2 - 12*a**2 - 6*b**2*c**2*x**2)/(12*d**2
*x*(c*x + 1))
```

3.110 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 480

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx = \frac{b^2c^2}{2d^2(1+cx)} - \frac{b^2c^2\operatorname{arctanh}(cx)}{2d^2} - \frac{bc(a+b\operatorname{arctanh}(cx))}{d^2x} + \frac{bc^2(a+b\operatorname{arctanh}(cx))}{d^2(1+cx)} - \frac{2c^2(a+b\operatorname{arctanh}(cx))^2}{d^2} - \frac{(a+b\operatorname{arctanh}(cx))^2}{2d^2x^2} + \frac{2c(a+b\operatorname{arctanh}(cx))^2}{d^2x} + \frac{c^2(a+b\operatorname{arctanh}(cx))^2}{d^2(1+cx)} + \frac{6c^2(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} + \frac{3c^2(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b^2c^2\log(1-c^2x^2)}{2d^2} - \frac{4bc^2(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d^2} - \frac{3bc^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d^2} + \frac{3bc^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d^2} - \frac{3bc^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^2} + \frac{2b^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d^2} + \frac{3b^2c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^2} - \frac{3b^2c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d^2} - \frac{3b^2c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d^2}$$

output
```
1/2*b^2*c^2/d^2/(c*x+1)-1/2*b^2*c^2*arctanh(c*x)/d^2-b*c*(a+b*arctanh(c*x)
)/d^2/x+b*c^2*(a+b*arctanh(c*x))/d^2/(c*x+1)-2*c^2*(a+b*arctanh(c*x))^2/d^
2-1/2*(a+b*arctanh(c*x))^2/d^2/x^2+2*c*(a+b*arctanh(c*x))^2/d^2/x+c^2*(a+b
*arctanh(c*x))^2/d^2/(c*x+1)-6*c^2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x
+1))/d^2+b^2*c^2*ln(x)/d^2+3*c^2*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2-1/
2*b^2*c^2*ln(-c^2*x^2+1)/d^2-4*b*c^2*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d^
2-3*b*c^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2+3*b*c^2*(a+b*arct
anh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2-3*b*c^2*(a+b*arctanh(c*x))*polylog(
2,1-2/(c*x+1))/d^2+2*b^2*c^2*polylog(2,-1+2/(c*x+1))/d^2+3/2*b^2*c^2*polyl
og(3,1-2/(-c*x+1))/d^2-3/2*b^2*c^2*polylog(3,-1+2/(-c*x+1))/d^2-3/2*b^2*c^
2*polylog(3,1-2/(c*x+1))/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx$$

$$= \frac{-\frac{4a^2}{x^2}+\frac{16a^2c}{x}+\frac{8a^2c^2}{1+cx}+24a^2c^2\log(x)-24a^2c^2\log(1+cx)+b^2c^2\left(i\pi^3-\frac{8\text{arctanh}(cx)}{cx}-12\text{arctanh}(cx)^2\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

output
```
((-4*a^2)/x^2 + (16*a^2*c)/x + (8*a^2*c^2)/(1 + c*x) + 24*a^2*c^2*Log[x] -
 24*a^2*c^2*Log[1 + c*x] + b^2*c^2*(I*Pi^3 - (8*ArcTanh[c*x])/(c*x) - 12*A
rcTanh[c*x]^2 - (4*ArcTanh[c*x]^2)/(c^2*x^2) + (16*ArcTanh[c*x]^2)/(c*x) -
 16*ArcTanh[c*x]^3 + 2*Cosh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*Cosh[2*ArcTan
h[c*x]] + 4*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 32*ArcTanh[c*x]*Log[1 -
E^(-2*ArcTanh[c*x])] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 8*L
og[(c*x)/Sqrt[1 - c^2*x^2]] + 16*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*ArcT
anh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])
] - 2*Sinh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 4*ArcTa
nh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + (4*a*b*(-6*c^2*x^2*PolyLog[2, E^(-2*ArcT
anh[c*x])] + c*x*(-2 + c*x*Cosh[2*ArcTanh[c*x]] - 8*c*x*Log[(c*x)/Sqrt[1 -
 c^2*x^2]] - c*x*Sinh[2*ArcTanh[c*x]]) + 2*ArcTanh[c*x]*(-1 + 4*c*x + c^2*
x^2 + c^2*x^2*Cosh[2*ArcTanh[c*x]] + 6*c^2*x^2*Log[1 - E^(-2*ArcTanh[c*x])
] - c^2*x^2*Sinh[2*ArcTanh[c*x]])))/x^2)/(8*d^2)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(cdx+d)^2}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(-\frac{3c^3(a+b\text{arctanh}(cx))^2}{d^2(cx+1)}-\frac{c^3(a+b\text{arctanh}(cx))^2}{d^2(cx+1)^2}+\frac{3c^2(a+b\text{arctanh}(cx))^2}{d^2x}+\frac{(a+b\text{arctanh}(cx))^2}{d^2x^3}-\frac{2c(\right.$$

$$\Big\downarrow \text{ 2009}$$

$$
-\frac{3bc^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{d^2}+\frac{3bc^2\operatorname{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\operatorname{arctanh}(cx))}{d^2}-\frac{3bc^2\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{d^2}+\frac{c^2(a+b\operatorname{arctanh}(cx))^2}{d^2(cx+1)}-\frac{2c^2(a+b\operatorname{arctanh}(cx))^2}{d^2}+\frac{bc^2(a+b\operatorname{arctanh}(cx))}{d^2(cx+1)}+\frac{6c^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))^2}{d^2}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))^2}{d^2}-\frac{4bc^2\log\left(2-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{d^2}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2d^2x^2}+\frac{2c(a+b\operatorname{arctanh}(cx))^2}{d^2x}-\frac{bc(a+b\operatorname{arctanh}(cx))}{d^2x}-\frac{b^2c^2\operatorname{arctanh}(cx)}{2d^2}+\frac{2b^2c^2\operatorname{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d^2}+\frac{3b^2c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^2}-\frac{3b^2c^2\operatorname{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d^2}-\frac{3b^2c^2\operatorname{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d^2}-\frac{b^2c^2\log\left(1-c^2x^2\right)}{2d^2}+\frac{b^2c^2}{2d^2(cx+1)}+\frac{b^2c^2\log(x)}{d^2}
$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

output
```
(b^2*c^2)/(2*d^2*(1 + c*x)) - (b^2*c^2*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b
*ArcTanh[c*x]))/(d^2*x) + (b*c^2*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (
2*c^2*(a + b*ArcTanh[c*x])^2)/d^2 - (a + b*ArcTanh[c*x])^2/(2*d^2*x^2) + (
2*c*(a + b*ArcTanh[c*x])^2)/(d^2*x) + (c^2*(a + b*ArcTanh[c*x])^2)/(d^2*(1
 + c*x)) + (6*c^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 + (
b^2*c^2*Log[x])/d^2 + (3*c^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2
- (b^2*c^2*Log[1 - c^2*x^2])/(2*d^2) - (4*b*c^2*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)])/d^2 - (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)])/d^2 + (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/
d^2 - (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 + (2*
b^2*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/d^2 + (3*b^2*c^2*PolyLog[3, 1 - 2/(1
 - c*x)])/(2*d^2) - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^2) - (3*
b^2*c^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.03 (sec) , antiderivative size = 1572, normalized size of antiderivative = 3.28

method	result	size
derivativedivides	Expression too large to display	1572
default	Expression too large to display	1572
parts	Expression too large to display	1577

input
```
int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/d^2*(-1/2/c^2/x^2+2/c/x+3*ln(c*x)+1/(c*x+1)-3*ln(c*x+1))+b^2/d^2*
(3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*arctanh(c*x)*polylog(
2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(
1/2))+6*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3*arctanh(c*x)
^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))
*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(
c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+
1)^2/(c^2*x^2-1)))*arctanh(c*x)^2+3*arctanh(c*x)^2*ln(c*x)-3*arctanh(c*x)^
2*ln(c*x+1)-4*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+ln((c*x+1)/(-c
^2*x^2+1)^(1/2)-1)-1/2*arctanh(c*x)^2/c^2/x^2-2*arctanh(c*x)^3+4*dilog((c*
x+1)/(-c^2*x^2+1)^(1/2))-4*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-1/4/(c*x+1)
*(c*x-1)+2*arctanh(c*x)^2-3/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn
(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-
3/2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+ln(1+(c*x+1)/(-c^2*x^2+1)^(
1/2))-6*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*polylog(3,(c*x+1)/(-c^2*x
^2+1)^(1/2))+2*arctanh(c*x)^2/c/x-3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+3/
2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^5 + 2*
c*d^2*x^4 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx = \frac{\int \frac{a^2}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

input
```
integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d)**2,x)
```

output
```
(Integral(a**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b**2*atanh(c*x
)**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(2*a*b*atanh(c*x)/(c**2*x
**5 + 2*c*x**4 + x**3), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x -
 1)/(c*d^2*x^3 + d^2*x^2)) + 1/8*(6*b^2*c^2*x^2 + 3*b^2*c*x - b^2 - 6*(b^2
*c^3*x^3 + b^2*c^2*x^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x^3 + d^2*x^2
) + integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(
c*x + 1) - (6*b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 2*b^2*c^2*x^2 - 4*a*b + (4*a*b
*c - b^2*c)*x - 2*(3*b^2*c^5*x^5 + 6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x
 + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^6 + c^2*d^2*x^5 - c*d^2*x^
4 - d^2*x^3), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2}{x^3\,(d+c\,d\,x)^2}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)^2),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^3*(d + c*d*x)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3(d+cdx)^2}\,dx=\text{Too large to display}$$

input
```
int((a+b*atanh(c*x))^2/x^3/(c*d*x+d)^2,x)
```

output

```
( - 2*atanh(c*x)**3*b**2*c**3*x**3 - 2*atanh(c*x)**3*b**2*c**2*x**2 - 6*at
anh(c*x)**2*a*b*c**3*x**3 - 6*atanh(c*x)**2*a*b*c**2*x**2 - 6*atanh(c*x)**
2*b**2*c**3*x**3 + 6*atanh(c*x)**2*b**2*c*x + 2*atanh(c*x)**2*b**2 - 12*at
anh(c*x)*a*b*c**3*x**3 + 12*atanh(c*x)*a*b*c*x + 4*atanh(c*x)*a*b - 12*ata
nh(c*x)*b**2*c**3*x**3 + 6*atanh(c*x)*b**2*c*x - 2*atanh(c*x)*b**2 - 24*in
t(atanh(c*x)/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*a*b*c*x**3 - 24*in
t(atanh(c*x)/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*a*b*x**2 + 4*int(a
tanh(c*x)/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*b**2*c*x**3 + 4*int(a
tanh(c*x)/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*b**2*x**2 - 12*int(at
anh(c*x)**2/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*b**2*c*x**3 - 12*in
t(atanh(c*x)**2/(c**3*x**6 + c**2*x**5 - c*x**4 - x**3),x)*b**2*x**2 + log
(c*x - 1)*a*b*c**3*x**3 + log(c*x - 1)*a*b*c**2*x**2 - 2*log(c*x - 1)*b**2
*c**3*x**3 - 2*log(c*x - 1)*b**2*c**2*x**2 - 24*log(c*x + 1)*a**2*c**3*x**
3 - 24*log(c*x + 1)*a**2*c**2*x**2 + 7*log(c*x + 1)*a*b*c**3*x**3 + 7*log(
c*x + 1)*a*b*c**2*x**2 + 10*log(c*x + 1)*b**2*c**3*x**3 + 10*log(c*x + 1)*
b**2*c**2*x**2 + 24*log(x)*a**2*c**3*x**3 + 24*log(x)*a**2*c**2*x**2 - 8*l
og(x)*a*b*c**3*x**3 - 8*log(x)*a*b*c**2*x**2 - 8*log(x)*b**2*c**3*x**3 - 8
*log(x)*b**2*c**2*x**2 - 24*a**2*c**3*x**3 + 12*a**2*c*x - 4*a**2 - 6*a*b*
c**3*x**3 + 4*a*b*c*x - 2*b**2*c*x)/(8*d**2*x**2*(c*x + 1))
```

3.111 $\int \frac{x^4(a+b\,\mathrm{arctanh}(cx))^2}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 408

$$
\begin{aligned}
\int \frac{x^4(a+b\,\mathrm{arctanh}(cx))^2}{(d+cdx)^3}\,dx = {} & \frac{abx}{c^4d^3} - \frac{b^2}{16c^5d^3(1+cx)^2} + \frac{29b^2}{16c^5d^3(1+cx)} \\
& - \frac{29b^2\mathrm{arctanh}(cx)}{16c^5d^3} + \frac{b^2x\mathrm{arctanh}(cx)}{c^4d^3} \\
& - \frac{b(a+b\,\mathrm{arctanh}(cx))}{4c^5d^3(1+cx)^2} + \frac{15b(a+b\,\mathrm{arctanh}(cx))}{4c^5d^3(1+cx)} \\
& - \frac{43(a+b\,\mathrm{arctanh}(cx))^2}{8c^5d^3} \\
& - \frac{3x(a+b\,\mathrm{arctanh}(cx))^2}{c^4d^3} + \frac{x^2(a+b\,\mathrm{arctanh}(cx))^2}{2c^3d^3} \\
& - \frac{(a+b\,\mathrm{arctanh}(cx))^2}{2c^5d^3(1+cx)^2} + \frac{4(a+b\,\mathrm{arctanh}(cx))^2}{c^5d^3(1+cx)} \\
& + \frac{6b(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^5d^3} \\
& - \frac{6(a+b\,\mathrm{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^5d^3} \\
& + \frac{b^2\log(1-c^2x^2)}{2c^5d^3} + \frac{3b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^5d^3} \\
& + \frac{6b(a+b\,\mathrm{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^5d^3} \\
& + \frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{c^5d^3}
\end{aligned}
$$

output
```
a*b*x/c^4/d^3-1/16*b^2/c^5/d^3/(c*x+1)^2+29/16*b^2/c^5/d^3/(c*x+1)-29/16*b
^2*arctanh(c*x)/c^5/d^3+b^2*x*arctanh(c*x)/c^4/d^3-1/4*b*(a+b*arctanh(c*x)
)/c^5/d^3/(c*x+1)^2+15/4*b*(a+b*arctanh(c*x))/c^5/d^3/(c*x+1)-43/8*(a+b*ar
ctanh(c*x))^2/c^5/d^3-3*x*(a+b*arctanh(c*x))^2/c^4/d^3+1/2*x^2*(a+b*arctan
h(c*x))^2/c^3/d^3-1/2*(a+b*arctanh(c*x))^2/c^5/d^3/(c*x+1)^2+4*(a+b*arctan
h(c*x))^2/c^5/d^3/(c*x+1)+6*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^5/d^3-6*
(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/c^5/d^3+1/2*b^2*ln(-c^2*x^2+1)/c^5/d^3+
3*b^2*polylog(2,1-2/(-c*x+1))/c^5/d^3+6*b*(a+b*arctanh(c*x))*polylog(2,1-2
/(c*x+1))/c^5/d^3+3*b^2*polylog(3,1-2/(c*x+1))/c^5/d^3
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.03

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$= \frac{-48a^2cx+8a^2c^2x^2-\frac{8a^2}{(1+cx)^2}+\frac{64a^2}{1+cx}+96a^2\log(1+cx)+ab\big(16cx+28\cosh(2\text{arctanh}(cx))-\cosh(4\text{arc}}{}$$

input
```
Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
(-48*a^2*c*x + 8*a^2*c^2*x^2 - (8*a^2)/(1 + c*x)^2 + (64*a^2)/(1 + c*x) +
96*a^2*Log[1 + c*x] + a*b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTa
nh[c*x]] - 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*
Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x
+ 4*c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 +
E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])) +
16*b^2*((-3 + 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + (56*Cosh[
2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[1 - c^2*x^2] + 192*PolyLog
[3, -E^(-2*ArcTanh[c*x])] - 56*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]]
 + 4*ArcTanh[c*x]*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]]
 + 96*Log[1 + E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcT
anh[c*x]]) + 8*ArcTanh[c*x]^2*(20 - 24*c*x + 4*c^2*x^2 + 14*Cosh[2*ArcTanh
[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh[c*x])] - 14*Sinh[
2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]]))/64))/(16*c^5*d^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(cdx+d)^3}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{6(a+b\text{arctanh}(cx))^2}{c^4d^3(cx+1)}-\frac{4(a+b\text{arctanh}(cx))^2}{c^4d^3(cx+1)^2}-\frac{3(a+b\text{arctanh}(cx))^2}{c^4d^3}+\frac{(a+b\text{arctanh}(cx))^2}{c^4d^3(cx+1)^3}+\frac{x(a+b\text{arc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^5d^3}+\frac{15b(a+b\text{arctanh}(cx))}{4c^5d^3(cx+1)}-\frac{b(a+b\text{arctanh}(cx))}{4c^5d^3(cx+1)^2}+\frac{4(a+b\text{arctanh}(cx))^2}{c^5d^3(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{2c^5d^3(cx+1)^2}-\frac{43(a+b\text{arctanh}(cx))^2}{8c^5d^3}+\frac{6b\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c^5d^3}-\frac{6\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c^5d^3}-\frac{3x(a+b\text{arctanh}(cx))^2}{c^4d^3}+\frac{x^2(a+b\text{arctanh}(cx))^2}{2c^3d^3}+\frac{abx}{c^4d^3}-\frac{29b^2\text{arctanh}(cx)}{16c^5d^3}+\frac{b^2x\text{arctanh}(cx)}{c^4d^3}+\frac{3b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^5d^3}+\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{29b^2}{16c^5d^3(cx+1)}-\frac{b^2}{16c^5d^3(cx+1)^2}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^5d^3}$$

input
```
Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
(a*b*x)/(c^4*d^3) - b^2/(16*c^5*d^3*(1 + c*x)^2) + (29*b^2)/(16*c^5*d^3*(1
 + c*x)) - (29*b^2*ArcTanh[c*x])/(16*c^5*d^3) + (b^2*x*ArcTanh[c*x])/(c^4*
d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c^5*d^3*(1 + c*x)^2) + (15*b*(a + b*Arc
Tanh[c*x]))/(4*c^5*d^3*(1 + c*x)) - (43*(a + b*ArcTanh[c*x])^2)/(8*c^5*d^3
) - (3*x*(a + b*ArcTanh[c*x])^2)/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/
(2*c^3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*A
rcTanh[c*x])^2)/(c^5*d^3*(1 + c*x)) + (6*b*(a + b*ArcTanh[c*x])*Log[2/(1 -
 c*x)])/(c^5*d^3) - (6*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^5*d^3)
+ (b^2*Log[1 - c^2*x^2])/(2*c^5*d^3) + (3*b^2*PolyLog[2, 1 - 2/(1 - c*x)])
/(c^5*d^3) + (6*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d
^3) + (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.48 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.62

method	result	size
derivativedivides	Expression too large to display	1069
default	Expression too large to display	1069
parts	Expression too large to display	1082

input
```
int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a^2/d^3*(1/2*c^2*x^2-3*c*x-1/2/(c*x+1)^2+4/(c*x+1)+6*ln(c*x+1))+b^2
/d^3*(3*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2+
6*arctanh(c*x)^2*ln(c*x+1)+6*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2
))+6*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+6*dilog(1+I*(c*x+1)/(
-c^2*x^2+1)^(1/2))+6*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3*I*Pi*csgn(I*(
c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1
)))^2*arctanh(c*x)^2-3*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c
*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-3*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1
-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^
2-6*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
2*arctanh(c*x)^2-1/64/(c*x+1)^2*(c*x-1)^2+4*arctanh(c*x)^3+1/2*arctanh(c*x
)^2*c^2*x^2-3*arctanh(c*x)^2*c*x-1/16*arctanh(c*x)*(c*x-1)^2/(c*x+1)^2-7/8
/(c*x+1)*(c*x-1)-43/8*arctanh(c*x)^2+3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+
(c*x+1)*arctanh(c*x)-ln(1+(c*x+1)^2/(-c^2*x^2+1))-6*arctanh(c*x)*polylog(2
,-(c*x+1)^2/(-c^2*x^2+1))-3*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2
/(c^2*x^2-1)))^3*arctanh(c*x)^2-3*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arc
tanh(c*x)^2-12*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-6*ln(2)*arcta
nh(c*x)^2-7/4*arctanh(c*x)*(c*x-1)/(c*x+1)+4/(c*x+1)*arctanh(c*x)^2-1/2/(c
*x+1)^2*arctanh(c*x)^2)+2*b*a/d^3*(1/2*arctanh(c*x)*c^2*x^2-3*arctanh(c...
```

Fricas [F]

$$\int \frac{x^4(a + b\text{arctanh}(cx))^2}{(d + cdx)^3} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)^2 x^4}{(cdx + d)^3} \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^3*
d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{\int \frac{a^2x^4}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^4\,\text{atanh}^2\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^4\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b
**2*x**4*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integra
l(2*a*b*x**4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x^4}{(cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*a^2*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + (c*x^2 - 6*x)
/(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3)) + 1/8*(b^2*c^4*x^4 - 4*b^2*c^3*x^3
 - 11*b^2*c^2*x^2 + 2*b^2*c*x + 7*b^2 + 12*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)
*log(c*x + 1))*log(-c*x + 1)^2/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - int
egrate(-1/4*((b^2*c^5*x^5 - b^2*c^4*x^4)*log(c*x + 1)^2 + 4*(a*b*c^5*x^5 -
 a*b*c^4*x^4)*log(c*x + 1) + (15*b^2*c^3*x^3 + 9*b^2*c^2*x^2 - (4*a*b*c^5
+ b^2*c^5)*x^5 + (4*a*b*c^4 + 3*b^2*c^4)*x^4 - 9*b^2*c*x - 7*b^2 - 2*(b^2*
c^5*x^5 - b^2*c^4*x^4 + 6*b^2*c^3*x^3 + 18*b^2*c^2*x^2 + 18*b^2*c*x + 6*b^
2)*log(c*x + 1))*log(-c*x + 1))/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x
 - c^4*d^3), x)
```

Giac [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^4}{(cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{x^4\,(a+b\,\text{atanh}(c\,x))^2}{(d+c\,d\,x)^3}\,dx$$

input
```
int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

output
```
int((x^4*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$=\frac{4\left(\int \frac{atanh(cx)x^4}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^7x^2+8\left(\int \frac{atanh(cx)x^4}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^6x+4\left(\int \frac{atanh(cx)x^4}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^5+2\left(\int\right.}{}$$

input
```
int(x^4*(a+b*atanh(c*x))^2/(c*d*x+d)^3,x)
```

output
```
(4*int((atanh(c*x)*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*a*b*c**7
*x**2 + 8*int((atanh(c*x)*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*a
*b*c**6*x + 4*int((atanh(c*x)*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),
x)*a*b*c**5 + 2*int((atanh(c*x)**2*x**4)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x
+ 1),x)*b**2*c**7*x**2 + 4*int((atanh(c*x)**2*x**4)/(c**3*x**3 + 3*c**2*x*
*2 + 3*c*x + 1),x)*b**2*c**6*x + 2*int((atanh(c*x)**2*x**4)/(c**3*x**3 + 3
*c**2*x**2 + 3*c*x + 1),x)*b**2*c**5 + 12*log(c*x + 1)*a**2*c**2*x**2 + 24
*log(c*x + 1)*a**2*c*x + 12*log(c*x + 1)*a**2 + a**2*c**4*x**4 - 4*a**2*c*
*3*x**3 - 12*a**2*c**2*x**2 + 6*a**2)/(2*c**5*d**3*(c**2*x**2 + 2*c*x + 1)
)
```

3.112 $\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 337

$$\begin{aligned}\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx &= \frac{b^2}{16c^4d^3(1+cx)^2}-\frac{21b^2}{16c^4d^3(1+cx)}+\frac{21b^2\operatorname{arctanh}(cx)}{16c^4d^3}\\&\quad+\frac{b(a+b\operatorname{arctanh}(cx))}{4c^4d^3(1+cx)^2}-\frac{11b(a+b\operatorname{arctanh}(cx))}{4c^4d^3(1+cx)}\\&\quad+\frac{19(a+b\operatorname{arctanh}(cx))^2}{8c^4d^3}+\frac{x(a+b\operatorname{arctanh}(cx))^2}{c^3d^3}\\&\quad+\frac{(a+b\operatorname{arctanh}(cx))^2}{2c^4d^3(1+cx)^2}-\frac{3(a+b\operatorname{arctanh}(cx))^2}{c^4d^3(1+cx)}\\&\quad-\frac{2b(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^4d^3}\\&\quad+\frac{3(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^4d^3}\\&\quad-\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^3}\\&\quad-\frac{3b(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^4d^3}\\&\quad-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^4d^3}\end{aligned}$$

output
```
1/16*b^2/c^4/d^3/(c*x+1)^2-21/16*b^2/c^4/d^3/(c*x+1)+21/16*b^2*arctanh(c*x
)/c^4/d^3+1/4*b*(a+b*arctanh(c*x))/c^4/d^3/(c*x+1)^2-11/4*b*(a+b*arctanh(c
*x))/c^4/d^3/(c*x+1)+19/8*(a+b*arctanh(c*x))^2/c^4/d^3+x*(a+b*arctanh(c*x)
)^2/c^3/d^3+1/2*(a+b*arctanh(c*x))^2/c^4/d^3/(c*x+1)^2-3*(a+b*arctanh(c*x)
)^2/c^4/d^3/(c*x+1)-2*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4/d^3+3*(a+b*a
rctanh(c*x))^2*ln(2/(c*x+1))/c^4/d^3-b^2*polylog(2,1-2/(-c*x+1))/c^4/d^3-3
*b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/c^4/d^3-3/2*b^2*polylog(3,1-2
/(c*x+1))/c^4/d^3
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.24

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{64a^2cx+\frac{32a^2}{(1+cx)^2}-\frac{192a^2}{1+cx}-192a^2\log(1+cx)+4ab\big(-20\cosh(2\text{arctanh}(cx))+\cosh(4\text{arctanh}(cx))+16}{}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
(64*a^2*c*x + (32*a^2)/(1 + c*x)^2 - (192*a^2)/(1 + c*x) - 192*a^2*Log[1 +
 c*x] + 4*a*b*(-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1
- c^2*x^2] - 48*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]]
 + 4*ArcTanh[c*x]*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]]
+ 24*Log[1 + E^(-2*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTa
nh[c*x]]) - Sinh[4*ArcTanh[c*x]]) + b^2*(-64*ArcTanh[c*x]^2 + 64*c*x*ArcTa
nh[c*x]^2 - 40*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]]
 - 80*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 4*ArcTa
nh[c*x]*Cosh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Cosh[4*ArcTanh[c*x]] - 128
*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 192*ArcTanh[c*x]^2*Log[1 + E^
(-2*ArcTanh[c*x])] - 64*(-1 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*
x])] - 96*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*
ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]
 - Sinh[4*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 8*ArcTanh[
c*x]^2*Sinh[4*ArcTanh[c*x]]))/(64*c^4*d^3)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(cdx+d)^3}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(-\frac{3(a+b\text{arctanh}(cx))^2}{c^3d^3(cx+1)}+\frac{3(a+b\text{arctanh}(cx))^2}{c^3d^3(cx+1)^2}+\frac{(a+b\text{arctanh}(cx))^2}{c^3d^3}-\frac{(a+b\text{arctanh}(cx))^2}{c^3d^3(cx+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{c^4d^3}-\frac{11b(a+b\text{arctanh}(cx))}{4c^4d^3(cx+1)}+\frac{b(a+b\text{arctanh}(cx))}{4c^4d^3(cx+1)^2}-\frac{3(a+b\text{arctanh}(cx))^2}{c^4d^3(cx+1)}+\frac{(a+b\text{arctanh}(cx))^2}{2c^4d^3(cx+1)^2}+\frac{19(a+b\text{arctanh}(cx))^2}{8c^4d^3}-\frac{2b\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c^4d^3}+\frac{3\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{c^4d^3}+\frac{x(a+b\text{arctanh}(cx))^2}{c^3d^3}+\frac{21b^2\text{arctan}(cx)}{16c^4d^3}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^3}-\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{21b^2}{16c^4d^3(cx+1)}+\frac{b^2}{16c^4d^3(cx+1)^2}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
b^2/(16*c^4*d^3*(1 + c*x)^2) - (21*b^2)/(16*c^4*d^3*(1 + c*x)) + (21*b^2*A
rcTanh[c*x])/(16*c^4*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)^
2) - (11*b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)) + (19*(a + b*ArcTan
h[c*x])^2)/(8*c^4*d^3) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d^3) + (a + b*Arc
Tanh[c*x])^2/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^3
*(1 + c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^4*d^3) + (3*(
a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^3) - (b^2*PolyLog[2, 1 - 2/
(1 - c*x)])/(c^4*d^3) - (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*
x)])/(c^4*d^3) - (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.48 (sec) , antiderivative size = 2771, normalized size of antiderivative = 8.22

method	result	size
parts	Expression too large to display	2771
derivativedivides	Expression too large to display	2772
default	Expression too large to display	2772

input
```
int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output

```
a^2/d^3*(1/c^3*x-3/c^4/(c*x+1)-3/c^4*ln(c*x+1)+1/2/c^4/(c*x+1)^2)+b^2/d^3/
c^4*(-3*arctanh(c*x)^2*ln(c*x+1)+3/4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/
(c^2*x^2-1)))*(2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-2*arctanh(c*x)^
2+polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))-19/8*arctanh(c*x)*ln(1+(c*x+1)^2/(-c
^2*x^2+1))+3/8*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*arctanh
(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*ln(2)*polylog(2,-(c*x+1)^2/(-
c^2*x^2+1))+3*ln(2)*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3*ln(2)*dilog(1-
I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/
8*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/64/(c*x+1)^2*(c*x-1)^2-2*arctanh
(c*x)^3+arctanh(c*x)^2*c*x+1/16*arctanh(c*x)*(c*x-1)^2/(c*x+1)^2+5/8/(c*x+
1)*(c*x-1)+19/8*arctanh(c*x)^2-19/16*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/
2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(
-c^2*x^2+1))+3*ln(2)*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3*ln(
2)*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3*ln(2)*arctanh(c*x)*ln
(1+(c*x+1)^2/(-c^2*x^2+1))-3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(
c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x
^2-1)))*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)*ln(1
-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog
(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)...
```

Fricas [F]

$$\int \frac{x^3(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^3}{(cdx+d)^3}\,dx$$

input

```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output

```
integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^3*
d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{\int \frac{a^2x^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b
**2*x**3*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integra
l(2*a*b*x**3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2x^3}{(cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*a^2*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - 2*x/(c^3*d^3
) + 6*log(c*x + 1)/(c^4*d^3)) + 1/8*(2*b^2*c^3*x^3 + 4*b^2*c^2*x^2 - 4*b^2
*c*x - 5*b^2 - 6*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*log(c*x + 1))*log(-c*x +
1)^2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - integrate(-1/4*((b^2*c^4*x^4
- b^2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1)
 - (2*(2*a*b*c^4 + b^2*c^4)*x^4 - 9*b^2*c*x - 2*(2*a*b*c^3 - 3*b^2*c^3)*x^
3 - 5*b^2 + 2*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 - 9*b^2*c^2*x^2 - 9*b^2*c*x - 3
*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^
3*x - c^3*d^3), x)
```

Giac [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2x^3}{(cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{atanh}(c\,x)\right)^2}{\left(d+c\,d\,x\right)^3}\,dx$$

input
```
int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

output
```
int((x^3*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$=\frac{4\left(\int \frac{atanh(cx)x^3}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^6x^2+8\left(\int \frac{atanh(cx)x^3}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^5x+4\left(\int \frac{atanh(cx)x^3}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^4+2\left(\int\right.}{}$$

input
```
int(x^3*(a+b*atanh(c*x))^2/(c*d*x+d)^3,x)
```

output
```
(4*int((atanh(c*x)*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*a*b*c**6
*x**2 + 8*int((atanh(c*x)*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*a
*b*c**5*x + 4*int((atanh(c*x)*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),
x)*a*b*c**4 + 2*int((atanh(c*x)**2*x**3)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x
+ 1),x)*b**2*c**6*x**2 + 4*int((atanh(c*x)**2*x**3)/(c**3*x**3 + 3*c**2*x*
*2 + 3*c*x + 1),x)*b**2*c**5*x + 2*int((atanh(c*x)**2*x**3)/(c**3*x**3 + 3
*c**2*x**2 + 3*c*x + 1),x)*b**2*c**4 - 6*log(c*x + 1)*a**2*c**2*x**2 - 12*
log(c*x + 1)*a**2*c*x - 6*log(c*x + 1)*a**2 + 2*a**2*c**3*x**3 + 6*a**2*c*
*2*x**2 - 3*a**2)/(2*c**4*d**3*(c**2*x**2 + 2*c*x + 1))
```

3.113 $\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 265

$$
\begin{aligned}
\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = & -\frac{b^2}{16c^3d^3(1+cx)^2}+\frac{13b^2}{16c^3d^3(1+cx)}-\frac{13b^2\text{arctanh}(cx)}{16c^3d^3} \\
& -\frac{b(a+b\text{arctanh}(cx))}{4c^3d^3(1+cx)^2}+\frac{7b(a+b\text{arctanh}(cx))}{4c^3d^3(1+cx)} \\
& -\frac{7(a+b\text{arctanh}(cx))^2}{8c^3d^3}-\frac{(a+b\text{arctanh}(cx))^2}{2c^3d^3(1+cx)^2} \\
& +\frac{2(a+b\text{arctanh}(cx))^2}{c^3d^3(1+cx)}-\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{c^3d^3} \\
& +\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{c^3d^3} \\
& +\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c^3d^3}
\end{aligned}
$$

output
```
-1/16*b^2/c^3/d^3/(c*x+1)^2+13/16*b^2/c^3/d^3/(c*x+1)-13/16*b^2*arctanh(c*
x)/c^3/d^3-1/4*b*(a+b*arctanh(c*x))/c^3/d^3/(c*x+1)^2+7/4*b*(a+b*arctanh(c
*x))/c^3/d^3/(c*x+1)-7/8*(a+b*arctanh(c*x))^2/c^3/d^3-1/2*(a+b*arctanh(c*x
))^2/c^3/d^3/(c*x+1)^2+2*(a+b*arctanh(c*x))^2/c^3/d^3/(c*x+1)-(a+b*arctanh
(c*x))^2*ln(2/(c*x+1))/c^3/d^3+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))
/c^3/d^3+1/2*b^2*polylog(3,1-2/(c*x+1))/c^3/d^3
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$= \frac{-\frac{8a^2}{(1+cx)^2}+\frac{32a^2}{1+cx}+16a^2\log(1+cx)+16b^2\left(\text{arctanh}(cx)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-}\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
((-8*a^2)/(1 + c*x)^2 + (32*a^2)/(1 + c*x) + 16*a^2*Log[1 + c*x] + 16*b^2*
(ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh
[c*x])]/2 + ((-Cosh[2*ArcTanh[c*x]] + Sinh[2*ArcTanh[c*x]])*(-24 + Cosh[2*
ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-12 + Cosh[2*ArcTanh[c*x]] - Sinh[2*ArcTan
h[c*x]]) - Sinh[2*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*(-6 + Cosh[2*ArcTanh[c*
x]]*(1 + 8*Log[1 + E^(-2*ArcTanh[c*x])]) + (-1 + 8*Log[1 + E^(-2*ArcTanh[c
*x])])*Sinh[2*ArcTanh[c*x]])))/64) + a*b*(12*Cosh[2*ArcTanh[c*x]] - Cosh[4
*ArcTanh[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c
*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*x]] - Cos
h[4*ArcTanh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcTanh[c*x]
] + Sinh[4*ArcTanh[c*x]])))/(16*c^3*d^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(cdx+d)^3}\,dx$$
$$\Big\downarrow \ 6502$$
$$\int \left(\frac{(a+b\text{arctanh}(cx))^2}{c^2d^3(cx+1)}-\frac{2(a+b\text{arctanh}(cx))^2}{c^2d^3(cx+1)^2}+\frac{(a+b\text{arctanh}(cx))^2}{c^2d^3(cx+1)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{c^3d^3}+\frac{7b(a+b\operatorname{arctanh}(cx))}{4c^3d^3(cx+1)}-\frac{b(a+b\operatorname{arctanh}(cx))}{4c^3d^3(cx+1)^2}+\frac{2(a+b\operatorname{arctanh}(cx))^2}{c^3d^3(cx+1)}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2c^3d^3(cx+1)^2}-\frac{7(a+b\operatorname{arctanh}(cx))^2}{8c^3d^3}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))^2}{c^3d^3}-\frac{13b^2\operatorname{arctanh}(cx)}{16c^3d^3}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{13b^2}{16c^3d^3(cx+1)}-\frac{b^2}{16c^3d^3(cx+1)^2}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
-1/16*b^2/(c^3*d^3*(1 + c*x)^2) + (13*b^2)/(16*c^3*d^3*(1 + c*x)) - (13*b^
2*ArcTanh[c*x])/(16*c^3*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*
x)^2) + (7*b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)) - (7*(a + b*ArcTa
nh[c*x])^2)/(8*c^3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^3*d^3*(1 + c*x)^2) +
 (2*(a + b*ArcTanh[c*x])^2)/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*
Log[2/(1 + c*x)])/(c^3*d^3) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1
+ c*x)])/(c^3*d^3) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.52 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.08

method	result	size
derivativedivides	Expression too large to display	815
default	Expression too large to display	815
parts	Expression too large to display	827

input
```
int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^2/d^3*(-1/2/(c*x+1)^2+2/(c*x+1)+ln(c*x+1))+b^2/d^3*(2/(c*x+1)*arc
tanh(c*x)^2+arctanh(c*x)^2*ln(c*x+1)-1/2/(c*x+1)^2*arctanh(c*x)^2-2*arctan
h(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3*arctanh(c*x)^3-1/8*(7+4*I*Pi*c
sgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+8*I*Pi*c
sgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+4*I*Pi*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))^3-4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-4*I*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csg
n(I/(1-(c*x+1)^2/(c^2*x^2-1)))+4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x
+1)^2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))+4*I*Pi*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+8*ln(2))*arctanh(c*x)^2-arc
tanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(-c^
2*x^2+1))-1/16*arctanh(c*x)*(c*x-1)^2/(c*x+1)^2-1/64/(c*x+1)^2*(c*x-1)^2-3
/4*arctanh(c*x)*(c*x-1)/(c*x+1)-3/8/(c*x+1)*(c*x-1))+2/d^3*a*b*(-1/2/(c*x+
1)^2*arctanh(c*x)+2/(c*x+1)*arctanh(c*x)+arctanh(c*x)*ln(c*x+1)-1/4*ln(c*x
+1)^2+1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/2*dilog(1/2*c*x+1
/2)-1/8/(c*x+1)^2+7/8/(c*x+1)-7/16*ln(c*x+1)+7/16*ln(c*x-1)))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2x^2}{(cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^3*
d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{\int \frac{a^2x^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b
**2*x**2*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integra
l(2*a*b*x**2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2x^2}{(cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*a^2*((4*c*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1
)/(c^3*d^3)) + 1/8*(4*b^2*c*x + 3*b^2 + 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*
log(c*x + 1))*log(-c*x + 1)^2/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - inte
grate(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 -
a*b*c^2*x^2)*log(c*x + 1) - (4*a*b*c^3*x^3 + 7*b^2*c*x - 4*(a*b*c^2 - b^2*
c^2)*x^2 + 3*b^2 + 2*(2*b^2*c^3*x^3 + 2*b^2*c^2*x^2 + 3*b^2*c*x + b^2)*log
(c*x + 1))*log(-c*x + 1))/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2
*d^3), x)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^2}{(cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx=\int \frac{x^2\,(a+b\,\text{atanh}(c\,x))^2}{(d+c\,d\,x)^3}\,dx$$

input
```
int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

output
```
int((x^2*(a + b*atanh(c*x))^2)/(d + c*d*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$= \frac{4\left(\int \frac{atanh(cx)x^2}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^5x^2+8\left(\int \frac{atanh(cx)x^2}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^4x+4\left(\int \frac{atanh(cx)x^2}{c^3x^3+3c^2x^2+3cx+1}dx\right)ab\,c^3+2\left(\int\right.}{}$$

input
```
int(x^2*(a+b*atanh(c*x))^2/(c*d*x+d)^3,x)
```

output
```
(4*int((atanh(c*x)*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*a*b*c**5
*x**2 + 8*int((atanh(c*x)*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),x)*a
*b*c**4*x + 4*int((atanh(c*x)*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1),
x)*a*b*c**3 + 2*int((atanh(c*x)**2*x**2)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x
+ 1),x)*b**2*c**5*x**2 + 4*int((atanh(c*x)**2*x**2)/(c**3*x**3 + 3*c**2*x*
*2 + 3*c*x + 1),x)*b**2*c**4*x + 2*int((atanh(c*x)**2*x**2)/(c**3*x**3 + 3
*c**2*x**2 + 3*c*x + 1),x)*b**2*c**3 + 2*log(c*x + 1)*a**2*c**2*x**2 + 4*l
og(c*x + 1)*a**2*c*x + 2*log(c*x + 1)*a**2 - 2*a**2*c**2*x**2 + a**2)/(2*c
**3*d**3*(c**2*x**2 + 2*c*x + 1))
```

3.114 $\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{5b^2\text{arctanh}(cx)}{16c^2d^3} + \frac{b(a+b\text{arctanh}(cx))}{4c^2d^3(1+cx)^2} - \frac{3b(a+b\text{arctanh}(cx))}{4c^2d^3(1+cx)} - \frac{(a+b\text{arctanh}(cx))^2}{8c^2d^3} + \frac{x^2(a+b\text{arctanh}(cx))^2}{2d^3(1+cx)^2}$$

output
```
1/16*b^2/c^2/d^3/(c*x+1)^2-5/16*b^2/c^2/d^3/(c*x+1)+5/16*b^2*arctanh(c*x)/
c^2/d^3+1/4*b*(a+b*arctanh(c*x))/c^2/d^3/(c*x+1)^2-3/4*b*(a+b*arctanh(c*x)
)/c^2/d^3/(c*x+1)-1/8*(a+b*arctanh(c*x))^2/c^2/d^3+1/2*x^2*(a+b*arctanh(c*
x))^2/d^3/(c*x+1)^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{2(8a^2+4ab+b^2)-2(16a^2+12ab+5b^2)(1+cx)-8b(b(2+3cx)+a(4+8cx))\text{arctanh}(cx)+4b^2(-1}{32c^2d^3(1}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
(2*(8*a^2 + 4*a*b + b^2) - 2*(16*a^2 + 12*a*b + 5*b^2)*(1 + c*x) - 8*b*(b*
(2 + 3*c*x) + a*(4 + 8*c*x))*ArcTanh[c*x] + 4*b^2*(-1 - 2*c*x + 3*c^2*x^2)
*ArcTanh[c*x]^2 - b*(12*a + 5*b)*(1 + c*x)^2*Log[1 - c*x] + b*(12*a + 5*b)
*(1 + c*x)^2*Log[1 + c*x])/(32*c^2*d^3*(1 + c*x)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6500, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(cdx+d)^3}\,dx$$

$$\Big\downarrow \text{6500}$$

$$\frac{x^2(a+b\text{arctanh}(cx))^2}{2d^3(cx+1)^2} - 2bc\int \left(\frac{a+b\text{arctanh}(cx)}{8c^2d^3(1-c^2x^2)} - \frac{3(a+b\text{arctanh}(cx))}{8c^2d^3(cx+1)^2} + \frac{a+b\text{arctanh}(cx)}{4c^2d^3(cx+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(a+b\text{arctanh}(cx))^2}{2d^3(cx+1)^2}-$$
$$2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{16bc^3d^3}+\frac{3(a+b\text{arctanh}(cx))}{8c^3d^3(cx+1)}-\frac{a+b\text{arctanh}(cx)}{8c^3d^3(cx+1)^2}-\frac{5b\text{arctanh}(cx)}{32c^3d^3}+\frac{5b}{32c^3d^3(cx+1)}-\frac{}{32c}\right.$$

input
```
Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

output
```
(x^2*(a + b*ArcTanh[c*x])^2)/(2*d^3*(1 + c*x)^2) - 2*b*c*(-1/32*b/(c^3*d^3
*(1 + c*x)^2) + (5*b)/(32*c^3*d^3*(1 + c*x)) - (5*b*ArcTanh[c*x])/(32*c^3*
d^3) - (a + b*ArcTanh[c*x])/(8*c^3*d^3*(1 + c*x)^2) + (3*(a + b*ArcTanh[c*
x]))/(8*c^3*d^3*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(16*b*c^3*d^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6500
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Si
mp[(a + b*ArcTanh[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b
*ArcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && N
eQ[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{-4b^2cx\operatorname{arctanh}(cx)^2+8ab\,c^2x^2+4abcx+8a^2c^2x^2+4b^2x^2c^2+3b^2cx+6b^2c^2x^2\operatorname{arctanh}(cx)^2-2c\,b^2\operatorname{arctanh}(cx)x-3b^2 a}{16d^3(cx-}$
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{2(cx+1)^2}-\frac{1}{cx+1}\right)}{d^3}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}+\frac{\operatorname{arctanh}(cx)^2}{2(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)\ln(cx-1)}{8}+\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{4(cx+1)}+\frac{3\operatorname{arctanh}(}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{2(cx+1)^2}-\frac{1}{cx+1}\right)}{d^3}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}+\frac{\operatorname{arctanh}(cx)^2}{2(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)\ln(cx-1)}{8}+\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{4(cx+1)}+\frac{3\operatorname{arctanh}(}{}\right.}{}}{}$
parts	$\frac{a^2\left(-\frac{1}{c^2(cx+1)}+\frac{1}{2c^2(cx+1)^2}\right)}{d^3}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}+\frac{\operatorname{arctanh}(cx)^2}{2(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)\ln(cx-1)}{8}+\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{4(cx+1)}\right.}{}$
orering	$\frac{(cx+1)(15c^6x^6-40c^5x^5+47c^4x^4-10x^3c^3-21c^2x^2+12cx-3)(a+b\operatorname{arctanh}(cx))^2}{32x^2c^4(cdx+d)^3}+\frac{(cx-1)(cx+1)^2(15c^4x^4-20x^3c}{}$
risch	$\frac{b^2(3c^2x^2-2cx-1)\ln(cx+1)^2}{32c^2d^3(cx+1)^2}-\frac{b(3b\,c^2x^2\ln(-cx+1)-2bcx\ln(-cx+1)+16acx+6bcx-b\ln(-cx+1)+8a+4b)\ln(cx+}{16c^2d^3(cx+1)^2}$

input
```
int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-4*b^2*c*x*arctanh(c*x)^2+8*a*b*c^2*x^2+4*a*b*c*x+8*a^2*c^2*x^2+4*b^
2*x^2*c^2+3*b^2*c*x+6*b^2*c^2*x^2*arctanh(c*x)^2-2*c*b^2*arctanh(c*x)*x-3*
b^2*arctanh(c*x)+5*b^2*arctanh(c*x)*c^2*x^2+12*arctanh(c*x)*a*b*c^2*x^2-8*
arctanh(c*x)*a*b*c*x-2*b^2*arctanh(c*x)^2-4*arctanh(c*x)*a*b)/d^3/(c*x+1)^
2/c^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = -\frac{2\,(16\,a^2+12\,ab+5\,b^2)cx-(3\,b^2c^2x^2-2\,b^2cx-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-((12\,ab+5}{32\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
-1/32*(2*(16*a^2 + 12*a*b + 5*b^2)*c*x - (3*b^2*c^2*x^2 - 2*b^2*c*x - b^2)
*log(-(c*x + 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((12*a*b + 5*b^2)
*c^2*x^2 - 2*(4*a*b + b^2)*c*x - 4*a*b - 3*b^2)*log(-(c*x + 1)/(c*x - 1)))
/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{\int \frac{a^2x}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x\,\text{atanh}^2\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

input
```
integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2*x/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2
*x*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*
b*x*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(143) = 286$.

Time = 0.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.73

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = -\frac{(2\,cx+1)b^2\,\text{artanh}\,(cx)^2}{2\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)} - \frac{1}{8}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3} - \frac{3\,\log\,(cx+1)}{c^3d^3} + \frac{3\,\log\,(cx-1)}{c^3d^3}\right) + \frac{8\,(2\,cx+1)\,\text{artanh}\,(cx)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}\right)ab - \frac{1}{32}\left(4\,c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3} - \frac{3\,\log\,(cx+1)}{c^3d^3} + \frac{3\,\log\,(cx-1)}{c^3d^3}\right)\text{artanh}\,(cx) + \frac{(3\,(c^2x^2+2\,cx+\right. - \frac{(2\,cx+1)a^2}{2\,(c^4d^3x^2+2\,c^3d^3x+c^2d^3)}$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*c*x + 1)*b^2*arctanh(c*x)^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
- 1/8*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x
+ 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c
^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*a*b - 1/32*(4*c*(2*(3*c*x + 2)/(c^5*d
^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1
)/(c^3*d^3))*arctanh(c*x) + (3*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + 3*(c
^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 + 10*c*x - (5*c^2*x^2 + 10*c*x + 6*(c^2
*x^2 + 2*c*x + 1)*log(c*x - 1) + 5)*log(c*x + 1) + 5*(c^2*x^2 + 2*c*x + 1)
*log(c*x - 1) + 8)*c^2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3))*b^2 - 1/2*(2
*c*x + 1)*a^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.44

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{1}{64}\,c\left(\frac{2\left(\frac{2\,(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^3d^3}+\frac{2\left(\frac{8\,(cx+1)ab}{cx-1}+4\,ab+\frac{4\,(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^2\log\left(-\frac{cx}{cx}\right.}{(cx+1)^2c^3d^3}\right.$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
1/64*c*(2*(2*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*
x - 1))^2/((c*x + 1)^2*c^3*d^3) + 2*(8*(c*x + 1)*a*b/(c*x - 1) + 4*a*b + 4
*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))/((c*
x + 1)^2*c^3*d^3) + (16*(c*x + 1)*a^2/(c*x - 1) + 8*a^2 + 16*(c*x + 1)*a*b
/(c*x - 1) + 4*a*b + 8*(c*x + 1)*b^2/(c*x - 1) + b^2)*(c*x - 1)^2/((c*x +
1)^2*c^3*d^3))
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.58

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx =$$
$$-\frac{16\,a\,b+17\,b^2\,\ln(c\,x+1)-17\,b^2\,\ln(1-c\,x)+b^2\ln(c\,x+1)^2+b^2\ln(1-c\,x)^2-28\,b^2\operatorname{atanh}(c\,x)+}{}$$

input
```
int((x*(a + b*atanh(c*x))^2)/(d + c*d*x)^3,x)
```

output
```
-(16*a*b + 17*b^2*log(c*x + 1) - 17*b^2*log(1 - c*x) + b^2*log(c*x + 1)^2
+ b^2*log(1 - c*x)^2 - 28*b^2*atanh(c*x) + 16*a^2 + 8*b^2 + 16*a*b*log(c*x
 + 1) - 16*a*b*log(1 - c*x) - 2*b^2*log(c*x + 1)*log(1 - c*x) - 24*a*b*ata
nh(c*x) + 32*a^2*c*x + 10*b^2*c*x + 30*b^2*c*x*log(c*x + 1) - 30*b^2*c*x*l
og(1 - c*x) - 3*b^2*c^2*x^2*log(c*x + 1)^2 - 3*b^2*c^2*x^2*log(1 - c*x)^2
- 28*b^2*c^2*x^2*atanh(c*x) + 2*b^2*c*x*log(c*x + 1)^2 + 2*b^2*c*x*log(1 -
 c*x)^2 - 56*b^2*c*x*atanh(c*x) + 9*b^2*c^2*x^2*log(c*x + 1) - 9*b^2*c^2*x
^2*log(1 - c*x) + 24*a*b*c*x + 32*a*b*c*x*log(c*x + 1) - 32*a*b*c*x*log(1
- c*x) - 4*b^2*c*x*log(c*x + 1)*log(1 - c*x) - 24*a*b*c^2*x^2*atanh(c*x) -
 48*a*b*c*x*atanh(c*x) + 6*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x))/(32*c^2*
d^3*(c*x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.91

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$=\frac{12atanh(cx)^2\,b^2c^2x^2-8atanh(cx)^2\,b^2cx-4atanh(cx)^2\,b^2+32atanh(cx)\,ab\,c^2x^2+12atanh(cx)\,b^2c^2x^2-}{}$$

input
```
int(x*(a+b*atanh(c*x))^2/(c*d*x+d)^3,x)
```

output
```
(12*atanh(c*x)**2*b**2*c**2*x**2 - 8*atanh(c*x)**2*b**2*c*x - 4*atanh(c*x)
**2*b**2 + 32*atanh(c*x)*a*b*c**2*x**2 + 12*atanh(c*x)*b**2*c**2*x**2 - 4*
atanh(c*x)*b**2 + 4*log(c*x - 1)*a*b*c**2*x**2 + 8*log(c*x - 1)*a*b*c*x +
4*log(c*x - 1)*a*b + log(c*x - 1)*b**2*c**2*x**2 + 2*log(c*x - 1)*b**2*c*x
 + log(c*x - 1)*b**2 - 4*log(c*x + 1)*a*b*c**2*x**2 - 8*log(c*x + 1)*a*b*c
*x - 4*log(c*x + 1)*a*b - log(c*x + 1)*b**2*c**2*x**2 - 2*log(c*x + 1)*b**
2*c*x - log(c*x + 1)*b**2 + 16*a**2*c**2*x**2 + 12*a*b*c**2*x**2 - 4*a*b +
 5*b**2*c**2*x**2 - 3*b**2)/(32*c**2*d**3*(c**2*x**2 + 2*c*x + 1))
```

3.115 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{(d+cdx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx=-\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}+\frac{3b^2\text{arctanh}(cx)}{16cd^3}-\frac{b(a+b\text{arctanh}(cx))}{4cd^3(1+cx)^2}-\frac{b(a+b\text{arctanh}(cx))}{4cd^3(1+cx)}+\frac{(a+b\text{arctanh}(cx))^2}{8cd^3}-\frac{(a+b\text{arctanh}(cx))^2}{2cd^3(1+cx)^2}$$

output
```
-1/16*b^2/c/d^3/(c*x+1)^2-3/16*b^2/c/d^3/(c*x+1)+3/16*b^2*arctanh(c*x)/c/d
^3-1/4*b*(a+b*arctanh(c*x))/c/d^3/(c*x+1)^2-1/4*b*(a+b*arctanh(c*x))/c/d^3
/(c*x+1)+1/8*(a+b*arctanh(c*x))^2/c/d^3-1/2*(a+b*arctanh(c*x))^2/c/d^3/(c*
x+1)^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{-8a^2-4ab-b^2}{16cd^3(1+cx)^2} - \frac{b(4a+3b)}{16cd^3(1+cx)} - \frac{b(4a+2b+bcx)\text{arctanh}(cx)}{4cd^3(1+cx)^2} + \frac{b^2(-3+2cx+c^2x^2)\,\text{arctanh}(cx)^2}{8cd^3(1+cx)^2} + \frac{(-4ab-3b^2)\log(1-cx)}{32cd^3} + \frac{(4ab+3b^2)\log(1+cx)}{32cd^3}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

output
```
(-8*a^2 - 4*a*b - b^2)/(16*c*d^3*(1 + c*x)^2) - (b*(4*a + 3*b))/(16*c*d^3*
(1 + c*x)) - (b*(4*a + 2*b + b*c*x)*ArcTanh[c*x])/(4*c*d^3*(1 + c*x)^2) +
(b^2*(-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^2)/(8*c*d^3*(1 + c*x)^2) + ((-4*a
*b - 3*b^2)*Log[1 - c*x])/(32*c*d^3) + ((4*a*b + 3*b^2)*Log[1 + c*x])/(32*
c*d^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(cdx+d)^3}\,dx$$

$$\Big\downarrow \ 6480$$

$$\frac{b\int \left(\frac{a+b\text{arctanh}(cx)}{4d^2(1-c^2x^2)}+\frac{a+b\text{arctanh}(cx)}{4d^2(cx+1)^2}+\frac{a+b\text{arctanh}(cx)}{2d^2(cx+1)^3}\right)dx}{d} - \frac{(a+b\text{arctanh}(cx))^2}{2cd^3(cx+1)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\left(\frac{(a+b\operatorname{arctanh}(cx))^2}{8bcd^2}-\frac{a+b\operatorname{arctanh}(cx)}{4cd^2(cx+1)}-\frac{a+b\operatorname{arctanh}(cx)}{4cd^2(cx+1)^2}+\frac{3b\operatorname{arctanh}(cx)}{16cd^2}-\frac{3b}{16cd^2(cx+1)}-\frac{b}{16cd^2(cx+1)^2}\right)}{d}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2cd^3(cx+1)^2}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x])^2/(c*d^3*(1 + c*x)^2) + (b*(-1/16*b/(c*d^2*(1 +
c*x)^2) - (3*b)/(16*c*d^2*(1 + c*x)) + (3*b*ArcTanh[c*x])/(16*c*d^2) - (a
+ b*ArcTanh[c*x])/(4*c*d^2*(1 + c*x)^2) - (a + b*ArcTanh[c*x])/(4*c*d^2*(1
 + c*x)) + (a + b*ArcTanh[c*x])^2/(8*b*c*d^2)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{4b^2cx\operatorname{arctanh}(cx)^2+16a^2cx+8ab\,c^2x^2+12abcx+8a^2c^2x^2+4b^2x^2c^2+5b^2cx+2b^2c^2x^2\operatorname{arctanh}(cx)^2+2c\,b^2\operatorname{arctanh}(cx)}{16d^3}$
derivativedivides	$-\frac{a^2}{2d^3(cx+1)^2}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{2(cx+1)^2}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{8}-\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{\operatorname{arctanh}(cx)}{4(cx+1)}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\ln(cx-1)\ln\left(\frac{cx}{2}\right.}{16}\right.}{}$
default	$-\frac{a^2}{2d^3(cx+1)^2}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{2(cx+1)^2}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{8}-\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{\operatorname{arctanh}(cx)}{4(cx+1)}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\ln(cx-1)\ln\left(\frac{cx}{2}\right.}{16}\right.}{}$
parts	$-\frac{a^2}{2d^3(cx+1)^2c}+\frac{b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{2(cx+1)^2}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{8}-\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{\operatorname{arctanh}(cx)}{4(cx+1)}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\ln(cx-1)}{}\right.}{}$
orering	$\frac{(cx+1)(9c^4x^4-18x^3c^3-7c^2x^2+37cx-21)(a+b\operatorname{arctanh}(cx))^2}{16c(cdx+d)^3}+\frac{(cx+1)^2(cx-1)(6x^3c^3-6c^2x^2-11cx+10)\left(\frac{2(a+b}{(cdx+}\right.}{16c^2}$
risch	$\frac{b^2(c^2x^2+2cx-3)\ln(cx+1)^2}{32d^3(cx+1)^2c}-\frac{b(b\,c^2x^2\ln(-cx+1)+2bcx\ln(-cx+1)+2bcx-3b\ln(-cx+1)+8a+4b)\ln(cx+1)}{16d^3(cx+1)^2c}+\frac{b^2}{}$

input
```
int((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*b^2*c*x*arctanh(c*x)^2+16*a^2*c*x+8*a*b*c^2*x^2+12*a*b*c*x+8*a^2*c
^2*x^2+4*b^2*x^2*c^2+5*b^2*c*x+2*b^2*c^2*x^2*arctanh(c*x)^2+2*c*b^2*arctan
h(c*x)*x-5*b^2*arctanh(c*x)+3*b^2*arctanh(c*x)*c^2*x^2+4*arctanh(c*x)*a*b*
c^2*x^2+8*arctanh(c*x)*a*b*c*x-6*b^2*arctanh(c*x)^2-12*arctanh(c*x)*a*b)/d
^3/(c*x+1)^2/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx =$$
$$-\frac{2\,(4\,ab+3\,b^2)cx-(b^2c^2x^2+2\,b^2cx-3\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-((4\,ab+3\,b^2)c^2x^2+}{32\,(c^3d^3x^2+2\,c^2d^3x+cd^3)}$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
-1/32*(2*(4*a*b + 3*b^2)*c*x - (b^2*c^2*x^2 + 2*b^2*c*x - 3*b^2)*log(-(c*x
 + 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((4*a*b + 3*b^2)*c^2*x^2 +
2*(4*a*b + b^2)*c*x - 12*a*b - 5*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^3*d^3*
x^2 + 2*c^2*d^3*x + c*d^3)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(d + cdx)^3}\, dx = \frac{\int \frac{a^2}{c^3x^3+3c^2x^2+3cx+1}\, dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\, dx + \int \frac{2ab \operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\, dx}{d^3}$$

input
```
integrate((a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*a
tanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*at
anh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(143) = 286$.

Time = 0.04 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.54

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(d + cdx)^3}\, dx = -\frac{1}{8}\left(c\left(\frac{2(cx+2)}{c^4d^3x^2 + 2c^3d^3x + c^2d^3} - \frac{\log(cx+1)}{c^2d^3} + \frac{\log(cx-1)}{c^2d^3}\right) + \frac{8 \operatorname{artanh}(cx)}{c^3d^3x^2 + 2c^2d^3x + cd^3}\right)ab - \frac{1}{32}\left(4c\left(\frac{2(cx+2)}{c^4d^3x^2 + 2c^3d^3x + c^2d^3} - \frac{\log(cx+1)}{c^2d^3} + \frac{\log(cx-1)}{c^2d^3}\right)\operatorname{artanh}(cx) + \frac{((c^2x^2 + 2cx + 1)\log}{} \right. - \frac{b^2 \operatorname{artanh}(cx)^2}{2(c^3d^3x^2 + 2c^2d^3x + cd^3)} - \frac{a^2}{2(c^3d^3x^2 + 2c^2d^3x + cd^3)}$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/8*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/
(c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*
d^3*x + c*d^3))*a*b - 1/32*(4*c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x +
c^2*d^3) - log(c*x + 1)/(c^2*d^3) + log(c*x - 1)/(c^2*d^3))*arctanh(c*x) +
 ((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1
)^2 + 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) +
3)*log(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*d^3*x
^2 + 2*c^4*d^3*x + c^3*d^3))*b^2 - 1/2*b^2*arctanh(c*x)^2/(c^3*d^3*x^2 + 2
*c^2*d^3*x + c*d^3) - 1/2*a^2/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.48

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx = \frac{1}{64}c\left(\frac{2\left(\frac{2(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^2c^2d^3}+\frac{2\left(\frac{8(cx+1)ab}{cx-1}-4ab+\frac{4(cx+1)b^2}{cx-1}-b^2\right)(cx-1)^2\log\left(-\frac{cx}{cx}\right.}{(cx+1)^2c^2d^3}\right.$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
1/64*c*(2*(2*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*
x - 1))^2/((c*x + 1)^2*c^2*d^3) + 2*(8*(c*x + 1)*a*b/(c*x - 1) - 4*a*b + 4
*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))/((c*
x + 1)^2*c^2*d^3) + (16*(c*x + 1)*a^2/(c*x - 1) - 8*a^2 + 16*(c*x + 1)*a*b
/(c*x - 1) - 4*a*b + 8*(c*x + 1)*b^2/(c*x - 1) - b^2)*(c*x - 1)^2/((c*x +
1)^2*c^2*d^3))
```

Mupad [B] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.38

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$= \frac{11\,b^2\,\ln(1-c\,x)-11\,b^2\,\ln(c\,x+1)-16\,a\,b-3\,b^2\ln(c\,x+1)^2-3\,b^2\ln(1-c\,x)^2+12\,b^2\,\mathrm{atanh}(c\,x)-}{}$$

input
```
int((a + b*atanh(c*x))^2/(d + c*d*x)^3,x)
```

output
```
(11*b^2*log(1 - c*x) - 11*b^2*log(c*x + 1) - 16*a*b - 3*b^2*log(c*x + 1)^2
 - 3*b^2*log(1 - c*x)^2 + 12*b^2*atanh(c*x) - 16*a^2 - 8*b^2 - 16*a*b*log(
c*x + 1) + 16*a*b*log(1 - c*x) + 6*b^2*log(c*x + 1)*log(1 - c*x) + 8*a*b*a
tanh(c*x) - 6*b^2*c*x - 10*b^2*c*x*log(c*x + 1) + 10*b^2*c*x*log(1 - c*x)
+ b^2*c^2*x^2*log(c*x + 1)^2 + b^2*c^2*x^2*log(1 - c*x)^2 + 12*b^2*c^2*x^2
*atanh(c*x) + 2*b^2*c*x*log(c*x + 1)^2 + 2*b^2*c*x*log(1 - c*x)^2 + 24*b^2
*c*x*atanh(c*x) - 3*b^2*c^2*x^2*log(c*x + 1) + 3*b^2*c^2*x^2*log(1 - c*x)
- 8*a*b*c*x - 4*b^2*c*x*log(c*x + 1)*log(1 - c*x) + 8*a*b*c^2*x^2*atanh(c*
x) + 16*a*b*c*x*atanh(c*x) - 2*b^2*c^2*x^2*log(c*x + 1)*log(1 - c*x))/(32*
c*d^3*(c*x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.83

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{(d+cdx)^3}\,dx$$
$$= \frac{4atanh(cx)^2\,b^2c^2x^2+8atanh(cx)^2\,b^2cx-12atanh(cx)^2\,b^2-32atanh(cx)\,ab+4atanh(cx)\,b^2c^2x^2-12at}{}$$

input
```
int((a+b*atanh(c*x))^2/(c*d*x+d)^3,x)
```

output

```
(4*atanh(c*x)**2*b**2*c**2*x**2 + 8*atanh(c*x)**2*b**2*c*x - 12*atanh(c*x)
**2*b**2 - 32*atanh(c*x)*a*b + 4*atanh(c*x)*b**2*c**2*x**2 - 12*atanh(c*x)
*b**2 - 4*log(c*x - 1)*a*b*c**2*x**2 - 8*log(c*x - 1)*a*b*c*x - 4*log(c*x
- 1)*a*b - log(c*x - 1)*b**2*c**2*x**2 - 2*log(c*x - 1)*b**2*c*x - log(c*x
 - 1)*b**2 + 4*log(c*x + 1)*a*b*c**2*x**2 + 8*log(c*x + 1)*a*b*c*x + 4*log
(c*x + 1)*a*b + log(c*x + 1)*b**2*c**2*x**2 + 2*log(c*x + 1)*b**2*c*x + lo
g(c*x + 1)*b**2 - 16*a**2 + 4*a*b*c**2*x**2 - 12*a*b + 3*b**2*c**2*x**2 -
5*b**2)/(32*c*d**3*(c**2*x**2 + 2*c*x + 1))
```

3.116 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x(d+cdx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 362

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x(d+cdx)^3}\,dx = \frac{b^2}{16d^3(1+cx)^2}+\frac{11b^2}{16d^3(1+cx)} - \frac{11b^2\operatorname{arctanh}(cx)}{16d^3}+\frac{b(a+b\operatorname{arctanh}(cx))}{4d^3(1+cx)^2} + \frac{5b(a+b\operatorname{arctanh}(cx))}{4d^3(1+cx)}-\frac{5(a+b\operatorname{arctanh}(cx))^2}{8d^3} + \frac{(a+b\operatorname{arctanh}(cx))^2}{2d^3(1+cx)^2}+\frac{(a+b\operatorname{arctanh}(cx))^2}{d^3(1+cx)} + \frac{2(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)}{d^3} + \frac{(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d^3} - \frac{b(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d^3} + \frac{b(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d^3} - \frac{b(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^3} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^3}-\frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d^3} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d^3}$$

output
```
1/16*b^2/d^3/(c*x+1)^2+11/16*b^2/d^3/(c*x+1)-11/16*b^2*arctanh(c*x)/d^3+1/
4*b*(a+b*arctanh(c*x))/d^3/(c*x+1)^2+5/4*b*(a+b*arctanh(c*x))/d^3/(c*x+1)-
5/8*(a+b*arctanh(c*x))^2/d^3+1/2*(a+b*arctanh(c*x))^2/d^3/(c*x+1)^2+(a+b*a
rctanh(c*x))^2/d^3/(c*x+1)-2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d
^3+(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^3-b*(a+b*arctanh(c*x))*polylog(2,1
-2/(-c*x+1))/d^3+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^3-b*(a+b*
arctanh(c*x))*polylog(2,1-2/(c*x+1))/d^3+1/2*b^2*polylog(3,1-2/(-c*x+1))/d
^3-1/2*b^2*polylog(3,-1+2/(-c*x+1))/d^3-1/2*b^2*polylog(3,1-2/(c*x+1))/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^3}\,dx$$
$$= \frac{\frac{96a^2}{(1+cx)^2}+\frac{192a^2}{1+cx}+192a^2\log(cx)-192a^2\log(1+cx)+12ab\big(12\cosh(2\text{arctanh}(cx))+\cosh(4\text{arctanh}(cx)\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

output
```
((96*a^2)/(1 + c*x)^2 + (192*a^2)/(1 + c*x) + 192*a^2*Log[c*x] - 192*a^2*L
og[1 + c*x] + 12*a*b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*
PolyLog[2, E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]
*(6*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[
c*x])] - 6*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c
*x]]) + b^2*((8*I)*Pi^3 - 128*ArcTanh[c*x]^3 + 72*Cosh[2*ArcTanh[c*x]] + 1
44*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 144*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c
*x]] + 3*Cosh[4*ArcTanh[c*x]] + 12*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] + 24*
ArcTanh[c*x]^2*Cosh[4*ArcTanh[c*x]] + 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcT
anh[c*x])] + 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 96*PolyLog[
3, E^(2*ArcTanh[c*x])] - 72*Sinh[2*ArcTanh[c*x]] - 144*ArcTanh[c*x]*Sinh[2
*ArcTanh[c*x]] - 144*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - 3*Sinh[4*ArcTan
h[c*x]] - 12*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 24*ArcTanh[c*x]^2*Sinh[4*
ArcTanh[c*x]]))/(192*d^3)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(cdx+d)^3}\,dx$$

$\big\downarrow$ 6502

$$\int \left(\frac{(a+b\text{arctanh}(cx))^2}{d^3x}-\frac{c(a+b\text{arctanh}(cx))^2}{d^3(cx+1)}-\frac{c(a+b\text{arctanh}(cx))^2}{d^3(cx+1)^2}-\frac{c(a+b\text{arctanh}(cx))^2}{d^3(cx+1)^3}\right)dx$$

$\big\downarrow$ 2009

$$-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{d^3}+\frac{b\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))}{d^3}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^3}+\frac{5b(a+b\text{arctanh}(cx))}{4d^3(cx+1)}+\frac{b(a+b\text{arctanh}(cx))}{4d^3(cx+1)^2}+\frac{(a+b\text{arctanh}(cx))^2}{d^3(cx+1)}+\frac{(a+b\text{arctanh}(cx))^2}{2d^3(cx+1)^2}-\frac{5(a+b\text{arctanh}(cx))^2}{8d^3}+\frac{2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{d^3}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d^3}-\frac{11b^2\text{arctanh}(cx)}{16d^3}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^3}-\frac{b^2\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d^3}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{11b^2}{16d^3(cx+1)}+\frac{b^2}{16d^3(cx+1)^2}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

output
```
b^2/(16*d^3*(1 + c*x)^2) + (11*b^2)/(16*d^3*(1 + c*x)) - (11*b^2*ArcTanh[c
*x])/(16*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) + (5*b*(a + b
*ArcTanh[c*x]))/(4*d^3*(1 + c*x)) - (5*(a + b*ArcTanh[c*x])^2)/(8*d^3) + (
a + b*ArcTanh[c*x])^2/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])^2/(d^3*(1
 + c*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 + ((a +
 b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog
[2, 1 - 2/(1 - c*x)])/d^3 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
 c*x)])/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 + (
b^2*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^3) - (b^2*PolyLog[3, -1 + 2/(1 - c*x
)])/(2*d^3) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.71 (sec) , antiderivative size = 1321, normalized size of antiderivative = 3.65

method	result	size
parts	Expression too large to display	1321
derivativedivides	Expression too large to display	1323
default	Expression too large to display	1323

input
```
int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^3*(1/2/(c*x+1)^2+1/(c*x+1)-ln(c*x+1)+ln(x))+b^2/d^3*(arctanh(c*x)^2*
ln(c*x)+1/2/(c*x+1)^2*arctanh(c*x)^2+1/(c*x+1)*arctanh(c*x)^2-arctanh(c*x)
^2*ln(c*x+1)+2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-2/3*arctanh(c
*x)^3+1/8*(4*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-
1)))^3-4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
1-(c*x+1)^2/(c^2*x^2-1)))^2+4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-4*I*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c
^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))-4*I*Pi*csgn(I*(-(c*x+1)^2/(c
^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))
^2+4*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I
/(1-(c*x+1)^2/(c^2*x^2-1)))-4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(
I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+4*I*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+4*I*Pi*csgn(I*(c*x+1)/(-c
^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+4*I*Pi*csgn(I/(1-(c*x+1)^
2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2
*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))+8*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(
1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-5+8*ln(2))*arctanh(c*x)^2-arctanh(c*
x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)
^(1/2))+2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(
c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^4 + 3*
c^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x(d + cdx)^3}\,dx = \frac{\int \frac{a^2}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

input
```
integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b**
2*atanh(c*x)**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(2*
a*b*atanh(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x(d + cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{(cdx + d)^3x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*a^2*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3
+ 2*log(x)/d^3) + 1/8*(2*b^2*c*x + 3*b^2 - 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^
2)*log(c*x + 1))*log(-c*x + 1)^2/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) + integra
te(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) -
(2*b^2*c^3*x^3 + 5*b^2*c^2*x^2 - 4*a*b + (4*a*b*c + 3*b^2*c)*x - 2*(b^2*c^
4*x^4 + 3*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x + 1))/
(c^4*d^3*x^5 + 2*c^3*d^3*x^4 - 2*c*d^3*x^2 - d^3*x), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^3}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^2}{x\,(d+c\,d\,x)^3}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^3),x)
```

output
```
int((a + b*atanh(c*x))^2/(x*(d + c*d*x)^3), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+cdx)^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*atanh(c*x))^2/x/(c*d*x+d)^3,x)
```

output

```
( - 16*atanh(c*x)**3*b**2*c**2*x**2 - 32*atanh(c*x)**3*b**2*c*x - 16*atanh
(c*x)**3*b**2 - 48*atanh(c*x)**2*a*b*c**2*x**2 - 96*atanh(c*x)**2*a*b*c*x
- 48*atanh(c*x)**2*a*b - 36*atanh(c*x)**2*b**2*c**2*x**2 - 24*atanh(c*x)**
2*b**2*c*x + 60*atanh(c*x)**2*b**2 - 48*atanh(c*x)*a*b*c**2*x**2 + 144*ata
nh(c*x)*a*b - 36*atanh(c*x)*b**2*c**2*x**2 + 60*atanh(c*x)*b**2 - 384*int(
atanh(c*x)/(c**4*x**5 + 2*c**3*x**4 - 2*c*x**2 - x),x)*a*b*c**2*x**2 - 768
*int(atanh(c*x)/(c**4*x**5 + 2*c**3*x**4 - 2*c*x**2 - x),x)*a*b*c*x - 384*
int(atanh(c*x)/(c**4*x**5 + 2*c**3*x**4 - 2*c*x**2 - x),x)*a*b - 192*int(a
tanh(c*x)**2/(c**4*x**5 + 2*c**3*x**4 - 2*c*x**2 - x),x)*b**2*c**2*x**2 -
384*int(atanh(c*x)**2/(c**4*x**5 + 2*c**3*x**4 - 2*c*x**2 - x),x)*b**2*c*x
 - 192*int(atanh(c*x)**2/(c**4*x**5 + 2*c**3*x**4 - 2*c*x**2 - x),x)*b**2
+ 12*log(c*x - 1)*a*b*c**2*x**2 + 24*log(c*x - 1)*a*b*c*x + 12*log(c*x - 1
)*a*b + 3*log(c*x - 1)*b**2*c**2*x**2 + 6*log(c*x - 1)*b**2*c*x + 3*log(c*
x - 1)*b**2 - 192*log(c*x + 1)*a**2*c**2*x**2 - 384*log(c*x + 1)*a**2*c*x
- 192*log(c*x + 1)*a**2 - 12*log(c*x + 1)*a*b*c**2*x**2 - 24*log(c*x + 1)*
a*b*c*x - 12*log(c*x + 1)*a*b - 3*log(c*x + 1)*b**2*c**2*x**2 - 6*log(c*x
+ 1)*b**2*c*x - 3*log(c*x + 1)*b**2 + 192*log(x)*a**2*c**2*x**2 + 384*log(
x)*a**2*c*x + 192*log(x)*a**2 - 96*a**2*c**2*x**2 + 192*a**2 - 36*a*b*c**2
*x**2 + 60*a*b - 21*b**2*c**2*x**2 + 27*b**2)/(192*d**3*(c**2*x**2 + 2*c*x
 + 1))
```

3.117 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 448

$$\int \frac{(a+b\,\text{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx = -\frac{b^2c}{16d^3(1+cx)^2}-\frac{19b^2c}{16d^3(1+cx)}+\frac{19b^2c\,\text{arctanh}(cx)}{16d^3} - \frac{bc(a+b\,\text{arctanh}(cx))}{4d^3(1+cx)^2}-\frac{9bc(a+b\,\text{arctanh}(cx))}{4d^3(1+cx)} + \frac{17c(a+b\,\text{arctanh}(cx))^2}{8d^3}-\frac{(a+b\,\text{arctanh}(cx))^2}{d^3x} - \frac{c(a+b\,\text{arctanh}(cx))^2}{2d^3(1+cx)^2}-\frac{2c(a+b\,\text{arctanh}(cx))^2}{d^3(1+cx)} - \frac{6c(a+b\,\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)}{d^3} - \frac{3c(a+b\,\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d^3} + \frac{2bc(a+b\,\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d^3} + \frac{3bc(a+b\,\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d^3} - \frac{3bc(a+b\,\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d^3} + \frac{3bc(a+b\,\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^3} - \frac{b^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d^3}-\frac{3b^2c\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^3} + \frac{3b^2c\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d^3} + \frac{3b^2c\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d^3}$$

output
```
-1/16*b^2*c/d^3/(c*x+1)^2-19/16*b^2*c/d^3/(c*x+1)+19/16*b^2*c*arctanh(c*x)
/d^3-1/4*b*c*(a+b*arctanh(c*x))/d^3/(c*x+1)^2-9/4*b*c*(a+b*arctanh(c*x))/d
^3/(c*x+1)+17/8*c*(a+b*arctanh(c*x))^2/d^3-(a+b*arctanh(c*x))^2/d^3/x-1/2*
c*(a+b*arctanh(c*x))^2/d^3/(c*x+1)^2-2*c*(a+b*arctanh(c*x))^2/d^3/(c*x+1)+
6*c*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d^3-3*c*(a+b*arctanh(c*x))
^2*ln(2/(c*x+1))/d^3+2*b*c*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))/d^3+3*b*c*(a
+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^3-3*b*c*(a+b*arctanh(c*x))*poly
log(2,-1+2/(-c*x+1))/d^3+3*b*c*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d
^3-b^2*c*polylog(2,-1+2/(c*x+1))/d^3-3/2*b^2*c*polylog(3,1-2/(-c*x+1))/d^3
+3/2*b^2*c*polylog(3,-1+2/(-c*x+1))/d^3+3/2*b^2*c*polylog(3,1-2/(c*x+1))/d
^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx$$

$$= \frac{-\frac{64a^2}{x}-\frac{32a^2c}{(1+cx)^2}-\frac{128a^2c}{1+cx}-192a^2c\log(x)+192a^2c\log(1+cx)+b^2c\Big(-8i\pi^3+64\text{arctanh}(cx)^2-\frac{64\text{arcta}}{cx}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

output
```
((-64*a^2)/x - (32*a^2*c)/(1 + c*x)^2 - (128*a^2*c)/(1 + c*x) - 192*a^2*c*
Log[x] + 192*a^2*c*Log[1 + c*x] + b^2*c*((-8*I)*Pi^3 + 64*ArcTanh[c*x]^2 -
 (64*ArcTanh[c*x]^2)/(c*x) + 128*ArcTanh[c*x]^3 - 40*Cosh[2*ArcTanh[c*x]]
- 80*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]^2*Cosh[2*ArcTanh[
c*x]] - Cosh[4*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] - 8*Arc
Tanh[c*x]^2*Cosh[4*ArcTanh[c*x]] + 128*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[
c*x])] - 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 64*PolyLog[2, E^
(-2*ArcTanh[c*x])] - 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 96*
PolyLog[3, E^(2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]
*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] + Sinh[4*Ar
cTanh[c*x]] + 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Sinh[
4*ArcTanh[c*x]]) + (4*a*b*(48*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*x*(-
20*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[(c*x)/Sqrt[1 - c^2
*x^2]] + 20*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]]) - 4*ArcTanh[c*x]*
(8 + 10*c*x*Cosh[2*ArcTanh[c*x]] + c*x*Cosh[4*ArcTanh[c*x]] + 24*c*x*Log[1
 - E^(-2*ArcTanh[c*x])] - 10*c*x*Sinh[2*ArcTanh[c*x]] - c*x*Sinh[4*ArcTanh
[c*x]])))/x)/(64*d^3)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(cdx+d)^3}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{3c^2(a+b\text{arctanh}(cx))^2}{d^3(cx+1)}+\frac{2c^2(a+b\text{arctanh}(cx))^2}{d^3(cx+1)^2}+\frac{c^2(a+b\text{arctanh}(cx))^2}{d^3(cx+1)^3}+\frac{(a+b\text{arctanh}(cx))^2}{d^3x^2}-\frac{3c(a$$

$$\Big\downarrow \text{2009}$$

$$\frac{3bc\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{d^3}-\frac{3bc\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))}{d^3}+\frac{3bc\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^3}-\frac{9bc(a+b\text{arctanh}(cx))}{4d^3(cx+1)}-\frac{bc(a+b\text{arctanh}(cx))}{4d^3(cx+1)^2}-\frac{(a+b\text{arctanh}(cx))^2}{d^3x}-\frac{2c(a+b\text{arctanh}(cx))^2}{d^3(cx+1)}-\frac{c(a+b\text{arctanh}(cx))^2}{2d^3(cx+1)^2}+\frac{17c(a+b\text{arctanh}(cx))^2}{8d^3}-\frac{6c\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{d^3}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^3}-\frac{3c\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d^3}+\frac{19b^2c\text{arctanh}(cx)}{16d^3}-\frac{b^2c\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d^3}-\frac{3b^2c\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^3}+\frac{3b^2c\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d^3}+\frac{3b^2c\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d^3}-\frac{19b^2c}{16d^3(cx+1)}-\frac{b^2c}{16d^3(cx+1)^2}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

output
```
-1/16*(b^2*c)/(d^3*(1 + c*x)^2) - (19*b^2*c)/(16*d^3*(1 + c*x)) + (19*b^2*
c*ArcTanh[c*x])/(16*d^3) - (b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2)
- (9*b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)) + (17*c*(a + b*ArcTanh[c*
x])^2)/(8*d^3) - (a + b*ArcTanh[c*x])^2/(d^3*x) - (c*(a + b*ArcTanh[c*x])^
2)/(2*d^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x])^2)/(d^3*(1 + c*x)) - (6
*c*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 - (3*c*(a + b*ArcT
anh[c*x])^2*Log[2/(1 + c*x)])/d^3 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/
(1 + c*x)])/d^3 + (3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])
/d^3 - (3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^3 + (3*
b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 - (b^2*c*PolyLog
[2, -1 + 2/(1 + c*x)])/d^3 - (3*b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^3)
 + (3*b^2*c*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*PolyLog[3, 1
- 2/(1 + c*x)])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.03 (sec) , antiderivative size = 4301, normalized size of antiderivative = 9.60

method	result	size
parts	Expression too large to display	4301
derivativedivides	Expression too large to display	4346
default	Expression too large to display	4346

input
```
int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^3*(-1/2/(c*x+1)^2*c-2*c/(c*x+1)+3*c*ln(c*x+1)-1/x-3*c*ln(x))+b^2/d^3
*c*(-3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*arctanh(c*x)*poly
log(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+
1)^(1/2))-6*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*arctanh(
c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)
^2/(c^2*x^2-1)))*(arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+(c
*x+1)/(-c^2*x^2+1)^(1/2))-dilog((c*x+1)/(-c^2*x^2+1)^(1/2)))-3*arctanh(c*x
)^2*ln(c*x)+3*ln(2)*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-3*ln(2)*dilog(1+(c*x
+1)/(-c^2*x^2+1)^(1/2))+3*ln(2)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*ln
(2)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*arctanh(c*x)^2*ln(c*x+1)-1/8*
arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*ln(1+(c*x+1)/
(-c^2*x^2+1)^(1/2))-3/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^
2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*x)*ln(1-(c*
x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-arc
tanh(c*x)^2+polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+polylog(2,-(c*x+1)/(-c^2
*x^2+1)^(1/2)))+3/2*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^
2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*(arctanh(c*x)*ln(1-(c*x+1)/(-c^
2*x^2+1)^(1/2))+arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)
^2+polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+polylog(2,-(c*x+1)/(-c^2*x^2+1...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^5 + 3*
c^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2(d + cdx)^3}\,dx = \frac{\int \frac{a^2}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

input
```
integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**3,x)
```

output
```
(Integral(a**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(
b**2*atanh(c*x)**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integ
ral(2*a*b*atanh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2(d + cdx)^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{(cdx + d)^3 x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*a^2*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*
c*log(c*x + 1)/d^3 + 6*c*log(x)/d^3) - 1/8*(6*b^2*c^2*x^2 + 9*b^2*c*x + 2*
b^2 - 6*(b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c*x + 1
)^2/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - integrate(-1/4*((b^2*c*x - b^2)*
log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 15*b^2*
c^3*x^3 + 11*b^2*c^2*x^2 + 4*a*b - 2*(2*a*b*c - b^2*c)*x - 2*(3*b^2*c^5*x^
5 + 9*b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x
 + 1))*log(-c*x + 1))/(c^4*d^3*x^6 + 2*c^3*d^3*x^5 - 2*c*d^3*x^3 - d^3*x^2
), x)
```

Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2}{(cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx=\int \frac{(a+b\operatorname{atanh}(c\,x))^2}{x^2\,(d+c\,d\,x)^3}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^3),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^2*(d + c*d*x)^3), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+cdx)^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*atanh(c*x))^2/x^2/(c*d*x+d)^3,x)
```

output
```
( - 16*atanh(c*x)**3*b**2*c**3*x**3 - 32*atanh(c*x)**3*b**2*c**2*x**2 - 16
*atanh(c*x)**3*b**2*c*x - 48*atanh(c*x)**2*a*b*c**3*x**3 - 96*atanh(c*x)**
2*a*b*c**2*x**2 - 48*atanh(c*x)**2*a*b*c*x - 60*atanh(c*x)**2*b**2*c**3*x*
*3 - 72*atanh(c*x)**2*b**2*c**2*x**2 + 36*atanh(c*x)**2*b**2*c*x + 64*atan
h(c*x)**2*b**2 - 48*atanh(c*x)*a*b*c**3*x**3 + 144*atanh(c*x)*a*b*c*x + 12
8*atanh(c*x)*a*b - 60*atanh(c*x)*b**2*c**3*x**3 + 132*atanh(c*x)*b**2*c*x
+ 64*atanh(c*x)*b**2 - 384*int(atanh(c*x)/(c**4*x**6 + 2*c**3*x**5 - 2*c*x
**3 - x**2),x)*a*b*c**2*x**3 - 768*int(atanh(c*x)/(c**4*x**6 + 2*c**3*x**5
 - 2*c*x**3 - x**2),x)*a*b*c*x**2 - 384*int(atanh(c*x)/(c**4*x**6 + 2*c**3
*x**5 - 2*c*x**3 - x**2),x)*a*b*x - 64*int(atanh(c*x)/(c**4*x**6 + 2*c**3*
x**5 - 2*c*x**3 - x**2),x)*b**2*c**2*x**3 - 128*int(atanh(c*x)/(c**4*x**6
+ 2*c**3*x**5 - 2*c*x**3 - x**2),x)*b**2*c*x**2 - 64*int(atanh(c*x)/(c**4*
x**6 + 2*c**3*x**5 - 2*c*x**3 - x**2),x)*b**2*x - 192*int(atanh(c*x)**2/(c
**4*x**6 + 2*c**3*x**5 - 2*c*x**3 - x**2),x)*b**2*c**2*x**3 - 384*int(atan
h(c*x)**2/(c**4*x**6 + 2*c**3*x**5 - 2*c*x**3 - x**2),x)*b**2*c*x**2 - 192
*int(atanh(c*x)**2/(c**4*x**6 + 2*c**3*x**5 - 2*c*x**3 - x**2),x)*b**2*x +
 28*log(c*x - 1)*a*b*c**3*x**3 + 56*log(c*x - 1)*a*b*c**2*x**2 + 28*log(c*
x - 1)*a*b*c*x + 17*log(c*x - 1)*b**2*c**3*x**3 + 34*log(c*x - 1)*b**2*c**
2*x**2 + 17*log(c*x - 1)*b**2*c*x + 384*log(c*x + 1)*a**2*c**3*x**3 + 768*
log(c*x + 1)*a**2*c**2*x**2 + 384*log(c*x + 1)*a**2*c*x + 100*log(c*x +...
```

3.118 $\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 176

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx = -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)} +\frac{11b^2\text{arctanh}(cx)}{144c}-\frac{b(a+b\text{arctanh}(cx))}{9c(1+cx)^3} -\frac{b(a+b\text{arctanh}(cx))}{12c(1+cx)^2}-\frac{b(a+b\text{arctanh}(cx))}{12c(1+cx)} +\frac{(a+b\text{arctanh}(cx))^2}{24c}-\frac{(a+b\text{arctanh}(cx))^2}{3c(1+cx)^3}$$

output
```
-1/54*b^2/c/(c*x+1)^3-5/144*b^2/c/(c*x+1)^2-11/144*b^2/c/(c*x+1)+11/144*b^
2*arctanh(c*x)/c-1/9*b*(a+b*arctanh(c*x))/c/(c*x+1)^3-1/12*b*(a+b*arctanh(
c*x))/c/(c*x+1)^2-1/12*b*(a+b*arctanh(c*x))/c/(c*x+1)+1/24*(a+b*arctanh(c*
x))^2/c-1/3*(a+b*arctanh(c*x))^2/c/(c*x+1)^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx = -\frac{16(18a^2+6ab+b^2)+6b(12a+5b)(1+cx)+6b(12a+11b)(1+cx)^2+24b(24a+b(10+9cx+3c^2x$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

output
```
-1/864*(16*(18*a^2 + 6*a*b + b^2) + 6*b*(12*a + 5*b)*(1 + c*x) + 6*b*(12*a
 + 11*b)*(1 + c*x)^2 + 24*b*(24*a + b*(10 + 9*c*x + 3*c^2*x^2))*ArcTanh[c*
x] - 36*b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + 3*b*(12*a
+ 11*b)*(1 + c*x)^3*Log[1 - c*x] - 3*b*(12*a + 11*b)*(1 + c*x)^3*Log[1 + c
*x])/(c*(1 + c*x)^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(cx+1)^4}dx$$

$$\Big\downarrow \text{6480}$$

$$\frac{2}{3}b\int \left(\frac{a+b\text{arctanh}(cx)}{8(1-c^2x^2)}+\frac{a+b\text{arctanh}(cx)}{8(cx+1)^2}+\frac{a+b\text{arctanh}(cx)}{4(cx+1)^3}+\frac{a+b\text{arctanh}(cx)}{2(cx+1)^4}\right)dx-\frac{(a+b\text{arctanh}(cx))^2}{3c(cx+1)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}b\left(\frac{(a+b\text{arctanh}(cx))^2}{16bc}-\frac{a+b\text{arctanh}(cx)}{8c(cx+1)}-\frac{a+b\text{arctanh}(cx)}{8c(cx+1)^2}-\frac{a+b\text{arctanh}(cx)}{6c(cx+1)^3}+\frac{11b\text{arctanh}(cx)}{96c}-\frac{\quad}{96c}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{3c(cx+1)^3}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x])^2/(c*(1 + c*x)^3) + (2*b*(-1/36*b/(c*(1 + c*x)^3
) - (5*b)/(96*c*(1 + c*x)^2) - (11*b)/(96*c*(1 + c*x)) + (11*b*ArcTanh[c*x
])/(96*c) - (a + b*ArcTanh[c*x])/(6*c*(1 + c*x)^3) - (a + b*ArcTanh[c*x])/
(8*c*(1 + c*x)^2) - (a + b*ArcTanh[c*x])/(8*c*(1 + c*x)) + (a + b*ArcTanh[
c*x])^2/(16*b*c)))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.43

method	result
parallelrisch	$-\frac{-54b^2cx\,\text{arctanh}(cx)^2-432a^2cx-144a^2c^3x^3-324ab\,c^2x^2-252abcx-432a^2c^2x^2-135b^2x^2c^2-56b^2x^3c^3-87b^2cx-1}{}$
derivativedivides	$\frac{-\frac{a^2}{3(cx+1)^3}+b^2\left(-\frac{\text{arctanh}(cx)^2}{3(cx+1)^3}-\frac{\text{arctanh}(cx)\ln(cx-1)}{24}-\frac{\text{arctanh}(cx)}{9(cx+1)^3}-\frac{\text{arctanh}(cx)}{12(cx+1)^2}-\frac{\text{arctanh}(cx)}{12(cx+1)}+\frac{\text{arctanh}(cx)\ln(cx+1)}{24}-\ln(\right.}{}$
default	$\frac{-\frac{a^2}{3(cx+1)^3}+b^2\left(-\frac{\text{arctanh}(cx)^2}{3(cx+1)^3}-\frac{\text{arctanh}(cx)\ln(cx-1)}{24}-\frac{\text{arctanh}(cx)}{9(cx+1)^3}-\frac{\text{arctanh}(cx)}{12(cx+1)^2}-\frac{\text{arctanh}(cx)}{12(cx+1)}+\frac{\text{arctanh}(cx)\ln(cx+1)}{24}-\ln(\right.}{}$
parts	$-\frac{a^2}{3(cx+1)^3c}+\frac{b^2\left(-\frac{\text{arctanh}(cx)^2}{3(cx+1)^3}-\frac{\text{arctanh}(cx)\ln(cx-1)}{24}-\frac{\text{arctanh}(cx)}{9(cx+1)^3}-\frac{\text{arctanh}(cx)}{12(cx+1)^2}-\frac{\text{arctanh}(cx)}{12(cx+1)}+\frac{\text{arctanh}(cx)\ln(cx+1)}{24}\right.}{}$
orering	$\frac{(135c^5x^5-9c^4x^4-324x^3c^3+98c^2x^2+337cx-237)(a+b\,\text{arctanh}(cx))^2}{216(cx+1)^3c}+\frac{(cx+1)^2(cx-1)(270c^4x^4+231x^3c^3-447c^2}{}$
risch	$\frac{b^2(x^3c^3+3c^2x^2+3cx-7)\ln(cx+1)^2}{96(cx+1)^3c}-\frac{b(3b\,x^3\ln(-cx+1)c^3+9b\,c^2x^2\ln(-cx+1)+6b\,c^2x^2+9bcx\ln(-cx+1)+18bcx-}{144(cx+1)^3c}$

input
```
int((a+b*arctanh(c*x))^2/(c*x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/432*(-54*b^2*c*x*arctanh(c*x)^2-432*a^2*c*x-144*a^2*c^3*x^3-324*a*b*c^2
*x^2-252*a*b*c*x-432*a^2*c^2*x^2-135*b^2*x^2*c^2-56*b^2*x^3*c^3-87*b^2*c*x
-18*b^2*c^3*x^3*arctanh(c*x)^2-54*b^2*c^2*x^2*arctanh(c*x)^2-33*arctanh(c*
x)*b^2*c^3*x^3+9*c*b^2*arctanh(c*x)*x+87*b^2*arctanh(c*x)-63*b^2*arctanh(c
*x)*c^2*x^2-36*arctanh(c*x)*a*b*c^3*x^3-108*arctanh(c*x)*a*b*c^2*x^2-108*a
rctanh(c*x)*a*b*c*x-120*a*b*c^3*x^3+126*b^2*arctanh(c*x)^2+252*arctanh(c*x
)*a*b)/(c*x+1)^3/c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.15

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx =$$

$$-\frac{6\,(12\,ab+11\,b^2)c^2x^2+54\,(4\,ab+3\,b^2)cx-9\,(b^2c^3x^3+3\,b^2c^2x^2+3\,b^2cx-7\,b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2+288\,a^2}{864\,(c^4x^3}$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="fricas")
```

output
```
-1/864*(6*(12*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b + 3*b^2)*c*x - 9*(b^2*c^3*
x^3 + 3*b^2*c^2*x^2 + 3*b^2*c*x - 7*b^2)*log(-(c*x + 1)/(c*x - 1))^2 + 288
*a^2 + 240*a*b + 112*b^2 - 3*((12*a*b + 11*b^2)*c^3*x^3 + 3*(12*a*b + 7*b^
2)*c^2*x^2 + 3*(12*a*b - b^2)*c*x - 84*a*b - 29*b^2)*log(-(c*x + 1)/(c*x -
 1)))/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(1 + cx)^4}\,dx = \int \frac{(a + b\,\text{atanh}\,(cx))^2}{(cx + 1)^4}\,dx$$

input
```
integrate((a+b*atanh(c*x))**2/(c*x+1)**4,x)
```

output
```
Integral((a + b*atanh(c*x))**2/(c*x + 1)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(158) = 316$.

Time = 0.05 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.53

$$\int \frac{(a + b\text{arctanh}(cx))^2}{(1 + cx)^4}\,dx = -\frac{1}{72}\left(c\left(\frac{2\,(3\,c^2x^2 + 9\,cx + 10)}{c^5x^3 + 3\,c^4x^2 + 3\,c^3x + c^2} - \frac{3\,\log\,(cx + 1)}{c^2} + \frac{3\,\log\,(cx - 1)}{c^2}\right) + \frac{48\,\text{artanh}\,(cx)}{c^4x^3 + 3\,c^3x^2 + 3\,c^2x + c}\right)a - \frac{1}{864}\left(12\,c\left(\frac{2\,(3\,c^2x^2 + 9\,cx + 10)}{c^5x^3 + 3\,c^4x^2 + 3\,c^3x + c^2} - \frac{3\,\log\,(cx + 1)}{c^2} + \frac{3\,\log\,(cx - 1)}{c^2}\right)\text{artanh}\,(cx) + \frac{(66\,c^2x^2 + 9}{} \right. - \frac{b^2\,\text{artanh}\,(cx)^2}{3\,(c^4x^3 + 3\,c^3x^2 + 3\,c^2x + c)} - \frac{a^2}{3\,(c^4x^3 + 3\,c^3x^2 + 3\,c^2x + c)}$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="maxima")
```

output
```
-1/72*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
 - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3
*c^3*x^2 + 3*c^2*x + c))*a*b - 1/864*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^
5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c
^2)*arctanh(c*x) + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c
*x + 1)^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x -
 3*(11*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)
*log(c*x - 1) + 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*lo
g(c*x - 1) + 112)*c^2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*b^2 - 1/3*b^2
*arctanh(c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^2/(c^4*x^3 + 3
*c^3*x^2 + 3*c^2*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) = 316$.

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx$$
$$= \frac{1}{1728}c\left(\frac{18\left(\frac{3(cx+1)^2b^2}{(cx-1)^2}-\frac{3(cx+1)b^2}{cx-1}+b^2\right)(cx-1)^3\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)^3c^2}+\frac{6\left(\frac{36(cx+1)^2ab}{(cx-1)^2}-\frac{36(cx+1)ab}{cx-1}+12ab+18\right.}{}\right.$$

input
```
integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="giac")
```

output
```
1/1728*c*(18*(3*(c*x + 1)^2*b^2/(c*x - 1)^2 - 3*(c*x + 1)*b^2/(c*x - 1) +
b^2)*(c*x - 1)^3*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^3*c^2) + 6*(36*(c*
x + 1)^2*a*b/(c*x - 1)^2 - 36*(c*x + 1)*a*b/(c*x - 1) + 12*a*b + 18*(c*x +
 1)^2*b^2/(c*x - 1)^2 - 9*(c*x + 1)*b^2/(c*x - 1) + 2*b^2)*(c*x - 1)^3*log
(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (216*(c*x + 1)^2*a^2/(c*x - 1)^
2 - 216*(c*x + 1)*a^2/(c*x - 1) + 72*a^2 + 216*(c*x + 1)^2*a*b/(c*x - 1)^2
 - 108*(c*x + 1)*a*b/(c*x - 1) + 24*a*b + 108*(c*x + 1)^2*b^2/(c*x - 1)^2
- 27*(c*x + 1)*b^2/(c*x - 1) + 4*b^2)*(c*x - 1)^3/((c*x + 1)^3*c^2))
```

Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.83

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx$$
$$= \ln(1-c\,x)\left(\ln(c\,x+1)\left(\frac{b^2}{3\,c\,(2\,c^3\,x^3+6\,c^2\,x^2+6\,c\,x+2)} - \frac{b^2\,(c^3\,x^3+3\,c^2\,x^2+3\,c\,x+1)}{24\,c\,(2\,c^3\,x^3+6\,c^2\,x^2+6\,c\,x+2)}\right) + \frac{b^2}{3\,c\,(6\,c^3\,x^3+18\,c^2\,x^2+18\,c\,x+6)} + \frac{b\,(6\,a-b)}{3\,c\,(6\,c^3\,x^3+18\,c^2\,x^2+18\,c\,x+6)} + \frac{b^2\,(11\,c^3\,x^3+45\,c^2\,x^2+69\,c\,x+51)}{48\,c\,(6\,c^3\,x^3+18\,c^2\,x^2+18\,c\,x+6)}\right) - \frac{x\,(27\,b^2+36\,a\,b)+x^2\,(11\,c\,b^2+12\,a\,c\,b)+\frac{8\,(18\,a^2+15\,a\,b+7\,b^2)}{3\,c}}{144\,c^3\,x^3+432\,c^2\,x^2+432\,c\,x+144} + \ln(c\,x+1)^2\left(\frac{b^2}{96\,c} - \frac{b^2}{12\,c^2\,\left(3\,x+3\,c\,x^2+\frac{1}{c}+c^2\,x^3\right)}\right) + \ln(1-c\,x)^2\left(\frac{b^2}{96\,c} - \frac{b^2}{3\,c\,(4\,c^3\,x^3+12\,c^2\,x^2+12\,c\,x+4)}\right) - \frac{\ln(c\,x+1)\,\left(\frac{7\,b^2}{96\,c^2}+\frac{5\,b^2\,x^2}{32}+\frac{23\,b^2\,x}{96\,c}+\frac{11\,b^2\,c\,x^3}{288}+\frac{b\,(16\,a+5\,b)}{48\,c^2}\right)}{3\,x+3\,c\,x^2+\frac{1}{c}+c^2\,x^3} - \frac{b\,\text{atan}(c\,x\,1\text{i})\,(6\,a+11\,b)\,1\text{i}}{72\,c}$$

input
```
int((a + b*atanh(c*x))^2/(c*x + 1)^4,x)
```

output
```
log(1 - c*x)*(log(c*x + 1)*(b^2/(3*c*(6*c*x + 6*c^2*x^2 + 2*c^3*x^3 + 2))
- (b^2*(3*c*x + 3*c^2*x^2 + c^3*x^3 + 1))/(24*c*(6*c*x + 6*c^2*x^2 + 2*c^3
*x^3 + 2))) + b^2/(3*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6)) + (b*(6*a -
b))/(3*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6)) + (b^2*(69*c*x + 45*c^2*x^
2 + 11*c^3*x^3 + 51))/(48*c*(18*c*x + 18*c^2*x^2 + 6*c^3*x^3 + 6))) - (x*(
36*a*b + 27*b^2) + x^2*(11*b^2*c + 12*a*b*c) + (8*(15*a*b + 18*a^2 + 7*b^2
))/(3*c))/(432*c*x + 432*c^2*x^2 + 144*c^3*x^3 + 144) + log(c*x + 1)^2*(b^
2/(96*c) - b^2/(12*c^2*(3*x + 3*c*x^2 + 1/c + c^2*x^3))) + log(1 - c*x)^2*
(b^2/(96*c) - b^2/(3*c*(12*c*x + 12*c^2*x^2 + 4*c^3*x^3 + 4))) - (log(c*x
+ 1)*((7*b^2)/(96*c^2) + (5*b^2*x^2)/32 + (23*b^2*x)/(96*c) + (11*b^2*c*x^
3)/288 + (b*(16*a + 5*b))/(48*c^2)))/(3*x + 3*c*x^2 + 1/c + c^2*x^3) - (b*
atan(c*x*1i)*(6*a + 11*b)*1i)/(72*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.28

$$\int \frac{(a+b\text{arctanh}(cx))^2}{(1+cx)^4}\,dx$$
$$= \frac{-96b^2cx-576atanh(cx)\,ab-36\log(cx-1)\,ab+36\log(cx+1)\,ab+24atanh(cx)\,b^2c^3x^3-144atanh(c\cdots}{}$$

input
```
int((a+b*atanh(c*x))^2/(c*x+1)^4,x)
```

output
```
(36*atanh(c*x)**2*b**2*c**3*x**3 + 108*atanh(c*x)**2*b**2*c**2*x**2 + 108*
atanh(c*x)**2*b**2*c*x - 252*atanh(c*x)**2*b**2 - 576*atanh(c*x)*a*b + 24*
atanh(c*x)*b**2*c**3*x**3 - 144*atanh(c*x)*b**2*c*x - 216*atanh(c*x)*b**2
- 36*log(c*x - 1)*a*b*c**3*x**3 - 108*log(c*x - 1)*a*b*c**2*x**2 - 108*log
(c*x - 1)*a*b*c*x - 36*log(c*x - 1)*a*b - 21*log(c*x - 1)*b**2*c**3*x**3 -
 63*log(c*x - 1)*b**2*c**2*x**2 - 63*log(c*x - 1)*b**2*c*x - 21*log(c*x -
1)*b**2 + 36*log(c*x + 1)*a*b*c**3*x**3 + 108*log(c*x + 1)*a*b*c**2*x**2 +
 108*log(c*x + 1)*a*b*c*x + 36*log(c*x + 1)*a*b + 21*log(c*x + 1)*b**2*c**
3*x**3 + 63*log(c*x + 1)*b**2*c**2*x**2 + 63*log(c*x + 1)*b**2*c*x + 21*lo
g(c*x + 1)*b**2 - 288*a**2 + 24*a*b*c**3*x**3 - 144*a*b*c*x - 216*a*b + 22
*b**2*c**3*x**3 - 96*b**2*c*x - 90*b**2)/(864*c*(c**3*x**3 + 3*c**2*x**2 +
 3*c*x + 1))
```

3.119 $\int \frac{\operatorname{arctanh}(ax)^2}{cx-acx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx-acx^2}\,dx = \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c} - \frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{2c}$$

output
```
arctanh(a*x)^2*ln(2-2/(-a*x+1))/c+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))/c-
1/2*polylog(3,-1+2/(-a*x+1))/c
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx-acx^2}\,dx = \frac{\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)}{c} + \frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)}{c} - \frac{\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)}{2c}$$

input
```
Integrate[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

output
```
(ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])])/c + (ArcTanh[a*x]*PolyLog[2,
E^(2*ArcTanh[a*x])])/c - PolyLog[3, E^(2*ArcTanh[a*x])]/(2*c)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 6494, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx-acx^2}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(c-acx)}dx$$

$$\Big\downarrow \text{6494}$$

$$\frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{2a\int \frac{\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{6620}$$

$$\frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{2a\left(\frac{1}{2}\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)}{c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{2a\left(\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{4a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)}{c}$$

input
```
Int[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

output
```
(ArcTanh[a*x]^2*Log[2 - 2/(1 - a*x)])/c - (2*a*(-1/2*(ArcTanh[a*x]*PolyLog
[2, -1 + 2/(1 - a*x)])/a + PolyLog[3, -1 + 2/(1 - a*x)]/(4*a)))/c
```

Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.02 (sec) , antiderivative size = 647, normalized size of antiderivative = 9.66

method	result
derivativedivides	$\frac{a \operatorname{arctanh}(ax)^2 \ln(ax)}{c} - \frac{a \operatorname{arctanh}(ax)^2 \ln(ax-1)}{c} + \frac{2a\left(\left(-2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2 + 2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3 + i\pi \operatorname{csgn}\left(i\left(-\cdots\right.\right.\right.\right.}{\cdots}$
default	$\frac{a \operatorname{arctanh}(ax)^2 \ln(ax)}{c} - \frac{a \operatorname{arctanh}(ax)^2 \ln(ax-1)}{c} + \frac{2a\left(\left(-2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2 + 2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3 + i\pi \operatorname{csgn}\left(i\left(-\cdots\right.\right.\right.\right.}{\cdots}$
parts	Expression too large to display

input
```
int(arctanh(a*x)^2/(-a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a/c*arctanh(a*x)^2*ln(a*x)-a/c*arctanh(a*x)^2*ln(a*x-1)+2*a/c*(1/4*(-
2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2
*x^2-1)+1))^3+I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(
a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1
))-I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-
1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*c
sgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I
*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+2*I*Pi+2*ln(2))*
arctanh(a*x)^2-1/2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/2*arctanh
(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*polylog(2,(a*x+1)/(-
a^2*x^2+1)^(1/2))-polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*arctanh(a*x)^2
*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))-polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx - acx^2}\, dx = \frac{\log\left(\frac{2\,ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 2\,\mathrm{Li}_2\left(-\frac{2\,ax}{ax-1} + 1\right)\log\left(-\frac{ax+1}{ax-1}\right) - 2\,\mathrm{polylog}\left(3, -\frac{ax+1}{ax-1}\right)}{4\,c}$$

input
```
integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="fricas")
```

output
```
1/4*(log(2*a*x/(a*x - 1))*log(-(a*x + 1)/(a*x - 1))^2 + 2*dilog(-2*a*x/(a*
x - 1) + 1)*log(-(a*x + 1)/(a*x - 1)) - 2*polylog(3, -(a*x + 1)/(a*x - 1))
)/c
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx - acx^2}\, dx = -\frac{\int \frac{\operatorname{atanh}^2(ax)}{ax^2 - x}\, dx}{c}$$

input
```
integrate(atanh(a*x)**2/(-a*c*x**2+c*x),x)
```

output
```
-Integral(atanh(a*x)**2/(a*x**2 - x), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx - acx^2}\, dx = \int -\frac{\operatorname{artanh}(ax)^2}{acx^2 - cx}\, dx$$

input
```
integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="maxima")
```

output
```
-1/12*log(-a*x + 1)^3/c + 1/4*integrate(-(log(a*x + 1)^2 - 2*log(a*x + 1)*
log(-a*x + 1))/(a*c*x^2 - c*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx - acx^2}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{acx^2 - cx}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/(a*c*x^2 - c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx - acx^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{c\,x - a\,c\,x^2}\,dx$$

input
```
int(atanh(a*x)^2/(c*x - a*c*x^2),x)
```

output
```
int(atanh(a*x)^2/(c*x - a*c*x^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{cx - acx^2}\,dx = \frac{atanh(ax)^3 - 3\left(\int \frac{atanh(ax)^2}{a^2x^3 - x}dx\right)}{3c}$$

input
```
int(atanh(a*x)^2/(-a*c*x^2+c*x),x)
```

output
```
(atanh(a*x)**3 - 3*int(atanh(a*x)**2/(a**2*x**3 - x),x))/(3*c)
```

3.120 $\int (1+cx)^3 (a+b\operatorname{arctanh}(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 306

$$\int (1+cx)^3(a+b\operatorname{arctanh}(cx))^3\,dx = 3ab^2x+\frac{b^3x}{4}-\frac{b^3\operatorname{arctanh}(cx)}{4c} +3b^3x\operatorname{arctanh}(cx)+\frac{1}{4}b^2cx^2(a+b\operatorname{arctanh}(cx)) +\frac{4b(a+b\operatorname{arctanh}(cx))^2}{c} +\frac{21}{4}bx(a+b\operatorname{arctanh}(cx))^2 +\frac{3}{2}bcx^2(a+b\operatorname{arctanh}(cx))^2 +\frac{1}{4}bc^2x^3(a+b\operatorname{arctanh}(cx))^2 +\frac{(1+cx)^4(a+b\operatorname{arctanh}(cx))^3}{4c} -\frac{11b^2(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c} -\frac{6b(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{c} +\frac{3b^3\log(1-c^2x^2)}{2c}-\frac{11b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c} -\frac{6b^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c} +\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}$$

output
```
3*a*b^2*x+1/4*b^3*x-1/4*b^3*arctanh(c*x)/c+3*b^3*x*arctanh(c*x)+1/4*b^2*c*
x^2*(a+b*arctanh(c*x))+4*b*(a+b*arctanh(c*x))^2/c+21/4*b*x*(a+b*arctanh(c*
x))^2+3/2*b*c*x^2*(a+b*arctanh(c*x))^2+1/4*b*c^2*x^3*(a+b*arctanh(c*x))^2+
1/4*(c*x+1)^4*(a+b*arctanh(c*x))^3/c-11*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+
1))/c-6*b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c+3/2*b^3*ln(-c^2*x^2+1)/c-1
1/2*b^3*polylog(2,1-2/(-c*x+1))/c-6*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(
-c*x+1))/c+3*b^3*polylog(3,1-2/(-c*x+1))/c
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 644 vs. $2(306) = 612$.

Time = 1.93 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.10

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx$$
$$= \frac{-2ab^2 + 8a^3cx + 42a^2bcx + 24ab^2cx + 2b^3cx + 12a^3c^2x^2 + 12a^2bc^2x^2 + 2ab^2c^2x^2 + 8a^3c^3x^3 + 2a^2bc^3x^3}{}$$

input
```
Integrate[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(-2*a*b^2 + 8*a^3*c*x + 42*a^2*b*c*x + 24*a*b^2*c*x + 2*b^3*c*x + 12*a^3*c
^2*x^2 + 12*a^2*b*c^2*x^2 + 2*a*b^2*c^2*x^2 + 8*a^3*c^3*x^3 + 2*a^2*b*c^3*
x^3 + 2*a^3*c^4*x^4 - 24*a*b^2*ArcTanh[c*x] - 2*b^3*ArcTanh[c*x] + 24*a^2*
b*c*x*ArcTanh[c*x] + 84*a*b^2*c*x*ArcTanh[c*x] + 24*b^3*c*x*ArcTanh[c*x] +
 36*a^2*b*c^2*x^2*ArcTanh[c*x] + 24*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^2
*x^2*ArcTanh[c*x] + 24*a^2*b*c^3*x^3*ArcTanh[c*x] + 4*a*b^2*c^3*x^3*ArcTan
h[c*x] + 6*a^2*b*c^4*x^4*ArcTanh[c*x] - 90*a*b^2*ArcTanh[c*x]^2 - 56*b^3*A
rcTanh[c*x]^2 + 24*a*b^2*c*x*ArcTanh[c*x]^2 + 42*b^3*c*x*ArcTanh[c*x]^2 +
36*a*b^2*c^2*x^2*ArcTanh[c*x]^2 + 12*b^3*c^2*x^2*ArcTanh[c*x]^2 + 24*a*b^2
*c^3*x^3*ArcTanh[c*x]^2 + 2*b^3*c^3*x^3*ArcTanh[c*x]^2 + 6*a*b^2*c^4*x^4*A
rcTanh[c*x]^2 - 30*b^3*ArcTanh[c*x]^3 + 8*b^3*c*x*ArcTanh[c*x]^3 + 12*b^3*
c^2*x^2*ArcTanh[c*x]^3 + 8*b^3*c^3*x^3*ArcTanh[c*x]^3 + 2*b^3*c^4*x^4*ArcT
anh[c*x]^3 - 96*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 88*b^3*A
rcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 48*b^3*ArcTanh[c*x]^2*Log[1 + E
^(-2*ArcTanh[c*x])] + 45*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 44*a*
b^2*Log[1 - c^2*x^2] + 12*b^3*Log[1 - c^2*x^2] + 4*b^2*(12*a + 11*b + 12*b
*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 24*b^3*PolyLog[3, -E^(-2
*ArcTanh[c*x])])/(8*c)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx+1)^3(a+b\text{arctanh}(cx))^3\,dx$$

$$\Big\downarrow \text{6480}$$

$$\frac{(cx+1)^4(a+b\text{arctanh}(cx))^3}{4c}-$$
$$\frac{3}{4}b\int \left(-c^2x^2(a+b\text{arctanh}(cx))^2-4cx(a+b\text{arctanh}(cx))^2+\frac{8(cx+1)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}-7(a+b\text{arctanh}(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx+1)^4(a+b\text{arctanh}(cx))^3}{4c}-$$
$$\frac{3}{4}b\left(-\frac{1}{3}c^2x^3(a+b\text{arctanh}(cx))^2+\frac{8b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-2cx^2(a+b\text{arctanh}(cx))^2-\frac{1}{3}\right.$$

input
```
Int[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

output
```
((1 + c*x)^4*(a + b*ArcTanh[c*x])^3)/(4*c) - (3*b*(-4*a*b*x - (b^2*x)/3 +
(b^2*ArcTanh[c*x])/(3*c) - 4*b^2*x*ArcTanh[c*x] - (b*c*x^2*(a + b*ArcTanh[
c*x]))/3 - (16*(a + b*ArcTanh[c*x])^2)/(3*c) - 7*x*(a + b*ArcTanh[c*x])^2
- 2*c*x^2*(a + b*ArcTanh[c*x])^2 - (c^2*x^3*(a + b*ArcTanh[c*x])^2)/3 + (4
4*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c) + (8*(a + b*ArcTanh[c*x])
^2*Log[2/(1 - c*x)])/c - (2*b^2*Log[1 - c^2*x^2])/c + (22*b^2*PolyLog[2, 1
 - 2/(1 - c*x)])/(3*c) + (8*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c
*x)])/c - (4*b^2*PolyLog[3, 1 - 2/(1 - c*x)])/c))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 713, normalized size of antiderivative = 2.33

method	result
derivativedivides	$\frac{\frac{a^3(cx+1)^4}{4}+b^3\left(-6i\pi\operatorname{csgn}\left(\frac{i}{1-\frac{(cx+1)^2}{c^2x^2-1}}\right)^3\operatorname{arctanh}(cx)^2+6i\pi\operatorname{csgn}\left(\frac{i}{1-\frac{(cx+1)^2}{c^2x^2-1}}\right)^2\operatorname{arctanh}(cx)^2+\frac{3\operatorname{arctanh}(cx)^3c^2x^2}{2}\right.}{}$
default	$\frac{\frac{a^3(cx+1)^4}{4}+b^3\left(-6i\pi\operatorname{csgn}\left(\frac{i}{1-\frac{(cx+1)^2}{c^2x^2-1}}\right)^3\operatorname{arctanh}(cx)^2+6i\pi\operatorname{csgn}\left(\frac{i}{1-\frac{(cx+1)^2}{c^2x^2-1}}\right)^2\operatorname{arctanh}(cx)^2+\frac{3\operatorname{arctanh}(cx)^3c^2x^2}{2}\right.}{}$
parts	$\frac{a^3(cx+1)^4}{4c}+\frac{b^3\left(-6i\pi\operatorname{csgn}\left(\frac{i}{1-\frac{(cx+1)^2}{c^2x^2-1}}\right)^3\operatorname{arctanh}(cx)^2+6i\pi\operatorname{csgn}\left(\frac{i}{1-\frac{(cx+1)^2}{c^2x^2-1}}\right)^2\operatorname{arctanh}(cx)^2+\frac{3\operatorname{arctanh}(cx)^3c^2}{2}\right.}{}$

input
```
int((c*x+1)^3*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/4*a^3*(c*x+1)^4+b^3*(3/2*arctanh(c*x)^3*c^2*x^2-1/4-11*arctanh(c*x)
*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-11*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x
^2+1)^(1/2))-11*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-11*dilog(1-I*(c*x+1)
/(-c^2*x^2+1)^(1/2))+1/4*c*x+1/4*arctanh(c*x)^2*c^3*x^3+1/4*arctanh(c*x)^3
+3/2*arctanh(c*x)^2*c^2*x^2+21/4*arctanh(c*x)^2*c*x+6*arctanh(c*x)^2*ln(c*
x-1)+arctanh(c*x)^3*c*x+1/4*arctanh(c*x)^3*c^4*x^4+arctanh(c*x)^3*c^3*x^3-
6*I*Pi*arctanh(c*x)^2+4*arctanh(c*x)^2+3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1)
)+7/2*(c*x+1)*arctanh(c*x)-3*ln(1+(c*x+1)^2/(-c^2*x^2+1))-6*arctanh(c*x)*p
olylog(2,-(c*x+1)^2/(-c^2*x^2+1))-6*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))
^3*arctanh(c*x)^2+6*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^
2+1/4*(c*x-3)*(c*x+1)*arctanh(c*x)-6*ln(2)*arctanh(c*x)^2)+3*a*b^2*(1/4*ar
ctanh(c*x)^2*c^4*x^4+arctanh(c*x)^2*c^3*x^3+3/2*arctanh(c*x)^2*c^2*x^2+arc
tanh(c*x)^2*c*x+1/4*arctanh(c*x)^2+1/6*arctanh(c*x)*c^3*x^3+arctanh(c*x)*c
^2*x^2+7/2*arctanh(c*x)*c*x+4*arctanh(c*x)*ln(c*x-1)+1/12*(c*x-1)^2+7/6*c*
x-7/6+7/3*ln(c*x-1)+4/3*ln(c*x+1)+ln(c*x-1)^2-2*dilog(1/2*c*x+1/2)-2*ln(c*
x-1)*ln(1/2*c*x+1/2))+3*a^2*b*(1/4*arctanh(c*x)*c^4*x^4+arctanh(c*x)*c^3*x
^3+3/2*arctanh(c*x)*c^2*x^2+arctanh(c*x)*c*x+1/4*arctanh(c*x)+1/12*x^3*c^3
+1/2*c^2*x^2+7/4*c*x+2*ln(c*x-1)))
```

Fricas [F]

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx = \int (cx+1)^3(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(a^3*c^3*x^3 + 3*a^3*c^2*x^2 + 3*a^3*c*x + (b^3*c^3*x^3 + 3*b^3*c^
2*x^2 + 3*b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c^3*x^3 + 3*a*b^2
*c^2*x^2 + 3*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(a^2*b*c^3*x^3 + 3*a^2*
b*c^2*x^2 + 3*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F]

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}\,(cx))^3\,(cx+1)^3\,dx$$

input
```
integrate((c*x+1)**3*(a+b*atanh(c*x))**3,x)
```

output
```
Integral((a + b*atanh(c*x))**3*(c*x + 1)**3, x)
```

Maxima [F]

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx=\int (cx+1)^3(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*c^3*x^4 + a^3*c^2*x^3 + 1/8*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 +
3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b*c^3 + 3/2*(2*x^
3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*c^2 + 3/2*a^3*c
*x^2 + 9/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x -
 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a
^2*b/c - 1/32*((b^3*c^4*x^4 + 4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x -
15*b^3)*log(-c*x + 1)^3 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^
3 + 12*(3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(b^3*c^
4*x^4 + 4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x + b^3)*log(c*x + 1))*log
(-c*x + 1)^2)/c - integrate(-1/16*(2*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*
c*x - b^3)*log(c*x + 1)^3 + 12*(a*b^2*c^4*x^4 + 2*a*b^2*c^3*x^3 - 2*a*b^2*
c*x - a*b^2)*log(c*x + 1)^2 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3
)*x^3 + 12*(3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 -
2*b^3*c*x - b^3)*log(c*x + 1)^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(6*b^3*c^2
*x^2 + (8*a*b^2*c^4 + b^3*c^4)*x^4 + 4*(4*a*b^2*c^3 + b^3*c^3)*x^3 - 8*a*b
^2 + b^3 - 4*(4*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x - 1)
, x)
```

Giac [F]

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx=\int (cx+1)^3(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((c*x + 1)^3*(b*arctanh(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}(c\,x))^3\,(c\,x+1)^3\,dx$$

input
```
int((a + b*atanh(c*x))^3*(c*x + 1)^3,x)
```

output
```
int((a + b*atanh(c*x))^3*(c*x + 1)^3, x)
```

Reduce [F]

$$\int (1+cx)^3(a+b\text{arctanh}(cx))^3\,dx=\text{Too large to display}$$

input
```
int((c*x+1)^3*(a+b*atanh(c*x))^3,x)
```

output

```
(atanh(c*x)**3*b**3*c**4*x**4 + 4*atanh(c*x)**3*b**3*c**3*x**3 + 6*atanh(c
*x)**3*b**3*c**2*x**2 + 4*atanh(c*x)**3*b**3*c*x - 7*atanh(c*x)**3*b**3 +
3*atanh(c*x)**2*a*b**2*c**4*x**4 + 12*atanh(c*x)**2*a*b**2*c**3*x**3 + 18*
atanh(c*x)**2*a*b**2*c**2*x**2 + 12*atanh(c*x)**2*a*b**2*c*x - 21*atanh(c*
x)**2*a*b**2 + atanh(c*x)**2*b**3*c**3*x**3 + 6*atanh(c*x)**2*b**3*c**2*x*
*2 + 21*atanh(c*x)**2*b**3*c*x - 6*atanh(c*x)**2*b**3 + 3*atanh(c*x)*a**2*
b*c**4*x**4 + 12*atanh(c*x)*a**2*b*c**3*x**3 + 18*atanh(c*x)*a**2*b*c**2*x
**2 + 12*atanh(c*x)*a**2*b*c*x + 3*atanh(c*x)*a**2*b + 2*atanh(c*x)*a*b**2
*c**3*x**3 + 12*atanh(c*x)*a*b**2*c**2*x**2 + 42*atanh(c*x)*a*b**2*c*x + 3
2*atanh(c*x)*a*b**2 + atanh(c*x)*b**3*c**2*x**2 + 12*atanh(c*x)*b**3*c*x +
 11*atanh(c*x)*b**3 + 48*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*a*b**2*c**2
 + 44*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*b**3*c**2 + 24*int((atanh(c*x)
**2*x)/(c**2*x**2 - 1),x)*b**3*c**2 + 24*log(c**2*x - c)*a**2*b + 44*log(c
**2*x - c)*a*b**2 + 12*log(c**2*x - c)*b**3 + a**3*c**4*x**4 + 4*a**3*c**3
*x**3 + 6*a**3*c**2*x**2 + 4*a**3*c*x + a**2*b*c**3*x**3 + 6*a**2*b*c**2*x
**2 + 21*a**2*b*c*x + a*b**2*c**2*x**2 + 12*a*b**2*c*x + b**3*c*x)/(4*c)
```

3.121 $\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 240

$$\begin{aligned}\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx &= ab^2x+b^3x\text{arctanh}(cx)+\frac{5b(a+b\text{arctanh}(cx))^2}{2c}\\ &+3bx(a+b\text{arctanh}(cx))^2\\ &+\frac{1}{2}bcx^2(a+b\text{arctanh}(cx))^2\\ &+\frac{(1+cx)^3(a+b\text{arctanh}(cx))^3}{3c}\\ &-\frac{6b^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c}\\ &-\frac{4b(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{c}\\ &+\frac{b^3\log(1-c^2x^2)}{2c}-\frac{3b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}\\ &-\frac{4b^2(a+b\text{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}\\ &+\frac{2b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}\end{aligned}$$

output
```
a*b^2*x+b^3*x*arctanh(c*x)+5/2*b*(a+b*arctanh(c*x))^2/c+3*b*x*(a+b*arctanh
(c*x))^2+1/2*b*c*x^2*(a+b*arctanh(c*x))^2+1/3*(c*x+1)^3*(a+b*arctanh(c*x))
^3/c-6*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c-4*b*(a+b*arctanh(c*x))^2*ln
(2/(-c*x+1))/c+1/2*b^3*ln(-c^2*x^2+1)/c-3*b^3*polylog(2,1-2/(-c*x+1))/c-4*
b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+2*b^3*polylog(3,1-2/(-c*x
+1))/c
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. $2(240) = 480$.

Time = 1.60 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.03

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx$$
$$= \frac{6a^3cx+18a^2bcx+6ab^2cx+6a^3c^2x^2+3a^2bc^2x^2+2a^3c^3x^3-6ab^2\text{arctanh}(cx)+18a^2bcx\text{arctanh}(cx)+}{}$$

input
```
Integrate[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(6*a^3*c*x + 18*a^2*b*c*x + 6*a*b^2*c*x + 6*a^3*c^2*x^2 + 3*a^2*b*c^2*x^2
+ 2*a^3*c^3*x^3 - 6*a*b^2*ArcTanh[c*x] + 18*a^2*b*c*x*ArcTanh[c*x] + 36*a*
b^2*c*x*ArcTanh[c*x] + 6*b^3*c*x*ArcTanh[c*x] + 18*a^2*b*c^2*x^2*ArcTanh[c
*x] + 6*a*b^2*c^2*x^2*ArcTanh[c*x] + 6*a^2*b*c^3*x^3*ArcTanh[c*x] - 42*a*b
^2*ArcTanh[c*x]^2 - 21*b^3*ArcTanh[c*x]^2 + 18*a*b^2*c*x*ArcTanh[c*x]^2 +
18*b^3*c*x*ArcTanh[c*x]^2 + 18*a*b^2*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^2*x^
2*ArcTanh[c*x]^2 + 6*a*b^2*c^3*x^3*ArcTanh[c*x]^2 - 14*b^3*ArcTanh[c*x]^3
+ 6*b^3*c*x*ArcTanh[c*x]^3 + 6*b^3*c^2*x^2*ArcTanh[c*x]^3 + 2*b^3*c^3*x^3*
ArcTanh[c*x]^3 - 48*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 36*b
^3*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^3*ArcTanh[c*x]^2*Log[1
 + E^(-2*ArcTanh[c*x])] + 21*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 1
8*a*b^2*Log[1 - c^2*x^2] + 3*b^3*Log[1 - c^2*x^2] + 6*b^2*(4*a + 3*b + 4*b
*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 12*b^3*PolyLog[3, -E^(-2
*ArcTanh[c*x])])/(6*c)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx+1)^2(a+b\text{arctanh}(cx))^3\,dx$$

$$\Big\downarrow \text{6480}$$

$$\frac{(cx+1)^3(a+b\text{arctanh}(cx))^3}{3c}- b\int \left(-cx(a+b\text{arctanh}(cx))^2+\frac{4(cx+1)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}-3(a+b\text{arctanh}(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx+1)^3(a+b\text{arctanh}(cx))^3}{3c}- b\left(\frac{4b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-\frac{1}{2}cx^2(a+b\text{arctanh}(cx))^2-3x(a+b\text{arctanh}(cx))^2-\frac{5(a+b\right.$$

input
```
Int[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

output
```
((1 + c*x)^3*(a + b*ArcTanh[c*x])^3)/(3*c) - b*(-(a*b*x) - b^2*x*ArcTanh[c
*x] - (5*(a + b*ArcTanh[c*x])^2)/(2*c) - 3*x*(a + b*ArcTanh[c*x])^2 - (c*x
^2*(a + b*ArcTanh[c*x])^2)/2 + (6*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])
/c + (4*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - (b^2*Log[1 - c^2*x^2]
)/(2*c) + (3*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/c + (4*b*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 - c*x)])/c - (2*b^2*PolyLog[3, 1 - 2/(1 - c*x)])/c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.54

method	result
derivativedivides	$\frac{\frac{(cx+1)^3 a^3}{3}+b^3\left(\frac{\operatorname{arctanh}(cx)^3 c^3 x^3}{3}+\operatorname{arctanh}(cx)^3 c^2 x^2+\operatorname{arctanh}(cx)^3 cx+\frac{\operatorname{arctanh}(cx)^3}{3}+\frac{\operatorname{arctanh}(cx)^2 c^2 x^2}{2}+3\operatorname{arctanh}(cx\right.}{}$
default	$\frac{\frac{(cx+1)^3 a^3}{3}+b^3\left(\frac{\operatorname{arctanh}(cx)^3 c^3 x^3}{3}+\operatorname{arctanh}(cx)^3 c^2 x^2+\operatorname{arctanh}(cx)^3 cx+\frac{\operatorname{arctanh}(cx)^3}{3}+\frac{\operatorname{arctanh}(cx)^2 c^2 x^2}{2}+3\operatorname{arctanh}(cx\right.}{}$
parts	$\frac{a^3(cx+1)^3}{3c}+\frac{b^3\left(\frac{\operatorname{arctanh}(cx)^3 c^3 x^3}{3}+\operatorname{arctanh}(cx)^3 c^2 x^2+\operatorname{arctanh}(cx)^3 cx+\frac{\operatorname{arctanh}(cx)^3}{3}+\frac{\operatorname{arctanh}(cx)^2 c^2 x^2}{2}+3\operatorname{arctanh}\right.}{}$

input
```
int((c*x+1)^2*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*(c*x+1)^3*a^3+b^3*(1/3*arctanh(c*x)^3*c^3*x^3+arctanh(c*x)^3*c^2*
x^2+arctanh(c*x)^3*c*x+1/3*arctanh(c*x)^3+1/2*arctanh(c*x)^2*c^2*x^2+3*arc
tanh(c*x)^2*c*x+4*arctanh(c*x)^2*ln(c*x-1)-4*arctanh(c*x)*polylog(2,-(c*x+
1)^2/(-c^2*x^2+1))+2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-4*I*Pi*arctanh(c*x
)^2-6*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6*dilog(1-I*(c*x+1)/(-c^2*x^2+
1)^(1/2))+5/2*arctanh(c*x)^2+4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^2*ar
ctanh(c*x)^2+(c*x+1)*arctanh(c*x)-4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))
^3*arctanh(c*x)^2-ln(1+(c*x+1)^2/(-c^2*x^2+1))-4*ln(2)*arctanh(c*x)^2-6*ar
ctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6*arctanh(c*x)*ln(1-I*(c*x+1
)/(-c^2*x^2+1)^(1/2)))+3*a*b^2*(1/3*arctanh(c*x)^2*c^3*x^3+arctanh(c*x)^2*
c^2*x^2+arctanh(c*x)^2*c*x+1/3*arctanh(c*x)^2+1/3*arctanh(c*x)*c^2*x^2+2*a
rctanh(c*x)*c*x+8/3*arctanh(c*x)*ln(c*x-1)+1/3*c*x-1/3+7/6*ln(c*x-1)+5/6*l
n(c*x+1)+2/3*ln(c*x-1)^2-4/3*dilog(1/2*c*x+1/2)-4/3*ln(c*x-1)*ln(1/2*c*x+1
/2))+3*a^2*b*(1/3*arctanh(c*x)*c^3*x^3+arctanh(c*x)*c^2*x^2+arctanh(c*x)*c
*x+1/3*arctanh(c*x)+1/6*c^2*x^2+c*x+4/3*ln(c*x-1)))
```

Fricas [F]

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx=\int (cx+1)^2(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(a^3*c^2*x^2 + 2*a^3*c*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^3)*arctanh
(c*x)^3 + a^3 + 3*(a*b^2*c^2*x^2 + 2*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3
*(a^2*b*c^2*x^2 + 2*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F]

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}\,(cx))^3\,(cx+1)^2\,dx$$

input
```
integrate((c*x+1)**2*(a+b*atanh(c*x))**3,x)
```

output
```
Integral((a + b*atanh(c*x))**3*(c*x + 1)**2, x)
```

Maxima [F]

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx=\int (cx+1)^2(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*c^2*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/
c^4))*a^2*b*c^2 + a^3*c*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c
*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x)
 + log(-c^2*x^2 + 1))*a^2*b/c - 1/24*((b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3
*c*x - 7*b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + (6*a*b^2*c^2 + b^3*c^
2)*x^2 + 6*(a*b^2*c + b^3*c)*x + (b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x
+ b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^3*x^3 + b
^3*c^2*x^2 - b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 + a*b^2*c^2*
x^2 - a*b^2*c*x - a*b^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*(6*a*b^2*c^
2 + b^3*c^2)*x^2 + 3*(b^3*c^3*x^3 + b^3*c^2*x^2 - b^3*c*x - b^3)*log(c*x +
 1)^2 + 12*(a*b^2*c + b^3*c)*x + 2*((6*a*b^2*c^3 + b^3*c^3)*x^3 - 6*a*b^2
+ b^3 + 3*(2*a*b^2*c^2 + b^3*c^2)*x^2 - 3*(2*a*b^2*c - b^3*c)*x)*log(c*x +
 1))*log(-c*x + 1))/(c*x - 1), x)
```

Giac [F]

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx=\int (cx+1)^2(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((c*x + 1)^2*(b*arctanh(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}(c\,x))^3\,(c\,x+1)^2\,dx$$

input
```
int((a + b*atanh(c*x))^3*(c*x + 1)^2,x)
```

output
```
int((a + b*atanh(c*x))^3*(c*x + 1)^2, x)
```

Reduce [F]

$$\int (1+cx)^2(a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{-6atanh(cx)^3\,b^3+6atanh(cx)\,b^3+48\left(\int \frac{atanh(cx)x}{c^2x^2-1}dx\right)a\,b^2c^2+2atanh(cx)^3\,b^3c^3x^3+6atanh(cx)^3\,b^3cx+}{}$$

input
```
int((c*x+1)^2*(a+b*atanh(c*x))^3,x)
```

output
```
(2*atanh(c*x)**3*b**3*c**3*x**3 + 6*atanh(c*x)**3*b**3*c**2*x**2 + 6*atanh
(c*x)**3*b**3*c*x - 6*atanh(c*x)**3*b**3 + 6*atanh(c*x)**2*a*b**2*c**3*x**
3 + 18*atanh(c*x)**2*a*b**2*c**2*x**2 + 18*atanh(c*x)**2*a*b**2*c*x - 18*a
tanh(c*x)**2*a*b**2 + 3*atanh(c*x)**2*b**3*c**2*x**2 + 18*atanh(c*x)**2*b*
*3*c*x - 3*atanh(c*x)**2*b**3 + 6*atanh(c*x)*a**2*b*c**3*x**3 + 18*atanh(c
*x)*a**2*b*c**2*x**2 + 18*atanh(c*x)*a**2*b*c*x + 6*atanh(c*x)*a**2*b + 6*
atanh(c*x)*a*b**2*c**2*x**2 + 36*atanh(c*x)*a*b**2*c*x + 30*atanh(c*x)*a*b
**2 + 6*atanh(c*x)*b**3*c*x + 6*atanh(c*x)*b**3 + 48*int((atanh(c*x)*x)/(c
**2*x**2 - 1),x)*a*b**2*c**2 + 36*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*b*
*3*c**2 + 24*int((atanh(c*x)**2*x)/(c**2*x**2 - 1),x)*b**3*c**2 + 24*log(c
**2*x - c)*a**2*b + 36*log(c**2*x - c)*a*b**2 + 6*log(c**2*x - c)*b**3 + 2
*a**3*c**3*x**3 + 6*a**3*c**2*x**2 + 6*a**3*c*x + 3*a**2*b*c**2*x**2 + 18*
a**2*b*c*x + 6*a*b**2*c*x)/(6*c)
```

3.122 $\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 191

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx = \frac{3b(a+b\text{arctanh}(cx))^2}{2c}+\frac{3}{2}bx(a+b\text{arctanh}(cx))^2+\frac{(1+cx)^2(a+b\text{arctanh}(cx))^3}{2c}-\frac{3b^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3b(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}-\frac{3b^2(a+b\text{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}$$

output
```
3/2*b*(a+b*arctanh(c*x))^2/c+3/2*b*x*(a+b*arctanh(c*x))^2+1/2*(c*x+1)^2*(a
+b*arctanh(c*x))^3/c-3*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c-3*b*(a+b*ar
ctanh(c*x))^2*ln(2/(-c*x+1))/c-3/2*b^3*polylog(2,1-2/(-c*x+1))/c-3*b^2*(a+
b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*polylog(3,1-2/(-c*x+1))/
c
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.75

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx$$
$$= \frac{4a^3cx+6a^2bcx+2a^3c^2x^2+12a^2bcx\text{arctanh}(cx)+12ab^2cx\text{arctanh}(cx)+6a^2bc^2x^2\text{arctanh}(cx)-18ab^2}{}$$

input
```
Integrate[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(4*a^3*c*x + 6*a^2*b*c*x + 2*a^3*c^2*x^2 + 12*a^2*b*c*x*ArcTanh[c*x] + 12*
a*b^2*c*x*ArcTanh[c*x] + 6*a^2*b*c^2*x^2*ArcTanh[c*x] - 18*a*b^2*ArcTanh[c
*x]^2 - 6*b^3*ArcTanh[c*x]^2 + 12*a*b^2*c*x*ArcTanh[c*x]^2 + 6*b^3*c*x*Arc
Tanh[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTanh[c*x]^2 - 6*b^3*ArcTanh[c*x]^3 + 4*b^
3*c*x*ArcTanh[c*x]^3 + 2*b^3*c^2*x^2*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x
]*Log[1 + E^(-2*ArcTanh[c*x])] - 12*b^3*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh
[c*x])] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 9*a^2*b*Log
[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 6*a*b^2*Log[1 - c^2*x^2] + 6*b^2*(2*a +
 b + 2*b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3,
 -E^(-2*ArcTanh[c*x])])/(4*c)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx+1)(a+b\text{arctanh}(cx))^3\,dx$$
$$\Big\downarrow \text{6480}$$
$$\frac{(cx+1)^2(a+b\text{arctanh}(cx))^3}{2c} - \frac{3}{2}b\int \left(\frac{2(cx+1)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}-(a+b\text{arctanh}(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx+1)^2(a+b\text{arctanh}(cx))^3}{2c}-$$
$$\frac{3}{2}b\left(\frac{2b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-x(a+b\text{arctanh}(cx))^2-\frac{(a+b\text{arctanh}(cx))^2}{c}+\frac{2b\log\left(\frac{2}{1-cx}\right.}{}\right.$$

input
```
Int[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

output
```
((1 + c*x)^2*(a + b*ArcTanh[c*x])^3)/(2*c) - (3*b*(-((a + b*ArcTanh[c*x])^
2/c) - x*(a + b*ArcTanh[c*x])^2 + (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x
)])/c + (2*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c + (b^2*PolyLog[2, 1
- 2/(1 - c*x)])/c + (2*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])
/c - (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/c))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 3401, normalized size of antiderivative = 17.81

method	result	size
derivativedivides	Expression too large to display	3401
default	Expression too large to display	3401
parts	Expression too large to display	3402

input
```
int((c*x+1)*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^3*(1/2*c^2*x^2+c*x)+b^3*(3/8*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(
c^2*x^2-1)))*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)
*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+
dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))+1/2*arctanh(c*x)^3*c^2*x^2-3/8*I*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)
^(1/2))+arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x+1)/
(-c^2*x^2+1)^(1/2))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-9/8*I*Pi*csgn(I
/(1-(c*x+1)^2/(c^2*x^2-1)))^3*(2*arctanh(c*x)^2-2*arctanh(c*x)*ln(1+(c*x+1
)^2/(-c^2*x^2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))-9/4*I*Pi*arctanh(c*x
)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-9/4*I*Pi*arctanh(c*x)*ln(1-I*(c*x+1)/
(-c^2*x^2+1)^(1/2))+9/4*I*Pi*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))+9/8
*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^2*(2*arctanh(c*x)^2-2*arctanh(c*x)
*ln(1+(c*x+1)^2/(-c^2*x^2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))-9/4*I*Pi
*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3*(arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^
2+1)^(1/2))+arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+dilog(1+I*(c*x
+1)/(-c^2*x^2+1)^(1/2))+dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-3/16*I*Pi*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))^3*(2*arctanh(c*x)^2-2*arctanh(c*x)*ln(1+(c*x+
1)^2/(-c^2*x^2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))-3/16*I*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*(2*arctanh(c*x)^2-2*ar...
```

Fricas [F]

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx = \int (cx+1)(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(a^3*c*x + (b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c*x + a
*b^2)*arctanh(c*x)^2 + 3*(a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F]

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx = \int (a+b\,\text{atanh}\,(cx))^3\,(cx+1)\,\,dx$$

input
```
integrate((c*x+1)*(a+b*atanh(c*x))**3,x)
```

output
```
Integral((a + b*atanh(c*x))**3*(c*x + 1), x)
```

Maxima [F]

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx = \int (cx+1)(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*c*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
log(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*a^2*b/c - 1/16*((b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-c*x + 1)^3
- 3*(2*a*b^2*c^2*x^2 + 2*(2*a*b^2*c + b^3*c)*x + (b^3*c^2*x^2 + 2*b^3*c*x
+ b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^2*x^2 - b
^3)*log(c*x + 1)^3 + 6*(a*b^2*c^2*x^2 - a*b^2)*log(c*x + 1)^2 - 3*(2*a*b^2
*c^2*x^2 + (b^3*c^2*x^2 - b^3)*log(c*x + 1)^2 + 2*(2*a*b^2*c + b^3*c)*x +
(2*b^3*c*x - 4*a*b^2 + b^3 + (4*a*b^2*c^2 + b^3*c^2)*x^2)*log(c*x + 1))*lo
g(-c*x + 1))/(c*x - 1), x)
```

Giac [F]

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx=\int (cx+1)(b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((c*x + 1)*(b*arctanh(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}(c\,x))^3\,(c\,x+1)\,dx$$

input
```
int((a + b*atanh(c*x))^3*(c*x + 1),x)
```

output
```
int((a + b*atanh(c*x))^3*(c*x + 1), x)
```

Reduce [F]

$$\int (1+cx)(a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{atanh(cx)^3\,b^3c^2x^2+2atanh(cx)^3\,b^3cx-atanh(cx)^3\,b^3+3atanh(cx)^2\,a\,b^2c^2x^2+6atanh(cx)^2\,a\,b^2cx-3a}{}$$

input
```
int((c*x+1)*(a+b*atanh(c*x))^3,x)
```

output
```
(atanh(c*x)**3*b**3*c**2*x**2 + 2*atanh(c*x)**3*b**3*c*x - atanh(c*x)**3*b
**3 + 3*atanh(c*x)**2*a*b**2*c**2*x**2 + 6*atanh(c*x)**2*a*b**2*c*x - 3*at
anh(c*x)**2*a*b**2 + 3*atanh(c*x)**2*b**3*c*x + 3*atanh(c*x)*a**2*b*c**2*x
**2 + 6*atanh(c*x)*a**2*b*c*x + 3*atanh(c*x)*a**2*b + 6*atanh(c*x)*a*b**2*
c*x + 6*atanh(c*x)*a*b**2 + 12*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*a*b**
2*c**2 + 6*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*b**3*c**2 + 6*int((atanh(
c*x)**2*x)/(c**2*x**2 - 1),x)*b**3*c**2 + 6*log(c**2*x - c)*a**2*b + 6*log
(c**2*x - c)*a*b**2 + a**3*c**2*x**2 + 2*a**3*c*x + 3*a**2*b*c*x)/(2*c)
```

3.123 $\int \frac{(a+b\textbf{arctanh}(cx))^3}{1+cx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{1+cx}\,dx = -\frac{(a+b\operatorname{arctanh}(cx))^3\log\left(\frac{2}{1+cx}\right)}{c} + \frac{3b(a+b\operatorname{arctanh}(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c} + \frac{3b^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2c} + \frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+cx}\right)}{4c}$$

output
```
-(a+b*arctanh(c*x))^3*ln(2/(c*x+1))/c+3/2*b*(a+b*arctanh(c*x))^2*polylog(2
,1-2/(c*x+1))/c+3/2*b^2*(a+b*arctanh(c*x))*polylog(3,1-2/(c*x+1))/c+3/4*b^
3*polylog(4,1-2/(c*x+1))/c
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx = \frac{-12a^2b\text{arctanh}(cx)\log\left(1+e^{-2\text{arctanh}(cx)}\right)-12ab^2\text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right)-4b^3\text{arctanh}(cx)}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

output
```
(-12*a^2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 12*a*b^2*ArcTanh[c*
x]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 4*b^3*ArcTanh[c*x]^3*Log[1 + E^(-2*Arc
Tanh[c*x])] + 4*a^3*Log[1 + c*x] + 6*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, -
E^(-2*ArcTanh[c*x])] + 6*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -E^(-2*ArcTan
h[c*x])] + 3*b^3*PolyLog[4, -E^(-2*ArcTanh[c*x])])/(4*c)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6470, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{cx+1}\,dx$$

$$\Big\downarrow \text{6470}$$

$$3b\int \frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^3}{c}$$

$$\Big\downarrow \text{6618}$$

$$3b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{2c}-b\int\frac{(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{1-c^2x^2}dx\right)-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^3}{c}$$

$$\Big\downarrow \text{6622}$$

$$3b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{2c}-b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{1-c^2x^2}dx-\frac{\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)(}{2c}\right.\right.$$
$$\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^3}{c}$$

$$\Big\downarrow \text{7164}$$

$$3b\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{2c}-b\left(-\frac{\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{b\,\text{PolyLog}\left(\right.}{}\right.\right.$$
$$\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^3}{c}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

output
```
-(((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/c) + 3*b*(((a + b*ArcTanh[c*x]
)^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) - b*(-1/2*((a + b*ArcTanh[c*x])*Pol
yLog[3, 1 - 2/(1 + c*x)])/c - (b*PolyLog[4, 1 - 2/(1 + c*x)])/(4*c)))
```

Defintions of rubi rules used

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 1215, normalized size of antiderivative = 10.95

method	result	size
derivativedivides	Expression too large to display	1215
default	Expression too large to display	1215
parts	Expression too large to display	1223

input
```
int((a+b*arctanh(c*x))^3/(c*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^3*ln(c*x+1)+b^3*(ln(c*x+1)*arctanh(c*x)^3-2*arctanh(c*x)^3*ln((c*x+
1)/(-c^2*x^2+1)^(1/2))+1/2*arctanh(c*x)^4-3/2*arctanh(c*x)^2*polylog(2,-(c
*x+1)^2/(-c^2*x^2+1))+3/2*arctanh(c*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-
3/4*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))-1/2*(I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1
))^3-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c
*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))
+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I/(1-
(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^
2*x^2-1)))^3+2*ln(2))*arctanh(c*x)^3)+3*a*b^2*(arctanh(c*x)^2*ln(c*x+1)-2*
arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3*arctanh(c*x)^3-1/2*(I*Pi
*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+2*I*Pi
*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+I*Pi*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))^3-I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(
c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/
(1-(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/
(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))+I*Pi*csgn(I*(c*x+1)^2...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{cx+1}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)/(c*x + 1), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx=\int \frac{(a+b\,\text{atanh}\,(cx))^3}{cx+1}\,dx$$

input
```
integrate((a+b*atanh(c*x))**3/(c*x+1),x)
```

output
```
Integral((a + b*atanh(c*x))**3/(c*x + 1), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{cx+1}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="maxima")
```

output
```
-1/8*b^3*log(c*x + 1)*log(-c*x + 1)^3/c + a^3*log(c*x + 1)/c + integrate(1
/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2
+ 6*(b^3*c*x*log(c*x + 1) + a*b^2*c*x - a*b^2)*log(-c*x + 1)^2 + 12*(a^2*b
*c*x - a^2*b)*log(c*x + 1) - 3*(4*a^2*b*c*x - 4*a^2*b + (b^3*c*x - b^3)*lo
g(c*x + 1)^2 + 4*(a*b^2*c*x - a*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*x^2
 - 1), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{cx+1}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3/(c*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x))^3}{c\,x+1}\,dx$$

input
```
int((a + b*atanh(c*x))^3/(c*x + 1),x)
```

output
```
int((a + b*atanh(c*x))^3/(c*x + 1), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{1+cx}\,dx = \frac{3\left(\int \frac{atanh(cx)}{cx+1}dx\right)a^2bc+\left(\int \frac{atanh(cx)^3}{cx+1}dx\right)b^3c+3\left(\int \frac{atanh(cx)^2}{cx+1}dx\right)a\,b^2c+\log(cx+1)\,a^3}{c}$$

input
```
int((a+b*atanh(c*x))^3/(c*x+1),x)
```

output
```
(3*int(atanh(c*x)/(c*x + 1),x)*a**2*b*c + int(atanh(c*x)**3/(c*x + 1),x)*b
**3*c + 3*int(atanh(c*x)**2/(c*x + 1),x)*a*b**2*c + log(c*x + 1)*a**3)/c
```

3.124 $\int \frac{(a+b\,\textbf{arctanh}(cx))^3}{(1+cx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^2}\,dx=-\frac{3b^3}{4c(1+cx)}+\frac{3b^3\text{arctanh}(cx)}{4c}-\frac{3b^2(a+b\text{arctanh}(cx))}{2c(1+cx)}+\frac{3b(a+b\text{arctanh}(cx))^2}{4c}-\frac{3b(a+b\text{arctanh}(cx))^2}{2c(1+cx)}+\frac{(a+b\text{arctanh}(cx))^3}{2c}-\frac{(a+b\text{arctanh}(cx))^3}{c(1+cx)}$$

output
```
-3/4*b^3/c/(c*x+1)+3/4*b^3*arctanh(c*x)/c-3/2*b^2*(a+b*arctanh(c*x))/c/(c*
x+1)+3/4*b*(a+b*arctanh(c*x))^2/c-3/2*b*(a+b*arctanh(c*x))^2/c/(c*x+1)+1/2
*(a+b*arctanh(c*x))^3/c-(a+b*arctanh(c*x))^3/c/(c*x+1)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^2}\,dx$$
$$=\frac{-8a^3-12a^2b-12ab^2-6b^3-12b(2a^2+2ab+b^2)\,\text{arctanh}(cx)+6b^2(2a+b)(-1+cx)\text{arctanh}(cx)^2+}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

output
```
(-8*a^3 - 12*a^2*b - 12*a*b^2 - 6*b^3 - 12*b*(2*a^2 + 2*a*b + b^2)*ArcTanh
[c*x] + 6*b^2*(2*a + b)*(-1 + c*x)*ArcTanh[c*x]^2 + 4*b^3*(-1 + c*x)*ArcTa
nh[c*x]^3 - 3*b*(2*a^2 + 2*a*b + b^2)*(1 + c*x)*Log[1 - c*x] + 6*a^2*b*Log
[1 + c*x] + 6*a*b^2*Log[1 + c*x] + 3*b^3*Log[1 + c*x] + 6*a^2*b*c*x*Log[1
+ c*x] + 6*a*b^2*c*x*Log[1 + c*x] + 3*b^3*c*x*Log[1 + c*x])/(8*c*(1 + c*x)
)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(cx+1)^2}\,dx$$

$$\Big\downarrow \text{6480}$$

$$3b\int \left(\frac{(a+b\text{arctanh}(cx))^2}{2\,(1-c^2x^2)}+\frac{(a+b\text{arctanh}(cx))^2}{2(cx+1)^2}\right)dx-\frac{(a+b\text{arctanh}(cx))^3}{c(cx+1)}$$

$$\Big\downarrow \text{2009}$$

$$3b\left(\frac{(a+b\text{arctanh}(cx))^3}{6bc}+\frac{(a+b\text{arctanh}(cx))^2}{4c}-\frac{(a+b\text{arctanh}(cx))^2}{2c(cx+1)}-\frac{b(a+b\text{arctanh}(cx))}{2c(cx+1)}+\frac{b^2\text{arctanh}(cx)}{4c}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{c(cx+1)}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

output
```
-((a + b*ArcTanh[c*x])^3/(c*(1 + c*x))) + 3*b*(-1/4*b^2/(c*(1 + c*x)) + (b
^2*ArcTanh[c*x])/(4*c) - (b*(a + b*ArcTanh[c*x]))/(2*c*(1 + c*x)) + (a + b
*ArcTanh[c*x])^2/(4*c) - (a + b*ArcTanh[c*x])^2/(2*c*(1 + c*x)) + (a + b*A
rcTanh[c*x])^3/(6*b*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\frac{-2 \operatorname{arctanh}(cx)^3 b^3 cx-6 \operatorname{arctanh}(cx)^2 a b^2 cx-3 b^3 \operatorname{arctanh}(cx)^2 cx-6 \operatorname{arctanh}(cx) a^2 bcx-6 a b^2 \operatorname{arctanh}(cx) cx-3 \text{ arc}}{}$
risch	$\frac{b^3(cx-1)\ln(cx+1)^3}{16(cx+1)c}+\frac{3b^2(-bcx\ln(-cx+1)+2acx+bcx+b\ln(-cx+1)-2a-b)\ln(cx+1)^2}{16(cx+1)c}-\frac{3b\left(-b^2cx\ln(-cx+1)^2\right.}{}$
orering	$\frac{3(5c^4x^4-4x^3c^3-4c^2x^2+4cx-1)(a+b \operatorname{arctanh}(cx))^3}{4c(cx+1)^2}+\frac{(cx-1)(cx+1)^2(21c^2x^2-24cx+5)\left(\frac{3(a+b \operatorname{arctanh}(cx))^2 bc}{(cx+1)^2(-c^2x^2+1)}-\right.}{4c^2}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+b*arctanh(c*x))^3/(c*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*arctanh(c*x)^3*b^3*c*x-6*arctanh(c*x)^2*a*b^2*c*x-3*b^3*arctanh(c
*x)^2*c*x-6*arctanh(c*x)*a^2*b*c*x-6*a*b^2*arctanh(c*x)*c*x-3*arctanh(c*x)
*b^3*c*x+2*arctanh(c*x)^3*b^3-4*a^3*c*x-6*a^2*b*c*x-6*a*b^2*c*x-3*b^3*c*x+
6*arctanh(c*x)^2*a*b^2+3*b^3*arctanh(c*x)^2+6*arctanh(c*x)*a^2*b+6*a*b^2*a
rctanh(c*x)+3*arctanh(c*x)*b^3)/(c*x+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.15

$$\int \frac{(a + b\text{arctanh}(cx))^3}{(1 + cx)^2}\,dx$$
$$= \frac{(b^3cx - b^3)\log\left(-\frac{cx+1}{cx-1}\right)^3 - 16\,a^3 - 24\,a^2b - 24\,ab^2 - 12\,b^3 - 3\,(2\,ab^2 + b^3 - (2\,ab^2 + b^3)cx)\log\left(-\frac{cx+1}{cx-1}\right)^2}{16\,(c^2x + c)}$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="fricas")
```

output
```
1/16*((b^3*c*x - b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 16*a^3 - 24*a^2*b - 24
*a*b^2 - 12*b^3 - 3*(2*a*b^2 + b^3 - (2*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(
c*x - 1))^2 - 6*(2*a^2*b + 2*a*b^2 + b^3 - (2*a^2*b + 2*a*b^2 + b^3)*c*x)*
log(-(c*x + 1)/(c*x - 1)))/(c^2*x + c)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{(1 + cx)^2}\,dx = \int \frac{(a + b\,\text{atanh}\,(cx))^3}{(cx + 1)^2}\,dx$$

input
```
integrate((a+b*atanh(c*x))**3/(c*x+1)**2,x)
```

output
```
Integral((a + b*atanh(c*x))**3/(c*x + 1)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(127) = 254$.

Time = 0.05 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.81

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="maxima")
```

output
```
-b^3*arctanh(c*x)^3/(c^2*x + c) - 3/4*(c*(2/(c^3*x + c^2) - log(c*x + 1)/c
^2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*a^2*b - 3/8*(4*c*(2/(
c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c*x +
 1)*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x
 - 1) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*x + c^3))
*a*b^2 - 1/16*(12*c*(2/(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2
)*arctanh(c*x)^2 - (((c*x + 1)*log(c*x + 1)^3 - (c*x + 1)*log(c*x - 1)^3 -
 3*(c*x + (c*x + 1)*log(c*x - 1) + 1)*log(c*x + 1)^2 - 3*(c*x + 1)*log(c*x
 - 1)^2 + 3*((c*x + 1)*log(c*x - 1)^2 + 2*c*x + 2*(c*x + 1)*log(c*x - 1) +
 2)*log(c*x + 1) - 6*(c*x + 1)*log(c*x - 1) - 12)*c^2/(c^5*x + c^4) - 6*((
c*x + 1)*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*lo
g(c*x - 1) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c*arctanh(c*x
)/(c^4*x + c^3))*c)*b^3 - 3*a*b^2*arctanh(c*x)^2/(c^2*x + c) - a^3/(c^2*x
+ c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^2}\,dx = \frac{1}{16}\left(\frac{(cx-1)b^3\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)c^2} + \frac{3\,(2\,ab^2+b^3)(cx-1)\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2} + \frac{6\,(2\,a^2b+2\,ab^2+b^3)(cx-1)\log\left(\right.}{(cx+1)c^2}\right.$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="giac")
```

output
```
1/16*((c*x - 1)*b^3*log(-(c*x + 1)/(c*x - 1))^3/((c*x + 1)*c^2) + 3*(2*a*b
^2 + b^3)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2) + 6*(2*a^2
*b + 2*a*b^2 + b^3)*(c*x - 1)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2) +
2*(4*a^3 + 6*a^2*b + 6*a*b^2 + 3*b^3)*(c*x - 1)/((c*x + 1)*c^2))*c
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.19

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{(1+cx)^2}\,dx = \ln(1-c\,x)\left(\ln(c\,x+1)\left(\frac{3\,b^3\,x+\frac{3\,b^2\,(4\,a+b)}{c}}{8\,c\,x+8}-\frac{3\,(b^3+a\,b^2)}{4\,c}+\frac{6\,b^3}{c\,(8\,c\,x+8)}\right)+\frac{3\,b^3\,x+\frac{3\,b^2\,(4\,a+b)}{c}}{8\,c\,x+8}-\ln(c\,x+1)^2\left(\frac{3\,b^3}{16\,c}-\frac{3\,b^3}{c\,(8\,c\,x+8)}\right)-\frac{3\,b^3\,x+\frac{3\,(-4\,a^2\,b+4\,a\,b^2+5\,b^3)}{c}}{8\,c\,x+8}+\frac{6\,b^3}{c\,(8\,c\,x+8)}+\frac{3\,(8\,c\,x+24)\,(b^3+a\,b^2)}{4\,c\,(8\,c\,x+8)}\right)-\ln(1-c\,x)^2\left(\frac{3\,b^3}{c\,(8\,c\,x+8)}-\frac{3\,(b^3+a\,b^2)}{8\,c}-\ln(c\,x+1)\left(\frac{3\,b^3}{16\,c}-\frac{3\,b^3}{c\,(8\,c\,x+8)}\right)+\frac{3\,b^3\,(8\,c\,x+24)}{16\,c\,(8\,c\,x+8)}+\frac{3\,b^2\,(2\,a-b)}{c\,(8\,c\,x+8)}\right)-\ln(c\,x+1)^2\left(\frac{\frac{3\,b^3\,x}{16\,c}+\frac{3\,b^2\,(4\,a+3\,b)}{16\,c^2}}{x+\frac{1}{c}}-\frac{3\,b^2\,(a+b)}{8\,c}\right)-\ln(1-c\,x)^3\left(\frac{b^3}{16\,c}-\frac{b^3}{c\,(8\,c\,x+8)}\right)+\ln(c\,x+1)^3\left(\frac{b^3}{16\,c}-\frac{b^3}{8\,c^2\,\left(x+\frac{1}{c}\right)}\right)-\frac{\ln(c\,x+1)\left(\frac{3\,b\,(2\,a^2+3\,a\,b+2\,b^2)}{4\,c^2}+\frac{3\,b^2\,x\,(a+b)}{4\,c}\right)}{x+\frac{1}{c}}-\frac{4\,a^3+6\,a^2\,b+6\,a\,b^2+3\,b^3}{2\,c\,(2\,c\,x+2)}-\frac{b\,\mathrm{atan}(c\,x\,1\mathrm{i})\,(2\,a^2+4\,a\,b+3\,b^2)\,3\mathrm{i}}{4\,c}$$

input
```
int((a + b*atanh(c*x))^3/(c*x + 1)^2,x)
```

output
```
log(1 - c*x)*(log(c*x + 1)*((3*b^3*x + (3*b^2*(4*a + b))/c)/(8*c*x + 8) -
(3*(a*b^2 + b^3))/(4*c) + (6*b^3)/(c*(8*c*x + 8))) + (3*b^3*x + (3*b^2*(4*
a + b))/c)/(8*c*x + 8) - log(c*x + 1)^2*((3*b^3)/(16*c) - (3*b^3)/(c*(8*c*
x + 8))) - (3*b^3*x + (3*(4*a*b^2 - 4*a^2*b + 5*b^3))/c)/(8*c*x + 8) + (6*
b^3)/(c*(8*c*x + 8)) + (3*(8*c*x + 24)*(a*b^2 + b^3))/(4*c*(8*c*x + 8))) -
 log(1 - c*x)^2*((3*b^3)/(c*(8*c*x + 8)) - (3*(a*b^2 + b^3))/(8*c) - log(c
*x + 1)*((3*b^3)/(16*c) - (3*b^3)/(c*(8*c*x + 8))) + (3*b^3*(8*c*x + 24))/
(16*c*(8*c*x + 8)) + (3*b^2*(2*a - b))/(c*(8*c*x + 8))) - log(c*x + 1)^2*(
((3*b^3*x)/(16*c) + (3*b^2*(4*a + 3*b))/(16*c^2))/(x + 1/c) - (3*b^2*(a +
b))/(8*c)) - log(1 - c*x)^3*(b^3/(16*c) - b^3/(c*(8*c*x + 8))) + log(c*x +
 1)^3*(b^3/(16*c) - b^3/(8*c^2*(x + 1/c))) - (log(c*x + 1)*((3*b*(3*a*b +
2*a^2 + 2*b^2))/(4*c^2) + (3*b^2*x*(a + b))/(4*c)))/(x + 1/c) - (6*a*b^2 +
 6*a^2*b + 4*a^3 + 3*b^3)/(2*c*(2*c*x + 2)) - (b*atan(c*x*1i)*(4*a*b + 2*a
^2 + 3*b^2)*3i)/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.19

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^2}\,dx$$
$$= \frac{6\log(cx-1)\,a^2b+6\log(cx-1)\,a\,b^2-6\log(cx+1)\,a^2b-6\log(cx+1)\,a\,b^2-4atanh(cx)^3\,b^3+6\log(cx$$

input
```
int((a+b*atanh(c*x))^3/(c*x+1)^2,x)
```

output
```
(4*atanh(c*x)**3*b**3*c*x - 4*atanh(c*x)**3*b**3 + 12*atanh(c*x)**2*a*b**2
*c*x - 12*atanh(c*x)**2*a*b**2 + 6*atanh(c*x)**2*b**3*c*x - 6*atanh(c*x)**
2*b**3 + 24*atanh(c*x)*a**2*b*c*x + 24*atanh(c*x)*a*b**2*c*x + 12*atanh(c*
x)*b**3*c*x + 6*log(c*x - 1)*a**2*b*c*x + 6*log(c*x - 1)*a**2*b + 6*log(c*
x - 1)*a*b**2*c*x + 6*log(c*x - 1)*a*b**2 + 3*log(c*x - 1)*b**3*c*x + 3*lo
g(c*x - 1)*b**3 - 6*log(c*x + 1)*a**2*b*c*x - 6*log(c*x + 1)*a**2*b - 6*lo
g(c*x + 1)*a*b**2*c*x - 6*log(c*x + 1)*a*b**2 - 3*log(c*x + 1)*b**3*c*x -
3*log(c*x + 1)*b**3 + 8*a**3*c*x + 12*a**2*b*c*x + 12*a*b**2*c*x + 6*b**3*
c*x)/(8*c*(c*x + 1))
```

3.125 $\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 208

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx = -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}+\frac{21b^3\text{arctanh}(cx)}{64c}-\frac{3b^2(a+b\text{arctanh}(cx))}{16c(1+cx)^2}-\frac{9b^2(a+b\text{arctanh}(cx))}{16c(1+cx)}+\frac{9b(a+b\text{arctanh}(cx))^2}{32c}-\frac{3b(a+b\text{arctanh}(cx))^2}{8c(1+cx)^2}-\frac{3b(a+b\text{arctanh}(cx))^2}{8c(1+cx)}+\frac{(a+b\text{arctanh}(cx))^3}{8c}-\frac{(a+b\text{arctanh}(cx))^3}{2c(1+cx)^2}$$

output
```
-3/64*b^3/c/(c*x+1)^2-21/64*b^3/c/(c*x+1)+21/64*b^3*arctanh(c*x)/c-3/16*b^
2*(a+b*arctanh(c*x))/c/(c*x+1)^2-9/16*b^2*(a+b*arctanh(c*x))/c/(c*x+1)+9/3
2*b*(a+b*arctanh(c*x))^2/c-3/8*b*(a+b*arctanh(c*x))^2/c/(c*x+1)^2-3/8*b*(a
+b*arctanh(c*x))^2/c/(c*x+1)+1/8*(a+b*arctanh(c*x))^3/c-1/2*(a+b*arctanh(c
*x))^3/c/(c*x+1)^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx$$
$$= \frac{-2(32a^3+24a^2b+12ab^2+3b^3)-6b(8a^2+12ab+7b^2)(1+cx)-24b(8a^2+4ab(2+cx)+b^2(4+3cx}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

output
```
(-2*(32*a^3 + 24*a^2*b + 12*a*b^2 + 3*b^3) - 6*b*(8*a^2 + 12*a*b + 7*b^2)*
(1 + c*x) - 24*b*(8*a^2 + 4*a*b*(2 + c*x) + b^2*(4 + 3*c*x))*ArcTanh[c*x]
+ 12*b^2*(-1 + c*x)*(4*a*(3 + c*x) + b*(5 + 3*c*x))*ArcTanh[c*x]^2 + 16*b^
3*(-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^3 - 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1
+ c*x)^2*Log[1 - c*x] + 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*x)^2*Log[1 + c
*x])/(128*c*(1 + c*x)^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(cx+1)^3}\,dx$$
$$\Big\downarrow \text{6480}$$
$$\frac{3}{2}b\int \left(\frac{(a+b\text{arctanh}(cx))^2}{4(1-c^2x^2)}+\frac{(a+b\text{arctanh}(cx))^2}{4(cx+1)^2}+\frac{(a+b\text{arctanh}(cx))^2}{2(cx+1)^3}\right)dx-\frac{(a+b\text{arctanh}(cx))^3}{2c(cx+1)^2}$$
$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}b\left(\frac{(a+b\text{arctanh}(cx))^3}{12bc}+\frac{3(a+b\text{arctanh}(cx))^2}{16c}-\frac{(a+b\text{arctanh}(cx))^2}{4c(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{4c(cx+1)^2}-\frac{3b(a+b\text{arcta}}{8c(cx+}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{2c(cx+1)^2}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x])^3/(c*(1 + c*x)^2) + (3*b*(-1/32*b^2/(c*(1 + c*x)
^2) - (7*b^2)/(32*c*(1 + c*x)) + (7*b^2*ArcTanh[c*x])/(32*c) - (b*(a + b*A
rcTanh[c*x]))/(8*c*(1 + c*x)^2) - (3*b*(a + b*ArcTanh[c*x]))/(8*c*(1 + c*x
)) + (3*(a + b*ArcTanh[c*x])^2)/(16*c) - (a + b*ArcTanh[c*x])^2/(4*c*(1 +
c*x)^2) - (a + b*ArcTanh[c*x])^2/(4*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/
(12*b*c)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.57

method	result
parallelrisch	$-\frac{-48a^2b\,c^2x^2-72a^2bcx-12b^3\,\text{arctanh}(cx)^2cx-18b^3\,\text{arctanh}(cx)^2c^2x^2-24b^3c^2x^2-32a^3c^2x^2-64a^3cx+72\,\text{arctanh}(}{}$
orering	$-\frac{(105c^5x^5-231c^4x^4+18x^3c^3+280c^2x^2-253cx+81)(a+b\,\text{arctanh}(cx))^3}{64(cx+1)^2c}-\frac{(cx-1)(cx+1)^2(105c^4x^4-154x^3c^3-88}{}$
risch	$\frac{b^3(c^2x^2+2cx-3)\ln(cx+1)^3}{64(cx+1)^2c}+\frac{3b^2(-2b\,c^2x^2\ln(-cx+1)+4a\,c^2x^2+3b\,c^2x^2-4bcx\ln(-cx+1)+8acx+2bcx+6b\ln(-c}{128(cx+1)^2c}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+b*arctanh(c*x))^3/(c*x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-48*a^2*b*c^2*x^2-72*a^2*b*c*x-12*b^3*arctanh(c*x)^2*c*x-18*b^3*arc
tanh(c*x)^2*c^2*x^2-24*b^3*c^2*x^2-32*a^3*c^2*x^2-64*a^3*c*x+72*arctanh(c*
x)*a^2*b-21*arctanh(c*x)*b^3*c^2*x^2-6*arctanh(c*x)*b^3*c*x-27*b^3*c*x-48*
a*b^2*c^2*x^2-60*a*b^2*c*x+24*arctanh(c*x)^3*b^3+30*b^3*arctanh(c*x)^2+27*
arctanh(c*x)*b^3+72*arctanh(c*x)^2*a*b^2-24*arctanh(c*x)^2*a*b^2*c^2*x^2-2
4*arctanh(c*x)*a^2*b*c^2*x^2-48*arctanh(c*x)^2*a*b^2*c*x-48*arctanh(c*x)*a
^2*b*c*x-36*a*b^2*arctanh(c*x)*c^2*x^2-24*a*b^2*arctanh(c*x)*c*x+60*a*b^2*
arctanh(c*x)-8*arctanh(c*x)^3*b^3*c^2*x^2-16*arctanh(c*x)^3*b^3*c*x)/(c*x+
1)^2/c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx$$
$$=\frac{2\,(b^3c^2x^2+2\,b^3cx-3\,b^3)\log\left(-\frac{cx+1}{cx-1}\right)^3-64\,a^3-96\,a^2b-96\,ab^2-48\,b^3-6\,(8\,a^2b+12\,ab^2+7\,b^3)cx+}{}$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="fricas")
```

output
```
1/128*(2*(b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 6
4*a^3 - 96*a^2*b - 96*a*b^2 - 48*b^3 - 6*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x
+ 3*((4*a*b^2 + 3*b^3)*c^2*x^2 - 12*a*b^2 - 5*b^3 + 2*(4*a*b^2 + b^3)*c*x)
*log(-(c*x + 1)/(c*x - 1))^2 + 3*((8*a^2*b + 12*a*b^2 + 7*b^3)*c^2*x^2 - 2
4*a^2*b - 20*a*b^2 - 9*b^3 + 2*(8*a^2*b + 4*a*b^2 + b^3)*c*x)*log(-(c*x +
1)/(c*x - 1)))/(c^3*x^2 + 2*c^2*x + c)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx = \int \frac{(a+b\,\text{atanh}\,(cx))^3}{(cx+1)^3}\,dx$$

input
```
integrate((a+b*atanh(c*x))**3/(c*x+1)**3,x)
```

output
```
Integral((a + b*atanh(c*x))**3/(c*x + 1)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(188) = 376.

Time = 0.06 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.83

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="maxima")
```

output
```
-1/2*b^3*arctanh(c*x)^3/(c^3*x^2 + 2*c^2*x + c) - 3/16*(c*(2*(c*x + 2)/(c^
4*x^2 + 2*c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*arctanh(
c*x)/(c^3*x^2 + 2*c^2*x + c))*a^2*b - 3/32*(4*c*(2*(c*x + 2)/(c^4*x^2 + 2*
c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c^2*x
^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 + 6*
c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c
*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*x^2 + 2*c^4*x
 + c^3))*a*b^2 - 1/128*(24*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3*x + c^2) - log(
c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x)^2 - ((2*(c^2*x^2 + 2*c*x + 1
)*log(c*x + 1)^3 - 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^3 - 3*(3*c^2*x^2 +
 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1)^2 - 9*(c^2
*x^2 + 2*c*x + 1)*log(c*x - 1)^2 - 42*c*x + 3*(7*c^2*x^2 + 2*(c^2*x^2 + 2*
c*x + 1)*log(c*x - 1)^2 + 14*c*x + 6*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) +
7)*log(c*x + 1) - 21*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) - 48)*c^2/(c^6*x^2
 + 2*c^5*x + c^4) - 12*((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 +
2*c*x + 1)*log(c*x - 1)^2 + 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*
x + 1)*log(c*x - 1) + 3)*log(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x -
1) + 8)*c*arctanh(c*x)/(c^5*x^2 + 2*c^4*x + c^3))*c)*b^3 - 3/2*a*b^2*arcta
nh(c*x)^2/(c^3*x^2 + 2*c^2*x + c) - 1/2*a^3/(c^3*x^2 + 2*c^2*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx$$
$$=\frac{1}{256}\left(\frac{4\left(\frac{2\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\frac{cx+1}{cx-1}\right)^3}{(cx+1)^2c^2}+\frac{6\left(\frac{8\,(cx+1)ab^2}{cx-1}-4\,ab^2+\frac{4\,(cx+1)b^3}{cx-1}-b^3\right)(cx-1)^2\log\left(-\right.}{(cx+1)^2c^2}\right.$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="giac")
```

output
```
1/256*(4*(2*(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(c*x + 1)/(c*x
 - 1))^3/((c*x + 1)^2*c^2) + 6*(8*(c*x + 1)*a*b^2/(c*x - 1) - 4*a*b^2 + 4*
(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*log(-(c*x + 1)/(c*x - 1))^2/((c
*x + 1)^2*c^2) + 6*(16*(c*x + 1)*a^2*b/(c*x - 1) - 8*a^2*b + 16*(c*x + 1)*
a*b^2/(c*x - 1) - 4*a*b^2 + 8*(c*x + 1)*b^3/(c*x - 1) - b^3)*(c*x - 1)^2*l
og(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2*c^2) + (64*(c*x + 1)*a^3/(c*x - 1) -
 32*a^3 + 96*(c*x + 1)*a^2*b/(c*x - 1) - 24*a^2*b + 96*(c*x + 1)*a*b^2/(c*
x - 1) - 12*a*b^2 + 48*(c*x + 1)*b^3/(c*x - 1) - 3*b^3)*(c*x - 1)^2/((c*x
+ 1)^2*c^2))*c
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.47

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*atanh(c*x))^3/(c*x + 1)^3,x)
```

output
```
(102*b^3*log(1 - c*x) - 102*b^3*log(c*x + 1) - 96*a*b^2 - 96*a^2*b - 15*b^
3*log(c*x + 1)^2 - 6*b^3*log(c*x + 1)^3 - 15*b^3*log(1 - c*x)^2 + 6*b^3*lo
g(1 - c*x)^3 + 150*b^3*atanh(c*x) - 64*a^3 - 48*b^3 + 144*a*b^2*atanh(c*x)
 + 48*a^2*b*atanh(c*x) + 30*b^3*log(c*x + 1)*log(1 - c*x) - 132*a*b^2*log(
c*x + 1) - 96*a^2*b*log(c*x + 1) + 132*a*b^2*log(1 - c*x) + 96*a^2*b*log(1
 - c*x) - 18*b^3*log(c*x + 1)*log(1 - c*x)^2 + 18*b^3*log(c*x + 1)^2*log(1
 - c*x) - 36*a*b^2*log(c*x + 1)^2 - 36*a*b^2*log(1 - c*x)^2 - 42*b^3*c*x -
 144*b^3*c*x*log(c*x + 1) + 144*b^3*c*x*log(1 - c*x) + 9*b^3*c^2*x^2*log(c
*x + 1)^2 + 2*b^3*c^2*x^2*log(c*x + 1)^3 + 9*b^3*c^2*x^2*log(1 - c*x)^2 -
2*b^3*c^2*x^2*log(1 - c*x)^3 + 150*b^3*c^2*x^2*atanh(c*x) - 72*a*b^2*c*x -
 48*a^2*b*c*x + 6*b^3*c*x*log(c*x + 1)^2 + 4*b^3*c*x*log(c*x + 1)^3 + 6*b^
3*c*x*log(1 - c*x)^2 - 4*b^3*c*x*log(1 - c*x)^3 + 72*a*b^2*log(c*x + 1)*lo
g(1 - c*x) + 300*b^3*c*x*atanh(c*x) - 54*b^3*c^2*x^2*log(c*x + 1) + 54*b^3
*c^2*x^2*log(1 - c*x) - 12*b^3*c*x*log(c*x + 1)*log(1 - c*x) - 36*a*b^2*c^
2*x^2*log(c*x + 1) + 36*a*b^2*c^2*x^2*log(1 - c*x) + 6*b^3*c^2*x^2*log(c*x
 + 1)*log(1 - c*x)^2 - 6*b^3*c^2*x^2*log(c*x + 1)^2*log(1 - c*x) - 120*a*b
^2*c*x*log(c*x + 1) + 120*a*b^2*c*x*log(1 - c*x) + 12*b^3*c*x*log(c*x + 1)
*log(1 - c*x)^2 - 12*b^3*c*x*log(c*x + 1)^2*log(1 - c*x) + 12*a*b^2*c^2*x^
2*log(c*x + 1)^2 + 12*a*b^2*c^2*x^2*log(1 - c*x)^2 + 144*a*b^2*c^2*x^2*ata
nh(c*x) + 48*a^2*b*c^2*x^2*atanh(c*x) + 24*a*b^2*c*x*log(c*x + 1)^2 + 2...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.51

$$\int \frac{(a + b\text{arctanh}(cx))^3}{(1 + cx)^3}\, dx$$
$$= \frac{-24\log(cx-1)\,a^2b - 12\log(cx-1)\,a\,b^2 + 24\log(cx+1)\,a^2b + 12\log(cx+1)\,a\,b^2 - 48atanh(cx)^3\,b^3 - }{}$$

input
```
int((a+b*atanh(c*x))^3/(c*x+1)^3,x)
```

output

```
(16*atanh(c*x)**3*b**3*c**2*x**2 + 32*atanh(c*x)**3*b**3*c*x - 48*atanh(c*
x)**3*b**3 + 48*atanh(c*x)**2*a*b**2*c**2*x**2 + 96*atanh(c*x)**2*a*b**2*c
*x - 144*atanh(c*x)**2*a*b**2 + 36*atanh(c*x)**2*b**3*c**2*x**2 + 24*atanh
(c*x)**2*b**3*c*x - 60*atanh(c*x)**2*b**3 - 192*atanh(c*x)*a**2*b + 48*ata
nh(c*x)*a*b**2*c**2*x**2 - 144*atanh(c*x)*a*b**2 + 36*atanh(c*x)*b**3*c**2
*x**2 - 60*atanh(c*x)*b**3 - 24*log(c*x - 1)*a**2*b*c**2*x**2 - 48*log(c*x
 - 1)*a**2*b*c*x - 24*log(c*x - 1)*a**2*b - 12*log(c*x - 1)*a*b**2*c**2*x*
*2 - 24*log(c*x - 1)*a*b**2*c*x - 12*log(c*x - 1)*a*b**2 - 3*log(c*x - 1)*
b**3*c**2*x**2 - 6*log(c*x - 1)*b**3*c*x - 3*log(c*x - 1)*b**3 + 24*log(c*
x + 1)*a**2*b*c**2*x**2 + 48*log(c*x + 1)*a**2*b*c*x + 24*log(c*x + 1)*a**
2*b + 12*log(c*x + 1)*a*b**2*c**2*x**2 + 24*log(c*x + 1)*a*b**2*c*x + 12*l
og(c*x + 1)*a*b**2 + 3*log(c*x + 1)*b**3*c**2*x**2 + 6*log(c*x + 1)*b**3*c
*x + 3*log(c*x + 1)*b**3 - 64*a**3 + 24*a**2*b*c**2*x**2 - 72*a**2*b + 36*
a*b**2*c**2*x**2 - 60*a*b**2 + 21*b**3*c**2*x**2 - 27*b**3)/(128*c*(c**2*x
**2 + 2*c*x + 1))
```

3.126 $\int \frac{(a+b\operatorname{arctanh}(cx))^3}{(1+cx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 275

$$\begin{aligned}\int \frac{(a+b\operatorname{arctanh}(cx))^3}{(1+cx)^4}\,dx &= -\frac{b^3}{108c(1+cx)^3}-\frac{19b^3}{576c(1+cx)^2}-\frac{85b^3}{576c(1+cx)}\\ &\quad+\frac{85b^3\operatorname{arctanh}(cx)}{576c}-\frac{b^2(a+b\operatorname{arctanh}(cx))}{18c(1+cx)^3}\\ &\quad-\frac{5b^2(a+b\operatorname{arctanh}(cx))}{48c(1+cx)^2}-\frac{11b^2(a+b\operatorname{arctanh}(cx))}{48c(1+cx)}\\ &\quad+\frac{11b(a+b\operatorname{arctanh}(cx))^2}{96c}-\frac{b(a+b\operatorname{arctanh}(cx))^2}{6c(1+cx)^3}\\ &\quad-\frac{b(a+b\operatorname{arctanh}(cx))^2}{8c(1+cx)^2}-\frac{b(a+b\operatorname{arctanh}(cx))^2}{8c(1+cx)}\\ &\quad+\frac{(a+b\operatorname{arctanh}(cx))^3}{24c}-\frac{(a+b\operatorname{arctanh}(cx))^3}{3c(1+cx)^3}\end{aligned}$$

output

```
-1/108*b^3/c/(c*x+1)^3-19/576*b^3/c/(c*x+1)^2-85/576*b^3/c/(c*x+1)+85/576*
b^3*arctanh(c*x)/c-1/18*b^2*(a+b*arctanh(c*x))/c/(c*x+1)^3-5/48*b^2*(a+b*a
rctanh(c*x))/c/(c*x+1)^2-11/48*b^2*(a+b*arctanh(c*x))/c/(c*x+1)+11/96*b*(a
+b*arctanh(c*x))^2/c-1/6*b*(a+b*arctanh(c*x))^2/c/(c*x+1)^3-1/8*b*(a+b*arc
tanh(c*x))^2/c/(c*x+1)^2-1/8*b*(a+b*arctanh(c*x))^2/c/(c*x+1)+1/24*(a+b*ar
ctanh(c*x))^3/c-1/3*(a+b*arctanh(c*x))^3/c/(c*x+1)^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^4}\,dx =$$
$$-\frac{32(36a^3+18a^2b+6ab^2+b^3)+6b(72a^2+60ab+19b^2)(1+cx)+6b(72a^2+132ab+85b^2)(1+cx)^2}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

output
```
-1/3456*(32*(36*a^3 + 18*a^2*b + 6*a*b^2 + b^3) + 6*b*(72*a^2 + 60*a*b + 1
9*b^2)*(1 + c*x) + 6*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^2 + 24*b*(144
*a^2 + 12*a*b*(10 + 9*c*x + 3*c^2*x^2) + b^2*(56 + 81*c*x + 33*c^2*x^2))*A
rcTanh[c*x] - 36*b^2*(-1 + c*x)*(12*a*(7 + 4*c*x + c^2*x^2) + b*(29 + 32*c
*x + 11*c^2*x^2))*ArcTanh[c*x]^2 - 144*b^3*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x
^3)*ArcTanh[c*x]^3 + 3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 - c
*x] - 3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 + c*x])/(c*(1 + c*
x)^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(cx+1)^4}dx$$
$$\Big\downarrow \text{6480}$$
$$b\int \left(\frac{(a+b\text{arctanh}(cx))^2}{8(1-c^2x^2)}+\frac{(a+b\text{arctanh}(cx))^2}{8(cx+1)^2}+\frac{(a+b\text{arctanh}(cx))^2}{4(cx+1)^3}+\frac{(a+b\text{arctanh}(cx))^2}{2(cx+1)^4}\right)dx-\frac{(a+b\text{arctanh}(cx))^3}{3c(cx+1)^3}$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{(a+b\text{arctanh}(cx))^3}{24bc}+\frac{11(a+b\text{arctanh}(cx))^2}{96c}-\frac{(a+b\text{arctanh}(cx))^2}{8c(cx+1)}-\frac{(a+b\text{arctanh}(cx))^2}{8c(cx+1)^2}-\frac{(a+b\text{arctanh}}{6c(cx+}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3c(cx+1)^3}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x])^3/(c*(1 + c*x)^3) + b*(-1/108*b^2/(c*(1 + c*x)^3
) - (19*b^2)/(576*c*(1 + c*x)^2) - (85*b^2)/(576*c*(1 + c*x)) + (85*b^2*Ar
cTanh[c*x])/(576*c) - (b*(a + b*ArcTanh[c*x]))/(18*c*(1 + c*x)^3) - (5*b*(
a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)^2) - (11*b*(a + b*ArcTanh[c*x]))/(48*
c*(1 + c*x)) + (11*(a + b*ArcTanh[c*x])^2)/(96*c) - (a + b*ArcTanh[c*x])^2
/(6*c*(1 + c*x)^3) - (a + b*ArcTanh[c*x])^2/(8*c*(1 + c*x)^2) - (a + b*Arc
Tanh[c*x])^2/(8*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(24*b*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.72

method	result
parallelrisch	$-\frac{-720a^2b\,x^3c^3-1944a^2b\,c^2x^2-1512a^2bcx+54b^3\operatorname{arctanh}(cx)^2cx-378b^3\operatorname{arctanh}(cx)^2c^2x^2-198b^3\operatorname{arctanh}(cx)^2c^3x^3}{}$
orering	$-\frac{(9840c^6x^6+915c^5x^5-19487c^4x^4+1928x^3c^3+9396c^2x^2-5643cx+3051)(a+b\operatorname{arctanh}(cx))^3}{2592(cx+1)^3c}-\frac{(cx-1)(cx+1)^2(7}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*arctanh(c*x))^3/(c*x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/1728*(-720*a^2*b*x^3*c^3-1944*a^2*b*c^2*x^2-1512*a^2*b*c*x+54*b^3*arcta
nh(c*x)^2*c*x-378*b^3*arctanh(c*x)^2*c^2*x^2-198*b^3*arctanh(c*x)^2*c^3*x^
3-729*b^3*c^2*x^2-576*a^3*c^3*x^3-1728*a^3*c^2*x^2-1728*a^3*c*x+1512*arcta
nh(c*x)*a^2*b-369*arctanh(c*x)*b^3*c^2*x^2+207*arctanh(c*x)*b^3*c*x-417*b^
3*c*x-1620*a*b^2*c^2*x^2-1044*a*b^2*c*x+504*arctanh(c*x)^3*b^3+522*b^3*arc
tanh(c*x)^2+417*arctanh(c*x)*b^3-328*b^3*c^3*x^3+1512*arctanh(c*x)^2*a*b^2
-672*a*b^2*c^3*x^3-216*arctanh(c*x)^2*a*b^2*c^3*x^3-216*arctanh(c*x)*a^2*b
*c^3*x^3-648*arctanh(c*x)^2*a*b^2*c^2*x^2-648*arctanh(c*x)*a^2*b*c^2*x^2-6
48*arctanh(c*x)^2*a*b^2*c*x-648*arctanh(c*x)*a^2*b*c*x-396*a*b^2*arctanh(c
*x)*c^3*x^3-756*a*b^2*arctanh(c*x)*c^2*x^2+108*a*b^2*arctanh(c*x)*c*x-255*
x^3*arctanh(c*x)*b^3*c^3+1044*a*b^2*arctanh(c*x)-72*arctanh(c*x)^3*b^3*c^3
*x^3-216*arctanh(c*x)^3*b^3*c^2*x^2-216*arctanh(c*x)^3*b^3*c*x)/(c*x+1)^3/
c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{(1+cx)^4}\,dx =$$
$$-\frac{6\,(72\,a^2b+132\,ab^2+85\,b^3)c^2x^2-18\,(b^3c^3x^3+3\,b^3c^2x^2+3\,b^3cx-7\,b^3)\log\left(-\frac{cx+1}{cx-1}\right)^3+1152\,a^3+144}{}$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="fricas")
```

output
```
-1/3456*(6*(72*a^2*b + 132*a*b^2 + 85*b^3)*c^2*x^2 - 18*(b^3*c^3*x^3 + 3*b
^3*c^2*x^2 + 3*b^3*c*x - 7*b^3)*log(-(c*x + 1)/(c*x - 1))^3 + 1152*a^3 + 1
440*a^2*b + 1344*a*b^2 + 656*b^3 + 162*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x -
9*((12*a*b^2 + 11*b^3)*c^3*x^3 + 3*(12*a*b^2 + 7*b^3)*c^2*x^2 - 84*a*b^2 -
 29*b^3 + 3*(12*a*b^2 - b^3)*c*x)*log(-(c*x + 1)/(c*x - 1))^2 - 3*((72*a^2
*b + 132*a*b^2 + 85*b^3)*c^3*x^3 + 3*(72*a^2*b + 84*a*b^2 + 41*b^3)*c^2*x^
2 - 504*a^2*b - 348*a*b^2 - 139*b^3 + 3*(72*a^2*b - 12*a*b^2 - 23*b^3)*c*x
)*log(-(c*x + 1)/(c*x - 1)))/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^4}\,dx = \int \frac{(a+b\,\text{atanh}\,(cx))^3}{(cx+1)^4}\,dx$$

input
```
integrate((a+b*atanh(c*x))**3/(c*x+1)**4,x)
```

output
```
Integral((a + b*atanh(c*x))**3/(c*x + 1)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(249) = 498$.

Time = 0.07 (sec) , antiderivative size = 1085, normalized size of antiderivative = 3.95

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="maxima")
```

output
```
-1/3*b^3*arctanh(c*x)^3/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/48*(c*(2*(
3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x
+ 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c^3*x^2 + 3*
c^2*x + c))*a^2*b - 1/288*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c
^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*arctanh
(c*x) + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 +
 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*
x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x -
1) + 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)
+ 112)*c^2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*a*b^2 - 1/3456*(72*c*(2*
(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x
 + 1)/c^2 + 3*log(c*x - 1)/c^2)*arctanh(c*x)^2 + ((510*c^2*x^2 - 18*(c^3*x
^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^3 + 18*(c^3*x^3 + 3*c^2*x^2 + 3*c
*x + 1)*log(c*x - 1)^3 + 9*(11*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3
+ 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 11)*log(c*x + 1)^2 + 99*(c^3*x^3 +
 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 1134*c*x - 3*(85*c^3*x^3 + 255*c^
2*x^2 + 18*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 255*c*x + 66
*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 85)*log(c*x + 1) + 255*(
c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 656)*c^2/(c^7*x^3 + 3*c^6*
x^2 + 3*c^5*x + c^4) + 12*(66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(249) = 498$.

Time = 0.14 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.02

$$\int \frac{(a + b\operatorname{arctanh}(cx))^3}{(1 + cx)^4} \, dx$$
$$= \frac{1}{6912} \left(\frac{36 \left(\frac{3 (cx+1)^2 b^3}{(cx-1)^2} - \frac{3 (cx+1) b^3}{cx-1} + b^3 \right) (cx-1)^3 \log \left(-\frac{cx+1}{cx-1} \right)^3}{(cx+1)^3 c^2} + \frac{18 \left(\frac{36 (cx+1)^2 ab^2}{(cx-1)^2} - \frac{36 (cx+1) ab^2}{cx-1} + 12\, ab^2 + \right.}{}\right.$$

input
```
integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="giac")
```

output
```
1/6912*(36*(3*(c*x + 1)^2*b^3/(c*x - 1)^2 - 3*(c*x + 1)*b^3/(c*x - 1) + b^
3)*(c*x - 1)^3*log(-(c*x + 1)/(c*x - 1))^3/((c*x + 1)^3*c^2) + 18*(36*(c*x
 + 1)^2*a*b^2/(c*x - 1)^2 - 36*(c*x + 1)*a*b^2/(c*x - 1) + 12*a*b^2 + 18*(
c*x + 1)^2*b^3/(c*x - 1)^2 - 9*(c*x + 1)*b^3/(c*x - 1) + 2*b^3)*(c*x - 1)^
3*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^3*c^2) + 6*(216*(c*x + 1)^2*a^2*b
/(c*x - 1)^2 - 216*(c*x + 1)*a^2*b/(c*x - 1) + 72*a^2*b + 216*(c*x + 1)^2*
a*b^2/(c*x - 1)^2 - 108*(c*x + 1)*a*b^2/(c*x - 1) + 24*a*b^2 + 108*(c*x +
1)^2*b^3/(c*x - 1)^2 - 27*(c*x + 1)*b^3/(c*x - 1) + 4*b^3)*(c*x - 1)^3*log
(-(c*x + 1)/(c*x - 1))/((c*x + 1)^3*c^2) + (864*(c*x + 1)^2*a^3/(c*x - 1)^
2 - 864*(c*x + 1)*a^3/(c*x - 1) + 288*a^3 + 1296*(c*x + 1)^2*a^2*b/(c*x -
1)^2 - 648*(c*x + 1)*a^2*b/(c*x - 1) + 144*a^2*b + 1296*(c*x + 1)^2*a*b^2/
(c*x - 1)^2 - 324*(c*x + 1)*a*b^2/(c*x - 1) + 48*a*b^2 + 648*(c*x + 1)^2*b
^3/(c*x - 1)^2 - 81*(c*x + 1)*b^3/(c*x - 1) + 8*b^3)*(c*x - 1)^3/((c*x + 1
)^3*c^2))*c
```

Mupad [B] (verification not implemented)

Time = 7.11 (sec) , antiderivative size = 1304, normalized size of antiderivative = 4.74

$$\int \frac{(a + b\text{arctanh}(cx))^3}{(1 + cx)^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*atanh(c*x))^3/(c*x + 1)^4,x)
```

output
```
(1398*b^3*log(1 - c*x) - 1398*b^3*log(c*x + 1) - 1344*a*b^2 - 1440*a^2*b -
 261*b^3*log(c*x + 1)^2 - 126*b^3*log(c*x + 1)^3 - 261*b^3*log(1 - c*x)^2
+ 126*b^3*log(1 - c*x)^3 + 1962*b^3*atanh(c*x) - 1152*a^3 - 656*b^3 + 1584
*a*b^2*atanh(c*x) + 432*a^2*b*atanh(c*x) + 522*b^3*log(c*x + 1)*log(1 - c*
x) - 1836*a*b^2*log(c*x + 1) - 1728*a^2*b*log(c*x + 1) + 1836*a*b^2*log(1
- c*x) + 1728*a^2*b*log(1 - c*x) - 378*b^3*log(c*x + 1)*log(1 - c*x)^2 + 3
78*b^3*log(c*x + 1)^2*log(1 - c*x) - 510*b^3*c^2*x^2 - 756*a*b^2*log(c*x +
 1)^2 - 756*a*b^2*log(1 - c*x)^2 - 1134*b^3*c*x - 3150*b^3*c*x*log(c*x + 1
) + 3150*b^3*c*x*log(1 - c*x) - 792*a*b^2*c^2*x^2 - 432*a^2*b*c^2*x^2 + 18
9*b^3*c^2*x^2*log(c*x + 1)^2 + 54*b^3*c^2*x^2*log(c*x + 1)^3 + 189*b^3*c^2
*x^2*log(1 - c*x)^2 - 54*b^3*c^2*x^2*log(1 - c*x)^3 + 99*b^3*c^3*x^3*log(c
*x + 1)^2 + 18*b^3*c^3*x^3*log(c*x + 1)^3 + 99*b^3*c^3*x^3*log(1 - c*x)^2
- 18*b^3*c^3*x^3*log(1 - c*x)^3 + 5886*b^3*c^2*x^2*atanh(c*x) + 1962*b^3*c
^3*x^3*atanh(c*x) - 1944*a*b^2*c*x - 1296*a^2*b*c*x - 27*b^3*c*x*log(c*x +
 1)^2 + 54*b^3*c*x*log(c*x + 1)^3 - 27*b^3*c*x*log(1 - c*x)^2 - 54*b^3*c*x
*log(1 - c*x)^3 + 1512*a*b^2*log(c*x + 1)*log(1 - c*x) + 5886*b^3*c*x*atan
h(c*x) - 2574*b^3*c^2*x^2*log(c*x + 1) + 2574*b^3*c^2*x^2*log(1 - c*x) - 7
26*b^3*c^3*x^3*log(c*x + 1) + 726*b^3*c^3*x^3*log(1 - c*x) + 54*b^3*c*x*lo
g(c*x + 1)*log(1 - c*x) - 1620*a*b^2*c^2*x^2*log(c*x + 1) + 1620*a*b^2*c^2
*x^2*log(1 - c*x) - 396*a*b^2*c^3*x^3*log(c*x + 1) + 396*a*b^2*c^3*x^3*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.67

$$\int \frac{(a+b\text{arctanh}(cx))^3}{(1+cx)^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*atanh(c*x))^3/(c*x+1)^4,x)
```

output

```
(144*atanh(c*x)**3*b**3*c**3*x**3 + 432*atanh(c*x)**3*b**3*c**2*x**2 + 432
*atanh(c*x)**3*b**3*c*x - 1008*atanh(c*x)**3*b**3 + 432*atanh(c*x)**2*a*b*
*2*c**3*x**3 + 1296*atanh(c*x)**2*a*b**2*c**2*x**2 + 1296*atanh(c*x)**2*a*
b**2*c*x - 3024*atanh(c*x)**2*a*b**2 + 396*atanh(c*x)**2*b**3*c**3*x**3 +
756*atanh(c*x)**2*b**3*c**2*x**2 - 108*atanh(c*x)**2*b**3*c*x - 1044*atanh
(c*x)**2*b**3 - 3456*atanh(c*x)*a**2*b + 288*atanh(c*x)*a*b**2*c**3*x**3 -
 1728*atanh(c*x)*a*b**2*c*x - 2592*atanh(c*x)*a*b**2 + 264*atanh(c*x)*b**3
*c**3*x**3 - 1152*atanh(c*x)*b**3*c*x - 1080*atanh(c*x)*b**3 - 216*log(c*x
 - 1)*a**2*b*c**3*x**3 - 648*log(c*x - 1)*a**2*b*c**2*x**2 - 648*log(c*x -
 1)*a**2*b*c*x - 216*log(c*x - 1)*a**2*b - 252*log(c*x - 1)*a*b**2*c**3*x*
*3 - 756*log(c*x - 1)*a*b**2*c**2*x**2 - 756*log(c*x - 1)*a*b**2*c*x - 252
*log(c*x - 1)*a*b**2 - 123*log(c*x - 1)*b**3*c**3*x**3 - 369*log(c*x - 1)*
b**3*c**2*x**2 - 369*log(c*x - 1)*b**3*c*x - 123*log(c*x - 1)*b**3 + 216*l
og(c*x + 1)*a**2*b*c**3*x**3 + 648*log(c*x + 1)*a**2*b*c**2*x**2 + 648*log
(c*x + 1)*a**2*b*c*x + 216*log(c*x + 1)*a**2*b + 252*log(c*x + 1)*a*b**2*c
**3*x**3 + 756*log(c*x + 1)*a*b**2*c**2*x**2 + 756*log(c*x + 1)*a*b**2*c*x
 + 252*log(c*x + 1)*a*b**2 + 123*log(c*x + 1)*b**3*c**3*x**3 + 369*log(c*x
 + 1)*b**3*c**2*x**2 + 369*log(c*x + 1)*b**3*c*x + 123*log(c*x + 1)*b**3 -
 1152*a**3 + 144*a**2*b*c**3*x**3 - 864*a**2*b*c*x - 1296*a**2*b + 264*a*b
**2*c**3*x**3 - 1152*a*b**2*c*x - 1080*a*b**2 + 170*b**3*c**3*x**3 - 62...
```

3.127 $\int \frac{x^2 \operatorname{arctanh}(ax)^3}{c+acx} dx$

Optimal result

Integrand size = 18, antiderivative size = 309

$$\begin{aligned}\int \frac{x^2 \operatorname{arctanh}(ax)^3}{c+acx} dx = {} & \frac{3\operatorname{arctanh}(ax)^2}{2a^3c} + \frac{3x\operatorname{arctanh}(ax)^2}{2a^2c} - \frac{3\operatorname{arctanh}(ax)^3}{2a^3c} \\ & - \frac{x\operatorname{arctanh}(ax)^3}{a^2c} + \frac{x^2\operatorname{arctanh}(ax)^3}{2ac} \\ & - \frac{3\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3c} + \frac{3\operatorname{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3c} \\ & - \frac{\operatorname{arctanh}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^3c} - \frac{3\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^3c} \\ & + \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c} \\ & + \frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+ax}\right)}{2a^3c} \\ & - \frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3c} \\ & + \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+ax}\right)}{2a^3c} \\ & + \frac{3\operatorname{PolyLog}\left(4,1-\frac{2}{1+ax}\right)}{4a^3c}\end{aligned}$$

output
```
3/2*arctanh(a*x)^2/a^3/c+3/2*x*arctanh(a*x)^2/a^2/c-3/2*arctanh(a*x)^3/a^3
/c-x*arctanh(a*x)^3/a^2/c+1/2*x^2*arctanh(a*x)^3/a/c-3*arctanh(a*x)*ln(2/(
-a*x+1))/a^3/c+3*arctanh(a*x)^2*ln(2/(-a*x+1))/a^3/c-arctanh(a*x)^3*ln(2/(
a*x+1))/a^3/c-3/2*polylog(2,1-2/(-a*x+1))/a^3/c+3*arctanh(a*x)*polylog(2,1
-2/(-a*x+1))/a^3/c+3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))/a^3/c-3/2*pol
ylog(3,1-2/(-a*x+1))/a^3/c+3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a^3/c+3
/4*polylog(4,1-2/(a*x+1))/a^3/c
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.56

$$\int \frac{x^2\text{arctanh}(ax)^3}{c+acx}\,dx$$
$$= \frac{-6\text{arctanh}(ax)^2 + 6ax\text{arctanh}(ax)^2 + 2\text{arctanh}(ax)^3 - 4ax\text{arctanh}(ax)^3 + 2a^2x^2\text{arctanh}(ax)^3 - 12\text{arc}}{}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

output
```
(-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]^3 - 4*a*x*ArcTa
nh[a*x]^3 + 2*a^2*x^2*ArcTanh[a*x]^3 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTa
nh[a*x])] + 12*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x
]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])^2*PolyLog[2, -E^(
-2*ArcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])]
 + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a^3*c)
```

Rubi [A] (verified)

Time = 4.02 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {6492, 27, 6452, 6492, 6436, 6470, 6542, 6436, 6510, 6546, 6470, 2849, 2752, 6618, 6620, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^3}{acx+c}dx$$

$\Big\downarrow$ 6492

$$\frac{\int x\text{arctanh}(ax)^3dx}{ac}-\frac{\int \frac{x\text{arctanh}(ax)^3}{c(ax+1)}dx}{a}$$

$\Big\downarrow$ 27

$$\frac{\int x\text{arctanh}(ax)^3dx}{ac}-\frac{\int \frac{x\text{arctanh}(ax)^3}{ax+1}dx}{ac}$$

$\Big\downarrow$ 6452

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}dx}{ac}-\frac{\int \frac{x\text{arctanh}(ax)^3}{ax+1}dx}{ac}$$

$\Big\downarrow$ 6492

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}dx}{ac}-\frac{\frac{\int \text{arctanh}(ax)^3dx}{a}-\frac{\int \frac{\text{arctanh}(ax)^3}{ax+1}dx}{a}}{ac}$$

$\Big\downarrow$ 6436

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}dx}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a}-\frac{\int \frac{\text{arctanh}(ax)^3}{ax+1}dx}{a}}{ac}$$

$\Big\downarrow$ 6470

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}dx}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a}-\frac{3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{a}}{ac}$$

$\Big\downarrow$ 6542

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)^2dx}{a^2}\right)}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a}-\frac{3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{a}}{ac}$$

$\Big\downarrow$ 6436

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\int\frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)^2-2a\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}\right)}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a}-\frac{3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{a}}{ac}$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}\right)}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a}-\frac{3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{a}}{ac}$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\int\frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right)}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\int\frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{a}}{ac}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right)}{ac}-\frac{\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\text{ar}}{a}}{ac}$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{ac}$$

$$\frac{\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\text{ar}}{a}}{ac}$$

$\downarrow$ 2752

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{ac}$$

$$\frac{\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\text{ar}}{a}}{ac}$$

$\downarrow$ 6618

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{ac}$$

$$\frac{\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right.}{2a}\right.}{}}{ac}$$

$\downarrow$ 6620

$$\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.$$

$$\frac{\quad}{ac}$$

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3}{\quad}$$

$$\frac{\quad}{ac}$$

$\downarrow$ 6622

$$\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.$$

$$\frac{\quad}{ac}$$

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3}{\quad}$$

$$\frac{\quad}{ac}$$

$\downarrow$ 7164

$$\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.$$

$$\frac{\quad}{ac}$$

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a}-\frac{3\left(\text{arc}\right.}{\quad}$$

$$\frac{\quad}{ac}$$

input
```
Int[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

output
```
((x^2*ArcTanh[a*x]^3)/2 - (3*a*(ArcTanh[a*x]^3/(3*a^3) - (x*ArcTanh[a*x]^2
 - 2*a*(-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + Pol
yLog[2, 1 - 2/(1 - a*x)]/(2*a))/a))/a^2))/2)/(a*c) - ((x*ArcTanh[a*x]^3 -
3*a*(-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-
1/2*(ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a
*x)]/(4*a)))/a))/a - (-((ArcTanh[a*x]^3*Log[2/(1 + a*x)])/a) + 3*((ArcTanh
[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, 1 -
2/(1 + a*x)])/(2*a) + PolyLog[4, 1 - 2/(1 + a*x)]/(4*a)))/a)/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2(ax\ \operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)+3)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4}{2c}-\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}-\frac{3\operatorname{arctanh}(ax)^2\operatorname{polylog}}{2c}$
default	$\frac{\operatorname{arctanh}(ax)^2(ax\ \operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)+3)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4}{2c}-\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}-\frac{3\operatorname{arctanh}(ax)^2\operatorname{polylog}}{2c}$

input
```
int(x^2*arctanh(a*x)^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/2/c*arctanh(a*x)^2*(a*x*arctanh(a*x)-arctanh(a*x)+3)*(a*x-1)+1/2/
c*arctanh(a*x)^4-1/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)-3/2/c*arc
tanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2/c*arctanh(a*x)*polylog(
3,-(a*x+1)^2/(-a^2*x^2+1))-3/4/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+3/c*ar
ctanh(a*x)^2-3/c*arctanh(a*x)*ln((a*x+1)^2/(-a^2*x^2+1)+1)-3/2/c*polylog(2
,-(a*x+1)^2/(-a^2*x^2+1))-2/c*arctanh(a*x)^3+3/c*arctanh(a*x)^2*ln((a*x+1)
^2/(-a^2*x^2+1)+1)+3/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2
/c*polylog(3,-(a*x+1)^2/(-a^2*x^2+1)))
```

Fricas [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{c+acx}\,dx=\int \frac{x^2\,\text{artanh}\,(ax)^3}{acx+c}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

output
```
integral(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{c+acx}\,dx=\frac{\int \frac{x^2\,\text{atanh}^3\,(ax)}{ax+1}\,dx}{c}$$

input
```
integrate(x**2*atanh(a*x)**3/(a*c*x+c),x)
```

output
```
Integral(x**2*atanh(a*x)**3/(a*x + 1), x)/c
```

Maxima [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{c+acx}\,dx=\int \frac{x^2\,\text{artanh}\,(ax)^3}{acx+c}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

output
```
-1/16*(a^2*x^2 - 2*a*x + 2*log(a*x + 1))*log(-a*x + 1)^3/(a^3*c) + 1/8*int
egrate(1/2*(2*(a^3*x^3 - a^2*x^2)*log(a*x + 1)^3 - 6*(a^3*x^3 - a^2*x^2)*l
og(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^3 - a^2*x^2 - 2*a*x + 2*(a^3*x^3 -
a^2*x^2 + a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*c*x^2 - a^2*c), x)
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{c+acx}\,dx=\int \frac{x^2\,\text{artanh}\,(ax)^3}{acx+c}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arctanh}(ax)^3}{c+acx}\,dx=\int \frac{x^2\,\text{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

input
```
int((x^2*atanh(a*x)^3)/(c + a*c*x),x)
```

output
```
int((x^2*atanh(a*x)^3)/(c + a*c*x), x)
```

Reduce [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{c+acx} \, dx = \frac{\int \frac{atanh(ax)^3 x^2}{ax+1} dx}{c}$$

input

```
int(x^2*atanh(a*x)^3/(a*c*x+c),x)
```

output

```
int((atanh(a*x)**3*x**2)/(a*x + 1),x)/c
```

3.128 $\int \frac{x\textbf{arctanh}(ax)^3}{c+acx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 205

$$\int \frac{x\text{arctanh}(ax)^3}{c+acx}\,dx = \frac{\text{arctanh}(ax)^3}{a^2c}+\frac{x\text{arctanh}(ax)^3}{ac}-\frac{3\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c}-\frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2c}-\frac{3\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1+ax}\right)}{2a^2c}+\frac{3\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2c}-\frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1+ax}\right)}{2a^2c}-\frac{3\,\text{PolyLog}\left(4,1-\frac{2}{1+ax}\right)}{4a^2c}$$

output
```
arctanh(a*x)^3/a^2/c+x*arctanh(a*x)^3/a/c-3*arctanh(a*x)^2*ln(2/(-a*x+1))/
a^2/c+arctanh(a*x)^3*ln(2/(a*x+1))/a^2/c-3*arctanh(a*x)*polylog(2,1-2/(-a*
x+1))/a^2/c-3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1))/a^2/c+3/2*polylog(3,
1-2/(-a*x+1))/a^2/c-3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a^2/c-3/4*poly
log(4,1-2/(a*x+1))/a^2/c
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.61

$$\int \frac{x\text{arctanh}(ax)^3}{c+acx}\,dx = \frac{-\text{arctanh}(ax)^3 + ax\text{arctanh}(ax)^3 - 3\text{arctanh}(ax)^2\log\left(1+e^{-2\text{arctanh}(ax)}\right) + \text{arctanh}(ax)^3\log\left(1+e^{-2\text{ar}}\right.}{}$$

input

```
Integrate[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

output

```
(-ArcTanh[a*x]^3 + a*x*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*Log[1 + E^(-2*Arc
Tanh[a*x])] + ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - (3*(-2 + ArcTa
nh[a*x])*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])])/2 - (3*(-1 + ArcTa
nh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])])/2 - (3*PolyLog[4, -E^(-2*ArcTan
h[a*x])])/4)/(a^2*c)
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6492, 27, 6436, 6470, 6546, 6470, 6618, 6620, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^3}{acx+c}\,dx$$

$$\Big\downarrow \text{ 6492}$$

$$\frac{\int \text{arctanh}(ax)^3dx}{ac} - \frac{\int \frac{\text{arctanh}(ax)^3}{c(ax+1)}dx}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \text{arctanh}(ax)^3dx}{ac} - \frac{\int \frac{\text{arctanh}(ax)^3}{ax+1}dx}{ac}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{ac}-\frac{\int \frac{\text{arctanh}(ax)^3}{ax+1}dx}{ac}$$

$\big\downarrow$ 6470

$$\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{ac}-\frac{3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

$\big\downarrow$ 6546

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\int \frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{ac}-\frac{3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

$\big\downarrow$ 6470

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{ac}-\frac{3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

$\big\downarrow$ 6618

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{ac}-\frac{3\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}-\int \frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

$\big\downarrow$ 6620

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{ac}-\frac{3\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}-\int \frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

$$\Big\downarrow \text{6622}$$

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{ac}-\frac{3\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2a}\right)-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

$$\Big\downarrow \text{7164}$$

$$\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{ac}-\frac{3\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2a}+\frac{\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a}\right)-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{a}}{ac}$$

input
```
Int[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

output
```
(x*ArcTanh[a*x]^3 - 3*a*(-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[2/
(1 - a*x)])/a - 2*(-1/2*(ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + Pol
yLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a))/(a*c) - (-((ArcTanh[a*x]^3*Log[2/(1 +
 a*x)])/a) + 3*((ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/(2*a) + (ArcT
anh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*a) + PolyLog[4, 1 - 2/(1 + a*x)]/
(4*a)))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.33 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.59

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^3 ax}{c} - \frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c} - \frac{3\left(-\frac{\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2} + \frac{\operatorname{arctanh}(ax) \operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2} - \cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^3 ax}{c} - \frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c} - \frac{3\left(-\frac{\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2} + \frac{\operatorname{arctanh}(ax) \operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2} - \cdots\right.}{\cdots}}{\cdots}$
parts	$\frac{x \operatorname{arctanh}(ax)^3}{ac} - \frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c\,a^2} - \frac{3a\left(-\frac{\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2a^3} + \frac{\operatorname{arctanh}(ax) \operatorname{polylog}\left(3,-\frac{(a\cdots}{-a^2\cdots}\right)}{2a^3}\cdots\right.}{\cdots}$

input
```
int(x*arctanh(a*x)^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/c*arctanh(a*x)^3*a*x-1/c*arctanh(a*x)^3*ln(a*x+1)-3/c*(-1/2*arcta
nh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*arctanh(a*x)*polylog(3,-(
a*x+1)^2/(-a^2*x^2+1))-1/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-2/3*arctanh(
a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*arctanh(a*x)^4-1/6*I*Pi*csgn(I/(
-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x
^2-1)+1))^2*arctanh(a*x)^3+1/6*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x
^2-1)+1))*arctanh(a*x)^3+arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+arcta
nh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/3*arctanh(a*x)^3-1/2*polylog(
3,-(a*x+1)^2/(-a^2*x^2+1))-1/3*ln(2)*arctanh(a*x)^3+1/6*I*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^
2*arctanh(a*x)^3-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^
2-1)+1))^3*arctanh(a*x)^3-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh
(a*x)^3-1/6*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*arctanh(a*x)^3-1/3*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3))
```

Fricas [F]

$$\int \frac{x\text{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{x\,\text{artanh}\left(ax\right)^3}{acx+c}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

output
```
integral(x*arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^3}{c+acx}\,dx = \frac{\int \frac{x\,\text{atanh}^3\left(ax\right)}{ax+1}\,dx}{c}$$

input
```
integrate(x*atanh(a*x)**3/(a*c*x+c),x)
```

output
```
Integral(x*atanh(a*x)**3/(a*x + 1), x)/c
```

Maxima [F]

$$\int \frac{x\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{x\,\operatorname{artanh}\left(ax\right)^3}{acx+c}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

output
```
-1/8*(a*x - log(a*x + 1))*log(-a*x + 1)^3/(a^2*c) + 1/8*integrate(((a^2*x^
2 - a*x)*log(a*x + 1)^3 - 3*(a^2*x^2 - a*x)*log(a*x + 1)^2*log(-a*x + 1) +
 3*(a^2*x^2 + a*x + (a^2*x^2 - 2*a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(
a^3*c*x^2 - a*c), x)
```

Giac [F]

$$\int \frac{x\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{x\,\operatorname{artanh}\left(ax\right)^3}{acx+c}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

output
```
integrate(x*arctanh(a*x)^3/(a*c*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{x\,\operatorname{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

input
```
int((x*atanh(a*x)^3)/(c + a*c*x),x)
```

output

```
int((x*atanh(a*x)^3)/(c + a*c*x), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^3}{c+acx}\,dx=\frac{\int \frac{atanh(ax)^3x}{ax+1}dx}{c}$$

input

```
int(x*atanh(a*x)^3/(a*c*x+c),x)
```

output

```
int((atanh(a*x)**3*x)/(a*x + 1),x)/c
```

3.129 $\int \frac{\text{arctanh}(ax)^3}{c+acx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{\text{arctanh}(ax)^3}{c+acx}\,dx = -\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac} + \frac{3\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1+ax}\right)}{2ac} + \frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1+ax}\right)}{2ac} + \frac{3\,\text{PolyLog}\left(4,1-\frac{2}{1+ax}\right)}{4ac}$$

output
```
-arctanh(a*x)^3*ln(2/(a*x+1))/a/c+3/2*arctanh(a*x)^2*polylog(2,1-2/(a*x+1)
)/a/c+3/2*arctanh(a*x)*polylog(3,1-2/(a*x+1))/a/c+3/4*polylog(4,1-2/(a*x+1
))/a/c
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{\text{arctanh}(ax)^3}{c+acx}\,dx = \frac{-4\text{arctanh}(ax)^3\log\left(1+e^{-2\text{arctanh}(ax)}\right) + 6\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right) + 6\text{arctanh}(ax)\,\text{Po}}{4ac}$$

input
```
Integrate[ArcTanh[a*x]^3/(c + a*c*x),x]
```

output
```
(-4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]^2*PolyLog
[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])
] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a*c)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6470, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{acx+c}dx$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx}{c}-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{ac}$$

$$\Big\downarrow \text{ 6618}$$

$$\frac{3\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}-\int \frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)}{c}-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{ac}$$

$$\Big\downarrow \text{ 6622}$$

$$\frac{3\left(-\frac{1}{2}\int \frac{\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2a}\right)}{c}-\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{ac}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{3\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a}\right)}{c}-\frac{\operatorname{arctanh}(ax)^3\log\left(\frac{2}{ax+1}\right)}{ac}$$

input
```
Int[ArcTanh[a*x]^3/(c + a*c*x),x]
```

output
```
-((ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a*c)) + (3*((ArcTanh[a*x]^2*PolyLog[2
, 1 - 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*
a) + PolyLog[4, 1 - 2/(1 + a*x)]/(4*a)))/c
```

Defintions of rubi rules used

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 593, normalized size of antiderivative = 5.70

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c} - \frac{3\left(\frac{2\operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3} - \frac{\operatorname{arctanh}(ax)^4}{6} + \frac{\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2, -\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2} - \frac{\operatorname{arctanh}(a}{}\right.}{}}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{c} - \frac{3\left(\frac{2\operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3} - \frac{\operatorname{arctanh}(ax)^4}{6} + \frac{\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2, -\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2} - \frac{\operatorname{arctanh}(a}{}\right.}{}}{}$
parts	$\frac{\ln(ax+1)\operatorname{arctanh}(ax)^3}{ac} - \frac{3\left(\frac{2\operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3a} - \frac{\operatorname{arctanh}(ax)^4}{6a} + \frac{\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2, -\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2a} - \frac{\operatorname{arc}}{}\right.}{}$

input
```
int(arctanh(a*x)^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/c*arctanh(a*x)^3*ln(a*x+1)-3/c*(2/3*arctanh(a*x)^3*ln((a*x+1)/(-a^2
*x^2+1)^(1/2))-1/6*arctanh(a*x)^4+1/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/
(-a^2*x^2+1))-1/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+1/4*poly
log(4,-(a*x+1)^2/(-a^2*x^2+1))+1/6*(-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/
(a^2*x^2-1)+1))+I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^
3-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1
)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x
^2-1)+1))^3+2*ln(2))*arctanh(a*x)^3))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{acx+c}\,dx$$

input
```
integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \frac{\int \frac{\operatorname{atanh}^3(ax)}{ax+1}\,dx}{c}$$

input
```
integrate(atanh(a*x)**3/(a*c*x+c),x)
```

output
```
Integral(atanh(a*x)**3/(a*x + 1), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{acx+c}\,dx$$

input
```
integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

output
```
-1/8*log(a*x + 1)*log(-a*x + 1)^3/(a*c) + 1/8*integrate((6*a*x*log(a*x + 1
)*log(-a*x + 1)^2 + (a*x - 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*
log(-a*x + 1))/(a^2*c*x^2 - c), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{acx+c}\,dx$$

input
```
integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(a*c*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{c+a\,c\,x}\,dx$$

input
```
int(atanh(a*x)^3/(c + a*c*x),x)
```

output
```
int(atanh(a*x)^3/(c + a*c*x), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{c+acx}\,dx = \frac{\int \frac{atanh(ax)^3}{ax+1}dx}{c}$$

input
```
int(atanh(a*x)^3/(a*c*x+c),x)
```

output
```
int(atanh(a*x)**3/(a*x + 1),x)/c
```

3.130 $\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx = \frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)}{4c}$$

output
```
arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3/2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1)
)/c-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*polylog(4,-1+2/(a*x+1))
/c
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx = \frac{\pi^4 - 32\operatorname{arctanh}(ax)^4 + 64\operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + 96\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)}{64c}$$

input
```
Integrate[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

output
```
(Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])]
+ 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyL
og[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6494, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(acx+c)}\,dx$$

$$\Big\downarrow \text{6494}$$

$$\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c} - \frac{3a\int \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{6618}$$

$$\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c} - \frac{3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a} - \int \frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \text{6622}$$

$$\frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}\right)}{c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{3a\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)}{c}$$

input
```
Int[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

output
```
(ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a*((ArcTanh[a*x]^2*PolyLog[2,
 -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2
*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a)))/c
```

Defintions of rubi rules used

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 1094, normalized size of antiderivative = 11.76

method	result	size
derivativedivides	Expression too large to display	1094
default	Expression too large to display	1094
parts	Expression too large to display	1481

input
```
int(arctanh(a*x)^3/x/(a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*arctanh(a*x)^3*ln(a*x)-1/c*arctanh(a*x)^3*ln(a*x+1)-3/c*(-2/3*arctanh(
a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*arctanh(a*x)^4-1/6*(I*Pi*csgn(I*
(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*
(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*
(a*x+1)^2/(a^2*x^2-1))-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^
2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x
+1)^2/(a^2*x^2-1)+1))-I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*
x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(-(a*x+1)^
2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a
^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-
1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1)
)*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+I*Pi*csgn(I/(-(a*x
+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)
+1))^2+2*ln(2))*arctanh(a*x)^3+1/3*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1
)-1)-1/3*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)
^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*polylog(2,(a*x+1)/(-...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{(acx+c)x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx = \frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^2+x}\,dx}{c}$$

input
```
integrate(atanh(a*x)**3/x/(a*c*x+c),x)
```

output
```
Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="maxima")
```

output
```
1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1
)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x +
 1)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((a*c*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx=\int \frac{\operatorname{atanh}(a\,x)^3}{x\,\left(c+a\,c\,x\right)}\,dx$$

input
```
int(atanh(a*x)^3/(x*(c + a*c*x)),x)
```

output
```
int(atanh(a*x)^3/(x*(c + a*c*x)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(c+acx)}\,dx=\frac{-atanh(ax)^4-4\left(\int \frac{atanh(ax)^3}{a^2x^3-x}dx\right)}{4c}$$

input
```
int(atanh(a*x)^3/x/(a*c*x+c),x)
```

output
```
( - atanh(a*x)**4 - 4*int(atanh(a*x)**3/(a**2*x**3 - x),x))/(4*c)
```

3.131 $\int \frac{\operatorname{arctanh}(ax)^3}{cx+acx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx+acx^2}\,dx = \frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)}{4c}$$

output
```
arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3/2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1)
)/c-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3/4*polylog(4,-1+2/(a*x+1))
/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx+acx^2}\,dx = \frac{\pi^4-32\operatorname{arctanh}(ax)^4+64\operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)+96\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)}{64c}$$

input
```
Integrate[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

output
```
(Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])]
+ 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyL
og[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2026, 6494, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{acx^2+cx}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(acx+c)}dx$$

$$\Big\downarrow \text{6494}$$

$$\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{3a\int \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{6618}$$

$$\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \text{ 6622}$$

$$\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{3a\left(-\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}\right)}{c}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)}{c}-\frac{3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)}{c}$$

input
```
Int[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

output
```
(ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a*((ArcTanh[a*x]^2*PolyLog[2,
 -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2
*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a)))/c
```

Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 1101, normalized size of antiderivative = 11.84

method	result	size
derivativedivides	Expression too large to display	1101
default	Expression too large to display	1101
parts	Expression too large to display	1481

input
```
int(arctanh(a*x)^3/(a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a/c*arctanh(a*x)^3*ln(a*x)-a/c*arctanh(a*x)^3*ln(a*x+1)-3*a/c*(-2/3*a
rctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*arctanh(a*x)^4-1/6*(I*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*
csgn(I*(a*x+1)^2/(a^2*x^2-1))-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2-I*Pi*csgn(I/(-(a*x+1)^2/
(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/(-(a*x+1)^2/(a^2*x^2-1)+1))-I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(
I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(-(
a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+
1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+I*Pi*csgn(I*(-(a*x+1)^2/(a
^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^
2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+I*Pi*csgn(I
/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2
*x^2-1)+1))^2+2*ln(2))*arctanh(a*x)^3+1/3*arctanh(a*x)^3*ln((a*x+1)^2/(-a^
2*x^2+1)-1)-1/3*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*
x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(3,-(a*x
+1)/(-a^2*x^2+1)^(1/2))-2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arcta
nh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*polylog(2,(a*...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx + acx^2} \, dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{acx^2 + cx} \, dx$$

input
```
integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx + acx^2}\,dx = \frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^2+x}\,dx}{c}$$

input
```
integrate(atanh(a*x)**3/(a*c*x**2+c*x),x)
```

output
```
Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx + acx^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{acx^2 + cx}\,dx$$

input
```
integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="maxima")
```

output
```
1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1
)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x +
 1)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx + acx^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{acx^2 + cx}\,dx$$

input
```
integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx + acx^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{a\,c\,x^2 + c\,x}\,dx$$

input
```
int(atanh(a*x)^3/(c*x + a*c*x^2),x)
```

output
```
int(atanh(a*x)^3/(c*x + a*c*x^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{cx + acx^2}\,dx = \frac{-atanh(ax)^4 - 4\left(\int \frac{atanh(ax)^3}{a^2x^3 - x}dx\right)}{4c}$$

input
```
int(atanh(a*x)^3/(a*c*x^2+c*x),x)
```

output
```
( - atanh(a*x)**4 - 4*int(atanh(a*x)**3/(a**2*x**3 - x),x))/(4*c)
```

3.132 $\int \frac{\operatorname{arctanh}(ax)^3}{x^2(c+acx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 191

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^2(c+acx)}\,dx &= \frac{a\operatorname{arctanh}(ax)^3}{c}-\frac{\operatorname{arctanh}(ax)^3}{cx}+\frac{3a\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c}\\&\quad-\frac{a\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c}\\&\quad-\frac{3a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{c}\\&\quad+\frac{3a\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c}\\&\quad-\frac{3a\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c}\\&\quad+\frac{3a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c}\\&\quad+\frac{3a\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)}{4c}\end{aligned}$$

output
```
a*arctanh(a*x)^3/c-arctanh(a*x)^3/c/x+3*a*arctanh(a*x)^2*ln(2-2/(a*x+1))/c
-a*arctanh(a*x)^3*ln(2-2/(a*x+1))/c-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1
))/c+3/2*a*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3/2*a*polylog(3,-1+2/(
a*x+1))/c+3/2*a*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3/4*a*polylog(4,-1+
2/(a*x+1))/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(c+acx)}\,dx$$
$$= \frac{a\left(\frac{i\pi^3}{8}-\frac{\pi^4}{64}-\operatorname{arctanh}(ax)^3-\frac{\operatorname{arctanh}(ax)^3}{ax}+\frac{1}{2}\operatorname{arctanh}(ax)^4+3\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)-\operatorname{ar}\right.}{}$$

input
```
Integrate[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

output
```
(a*((I/8)*Pi^3 - Pi^4/64 - ArcTanh[a*x]^3 - ArcTanh[a*x]^3/(a*x) + ArcTanh
[a*x]^4/2 + 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^3*
Log[1 - E^(2*ArcTanh[a*x])] - (3*(-2 + ArcTanh[a*x])*ArcTanh[a*x]*PolyLog[
2, E^(2*ArcTanh[a*x])])/2 + (3*(-1 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh
[a*x])])/2 - (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4))/c
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6496, 27, 6452, 6494, 6550, 6494, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(acx+c)}\,dx$$

$$\Big\downarrow \text{ 6496}$$

$$\frac{\int \frac{\operatorname{arctanh}(ax)^3}{x^2}dx}{c}-a\int \frac{\operatorname{arctanh}(ax)^3}{cx(ax+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{x^2}dx}{c}-\frac{a\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{x}}{c}-\frac{a\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx}{c}$$

$$\Big\downarrow \text{6494}$$

$$\frac{3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{x}}{c}-\frac{a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \text{6550}$$

$$\frac{3a\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)-\frac{\text{arctanh}(ax)^3}{x}}{c}-\frac{a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \text{6494}$$

$$\frac{3a\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\text{arctanh}(ax)^3}{x}}{c}-\frac{a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \text{6618}$$

$$\frac{3a\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{}{a}\right.\right.}{c}$$

$$\frac{a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int \frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)\right)}{c}$$

$$\Big\downarrow \text{6622}$$

$$\frac{3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{}{a}\right.\right.}{c}$$

$$\frac{a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)}{}\right.\right.}{c}$$

$\downarrow$ 7164

$$\frac{3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)}{c}$$

$$\frac{a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}(}{4}\right.\right.}{c}$$

input
```
Int[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

output
```
(-(ArcTanh[a*x]^3/x) + 3*a*(ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1
 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + PolyLo
g[3, -1 + 2/(1 + a*x)]/(4*a))))/c - (a*(ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)
] - 3*a*((ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*
x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a
))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.66 (sec) , antiderivative size = 1339, normalized size of antiderivative = 7.01

method	result	size
derivativedivides	Expression too large to display	1339
default	Expression too large to display	1339
parts	Expression too large to display	1717

input
```
int(arctanh(a*x)^3/x^2/(a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c*arctanh(a*x)^3/a/x-1/c*arctanh(a*x)^3*ln(a*x)+1/c*arctanh(a*x)^3*l
n(a*x+1)-3/c*(-1/6*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*ar
ctanh(a*x)^3+1/6*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^
2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1
)+1))*arctanh(a*x)^3-1/6*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^3+1/6*I*Pi*csgn(
I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^
2*x^2-1)+1))^2*arctanh(a*x)^3+2*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*po
lylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2/3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x
^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,(a*x+1)/
(-a^2*x^2+1)^(1/2))+1/3*ln(2)*arctanh(a*x)^3-1/6*arctanh(a*x)^4+1/3*arctan
h(a*x)^3-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a
*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*polylog(2,-(a*x+1
)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*poly
log(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^
2+1)-1)+1/3*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*arctanh(a*
x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^
2+1)^(1/2))-arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/6*I*Pi*cs...
```

Fricas [F]

$$\int \frac{\text{arctanh}(ax)^3}{x^2(c+acx)}\,dx = \int \frac{\text{artanh}\,(ax)^3}{(acx+c)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a*c*x^3 + c*x^2), x)
```

Sympy [F]

$$\int \frac{\text{arctanh}(ax)^3}{x^2(c+acx)}\,dx = \frac{\int \frac{\text{atanh}^3\,(ax)}{ax^3+x^2}\,dx}{c}$$

input
```
integrate(atanh(a*x)**3/x**2/(a*c*x+c),x)
```

output
```
Integral(atanh(a*x)**3/(a*x**3 + x**2), x)/c
```

Maxima [F]

$$\int \frac{\text{arctanh}(ax)^3}{x^2(c+acx)}\,dx = \int \frac{\text{artanh}\,(ax)^3}{(acx+c)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="maxima")
```

output
```
-1/8*(a*x*log(a*x + 1) - 1)*log(-a*x + 1)^3/(c*x) + 1/8*integrate(((a*x -
1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2
+ a*x - (a^3*x^3 + a^2*x^2 + a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^2*
c*x^4 - c*x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(c+acx)}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((a*c*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(c+acx)}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,(c+a\,c\,x)}\,dx$$

input
```
int(atanh(a*x)^3/(x^2*(c + a*c*x)),x)
```

output
```
int(atanh(a*x)^3/(x^2*(c + a*c*x)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(c+acx)}\,dx = \frac{\int \frac{atanh(ax)^3}{a\,x^3+x^2}dx}{c}$$

input
```
int(atanh(a*x)^3/x^2/(a*c*x+c),x)
```

output
```
int(atanh(a*x)**3/(a*x**3 + x**2),x)/c
```

3.133 $\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 305

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx &= \frac{3a^2\operatorname{arctanh}(ax)^2}{2c} - \frac{3a\operatorname{arctanh}(ax)^2}{2cx} - \frac{a^2\operatorname{arctanh}(ax)^3}{2c} \\ &\quad - \frac{\operatorname{arctanh}(ax)^3}{2cx^2} + \frac{a\operatorname{arctanh}(ax)^3}{cx} + \frac{3a^2\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c} \\ &\quad - \frac{3a^2\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} \\ &\quad + \frac{a^2\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3a^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c} \\ &\quad + \frac{3a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{c} \\ &\quad - \frac{3a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{2c} \\ &\quad + \frac{3a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c} \\ &\quad - \frac{3a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{2c} \\ &\quad - \frac{3a^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)}{4c}\end{aligned}$$

output
```
3/2*a^2*arctanh(a*x)^2/c-3/2*a*arctanh(a*x)^2/c/x-1/2*a^2*arctanh(a*x)^3/c
-1/2*arctanh(a*x)^3/c/x^2+a*arctanh(a*x)^3/c/x+3*a^2*arctanh(a*x)*ln(2-2/(
a*x+1))/c-3*a^2*arctanh(a*x)^2*ln(2-2/(a*x+1))/c+a^2*arctanh(a*x)^3*ln(2-2
/(a*x+1))/c-3/2*a^2*polylog(2,-1+2/(a*x+1))/c+3*a^2*arctanh(a*x)*polylog(2
,-1+2/(a*x+1))/c-3/2*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c+3/2*a^2*
polylog(3,-1+2/(a*x+1))/c-3/2*a^2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c-3
/4*a^2*polylog(4,-1+2/(a*x+1))/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \frac{a^2\left(-8i\pi^3+\pi^4+96\operatorname{arctanh}(ax)^2-\frac{96\operatorname{arctanh}(ax)^2}{ax}+96\operatorname{arctanh}(ax)^3-\frac{32\operatorname{arctanh}(ax)^3}{a^2x^2}+\frac{64\operatorname{arctanh}(ax)^3}{ax}\right.}{}$$

input
```
Integrate[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

output
```
(a^2*((-8*I)*Pi^3 + Pi^4 + 96*ArcTanh[a*x]^2 - (96*ArcTanh[a*x]^2)/(a*x) +
 96*ArcTanh[a*x]^3 - (32*ArcTanh[a*x]^3)/(a^2*x^2) + (64*ArcTanh[a*x]^3)/(
a*x) - 32*ArcTanh[a*x]^4 + 192*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] -
 192*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1
- E^(2*ArcTanh[a*x])] - 96*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*(-2 + ArcT
anh[a*x])*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 96*PolyLog[3, E^(2
*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*Poly
Log[4, E^(2*ArcTanh[a*x])]))/(64*c)
```

Rubi [A] (verified)

Time = 3.54 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6496, 27, 6452, 6496, 6452, 6494, 6544, 6452, 6510, 6550, 6494, 2897, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x^3(acx+c)}dx$$

$$\Big\downarrow \text{6496}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{x^3}dx}{c}-a\int \frac{\text{arctanh}(ax)^3}{cx^2(ax+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{x^3}dx}{c}-\frac{a\int \frac{\text{arctanh}(ax)^3}{x^2(ax+1)}dx}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{3}{2}a\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\int \frac{\text{arctanh}(ax)^3}{x^2(ax+1)}dx}{c}$$

$$\Big\downarrow \text{6496}$$

$$\frac{\frac{3}{2}a\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(\int \frac{\text{arctanh}(ax)^3}{x^2}dx-a\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx\right)}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{3}{2}a\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-a\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx-\frac{\text{arctanh}(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{6494}$$

$$\frac{\frac{3}{2}a\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{6544}$$

$$\frac{\frac{3}{2}a\left(a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^2}{x^2}dx\right)-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{3}{2}a\left(a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+2a\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^2}{x}\right)-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{6510}$$

$$\frac{\frac{3}{2}a\left(2a\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}\right)-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx-a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{6550}$$

$$\frac{\frac{3}{2}a\left(2a\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}\right)-\frac{\text{arctanh}(ax)^3}{2x^2}}{c}-\frac{a\left(-a\left(\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)+3a\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax\right.\right.}{c}$$

$$\Big\downarrow \text{6494}$$

$$\frac{\frac{3}{2}a\left(2a\left(-a\int\frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)}{x}\right.}{c}$$

$$\frac{a\left(3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)-a\left(\text{arctanh}(ax)^3\text{lo}\right.\right.}{c}$$

$\Big\downarrow$ 2897

$$\frac{\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}}{x}\right.}{c}$$

$$\frac{a\left(3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)-a\left(\text{arctanh}(ax)^3\text{lo}\right.\right.}{c}$$

$\Big\downarrow$ 6618

$$\frac{\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}}{x}\right.}{c}$$

$$a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.$$

$\Big\downarrow$ 6622

$$\frac{\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}}{x}\right.}{c}$$

$$a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.$$

$\Big\downarrow$ 7164

$$\frac{\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}}{x}\right.}{c}$$

$$a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right.\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

output
```
(-1/2*ArcTanh[a*x]^3/x^2 + (3*a*(-(ArcTanh[a*x]^2/x) + (a*ArcTanh[a*x]^3)/
3 + 2*a*(ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2,
 -1 + 2/(1 + a*x)]/2)))/2)/c - (a*(-(ArcTanh[a*x]^3/x) + 3*a*(ArcTanh[a*x]
^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2,
 -1 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a))) - a*(ArcT
anh[a*x]^3*Log[2 - 2/(1 + a*x)] - 3*a*((ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(
1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*a) + Pol
yLog[4, -1 + 2/(1 + a*x)]/(4*a)))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(287) = 574$.

Time = 8.99 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.97

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)-3ax-\operatorname{arctanh}(ax))(ax-1)}{2c\,a^2x^2}-\frac{\operatorname{arctanh}(ax)^4}{2c}+\frac{\operatorname{arctanh}(ax)^3\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)-3ax-\operatorname{arctanh}(ax))(ax-1)}{2c\,a^2x^2}-\frac{\operatorname{arctanh}(ax)^4}{2c}+\frac{\operatorname{arctanh}(ax)^3\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}\right.$

input
```
int(arctanh(a*x)^3/x^3/(a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/c*arctanh(a*x)^2*(a*x*arctanh(a*x)-3*a*x-arctanh(a*x))*(a*x-1)/a
^2/x^2-1/2/c*arctanh(a*x)^4+1/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+3/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh
(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,(a*x+1)/(-a^2*x^
2+1)^(1/2))+1/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctan
h(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(3
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3
/c*arctanh(a*x)^2+3/c*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^
2+1)^(1/2))+3/c*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2/c*arctanh(a*x)^3-
3/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
-3/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6/c*arctanh(a*x)*poly
log(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a*c*x^4 + c*x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^4+x^3}\,dx}{c}$$

input
```
integrate(atanh(a*x)**3/x**3/(a*c*x+c),x)
```

output
```
Integral(atanh(a*x)**3/(a*x**4 + x**3), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="maxima")
```

output
```
1/16*(2*a^2*x^2*log(a*x + 1) - 2*a*x + 1)*log(-a*x + 1)^3/(c*x^2) - 1/8*in
tegrate(-1/2*(2*(a*x - 1)*log(a*x + 1)^3 - 6*(a*x - 1)*log(a*x + 1)^2*log(
-a*x + 1) + 3*(2*a^3*x^3 + a^2*x^2 - a*x - 2*(a^4*x^4 + a^3*x^3 - a*x + 1)
*log(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x^5 - c*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{(acx+c)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((a*c*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\left(c+a\,c\,x\right)}\,dx$$

input
```
int(atanh(a*x)^3/(x^3*(c + a*c*x)),x)
```

output
```
int(atanh(a*x)^3/(x^3*(c + a*c*x)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(c+acx)}\,dx = \frac{\int \frac{atanh(ax)^3}{a\,x^4+x^3}dx}{c}$$

input
```
int(atanh(a*x)^3/x^3/(a*c*x+c),x)
```

output
```
int(atanh(a*x)**3/(a*x**4 + x**3),x)/c
```

3.134 $\int \frac{x^2 \mathbf{arctanh}(ax)^4}{c-acx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 384

$$\int \frac{x^2\text{arctanh}(ax)^4}{c-acx}\,dx = -\frac{2\text{arctanh}(ax)^3}{a^3c} - \frac{2x\text{arctanh}(ax)^3}{a^2c} - \frac{\text{arctanh}(ax)^4}{2a^3c} - \frac{x\text{arctanh}(ax)^4}{a^2c} - \frac{x^2\text{arctanh}(ax)^4}{2ac} + \frac{6\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3c} + \frac{4\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^3c} + \frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^3c} + \frac{6\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c} + \frac{6\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c} + \frac{2\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c} - \frac{3\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^3c} - \frac{6\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^3c} - \frac{3\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^3c} + \frac{3\,\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^3c} + \frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^3c} - \frac{3\,\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^3c}$$

output

```
-2*arctanh(a*x)^3/a^3/c-2*x*arctanh(a*x)^3/a^2/c-1/2*arctanh(a*x)^4/a^3/c-
x*arctanh(a*x)^4/a^2/c-1/2*x^2*arctanh(a*x)^4/a/c+6*arctanh(a*x)^2*ln(2/(-
a*x+1))/a^3/c+4*arctanh(a*x)^3*ln(2/(-a*x+1))/a^3/c+arctanh(a*x)^4*ln(2/(-
a*x+1))/a^3/c+6*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^3/c+6*arctanh(a*x)^
2*polylog(2,1-2/(-a*x+1))/a^3/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a
^3/c-3*polylog(3,1-2/(-a*x+1))/a^3/c-6*arctanh(a*x)*polylog(3,1-2/(-a*x+1)
)/a^3/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a^3/c+3*polylog(4,1-2/(-a
*x+1))/a^3/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+1))/a^3/c-3/2*polylog(5,1-
2/(-a*x+1))/a^3/c
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.61

$$\int \frac{x^2\text{arctanh}(ax)^4}{c-acx}\,dx = -\frac{-2\text{arctanh}(ax)^3+2ax\text{arctanh}(ax)^3-\text{arctanh}(ax)^4+ax\text{arctanh}(ax)^4-\frac{1}{2}(1-a^2x^2)\,\text{arctanh}(ax)^4-}{}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

output
```
-((-2*ArcTanh[a*x]^3 + 2*a*x*ArcTanh[a*x]^3 - ArcTanh[a*x]^4 + a*x*ArcTanh
[a*x]^4 - ((1 - a^2*x^2)*ArcTanh[a*x]^4)/2 - (2*ArcTanh[a*x]^5)/5 - 6*ArcT
anh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*A
rcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*ArcTanh[a*
x]*(3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, -E^(-2*ArcTanh[a*x])]
 + 3*(1 + ArcTanh[a*x])^2*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -
E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])] + (
3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a^3*c))
```

Rubi [A] (verified)

Time = 3.96 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {6492, 27, 6452, 6492, 6436, 6470, 6542, 6436, 6510, 6546, 6470, 6620, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^4}{c-acx}\,dx$$

$$\Big\downarrow \; 6492$$

$$\frac{\int \frac{x\text{arctanh}(ax)^4}{c(1-ax)}dx}{a}-\frac{\int x\text{arctanh}(ax)^4dx}{ac}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x\text{arctanh}(ax)^4}{1-ax}dx}{ac}-\frac{\int x\text{arctanh}(ax)^4dx}{ac}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^4}{1-ax}dx}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}dx}{ac}$$

$$\Big\downarrow \text{6492}$$

$$\frac{\frac{\int \frac{\text{arctanh}(ax)^4}{1-ax}dx}{a}-\frac{\int \text{arctanh}(ax)^4dx}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}dx}{ac}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\frac{\int \frac{\text{arctanh}(ax)^4}{1-ax}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}dx}{ac}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}dx}{ac}$$

$$\Big\downarrow \text{6542}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)^3dx}{a^2}\right)}{ac}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}\right)}{ac}$$

$$\Big\downarrow \text{6510}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\int\frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}\right)}{ac}$$

$\downarrow$ 6546

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\left(\frac{\int\frac{\text{arctanh}(ax)^3}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}\right)}{a}}{ac}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\int\frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a^2}\right)}{ac}$$

$\downarrow$ 6470

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{x\text{arctanh}(ax)^4-4a\left(\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-a\right.}}{1-a^2x^2}}{a}\right.}{a}}{ac}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctar}}{3}\right.}{a^2}\right.}{ac}$$

$\downarrow$ 6620

$$\frac{\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\int\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{x\text{arctanh}(ax)^4-4a\left(\frac{\text{ar}\ldots}{\ldots}\right.}{ac}}{\ldots}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}\ldots}{\ldots}\right.}{a}\right.}{a^2}\right.}{ac}$$

$\downarrow$ 6624

$$\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,1-\ldots\right.}{2a}\right.}{a}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}\ldots}{\ldots}\right.}{a}\right.}{a^2}\right.}{ac}$$

$\downarrow$ 6624

$$\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{2a}\right)-\frac{\text{arct}\ldots}{\ldots}\right.}{a}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}\ldots}{\ldots}\right.}{a}\right.}{a^2}\right.}{ac}$$

$\downarrow$ 7164

$$\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{arctanh}(ax)\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{4a}\right)-\frac{\text{arctanh}(}{}\right.}{a}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^4-2a\left(\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\text{P}}{}\right.}{a}\right.}{a^2}\right.}{ac}$$

input
```
Int[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

output
```
-(((x^2*ArcTanh[a*x]^4)/2 - 2*a*(ArcTanh[a*x]^4/(4*a^3) - (x*ArcTanh[a*x]^
3 - 3*a*(-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a -
2*(-1/2*(ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1
 - a*x)]/(4*a)))/a))/a^2))/(a*c)) + (-((x*ArcTanh[a*x]^4 - 4*a*(-1/4*ArcTa
nh[a*x]^4/a^2 + ((ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a - 3*(-1/2*(ArcTanh[a*
x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/a + (ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 -
a*x)])/(2*a) - PolyLog[4, 1 - 2/(1 - a*x)]/(4*a)))/a))/a) + ((ArcTanh[a*x]
^4*Log[2/(1 - a*x)])/a - 4*(-1/2*(ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x
)])/a + (3*((ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a) - (ArcTanh[
a*x]*PolyLog[4, 1 - 2/(1 - a*x)])/(2*a) + PolyLog[5, 1 - 2/(1 - a*x)]/(4*a
)))/2))/a)/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^3(ax\operatorname{arctanh}(ax)+3\operatorname{arctanh}(ax)+4)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}+\frac{2\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{c}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^3(ax\operatorname{arctanh}(ax)+3\operatorname{arctanh}(ax)+4)(ax-1)}{2c}+\frac{\operatorname{arctanh}(ax)^4\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)}{c}+\frac{2\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{c}}{}$

input
```
int(x^2*arctanh(a*x)^4/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/2/c*arctanh(a*x)^3*(a*x*arctanh(a*x)+3*arctanh(a*x)+4)*(a*x-1)+1
/c*arctanh(a*x)^4*ln((a*x+1)^2/(-a^2*x^2+1)+1)+2/c*arctanh(a*x)^3*polylog(
2,-(a*x+1)^2/(-a^2*x^2+1))-3/c*arctanh(a*x)^2*polylog(3,-(a*x+1)^2/(-a^2*x
^2+1))+3/c*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/2/c*polylog(5
,-(a*x+1)^2/(-a^2*x^2+1))-4/c*arctanh(a*x)^3+6/c*arctanh(a*x)^2*ln((a*x+1)
^2/(-a^2*x^2+1)+1)+6/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/c
*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-2/c*arctanh(a*x)^4+4/c*arctanh(a*x)^3*
ln((a*x+1)^2/(-a^2*x^2+1)+1)+6/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2
*x^2+1))-6/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/c*polylog(4
,-(a*x+1)^2/(-a^2*x^2+1)))
```

Fricas [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)^4}{c-acx}\,dx=\int -\frac{x^2\operatorname{artanh}(ax)^4}{acx-c}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{x^2 \text{arctanh}(ax)^4}{c-acx}\,dx = -\frac{\int \frac{x^2 \operatorname{atanh}^4(ax)}{ax-1}\,dx}{c}$$

input
```
integrate(x**2*atanh(a*x)**4/(-a*c*x+c),x)
```

output
```
-Integral(x**2*atanh(a*x)**4/(a*x - 1), x)/c
```

Maxima [F]

$$\int \frac{x^2 \text{arctanh}(ax)^4}{c-acx}\,dx = \int -\frac{x^2 \operatorname{artanh}(ax)^4}{acx-c}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/320*(4*log(-a*x + 1)^5 + 5*(2*log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 6*l
og(-a*x + 1)^2 - 6*log(-a*x + 1) + 3)*(a*x - 1)^2 + 40*(log(-a*x + 1)^4 -
4*log(-a*x + 1)^3 + 12*log(-a*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))
/(a^3*c) + 1/16*integrate(-(x^2*log(a*x + 1)^4 - 4*x^2*log(a*x + 1)^3*log(
-a*x + 1) + 6*x^2*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*x^2*log(a*x + 1)*log(
-a*x + 1)^3)/(a*c*x - c), x)
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)^4}{c-acx}\,dx = \int -\frac{x^2 \operatorname{artanh}(ax)^4}{acx-c}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arctanh}(ax)^4}{c-acx}\,dx=\int \frac{x^2\,\mathrm{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

input
```
int((x^2*atanh(a*x)^4)/(c - a*c*x),x)
```

output
```
int((x^2*atanh(a*x)^4)/(c - a*c*x), x)
```

Reduce [F]

$$\int \frac{x^2\text{arctanh}(ax)^4}{c-acx}\,dx=-\frac{\int \frac{atanh(ax)^4x^2}{ax-1}dx}{c}$$

input
```
int(x^2*atanh(a*x)^4/(-a*c*x+c),x)
```

output
```
( - int((atanh(a*x)**4*x**2)/(a*x - 1),x))/c
```

3.135 $\int \frac{x\textbf{arctanh}(ax)^4}{c-acx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 261

$$\begin{aligned}\int \frac{x\operatorname{arctanh}(ax)^4}{c-acx}\,dx = & -\frac{\operatorname{arctanh}(ax)^4}{a^2c}-\frac{x\operatorname{arctanh}(ax)^4}{ac} \\ & +\frac{4\operatorname{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c}+\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} \\ & +\frac{6\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2c} \\ & +\frac{2\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2c} \\ & -\frac{6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^2c} \\ & -\frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^2c} \\ & +\frac{3\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^2c}+\frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^2c} \\ & -\frac{3\operatorname{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^2c}\end{aligned}$$

output
```
-arctanh(a*x)^4/a^2/c-x*arctanh(a*x)^4/a/c+4*arctanh(a*x)^3*ln(2/(-a*x+1))
/a^2/c+arctanh(a*x)^4*ln(2/(-a*x+1))/a^2/c+6*arctanh(a*x)^2*polylog(2,1-2/
(-a*x+1))/a^2/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))/a^2/c-6*arctanh(a
*x)*polylog(3,1-2/(-a*x+1))/a^2/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))
/a^2/c+3*polylog(4,1-2/(-a*x+1))/a^2/c+3*arctanh(a*x)*polylog(4,1-2/(-a*x+
1))/a^2/c-3/2*polylog(5,1-2/(-a*x+1))/a^2/c
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.66

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx = -\frac{-\text{arctanh}(ax)^4+ax\text{arctanh}(ax)^4-\frac{2}{5}\text{arctanh}(ax)^5-4\text{arctanh}(ax)^3\log\left(1+e^{-2\text{arctanh}(ax)}\right)-\text{arctanl}}{}$$

input
```
Integrate[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

output
```
-((-ArcTanh[a*x]^4 + a*x*ArcTanh[a*x]^4 - (2*ArcTanh[a*x]^5)/5 - 4*ArcTanh
[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTan
h[a*x])] + 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, -E^(-2*ArcTanh[a
*x])] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])]
 + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*A
rcTanh[a*x])] + (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a^2*c))
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {6492, 27, 6436, 6470, 6546, 6470, 6620, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx$$

$$\Big\downarrow \text{6492}$$

$$\frac{\int \frac{\text{arctanh}(ax)^4}{c(1-ax)}dx}{a}-\frac{\int \text{arctanh}(ax)^4dx}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{arctanh}(ax)^4}{1-ax}dx}{ac}-\frac{\int \text{arctanh}(ax)^4dx}{ac}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\int \frac{\text{arctanh}(ax)^4}{1-ax}dx}{ac}-\frac{x\text{arctanh}(ax)^4-4a\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{ac}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{ac}-\frac{x\text{arctanh}(ax)^4-4a\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{ac}$$

$$\Big\downarrow \text{6546}$$

$$\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{ac}-\frac{x\text{arctanh}(ax)^4-4a\left(\frac{\int \frac{\text{arctanh}(ax)^3}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}\right)}{ac}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{ac}-\frac{x\text{arctanh}(ax)^4-4a\left(\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}\right)}{ac}$$

$$\Big\downarrow \text{6620}$$

$$\frac{\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\int\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{ac}-$$

$$\frac{x\operatorname{arctanh}(ax)^4-4a\left(\frac{\frac{\operatorname{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-a\right.}{ac}$$

$\downarrow$ 6624

$$\frac{\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\operatorname{arctanh}(a}{}\right.}{ac}$$

$$\frac{x\operatorname{arctanh}(ax)^4-4a\left(\frac{\frac{\operatorname{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{Po}}{}\right.}{a}\right.}{ac}$$

$\downarrow$ 6624

$$\frac{\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,1\right.}{2a}\right.\right.}{ac}$$

$$\frac{x\operatorname{arctanh}(ax)^4-4a\left(\frac{\frac{\operatorname{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{Po}}{}\right.}{a}\right.}{ac}$$

$\downarrow$ 7164

$$\frac{\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a}-4\left(\frac{3}{2}\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{2a}+\frac{\operatorname{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{4a}\right.\right.}{ac}$$

$$\frac{x\operatorname{arctanh}(ax)^4-4a\left(\frac{\frac{\operatorname{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\operatorname{PolyLog}\left(\right.}{}\right.}{a}\right.}{ac}$$

input
```
Int[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

output
```
-((x*ArcTanh[a*x]^4 - 4*a*(-1/4*ArcTanh[a*x]^4/a^2 + ((ArcTanh[a*x]^3*Log[
2/(1 - a*x)])/a - 3*(-1/2*(ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/a +
 (ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a) - PolyLog[4, 1 - 2/(1 -
a*x)]/(4*a)))/a))/(a*c)) + ((ArcTanh[a*x]^4*Log[2/(1 - a*x)])/a - 4*(-1/2*
(ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x)])/a + (3*((ArcTanh[a*x]^2*PolyL
og[3, 1 - 2/(1 - a*x)])/(2*a) - (ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 - a*x)])
/(2*a) + PolyLog[5, 1 - 2/(1 - a*x)]/(4*a)))/2))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6492
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])
^p, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTanh[c*x])^p/(d +
e*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2
- e^2, 0] && GtQ[m, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^4 ax}{c}-\frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{c}+\frac{2\operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)-3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right)+3}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^4 ax}{c}-\frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{c}+\frac{2\operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)-3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(3,-\frac{(ax+1)^2}{-a^2x^2+1}\right)+3}{}}{}$
parts	$-\frac{x\operatorname{arctanh}(ax)^4}{ac}-\frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{c\,a^2}+\frac{4a\left(\frac{\operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)}{2a^3}-\frac{3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(3,-\right.}{4a^3}\right.}{}$

input
```
int(x*arctanh(a*x)^4/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/c*arctanh(a*x)^4*a*x-1/c*arctanh(a*x)^4*ln(a*x-1)+4/c*(1/2*arcta
nh(a*x)^3*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/4*arctanh(a*x)^2*polylog(3,
-(a*x+1)^2/(-a^2*x^2+1))+3/4*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1
))-3/8*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))+1/4*I*Pi*arctanh(a*x)^4-1/4*I*Pi
*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^4+1/4*I*Pi*csgn(I/(-(a*
x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^4+1/4*ln(2)*arctanh(a*x)^4-1/4*arcta
nh(a*x)^4+3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+arctanh(a*x)^3*ln((a*x+1)
^2/(-a^2*x^2+1)+1)+3/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3
/2*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))))
```

Fricas [F]

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx=\int -\frac{x\,\text{artanh}\,(ax)^4}{acx-c}\,dx$$

input
```
integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx=-\frac{\int \frac{x\,\text{atanh}^4\,(ax)}{ax-1}\,dx}{c}$$

input
```
integrate(x*atanh(a*x)**4/(-a*c*x+c),x)
```

output
```
-Integral(x*atanh(a*x)**4/(a*x - 1), x)/c
```

Maxima [F]

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx = \int -\frac{x\,\text{artanh}\,(ax)^4}{acx-c}\,dx$$

input
```
integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/80*(log(-a*x + 1)^5 + 5*(log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 12*log(-
a*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a^2*c) + 1/16*integrate(-(
x*log(a*x + 1)^4 - 4*x*log(a*x + 1)^3*log(-a*x + 1) + 6*x*log(a*x + 1)^2*l
og(-a*x + 1)^2 - 4*x*log(a*x + 1)*log(-a*x + 1)^3)/(a*c*x - c), x)
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx = \int -\frac{x\,\text{artanh}\,(ax)^4}{acx-c}\,dx$$

input
```
integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx = \int \frac{x\,\text{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

input
```
int((x*atanh(a*x)^4)/(c - a*c*x),x)
```

output
```
int((x*atanh(a*x)^4)/(c - a*c*x), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^4}{c-acx}\,dx = -\frac{\int \frac{atanh(ax)^4x}{ax-1}dx}{c}$$

input
```
int(x*atanh(a*x)^4/(-a*c*x+c),x)
```

output
```
( - int((atanh(a*x)**4*x)/(a*x - 1),x))/c
```

3.136 $\int \frac{\text{arctanh}(ax)^4}{c-acx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{\text{arctanh}(ax)^4}{c-acx}\,dx = \frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{ac} - \frac{3\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{ac} + \frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{ac} - \frac{3\,\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2ac}$$

output
```
arctanh(a*x)^4*ln(2/(-a*x+1))/a/c+2*arctanh(a*x)^3*polylog(2,1-2/(-a*x+1))
/a/c-3*arctanh(a*x)^2*polylog(3,1-2/(-a*x+1))/a/c+3*arctanh(a*x)*polylog(4
,1-2/(-a*x+1))/a/c-3/2*polylog(5,1-2/(-a*x+1))/a/c
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\text{arctanh}(ax)^4}{c-acx}\,dx = -\frac{-\frac{2}{5}\text{arctanh}(ax)^5 - \text{arctanh}(ax)^4\log\left(1+e^{-2\text{arctanh}(ax)}\right) + 2\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right)}{}$$

input
```
Integrate[ArcTanh[a*x]^4/(c - a*c*x),x]
```

output
```
-(((-2*ArcTanh[a*x]^5)/5 - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2
*ArcTanh[a*x]^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]^2*PolyLo
g[3, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x]
)] + (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a*c))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6470, 6620, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^4}{c-acx}dx$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac}-\frac{4\int \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{ 6620}$$

$$\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac}-\frac{4\left(\frac{3}{2}\int \frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{c}$$

$$\Big\downarrow \text{ 6624}$$

$$\frac{\text{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac}-\frac{4\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\int \frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{c}$$

$\big\downarrow$ 6624

$$\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac}-$$

$$\frac{4\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{2a}\right)-\frac{\operatorname{arctanh}(ax)^3}{}\right.}{c}$$

$\big\downarrow$ 7164

$$\frac{\operatorname{arctanh}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac}-$$

$$\frac{4\left(\frac{3}{2}\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{2a}+\frac{\operatorname{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{4a}\right)-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLog}}{2a}\right.}{c}$$

input
```
Int[ArcTanh[a*x]^4/(c - a*c*x),x]
```

output
```
(ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a*c) - (4*(-1/2*(ArcTanh[a*x]^3*PolyLog
[2, 1 - 2/(1 - a*x)])/a + (3*((ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])
/(2*a) - (ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 - a*x)])/(2*a) + PolyLog[5, 1 -
 2/(1 - a*x)]/(4*a)))/2))/c
```

Defintions of rubi rules used

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{\left(-i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2+i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3+i\pi+\ln(2)\right)\operatorname{arctanh}(ax)^4+2\operatorname{arctanh}(ax)^3\,\mathrm{po}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{\left(-i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2+i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3+i\pi+\ln(2)\right)\operatorname{arctanh}(ax)^4+2\operatorname{arctanh}(ax)^3\,\mathrm{po}}{}$
parts	$-\frac{\ln(ax-1)\operatorname{arctanh}(ax)^4}{ac}+\frac{\frac{\left(-i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2+i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3+i\pi+\ln(2)\right)\operatorname{arctanh}(ax)^4}{a}+\frac{2\operatorname{arctanh}(}{}}{}$

input
```
int(arctanh(a*x)^4/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/c*arctanh(a*x)^4*ln(a*x-1)+4/c*(1/4*(-I*Pi*csgn(I/(-(a*x+1)^2/(a^2
*x^2-1)+1))^2+I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+I*Pi+ln(2))*arctan
h(a*x)^4+1/2*arctanh(a*x)^3*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/4*arctanh
(a*x)^2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4*arctanh(a*x)*polylog(4,-(a*
x+1)^2/(-a^2*x^2+1))-3/8*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{c - acx}\,dx = \int -\frac{\operatorname{artanh}\left(ax\right)^4}{acx - c}\,dx$$

input
```
integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{c - acx}\,dx = -\frac{\int \frac{\operatorname{atanh}^4\left(ax\right)}{ax-1}\,dx}{c}$$

input
```
integrate(atanh(a*x)**4/(-a*c*x+c),x)
```

output
```
-Integral(atanh(a*x)**4/(a*x - 1), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{c - acx}\,dx = \int -\frac{\operatorname{artanh}\left(ax\right)^4}{acx - c}\,dx$$

input
```
integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/80*log(-a*x + 1)^5/(a*c) + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x
+ 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*l
og(-a*x + 1)^3)/(a*c*x - c), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{c-acx}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{acx-c}\,dx$$

input
```
integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^4/(a*c*x - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^4}{c-acx}\,dx = \int \frac{\operatorname{atanh}(a\,x)^4}{c-a\,c\,x}\,dx$$

input
```
int(atanh(a*x)^4/(c - a*c*x),x)
```

output
```
int(atanh(a*x)^4/(c - a*c*x), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{c-acx}\,dx = -\frac{\int \frac{atanh(ax)^4}{ax-1}dx}{c}$$

input
```
int(atanh(a*x)^4/(-a*c*x+c),x)
```

output
```
( - int(atanh(a*x)**4/(a*x - 1),x))/c
```

3.137 $\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)} dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)}\,dx = \frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c} - \frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{c} + \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,-1+\frac{2}{1-ax}\right)}{c} - \frac{3\operatorname{PolyLog}\left(5,-1+\frac{2}{1-ax}\right)}{2c}$$

output
```
arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1)
)/c-3*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c+3*arctanh(a*x)*polylog(4,-
1+2/(-a*x+1))/c-3/2*polylog(5,-1+2/(-a*x+1))/c
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)}\,dx = \frac{\operatorname{arctanh}(ax)^4\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)}{c} + \frac{2\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)}{c} - \frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)}{c} + \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,e^{2\operatorname{arctanh}(ax)}\right)}{c} - \frac{3\operatorname{PolyLog}\left(5,e^{2\operatorname{arctanh}(ax)}\right)}{2c}$$

input
```
Integrate[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

output
```
(ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog
[2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x
])])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5,
 E^(2*ArcTanh[a*x])])/(2*c)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6494, 6620, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)}\,dx$$

$$\Big\downarrow \text{6494}$$

$$\frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{4a\int \frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{c}$$

$\Big\downarrow$ 6620

$$\frac{\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\int\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)}{c}$$

$\Big\downarrow$ 6624

$$\frac{\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)}{c}$$

$\Big\downarrow$ 6624

$$\frac{\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{2a}\right)-\text{arctanh}(ax)}{c}$$

$\Big\downarrow$ 7164

$$\frac{\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{2a}+\frac{\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{4a}\right)-\frac{\text{arctanh}(ax)^3\,\text{PolyLo}}{2a}\right.}{c}$$

input
```
Int[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

output
```
(ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c - (4*a*(-1/2*(ArcTanh[a*x]^3*PolyL
og[2, -1 + 2/(1 - a*x)])/a + (3*((ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*
x)])/(2*a) - (ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/(2*a) + PolyLog[5
, -1 + 2/(1 - a*x)]/(4*a)))/2))/c
```

Defintions of rubi rules used

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.15 (sec) , antiderivative size = 754, normalized size of antiderivative = 6.39

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^4 \ln(ax)}{c} - \frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{c} + \frac{\left(-2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2 + 2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3 + i\pi \operatorname{csgn}\left(\right.\right.}{}$
default	$\frac{\operatorname{arctanh}(ax)^4 \ln(ax)}{c} - \frac{\operatorname{arctanh}(ax)^4 \ln(ax-1)}{c} + \frac{\left(-2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2 + 2i\pi \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3 + i\pi \operatorname{csgn}\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int(arctanh(a*x)^4/x/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*arctanh(a*x)^4*ln(a*x)-1/c*arctanh(a*x)^4*ln(a*x-1)+4/c*(1/8*(-2*I*Pi*
csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)
+1))^3+I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2
-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))-I*Pi
*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a
*x+1)^2/(a^2*x^2-1)+1))^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(
-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(-(a*x
+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+2*I*Pi+2*ln(2))*arctanh
(a*x)^4-1/4*arctanh(a*x)^4*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/4*arctanh(a*x)^4
*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*polylog(2,-(a*x+1)/(-a^2*
x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*ar
ctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(5,-(a*x+1)/(-a
^2*x^2+1)^(1/2))+1/4*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arcta
nh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(3
,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)
^(1/2))-6*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)}\,dx = \frac{\log\left(\frac{2\,ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 4\,\mathrm{Li}_2\left(-\frac{2\,ax}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 - 12\,\log\left(-\frac{ax+1}{ax-1}\right)^2\mathrm{polylog}\left(3,-\frac{ax+1}{ax-1}\right) + 24\,\log}{16\,c}$$

input
```
integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="fricas")
```

output
```
1/16*(log(2*a*x/(a*x - 1))*log(-(a*x + 1)/(a*x - 1))^4 + 4*dilog(-2*a*x/(a
*x - 1) + 1)*log(-(a*x + 1)/(a*x - 1))^3 - 12*log(-(a*x + 1)/(a*x - 1))^2*
polylog(3, -(a*x + 1)/(a*x - 1)) + 24*log(-(a*x + 1)/(a*x - 1))*polylog(4,
 -(a*x + 1)/(a*x - 1)) - 24*polylog(5, -(a*x + 1)/(a*x - 1)))/c
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)}\,dx = -\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^2-x}\,dx}{c}$$

input
```
integrate(atanh(a*x)**4/x/(-a*c*x+c),x)
```

output
```
-Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c-acx)}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x}\,dx$$

input
```
integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)
^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-
a*x + 1)^3)/(a*c*x^2 - c*x), x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)^4}{x(c-acx)}\,dx = \int -\frac{\text{artanh}\,(ax)^4}{(acx-c)x}\,dx$$

input
```
integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^4/((a*c*x - c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)^4}{x(c-acx)}\,dx = \int \frac{\text{atanh}(a\,x)^4}{x\,(c-a\,c\,x)}\,dx$$

input
```
int(atanh(a*x)^4/(x*(c - a*c*x)),x)
```

output
```
int(atanh(a*x)^4/(x*(c - a*c*x)), x)
```

Reduce [F]

$$\int \frac{\text{arctanh}(ax)^4}{x(c-acx)}\,dx = \frac{atanh(ax)^5 - 5\left(\int \frac{atanh(ax)^4}{a^2x^3-x}dx\right)}{5c}$$

input
```
int(atanh(a*x)^4/x/(-a*c*x+c),x)
```

output
```
(atanh(a*x)**5 - 5*int(atanh(a*x)**4/(a**2*x**3 - x),x))/(5*c)
```

3.138 $\int \frac{\operatorname{arctanh}(ax)^4}{cx-acx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx-acx^2}\,dx = \frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c} - \frac{3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{c} + \frac{3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,-1+\frac{2}{1-ax}\right)}{c} - \frac{3\operatorname{PolyLog}\left(5,-1+\frac{2}{1-ax}\right)}{2c}$$

output
```
arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1)
)/c-3*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c+3*arctanh(a*x)*polylog(4,-
1+2/(-a*x+1))/c-3/2*polylog(5,-1+2/(-a*x+1))/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx - acx^2}\,dx = \frac{\operatorname{arctanh}(ax)^4 \log\left(1 - e^{2\operatorname{arctanh}(ax)}\right)}{c} + \frac{2\operatorname{arctanh}(ax)^3 \operatorname{PolyLog}\left(2, e^{2\operatorname{arctanh}(ax)}\right)}{c} - \frac{3\operatorname{arctanh}(ax)^2 \operatorname{PolyLog}\left(3, e^{2\operatorname{arctanh}(ax)}\right)}{c} + \frac{3\operatorname{arctanh}(ax) \operatorname{PolyLog}\left(4, e^{2\operatorname{arctanh}(ax)}\right)}{c} - \frac{3 \operatorname{PolyLog}\left(5, e^{2\operatorname{arctanh}(ax)}\right)}{2c}$$

input
```
Integrate[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

output
```
(ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog
[2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x
])])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5,
 E^(2*ArcTanh[a*x])])/(2*c)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2026, 6494, 6620, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx - acx^2}\,dx$$

$$\Big\downarrow \; 2026$$

$$\int \frac{\operatorname{arctanh}(ax)^4}{x(c - acx)}dx$$

$$\Big\downarrow \; 6494$$

$$\frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\int\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \; 6620$$

$$\frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\int\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)}{c}$$

$$\Big\downarrow \; 6624$$

$$\frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx\right)-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)}{c}$$

$$\Big\downarrow \; 6624$$

$$\frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{2a}\right)-\frac{\operatorname{arctanh}(ax)}{}\right.}{c}$$

$$\Big\downarrow \; 7164$$

$$\frac{\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{4a\left(\frac{3}{2}\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{4a}\right)-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLo}}{2a}\right.}{c}$$

input
```
Int[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

output
```
(ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c - (4*a*(-1/2*(ArcTanh[a*x]^3*PolyL
og[2, -1 + 2/(1 - a*x)])/a + (3*((ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*
x)])/(2*a) - (ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/(2*a) + PolyLog[5
, -1 + 2/(1 - a*x)]/(4*a)))/2))/c
```

Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 761, normalized size of antiderivative = 6.45

method	result
derivativedivides	$\frac{\frac{a\,\mathrm{arctanh}(ax)^4\ln(ax)}{c}-\frac{a\,\mathrm{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{4a\left(\left(-2i\pi\,\mathrm{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2+2i\pi\,\mathrm{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3+i\pi\,\mathrm{csgn}\left(i\left(-\right.\right.\right.\right.}{}}{}$
default	$\frac{\frac{a\,\mathrm{arctanh}(ax)^4\ln(ax)}{c}-\frac{a\,\mathrm{arctanh}(ax)^4\ln(ax-1)}{c}+\frac{4a\left(\left(-2i\pi\,\mathrm{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^2+2i\pi\,\mathrm{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right)^3+i\pi\,\mathrm{csgn}\left(i\left(-\right.\right.\right.\right.}{}}{}$
parts	Expression too large to display

input
```
int(arctanh(a*x)^4/(-a*c*x^2+c*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a/c*arctanh(a*x)^4*ln(a*x)-a/c*arctanh(a*x)^4*ln(a*x-1)+4*a/c*(1/8*(-
2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2
*x^2-1)+1))^3+I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(
a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1
))-I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-
1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*c
sgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I
*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+2*I*Pi+2*ln(2))*
arctanh(a*x)^4-1/4*arctanh(a*x)^4*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/4*arctanh
(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2
))+6*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(5,-(a*x
+1)/(-a^2*x^2+1)^(1/2))+1/4*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2)
)+arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*po
lylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(4,(a*x+1)/(-a^2
*x^2+1)^(1/2))-6*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx - acx^2} \, dx = \frac{\log\left(\frac{2\,ax}{ax-1}\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 4\,\mathrm{Li}_2\left(-\frac{2\,ax}{ax-1}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 - 12\,\log\left(-\frac{ax+1}{ax-1}\right)^2\mathrm{polylog}\left(3,-\frac{ax+1}{ax-1}\right) + 24\,\log}{16\,c}$$

input
```
integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="fricas")
```

output
```
1/16*(log(2*a*x/(a*x - 1))*log(-(a*x + 1)/(a*x - 1))^4 + 4*dilog(-2*a*x/(a
*x - 1) + 1)*log(-(a*x + 1)/(a*x - 1))^3 - 12*log(-(a*x + 1)/(a*x - 1))^2*
polylog(3, -(a*x + 1)/(a*x - 1)) + 24*log(-(a*x + 1)/(a*x - 1))*polylog(4,
 -(a*x + 1)/(a*x - 1)) - 24*polylog(5, -(a*x + 1)/(a*x - 1)))/c
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx - acx^2} \, dx = -\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^2-x}\,dx}{c}$$

input
```
integrate(atanh(a*x)**4/(-a*c*x**2+c*x),x)
```

output
```
-Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx - acx^2} \, dx = \int -\frac{\operatorname{artanh}(ax)^4}{acx^2 - cx} \, dx$$

input
```
integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="maxima")
```

output
```
-1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)
^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-
a*x + 1)^3)/(a*c*x^2 - c*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx-acx^2}\,dx=\int -\frac{\operatorname{artanh}(ax)^4}{acx^2-cx}\,dx$$

input
```
integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx-acx^2}\,dx=\int \frac{\operatorname{atanh}(a\,x)^4}{c\,x-a\,c\,x^2}\,dx$$

input
```
int(atanh(a*x)^4/(c*x - a*c*x^2),x)
```

output
```
int(atanh(a*x)^4/(c*x - a*c*x^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{cx-acx^2}\,dx=\frac{atanh(ax)^5-5\left(\int \frac{atanh(ax)^4}{a^2x^3-x}dx\right)}{5c}$$

input
```
int(atanh(a*x)^4/(-a*c*x^2+c*x),x)
```

output
```
(atanh(a*x)**5 - 5*int(atanh(a*x)**4/(a**2*x**3 - x),x))/(5*c)
```

3.139 $\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx &= \frac{a\operatorname{arctanh}(ax)^4}{c} - \frac{\operatorname{arctanh}(ax)^4}{cx} + \frac{a\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} \\ &\quad + \frac{4a\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} \\ &\quad + \frac{2a\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c} \\ &\quad - \frac{6a\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{c} \\ &\quad - \frac{3a\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{c} \\ &\quad - \frac{6a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{c} \\ &\quad + \frac{3a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,-1+\frac{2}{1-ax}\right)}{c} \\ &\quad - \frac{3a\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)}{c} - \frac{3a\operatorname{PolyLog}\left(5,-1+\frac{2}{1-ax}\right)}{2c}\end{aligned}$$

output
```
a*arctanh(a*x)^4/c-arctanh(a*x)^4/c/x+a*arctanh(a*x)^4*ln(2-2/(-a*x+1))/c+
4*a*arctanh(a*x)^3*ln(2-2/(a*x+1))/c+2*a*arctanh(a*x)^3*polylog(2,-1+2/(-a
*x+1))/c-6*a*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))/c-3*a*arctanh(a*x)^2*p
olylog(3,-1+2/(-a*x+1))/c-6*a*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3*a*a
rctanh(a*x)*polylog(4,-1+2/(-a*x+1))/c-3*a*polylog(4,-1+2/(a*x+1))/c-3/2*a
*polylog(5,-1+2/(-a*x+1))/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx = -\frac{a\left(-\frac{\pi^4}{16}+\frac{i\pi^5}{160}+\operatorname{arctanh}(ax)^4+\frac{\operatorname{arctanh}(ax)^4}{ax}-4\operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)-\operatorname{arctanh}(ax)^4\log\right.}{}$$

input
```
Integrate[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

output
```
-((a*(-1/16*Pi^4 + (I/160)*Pi^5 + ArcTanh[a*x]^4 + ArcTanh[a*x]^4/(a*x) -
4*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2
*ArcTanh[a*x])] - 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, E^(2*ArcT
anh[a*x])] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x
])] - 3*PolyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*Ar
cTanh[a*x])] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6496, 27, 6452, 6494, 6550, 6494, 6618, 6620, 6622, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}dx$$

$$\Big\downarrow \ 6496$$

$$\frac{\int \frac{\operatorname{arctanh}(ax)^4}{x^2}dx}{c}+a\int \frac{\operatorname{arctanh}(ax)^4}{cx(1-ax)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\operatorname{arctanh}(ax)^4}{x^2}dx}{c}+\frac{a\int \frac{\operatorname{arctanh}(ax)^4}{x(1-ax)}dx}{c}$$

$$\Big\downarrow \ 6452$$

$$\frac{4a\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^4}{x}}{c}+\frac{a\int \frac{\operatorname{arctanh}(ax)^4}{x(1-ax)}dx}{c}$$

$$\Big\downarrow \ 6494$$

$$\frac{4a\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^4}{x}}{c}+\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \ 6550$$

$$\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)}{c}+\frac{4a\left(\int \frac{\operatorname{arctanh}(ax)^3}{x(ax+1)}dx+\frac{1}{4}\operatorname{arctanh}(ax)^4\right)-\frac{\operatorname{arctanh}(ax)^4}{x}}{c}$$

$$\Big\downarrow \ 6494$$

$$\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)}{c}+\frac{4a\left(-3a\int \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\operatorname{arctanh}(ax)^4}{x}}{c}$$

$$\Big\downarrow \ 6618$$

$$\frac{4a\left(-3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3}{c}$$

$$\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int\frac{\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)}{c}$$

$$\Big\downarrow \text{6620}$$

$$\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\int\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)\right)}{c}$$

$$\frac{4a\left(-3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3}{c}$$

$$\Big\downarrow \text{6622}$$

$$\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\int\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right)\right)}{c}$$

$$\frac{4a\left(-3a\left(-\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}\right)+\frac{1}{4}\operatorname{arcta}}{c}$$

$$\Big\downarrow \text{6624}$$

$$\frac{a\left(\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx\right)-\right.\right.}{c}$$

$$\frac{4a\left(-3a\left(-\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}\right)+\frac{1}{4}\operatorname{arcta}}{c}$$

$$\Big\downarrow \text{6624}$$

$$\frac{4a\left(-3a\left(-\frac{1}{2}\int \frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}\right)+\frac{1}{4}\text{arcta}\right.}{c}$$

$$\frac{a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\frac{\text{arctanh}(a}{}\right.\right.\right.}{c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{4a\left(-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{4}\text{arctanh}(ax)^4\right.}{c}$$

$$\frac{a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(4,\frac{2}{1-ax}-1\right)}{2a}+\frac{\text{PolyL}}{}\right.\right.\right.}{c}$$

input
```
Int[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

output
```
(-(ArcTanh[a*x]^4/x) + 4*a*(ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 - 2/(1
 + a*x)] - 3*a*((ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + (Arc
Tanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 + a*x
)]/(4*a))))/c + (a*(ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)] - 4*a*(-1/2*(ArcTa
nh[a*x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/a + (3*((ArcTanh[a*x]^2*PolyLog[3,
 -1 + 2/(1 - a*x)])/(2*a) - (ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/(2
*a) + PolyLog[5, -1 + 2/(1 - a*x)]/(4*a)))/2)))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(237) = 474$.

Time = 1.46 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.40

method	result
derivativedivides	$a\left(\frac{\operatorname{arctanh}(ax)^4(ax-1)}{cax}+\frac{\operatorname{arctanh}(ax)^4\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}+\frac{4\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}-\frac{12\operatorname{ar}}{}\right.$
default	$a\left(\frac{\operatorname{arctanh}(ax)^4(ax-1)}{cax}+\frac{\operatorname{arctanh}(ax)^4\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}+\frac{4\operatorname{arctanh}(ax)^3\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c}-\frac{12\operatorname{ar}}{}\right.$

input
```
int(arctanh(a*x)^4/x^2/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*(1/c*arctanh(a*x)^4/a/x*(a*x-1)+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^
2+1)^(1/2))+4/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*
arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*pol
ylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)
^(1/2))-24/c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2/c*arctanh(a*x)^4+4/c
*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)^2*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2
*x^2+1)^(1/2))+24/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)
^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)^2*polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2))-24/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+24/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx=\int -\frac{\operatorname{artanh}\left(ax\right)^4}{(acx-c)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^4/(a*c*x^3 - c*x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx=-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^3-x^2}\,dx}{c}$$

input
```
integrate(atanh(a*x)**4/x**2/(-a*c*x+c),x)
```

output
```
-Integral(atanh(a*x)**4/(a*x**3 - x**2), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/80*(a*x*log(-a*x + 1)^5 + 5*log(-a*x + 1)^4)/(c*x) + 1/16*integrate(-(l
og(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x
 + 1)^2 - 4*(a*x + log(a*x + 1))*log(-a*x + 1)^3)/(a*c*x^3 - c*x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^4/((a*c*x - c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx = \int \frac{\operatorname{atanh}(a\,x)^4}{x^2\,\left(c-a\,c\,x\right)}\,dx$$

input
```
int(atanh(a*x)^4/(x^2*(c - a*c*x)),x)
```

output
```
int(atanh(a*x)^4/(x^2*(c - a*c*x)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^2(c-acx)}\,dx = -\frac{\int \frac{atanh(ax)^4}{a\,x^3-x^2}dx}{c}$$

input
```
int(atanh(a*x)^4/x^2/(-a*c*x+c),x)
```

output
```
( - int(atanh(a*x)**4/(a*x**3 - x**2),x))/c
```

3.140 $\int \frac{\text{arctanh}(ax)^4}{x^3(c-acx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 380

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx = \frac{2a^2\operatorname{arctanh}(ax)^3}{c} - \frac{2a\operatorname{arctanh}(ax)^3}{cx} + \frac{3a^2\operatorname{arctanh}(ax)^4}{2c} - \frac{\operatorname{arctanh}(ax)^4}{2cx^2} - \frac{a\operatorname{arctanh}(ax)^4}{cx} + \frac{a^2\operatorname{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{6a^2\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{4a^2\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{2a^2\operatorname{arctanh}(ax)^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)}{c} - \frac{6a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{c} - \frac{6a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)}{c} - \frac{3a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)}{c} - \frac{3a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{c} - \frac{6a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)}{c} + \frac{3a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(4,-1+\frac{2}{1-ax}\right)}{c} - \frac{3a^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)}{c} - \frac{3a^2\operatorname{PolyLog}\left(5,-1+\frac{2}{1-ax}\right)}{2c}$$

output
```
2*a^2*arctanh(a*x)^3/c-2*a*arctanh(a*x)^3/c/x+3/2*a^2*arctanh(a*x)^4/c-1/2
*arctanh(a*x)^4/c/x^2-a*arctanh(a*x)^4/c/x+a^2*arctanh(a*x)^4*ln(2-2/(-a*x
+1))/c+6*a^2*arctanh(a*x)^2*ln(2-2/(a*x+1))/c+4*a^2*arctanh(a*x)^3*ln(2-2/
(a*x+1))/c+2*a^2*arctanh(a*x)^3*polylog(2,-1+2/(-a*x+1))/c-6*a^2*arctanh(a
*x)*polylog(2,-1+2/(a*x+1))/c-6*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))
/c-3*a^2*arctanh(a*x)^2*polylog(3,-1+2/(-a*x+1))/c-3*a^2*polylog(3,-1+2/(a
*x+1))/c-6*a^2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))/c+3*a^2*arctanh(a*x)*p
olylog(4,-1+2/(-a*x+1))/c-3*a^2*polylog(4,-1+2/(a*x+1))/c-3/2*a^2*polylog(
5,-1+2/(-a*x+1))/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx =$$
$$-\frac{a^2\left(-\frac{i\pi^3}{4}-\frac{\pi^4}{16}+\frac{i\pi^5}{160}+2\operatorname{arctanh}(ax)^3+\frac{2\operatorname{arctanh}(ax)^3}{ax}+\frac{1}{2}\operatorname{arctanh}(ax)^4+\frac{\operatorname{arctanh}(ax)^4}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)^4}{ax}\right.}{}$$

input
```
Integrate[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

output
```
-((a^2*((-1/4*I)*Pi^3 - Pi^4/16 + (I/160)*Pi^5 + 2*ArcTanh[a*x]^3 + (2*Arc
Tanh[a*x]^3)/(a*x) + ArcTanh[a*x]^4/2 + ArcTanh[a*x]^4/(2*a^2*x^2) + ArcTa
nh[a*x]^4/(a*x) - 6*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 4*ArcTanh
[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[
a*x])] - 2*ArcTanh[a*x]*(3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, E
^(2*ArcTanh[a*x])] + 3*(1 + ArcTanh[a*x])^2*PolyLog[3, E^(2*ArcTanh[a*x])]
 - 3*PolyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTa
nh[a*x])] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

Rubi [A] (verified)

Time = 4.50 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.895, Rules used = {6496, 27, 6452, 6496, 6452, 6494, 6544, 6452, 6510, 6550, 6494, 6618, 6620, 6622, 6624, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}dx$$

$$\Big\downarrow \text{ 6496}$$

$$\frac{\int \frac{\text{arctanh}(ax)^4}{x^3}dx}{c}+a\int \frac{\text{arctanh}(ax)^4}{cx^2(1-ax)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{arctanh}(ax)^4}{x^3}dx}{c}+\frac{a\int \frac{\text{arctanh}(ax)^4}{x^2(1-ax)}dx}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{2a\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+\frac{a\int \frac{\text{arctanh}(ax)^4}{x^2(1-ax)}dx}{c}$$

$$\Big\downarrow \text{6496}$$

$$\frac{2a\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+\frac{a\left(\int \frac{\text{arctanh}(ax)^4}{x^2}dx+a\int \frac{\text{arctanh}(ax)^4}{x(1-ax)}dx\right)}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{2a\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+\frac{a\left(4a\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx+a\int \frac{\text{arctanh}(ax)^4}{x(1-ax)}dx-\frac{\text{arctanh}(ax)^4}{x}\right)}{c}$$

$$\Big\downarrow \text{6494}$$

$$\frac{2a\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+\frac{a\left(4a\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx+a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^4}{x}\right)}{c}$$

$$\Big\downarrow \text{6544}$$

$$\frac{2a\left(a^2\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^3}{x^2}dx\right)-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+\frac{a\left(4a\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx+a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^4}{x}\right)}{c}$$

$$\Big\downarrow \text{6452}$$

$$\frac{2a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx+a^2\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3}{x}\right)-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+$$

$$\frac{a\left(4a\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx+a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^4}{x}\right)}{c}$$

$$\Big\downarrow \text{6510}$$

$$\frac{2a\left(3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}\right)-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}+$$

$$\frac{a\left(4a\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx+a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\frac{\text{arctanh}(ax)^4}{x}\right)}{c}$$

$$\Big\downarrow \text{6550}$$

$$\frac{a\left(a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)+4a\left(\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx+\frac{1}{4}\text{arctanh}(ax)\right.\right.}{c}$$

$$\frac{2a\left(3a\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}\right)-\frac{\text{arctanh}(ax)^4}{2x^2}}{c}$$

$$\Big\downarrow \text{6494}$$

$$\frac{2a\left(3a\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{4}a\text{arctanh}(ax)^4\right.}{c}$$

$$\frac{a\left(a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\int \frac{\text{arctanh}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)+4a\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right.\right.}{c}$$

$$\Big\downarrow \text{6618}$$

$$\frac{2a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.}{c}$$

$$\frac{a\left(4a\left(-3a\left(\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int \frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(a\right.\right.}{c}$$

$$\Big\downarrow \text{6620}$$

$$\frac{2a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.}{a\left(a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\int\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right.\right.\right.}$$

$\downarrow$ 6622

$$\frac{2a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.}{a\left(a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\int\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3\,\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{2a}\right.\right.\right.}$$

$\downarrow$ 6624

$$\frac{2a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.}{a\left(a\left(\text{arctanh}(ax)^4\log\left(2-\frac{2}{1-ax}\right)-4a\left(\frac{3}{2}\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2a}-\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{1-a^2x^2}dx\right.\right.\right.\right.}$$

$\downarrow$ 6624

$$\frac{2a\left(3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2\right.\right.\right.}{a\left(4a\left(-3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}\right)+\frac{1}{4}\text{ar}\right.\right.}$$

$\downarrow$ 7164

$$\frac{2a\left(3a\left(-2a\left(\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+\cdots}\right.\right.\right.}{c}$$

$$\frac{a\left(4a\left(-3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{4}\operatorname{arctanh}(a\cdots\right.\right.}{\cdots}$$

input
```
Int[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

output
```
(-1/2*ArcTanh[a*x]^4/x^2 + 2*a*(-(ArcTanh[a*x]^3/x) + (a*ArcTanh[a*x]^4)/4
 + 3*a*(ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((Arc
Tanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x
)]/(4*a)))))/c + (a*(-(ArcTanh[a*x]^4/x) + 4*a*(ArcTanh[a*x]^4/4 + ArcTanh
[a*x]^3*Log[2 - 2/(1 + a*x)] - 3*a*((ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 +
 a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*a) + PolyLo
g[4, -1 + 2/(1 + a*x)]/(4*a))) + a*(ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)] -
4*a*(-1/2*(ArcTanh[a*x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/a + (3*((ArcTanh[a
*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/(2*a) - (ArcTanh[a*x]*PolyLog[4, -1 +
2/(1 - a*x)])/(2*a) + PolyLog[5, -1 + 2/(1 - a*x)]/(4*a)))/2))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6496
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
 x] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTanh[c*x])^p/(d + e*x)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
0] && LtQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(374) = 748$.

Time = 4.69 (sec) , antiderivative size = 779, normalized size of antiderivative = 2.05

method	result
derivativedivides	$a^2 \left(\frac{\operatorname{arctanh}(ax)^3 (3ax \operatorname{arctanh}(ax) + \operatorname{arctanh}(ax) + 4ax)(ax-1)}{2c a^2 x^2} + \frac{4 \operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2, \frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c} + \frac{24 \mathrm{p}}{}\right.$
default	$a^2 \left(\frac{\operatorname{arctanh}(ax)^3 (3ax \operatorname{arctanh}(ax) + \operatorname{arctanh}(ax) + 4ax)(ax-1)}{2c a^2 x^2} + \frac{4 \operatorname{arctanh}(ax)^3 \operatorname{polylog}\left(2, \frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{c} + \frac{24 \mathrm{p}}{}\right.$

input
```
int(arctanh(a*x)^4/x^3/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2/c*arctanh(a*x)^3*(3*a*x*arctanh(a*x)+arctanh(a*x)+4*a*x)*(a*x-1)/
a^2/x^2+4/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*poly
log(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-12/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*polylog(3,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2*
x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*
arctanh(a*x)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*
polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1
)/(-a^2*x^2+1)^(1/2))-2/c*arctanh(a*x)^4-4/c*arctanh(a*x)^3+6/c*arctanh(a*
x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))+1/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*ar
ctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)^2*polylog(
2,(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^
2+1)^(1/2))+12/c*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/
c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+12/c*arctanh(a*x)*polylog(2,-(a*x+1)/(
-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c
*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^4/(a*c*x^4 - c*x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx = -\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^4-x^3}\,dx}{c}$$

input
```
integrate(atanh(a*x)**4/x**3/(-a*c*x+c),x)
```

output
```
-Integral(atanh(a*x)**4/(a*x**4 - x**3), x)/c
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/160*(2*a^2*x^2*log(-a*x + 1)^5 + 5*(2*a*x + 1)*log(-a*x + 1)^4)/(c*x^2)
 + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*lo
g(a*x + 1)^2*log(-a*x + 1)^2 - 2*(2*a^2*x^2 + a*x + 2*log(a*x + 1))*log(-a
*x + 1)^3)/(a*c*x^4 - c*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx = \int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^4/((a*c*x - c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx=\int \frac{\mathrm{atanh}(a\,x)^4}{x^3\,\left(c-a\,c\,x\right)}\,dx$$

input
```
int(atanh(a*x)^4/(x^3*(c - a*c*x)),x)
```

output
```
int(atanh(a*x)^4/(x^3*(c - a*c*x)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^4}{x^3(c-acx)}\,dx=-\frac{\int \frac{atanh(ax)^4}{a\,x^4-x^3}dx}{c}$$

input
```
int(atanh(a*x)^4/x^3/(-a*c*x+c),x)
```

output
```
( - int(atanh(a*x)**4/(a*x**4 - x**3),x))/c
```

3.141 $\int \frac{x}{(c+acx)\text{arctanh}(ax)} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)} dx = \text{Int}\left(\frac{x}{(c+acx)\text{arctanh}(ax)}, x\right)$$

output
```
Defer(Int)(x/(a*c*x+c)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)} dx = \int \frac{x}{(c+acx)\text{arctanh}(ax)} dx$$

input
```
Integrate[x/((c + a*c*x)*ArcTanh[a*x]),x]
```

output
```
Integrate[x/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arctanh}(ax)(acx+c)}\,dx$$

$$\Big\downarrow \; 6651$$

$$\int \frac{x}{\text{arctanh}(ax)(acx+c)}dx$$

input
```
Int[x/((c + a*c*x)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(acx+c)\,\text{arctanh}\,(ax)}dx$$

input
```
int(x/(a*c*x+c)/arctanh(a*x),x)
```

output
```
int(x/(a*c*x+c)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{x}{(acx+c)\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(x/((a*c*x + c)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)}\,dx = \frac{\int \frac{x}{ax\,\text{atanh}\,(ax)+\text{atanh}\,(ax)}\,dx}{c}$$

input
```
integrate(x/(a*c*x+c)/atanh(a*x),x)
```

output
```
Integral(x/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{x}{(acx+c)\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{x}{(acx+c)\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{x}{\text{atanh}\,(a\,x)\,\,(c+a\,c\,x)}\,dx$$

input
```
int(x/(atanh(a*x)*(c + a*c*x)),x)
```

output
```
int(x/(atanh(a*x)*(c + a*c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)}\,dx = \frac{\int \frac{x}{atanh(ax)ax+atanh(ax)}dx}{c}$$

input
```
int(x/(a*c*x+c)/atanh(a*x),x)
```

output
```
int(x/(atanh(a*x)*a*x + atanh(a*x)),x)/c
```

3.142 $\int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{1}{(c+acx)\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(1/(a*c*x+c)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx$$

input
```
Integrate[1/((c + a*c*x)*ArcTanh[a*x]),x]
```

output
```
Integrate[1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arctanh}(ax)(acx+c)} \, dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{1}{\operatorname{arctanh}(ax)(acx+c)}dx$$

input
```
Int[1/((c + a*c*x)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{(acx+c)\operatorname{arctanh}(ax)}dx$$

input
```
int(1/(a*c*x+c)/arctanh(a*x),x)
```

output
```
int(1/(a*c*x+c)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\mathrm{arctanh}(ax)}\,dx=\int \frac{1}{(acx+c)\,\mathrm{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a*c*x + c)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\mathrm{arctanh}(ax)}\,dx=\frac{\int \frac{1}{ax\,\mathrm{atanh}\,(ax)+\mathrm{atanh}\,(ax)}\,dx}{c}$$

input
```
integrate(1/(a*c*x+c)/atanh(a*x),x)
```

output
```
Integral(1/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\mathrm{arctanh}(ax)}\,dx=\int \frac{1}{(acx+c)\,\mathrm{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx=\int \frac{1}{(acx+c)\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx=\int \frac{1}{\text{atanh}\,(a\,x)\,\left(c+a\,c\,x\right)}\,dx$$

input
```
int(1/(atanh(a*x)*(c + a*c*x)),x)
```

output
```
int(1/(atanh(a*x)*(c + a*c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)}\,dx = \frac{\left(\int \frac{x}{atanh(ax)a^2x^2-atanh(ax)}dx\right)a^2+\log(atanh(ax))}{ac}$$

input
```
int(1/(a*c*x+c)/atanh(a*x),x)
```

output
```
(int(x/(atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**2 + log(atanh(a*x)))/(a*c
)
```

3.143 $\int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{1}{x(c+acx)\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(1/x/(a*c*x+c)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(ax)(acx+c)} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{x\text{arctanh}(ax)(acx+c)}dx$$

input
```
Int[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(acx+c)\,\text{arctanh}\,(ax)}dx$$

input
```
int(1/x/(a*c*x+c)/arctanh(a*x),x)
```

output
```
int(1/x/(a*c*x+c)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(c+acx)\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(acx+c)x\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a*c*x^2 + c*x)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\operatorname{arctanh}(ax)}\,dx = \frac{\int \frac{1}{ax^2\operatorname{atanh}(ax)+x\operatorname{atanh}(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a*c*x+c)/atanh(a*x),x)
```

output
```
Integral(1/(a*x**2*atanh(a*x) + x*atanh(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(acx+c)x\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{1}{(acx+c)x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx = \int \frac{1}{x\,\text{atanh}\,(a\,x)\,\,(c+a\,c\,x)}\,dx$$

input
```
int(1/(x*atanh(a*x)*(c + a*c*x)),x)
```

output
```
int(1/(x*atanh(a*x)*(c + a*c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)}\,dx = \frac{-\left(\int \frac{1}{atanh(ax)a^2x^3-atanh(ax)x}dx\right)-\log(atanh(ax))}{c}$$

input
```
int(1/x/(a*c*x+c)/atanh(a*x),x)
```

output
```
( - (int(1/(atanh(a*x)*a**2*x**3 - atanh(a*x)*x),x) + log(atanh(a*x))))/c
```

3.144 $\int \frac{x}{(c+acx)\operatorname{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{(c+acx)\operatorname{arctanh}(ax)^2} dx = \operatorname{Int}\left(\frac{x}{(c+acx)\operatorname{arctanh}(ax)^2}, x\right)$$

output
```
Defer(Int)(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\operatorname{arctanh}(ax)^2} dx = \int \frac{x}{(c+acx)\operatorname{arctanh}(ax)^2} dx$$

input
```
Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arctanh}(ax)^2(acx+c)}\,dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{x}{\operatorname{arctanh}(ax)^2(acx+c)}dx$$

input
```
Int[x/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(acx+c)\operatorname{arctanh}(ax)^2}dx$$

input
```
int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

output
```
int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{x}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)^2}\,dx = \frac{\int \frac{x}{ax\,\text{atanh}^2\,(ax)+\text{atanh}^2\,(ax)}\,dx}{c}$$

input
```
integrate(x/(a*c*x+c)/atanh(a*x)**2,x)
```

output
```
Integral(x/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{x}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*(a*x^2 - x)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + integrate(-2*(2*a*x
 - 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{x}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{x}{\text{atanh}\,(a\,x)^2\,\,(c+a\,c\,x)}\,dx$$

input
```
int(x/(atanh(a*x)^2*(c + a*c*x)),x)
```

output
```
int(x/(atanh(a*x)^2*(c + a*c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x}{(c+acx)\text{arctanh}(ax)^2}\,dx = \frac{\int \frac{x}{atanh(ax)^2ax+atanh(ax)^2}dx}{c}$$

input
```
int(x/(a*c*x+c)/atanh(a*x)^2,x)
```

output
```
int(x/(atanh(a*x)**2*a*x + atanh(a*x)**2),x)/c
```

3.145 $\int \frac{1}{(c+acx)\mathrm{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{(c+acx)\mathrm{arctanh}(ax)^2} dx = \mathrm{Int}\left(\frac{1}{(c+acx)\mathrm{arctanh}(ax)^2}, x\right)$$

output
```
Defer(Int)(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\mathrm{arctanh}(ax)^2} dx = \int \frac{1}{(c+acx)\mathrm{arctanh}(ax)^2} dx$$

input
```
Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arctanh}(ax)^2(acx+c)} \, dx$$

$$\Big\downarrow \ 6651$$

$$\int \frac{1}{\text{arctanh}(ax)^2(acx+c)}dx$$

input
```
Int[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{(acx+c)\,\text{arctanh}\,(ax)^2}dx$$

input
```
int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

output
```
int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)^2}\,dx = \frac{\int \frac{1}{ax\,\text{atanh}^2\,(ax)+\text{atanh}^2\,(ax)}\,dx}{c}$$

input
```
integrate(1/(a*c*x+c)/atanh(a*x)**2,x)
```

output
```
Integral(1/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.87

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*(a*x - 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + 2*integrate(-1/(c*log
(a*x + 1) - c*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{\text{atanh}\,(a\,x)^2\,(c+a\,c\,x)}\,dx$$

input
```
int(1/(atanh(a*x)^2*(c + a*c*x)),x)
```

output
```
int(1/(atanh(a*x)^2*(c + a*c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int \frac{1}{(c+acx)\text{arctanh}(ax)^2}\,dx = \frac{atanh(ax)\left(\int \frac{x}{atanh(ax)^2a^2x^2-atanh(ax)^2}dx\right)a^2-1}{atanh\,(ax)\,ac}$$

input
```
int(1/(a*c*x+c)/atanh(a*x)^2,x)
```

output
```
(atanh(a*x)*int(x/(atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**2 - 1)/(
atanh(a*x)*a*c)
```

3.146 $\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \text{Int}\left(\frac{1}{x(c+acx)\text{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(ax)^2(acx+c)}\,dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{x\text{arctanh}(ax)^2(acx+c)}dx$$

input
```
Int[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(acx+c)\operatorname{arctanh}\,(ax)^2}dx$$

input
```
int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

output
```
int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(acx+c)x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a*c*x^2 + c*x)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \frac{\int \frac{1}{ax^2\,\text{atanh}^2\,(ax)+x\,\text{atanh}^2\,(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a*c*x+c)/atanh(a*x)**2,x)
```

output
```
Integral(1/(a*x**2*atanh(a*x)**2 + x*atanh(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(acx+c)x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*(a*x - 1)/(a*c*x*log(a*x + 1) - a*c*x*log(-a*x + 1)) + 2*integrate(-1/(a
*c*x^2*log(a*x + 1) - a*c*x^2*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(acx+c)x\,\text{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a*c*x + c)*x*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \int \frac{1}{x\,\text{atanh}\left(a\,x\right)^2\,\left(c+a\,c\,x\right)}\,dx$$

input
```
int(1/(x*atanh(a*x)^2*(c + a*c*x)),x)
```

output
```
int(1/(x*atanh(a*x)^2*(c + a*c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x(c+acx)\text{arctanh}(ax)^2}\,dx = \frac{-atanh(ax)\left(\int \frac{1}{atanh(ax)^2a^2x^3-atanh(ax)^2x}dx\right)+1}{atanh\,(ax)\,c}$$

input
```
int(1/x/(a*c*x+c)/atanh(a*x)^2,x)
```

output
```
( - atanh(a*x)*int(1/(atanh(a*x)**2*a**2*x**3 - atanh(a*x)**2*x),x) + 1)/(
atanh(a*x)*c)
```

3.147 $\int \frac{x^3(a+b\text{arctanh}(cx))}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 275

$$\begin{aligned}\int \frac{x^3(a+b\text{arctanh}(cx))}{d+ex}\,dx &= \frac{ad^2x}{e^3}-\frac{bdx}{2ce^2}+\frac{bx^2}{6ce}+\frac{bd\text{arctanh}(cx)}{2c^2e^2}+\frac{bd^2x\text{arctanh}(cx)}{e^3}\\ &\quad-\frac{dx^2(a+b\text{arctanh}(cx))}{2e^2}+\frac{x^3(a+b\text{arctanh}(cx))}{3e}\\ &\quad+\frac{d^3(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{e^4}\\ &\quad-\frac{d^3(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^4}\\ &\quad+\frac{bd^2\log(1-c^2x^2)}{2ce^3}+\frac{b\log(1-c^2x^2)}{6c^3e}\\ &\quad-\frac{bd^3\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2e^4}\\ &\quad+\frac{bd^3\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e^4}\end{aligned}$$

output
```
a*d^2*x/e^3-1/2*b*d*x/c/e^2+1/6*b*x^2/c/e+1/2*b*d*arctanh(c*x)/c^2/e^2+b*d
^2*x*arctanh(c*x)/e^3-1/2*d*x^2*(a+b*arctanh(c*x))/e^2+1/3*x^3*(a+b*arctan
h(c*x))/e+d^3*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^4-d^3*(a+b*arctanh(c*x))*
ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^4+1/2*b*d^2*ln(-c^2*x^2+1)/c/e^3+1/6*b*l
n(-c^2*x^2+1)/c^3/e-1/2*b*d^3*polylog(2,1-2/(c*x+1))/e^4+1/2*b*d^3*polylog
(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.47 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.72

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+ex}\,dx$$

$$= \frac{-\frac{be^3}{c^3}+6ad^2ex-\frac{3bde^2x}{c}-3ade^2x^2+\frac{be^3x^2}{c}+2ae^3x^3+\frac{3bde^2\text{arctanh}(cx)}{c^2}-3ibd^3\pi\text{arctanh}(cx)+6bd^2ex\text{arc}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

output
```
(-((b*e^3)/c^3) + 6*a*d^2*e*x - (3*b*d*e^2*x)/c - 3*a*d*e^2*x^2 + (b*e^3*x
^2)/c + 2*a*e^3*x^3 + (3*b*d*e^2*ArcTanh[c*x])/c^2 - (3*I)*b*d^3*Pi*ArcTan
h[c*x] + 6*b*d^2*e*x*ArcTanh[c*x] - 3*b*d*e^2*x^2*ArcTanh[c*x] + 2*b*e^3*x
^3*ArcTanh[c*x] - 6*b*d^3*ArcTanh[(c*d)/e]*ArcTanh[c*x] + 3*b*d^3*ArcTanh[
c*x]^2 - (3*b*d^2*e*ArcTanh[c*x]^2)/c + (3*b*d^2*Sqrt[1 - (c^2*d^2)/e^2]*e
*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c*d)/e]) + 6*b*d^3*ArcTanh[c*x]*Log[1 + E^(
-2*ArcTanh[c*x])] + (3*I)*b*d^3*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 6*b*d^3*A
rcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*b*d^
3*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*a*d^3
*Log[d + e*x] + (3*b*d^2*e*Log[1 - c^2*x^2])/c + (b*e^3*Log[1 - c^2*x^2])/
c^3 + ((3*I)/2)*b*d^3*Pi*Log[1 - c^2*x^2] + 6*b*d^3*ArcTanh[(c*d)/e]*Log[I
*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 3*b*d^3*PolyLog[2, -E^(-2*ArcTan
h[c*x])] + 3*b*d^3*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(
6*e^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{arctanh}(cx))}{d+ex}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(-\frac{d^3(a+b\text{arctanh}(cx))}{e^3(d+ex)}+\frac{d^2(a+b\text{arctanh}(cx))}{e^3}-\frac{dx(a+b\text{arctanh}(cx))}{e^2}+\frac{x^2(a+b\text{arctanh}(cx))}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^3\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e^4}-\frac{d^3(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^4}-\frac{dx^2(a+b\text{arctanh}(cx))}{2e^2}+\frac{x^3(a+b\text{arctanh}(cx))}{3e}+\frac{ad^2x}{e^3}+\frac{bd\text{arctanh}(cx)}{2c^2e^2}+\frac{bd^2x\text{arctanh}(cx)}{e^3}+\frac{bd^2\log(1-c^2x^2)}{2ce^3}+\frac{b\log(1-c^2x^2)}{6c^3e}-\frac{bd^3\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^4}+\frac{bd^3\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^4}-\frac{bdx}{2ce^2}+\frac{bx^2}{6ce}$$

input
```
Int[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

output
```
(a*d^2*x)/e^3 - (b*d*x)/(2*c*e^2) + (b*x^2)/(6*c*e) + (b*d*ArcTanh[c*x])/(
2*c^2*e^2) + (b*d^2*x*ArcTanh[c*x])/e^3 - (d*x^2*(a + b*ArcTanh[c*x]))/(2*
e^2) + (x^3*(a + b*ArcTanh[c*x]))/(3*e) + (d^3*(a + b*ArcTanh[c*x])*Log[2/
(1 + c*x)])/e^4 - (d^3*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)
*(1 + c*x))])/e^4 + (b*d^2*Log[1 - c^2*x^2])/(2*c*e^3) + (b*Log[1 - c^2*x^
2])/(6*c^3*e) - (b*d^3*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^4) + (b*d^3*PolyL
og[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.17

method	result
parts	$\frac{a x^3}{3e} - \frac{a d x^2}{2e^2} + \frac{a d^2 x}{e^3} - \frac{a d^3 \ln(ex+d)}{e^4} + \frac{b\left(\frac{c^4 \operatorname{arctanh}(cx)x^3}{3e} - \frac{c^4 \operatorname{arctanh}(cx)x^2 d}{2e^2} + \frac{c^4 \operatorname{arctanh}(cx)x d^2}{e^3} - \frac{c^4 \operatorname{arctanh}(c\cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a c^4 d^2 x}{e^3} - \frac{a c^4 d x^2}{2e^2} + \frac{a c^4 x^3}{3e} - \frac{a c^4 d^3 \ln(cex+cd)}{e^4} + bc\left(\frac{\operatorname{arctanh}(cx)c^3 d^2 x}{e^3} - \frac{\operatorname{arctanh}(cx)c^3 d x^2}{2e^2} + \frac{\operatorname{arctanh}(cx)c^3 x^3}{3e} - \frac{\operatorname{arctanh}(cx)c^3\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{a c^4 d^2 x}{e^3} - \frac{a c^4 d x^2}{2e^2} + \frac{a c^4 x^3}{3e} - \frac{a c^4 d^3 \ln(cex+cd)}{e^4} + bc\left(\frac{\operatorname{arctanh}(cx)c^3 d^2 x}{e^3} - \frac{\operatorname{arctanh}(cx)c^3 d x^2}{2e^2} + \frac{\operatorname{arctanh}(cx)c^3 x^3}{3e} - \frac{\operatorname{arctanh}(cx)c^3\cdots}{\cdots}\right.}{\cdots}$
risch	$-\frac{bd\ln(cx+1)x^2}{4e^2} + \frac{bd\ln(cx+1)}{4c^2e^2} + \frac{b\ln(cx+1)x d^2}{2e^3} + \frac{b\ln(cx+1)d^2}{2c e^3} - \frac{b d^3 \ln(cx+1)\ln\left(\frac{(cx+1)e+cd-e}{cd-e}\right)}{2e^4} - \frac{bd\ln\cdots}{4\cdots}$

input
```
int(x^3*(a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/e*x^3-1/2*a/e^2*d*x^2+a*d^2*x/e^3-a*d^3/e^4*ln(e*x+d)+b/c^4*(1/3*c^4
*arctanh(c*x)/e*x^3-1/2*c^4*arctanh(c*x)/e^2*x^2*d+c^4*arctanh(c*x)/e^3*x*
d^2-c^4*arctanh(c*x)*d^3/e^4*ln(c*e*x+c*d)-c/e*(1/e^2*c^3*d^3*(1/2/e*(dilo
g((c*e*x-e)/(-c*d-e))+ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e)))-1/2/e*(dilog((
c*e*x+e)/(-c*d+e))+ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))))-1/6/e^2*(-5*c*d*
(c*e*x+c*d)+(c*e*x+c*d)^2+(3*c^2*d^2-3/2*c*d*e+e^2)*ln(c*e*x-e)+(3*c^2*d^2
+3/2*c*d*e+e^2)*ln(c*e*x+e))))
```

Fricas [F]

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + ex}\, dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^3}{ex + d}\, dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arctanh(c*x) + a*x^3)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + ex}\, dx = \int \frac{x^3(a + b\,\text{atanh}\,(cx))}{d + ex}\, dx$$

input
```
integrate(x**3*(a+b*atanh(c*x))/(e*x+d),x)
```

output
```
Integral(x**3*(a + b*atanh(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + ex} \, dx = \int \frac{(b \text{artanh}(cx) + a)x^3}{ex + d} \, dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
-1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) +
1/2*b*integrate(x^3*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + ex} \, dx = \int \frac{(b \text{artanh}(cx) + a)x^3}{ex + d} \, dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^3/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{arctanh}(cx))}{d + ex} \, dx = \int \frac{x^3 (a + b \,\text{atanh}(c\,x))}{d + e\,x} \, dx$$

input
```
int((x^3*(a + b*atanh(c*x)))/(d + e*x),x)
```

output
```
int((x^3*(a + b*atanh(c*x)))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx$$
$$= \frac{6\left(\int \frac{atanh(cx)x^3}{ex+d}dx\right)b\,e^4 - 6\log(ex+d)\,a\,d^3 + 6a\,d^2ex - 3ad\,e^2x^2 + 2a\,e^3x^3}{6e^4}$$

input
```
int(x^3*(a+b*atanh(c*x))/(e*x+d),x)
```

output
```
(6*int((atanh(c*x)*x**3)/(d + e*x),x)*b*e**4 - 6*log(d + e*x)*a*d**3 + 6*a
*d**2*e*x - 3*a*d*e**2*x**2 + 2*a*e**3*x**3)/(6*e**4)
```

3.148 $\int \frac{x^2(a+b\text{arctanh}(cx))}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 214

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+ex}\,dx = -\frac{adx}{e^2}+\frac{bx}{2ce}-\frac{b\text{arctanh}(cx)}{2c^2e}-\frac{bdx\text{arctanh}(cx)}{e^2}+\frac{x^2(a+b\text{arctanh}(cx))}{2e}-\frac{d^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{e^3}+\frac{d^2(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^3}-\frac{bd\log(1-c^2x^2)}{2ce^2}+\frac{bd^2\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2e^3}-\frac{bd^2\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e^3}$$

output
```
-a*d*x/e^2+1/2*b*x/c/e-1/2*b*arctanh(c*x)/c^2/e-b*d*x*arctanh(c*x)/e^2+1/2
*x^2*(a+b*arctanh(c*x))/e-d^2*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^3+d^2*(a+
b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3-1/2*b*d*ln(-c^2*x^2+1)
/c/e^2+1/2*b*d^2*polylog(2,1-2/(c*x+1))/e^3-1/2*b*d^2*polylog(2,1-2*c*(e*x
+d)/(c*d+e)/(c*x+1))/e^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.84

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+ex}\,dx$$
$$= \frac{-2adex+\frac{be^2x}{c}+ae^2x^2-\frac{be^2\text{arctanh}(cx)}{c^2}+ibd^2\pi\text{arctanh}(cx)-2bdex\text{arctanh}(cx)+be^2x^2\text{arctanh}(cx)+}{}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

output
```
(-2*a*d*e*x + (b*e^2*x)/c + a*e^2*x^2 - (b*e^2*ArcTanh[c*x])/c^2 + I*b*d^2
*Pi*ArcTanh[c*x] - 2*b*d*e*x*ArcTanh[c*x] + b*e^2*x^2*ArcTanh[c*x] + 2*b*d
^2*ArcTanh[(c*d)/e]*ArcTanh[c*x] - b*d^2*ArcTanh[c*x]^2 + (b*d*e*ArcTanh[c
*x]^2)/c - (b*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c*
d)/e]) - 2*b*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - I*b*d^2*Pi*Lo
g[1 + E^(2*ArcTanh[c*x])] + 2*b*d^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTan
h[(c*d)/e] + ArcTanh[c*x]))] + 2*b*d^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh
[(c*d)/e] + ArcTanh[c*x]))] + 2*a*d^2*Log[d + e*x] - (b*d*e*Log[1 - c^2*x^
2])/c - (I/2)*b*d^2*Pi*Log[1 - c^2*x^2] - 2*b*d^2*ArcTanh[(c*d)/e]*Log[I*S
inh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + b*d^2*PolyLog[2, -E^(-2*ArcTanh[c*
x])] - b*d^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{d+ex}\,dx$$
$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{d^2(a+b\text{arctanh}(cx))}{e^2(d+ex)}-\frac{d(a+b\text{arctanh}(cx))}{e^2}+\frac{x(a+b\text{arctanh}(cx))}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d^2\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e^3}+\frac{d^2(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}+\frac{x^2(a+b\text{arctanh}(cx))}{2e}-\frac{adx}{e^2}-\frac{b\text{arctanh}(cx)}{2c^2e}-\frac{bdx\text{arctanh}(cx)}{e^2}-\frac{bd\log(1-c^2x^2)}{2ce^2}+\frac{bd^2\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^3}-\frac{bd^2\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^3}+\frac{bx}{2ce}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

output
```
-((a*d*x)/e^2) + (b*x)/(2*c*e) - (b*ArcTanh[c*x])/(2*c^2*e) - (b*d*x*ArcTa
nh[c*x])/e^2 + (x^2*(a + b*ArcTanh[c*x]))/(2*e) - (d^2*(a + b*ArcTanh[c*x]
)*Log[2/(1 + c*x)])/e^3 + (d^2*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((
c*d + e)*(1 + c*x))])/e^3 - (b*d*Log[1 - c^2*x^2])/(2*c*e^2) + (b*d^2*Poly
Log[2, 1 - 2/(1 + c*x)])/(2*e^3) - (b*d^2*PolyLog[2, 1 - (2*c*(d + e*x))/(
(c*d + e)*(1 + c*x))])/(2*e^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.22

method	result
parts	$\frac{a\,x^{2}}{2e}-\frac{adx}{e^{2}}+\frac{a\,d^{2}\ln(ex+d)}{e^{3}}+\frac{b\left(\frac{c^{3}\operatorname{arctanh}(cx)x^{2}}{2e}-\frac{c^{3}\operatorname{arctanh}(cx)dx}{e^{2}}+\frac{c^{3}\operatorname{arctanh}(cx)d^{2}\ln(cex+cd)}{e^{3}}-\frac{c\left(-\frac{cex+cd+(-}{}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^{3}dx}{e^{2}}+\frac{a\,c^{3}x^{2}}{2e}+\frac{a\,c^{3}d^{2}\ln(cex+cd)}{e^{3}}+bc\left(-\frac{\operatorname{arctanh}(cx)c^{2}dx}{e^{2}}+\frac{\operatorname{arctanh}(cx)c^{2}x^{2}}{2e}+\frac{\operatorname{arctanh}(cx)c^{2}d^{2}\ln(cex+cd)}{e^{3}}-\frac{-\frac{cd+cex+(}{}}{}\right.}{}$
default	$\frac{-\frac{a\,c^{3}dx}{e^{2}}+\frac{a\,c^{3}x^{2}}{2e}+\frac{a\,c^{3}d^{2}\ln(cex+cd)}{e^{3}}+bc\left(-\frac{\operatorname{arctanh}(cx)c^{2}dx}{e^{2}}+\frac{\operatorname{arctanh}(cx)c^{2}x^{2}}{2e}+\frac{\operatorname{arctanh}(cx)c^{2}d^{2}\ln(cex+cd)}{e^{3}}-\frac{-\frac{cd+cex+(}{}}{}\right.}{}$
risch	$-\frac{b\ln(-cx+1)d}{2c\,e^{2}}-\frac{b\,d^{2}\ln(-cx+1)\ln\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2e^{3}}+\frac{b\ln(-cx+1)xd}{2e^{2}}+\frac{bx}{2ce}+\frac{b\ln(cx+1)x^{2}}{4e}-\frac{b\ln(cx+1)}{4c^{2}e}$

input
```
int(x^2*(a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*x^2-a/e^2*d*x+a*d^2/e^3*ln(e*x+d)+b/c^3*(1/2*c^3*arctanh(c*x)/e*x^
2-c^3*arctanh(c*x)/e^2*d*x+c^3*arctanh(c*x)*d^2/e^3*ln(c*e*x+c*d)-c/e*(-1/
2/e*(c*e*x+c*d+(-c*d+1/2*e)*ln(c*e*x-e)+(-c*d-1/2*e)*ln(c*e*x+e))-1/e*c^2*
d^2*(1/2/e*(dilog((c*e*x-e)/(-c*d-e))+ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))
)-1/2/e*(dilog((c*e*x+e)/(-c*d+e))+ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e)))))
)
```

Fricas [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{d + ex}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^2}{ex + d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctanh(c*x) + a*x^2)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{d + ex}\,dx = \int \frac{x^2(a + b\,\text{atanh}\,(cx))}{d + ex}\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x))/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*atanh(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))}{d + ex}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x^2}{ex + d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/2*b*integrate(x^2
*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx = \int \frac{x^2\,(a+b\operatorname{atanh}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x^2*(a + b*atanh(c*x)))/(d + e*x),x)
```

output
```
int((x^2*(a + b*atanh(c*x)))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx = \frac{2\left(\int \frac{atanh(cx)x^2}{ex+d}dx\right)b\,e^3+2\log(ex+d)\,a\,d^2-2adex+a\,e^2x^2}{2e^3}$$

input
```
int(x^2*(a+b*atanh(c*x))/(e*x+d),x)
```

output
```
(2*int((atanh(c*x)*x**2)/(d + e*x),x)*b*e**3 + 2*log(d + e*x)*a*d**2 - 2*a
*d*e*x + a*e**2*x**2)/(2*e**3)
```

3.149 $\int \frac{x(a+b\text{arctanh}(cx))}{d+ex}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 156

$$\begin{aligned}\int \frac{x(a+b\text{arctanh}(cx))}{d+ex}\,dx &= \frac{ax}{e}+\frac{bx\text{arctanh}(cx)}{e}+\frac{d(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{e^2}\\ &\quad-\frac{d(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2}\\ &\quad+\frac{b\log(1-c^2x^2)}{2ce}-\frac{bd\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2e^2}\\ &\quad+\frac{bd\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e^2}\end{aligned}$$

output

```
a*x/e+b*x*arctanh(c*x)/e+d*(a+b*arctanh(c*x))*ln(2/(c*x+1))/e^2-d*(a+b*arc
tanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2+1/2*b*ln(-c^2*x^2+1)/c/e-1/
2*b*d*polylog(2,1-2/(c*x+1))/e^2+1/2*b*d*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(
c*x+1))/e^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.02

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx = \frac{2aex-2ad\log(d+ex)+\frac{b\left(-icd\pi\operatorname{arctanh}(cx)+2cex\operatorname{arctanh}(cx)-2cd\operatorname{arctanh}\left(\frac{cd}{e}\right)\operatorname{arctanh}(cx)+cd\operatorname{arctanh}(cx)^2-e\ldots\right.}{}}{}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

output
```
(2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*e*x*Arc
Tanh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*A
rcTanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)
/e] + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E
^(2*ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e
] + ArcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] +
 ArcTanh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*
c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*Po
lyLog[2, -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] +
ArcTanh[c*x]))]))/c)/(2*e^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx$$

$$\Big\downarrow \; 6502$$

$$\int \left(\frac{a+b\text{arctanh}(cx)}{e}-\frac{d(a+b\text{arctanh}(cx))}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e^2}-\frac{d(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}+\frac{ax}{e}+\frac{bx\text{arctanh}(cx)}{e}+\frac{b\log\left(1-c^2x^2\right)}{2ce}-\frac{bd\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^2}+\frac{bd\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^2}$$

input
```
Int[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

output
```
(a*x)/e + (b*x*ArcTanh[c*x])/e + (d*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])
/e^2 - (d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])
/e^2 + (b*Log[1 - c^2*x^2])/(2*c*e) - (b*d*PolyLog[2, 1 - 2/(1 + c*x)])/(2
*e^2) + (b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^2
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.37

method	result
parts	$\frac{ax}{e}-\frac{ad\ln(ex+d)}{e^2}+\frac{b\left(\frac{c^2\operatorname{arctanh}(cx)x}{e}-\frac{c^2\operatorname{arctanh}(cx)d\ln(cex+cd)}{e^2}-\frac{c\left(-\frac{\ln\left(c^2d^2-2cd(cex+cd)-e^2+(cex+cd)^2\right)}{2}+cd\left(\cdots\right.\right.}{\cdots}\right.}{c^2}$
derivativedivides	$\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+cd)}{e^2}+bc\left(\frac{\operatorname{arctanh}(cx)cx}{e}-\frac{\operatorname{arctanh}(cx)dc\ln(cex+cd)}{e^2}-\frac{-\frac{\ln\left(c^2d^2-2cd(cex+cd)-e^2+(cex+cd)^2\right)}{2}+cd\left(\frac{\operatorname{dilog}\cdots}{\cdots}\right.}{\cdots}\right.}{c^2}$
default	$\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+cd)}{e^2}+bc\left(\frac{\operatorname{arctanh}(cx)cx}{e}-\frac{\operatorname{arctanh}(cx)dc\ln(cex+cd)}{e^2}-\frac{-\frac{\ln\left(c^2d^2-2cd(cex+cd)-e^2+(cex+cd)^2\right)}{2}+cd\left(\frac{\operatorname{dilog}\cdots}{\cdots}\right.}{\cdots}\right.}{c^2}$
risch	$-\frac{b\ln(-cx+1)x}{2e}+\frac{b\ln(-cx+1)}{2ce}-\frac{b}{ce}+\frac{bd\operatorname{dilog}\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2e^2}+\frac{bd\ln(-cx+1)\ln\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2e^2}+\frac{ax}{e}\cdots$

input
```
int(x*(a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a/e*x-a*d/e^2*ln(e*x+d)+b/c^2*(c^2*arctanh(c*x)/e*x-c^2*arctanh(c*x)*d/e^2
*ln(c*e*x+c*d)-c/e*(-1/2*ln(c^2*d^2-2*c*d*(c*e*x+c*d)-e^2+(c*e*x+c*d)^2)+c
*d*(1/2/e*(dilog((c*e*x-e)/(-c*d-e))+ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e)))
-1/2/e*(dilog((c*e*x+e)/(-c*d+e))+ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))))))
```

Fricas [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x*arctanh(c*x) + a*x)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x(a + b\text{arctanh}(cx))}{d + ex}\,dx = \int \frac{x(a + b\,\text{atanh}\,(cx))}{d + ex}\,dx$$

input
```
integrate(x*(a+b*atanh(c*x))/(e*x+d),x)
```

output
```
Integral(x*(a + b*atanh(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x(a + b\text{arctanh}(cx))}{d + ex}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x}{ex + d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
a*(x/e - d*log(e*x + d)/e^2) + 1/2*b*integrate(x*(log(c*x + 1) - log(-c*x
+ 1))/(e*x + d), x)
```

Giac [F]

$$\int \frac{x(a + b\text{arctanh}(cx))}{d + ex}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)x}{ex + d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*x/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx=\int \frac{x\,(a+b\,\operatorname{atanh}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x*(a + b*atanh(c*x)))/(d + e*x),x)
```

output
```
int((x*(a + b*atanh(c*x)))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))}{d+ex}\,dx=\frac{\left(\int \frac{atanh(cx)x}{ex+d}dx\right)b\,e^2-\log(ex+d)\,ad+aex}{e^2}$$

input
```
int(x*(a+b*atanh(c*x))/(e*x+d),x)
```

output
```
(int((atanh(c*x)*x)/(d + e*x),x)*b*e**2 - log(d + e*x)*a*d + a*e*x)/e**2
```

3.150 $\int \frac{a+b\text{arctanh}(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{a+b\text{arctanh}(cx)}{d+ex}\,dx = -\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2e} - \frac{b\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e}$$

output
```
-(a+b*arctanh(c*x))*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d
+e)/(c*x+1))/e+1/2*b*polylog(2,1-2/(c*x+1))/e-1/2*b*polylog(2,1-2*c*(e*x+d
)/(c*d+e)/(c*x+1))/e
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.25

$$\int \frac{a + b\text{arctanh}(cx)}{d + ex}\,dx = \frac{a\log(d+ex) + b\text{arctanh}(cx)\left(\frac{1}{2}\log\left(1-c^2x^2\right) + \log\left(i\sinh\left(\text{arctanh}\left(\frac{cd}{e}\right) + \text{arctanh}(cx)\right)\right)\right) - \frac{1}{2}ib\Big(-\frac{1}{4}i(}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

output
```
(a*Log[d + e*x] + b*ArcTanh[c*x]*(Log[1 - c^2*x^2]/2 + Log[I*Sinh[ArcTanh[
(c*d)/e] + ArcTanh[c*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c*x])^2
 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*Log[1
 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[1 - E
^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Log[2
/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I)*Si
nh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])] -
 I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/e
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arctanh}(cx)}{d + ex}\,dx$$

$$\Big\downarrow \; 6472$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{1-c^2x^2}dx}{e}+\frac{bc\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-c^2x^2}dx}{e}+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e}$$

$\Big\downarrow$ 2849

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{1-c^2x^2}dx}{e}+\frac{b\int \frac{\log\left(\frac{2}{cx+1}\right)}{1-\frac{2}{cx+1}}d\frac{1}{cx+1}}{e}+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e}$$

$\Big\downarrow$ 2752

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{1-c^2x^2}dx}{e}+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e}$$

$\Big\downarrow$ 2897

$$\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

output
```
-(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])*Log[(
2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*PolyLog[2, 1 - 2/(1 + c*x)])
/(2*e) - (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6472
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\ln(cex+cd)\operatorname{arctanh}(cx)}{e}+\frac{b\ln(cex+cd)\ln\left(\frac{cex-e}{-cd-e}\right)}{2e}+\frac{b\operatorname{dilog}\left(\frac{cex-e}{-cd-e}\right)}{2e}-\frac{b\ln(cex+cd)\ln\left(\frac{cex+\cdots}{-cd+\cdots}\right.}{2e}\cdots$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\ln(cex+cd)\operatorname{arctanh}(cx)}{e}-\frac{-\frac{e\left(\operatorname{dilog}\left(\frac{cex-e}{-cd-e}\right)+\ln(cex+cd)\ln\left(\frac{cex-e}{-cd-e}\right)\right)}{2}+\frac{e\left(\operatorname{dilog}\left(\frac{cex+e}{-cd+e}\right)+\ln(cex+cd)\ln\left(\cdots\right)\right)}{2}}{e^2}\cdots\right)}{c}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\ln(cex+cd)\operatorname{arctanh}(cx)}{e}-\frac{-\frac{e\left(\operatorname{dilog}\left(\frac{cex-e}{-cd-e}\right)+\ln(cex+cd)\ln\left(\frac{cex-e}{-cd-e}\right)\right)}{2}+\frac{e\left(\operatorname{dilog}\left(\frac{cex+e}{-cd+e}\right)+\ln(cex+cd)\ln\left(\cdots\right)\right)}{2}}{e^2}\cdots\right)}{c}$
risch	$-\frac{b\operatorname{dilog}\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2e}-\frac{b\ln(-cx+1)\ln\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2e}+\frac{a\ln((-cx+1)e-cd-e)}{e}+\frac{b\operatorname{dilog}\left(\frac{(cx+1)e+cd\cdots}{cd-e}\right.}{2e}\cdots$

input
```
int((a+b*arctanh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b*ln(c*e*x+c*d)/e*arctanh(c*x)+1/2*b/e*ln(c*e*x+c*d)*ln((c*e
*x-e)/(-c*d-e))+1/2*b/e*dilog((c*e*x-e)/(-c*d-e))-1/2*b/e*ln(c*e*x+c*d)*ln
((c*e*x+e)/(-c*d+e))-1/2*b/e*dilog((c*e*x+e)/(-c*d+e))
```

Fricas [F]

$$\int \frac{a + b\text{arctanh}(cx)}{d + ex} \, dx = \int \frac{b \operatorname{artanh}(cx) + a}{ex + d} \, dx$$

input
```
integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{d + ex} \, dx = \int \frac{a + b \operatorname{atanh}(cx)}{d + ex} \, dx$$

input
```
integrate((a+b*atanh(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\text{arctanh}(cx)}{d+ex}\,dx=\int \frac{b\,\text{artanh}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x + d), x) + a*log(e*x +
 d)/e
```

Giac [F]

$$\int \frac{a+b\text{arctanh}(cx)}{d+ex}\,dx=\int \frac{b\,\text{artanh}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{d+ex}\,dx=\int \frac{a+b\,\text{atanh}(c\,x)}{d+e\,x}\,dx$$

input
```
int((a + b*atanh(c*x))/(d + e*x),x)
```

output
```
int((a + b*atanh(c*x))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{a + b\text{arctanh}(cx)}{d + ex} \, dx = \frac{\left(\int \frac{atanh(cx)}{ex+d}dx\right) be + \log(ex + d) \, a}{e}$$

input
```
int((a+b*atanh(c*x))/(e*x+d),x)
```

output
```
(int(atanh(c*x)/(d + e*x),x)*b*e + log(d + e*x)*a)/e
```

3.151 $\int \frac{a+b\text{arctanh}(cx)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx)}{x(d+ex)}\,dx &= \frac{a\log(x)}{d}+\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d}\\ &\quad -\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}\\ &\quad -\frac{b\,\text{PolyLog}(2,-cx)}{2d}+\frac{b\,\text{PolyLog}(2,cx)}{2d}\\ &\quad -\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2d}\end{aligned}$$

output
```
a*ln(x)/d+(a+b*arctanh(c*x))*ln(2/(c*x+1))/d-(a+b*arctanh(c*x))*ln(2*c*(e*
x+d)/(c*d+e)/(c*x+1))/d-1/2*b*polylog(2,-c*x)/d+1/2*b*polylog(2,c*x)/d-1/2
*b*polylog(2,1-2/(c*x+1))/d+1/2*b*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))
/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.99

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+ex)}\,dx$$

$$= \frac{2ad\log(x) - 2ad\log(d+ex) + \frac{b\left(-icd\pi\text{arctanh}(cx)-2cd\text{arctanh}\left(\frac{cd}{e}\right)\text{arctanh}(cx)+cd\text{arctanh}(cx)^2-e\text{arctanh}(cx)\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```

output
```
(2*a*d*Log[x] - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] - 2*c*d*
ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2 + (S
qrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*ArcTan
h[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c*x])]
 - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))
] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] +
 (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh
[(c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*d*Pol
yLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c)/(2*d^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+ex)}\,dx$$

$$\Big\downarrow \; 6502$$

$$\int \left(\frac{a+b\text{arctanh}(cx)}{dx}-\frac{e(a+b\text{arctanh}(cx))}{d(d+ex)}\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d}+\frac{a\log(x)}{d}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}-\frac{b\,\text{PolyLog}(2,-cx)}{2d}+\frac{b\,\text{PolyLog}(2,cx)}{2d}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh
[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d - (b*PolyLog[2, -(c*x
)])/(2*d) + (b*PolyLog[2, c*x])/(2*d) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2
*d) + (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

method	result
risch	$\frac{b\,\mathrm{dilog}(-cx+1)}{2d}+\frac{b\,\mathrm{dilog}\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2d}+\frac{b\ln(-cx+1)\ln\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2d}+\frac{a\ln(-cx)}{d}-\frac{a\ln((-cx+1)e\cdots}{d}$
parts	$\frac{a\ln(x)}{d}-\frac{a\ln(ex+d)}{d}+b\Bigg(\frac{\operatorname{arctanh}(cx)\ln(cx)}{d}-\frac{\operatorname{arctanh}(cx)\ln(cex+cd)}{d}-c\Bigg(\frac{e\big(\mathrm{dilog}\big(\frac{cex-e}{-cd-e}\big)+\ln(cex+cd)\ln(\cdots}{2}$
derivativedivides	$-\frac{a\ln(cex+cd)}{d}+\frac{a\ln(cx)}{d}+bc\Bigg(-\frac{\operatorname{arctanh}(cx)\ln(cex+cd)}{dc}+\frac{\operatorname{arctanh}(cx)\ln(cx)}{dc}-\frac{e\big(\mathrm{dilog}\big(\frac{cex-e}{-cd-e}\big)+\ln(cex+\cdots}{2}$
default	$-\frac{a\ln(cex+cd)}{d}+\frac{a\ln(cx)}{d}+bc\Bigg(-\frac{\operatorname{arctanh}(cx)\ln(cex+cd)}{dc}+\frac{\operatorname{arctanh}(cx)\ln(cx)}{dc}-\frac{e\big(\mathrm{dilog}\big(\frac{cex-e}{-cd-e}\big)+\ln(cex+\cdots}{2}$

input
```
int((a+b*arctanh(c*x))/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d*dilog(-c*x+1)+1/2*b/d*dilog(((-c*x+1)*e-c*d-e)/(-c*d-e))+1/2*b/d*l
n(-c*x+1)*ln(((-c*x+1)*e-c*d-e)/(-c*d-e))+a/d*ln(-c*x)-a/d*ln((-c*x+1)*e-c
*d-e)-1/2*b/d*dilog(c*x+1)-1/2*b/d*dilog(((c*x+1)*e+c*d-e)/(c*d-e))-1/2*b/
d*ln(c*x+1)*ln(((c*x+1)*e+c*d-e)/(c*d-e))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x(d+ex)}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + ex)}\,dx = \int \frac{a + b\,\text{atanh}\,(cx)}{x\,(d + ex)}\,dx$$

input
```
integrate((a+b*atanh(c*x))/x/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + ex)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="maxima")
```

output
```
-a*(log(e*x + d)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x
+ 1))/(e*x^2 + d*x), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x(d + ex)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+ex)}\,dx = \int \frac{a+b\,\text{atanh}(c\,x)}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x*(d + e*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{a+b\text{arctanh}(cx)}{x(d+ex)}\,dx = \frac{-atanh(cx)^2\,bcd-2\left(\int \frac{atanh(cx)}{c^2e\,x^4+c^2d\,x^3-e\,x^2-dx}dx\right)bde-2\left(\int \frac{atanh(cx)}{c^2e\,x^3+c^2d\,x^2-ex-d}dx\right)b\,c^2d^2-2\log(ex+d)\,ae -}{2de}$$

input
```
int((a+b*atanh(c*x))/x/(e*x+d),x)
```

output
```
( - atanh(c*x)**2*b*c*d - 2*int(atanh(c*x)/(c**2*d*x**3 + c**2*e*x**4 - d*
x - e*x**2),x)*b*d*e - 2*int(atanh(c*x)/(c**2*d*x**2 + c**2*e*x**3 - d - e
*x),x)*b*c**2*d**2 - 2*log(d + e*x)*a*e + 2*log(x)*a*e)/(2*d*e)
```

3.152 $\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\begin{aligned}\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+ex)}\,dx = &-\frac{a+b\operatorname{arctanh}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{ae\log(x)}{d^2}\\ &-\frac{e(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^2}\\ &+\frac{e(a+b\operatorname{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d^2}\\ &-\frac{bc\log(1-c^2x^2)}{2d}+\frac{be\operatorname{PolyLog}(2,-cx)}{2d^2}-\frac{be\operatorname{PolyLog}(2,cx)}{2d^2}\\ &+\frac{be\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d^2}-\frac{be\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2d^2}\end{aligned}$$

output
```
-(a+b*arctanh(c*x))/d/x+b*c*ln(x)/d-a*e*ln(x)/d^2-e*(a+b*arctanh(c*x))*ln(
2/(c*x+1))/d^2+e*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2-1/
2*b*c*ln(-c^2*x^2+1)/d+1/2*b*e*polylog(2,-c*x)/d^2-1/2*b*e*polylog(2,c*x)/
d^2+1/2*b*e*polylog(2,1-2/(c*x+1))/d^2-1/2*b*e*polylog(2,1-2*c*(e*x+d)/(c*
d+e)/(c*x+1))/d^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.80

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+ex)}\,dx =$$

$$-\frac{\frac{2ad^2}{x} - ibde\pi\text{arctanh}(cx) + \frac{2bd^2\text{arctanh}(cx)}{x} - 2bde\text{arctanh}\left(\frac{cd}{e}\right)\text{arctanh}(cx) + bde\text{arctanh}(cx)^2 - \frac{be^2\text{ar}}{}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

output
```
-1/2*((2*a*d^2)/x - I*b*d*e*Pi*ArcTanh[c*x] + (2*b*d^2*ArcTanh[c*x])/x - 2
*b*d*e*ArcTanh[(c*d)/e]*ArcTanh[c*x] + b*d*e*ArcTanh[c*x]^2 - (b*e^2*ArcTa
nh[c*x]^2)/c + (b*Sqrt[1 - (c^2*d^2)/e^2]*e^2*ArcTanh[c*x]^2)/(c*E^ArcTanh
[(c*d)/e]) + 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*b*d*e*P
i*Log[1 + E^(2*ArcTanh[c*x])] - 2*b*d*e*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(Ar
cTanh[(c*d)/e] + ArcTanh[c*x]))] - 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*(Arc
Tanh[(c*d)/e] + ArcTanh[c*x]))] + 2*a*d*e*Log[x] - 2*a*d*e*Log[d + e*x] -
2*b*c*d^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + (I/2)*b*d*e*Pi*Log[1 - c^2*x^2] +
 2*b*d*e*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - b
*d*e*PolyLog[2, E^(-2*ArcTanh[c*x])] + b*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*
d)/e] + ArcTanh[c*x]))])/d^3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(\frac{e^2(a+b\text{arctanh}(cx))}{d^2(d+ex)}-\frac{e(a+b\text{arctanh}(cx))}{d^2x}+\frac{a+b\text{arctanh}(cx)}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}+\frac{e(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^2}-\frac{a+b\text{arctanh}(cx)}{dx}-\frac{ae\log(x)}{d^2}-\frac{bc\log\left(1-c^2x^2\right)}{2d}+\frac{be\,\text{PolyLog}(2,-cx)}{2d^2}-\frac{be\,\text{PolyLog}(2,cx)}{2d^2}+\frac{be\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}-\frac{be\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d^2}+\frac{bc\log(x)}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

output
```
-((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a
+ b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])*Log[(2*c
*(d + e*x))/((c*d + e)*(1 + c*x))])/d^2 - (b*c*Log[1 - c^2*x^2])/(2*d) + (
b*e*PolyLog[2, -(c*x)])/(2*d^2) - (b*e*PolyLog[2, c*x])/(2*d^2) + (b*e*Pol
yLog[2, 1 - 2/(1 + c*x)])/(2*d^2) - (b*e*PolyLog[2, 1 - (2*c*(d + e*x))/((
c*d + e)*(1 + c*x))])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.34

method	result
parts	$a\left(-\frac{1}{dx}-\frac{e\ln(x)}{d^2}+\frac{e\ln(ex+d)}{d^2}\right)+bc\left(-\frac{\operatorname{arctanh}(cx)}{dcx}-\frac{\operatorname{arctanh}(cx)e\ln(cx)}{c\,d^2}+\frac{\operatorname{arctanh}(cx)e\ln(cex+cd)}{c\,d^2}\right.$
derivativedivides	$c\left(\frac{ae\ln(cex+cd)}{c\,d^2}-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+bc\left(\frac{\operatorname{arctanh}(cx)e\ln(cex+cd)}{d^2c^2}-\frac{\operatorname{arctanh}(cx)}{d\,c^2x}-\frac{\operatorname{arctanh}(cx)e\ln(cx)}{d^2c^2}\right.\right.$
default	$c\left(\frac{ae\ln(cex+cd)}{c\,d^2}-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+bc\left(\frac{\operatorname{arctanh}(cx)e\ln(cex+cd)}{d^2c^2}-\frac{\operatorname{arctanh}(cx)}{d\,c^2x}-\frac{\operatorname{arctanh}(cx)e\ln(cx)}{d^2c^2}\right.\right.$
risch	$-\frac{be\operatorname{dilog}(-cx+1)}{2d^2}-\frac{be\operatorname{dilog}\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2d^2}-\frac{be\ln(-cx+1)\ln\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2d^2}+\frac{cb\ln(-cx)}{2d}-\frac{cb\ln(-c}{2d}$

input
```
int((a+b*arctanh(c*x))/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x-e/d^2*ln(x)+e/d^2*ln(e*x+d))+b*c*(-arctanh(c*x)/d/c/x-1/c*arctan
h(c*x)*e/d^2*ln(c*x)+1/c*arctanh(c*x)*e/d^2*ln(c*e*x+c*d)-c*(1/d/c*(1/2*ln
(c*x-1)-ln(c*x)+1/2*ln(c*x+1))+1/d^2/c^2*e*(-1/2*dilog(c*x)-1/2*dilog(c*x+
1)-1/2*ln(c*x)*ln(c*x+1))-1/d^2/c^2*(1/2*e*(dilog((c*e*x-e)/(-c*d-e))+ln(c
*e*x+c*d)*ln((c*e*x-e)/(-c*d-e)))-1/2*e*(dilog((c*e*x+e)/(-c*d+e))+ln(c*e*
x+c*d)*ln((c*e*x+e)/(-c*d+e))))))
```

Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^2(d+ex)}\,dx=\int \frac{b\operatorname{artanh}(cx)+a}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(e*x^3 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + ex)}\,dx = \int \frac{a + b\,\text{atanh}\,(cx)}{x^2\,(d + ex)}\,dx$$

input
```
integrate((a+b*atanh(c*x))/x**2/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + ex)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*b*integrate((log(c*x
 + 1) - log(-c*x + 1))/(e*x^3 + d*x^2), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^2(d + ex)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atanh}(c\,x)}{x^2\,(d+e\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^2*(d + e*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2(d+ex)}\,dx = \frac{-\left(\int \frac{atanh(cx)}{c^2e\,x^5+c^2d\,x^4-e\,x^3-d\,x^2}dx\right)b\,d^2x+\left(\int \frac{atanh(cx)}{c^2e\,x^3+c^2d\,x^2-ex-d}dx\right)b\,c^2d^2x+\log(ex+d)\,aex-\log(x)\,aex-a}{d^2x}$$

input
```
int((a+b*atanh(c*x))/x^2/(e*x+d),x)
```

output
```
( - int(atanh(c*x)/(c**2*d*x**4 + c**2*e*x**5 - d*x**2 - e*x**3),x)*b*d**2
*x + int(atanh(c*x)/(c**2*d*x**2 + c**2*e*x**3 - d - e*x),x)*b*c**2*d**2*x
 + log(d + e*x)*a*e*x - log(x)*a*e*x - a*d)/(d**2*x)
```

3.153 $\int \frac{a+b\text{arctanh}(cx)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 261

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx)}{x^3(d+ex)}\,dx =& -\frac{bc}{2dx}+\frac{bc^2\text{arctanh}(cx)}{2d}-\frac{a+b\text{arctanh}(cx)}{2dx^2} \\ &+\frac{e(a+b\text{arctanh}(cx))}{d^2x}-\frac{bce\log(x)}{d^2} \\ &+\frac{ae^2\log(x)}{d^3}+\frac{e^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1+cx}\right)}{d^3} \\ &-\frac{e^2(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d^3} \\ &+\frac{bce\log(1-c^2x^2)}{2d^2}-\frac{be^2\text{PolyLog}(2,-cx)}{2d^3} \\ &+\frac{be^2\text{PolyLog}(2,cx)}{2d^3}-\frac{be^2\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2d^3} \\ &+\frac{be^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2d^3}\end{aligned}$$

output
```
-1/2*b*c/d/x+1/2*b*c^2*arctanh(c*x)/d-1/2*(a+b*arctanh(c*x))/d/x^2+e*(a+b*
arctanh(c*x))/d^2/x-b*c*e*ln(x)/d^2+a*e^2*ln(x)/d^3+e^2*(a+b*arctanh(c*x))
*ln(2/(c*x+1))/d^3-e^2*(a+b*arctanh(c*x))*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/
d^3+1/2*b*c*e*ln(-c^2*x^2+1)/d^2-1/2*b*e^2*polylog(2,-c*x)/d^3+1/2*b*e^2*p
olylog(2,c*x)/d^3-1/2*b*e^2*polylog(2,1-2/(c*x+1))/d^3+1/2*b*e^2*polylog(2
,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.67

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+ex)}\,dx = -\frac{a}{2dx^2}+\frac{ae}{d^2x}+\frac{ae^2\log(x)}{d^3}-\frac{ae^2\log(d+ex)}{d^3}$$
$$-\frac{b\left(\frac{c^2d^3}{x}+icde^2\pi\text{arctanh}(cx)-\frac{2cd^2e\text{arctanh}(cx)}{x}+\frac{cd^3(1-c^2x^2)\text{arctanh}(cx)}{x^2}+2cde^2\text{arctanh}\left(\frac{cd}{e}\right)\text{arctanh}(c\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

output
```
-1/2*a/(d*x^2) + (a*e)/(d^2*x) + (a*e^2*Log[x])/d^3 - (a*e^2*Log[d + e*x])
/d^3 - (b*((c^2*d^3)/x + I*c*d*e^2*Pi*ArcTanh[c*x] - (2*c*d^2*e*ArcTanh[c*
x])/x + (c*d^3*(1 - c^2*x^2)*ArcTanh[c*x])/x^2 + 2*c*d*e^2*ArcTanh[(c*d)/e
]*ArcTanh[c*x] - c*d*e^2*ArcTanh[c*x]^2 + e^3*ArcTanh[c*x]^2 - (Sqrt[1 - (
c^2*d^2)/e^2]*e^3*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e^2*ArcTanh[c
*x]*Log[1 - E^(-2*ArcTanh[c*x])] - I*c*d*e^2*Pi*Log[1 + E^(2*ArcTanh[c*x])
] + 2*c*d*e^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))] + 2*c*d*e^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[
c*x]))] + I*c*d*e^2*Pi*Log[1/Sqrt[1 - c^2*x^2]] + 2*c^2*d^2*e*Log[(c*x)/Sq
rt[1 - c^2*x^2]] - 2*c*d*e^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e]
+ ArcTanh[c*x]]] + c*d*e^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - c*d*e^2*PolyL
og[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(2*c*d^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(-\frac{e^3(a+b\text{arctanh}(cx))}{d^3(d+ex)}+\frac{e^2(a+b\text{arctanh}(cx))}{d^3x}-\frac{e(a+b\text{arctanh}(cx))}{d^2x^2}+\frac{a+b\text{arctanh}(cx)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^3}-\frac{e^2(a+b\text{arctanh}(cx))\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^3}+\frac{e(a+b\text{arctanh}(cx))}{d^2x}-\frac{a+b\text{arctanh}(cx)}{2dx^2}+\frac{ae^2\log(x)}{d^3}+\frac{bc^2\text{arctanh}(cx)}{2d}+\frac{bce\log(1-c^2x^2)}{2d^2}-\frac{be^2\text{PolyLog}(2,-cx)}{2d^3}+\frac{be^2\text{PolyLog}(2,cx)}{2d^3}-\frac{be^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{be^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d^3}-\frac{bce\log(x)}{d^2}-\frac{bc}{2dx}$$

input
```
Int[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

output
```
-1/2*(b*c)/(d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*
x^2) + (e*(a + b*ArcTanh[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[
x])/d^3 + (e^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (e^2*(a + b*Ar
cTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d^3 + (b*c*e*Log[1
 - c^2*x^2])/(2*d^2) - (b*e^2*PolyLog[2, -(c*x)])/(2*d^3) + (b*e^2*PolyLog
[2, c*x])/(2*d^3) - (b*e^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3) + (b*e^2*P
olyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.23

method	result
parts	$a\left(-\frac{1}{2d\,x^2}+\frac{e^2\ln(x)}{d^3}+\frac{e}{d^2x}-\frac{e^2\ln(ex+d)}{d^3}\right)+b\,c^2\left(-\frac{\operatorname{arctanh}(cx)}{2d\,c^2x^2}+\frac{\operatorname{arctanh}(cx)e^2\ln(cx)}{c^2d^3}+\frac{\operatorname{arctanh}(\cdots}{c^2d^2x}\right.$
derivativedivides	$c^2\left(-\frac{a\,e^2\ln(cex+cd)}{c^2d^3}-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}+bc\left(-\frac{\operatorname{arctanh}(cx)e^2\ln(cex+cd)}{d^3c^3}-\frac{\operatorname{arctanh}(cx)\cdots}{2d\,c^3x^2}\right.\right.$
default	$c^2\left(-\frac{a\,e^2\ln(cex+cd)}{c^2d^3}-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}+bc\left(-\frac{\operatorname{arctanh}(cx)e^2\ln(cex+cd)}{d^3c^3}-\frac{\operatorname{arctanh}(cx)\cdots}{2d\,c^3x^2}\right.\right.$
risch	$\frac{c^2b\ln(-cx)}{4d}-\frac{bc}{2dx}-\frac{c^2b\ln(-cx+1)}{4d}+\frac{b\ln(-cx+1)}{4d\,x^2}+\frac{b\,e^2\,\mathrm{dilog}(-cx+1)}{2d^3}+\frac{b\,e^2\,\mathrm{dilog}\left(\frac{(-cx+1)e-cd-e}{-cd-e}\right)}{2d^3}+\cdots$

input
```
int((a+b*arctanh(c*x))/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x-e^2/d^3*ln(e*x+d))+b*c^2*(-1/2*arctanh
(c*x)/d/c^2/x^2+1/c^2*arctanh(c*x)*e^2/d^3*ln(c*x)+1/c^2*arctanh(c*x)*e/d^
2/x-1/c^2*arctanh(c*x)*e^2/d^3*ln(c*e*x+c*d)-1/2*c*(-2/d^3/c^3*e^2*(-1/2*d
ilog(c*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1))+2/d^3/c^3*e*(1/2*e*(dilo
g((c*e*x-e)/(-c*d-e))+ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e)))-1/2*e*(dilog((
c*e*x+e)/(-c*d+e))+ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))))-1/c^2/d^2*(-d/x-
2*e*ln(c*x)+(-1/2*c*d+e)*ln(c*x-1)+(1/2*c*d+e)*ln(c*x+1))))
```

Fricas [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + ex)} \, dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/(e*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + ex)} \, dx = \int \frac{a + b\,\text{atanh}\,(cx)}{x^3\,(d + ex)} \, dx$$

input
```
integrate((a+b*atanh(c*x))/x**3/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + ex)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
 + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x^4 + d*x^3), x)
```

Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + ex)}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{(ex + d)x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + ex)}\,dx = \int \frac{a + b\,\text{atanh}(c\,x)}{x^3\,(d + e\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))/(x^3*(d + e*x)),x)
```

output
```
int((a + b*atanh(c*x))/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x^3(d + ex)}\,dx$$
$$= \frac{atanh(cx)^2\,bcde\,x^2 + atanh(cx)\,b\,c^2d^2x^2 - atanh(cx)\,b\,d^2 + 2\left(\int \frac{atanh(cx)}{c^2e\,x^5+c^2d\,x^4-e\,x^3-d\,x^2}dx\right)b\,d^2e\,x^2 + 2\left(\int \right.}{2d^3x^2}$$

input
```
int((a+b*atanh(c*x))/x^3/(e*x+d),x)
```

output
```
(atanh(c*x)**2*b*c*d*e*x**2 + atanh(c*x)*b*c**2*d**2*x**2 - atanh(c*x)*b*d
**2 + 2*int(atanh(c*x)/(c**2*d*x**4 + c**2*e*x**5 - d*x**2 - e*x**3),x)*b*
d**2*e*x**2 + 2*int((atanh(c*x)*x)/(c**2*d*x**2 + c**2*e*x**3 - d - e*x),x
)*b*c**2*d*e**2*x**2 - 2*log(d + e*x)*a*e**2*x**2 + 2*log(x)*a*e**2*x**2 -
 a*d**2 + 2*a*d*e*x - b*c*d**2*x)/(2*d**3*x**2)
```

3.154 $\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 385

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx = \frac{abx}{ce}+\frac{b^2x\operatorname{arctanh}(cx)}{ce}-\frac{d(a+b\operatorname{arctanh}(cx))^2}{ce^2} - \frac{(a+b\operatorname{arctanh}(cx))^2}{2c^2e}-\frac{dx(a+b\operatorname{arctanh}(cx))^2}{e^2} + \frac{x^2(a+b\operatorname{arctanh}(cx))^2}{2e} + \frac{2bd(a+b\operatorname{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{ce^2} - \frac{d^2(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{e^3} + \frac{d^2(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^3} + \frac{b^2\log(1-c^2x^2)}{2c^2e}+\frac{b^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{ce^2} + \frac{bd^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{e^3} - \frac{bd^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^3} + \frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2e^3} - \frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e^3}$$

output

```
a*b*x/c/e+b^2*x*arctanh(c*x)/c/e-d*(a+b*arctanh(c*x))^2/c/e^2-1/2*(a+b*arc
tanh(c*x))^2/c^2/e-d*x*(a+b*arctanh(c*x))^2/e^2+1/2*x^2*(a+b*arctanh(c*x))
^2/e+2*b*d*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c/e^2-d^2*(a+b*arctanh(c*x))^
2*ln(2/(c*x+1))/e^3+d^2*(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1
))/e^3+1/2*b^2*ln(-c^2*x^2+1)/c^2/e+b^2*d*polylog(2,1-2/(-c*x+1))/c/e^2+b*
d^2*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e^3-b*d^2*(a+b*arctanh(c*x))
*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3+1/2*b^2*d^2*polylog(3,1-2/(c
*x+1))/e^3-1/2*b^2*d^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.27 (sec) , antiderivative size = 1414, normalized size of antiderivative = 3.67

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+ex}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

output
```
-((a^2*d*x)/e^2) + (a^2*x^2)/(2*e) + (a^2*d^2*Log[d + e*x])/e^3 + (a*b*(c*
e^2*x + I*c^2*d^2*Pi*ArcTanh[c*x] - 2*c^2*d*e*x*ArcTanh[c*x] - e^2*(1 - c^
2*x^2)*ArcTanh[c*x] + 2*c^2*d^2*ArcTanh[(c*d)/e]*ArcTanh[c*x] - c^2*d^2*Ar
cTanh[c*x]^2 + c*d*e*ArcTanh[c*x]^2 - (c*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTa
nh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c^2*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTa
nh[c*x])] - I*c^2*d^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c^2*d^2*ArcTanh[(
c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*ArcT
anh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c*d*e*Log[1
/Sqrt[1 - c^2*x^2]] + I*c^2*d^2*Pi*Log[1/Sqrt[1 - c^2*x^2]] - 2*c^2*d^2*Ar
cTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + c^2*d^2*Poly
Log[2, -E^(-2*ArcTanh[c*x])] - c^2*d^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e]
+ ArcTanh[c*x]))]))/(c^2*e^3) + (b^2*((6*c*e^2*x*ArcTanh[c*x] + 6*c*d*e*Ar
cTanh[c*x]^2 - 6*c^2*d*e*x*ArcTanh[c*x]^2 - 3*e^2*(1 - c^2*x^2)*ArcTanh[c*
x]^2 - 2*c^2*d^2*ArcTanh[c*x]^3 + 2*c*d*e*ArcTanh[c*x]^3 + 12*c*d*e*ArcTan
h[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 6*c^2*d^2*ArcTanh[c*x]^2*Log[1 + E^(
-2*ArcTanh[c*x])] - 6*e^2*Log[1/Sqrt[1 - c^2*x^2]] + 6*c*d*(-e + c*d*ArcTa
nh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*c^2*d^2*PolyLog[3, -E^(-2*Ar
cTanh[c*x])])/(6*e^3) - (c*d*(-(c*d) + e)*(c*d + e)*(-6*c*d*ArcTanh[c*x]^3
 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^3)/E^Arc
Tanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 - (Sqrt[c*d + e]*E^ArcTanh[c...
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+ex}dx$$
$$\Big\downarrow \text{6502}$$
$$\int \left(\frac{d^2(a+b\text{arctanh}(cx))^2}{e^2(d+ex)}-\frac{d(a+b\text{arctanh}(cx))^2}{e^2}+\frac{x(a+b\text{arctanh}(cx))^2}{e}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{(a+b\text{arctanh}(cx))^2}{2c^2e}+\frac{bd^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e^3}-\frac{bd^2(a+b\text{arctanh}(cx))\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e^3}-\frac{d^2\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{e^3}+\frac{d^2(a+b\text{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}-\frac{dx(a+b\text{arctanh}(cx))^2}{e^2}-\frac{d(a+b\text{arctanh}(cx))^2}{ce^2}+\frac{2bd\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{ce^2}+\frac{x^2(a+b\text{arctanh}(cx))^2}{2e}+\frac{abx}{ce}+\frac{b^2x\text{arctanh}(cx)}{ce}+\frac{b^2\log(1-c^2x^2)}{2c^2e}+\frac{b^2d^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2e^3}-\frac{b^2d^2\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^3}+\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{ce^2}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

output
```
(a*b*x)/(c*e) + (b^2*x*ArcTanh[c*x])/(c*e) - (d*(a + b*ArcTanh[c*x])^2)/(c
*e^2) - (a + b*ArcTanh[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTanh[c*x])^2)/e^
2 + (x^2*(a + b*ArcTanh[c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/(c*e^2) - (d^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e^3 +
 (d^2*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e
^3 + (b^2*Log[1 - c^2*x^2])/(2*c^2*e) + (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)]
)/(c*e^2) + (b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^3 -
 (b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1
+ c*x))])/e^3 + (b^2*d^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e^3) - (b^2*d^2*P
olyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.75 (sec) , antiderivative size = 1573, normalized size of antiderivative = 4.09

method	result	size
parts	Expression too large to display	1573
derivativedivides	Expression too large to display	1577
default	Expression too large to display	1577

input
```
int(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2/e*x^2-a^2/e^2*d*x+a^2*d^2/e^3*ln(e*x+d)+b^2/c^3*(1/2*c^3*arctanh(c
*x)^2/e*x^2-c^3*arctanh(c*x)^2/e^2*d*x+c^3*arctanh(c*x)^2*d^2/e^3*ln(c*e*x
+c*d)-2*c*(1/4/e*arctanh(c*x)^2+1/2/e^2*d*c*arctanh(c*x)^2-1/4*I/e^3*c^2*d
^2*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-
1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e
*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-1/e
^2*c*d*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/e^2*c*d*dilog(1-I*(c*x+1)/(
-c^2*x^2+1)^(1/2))-1/2*(c*x+1)*arctanh(c*x)/e+1/2/e*ln(1+(c*x+1)^2/(-c^2*x
^2+1))-1/e^2*c*d*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/e^2*c*d
*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/4*I/e^3*c^2*d^2*Pi*csgn
(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)
^2/(c^2*x^2-1)))^3*arctanh(c*x)^2+1/4*I/e^3*c^2*d^2*Pi*csgn(I/(1-(c*x+1)^2
/(c^2*x^2-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^
2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+1/4*I/e^3*c^2*d^2*Pi*
csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(
I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^
2/(c^2*x^2-1)))^2*arctanh(c*x)^2+1/2*d^2*c^2/e^3*arctanh(c*x)*polylog(2,-(
c*x+1)^2/(-c^2*x^2+1))-1/4*d^2*c^2/e^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+
1/2*d^2*c^2/e^3*arctanh(c*x)^2*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1
)^2/(-c^2*x^2+1)-1))-1/2*d^2*c^2/e^2/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e...
```

Fricas [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(e*x
+ d), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))^2}{d + ex} dx = \int \frac{x^2(a + b\,\text{atanh}\,(cx))^2}{d + ex} dx$$

input
```
integrate(x**2*(a+b*atanh(c*x))**2/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*atanh(c*x))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))^2}{d + ex} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2 x^2}{ex + d} dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/8*(b^2*e*x^2 -
2*b^2*d*x)*log(-c*x + 1)^2/e^2 - integrate(-1/4*((b^2*c*e^2*x^3 - b^2*e^2*
x^2)*log(c*x + 1)^2 + 4*(a*b*c*e^2*x^3 - a*b*e^2*x^2)*log(c*x + 1) + (2*b^
2*c*d^2*x - (4*a*b*c*e^2 + b^2*c*e^2)*x^3 + (b^2*c*d*e + 4*a*b*e^2)*x^2 -
2*(b^2*c*e^2*x^3 - b^2*e^2*x^2)*log(c*x + 1))*log(-c*x + 1))/(c*e^3*x^2 -
d*e^2 + (c*d*e^2 - e^3)*x), x)
```

Giac [F]

$$\int \frac{x^2(a + b\text{arctanh}(cx))^2}{d + ex} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2 x^2}{ex + d} dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{d+e\,x} \,d x$$

input
```
int((x^2*(a + b*atanh(c*x))^2)/(d + e*x),x)
```

output
```
int((x^2*(a + b*atanh(c*x))^2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))^2}{d+ex}\,dx = \frac{4\left(\int \frac{atanh(cx)x^2}{ex+d}dx\right)ab\,e^3+2\left(\int \frac{atanh(cx)^2x^2}{ex+d}dx\right)b^2e^3+2\log(ex+d)\,a^2d^2-2a^2dex+a^2e^2x^2}{2e^3}$$

input
```
int(x^2*(a+b*atanh(c*x))^2/(e*x+d),x)
```

output
```
(4*int((atanh(c*x)*x**2)/(d + e*x),x)*a*b*e**3 + 2*int((atanh(c*x)**2*x**2
)/(d + e*x),x)*b**2*e**3 + 2*log(d + e*x)*a**2*d**2 - 2*a**2*d*e*x + a**2*
e**2*x**2)/(2*e**3)
```

3.155 $\int \frac{x(a+b\,\mathrm{arctanh}(cx))^2}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 279

$$\begin{aligned}\int \frac{x(a+b\,\mathrm{arctanh}(cx))^2}{d+ex}\,dx &= \frac{(a+b\,\mathrm{arctanh}(cx))^2}{ce}+\frac{x(a+b\,\mathrm{arctanh}(cx))^2}{e}\\
&-\frac{2b(a+b\,\mathrm{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{ce}\\
&+\frac{d(a+b\,\mathrm{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{e^2}\\
&-\frac{d(a+b\,\mathrm{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2}\\
&-\frac{b^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{ce}\\
&-\frac{bd(a+b\,\mathrm{arctanh}(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{e^2}\\
&+\frac{bd(a+b\,\mathrm{arctanh}(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2}\\
&-\frac{b^2d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2e^2}\\
&+\frac{b^2d\,\mathrm{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e^2}\end{aligned}$$

output
```
(a+b*arctanh(c*x))^2/c/e+x*(a+b*arctanh(c*x))^2/e-2*b*(a+b*arctanh(c*x))*l
n(2/(-c*x+1))/c/e+d*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e^2-d*(a+b*arctanh(
c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2-b^2*polylog(2,1-2/(-c*x+1))/c/
e-b*d*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e^2+b*d*(a+b*arctanh(c*x))
*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2-1/2*b^2*d*polylog(3,1-2/(c*x
+1))/e^2+1/2*b^2*d*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.53 (sec) , antiderivative size = 1153, normalized size of antiderivative = 4.13

$$\int \frac{x(a + b\text{arctanh}(cx))^2}{d + ex} \, dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

output
```
(6*a^2*e*x - 6*a^2*d*Log[d + e*x] + (6*a*b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c
*e*x*ArcTanh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]
^2 - e*ArcTanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTa
nh[(c*d)/e] + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*L
og[1 + E^(2*ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh
[(c*d)/e] + ArcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c
*d)/e] + ArcTanh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x
^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]]
- c*d*PolyLog[2, -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*
d)/e] + ArcTanh[c*x]))]))/c + (b^2*(-6*e*ArcTanh[c*x]^2 + 6*c*e*x*ArcTanh[
c*x]^2 + 8*c*d*ArcTanh[c*x]^3 - 4*e*ArcTanh[c*x]^3 + (4*Sqrt[1 - (c^2*d^2)
/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - 12*e*ArcTanh[c*x]*Log[1 + E^(
-2*ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (6
*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] + 6*c*
d*ArcTanh[c*x]^2*Log[1 - (Sqrt[c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]]
+ 6*c*d*ArcTanh[c*x]^2*Log[1 + (Sqrt[c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d)
+ e]] - 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])]
- 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*
d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 12*c*d
*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh...
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+ex}dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{(a+b\text{arctanh}(cx))^2}{e}-\frac{d(a+b\text{arctanh}(cx))^2}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bd\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e^2}+\frac{bd(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e^2}+\frac{d\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{e^2}-\frac{d(a+b\text{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}+\frac{x(a+b\text{arctanh}(cx))^2}{e}+\frac{(a+b\text{arctanh}(cx))^2}{ce}-\frac{2b\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{ce}-\frac{b^2d\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2e^2}+\frac{b^2d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e^2}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{ce}$$

input
```
Int[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

output
```
(a + b*ArcTanh[c*x])^2/(c*e) + (x*(a + b*ArcTanh[c*x])^2)/e - (2*b*(a + b*
ArcTanh[c*x])*Log[2/(1 - c*x)])/(c*e) + (d*(a + b*ArcTanh[c*x])^2*Log[2/(1
 + c*x)])/e^2 - (d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(
1 + c*x))])/e^2 - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e) - (b*d*(a + b*Ar
cTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^2 + (b*d*(a + b*ArcTanh[c*x])*P
olyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^2 - (b^2*d*PolyLog
[3, 1 - 2/(1 + c*x)])/(2*e^2) + (b^2*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*
d + e)*(1 + c*x))])/(2*e^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.59 (sec) , antiderivative size = 13674, normalized size of antiderivative = 49.01

method	result	size
derivativedivides	Expression too large to display	13674
default	Expression too large to display	13674
parts	Expression too large to display	13680

input
```
int(x*(a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(e*x + d),
x)
```

Sympy [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+ex}\,dx = \int \frac{x(a+b\,\text{atanh}\,(cx))^2}{d+ex}\,dx$$

input
```
integrate(x*(a+b*atanh(c*x))**2/(e*x+d),x)
```

output
```
Integral(x*(a + b*atanh(c*x))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/4*b^2*x*log(-c*x + 1)^2/e + a^2*(x/e - d*log(e*x + d)/e^2) - integrate(-
1/4*((b^2*c*e*x^2 - b^2*e*x)*log(c*x + 1)^2 + 4*(a*b*c*e*x^2 - a*b*e*x)*lo
g(c*x + 1) - 2*((2*a*b*c*e + b^2*c*e)*x^2 + (b^2*c*d - 2*a*b*e)*x + (b^2*c
*e*x^2 - b^2*e*x)*log(c*x + 1))*log(-c*x + 1))/(c*e^2*x^2 - d*e + (c*d*e -
 e^2)*x), x)
```

Giac [F]

$$\int \frac{x(a+b\text{arctanh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^2x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{x\,(a+b\,\mathrm{atanh}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x*(a + b*atanh(c*x))^2)/(d + e*x),x)
```

output
```
int((x*(a + b*atanh(c*x))^2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx = \frac{2\left(\int \frac{atanh(cx)x}{ex+d}dx\right)ab\,e^2+\left(\int \frac{atanh(cx)^2x}{ex+d}dx\right)b^2e^2-\log(ex+d)\,a^2d+a^2ex}{e^2}$$

input
```
int(x*(a+b*atanh(c*x))^2/(e*x+d),x)
```

output
```
(2*int((atanh(c*x)*x)/(d + e*x),x)*a*b*e**2 + int((atanh(c*x)**2*x)/(d + e
*x),x)*b**2*e**2 - log(d + e*x)*a**2*d + a**2*e*x)/e**2
```

3.156 $\int \frac{(a+b\text{arctanh}(cx))^2}{d+ex} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 188

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+ex} \, dx = -\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{e} + \frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b(a+b\text{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{e} - \frac{b(a+b\text{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2e} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e}$$

output
```
-(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/e+(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/
(c*d+e)/(c*x+1))/e+b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/e-b*(a+b*ar
ctanh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*b^2*polylog(3,1
-2/(c*x+1))/e-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.58 (sec) , antiderivative size = 1055, normalized size of antiderivative = 5.61

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

output
```
(6*a^2*Log[d + e*x] + 6*a*b*ArcTanh[c*x]*(Log[1 - c^2*x^2] + 2*Log[I*Sinh[
ArcTanh[(c*d)/e] + ArcTanh[c*x]]]) - (6*I)*a*b*((-1/4*I)*(Pi - (2*I)*ArcTa
nh[c*x])^2 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c
*x])*Log[1 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])
*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c
*x])*Log[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Lo
g[(2*I)*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTan
h[c*x])] - I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]) + (b^2*
(-8*c*d*ArcTanh[c*x]^3 + 4*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e
*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*A
rcTanh[c*x])] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTa
nh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 - (Sqrt[c*d + e]*E^ArcTanh[c*x])/
Sqrt[-(c*d) + e]] - 6*c*d*ArcTanh[c*x]^2*Log[1 + (Sqrt[c*d + e]*E^ArcTanh[
c*x])/Sqrt[-(c*d) + e]] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + Arc
Tanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTan
h[c*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*
d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*
c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTan
h[c*x])))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/...
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6474}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{ 6474}$$

$$-\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e}+\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{e}-\frac{\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{e}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2e}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

output
```
-(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^2*L
og[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*(a + b*ArcTanh[c*x])*Pol
yLog[2, 1 - 2/(1 + c*x)])/e - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*
(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(
2*e) - (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Defintions of rubi rules used

rule 6474
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.00 (sec) , antiderivative size = 1087, normalized size of antiderivative = 5.78

method	result	size
derivativedivides	Expression too large to display	1087
default	Expression too large to display	1087
parts	Expression too large to display	1090

input
```
int((a+b*arctanh(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*ln(c*e*x+c*d)/e+b^2*c*(ln(c*e*x+c*d)/e*arctanh(c*x)^2-2/e*(1/2*
arctanh(c*x)^2*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)
-1))-1/4*I*Pi*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2
-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))*(csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+
e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))-csgn(I*(d
*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c
^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))-csgn(I*(d*c*(1-(c*x+1)^2/(c^
2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2
-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))+csgn(I*(d*c*
(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*
x^2-1)))^2)*arctanh(c*x)^2+1/2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2
+1))-1/4*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/2/(c*d+e)*e*arctanh(c*x)^2*l
n(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2/(c*d+e)*e*arctanh(c*x)*po
lylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/4/(c*d+e)*e*polylog(3,(
c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2/(c*d+e)*d*c*arctanh(c*x)^2*ln(
1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2/(c*d+e)*d*c*arctanh(c*x)*po
lylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/4/(c*d+e)*d*c*polylog(3
,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))))+2*a*b*c*(ln(c*e*x+c*d)/e*arcta
nh(c*x)-1/e^2*(-1/2*e*(dilog((c*e*x-e)/(-c*d-e))+ln(c*e*x+c*d)*ln((c*e*x-e
)/(-c*d-e)))+1/2*e*(dilog((c*e*x+e)/(-c*d+e))+ln(c*e*x+c*d)*ln((c*e*x+e...
```

Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(a+b\,\text{atanh}\,(cx))^2}{d+ex}\,dx$$

input
```
integrate((a+b*atanh(c*x))**2/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))**2/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*log(e*x + d)/e + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/(e
*x + d) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx=\int \frac{(a+b\,\mathrm{atanh}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(d + e*x),x)
```

output
```
int((a + b*atanh(c*x))^2/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{d+ex}\,dx=\frac{2\left(\int \frac{atanh(cx)}{ex+d}dx\right)abe+\left(\int \frac{atanh(cx)^2}{ex+d}dx\right)b^2e+\log(ex+d)\,a^2}{e}$$

input
```
int((a+b*atanh(c*x))^2/(e*x+d),x)
```

output
```
(2*int(atanh(c*x)/(d + e*x),x)*a*b*e + int(atanh(c*x)**2/(d + e*x),x)*b**2
*e + log(d + e*x)*a**2)/e
```

3.157 $\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 319

$$
\begin{aligned}
\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+ex)}\,dx =& \frac{2(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)}{d} \\
&+\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d} \\
&-\frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} \\
&-\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d} \\
&+\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d} \\
&-\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d} \\
&+\frac{b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} \\
&+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d} \\
&-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d} \\
&+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2d}
\end{aligned}
$$

output
```
-2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))/d+(a+b*arctanh(c*x))^2*ln(2
/(c*x+1))/d-(a+b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d-b*(a+b*
arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d+b*(a+b*arctanh(c*x))*polylog(2,-1+
2/(-c*x+1))/d-b*(a+b*arctanh(c*x))*polylog(2,1-2/(c*x+1))/d+b*(a+b*arctanh
(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d+1/2*b^2*polylog(3,1-2/(-
c*x+1))/d-1/2*b^2*polylog(3,-1+2/(-c*x+1))/d-1/2*b^2*polylog(3,1-2/(c*x+1)
)/d+1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.73 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.61

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

output
```
(a^2*Log[x])/d - (a^2*Log[d + e*x])/d + (a*b*((-I)*c*d*Pi*ArcTanh[c*x] - 2
*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2
 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*A
rcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c
*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]
))] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[Ar
cTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*
d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(c*d^2) + (b^2*(I
*c*d*Pi^3 - 8*c*d*ArcTanh[c*x]^3 - 8*e*ArcTanh[c*x]^3 + 24*c*d*ArcTanh[c*x
]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*ArcT
anh[c*x])] - 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] - (24*(c*d - e)*(c*d +
e)*(-6*c*d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2
]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^
(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 - (Sqrt[
c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]] - 6*c*d*ArcTanh[c*x]^2*Log[1 +
(Sqrt[c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]] + 6*c*d*ArcTanh[c*x]^2*Lo
g[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 +
E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(
ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*...
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(\frac{(a+b\text{arctanh}(cx))^2}{dx}-\frac{e(a+b\text{arctanh}(cx))^2}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d}-\frac{(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{d}+\frac{b\operatorname{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\operatorname{arctanh}(cx))}{d}-\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))}{d}+\frac{2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))^2}{d}+\frac{\log\left(\frac{2}{cx+1}\right)(a+b\operatorname{arctanh}(cx))^2}{d}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

output
```
(2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d + ((a + b*ArcTanh[c*
x])^2*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((
c*d + e)*(1 + c*x))])/d - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*
x)])/d + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d - (b*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d + (b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + (b^2*PolyLog[3,
 1 - 2/(1 - c*x)])/(2*d) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d) - (b^2
*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/
((c*d + e)*(1 + c*x))])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 1679, normalized size of antiderivative = 5.26

method	result	size
parts	Expression too large to display	1679
derivativedivides	Expression too large to display	1689
default	Expression too large to display	1689

input
```
int((a+b*arctanh(c*x))^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*ln(x)-a^2/d*ln(e*x+d)+b^2*(arctanh(c*x)^2/d*ln(c*x)-arctanh(c*x)^2/d
*ln(c*e*x+c*d)-2*c*(-1/2/d/c*arctanh(c*x)^2*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+
1))+e*((c*x+1)^2/(-c^2*x^2+1)-1))-1/4*I*Pi*(csgn(I/(1-(c*x+1)^2/(c^2*x^2-1
)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(
1-(c*x+1)^2/(c^2*x^2-1)))-csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(d*c*(1
-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I*(d*c*(1-(c*x
+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1))
)-csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(
c*x+1)^2/(c^2*x^2-1)))^2+csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(d*c*(1-
(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2
-1)))^2-csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-
(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+
1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))^2+csgn(I*(-(c*x+1)^2/(c^2*
x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3-csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1
))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))^3)*arctanh(c*x
)^2/c/d+1/2/d/c*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-1/2/d/c*arctan
h(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-1/d/c*arctanh(c*x)*polylog(2,(c*
x+1)/(-c^2*x^2+1)^(1/2))+1/d/c*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2/d
/c*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-1/d/c*arctanh(c*x)*p...
```

Fricas [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x(d + ex)}\, dx = \int \frac{(b\operatorname{artanh}(cx) + a)^2}{(ex + d)x}\, dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x(d + ex)}\, dx = \int \frac{(a + b\operatorname{atanh}(cx))^2}{x(d + ex)}\, dx$$

input
```
integrate((a+b*atanh(c*x))**2/x/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))**2/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x(d + ex)}\, dx = \int \frac{(b\operatorname{artanh}(cx) + a)^2}{(ex + d)x}\, dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

output
```
-a^2*(log(e*x + d)/d - log(x)/d) + integrate(1/4*b^2*(log(c*x + 1) - log(-
c*x + 1))^2/(e*x^2 + d*x) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x^2 + d*
x), x)
```

Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x(d+ex)}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x(d+ex)}\,dx = \int \frac{(a+b\operatorname{atanh}(c\,x))^2}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x*(d + e*x)),x)
```

output
```
int((a + b*atanh(c*x))^2/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x(d+ex)}\,dx = \frac{-atanh(cx)^3 b^2cd - 3atanh(cx)^2 abcd - 6\left(\int \frac{atanh(cx)}{c^2ex^4+c^2dx^3-ex^2-dx}dx\right) abde - 6\left(\int \frac{atanh(cx)}{c^2ex^3+c^2dx^2-ex-d}dx\right) ab\,c}{3}$$

input
```
int((a+b*atanh(c*x))^2/x/(e*x+d),x)
```

output

```
( - atanh(c*x)**3*b**2*c*d - 3*atanh(c*x)**2*a*b*c*d - 6*int(atanh(c*x)/(c
**2*d*x**3 + c**2*e*x**4 - d*x - e*x**2),x)*a*b*d*e - 6*int(atanh(c*x)/(c*
*2*d*x**2 + c**2*e*x**3 - d - e*x),x)*a*b*c**2*d**2 - 3*int(atanh(c*x)**2/
(c**2*d*x**3 + c**2*e*x**4 - d*x - e*x**2),x)*b**2*d*e - 3*int(atanh(c*x)*
*2/(c**2*d*x**2 + c**2*e*x**3 - d - e*x),x)*b**2*c**2*d**2 - 3*log(d + e*x
)*a**2*e + 3*log(x)*a**2*e)/(3*d*e)
```

3.158 $\int \frac{(a+b\,\mathbf{arctanh}(cx))^2}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 412

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(d+ex)}\,dx = \frac{c(a+b\operatorname{arctanh}(cx))^2}{d} - \frac{(a+b\operatorname{arctanh}(cx))^2}{dx} - \frac{2e(a+b\operatorname{arctanh}(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)}{d^2} - \frac{e(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d^2} + \frac{2bc(a+b\operatorname{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)}{d} + \frac{be(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d^2} - \frac{be(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d^2} + \frac{be(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d^2} - \frac{b^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)}{d} - \frac{be(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d^2} - \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d^2} - \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2d^2}$$

output
```
c*(a+b*arctanh(c*x))^2/d-(a+b*arctanh(c*x))^2/d/x+2*e*(a+b*arctanh(c*x))^2
*arctanh(-1+2/(-c*x+1))/d^2-e*(a+b*arctanh(c*x))^2*ln(2/(c*x+1))/d^2+e*(a+
b*arctanh(c*x))^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2+2*b*c*(a+b*arctanh(c
*x))*ln(2-2/(c*x+1))/d+b*e*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/d^2-
b*e*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))/d^2+b*e*(a+b*arctanh(c*x))
*polylog(2,1-2/(c*x+1))/d^2-b^2*c*polylog(2,-1+2/(c*x+1))/d-b*e*(a+b*arcta
nh(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2-1/2*b^2*e*polylog(3,
1-2/(-c*x+1))/d^2+1/2*b^2*e*polylog(3,-1+2/(-c*x+1))/d^2+1/2*b^2*e*polylog
(3,1-2/(c*x+1))/d^2-1/2*b^2*e*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 1305, normalized size of antiderivative = 3.17

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2(d + ex)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

output
```
-(a^2/(d*x)) - (a^2*e*Log[x])/d^2 + (a^2*e*Log[d + e*x])/d^2 + (a*b*(I*c*d
*e*Pi*ArcTanh[c*x] - (2*c*d^2*ArcTanh[c*x])/x + 2*c*d*e*ArcTanh[(c*d)/e]*A
rcTanh[c*x] - c*d*e*ArcTanh[c*x]^2 + e^2*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*d
^2)/e^2]*e^2*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e*ArcTanh[c*x]*Log
[1 - E^(-2*ArcTanh[c*x])] - I*c*d*e*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c*d
*e*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*
c*d*e*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c
^2*d^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] - (I/2)*c*d*e*Pi*Log[1 - c^2*x^2] - 2*
c*d*e*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + c*d*
e*PolyLog[2, E^(-2*ArcTanh[c*x])] - c*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/
e] + ArcTanh[c*x]))]))/(c*d^3) + (b^2*((-I)*c*d*e*Pi^3 + 24*c^2*d^2*ArcTan
h[c*x]^2 - (24*c*d^2*ArcTanh[c*x]^2)/x + 8*c*d*e*ArcTanh[c*x]^3 + 8*e^2*Ar
cTanh[c*x]^3 + 48*c^2*d^2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*c
*d*e*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^2*d^2*PolyLog[2, E^
(-2*ArcTanh[c*x])] - 24*c*d*e*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])]
+ 12*c*d*e*PolyLog[3, E^(2*ArcTanh[c*x])]))/(24*c*d^3) + (b^2*(c*d - e)*e*
(c*d + e)*(-6*c*d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d
^2)/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*
Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 -
 (Sqrt[c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]] - 6*c*d*ArcTanh[c*x]^...
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{ 6502}$$

$$\int \left(\frac{e^2(a+b\text{arctanh}(cx))^2}{d^2(d+ex)}-\frac{e(a+b\text{arctanh}(cx))^2}{d^2x}+\frac{(a+b\text{arctanh}(cx))^2}{dx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{be\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{be\,\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)(a+b\text{arctanh}(cx))}{d^2}+\frac{be\,\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d^2}-\frac{be(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d^2}-\frac{2e\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{d^2}-\frac{e\log\left(\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))^2}{d^2}+\frac{e(a+b\text{arctanh}(cx))^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d^2}+\frac{c(a+b\text{arctanh}(cx))^2}{d}-\frac{(a+b\text{arctanh}(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))}{d}-\frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d^2}+\frac{b^2e\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d^2}+\frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d^2}-\frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d^2}-\frac{b^2c\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

output
```
(c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) - (2*e*(a + b*
ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (e*(a + b*ArcTanh[c*x])^2*
Log[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + e)*(1 + c*x))])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)]
)/d + (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2 - (b*e*(a
 + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (b*e*(a + b*ArcTanh
[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1 + 2/(1 + c*
x)])/d - (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) + (b^2*e
*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + c*x)
])/(2*d^2) - (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])
/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.72 (sec) , antiderivative size = 16456, normalized size of antiderivative = 39.94

method	result	size
parts	Expression too large to display	16456
derivativedivides	Expression too large to display	16499
default	Expression too large to display	16499

input
```
int((a+b*arctanh(c*x))^2/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+ex)}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x^3 + d*x^2),
x)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2(d + ex)}\,dx = \int \frac{(a + b\,\text{atanh}\,(cx))^2}{x^2\,(d + ex)}\,dx$$

input
```
integrate((a+b*atanh(c*x))**2/x**2/(e*x+d),x)
```

output
```
Integral((a + b*atanh(c*x))**2/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2(d + ex)}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{(ex + d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/4*b^2*log(-c*x + 1)^
2/(d*x) - integrate(-1/4*((b^2*c*d*x - b^2*d)*log(c*x + 1)^2 + 4*(a*b*c*d*
x - a*b*d)*log(c*x + 1) + 2*(b^2*c*e*x^2 + 2*a*b*d - (2*a*b*c*d - b^2*c*d)
*x - (b^2*c*d*x - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c*d*e*x^4 - d^2*x^2
 + (c*d^2 - d*e)*x^3), x)
```

Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2(d + ex)}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{(ex + d)x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+ex)}\,dx=\int \frac{\left(a+b\,\text{atanh}(c\,x)\right)^2}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*atanh(c*x))^2/(x^2*(d + e*x)),x)
```

output
```
int((a + b*atanh(c*x))^2/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(d+ex)}\,dx$$
$$=\frac{-2\left(\int \frac{atanh(cx)}{c^2e\,x^5+c^2d\,x^4-e\,x^3-d\,x^2}dx\right)ab\,d^2x+2\left(\int \frac{atanh(cx)}{c^2e\,x^3+c^2d\,x^2-ex-d}dx\right)ab\,c^2d^2x-\left(\int \frac{atanh(cx)^2}{c^2e\,x^5+c^2d\,x^4-e\,x^3-d\,x^2}dx\right)b}{d^2x}$$

input
```
int((a+b*atanh(c*x))^2/x^2/(e*x+d),x)
```

output
```
( - 2*int(atanh(c*x)/(c**2*d*x**4 + c**2*e*x**5 - d*x**2 - e*x**3),x)*a*b*
d**2*x + 2*int(atanh(c*x)/(c**2*d*x**2 + c**2*e*x**3 - d - e*x),x)*a*b*c**
2*d**2*x - int(atanh(c*x)**2/(c**2*d*x**4 + c**2*e*x**5 - d*x**2 - e*x**3)
,x)*b**2*d**2*x + int(atanh(c*x)**2/(c**2*d*x**2 + c**2*e*x**3 - d - e*x),
x)*b**2*c**2*d**2*x + log(d + e*x)*a**2*e*x - log(x)*a**2*e*x - a**2*d)/(d
**2*x)
```

3.159 $\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 275

$$\begin{aligned}\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx &= \frac{2\operatorname{arctanh}(cx)^2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)}{d}+\frac{\operatorname{arctanh}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d}\\
&\quad-\frac{\operatorname{arctanh}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}\\
&\quad-\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d}\\
&\quad+\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right)}{d}\\
&\quad-\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{d}\\
&\quad+\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d}\\
&\quad+\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)}{2d}\\
&\quad-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2d}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2d}\end{aligned}$$

output
```
-2*arctanh(c*x)^2*arctanh(-1+2/(-c*x+1))/d+arctanh(c*x)^2*ln(2/(c*x+1))/d-
arctanh(c*x)^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/d-arctanh(c*x)*polylog(2,1-
2/(-c*x+1))/d+arctanh(c*x)*polylog(2,-1+2/(-c*x+1))/d-arctanh(c*x)*polylog
(2,1-2/(c*x+1))/d+arctanh(c*x)*polylog(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d+
1/2*polylog(3,1-2/(-c*x+1))/d-1/2*polylog(3,-1+2/(-c*x+1))/d-1/2*polylog(3
,1-2/(c*x+1))/d+1/2*polylog(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.09

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
Integrate[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

output
```
(I*c*d*Pi^3 - 8*c*d*ArcTanh[c*x]^3 - 8*e*ArcTanh[c*x]^3 + 24*c*d*ArcTanh[c
*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*Ar
cTanh[c*x])] - 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] - (24*(c*d - e)*(c*d
+ e)*(-6*c*d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e
^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(
E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 - (Sqr
t[c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]] - 6*c*d*ArcTanh[c*x]^2*Log[1
+ (Sqrt[c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]] + 6*c*d*ArcTanh[c*x]^2*
Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1
+ E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2
*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x
]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)
/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*
x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c
*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log
[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c
*d)/e] + ArcTanh[c*x]]] - 12*c*d*ArcTanh[c*x]*PolyLog[2, -((Sqrt[c*d + e]*
E^ArcTanh[c*x])/Sqrt[-(c*d) + e])] - 12*c*d*ArcTanh[c*x]*PolyLog[2, (Sqrt[
c*d + e]*E^ArcTanh[c*x])/Sqrt[-(c*d) + e]] + 12*c*d*ArcTanh[c*x]*PolyLog[2
, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2...
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6502, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{6502}$$

$$\int \left(\frac{\operatorname{arctanh}(cx)^2}{dx}-\frac{e\operatorname{arctanh}(cx)^2}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{d}-\frac{\operatorname{arctanh}(cx)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{d}+\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,\frac{2}{1-cx}-1\right)}{d}-\frac{\operatorname{arctanh}(cx)\operatorname{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d}+\frac{2\operatorname{arctanh}\left(1-\frac{2}{1-cx}\right)\operatorname{arctanh}(cx)^2}{d}+\frac{\operatorname{arctanh}(cx)^2\log\left(\frac{2}{cx+1}\right)}{d}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2d}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{2d}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2d}$$

input
```
Int[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

output
```
(2*ArcTanh[c*x]^2*ArcTanh[1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]^2*Log[2/(1 +
 c*x)])/d - (ArcTanh[c*x]^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d
- (ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]*PolyLog[2,
-1 + 2/(1 - c*x)])/d - (ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/d + (Arc
Tanh[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + PolyL
og[3, 1 - 2/(1 - c*x)]/(2*d) - PolyLog[3, -1 + 2/(1 - c*x)]/(2*d) - PolyLo
g[3, 1 - 2/(1 + c*x)]/(2*d) + PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1
 + c*x))]/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6502
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.32 (sec) , antiderivative size = 1466, normalized size of antiderivative = 5.33

method	result	size
derivativedivides	Expression too large to display	1466
default	Expression too large to display	1466
parts	Expression too large to display	2304

input
```
int(arctanh(c*x)^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(c*x)^2/d*ln(c*e*x+c*d)+arctanh(c*x)^2/d*ln(c*x)-2*c*(-1/2/d/c*arc
tanh(c*x)^2*ln(d*c*(1+(c*x+1)^2/(-c^2*x^2+1))+e*((c*x+1)^2/(-c^2*x^2+1)-1)
)+1/4*I/d/c*Pi*(csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(d*c*(1-(c*x+1)^2
/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2
*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))-csgn(I/(
1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1
)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))^2-csgn(I/(1-(c*x+1)^2/(c^2*
x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)
-1)/(1-(c*x+1)^2/(c^2*x^2-1)))+csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-
(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-csgn(I*(d*c*(1-(c*x+
1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1)))*csgn(I*(d*c*(1-(c*x+1)^2/
(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+1)^2/(c^2*x^2-1)))^2+cs
gn(I*(d*c*(1-(c*x+1)^2/(c^2*x^2-1))+e*(-(c*x+1)^2/(c^2*x^2-1)-1))/(1-(c*x+
1)^2/(c^2*x^2-1)))^3+csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2
/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-csgn(I*(-(c*x+1)^2/(c^2*x^2-1
)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3)*arctanh(c*x)^2+1/2/d/c*arctanh(c*x)^2*l
n((c*x+1)^2/(-c^2*x^2+1)-1)-1/2/d/c*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+
1)^(1/2))-1/d/c*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+1/d/c*p
olylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2/d/c*arctanh(c*x)^2*ln(1+(c*x+1)/(
-c^2*x^2+1)^(1/2))-1/d/c*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \int \frac{\operatorname{artanh}\left(cx\right)^2}{(ex+d)x}\,dx$$

input
```
integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arctanh(c*x)^2/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \int \frac{\operatorname{atanh}^2(cx)}{x(d+ex)}\,dx$$

input
```
integrate(atanh(c*x)**2/x/(e*x+d),x)
```

output
```
Integral(atanh(c*x)**2/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \int \frac{\operatorname{artanh}(cx)^2}{(ex+d)x}\,dx$$

input
```
integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \int \frac{\operatorname{artanh}(cx)^2}{(ex+d)x}\,dx$$

input
```
integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \int \frac{\operatorname{atanh}(c\,x)^2}{x\,(d+e\,x)}\,dx$$

input
```
int(atanh(c*x)^2/(x*(d + e*x)),x)
```

output
```
int(atanh(c*x)^2/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(cx)^2}{x(d+ex)}\,dx = \frac{-atanh(cx)^3 c - 3\left(\int \frac{atanh(cx)^2}{c^2 e\,x^4+c^2 d\,x^3-e\,x^2-dx}dx\right) e - 3\left(\int \frac{atanh(cx)^2}{c^2 e\,x^3+c^2 d\,x^2-ex-d}dx\right) c^2 d}{3e}$$

input
```
int(atanh(c*x)^2/x/(e*x+d),x)
```

output
```
( - atanh(c*x)**3*c - 3*int(atanh(c*x)**2/(c**2*d*x**3 + c**2*e*x**4 - d*x
 - e*x**2),x)*e - 3*int(atanh(c*x)**2/(c**2*d*x**2 + c**2*e*x**3 - d - e*x
),x)*c**2*d)/(3*e)
```

3.160 $\int \frac{1}{(d+ex)(a+b\arctan(cx))} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))} dx = \text{Int}\left(\frac{1}{(d+ex)(a+b\arctan(cx))}, x\right)$$

output
```
Defer(Int)(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))} dx = \int \frac{1}{(d+ex)(a+b\arctan(cx))} dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\,(a+b\arctan(cx))}dx$$

input
```
int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

output
```
int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x + a*d + (b*e*x + b*d)*arctan(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(a+b\operatorname{atan}(cx))(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
```

output
```
Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{atan}\,(c\,x))\,\,(d+e\,x)}\,dx$$

input
```
int(1/((a + b*atan(c*x))*(d + e*x)),x)
```

output
```
int(1/((a + b*atan(c*x))*(d + e*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{atan\,(cx)\,bd + atan\,(cx)\,bex + ad + aex}dx$$

input
```
int(1/(e*x+d)/(a+b*atan(c*x)),x)
```

output
```
int(1/(atan(c*x)*b*d + atan(c*x)*b*e*x + a*d + a*e*x),x)
```

3.161 $\int x^4(1-a^2x^2)\,\mathbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int x^4(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\text{arctanh}(ax) - \frac{1}{7}a^2x^7\text{arctanh}(ax)+\frac{\log(1-a^2x^2)}{35a^5}$$

output
```
1/35*x^2/a^3+1/70*x^4/a-1/42*a*x^6+1/5*x^5*arctanh(a*x)-1/7*a^2*x^7*arctan
h(a*x)+1/35*ln(-a^2*x^2+1)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int x^4(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\text{arctanh}(ax) - \frac{1}{7}a^2x^7\text{arctanh}(ax)+\frac{\log(1-a^2x^2)}{35a^5}$$

input
```
Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*A
rcTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6576, 6452, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^4\operatorname{arctanh}(ax)dx-a^2\int x^6\operatorname{arctanh}(ax)dx$$

$$\Big\downarrow \text{6452}$$

$$-a^2\left(\frac{1}{7}x^7\operatorname{arctanh}(ax)-\frac{1}{7}a\int\frac{x^7}{1-a^2x^2}dx\right)-\frac{1}{5}a\int\frac{x^5}{1-a^2x^2}dx+\frac{1}{5}x^5\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{243}$$

$$-a^2\left(\frac{1}{7}x^7\operatorname{arctanh}(ax)-\frac{1}{14}a\int\frac{x^6}{1-a^2x^2}dx^2\right)-\frac{1}{10}a\int\frac{x^4}{1-a^2x^2}dx^2+\frac{1}{5}x^5\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{10}a\int\left(-\frac{x^2}{a^2}-\frac{1}{a^4(a^2x^2-1)}-\frac{1}{a^4}\right)dx^2-a^2\left(\frac{1}{7}x^7\operatorname{arctanh}(ax)-\frac{1}{14}a\int\left(-\frac{x^4}{a^2}-\frac{x^2}{a^4}-\frac{1}{a^6(a^2x^2-1)}-\frac{1}{a^6}\right)dx^2\right)+\frac{1}{5}x^5\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)-a^2\left(\frac{1}{7}x^7\operatorname{arctanh}(ax)-\frac{1}{14}a\left(-\frac{x^2}{a^6}-\frac{x^4}{2a^4}-\frac{x^6}{3a^2}-\frac{\log(1-a^2x^2)}{a^8}\right)\right)+\frac{1}{5}x^5\operatorname{arctanh}(ax)$$

input
```
Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
(x^5*ArcTanh[a*x])/5 - (a*(-(x^2/a^4) - x^4/(2*a^2) - Log[1 - a^2*x^2]/a^6
))/10 - a^2*((x^7*ArcTanh[a*x])/7 - (a*(-(x^2/a^6) - x^4/(2*a^4) - x^6/(3*
a^2) - Log[1 - a^2*x^2]/a^8))/14)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{a^2x^7\operatorname{arctanh}(ax)}{7}+\frac{x^5\operatorname{arctanh}(ax)}{5}-\frac{a\left(\frac{\frac{5}{3}a^4x^6-a^2x^4-2x^2}{2a^4}-\frac{\ln\left(a^2x^2-1\right)}{a^6}\right)}{35}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}-\frac{a^6x^6}{42}+\frac{a^4x^4}{70}+\frac{a^2x^2}{35}+\frac{\ln(ax-1)}{35}+\frac{\ln(ax+1)}{35}}{a^5}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}-\frac{a^6x^6}{42}+\frac{a^4x^4}{70}+\frac{a^2x^2}{35}+\frac{\ln(ax-1)}{35}+\frac{\ln(ax+1)}{35}}{a^5}$
parallelrisch	$-\frac{30\operatorname{arctanh}(ax)a^7x^7+5a^6x^6-42\operatorname{arctanh}(ax)a^5x^5-3a^4x^4-6-6a^2x^2-12\ln(ax-1)-12\operatorname{arctanh}(ax)}{210a^5}$
risch	$\left(-\frac{1}{14}a^2x^7+\frac{1}{10}x^5\right)\ln(ax+1)+\frac{a^2x^7\ln(-ax+1)}{14}-\frac{a\,x^6}{42}-\frac{x^5\ln(-ax+1)}{10}+\frac{x^4}{70a}+\frac{x^2}{35a^3}+\frac{\ln(a^2}{35}$
meijerg	$-\frac{-\frac{x^2a^2\left(4a^4x^4+6a^2x^2+12\right)}{42}-\frac{2x^8a^8\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(-a^2x^2+1\right)}{7}}{4a^5}-\frac{-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\frac{2a^6x^6\left(\ln\left(1-\right.\right.}{}}{}$

input
```
int(x^4*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2*x^7*arctanh(a*x)+1/5*x^5*arctanh(a*x)-1/35*a*(1/2/a^4*(5/3*a^4*x^
6-a^2*x^4-2*x^2)-1/a^6*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx$$
$$=-\frac{5\,a^6x^6-3\,a^4x^4-6\,a^2x^2+3\,(5\,a^7x^7-7\,a^5x^5)\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(a^2x^2-1\right)}{210\,a^5}$$

input
```
integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/210*(5*a^6*x^6 - 3*a^4*x^4 - 6*a^2*x^2 + 3*(5*a^7*x^7 - 7*a^5*x^5)*log(
-(a*x + 1)/(a*x - 1)) - 6*log(a^2*x^2 - 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = \begin{cases} -\frac{a^2x^7\operatorname{atanh}(ax)}{7} - \frac{ax^6}{42} + \frac{x^5\operatorname{atanh}(ax)}{5} + \frac{x^4}{70a} + \frac{x^2}{35a^3} + \frac{2\log\left(x-\frac{1}{a}\right)}{35a^5} + \frac{2\operatorname{atanh}(ax)}{35a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(-a**2*x**2+1)*atanh(a*x),x)
```

output
```
Piecewise((-a**2*x**7*atanh(a*x)/7 - a*x**6/42 + x**5*atanh(a*x)/5 + x**4/
(70*a) + x**2/(35*a**3) + 2*log(x - 1/a)/(35*a**5) + 2*atanh(a*x)/(35*a**5
), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = -\frac{1}{210}\,a\left(\frac{5\,a^4x^6 - 3\,a^2x^4 - 6\,x^2}{a^4} - \frac{6\,\log(ax+1)}{a^6} - \frac{6\,\log(ax-1)}{a^6}\right) - \frac{1}{35}\left(5\,a^2x^7 - 7\,x^5\right)\operatorname{artanh}(ax)$$

input
```
integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/210*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*log
(a*x - 1)/a^6) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 335, normalized size of antiderivative = 4.65

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx$$
$$= \frac{2}{105}a\left(\frac{3\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6} - \frac{3\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6} - \frac{\frac{3(ax+1)^5}{(ax-1)^5}+\frac{36(ax+1)^4}{(ax-1)^4}+\frac{2(ax+1)^3}{(ax-1)^3}+\frac{36(ax+1)^2}{(ax-1)^2}+\frac{3(ax+1)}{ax-1}}{a^6\left(\frac{ax+1}{ax-1}-1\right)^6} - \right.$$

input
```
integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

output
```
2/105*a*(3*log(abs(-a*x - 1)/abs(a*x - 1))/a^6 - 3*log(abs(-(a*x + 1)/(a*x
 - 1) + 1))/a^6 - (3*(a*x + 1)^5/(a*x - 1)^5 + 36*(a*x + 1)^4/(a*x - 1)^4
+ 2*(a*x + 1)^3/(a*x - 1)^3 + 36*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x + 1)/(a*
x - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^6) - 3*(35*(a*x + 1)^5/(a*x - 1)^5
+ 35*(a*x + 1)^4/(a*x - 1)^4 + 70*(a*x + 1)^3/(a*x - 1)^3 + 14*(a*x + 1)^2
/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1
)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)
*a/(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^7))
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = \frac{\frac{\ln(a^2x^2-1)}{35}+\frac{a^2x^2}{35}+\frac{a^4x^4}{70}}{a^5} - \frac{a\,x^6}{42} + \frac{x^5\,\mathrm{atanh}(a\,x)}{5} - \frac{a^2\,x^7\,\mathrm{atanh}(a\,x)}{7}$$

input
```
int(-x^4*atanh(a*x)*(a^2*x^2 - 1),x)
```

output
```
(log(a^2*x^2 - 1)/35 + (a^2*x^2)/35 + (a^4*x^4)/70)/a^5 - (a*x^6)/42 + (x^
5*atanh(a*x))/5 - (a^2*x^7*atanh(a*x))/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = \frac{-30atanh(ax)\,a^7x^7+42atanh(ax)\,a^5x^5+12atanh(ax)+12\log(a^2x-a)-5a^6x^6+3a^4x^4+6a^2x^2}{210a^5}$$

input
```
int(x^4*(-a^2*x^2+1)*atanh(a*x),x)
```

output
```
( - 30*atanh(a*x)*a**7*x**7 + 42*atanh(a*x)*a**5*x**5 + 12*atanh(a*x) + 12
*log(a**2*x - a) - 5*a**6*x**6 + 3*a**4*x**4 + 6*a**2*x**2)/(210*a**5)
```

3.162 $\int x^3(1-a^2x^2)\,\textbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}-\frac{\text{arctanh}(ax)}{12a^4}+\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{6}a^2x^6\text{arctanh}(ax)$$

output
```
1/12*x/a^3+1/36*x^3/a-1/30*a*x^5-1/12*arctanh(a*x)/a^4+1/4*x^4*arctanh(a*x
)-1/6*a^2*x^6*arctanh(a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}+\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{6}a^2x^6\text{arctanh}(ax)+\frac{\log(1-ax)}{24a^4}-\frac{\log(1+ax)}{24a^4}$$

input
```
Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*Arc
Tanh[a*x])/6 + Log[1 - a*x]/(24*a^4) - Log[1 + a*x]/(24*a^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6576, 6452, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(1-a^2 x^2\right) \text{arctanh}(ax) \, dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^3 \text{arctanh}(ax)dx - a^2 \int x^5 \text{arctanh}(ax)dx$$

$$\Big\downarrow \text{6452}$$

$$-a^2 \left(\frac{1}{6} x^6 \text{arctanh}(ax) - \frac{1}{6} a \int \frac{x^6}{1-a^2x^2}dx\right) - \frac{1}{4} a \int \frac{x^4}{1-a^2x^2}dx + \frac{1}{4} x^4 \text{arctanh}(ax)$$

$$\Big\downarrow \text{254}$$

$$-\frac{1}{4} a \int \left(-\frac{x^2}{a^2} + \frac{1}{a^4 \left(1-a^2x^2\right)} - \frac{1}{a^4}\right) dx - a^2 \left(\frac{1}{6} x^6 \text{arctanh}(ax) - \frac{1}{6} a \int \left(-\frac{x^4}{a^2} - \frac{x^2}{a^4} + \frac{1}{a^6 \left(1-a^2x^2\right)} - \frac{1}{a^6}\right) dx\right) + \frac{1}{4} x^4 \text{arctanh}(ax)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4} a \left(\frac{\text{arctanh}(ax)}{a^5} - \frac{x}{a^4} - \frac{x^3}{3a^2}\right) - a^2 \left(\frac{1}{6} x^6 \text{arctanh}(ax) - \frac{1}{6} a \left(\frac{\text{arctanh}(ax)}{a^7} - \frac{x}{a^6} - \frac{x^3}{3a^4} - \frac{x^5}{5a^2}\right)\right) + \frac{1}{4} x^4 \text{arctanh}(ax)$$

input
```
Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
(x^4*ArcTanh[a*x])/4 - (a*(-(x/a^4) - x^3/(3*a^2) + ArcTanh[a*x]/a^5))/4 -
 a^2*((x^6*ArcTanh[a*x])/6 - (a*(-(x/a^6) - x^3/(3*a^4) - x^5/(5*a^2) + Ar
cTanh[a*x]/a^7))/6)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result
parallelrisch	$-\frac{30 \operatorname{arctanh}(ax)a^6x^6+6a^5x^5-45a^4x^4 \operatorname{arctanh}(ax)-5a^3x^3-15ax+15 \operatorname{arctanh}(ax)}{180a^4}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^6x^6}{6}+\frac{a^4x^4 \operatorname{arctanh}(ax)}{4}-\frac{a^5x^5}{30}+\frac{a^3x^3}{36}+\frac{ax}{12}+\frac{\ln(ax-1)}{24}-\frac{\ln(ax+1)}{24}}{a^4}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^6x^6}{6}+\frac{a^4x^4 \operatorname{arctanh}(ax)}{4}-\frac{a^5x^5}{30}+\frac{a^3x^3}{36}+\frac{ax}{12}+\frac{\ln(ax-1)}{24}-\frac{\ln(ax+1)}{24}}{a^4}$
parts	$-\frac{a^2x^6 \operatorname{arctanh}(ax)}{6}+\frac{x^4 \operatorname{arctanh}(ax)}{4}-\frac{a\left(\frac{\frac{2}{5}a^4x^5-\frac{1}{3}a^2x^3-x}{a^4}+\frac{\ln(ax+1)}{2a^5}-\frac{\ln(ax-1)}{2a^5}\right)}{12}$
risch	$\left(-\frac{1}{12}a^2x^6+\frac{1}{8}x^4\right)\ln(ax+1)+\frac{a^2x^6\ln(-ax+1)}{12}-\frac{a\,x^5}{30}-\frac{x^4\ln(-ax+1)}{8}+\frac{x^3}{36a}+\frac{x}{12a^3}+\frac{\ln(-ax}{24a}$
orering	$\frac{(15a^6x^6-22a^4x^4-15a^2x^2+15)(-a^2x^2+1) \operatorname{arctanh}(ax)}{45a^4(a^2x^2-1)}-\frac{(6a^4x^4-5a^2x^2-15)(3x^2(-a^2x^2+1) \operatorname{arctanh}(ax)-2x^4a}{180x^2a^4}$
meijerg	$-\frac{i\left(-\frac{2ixa(21a^4x^4+35a^2x^2+105)}{315}-\frac{ixa(-7a^6x^6+7)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{21\sqrt{a^2x^2}}\right)}{4a^4}-\frac{i\left(\frac{ixa(5a^2x^2+15)}{15}+\frac{ixa(-5a^4}{}\right.}{}$

input
```
int(x^3*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/180*(30*arctanh(a*x)*a^6*x^6+6*a^5*x^5-45*a^4*x^4*arctanh(a*x)-5*a^3*x^
3-15*a*x+15*arctanh(a*x))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = -\frac{12\,a^5x^5-10\,a^3x^3-30\,ax+15\,(2\,a^6x^6-3\,a^4x^4+1)\log\left(-\frac{ax+1}{ax-1}\right)}{360\,a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/360*(12*a^5*x^5 - 10*a^3*x^3 - 30*a*x + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*
log(-(a*x + 1)/(a*x - 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)\,dx$$
$$=\begin{cases} -\frac{a^2x^6\operatorname{atanh}(ax)}{6}-\frac{ax^5}{30}+\frac{x^4\operatorname{atanh}(ax)}{4}+\frac{x^3}{36a}+\frac{x}{12a^3}-\frac{\operatorname{atanh}(ax)}{12a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(-a**2*x**2+1)*atanh(a*x),x)
```

output
```
Piecewise((-a**2*x**6*atanh(a*x)/6 - a*x**5/30 + x**4*atanh(a*x)/4 + x**3/
(36*a) + x/(12*a**3) - atanh(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)\,dx$$
$$=-\frac{1}{360}\,a\left(\frac{2\,(6\,a^4x^5-5\,a^2x^3-15\,x)}{a^4}+\frac{15\,\log(ax+1)}{a^5}-\frac{15\,\log(ax-1)}{a^5}\right)-\frac{1}{12}\,(2\,a^2x^6-3\,x^4)\operatorname{artanh}(ax)$$

input
```
integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/360*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*
log(a*x - 1)/a^5) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.60

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)\,dx = -\frac{1}{45}a\left(\frac{\frac{45\,(ax+1)^3}{(ax-1)^3}-\frac{25\,(ax+1)^2}{(ax-1)^2}+\frac{35\,(ax+1)}{ax-1}-7}{a^5\left(\frac{ax+1}{ax-1}-1\right)^5}+\frac{30\left(\frac{3\,(ax+1)^4}{(ax-1)^4}+\frac{2\,(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)^2}{(ax-1)^2}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^5\left(\frac{ax+1}{ax-1}-1\right)^6}\right)$$

input
```
integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

output
```
-1/45*a*((45*(a*x + 1)^3/(a*x - 1)^3 - 25*(a*x + 1)^2/(a*x - 1)^2 + 35*(a*
x + 1)/(a*x - 1) - 7)/(a^5*((a*x + 1)/(a*x - 1) - 1)^5) + 30*(3*(a*x + 1)^
4/(a*x - 1)^4 + 2*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2)*log
(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x +
 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^5*((a*x + 1)/(a*x
- 1) - 1)^6))
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \frac{\frac{a\,x}{12}-\frac{\text{atanh}(a\,x)}{12}+\frac{a^3\,x^3}{36}}{a^4}-\frac{a\,x^5}{30}+\frac{x^4\,\text{atanh}(a\,x)}{4}-\frac{a^2\,x^6\,\text{atanh}(a\,x)}{6}$$

input
```
int(-x^3*atanh(a*x)*(a^2*x^2 - 1),x)
```

output
```
((a*x)/12 - atanh(a*x)/12 + (a^3*x^3)/36)/a^4 - (a*x^5)/30 + (x^4*atanh(a*
x))/4 - (a^2*x^6*atanh(a*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)\,dx$$
$$= \frac{-30atanh(ax)\,a^6x^6+45atanh(ax)\,a^4x^4-15atanh(ax)-6a^5x^5+5a^3x^3+15ax}{180a^4}$$

input
```
int(x^3*(-a^2*x^2+1)*atanh(a*x),x)
```

output
```
( - 30*atanh(a*x)*a**6*x**6 + 45*atanh(a*x)*a**4*x**4 - 15*atanh(a*x) - 6*
a**5*x**5 + 5*a**3*x**3 + 15*a*x)/(180*a**4)
```

3.163 $\int x^2(1-a^2x^2)\,\mathbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)\,dx=\frac{x^2}{15a}-\frac{ax^4}{20}+\frac{1}{3}x^3\operatorname{arctanh}(ax)-\frac{1}{5}a^2x^5\operatorname{arctanh}(ax)+\frac{\log(1-a^2x^2)}{15a^3}$$

output
```
1/15*x^2/a-1/20*a*x^4+1/3*x^3*arctanh(a*x)-1/5*a^2*x^5*arctanh(a*x)+1/15*l
n(-a^2*x^2+1)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)\,dx=\frac{x^2}{15a}-\frac{ax^4}{20}+\frac{1}{3}x^3\operatorname{arctanh}(ax)-\frac{1}{5}a^2x^5\operatorname{arctanh}(ax)+\frac{\log(1-a^2x^2)}{15a^3}$$

input
```
Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5
+ Log[1 - a^2*x^2]/(15*a^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6576, 6452, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (1-a^2x^2) \operatorname{arctanh}(ax) \, dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^2 \operatorname{arctanh}(ax)dx - a^2 \int x^4 \operatorname{arctanh}(ax)dx$$

$$\Big\downarrow \text{6452}$$

$$-a^2 \left(\frac{1}{5}x^5 \operatorname{arctanh}(ax) - \frac{1}{5}a \int \frac{x^5}{1-a^2x^2}dx\right) - \frac{1}{3}a \int \frac{x^3}{1-a^2x^2}dx + \frac{1}{3}x^3 \operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{243}$$

$$-a^2 \left(\frac{1}{5}x^5 \operatorname{arctanh}(ax) - \frac{1}{10}a \int \frac{x^4}{1-a^2x^2}dx^2\right) - \frac{1}{6}a \int \frac{x^2}{1-a^2x^2}dx^2 + \frac{1}{3}x^3 \operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{6}a \int \left(-\frac{1}{a^2} - \frac{1}{a^2 (a^2x^2-1)}\right) dx^2 - a^2 \left(\frac{1}{5}x^5 \operatorname{arctanh}(ax) - \frac{1}{10}a \int \left(-\frac{x^2}{a^2} - \frac{1}{a^4 (a^2x^2-1)} - \frac{1}{a^4}\right) dx^2\right) + \frac{1}{3}x^3 \operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}a \left(-\frac{x^2}{a^2} - \frac{\log(1-a^2x^2)}{a^4}\right) - a^2 \left(\frac{1}{5}x^5 \operatorname{arctanh}(ax) - \frac{1}{10}a \left(-\frac{x^2}{a^4} - \frac{x^4}{2a^2} - \frac{\log(1-a^2x^2)}{a^6}\right)\right) + \frac{1}{3}x^3 \operatorname{arctanh}(ax)$$

input
```
Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
(x^3*ArcTanh[a*x])/3 - (a*(-(x^2/a^2) - Log[1 - a^2*x^2]/a^4))/6 - a^2*((x
^5*ArcTanh[a*x])/5 - (a*(-(x^2/a^4) - x^4/(2*a^2) - Log[1 - a^2*x^2]/a^6))
/10)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
parallelrisch	$-\frac{12 \operatorname{arctanh}(ax) a^{5} x^{5}+3 a^{4} x^{4}-20 a^{3} x^{3} \operatorname{arctanh}(ax)-4 a^{2} x^{2}-8 \ln(ax-1)-8 \operatorname{arctanh}(ax)}{60 a^{3}}$
parts	$-\frac{a^{2} x^{5} \operatorname{arctanh}(ax)}{5}+\frac{x^{3} \operatorname{arctanh}(ax)}{3}-\frac{a\left(\frac{\frac{3}{2} a^{2} x^{4}-2 x^{2}}{2 a^{2}}-\frac{\ln\left(a^{2} x^{2}-1\right)}{a^{4}}\right)}{15}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax) a^{5} x^{5}}{5}+\frac{a^{3} x^{3} \operatorname{arctanh}(ax)}{3}-\frac{a^{4} x^{4}}{20}+\frac{a^{2} x^{2}}{15}+\frac{\ln(ax-1)}{15}+\frac{\ln(ax+1)}{15}}{a^{3}}$
default	$\frac{-\frac{\operatorname{arctanh}(ax) a^{5} x^{5}}{5}+\frac{a^{3} x^{3} \operatorname{arctanh}(ax)}{3}-\frac{a^{4} x^{4}}{20}+\frac{a^{2} x^{2}}{15}+\frac{\ln(ax-1)}{15}+\frac{\ln(ax+1)}{15}}{a^{3}}$
risch	$\left(-\frac{1}{10} a^{2} x^{5}+\frac{1}{6} x^{3}\right) \ln(ax+1)+\frac{a^{2} x^{5} \ln(-ax+1)}{10}-\frac{a x^{4}}{20}-\frac{x^{3} \ln(-ax+1)}{6}+\frac{x^{2}}{15 a}+\frac{\ln\left(a^{2} x^{2}-1\right)}{15 a^{3}}-$
meijerg	$\frac{-\frac{a^{2} x^{2}\left(3 a^{2} x^{2}+6\right)}{15}+\frac{2 a^{6} x^{6}\left(\ln\left(1-\sqrt{a^{2} x^{2}}\right)-\ln\left(1+\sqrt{a^{2} x^{2}}\right)\right)}{5 \sqrt{a^{2} x^{2}}}-\frac{2 \ln\left(-a^{2} x^{2}+1\right)}{5}}{4 a^{3}}+\frac{\frac{2 a^{2} x^{2}}{3}-\frac{2 a^{4} x^{4}\left(\ln\left(1-\sqrt{a^{2} x^{2}}\right)-\ln\left(1+\sqrt{a^{2}\right.\right.}{3 \sqrt{a^{2} x^{2}}}}{4 a^{3}}$

input
```
int(x^2*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(12*arctanh(a*x)*a^5*x^5+3*a^4*x^4-20*a^3*x^3*arctanh(a*x)-4*a^2*x^2
-8*ln(a*x-1)-8*arctanh(a*x))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int x^{2}\left(1-a^{2} x^{2}\right) \operatorname{arctanh}(ax)\, dx = -\frac{3 a^{4} x^{4}-4 a^{2} x^{2}+2\left(3 a^{5} x^{5}-5 a^{3} x^{3}\right) \log\left(-\frac{ax+1}{ax-1}\right)-4 \log\left(a^{2} x^{2}-1\right)}{60 a^{3}}$$

input
```
integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/60*(3*a^4*x^4 - 4*a^2*x^2 + 2*(3*a^5*x^5 - 5*a^3*x^3)*log(-(a*x + 1)/(a
*x - 1)) - 4*log(a^2*x^2 - 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int x^2(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \begin{cases} -\frac{a^2x^5\,\text{atanh}\,(ax)}{5} - \frac{ax^4}{20} + \frac{x^3\,\text{atanh}\,(ax)}{3} + \frac{x^2}{15a} + \frac{2\log\left(x-\frac{1}{a}\right)}{15a^3} + \frac{2\,\text{atanh}\,(ax)}{15a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(-a**2*x**2+1)*atanh(a*x),x)
```

output
```
Piecewise((-a**2*x**5*atanh(a*x)/5 - a*x**4/20 + x**3*atanh(a*x)/3 + x**2/
(15*a) + 2*log(x - 1/a)/(15*a**3) + 2*atanh(a*x)/(15*a**3), Ne(a, 0)), (0,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int x^2(1-a^2x^2)\,\text{arctanh}(ax)\,dx = -\frac{1}{60}\,a\left(\frac{3\,a^2x^4-4\,x^2}{a^2} - \frac{4\,\log\,(ax+1)}{a^4} - \frac{4\,\log\,(ax-1)}{a^4}\right) - \frac{1}{15}\left(3\,a^2x^5-5\,x^3\right)\text{artanh}\,(ax)$$

input
```
integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/60*a*((3*a^2*x^4 - 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4
) - 1/15*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.32

$$\int x^2(1-a^2x^2)\,\text{arctanh}(ax)\,dx$$
$$= \frac{2}{15}a\left(\frac{\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4} - \frac{\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4} - \frac{\frac{(ax+1)^3}{(ax-1)^3} + \frac{4(ax+1)^2}{(ax-1)^2} + \frac{ax+1}{ax-1}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^4} - \frac{\left(\frac{15(ax+1)^3}{(ax-1)^3} + \frac{5(ax+1)^2}{(ax-1)^2} + \frac{5(ax+1)}{ax-1}\right.}{a^4\left(\frac{ax+1}{ax-1}\right.}\right.$$

input
```
integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

output
```
2/15*a*(log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - log(abs(-(a*x + 1)/(a*x - 1)
 + 1))/a^4 - ((a*x + 1)^3/(a*x - 1)^3 + 4*(a*x + 1)^2/(a*x - 1)^2 + (a*x +
 1)/(a*x - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^4) - (15*(a*x + 1)^3/(a*x -
1)^3 + 5*(a*x + 1)^2/(a*x - 1)^2 + 5*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*
x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x
- 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)
^5))
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int x^2(1-a^2x^2)\,\text{arctanh}(ax)\,dx = \frac{\frac{\ln(a^2x^2-1)}{15} + \frac{a^2x^2}{15}}{a^3} - \frac{a\,x^4}{20} + \frac{x^3\,\text{atanh}(a\,x)}{3} - \frac{a^2\,x^5\,\text{atanh}(a\,x)}{5}$$

input
```
int(-x^2*atanh(a*x)*(a^2*x^2 - 1),x)
```

output
```
(log(a^2*x^2 - 1)/15 + (a^2*x^2)/15)/a^3 - (a*x^4)/20 + (x^3*atanh(a*x))/3
 - (a^2*x^5*atanh(a*x))/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)\,dx$$
$$=\frac{-12atanh(ax)\,a^5x^5+20atanh(ax)\,a^3x^3+8atanh(ax)+8\log(a^2x-a)-3a^4x^4+4a^2x^2}{60a^3}$$

input
```
int(x^2*(-a^2*x^2+1)*atanh(a*x),x)
```

output
```
( - 12*atanh(a*x)*a**5*x**5 + 20*atanh(a*x)*a**3*x**3 + 8*atanh(a*x) + 8*l
og(a**2*x - a) - 3*a**4*x**4 + 4*a**2*x**2)/(60*a**3)
```

3.164 $\int x(1-a^2x^2)\,\textbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx=\frac{x}{4a}-\frac{ax^3}{12}-\frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{4a^2}$$

output
```
1/4*x/a-1/12*a*x^3-1/4*(-a^2*x^2+1)^2*arctanh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx=\frac{x}{4a}-\frac{ax^3}{12}+\frac{1}{2}x^2\operatorname{arctanh}(ax)-\frac{1}{4}a^2x^4\operatorname{arctanh}(ax)+\frac{\log(1-ax)}{8a^2}-\frac{\log(1+ax)}{8a^2}$$

input
```
Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
x/(4*a) - (a*x^3)/12 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTanh[a*x])/4 + L
og[1 - a*x]/(8*a^2) - Log[1 + a*x]/(8*a^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(1-a^2x^2)\,\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int (1-a^2x^2)\,dx}{4a}-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{4a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x-\frac{a^2x^3}{3}}{4a}-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{4a^2}$$

input
```
Int[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
(x - (a^2*x^3)/3)/(4*a) - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{x^{4} a^{2} \operatorname{arctanh}\left(a x\right)}{4}+\frac{\operatorname{arctanh}\left(a x\right) x^{2}}{2}-\frac{\operatorname{arctanh}\left(a x\right)}{4 a^{2}}+\frac{-\frac{1}{3} a^{2} x^{3}+x}{4 a}$
derivativedivides	$\frac{-\frac{a^{4} x^{4} \operatorname{arctanh}\left(a x\right)}{4}+\frac{a^{2} x^{2} \operatorname{arctanh}\left(a x\right)}{2}-\frac{\operatorname{arctanh}\left(a x\right)}{4}-\frac{a^{3} x^{3}}{12}+\frac{a x}{4}}{a^{2}}$
default	$\frac{-\frac{a^{4} x^{4} \operatorname{arctanh}\left(a x\right)}{4}+\frac{a^{2} x^{2} \operatorname{arctanh}\left(a x\right)}{2}-\frac{\operatorname{arctanh}\left(a x\right)}{4}-\frac{a^{3} x^{3}}{12}+\frac{a x}{4}}{a^{2}}$
parallelrisch	$-\frac{3 a^{4} x^{4} \operatorname{arctanh}\left(a x\right)+a^{3} x^{3}-6 a^{2} x^{2} \operatorname{arctanh}\left(a x\right)-3 a x+3 \operatorname{arctanh}\left(a x\right)}{12 a^{2}}$
risch	$-\frac{\left(a^{2} x^{2}-1\right)^{2} \ln \left(a x+1\right)}{8 a^{2}}+\frac{a^{2} x^{4} \ln \left(-a x+1\right)}{8}-\frac{a\,x^{3}}{12}-\frac{x^{2} \ln \left(-a x+1\right)}{4}+\frac{x}{4 a}+\frac{\ln \left(a x-1\right)}{8 a^{2}}$
orering	$\frac{\left(3 a^{4} x^{4}-8 a^{2} x^{2}+3\right) \left(-a^{2} x^{2}+1\right) \operatorname{arctanh}\left(a x\right)}{6 a^{2} \left(a^{2} x^{2}-1\right)}-\frac{\left(a^{2} x^{2}-3\right) \left(\left(-a^{2} x^{2}+1\right) \operatorname{arctanh}\left(a x\right)-2 a^{2} x^{2} \operatorname{arctanh}\left(a x\right)+a x \right)}{12 a^{2}}$
meijerg	$\frac{i \left(\frac{i x a \left(5 a^{2} x^{2}+15\right)}{15}+\frac{i x a \left(-5 a^{4} x^{4}+5\right) \left(\ln \left(1-\sqrt{a^{2} x^{2}}\right)-\ln \left(1+\sqrt{a^{2} x^{2}}\right)\right)}{10 \sqrt{a^{2} x^{2}}}\right)}{4 a^{2}}+\frac{i \left(-2 i x a+2 i \left(-a x+1\right) \left(a x+1\right) \operatorname{arctanh}\left(a x\right)\right)}{4 a^{2}}$

input
```
int(x*(-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4*a^2*arctanh(a*x)+1/2*arctanh(a*x)*x^2-1/4*arctanh(a*x)/a^2+1/4*(-
1/3*a^2*x^3+x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int x\left(1-a^{2} x^{2}\right) \operatorname{arctanh}(a x)\, dx=-\frac{2\, a^{3} x^{3}-6\, a x+3\, \left(a^{4} x^{4}-2\, a^{2} x^{2}+1\right) \log \left(-\frac{a x+1}{a x-1}\right)}{24\, a^{2}}$$

input
```
integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/24*(2*a^3*x^3 - 6*a*x + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = \begin{cases} -\frac{a^2x^4\operatorname{atanh}(ax)}{4} - \frac{ax^3}{12} + \frac{x^2\operatorname{atanh}(ax)}{2} + \frac{x}{4a} - \frac{\operatorname{atanh}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*(-a**2*x**2+1)*atanh(a*x),x)
```

output
```
Piecewise((-a**2*x**4*atanh(a*x)/4 - a*x**3/12 + x**2*atanh(a*x)/2 + x/(4*
a) - atanh(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = -\frac{(a^2x^2-1)^2\operatorname{artanh}(ax)}{4a^2} - \frac{a^2x^3-3x}{12a}$$

input
```
integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/4*(a^2*x^2 - 1)^2*arctanh(a*x)/a^2 - 1/12*(a^2*x^3 - 3*x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(33) = 66.

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.00

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = -\frac{1}{3}\,a\left(\frac{\frac{3\,(ax+1)}{ax-1}-1}{a^3\left(\frac{ax+1}{ax-1}-1\right)^3}+\frac{6\,(ax+1)^2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^2a^3\left(\frac{ax+1}{ax-1}-1\right)^4}\right)$$

input
```
integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

output
```
-1/3*a*((3*(a*x + 1)/(a*x - 1) - 1)/(a^3*((a*x + 1)/(a*x - 1) - 1)^3) + 6*
(a*x + 1)^2*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a)
+ 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x
- 1)^2*a^3*((a*x + 1)/(a*x - 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = \frac{x^2\operatorname{atanh}(a\,x)}{2}-\frac{\frac{\operatorname{atanh}(a\,x)}{4}-\frac{a\,x}{4}}{a^2}-\frac{a\,x^3}{12}-\frac{a^2\,x^4\operatorname{atanh}(a\,x)}{4}$$

input
```
int(-x*atanh(a*x)*(a^2*x^2 - 1),x)
```

output
```
(x^2*atanh(a*x))/2 - (atanh(a*x)/4 - (a*x)/4)/a^2 - (a*x^3)/12 - (a^2*x^4*
atanh(a*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)\,dx = \frac{-3atanh(ax)\,a^4x^4+6atanh(ax)\,a^2x^2-3atanh(ax)-a^3x^3+3ax}{12a^2}$$

input
```
int(x*(-a^2*x^2+1)*atanh(a*x),x)
```

output
```
( - 3*atanh(a*x)*a**4*x**4 + 6*atanh(a*x)*a**2*x**2 - 3*atanh(a*x) - a**3*
x**3 + 3*a*x)/(12*a**2)
```

3.165 $\int (1-a^2x^2) \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)\,dx = \frac{1-a^2x^2}{6a} + \frac{2}{3}x\operatorname{arctanh}(ax) + \frac{1}{3}x(1-a^2x^2)\operatorname{arctanh}(ax) + \frac{\log(1-a^2x^2)}{3a}$$

output
```
1/6*(-a^2*x^2+1)/a+2/3*x*arctanh(a*x)+1/3*x*(-a^2*x^2+1)*arctanh(a*x)+1/3*
ln(-a^2*x^2+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)\,dx = -\frac{ax^2}{6} + x\operatorname{arctanh}(ax) - \frac{1}{3}a^2x^3\operatorname{arctanh}(ax) + \frac{\log(1-a^2x^2)}{3a}$$

input
```
Integrate[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
-1/6*(a*x^2) + x*ArcTanh[a*x] - (a^2*x^3*ArcTanh[a*x])/3 + Log[1 - a^2*x^2
]/(3*a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6504, 6436, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^2x^2)\,\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6504}$$

$$\frac{2}{3}\int \text{arctanh}(ax)dx+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}$$

$$\Big\downarrow \text{6436}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}$$

input
```
Int[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

output
```
(1 - a^2*x^2)/(6*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*x
] + Log[1 - a^2*x^2]/(2*a)))/3
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

method	result
parts	$-\frac{x^{3} a^{2} \operatorname{arctanh}\left(a x\right)}{3}+x \,\operatorname{arctanh}\left(a x\right)-\frac{a \left(\frac{x^{2}}{2}-\frac{\ln\left(a^{2} x^{2}-1\right)}{a^{2}}\right)}{3}$
parallelrisch	$-\frac{2 a^{3} x^{3} \operatorname{arctanh}\left(a x\right)+a^{2} x^{2}-6 a x \,\operatorname{arctanh}\left(a x\right)-4 \ln\left(a x-1\right)-4 \,\operatorname{arctanh}\left(a x\right)}{6 a}$
derivativedivides	$\frac{-\frac{a^{3} x^{3} \operatorname{arctanh}\left(a x\right)}{3}+a x \,\operatorname{arctanh}\left(a x\right)-\frac{a^{2} x^{2}}{6}+\frac{\ln\left(a x-1\right)}{3}+\frac{\ln\left(a x+1\right)}{3}}{a}$
default	$\frac{-\frac{a^{3} x^{3} \operatorname{arctanh}\left(a x\right)}{3}+a x \,\operatorname{arctanh}\left(a x\right)-\frac{a^{2} x^{2}}{6}+\frac{\ln\left(a x-1\right)}{3}+\frac{\ln\left(a x+1\right)}{3}}{a}$
risch	$\left(-\frac{1}{6} a^{2} x^{3}+\frac{1}{2} x \right) \ln\left(a x+1\right)+\frac{a^{2} x^{3} \ln\left(-a x+1\right)}{6}-\frac{a \,x^{2}}{6}-\frac{x \ln\left(-a x+1\right)}{2}+\frac{\ln\left(a^{2} x^{2}-1\right)}{3 a}$
meijerg	$-\frac{\frac{2 a^{2} x^{2} \left(\ln\left(1-\sqrt{a^{2} x^{2}}\right)-\ln\left(1+\sqrt{a^{2} x^{2}}\right)\right)}{\sqrt{a^{2} x^{2}}}-2 \ln\left(-a^{2} x^{2}+1\right)}{4 a}-\frac{\frac{2 a^{2} x^{2}}{3}-\frac{2 a^{4} x^{4} \left(\ln\left(1-\sqrt{a^{2} x^{2}}\right)-\ln\left(1+\sqrt{a^{2} x^{2}}\right)\right)}{3 \sqrt{a^{2} x^{2}}}+\frac{2 \ln\left(-a^{2} x^{2}\right.}{3}}{4 a}$

input
```
int((-a^2*x^2+1)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*a^2*arctanh(a*x)+x*arctanh(a*x)-1/3*a*(1/2*x^2-1/a^2*ln(a^2*x^2-1
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int (1-a^2x^2)\,\text{arctanh}(ax)\,dx = -\frac{a^2x^2+(a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-2\,\log\left(a^2x^2-1\right)}{6\,a}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/6*(a^2*x^2 + (a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 2*log(a^2*x^
2 - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int (1-a^2x^2)\,\text{arctanh}(ax)\,dx = \begin{cases} -\frac{a^2x^3\,\text{atanh}\,(ax)}{3}-\frac{ax^2}{6}+x\,\text{atanh}\,(ax)+\frac{2\log\left(x-\frac{1}{a}\right)}{3a}+\frac{2\,\text{atanh}\,(ax)}{3a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x),x)
```

output
```
Piecewise((-a**2*x**3*atanh(a*x)/3 - a*x**2/6 + x*atanh(a*x) + 2*log(x - 1
/a)/(3*a) + 2*atanh(a*x)/(3*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)\, dx = -\frac{1}{6}\left(x^2 - \frac{2\log(ax+1)}{a^2} - \frac{2\log(ax-1)}{a^2}\right)a - \frac{1}{3}\left(a^2x^3 - 3x\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/6*(x^2 - 2*log(a*x + 1)/a^2 - 2*log(a*x - 1)/a^2)*a - 1/3*(a^2*x^3 - 3*
x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(54) = 108.

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.17

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)\, dx = \frac{2}{3}a\left(\frac{\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2} - \frac{\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2} - \frac{\left(\frac{3(ax+1)}{ax-1}-1\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)^3} - \frac{ax+1}{(ax-1)a^2\left(\frac{ax+1}{ax-1}-1\right)}\right.$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

output
```
2/3*a*(log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - log(abs(-(a*x + 1)/(a*x - 1)
+ 1))/a^2 - (3*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/
((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a
/(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^3) - (a*x + 1)/((a*x
- 1)*a^2*((a*x + 1)/(a*x - 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)\,dx = x\,\operatorname{atanh}(a\,x)-\frac{a\,x^2}{6}+\frac{\ln\left(a^2\,x^2-1\right)}{3\,a}-\frac{a^2\,x^3\,\operatorname{atanh}(a\,x)}{3}$$

input
```
int(-atanh(a*x)*(a^2*x^2 - 1),x)
```

output
```
x*atanh(a*x) - (a*x^2)/6 + log(a^2*x^2 - 1)/(3*a) - (a^2*x^3*atanh(a*x))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)\,dx = \frac{-2atanh(ax)\,a^3x^3+6atanh(ax)\,ax+4atanh(ax)+4\log(a^2x-a)-a^2x^2}{6a}$$

input
```
int((-a^2*x^2+1)*atanh(a*x),x)
```

output
```
( - 2*atanh(a*x)*a**3*x**3 + 6*atanh(a*x)*a*x + 4*atanh(a*x) + 4*log(a**2*
x - a) - a**2*x**2)/(6*a)
```

3.166 $\int \frac{(1-a^2x^2)\text{arctanh}(ax)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x}\,dx = -\frac{ax}{2}+\frac{1}{2}\text{arctanh}(ax)-\frac{1}{2}a^2x^2\text{arctanh}(ax)-\frac{\text{PolyLog}(2,-ax)}{2}+\frac{\text{PolyLog}(2,ax)}{2}$$

output
```
-1/2*a*x+1/2*arctanh(a*x)-1/2*a^2*x^2*arctanh(a*x)-1/2*polylog(2,-a*x)+1/2
*polylog(2,a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x}\,dx = -\frac{ax}{2}-\frac{1}{2}a^2x^2\text{arctanh}(ax)-\frac{1}{4}\log(1-ax)+\frac{1}{4}\log(1+ax)+\frac{1}{2}(-\text{PolyLog}(2,-ax)+\text{PolyLog}(2,ax))$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

output
```
-1/2*(a*x) - (a^2*x^2*ArcTanh[a*x])/2 - Log[1 - a*x]/4 + Log[1 + a*x]/4 +
(-PolyLog[2, -(a*x)] + PolyLog[2, a*x])/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6576, 6446, 6452, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\text{arctanh}(ax)}{x}dx - a^2\int x\text{arctanh}(ax)dx$$

$$\Big\downarrow \text{6446}$$

$$a^2(-\int x\text{arctanh}(ax)dx) - \frac{\text{PolyLog}(2,-ax)}{2} + \frac{\text{PolyLog}(2,ax)}{2}$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax) - \frac{1}{2}a\int \frac{x^2}{1-a^2x^2}dx\right)\right) - \frac{\text{PolyLog}(2,-ax)}{2} + \frac{\text{PolyLog}(2,ax)}{2}$$

$$\Big\downarrow \text{262}$$

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax) - \frac{1}{2}a\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2} - \frac{x}{a^2}\right)\right)\right) - \frac{\text{PolyLog}(2,-ax)}{2} + \frac{\text{PolyLog}(2,ax)}{2}$$

$$\Big\downarrow \text{219}$$

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax) - \frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3} - \frac{x}{a^2}\right)\right)\right) - \frac{\text{PolyLog}(2,-ax)}{2} + \frac{\text{PolyLog}(2,ax)}{2}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

output
```
-(a^2*((x^2*ArcTanh[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2)) - Poly
Log[2, -(a*x)]/2 + PolyLog[2, a*x]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

method	result
derivativedivides	$-\frac{a^2x^2\,\operatorname{arctanh}(ax)}{2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1}{2}$
default	$-\frac{a^2x^2\,\operatorname{arctanh}(ax)}{2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1}{2}$
risch	$\frac{(-ax+1)^2\ln(-ax+1)}{4}-\frac{ax}{2}-\frac{(-ax+1)\ln(-ax+1)}{2}+\frac{\operatorname{dilog}(-ax+1)}{2}-\frac{(ax+1)^2\ln(ax+1)}{4}+\frac{(ax+1)\ln(ax+}{2}$
meijerg	$-\frac{i\left(\frac{2iax\,\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-\frac{2iax\,\operatorname{polylog}\left(2,-\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}-\frac{i(-2ixa+2i(-ax+1)(ax+1)\operatorname{arctanh}(ax))}{4}$
parts	$-\frac{a^2x^2\,\operatorname{arctanh}(ax)}{2}+\operatorname{arctanh}(ax)\ln(x)-\frac{a\left(x-\frac{\ln(ax+1)}{2a}+\frac{\ln(ax-1)}{2a}+\frac{\operatorname{dilog}(ax+1)}{a}+\frac{\ln(x)\ln(ax+1)}{a}-\frac{(\ln(x)-}{}\right.}{2}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*a*x-1/4*ln(a*x-1)+1/4*l
n(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x}\,dx=\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x}\,dx = -\int \left(-\frac{\operatorname{atanh}(ax)}{x}\right)dx - \int a^2x\operatorname{atanh}(ax)\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)/x,x)
```

output
```
-Integral(-atanh(a*x)/x, x) - Integral(a**2*x*atanh(a*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x}\,dx = -\frac{1}{4}a\left(2x + \frac{2(\log(ax+1)\log(x) + \operatorname{Li}_2(-ax))}{a} - \frac{2(\log(-ax+1)\log(x) + \operatorname{Li}_2(ax))}{a} - \frac{\log(ax+1)}{a} + \right. - \frac{1}{2}\left(a^2x^2 - \log(x^2)\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x + 2*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 2*(log(-a*x + 1)*l
og(x) + dilog(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) - 1/2*(a^2*x^2 -
log(x^2))*arctanh(a*x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x}\,dx = \int -\frac{(a^2x^2-1)\,\mathrm{artanh}\,(ax)}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x}\,dx = -\int \frac{\mathrm{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}{x}\,dx$$

input
```
int(-(atanh(a*x)*(a^2*x^2 - 1))/x,x)
```

output
```
-int((atanh(a*x)*(a^2*x^2 - 1))/x, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x}\,dx = -\frac{atanh(ax)\,a^2x^2}{2} + \frac{atanh(ax)}{2} + \int \frac{atanh(ax)}{x}dx - \frac{ax}{2}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)/x,x)
```

output
```
( - atanh(a*x)*a**2*x**2 + atanh(a*x) + 2*int(atanh(a*x)/x,x) - a*x)/2
```

3.167 $\int \frac{(1-a^2x^2)\mathbf{arctanh}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^2}\,dx = -\frac{\text{arctanh}(ax)}{x} - a^2x\text{arctanh}(ax) + a\log(x) - a\log\left(1-a^2x^2\right)$$

output
```
-arctanh(a*x)/x-a^2*x*arctanh(a*x)+a*ln(x)-a*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^2}\,dx = -\frac{\text{arctanh}(ax)}{x} - a^2x\text{arctanh}(ax) + a\log(x) - a\log\left(1-a^2x^2\right)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]
```

output
```
-(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6576, 6436, 240, 6452, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^2}\,dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int \frac{\text{arctanh}(ax)}{x^2}dx - a^2\int \text{arctanh}(ax)dx$$

$$\Big\downarrow \text{ 6436}$$

$$\int \frac{\text{arctanh}(ax)}{x^2}dx - a^2\left(x\text{arctanh}(ax) - a\int \frac{x}{1-a^2x^2}dx\right)$$

$$\Big\downarrow \text{ 240}$$

$$\int \frac{\text{arctanh}(ax)}{x^2}dx - a^2\left(\frac{\log(1-a^2x^2)}{2a} + x\text{arctanh}(ax)\right)$$

$$\Big\downarrow \text{ 6452}$$

$$a\int \frac{1}{x(1-a^2x^2)}dx - \left(a^2\left(\frac{\log(1-a^2x^2)}{2a} + x\text{arctanh}(ax)\right)\right) - \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2 - \left(a^2\left(\frac{\log(1-a^2x^2)}{2a} + x\text{arctanh}(ax)\right)\right) - \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2 + \int \frac{1}{x^2}dx^2\right) - \left(a^2\left(\frac{\log(1-a^2x^2)}{2a} + x\text{arctanh}(ax)\right)\right) - \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2 + \log(x^2)\right) - \left(a^2\left(\frac{\log(1-a^2x^2)}{2a} + x\text{arctanh}(ax)\right)\right) - \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \; 16$$

$$-\left(a^2\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]
```

output
```
-(ArcTanh[a*x]/x) + (a*(Log[x^2] - Log[1 - a^2*x^2]))/2 - a^2*(x*ArcTanh[a
*x] + Log[1 - a^2*x^2]/(2*a))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result
parts	$-a^{2}x \,\operatorname{arctanh}\left(a x \right)-\frac{\operatorname{arctanh}\left(a x \right)}{x}-a \left(-\ln \left(x \right)+\ln \left(a x +1\right)+\ln \left(a x -1\right)\right)$
derivativedivides	$a \left(-a x \,\operatorname{arctanh}\left(a x \right)-\frac{\operatorname{arctanh}\left(a x \right)}{a x}+\ln \left(a x \right)-\ln \left(a x -1\right)-\ln \left(a x +1\right)\right)$
default	$a \left(-a x \,\operatorname{arctanh}\left(a x \right)-\frac{\operatorname{arctanh}\left(a x \right)}{a x}+\ln \left(a x \right)-\ln \left(a x -1\right)-\ln \left(a x +1\right)\right)$
parallelrisch	$\frac{-a^{2} x^{2} \operatorname{arctanh}\left(a x \right)+a \ln \left(x \right) x -2 \ln \left(a x -1\right) a x -2 a x \,\operatorname{arctanh}\left(a x \right)-\operatorname{arctanh}\left(a x \right)}{x}$
risch	$-\frac{\left(a^{2} x^{2}+1\right) \ln \left(a x +1\right)}{2 x}+\frac{x^{2} \ln \left(-a x +1\right) a^{2}+2 a \ln \left(x \right) x -2 a \ln \left(a^{2} x^{2}-1\right) x +\ln \left(-a x +1\right)}{2 x}$
meijerg	$\frac{a \left(4 \ln \left(x \right)+4 \ln \left(i a \right)+\frac{2 \ln \left(1-\sqrt{a^{2} x^{2}}\right)-2 \ln \left(1+\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(-a^{2} x^{2}+1\right)\right)}{4}+\frac{a \left(\frac{2 a^{2} x^{2} \left(\ln \left(1-\sqrt{a^{2} x^{2}}\right)-\ln \left(1+\sqrt{a^{2} x^{2}}\right)\right)}{\sqrt{a^{2} x^{2}}}-\right.}{4}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*x*arctanh(a*x)-arctanh(a*x)/x-a*(-ln(x)+ln(a*x+1)+ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^2}\,dx = -\frac{2\,ax\log\left(a^2x^2-1\right)-2\,ax\log\left(x\right)+\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + (a^2*x^2 + 1)*log(-(a*x + 1)
/(a*x - 1)))/x
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^2}\,dx = \begin{cases} -a^2x\operatorname{atanh}\left(ax\right)+a\log\left(x\right)-2a\log\left(x-\frac{1}{a}\right)-2a\operatorname{atanh}\left(ax\right)-\frac{\operatorname{atanh}\left(ax\right)}{x} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)/x**2,x)
```

output
```
Piecewise((-a**2*x*atanh(a*x) + a*log(x) - 2*a*log(x - 1/a) - 2*a*atanh(a*
x) - atanh(a*x)/x, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^2}\,dx = -a(\log(ax+1)+\log(ax-1)-\log(x)) - \left(a^2x+\frac{1}{x}\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

output
```
-a*(log(a*x + 1) + log(a*x - 1) - log(x)) - (a^2*x + 1/x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(38) = 76.

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.82

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^2}\,dx = -a\left(\frac{2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{\frac{(ax+1)^2}{(ax-1)^2}-1}+\log\left(\frac{(ax+1)^2}{(ax-1)^2}\right)-\log\left(\left|\frac{(ax+1)^2}{(ax-1)^2}-1\right|\right)\right)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="giac")
```

output
```
-a*(2*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(
a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)^2
/(a*x - 1)^2 - 1) + log((a*x + 1)^2/(a*x - 1)^2) - log(abs((a*x + 1)^2/(a*
x - 1)^2 - 1)))
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^2}\,dx = a\,\ln(x) - a\,\ln\left(a^2\,x^2-1\right) - \frac{\text{atanh}(a\,x)}{x} - a^2\,x\,\text{atanh}(a\,x)$$

input
```
int(-(atanh(a*x)*(a^2*x^2 - 1))/x^2,x)
```

output
```
a*log(x) - a*log(a^2*x^2 - 1) - atanh(a*x)/x - a^2*x*atanh(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^2}\,dx = \frac{-atanh(ax)\,a^2x^2 - 2atanh(ax)\,ax - atanh(ax) - 2\log(a^2x-a)\,ax + \log(x)\,ax}{x}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)/x^2,x)
```

output
```
( - atanh(a*x)*a**2*x**2 - 2*atanh(a*x)*a*x - atanh(a*x) - 2*log(a**2*x -
a)*a*x + log(x)*a*x)/x
```

3.168 $\int \frac{(1-a^2x^2)\textbf{arctanh}(ax)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^3}\,dx = -\frac{a}{2x}+\frac{1}{2}a^2\operatorname{arctanh}(ax)-\frac{\operatorname{arctanh}(ax)}{2x^2} +\frac{1}{2}a^2\operatorname{PolyLog}(2,-ax)-\frac{1}{2}a^2\operatorname{PolyLog}(2,ax)$$

output
```
-1/2*a/x+1/2*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1/2*a^2*polylog(2,-a*x)
-1/2*a^2*polylog(2,a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^3}\,dx = -\frac{a}{2x}-\frac{\operatorname{arctanh}(ax)}{2x^2}-\frac{1}{4}a^2\log(1-ax)+\frac{1}{4}a^2\log(1+ax) -\frac{1}{2}a^2(-\operatorname{PolyLog}(2,-ax)+\operatorname{PolyLog}(2,ax))$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

output
```
-1/2*a/x - ArcTanh[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x]
)/4 - (a^2*(-PolyLog[2, -(a*x)] + PolyLog[2, a*x]))/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6576, 6446, 6452, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^3}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\text{arctanh}(ax)}{x^3}dx - a^2\int \frac{\text{arctanh}(ax)}{x}dx$$

$$\Big\downarrow \text{6446}$$

$$\int \frac{\text{arctanh}(ax)}{x^3}dx - a^2\left(\frac{\text{PolyLog}(2,ax)}{2}-\frac{\text{PolyLog}(2,-ax)}{2}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}a\int \frac{1}{x^2\,(1-a^2x^2)}dx - \left(a^2\left(\frac{\text{PolyLog}(2,ax)}{2}-\frac{\text{PolyLog}(2,-ax)}{2}\right)\right) - \frac{\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx - \frac{1}{x}\right) - \left(a^2\left(\frac{\text{PolyLog}(2,ax)}{2}-\frac{\text{PolyLog}(2,-ax)}{2}\right)\right) - \frac{\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$-\left(a^2\left(\frac{\text{PolyLog}(2,ax)}{2}-\frac{\text{PolyLog}(2,-ax)}{2}\right)\right) - \frac{\text{arctanh}(ax)}{2x^2} + \frac{1}{2}a\left(a\text{arctanh}(ax) - \frac{1}{x}\right)$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

output
```
-1/2*ArcTanh[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 - a^2*(-1/2*PolyL
og[2, -(a*x)] + PolyLog[2, a*x]/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

method	result
derivativedivides	$a^2\left(-\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}-\frac{1}{2ax}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}+\frac{\operatorname{dilog}(ax)}{2}+\frac{\operatorname{dilog}(ax+}{2}\right.$
default	$a^2\left(-\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}-\frac{1}{2ax}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}+\frac{\operatorname{dilog}(ax)}{2}+\frac{\operatorname{dilog}(ax+}{2}\right.$
risch	$-\frac{a^2\operatorname{dilog}(-ax+1)}{2}+\frac{a^2\ln(-ax)}{4}-\frac{a}{2x}-\frac{a^2\ln(-ax+1)}{4}+\frac{\ln(-ax+1)}{4x^2}+\frac{a^2\operatorname{dilog}(ax+1)}{2}-\frac{a^2\ln(ax)}{4}+\frac{a}{}$
meijerg	$\frac{ia^2\left(\frac{2i}{xa}+\frac{2i(-ax+1)(ax+1)\operatorname{arctanh}(ax)}{x^2a^2}\right)}{4}+\frac{ia^2\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-\frac{2iax\operatorname{polylog}\left(2,-\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}$
parts	$-\frac{\operatorname{arctanh}(ax)}{2x^2}-\operatorname{arctanh}(ax)a^2\ln(x)-\frac{a\left(-\frac{a\ln(ax+1)}{2}+\frac{a\ln(ax-1)}{2}+\frac{1}{x}+2a^2\left(-\frac{\operatorname{dilog}(ax+1)}{2a}-\frac{\ln(x)\ln(ax+}{2a}\right.\right.}{2}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/a^2/x^2-1/2/a/x-1/4*ln(a*x-1)+
1/4*ln(a*x+1)+1/2*dilog(a*x)+1/2*dilog(a*x+1)+1/2*ln(a*x)*ln(a*x+1))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^3}\,dx=\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^3}\,dx = -\int \left(-\frac{\operatorname{atanh}(ax)}{x^3}\right)dx - \int \frac{a^2 \operatorname{atanh}(ax)}{x}\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)/x**3,x)
```

output
```
-Integral(-atanh(a*x)/x**3, x) - Integral(a**2*atanh(a*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^3}\,dx$$
$$= \frac{1}{4}\Big(2\left(\log(ax+1)\log(x) + \operatorname{Li}_2(-ax)\right)a - 2\left(\log(-ax+1)\log(x) + \operatorname{Li}_2(ax)\right)a + a\log(ax+1) - a\log($$
$$- \frac{1}{2}\left(a^2\log(x^2) + \frac{1}{x^2}\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/4*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + d
ilog(a*x))*a + a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*(a^2*log(x^2
) + 1/x^2)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(44) = 88$.

Time = 1.15 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.89

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^3}\,dx$$

$$= a^2\left(\frac{\log\left(\frac{(ax+1)^2}{(ax-1)^2}\right)}{a} - \frac{\log\left(\left|\frac{(ax+1)^2}{(ax-1)^2}-1\right|\right)}{a} + \frac{\frac{(ax+1)^2}{(ax-1)^2}-2}{a\left(\frac{(ax+1)^2}{(ax-1)^2}-1\right)} - \frac{2\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}-1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}+1}\right)}{a\left(\frac{(ax+1)^2}{(ax-1)^2}-1\right)^2}\right)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="giac")
```

output
```
a^2*(log((a*x + 1)^2/(a*x - 1)^2)/a - log(abs((a*x + 1)^2/(a*x - 1)^2 - 1)
)/a + ((a*x + 1)^2/(a*x - 1)^2 - 2)/(a*((a*x + 1)^2/(a*x - 1)^2 - 1)) - 2*
log(-(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(a*x - 1) + 1)/((a*
x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*
x - 1) - a) - 1)) - 1)/(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/(
a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1
)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + 1))/(a*((a*x + 1)^2/(a*x - 1)^2 - 1)
^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^3}\,dx = -\int \frac{\text{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}{x^3}\,dx$$

input
```
int(-(atanh(a*x)*(a^2*x^2 - 1))/x^3,x)
```

output
```
-int((atanh(a*x)*(a^2*x^2 - 1))/x^3, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^3}\,dx = \frac{atanh(ax)\,a^2x^2 - atanh(ax) - 2\left(\int \frac{atanh(ax)}{x}dx\right)a^2x^2 - ax}{2x^2}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)/x^3,x)
```

output
```
(atanh(a*x)*a**2*x**2 - atanh(a*x) - 2*int(atanh(a*x)/x,x)*a**2*x**2 - a*x
)/(2*x**2)
```

3.169 $\int \frac{(1-a^2x^2)\textbf{arctanh}(ax)}{x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^4}\,dx = -\frac{a}{6x^2}-\frac{\mathrm{arctanh}(ax)}{3x^3}+\frac{a^2\mathrm{arctanh}(ax)}{x} - \frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log\left(1-a^2x^2\right)$$

output
```
-1/6*a/x^2-1/3*arctanh(a*x)/x^3+a^2*arctanh(a*x)/x-2/3*a^3*ln(x)+1/3*a^3*l
n(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^4}\,dx = -\frac{a}{6x^2}-\frac{\mathrm{arctanh}(ax)}{3x^3}+\frac{a^2\mathrm{arctanh}(ax)}{x} - \frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log\left(1-a^2x^2\right)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

output
```
-1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/
3 + (a^3*Log[1 - a^2*x^2])/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6576, 6452, 243, 47, 14, 16, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^4}dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\text{arctanh}(ax)}{x^4}dx-a^2\int \frac{\text{arctanh}(ax)}{x^2}dx$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(a\int \frac{1}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\left(a^2\left(\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\int \frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{47}$$

$$-\left(a^2\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\int \frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$-\left(a^2\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log(x^2)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\int \frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}a\int \frac{1}{x^4\left(1-a^2x^2\right)}dx^2-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)-\frac{\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \ 54$$

$$\frac{1}{6}a\int \left(-\frac{a^4}{a^2x^2-1}+\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)-\frac{\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \ 2009$$

$$-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)-\frac{\text{arctanh}(ax)}{3x^3}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

output
```
-1/3*ArcTanh[a*x]/x^3 - a^2*(-(ArcTanh[a*x]/x) + (a*(Log[x^2] - Log[1 - a^
2*x^2]))/2) + (a*(-x^(-2) + a^2*Log[x^2] - a^2*Log[1 - a^2*x^2]))/6
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

method	result
derivativedivides	$a^3\left(\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{3a^3x^3}-\frac{1}{6a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(ax-1)}{3}+\frac{\ln(ax+1)}{3}\right)$
default	$a^3\left(\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{3a^3x^3}-\frac{1}{6a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(ax-1)}{3}+\frac{\ln(ax+1)}{3}\right)$
parts	$-\frac{\operatorname{arctanh}(ax)}{3x^3}+\frac{a^2\operatorname{arctanh}(ax)}{x}-\frac{a\left(\frac{1}{2x^2}+2a^2\ln(x)-a^2\ln(ax+1)-a^2\ln(ax-1)\right)}{3}$
parallelrisch	$-\frac{4\ln(x)a^3x^3-4\ln(ax-1)x^3a^3-4a^3x^3\operatorname{arctanh}(ax)+a^3x^3-6a^2x^2\operatorname{arctanh}(ax)+ax+2\operatorname{arctanh}(ax)}{6x^3}$
risch	$\frac{(3a^2x^2-1)\ln(ax+1)}{6x^3}-\frac{4\ln(x)a^3x^3-2\ln(-a^2x^2+1)a^3x^3+3x^2\ln(-ax+1)a^2+ax-\ln(-ax+1)}{6x^3}$
meijerg	$-\frac{a^3\left(\frac{2}{a^2x^2}+\frac{4}{9}-\frac{4\ln(x)}{3}-\frac{4\ln(ia)}{3}-\frac{2\left(10a^2x^2+30\right)}{45a^2x^2}-\frac{2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3a^2x^2\sqrt{a^2x^2}}+\frac{2\ln\left(-a^2x^2+1\right)}{3}\right)}{4}-\frac{a^3\left(4\ln(x)\right.}{}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(arctanh(a*x)/a/x-1/3*arctanh(a*x)/a^3/x^3-1/6/a^2/x^2-2/3*ln(a*x)+1/3
*ln(a*x-1)+1/3*ln(a*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^4}\,dx = \frac{2\,a^3x^3\log\left(a^2x^2-1\right)-4\,a^3x^3\log\left(x\right)-ax+\left(3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*a^3*x^3*log(a^2*x^2 - 1) - 4*a^3*x^3*log(x) - a*x + (3*a^2*x^2 - 1)
*log(-(a*x + 1)/(a*x - 1)))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^4}\,dx$$
$$= \begin{cases} -\frac{2a^3\log(x)}{3} + \frac{2a^3\log\left(x-\frac{1}{a}\right)}{3} + \frac{2a^3\operatorname{atanh}(ax)}{3} + \frac{a^2\operatorname{atanh}(ax)}{x} - \frac{a}{6x^2} - \frac{\operatorname{atanh}(ax)}{3x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)/x**4,x)
```

output
```
Piecewise((-2*a**3*log(x)/3 + 2*a**3*log(x - 1/a)/3 + 2*a**3*atanh(a*x)/3
+ a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3), Ne(a, 0)), (0, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^4}\,dx = \frac{1}{6}\left(2\,a^2\log\left(a^2x^2-1\right) - 2\,a^2\log\left(x^2\right) - \frac{1}{x^2}\right)a + \frac{(3\,a^2x^2-1)\operatorname{artanh}(ax)}{3\,x^3}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

output
```
1/6*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x^2) - 1/x^2)*a + 1/3*(3*a^2*x^2 -
 1)*arctanh(a*x)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(50) = 100$.

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.52

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^4}\,dx$$
$$= \frac{2}{3}\left(a^2\log\left(\frac{|-ax-1|}{|ax-1|}\right) - a^2\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right) + \frac{(ax+1)a^2}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2} - \frac{\left(\frac{3\,(ax+1)a^2}{ax-1}+a^2\right)\log\left(-\cdots\right.}{\left(\frac{ax+1}{ax-1}+1\right)^3}\right.$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="giac")
```

output
```
2/3*(a^2*log(abs(-a*x - 1)/abs(a*x - 1)) - a^2*log(abs(-(a*x + 1)/(a*x - 1
) - 1)) + (a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2) - (3*(a*x
+ 1)*a^2/(a*x - 1) + a^2)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(
a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a
) - 1))/((a*x + 1)/(a*x - 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^4}\,dx = \frac{a^3\,\ln\left(a^2\,x^2-1\right)}{3} - \frac{a}{6\,x^2} - \frac{\mathrm{atanh}(a\,x)}{3\,x^3} - \frac{2\,a^3\,\ln(x)}{3} + \frac{a^2\,\mathrm{atanh}(a\,x)}{x}$$

input
```
int(-(atanh(a*x)*(a^2*x^2 - 1))/x^4,x)
```

output
```
(a^3*log(a^2*x^2 - 1))/3 - a/(6*x^2) - atanh(a*x)/(3*x^3) - (2*a^3*log(x))
/3 + (a^2*atanh(a*x))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^4}\,dx = \frac{4atanh(ax)\,a^3x^3+6atanh(ax)\,a^2x^2-2atanh(ax)+4\log(a^2x-a)\,a^3x^3-4\log(x)\,a^3x^3-ax}{6x^3}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)/x^4,x)
```

output
```
(4*atanh(a*x)*a**3*x**3 + 6*atanh(a*x)*a**2*x**2 - 2*atanh(a*x) + 4*log(a*
*2*x - a)*a**3*x**3 - 4*log(x)*a**3*x**3 - a*x)/(6*x**3)
```

3.170 $\int \frac{(1-a^2x^2)\text{arctanh}(ax)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^5}\,dx = -\frac{a}{12x^3}+\frac{a^3}{4x}-\frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{4x^4}$$

output
```
-1/12*a/x^3+1/4*a^3/x-1/4*(-a^2*x^2+1)^2*arctanh(a*x)/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^5}\,dx = -\frac{a}{12x^3}+\frac{a^3}{4x}-\frac{\operatorname{arctanh}(ax)}{4x^4}+\frac{a^2\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{8}a^4\log(1-ax)-\frac{1}{8}a^4\log(1+ax)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

output
```
-1/12*a/x^3 + a^3/(4*x) - ArcTanh[a*x]/(4*x^4) + (a^2*ArcTanh[a*x])/(2*x^2
) + (a^4*Log[1 - a*x])/8 - (a^4*Log[1 + a*x])/8
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6570, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^5}\,dx$$

$$\Big\downarrow \text{6570}$$

$$\frac{1}{4}a\int \frac{1-a^2x^2}{x^4}dx-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{4x^4}$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{4}a\int \left(\frac{1}{x^4}-\frac{a^2}{x^2}\right)dx-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a\left(\frac{a^2}{x}-\frac{1}{3x^3}\right)-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{4x^4}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

output
```
(a*(-1/3*1/x^3 + a^2/x))/4 - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*x^4)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

method	result
parallelrisch	$-\frac{3a^4x^4\operatorname{arctanh}(ax)-3a^3x^3-6a^2x^2\operatorname{arctanh}(ax)+ax+3\operatorname{arctanh}(ax)}{12x^4}$
derivativedivides	$a^4\left(-\frac{\operatorname{arctanh}(ax)}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\frac{1}{4ax}-\frac{1}{12a^3x^3}+\frac{\ln(ax-1)}{8}-\frac{\ln(ax+1)}{8}\right)$
default	$a^4\left(-\frac{\operatorname{arctanh}(ax)}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\frac{1}{4ax}-\frac{1}{12a^3x^3}+\frac{\ln(ax-1)}{8}-\frac{\ln(ax+1)}{8}\right)$
parts	$-\frac{\operatorname{arctanh}(ax)}{4x^4}+\frac{a^2\operatorname{arctanh}(ax)}{2x^2}-\frac{a\left(\frac{1}{3x^3}-\frac{a^2}{x}+\frac{a^3\ln(ax+1)}{2}-\frac{a^3\ln(ax-1)}{2}\right)}{4}$
risch	$\frac{(2a^2x^2-1)\ln(ax+1)}{8x^4}+\frac{3x^4\ln(-ax+1)a^4-3\ln(-ax-1)a^4x^4+6a^3x^3-6x^2\ln(-ax+1)a^2-2ax+3\ln(-ax+1)}{24x^4}$
orering	$\frac{(3a^4x^4-6a^2x^2+2)(-a^2x^2+1)\operatorname{arctanh}(ax)}{3x^4(a^2x^2-1)}+\frac{(3a^2x^2-1)x^2\left(-\frac{2a^2\operatorname{arctanh}(ax)}{x^4}+\frac{a}{x^5}-\frac{5(-a^2x^2+1)\operatorname{arctanh}(ax)}{x^6}\right)}{12}$
meijerg	$-\frac{ia^4\left(-\frac{i}{3x^3a^3}-\frac{i}{xa}+\frac{4i\left(-\frac{3a^4x^4}{8}+\frac{3}{8}\right)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3x^3a^3\sqrt{a^2x^2}}\right)}{4}-\frac{ia^4\left(\frac{2i}{xa}+\frac{2i(-ax+1)(ax+1)\operatorname{arctanh}(ax)}{x^2a^2}\right)}{4}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*a^4*x^4*arctanh(a*x)-3*a^3*x^3-6*a^2*x^2*arctanh(a*x)+a*x+3*arcta
nh(a*x))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^5}\,dx = \frac{6\,a^3x^3-2\,ax-3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{24\,x^4}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

output
```
1/24*(6*a^3*x^3 - 2*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1)))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^5}\,dx = -\frac{a^4\operatorname{atanh}(ax)}{4}+\frac{a^3}{4x}+\frac{a^2\operatorname{atanh}(ax)}{2x^2}-\frac{a}{12x^3}-\frac{\operatorname{atanh}(ax)}{4x^4}$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)/x**5,x)
```

output
```
-a**4*atanh(a*x)/4 + a**3/(4*x) + a**2*atanh(a*x)/(2*x**2) - a/(12*x**3) -
 atanh(a*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^5}\,dx = -\frac{1}{24}\left(3\,a^3\log(ax+1)-3\,a^3\log(ax-1)-\frac{2\,(3\,a^2x^2-1)}{x^3}\right)a + \frac{(2\,a^2x^2-1)\operatorname{artanh}(ax)}{4\,x^4}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

output
```
-1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 - 1)/x^3)*a
+ 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^5}\,dx$$
$$= -\frac{1}{3}a\left(\frac{a^3\left(\frac{3\,(ax+1)}{ax-1}+1\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}+\frac{6\,(ax+1)^2a^3\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^2\left(\frac{ax+1}{ax-1}+1\right)^4}\right)$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="giac")
```

output
```
-1/3*a*(a^3*(3*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)/(a*x - 1) + 1)^3 + 6*(a
*x + 1)^2*a^3*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a
) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*
x - 1)^2*((a*x + 1)/(a*x - 1) + 1)^4))
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^5}\,dx = \frac{a^3}{4\,x} - \frac{\text{atanh}(a\,x)}{4\,x^4} - \frac{a}{12\,x^3} + \frac{a^5\,\text{atan}\left(\frac{a^2\,x}{\sqrt{-a^2}}\right)}{4\,\sqrt{-a^2}} + \frac{a^2\,\text{atanh}(a\,x)}{2\,x^2}$$

input
```
int(-(atanh(a*x)*(a^2*x^2 - 1))/x^5,x)
```

output
```
a^3/(4*x) - atanh(a*x)/(4*x^4) - a/(12*x^3) + (a^5*atan((a^2*x)/(-a^2)^(1/
2)))/(4*(-a^2)^(1/2)) + (a^2*atanh(a*x))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^5}\,dx = \frac{-3atanh(ax)\,a^4x^4 + 6atanh(ax)\,a^2x^2 - 3atanh(ax) + 3a^3x^3 - ax}{12x^4}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)/x^5,x)
```

output
```
( - 3*atanh(a*x)*a**4*x**4 + 6*atanh(a*x)*a**2*x**2 - 3*atanh(a*x) + 3*a**
3*x**3 - a*x)/(12*x**4)
```

3.171 $\int \frac{(1-a^2x^2)\textbf{arctanh}(ax)}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^6}\,dx = -\frac{a}{20x^4}+\frac{a^3}{15x^2}-\frac{\mathrm{arctanh}(ax)}{5x^5}+\frac{a^2\mathrm{arctanh}(ax)}{3x^3} - \frac{2}{15}a^5\log(x)+\frac{1}{15}a^5\log\left(1-a^2x^2\right)$$

output
```
-1/20*a/x^4+1/15*a^3/x^2-1/5*arctanh(a*x)/x^5+1/3*a^2*arctanh(a*x)/x^3-2/1
5*a^5*ln(x)+1/15*a^5*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)}{x^6}\,dx = -\frac{a}{20x^4}+\frac{a^3}{15x^2}-\frac{\mathrm{arctanh}(ax)}{5x^5}+\frac{a^2\mathrm{arctanh}(ax)}{3x^3} - \frac{2}{15}a^5\log(x)+\frac{1}{15}a^5\log\left(1-a^2x^2\right)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

output
```
-1/20*a/x^4 + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*
x^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6576, 6452, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^6}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\text{arctanh}(ax)}{x^6}dx-a^2\int \frac{\text{arctanh}(ax)}{x^4}dx$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{3x^3}\right)\right)+\frac{1}{5}a\int \frac{1}{x^5(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$-\left(a^2\left(\frac{1}{6}a\int \frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}\right)\right)+\frac{1}{10}a\int \frac{1}{x^6(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{54}$$

$$-\left(a^2\left(\frac{1}{6}a\int \left(-\frac{a^4}{a^2x^2-1}+\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\text{arctanh}(ax)}{3x^3}\right)\right)+\frac{1}{10}a\int \left(-\frac{a^6}{a^2x^2-1}+\frac{a^4}{x^2}+\frac{a^2}{x^4}+\frac{1}{x^6}\right)dx^2-\frac{\text{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$-\left(a^2\left(\frac{1}{6}a\left(a^2\log(x^2)-a^2\log(1-a^2x^2)-\frac{1}{x^2}\right)-\frac{\text{arctanh}(ax)}{3x^3}\right)\right)+\frac{1}{10}a\left(a^4\log(x^2)-\frac{a^2}{x^2}-a^4\log(1-a^2x^2)-\frac{1}{2x^4}\right)-\frac{\text{arctanh}(ax)}{5x^5}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

output
```
-1/5*ArcTanh[a*x]/x^5 + (a*(-1/2*1/x^4 - a^2/x^2 + a^4*Log[x^2] - a^4*Log[
1 - a^2*x^2]))/10 - a^2*(-1/3*ArcTanh[a*x]/x^3 + (a*(-x^(-2) + a^2*Log[x^2
] - a^2*Log[1 - a^2*x^2]))/6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result
derivativedivides	$a^{5}\left(-\frac{\operatorname{arctanh}(ax)}{5a^{5}x^{5}}+\frac{\operatorname{arctanh}(ax)}{3a^{3}x^{3}}-\frac{1}{20a^{4}x^{4}}+\frac{1}{15a^{2}x^{2}}-\frac{2\ln(ax)}{15}+\frac{\ln(ax-1)}{15}+\frac{\ln(ax+1)}{15}\right)$
default	$a^{5}\left(-\frac{\operatorname{arctanh}(ax)}{5a^{5}x^{5}}+\frac{\operatorname{arctanh}(ax)}{3a^{3}x^{3}}-\frac{1}{20a^{4}x^{4}}+\frac{1}{15a^{2}x^{2}}-\frac{2\ln(ax)}{15}+\frac{\ln(ax-1)}{15}+\frac{\ln(ax+1)}{15}\right)$
parts	$\frac{a^{2}\operatorname{arctanh}(ax)}{3x^{3}}-\frac{\operatorname{arctanh}(ax)}{5x^{5}}-\frac{a\left(\frac{3}{4x^{4}}-\frac{a^{2}}{x^{2}}+2a^{4}\ln(x)-a^{4}\ln(ax+1)-a^{4}\ln(ax-1)\right)}{15}$
parallelrisch	$-\frac{8\ln(x)a^{5}x^{5}-8\ln(ax-1)x^{5}a^{5}-8\operatorname{arctanh}(ax)a^{5}x^{5}-4a^{5}x^{5}-4a^{3}x^{3}-20a^{2}x^{2}\operatorname{arctanh}(ax)+3ax+12\operatorname{arctanh}(ax)}{60x^{5}}$
risch	$\frac{(5a^{2}x^{2}-3)\ln(ax+1)}{30x^{5}}-\frac{8\ln(x)a^{5}x^{5}-4\ln(-a^{2}x^{2}+1)a^{5}x^{5}-4a^{3}x^{3}+10x^{2}\ln(-ax+1)a^{2}+3ax-6\ln(-ax+1)}{60x^{5}}$
meijerg	$\frac{a^{5}\left(-\frac{1}{a^{4}x^{4}}-\frac{2}{3a^{2}x^{2}}-\frac{4}{25}+\frac{4\ln(x)}{5}+\frac{4\ln(ia)}{5}+\frac{\frac{4}{25}a^{4}x^{4}+\frac{4}{15}a^{2}x^{2}+\frac{4}{5}}{a^{4}x^{4}}+\frac{\frac{2\ln\left(1-\sqrt{a^{2}x^{2}}\right)}{5}-\frac{2\ln\left(1+\sqrt{a^{2}x^{2}}\right)}{5}}{a^{4}x^{4}\sqrt{a^{2}x^{2}}}-\frac{2\ln\left(-a^{2}x^{2}+1\right)}{5}\right)}{4}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/5*arctanh(a*x)/a^5/x^5+1/3*arctanh(a*x)/a^3/x^3-1/20/a^4/x^4+1/15/
a^2/x^2-2/15*ln(a*x)+1/15*ln(a*x-1)+1/15*ln(a*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^6}\,dx = \frac{4\,a^{5}x^{5}\log\left(a^{2}x^{2}-1\right)-8\,a^{5}x^{5}\log\left(x\right)+4\,a^{3}x^{3}-3\,ax+2\left(5\,a^{2}x^{2}-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,x^{5}}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

output
```
1/60*(4*a^5*x^5*log(a^2*x^2 - 1) - 8*a^5*x^5*log(x) + 4*a^3*x^3 - 3*a*x +
2*(5*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^6}\,dx = \begin{cases} -\frac{2a^5\log(x)}{15} + \frac{2a^5\log\left(x-\frac{1}{a}\right)}{15} + \frac{2a^5\operatorname{atanh}(ax)}{15} + \frac{a^3}{15x^2} + \frac{a^2\operatorname{atanh}(ax)}{3x^3} - \frac{a}{20x^4} - \frac{\operatorname{atanh}(ax)}{5x^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)/x**6,x)
```

output
```
Piecewise((-2*a**5*log(x)/15 + 2*a**5*log(x - 1/a)/15 + 2*a**5*atanh(a*x)/
15 + a**3/(15*x**2) + a**2*atanh(a*x)/(3*x**3) - a/(20*x**4) - atanh(a*x)/
(5*x**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{x^6}\,dx = \frac{1}{60}\left(4\,a^4\log\left(a^2x^2-1\right) - 4\,a^4\log\left(x^2\right) + \frac{4\,a^2x^2-3}{x^4}\right)a + \frac{(5\,a^2x^2-3)\operatorname{artanh}(ax)}{15\,x^5}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

output
```
1/60*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 - 3)/x^4)*a + 1
/15*(5*a^2*x^2 - 3)*arctanh(a*x)/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(59) = 118$.

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.96

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^6}\,dx$$
$$= \frac{2}{15}\left(a^4\log\left(\frac{|-ax-1|}{|ax-1|}\right) - a^4\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right) + \frac{\frac{(ax+1)^3a^4}{(ax-1)^3} - \frac{4(ax+1)^2a^4}{(ax-1)^2} + \frac{(ax+1)a^4}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^4} - \frac{\left(\frac{15(ax+1)^3a^4}{(ax-1)^3} - \right.}{}\right.$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="giac")
```

output
```
2/15*(a^4*log(abs(-a*x - 1)/abs(a*x - 1)) - a^4*log(abs(-(a*x + 1)/(a*x -
1) - 1)) + ((a*x + 1)^3*a^4/(a*x - 1)^3 - 4*(a*x + 1)^2*a^4/(a*x - 1)^2 +
(a*x + 1)*a^4/(a*x - 1))/((a*x + 1)/(a*x - 1) + 1)^4 - (15*(a*x + 1)^3*a^4
/(a*x - 1)^3 - 5*(a*x + 1)^2*a^4/(a*x - 1)^2 + 5*(a*x + 1)*a^4/(a*x - 1) +
 a^4)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(
a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)/(
a*x - 1) + 1)^5)*a
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^6}\,dx = \frac{a^5\,\ln\left(a^2\,x^2-1\right)}{15} - \frac{\frac{\text{atanh}(a\,x)}{5} + \frac{a\,x}{20} - \frac{a^3\,x^3}{15} - \frac{a^2\,x^2\,\text{atanh}(a\,x)}{3}}{x^5} - \frac{2\,a^5\,\ln(x)}{15}$$

input
```
int(-(atanh(a*x)*(a^2*x^2 - 1))/x^6,x)
```

output
```
(a^5*log(a^2*x^2 - 1))/15 - (atanh(a*x)/5 + (a*x)/20 - (a^3*x^3)/15 - (a^2
*x^2*atanh(a*x))/3)/x^5 - (2*a^5*log(x))/15
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^6}\,dx$$
$$= \frac{8atanh(ax)\,a^5x^5+20atanh(ax)\,a^2x^2-12atanh(ax)+8\log(a^2x-a)\,a^5x^5-8\log(x)\,a^5x^5+4a^3x^3-3a}{60x^5}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)/x^6,x)
```

output
```
(8*atanh(a*x)*a**5*x**5 + 20*atanh(a*x)*a**2*x**2 - 12*atanh(a*x) + 8*log(
a**2*x - a)*a**5*x**5 - 8*log(x)*a**5*x**5 + 4*a**3*x**3 - 3*a*x)/(60*x**5
)
```

3.172 $\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = \frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{4\operatorname{arctanh}(ax)}{105a^5}+\frac{2x^2\operatorname{arctanh}(ax)}{35a^3}+\frac{x^4\operatorname{arctanh}(ax)}{35a}-\frac{1}{21}ax^6\operatorname{arctanh}(ax)+\frac{2\operatorname{arctanh}(ax)^2}{35a^5}+\frac{1}{5}x^5\operatorname{arctanh}(ax)^2-\frac{1}{7}a^2x^7\operatorname{arctanh}(ax)^2-\frac{4\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{35a^5}-\frac{2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a^5}$$

output
```
4/105*x/a^4-2/315*x^3/a^2-1/105*x^5-4/105*arctanh(a*x)/a^5+2/35*x^2*arctan
h(a*x)/a^3+1/35*x^4*arctanh(a*x)/a-1/21*a*x^6*arctanh(a*x)+2/35*arctanh(a*
x)^2/a^5+1/5*x^5*arctanh(a*x)^2-1/7*a^2*x^7*arctanh(a*x)^2-4/35*arctanh(a*
x)*ln(2/(-a*x+1))/a^5-2/35*polylog(2,1-2/(-a*x+1))/a^5
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

$$\int x^4(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx = -\frac{-12ax+2a^3x^3+3a^5x^5+9(2-7a^5x^5+5a^7x^7)\,\text{arctanh}(ax)^2+3\text{arctanh}(ax)\left(4-6a^2x^2-3a^4x^4+\right.}{315a^5}$$

input
```
Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
-1/315*(-12*a*x + 2*a^3*x^3 + 3*a^5*x^5 + 9*(2 - 7*a^5*x^5 + 5*a^7*x^7)*Ar
cTanh[a*x]^2 + 3*ArcTanh[a*x]*(4 - 6*a^2*x^2 - 3*a^4*x^4 + 5*a^6*x^6 + 12*
Log[1 + E^(-2*ArcTanh[a*x])]) - 18*PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^5
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 411 vs. 2(162) = 324.

Time = 2.67 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.54, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6576, 6452, 6542, 6452, 254, 2009, 6542, 6452, 254, 262, 219, 2009, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^4\text{arctanh}(ax)^2dx-a^2\int x^6\text{arctanh}(ax)^2dx$$

$$\Big\downarrow \text{6452}$$

$$-a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\int\frac{x^7\text{arctanh}(ax)}{1-a^2x^2}dx\right)-\frac{2}{5}a\int\frac{x^5\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6542}$$

$$-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^3\text{arctanh}(ax)dx}{a^2}\right)-a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\int \frac{x^5\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^5\text{arctanh}(ax)dx}{a^2}\right)\right)+\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6452}$$

$$-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \frac{x^4}{1-a^2x^2}dx}{a^2}\right)-a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\int \frac{x^5\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax)-\frac{1}{6}a\int \frac{x^6}{1-a^2x^2}dx}{a^2}\right)\right)+\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{254}$$

$$-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}\right)-a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\int \frac{x^5\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax)-\frac{1}{6}a\int \left(-\frac{x^4}{a^2}-\frac{x^2}{a^4}+\frac{1}{a^6(1-a^2x^2)}-\frac{1}{a^6}\right)dx}{a^2}\right)\right)+\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)-a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\int \frac{x^5\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax)-\frac{1}{6}a\left(\frac{\text{arctanh}(ax)}{a^7}-\frac{x}{a^6}-\frac{x^3}{3a^4}-\frac{x^5}{5a^2}\right)}{a^2}\right)\right)+\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6542}$$

$$-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^3\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax)-\frac{1}{6}a\left(\frac{\text{arctanh}(ax)}{a^7}-\frac{x}{a^6}-\right.}{a^2}\right.\right.$$
$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$\downarrow$ 6452

$$-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int \frac{x^2}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \frac{x^4}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax)-\frac{1}{6}a\left(\frac{\text{arctanh}}{a^7}\right.}{a^2}\right.\right.$$
$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$\downarrow$ 254

$$-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int \frac{x^2}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax}{}\right.\right.$$
$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$\downarrow$ 262

$$-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int\left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax}{}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\downarrow \; 219$$

$$-a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int\left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(a}{}\right.\right.$$

$$\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\downarrow \; 2009$$

$$-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right.}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{6}x^6\text{arctanh}(ax)-}{}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\downarrow \; 6542$$

$$-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\,\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right.}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\int\frac{x\,\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\,\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{6}}{}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow\ 6452$$

$$-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\,\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right.}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\int\frac{x\,\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int\frac{x^2}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}\right.}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow\ 262$$

$$-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\,\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right.}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\int\frac{x\,\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\int\frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}}{a^5}\right.}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow\ 219$$

$$-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right.}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctan}}{a^5}\right.}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6546}$$

$$-\frac{2}{5}a\left(\frac{\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(a}{a^5}\right.}{a^2}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(a}{}}{a^2}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6470}$$

$$-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax}{}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right.}{a^2}}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{2849}$$

$$-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}\cdots}{\cdots}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}\cdots\right.}{a^2}}{a^2}}{a^2}\cdots\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

$\downarrow$ 2752

$$-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}\cdots}{\cdots}\right.$$

$$a^2\left(\frac{1}{7}x^7\text{arctanh}(ax)^2-\frac{2}{7}a\left(\frac{\frac{\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}\cdots\right.}{a^2}}{a^2}}{a^2}\cdots\right.\right.$$

$$\frac{1}{5}x^5\text{arctanh}(ax)^2$$

input
```
Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
(x^5*ArcTanh[a*x]^2)/5 - (2*a*(-(((x^4*ArcTanh[a*x])/4 - (a*(-(x/a^4) - x^
3/(3*a^2) + ArcTanh[a*x]/a^5))/4)/a^2) + (-(((x^2*ArcTanh[a*x])/2 - (a*(-(
x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2) + (-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh
[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2)/a^2
))/5 - a^2*((x^7*ArcTanh[a*x]^2)/7 - (2*a*(-(((x^6*ArcTanh[a*x])/6 - (a*(-
(x/a^6) - x^3/(3*a^4) - x^5/(5*a^2) + ArcTanh[a*x]/a^7))/6)/a^2) + (-(((x^
4*ArcTanh[a*x])/4 - (a*(-(x/a^4) - x^3/(3*a^2) + ArcTanh[a*x]/a^5))/4)/a^2
) + (-(((x^2*ArcTanh[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2) +
 (-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2
, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2)/a^2)/a^2))/7)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.23

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}+\frac{a^4 x^4 \operatorname{arctanh}(ax)}{35}+\frac{2a^2 x^2 \operatorname{arctanh}(ax)}{35}+\frac{2 \operatorname{arctanh}(ax)\ln(ax-1)}{35}$
default	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}+\frac{a^4 x^4 \operatorname{arctanh}(ax)}{35}+\frac{2a^2 x^2 \operatorname{arctanh}(ax)}{35}+\frac{2 \operatorname{arctanh}(ax)\ln(ax-1)}{35}$
parts	$-\frac{a^2 x^7 \operatorname{arctanh}(ax)^2}{7}+\frac{x^5 \operatorname{arctanh}(ax)^2}{5}-\frac{a\,x^6 \operatorname{arctanh}(ax)}{21}+\frac{x^4 \operatorname{arctanh}(ax)}{35a}+\frac{2x^2 \operatorname{arctanh}(ax)}{35a^3}+\frac{2 \operatorname{arctan}}{}$
risch	$-\frac{\left(\left(-\frac{1}{25}+\frac{\ln(ax+1)}{5}\right)(ax+1)^5+\left(\frac{1}{4}-\ln(ax+1)\right)(ax+1)^4+\left(-\frac{2}{3}+2\ln(ax+1)\right)(ax+1)^3+(1-2\ln(ax+1))(ax+1)^2+(-1}{2a^5}$

input
```
int(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-1/7*arctanh(a*x)^2*a^7*x^7+1/5*arctanh(a*x)^2*a^5*x^5-1/21*arctanh
(a*x)*a^6*x^6+1/35*a^4*x^4*arctanh(a*x)+2/35*a^2*x^2*arctanh(a*x)+2/35*arc
tanh(a*x)*ln(a*x-1)+2/35*arctanh(a*x)*ln(a*x+1)+1/70*ln(a*x-1)^2-2/35*dilo
g(1/2*a*x+1/2)-1/35*ln(a*x-1)*ln(1/2*a*x+1/2)-1/70*ln(a*x+1)^2+1/35*(ln(a*
x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/105*a^5*x^5-2/315*a^3*x^3+4/105*a
*x+2/105*ln(a*x-1)-2/105*ln(a*x+1))
```

Fricas [F]

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx=\int -\left(a^2x^2-1\right)x^4\operatorname{artanh}\left(ax\right)^2\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^6 - x^4)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = -\int\left(-x^4\operatorname{atanh}^2(ax)\right)\,dx - \int a^2x^6\operatorname{atanh}^2(ax)\,dx$$

input
```
integrate(x**4*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

output
```
-Integral(-x**4*atanh(a*x)**2, x) - Integral(a**2*x**6*atanh(a*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17

$$\int x^4(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx =$$
$$-\frac{1}{630}a^2\left(\frac{6a^5x^5+4a^3x^3-24ax+9\log(ax+1)^2-18\log(ax+1)\log(ax-1)-9\log(ax-1)^2-}{a^7}\right.$$
$$-\frac{1}{105}a\left(\frac{5a^4x^6-3a^2x^4-6x^2}{a^4}-\frac{6\log(ax+1)}{a^6}-\frac{6\log(ax-1)}{a^6}\right)\operatorname{artanh}(ax)$$
$$-\frac{1}{35}\left(5a^2x^7-7x^5\right)\operatorname{artanh}(ax)^2$$

input
```
integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-1/630*a^2*((6*a^5*x^5 + 4*a^3*x^3 - 24*a*x + 9*log(a*x + 1)^2 - 18*log(a*
x + 1)*log(a*x - 1) - 9*log(a*x - 1)^2 - 12*log(a*x - 1))/a^7 + 36*(log(a*
x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 + 12*log(a*x + 1)/a
^7) - 1/105*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 -
6*log(a*x - 1)/a^6)*arctanh(a*x) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)^2
```

Giac [F]

$$\int x^4 (1-a^2x^2) \operatorname{arctanh}(ax)^2 \, dx = \int -(a^2x^2-1)x^4 \operatorname{artanh}(ax)^2 \, dx$$

input
```
integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*x^4*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (1-a^2x^2) \operatorname{arctanh}(ax)^2 \, dx = -\int x^4 \operatorname{atanh}(a\,x)^2 \left(a^2\,x^2-1\right) \, dx$$

input
```
int(-x^4*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

output
```
-int(x^4*atanh(a*x)^2*(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int x^4 (1-a^2x^2) \operatorname{arctanh}(ax)^2 \, dx$$
$$= \frac{-45 atanh(ax)^2 a^7x^7 + 63 atanh(ax)^2 a^5x^5 - 18 atanh(ax)^2 ax - 15 atanh(ax) a^6x^6 + 9 atanh(ax) a^4x^4 +}{315a^5}$$

input
```
int(x^4*(-a^2*x^2+1)*atanh(a*x)^2,x)
```

output
```
( - 45*atanh(a*x)**2*a**7*x**7 + 63*atanh(a*x)**2*a**5*x**5 - 18*atanh(a*x
)**2*a*x - 15*atanh(a*x)*a**6*x**6 + 9*atanh(a*x)*a**4*x**4 + 18*atanh(a*x
)*a**2*x**2 - 12*atanh(a*x) + 18*int(atanh(a*x)**2,x)*a - 3*a**5*x**5 - 2*
a**3*x**3 + 12*a*x)/(315*a**5)
```

3.173 $\int x^3(1-a^2x^2)\,\textbf{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = -\frac{x^2}{180a^2}-\frac{x^4}{60}+\frac{x\operatorname{arctanh}(ax)}{6a^3}+\frac{x^3\operatorname{arctanh}(ax)}{18a}-\frac{1}{15}ax^5\operatorname{arctanh}(ax)-\frac{\operatorname{arctanh}(ax)^2}{12a^4}+\frac{1}{4}x^4\operatorname{arctanh}(ax)^2-\frac{1}{6}a^2x^6\operatorname{arctanh}(ax)^2+\frac{7\log(1-a^2x^2)}{90a^4}$$

output
```
-1/180*x^2/a^2-1/60*x^4+1/6*x*arctanh(a*x)/a^3+1/18*x^3*arctanh(a*x)/a-1/1
5*a*x^5*arctanh(a*x)-1/12*arctanh(a*x)^2/a^4+1/4*x^4*arctanh(a*x)^2-1/6*a^
2*x^6*arctanh(a*x)^2+7/90*ln(-a^2*x^2+1)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = -\frac{a^2x^2+3a^4x^4+2ax(-15-5a^2x^2+6a^4x^4)\operatorname{arctanh}(ax)+15(1-3a^4x^4+2a^6x^6)\operatorname{arctanh}(ax)^2-14\,\mathrm{l}}{180a^4}$$

input
```
Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
-1/180*(a^2*x^2 + 3*a^4*x^4 + 2*a*x*(-15 - 5*a^2*x^2 + 6*a^4*x^4)*ArcTanh[
a*x] + 15*(1 - 3*a^4*x^4 + 2*a^6*x^6)*ArcTanh[a*x]^2 - 14*Log[1 - a^2*x^2]
)/a^4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 297 vs. 2(116) = 232.

Time = 2.44 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.56, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {6576, 6452, 6542, 6452, 243, 49, 2009, 6542, 6436, 240, 6452, 243, 49, 2009, 6510, 6542, 6436, 240, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^3\operatorname{arctanh}(ax)^2dx - a^2\int x^5\operatorname{arctanh}(ax)^2dx$$

$$\Big\downarrow \text{6452}$$

$$-a^2\left(\frac{1}{6}x^6\operatorname{arctanh}(ax)^2-\frac{1}{3}a\int\frac{x^6\operatorname{arctanh}(ax)}{1-a^2x^2}dx\right)-\frac{1}{2}a\int\frac{x^4\operatorname{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{4}x^4\operatorname{arctanh}(ax)^2$$

$$\Big\downarrow \text{6542}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\text{arctanh}(ax)dx}{a^2}\right)-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\int \frac{x^4\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^4\text{arctanh}(ax)dx}{a^2}\right)\right)+\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6452}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{3}a\int \frac{x^3}{1-a^2x^2}dx}{a^2}\right)-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\int \frac{x^4\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{5}a\int \frac{x^5}{1-a^2x^2}dx}{a^2}\right)\right)+\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\int \frac{x^2}{1-a^2x^2}dx^2}{a^2}\right)-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\int \frac{x^4\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\int \frac{x^4}{1-a^2x^2}dx^2}{a^2}\right)\right)+\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\int \left(-\frac{1}{a^2}-\frac{1}{a^2(a^2x^2-1)}\right)dx^2}{a^2}\right)-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\int \frac{x^4\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\int \left(-\frac{x^2}{a^2}-\frac{1}{a^4(a^2x^2-1)}-\frac{1}{a^4}\right)dx^2}{a^2}\right)\right)+\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a\left(\frac{\int\frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\int\frac{x^4\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)\right)+$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6542}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1}{}\right.}{a^2}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6436}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)-a\int\frac{x}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1}{}\right.}{a^2}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{240}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1}{}\right.}{a^2}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{ 6452}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{3}a\int \frac{x^3}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\right.}{a^2}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\int \frac{x^2}{1-a^2x^2}dx^2}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\right.}{a^2}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$
$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\int\left(-\frac{1}{a^2}-\frac{1}{a^2(a^2x^2-1)}\right)dx^2}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-}{}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}a\left(\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{10}a\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6510}$$

$$-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arctanh}(ax)-\frac{1}{1}\right.\right.$$

$$\frac{1}{2}a\left(\frac{\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)+$$

$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6542}$$

$$-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\text{arc}}{}\right.\right.$$

$$\frac{1}{2}a\left(\frac{\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)+$$

$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6436}$$

$$-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)-a\int\frac{x}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{2}a\left(\frac{\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)+$$

$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$\Big\downarrow$ 240

$$-a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{2}a\left(\frac{\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)+$$

$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

$\Big\downarrow$ 6510

$$-\frac{1}{2}a\left(\frac{\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{6}x^6\text{arctanh}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\text{arctanh}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}\cdots}{}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(ax)^2$$

input
```
Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
(x^4*ArcTanh[a*x]^2)/4 - (a*(-(((x^3*ArcTanh[a*x])/3 - (a*(-(x^2/a^2) - Lo
g[1 - a^2*x^2]/a^4))/6)/a^2) + (ArcTanh[a*x]^2/(2*a^3) - (x*ArcTanh[a*x] +
 Log[1 - a^2*x^2]/(2*a))/a^2)/a^2))/2 - a^2*((x^6*ArcTanh[a*x]^2)/6 - (a*(
-(((x^5*ArcTanh[a*x])/5 - (a*(-(x^2/a^4) - x^4/(2*a^2) - Log[1 - a^2*x^2]/
a^6))/10)/a^2) + (-(((x^3*ArcTanh[a*x])/3 - (a*(-(x^2/a^2) - Log[1 - a^2*x
^2]/a^4))/6)/a^2) + (ArcTanh[a*x]^2/(2*a^3) - (x*ArcTanh[a*x] + Log[1 - a^
2*x^2]/(2*a))/a^2)/a^2)/a^2))/3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result
parallelrisch	$-\frac{30 \operatorname{arctanh}(ax)^2 a^6 x^6+12 \operatorname{arctanh}(ax) a^5 x^5-45 a^4 x^4 \operatorname{arctanh}(ax)^2+3 a^4 x^4-10 a^3 x^3 \operatorname{arctanh}(ax)+1+a^2 x^2-30 a x \text{ a}}{180 a^4}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2 a^6 x^6}{6}+\frac{a^4 x^4 \operatorname{arctanh}(ax)^2}{4}-\frac{\operatorname{arctanh}(ax) a^5 x^5}{15}+\frac{a^3 x^3 \operatorname{arctanh}(ax)}{18}+\frac{a x \operatorname{arctanh}(ax)}{6}+\frac{\operatorname{arctanh}(ax) \ln(ax-1)}{12}-\text{arc}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2 a^6 x^6}{6}+\frac{a^4 x^4 \operatorname{arctanh}(ax)^2}{4}-\frac{\operatorname{arctanh}(ax) a^5 x^5}{15}+\frac{a^3 x^3 \operatorname{arctanh}(ax)}{18}+\frac{a x \operatorname{arctanh}(ax)}{6}+\frac{\operatorname{arctanh}(ax) \ln(ax-1)}{12}-\text{arc}}{}$
parts	$-\frac{a^2 x^6 \operatorname{arctanh}(ax)^2}{6}+\frac{x^4 \operatorname{arctanh}(ax)^2}{4}-\frac{a x^5 \operatorname{arctanh}(ax)}{15}+\frac{x^3 \operatorname{arctanh}(ax)}{18a}+\frac{x \operatorname{arctanh}(ax)}{6a^3}+\frac{\operatorname{arctanh}(a}{1}$
risch	$-\frac{(2a^6x^6-3a^4x^4+1)\ln(ax+1)^2}{48a^4}+\frac{(30a^6x^6\ln(-ax+1)-12a^5x^5-45x^4\ln(-ax+1)a^4+10a^3x^3+30ax+15\ln(-ax+}{360a^4}$

input
```
int(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/180*(30*arctanh(a*x)^2*a^6*x^6+12*arctanh(a*x)*a^5*x^5-45*a^4*x^4*arcta
nh(a*x)^2+3*a^4*x^4-10*a^3*x^3*arctanh(a*x)+1+a^2*x^2-30*a*x*arctanh(a*x)+
15*arctanh(a*x)^2-28*ln(a*x-1)-28*arctanh(a*x))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = -\frac{12\,a^4x^4+4\,a^2x^2+15\,(2\,a^6x^6-3\,a^4x^4+1)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(6\,a^5x^5-5\,a^3x^3-15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-}{720\,a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-1/720*(12*a^4*x^4 + 4*a^2*x^2 + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*log(-(a*x
+ 1)/(a*x - 1))^2 + 4*(6*a^5*x^5 - 5*a^3*x^3 - 15*a*x)*log(-(a*x + 1)/(a*x
 - 1)) - 56*log(a^2*x^2 - 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = \begin{cases} -\frac{a^2x^6\operatorname{atanh}^2(ax)}{6}-\frac{ax^5\operatorname{atanh}(ax)}{15}+\frac{x^4\operatorname{atanh}^2(ax)}{4}-\frac{x^4}{60}+\frac{x^3\operatorname{atanh}(ax)}{18a}-\frac{x^2}{180a^2}+\frac{x\operatorname{atanh}(ax)}{6a^3}+\frac{7\log\left(x-\frac{1}{a}\right)}{45a^4}-\frac{\operatorname{atanh}^2}{12a} \\ 0 \end{cases}$$

input
```
integrate(x**3*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

output
```
Piecewise((-a**2*x**6*atanh(a*x)**2/6 - a*x**5*atanh(a*x)/15 + x**4*atanh(
a*x)**2/4 - x**4/60 + x**3*atanh(a*x)/(18*a) - x**2/(180*a**2) + x*atanh(a
*x)/(6*a**3) + 7*log(x - 1/a)/(45*a**4) - atanh(a*x)**2/(12*a**4) + 7*atan
h(a*x)/(45*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$
$$=-\frac{1}{180}a\left(\frac{2(6a^4x^5-5a^2x^3-15x)}{a^4}+\frac{15\log(ax+1)}{a^5}-\frac{15\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)$$
$$-\frac{1}{12}(2a^2x^6-3x^4)\operatorname{artanh}(ax)^2$$
$$-\frac{12a^4x^4+4a^2x^2+2(15\log(ax-1)-28)\log(ax+1)-15\log(ax+1)^2-15\log(ax-1)^2-56\log}{720a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-1/180*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*
log(a*x - 1)/a^5)*arctanh(a*x) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)^2 -
 1/720*(12*a^4*x^4 + 4*a^2*x^2 + 2*(15*log(a*x - 1) - 28)*log(a*x + 1) - 1
5*log(a*x + 1)^2 - 15*log(a*x - 1)^2 - 56*log(a*x - 1))/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(98) = 196$.

Time = 0.13 (sec) , antiderivative size = 522, normalized size of antiderivative = 4.50

$$\int x^3(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx =$$
$$-\frac{1}{45}\left(\frac{15\left(\frac{3(ax+1)^4}{(ax-1)^4}+\frac{2(ax+1)^3}{(ax-1)^3}+\frac{3(ax+1)^2}{(ax-1)^2}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^6a^5}{(ax-1)^6}-\frac{6(ax+1)^5a^5}{(ax-1)^5}+\frac{15(ax+1)^4a^5}{(ax-1)^4}-\frac{20(ax+1)^3a^5}{(ax-1)^3}+\frac{15(ax+1)^2a^5}{(ax-1)^2}-\frac{6(ax+1)a^5}{ax-1}+a^5}+\frac{\left(\frac{45(ax+}{(ax-1}\right.}{\frac{(ax+1)^5a^5}{(ax-1)^5}-\frac{5(a}{(a}}\right.$$

input
```
integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
-1/45*(15*(3*(a*x + 1)^4/(a*x - 1)^4 + 2*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x
+ 1)^2/(a*x - 1)^2)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^6*a^5/(a*x - 1)
^6 - 6*(a*x + 1)^5*a^5/(a*x - 1)^5 + 15*(a*x + 1)^4*a^5/(a*x - 1)^4 - 20*(
a*x + 1)^3*a^5/(a*x - 1)^3 + 15*(a*x + 1)^2*a^5/(a*x - 1)^2 - 6*(a*x + 1)*
a^5/(a*x - 1) + a^5) + (45*(a*x + 1)^3/(a*x - 1)^3 - 25*(a*x + 1)^2/(a*x -
 1)^2 + 35*(a*x + 1)/(a*x - 1) - 7)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^5
*a^5/(a*x - 1)^5 - 5*(a*x + 1)^4*a^5/(a*x - 1)^4 + 10*(a*x + 1)^3*a^5/(a*x
 - 1)^3 - 10*(a*x + 1)^2*a^5/(a*x - 1)^2 + 5*(a*x + 1)*a^5/(a*x - 1) - a^5
) + (7*(a*x + 1)^3/(a*x - 1)^3 - 2*(a*x + 1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(
a*x - 1))/((a*x + 1)^4*a^5/(a*x - 1)^4 - 4*(a*x + 1)^3*a^5/(a*x - 1)^3 + 6
*(a*x + 1)^2*a^5/(a*x - 1)^2 - 4*(a*x + 1)*a^5/(a*x - 1) + a^5) + 7*log(-(
a*x + 1)/(a*x - 1) + 1)/a^5 - 7*log(-(a*x + 1)/(a*x - 1))/a^5)*a
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx =$$
$$-\frac{a^2\,x^2-14\,\ln\left(a^2\,x^2-1\right)+3\,a^4\,x^4+15\,\text{atanh}(a\,x)^2-10\,a^3\,x^3\,\text{atanh}(a\,x)+12\,a^5\,x^5\,\text{atanh}(a\,x)-30\,a}{180\,a^4}$$

input
```
int(-x^3*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

output
```
-(a^2*x^2 - 14*log(a^2*x^2 - 1) + 3*a^4*x^4 + 15*atanh(a*x)^2 - 10*a^3*x^3
*atanh(a*x) + 12*a^5*x^5*atanh(a*x) - 30*a*x*atanh(a*x) - 45*a^4*x^4*atanh
(a*x)^2 + 30*a^6*x^6*atanh(a*x)^2)/(180*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int x^3(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx$$
$$=\frac{-30atanh(ax)^2\,a^6x^6+45atanh(ax)^2\,a^4x^4-15atanh(ax)^2-12atanh(ax)\,a^5x^5+10atanh(ax)\,a^3x^3+30}{180a^4}$$

input
```
int(x^3*(-a^2*x^2+1)*atanh(a*x)^2,x)
```

output
```
( - 30*atanh(a*x)**2*a**6*x**6 + 45*atanh(a*x)**2*a**4*x**4 - 15*atanh(a*x
)**2 - 12*atanh(a*x)*a**5*x**5 + 10*atanh(a*x)*a**3*x**3 + 30*atanh(a*x)*a
*x + 28*atanh(a*x) + 28*log(a**2*x - a) - 3*a**4*x**4 - a**2*x**2)/(180*a*
*4)
```

3.174 $\int x^2(1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int x^2(1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx = \frac{x}{30a^2}-\frac{x^3}{30}-\frac{\mathrm{arctanh}(ax)}{30a^3}+\frac{2x^2\mathrm{arctanh}(ax)}{15a} -\frac{1}{10}ax^4\mathrm{arctanh}(ax)+\frac{2\mathrm{arctanh}(ax)^2}{15a^3} +\frac{1}{3}x^3\mathrm{arctanh}(ax)^2-\frac{1}{5}a^2x^5\mathrm{arctanh}(ax)^2 -\frac{4\mathrm{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{15a^3}-\frac{2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a^3}$$

output
```
1/30*x/a^2-1/30*x^3-1/30*arctanh(a*x)/a^3+2/15*x^2*arctanh(a*x)/a-1/10*a*x
^4*arctanh(a*x)+2/15*arctanh(a*x)^2/a^3+1/3*x^3*arctanh(a*x)^2-1/5*a^2*x^5
*arctanh(a*x)^2-4/15*arctanh(a*x)*ln(2/(-a*x+1))/a^3-2/15*polylog(2,1-2/(-
a*x+1))/a^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx =$$
$$-\frac{-ax+a^3x^3+2(2-5a^3x^3+3a^5x^5)\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\left(1-4a^2x^2+3a^4x^4+8\log\left(1+e^{-2\right.\right.}}{30a^3}$$

input
```
Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
-1/30*(-(a*x) + a^3*x^3 + 2*(2 - 5*a^3*x^3 + 3*a^5*x^5)*ArcTanh[a*x]^2 + A
rcTanh[a*x]*(1 - 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^(-2*ArcTanh[a*x])]) -
 4*PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 297 vs. 2(138) = 276.

Time = 1.88 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6576, 6452, 6542, 6452, 254, 262, 219, 2009, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^2\operatorname{arctanh}(ax)^2dx-a^2\int x^4\operatorname{arctanh}(ax)^2dx$$

$$\Big\downarrow \text{6452}$$

$$-a^2\left(\frac{1}{5}x^5\operatorname{arctanh}(ax)^2-\frac{2}{5}a\int\frac{x^5\operatorname{arctanh}(ax)}{1-a^2x^2}dx\right)-\frac{2}{3}a\int\frac{x^3\operatorname{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{3}x^3\operatorname{arctanh}(ax)^2$$

$$\Big\downarrow \text{6542}$$

$$-\frac{2}{3}a\left(\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\text{arctanh}(ax)dx}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^3\text{arctanh}(ax)dx}{a^2}\right)\right)+\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6452}$$

$$-\frac{2}{3}a\left(\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int \frac{x^2}{1-a^2x^2}dx}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \frac{x^4}{1-a^2x^2}dx}{a^2}\right)\right)+\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{254}$$

$$-\frac{2}{3}a\left(\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int \frac{x^2}{1-a^2x^2}dx}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}\right)\right)+\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{262}$$

$$-\frac{2}{3}a\left(\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int \left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}\right)\right)+\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{219}$$

$$-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\int\frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\int\left(-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}-\frac{1}{a^4}\right)dx}{a^2}\right)\right)-\frac{2}{3}a\left(\frac{\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)+\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 2009

$$-\frac{2}{3}a\left(\frac{\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\int\frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)\right)+\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 6542

$$-\frac{2}{3}a\left(\frac{\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\text{arctanh}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\ldots\right.}{a^2}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 6452

$$-\frac{2}{3}a\left(\frac{\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int\frac{x^2}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}\ldots}{a^5}\right.}{a^2}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 262

$$-\frac{2}{3}a\left(\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\text{ar}\ldots\right.}{a^2}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 219

$$-\frac{2}{3}a\left(\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctanh}(ax)-\frac{1}{4}a\left(\ldots\right.}{a\ldots}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 6546

$$-\frac{2}{3}a\left(\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\text{arctan}\ldots}{\ldots}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$\downarrow$ 6470

$$-\frac{2}{3}a\left(\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\right.}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$$\Big\downarrow \ 2849$$

$$-\frac{2}{3}a\left(\frac{\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}}{a^3}\right.}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

$$\Big\downarrow \ 2752$$

$$-\frac{2}{3}a\left(\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)-$$

$$a^2\left(\frac{1}{5}x^5\text{arctanh}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}}{a^3}\right.}{a^2}}{a^2}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(ax)^2$$

input
```
Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
(x^3*ArcTanh[a*x]^2)/3 - (2*a*(-(((x^2*ArcTanh[a*x])/2 - (a*(-(x/a^2) + Ar
cTanh[a*x]/a^3))/2)/a^2) + (-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2
/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2))/3 - a^2*((x^5
*ArcTanh[a*x]^2)/5 - (2*a*(-(((x^4*ArcTanh[a*x])/4 - (a*(-(x/a^4) - x^3/(3
*a^2) + ArcTanh[a*x]/a^5))/4)/a^2) + (-(((x^2*ArcTanh[a*x])/2 - (a*(-(x/a^
2) + ArcTanh[a*x]/a^3))/2)/a^2) + (-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x
]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2)/a^2))/5
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}-\frac{a^4 x^4 \operatorname{arctanh}(ax)}{10}+\frac{2a^2x^2 \operatorname{arctanh}(ax)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(a}{15}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}-\frac{a^4 x^4 \operatorname{arctanh}(ax)}{10}+\frac{2a^2x^2 \operatorname{arctanh}(ax)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{15}+\frac{2\operatorname{arctanh}(ax)\ln(a}{15}}{}$
parts	$-\frac{a^2x^5 \operatorname{arctanh}(ax)^2}{5}+\frac{x^3 \operatorname{arctanh}(ax)^2}{3}-\frac{a x^4 \operatorname{arctanh}(ax)}{10}+\frac{2x^2 \operatorname{arctanh}(ax)}{15a}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{15a^3}+$
risch	$-\frac{x^3}{30}-\frac{443}{3375a^3}-\frac{a\ln(ax+1)x^4}{20}-\frac{\ln(ax+1)x^2}{60a}-\frac{\ln(ax+1)x}{6a^2}-\frac{a^2\ln(ax+1)^2x^5}{20}-\frac{x^3\ln(-ax+1)}{18}-\frac{7\ln(a}{45}$

input
```
int(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/5*arctanh(a*x)^2*a^5*x^5+1/3*arctanh(a*x)^2*a^3*x^3-1/10*a^4*x^4
*arctanh(a*x)+2/15*a^2*x^2*arctanh(a*x)+2/15*arctanh(a*x)*ln(a*x-1)+2/15*a
rctanh(a*x)*ln(a*x+1)+1/30*ln(a*x-1)^2-2/15*dilog(1/2*a*x+1/2)-1/15*ln(a*x
-1)*ln(1/2*a*x+1/2)+1/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/30
*ln(a*x+1)^2-1/30*a^3*x^3+1/30*a*x+1/60*ln(a*x-1)-1/60*ln(a*x+1))
```

Fricas [F]

$$\int x^2\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2\,dx=\int -\left(a^2x^2-1\right)x^2\operatorname{artanh}\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^4 - x^2)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^2(1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx = -\int \left(-x^2\,\mathrm{atanh}^2\left(ax\right)\right)\,dx - \int a^2x^4\,\mathrm{atanh}^2\left(ax\right)dx$$

input
```
integrate(x**2*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

output
```
-Integral(-x**2*atanh(a*x)**2, x) - Integral(a**2*x**4*atanh(a*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

$$\int x^2(1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx =$$
$$-\frac{1}{60}\,a^2\left(\frac{2\,a^3x^3-2\,ax+2\,\log\left(ax+1\right)^2-4\,\log\left(ax+1\right)\log\left(ax-1\right)-2\,\log\left(ax-1\right)^2-\log\left(ax-1\right)}{a^5}\right.$$
$$-\frac{1}{30}\,a\left(\frac{3\,a^2x^4-4\,x^2}{a^2}-\frac{4\,\log\left(ax+1\right)}{a^4}-\frac{4\,\log\left(ax-1\right)}{a^4}\right)\mathrm{artanh}\left(ax\right)$$
$$-\frac{1}{15}\left(3\,a^2x^5-5\,x^3\right)\mathrm{artanh}\left(ax\right)^2$$

input
```
integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-1/60*a^2*((2*a^3*x^3 - 2*a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x
- 1) - 2*log(a*x - 1)^2 - log(a*x - 1))/a^5 + 8*(log(a*x - 1)*log(1/2*a*x
+ 1/2) + dilog(-1/2*a*x + 1/2))/a^5 + log(a*x + 1)/a^5) - 1/30*a*((3*a^2*x
^4 - 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)*arctanh(a*x) -
1/15*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)^2
```

Giac [F]

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = \int -(a^2x^2-1)x^2\operatorname{artanh}(ax)^2\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*x^2*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = -\int x^2\operatorname{atanh}(a\,x)^2\left(a^2\,x^2-1\right)\,dx$$

input
```
int(-x^2*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

output
```
-int(x^2*atanh(a*x)^2*(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int x^2(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$
$$= \frac{-6atanh(ax)^2a^5x^5+10atanh(ax)^2a^3x^3-4atanh(ax)^2ax-3atanh(ax)a^4x^4+4atanh(ax)a^2x^2-ata}{30a^3}$$

input
```
int(x^2*(-a^2*x^2+1)*atanh(a*x)^2,x)
```

output
```
( - 6*atanh(a*x)**2*a**5*x**5 + 10*atanh(a*x)**2*a**3*x**3 - 4*atanh(a*x)*
*2*a*x - 3*atanh(a*x)*a**4*x**4 + 4*atanh(a*x)*a**2*x**2 - atanh(a*x) + 4*
int(atanh(a*x)**2,x)*a - a**3*x**3 + a*x)/(30*a**3)
```

3.175 $\int x(1-a^2x^2)\,\mathbf{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx=\frac{1-a^2x^2}{12a^2}+\frac{x\operatorname{arctanh}(ax)}{3a}+\frac{x(1-a^2x^2)\operatorname{arctanh}(ax)}{6a}-\frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{4a^2}+\frac{\log(1-a^2x^2)}{6a^2}$$

output
```
1/12*(-a^2*x^2+1)/a^2+1/3*x*arctanh(a*x)/a+1/6*x*(-a^2*x^2+1)*arctanh(a*x)
/a-1/4*(-a^2*x^2+1)^2*arctanh(a*x)^2/a^2+1/6*ln(-a^2*x^2+1)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{-a^2x^2+(6ax-2a^3x^3)\operatorname{arctanh}(ax)-3(-1+a^2x^2)^2\operatorname{arctanh}(ax)^2+2\log(1-a^2x^2)}{12a^2}$$

input
```
Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
(-(a^2*x^2) + (6*a*x - 2*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^2 + 2*Log[1 - a^2*x^2])/(12*a^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{6556, 6504, 6436, 240\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-a^2x^2\right)\text{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int \left(1-a^2x^2\right)\text{arctanh}(ax)dx}{2a}-\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{6504}$$

$$\frac{\frac{2}{3}\int \text{arctanh}(ax)dx+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}}{2a}-\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\frac{2}{3}\left(x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}}{2a}-\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}}{2a}-\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}{4a^2}$$

input
```
Int[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
-1/4*((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/a^2 + ((1 - a^2*x^2)/(6*a) + (x*(1 -
 a^2*x^2)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*x] + Log[1 - a^2*x^2]/(2*a)))/
3)/(2*a)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{3a^4x^4\operatorname{arctanh}(ax)^2+2a^3x^3\operatorname{arctanh}(ax)-6a^2x^2\operatorname{arctanh}(ax)^2+a^2x^2-6ax\operatorname{arctanh}(ax)+3\operatorname{arctanh}(ax)^2-4\ln(ax-}{12a^2}$
derivativedivides	$\frac{-\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}+\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{4}-\frac{a^3x^3\operatorname{arctanh}(ax)}{6}+\frac{ax\operatorname{arctanh}(ax)}{2}-\frac{a^2x^2}{12}+\frac{\ln(ax-1)}{6}+\frac{\ln(ax+1)}{6}}{a^2}$
default	$\frac{-\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}+\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{4}-\frac{a^3x^3\operatorname{arctanh}(ax)}{6}+\frac{ax\operatorname{arctanh}(ax)}{2}-\frac{a^2x^2}{12}+\frac{\ln(ax-1)}{6}+\frac{\ln(ax+1)}{6}}{a^2}$
parts	$-\frac{x^4a^2\operatorname{arctanh}(ax)^2}{4}+\frac{\operatorname{arctanh}(ax)^2x^2}{2}-\frac{\operatorname{arctanh}(ax)^2}{4a^2}+\frac{-\frac{a^3x^3\operatorname{arctanh}(ax)}{3}+ax\operatorname{arctanh}(ax)-\frac{a^2x^2}{6}+\frac{\ln(ax-1}{3}}{2a^2}$
risch	$-\frac{(a^2x^2-1)^2\ln(ax+1)^2}{16a^2}+\frac{(3x^4\ln(-ax+1)a^4-2a^3x^3-6x^2\ln(-ax+1)a^2+6ax+3\ln(-ax+1))\ln(ax+1)}{24a^2}-\frac{\ln(-a}{}$

input
```
int(x*(-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*a^4*x^4*arctanh(a*x)^2+2*a^3*x^3*arctanh(a*x)-6*a^2*x^2*arctanh(a
*x)^2+a^2*x^2-6*a*x*arctanh(a*x)+3*arctanh(a*x)^2-4*ln(a*x-1)-4*arctanh(a*
x))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx = -\frac{4\,a^2x^2+3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,(a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-8\,\log\left(a^2x^2-1\right)}{48\,a^2}$$

input
```
integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
 + 4*(a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 8*log(a^2*x^2 - 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$
$$=\begin{cases}-\frac{a^2x^4\operatorname{atanh}^2(ax)}{4}-\frac{ax^3\operatorname{atanh}(ax)}{6}+\frac{x^2\operatorname{atanh}^2(ax)}{2}-\frac{x^2}{12}+\frac{x\operatorname{atanh}(ax)}{2a}+\frac{\log\left(x-\frac{1}{a}\right)}{3a^2}-\frac{\operatorname{atanh}^2(ax)}{4a^2}+\frac{\operatorname{atanh}(ax)}{3a^2} & \text{for } a\\ 0 & \text{othe}\end{cases}$$

input
```
integrate(x*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

output
```
Piecewise((-a**2*x**4*atanh(a*x)**2/4 - a*x**3*atanh(a*x)/6 + x**2*atanh(a
*x)**2/2 - x**2/12 + x*atanh(a*x)/(2*a) + log(x - 1/a)/(3*a**2) - atanh(a*
x)**2/(4*a**2) + atanh(a*x)/(3*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

$$\int x(1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx$$
$$=-\frac{(a^2x^2-1)^2\operatorname{artanh}(ax)^2}{4a^2}-\frac{\left(x^2-\frac{2\log(ax+1)}{a^2}-\frac{2\log(ax-1)}{a^2}\right)a+2(a^2x^3-3x)\operatorname{artanh}(ax)}{12a}$$

input
```
integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(a^2*x^2 - 1)^2*arctanh(a*x)^2/a^2 - 1/12*((x^2 - 2*log(a*x + 1)/a^2
- 2*log(a*x - 1)/a^2)*a + 2*(a^2*x^3 - 3*x)*arctanh(a*x))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(82) = 164$.

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.21

$$\int x(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx = -\frac{1}{3}a\left(\frac{\left(\frac{3\,(ax+1)}{ax-1}-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^3a^3}{(ax-1)^3}-\frac{3\,(ax+1)^2a^3}{(ax-1)^2}+\frac{3\,(ax+1)a^3}{ax-1}-a^3}+\frac{3\,(ax+1)^2\log\left(-\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^4a^3}{(ax-1)^4}-\frac{4\,(ax+1)^3a^3}{(ax-1)^3}+\frac{6\,(ax+1)^2a^3}{(ax-1)^2}-\frac{4\,(ax+1)a^3}{ax-1}+a^3\right)(a$$

input
```
integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
-1/3*a*((3*(a*x + 1)/(a*x - 1) - 1)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^3
*a^3/(a*x - 1)^3 - 3*(a*x + 1)^2*a^3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x -
1) - a^3) + 3*(a*x + 1)^2*log(-(a*x + 1)/(a*x - 1))^2/(((a*x + 1)^4*a^3/(a
*x - 1)^4 - 4*(a*x + 1)^3*a^3/(a*x - 1)^3 + 6*(a*x + 1)^2*a^3/(a*x - 1)^2
- 4*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)^2) + (a*x + 1)/(((a*x + 1)^2*
a^3/(a*x - 1)^2 - 2*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)) + log(-(a*x
+ 1)/(a*x - 1) + 1)/a^3 - log(-(a*x + 1)/(a*x - 1))/a^3)
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx = \frac{x^2\,\text{atanh}(a\,x)^2}{2}-\frac{\text{atanh}(a\,x)^2}{4\,a^2}-\frac{x^2}{12}+\frac{\ln\left(a^2\,x^2-1\right)}{6\,a^2}+\frac{x\,\text{atanh}(a\,x)}{2\,a}-\frac{a\,x^3\,\text{atanh}(a\,x)}{6}-\frac{a^2\,x^4\,\text{atanh}(a\,x)^2}{4}$$

input
```
int(-x*atanh(a*x)^2*(a^2*x^2 - 1),x)
```

output
```
(x^2*atanh(a*x)^2)/2 - atanh(a*x)^2/(4*a^2) - x^2/12 + log(a^2*x^2 - 1)/(6
*a^2) + (x*atanh(a*x))/(2*a) - (a*x^3*atanh(a*x))/6 - (a^2*x^4*atanh(a*x)^
2)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int x(1-a^2x^2)\,\text{arctanh}(ax)^2\,dx$$
$$= \frac{-3atanh(ax)^2\,a^4x^4+6atanh(ax)^2\,a^2x^2-3atanh(ax)^2-2atanh(ax)\,a^3x^3+6atanh(ax)\,ax+4atanh(a}{12a^2}$$

input
```
int(x*(-a^2*x^2+1)*atanh(a*x)^2,x)
```

output
```
( - 3*atanh(a*x)**2*a**4*x**4 + 6*atanh(a*x)**2*a**2*x**2 - 3*atanh(a*x)**
2 - 2*atanh(a*x)*a**3*x**3 + 6*atanh(a*x)*a*x + 4*atanh(a*x) + 4*log(a**2*
x - a) - a**2*x**2)/(12*a**2)
```

3.176 $\int (1-a^2x^2) \operatorname{arctanh}(ax)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)^2 \, dx = -\frac{x}{3} + \frac{(1-a^2x^2)\operatorname{arctanh}(ax)}{3a} + \frac{2\operatorname{arctanh}(ax)^2}{3a} + \frac{2}{3}x\operatorname{arctanh}(ax)^2 + \frac{1}{3}x(1-a^2x^2)\operatorname{arctanh}(ax)^2 - \frac{4\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{3a} - \frac{2\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{3a}$$

output
```
-1/3*x+1/3*(-a^2*x^2+1)*arctanh(a*x)/a+2/3*arctanh(a*x)^2/a+2/3*x*arctanh(
a*x)^2+1/3*x*(-a^2*x^2+1)*arctanh(a*x)^2-4/3*arctanh(a*x)*ln(2/(-a*x+1))/a
-2/3*polylog(2,1-2/(-a*x+1))/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)^2 \, dx = -\frac{ax + (-1+ax)^2(2+ax)\operatorname{arctanh}(ax)^2 + \operatorname{arctanh}(ax)\left(-1+a^2x^2+4\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right)\right) - 2\operatorname{Pol}}{3a}$$

input
```
Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
-1/3*(a*x + (-1 + a*x)^2*(2 + a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-1 + a^2
*x^2 + 4*Log[1 + E^(-2*ArcTanh[a*x])]) - 2*PolyLog[2, -E^(-2*ArcTanh[a*x])
])/a
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6506, 24, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)\text{arctanh}(ax)^2dx$$

$$\Big\downarrow \text{ 6506}$$

$$\frac{2}{3}\int \text{arctanh}(ax)^2dx-\frac{\int 1dx}{3}+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2}{3}\int \text{arctanh}(ax)^2dx+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}-\frac{x}{3}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}-\frac{x}{3}$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}-\frac{x}{3}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^2+\frac{(1-a^2x^2)\,\text{arctanh}(ax)}{3a}-\frac{x}{3}$$

$\Big\downarrow$ 2849

$$\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^2+\frac{(1-a^2x^2)\,\text{arctanh}(ax)}{3a}-\frac{x}{3}$$

$\Big\downarrow$ 2752

$$\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^2+\frac{(1-a^2x^2)\,\text{arctanh}(ax)}{3a}-\frac{x}{3}$$

input
```
Int[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

output
```
-1/3*x + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x
]^2)/3 + (2*(x*ArcTanh[a*x]^2 - 2*a*(-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a
*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\operatorname{arctanh}(ax)^2 ax-\frac{a^2 x^2 \operatorname{arctanh}(ax)}{3}+\frac{2 \operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2 \operatorname{arctanh}(ax)\ln(ax+1)}{3}-\frac{ax}{3}-\frac{\ln(ax-1)}{6}}{a}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\operatorname{arctanh}(ax)^2 ax-\frac{a^2 x^2 \operatorname{arctanh}(ax)}{3}+\frac{2 \operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2 \operatorname{arctanh}(ax)\ln(ax+1)}{3}-\frac{ax}{3}-\frac{\ln(ax-1)}{6}}{a}$
parts	$-\frac{x^3 a^2 \operatorname{arctanh}(ax)^2}{3}+x \operatorname{arctanh}(ax)^2-\frac{x^2 \operatorname{arctanh}(ax) a}{3}+\frac{2 \operatorname{arctanh}(ax)\ln(ax-1)}{3a}+\frac{2 \operatorname{arctanh}(ax)\ln}{3a}$
risch	$-\frac{x}{3}+\frac{a\ln(-ax+1)x^2}{6}+\frac{\ln(-ax+1)\ln(ax+1)}{6a}-\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{3a}+\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{3a}-\frac{a\ln(ax-}{6}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/3*arctanh(a*x)^2*a^3*x^3+arctanh(a*x)^2*a*x-1/3*a^2*x^2*arctanh(a*
x)+2/3*arctanh(a*x)*ln(a*x-1)+2/3*arctanh(a*x)*ln(a*x+1)-1/3*a*x-1/6*ln(a*
x-1)+1/6*ln(a*x+1)+1/6*ln(a*x-1)^2-2/3*dilog(1/2*a*x+1/2)-1/3*ln(a*x-1)*ln
(1/2*a*x+1/2)-1/6*ln(a*x+1)^2+1/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+
1/2))
```

Fricas [F]

$$\int (1-a^2x^2)\operatorname{arctanh}(ax)^2\,dx=\int -\left(a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^2\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2\,dx = -\int a^2x^2 \operatorname{atanh}^2(ax)\,dx - \int \left(-\operatorname{atanh}^2(ax)\right)\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2,x)
```

output
```
-Integral(a**2*x**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25

$$\int \left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2\,dx =$$
$$-\frac{1}{6}a^2\left(\frac{2\,ax+\log(ax+1)^2-2\,\log(ax+1)\log(ax-1)-\log(ax-1)^2+\log(ax-1)}{a^3}+\frac{4\,(\log(ax-$$
$$-\frac{1}{3}\left(x^2-\frac{2\,\log(ax+1)}{a^2}-\frac{2\,\log(ax-1)}{a^2}\right)a\operatorname{artanh}(ax)$$
$$-\frac{1}{3}\left(a^2x^3-3\,x\right)\operatorname{artanh}(ax)^2$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-1/6*a^2*((2*a*x + log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x
- 1)^2 + log(a*x - 1))/a^3 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1
/2*a*x + 1/2))/a^3 - log(a*x + 1)/a^3) - 1/3*(x^2 - 2*log(a*x + 1)/a^2 - 2
*log(a*x - 1)/a^2)*a*arctanh(a*x) - 1/3*(a^2*x^3 - 3*x)*arctanh(a*x)^2
```

Giac [F]

$$\int (1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx = \int -\left(a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx = -\int \mathrm{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)\,dx$$

input
```
int(-atanh(a*x)^2*(a^2*x^2 - 1),x)
```

output
```
-int(atanh(a*x)^2*(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int (1-a^2x^2)\,\mathrm{arctanh}(ax)^2\,dx = \frac{-atanh(ax)^2\,a^3x^3+3atanh(ax)^2\,ax-atanh(ax)\,a^2x^2+atanh(ax)+4\left(\int \frac{atanh(ax)x}{a^2x^2-1}dx\right)a^2-ax}{3a}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2,x)
```

output
```
( - atanh(a*x)**2*a**3*x**3 + 3*atanh(a*x)**2*a*x - atanh(a*x)*a**2*x**2 +
 atanh(a*x) + 4*int((atanh(a*x)*x)/(a**2*x**2 - 1),x)*a**2 - a*x)/(3*a)
```

3.177 $\int \frac{(1-a^2x^2)\text{arctanh}(ax)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\begin{aligned}\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x}\,dx &= -ax\text{arctanh}(ax)+\frac{1}{2}\text{arctanh}(ax)^2-\frac{1}{2}a^2x^2\text{arctanh}(ax)^2\\ &\quad+2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)\\ &\quad-\frac{1}{2}\log\left(1-a^2x^2\right)\\ &\quad-\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\\ &\quad+\text{arctanh}(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)\\ &\quad+\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)\\ &\quad-\frac{1}{2}\text{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)\end{aligned}$$

output

```
-a*x*arctanh(a*x)+1/2*arctanh(a*x)^2-1/2*a^2*x^2*arctanh(a*x)^2-2*arctanh(
a*x)^2*arctanh(-1+2/(-a*x+1))-1/2*ln(-a^2*x^2+1)-arctanh(a*x)*polylog(2,1-
2/(-a*x+1))+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*polylog(3,1-2/(-a*x+
1))-1/2*polylog(3,-1+2/(-a*x+1))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)^2}{x}\,dx = -ax\mathrm{arctanh}(ax) - \frac{1}{2}\left(-1+a^2x^2\right)\mathrm{arctanh}(ax)^2 - \frac{2}{3}\mathrm{arctanh}(ax)^3 - \mathrm{arctanh}(ax)^2\log\left(1+e^{-2\mathrm{arctanh}(ax)}\right) + \mathrm{arctanh}(ax)^2\log\left(1-e^{2\mathrm{arctanh}(ax)}\right) - \frac{1}{2}\log\left(1-a^2x^2\right) + \mathrm{arctanh}(ax)\,\mathrm{PolyLog}\left(2,-e^{-2\mathrm{arctanh}(ax)}\right) + \mathrm{arctanh}(ax)\,\mathrm{PolyLog}\left(2,e^{2\mathrm{arctanh}(ax)}\right) + \frac{1}{2}\,\mathrm{PolyLog}\left(3,-e^{-2\mathrm{arctanh}(ax)}\right) - \frac{1}{2}\,\mathrm{PolyLog}\left(3,e^{2\mathrm{arctanh}(ax)}\right)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

output
```
-(a*x*ArcTanh[a*x]) - ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/2 - (2*ArcTanh[a*x]^
3)/3 - ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]^2*Log[1
- E^(2*ArcTanh[a*x])] - Log[1 - a^2*x^2]/2 + ArcTanh[a*x]*PolyLog[2, -E^(-
2*ArcTanh[a*x])] + ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + PolyLog[3
, -E^(-2*ArcTanh[a*x])]/2 - PolyLog[3, E^(2*ArcTanh[a*x])]/2
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6576, 6448, 6452, 6542, 6436, 240, 6510, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)^2}{x}\,dx$$

$$\Big\downarrow \; 6576$$

$$\int \frac{\text{arctanh}(ax)^2}{x}dx - a^2\int x\text{arctanh}(ax)^2dx$$

$$\Big\downarrow \text{6448}$$

$$-4a\int \frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx + a^2\left(-\int x\text{arctanh}(ax)^2dx\right) + 2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx\right)\right) - 4a\int \frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx + 2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$$\Big\downarrow \text{6542}$$

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int \text{arctanh}(ax)dx}{a^2}\right)\right)\right) - 4a\int \frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx + 2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$$\Big\downarrow \text{6436}$$

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2} - \frac{x\text{arctanh}(ax) - a\int \frac{x}{1-a^2x^2}dx}{a^2}\right)\right)\right) - 4a\int \frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx + 2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$$\Big\downarrow \text{240}$$

$$-4a\int \frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx - \left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{\log(1-a^2x^2)}{2a} + x\text{arctanh}(ax)}{a^2}\right)\right)\right) + 2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$$\Big\downarrow \text{6510}$$

$$-4a\int \frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2-a\left(\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)\right)\right)+2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$\downarrow$ 6614

$$-4a\left(\frac{1}{2}\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{1}{2}\int \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2-a\left(\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)\right)\right)+2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$\downarrow$ 6620

$$-4a\left(\frac{1}{2}\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{1-ax}-\right.}{1-a^2x^2}\right.\right.$$

$$\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2-a\left(\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)\right)\right)+2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

$\downarrow$ 7164

$$-\left(a^2\left(\frac{1}{2}x^2\text{arctanh}(ax)^2-a\left(\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)\right)\right)-$$

$$4a\left(\frac{1}{2}\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}\right)+\frac{1}{2}\left(\frac{\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{4a}-\frac{\text{arctan}}{}\right.\right.$$

$$2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

output
```
2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - a^2*((x^2*ArcTanh[a*x]^2)/2 -
a*(ArcTanh[a*x]^2/(2*a^3) - (x*ArcTanh[a*x] + Log[1 - a^2*x^2]/(2*a))/a^2)
) - 4*a*(((ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a) - PolyLog[3, 1
- 2/(1 - a*x)]/(4*a))/2 + (-1/2*(ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)]
)/a + PolyLog[3, -1 + 2/(1 - a*x)]/(4*a))/2)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.26 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.54

method	result
derivativedivides	$-\frac{a^{2} x^{2} \operatorname{arctanh}(a x)^{2}}{2}+\operatorname{arctanh}(a x)^{2} \ln (a x)-\operatorname{arctanh}(a x)^{2} \ln \left(\frac{(a x+1)^{2}}{-a^{2} x^{2}+1}-1\right)+\operatorname{arctanh}$
default	$-\frac{a^{2} x^{2} \operatorname{arctanh}(a x)^{2}}{2}+\operatorname{arctanh}(a x)^{2} \ln (a x)-\operatorname{arctanh}(a x)^{2} \ln \left(\frac{(a x+1)^{2}}{-a^{2} x^{2}+1}-1\right)+\operatorname{arctanh}$
parts	Expression too large to display

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*x^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+
1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arc
tanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*
x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*
x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1
)^(1/2))-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a
*x+1)^2/(-a^2*x^2+1))+ln((a*x+1)^2/(-a^2*x^2+1)+1)+1/2*I*Pi*csgn(I*(-(a*x+
1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2
/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*x)^2-(a*x+1)*arctanh
(a*x)+1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1
))^3*arctanh(a*x)^2+1/2*arctanh(a*x)^2-1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^
2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*a
rctanh(a*x)^2-1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)
^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x}\,dx = \int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x}\,dx = -\int \left(-\frac{\operatorname{atanh}^2(ax)}{x}\right)dx - \int a^2x\operatorname{atanh}^2(ax)\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2/x,x)
```

output
```
-Integral(-atanh(a*x)**2/x, x) - Integral(a**2*x*atanh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x}\,dx = \int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="maxima")
```

output
```
-1/8*a^2*x^2*log(-a*x + 1)^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x +
1)*log(a*x + 1)^2 - (a^3*x^3 + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1
))*log(-a*x + 1))/(a*x^2 - x), x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x}\,dx = \int -\frac{(a^2x^2-1)\,\text{artanh}\,(ax)^2}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x}\,dx = -\int \frac{\text{atanh}(a\,x)^2\,(a^2\,x^2-1)}{x}\,dx$$

input
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x,x)
```

output
```
-int((atanh(a*x)^2*(a^2*x^2 - 1))/x, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x}\,dx = -\frac{atanh(ax)^2\,a^2x^2}{2} + \frac{atanh(ax)^2}{2} - atanh(ax)\,ax - atanh(ax) + \int \frac{atanh(ax)^2}{x}dx - \log(a^2x-a)$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2/x,x)
```

output
```
( - atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2 - 2*atanh(a*x)*a*x - 2*atanh(a
*x) + 2*int(atanh(a*x)**2/x,x) - 2*log(a**2*x - a))/2
```

3.178 $\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^2}\,dx = -\frac{\operatorname{arctanh}(ax)^2}{x} - a^2x\operatorname{arctanh}(ax)^2 + 2a\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) + a\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right) - a\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output

```
-arctanh(a*x)^2/x-a^2*x*arctanh(a*x)^2+2*a*arctanh(a*x)*ln(2/(-a*x+1))+2*a
*arctanh(a*x)*ln(2-2/(a*x+1))+a*polylog(2,1-2/(-a*x+1))-a*polylog(2,-1+2/(
a*x+1))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^2}\,dx = -a\operatorname{arctanh}(ax)\left(-\operatorname{arctanh}(ax)+ax\operatorname{arctanh}(ax)-2\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right)\right) - a\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) + a\left(\operatorname{arctanh}(ax)\left(\operatorname{arctanh}(ax)-\frac{\operatorname{arctanh}(ax)}{ax}+2\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)\right)-\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

output
```
-(a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcTa
nh[a*x])])) - a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + a*(ArcTanh[a*x]*(ArcTan
h[a*x] - ArcTanh[a*x]/(a*x) + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2,
 E^(-2*ArcTanh[a*x])])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6576, 6436, 6452, 6546, 6470, 2849, 2752, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^2}\,dx$$

$$\Big\downarrow \; 6576$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2}dx - a^2\int \operatorname{arctanh}(ax)^2dx$$

$$\Big\downarrow \ 6436$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2}dx - a^2\left(x\operatorname{arctanh}(ax)^2 - 2a\int \frac{x\operatorname{arctanh}(ax)}{1-a^2x^2}dx\right)$$

$$\Big\downarrow \ 6452$$

$$-\left(a^2\left(x\operatorname{arctanh}(ax)^2 - 2a\int \frac{x\operatorname{arctanh}(ax)}{1-a^2x^2}dx\right)\right) + 2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx - \frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \ 6546$$

$$2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx - \left(a^2\left(x\operatorname{arctanh}(ax)^2 - 2a\left(\frac{\int \frac{\operatorname{arctanh}(ax)}{1-ax}dx}{a} - \frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right) - \frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \ 6470$$

$$2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx - \left(a^2\left(x\operatorname{arctanh}(ax)^2 - 2a\left(\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a} - \int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a} - \frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right) - \frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \ 2849$$

$$2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx - \left(a^2\left(x\operatorname{arctanh}(ax)^2 - 2a\left(\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a} + \frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a} - \frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right) - \frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \ 2752$$

$$2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx - \left(a^2\left(x\operatorname{arctanh}(ax)^2 - 2a\left(\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a} + \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a} - \frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right) - \frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6550}$$

$$2a\left(\int \frac{\operatorname{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\operatorname{arctanh}(ax)^2\right)-\left(a^2\left(x\operatorname{arctanh}(ax)^2-2a\left(\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6494}$$

$$2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\left(a^2\left(x\operatorname{arctanh}(ax)^2-2a\left(\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{2897}$$

$$-\left(a^2\left(x\operatorname{arctanh}(ax)^2-2a\left(\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\operatorname{arctanh}(ax)^2}{2a^2}\right)\right)\right)+2a\left(\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{x}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

output
```
-(ArcTanh[a*x]^2/x) - a^2*(x*ArcTanh[a*x]^2 - 2*a*(-1/2*ArcTanh[a*x]^2/a^2
 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))
/a)) + 2*a*(ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog
[2, -1 + 2/(1 + a*x)]/2)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

method	result
derivativedivides	$a\left(-\operatorname{arctanh}(ax)^2 ax-\frac{\operatorname{arctanh}(ax)^2}{ax}+2\operatorname{arctanh}(ax)\ln(ax)-2\operatorname{arctanh}(ax)\ln(ax -$
default	$a\left(-\operatorname{arctanh}(ax)^2 ax-\frac{\operatorname{arctanh}(ax)^2}{ax}+2\operatorname{arctanh}(ax)\ln(ax)-2\operatorname{arctanh}(ax)\ln(ax -$
parts	$-a^2x\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{x}+2a\operatorname{arctanh}(ax)\ln(ax)-2a\operatorname{arctanh}(ax)\ln(ax -$

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-arctanh(a*x)^2*a*x-arctanh(a*x)^2/a/x+2*arctanh(a*x)*ln(a*x)-2*arctanh
(a*x)*ln(a*x-1)-2*arctanh(a*x)*ln(a*x+1)-dilog(a*x)-dilog(a*x+1)-ln(a*x)*l
n(a*x+1)-1/2*ln(a*x-1)^2+2*dilog(1/2*a*x+1/2)+ln(a*x-1)*ln(1/2*a*x+1/2)+1/
2*ln(a*x+1)^2-(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^2}\,dx = \int -\frac{(a^2x^2-1)\,\text{artanh}\,(ax)^2}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^2}\,dx = -\int a^2\,\text{atanh}^2\,(ax)\,dx - \int \left(-\frac{\text{atanh}^2\,(ax)}{x^2}\right)dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2/x**2,x)
```

output
```
-Integral(a**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.63

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^2}\,dx$$
$$= \frac{1}{2}a^2\left(\frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right) + \text{Li}_2(\right.}{a}\right.$$
$$- 2\,a(\log(ax+1) + \log(ax-1) - \log(x))\,\text{artanh}\,(ax) - \left(a^2x + \frac{1}{x}\right)\text{artanh}\,(ax)^2$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/2*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 2*(log(
a*x + 1)*log(x) + dilog(-a*x))/a + 2*(log(-a*x + 1)*log(x) + dilog(a*x))/a
) - 2*a*(log(a*x + 1) + log(a*x - 1) - log(x))*arctanh(a*x) - (a^2*x + 1/x
)*arctanh(a*x)^2
```

Giac [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^2}\,dx = \int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^2}\,dx = \int -\frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)}{x^2}\,dx$$

input
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^2,x)
```

output
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^2, x)
```

Reduce [F]

$$\int \frac{(1 - a^2x^2) \operatorname{arctanh}(ax)^2}{x^2} \, dx$$
$$= \frac{-atanh(ax)^2 - \left(\int atanh(ax)^2 \, dx\right) a^2x - 2\left(\int \frac{atanh(ax)}{a^2x^3 - x} dx\right) ax}{x}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2/x^2,x)
```

output
```
( - atanh(a*x)**2 - int(atanh(a*x)**2,x)*a**2*x - 2*int(atanh(a*x)/(a**2*x
**3 - x),x)*a*x)/x
```

3.179 $\int \frac{(1-a^2x^2)\text{arctanh}(ax)^2}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 172

$$\begin{aligned}\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^3}\,dx =& -\frac{a\text{arctanh}(ax)}{x}+\frac{1}{2}a^2\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)^2}{2x^2}\\ &-2a^2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)\\ &+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)\\ &+a^2\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\\ &-a^2\text{arctanh}(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)\\ &-\frac{1}{2}a^2\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)\\ &+\frac{1}{2}a^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)\end{aligned}$$

output
```
-a*arctanh(a*x)/x+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+2*a^2*arct
anh(a*x)^2*arctanh(-1+2/(-a*x+1))+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+a^2*arc
tanh(a*x)*polylog(2,1-2/(-a*x+1))-a^2*arctanh(a*x)*polylog(2,-1+2/(-a*x+1)
)-1/2*a^2*polylog(3,1-2/(-a*x+1))+1/2*a^2*polylog(3,-1+2/(-a*x+1))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.11

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^3}\,dx = -\frac{a\operatorname{arctanh}(ax)}{x} + \frac{(-1+a^2x^2)\operatorname{arctanh}(ax)^2}{2x^2} + \frac{2}{3}a^2\operatorname{arctanh}(ax)^3 + a^2\operatorname{arctanh}(ax)^2\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right) - a^2\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + a^2\log(x) - \frac{1}{2}a^2\log\left(1-a^2x^2\right) - a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) - a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - \frac{1}{2}a^2\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(ax)}\right) + \frac{1}{2}a^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

output
```
-((a*ArcTanh[a*x])/x) + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) + (2*a^2*A
rcTanh[a*x]^3)/3 + a^2*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - a^2*A
rcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + a^2*Log[x] - (a^2*Log[1 - a^2*
x^2])/2 - a^2*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - a^2*ArcTanh[
a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - (a^2*PolyLog[3, -E^(-2*ArcTanh[a*x])
])/2 + (a^2*PolyLog[3, E^(2*ArcTanh[a*x])])/2
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6576, 6448, 6452, 6544, 6452, 243, 47, 14, 16, 6510, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3}dx - a^2\int \frac{\operatorname{arctanh}(ax)^2}{x}dx$$

$$\Big\downarrow \text{6448}$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3}dx - a^2\left(2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6544}$$

$$-\left(a^2\left(2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx+\int \frac{\operatorname{arctanh}(ax)}{x^2}dx\right)-\frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx+a\int \frac{1}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)}{x}\right)-\frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$-\left(a^2\left(2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int \frac{\operatorname{arctanh}(ax)\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2-\frac{\operatorname{arctanh}(ax)}{x}\right)-\frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int\frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(a^2\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int\frac{1}{1-a^2x^2}dx^2+\int\frac{1}{x^2}dx^2\right)-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int\frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(a^2\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int\frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int\frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(a^2\int\frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\int\frac{\text{arctanh}(ax)\text{arctanh}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6614}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\left(\frac{1}{2}\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{1}{2}\int\frac{\text{arctanh}(ax)\log}{1-a^2x^2}\right.\right.\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6620}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\left(\frac{1}{2}\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\right.}{1-a}\right.\right.\right.\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{7164}$$

$$-\left(a^2\left(2\text{arctanh}(ax)^2\text{arctanh}\left(1-\frac{2}{1-ax}\right)-4a\left(\frac{1}{2}\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\right.}{4a}\right.\right.\right.\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

output
```
-1/2*ArcTanh[a*x]^2/x^2 + a*(-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + (a
*(Log[x^2] - Log[1 - a^2*x^2]))/2) - a^2*(2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(
1 - a*x)] - 4*a*(((ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a) - PolyL
og[3, 1 - 2/(1 - a*x)]/(4*a))/2 + (-1/2*(ArcTanh[a*x]*PolyLog[2, -1 + 2/(1
 - a*x)])/a + PolyLog[3, -1 + 2/(1 - a*x)]/(4*a))/2))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.06 (sec) , antiderivative size = 736, normalized size of antiderivative = 4.28

method	result
derivativedivides	$a^2 \left(-\operatorname{arctanh}(ax)^2 \ln(ax) - \frac{\operatorname{arctanh}(ax)^2}{2a^2x^2} - \frac{\left(ax-\sqrt{-a^2x^2+1}+1\right)\operatorname{arctanh}(ax)}{2ax} - \frac{\operatorname{arctanh}(ax)\left(ax+\right.}{2a} \right.$
default	$a^2 \left(-\operatorname{arctanh}(ax)^2 \ln(ax) - \frac{\operatorname{arctanh}(ax)^2}{2a^2x^2} - \frac{\left(ax-\sqrt{-a^2x^2+1}+1\right)\operatorname{arctanh}(ax)}{2ax} - \frac{\operatorname{arctanh}(ax)\left(ax+\right.}{2a} \right.$
parts	Expression too large to display

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^2/a^2/x^2-1/2*(a*x-(-a^2*x^2
+1)^(1/2)+1)/a/x*arctanh(a*x)-1/2*arctanh(a*x)*(a*x+(-a^2*x^2+1)^(1/2)+1)/
a/x+1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^
2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*csgn(I*(-(a*
x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)
^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*x)^2+1/2*arctanh(a
*x)^2-1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1
))^3*arctanh(a*x)^2+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+ln(1+(a*x+1)/(-a^2*x^
2+1)^(1/2))+1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2
/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+arctanh(a*x)^
2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1
/2))-2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,(a*x
+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*
arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(
-a^2*x^2+1)^(1/2))+arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*pol
ylog(3,-(a*x+1)^2/(-a^2*x^2+1)))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^3}\,dx = \int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^3}\,dx = -\int \left(-\frac{\operatorname{atanh}^2(ax)}{x^3}\right)dx - \int \frac{a^2\operatorname{atanh}^2(ax)}{x}\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2/x**3,x)
```

output
```
-Integral(-atanh(a*x)**2/x**3, x) - Integral(a**2*atanh(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^3}\,dx = \int -\frac{(a^2x^2-1)\,\text{artanh}\,(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
-1/8*log(-a*x + 1)^2/x^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1)*l
og(a*x + 1)^2 - (a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))*log(-
a*x + 1))/(a*x^4 - x^3), x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^3}\,dx = \int -\frac{(a^2x^2-1)\,\text{artanh}\,(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^3}\,dx = -\int \frac{\text{atanh}(a\,x)^2\,(a^2\,x^2-1)}{x^3}\,dx$$

input
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^3,x)
```

output
```
-int((atanh(a*x)^2*(a^2*x^2 - 1))/x^3, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^3}\,dx$$
$$= \frac{atanh(ax)^2 a^2x^2 - atanh(ax)^2 - 2atanh(ax)\,a^2x^2 - 2atanh(ax)\,ax - 2\left(\int \frac{atanh(ax)^2}{x}dx\right)a^2x^2 - 2\log(a^2x}{2x^2}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2/x^3,x)
```

output
```
(atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2 - 2*atanh(a*x)*a**2*x**2 - 2*atan
h(a*x)*a*x - 2*int(atanh(a*x)**2/x,x)*a**2*x**2 - 2*log(a**2*x - a)*a**2*x
**2 + 2*log(x)*a**2*x**2)/(2*x**2)
```

3.180 $\int \frac{(1-a^2x^2)\text{arctanh}(ax)^2}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\begin{aligned}\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^4}\,dx = &-\frac{a^2}{3x}+\frac{1}{3}a^3\text{arctanh}(ax)-\frac{a\text{arctanh}(ax)}{3x^2}\\ &-\frac{2}{3}a^3\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)^2}{3x^3}+\frac{a^2\text{arctanh}(ax)^2}{x}\\ &-\frac{4}{3}a^3\text{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)\\ &+\frac{2}{3}a^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-1/3*a^2/x+1/3*a^3*arctanh(a*x)-1/3*a*arctanh(a*x)/x^2-2/3*a^3*arctanh(a*x
)^2-1/3*arctanh(a*x)^2/x^3+a^2*arctanh(a*x)^2/x-4/3*a^3*arctanh(a*x)*ln(2-
2/(a*x+1))+2/3*a^3*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^4}\,dx$$
$$= \frac{-a^2x^2-(-1+ax)^2(1+2ax)\text{arctanh}(ax)^2+\text{arctanh}(ax)\left(-ax+a^3x^3-4a^3x^3\log\left(1-e^{-2\text{arctanh}(ax)}\right)\right.}{3x^3}$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

output
```
(-(a^2*x^2) - (-1 + a*x)^2*(1 + 2*a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-(a*
x) + a^3*x^3 - 4*a^3*x^3*Log[1 - E^(-2*ArcTanh[a*x])]) + 2*a^3*x^3*PolyLog
[2, E^(-2*ArcTanh[a*x])])/(3*x^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.40, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6576, 6452, 6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^4}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^4}dx-a^2\int \frac{\text{arctanh}(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{6452}$$

$$-\left(a^2\left(2a\int \frac{\text{arctanh}(ax)}{x\,(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^2}{x}\right)\right)+\frac{2}{3}a\int \frac{\text{arctanh}(ax)}{x^3\,(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6544}$$

$$-\left(a^2\left(2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)+\frac{2}{3}a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx+\int \frac{\operatorname{arctanh}(ax)}{x^3}dx\right)-\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

$\Big\downarrow$ 6452

$$-\left(a^2\left(2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)+\frac{2}{3}a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)}{2x^2}\right)-\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

$\Big\downarrow$ 264

$$-\left(a^2\left(2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)+\frac{2}{3}a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{\operatorname{arctanh}(ax)}{2x^2}\right)-\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

$\Big\downarrow$ 219

$$-\left(a^2\left(2a\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)+\frac{2}{3}a\left(a^2\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\operatorname{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

$\Big\downarrow$ 6550

$$\frac{2}{3}a\left(a^2\left(\int \frac{\operatorname{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\operatorname{arctanh}(ax)^2\right)-\frac{\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\operatorname{arctanh}(ax)-\frac{1}{x}\right)\right)-\left(a^2\left(2a\left(\int \frac{\operatorname{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\operatorname{arctanh}(ax)^2\right)-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

$\Big\downarrow$ 6494

$$-\left(a^2\left(2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)+\frac{2}{3}a\left(a^2\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\ldots\right.\right.$$
$$\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

$\Big\downarrow$ 2897

$$\frac{2}{3}a\left(a^2\left(\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(\right.\right.$$
$$\left(a^2\left(2a\left(\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{x}\right)\right)-$$
$$\frac{\operatorname{arctanh}(ax)^2}{3x^3}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

output
```
-1/3*ArcTanh[a*x]^2/x^3 - a^2*(-(ArcTanh[a*x]^2/x) + 2*a*(ArcTanh[a*x]^2/2
 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)) +
(2*a*(-1/2*ArcTanh[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 + a^2*(ArcT
anh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 +
a*x)]/2)))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.71

method	result
derivativedivides	$a^3\left(\frac{\operatorname{arctanh}(ax)^2}{ax}-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{3a^2x^2}-\frac{4\operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2\operatorname{ar}}{}\right.$
default	$a^3\left(\frac{\operatorname{arctanh}(ax)^2}{ax}-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{3a^2x^2}-\frac{4\operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{2\operatorname{arctanh}(ax)\ln(ax-1)}{3}+\frac{2\operatorname{ar}}{}\right.$
parts	$-\frac{\operatorname{arctanh}(ax)^2}{3x^3}+\frac{a^2\operatorname{arctanh}(ax)^2}{x}-\frac{a\operatorname{arctanh}(ax)}{3x^2}-\frac{4a^3\operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{2a^3\operatorname{arctanh}(ax)\ln(ax-1)}{3}+$

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(arctanh(a*x)^2/a/x-1/3*arctanh(a*x)^2/a^3/x^3-1/3*arctanh(a*x)/a^2/x^
2-4/3*arctanh(a*x)*ln(a*x)+2/3*arctanh(a*x)*ln(a*x-1)+2/3*arctanh(a*x)*ln(
a*x+1)-1/3/a/x-1/6*ln(a*x-1)+1/6*ln(a*x+1)+2/3*dilog(a*x)+2/3*dilog(a*x+1)
+2/3*ln(a*x)*ln(a*x+1)+1/6*ln(a*x-1)^2-2/3*dilog(1/2*a*x+1/2)-1/3*ln(a*x-1
)*ln(1/2*a*x+1/2)-1/6*ln(a*x+1)^2+1/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*
a*x+1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^4}\,dx=\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^4}\,dx = -\int \left(-\frac{\text{atanh}^2\,(ax)}{x^4}\right)dx - \int \frac{a^2\,\text{atanh}^2\,(ax)}{x^2}\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2/x**4,x)
```

output
```
-Integral(-atanh(a*x)**2/x**4, x) - Integral(a**2*atanh(a*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.62

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^4}\,dx =$$
$$-\frac{1}{6}\left(4\left(\log\,(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a-4\,(\log\,(ax+1)\log\,(x)+\text{Li}_2(-ax))a+\right.$$
$$+\frac{1}{3}\left(2\,a^2\log\,(a^2x^2-1)-2\,a^2\log\,(x^2)-\frac{1}{x^2}\right)a\,\text{artanh}\,(ax)$$
$$+\frac{(3\,a^2x^2-1)\,\text{artanh}\,(ax)^2}{3\,x^3}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
-1/6*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(l
og(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x)
)*a - a*log(a*x + 1) + a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*
x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 + 2)/x)*a^2 + 1/3*(2*a^2*log(a^2*
x^2 - 1) - 2*a^2*log(x^2) - 1/x^2)*a*arctanh(a*x) + 1/3*(3*a^2*x^2 - 1)*ar
ctanh(a*x)^2/x^3
```

Giac [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^4}\,dx=\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^4}\,dx=-\int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)}{x^4}\,dx$$

input
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^4,x)
```

output
```
-int((atanh(a*x)^2*(a^2*x^2 - 1))/x^4, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^4}\,dx$$
$$=\frac{3atanh(ax)^2\,a^2x^2-atanh(ax)^2+3atanh(ax)\,a^3x^3-3atanh(ax)\,ax+4\left(\int\frac{atanh(ax)}{a^2x^5-x^3}dx\right)a\,x^3-3a^2x^2}{3x^3}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2/x^4,x)
```

output
```
(3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2 + 3*atanh(a*x)*a**3*x**3 - 3*at
anh(a*x)*a*x + 4*int(atanh(a*x)/(a**2*x**5 - x**3),x)*a*x**3 - 3*a**2*x**2
)/(3*x**3)
```

3.181 $\int \frac{(1-a^2x^2)\mathbf{arctanh}(ax)^2}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} - \frac{a\text{arctanh}(ax)}{6x^3} + \frac{a^3\text{arctanh}(ax)}{2x} - \frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4} - \frac{1}{3}a^4\log(x) + \frac{1}{6}a^4\log\left(1-a^2x^2\right)$$

output
```
-1/12*a^2/x^2-1/6*a*arctanh(a*x)/x^3+1/2*a^3*arctanh(a*x)/x-1/4*(-a^2*x^2+
1)^2*arctanh(a*x)^2/x^4-1/3*a^4*ln(x)+1/6*a^4*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx = \frac{-a^2x^2 + (-2ax+6a^3x^3)\,\text{arctanh}(ax) - 3(-1+a^2x^2)^2\,\text{arctanh}(ax)^2 - 4a^4x^4\log(x) + 2a^4x^4\log(1-a^2}{12x^4}$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

output
```
(-(a^2*x^2) + (-2*a*x + 6*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTa
nh[a*x]^2 - 4*a^4*x^4*Log[x] + 2*a^4*x^4*Log[1 - a^2*x^2])/(12*x^4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6570, 6576, 6452, 243, 47, 14, 16, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx$$

$$\Big\downarrow \text{ 6570}$$

$$\frac{1}{2}a\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)}{x^4}dx-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{ 6576}$$

$$\frac{1}{2}a\left(\int \frac{\text{arctanh}(ax)}{x^4}dx-a^2\int \frac{\text{arctanh}(ax)}{x^2}dx\right)-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}a\left(-\left(a^2\left(a\int \frac{1}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{3x^3}\right)-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\left(-\left(a^2\left(\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\int \frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}\right)-\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}a\left(-\left(a^2\left(\frac{1}{2}a\left(a^2\int\frac{1}{1-a^2x^2}dx^2+\int\frac{1}{x^2}dx^2\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\int\frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}\right)$$
$$\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}a\left(-\left(a^2\left(\frac{1}{2}a\left(a^2\int\frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\int\frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{3x^3}\right)-$$
$$\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\int\frac{1}{x^4(1-a^2x^2)}dx^2-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)-\frac{\text{arctanh}(ax)}{3x^3}\right)-$$
$$\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\int\left(-\frac{a^4}{a^2x^2-1}+\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)-\frac{\text{arctanh}(a}{3x^3}$$
$$\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\left(-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)-\text{ar}$$
$$\frac{(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{4x^4}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

output
```
-1/4*((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4 + (a*(-1/3*ArcTanh[a*x]/x^3 - a^
2*(-(ArcTanh[a*x]/x) + (a*(Log[x^2] - Log[1 - a^2*x^2]))/2) + (a*(-x^(-2)
+ a^2*Log[x^2] - a^2*Log[1 - a^2*x^2]))/6))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27

method	result
parallelrisch	$-\frac{3a^4x^4\operatorname{arctanh}(ax)^2+4\ln(x)a^4x^4-4\ln(ax-1)x^4a^4-4a^4x^4\operatorname{arctanh}(ax)+a^4x^4-6a^3x^3\operatorname{arctanh}(ax)-6a^2x^2\operatorname{arctanh}\cdots}{12x^4}$
derivativedivides	$a^4\left(-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)}{2ax}-\frac{\operatorname{arctanh}(ax)}{6a^3x^3}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)\cdots}{4}\right.$
default	$a^4\left(-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}+\frac{\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{\operatorname{arctanh}(ax)}{2ax}-\frac{\operatorname{arctanh}(ax)}{6a^3x^3}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)\cdots}{4}\right.$
parts	$-\frac{\operatorname{arctanh}(ax)^2}{4x^4}+\frac{a^2\operatorname{arctanh}(ax)^2}{2x^2}+\frac{a^3\operatorname{arctanh}(ax)}{2x}-\frac{a\operatorname{arctanh}(ax)}{6x^3}+\frac{a^4\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{a^4\operatorname{arcta}\cdots}{}$
risch	$-\frac{(a^4x^4-2a^2x^2+1)\ln(ax+1)^2}{16x^4}+\frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-2ax+3\ln(-ax+1))\ln(ax+1)}{24x^4}-\cdots$

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*a^4*x^4*arctanh(a*x)^2+4*ln(x)*a^4*x^4-4*ln(a*x-1)*x^4*a^4-4*a^4*
x^4*arctanh(a*x)+a^4*x^4-6*a^3*x^3*arctanh(a*x)-6*a^2*x^2*arctanh(a*x)^2+a
^2*x^2+2*a*x*arctanh(a*x)+3*arctanh(a*x)^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int\frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^5}\,dx$$
$$=\frac{8\,a^4x^4\log\left(a^2x^2-1\right)-16\,a^4x^4\log\left(x\right)-4\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^3x^3-ax\right)\log\cdots}{48\,x^4}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/48*(8*a^4*x^4*log(a^2*x^2 - 1) - 16*a^4*x^4*log(x) - 4*a^2*x^2 - 3*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 - a*x)*log
(-(a*x + 1)/(a*x - 1)))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx$$
$$= \begin{cases} -\frac{a^4\log(x)}{3} + \frac{a^4\log\left(x-\frac{1}{a}\right)}{3} - \frac{a^4\,\text{atanh}^2(ax)}{4} + \frac{a^4\,\text{atanh}(ax)}{3} + \frac{a^3\,\text{atanh}(ax)}{2x} + \frac{a^2\,\text{atanh}^2(ax)}{2x^2} - \frac{a^2}{12x^2} - \frac{a\,\text{atanh}(ax)}{6x^3} - \text{ata} \\ 0 \end{cases}$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2/x**5,x)
```

output
```
Piecewise((-a**4*log(x)/3 + a**4*log(x - 1/a)/3 - a**4*atanh(a*x)**2/4 + a
**4*atanh(a*x)/3 + a**3*atanh(a*x)/(2*x) + a**2*atanh(a*x)**2/(2*x**2) - a
**2/(12*x**2) - a*atanh(a*x)/(6*x**3) - atanh(a*x)**2/(4*x**4), Ne(a, 0)),
 (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(76) = 152.

Time = 0.03 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.84

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx =$$
$$-\frac{1}{48}\left(16\,a^2\log(x) - \frac{3\,a^2x^2\log(ax+1)^2 + 3\,a^2x^2\log(ax-1)^2 + 8\,a^2x^2\log(ax-1) - 2\,(3\,a^2x^2\log(a\right.}{x^2}$$
$$-\frac{1}{12}\left(3\,a^3\log(ax+1) - 3\,a^3\log(ax-1) - \frac{2\,(3\,a^2x^2-1)}{x^3}\right)a\,\text{artanh}(ax)$$
$$+\frac{(2\,a^2x^2-1)\,\text{artanh}(ax)^2}{4\,x^4}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/48*(16*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^
2 + 8*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 4*a^2*x^2)*log(a*
x + 1) - 4)/x^2)*a^2 - 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(
3*a^2*x^2 - 1)/x^3)*a*arctanh(a*x) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/x^
4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(76) = 152$.

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.17

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx =$$

$$-\frac{1}{3}\left(a^3\log\left(-\frac{ax+1}{ax-1}-1\right)-a^3\log\left(-\frac{ax+1}{ax-1}\right)+\frac{3\,(ax+1)^2a^3\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax-1)^2\left(\frac{(ax+1)^4}{(ax-1)^4}+\frac{4\,(ax+1)^3}{(ax-1)^3}+\frac{6\,(ax+1)^2}{(ax-1)^2}+\frac{4\,(ax+1)}{ax-1}\right.}\right.$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

output
```
-1/3*(a^3*log(-(a*x + 1)/(a*x - 1) - 1) - a^3*log(-(a*x + 1)/(a*x - 1)) +
3*(a*x + 1)^2*a^3*log(-(a*x + 1)/(a*x - 1))^2/((a*x - 1)^2*((a*x + 1)^4/(a
*x - 1)^4 + 4*(a*x + 1)^3/(a*x - 1)^3 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x
 + 1)/(a*x - 1) + 1)) - (a*x + 1)*a^3/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2
+ 2*(a*x + 1)/(a*x - 1) + 1)) + (3*(a*x + 1)*a^3/(a*x - 1) + a^3)*log(-(a*
x + 1)/(a*x - 1))/((a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2 + 3
*(a*x + 1)/(a*x - 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.76

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx = \ln(1-a\,x)^2\left(\frac{\frac{a^2\,x^2}{2}-\frac{1}{4}}{4\,x^4}-\frac{a^4}{16}\right)-\ln(1-a\,x)\left(\ln(a\,x+1)\left(\frac{\frac{a^2\,x^2}{2}-\frac{1}{4}}{2\,x^4}-\frac{a^4}{8}\right)+\frac{3\,a^5\,x-2\,a^4}{24\,a^3\,x^3}-\frac{3\,x\,a^5+2\,a^4}{24\,a^3\,x^3}-\frac{a\,(22\,a^3\,x^3-12\,a^2\,x^2+6\,a\,x-4)}{96\,x^3}+\frac{a\,(44\,a^3\,x^3+24\,a^2\,x^2+12\,a\,x+8)}{192\,x^3}\right)-\frac{a^4\,\ln(x)}{3}+\ln(a\,x+1)^2\left(\frac{\frac{a^2\,x^2}{8}-\frac{1}{16}}{x^4}-\frac{a^4}{16}\right)+\frac{a^4\,\ln(a^2\,x^2-1)}{6}-\frac{a^2}{12\,x^2}+\frac{a\,\ln(a\,x+1)\left(\frac{a^2\,x^2}{4}-\frac{1}{12}\right)}{x^3}$$

input
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^5,x)
```

output
```
log(1 - a*x)^2*(((a^2*x^2)/2 - 1/4)/(4*x^4) - a^4/16) - log(1 - a*x)*(log(
a*x + 1)*(((a^2*x^2)/2 - 1/4)/(2*x^4) - a^4/8) + (3*a^5*x - 2*a^4)/(24*a^3
*x^3) - (3*a^5*x + 2*a^4)/(24*a^3*x^3) - (a*(6*a*x - 12*a^2*x^2 + 22*a^3*x
^3 - 4))/(96*x^3) + (a*(12*a*x + 24*a^2*x^2 + 44*a^3*x^3 + 8))/(192*x^3))
- (a^4*log(x))/3 + log(a*x + 1)^2*(((a^2*x^2)/8 - 1/16)/x^4 - a^4/16) + (a
^4*log(a^2*x^2 - 1))/6 - a^2/(12*x^2) + (a*log(a*x + 1)*((a^2*x^2)/4 - 1/1
2))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^5}\,dx = \frac{-3atanh(ax)^2\,a^4x^4+6atanh(ax)^2\,a^2x^2-3atanh(ax)^2+4atanh(ax)\,a^4x^4+6atanh(ax)\,a^3x^3-2atanh(}{12x^4}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2/x^5,x)
```

output
```
( - 3*atanh(a*x)**2*a**4*x**4 + 6*atanh(a*x)**2*a**2*x**2 - 3*atanh(a*x)**
2 + 4*atanh(a*x)*a**4*x**4 + 6*atanh(a*x)*a**3*x**3 - 2*atanh(a*x)*a*x + 4
*log(a**2*x - a)*a**4*x**4 - 4*log(x)*a**4*x**4 - a**2*x**2)/(12*x**4)
```

3.182 $\int \frac{(1-a^2x^2)\textbf{arctanh}(ax)^2}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\begin{aligned}\int \frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{x^6}\,dx = &-\frac{a^2}{30x^3}+\frac{a^4}{30x}-\frac{1}{30}a^5\text{arctanh}(ax)-\frac{a\text{arctanh}(ax)}{10x^4}\\&+\frac{2a^3\text{arctanh}(ax)}{15x^2}-\frac{2}{15}a^5\text{arctanh}(ax)^2\\&-\frac{\text{arctanh}(ax)^2}{5x^5}+\frac{a^2\text{arctanh}(ax)^2}{3x^3}\\&-\frac{4}{15}a^5\text{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)\\&+\frac{2}{15}a^5\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-1/30*a^2/x^3+1/30*a^4/x-1/30*a^5*arctanh(a*x)-1/10*a*arctanh(a*x)/x^4+2/1
5*a^3*arctanh(a*x)/x^2-2/15*a^5*arctanh(a*x)^2-1/5*arctanh(a*x)^2/x^5+1/3*
a^2*arctanh(a*x)^2/x^3-4/15*a^5*arctanh(a*x)*ln(2-2/(a*x+1))+2/15*a^5*poly
log(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^6}\,dx$$
$$= \frac{a^2x^2(-1+a^2x^2)-2(3-5a^2x^2+2a^5x^5)\operatorname{arctanh}(ax)^2-ax\operatorname{arctanh}(ax)\left(3-4a^2x^2+a^4x^4+8a^4x^4\log\right.}{30x^5}$$

input
```
Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

output
```
(a^2*x^2*(-1 + a^2*x^2) - 2*(3 - 5*a^2*x^2 + 2*a^5*x^5)*ArcTanh[a*x]^2 - a
*x*ArcTanh[a*x]*(3 - 4*a^2*x^2 + a^4*x^4 + 8*a^4*x^4*Log[1 - E^(-2*ArcTanh
[a*x])]) + 4*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*x^5)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.71, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6576, 6452, 6544, 6452, 264, 219, 264, 219, 6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^6}dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^6}dx-a^2\int \frac{\operatorname{arctanh}(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{5}a\int \frac{\operatorname{arctanh}(ax)}{x^5(1-a^2x^2)}dx-\left(a^2\left(\frac{2}{3}a\int \frac{\operatorname{arctanh}(ax)}{x^3(1-a^2x^2)}dx-\frac{\operatorname{arctanh}(ax)^2}{3x^3}\right)\right)-\frac{\operatorname{arctanh}(ax)^2}{5x^5}$$

$$\Big\downarrow \text{ 6544}$$

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int\frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\int\frac{\text{arctanh}(ax)}{x^3}dx\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\int\frac{\text{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}dx+\int\frac{\text{arctanh}(ax)}{x^5}dx\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$$\Big\downarrow \text{ 6452}$$

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int\frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\int\frac{1}{x^2\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\int\frac{\text{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}dx+\frac{1}{4}a\int\frac{1}{x^4\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{4x^4}\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$$\Big\downarrow \text{ 264}$$

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int\frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\left(a^2\int\frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{\text{arctanh}(ax)}{2x^2}\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\int\frac{\text{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}dx+\frac{1}{4}a\left(a^2\int\frac{1}{x^2\left(1-a^2x^2\right)}dx-\frac{1}{3x^3}\right)-\frac{\text{arctanh}(ax)}{4x^4}\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$$\Big\downarrow \text{ 219}$$

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int\frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\int\frac{\text{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}dx+\frac{1}{4}a\left(a^2\int\frac{1}{x^2\left(1-a^2x^2\right)}dx-\frac{1}{3x^3}\right)-\frac{\text{arctanh}(ax)}{4x^4}\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$$\Big\downarrow \text{ 264}$$

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int\frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\int\frac{\text{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}dx+\frac{1}{4}a\left(a^2\left(a^2\int\frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\text{arctanh}(ax)}{4x^4}\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$$\Big\downarrow \text{ 219}$$

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\int \frac{\text{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}dx+\frac{1}{4}a\left(a^2\left(a\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\text{arctanh}(ax)}{4x^4}\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\Big\downarrow$ 6544

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\int \frac{\text{arctanh}(ax)}{x^3}dx\right)+\frac{1}{4}a\left(a^2\left(a\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\text{arctanh}(ax)}{4x^4}\right)-\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\Big\downarrow$ 6452

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}\right)+\frac{1}{4}a\left(a^2\left(a\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)\right.\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\Big\downarrow$ 264

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{\text{arctanh}(ax)}{2x^2}\right)+\frac{1}{4}a\left(a^2\left(a\text{arctanh}(ax)-\frac{1}{x}\right)-\right.\right.\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\Big\downarrow$ 219

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(ax)^2}{3x^3}\right)\right)+\frac{2}{5}a\left(a^2\left(a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)+\frac{1}{4}a\left(a^2\left(a\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{1}{3x^3}\right.\right.\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\Big\downarrow$ 6550

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)-\frac{\text{arctanh}(a}{3x^3}\right.\right.$$
$$\frac{2}{5}a\left(a^2\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right)+\frac{1}{4}a\left(a^2\left(a\text{arct}\right.\right.\right.$$
$$\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\downarrow$ 6494

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}\right.\right.\right.$$
$$\frac{2}{5}a\left(a^2\left(a^2\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}\right.\right.$$
$$\frac{\text{arctanh}(ax)^2}{5x^5}$$

$\downarrow$ 2897

$$-\left(a^2\left(\frac{2}{3}a\left(a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\text{arctanh}(ax}{2x^2}\right.\right.\right.$$
$$\frac{2}{5}a\left(a^2\left(a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\right.\right.$$
$$\frac{\text{arctanh}(ax)^2}{5x^5}$$

input
```
Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

output
```
-1/5*ArcTanh[a*x]^2/x^5 - a^2*(-1/3*ArcTanh[a*x]^2/x^3 + (2*a*(-1/2*ArcTan
h[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 + a^2*(ArcTanh[a*x]^2/2 + Ar
cTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)))/3) + (
2*a*(-1/4*ArcTanh[a*x]/x^4 + (a*(-1/3*1/x^3 + a^2*(-x^(-1) + a*ArcTanh[a*x
])))/4 + a^2*(-1/2*ArcTanh[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 + a
^2*(ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 +
 2/(1 + a*x)]/2))))/5
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.53

method	result
derivativedivides	$a^5\left(-\frac{\operatorname{arctanh}(ax)^2}{5a^5x^5}+\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{10a^4x^4}+\frac{2\operatorname{arctanh}(ax)}{15a^2x^2}-\frac{4\operatorname{arctanh}(ax)\ln(ax)}{15}+\frac{2\operatorname{arctanh}(a\right.$
default	$a^5\left(-\frac{\operatorname{arctanh}(ax)^2}{5a^5x^5}+\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{10a^4x^4}+\frac{2\operatorname{arctanh}(ax)}{15a^2x^2}-\frac{4\operatorname{arctanh}(ax)\ln(ax)}{15}+\frac{2\operatorname{arctanh}(a\right.$
parts	$\frac{a^2\operatorname{arctanh}(ax)^2}{3x^3}-\frac{\operatorname{arctanh}(ax)^2}{5x^5}-\frac{a\operatorname{arctanh}(ax)}{10x^4}+\frac{2a^3\operatorname{arctanh}(ax)}{15x^2}-\frac{4a^5\operatorname{arctanh}(ax)\ln(ax)}{15}+\frac{2a^5\operatorname{arctan}}{}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/5*arctanh(a*x)^2/a^5/x^5+1/3*arctanh(a*x)^2/a^3/x^3-1/10*arctanh(a
*x)/a^4/x^4+2/15*arctanh(a*x)/a^2/x^2-4/15*arctanh(a*x)*ln(a*x)+2/15*arcta
nh(a*x)*ln(a*x-1)+2/15*arctanh(a*x)*ln(a*x+1)-1/30/a^3/x^3+1/30/a/x+1/60*l
n(a*x-1)-1/60*ln(a*x+1)+2/15*dilog(a*x)+2/15*dilog(a*x+1)+2/15*ln(a*x)*ln(
a*x+1)+1/30*ln(a*x-1)^2-2/15*dilog(1/2*a*x+1/2)-1/15*ln(a*x-1)*ln(1/2*a*x+
1/2)+1/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/30*ln(a*x+1)^2)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)^2}{x^6}\,dx = \int -\frac{(a^2x^2-1)\,\mathrm{artanh}\left(ax\right)^2}{x^6}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)^2}{x^6}\,dx = -\int \left(-\frac{\mathrm{atanh}^2\left(ax\right)}{x^6}\right)dx - \int \frac{a^2\,\mathrm{atanh}^2\left(ax\right)}{x^4}\,dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**2/x**6,x)
```

output
```
-Integral(-atanh(a*x)**2/x**6, x) - Integral(a**2*atanh(a*x)**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int \frac{(1-a^2x^2)\,\mathrm{arctanh}(ax)^2}{x^6}\,dx = \\
&-\frac{1}{60}\left(8\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a^3-8\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a^3\right. \\
&+\frac{1}{30}\left(4\,a^4\log\left(a^2x^2-1\right)-4\,a^4\log\left(x^2\right)+\frac{4\,a^2x^2-3}{x^4}\right)a\,\mathrm{artanh}\left(ax\right) \\
&+\frac{(5\,a^2x^2-3)\,\mathrm{artanh}\left(ax\right)^2}{15\,x^5}
\end{aligned}$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

output
```
-1/60*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 8
*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 8*(log(-a*x + 1)*log(x) + dilog
(a*x))*a^3 + a^3*log(a*x + 1) - a^3*log(a*x - 1) + 2*(a^3*x^3*log(a*x + 1)
^2 - 2*a^3*x^3*log(a*x + 1)*log(a*x - 1) - a^3*x^3*log(a*x - 1)^2 - a^2*x^
2 + 1)/x^3)*a^2 + 1/30*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x
^2 - 3)/x^4)*a*arctanh(a*x) + 1/15*(5*a^2*x^2 - 3)*arctanh(a*x)^2/x^5
```

Giac [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^6}\,dx = \int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^6}\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^6}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)}{x^6}\,dx$$

input
```
int(-(atanh(a*x)^2*(a^2*x^2 - 1))/x^6,x)
```

output
```
-int((atanh(a*x)^2*(a^2*x^2 - 1))/x^6, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{x^6}\,dx$$
$$= \frac{10atanh(ax)^2 a^2x^2 - 6atanh(ax)^2 - atanh(ax)\,a^5x^5 + 4atanh(ax)\,a^3x^3 - 3atanh(ax)\,ax + 8\left(\int \frac{atanh(ax)}{a^2x^3-x}\right.}{30x^5}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^2/x^6,x)
```

output
```
(10*atanh(a*x)**2*a**2*x**2 - 6*atanh(a*x)**2 - atanh(a*x)*a**5*x**5 + 4*a
tanh(a*x)*a**3*x**3 - 3*atanh(a*x)*a*x + 8*int(atanh(a*x)/(a**2*x**3 - x),
x)*a**5*x**5 + a**4*x**4 - a**2*x**2)/(30*x**5)
```

3.183 $\int (1 - a^2x^2) \operatorname{arctanh}(ax)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 157

$$\begin{aligned}\int (1-a^2x^2) \operatorname{arctanh}(ax)^3 \, dx = & -x\operatorname{arctanh}(ax) + \frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{2a} \\ & + \frac{2\operatorname{arctanh}(ax)^3}{3a} + \frac{2}{3}x\operatorname{arctanh}(ax)^3 \\ & + \frac{1}{3}x(1-a^2x^2)\operatorname{arctanh}(ax)^3 \\ & - \frac{2\operatorname{arctanh}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} - \frac{\log(1-a^2x^2)}{2a} \\ & - \frac{2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2, 1-\frac{2}{1-ax}\right)}{a} \\ & + \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{1-ax}\right)}{a}\end{aligned}$$

output
```
-x*arctanh(a*x)+1/2*(-a^2*x^2+1)*arctanh(a*x)^2/a+2/3*arctanh(a*x)^3/a+2/3
*x*arctanh(a*x)^3+1/3*x*(-a^2*x^2+1)*arctanh(a*x)^3-2*arctanh(a*x)^2*ln(2/
(-a*x+1))/a-1/2*ln(-a^2*x^2+1)/a-2*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+
polylog(3,1-2/(-a*x+1))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int (1-a^2x^2)\,\text{arctanh}(ax)^3\,dx = -\frac{6ax\text{arctanh}(ax)-3\text{arctanh}(ax)^2+3a^2x^2\text{arctanh}(ax)^2+4\text{arctanh}(ax)^3-6ax\text{arctanh}(ax)^3+2a^3x^3}{}$$

input
```
Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

output
```
-1/6*(6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 + 4
*ArcTanh[a*x]^3 - 6*a*x*ArcTanh[a*x]^3 + 2*a^3*x^3*ArcTanh[a*x]^3 + 12*Arc
Tanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + 3*Log[1 - a^2*x^2] - 12*ArcTanh
[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6*PolyLog[3, -E^(-2*ArcTanh[a*x])
])/a
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6506, 6436, 240, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^2x^2)\,\text{arctanh}(ax)^3\,dx$$

$$\Big\downarrow \text{6506}$$

$$-\int \text{arctanh}(ax)dx+\frac{2}{3}\int \text{arctanh}(ax)^3dx+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}$$

$$\Big\downarrow \text{6436}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx\right)+a\int\frac{x}{1-a^2x^2}dx+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}-x\text{arctanh}(ax)$$

$$\Big\downarrow \text{240}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}-\frac{\log(1-a^2x^2)}{2a}-x\text{arctanh}(ax)$$

$$\Big\downarrow \text{6546}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\int\frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}-\frac{\log(1-a^2x^2)}{2a}-x\text{arctanh}(ax)$$

$$\Big\downarrow \text{6470}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}-\frac{\log(1-a^2x^2)}{2a}-x\text{arctanh}(ax)$$

$$\Big\downarrow \text{6620}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}\right.\right.$$
$$\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}-\frac{\log(1-a^2x^2)}{2a}-x\text{arctanh}(ax)$$

$$\Big\downarrow \text{7164}$$

$$\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arct}}{}\right.\right.$$

$$\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}-\frac{\log(1-a^2x^2)}{2a}-x\text{arctanh}(ax)$$

input
```
Int[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

output
```
-(x*ArcTanh[a*x]) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(2*a) + (x*(1 - a^2*x^2
)*ArcTanh[a*x]^3)/3 - Log[1 - a^2*x^2]/(2*a) + (2*(x*ArcTanh[a*x]^3 - 3*a*
(-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*
(ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]
/(4*a)))/a)))/3
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.94 (sec) , antiderivative size = 772, normalized size of antiderivative = 4.92

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^3 a^3 x^3}{3}+\operatorname{arctanh}(ax)^3 ax-\frac{a^2x^2 \operatorname{arctanh}(ax)^2}{2}+\operatorname{arctanh}(ax)^2 \ln(ax-1)+\operatorname{arctanh}(ax)^2 \ln(ax+1)-2 \operatorname{arctanh}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^3 a^3 x^3}{3}+\operatorname{arctanh}(ax)^3 ax-\frac{a^2x^2 \operatorname{arctanh}(ax)^2}{2}+\operatorname{arctanh}(ax)^2 \ln(ax-1)+\operatorname{arctanh}(ax)^2 \ln(ax+1)-2 \operatorname{arctanh}}{}$
parts	$-\frac{\operatorname{arctanh}(ax)^3 a^2 x^3}{3}+x \operatorname{arctanh}(ax)^3-\frac{a \operatorname{arctanh}(ax)^2 x^2}{2}+\frac{\operatorname{arctanh}(ax)^2 \ln(ax-1)}{a}+\frac{\operatorname{arctanh}(ax)^2 \ln}{a}$

input
```
int((-a^2*x^2+1)*arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/3*arctanh(a*x)^3*a^3*x^3+arctanh(a*x)^3*a*x-1/2*a^2*x^2*arctanh(a*
x)^2+arctanh(a*x)^2*ln(a*x-1)+arctanh(a*x)^2*ln(a*x+1)-2*arctanh(a*x)*poly
log(2,-(a*x+1)^2/(-a^2*x^2+1))+polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-2*arctan
h(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+2/3*arctanh(a*x)^3-1/2*I*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^
2+ln((a*x+1)^2/(-a^2*x^2+1)+1)-(a*x+1)*arctanh(a*x)-2*arctanh(a*x)^2*ln(2)
+1/2*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*
x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1))^3*arctanh(a*x)^2+1/2*arctanh(a*x)^2+I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-
1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-1/2*I*
Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)
^2-I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2
*arctanh(a*x)^2-I*Pi*arctanh(a*x)^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1)
)^3*arctanh(a*x)^2+1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1)
)*arctanh(a*x)^2)
```

Fricas [F]

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)^3 \, dx = \int -\left(a^2x^2-1\right) \operatorname{artanh}\left(ax\right)^3 \, dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Sympy [F]

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)^3 \, dx = -\int a^2x^2 \operatorname{atanh}^3\left(ax\right) dx - \int \left(-\operatorname{atanh}^3\left(ax\right)\right) \, dx$$

input
```
integrate((-a**2*x**2+1)*atanh(a*x)**3,x)
```

output
```
-Integral(a**2*x**2*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

Maxima [F]

$$\int (1-a^2x^2) \operatorname{arctanh}(ax)^3 \, dx = \int -\left(a^2x^2-1\right) \operatorname{artanh}\left(ax\right)^3 \, dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
1/48*(2*a^3*x^3 - 3*a^2*x^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*x + 1
))*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a
*x + 1) - 6)*(a*x - 1)/a + 1/864*(4*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2
 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*log(-a*x +
 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 - 3*log(-a
*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/3*(3*(a^3
*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1)^3 + (2*a^3*x^3 - 3*a^2*x^2 - 9*(a^3
*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2
)*log(a*x + 1))*log(-a*x + 1))/(a*x - 1), x)
```

Giac [F]

$$\int (1-a^2x^2)\operatorname{arctanh}(ax)^3\,dx=\int -\left(a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-a^2x^2)\operatorname{arctanh}(ax)^3\,dx=-\int \operatorname{atanh}(a\,x)^3\,\left(a^2\,x^2-1\right)\,dx$$

input
```
int(-atanh(a*x)^3*(a^2*x^2 - 1),x)
```

output
```
-int(atanh(a*x)^3*(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int (1 - a^2x^2) \operatorname{arctanh}(ax)^3 \, dx$$
$$= \frac{-2atanh(ax)^3 a^3x^3 + 6atanh(ax)^3 ax - 3atanh(ax)^2 a^2x^2 + 3atanh(ax)^2 - 6atanh(ax) ax - 6atanh(ax}{6a}$$

input
```
int((-a^2*x^2+1)*atanh(a*x)^3,x)
```

output
```
( - 2*atanh(a*x)**3*a**3*x**3 + 6*atanh(a*x)**3*a*x - 3*atanh(a*x)**2*a**2
*x**2 + 3*atanh(a*x)**2 - 6*atanh(a*x)*a*x - 6*atanh(a*x) + 12*int((atanh(
a*x)**2*x)/(a**2*x**2 - 1),x)*a**2 - 6*log(a**2*x - a))/(6*a)
```

3.184 $\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 193

$$\begin{aligned}\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx &= \text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right)\\
&\quad-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)\\
&\quad-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)\\
&\quad-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{\sqrt{2}+x}\right)\\
&\quad+\frac{1}{4}\text{PolyLog}\left(2,1+\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)\\
&\quad+\frac{1}{4}\text{PolyLog}\left(2,1-\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)\end{aligned}$$

output
```
arctanh(1/2*x*2^(1/2))*ln(2*2^(1/2)/(2^(1/2)+x))-1/2*arctanh(1/2*x*2^(1/2)
)*ln((-4+4*x)/(2-2^(1/2))/(2^(1/2)+x))-1/2*arctanh(1/2*x*2^(1/2))*ln(4*(1+
x)/(2+2^(1/2))/(2^(1/2)+x))-1/2*polylog(2,1-2*2^(1/2)/(2^(1/2)+x))+1/4*pol
ylog(2,1+4*(1-x)/(2-2^(1/2))/(2^(1/2)+x))+1/4*polylog(2,1-4*(1+x)/(2+2^(1/
2))/(2^(1/2)+x))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.20

$$\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = \frac{1}{4}\Bigg(-4\text{arcsinh}(1)\text{arctanh}(x) + 4\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1+e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) + 2\text{arcsinh}(1)\log\left(1+\left(-3+2\sqrt{2}\right)e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) - 2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1+\left(-3+2\sqrt{2}\right)e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) - 2\text{arcsinh}(1)\log\left(1-\left(3+2\sqrt{2}\right)e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) - 2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-\left(3+2\sqrt{2}\right)e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) - 2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) + \text{PolyLog}\left(2,\left(3-2\sqrt{2}\right)e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right) + \text{PolyLog}\left(2,\left(3+2\sqrt{2}\right)e^{-2\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}\right)\Bigg)$$

input
```
Integrate[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

output
```
(-4*ArcSinh[1]*ArcTanh[x] + 4*ArcTanh[x/Sqrt[2]]*Log[1 + E^(-2*ArcTanh[x/S
qrt[2]])] + 2*ArcSinh[1]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])
] - 2*ArcTanh[x/Sqrt[2]]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])
] - 2*ArcSinh[1]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*Arc
Tanh[x/Sqrt[2]]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*Poly
Log[2, -E^(-2*ArcTanh[x/Sqrt[2]])] + PolyLog[2, (3 - 2*Sqrt[2])/E^(2*ArcTa
nh[x/Sqrt[2]])] + PolyLog[2, (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])])/4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}dx$$

$$\Big\downarrow \text{6554}$$

$$\int \left(-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{2(x-1)}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{2(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)+\frac{1}{4}\text{PolyLog}\left(2,1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)$$

input
```
Int[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

output
```
ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Lo
g[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/2 - (ArcTanh[x/Sqrt[2]]*Log
[(4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/2 - PolyLog[2, 1 - (2*Sqrt[2]
)/(Sqrt[2] + x)]/2 + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] +
x))]/4 + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6554
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

method	result
parts	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{2}\right)\ln\left(-1+x\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{2}\right)\ln\left(1+x\right)}{2}-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,\ln\left(-1+x\right)\ln\left(\frac{\sqrt{2}-x}{\sqrt{2}-1}\right)}{4}-\frac{\sqrt{2}\,\ln\left(-1+x\right)\ln\left(\frac{\sqrt{2}+x}{1+\sqrt{2}}\right)}{4}\cdots\right.}{}$
derivativedivides	$-\frac{\ln\left(x^{2}-1\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\ln\left(\frac{\sqrt{2}\,x}{2}+1\right)\ln\left(x^{2}-1\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}\,x}{2}+1\right)\ln\left(\frac{\sqrt{2}-\sqrt{2}\,x}{2+\sqrt{2}}\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}\,x}{2}+1\right)\ln\left(\frac{\sqrt{2}+\sqrt{2}\,x}{-2+\sqrt{2}}\cdots\right.}{4}$
default	$-\frac{\ln\left(x^{2}-1\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\ln\left(\frac{\sqrt{2}\,x}{2}+1\right)\ln\left(x^{2}-1\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}\,x}{2}+1\right)\ln\left(\frac{\sqrt{2}-\sqrt{2}\,x}{2+\sqrt{2}}\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}\,x}{2}+1\right)\ln\left(\frac{\sqrt{2}+\sqrt{2}\,x}{-2+\sqrt{2}}\cdots\right.}{4}$
risch	$\frac{\left(\ln\left(1-\frac{\sqrt{2}\,x}{2}\right)-\ln\left(\frac{2-\sqrt{2}\,x}{2+\sqrt{2}}\right)\right)\ln\left(\frac{\sqrt{2}+\sqrt{2}\,x}{2+\sqrt{2}}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{2-\sqrt{2}\,x}{2+\sqrt{2}}\right)}{4}+\frac{\left(\ln\left(1-\frac{\sqrt{2}\,x}{2}\right)-\ln\left(\frac{2-\sqrt{2}\,x}{2-\sqrt{2}}\right)\right)\ln\left(\frac{\sqrt{2}\,x-\sqrt{2}}{2-\sqrt{2}}\right)}{4}-\cdots$

input
```
int(x*arctanh(1/2*2^(1/2)*x)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/2*2^(1/2)*x)*ln(-1+x)-1/2*arctanh(1/2*2^(1/2)*x)*ln(1+x)-1/
2*2^(1/2)*(1/4*2^(1/2)*ln(-1+x)*ln((2^(1/2)-x)/(2^(1/2)-1))-1/4*2^(1/2)*ln
(-1+x)*ln((2^(1/2)+x)/(1+2^(1/2)))+1/4*2^(1/2)*dilog((2^(1/2)-x)/(2^(1/2)-
1))-1/4*2^(1/2)*dilog((2^(1/2)+x)/(1+2^(1/2)))+1/4*2^(1/2)*ln(1+x)*ln((2^(
1/2)-x)/(1+2^(1/2)))-1/4*2^(1/2)*ln(1+x)*ln((2^(1/2)+x)/(2^(1/2)-1))+1/4*2
^(1/2)*dilog((2^(1/2)-x)/(1+2^(1/2)))-1/4*2^(1/2)*dilog((2^(1/2)+x)/(2^(1/
2)-1)))
```

Fricas [F]

$$\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx=\int -\frac{x\,\text{artanh}\left(\frac{1}{2}\sqrt{2}x\right)}{x^2-1}\,dx$$

input
```
integrate(x*arctanh(1/2*2^(1/2)*x)/(-x^2+1),x, algorithm="fricas")
```

output
```
integral(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx=-\int \frac{x\,\text{atanh}\left(\frac{\sqrt{2}x}{2}\right)}{x^2-1}\,dx$$

input
```
integrate(x*atanh(1/2*2**(1/2)*x)/(-x**2+1),x)
```

output
```
-Integral(x*atanh(sqrt(2)*x/2)/(x**2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.44

$$\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = -\frac{1}{2}\,\text{artanh}\left(\frac{1}{2}\,\sqrt{2}x\right)\log\left(x^2-1\right)-\frac{1}{4}\,\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)$$
$$+\frac{1}{8}\,\sqrt{2}\left(\sqrt{2}\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\sqrt{2}\left(\left(\log\left(2\,x+2\,\sqrt{2}\right)-\log\left(2\,x-2\,\sqrt{2}\right)\right)\log\left(x^2-1\right)-\right.\right.$$

input
```
integrate(x*arctanh(1/2*2^(1/2)*x)/(-x^2+1),x, algorithm="maxima")
```

output
```
-1/2*arctanh(1/2*sqrt(2)*x)*log(x^2 - 1) - 1/4*log(x^2 - 1)*log((x - sqrt(
2))/(x + sqrt(2))) + 1/8*sqrt(2)*(sqrt(2)*log(x^2 - 1)*log((x - sqrt(2))/(
x + sqrt(2))) + sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log
(x^2 - 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x
 - sqrt(2))*log((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(
x + sqrt(2))/(sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt
(2) - 1) + 1) - dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/
(sqrt(2) + 1)) - dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))
/(sqrt(2) - 1))))
```

Giac [F]

$$\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = \int -\frac{x\,\text{artanh}\left(\frac{1}{2}\,\sqrt{2}x\right)}{x^2-1}\,dx$$

input
```
integrate(x*arctanh(1/2*2^(1/2)*x)/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\mathrm{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = -\int \frac{x\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{x^2-1}\,dx$$

input
```
int(-(x*atanh((2^(1/2)*x)/2))/(x^2 - 1),x)
```

output
```
-int((x*atanh((2^(1/2)*x)/2))/(x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x\mathrm{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = -\left(\int \frac{atanh\left(\frac{\sqrt{2}\,x}{2}\right)x}{x^2-1}dx\right)$$

input
```
int(x*atanh(1/2*2^(1/2)*x)/(-x^2+1),x)
```

output
```
- int((atanh((sqrt(2)*x)/2)*x)/(x**2 - 1),x)
```

3.185 $\int \frac{x(1-a^2x^2)}{\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(1-a^2x^2)}{\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{x(1-a^2x^2)}{\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(1-a^2x^2)}{\text{arctanh}(ax)}\,dx = \int \frac{x(1-a^2x^2)}{\text{arctanh}(ax)}\,dx$$

input
```
Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
```

output
```
Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-a^2x^2)}{\text{arctanh}(ax)} \, dx$$
$$\Big\downarrow \ 6651$$
$$\int \frac{x(1-a^2x^2)}{\text{arctanh}(ax)}dx$$

input
```
Int[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(-a^2x^2+1)}{\operatorname{arctanh}(ax)}dx$$

input
```
int(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

output
```
int(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)}\,dx = \int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^3 - x)/arctanh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)}\,dx = -\int \left(-\frac{x}{\operatorname{atanh}(ax)}\right)dx - \int \frac{a^2x^3}{\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x*(-a**2*x**2+1)/atanh(a*x),x)
```

output
```
-Integral(-x/atanh(a*x), x) - Integral(a**2*x**3/atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)}\,dx = \int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*x^2 - 1)*x/arctanh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)}\,dx = \int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*x/arctanh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)}\,dx = -\int \frac{x\,(a^2\,x^2-1)}{\operatorname{atanh}(a\,x)}\,dx$$

input
```
int(-(x*(a^2*x^2 - 1))/atanh(a*x),x)
```

output
```
-int((x*(a^2*x^2 - 1))/atanh(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x^3}{atanh\,(ax)}dx\right)a^2 + \int \frac{x}{atanh\,(ax)}dx$$

input

```
int(x*(-a^2*x^2+1)/atanh(a*x),x)
```

output

```
- int(x**3/atanh(a*x),x)*a**2 + int(x/atanh(a*x),x)
```

3.186 $\int \frac{1-a^2x^2}{\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{1-a^2x^2}{\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)}\,dx = \int \frac{1-a^2x^2}{\text{arctanh}(ax)}\,dx$$

input
```
Integrate[(1 - a^2*x^2)/ArcTanh[a*x],x]
```

output
```
Integrate[(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)}dx$$

input
```
Int[(1 - a^2*x^2)/ArcTanh[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-a^2x^2+1}{\text{arctanh}\,(ax)}dx$$

input
```
int((-a^2*x^2+1)/arctanh(a*x),x)
```

output
```
int((-a^2*x^2+1)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)}\,dx = -\int \frac{a^2x^2}{\operatorname{atanh}\left(ax\right)}\,dx - \int \left(-\frac{1}{\operatorname{atanh}\left(ax\right)}\right)\,dx$$

input
```
integrate((-a**2*x**2+1)/atanh(a*x),x)
```

output
```
-Integral(a**2*x**2/atanh(a*x), x) - Integral(-1/atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*x^2 - 1)/arctanh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)}\,dx = -\int \frac{a^2\,x^2-1}{\operatorname{atanh}(a\,x)}\,dx$$

input
```
int(-(a^2*x^2 - 1)/atanh(a*x),x)
```

output
```
-int((a^2*x^2 - 1)/atanh(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)}\,dx = \frac{-\left(\int \frac{x^4}{atanh(ax)a^2x^2-atanh(ax)}dx\right)a^5+2\left(\int \frac{x^2}{atanh(ax)a^2x^2-atanh(ax)}dx\right)a^3+\log(atanh(ax))}{a}$$

input
```
int((-a^2*x^2+1)/atanh(a*x),x)
```

output
```
( - int(x**4/(atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**5 + 2*int(x**2/(ata
nh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**3 + log(atanh(a*x)))/a
```

3.187 $\int \frac{1-a^2x^2}{x\textbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{1-a^2x^2}{x\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)/x/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx = \int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx$$

input
```
Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
```

output
```
Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}dx$$

input
```
Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-a^2x^2+1}{x\,\text{arctanh}\,(ax)}dx$$

input
```
int((-a^2*x^2+1)/x/arctanh(a*x),x)
```

output
```
int((-a^2*x^2+1)/x/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx=\int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx=-\int \left(-\frac{1}{x\,\text{atanh}\,(ax)}\right)\,dx-\int \frac{a^2x}{\text{atanh}\,(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)/x/atanh(a*x),x)
```

output
```
-Integral(-1/(x*atanh(a*x)), x) - Integral(a**2*x/atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx=\int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx = \int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx = -\int \frac{a^2\,x^2-1}{x\,\text{atanh}\,(a\,x)}\,dx$$

input
```
int(-(a^2*x^2 - 1)/(x*atanh(a*x)),x)
```

output
```
-int((a^2*x^2 - 1)/(x*atanh(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)}\,dx = -\left(\int \frac{x}{atanh\,(ax)}dx\right)a^2 + \int \frac{1}{atanh\,(ax)\,x}dx$$

input
```
int((-a^2*x^2+1)/x/atanh(a*x),x)
```

output
```
- int(x/atanh(a*x),x)*a**2 + int(1/(atanh(a*x)*x),x)
```

3.188 $\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2} \, dx = \operatorname{Int}\left(\frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}, x\right)$$

output
```
Defer(Int)(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2} \, dx = \int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2} \, dx$$

input
```
Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

output
```
Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2} \, dx$$
$$\Big\downarrow \; 6651$$
$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}dx$$

input
```
Int[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(-a^2x^2+1)}{\operatorname{arctanh}\left(ax\right)^2}dx$$

input
```
int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

output
```
int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^3 - x)/arctanh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}\,dx = -\int \left(-\frac{x}{\operatorname{atanh}^2(ax)}\right)dx - \int \frac{a^2x^3}{\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x*(-a**2*x**2+1)/atanh(a*x)**2,x)
```

output
```
-Integral(-x/atanh(a*x)**2, x) - Integral(a**2*x**3/atanh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.78

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*(a^4*x^5 - 2*a^2*x^3 + x)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrat
e(-2*(5*a^4*x^4 - 6*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)*x/arctanh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}\,dx = -\int \frac{x\,(a^2\,x^2-1)}{\operatorname{atanh}(a\,x)^2}\,dx$$

input
```
int(-(x*(a^2*x^2 - 1))/atanh(a*x)^2,x)
```

output
```
-int((x*(a^2*x^2 - 1))/atanh(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{x(1-a^2x^2)}{\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{x^3}{atanh\left(ax\right)^2}dx\right)a^2 + \int \frac{x}{atanh\left(ax\right)^2}dx$$

input
```
int(x*(-a^2*x^2+1)/atanh(a*x)^2,x)
```

output
```
- int(x**3/atanh(a*x)**2,x)*a**2 + int(x/atanh(a*x)**2,x)
```

3.189 $\int \frac{1-a^2x^2}{\text{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)^2} dx = \text{Int}\left(\frac{1-a^2x^2}{\text{arctanh}(ax)^2}, x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)^2} dx = \int \frac{1-a^2x^2}{\text{arctanh}(ax)^2} dx$$

input
```
Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
```

output
```
Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)^2}dx$$

input
```
Int[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-a^2x^2+1}{\text{arctanh}\left(ax\right)^2}dx$$

input
```
int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1 - a^2x^2}{\operatorname{arctanh}(ax)^2} \, dx = \int -\frac{a^2x^2 - 1}{\operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1 - a^2x^2}{\operatorname{arctanh}(ax)^2} \, dx = -\int \frac{a^2x^2}{\operatorname{atanh}^2(ax)} \, dx - \int \left(-\frac{1}{\operatorname{atanh}^2(ax)}\right) dx$$

input
```
integrate((-a**2*x**2+1)/atanh(a*x)**2,x)
```

output
```
-Integral(a**2*x**2/atanh(a*x)**2, x) - Integral(-1/atanh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{1 - a^2x^2}{\operatorname{arctanh}(ax)^2} \, dx = \int -\frac{a^2x^2 - 1}{\operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrat
e(-8*(a^3*x^3 - a*x)/(log(a*x + 1) - log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^2}\,dx = -\int \frac{a^2\,x^2-1}{\operatorname{atanh}\left(a\,x\right)^2}\,dx$$

input
```
int(-(a^2*x^2 - 1)/atanh(a*x)^2,x)
```

output
```
-int((a^2*x^2 - 1)/atanh(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{x^2}{atanh\,(ax)^2}dx\right)a^2 + \int \frac{1}{atanh\,(ax)^2}dx$$

input
```
int((-a^2*x^2+1)/atanh(a*x)^2,x)
```

output
```
- int(x**2/atanh(a*x)**2,x)*a**2 + int(1/atanh(a*x)**2,x)
```

3.190 $\int \frac{1-a^2x^2}{x\mathbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx = \text{Int}\left(\frac{1-a^2x^2}{x\text{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
```

output
```
Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}dx$$

input
```
Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-a^2x^2+1}{x\operatorname{arctanh}(ax)^2}dx$$

input
```
int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx = \int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx = -\int \left(-\frac{1}{x\,\text{atanh}^2\,(ax)}\right)dx - \int \frac{a^2x}{\text{atanh}^2\,(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)/x/atanh(a*x)**2,x)
```

output
```
-Integral(-1/(x*atanh(a*x)**2), x) - Integral(a**2*x/atanh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.70

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx = \int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) - inte
grate(-2*(3*a^4*x^4 - 2*a^2*x^2 - 1)/(a*x^2*log(a*x + 1) - a*x^2*log(-a*x
+ 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx=\int -\frac{a^2x^2-1}{x\,\text{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx=-\int \frac{a^2\,x^2-1}{x\,\text{atanh}\left(a\,x\right)^2}\,dx$$

input
```
int(-(a^2*x^2 - 1)/(x*atanh(a*x)^2),x)
```

output
```
-int((a^2*x^2 - 1)/(x*atanh(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{1-a^2x^2}{x\text{arctanh}(ax)^2}\,dx = -\left(\int \frac{x}{atanh\,(ax)^2}dx\right)a^2 + \int \frac{1}{atanh\,(ax)^2\,x}dx$$

input
```
int((-a^2*x^2+1)/x/atanh(a*x)^2,x)
```

output
```
- int(x/atanh(a*x)**2,x)*a**2 + int(1/(atanh(a*x)**2*x),x)
```

3.191 $\int \frac{1-a^2x^2}{\mathbf{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3} \, dx = \operatorname{Int}\left(\frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}, x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3} \, dx = \int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3} \, dx$$

input
```
Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
```

output
```
Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)^3} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1-a^2x^2}{\text{arctanh}(ax)^3}dx$$

input
```
Int[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-a^2x^2+1}{\text{arctanh}\left(ax\right)^3}dx$$

input
```
int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

output
```
int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}\,(ax)^3}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}\,dx = -\int \frac{a^2x^2}{\operatorname{atanh}^3(ax)}\,dx - \int \left(-\frac{1}{\operatorname{atanh}^3(ax)}\right)dx$$

input
```
integrate((-a**2*x**2+1)/atanh(a*x)**3,x)
```

output
```
-Integral(a**2*x**2/atanh(a*x)**3, x) - Integral(-1/atanh(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 9.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}\,(ax)^3}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-2*(a^4*x^4 - 2*a^2*x^2 - 2*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) + 2*(
a^5*x^5 - 2*a^3*x^3 + a*x)*log(-a*x + 1) + 1)/(a*log(a*x + 1)^2 - 2*a*log(
a*x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2) + integrate(-4*(5*a^4*x^4 - 6*
a^2*x^2 + 1)/(log(a*x + 1) - log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{a^2x^2-1}{\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}\,dx = -\int \frac{a^2\,x^2-1}{\operatorname{atanh}\left(a\,x\right)^3}\,dx$$

input
```
int(-(a^2*x^2 - 1)/atanh(a*x)^3,x)
```

output
```
-int((a^2*x^2 - 1)/atanh(a*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1-a^2x^2}{\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{x^2}{atanh\,(ax)^3}dx\right)a^2 + \int \frac{1}{atanh\,(ax)^3}dx$$

input
```
int((-a^2*x^2+1)/atanh(a*x)^3,x)
```

output
```
- int(x**2/atanh(a*x)**3,x)*a**2 + int(1/atanh(a*x)**3,x)
```

3.192 $\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{4x^2}{315a^3} + \frac{2x^4}{315a} - \frac{11ax^6}{378} + \frac{a^3x^8}{72} + \frac{1}{5}x^5\operatorname{arctanh}(ax) - \frac{2}{7}a^2x^7\operatorname{arctanh}(ax) + \frac{1}{9}a^4x^9\operatorname{arctanh}(ax) + \frac{4\log(1-a^2x^2)}{315a^5}$$

output
```
4/315*x^2/a^3+2/315*x^4/a-11/378*a*x^6+1/72*a^3*x^8+1/5*x^5*arctanh(a*x)-2
/7*a^2*x^7*arctanh(a*x)+1/9*a^4*x^9*arctanh(a*x)+4/315*ln(-a^2*x^2+1)/a^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{4x^2}{315a^3} + \frac{2x^4}{315a} - \frac{11ax^6}{378} + \frac{a^3x^8}{72} + \frac{1}{5}x^5\operatorname{arctanh}(ax) - \frac{2}{7}a^2x^7\operatorname{arctanh}(ax) + \frac{1}{9}a^4x^9\operatorname{arctanh}(ax) + \frac{4\log(1-a^2x^2)}{315a^5}$$

input
```
Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5
*ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 +
 (4*Log[1 - a^2*x^2])/(315*a^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(1-a^2x^2\right)^2 \text{arctanh}(ax) \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4x^8\text{arctanh}(ax)-2a^2x^6\text{arctanh}(ax)+x^4\text{arctanh}(ax)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}a^4x^9\text{arctanh}(ax)+\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}-\frac{2}{7}a^2x^7\text{arctanh}(ax)+\frac{4\log\left(1-a^2x^2\right)}{315a^5}+\frac{1}{5}x^5\text{arctanh}(ax)-\frac{11ax^6}{378}+\frac{2x^4}{315a}$$

input
```
Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5
*ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 +
 (4*Log[1 - a^2*x^2])/(315*a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^4x^9\operatorname{arctanh}(ax)}{9}-\frac{2a^2x^7\operatorname{arctanh}(ax)}{7}+\frac{x^5\operatorname{arctanh}(ax)}{5}-\frac{a\left(-\frac{\frac{35}{4}a^6x^8-\frac{55}{3}a^4x^6+4a^2x^4+8x^2}{2a^4}-\frac{4\ln\left(a^2x^2-1\right)}{a^6}\right)}{315}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^9x^9}{9}-\frac{2\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^8x^8}{72}-\frac{11a^6x^6}{378}+\frac{2a^4x^4}{315}+\frac{4a^2x^2}{315}+\frac{4\ln(ax-1)}{315}+\frac{4\ln(ax+1)}{315}}{a^5}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^9x^9}{9}-\frac{2\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{\operatorname{arctanh}(ax)a^5x^5}{5}+\frac{a^8x^8}{72}-\frac{11a^6x^6}{378}+\frac{2a^4x^4}{315}+\frac{4a^2x^2}{315}+\frac{4\ln(ax-1)}{315}+\frac{4\ln(ax+1)}{315}}{a^5}$
parallelrisch	$-\frac{-840\operatorname{arctanh}(ax)a^9x^9-105a^8x^8+2160\operatorname{arctanh}(ax)a^7x^7+220a^6x^6-1512\operatorname{arctanh}(ax)a^5x^5-48a^4x^4-96-96a^2x^2}{7560a^5}$
risch	$\left(\frac{1}{18}a^4x^9-\frac{1}{7}a^2x^7+\frac{1}{10}x^5\right)\ln(ax+1)-\frac{a^4x^9\ln(-ax+1)}{18}+\frac{a^3x^8}{72}+\frac{a^2x^7\ln(-ax+1)}{7}-\frac{11a\,x^6}{378}-$
meijerg	$-\frac{-\frac{x^2a^2\left(15a^6x^6+20a^4x^4+30a^2x^2+60\right)}{270}+\frac{2x^{10}a^{10}\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{9\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{9}}{4a^5}-\frac{x^2a^2\left(4a^4x^4+6a^2\right.}{42}$

input
```
int(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*a^4*x^9*arctanh(a*x)-2/7*a^2*x^7*arctanh(a*x)+1/5*x^5*arctanh(a*x)-1/3
15*a*(-1/2/a^4*(35/4*a^6*x^8-55/3*a^4*x^6+4*a^2*x^4+8*x^2)-4/a^6*ln(a^2*x^
2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{105\,a^8x^8 - 220\,a^6x^6 + 48\,a^4x^4 + 96\,a^2x^2 + 12\,(35\,a^9x^9 - 90\,a^7x^7 + 63\,a^5x^5)\log\left(-\frac{ax+1}{ax-1}\right) + 96\,\log\left(a^2x^2 - \right.}{7560\,a^5}$$

input
```
integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/7560*(105*a^8*x^8 - 220*a^6*x^6 + 48*a^4*x^4 + 96*a^2*x^2 + 12*(35*a^9*x
^9 - 90*a^7*x^7 + 63*a^5*x^5)*log(-(a*x + 1)/(a*x - 1)) + 96*log(a^2*x^2 -
 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int x^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \begin{cases} \frac{a^4x^9\operatorname{atanh}(ax)}{9} + \frac{a^3x^8}{72} - \frac{2a^2x^7\operatorname{atanh}(ax)}{7} - \frac{11ax^6}{378} + \frac{x^5\operatorname{atanh}(ax)}{5} + \frac{2x^4}{315a} + \frac{4x^2}{315a^3} + \frac{8\log\left(x-\frac{1}{a}\right)}{315a^5} + \frac{8\operatorname{atanh}(ax)}{315a^5} & \text{for } a \\ 0 & \text{othe} \end{cases}$$

input
```
integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x),x)
```

output
```
Piecewise((a**4*x**9*atanh(a*x)/9 + a**3*x**8/72 - 2*a**2*x**7*atanh(a*x)/
7 - 11*a*x**6/378 + x**5*atanh(a*x)/5 + 2*x**4/(315*a) + 4*x**2/(315*a**3)
 + 8*log(x - 1/a)/(315*a**5) + 8*atanh(a*x)/(315*a**5), Ne(a, 0)), (0, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{1}{7560}\,a\left(\frac{105\,a^6x^8 - 220\,a^4x^6 + 48\,a^2x^4 + 96\,x^2}{a^4} + \frac{96\,\log(ax+1)}{a^6} + \frac{96\,\log(ax-1)}{a^6}\right) + \frac{1}{315}\left(35\,a^4x^9 - 90\,a^2x^7 + 63\,x^5\right)\operatorname{artanh}(ax)$$

input
```
integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/7560*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x^4 + 96*x^2)/a^4 + 96*log(a
*x + 1)/a^6 + 96*log(a*x - 1)/a^6) + 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x
^5)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(80) = 160.

Time = 0.13 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.99

$$\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{4}{945}\,a\left(\frac{6\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6} - \frac{6\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^6} - \frac{\frac{6\,(ax+1)^7}{(ax-1)^7} - \frac{45\,(ax+1)^6}{(ax-1)^6} - \frac{274\,(ax+1)^5}{(ax-1)^5} - \frac{214\,(ax+1)^4}{(ax-1)^4} - \frac{274\,(ax+}{(ax-1)}}{a^6\left(\frac{ax+1}{ax-1}-1\right)^8}\right.$$

input
```
integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

output
```
4/945*a*(6*log(abs(-a*x - 1)/abs(a*x - 1))/a^6 - 6*log(abs(-(a*x + 1)/(a*x
 - 1) + 1))/a^6 - (6*(a*x + 1)^7/(a*x - 1)^7 - 45*(a*x + 1)^6/(a*x - 1)^6
- 274*(a*x + 1)^5/(a*x - 1)^5 - 214*(a*x + 1)^4/(a*x - 1)^4 - 274*(a*x + 1
)^3/(a*x - 1)^3 - 45*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1))/(a^6
*((a*x + 1)/(a*x - 1) - 1)^8) + 6*(210*(a*x + 1)^6/(a*x - 1)^6 + 315*(a*x
+ 1)^5/(a*x - 1)^5 + 441*(a*x + 1)^4/(a*x - 1)^4 + 126*(a*x + 1)^3/(a*x -
1)^3 + 36*(a*x + 1)^2/(a*x - 1)^2 - 9*(a*x + 1)/(a*x - 1) + 1)*log(-(a*((a
*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x
 - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1
)^9))
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int x^4(1-a^2x^2)^2\operatorname{arctanh}(ax)\,dx = \frac{4\ln(a^2x^2-1)}{315a^5}-\frac{11ax^6}{378}+\ln(ax+1)\left(\frac{a^4x^9}{18}-\frac{a^2x^7}{7}+\frac{x^5}{10}\right)-\ln(1-ax)\left(\frac{a^4x^9}{18}-\frac{a^2x^7}{7}+\frac{x^5}{10}\right)+\frac{2x^4}{315a}+\frac{4x^2}{315a^3}+\frac{a^3x^8}{72}$$

input
```
int(x^4*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

output
```
(4*log(a^2*x^2 - 1))/(315*a^5) - (11*a*x^6)/378 + log(a*x + 1)*(x^5/10 - (
a^2*x^7)/7 + (a^4*x^9)/18) - log(1 - a*x)*(x^5/10 - (a^2*x^7)/7 + (a^4*x^9
)/18) + (2*x^4)/(315*a) + (4*x^2)/(315*a^3) + (a^3*x^8)/72
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^4 \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\, dx$$
$$= \frac{840 atanh(ax)\, a^9x^9 - 2160 atanh(ax)\, a^7x^7 + 1512 atanh(ax)\, a^5x^5 + 192 atanh(ax) + 192 \log(a^2x - a) +}{7560a^5}$$

input
```
int(x^4*(-a^2*x^2+1)^2*atanh(a*x),x)
```

output
```
(840*atanh(a*x)*a**9*x**9 - 2160*atanh(a*x)*a**7*x**7 + 1512*atanh(a*x)*a*
*5*x**5 + 192*atanh(a*x) + 192*log(a**2*x - a) + 105*a**8*x**8 - 220*a**6*
x**6 + 48*a**4*x**4 + 96*a**2*x**2)/(7560*a**5)
```

3.193 $\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}-\frac{\operatorname{arctanh}(ax)}{24a^4}+\frac{1}{4}x^4\operatorname{arctanh}(ax)-\frac{1}{3}a^2x^6\operatorname{arctanh}(ax)+\frac{1}{8}a^4x^8\operatorname{arctanh}(ax)$$

output
```
1/24*x/a^3+1/72*x^3/a-1/24*a*x^5+1/56*a^3*x^7-1/24*arctanh(a*x)/a^4+1/4*x^
4*arctanh(a*x)-1/3*a^2*x^6*arctanh(a*x)+1/8*a^4*x^8*arctanh(a*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

$$\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{x}{24a^3}+\frac{x^3}{72a}-\frac{ax^5}{24}+\frac{a^3x^7}{56}+\frac{1}{4}x^4\operatorname{arctanh}(ax)-\frac{1}{3}a^2x^6\operatorname{arctanh}(ax)+\frac{1}{8}a^4x^8\operatorname{arctanh}(ax)+\frac{\log(1-ax)}{48a^4}-\frac{\log(1+ax)}{48a^4}$$

input
```
Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 + (x^4*ArcTanh[a*x])/4
 - (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])/8 + Log[1 - a*x]/(48*
a^4) - Log[1 + a*x]/(48*a^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(1-a^2x^2\right)^2 \text{arctanh}(ax) \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4x^7\text{arctanh}(ax)-2a^2x^5\text{arctanh}(ax)+x^3\text{arctanh}(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^4x^8\text{arctanh}(ax)-\frac{\text{arctanh}(ax)}{24a^4}+\frac{a^3x^7}{56}+\frac{x}{24a^3}-\frac{1}{3}a^2x^6\text{arctanh}(ax)+\frac{1}{4}x^4\text{arctanh}(ax)-\frac{ax^5}{24}+\frac{x^3}{72a}$$

input
```
Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 - ArcTanh[a*x]/(24*a^4
) + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x
])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

method	result
parallelrisch	$-\frac{-63\,\text{arctanh}(ax)a^8x^8-9a^7x^7+168\,\text{arctanh}(ax)a^6x^6+21a^5x^5-126a^4x^4\,\text{arctanh}(ax)-7a^3x^3-21ax+21\,\text{arctanh}(a}{504a^4}$
derivativedivides	$\frac{\frac{\text{arctanh}(ax)a^8x^8}{8}-\frac{\text{arctanh}(ax)a^6x^6}{3}+\frac{a^4x^4\,\text{arctanh}(ax)}{4}+\frac{a^7x^7}{56}-\frac{a^5x^5}{24}+\frac{a^3x^3}{72}+\frac{ax}{24}+\frac{\ln(ax-1)}{48}-\frac{\ln(ax+1)}{48}}{a^4}$
default	$\frac{\frac{\text{arctanh}(ax)a^8x^8}{8}-\frac{\text{arctanh}(ax)a^6x^6}{3}+\frac{a^4x^4\,\text{arctanh}(ax)}{4}+\frac{a^7x^7}{56}-\frac{a^5x^5}{24}+\frac{a^3x^3}{72}+\frac{ax}{24}+\frac{\ln(ax-1)}{48}-\frac{\ln(ax+1)}{48}}{a^4}$
parts	$\frac{a^4x^8\,\text{arctanh}(ax)}{8}-\frac{a^2x^6\,\text{arctanh}(ax)}{3}+\frac{x^4\,\text{arctanh}(ax)}{4}-\frac{a\left(-\frac{\frac{3}{7}a^6x^7-a^4x^5+\frac{1}{3}a^2x^3+x}{a^4}+\frac{\ln(ax+1)}{2a^5}-\frac{\ln(ax-1)}{2a^5}\right)}{24}$
risch	$\left(\frac{1}{16}a^4x^8-\frac{1}{6}a^2x^6+\frac{1}{8}x^4\right)\ln(ax+1)-\frac{a^4x^8\ln(-ax+1)}{16}+\frac{a^3x^7}{56}+\frac{a^2x^6\ln(-ax+1)}{6}-\frac{a\,x^5}{24}-\frac{x^4\,\text{l}}{}$
orering	$\frac{(63a^8x^8-171a^6x^6+119a^4x^4+63a^2x^2-42)\left(-a^2x^2+1\right)^2\text{arctanh}(ax)}{252a^4(ax+1)(ax-1)(a^2x^2-1)}-\frac{(9a^6x^6-21a^4x^4+7a^2x^2+21)\left(3x^2\left(-a^2x^2+\right.\right.}{}$
meijerg	$-\frac{i\left(\frac{ixa\left(45a^6x^6+63a^4x^4+105a^2x^2+315\right)}{630}+\frac{ixa\left(-9a^8x^8+9\right)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{36\sqrt{a^2x^2}}\right)}{4a^4}-\frac{i\left(-\frac{2ixa\left(21a^4x^4+35a^2\right.}{315}\right.}{}$

input
```
int(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/504*(-63*arctanh(a*x)*a^8*x^8-9*a^7*x^7+168*arctanh(a*x)*a^6*x^6+21*a^5
*x^5-126*a^4*x^4*arctanh(a*x)-7*a^3*x^3-21*a*x+21*arctanh(a*x))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int x^3 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{18\,a^7x^7 - 42\,a^5x^5 + 14\,a^3x^3 + 42\,ax + 21\,(3\,a^8x^8 - 8\,a^6x^6 + 6\,a^4x^4 - 1)\log\left(-\frac{ax+1}{ax-1}\right)}{1008\,a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/1008*(18*a^7*x^7 - 42*a^5*x^5 + 14*a^3*x^3 + 42*a*x + 21*(3*a^8*x^8 - 8*
a^6*x^6 + 6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int x^3 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \begin{cases} \frac{a^4x^8 \operatorname{atanh}(ax)}{8} + \frac{a^3x^7}{56} - \frac{a^2x^6 \operatorname{atanh}(ax)}{3} - \frac{ax^5}{24} + \frac{x^4 \operatorname{atanh}(ax)}{4} + \frac{x^3}{72a} + \frac{x}{24a^3} - \frac{\operatorname{atanh}(ax)}{24a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x),x)
```

output
```
Piecewise((a**4*x**8*atanh(a*x)/8 + a**3*x**7/56 - a**2*x**6*atanh(a*x)/3
- a*x**5/24 + x**4*atanh(a*x)/4 + x**3/(72*a) + x/(24*a**3) - atanh(a*x)/(
24*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int x^3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)\,dx$$
$$=\frac{1}{1008}\,a\left(\frac{2\,(9\,a^6x^7-21\,a^4x^5+7\,a^2x^3+21\,x)}{a^4}-\frac{21\,\log\left(ax+1\right)}{a^5}+\frac{21\,\log\left(ax-1\right)}{a^5}\right)$$
$$+\frac{1}{24}\left(3\,a^4x^8-8\,a^2x^6+6\,x^4\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/1008*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x +
 1)/a^5 + 21*log(a*x - 1)/a^5) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6*x^4)*arct
anh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(71) = 142.

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.76

$$\int x^3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)\,dx$$
$$=\frac{4}{63}\,a\left(\frac{\frac{28\,(ax+1)^4}{(ax-1)^4}-\frac{7\,(ax+1)^3}{(ax-1)^3}+\frac{21\,(ax+1)^2}{(ax-1)^2}-\frac{7\,(ax+1)}{ax-1}+1}{a^5\left(\frac{ax+1}{ax-1}-1\right)^7}+\frac{84\left(\frac{(ax+1)^5}{(ax-1)^5}+\frac{(ax+1)^4}{(ax-1)^4}+\frac{(ax+1)^3}{(ax-1)^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}}\right.}{a^5\left(\frac{ax+1}{ax-1}-1\right)^8}\right.$$

input
```
integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

output
```
4/63*a*((28*(a*x + 1)^4/(a*x - 1)^4 - 7*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x
+ 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x - 1) + 1)/(a^5*((a*x + 1)/(a*x - 1)
- 1)^7) + 84*((a*x + 1)^5/(a*x - 1)^5 + (a*x + 1)^4/(a*x - 1)^4 + (a*x + 1
)^3/(a*x - 1)^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1)
- a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(
a^5*((a*x + 1)/(a*x - 1) - 1)^8))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int x^3(1-a^2x^2)^2\operatorname{arctanh}(ax)\,dx = \frac{x}{24\,a^3}-\frac{a\,x^5}{24}+\ln\left(a\,x+1\right)\,\left(\frac{a^4\,x^8}{16}-\frac{a^2\,x^6}{6}+\frac{x^4}{8}\right) - \ln\left(1-a\,x\right)\,\left(\frac{a^4\,x^8}{16}-\frac{a^2\,x^6}{6}+\frac{x^4}{8}\right) + \frac{x^3}{72\,a}+\frac{a^3\,x^7}{56}+\frac{\mathrm{atan}(a\,x\,1\mathrm{i})\;1\mathrm{i}}{24\,a^4}$$

input
```
int(x^3*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

output
```
x/(24*a^3) - (a*x^5)/24 + (atan(a*x*1i)*1i)/(24*a^4) + log(a*x + 1)*(x^4/8
 - (a^2*x^6)/6 + (a^4*x^8)/16) - log(1 - a*x)*(x^4/8 - (a^2*x^6)/6 + (a^4*
x^8)/16) + x^3/(72*a) + (a^3*x^7)/56
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int x^3(1-a^2x^2)^2\operatorname{arctanh}(ax)\,dx = \frac{63atanh(ax)\,a^8x^8-168atanh(ax)\,a^6x^6+126atanh(ax)\,a^4x^4-21atanh(ax)+9a^7x^7-21a^5x^5+7a^3x^3}{504a^4}$$

input
```
int(x^3*(-a^2*x^2+1)^2*atanh(a*x),x)
```

output

```
(63*atanh(a*x)*a**8*x**8 - 168*atanh(a*x)*a**6*x**6 + 126*atanh(a*x)*a**4*
x**4 - 21*atanh(a*x) + 9*a**7*x**7 - 21*a**5*x**5 + 7*a**3*x**3 + 21*a*x)/
(504*a**4)
```

3.194 $\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{4x^2}{105a} - \frac{9ax^4}{140} + \frac{a^3x^6}{42} + \frac{1}{3}x^3\operatorname{arctanh}(ax) - \frac{2}{5}a^2x^5\operatorname{arctanh}(ax) + \frac{1}{7}a^4x^7\operatorname{arctanh}(ax) + \frac{4\log(1-a^2x^2)}{105a^3}$$

output
```
4/105*x^2/a-9/140*a*x^4+1/42*a^3*x^6+1/3*x^3*arctanh(a*x)-2/5*a^2*x^5*arct
anh(a*x)+1/7*a^4*x^7*arctanh(a*x)+4/105*ln(-a^2*x^2+1)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{4x^2}{105a} - \frac{9ax^4}{140} + \frac{a^3x^6}{42} + \frac{1}{3}x^3\operatorname{arctanh}(ax) - \frac{2}{5}a^2x^5\operatorname{arctanh}(ax) + \frac{1}{7}a^4x^7\operatorname{arctanh}(ax) + \frac{4\log(1-a^2x^2)}{105a^3}$$

input
```
Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2
*a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])
/(105*a^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(1-a^2x^2\right)^2 \text{arctanh}(ax) \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4x^6\text{arctanh}(ax)-2a^2x^4\text{arctanh}(ax)+x^2\text{arctanh}(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}a^4x^7\text{arctanh}(ax)+\frac{a^3x^6}{42}-\frac{2}{5}a^2x^5\text{arctanh}(ax)+\frac{4\log\left(1-a^2x^2\right)}{105a^3}+\frac{1}{3}x^3\text{arctanh}(ax)-\frac{9ax^4}{140}+\frac{4x^2}{105a}$$

input
```
Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2
*a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])
/(105*a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
parallelrisch	$-\frac{-60 \operatorname{arctanh}(ax) a^7 x^7-10 a^6 x^6+168 \operatorname{arctanh}(ax) a^5 x^5+27 a^4 x^4-140 a^3 x^3 \operatorname{arctanh}(ax)-16 a^2 x^2-32 \ln(ax-1)-32}{420 a^3}$
parts	$\frac{a^4 x^7 \operatorname{arctanh}(ax)}{7}-\frac{2 a^2 x^5 \operatorname{arctanh}(ax)}{5}+\frac{x^3 \operatorname{arctanh}(ax)}{3}-\frac{a\left(-\frac{5 a^4 x^6-\frac{27}{2} a^2 x^4+8 x^2}{2 a^2}-\frac{4 \ln\left(a^2 x^2-1\right)}{a^4}\right)}{105}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax) a^7 x^7}{7}-\frac{2 \operatorname{arctanh}(ax) a^5 x^5}{5}+\frac{a^3 x^3 \operatorname{arctanh}(ax)}{3}+\frac{a^6 x^6}{42}-\frac{9 a^4 x^4}{140}+\frac{4 a^2 x^2}{105}+\frac{4 \ln(ax-1)}{105}+\frac{4 \ln(ax+1)}{105}}{a^3}$
default	$\frac{\frac{\operatorname{arctanh}(ax) a^7 x^7}{7}-\frac{2 \operatorname{arctanh}(ax) a^5 x^5}{5}+\frac{a^3 x^3 \operatorname{arctanh}(ax)}{3}+\frac{a^6 x^6}{42}-\frac{9 a^4 x^4}{140}+\frac{4 a^2 x^2}{105}+\frac{4 \ln(ax-1)}{105}+\frac{4 \ln(ax+1)}{105}}{a^3}$
risch	$\left(\frac{1}{14} a^4 x^7-\frac{1}{5} a^2 x^5+\frac{1}{6} x^3\right) \ln(ax+1)-\frac{a^4 x^7 \ln(-ax+1)}{14}+\frac{a^3 x^6}{42}+\frac{a^2 x^5 \ln(-ax+1)}{5}-\frac{9 a x^4}{140}-\frac{x^3}{}$
meijerg	$\frac{\frac{x^2 a^2\left(4 a^4 x^4+6 a^2 x^2+12\right)}{42}-\frac{2 x^8 a^8\left(\ln\left(1-\sqrt{a^2 x^2}\right)-\ln\left(1+\sqrt{a^2 x^2}\right)\right)}{7 \sqrt{a^2 x^2}}+\frac{2 \ln\left(-a^2 x^2+1\right)}{7}}{4 a^3}+\frac{-\frac{a^2 x^2\left(3 a^2 x^2+6\right)}{15}+\frac{2 a^6 x^6\left(\ln\left(1-\sqrt{}\right.\right.}{}}{}$

input
```
int(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/420*(-60*arctanh(a*x)*a^7*x^7-10*a^6*x^6+168*arctanh(a*x)*a^5*x^5+27*a^
4*x^4-140*a^3*x^3*arctanh(a*x)-16*a^2*x^2-32*ln(a*x-1)-32*arctanh(a*x))/a^
3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int x^2 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{10\,a^6x^6 - 27\,a^4x^4 + 16\,a^2x^2 + 2\,(15\,a^7x^7 - 42\,a^5x^5 + 35\,a^3x^3)\log\left(-\frac{ax+1}{ax-1}\right) + 16\,\log\left(a^2x^2-1\right)}{420\,a^3}$$

input
```
integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/420*(10*a^6*x^6 - 27*a^4*x^4 + 16*a^2*x^2 + 2*(15*a^7*x^7 - 42*a^5*x^5 +
 35*a^3*x^3)*log(-(a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int x^2 (1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \begin{cases} \frac{a^4x^7\operatorname{atanh}(ax)}{7} + \frac{a^3x^6}{42} - \frac{2a^2x^5\operatorname{atanh}(ax)}{5} - \frac{9ax^4}{140} + \frac{x^3\operatorname{atanh}(ax)}{3} + \frac{4x^2}{105a} + \frac{8\log\left(x-\frac{1}{a}\right)}{105a^3} + \frac{8\operatorname{atanh}(ax)}{105a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x),x)
```

output
```
Piecewise((a**4*x**7*atanh(a*x)/7 + a**3*x**6/42 - 2*a**2*x**5*atanh(a*x)/
5 - 9*a*x**4/140 + x**3*atanh(a*x)/3 + 4*x**2/(105*a) + 8*log(x - 1/a)/(10
5*a**3) + 8*atanh(a*x)/(105*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int x^2(1-a^2x^2)^2\,\mathrm{arctanh}(ax)\,dx = \frac{1}{420}\,a\left(\frac{10\,a^4x^6-27\,a^2x^4+16\,x^2}{a^2}+\frac{16\,\log\left(ax+1\right)}{a^4}+\frac{16\,\log\left(ax-1\right)}{a^4}\right) + \frac{1}{105}\left(15\,a^4x^7-42\,a^2x^5+35\,x^3\right)\mathrm{artanh}\left(ax\right)$$

input
```
integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/420*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 + 16
*log(a*x - 1)/a^4) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^3)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(72) = 144.

Time = 0.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.71

$$\int x^2(1-a^2x^2)^2\,\mathrm{arctanh}(ax)\,dx = \frac{4}{105}\,a\left(\frac{2\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4}-\frac{2\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^4}-\frac{\frac{2\,(ax+1)^5}{(ax-1)^5}-\frac{11\,(ax+1)^4}{(ax-1)^4}-\frac{22\,(ax+1)^3}{(ax-1)^3}-\frac{11\,(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^6}\right.$$

input
```
integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

output
```
4/105*a*(2*log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - 2*log(abs(-(a*x + 1)/(a*x
 - 1) + 1))/a^4 - (2*(a*x + 1)^5/(a*x - 1)^5 - 11*(a*x + 1)^4/(a*x - 1)^4
- 22*(a*x + 1)^3/(a*x - 1)^3 - 11*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a
*x - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^6) + 2*(70*(a*x + 1)^4/(a*x - 1)^4
 + 35*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x + 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(
a*x - 1) + 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a
) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4
*((a*x + 1)/(a*x - 1) - 1)^7))
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{x^3\,\mathrm{atanh}(a\,x)}{3} - \frac{9\,a\,x^4}{140} + \frac{4\,\ln\left(a^2\,x^2-1\right)}{105\,a^3} + \frac{4\,x^2}{105\,a} + \frac{a^3\,x^6}{42} - \frac{2\,a^2\,x^5\,\mathrm{atanh}(a\,x)}{5} + \frac{a^4\,x^7\,\mathrm{atanh}(a\,x)}{7}$$

input
```
int(x^2*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

output
```
(x^3*atanh(a*x))/3 - (9*a*x^4)/140 + (4*log(a^2*x^2 - 1))/(105*a^3) + (4*x
^2)/(105*a) + (a^3*x^6)/42 - (2*a^2*x^5*atanh(a*x))/5 + (a^4*x^7*atanh(a*x
))/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{60atanh(ax)\,a^7x^7 - 168atanh(ax)\,a^5x^5 + 140atanh(ax)\,a^3x^3 + 32atanh(ax) + 32\log(a^2x-a) + 10a^6x^6}{420a^3}$$

input
```
int(x^2*(-a^2*x^2+1)^2*atanh(a*x),x)
```

output
```
(60*atanh(a*x)*a**7*x**7 - 168*atanh(a*x)*a**5*x**5 + 140*atanh(a*x)*a**3*
x**3 + 32*atanh(a*x) + 32*log(a**2*x - a) + 10*a**6*x**6 - 27*a**4*x**4 +
16*a**2*x**2)/(420*a**3)
```

3.195 $\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 50

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{x}{6a} - \frac{ax^3}{9} + \frac{a^3x^5}{30} - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}{6a^2}$$

output
```
1/6*x/a-1/9*a*x^3+1/30*a^3*x^5-1/6*(-a^2*x^2+1)^3*arctanh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.86

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{x}{6a} - \frac{ax^3}{9} + \frac{a^3x^5}{30} + \frac{1}{2}x^2\operatorname{arctanh}(ax) - \frac{1}{2}a^2x^4\operatorname{arctanh}(ax) + \frac{1}{6}a^4x^6\operatorname{arctanh}(ax) + \frac{\log(1-ax)}{12a^2} - \frac{\log(1+ax)}{12a^2}$$

input
```
Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTa
nh[a*x])/2 + (a^4*x^6*ArcTanh[a*x])/6 + Log[1 - a*x]/(12*a^2) - Log[1 + a*
x]/(12*a^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6556, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-a^2x^2\right)^2\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int \left(1-a^2x^2\right)^2dx}{6a}-\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{6a^2}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(a^4x^4-2a^2x^2+1\right)dx}{6a}-\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{6a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x}{6a}-\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{6a^2}$$

input
```
Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)/(6*a) - ((1 - a^2*x^2)^3*ArcTanh[a*x])/(
6*a^2)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^6x^6}{6}-\frac{a^4x^4\operatorname{arctanh}(ax)}{2}+\frac{a^2x^2\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{9}+\frac{ax}{6}}{a^2}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^6x^6}{6}-\frac{a^4x^4\operatorname{arctanh}(ax)}{2}+\frac{a^2x^2\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{9}+\frac{ax}{6}}{a^2}$
parallelrisch	$-\frac{-15\operatorname{arctanh}(ax)a^6x^6-3a^5x^5+45a^4x^4\operatorname{arctanh}(ax)+10a^3x^3-45a^2x^2\operatorname{arctanh}(ax)-15ax+15\operatorname{arctanh}(ax)}{90a^2}$
parts	$\frac{a^4\operatorname{arctanh}(ax)x^6}{6}-\frac{x^4a^2\operatorname{arctanh}(ax)}{2}+\frac{\operatorname{arctanh}(ax)x^2}{2}-\frac{\operatorname{arctanh}(ax)}{6a^2}-\frac{-\frac{1}{5}a^4x^5+\frac{2}{3}a^2x^3-x}{6a}$
risch	$\frac{(a^2x^2-1)^3\ln(ax+1)}{12a^2}-\frac{a^4x^6\ln(-ax+1)}{12}+\frac{a^3x^5}{30}+\frac{a^2x^4\ln(-ax+1)}{4}-\frac{a\,x^3}{9}-\frac{x^2\ln(-ax+1)}{4}+\frac{x}{6a}+\frac{\ln(-}{1}$
orering	$\frac{(15a^6x^6-49a^4x^4+65a^2x^2-15)(-a^2x^2+1)^2\operatorname{arctanh}(ax)}{45a^2(ax+1)(ax-1)(a^2x^2-1)}-\frac{(3a^4x^4-10a^2x^2+15)\left((-a^2x^2+1)^2\operatorname{arctanh}(ax)-4x^2(\right.}{90a^2(ax+1)(ax}$
meijerg	$\frac{i\left(-\frac{2ixa(21a^4x^4+35a^2x^2+105)}{315}-\frac{ixa(-7a^6x^6+7)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{21\sqrt{a^2x^2}}\right)}{4a^2}+\frac{i\left(\frac{ixa(5a^2x^2+15)}{15}+\frac{ixa(-5a^4x^4}{}\right.}{}$

input
```
int(x*(-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/6*arctanh(a*x)*a^6*x^6-1/2*a^4*x^4*arctanh(a*x)+1/2*a^2*x^2*arcta
nh(a*x)-1/6*arctanh(a*x)+1/30*a^5*x^5-1/9*a^3*x^3+1/6*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{6\,a^5x^5-20\,a^3x^3+30\,ax+15\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)}{180\,a^2}$$

input
```
integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/180*(6*a^5*x^5 - 20*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x
^2 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \begin{cases} \frac{a^4x^6\operatorname{atanh}(ax)}{6}+\frac{a^3x^5}{30}-\frac{a^2x^4\operatorname{atanh}(ax)}{2}-\frac{ax^3}{9}+\frac{x^2\operatorname{atanh}(ax)}{2}+\frac{x}{6a}-\frac{\operatorname{atanh}(ax)}{6a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*(-a**2*x**2+1)**2*atanh(a*x),x)
```

output
```
Piecewise((a**4*x**6*atanh(a*x)/6 + a**3*x**5/30 - a**2*x**4*atanh(a*x)/2
- a*x**3/9 + x**2*atanh(a*x)/2 + x/(6*a) - atanh(a*x)/(6*a**2), Ne(a, 0)),
 (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{(a^2x^2-1)^3 \operatorname{artanh}(ax)}{6a^2} + \frac{3a^4x^5-10a^2x^3+15x}{90a}$$

input
```
integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/6*(a^2*x^2 - 1)^3*arctanh(a*x)/a^2 + 1/90*(3*a^4*x^5 - 10*a^2*x^3 + 15*x
)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(41) = 82.

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.52

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx$$

$$= \frac{8}{45}a\left(\frac{\frac{10(ax+1)^2}{(ax-1)^2}-\frac{5(ax+1)}{ax-1}+1}{a^3\left(\frac{ax+1}{ax-1}-1\right)^5} + \frac{30(ax+1)^3\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)}{(ax-1)^3a^3\left(\frac{ax+1}{ax-1}-1\right)^6}\right)$$

input
```
integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

output
```
8/45*a*((10*(a*x + 1)^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)/(a^3*((a*
x + 1)/(a*x - 1) - 1)^5) + 30*(a*x + 1)^3*log(-(a*((a*x + 1)/(a*x - 1) + 1
)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)
*a/(a*x - 1) - a) - 1))/((a*x - 1)^3*a^3*((a*x + 1)/(a*x - 1) - 1)^6))
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{x^2\,\mathrm{atanh}(a\,x)}{2} - \frac{\frac{\mathrm{atanh}(a\,x)}{6} - \frac{a\,x}{6}}{a^2} - \frac{a\,x^3}{9} + \frac{a^3\,x^5}{30} - \frac{a^2\,x^4\,\mathrm{atanh}(a\,x)}{2} + \frac{a^4\,x^6\,\mathrm{atanh}(a\,x)}{6}$$

input
```
int(x*atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

output
```
(x^2*atanh(a*x))/2 - (atanh(a*x)/6 - (a*x)/6)/a^2 - (a*x^3)/9 + (a^3*x^5)/
30 - (a^2*x^4*atanh(a*x))/2 + (a^4*x^6*atanh(a*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{15atanh(ax)\,a^6x^6 - 45atanh(ax)\,a^4x^4 + 45atanh(ax)\,a^2x^2 - 15atanh(ax) + 3a^5x^5 - 10a^3x^3 + 15ax}{90a^2}$$

input
```
int(x*(-a^2*x^2+1)^2*atanh(a*x),x)
```

output
```
(15*atanh(a*x)*a**6*x**6 - 45*atanh(a*x)*a**4*x**4 + 45*atanh(a*x)*a**2*x*
*2 - 15*atanh(a*x) + 3*a**5*x**5 - 10*a**3*x**3 + 15*a*x)/(90*a**2)
```

3.196 $\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax) \, dx = \frac{2(1-a^2x^2)}{15a} + \frac{(1-a^2x^2)^2}{20a} + \frac{8}{15}x\operatorname{arctanh}(ax) + \frac{4}{15}x(1-a^2x^2)\operatorname{arctanh}(ax) + \frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax) + \frac{4\log(1-a^2x^2)}{15a}$$

output
```
2/15*(-a^2*x^2+1)/a+1/20*(-a^2*x^2+1)^2/a+8/15*x*arctanh(a*x)+4/15*x*(-a^2
*x^2+1)*arctanh(a*x)+1/5*x*(-a^2*x^2+1)^2*arctanh(a*x)+4/15*ln(-a^2*x^2+1)
/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax) \, dx = -\frac{7ax^2}{30} + \frac{a^3x^4}{20} + x\operatorname{arctanh}(ax) - \frac{2}{3}a^2x^3\operatorname{arctanh}(ax) + \frac{1}{5}a^4x^5\operatorname{arctanh}(ax) + \frac{4\log(1-a^2x^2)}{15a}$$

input
```
Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(-7*a*x^2)/30 + (a^3*x^4)/20 + x*ArcTanh[a*x] - (2*a^2*x^3*ArcTanh[a*x])/3
 + (a^4*x^5*ArcTanh[a*x])/5 + (4*Log[1 - a^2*x^2])/(15*a)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6504, 6504, 6436, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^2 \text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{ 6504}$$

$$\frac{4}{5}\int \left(1-a^2x^2\right)\text{arctanh}(ax)dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^2}{20a}$$

$$\Big\downarrow \text{ 6504}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \text{arctanh}(ax)dx+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^2}{20a}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^2}{20a}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)+\frac{\left(1-a^2x^2\right)^2}{20a}$$

input
```
Int[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

output
```
(1 - a^2*x^2)^2/(20*a) + (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 + (4*((1 - a^2
*x^2)/(6*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*x] + Log[
1 - a^2*x^2]/(2*a)))/3))/5
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

method	result
parts	$\frac{\operatorname{arctanh}(ax)a^4x^5}{5}-\frac{2x^3a^2\operatorname{arctanh}(ax)}{3}+x\operatorname{arctanh}(ax)-\frac{a\left(-\frac{3a^2x^4}{4}+\frac{7x^2}{2}-\frac{4\ln\left(a^2x^2-1\right)}{a^2}\right)}{15}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)a^5x^5}{5}-\frac{2a^3x^3\operatorname{arctanh}(ax)}{3}+ax\operatorname{arctanh}(ax)+\frac{a^4x^4}{20}-\frac{7a^2x^2}{30}+\frac{4\ln(ax-1)}{15}+\frac{4\ln(ax+1)}{15}}{a}$
default	$\frac{\frac{\operatorname{arctanh}(ax)a^5x^5}{5}-\frac{2a^3x^3\operatorname{arctanh}(ax)}{3}+ax\operatorname{arctanh}(ax)+\frac{a^4x^4}{20}-\frac{7a^2x^2}{30}+\frac{4\ln(ax-1)}{15}+\frac{4\ln(ax+1)}{15}}{a}$
parallelrisch	$-\frac{-12\operatorname{arctanh}(ax)a^5x^5-3a^4x^4+40a^3x^3\operatorname{arctanh}(ax)+14a^2x^2-60ax\operatorname{arctanh}(ax)-32\ln(ax-1)-32\operatorname{arctanh}(ax)}{60a}$
risch	$\left(\frac{1}{10}a^4x^5-\frac{1}{3}a^2x^3+\frac{1}{2}x\right)\ln(ax+1)-\frac{a^4x^5\ln(-ax+1)}{10}+\frac{a^3x^4}{20}+\frac{a^2x^3\ln(-ax+1)}{3}-\frac{7a\,x^2}{30}-\frac{x\ln}{}$
meijerg	$-\frac{\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{\sqrt{a^2x^2}}-2\ln\left(-a^2x^2+1\right)}{4a}-\frac{-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\frac{2a^6x^6\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{5\sqrt{a^2x^2}}}{4a}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*arctanh(a*x)*a^4*x^5-2/3*x^3*a^2*arctanh(a*x)+x*arctanh(a*x)-1/15*a*(-
3/4*a^2*x^4+7/2*x^2-4/a^2*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\,dx = \frac{3\,a^4x^4-14\,a^2x^2+2\left(3\,a^5x^5-10\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+16\,\log\left(a^2x^2-1\right)}{60\,a}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/60*(3*a^4*x^4 - 14*a^2*x^2 + 2*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a
*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \begin{cases} \frac{a^4x^5 \operatorname{atanh}(ax)}{5} + \frac{a^3x^4}{20} - \frac{2a^2x^3 \operatorname{atanh}(ax)}{3} - \frac{7ax^2}{30} + x \operatorname{atanh}(ax) + \frac{8\log\left(x-\frac{1}{a}\right)}{15a} + \frac{8 \operatorname{atanh}(ax)}{15a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x),x)
```

output
```
Piecewise((a**4*x**5*atanh(a*x)/5 + a**3*x**4/20 - 2*a**2*x**3*atanh(a*x)/
3 - 7*a*x**2/30 + x*atanh(a*x) + 8*log(x - 1/a)/(15*a) + 8*atanh(a*x)/(15*
a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{1}{60}\left(3\,a^2x^4 - 14\,x^2 + \frac{16\,\log(ax+1)}{a^2} + \frac{16\,\log(ax-1)}{a^2}\right)a$$
$$+ \frac{1}{15}\left(3\,a^4x^5 - 10\,a^2x^3 + 15\,x\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/60*(3*a^2*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a +
1/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.45

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\,dx$$

$$= \frac{4}{15}\,a\left(\frac{2\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2} - \frac{2\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2} - \frac{\frac{2\,(ax+1)^3}{(ax-1)^3} - \frac{7\,(ax+1)^2}{(ax-1)^2} + \frac{2\,(ax+1)}{ax-1}}{a^2\left(\frac{ax+1}{ax-1}-1\right)^4} + \frac{2\left(\frac{10\,(ax+1)^2}{(ax-1)^2} - \frac{5\,(ax+1)}{ax-1} + \right.}{a^2\left(\frac{ax-}{ax-}\right.}\right.$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

output
```
4/15*a*(2*log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - 2*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^2 - (2*(a*x + 1)^3/(a*x - 1)^3 - 7*(a*x + 1)^2/(a*x - 1)^2 +
2*(a*x + 1)/(a*x - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^4) + 2*(10*(a*x + 1)
^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)*log(-(a*((a*x + 1)/(a*x - 1) +
 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x +
1)*a/(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\,dx = x\,\mathrm{atanh}(a\,x) - \frac{7\,a\,x^2}{30} + \frac{4\,\ln\left(a^2\,x^2-1\right)}{15\,a} + \frac{a^3\,x^4}{20} - \frac{2\,a^2\,x^3\,\mathrm{atanh}(a\,x)}{3} + \frac{a^4\,x^5\,\mathrm{atanh}(a\,x)}{5}$$

input
```
int(atanh(a*x)*(a^2*x^2 - 1)^2,x)
```

output
```
x*atanh(a*x) - (7*a*x^2)/30 + (4*log(a^2*x^2 - 1))/(15*a) + (a^3*x^4)/20 -
 (2*a^2*x^3*atanh(a*x))/3 + (a^4*x^5*atanh(a*x))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{12atanh(ax)\,a^5x^5 - 40atanh(ax)\,a^3x^3 + 60atanh(ax)\,ax + 32atanh(ax) + 32\log(a^2x-a) + 3a^4x^4 - 14}{60a}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x),x)
```

output
```
(12*atanh(a*x)*a**5*x**5 - 40*atanh(a*x)*a**3*x**3 + 60*atanh(a*x)*a*x + 3
2*atanh(a*x) + 32*log(a**2*x - a) + 3*a**4*x**4 - 14*a**2*x**2)/(60*a)
```

3.197 $\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} dx = -\frac{3ax}{4} + \frac{a^3x^3}{12} + \frac{3}{4}\operatorname{arctanh}(ax) - a^2x^2\operatorname{arctanh}(ax) + \frac{1}{4}a^4x^4\operatorname{arctanh}(ax) - \frac{\operatorname{PolyLog}(2,-ax)}{2} + \frac{\operatorname{PolyLog}(2,ax)}{2}$$

output
```
-3/4*a*x+1/12*a^3*x^3+3/4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+1/4*a^4*x^4*ar
ctanh(a*x)-1/2*polylog(2,-a*x)+1/2*polylog(2,a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} dx = -\frac{3ax}{4} + \frac{a^3x^3}{12} - a^2x^2\operatorname{arctanh}(ax) + \frac{1}{4}a^4x^4\operatorname{arctanh}(ax) - \frac{3}{8}\log(1-ax) + \frac{3}{8}\log(1+ax) + \frac{1}{2}(-\operatorname{PolyLog}(2,-ax) + \operatorname{PolyLog}(2,ax))$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

output
```
(-3*a*x)/4 + (a^3*x^3)/12 - a^2*x^2*ArcTanh[a*x] + (a^4*x^4*ArcTanh[a*x])/
4 - (3*Log[1 - a*x])/8 + (3*Log[1 + a*x])/8 + (-PolyLog[2, -(a*x)] + PolyL
og[2, a*x])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4x^3\operatorname{arctanh}(ax) - 2a^2x\operatorname{arctanh}(ax) + \frac{\operatorname{arctanh}(ax)}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^4x^4\operatorname{arctanh}(ax) + \frac{a^3x^3}{12} - a^2x^2\operatorname{arctanh}(ax) + \frac{3}{4}\operatorname{arctanh}(ax) - \frac{\operatorname{PolyLog}(2,-ax)}{2} + \frac{\operatorname{PolyLog}(2,ax)}{2} - \frac{3ax}{4}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

output
```
(-3*a*x)/4 + (a^3*x^3)/12 + (3*ArcTanh[a*x])/4 - a^2*x^2*ArcTanh[a*x] + (a
^4*x^4*ArcTanh[a*x])/4 - PolyLog[2, -(a*x)]/2 + PolyLog[2, a*x]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{a^4x^4\operatorname{arctanh}(ax)}{4} - a^2x^2\operatorname{arctanh}(ax) + \operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(a}{}$
default	$\frac{a^4x^4\operatorname{arctanh}(ax)}{4} - a^2x^2\operatorname{arctanh}(ax) + \operatorname{arctanh}(ax)\ln(ax) - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(a}{}$
parts	$\frac{a^4x^4\operatorname{arctanh}(ax)}{4} - a^2x^2\operatorname{arctanh}(ax) + \operatorname{arctanh}(ax)\ln(x) - \frac{a\left(\frac{2\operatorname{dilog}(ax+1)}{a} + \frac{2\ln(x)\ln(ax+1)}{a} - \frac{2(}{}\right.}{}$
risch	$\frac{(ax+1)^4\ln(ax+1)}{8} + \frac{a^3x^3}{12} - \frac{3ax}{4} - \frac{(ax+1)^3\ln(ax+1)}{2} + \frac{(ax+1)^2\ln(ax+1)}{4} + \frac{(ax+1)\ln(ax+1)}{2} - \frac{\operatorname{dilog}(ax}{2}$
meijerg	$-\frac{i\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}} - \frac{2iax\operatorname{polylog}\left(2,-\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4} - \frac{i\left(\frac{ixa\left(5a^2x^2+15\right)}{15} + \frac{ixa\left(-5a^4x^4+5\right)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{}\right)\right.}{10\sqrt{a^2x^2}}\right.}{4}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*x^4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*dil
og(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)+1/12*a^3*x^3-3/4*a*x-3/8*ln
(a*x-1)+3/8*ln(a*x+1)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)}{x} \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}{x} \, dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)/x,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.51

$$\begin{aligned} &\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx \\ &= \frac{1}{24}\left(2\,a^2x^3 - 18\,x - \frac{12\,(\log(ax+1)\log(x) + \operatorname{Li}_2(-ax))}{a} + \frac{12\,(\log(-ax+1)\log(x) + \operatorname{Li}_2(ax))}{a} + \frac{9\log\right. \\ &\quad + \frac{1}{4}\left(a^4x^4 - 4\,a^2x^2 + 2\,\log\left(x^2\right)\right) \operatorname{artanh}(ax) \end{aligned}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="maxima")
```

output
```
1/24*(2*a^2*x^3 - 18*x - 12*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 12*(lo
g(-a*x + 1)*log(x) + dilog(a*x))/a + 9*log(a*x + 1)/a - 9*log(a*x - 1)/a)*
a + 1/4*(a^4*x^4 - 4*a^2*x^2 + 2*log(x^2))*arctanh(a*x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)}{x} \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x} \, dx$$

input
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x,x)
```

output
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x} \, dx = \frac{atanh(ax)\,a^4x^4}{4} - atanh(ax)\,a^2x^2 + \frac{3atanh(ax)}{4} + \int \frac{atanh(ax)}{x} dx + \frac{a^3x^3}{12} - \frac{3ax}{4}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)/x,x)
```

output
```
(3*atanh(a*x)*a**4*x**4 - 12*atanh(a*x)*a**2*x**2 + 9*atanh(a*x) + 12*int(
atanh(a*x)/x,x) + a**3*x**3 - 9*a*x)/12
```

3.198 $\int \frac{(1-a^2x^2)^2\text{arctanh}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{x^2}\,dx = \frac{a^3x^2}{6} - \frac{\text{arctanh}(ax)}{x} - 2a^2x\text{arctanh}(ax) + \frac{1}{3}a^4x^3\text{arctanh}(ax) + a\log(x) - \frac{4}{3}a\log\left(1-a^2x^2\right)$$

output
```
1/6*a^3*x^2-arctanh(a*x)/x-2*a^2*x*arctanh(a*x)+1/3*a^4*x^3*arctanh(a*x)+a
*ln(x)-4/3*a*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{x^2}\,dx = \frac{a^3x^2}{6} - \frac{\text{arctanh}(ax)}{x} - 2a^2x\text{arctanh}(ax) + \frac{1}{3}a^4x^3\text{arctanh}(ax) + a\log(x) - \frac{4}{3}a\log\left(1-a^2x^2\right)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

output
```
(a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x
])/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6574}$$

$$\int \left(a^4x^2\operatorname{arctanh}(ax) - 2a^2\operatorname{arctanh}(ax) + \frac{\operatorname{arctanh}(ax)}{x^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^4x^3\operatorname{arctanh}(ax) + \frac{a^3x^2}{6} - 2a^2x\operatorname{arctanh}(ax) - \frac{4}{3}a\log(1-a^2x^2) - \frac{\operatorname{arctanh}(ax)}{x} + a\log(x)$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

output
```
(a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x
])/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

method	result
derivativedivides	$a\left(\frac{a^3x^3\,\mathrm{arctanh}(ax)}{3}-2ax\,\mathrm{arctanh}\left(ax\right)-\frac{\mathrm{arctanh}(ax)}{ax}+\frac{a^2x^2}{6}+\ln\left(ax\right)-\frac{4\ln(ax-1)}{3}-\frac{4\ln(ax\cdot}{3}\right.$
default	$a\left(\frac{a^3x^3\,\mathrm{arctanh}(ax)}{3}-2ax\,\mathrm{arctanh}\left(ax\right)-\frac{\mathrm{arctanh}(ax)}{ax}+\frac{a^2x^2}{6}+\ln\left(ax\right)-\frac{4\ln(ax-1)}{3}-\frac{4\ln(ax\cdot}{3}\right.$
parts	$\frac{a^4x^3\,\mathrm{arctanh}(ax)}{3}-2a^2x\,\mathrm{arctanh}\left(ax\right)-\frac{\mathrm{arctanh}(ax)}{x}-\frac{a\left(-\frac{a^2x^2}{2}-3\ln(x)+4\ln(ax+1)+4\ln(ax-1)\right)}{3}$
parallelrisch	$\frac{2a^4x^4\,\mathrm{arctanh}(ax)+a^3x^3-12a^2x^2\,\mathrm{arctanh}(ax)+6a\ln(x)x-16\ln(ax-1)ax-16ax\,\mathrm{arctanh}(ax)-6\,\mathrm{arctanh}(ax)}{6x}$
risch	$\frac{(a^4x^4-6a^2x^2-3)\ln(ax+1)}{6x}+\frac{-x^4\ln(-ax+1)a^4+a^3x^3+6x^2\ln(-ax+1)a^2+6a\ln(x)x-8a\ln(a^2x^2-1)x+3\ln(-ax\cdot}{6x}$
meijerg	$\frac{a\left(4\ln(x)+4\ln(ia)+\frac{2\ln\left(1-\sqrt{a^2x^2}\right)-2\ln\left(1+\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln\left(-a^2x^2+1\right)\right)}{4}+\frac{a\left(\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a\cdot}\right.\right.}{3\sqrt{a^2x^2}}\right.}{4}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*a^3*x^3*arctanh(a*x)-2*a*x*arctanh(a*x)-arctanh(a*x)/a/x+1/6*a^2*x^
2+ln(a*x)-4/3*ln(a*x-1)-4/3*ln(a*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int\frac{(1-a^2x^2)^2\,\mathrm{arctanh}(ax)}{x^2}\,dx = \frac{a^3x^3-8\,ax\log\left(a^2x^2-1\right)+6\,ax\log\left(x\right)+\left(a^4x^4-6\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="fricas")
```

output
```
1/6*(a^3*x^3 - 8*a*x*log(a^2*x^2 - 1) + 6*a*x*log(x) + (a^4*x^4 - 6*a^2*x^
2 - 3)*log(-(a*x + 1)/(a*x - 1)))/x
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^2} \, dx = \begin{cases} \frac{a^4x^3 \operatorname{atanh}(ax)}{3} + \frac{a^3x^2}{6} - 2a^2x \operatorname{atanh}(ax) + a\log(x) - \frac{8a\log\left(x-\frac{1}{a}\right)}{3} - \frac{8a\operatorname{atanh}(ax)}{3} - \frac{\operatorname{atanh}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)/x**2,x)
```

output
```
Piecewise((a**4*x**3*atanh(a*x)/3 + a**3*x**2/6 - 2*a**2*x*atanh(a*x) + a*
log(x) - 8*a*log(x - 1/a)/3 - 8*a*atanh(a*x)/3 - atanh(a*x)/x, Ne(a, 0)),
(0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^2} \, dx = \frac{1}{6}\left(a^2x^2 - 8\log(ax+1) - 8\log(ax-1) + 6\log(x)\right)a + \frac{1}{3}\left(a^4x^3 - 6a^2x - \frac{3}{x}\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="maxima")
```

output
```
1/6*(a^2*x^2 - 8*log(a*x + 1) - 8*log(a*x - 1) + 6*log(x))*a + 1/3*(a^4*x^
3 - 6*a^2*x - 3/x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.89

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^2}\,dx$$
$$=\frac{1}{3}\left(\left(\frac{3}{\frac{ax+1}{ax-1}+1}-\frac{\frac{3(ax+1)^2}{(ax-1)^2}-\frac{12(ax+1)}{ax-1}+5}{\left(\frac{ax+1}{ax-1}-1\right)^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)+\frac{2(ax+1)}{(ax-1)\left(\frac{ax+1}{ax-1}-1\right)^2}-8\log\left(\frac{|-\right.$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="giac")
```

output
```
1/3*((3/((a*x + 1)/(a*x - 1) + 1) - (3*(a*x + 1)^2/(a*x - 1)^2 - 12*(a*x +
 1)/(a*x - 1) + 5)/((a*x + 1)/(a*x - 1) - 1)^3)*log(-(a*((a*x + 1)/(a*x -
1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*
x + 1)*a/(a*x - 1) - a) - 1)) + 2*(a*x + 1)/((a*x - 1)*((a*x + 1)/(a*x - 1
) - 1)^2) - 8*log(abs(-a*x - 1)/abs(a*x - 1)) + 5*log(abs(-(a*x + 1)/(a*x
- 1) + 1)) + 3*log(abs(-(a*x + 1)/(a*x - 1) - 1)))*a
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^2}\,dx = a\ln(x)-\frac{4a\ln(a^2x^2-1)}{3}-\frac{\operatorname{atanh}(ax)}{x}+\frac{a^3x^2}{6}-2a^2x\operatorname{atanh}(ax)+\frac{a^4x^3\operatorname{atanh}(ax)}{3}$$

input
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^2,x)
```

output
```
a*log(x) - (4*a*log(a^2*x^2 - 1))/3 - atanh(a*x)/x + (a^3*x^2)/6 - 2*a^2*x
*atanh(a*x) + (a^4*x^3*atanh(a*x))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^2}\,dx$$
$$= \frac{2atanh(ax)\,a^4x^4 - 12atanh(ax)\,a^2x^2 - 16atanh(ax)\,ax - 6atanh(ax) - 16\log(a^2x-a)\,ax + 6\log(x)\,a}{6x}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)/x^2,x)
```

output
```
(2*atanh(a*x)*a**4*x**4 - 12*atanh(a*x)*a**2*x**2 - 16*atanh(a*x)*a*x - 6*
atanh(a*x) - 16*log(a**2*x - a)*a*x + 6*log(x)*a*x + a**3*x**3)/(6*x)
```

3.199 $\int \frac{(1-a^2x^2)^2\text{arctanh}(ax)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{x^3}\,dx = -\frac{a}{2x}+\frac{a^3x}{2}-\frac{\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{2}a^4x^2\operatorname{arctanh}(ax) + a^2\operatorname{PolyLog}(2,-ax)-a^2\operatorname{PolyLog}(2,ax)$$

output
```
-1/2*a/x+1/2*a^3*x-1/2*arctanh(a*x)/x^2+1/2*a^4*x^2*arctanh(a*x)+a^2*polyl
og(2,-a*x)-a^2*polylog(2,a*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{x^3}\,dx = -\frac{a}{2x}+\frac{a^3x}{2}-\frac{\operatorname{arctanh}(ax)}{2x^2}+\frac{1}{2}a^4x^2\operatorname{arctanh}(ax) - a^2(-\operatorname{PolyLog}(2,-ax)+\operatorname{PolyLog}(2,ax))$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

output
```
-1/2*a/x + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 - a
^2*(-PolyLog[2, -(a*x)] + PolyLog[2, a*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^3} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4 x \operatorname{arctanh}(ax) - \frac{2a^2 \operatorname{arctanh}(ax)}{x} + \frac{\operatorname{arctanh}(ax)}{x^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4x^2 \operatorname{arctanh}(ax) + \frac{a^3x}{2} + a^2 \operatorname{PolyLog}(2,-ax) - a^2 \operatorname{PolyLog}(2,ax) - \frac{\operatorname{arctanh}(ax)}{2x^2} - \frac{a}{2x}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

output
```
-1/2*a/x + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 + a
^2*PolyLog[2, -(a*x)] - a^2*PolyLog[2, a*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

method	result
derivativedivides	$a^{2}\Big(\frac{a^{2}x^{2}\operatorname{arctanh}(ax)}{2}-2\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2a^{2}x^{2}}+\operatorname{dilog}(ax)+\operatorname{dilog}(ax+1)+$
default	$a^{2}\Big(\frac{a^{2}x^{2}\operatorname{arctanh}(ax)}{2}-2\operatorname{arctanh}(ax)\ln(ax)-\frac{\operatorname{arctanh}(ax)}{2a^{2}x^{2}}+\operatorname{dilog}(ax)+\operatorname{dilog}(ax+1)+$
risch	$\frac{a^{4}\ln(ax+1)x^{2}}{4}+\frac{a^{3}x}{2}+a^{2}\operatorname{dilog}(ax+1)-\frac{a^{2}\ln(ax)}{4}-\frac{a}{2x}-\frac{\ln(ax+1)}{4x^{2}}-\frac{a^{4}\ln(-ax+1)x^{2}}{4}-a^{2}\,\mathrm{di}$
parts	$\frac{a^{4}x^{2}\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)}{2x^{2}}-2\operatorname{arctanh}(ax)a^{2}\ln(x)-\frac{a\Big(-a^{2}x+\frac{1}{x}+4a^{2}\Big(-\frac{\operatorname{dilog}(ax+1)}{2a}-\frac{\ln(x)\ln(}{2a}}{}$
meijerg	$\frac{ia^{2}\left(\frac{2i}{xa}+\frac{2i(-ax+1)(ax+1)\operatorname{arctanh}(ax)}{x^{2}a^{2}}\right)}{4}+\frac{ia^{2}(-2ixa+2i(-ax+1)(ax+1)\operatorname{arctanh}(ax))}{4}+\frac{ia^{2}\Bigg(\frac{2iax\operatorname{polylog}(2,\sqrt{a^{2}x}}{\sqrt{a^{2}x^{2}}}}{}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*a^2*x^2*arctanh(a*x)-2*arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/a^2/
x^2+dilog(a*x)+dilog(a*x+1)+ln(a*x)*ln(a*x+1)+1/2*a*x-1/2/a/x)
```

Fricas [F]

$$\int \frac{(1-a^{2}x^{2})^{2}\operatorname{arctanh}(ax)}{x^{3}}\,dx=\int \frac{(a^{2}x^{2}-1)^{2}\operatorname{artanh}(ax)}{x^{3}}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^3}\,dx = \int \frac{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}(ax)}{x^3}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)/x**3,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^3}\,dx$$
$$= \frac{1}{2}\left(2\left(\log(ax+1)\log(x) + \operatorname{Li}_2(-ax)\right)a - 2\left(\log(-ax+1)\log(x) + \operatorname{Li}_2(ax)\right)a + \frac{a^2x^2-1}{x}\right)a + \frac{1}{2}\left(a^4x^2 - 2\,a^2\log\left(x^2\right) - \frac{1}{x^2}\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + d
ilog(a*x))*a + (a^2*x^2 - 1)/x)*a + 1/2*(a^4*x^2 - 2*a^2*log(x^2) - 1/x^2)
*arctanh(a*x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^3}\,dx=\int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^3}\,dx=\int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x^3}\,dx$$

input
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^3,x)
```

output
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^3}\,dx =\frac{atanh(ax)\,a^4x^4-atanh(ax)-4\left(\int \frac{atanh(ax)}{x}dx\right)a^2x^2+a^3x^3-ax}{2x^2}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)/x^3,x)
```

output
```
(atanh(a*x)*a**4*x**4 - atanh(a*x) - 4*int(atanh(a*x)/x,x)*a**2*x**2 + a**
3*x**3 - a*x)/(2*x**2)
```

3.200 $\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)}{x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)}{x^4} \, dx = -\frac{a}{6x^2} - \frac{\text{arctanh}(ax)}{3x^3} + \frac{2a^2\text{arctanh}(ax)}{x} + a^4x\text{arctanh}(ax) - \frac{5}{3}a^3\log(x) + \frac{4}{3}a^3\log\left(1-a^2x^2\right)$$

output
```
-1/6*a/x^2-1/3*arctanh(a*x)/x^3+2*a^2*arctanh(a*x)/x+a^4*x*arctanh(a*x)-5/
3*a^3*ln(x)+4/3*a^3*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)}{x^4} \, dx = -\frac{a}{6x^2} - \frac{\text{arctanh}(ax)}{3x^3} + \frac{2a^2\text{arctanh}(ax)}{x} + a^4x\text{arctanh}(ax) - \frac{5}{3}a^3\log(x) + \frac{4}{3}a^3\log\left(1-a^2x^2\right)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

output
```
-1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh
[a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4\operatorname{arctanh}(ax) - \frac{2a^2\operatorname{arctanh}(ax)}{x^2} + \frac{\operatorname{arctanh}(ax)}{x^4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^4x\operatorname{arctanh}(ax) - \frac{5}{3}a^3\log(x) + \frac{2a^2\operatorname{arctanh}(ax)}{x} + \frac{4}{3}a^3\log\left(1-a^2x^2\right) - \frac{\operatorname{arctanh}(ax)}{3x^3} - \frac{a}{6x^2}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

output
```
-1/6*a/x^2 - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh
[a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

method	result
derivativedivides	$a^{3}\left(ax\ \operatorname{arctanh}\left(ax\right)+\frac{2\ \operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{3a^{3}x^{3}}-\frac{1}{6a^{2}x^{2}}-\frac{5\ln(ax)}{3}+\frac{4\ln(ax-1)}{3}+\frac{4\ln(ax+1)}{3}\cdots\right.$
default	$a^{3}\left(ax\ \operatorname{arctanh}\left(ax\right)+\frac{2\ \operatorname{arctanh}(ax)}{ax}-\frac{\operatorname{arctanh}(ax)}{3a^{3}x^{3}}-\frac{1}{6a^{2}x^{2}}-\frac{5\ln(ax)}{3}+\frac{4\ln(ax-1)}{3}+\frac{4\ln(ax+1)}{3}\cdots\right.$
parts	$a^{4}x\ \operatorname{arctanh}\left(ax\right)-\frac{\operatorname{arctanh}(ax)}{3x^{3}}+\frac{2a^{2}\operatorname{arctanh}(ax)}{x}-\frac{a\left(\frac{1}{2x^{2}}+5a^{2}\ln(x)-4a^{2}\ln(ax+1)-4a^{2}\ln(ax-1)\right)}{3}$
parallelrisch	$-\frac{-6a^{4}x^{4}\operatorname{arctanh}(ax)+10\ln(x)a^{3}x^{3}-16\ln(ax-1)x^{3}a^{3}-16a^{3}x^{3}\operatorname{arctanh}(ax)+a^{3}x^{3}-12a^{2}x^{2}\operatorname{arctanh}(ax)+ax+2\ \mathrm{ar}\cdots}{6x^{3}}$
risch	$\frac{\left(3a^{4}x^{4}+6a^{2}x^{2}-1\right)\ln(ax+1)}{6x^{3}}-\frac{3x^{4}\ln(-ax+1)a^{4}+10\ln(x)a^{3}x^{3}-8\ln\left(-a^{2}x^{2}+1\right)a^{3}x^{3}+6x^{2}\ln(-ax+1)a^{2}+ax-\ln(\cdots}{6x^{3}}$
meijerg	$-\frac{a^{3}\left(\frac{2}{a^{2}x^{2}}+\frac{4}{9}-\frac{4\ln(x)}{3}-\frac{4\ln(ia)}{3}-\frac{2\left(10a^{2}x^{2}+30\right)}{45a^{2}x^{2}}-\frac{2\left(\ln\left(1-\sqrt{a^{2}x^{2}}\right)-\ln\left(1+\sqrt{a^{2}x^{2}}\right)\right)}{3a^{2}x^{2}\sqrt{a^{2}x^{2}}}+\frac{2\ln\left(-a^{2}x^{2}+1\right)}{3}\right)}{4}-\frac{a^{3}\left(\frac{2a^{2}x^{2}\left(\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(a*x*arctanh(a*x)+2*arctanh(a*x)/a/x-1/3*arctanh(a*x)/a^3/x^3-1/6/a^2/
x^2-5/3*ln(a*x)+4/3*ln(a*x-1)+4/3*ln(a*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4} \, dx = \frac{8 a^3x^3 \log\left(a^2x^2-1\right) - 10 a^3x^3 \log\left(x\right) - ax + \left(3 a^4x^4 + 6 a^2x^2 - 1\right) \log\left(-\frac{ax+1}{ax-1}\right)}{6 x^3}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="fricas")
```

output
```
1/6*(8*a^3*x^3*log(a^2*x^2 - 1) - 10*a^3*x^3*log(x) - a*x + (3*a^4*x^4 + 6
*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1)))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4} \, dx = \begin{cases} a^4x \operatorname{atanh}(ax) - \frac{5a^3 \log(x)}{3} + \frac{8a^3 \log\left(x - \frac{1}{a}\right)}{3} + \frac{8a^3 \operatorname{atanh}(ax)}{3} + \frac{2a^2 \operatorname{atanh}(ax)}{x} - \frac{a}{6x^2} - \frac{\operatorname{atanh}(ax)}{3x^3} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)/x**4,x)
```

output
```
Piecewise((a**4*x*atanh(a*x) - 5*a**3*log(x)/3 + 8*a**3*log(x - 1/a)/3 + 8
*a**3*atanh(a*x)/3 + 2*a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3
), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4}\,dx = \frac{1}{6}\left(8\,a^2\log\left(ax+1\right)+8\,a^2\log\left(ax-1\right)-10\,a^2\log\left(x\right)-\frac{1}{x^2}\right)a + \frac{1}{3}\left(3\,a^4x+\frac{6\,a^2x^2-1}{x^3}\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="maxima")
```

output
```
1/6*(8*a^2*log(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a +
1/3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(60) = 120.

Time = 0.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.03

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4}\,dx = \frac{1}{3}\left(8\,a^2\log\left(\frac{|-ax-1|}{|ax-1|}\right)-3\,a^2\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)-5\,a^2\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\left(\frac{3\,a^2}{\frac{ax+1}{ax-1}-1}-\cdots\right.\right.$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="giac")
```

output
```
1/3*(8*a^2*log(abs(-a*x - 1)/abs(a*x - 1)) - 3*a^2*log(abs(-(a*x + 1)/(a*x
 - 1) + 1)) - 5*a^2*log(abs(-(a*x + 1)/(a*x - 1) - 1)) + (3*a^2/((a*x + 1)
/(a*x - 1) - 1) - (3*(a*x + 1)^2*a^2/(a*x - 1)^2 + 12*(a*x + 1)*a^2/(a*x -
 1) + 5*a^2)/((a*x + 1)/(a*x - 1) + 1)^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1
)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)
*a/(a*x - 1) - a) - 1)) + 2*(a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1)
+ 1)^2))*a
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4}\,dx = \frac{4\,a^3\,\ln\left(a^2\,x^2-1\right)}{3}-\frac{a}{6\,x^2}-\frac{\operatorname{atanh}(a\,x)}{3\,x^3}-\frac{5\,a^3\,\ln\left(x\right)}{3}+a^4\,x\,\operatorname{atanh}(a\,x)+\frac{2\,a^2\,\operatorname{atanh}(a\,x)}{x}$$

input
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^4,x)
```

output
```
(4*a^3*log(a^2*x^2 - 1))/3 - a/(6*x^2) - atanh(a*x)/(3*x^3) - (5*a^3*log(x
))/3 + a^4*x*atanh(a*x) + (2*a^2*atanh(a*x))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^4}\,dx = \frac{6atanh(ax)\,a^4x^4+16atanh(ax)\,a^3x^3+12atanh(ax)\,a^2x^2-2atanh(ax)+16\log(a^2x-a)\,a^3x^3-10\log}{6x^3}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)/x^4,x)
```

output
```
(6*atanh(a*x)*a**4*x**4 + 16*atanh(a*x)*a**3*x**3 + 12*atanh(a*x)*a**2*x**
2 - 2*atanh(a*x) + 16*log(a**2*x - a)*a**3*x**3 - 10*log(x)*a**3*x**3 - a*
x)/(6*x**3)
```

3.201 $\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)}{x^5} dx = -\frac{a}{12x^3} + \frac{3a^3}{4x} - \frac{3}{4}a^4\text{arctanh}(ax) - \frac{\text{arctanh}(ax)}{4x^4} + \frac{a^2\text{arctanh}(ax)}{x^2} - \frac{1}{2}a^4 \text{PolyLog}(2,-ax) + \frac{1}{2}a^4 \text{PolyLog}(2,ax)$$

output
```
-1/12*a/x^3+3/4*a^3/x-3/4*a^4*arctanh(a*x)-1/4*arctanh(a*x)/x^4+a^2*arctan
h(a*x)/x^2-1/2*a^4*polylog(2,-a*x)+1/2*a^4*polylog(2,a*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)}{x^5} dx = -\frac{a}{12x^3} + \frac{3a^3}{4x} - \frac{\text{arctanh}(ax)}{4x^4} + \frac{a^2\text{arctanh}(ax)}{x^2} + \frac{3}{8}a^4 \log(1-ax) - \frac{3}{8}a^4 \log(1+ax) + \frac{1}{2}a^4(-\text{PolyLog}(2,-ax) + \text{PolyLog}(2,ax))$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

output
```
-1/12*a/x^3 + (3*a^3)/(4*x) - ArcTanh[a*x]/(4*x^4) + (a^2*ArcTanh[a*x])/x^
2 + (3*a^4*Log[1 - a*x])/8 - (3*a^4*Log[1 + a*x])/8 + (a^4*(-PolyLog[2, -(
a*x)] + PolyLog[2, a*x]))/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{6574, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(\frac{a^4\operatorname{arctanh}(ax)}{x} - \frac{2a^2\operatorname{arctanh}(ax)}{x^3} + \frac{\operatorname{arctanh}(ax)}{x^5}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{4}a^4\operatorname{arctanh}(ax) - \frac{1}{2}a^4 \operatorname{PolyLog}(2,-ax) + \frac{1}{2}a^4 \operatorname{PolyLog}(2,ax) + \frac{3a^3}{4x} + \frac{a^2\operatorname{arctanh}(ax)}{x^2} - \frac{\operatorname{arctanh}(ax)}{4x^4} - \frac{a}{12x^3}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

output
```
-1/12*a/x^3 + (3*a^3)/(4*x) - (3*a^4*ArcTanh[a*x])/4 - ArcTanh[a*x]/(4*x^4
) + (a^2*ArcTanh[a*x])/x^2 - (a^4*PolyLog[2, -(a*x)])/2 + (a^4*PolyLog[2,
a*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

method	result
derivativedivides	$a^4\left(\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{a^2x^2}-\frac{\operatorname{arctanh}(ax)}{4a^4x^4}-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1}{2}\right.$
default	$a^4\left(\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{a^2x^2}-\frac{\operatorname{arctanh}(ax)}{4a^4x^4}-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1}{2}\right.$
parts	$-\frac{\operatorname{arctanh}(ax)}{4x^4}+\frac{a^2\operatorname{arctanh}(ax)}{x^2}+\operatorname{arctanh}(ax)a^4\ln(x)-\frac{a\left(-4a^4\left(-\frac{\operatorname{dilog}(ax+1)}{2a}-\frac{\ln(x)\ln(ax+1)}{2a}+\frac{(\ln(}{}\right.\right.}{}$
risch	$-\frac{a^4\operatorname{dilog}(ax+1)}{2}-\frac{a}{12x^3}+\frac{3a^4\ln(ax)}{8}+\frac{3a^3}{4x}-\frac{3a^4\ln(ax+1)}{8}-\frac{\ln(ax+1)}{8x^4}+\frac{a^2\ln(ax+1)}{2x^2}+\frac{a^4\operatorname{dilog}(-ax}{2}$
meijerg	$-\frac{ia^4\left(-\frac{i}{3x^3a^3}-\frac{i}{xa}+\frac{4i\left(-\frac{3a^4x^4}{8}+\frac{3}{8}\right)\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3x^3a^3\sqrt{a^2x^2}}\right)}{4}-\frac{ia^4\left(\frac{2iax\operatorname{polylog}\left(2,\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-\frac{2iax\operatorname{polylog}(}{\sqrt{a^2}}\right.}{4}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(arctanh(a*x)*ln(a*x)+arctanh(a*x)/a^2/x^2-1/4*arctanh(a*x)/a^4/x^4-1/
2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)-1/12/a^3/x^3+3/4/a/x+3
/8*ln(a*x-1)-3/8*ln(a*x+1))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5} dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)}{x^5} dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5} dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}{x^5} dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)/x**5,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5} dx = -\frac{1}{24}\Big(12(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax))a^3-12(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax))a^3+9a^3\log(ax+$$
$$+\frac{1}{4}\left(2a^4\log(x^2)+\frac{4a^2x^2-1}{x^4}\right)\operatorname{artanh}(ax)$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="maxima")
```

output
```
-1/24*(12*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 - 12*(log(-a*x + 1)*log(
x) + dilog(a*x))*a^3 + 9*a^3*log(a*x + 1) - 9*a^3*log(a*x - 1) - 2*(9*a^2*
x^2 - 1)/x^3)*a + 1/4*(2*a^4*log(x^2) + (4*a^2*x^2 - 1)/x^4)*arctanh(a*x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)}{x^5}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5}\,dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}{x^5}\,dx$$

input
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^5,x)
```

output
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^5}\,dx = \frac{-9atanh(ax)\,a^4x^4+12atanh(ax)\,a^2x^2-3atanh(ax)+12\left(\int \frac{atanh(ax)}{x}dx\right)a^4x^4+9a^3x^3-ax}{12x^4}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)/x^5,x)
```

output
```
( - 9*atanh(a*x)*a**4*x**4 + 12*atanh(a*x)*a**2*x**2 - 3*atanh(a*x) + 12*i
nt(atanh(a*x)/x,x)*a**4*x**4 + 9*a**3*x**3 - a*x)/(12*x**4)
```

3.202 $\int \frac{(1-a^2x^2)^2\text{arctanh}(ax)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{x^6}\,dx = -\frac{a}{20x^4}+\frac{7a^3}{30x^2}-\frac{\text{arctanh}(ax)}{5x^5}+\frac{2a^2\text{arctanh}(ax)}{3x^3}-\frac{a^4\text{arctanh}(ax)}{x}+\frac{8}{15}a^5\log(x)-\frac{4}{15}a^5\log\left(1-a^2x^2\right)$$

output
```
-1/20*a/x^4+7/30*a^3/x^2-1/5*arctanh(a*x)/x^5+2/3*a^2*arctanh(a*x)/x^3-a^4
*arctanh(a*x)/x+8/15*a^5*ln(x)-4/15*a^5*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)^2\,\text{arctanh}(ax)}{x^6}\,dx = -\frac{a}{20x^4}+\frac{7a^3}{30x^2}-\frac{\text{arctanh}(ax)}{5x^5}+\frac{2a^2\text{arctanh}(ax)}{3x^3}-\frac{a^4\text{arctanh}(ax)}{x}+\frac{8}{15}a^5\log(x)-\frac{4}{15}a^5\log\left(1-a^2x^2\right)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

output
```
-1/20*a/x^4 + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x
])/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2
*x^2])/15
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^6} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(\frac{a^4\operatorname{arctanh}(ax)}{x^2} - \frac{2a^2\operatorname{arctanh}(ax)}{x^4} + \frac{\operatorname{arctanh}(ax)}{x^6}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8}{15}a^5\log(x) - \frac{a^4\operatorname{arctanh}(ax)}{x} + \frac{7a^3}{30x^2} + \frac{2a^2\operatorname{arctanh}(ax)}{3x^3} - \frac{4}{15}a^5\log\left(1-a^2x^2\right) - \frac{\operatorname{arctanh}(ax)}{5x^5} - \frac{a}{20x^4}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

output

```
-1/20*a/x^4 + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x
])/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2
*x^2])/15
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result
derivativedivides	$a^5\left(-\frac{\operatorname{arctanh}(ax)}{ax}+\frac{2\operatorname{arctanh}(ax)}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{5a^5x^5}-\frac{1}{20a^4x^4}+\frac{7}{30a^2x^2}+\frac{8\ln(ax)}{15}-\frac{4\ln(ax-1)}{15}-\frac{4\ln(\right.$
default	$a^5\left(-\frac{\operatorname{arctanh}(ax)}{ax}+\frac{2\operatorname{arctanh}(ax)}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{5a^5x^5}-\frac{1}{20a^4x^4}+\frac{7}{30a^2x^2}+\frac{8\ln(ax)}{15}-\frac{4\ln(ax-1)}{15}-\frac{4\ln(\right.$
parts	$\frac{2a^2\operatorname{arctanh}(ax)}{3x^3}-\frac{a^4\operatorname{arctanh}(ax)}{x}-\frac{\operatorname{arctanh}(ax)}{5x^5}-\frac{a\left(\frac{3}{4x^4}-\frac{7a^2}{2x^2}-8a^4\ln(x)+4a^4\ln(ax+1)+4a^4\ln(ax-1)\right)}{15}$
parallelrisch	$\frac{32\ln(x)a^5x^5-32\ln(ax-1)x^5a^5-32\operatorname{arctanh}(ax)a^5x^5+14a^5x^5-60a^4x^4\operatorname{arctanh}(ax)+14a^3x^3+40a^2x^2\operatorname{arctanh}(ax)-}{60x^5}$
risch	$-\frac{(15a^4x^4-10a^2x^2+3)\ln(ax+1)}{30x^5}+\frac{32\ln(x)a^5x^5-16\ln(a^2x^2-1)a^5x^5+30x^4\ln(-ax+1)a^4+14a^3x^3-20x^2\ln(-ax}{60x^5}$
meijerg	$\frac{a^5\left(-\frac{1}{a^4x^4}-\frac{2}{3a^2x^2}-\frac{4}{25}+\frac{4\ln(x)}{5}+\frac{4\ln(ia)}{5}+\frac{\frac{4}{25}a^4x^4+\frac{4}{15}a^2x^2+\frac{4}{5}}{a^4x^4}+\frac{\frac{2\ln\left(1-\sqrt{a^2x^2}\right)}{5}-\frac{2\ln\left(1+\sqrt{a^2x^2}\right)}{5}}{a^4x^4\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{5}\right)}{4}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-arctanh(a*x)/a/x+2/3*arctanh(a*x)/a^3/x^3-1/5*arctanh(a*x)/a^5/x^5-1
/20/a^4/x^4+7/30/a^2/x^2+8/15*ln(a*x)-4/15*ln(a*x-1)-4/15*ln(a*x+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^6} dx = -\frac{16 a^5 x^5 \log(a^2x^2-1) - 32 a^5 x^5 \log(x) - 14 a^3 x^3 + 3 ax + 2 (15 a^4 x^4 - 10 a^2 x^2 + 3) \log\left(-\frac{ax+1}{ax-1}\right)}{60 x^5}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(16*a^5*x^5*log(a^2*x^2 - 1) - 32*a^5*x^5*log(x) - 14*a^3*x^3 + 3*a*
x + 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^6} dx = \begin{cases} \frac{8a^5 \log(x)}{15} - \frac{8a^5 \log\left(x-\frac{1}{a}\right)}{15} - \frac{8a^5 \operatorname{atanh}(ax)}{15} - \frac{a^4 \operatorname{atanh}(ax)}{x} + \frac{7a^3}{30x^2} + \frac{2a^2 \operatorname{atanh}(ax)}{3x^3} - \frac{a}{20x^4} - \frac{\operatorname{atanh}(ax)}{5x^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)/x**6,x)
```

output
```
Piecewise((8*a**5*log(x)/15 - 8*a**5*log(x - 1/a)/15 - 8*a**5*atanh(a*x)/1
5 - a**4*atanh(a*x)/x + 7*a**3/(30*x**2) + 2*a**2*atanh(a*x)/(3*x**3) - a/
(20*x**4) - atanh(a*x)/(5*x**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^6}\,dx = -\frac{1}{60}\left(16\,a^4\log\left(a^2x^2-1\right)-16\,a^4\log\left(x^2\right)-\frac{14\,a^2x^2-3}{x^4}\right)a - \frac{(15\,a^4x^4-10\,a^2x^2+3)\operatorname{artanh}(ax)}{15\,x^5}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="maxima")
```

output
```
-1/60*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2) - (14*a^2*x^2 - 3)/x^4)*a
 - 1/15*(15*a^4*x^4 - 10*a^2*x^2 + 3)*arctanh(a*x)/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(71) = 142.

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.19

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^6}\,dx = -\frac{4}{15}\left(2\,a^4\log\left(\frac{|-ax-1|}{|ax-1|}\right)-2\,a^4\log\left(\left|-\frac{ax+1}{ax-1}-1\right|\right)+\frac{\frac{2\,(ax+1)^3a^4}{(ax-1)^3}+\frac{7\,(ax+1)^2a^4}{(ax-1)^2}+\frac{2\,(ax+1)a^4}{ax-1}}{\left(\frac{ax+1}{ax-1}+1\right)^4}-\frac{2\left(10\right.}{}\right.$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="giac")
```

output
```
-4/15*(2*a^4*log(abs(-a*x - 1)/abs(a*x - 1)) - 2*a^4*log(abs(-(a*x + 1)/(a
*x - 1) - 1)) + (2*(a*x + 1)^3*a^4/(a*x - 1)^3 + 7*(a*x + 1)^2*a^4/(a*x -
1)^2 + 2*(a*x + 1)*a^4/(a*x - 1))/((a*x + 1)/(a*x - 1) + 1)^4 - 2*(10*(a*x
 + 1)^2*a^4/(a*x - 1)^2 + 5*(a*x + 1)*a^4/(a*x - 1) + a^4)*log(-(a*((a*x +
 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1
) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/((a*x + 1)/(a*x - 1) + 1)^5)*a
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{x^6}\,dx = \frac{8\,a^5\,\ln(x)}{15} - \frac{a}{20\,x^4} - \frac{\operatorname{atanh}(a\,x)}{5\,x^5} - \frac{4\,a^5\,\ln(a^2\,x^2-1)}{15} + \frac{7\,a^3}{30\,x^2} + \frac{2\,a^2\operatorname{atanh}(a\,x)}{3\,x^3} - \frac{a^4\operatorname{atanh}(a\,x)}{x}$$

input
```
int((atanh(a*x)*(a^2*x^2 - 1)^2)/x^6,x)
```

output
```
(8*a^5*log(x))/15 - a/(20*x^4) - atanh(a*x)/(5*x^5) - (4*a^5*log(a^2*x^2 -
 1))/15 + (7*a^3)/(30*x^2) + (2*a^2*atanh(a*x))/(3*x^3) - (a^4*atanh(a*x))
/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{x^6}\,dx$$
$$= \frac{-32 atanh(ax)\,a^5x^5 - 60 atanh(ax)\,a^4x^4 + 40 atanh(ax)\,a^2x^2 - 12 atanh(ax) - 32\log(a^2x-a)\,a^5x^5 + 32}{60x^5}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)/x^6,x)
```

output
```
( - 32*atanh(a*x)*a**5*x**5 - 60*atanh(a*x)*a**4*x**4 + 40*atanh(a*x)*a**2
*x**2 - 12*atanh(a*x) - 32*log(a**2*x - a)*a**5*x**5 + 32*log(x)*a**5*x**5
 + 14*a**3*x**3 - 3*a*x)/(60*x**5)
```

3.203 $\int x^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 202

$$\begin{aligned}\int x^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = {} & \frac{29x}{3780a^4} - \frac{67x^3}{11340a^2} - \frac{23x^5}{3780} + \frac{a^2x^7}{252} \\ & - \frac{29\operatorname{arctanh}(ax)}{3780a^5} + \frac{8x^2\operatorname{arctanh}(ax)}{315a^3} \\ & + \frac{4x^4\operatorname{arctanh}(ax)}{315a} - \frac{11}{189}ax^6\operatorname{arctanh}(ax) \\ & + \frac{1}{36}a^3x^8\operatorname{arctanh}(ax) + \frac{8\operatorname{arctanh}(ax)^2}{315a^5} \\ & + \frac{1}{5}x^5\operatorname{arctanh}(ax)^2 - \frac{2}{7}a^2x^7\operatorname{arctanh}(ax)^2 \\ & + \frac{1}{9}a^4x^9\operatorname{arctanh}(ax)^2 - \frac{16\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{315a^5} \\ & - \frac{8\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{315a^5}\end{aligned}$$

output
```
29/3780*x/a^4-67/11340*x^3/a^2-23/3780*x^5+1/252*a^2*x^7-29/3780*arctanh(a
*x)/a^5+8/315*x^2*arctanh(a*x)/a^3+4/315*x^4*arctanh(a*x)/a-11/189*a*x^6*a
rctanh(a*x)+1/36*a^3*x^8*arctanh(a*x)+8/315*arctanh(a*x)^2/a^5+1/5*x^5*arc
tanh(a*x)^2-2/7*a^2*x^7*arctanh(a*x)^2+1/9*a^4*x^9*arctanh(a*x)^2-16/315*a
rctanh(a*x)*ln(2/(-a*x+1))/a^5-8/315*polylog(2,1-2/(-a*x+1))/a^5
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int x^4\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{ax(87-67a^2x^2-69a^4x^4+45a^6x^6)+36(-8+63a^5x^5-90a^7x^7+35a^9x^9)\operatorname{arctanh}(ax)^2+3\operatorname{arctanh}(a}{}$$

input
```
Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(a*x*(87 - 67*a^2*x^2 - 69*a^4*x^4 + 45*a^6*x^6) + 36*(-8 + 63*a^5*x^5 - 9
0*a^7*x^7 + 35*a^9*x^9)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-29 + 96*a^2*x^2
+ 48*a^4*x^4 - 220*a^6*x^6 + 105*a^8*x^8 - 192*Log[1 + E^(-2*ArcTanh[a*x])
]) + 288*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(11340*a^5)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2\,dx$$
$$\Big\downarrow \text{6574}$$
$$\int \left(a^4x^8\operatorname{arctanh}(ax)^2-2a^2x^6\operatorname{arctanh}(ax)^2+x^4\operatorname{arctanh}(ax)^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{8\operatorname{arctanh}(ax)^2}{315a^5}-\frac{29\operatorname{arctanh}(ax)}{3780a^5}-\frac{16\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{315a^5}-\frac{8\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{315a^5}+\frac{1}{9}a^4x^9\operatorname{arctanh}(ax)^2+\frac{29x}{3780a^4}+\frac{1}{36}a^3x^8\operatorname{arctanh}(ax)+\frac{8x^2\operatorname{arctanh}(ax)}{315a^3}-\frac{2}{7}a^2x^7\operatorname{arctanh}(ax)^2+\frac{a^2x^7}{252}-\frac{67x^3}{11340a^2}-\frac{11}{189}ax^6\operatorname{arctanh}(ax)+\frac{1}{5}x^5\operatorname{arctanh}(ax)^2+\frac{4x^4\operatorname{arctanh}(ax)}{315a}-\frac{23x^5}{3780}$$

input
```
Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(29*x)/(3780*a^4) - (67*x^3)/(11340*a^2) - (23*x^5)/3780 + (a^2*x^7)/252 -
 (29*ArcTanh[a*x])/(3780*a^5) + (8*x^2*ArcTanh[a*x])/(315*a^3) + (4*x^4*Ar
cTanh[a*x])/(315*a) - (11*a*x^6*ArcTanh[a*x])/189 + (a^3*x^8*ArcTanh[a*x])
/36 + (8*ArcTanh[a*x]^2)/(315*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (2*a^2*x^7*A
rcTanh[a*x]^2)/7 + (a^4*x^9*ArcTanh[a*x]^2)/9 - (16*ArcTanh[a*x]*Log[2/(1
- a*x)])/(315*a^5) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(315*a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
 + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2 a^9 x^9}{9}-\frac{2\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax) a^8 x^8}{36}-\frac{11\operatorname{arctanh}(ax) a^6 x^6}{189}+\frac{4a^4x^4\operatorname{arctanh}(ax)}{315}+\frac{8}{}$
default	$\frac{\operatorname{arctanh}(ax)^2 a^9 x^9}{9}-\frac{2\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax) a^8 x^8}{36}-\frac{11\operatorname{arctanh}(ax) a^6 x^6}{189}+\frac{4a^4x^4\operatorname{arctanh}(ax)}{315}+\frac{8}{}$
parts	$\frac{a^4x^9\operatorname{arctanh}(ax)^2}{9}-\frac{2a^2x^7\operatorname{arctanh}(ax)^2}{7}+\frac{x^5\operatorname{arctanh}(ax)^2}{5}+\frac{a^3x^8\operatorname{arctanh}(ax)}{36}-\frac{11a\,x^6\operatorname{arctanh}(ax)}{189}+\frac{4a}{}$
risch	$-\frac{\left(\left(-\frac{1}{25}+\frac{\ln(ax+1)}{5}\right)(ax+1)^5+\left(\frac{1}{4}-\ln(ax+1)\right)(ax+1)^4+\left(-\frac{2}{3}+2\ln(ax+1)\right)(ax+1)^3+(1-2\ln(ax+1))(ax+1)^2+(-1}{2a^5}$

input
```
int(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/9*arctanh(a*x)^2*a^9*x^9-2/7*arctanh(a*x)^2*a^7*x^7+1/5*arctanh(a
*x)^2*a^5*x^5+1/36*arctanh(a*x)*a^8*x^8-11/189*arctanh(a*x)*a^6*x^6+4/315*
a^4*x^4*arctanh(a*x)+8/315*a^2*x^2*arctanh(a*x)+8/315*arctanh(a*x)*ln(a*x-
1)+8/315*arctanh(a*x)*ln(a*x+1)+1/252*a^7*x^7-23/3780*a^5*x^5-67/11340*a^3
*x^3+29/3780*a*x+29/7560*ln(a*x-1)-29/7560*ln(a*x+1)+2/315*ln(a*x-1)^2-8/3
15*dilog(1/2*a*x+1/2)-4/315*ln(a*x-1)*ln(1/2*a*x+1/2)+4/315*(ln(a*x+1)-ln(
1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-2/315*ln(a*x+1)^2)
```

Fricas [F]

$$\int x^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = \int \left(a^2x^2-1\right)^2 x^4 \operatorname{artanh}\left(ax\right)^2 \, dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^8 - 2*a^2*x^6 + x^4)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = \int x^4(ax-1)^2 \left(ax+1\right)^2 \operatorname{atanh}^2\left(ax\right) dx$$

input
```
integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

output
```
Integral(x**4*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

$$\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$$

$$= \frac{1}{22680} a^2 \Bigg(\frac{90 a^7 x^7 - 138 a^5 x^5 - 134 a^3 x^3 + 174 ax - 144 \log(ax+1)^2 + 288 \log(ax+1) \log(ax-1) +}{a^7}$$

$$+ \frac{1}{3780} a \left(\frac{105 a^6 x^8 - 220 a^4 x^6 + 48 a^2 x^4 + 96 x^2}{a^4} + \frac{96 \log(ax+1)}{a^6} + \frac{96 \log(ax-1)}{a^6} \right) \operatorname{artanh}(ax)$$

$$+ \frac{1}{315} (35 a^4 x^9 - 90 a^2 x^7 + 63 x^5) \operatorname{artanh}(ax)^2$$

input
```
integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
1/22680*a^2*((90*a^7*x^7 - 138*a^5*x^5 - 134*a^3*x^3 + 174*a*x - 144*log(a
*x + 1)^2 + 288*log(a*x + 1)*log(a*x - 1) + 144*log(a*x - 1)^2 + 87*log(a*
x - 1))/a^7 - 576*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2)
)/a^7 - 87*log(a*x + 1)/a^7) + 1/3780*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a
^2*x^4 + 96*x^2)/a^4 + 96*log(a*x + 1)/a^6 + 96*log(a*x - 1)/a^6)*arctanh(
a*x) + 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5)*arctanh(a*x)^2
```

Giac [F]

$$\int x^4 (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = \int (a^2x^2-1)^2 x^4 \operatorname{artanh}(ax)^2 \, dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*x^4*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(1-a^2x^2)^2\operatorname{arctanh}(ax)^2\,dx=\int x^4\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2\,dx$$

input
```
int(x^4*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

output
```
int(x^4*atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int x^4(1-a^2x^2)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{1260atanh(ax)^2a^9x^9-3240atanh(ax)^2a^7x^7+2268atanh(ax)^2a^5x^5-288atanh(ax)^2ax+315atanh(a}{}$$

input
```
int(x^4*(-a^2*x^2+1)^2*atanh(a*x)^2,x)
```

output
```
(1260*atanh(a*x)**2*a**9*x**9 - 3240*atanh(a*x)**2*a**7*x**7 + 2268*atanh(
a*x)**2*a**5*x**5 - 288*atanh(a*x)**2*a*x + 315*atanh(a*x)*a**8*x**8 - 660
*atanh(a*x)*a**6*x**6 + 144*atanh(a*x)*a**4*x**4 + 288*atanh(a*x)*a**2*x**
2 - 87*atanh(a*x) + 288*int(atanh(a*x)**2,x)*a + 45*a**7*x**7 - 69*a**5*x*
*5 - 67*a**3*x**3 + 87*a*x)/(11340*a**5)
```

3.204 $\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\begin{aligned}\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = &-\frac{5x^2}{504a^2}-\frac{x^4}{84}+\frac{a^2x^6}{168}+\frac{x\operatorname{arctanh}(ax)}{12a^3}\\ &+\frac{x^3\operatorname{arctanh}(ax)}{36a}-\frac{1}{12}ax^5\operatorname{arctanh}(ax)\\ &+\frac{1}{28}a^3x^7\operatorname{arctanh}(ax)-\frac{\operatorname{arctanh}(ax)^2}{24a^4}\\ &+\frac{1}{4}x^4\operatorname{arctanh}(ax)^2-\frac{1}{3}a^2x^6\operatorname{arctanh}(ax)^2\\ &+\frac{1}{8}a^4x^8\operatorname{arctanh}(ax)^2+\frac{2\log(1-a^2x^2)}{63a^4}\end{aligned}$$

output
```
-5/504*x^2/a^2-1/84*x^4+1/168*a^2*x^6+1/12*x*arctanh(a*x)/a^3+1/36*x^3*arc
tanh(a*x)/a-1/12*a*x^5*arctanh(a*x)+1/28*a^3*x^7*arctanh(a*x)-1/24*arctanh
(a*x)^2/a^4+1/4*x^4*arctanh(a*x)^2-1/3*a^2*x^6*arctanh(a*x)^2+1/8*a^4*x^8*
arctanh(a*x)^2+2/63*ln(-a^2*x^2+1)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{-5a^2x^2-6a^4x^4+3a^6x^6+2ax(21+7a^2x^2-21a^4x^4+9a^6x^6)\operatorname{arctanh}(ax)+21(-1+a^2x^2)^3(1+3a^2x}{504a^4}$$

input
```
Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(-5*a^2*x^2 - 6*a^4*x^4 + 3*a^6*x^6 + 2*a*x*(21 + 7*a^2*x^2 - 21*a^4*x^4 +
 9*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a^2*x^2)^3*(1 + 3*a^2*x^2)*ArcTanh[a*x
]^2 + 16*Log[1 - a^2*x^2])/(504*a^4)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx$$
$$\Big\downarrow \text{ 6574}$$
$$\int \left(a^4x^7\operatorname{arctanh}(ax)^2-2a^2x^5\operatorname{arctanh}(ax)^2+x^3\operatorname{arctanh}(ax)^2\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{8}a^4x^8\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{24a^4}+\frac{1}{28}a^3x^7\operatorname{arctanh}(ax)+\frac{x\operatorname{arctanh}(ax)}{12a^3}-\frac{1}{3}a^2x^6\operatorname{arctanh}(ax)^2+\frac{a^2x^6}{168}-\frac{5x^2}{504a^2}+\frac{2\log(1-a^2x^2)}{63a^4}-\frac{1}{12}ax^5\operatorname{arctanh}(ax)+\frac{1}{4}x^4\operatorname{arctanh}(ax)^2+\frac{x^3\operatorname{arctanh}(ax)}{36a}-\frac{x^4}{84}$$

input
```
Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(-5*x^2)/(504*a^2) - x^4/84 + (a^2*x^6)/168 + (x*ArcTanh[a*x])/(12*a^3) +
(x^3*ArcTanh[a*x])/(36*a) - (a*x^5*ArcTanh[a*x])/12 + (a^3*x^7*ArcTanh[a*x
])/28 - ArcTanh[a*x]^2/(24*a^4) + (x^4*ArcTanh[a*x]^2)/4 - (a^2*x^6*ArcTan
h[a*x]^2)/3 + (a^4*x^8*ArcTanh[a*x]^2)/8 + (2*Log[1 - a^2*x^2])/(63*a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

method	result
parallelrisch	$-\frac{-63\,\operatorname{arctanh}(ax)^2 a^8x^8-18\,\operatorname{arctanh}(ax)a^7x^7+168\,\operatorname{arctanh}(ax)^2a^6x^6-3a^6x^6+42\,\operatorname{arctanh}(ax)a^5x^5-126a^4x^4\,\text{arcta}}{5}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2a^8x^8}{8}-\frac{\operatorname{arctanh}(ax)^2a^6x^6}{3}+\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}+\frac{\operatorname{arctanh}(ax)a^7x^7}{28}-\frac{\operatorname{arctanh}(ax)a^5x^5}{12}+\frac{a^3x^3\operatorname{arctanh}(ax)}{36}+\frac{ax\ \text{arc}}{}}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2a^8x^8}{8}-\frac{\operatorname{arctanh}(ax)^2a^6x^6}{3}+\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}+\frac{\operatorname{arctanh}(ax)a^7x^7}{28}-\frac{\operatorname{arctanh}(ax)a^5x^5}{12}+\frac{a^3x^3\operatorname{arctanh}(ax)}{36}+\frac{ax\ \text{arc}}{}}{}$
parts	$\frac{a^4x^8\operatorname{arctanh}(ax)^2}{8}-\frac{a^2x^6\operatorname{arctanh}(ax)^2}{3}+\frac{x^4\operatorname{arctanh}(ax)^2}{4}+\frac{a^3x^7\operatorname{arctanh}(ax)}{28}-\frac{a\,x^5\operatorname{arctanh}(ax)}{12}+\frac{x^3\,\text{arc}}{}$
risch	$\frac{(3a^8x^8-8a^6x^6+6a^4x^4-1)\ln(ax+1)^2}{96a^4}-\frac{(63a^8x^8\ln(-ax+1)-18a^7x^7-168a^6x^6\ln(-ax+1)+42a^5x^5+126x^4\ln(-a}{1008a^4}$

input
```
int(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/504*(-63*arctanh(a*x)^2*a^8*x^8-18*arctanh(a*x)*a^7*x^7+168*arctanh(a*x
)^2*a^6*x^6-3*a^6*x^6+42*arctanh(a*x)*a^5*x^5-126*a^4*x^4*arctanh(a*x)^2+6
*a^4*x^4-14*a^3*x^3*arctanh(a*x)+5+5*a^2*x^2-42*a*x*arctanh(a*x)+21*arctan
h(a*x)^2-32*ln(a*x-1)-32*arctanh(a*x))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int x^3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{12\,a^6x^6-24\,a^4x^4-20\,a^2x^2+21\left(3\,a^8x^8-8\,a^6x^6+6\,a^4x^4-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(9\,a^7x^7-21\,a^5x^5+7\,a\right.}{2016\,a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 + 21*(3*a^8*x^8 - 8*a^6*x^6 +
 6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(9*a^7*x^7 - 21*a^5*x^5 +
7*a^3*x^3 + 21*a*x)*log(-(a*x + 1)/(a*x - 1)) + 64*log(a^2*x^2 - 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\begin{cases}\frac{a^4x^8\operatorname{atanh}^2(ax)}{8}+\frac{a^3x^7\operatorname{atanh}(ax)}{28}-\frac{a^2x^6\operatorname{atanh}^2(ax)}{3}+\frac{a^2x^6}{168}-\frac{ax^5\operatorname{atanh}(ax)}{12}+\frac{x^4\operatorname{atanh}^2(ax)}{4}-\frac{x^4}{84}+\frac{x^3\operatorname{atanh}(ax)}{36a}-\frac{5}{50\cdots}\\0\end{cases}$$

input
```
integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

output
```
Piecewise((a**4*x**8*atanh(a*x)**2/8 + a**3*x**7*atanh(a*x)/28 - a**2*x**6
*atanh(a*x)**2/3 + a**2*x**6/168 - a*x**5*atanh(a*x)/12 + x**4*atanh(a*x)*
*2/4 - x**4/84 + x**3*atanh(a*x)/(36*a) - 5*x**2/(504*a**2) + x*atanh(a*x)
/(12*a**3) + 4*log(x - 1/a)/(63*a**4) - atanh(a*x)**2/(24*a**4) + 4*atanh(
a*x)/(63*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int x^3(1-a^2x^2)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{1}{504}a\left(\frac{2\left(9a^6x^7-21a^4x^5+7a^2x^3+21x\right)}{a^4}-\frac{21\log(ax+1)}{a^5}+\frac{21\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)$$
$$+\frac{1}{24}\left(3a^4x^8-8a^2x^6+6x^4\right)\operatorname{artanh}(ax)^2$$
$$+\frac{12a^6x^6-24a^4x^4-20a^2x^2-2\left(21\log(ax-1)-32\right)\log(ax+1)+21\log(ax+1)^2+21\log(ax-}{2016a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
1/504*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x +
1)/a^5 + 21*log(a*x - 1)/a^5)*arctanh(a*x) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 +
 6*x^4)*arctanh(a*x)^2 + 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 - 2*
(21*log(a*x - 1) - 32)*log(a*x + 1) + 21*log(a*x + 1)^2 + 21*log(a*x - 1)^
2 + 64*log(a*x - 1))/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(132) = 264$.

Time = 0.14 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.38

$$\int x^3(1-a^2x^2)^2\operatorname{arctanh}(ax)^2\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
2/63*(84*((a*x + 1)^5/(a*x - 1)^5 + (a*x + 1)^4/(a*x - 1)^4 + (a*x + 1)^3/
(a*x - 1)^3)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^8*a^5/(a*x - 1)^8 - 8*
(a*x + 1)^7*a^5/(a*x - 1)^7 + 28*(a*x + 1)^6*a^5/(a*x - 1)^6 - 56*(a*x + 1
)^5*a^5/(a*x - 1)^5 + 70*(a*x + 1)^4*a^5/(a*x - 1)^4 - 56*(a*x + 1)^3*a^5/
(a*x - 1)^3 + 28*(a*x + 1)^2*a^5/(a*x - 1)^2 - 8*(a*x + 1)*a^5/(a*x - 1) +
 a^5) + 2*(28*(a*x + 1)^4/(a*x - 1)^4 - 7*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*
x + 1)^2/(a*x - 1)^2 - 7*(a*x + 1)/(a*x - 1) + 1)*log(-(a*x + 1)/(a*x - 1)
)/((a*x + 1)^7*a^5/(a*x - 1)^7 - 7*(a*x + 1)^6*a^5/(a*x - 1)^6 + 21*(a*x +
 1)^5*a^5/(a*x - 1)^5 - 35*(a*x + 1)^4*a^5/(a*x - 1)^4 + 35*(a*x + 1)^3*a^
5/(a*x - 1)^3 - 21*(a*x + 1)^2*a^5/(a*x - 1)^2 + 7*(a*x + 1)*a^5/(a*x - 1)
 - a^5) - (2*(a*x + 1)^5/(a*x - 1)^5 - 11*(a*x + 1)^4/(a*x - 1)^4 + 6*(a*x
 + 1)^3/(a*x - 1)^3 - 11*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1))/
((a*x + 1)^6*a^5/(a*x - 1)^6 - 6*(a*x + 1)^5*a^5/(a*x - 1)^5 + 15*(a*x + 1
)^4*a^5/(a*x - 1)^4 - 20*(a*x + 1)^3*a^5/(a*x - 1)^3 + 15*(a*x + 1)^2*a^5/
(a*x - 1)^2 - 6*(a*x + 1)*a^5/(a*x - 1) + a^5) - 2*log(-(a*x + 1)/(a*x - 1
) + 1)/a^5 + 2*log(-(a*x + 1)/(a*x - 1))/a^5)*a
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.42

$$\int x^3 (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = \frac{2\,\ln\left(a^2\,x^2-1\right)}{63\,a^4} - \ln\left(1-a\,x\right)^2\left(\frac{1}{96\,a^4}-\frac{x^4}{16}+\frac{a^2\,x^6}{12}-\frac{a^4\,x^8}{32}\right)-\frac{x^4}{84} - \ln\left(a\,x+1\right)^2\left(\frac{1}{96\,a^4}-\frac{x^4}{16}+\frac{a^2\,x^6}{12}-\frac{a^4\,x^8}{32}\right) - \ln\left(1-a\,x\right)\left(\frac{x}{24\,a^3} - \ln\left(a\,x+1\right)\left(\frac{1}{48\,a^4}-\frac{x^4}{8}+\frac{a^2\,x^6}{6}-\frac{a^4\,x^8}{16}\right) - \frac{a\,x^5}{24}+\frac{x^3}{72\,a}+\frac{a^3\,x^7}{56}\right)-\frac{5\,x^2}{504\,a^2}+\frac{a^2\,x^6}{168} + a\,\ln\left(a\,x+1\right)\left(\frac{x}{24\,a^4}-\frac{x^5}{24}+\frac{x^3}{72\,a^2}+\frac{a^2\,x^7}{56}\right)$$

input
```
int(x^3*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

output
```
(2*log(a^2*x^2 - 1))/(63*a^4) - log(1 - a*x)^2*(1/(96*a^4) - x^4/16 + (a^2
*x^6)/12 - (a^4*x^8)/32) - x^4/84 - log(a*x + 1)^2*(1/(96*a^4) - x^4/16 +
(a^2*x^6)/12 - (a^4*x^8)/32) - log(1 - a*x)*(x/(24*a^3) - log(a*x + 1)*(1/
(48*a^4) - x^4/8 + (a^2*x^6)/6 - (a^4*x^8)/16) - (a*x^5)/24 + x^3/(72*a) +
 (a^3*x^7)/56) - (5*x^2)/(504*a^2) + (a^2*x^6)/168 + a*log(a*x + 1)*(x/(24
*a^4) - x^5/24 + x^3/(72*a^2) + (a^2*x^7)/56)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.91

$$\int x^3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx$$
$$= \frac{63atanh(ax)^2 a^8x^8 - 168atanh(ax)^2 a^6x^6 + 126atanh(ax)^2 a^4x^4 - 21atanh(ax)^2 + 18atanh(ax)\,a^7x^7 -}{}$$

input
```
int(x^3*(-a^2*x^2+1)^2*atanh(a*x)^2,x)
```

output
```
(63*atanh(a*x)**2*a**8*x**8 - 168*atanh(a*x)**2*a**6*x**6 + 126*atanh(a*x)
**2*a**4*x**4 - 21*atanh(a*x)**2 + 18*atanh(a*x)*a**7*x**7 - 42*atanh(a*x)
*a**5*x**5 + 14*atanh(a*x)*a**3*x**3 + 42*atanh(a*x)*a*x + 32*atanh(a*x) +
 32*log(a**2*x - a) + 3*a**6*x**6 - 6*a**4*x**4 - 5*a**2*x**2)/(504*a**4)
```

3.205 $\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 178

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = -\frac{x}{210a^2} - \frac{17x^3}{630} + \frac{a^2x^5}{105} + \frac{\operatorname{arctanh}(ax)}{210a^3} + \frac{8x^2\operatorname{arctanh}(ax)}{105a} - \frac{9}{70}ax^4\operatorname{arctanh}(ax) + \frac{1}{21}a^3x^6\operatorname{arctanh}(ax) + \frac{8\operatorname{arctanh}(ax)^2}{105a^3} + \frac{1}{3}x^3\operatorname{arctanh}(ax)^2 - \frac{2}{5}a^2x^5\operatorname{arctanh}(ax)^2 + \frac{1}{7}a^4x^7\operatorname{arctanh}(ax)^2 - \frac{16\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{105a^3} - \frac{8\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{105a^3}$$

output
```
-1/210*x/a^2-17/630*x^3+1/105*a^2*x^5+1/210*arctanh(a*x)/a^3+8/105*x^2*arc
tanh(a*x)/a-9/70*a*x^4*arctanh(a*x)+1/21*a^3*x^6*arctanh(a*x)+8/105*arctan
h(a*x)^2/a^3+1/3*x^3*arctanh(a*x)^2-2/5*a^2*x^5*arctanh(a*x)^2+1/7*a^4*x^7
*arctanh(a*x)^2-16/105*arctanh(a*x)*ln(2/(-a*x+1))/a^3-8/105*polylog(2,1-2
/(-a*x+1))/a^3
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.68

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx$$
$$= \frac{ax(-3-17a^2x^2+6a^4x^4)+6(-8+35a^3x^3-42a^5x^5+15a^7x^7)\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\left(3+48a^2x^2\right.}{630a^3}$$

input
```
Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(a*x*(-3 - 17*a^2*x^2 + 6*a^4*x^4) + 6*(-8 + 35*a^3*x^3 - 42*a^5*x^5 + 15*
a^7*x^7)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(3 + 48*a^2*x^2 - 81*a^4*x^4 + 30*a
^6*x^6 - 96*Log[1 + E^(-2*ArcTanh[a*x])]) + 48*PolyLog[2, -E^(-2*ArcTanh[a
*x])])/(630*a^3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx$$
$$\Big\downarrow \; 6574$$
$$\int \left(a^4x^6\operatorname{arctanh}(ax)^2-2a^2x^4\operatorname{arctanh}(ax)^2+x^2\operatorname{arctanh}(ax)^2\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{7}a^4x^7\operatorname{arctanh}(ax)^2+\frac{1}{21}a^3x^6\operatorname{arctanh}(ax)+\frac{8\operatorname{arctanh}(ax)^2}{105a^3}+\frac{\operatorname{arctanh}(ax)}{210a^3}-\frac{16\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{105a^3}-\frac{8\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{105a^3}-\frac{2}{5}a^2x^5\operatorname{arctanh}(ax)^2+\frac{a^2x^5}{105}-\frac{x}{210a^2}-\frac{9}{70}ax^4\operatorname{arctanh}(ax)+\frac{1}{3}x^3\operatorname{arctanh}(ax)^2+\frac{8x^2\operatorname{arctanh}(ax)}{105a}-\frac{17x^3}{630}$$

input
```
Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
-1/210*x/a^2 - (17*x^3)/630 + (a^2*x^5)/105 + ArcTanh[a*x]/(210*a^3) + (8*
x^2*ArcTanh[a*x])/(105*a) - (9*a*x^4*ArcTanh[a*x])/70 + (a^3*x^6*ArcTanh[a
*x])/21 + (8*ArcTanh[a*x]^2)/(105*a^3) + (x^3*ArcTanh[a*x]^2)/3 - (2*a^2*x
^5*ArcTanh[a*x]^2)/5 + (a^4*x^7*ArcTanh[a*x]^2)/7 - (16*ArcTanh[a*x]*Log[2
/(1 - a*x)])/(105*a^3) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(105*a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}-\frac{2\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}-\frac{9a^4x^4\operatorname{arctanh}(ax)}{70}+\frac{8a^2x^2\operatorname{arctanh}(ax)}{105}+\frac{8\ a}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}-\frac{2\operatorname{arctanh}(ax)^2 a^5 x^5}{5}+\frac{\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\frac{\operatorname{arctanh}(ax) a^6 x^6}{21}-\frac{9a^4x^4\operatorname{arctanh}(ax)}{70}+\frac{8a^2x^2\operatorname{arctanh}(ax)}{105}+\frac{8\ a}{}$
parts	$\frac{a^4x^7\operatorname{arctanh}(ax)^2}{7}-\frac{2a^2x^5\operatorname{arctanh}(ax)^2}{5}+\frac{x^3\operatorname{arctanh}(ax)^2}{3}+\frac{a^3x^6\operatorname{arctanh}(ax)}{21}-\frac{9a\,x^4\operatorname{arctanh}(ax)}{70}+\frac{8x^2}{}$
risch	$-\frac{17x^3}{630}-\frac{177151}{2315250a^3}+\frac{a^2x^5}{105}-\frac{9a\ln(ax+1)x^4}{140}-\frac{19\ln(ax+1)x^2}{420a}-\frac{\ln(ax+1)x}{6a^2}-\frac{a^2\ln(ax+1)^2x^5}{10}-\frac{x^3\ln}{}$

input
```
int(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/7*arctanh(a*x)^2*a^7*x^7-2/5*arctanh(a*x)^2*a^5*x^5+1/3*arctanh(a
*x)^2*a^3*x^3+1/21*arctanh(a*x)*a^6*x^6-9/70*a^4*x^4*arctanh(a*x)+8/105*a^
2*x^2*arctanh(a*x)+8/105*arctanh(a*x)*ln(a*x-1)+8/105*arctanh(a*x)*ln(a*x+
1)+1/105*a^5*x^5-17/630*a^3*x^3-1/210*a*x-1/420*ln(a*x-1)+1/420*ln(a*x+1)+
2/105*ln(a*x-1)^2-8/105*dilog(1/2*a*x+1/2)-4/105*ln(a*x-1)*ln(1/2*a*x+1/2)
+4/105*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-2/105*ln(a*x+1)^2)
```

Fricas [F]

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = \int (a^2x^2-1)^2x^2 \operatorname{artanh}(ax)^2\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^6 - 2*a^2*x^4 + x^2)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = \int x^2(ax-1)^2(ax+1)^2 \operatorname{atanh}^2(ax)\,dx$$

input
```
integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

output
```
Integral(x**2*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int x^2 (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$$
$$= \frac{1}{1260} a^2 \left(\frac{12 a^5 x^5 - 34 a^3 x^3 - 6 ax - 24 \log(ax+1)^2 + 48 \log(ax+1) \log(ax-1) + 24 \log(ax-1)^2}{a^5} \right.$$
$$+ \frac{1}{210} a \left(\frac{10 a^4 x^6 - 27 a^2 x^4 + 16 x^2}{a^2} + \frac{16 \log(ax+1)}{a^4} + \frac{16 \log(ax-1)}{a^4} \right) \operatorname{artanh}(ax)$$
$$+ \frac{1}{105} \left(15 a^4 x^7 - 42 a^2 x^5 + 35 x^3\right) \operatorname{artanh}(ax)^2$$

input
```
integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
1/1260*a^2*((12*a^5*x^5 - 34*a^3*x^3 - 6*a*x - 24*log(a*x + 1)^2 + 48*log(
a*x + 1)*log(a*x - 1) + 24*log(a*x - 1)^2 - 3*log(a*x - 1))/a^5 - 96*(log(
a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 + 3*log(a*x + 1)/
a^5) + 1/210*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a
^4 + 16*log(a*x - 1)/a^4)*arctanh(a*x) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 +
35*x^3)*arctanh(a*x)^2
```

Giac [F]

$$\int x^2 (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = \int (a^2x^2-1)^2 x^2 \operatorname{artanh}(ax)^2 \, dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*x^2*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 dx = \int x^2 \operatorname{atanh}(a\,x)^2 \left(a^2\,x^2-1\right)^2 dx$$

input
```
int(x^2*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

output
```
int(x^2*atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int x^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 dx = \frac{90atanh(ax)^2 a^7x^7 - 252atanh(ax)^2 a^5x^5 + 210atanh(ax)^2 a^3x^3 - 48atanh(ax)^2 ax + 30atanh(ax)\, a^6x^6}{63}$$

input
```
int(x^2*(-a^2*x^2+1)^2*atanh(a*x)^2,x)
```

output
```
(90*atanh(a*x)**2*a**7*x**7 - 252*atanh(a*x)**2*a**5*x**5 + 210*atanh(a*x)
**2*a**3*x**3 - 48*atanh(a*x)**2*a*x + 30*atanh(a*x)*a**6*x**6 - 81*atanh(
a*x)*a**4*x**4 + 48*atanh(a*x)*a**2*x**2 + 3*atanh(a*x) + 48*int(atanh(a*x
)**2,x)*a + 6*a**5*x**5 - 17*a**3*x**3 - 3*a*x)/(630*a**3)
```

3.206 $\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx = \frac{2(1-a^2x^2)}{45a^2} + \frac{(1-a^2x^2)^2}{60a^2} + \frac{8x\operatorname{arctanh}(ax)}{45a} + \frac{4x(1-a^2x^2)\operatorname{arctanh}(ax)}{45a} + \frac{x(1-a^2x^2)^2\operatorname{arctanh}(ax)}{15a} - \frac{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}{6a^2} + \frac{4\log(1-a^2x^2)}{45a^2}$$

output
```
2/45*(-a^2*x^2+1)/a^2+1/60*(-a^2*x^2+1)^2/a^2+8/45*x*arctanh(a*x)/a+4/45*x
*(-a^2*x^2+1)*arctanh(a*x)/a+1/15*x*(-a^2*x^2+1)^2*arctanh(a*x)/a-1/6*(-a^
2*x^2+1)^3*arctanh(a*x)^2/a^2+4/45*ln(-a^2*x^2+1)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.59

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 dx = \frac{-14a^2x^2+3a^4x^4+4ax(15-10a^2x^2+3a^4x^4)\operatorname{arctanh}(ax)+30(-1+a^2x^2)^3\operatorname{arctanh}(ax)^2+16\log(1-a^2x^2)}{180a^2}$$

input
```
Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(-14*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 10*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x
] + 30*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2 + 16*Log[1 - a^2*x^2])/(180*a^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6556, 6504, 6504, 6436, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)dx}{3a} - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{6a^2}$$

$$\Big\downarrow \text{6504}$$

$$\frac{\frac{4}{5}\int (1-a^2x^2)\operatorname{arctanh}(ax)dx + \frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax) + \frac{(1-a^2x^2)^2}{20a}}{3a} - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{6a^2}$$

$$\Big\downarrow \text{6504}$$

$$\frac{\frac{4}{5}\left(\frac{2}{3}\int \text{arctanh}(ax)dx+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)+\frac{(1-a^2x^2)^2}{20a}}{3a}-\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{6a^2}$$

$$\Big\downarrow \; 6436$$

$$\frac{\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)+\frac{(1-}{}}{3a}-\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{6a^2}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)+\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)+\frac{(1-a}{20}}{3a}-\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{6a^2}$$

input
```
Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
-1/6*((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/a^2 + ((1 - a^2*x^2)^2/(20*a) + (x*(
1 - a^2*x^2)^2*ArcTanh[a*x])/5 + (4*((1 - a^2*x^2)/(6*a) + (x*(1 - a^2*x^2
)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*x] + Log[1 - a^2*x^2]/(2*a)))/3))/5)/(
3*a)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

method	result
parallelrisch	$-\frac{-30 \operatorname{arctanh}(ax)^2 a^6 x^6 - 12 \operatorname{arctanh}(ax) a^5 x^5 + 90 a^4 x^4 \operatorname{arctanh}(ax)^2 - 3 a^4 x^4 + 40 a^3 x^3 \operatorname{arctanh}(ax) - 90 a^2 x^2 \operatorname{arctanh}}{180 a^2}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2 a^6 x^6}{6} - \frac{a^4 x^4 \operatorname{arctanh}(ax)^2}{2} + \frac{a^2 x^2 \operatorname{arctanh}(ax)^2}{2} - \frac{\operatorname{arctanh}(ax)^2}{6} + \frac{\operatorname{arctanh}(ax) a^5 x^5}{15} - \frac{2 a^3 x^3 \operatorname{arctanh}(ax)}{9} + \frac{ax \operatorname{arctan}}{3}}{a^2}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2 a^6 x^6}{6} - \frac{a^4 x^4 \operatorname{arctanh}(ax)^2}{2} + \frac{a^2 x^2 \operatorname{arctanh}(ax)^2}{2} - \frac{\operatorname{arctanh}(ax)^2}{6} + \frac{\operatorname{arctanh}(ax) a^5 x^5}{15} - \frac{2 a^3 x^3 \operatorname{arctanh}(ax)}{9} + \frac{ax \operatorname{arctan}}{3}}{a^2}$
parts	$\frac{a^4 \operatorname{arctanh}(ax)^2 x^6}{6} - \frac{x^4 a^2 \operatorname{arctanh}(ax)^2}{2} + \frac{\operatorname{arctanh}(ax)^2 x^2}{2} - \frac{\operatorname{arctanh}(ax)^2}{6a^2} - \frac{-\frac{\operatorname{arctanh}(ax) a^5 x^5}{5} + \frac{2 a^3 x^3 \operatorname{arctan}}{3}}{}$
risch	$\frac{(a^2 x^2 - 1)^3 \ln(ax+1)^2}{24 a^2} - \frac{(15 a^6 x^6 \ln(-ax+1) - 6 a^5 x^5 - 45 x^4 \ln(-ax+1) a^4 + 20 a^3 x^3 + 45 x^2 \ln(-ax+1) a^2 - 30 ax - 15 \ln}{180 a^2}$

input
```
int(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/180*(-30*arctanh(a*x)^2*a^6*x^6-12*arctanh(a*x)*a^5*x^5+90*a^4*x^4*arct
anh(a*x)^2-3*a^4*x^4+40*a^3*x^3*arctanh(a*x)-90*a^2*x^2*arctanh(a*x)^2+14*
a^2*x^2-60*a*x*arctanh(a*x)+30*arctanh(a*x)^2-32*ln(a*x-1)-32*arctanh(a*x)
)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84

$$\int x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{6\,a^4x^4-28\,a^2x^2+15\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^5x^5-10\,a^3x^3+15\,ax\right)\log\left(-\frac{ax}{ax}\right.}{360\,a^2}$$

input
```
integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/360*(6*a^4*x^4 - 28*a^2*x^2 + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*l
og(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*x
 + 1)/(a*x - 1)) + 32*log(a^2*x^2 - 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2\,dx$$
$$=\begin{cases}\frac{a^4x^6\operatorname{atanh}^2(ax)}{6}+\frac{a^3x^5\operatorname{atanh}(ax)}{15}-\frac{a^2x^4\operatorname{atanh}^2(ax)}{2}+\frac{a^2x^4}{60}-\frac{2ax^3\operatorname{atanh}(ax)}{9}+\frac{x^2\operatorname{atanh}^2(ax)}{2}-\frac{7x^2}{90}+\frac{x\operatorname{atanh}(ax)}{3a}+\cdots\\0\end{cases}$$

input
```
integrate(x*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

output
```
Piecewise((a**4*x**6*atanh(a*x)**2/6 + a**3*x**5*atanh(a*x)/15 - a**2*x**4
*atanh(a*x)**2/2 + a**2*x**4/60 - 2*a*x**3*atanh(a*x)/9 + x**2*atanh(a*x)*
*2/2 - 7*x**2/90 + x*atanh(a*x)/(3*a) + 8*log(x - 1/a)/(45*a**2) - atanh(a
*x)**2/(6*a**2) + 8*atanh(a*x)/(45*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = \frac{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}{6\,a^2} + \frac{\left(3\,a^2x^4 - 14\,x^2 + \frac{16\,\log(ax+1)}{a^2} + \frac{16\,\log(ax-1)}{a^2}\right)a + 4\,(3\,a^4x^5 - 10\,a^2x^3 + 15\,x)\operatorname{artanh}(ax)}{180\,a}$$

input
```
integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
1/6*(a^2*x^2 - 1)^3*arctanh(a*x)^2/a^2 + 1/180*((3*a^2*x^4 - 14*x^2 + 16*l
og(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 4*(3*a^4*x^5 - 10*a^2*x^3 + 15*
x)*arctanh(a*x))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(119) = 238$.

Time = 0.13 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.43

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = \frac{4}{45}\,a\left(\frac{2\left(\frac{10\,(ax+1)^2}{(ax-1)^2} - \frac{5\,(ax+1)}{ax-1} + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^5a^3}{(ax-1)^5} - \frac{5\,(ax+1)^4a^3}{(ax-1)^4} + \frac{10\,(ax+1)^3a^3}{(ax-1)^3} - \frac{10\,(ax+1)^2a^3}{(ax-1)^2} + \frac{5\,(ax+1)a^3}{ax-1} - a^3} + \frac{}{\left(\frac{(ax+1)^6a^3}{(ax-1)^6} - \frac{6\,(ax+1)^5a^3}{(ax-1)^5} + \frac{15\,(a}{(a}\right.}\right.$$

input
```
integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
4/45*a*(2*(10*(a*x + 1)^2/(a*x - 1)^2 - 5*(a*x + 1)/(a*x - 1) + 1)*log(-(a
*x + 1)/(a*x - 1))/((a*x + 1)^5*a^3/(a*x - 1)^5 - 5*(a*x + 1)^4*a^3/(a*x -
 1)^4 + 10*(a*x + 1)^3*a^3/(a*x - 1)^3 - 10*(a*x + 1)^2*a^3/(a*x - 1)^2 +
5*(a*x + 1)*a^3/(a*x - 1) - a^3) + 30*(a*x + 1)^3*log(-(a*x + 1)/(a*x - 1)
)^2/(((a*x + 1)^6*a^3/(a*x - 1)^6 - 6*(a*x + 1)^5*a^3/(a*x - 1)^5 + 15*(a*
x + 1)^4*a^3/(a*x - 1)^4 - 20*(a*x + 1)^3*a^3/(a*x - 1)^3 + 15*(a*x + 1)^2
*a^3/(a*x - 1)^2 - 6*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)^3) - (2*(a*x
 + 1)^3/(a*x - 1)^3 - 7*(a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1))/(
(a*x + 1)^4*a^3/(a*x - 1)^4 - 4*(a*x + 1)^3*a^3/(a*x - 1)^3 + 6*(a*x + 1)^
2*a^3/(a*x - 1)^2 - 4*(a*x + 1)*a^3/(a*x - 1) + a^3) - 2*log(-(a*x + 1)/(a
*x - 1) + 1)/a^3 + 2*log(-(a*x + 1)/(a*x - 1))/a^3)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx = \frac{x^2\,\mathrm{atanh}(a\,x)^2}{2} - \frac{\mathrm{atanh}(a\,x)^2}{6\,a^2} - \frac{7\,x^2}{90} + \frac{4\,\ln\left(a^2\,x^2-1\right)}{45\,a^2} + \frac{a^2\,x^4}{60} + \frac{x\,\mathrm{atanh}(a\,x)}{3\,a} - \frac{2\,a\,x^3\,\mathrm{atanh}(a\,x)}{9} + \frac{a^3\,x^5\,\mathrm{atanh}(a\,x)}{15} - \frac{a^2\,x^4\,\mathrm{atanh}(a\,x)^2}{2} + \frac{a^4\,x^6\,\mathrm{atanh}(a\,x)^2}{6}$$

input
```
int(x*atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

output
```
(x^2*atanh(a*x)^2)/2 - atanh(a*x)^2/(6*a^2) - (7*x^2)/90 + (4*log(a^2*x^2
- 1))/(45*a^2) + (a^2*x^4)/60 + (x*atanh(a*x))/(3*a) - (2*a*x^3*atanh(a*x)
)/9 + (a^3*x^5*atanh(a*x))/15 - (a^2*x^4*atanh(a*x)^2)/2 + (a^4*x^6*atanh(
a*x)^2)/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$$
$$= \frac{30atanh(ax)^2 a^6x^6 - 90atanh(ax)^2 a^4x^4 + 90atanh(ax)^2 a^2x^2 - 30atanh(ax)^2 + 12atanh(ax) a^5x^5 - 40}{180a^2}$$

input
```
int(x*(-a^2*x^2+1)^2*atanh(a*x)^2,x)
```

output
```
(30*atanh(a*x)**2*a**6*x**6 - 90*atanh(a*x)**2*a**4*x**4 + 90*atanh(a*x)**
2*a**2*x**2 - 30*atanh(a*x)**2 + 12*atanh(a*x)*a**5*x**5 - 40*atanh(a*x)*a
**3*x**3 + 60*atanh(a*x)*a*x + 32*atanh(a*x) + 32*log(a**2*x - a) + 3*a**4
*x**4 - 14*a**2*x**2)/(180*a**2)
```

3.207 $\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 171

$$\begin{aligned}\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2\,dx =& -\frac{11x}{30}+\frac{a^2x^3}{30}+\frac{4(1-a^2x^2)\operatorname{arctanh}(ax)}{15a}\\ &+\frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{10a}+\frac{8\operatorname{arctanh}(ax)^2}{15a}\\ &+\frac{8}{15}x\operatorname{arctanh}(ax)^2+\frac{4}{15}x(1-a^2x^2)\operatorname{arctanh}(ax)^2\\ &+\frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2\\ &-\frac{16\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{15a}-\frac{8\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a}\end{aligned}$$

output
```
-11/30*x+1/30*a^2*x^3+4/15*(-a^2*x^2+1)*arctanh(a*x)/a+1/10*(-a^2*x^2+1)^2
*arctanh(a*x)/a+8/15*arctanh(a*x)^2/a+8/15*x*arctanh(a*x)^2+4/15*x*(-a^2*x
^2+1)*arctanh(a*x)^2+1/5*x*(-a^2*x^2+1)^2*arctanh(a*x)^2-16/15*arctanh(a*x
)*ln(2/(-a*x+1))/a-8/15*polylog(2,1-2/(-a*x+1))/a
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.58

$$\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 dx = \frac{ax(-11+a^2x^2)+2(-1+ax)^3(8+9ax+3a^2x^2)\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\left(11-14a^2x^2+3a^4x^4-\right.}{30a}$$

input
```
Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(a*x*(-11 + a^2*x^2) + 2*(-1 + a*x)^3*(8 + 9*a*x + 3*a^2*x^2)*ArcTanh[a*x]
^2 + ArcTanh[a*x]*(11 - 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^(-2*ArcTanh[
a*x])]) + 16*PolyLog[2, -E^(-2*ArcTanh[a*x])]))/(30*a)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6506, 2009, 6506, 24, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2 dx$$

$$\Big\downarrow \text{6506}$$

$$\frac{4}{5}\int (1-a^2x^2)\operatorname{arctanh}(ax)^2 dx-\frac{1}{10}\int (1-a^2x^2)dx+\frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2+\frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{10a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}\int (1-a^2x^2)\operatorname{arctanh}(ax)^2 dx+\frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2+\frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6506}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \text{arctanh}(ax)^2dx-\frac{\int 1dx}{3}+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \text{arctanh}(ax)^2dx+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}-\frac{x}{3}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6436}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)\text{arctanh}(ax)}{3a}-\frac{x}{3}\right)$$
$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6546}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)}{3}\right.$$
$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6470}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctan}\right.$$
$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arcta}$$

$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arc}$$

$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)$$

input
```
Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

output
```
(-x + (a^2*x^3)/3)/10 + ((1 - a^2*x^2)^2*ArcTanh[a*x])/(10*a) + (x*(1 - a^
2*x^2)^2*ArcTanh[a*x]^2)/5 + (4*(-1/3*x + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*
a) + (x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/3 + (2*(x*ArcTanh[a*x]^2 - 2*a*(-1/2
*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 -
2/(1 - a*x)]/(2*a))/a)))/3))/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\frac{2\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\operatorname{arctanh}(ax)^2 ax+\frac{a^4 x^4 \operatorname{arctanh}(ax)}{10}-\frac{7a^2 x^2 \operatorname{arctanh}(ax)}{15}+\frac{8 \operatorname{arctanh}(ax)\ln(ax-1)}{15}+$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\frac{2\operatorname{arctanh}(ax)^2 a^3 x^3}{3}+\operatorname{arctanh}(ax)^2 ax+\frac{a^4 x^4 \operatorname{arctanh}(ax)}{10}-\frac{7a^2 x^2 \operatorname{arctanh}(ax)}{15}+\frac{8 \operatorname{arctanh}(ax)\ln(ax-1)}{15}+$
parts	$\frac{\operatorname{arctanh}(ax)^2 a^4 x^5}{5}-\frac{2x^3 a^2 \operatorname{arctanh}(ax)^2}{3}+x\operatorname{arctanh}(ax)^2+\frac{a^3 \operatorname{arctanh}(ax)x^4}{10}-\frac{7x^2 \operatorname{arctanh}(ax)a}{15}+$
risch	$-\frac{11x}{30}+\frac{a^3\ln(ax+1)x^4}{20}-\frac{a^3\ln(-ax+1)x^4}{20}+\frac{a^2x^3}{30}+\frac{7a\ln(-ax+1)x^2}{30}+\frac{7\ln(-ax+1)\ln(ax+1)}{30a}-\frac{7\ln(-$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*arctanh(a*x)^2*a^5*x^5-2/3*arctanh(a*x)^2*a^3*x^3+arctanh(a*x)^2*
a*x+1/10*a^4*x^4*arctanh(a*x)-7/15*a^2*x^2*arctanh(a*x)+8/15*arctanh(a*x)*
ln(a*x-1)+8/15*arctanh(a*x)*ln(a*x+1)+1/30*a^3*x^3-11/30*a*x-11/60*ln(a*x-
1)+11/60*ln(a*x+1)+2/15*ln(a*x-1)^2-8/15*dilog(1/2*a*x+1/2)-4/15*ln(a*x-1)
*ln(1/2*a*x+1/2)+4/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-2/15*ln
(a*x+1)^2)
```

Fricas [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2\,dx=\int \left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^2\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2\,dx=\int (ax-1)^2\,(ax+1)^2\operatorname{atanh}^2(ax)\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2\,dx$$
$$=\frac{1}{60}a^2\left(\frac{2\,a^3x^3-22\,ax-8\,\log(ax+1)^2+16\,\log(ax+1)\log(ax-1)+8\,\log(ax-1)^2-11\,\log(ax-}{a^3}\right.$$
$$+\frac{1}{30}\left(3\,a^2x^4-14\,x^2+\frac{16\,\log(ax+1)}{a^2}+\frac{16\,\log(ax-1)}{a^2}\right)a\operatorname{artanh}(ax)$$
$$+\frac{1}{15}\left(3\,a^4x^5-10\,a^2x^3+15\,x\right)\operatorname{artanh}(ax)^2$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
1/60*a^2*((2*a^3*x^3 - 22*a*x - 8*log(a*x + 1)^2 + 16*log(a*x + 1)*log(a*x
 - 1) + 8*log(a*x - 1)^2 - 11*log(a*x - 1))/a^3 - 32*(log(a*x - 1)*log(1/2
*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 + 11*log(a*x + 1)/a^3) + 1/30*(3*
a^2*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a*arctanh(a*
x) + 1/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)^2
```

Giac [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2 \, dx = \int \left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^2 \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2 \, dx = \int \operatorname{atanh}(a\,x)^2 \left(a^2\,x^2-1\right)^2 \, dx$$

input
```
int(atanh(a*x)^2*(a^2*x^2 - 1)^2,x)
```

output
```
int(atanh(a*x)^2*(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2 \, dx$$
$$= \frac{6atanh(ax)^2 a^5x^5 - 20atanh(ax)^2 a^3x^3 + 30atanh(ax)^2 ax + 3atanh(ax) a^4x^4 - 14atanh(ax) a^2x^2 + 11}{30a}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2,x)
```

output
```
(6*atanh(a*x)**2*a**5*x**5 - 20*atanh(a*x)**2*a**3*x**3 + 30*atanh(a*x)**2
*a*x + 3*atanh(a*x)*a**4*x**4 - 14*atanh(a*x)*a**2*x**2 + 11*atanh(a*x) +
32*int((atanh(a*x)*x)/(a**2*x**2 - 1),x)*a**2 + a**3*x**3 - 11*a*x)/(30*a)
```

3.208 $\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 186

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} dx = \frac{a^2x^2}{12} - \frac{3}{2}ax\operatorname{arctanh}(ax) + \frac{1}{6}a^3x^3\operatorname{arctanh}(ax) + \frac{3}{4}\operatorname{arctanh}(ax)^2 - a^2x^2\operatorname{arctanh}(ax)^2 + \frac{1}{4}a^4x^4\operatorname{arctanh}(ax)^2 + 2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right) - \frac{2}{3}\log(1-a^2x^2) - \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right) + \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)$$

output
```
1/12*a^2*x^2-3/2*a*x*arctanh(a*x)+1/6*a^3*x^3*arctanh(a*x)+3/4*arctanh(a*x
)^2-a^2*x^2*arctanh(a*x)^2+1/4*a^4*x^4*arctanh(a*x)^2-2*arctanh(a*x)^2*arc
tanh(-1+2/(-a*x+1))-2/3*ln(-a^2*x^2+1)-arctanh(a*x)*polylog(2,1-2/(-a*x+1)
)+arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2*polylog(3,1-2/(-a*x+1))-1/2*po
lylog(3,-1+2/(-a*x+1))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} \, dx = & \frac{a^2x^2}{12} - 2ax\operatorname{arctanh}(ax) + \frac{1}{6}ax(3+a^2x^2)\operatorname{arctanh}(ax) \\ & - (-1+a^2x^2)\operatorname{arctanh}(ax)^2 \\ & + \frac{1}{4}(-1+a^4x^4)\operatorname{arctanh}(ax)^2 - \frac{2}{3}\operatorname{arctanh}(ax)^3 \\ & - \operatorname{arctanh}(ax)^2\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right) \\ & + \operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) - \frac{2}{3}\log\left(1-a^2x^2\right) \\ & + \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) \\ & + \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) \\ & + \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(ax)}\right) \\ & - \frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)\end{aligned}$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

output
```
(a^2*x^2)/12 - 2*a*x*ArcTanh[a*x] + (a*x*(3 + a^2*x^2)*ArcTanh[a*x])/6 - (
-1 + a^2*x^2)*ArcTanh[a*x]^2 + ((-1 + a^4*x^4)*ArcTanh[a*x]^2)/4 - (2*ArcT
anh[a*x]^3)/3 - ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]
^2*Log[1 - E^(2*ArcTanh[a*x])] - (2*Log[1 - a^2*x^2])/3 + ArcTanh[a*x]*Pol
yLog[2, -E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])
] + PolyLog[3, -E^(-2*ArcTanh[a*x])]/2 - PolyLog[3, E^(2*ArcTanh[a*x])]/2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(a^4x^3\operatorname{arctanh}(ax)^2 - 2a^2x\operatorname{arctanh}(ax)^2 + \frac{\operatorname{arctanh}(ax)^2}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^4x^4\operatorname{arctanh}(ax)^2 + \frac{1}{6}a^3x^3\operatorname{arctanh}(ax) - a^2x^2\operatorname{arctanh}(ax)^2 + \frac{a^2x^2}{12} - \frac{2}{3}\log(1-a^2x^2) - \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2, 1-\frac{2}{1-ax}\right) + \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2, \frac{2}{1-ax}-1\right) - \frac{3}{2}ax\operatorname{arctanh}(ax) + \frac{3}{4}\operatorname{arctanh}(ax)^2 + 2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3, 1-\frac{2}{1-ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3, \frac{2}{1-ax}-1\right)$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

output
```
(a^2*x^2)/12 - (3*a*x*ArcTanh[a*x])/2 + (a^3*x^3*ArcTanh[a*x])/6 + (3*ArcT
anh[a*x]^2)/4 - a^2*x^2*ArcTanh[a*x]^2 + (a^4*x^4*ArcTanh[a*x]^2)/4 + 2*Ar
cTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 - ArcTanh[a
*x]*PolyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)
] + PolyLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.38 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.94

method	result
derivativedivides	$\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}-a^2x^2\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)^2\ln(ax)-\operatorname{arctanh}(ax)^2\ln\left(\frac{(ax+1}{-a^2x^2}\right.$
default	$\frac{a^4x^4\operatorname{arctanh}(ax)^2}{4}-a^2x^2\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)^2\ln(ax)-\operatorname{arctanh}(ax)^2\ln\left(\frac{(ax+1}{-a^2x^2}\right.$
parts	Expression too large to display

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*x^4*arctanh(a*x)^2-a^2*x^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-a
rctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1+(a*x+1)/(-a
^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*p
olylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^
2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog
(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2
+1))+1/2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+4/3*ln((a*x+1)^2/(-a^2*x^2+1)+
1)-1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2
-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-(a*x+1)*arctanh(a*x)+1
/6*a*x-1/6-1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/
(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+3/4*arctanh(a*
x)^2+1/12*(a*x-1)^2+1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2
/(a^2*x^2-1)+1))^3*arctanh(a*x)^2+1/6*(a^2*x^2-4*a*x+7)*(a*x+1)*arctanh(a*
x)+1/2*(a*x-3)*(a*x+1)*arctanh(a*x)+1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1
)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)
/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*x)^2
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x}\,dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} \, dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x} \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="maxima")
```

output
```
1/16*(a^4*x^4 - 4*a^2*x^2)*log(-a*x + 1)^2 - 1/4*integrate(-1/2*(2*(a^5*x^
5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (a^5*x^5 -
 4*a^3*x^3 + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a
*x + 1))*log(-a*x + 1))/(a*x^2 - x), x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} \, dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x} \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x} \, dx = \int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2}{x} \, dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x}\,dx = \frac{atanh(ax)^2 a^4x^4}{4} - atanh(ax)^2 a^2x^2 + \frac{3atanh(ax)^2}{4} + \frac{atanh(ax)\,a^3x^3}{6} - \frac{3atanh(ax)\,ax}{2} - \frac{4atanh(ax)}{3} + \int \frac{atanh(ax)^2}{x}dx - \frac{4\log(a^2x-a)}{3} + \frac{a^2x^2}{12}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x,x)
```

output
```
(3*atanh(a*x)**2*a**4*x**4 - 12*atanh(a*x)**2*a**2*x**2 + 9*atanh(a*x)**2
+ 2*atanh(a*x)*a**3*x**3 - 18*atanh(a*x)*a*x - 16*atanh(a*x) + 12*int(atan
h(a*x)**2/x,x) - 16*log(a**2*x - a) + a**2*x**2)/12
```

3.209 $\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\begin{aligned}\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2} dx = {} & \frac{a^2x}{3} - \frac{1}{3}a\operatorname{arctanh}(ax) + \frac{1}{3}a^3x^2\operatorname{arctanh}(ax) \\ & - \frac{2}{3}a\operatorname{arctanh}(ax)^2 - \frac{\operatorname{arctanh}(ax)^2}{x} \\ & - 2a^2x\operatorname{arctanh}(ax)^2 + \frac{1}{3}a^4x^3\operatorname{arctanh}(ax)^2 \\ & + \frac{10}{3}a\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right) \\ & + 2a\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) \\ & + \frac{5}{3}a\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right) \\ & - a\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output

```
1/3*a^2*x-1/3*a*arctanh(a*x)+1/3*a^3*x^2*arctanh(a*x)-2/3*a*arctanh(a*x)^2
-arctanh(a*x)^2/x-2*a^2*x*arctanh(a*x)^2+1/3*a^4*x^3*arctanh(a*x)^2+10/3*a
*arctanh(a*x)*ln(2/(-a*x+1))+2*a*arctanh(a*x)*ln(2-2/(a*x+1))+5/3*a*polylo
g(2,1-2/(-a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.17

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2} dx = -2a\operatorname{arctanh}(ax) \left(-\operatorname{arctanh}(ax) + ax\operatorname{arctanh}(ax) - 2\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right)\right) - 2a\,\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) + \frac{1}{3}a\Big(ax - \operatorname{arctanh}(ax)^2 + ax\operatorname{arctanh}(ax)^2 - (1-a^2x^2)\operatorname{arctanh}(ax)(1+ax\operatorname{arctanh}(ax)) - 2\operatorname{arctanh}(ax)\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right) + \operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right)\Big) + a\left(\operatorname{arctanh}(ax)\left(\operatorname{arctanh}(ax) - \frac{\operatorname{arctanh}(ax)}{ax} + 2\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)\right) - \operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

output
```
-2*a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcT
anh[a*x])]) - 2*a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + (a*(a*x - ArcTanh[a*x
]^2 + a*x*ArcTanh[a*x]^2 - (1 - a^2*x^2)*ArcTanh[a*x]*(1 + a*x*ArcTanh[a*x
]) - 2*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] + PolyLog[2, -E^(-2*ArcTa
nh[a*x])]))/3 + a*(ArcTanh[a*x]*(ArcTanh[a*x] - ArcTanh[a*x]/(a*x) + 2*Log
[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2, E^(-2*ArcTanh[a*x])])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \text{arctanh}(ax)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 6574}$$

$$\int \left(a^4x^2\text{arctanh}(ax)^2-2a^2\text{arctanh}(ax)^2+\frac{\text{arctanh}(ax)^2}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^4x^3\text{arctanh}(ax)^2+\frac{1}{3}a^3x^2\text{arctanh}(ax)-2a^2x\text{arctanh}(ax)^2+\frac{a^2x}{3}-\frac{2}{3}a\text{arctanh}(ax)^2-\frac{1}{3}a\text{arctanh}(ax)-\frac{\text{arctanh}(ax)^2}{x}+\frac{10}{3}a\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)+2a\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)+\frac{5}{3}a\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

output
```
(a^2*x)/3 - (a*ArcTanh[a*x])/3 + (a^3*x^2*ArcTanh[a*x])/3 - (2*a*ArcTanh[a
*x]^2)/3 - ArcTanh[a*x]^2/x - 2*a^2*x*ArcTanh[a*x]^2 + (a^4*x^3*ArcTanh[a*
x]^2)/3 + (10*a*ArcTanh[a*x]*Log[2/(1 - a*x)])/3 + 2*a*ArcTanh[a*x]*Log[2
- 2/(1 + a*x)] + (5*a*PolyLog[2, 1 - 2/(1 - a*x)])/3 - a*PolyLog[2, -1 + 2
/(1 + a*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.30

method	result
derivativedivides	$a\left(\frac{\operatorname{arctanh}(ax)^2 a^3x^3}{3} - 2\operatorname{arctanh}(ax)^2 ax - \frac{\operatorname{arctanh}(ax)^2}{ax} + \frac{a^2x^2\operatorname{arctanh}(ax)}{3} + 2\operatorname{arctanh}(ax)\right.$
default	$a\left(\frac{\operatorname{arctanh}(ax)^2 a^3x^3}{3} - 2\operatorname{arctanh}(ax)^2 ax - \frac{\operatorname{arctanh}(ax)^2}{ax} + \frac{a^2x^2\operatorname{arctanh}(ax)}{3} + 2\operatorname{arctanh}(ax)\right.$
parts	$\frac{a^4x^3\operatorname{arctanh}(ax)^2}{3} - 2a^2x\operatorname{arctanh}(ax)^2 - \frac{\operatorname{arctanh}(ax)^2}{x} + \frac{a^3x^2\operatorname{arctanh}(ax)}{3} + 2a\operatorname{arctanh}(ax)$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*arctanh(a*x)^2*a^3*x^3-2*arctanh(a*x)^2*a*x-arctanh(a*x)^2/a/x+1/3*
a^2*x^2*arctanh(a*x)+2*arctanh(a*x)*ln(a*x)-8/3*arctanh(a*x)*ln(a*x-1)-8/3
*arctanh(a*x)*ln(a*x+1)+1/3*a*x+1/6*ln(a*x-1)-1/6*ln(a*x+1)-dilog(a*x)-dil
og(a*x+1)-ln(a*x)*ln(a*x+1)-2/3*ln(a*x-1)^2+8/3*dilog(1/2*a*x+1/2)+4/3*ln(
a*x-1)*ln(1/2*a*x+1/2)-4/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+2/
3*ln(a*x+1)^2)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2}\,dx = \int \frac{{\left(a^{2} x^{2} - 1\right)}^{2} \operatorname{artanh}\left(a x\right)^{2}}{x^{2}}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2}\,dx = \int \frac{\left(ax-1\right)^{2} \left(ax+1\right)^{2} \operatorname{atanh}^{2}{\left(ax \right)}}{x^{2}}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**2,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.28

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2}\,dx$$
$$= \frac{1}{6}\,a^{2}\left(\frac{2\,\left(ax + 2\,\log\left(ax+1\right)^{2} - 4\,\log\left(ax+1\right)\log\left(ax-1\right) - 2\,\log\left(ax-1\right)^{2}\right)}{a} + \frac{16\,\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\right.\right.}{}\right.$$
$$+ \frac{1}{3}\left(a^{2}x^{2} - 8\,\log\left(ax+1\right) - 8\,\log\left(ax-1\right) + 6\,\log\left(x\right)\right)a\operatorname{artanh}\left(ax\right)$$
$$+ \frac{1}{3}\left(a^{4}x^{3} - 6\,a^{2}x - \frac{3}{x}\right)\operatorname{artanh}\left(ax\right)^{2}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/6*a^2*(2*(a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x - 1) - 2*log(a
*x - 1)^2)/a + 16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2)
)/a - 6*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 6*(log(-a*x + 1)*log(x) +
dilog(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) + 1/3*(a^2*x^2 - 8*log(a*
x + 1) - 8*log(a*x - 1) + 6*log(x))*a*arctanh(a*x) + 1/3*(a^4*x^3 - 6*a^2*
x - 3/x)*arctanh(a*x)^2
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2} dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^2} dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2} dx = \int \frac{\operatorname{atanh}(a\,x)^2 (a^2\,x^2-1)^2}{x^2} dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^2,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^2}\,dx$$
$$= \frac{atanh(ax)^2 a^4x^4 - 6atanh(ax)^2 a^2x^2 - 3atanh(ax)^2 + atanh(ax)\,a^3x^3 - atanh(ax)\,ax - 6\left(\int \frac{atanh(ax)}{a^2x^3-x}dx\right.}{3x}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^2,x)
```

output
```
(atanh(a*x)**2*a**4*x**4 - 6*atanh(a*x)**2*a**2*x**2 - 3*atanh(a*x)**2 + a
tanh(a*x)*a**3*x**3 - atanh(a*x)*a*x - 6*int(atanh(a*x)/(a**2*x**3 - x),x)
*a*x - 10*int((atanh(a*x)*x)/(a**2*x**2 - 1),x)*a**3*x + a**2*x**2)/(3*x)
```

3.210 $\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3} dx = -\frac{a\operatorname{arctanh}(ax)}{x} + a^3x\operatorname{arctanh}(ax) - \frac{\operatorname{arctanh}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\operatorname{arctanh}(ax)^2 - 4a^2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right) + a^2\log(x) + 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right) - 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right) - a^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right) + a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)$$

output
```
-a*arctanh(a*x)/x+a^3*x*arctanh(a*x)-1/2*arctanh(a*x)^2/x^2+1/2*a^4*x^2*ar
ctanh(a*x)^2+4*a^2*arctanh(a*x)^2*arctanh(-1+2/(-a*x+1))+a^2*ln(x)+2*a^2*a
rctanh(a*x)*polylog(2,1-2/(-a*x+1))-2*a^2*arctanh(a*x)*polylog(2,-1+2/(-a*
x+1))-a^2*polylog(3,1-2/(-a*x+1))+a^2*polylog(3,-1+2/(-a*x+1))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.23

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3} dx = -\frac{a\operatorname{arctanh}(ax)}{x} + a^3x\operatorname{arctanh}(ax) + \frac{1}{2}a^2(-1+a^2x^2)\operatorname{arctanh}(ax)^2 + \frac{(-1+a^2x^2)\operatorname{arctanh}(ax)^2}{2x^2} + \frac{4}{3}a^2\operatorname{arctanh}(ax)^3 + 2a^2\operatorname{arctanh}(ax)^2\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right) - 2a^2\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + a^2\log(x) - 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) - 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - a^2\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(ax)}\right) + a^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

output
```
-((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] + (a^2*(-1 + a^2*x^2)*ArcTanh[a
*x]^2)/2 + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) + (4*a^2*ArcTanh[a*x]^3
)/3 + 2*a^2*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 2*a^2*ArcTanh[a*
x]^2*Log[1 - E^(2*ArcTanh[a*x])] + a^2*Log[x] - 2*a^2*ArcTanh[a*x]*PolyLog
[2, -E^(-2*ArcTanh[a*x])] - 2*a^2*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x
])] - a^2*PolyLog[3, -E^(-2*ArcTanh[a*x])] + a^2*PolyLog[3, E^(2*ArcTanh[a
*x])]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3} \, dx$$

$$\Big\downarrow \; 6574$$

$$\int \left(a^4 x \operatorname{arctanh}(ax)^2 - \frac{2a^2 \operatorname{arctanh}(ax)^2}{x} + \frac{\operatorname{arctanh}(ax)^2}{x^3}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}a^4x^2\operatorname{arctanh}(ax)^2 + a^3x\operatorname{arctanh}(ax) + 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2, 1-\frac{2}{1-ax}\right) - 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2, \frac{2}{1-ax}-1\right) - 4a^2\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right) - a^2\operatorname{PolyLog}\left(3, 1-\frac{2}{1-ax}\right) + a^2\operatorname{PolyLog}\left(3, \frac{2}{1-ax}-1\right) + a^2\log(x) - \frac{\operatorname{arctanh}(ax)^2}{2x^2} - \frac{a\operatorname{arctanh}(ax)}{x}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

output
```
-((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] - ArcTanh[a*x]^2/(2*x^2) + (a^4
*x^2*ArcTanh[a*x]^2)/2 - 4*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a
^2*Log[x] + 2*a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - 2*a^2*ArcTanh
[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] - a^2*PolyLog[3, 1 - 2/(1 - a*x)] + a^2
*PolyLog[3, -1 + 2/(1 - a*x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.24 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.81

method	result
derivativedivides	$a^2\left(\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2} - 2\operatorname{arctanh}(ax)^2\ln(ax) - \frac{\operatorname{arctanh}(ax)^2}{2a^2x^2} + 2\operatorname{arctanh}(ax)^2\ln\left(\frac{(ax+1)}{-a^2x^2+}\right.\right.$
default	$a^2\left(\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2} - 2\operatorname{arctanh}(ax)^2\ln(ax) - \frac{\operatorname{arctanh}(ax)^2}{2a^2x^2} + 2\operatorname{arctanh}(ax)^2\ln\left(\frac{(ax+1)}{-a^2x^2+}\right.\right.$
parts	Expression too large to display

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*a^2*x^2*arctanh(a*x)^2-2*arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^
2/a^2/x^2+2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-2*arctanh(a*x)^2*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-4*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))+4*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)^2*ln(1
-(a*x+1)/(-a^2*x^2+1)^(1/2))-4*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)
^(1/2))+4*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-
(a*x+1)^2/(-a^2*x^2+1))-polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-I*Pi*csgn(I*(-(
a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2-ln((a
*x+1)^2/(-a^2*x^2+1)+1)-1/2*(a*x-(-a^2*x^2+1)^(1/2)+1)/a/x*arctanh(a*x)-1/
2*arctanh(a*x)*(a*x+(-a^2*x^2+1)^(1/2)+1)/a/x+(a*x+1)*arctanh(a*x)+I*Pi*cs
gn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+
1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+ln(
1+(a*x+1)/(-a^2*x^2+1)^(1/2))+I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn
(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2
-I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1)
)*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*
x)^2)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^3}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**3,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
-1/16*(2*x^2*log(-a*x + 1) - a*((a*x^2 + 2*x)/a^2 + 2*log(a*x - 1)/a^3))*a
^4 - 1/2*a^4*integrate(x*log(a*x + 1)*log(-a*x + 1), x) + 1/4*a^3*integrat
e(a*x*log(a*x + 1)^2, x) + 1/4*a^3*integrate(log(a*x + 1)^2/(a^3*x^3), x)
+ 1/4*(a*x - (a*x - 1)*log(-a*x + 1) - 1)*a^2 - 1/2*a^2*integrate(log(a*x
+ 1)^2/x, x) + a^2*integrate(log(a*x + 1)*log(-a*x + 1)/x, x) - 1/4*a^2*in
tegrate(log(-a*x + 1)/x, x) - 1/4*(a*(log(a*x - 1) - log(x)) - log(-a*x +
1)/x)*a + 1/8*(a^4*x^4 - 1)*log(-a*x + 1)^2/x^2 - 1/2*integrate(log(a*x +
1)*log(-a*x + 1)/x^3, x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2}{x^3}\,dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^3,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^3}\,dx$$
$$= \frac{atanh(ax)^2\, a^4x^4 - atanh(ax)^2 + 2atanh(ax)\, a^3x^3 - 2atanh(ax)\, ax - 4\left(\int \frac{atanh(ax)^2}{x}dx\right) a^2x^2 + 2\log(x)\, a}{2x^2}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^3,x)
```

output
```
(atanh(a*x)**2*a**4*x**4 - atanh(a*x)**2 + 2*atanh(a*x)*a**3*x**3 - 2*atan
h(a*x)*a*x - 4*int(atanh(a*x)**2/x,x)*a**2*x**2 + 2*log(x)*a**2*x**2)/(2*x
**2)
```

3.211 $\int \frac{(1-a^2x^2)^2\textbf{arctanh}(ax)^2}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{x^4}\,dx = -\frac{a^2}{3x}+\frac{1}{3}a^3\operatorname{arctanh}(ax)-\frac{a\operatorname{arctanh}(ax)}{3x^2} -\frac{2}{3}a^3\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{3x^3} +\frac{2a^2\operatorname{arctanh}(ax)^2}{x}+a^4x\operatorname{arctanh}(ax)^2 -2a^3\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right) -\frac{10}{3}a^3\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) -a^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right) +\frac{5}{3}a^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output

```
-1/3*a^2/x+1/3*a^3*arctanh(a*x)-1/3*a*arctanh(a*x)/x^2-2/3*a^3*arctanh(a*x
)^2-1/3*arctanh(a*x)^2/x^3+2*a^2*arctanh(a*x)^2/x+a^4*x*arctanh(a*x)^2-2*a
^3*arctanh(a*x)*ln(2/(-a*x+1))-10/3*a^3*arctanh(a*x)*ln(2-2/(a*x+1))-a^3*p
olylog(2,1-2/(-a*x+1))+5/3*a^3*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4}\,dx = \frac{1}{3}\left(-\frac{a^2}{x}+a^3\operatorname{arctanh}(ax)-\frac{a\operatorname{arctanh}(ax)}{x^2} - 8a^3\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{x^3} + \frac{6a^2\operatorname{arctanh}(ax)^2}{x}+3a^4x\operatorname{arctanh}(ax)^2 - 10a^3\operatorname{arctanh}(ax)\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right) - 6a^3\operatorname{arctanh}(ax)\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right) + 3a^3\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) + 5a^3\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

output
```
(-(a^2/x) + a^3*ArcTanh[a*x] - (a*ArcTanh[a*x])/x^2 - 8*a^3*ArcTanh[a*x]^2
 - ArcTanh[a*x]^2/x^3 + (6*a^2*ArcTanh[a*x]^2)/x + 3*a^4*x*ArcTanh[a*x]^2
- 10*a^3*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 6*a^3*ArcTanh[a*x]*Lo
g[1 + E^(-2*ArcTanh[a*x])] + 3*a^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 5*a^
3*PolyLog[2, E^(-2*ArcTanh[a*x])])/3
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4}\,dx$$

$$\Big\downarrow \text{ 6574}$$

$$\int \left(a^4 \text{arctanh}(ax)^2 - \frac{2a^2 \text{arctanh}(ax)^2}{x^2} + \frac{\text{arctanh}(ax)^2}{x^4}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4 x \text{arctanh}(ax)^2 - \frac{2}{3}a^3 \text{arctanh}(ax)^2 + \frac{1}{3}a^3 \text{arctanh}(ax) - 2a^3 \text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right) - \frac{10}{3}a^3 \text{arctanh}(ax)\log\left(2 - \frac{2}{ax+1}\right) - a^3 \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right) + \frac{5}{3}a^3 \operatorname{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + \frac{2a^2 \text{arctanh}(ax)^2}{x} - \frac{a^2}{3x} - \frac{\text{arctanh}(ax)^2}{3x^3} - \frac{a \text{arctanh}(ax)}{3x^2}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

output
```
-1/3*a^2/x + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcT
anh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (2*a^2*ArcTanh[a*x]^2)/x + a^4*x*
ArcTanh[a*x]^2 - 2*a^3*ArcTanh[a*x]*Log[2/(1 - a*x)] - (10*a^3*ArcTanh[a*x
]*Log[2 - 2/(1 + a*x)])/3 - a^3*PolyLog[2, 1 - 2/(1 - a*x)] + (5*a^3*PolyL
og[2, -1 + 2/(1 + a*x)])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.25

method	result
derivativedivides	$a^3\left(\operatorname{arctanh}(ax)^2 ax+\frac{2\operatorname{arctanh}(ax)^2}{ax}-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{3a^2x^2}-\frac{10\operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{8\,\mathrm{a}}{}\right.$
default	$a^3\left(\operatorname{arctanh}(ax)^2 ax+\frac{2\operatorname{arctanh}(ax)^2}{ax}-\frac{\operatorname{arctanh}(ax)^2}{3a^3x^3}-\frac{\operatorname{arctanh}(ax)}{3a^2x^2}-\frac{10\operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{8\,\mathrm{a}}{}\right.$
parts	$a^4x\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{3x^3}+\frac{2a^2\operatorname{arctanh}(ax)^2}{x}-\frac{a\operatorname{arctanh}(ax)}{3x^2}-\frac{10a^3\operatorname{arctanh}(ax)\ln(ax)}{3}+\frac{8}{}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(arctanh(a*x)^2*a*x+2*arctanh(a*x)^2/a/x-1/3*arctanh(a*x)^2/a^3/x^3-1/
3*arctanh(a*x)/a^2/x^2-10/3*arctanh(a*x)*ln(a*x)+8/3*arctanh(a*x)*ln(a*x-1
)+8/3*arctanh(a*x)*ln(a*x+1)-1/3/a/x-1/6*ln(a*x-1)+1/6*ln(a*x+1)+5/3*dilog
(a*x)+5/3*dilog(a*x+1)+5/3*ln(a*x)*ln(a*x+1)+2/3*ln(a*x-1)^2-8/3*dilog(1/2
*a*x+1/2)-4/3*ln(a*x-1)*ln(1/2*a*x+1/2)+4/3*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln
(-1/2*a*x+1/2)-2/3*ln(a*x+1)^2)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{x^4}\,dx=\int \frac{(a^2x^2-1)^2\operatorname{artanh}(ax)^2}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4} dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^4} dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**4,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.22

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4} dx = -\frac{1}{6}\left(16\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-10\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)a + \frac{1}{3}\left(8a^2\log(ax+1)+8a^2\log(ax-1)-10a^2\log(x)-\frac{1}{x^2}\right)a \operatorname{artanh}(ax) + \frac{1}{3}\left(3a^4x+\frac{6a^2x^2-1}{x^3}\right)\operatorname{artanh}(ax)^2$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
-1/6*(16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 10*
(log(a*x + 1)*log(x) + dilog(-a*x))*a + 10*(log(-a*x + 1)*log(x) + dilog(a
*x))*a - a*log(a*x + 1) + a*log(a*x - 1) + 2*(2*a*x*log(a*x + 1)^2 - 4*a*x
*log(a*x + 1)*log(a*x - 1) - 2*a*x*log(a*x - 1)^2 + 1)/x)*a^2 + 1/3*(8*a^2
*log(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a*arctanh(a*x)
 + 1/3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)^2
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2}{x^4}\,dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^4,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^4}\,dx = \frac{6atanh(ax)^2\,a^2x^2 - atanh(ax)^2 + atanh(ax)\,a^3x^3 - atanh(ax)\,ax + 3\left(\int atanh(ax)^2\,dx\right)a^4x^3 + 10\left(\int \frac{at}{a}\right.}{3x^3}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^4,x)
```

output
```
(6*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2 + atanh(a*x)*a**3*x**3 - atanh(
a*x)*a*x + 3*int(atanh(a*x)**2,x)*a**4*x**3 + 10*int(atanh(a*x)/(a**2*x**3
 - x),x)*a**3*x**3 - a**2*x**2)/(3*x**3)
```

3.212 $\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 214

$$\begin{aligned}\int \frac{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{x^5}\,dx = &-\frac{a^2}{12x^2}-\frac{a\operatorname{arctanh}(ax)}{6x^3}+\frac{3a^3\operatorname{arctanh}(ax)}{2x}\\ &-\frac{3}{4}a^4\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{4x^4}+\frac{a^2\operatorname{arctanh}(ax)^2}{x^2}\\ &+2a^4\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)\\ &-\frac{4}{3}a^4\log(x)+\frac{2}{3}a^4\log\left(1-a^2x^2\right)\\ &-a^4\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\\ &+a^4\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ax}\right)\\ &+\frac{1}{2}a^4\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)\\ &-\frac{1}{2}a^4\operatorname{PolyLog}\left(3,-1+\frac{2}{1-ax}\right)\end{aligned}$$

output
```
-1/12*a^2/x^2-1/6*a*arctanh(a*x)/x^3+3/2*a^3*arctanh(a*x)/x-3/4*a^4*arctan
h(a*x)^2-1/4*arctanh(a*x)^2/x^4+a^2*arctanh(a*x)^2/x^2-2*a^4*arctanh(a*x)^
2*arctanh(-1+2/(-a*x+1))-4/3*a^4*ln(x)+2/3*a^4*ln(-a^2*x^2+1)-a^4*arctanh(
a*x)*polylog(2,1-2/(-a*x+1))+a^4*arctanh(a*x)*polylog(2,-1+2/(-a*x+1))+1/2
*a^4*polylog(3,1-2/(-a*x+1))-1/2*a^4*polylog(3,-1+2/(-a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5}\,dx = \frac{1}{24}\Bigg(2a^4 + ia^4\pi^3 - \frac{2a^2}{x^2} - \frac{4a\operatorname{arctanh}(ax)}{x^3} + \frac{36a^3\operatorname{arctanh}(ax)}{x} - 18a^4\operatorname{arctanh}(ax)^2 - \frac{6\operatorname{arctanh}(ax)^2}{x^4} + \frac{24a^2\operatorname{arctanh}(ax)^2}{x^2} - 16a^4\operatorname{arctanh}(ax)^3 - 24a^4\operatorname{arctanh}(ax)^2\log\left(1+e^{-2\operatorname{arctanh}(ax)}\right) + 24a^4\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) - 32a^4\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right) + 24a^4\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(ax)}\right) + 24a^4\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) + 12a^4\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(ax)}\right) - 12a^4\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)\Bigg)$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

output
```
(2*a^4 + I*a^4*Pi^3 - (2*a^2)/x^2 - (4*a*ArcTanh[a*x])/x^3 + (36*a^3*ArcTa
nh[a*x])/x - 18*a^4*ArcTanh[a*x]^2 - (6*ArcTanh[a*x]^2)/x^4 + (24*a^2*ArcT
anh[a*x]^2)/x^2 - 16*a^4*ArcTanh[a*x]^3 - 24*a^4*ArcTanh[a*x]^2*Log[1 + E^
(-2*ArcTanh[a*x])] + 24*a^4*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 3
2*a^4*Log[(a*x)/Sqrt[1 - a^2*x^2]] + 24*a^4*ArcTanh[a*x]*PolyLog[2, -E^(-2
*ArcTanh[a*x])] + 24*a^4*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 12*
a^4*PolyLog[3, -E^(-2*ArcTanh[a*x])] - 12*a^4*PolyLog[3, E^(2*ArcTanh[a*x]
)])/24
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5} \, dx$$

$$\Big\downarrow \text{6574}$$

$$\int \left(\frac{a^4\operatorname{arctanh}(ax)^2}{x}-\frac{2a^2\operatorname{arctanh}(ax)^2}{x^3}+\frac{\operatorname{arctanh}(ax)^2}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-a^4\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+a^4\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-ax}-1\right)-\frac{3}{4}a^4\operatorname{arctanh}(ax)^2+2a^4\operatorname{arctanh}(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1-ax}\right)+\frac{1}{2}a^4\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}a^4\operatorname{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\frac{4}{3}a^4\log(x)+\frac{3a^3\operatorname{arctanh}(ax)}{2x}+\frac{a^2\operatorname{arctanh}(ax)^2}{x^2}-\frac{a^2}{12x^2}+\frac{2}{3}a^4\log\left(1-a^2x^2\right)-\frac{\operatorname{arctanh}(ax)^2}{4x^4}-\frac{a\operatorname{arctanh}(ax)}{6x^3}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

output
```
-1/12*a^2/x^2 - (a*ArcTanh[a*x])/(6*x^3) + (3*a^3*ArcTanh[a*x])/(2*x) - (3
*a^4*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(4*x^4) + (a^2*ArcTanh[a*x]^2)/x^2
 + 2*a^4*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (4*a^4*Log[x])/3 + (2*a
^4*Log[1 - a^2*x^2])/3 - a^4*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] + a^
4*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + (a^4*PolyLog[3, 1 - 2/(1 - a
*x)])/2 - (a^4*PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.86 (sec) , antiderivative size = 1124, normalized size of antiderivative = 5.25

method	result	size
derivativedivides	Expression too large to display	1124
default	Expression too large to display	1124
parts	Expression too large to display	1571

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(arctanh(a*x)^2/a^2/x^2-1/4*arctanh(a*x)^2/a^4/x^4-1/24*(-(-a^2*x^2+1)
^(1/2)*a^2*x^2+5*a^3*x^3+3*a*x*(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)^(1/2)-3*a
*x+2)*arctanh(a*x)/a^3/x^3+1/8*(-a*x*(-a^2*x^2+1)^(1/2)+2*a^2*x^2+(-a^2*x^
2+1)^(1/2)+a*x-1)*arctanh(a*x)/a^2/x^2-1/24*((-a^2*x^2+1)^(1/2)*a^2*x^2+5*
a^3*x^3-3*a*x*(-a^2*x^2+1)^(1/2)+2*(-a^2*x^2+1)^(1/2)-3*a*x+2)*arctanh(a*x
)/a^3/x^3+1/8*(a*x*(-a^2*x^2+1)^(1/2)+2*a^2*x^2-(-a^2*x^2+1)^(1/2)+a*x-1)*
arctanh(a*x)/a^2/x^2-4/3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*
(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a
*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*x)^2-1/12/(a*
x-(-a^2*x^2+1)^(1/2)+1)*(-a^2*x^2+1)^(1/2)-2*polylog(3,-(a*x+1)/(-a^2*x^2+
1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*polylog(2,-
(a*x+1)^2/(-a^2*x^2+1))-3/4*arctanh(a*x)^2+2*arctanh(a*x)*polylog(2,-(a*x+
1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2)
)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*
Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh
(a*x)^2-4/3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+1/12/(a*x+(-a^2*x^2+1)^(1/2)+
1)*(-a^2*x^2+1)^(1/2)+1/24*(a*x-1)/((-a^2*x^2+1)^(1/2)+1)-1/24*(a*x-1)/((-
a^2*x^2+1)^(1/2)-1)+arctanh(a*x)^2*ln(a*x)+1/2*polylog(3,-(a*x+1)^2/(-a^2*
x^2+1))+5/8*(a*x-(-a^2*x^2+1)^(1/2)+1)/a/x*arctanh(a*x)+5/8*arctanh(a*x...
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^5}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^5, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5} \, dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^5} \, dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**5,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5} \, dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^5} \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
1/16*(4*a^2*x^2 - 1)*log(-a*x + 1)^2/x^4 - 1/4*integrate(-1/2*(2*(a^5*x^5
- a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (4*a^3*x^3 -
 a*x + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1
))*log(-a*x + 1))/(a*x^6 - x^5), x)
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5} \, dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^5} \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5} \, dx = \int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2}{x^5} \, dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^5,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^5} \, dx$$
$$= \frac{-9atanh(ax)^2\,a^4x^4 + 12atanh(ax)^2\,a^2x^2 - 3atanh(ax)^2 + 16atanh(ax)\,a^4x^4 + 18atanh(ax)\,a^3x^3 - 2ata}{12x^4}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^5,x)
```

output
```
( - 9*atanh(a*x)**2*a**4*x**4 + 12*atanh(a*x)**2*a**2*x**2 - 3*atanh(a*x)*
*2 + 16*atanh(a*x)*a**4*x**4 + 18*atanh(a*x)*a**3*x**3 - 2*atanh(a*x)*a*x
+ 12*int(atanh(a*x)**2/x,x)*a**4*x**4 + 16*log(a**2*x - a)*a**4*x**4 - 16*
log(x)*a**4*x**4 - a**2*x**2)/(12*x**4)
```

3.213 $\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6}\,dx = -\frac{a^2}{30x^3}+\frac{11a^4}{30x}-\frac{11}{30}a^5\operatorname{arctanh}(ax)-\frac{a\operatorname{arctanh}(ax)}{10x^4}+\frac{7a^3\operatorname{arctanh}(ax)}{15x^2}+\frac{8}{15}a^5\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{5x^5}+\frac{2a^2\operatorname{arctanh}(ax)^2}{3x^3}-\frac{a^4\operatorname{arctanh}(ax)^2}{x}+\frac{16}{15}a^5\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{8}{15}a^5\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
-1/30*a^2/x^3+11/30*a^4/x-11/30*a^5*arctanh(a*x)-1/10*a*arctanh(a*x)/x^4+7
/15*a^3*arctanh(a*x)/x^2+8/15*a^5*arctanh(a*x)^2-1/5*arctanh(a*x)^2/x^5+2/
3*a^2*arctanh(a*x)^2/x^3-a^4*arctanh(a*x)^2/x+16/15*a^5*arctanh(a*x)*ln(2-
2/(a*x+1))-8/15*a^5*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6} dx$$
$$= \frac{a^2x^2(-1+11a^2x^2)+2(-1+ax)^3(3+9ax+8a^2x^2)\operatorname{arctanh}(ax)^2+ax\operatorname{arctanh}(ax)\left(-3+14a^2x^2-11\right.}{30x^5}$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

output
```
(a^2*x^2*(-1 + 11*a^2*x^2) + 2*(-1 + a*x)^3*(3 + 9*a*x + 8*a^2*x^2)*ArcTan
h[a*x]^2 + a*x*ArcTanh[a*x]*(-3 + 14*a^2*x^2 - 11*a^4*x^4 + 32*a^4*x^4*Log
[1 - E^(-2*ArcTanh[a*x])]) - 16*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(
30*x^5)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6} dx$$
$$\Big\downarrow \text{6574}$$
$$\int \left(\frac{a^4\operatorname{arctanh}(ax)^2}{x^2}-\frac{2a^2\operatorname{arctanh}(ax)^2}{x^4}+\frac{\operatorname{arctanh}(ax)^2}{x^6}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{8}{15}a^5\text{arctanh}(ax)^2-\frac{11}{30}a^5\text{arctanh}(ax)+\frac{16}{15}a^5\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{8}{15}a^5\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^4\text{arctanh}(ax)^2}{x}+\frac{11a^4}{30x}+\frac{7a^3\text{arctanh}(ax)}{15x^2}+\frac{2a^2\text{arctanh}(ax)^2}{3x^3}-\frac{a^2}{30x^3}-\frac{\text{arctanh}(ax)^2}{5x^5}-\frac{a\text{arctanh}(ax)}{10x^4}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

output
```
-1/30*a^2/x^3 + (11*a^4)/(30*x) - (11*a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*
x])/(10*x^4) + (7*a^3*ArcTanh[a*x])/(15*x^2) + (8*a^5*ArcTanh[a*x]^2)/15 -
 ArcTanh[a*x]^2/(5*x^5) + (2*a^2*ArcTanh[a*x]^2)/(3*x^3) - (a^4*ArcTanh[a*
x]^2)/x + (16*a^5*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/15 - (8*a^5*PolyLog[2
, -1 + 2/(1 + a*x)])/15
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.48

method	result
derivativedivides	$a^5\left(-\frac{\text{arctanh}(ax)^2}{ax}+\frac{2\,\text{arctanh}(ax)^2}{3a^3x^3}-\frac{\text{arctanh}(ax)^2}{5a^5x^5}-\frac{\text{arctanh}(ax)}{10a^4x^4}+\frac{7\,\text{arctanh}(ax)}{15a^2x^2}+\frac{16\,\text{arctanh}(ax)\,\text{ln}}{15}\right.$
default	$a^5\left(-\frac{\text{arctanh}(ax)^2}{ax}+\frac{2\,\text{arctanh}(ax)^2}{3a^3x^3}-\frac{\text{arctanh}(ax)^2}{5a^5x^5}-\frac{\text{arctanh}(ax)}{10a^4x^4}+\frac{7\,\text{arctanh}(ax)}{15a^2x^2}+\frac{16\,\text{arctanh}(ax)\,\text{ln}}{15}\right.$
parts	$\frac{2a^2\,\text{arctanh}(ax)^2}{3x^3}-\frac{a^4\,\text{arctanh}(ax)^2}{x}-\frac{\text{arctanh}(ax)^2}{5x^5}-\frac{a\,\text{arctanh}(ax)}{10x^4}+\frac{7a^3\,\text{arctanh}(ax)}{15x^2}+\frac{16a^5\,\text{arctanh}(a}{15}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-arctanh(a*x)^2/a/x+2/3*arctanh(a*x)^2/a^3/x^3-1/5*arctanh(a*x)^2/a^5
/x^5-1/10*arctanh(a*x)/a^4/x^4+7/15*arctanh(a*x)/a^2/x^2+16/15*arctanh(a*x
)*ln(a*x)-8/15*arctanh(a*x)*ln(a*x-1)-8/15*arctanh(a*x)*ln(a*x+1)-1/30/a^3
/x^3+11/30/a/x+11/60*ln(a*x-1)-11/60*ln(a*x+1)-8/15*dilog(a*x)-8/15*dilog(
a*x+1)-8/15*ln(a*x)*ln(a*x+1)-2/15*ln(a*x-1)^2+8/15*dilog(1/2*a*x+1/2)+4/1
5*ln(a*x-1)*ln(1/2*a*x+1/2)-4/15*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1
/2)+2/15*ln(a*x+1)^2)
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^6}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^6, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6}\,dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^6}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**6,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.52

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6} dx$$
$$= \frac{1}{60}\left(32\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^3-32\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)a^3\right.$$
$$-\frac{1}{30}\left(16\,a^4\log\left(a^2x^2-1\right)-16\,a^4\log\left(x^2\right)-\frac{14\,a^2x^2-3}{x^4}\right)a\operatorname{artanh}(ax)$$
$$-\frac{(15\,a^4x^4-10\,a^2x^2+3)\operatorname{artanh}(ax)^2}{15\,x^5}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

output
```
1/60*(32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 3
2*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 32*(log(-a*x + 1)*log(x) + dil
og(a*x))*a^3 - 11*a^3*log(a*x + 1) + 11*a^3*log(a*x - 1) + 2*(4*a^3*x^3*lo
g(a*x + 1)^2 - 8*a^3*x^3*log(a*x + 1)*log(a*x - 1) - 4*a^3*x^3*log(a*x - 1
)^2 + 11*a^2*x^2 - 1)/x^3)*a^2 - 1/30*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*lo
g(x^2) - (14*a^2*x^2 - 3)/x^4)*a*arctanh(a*x) - 1/15*(15*a^4*x^4 - 10*a^2*
x^2 + 3)*arctanh(a*x)^2/x^5
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6} dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^6} dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6} \, dx = \int \frac{\operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^2}{x^6} \, dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^6,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^6, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^6} \, dx$$
$$= \frac{-30 atanh(ax)^2 a^4x^4 + 20 atanh(ax)^2 a^2x^2 - 6 atanh(ax)^2 - 11 atanh(ax) a^5x^5 + 14 atanh(ax) a^3x^3 - 3a}{30x^5}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^6,x)
```

output
```
( - 30*atanh(a*x)**2*a**4*x**4 + 20*atanh(a*x)**2*a**2*x**2 - 6*atanh(a*x)
**2 - 11*atanh(a*x)*a**5*x**5 + 14*atanh(a*x)*a**3*x**3 - 3*atanh(a*x)*a*x
 - 32*int(atanh(a*x)/(a**2*x**3 - x),x)*a**5*x**5 + 11*a**4*x**4 - a**2*x*
*2)/(30*x**5)
```

3.214 $\int \frac{(1-a^2x^2)^2\text{arctanh}(ax)^2}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7} dx = -\frac{a^2}{60x^4} + \frac{7a^4}{90x^2} - \frac{a\operatorname{arctanh}(ax)}{15x^5} + \frac{2a^3\operatorname{arctanh}(ax)}{9x^3} - \frac{a^5\operatorname{arctanh}(ax)}{3x} - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{6x^6} + \frac{8}{45}a^6\log(x) - \frac{4}{45}a^6\log(1-a^2x^2)$$

output
```
-1/60*a^2/x^4+7/90*a^4/x^2-1/15*a*arctanh(a*x)/x^5+2/9*a^3*arctanh(a*x)/x^
3-1/3*a^5*arctanh(a*x)/x-1/6*(-a^2*x^2+1)^3*arctanh(a*x)^2/x^6+8/45*a^6*ln
(x)-4/45*a^6*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7} dx = \frac{-4ax(3-10a^2x^2+15a^4x^4)\operatorname{arctanh}(ax) + 30(-1+a^2x^2)^3\operatorname{arctanh}(ax)^2 + a^2x^2(-3+14a^2x^2+32a^4x^4}{180x^6}$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

output
```
(-4*a*x*(3 - 10*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x] + 30*(-1 + a^2*x^2)^3*A
rcTanh[a*x]^2 + a^2*x^2*(-3 + 14*a^2*x^2 + 32*a^4*x^4*Log[x] - 16*a^4*x^4*
Log[1 - a^2*x^2]))/(180*x^6)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6570, 6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7} \, dx$$

$$\Big\downarrow \text{ 6570}$$

$$\frac{1}{3}a\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}{x^6}dx - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{6x^6}$$

$$\Big\downarrow \text{ 6574}$$

$$\frac{1}{3}a\int \left(\frac{\operatorname{arctanh}(ax)a^4}{x^2} - \frac{2\operatorname{arctanh}(ax)a^2}{x^4} + \frac{\operatorname{arctanh}(ax)}{x^6}\right)dx - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{6x^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a\left(\frac{8}{15}a^5\log(x) - \frac{a^4\operatorname{arctanh}(ax)}{x} + \frac{7a^3}{30x^2} + \frac{2a^2\operatorname{arctanh}(ax)}{3x^3} - \frac{4}{15}a^5\log(1-a^2x^2) - \frac{\operatorname{arctanh}(ax)}{5x^5} - \frac{a}{20x^4}\right) - \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{6x^6}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

output
```
-1/6*((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/x^6 + (a*(-1/20*a/x^4 + (7*a^3)/(30*
x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x])/(3*x^3) - (a^4*ArcTanh[
a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x^2])/15))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.32

method	result
parallelrisch	$\frac{30\,\text{arctanh}(ax)^2 a^6 x^6+32\ln(x) a^6 x^6-32\ln(ax-1) x^6 a^6-32\,\text{arctanh}(ax) a^6 x^6+14 a^6 x^6-60\,\text{arctanh}(ax) a^5 x^5-90 a^4 x^4}{180x}$
derivativedivides	$a^6\left(-\frac{\text{arctanh}(ax)^2}{6a^6x^6}-\frac{\text{arctanh}(ax)^2}{2a^2x^2}+\frac{\text{arctanh}(ax)^2}{2a^4x^4}-\frac{\text{arctanh}(ax)}{15a^5x^5}+\frac{2\,\text{arctanh}(ax)}{9a^3x^3}-\frac{\text{arctanh}(ax)}{3ax}-\text{ar}\right.$
default	$a^6\left(-\frac{\text{arctanh}(ax)^2}{6a^6x^6}-\frac{\text{arctanh}(ax)^2}{2a^2x^2}+\frac{\text{arctanh}(ax)^2}{2a^4x^4}-\frac{\text{arctanh}(ax)}{15a^5x^5}+\frac{2\,\text{arctanh}(ax)}{9a^3x^3}-\frac{\text{arctanh}(ax)}{3ax}-\text{ar}\right.$
parts	$\frac{a^2\,\text{arctanh}(ax)^2}{2x^4}-\frac{\text{arctanh}(ax)^2}{6x^6}-\frac{\text{arctanh}(ax)^2 a^4}{2x^2}-\frac{a\,\text{arctanh}(ax)}{15x^5}+\frac{2a^3\,\text{arctanh}(ax)}{9x^3}-\frac{a^5\,\text{arctanh}(ax)}{3x}-$
risch	$\frac{(a^6x^6-3a^4x^4+3a^2x^2-1)\ln(ax+1)^2}{24x^6}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-20a^3x^3+45x^2\ln(-ax+1}{180x^6}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/180*(30*arctanh(a*x)^2*a^6*x^6+32*ln(x)*a^6*x^6-32*ln(a*x-1)*x^6*a^6-32*
arctanh(a*x)*a^6*x^6+14*a^6*x^6-60*arctanh(a*x)*a^5*x^5-90*a^4*x^4*arctanh
(a*x)^2+14*a^4*x^4+40*a^3*x^3*arctanh(a*x)+90*a^2*x^2*arctanh(a*x)^2-3*a^2
*x^2-12*a*x*arctanh(a*x)-30*arctanh(a*x)^2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7} \, dx = -\frac{32\, a^6x^6 \log\left(a^2x^2-1\right) - 64\, a^6x^6 \log\left(x\right) - 28\, a^4x^4 + 6\, a^2x^2 - 15\left(a^6x^6 - 3\, a^4x^4 + 3\, a^2x^2 - 1\right) \log\left(-\frac{ax}{ax}\right.}{360\, x^6}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="fricas")
```

output
```
-1/360*(32*a^6*x^6*log(a^2*x^2 - 1) - 64*a^6*x^6*log(x) - 28*a^4*x^4 + 6*a
^2*x^2 - 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1)
)^2 + 4*(15*a^5*x^5 - 10*a^3*x^3 + 3*a*x)*log(-(a*x + 1)/(a*x - 1)))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.31

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7} \, dx = \begin{cases} \frac{8a^6 \log(x)}{45} - \frac{8a^6 \log\left(x-\frac{1}{a}\right)}{45} + \frac{a^6 \operatorname{atanh}^2(ax)}{6} - \frac{8a^6 \operatorname{atanh}(ax)}{45} - \frac{a^5 \operatorname{atanh}(ax)}{3x} - \frac{a^4 \operatorname{atanh}^2(ax)}{2x^2} + \frac{7a^4}{90x^2} + \frac{2a^3 \operatorname{atanh}(ax)}{9x^3} + \\ 0 \end{cases}$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**7,x)
```

output
```
Piecewise((8*a**6*log(x)/45 - 8*a**6*log(x - 1/a)/45 + a**6*atanh(a*x)**2/
6 - 8*a**6*atanh(a*x)/45 - a**5*atanh(a*x)/(3*x) - a**4*atanh(a*x)**2/(2*x
**2) + 7*a**4/(90*x**2) + 2*a**3*atanh(a*x)/(9*x**3) + a**2*atanh(a*x)**2/
(2*x**4) - a**2/(60*x**4) - a*atanh(a*x)/(15*x**5) - atanh(a*x)**2/(6*x**6
), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.66

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7}\,dx$$
$$= \frac{1}{360}\left(64\,a^4\log(x) - \frac{15\,a^4x^4\log(ax+1)^2 + 15\,a^4x^4\log(ax-1)^2 + 32\,a^4x^4\log(ax-1) - 28\,a^2x^2 - 2\,(\right.}{x^4}$$
$$+ \frac{1}{90}\left(15\,a^5\log(ax+1) - 15\,a^5\log(ax-1) - \frac{2\,(15\,a^4x^4 - 10\,a^2x^2 + 3)}{x^5}\right)a\,\operatorname{artanh}(ax)$$
$$- \frac{(3\,a^4x^4 - 3\,a^2x^2 + 1)\operatorname{artanh}(ax)^2}{6\,x^6}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="maxima")
```

output
```
1/360*(64*a^4*log(x) - (15*a^4*x^4*log(a*x + 1)^2 + 15*a^4*x^4*log(a*x - 1
)^2 + 32*a^4*x^4*log(a*x - 1) - 28*a^2*x^2 - 2*(15*a^4*x^4*log(a*x - 1) -
16*a^4*x^4)*log(a*x + 1) + 6)/x^4)*a^2 + 1/90*(15*a^5*log(a*x + 1) - 15*a^
5*log(a*x - 1) - 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)/x^5)*a*arctanh(a*x) - 1/6
*(3*a^4*x^4 - 3*a^2*x^2 + 1)*arctanh(a*x)^2/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(96) = 192.

Time = 0.13 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.89

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7}\,dx$$
$$= \frac{4}{45}\left(2\,a^5\log\left(-\frac{ax+1}{ax-1}-1\right) - 2\,a^5\log\left(-\frac{ax+1}{ax-1}\right) + \frac{30\,(ax+1)^3a^5\log\left(-\frac{a}{a}\right.}{(ax-1)^3\left(\frac{(ax+1)^6}{(ax-1)^6} + \frac{6\,(ax+1)^5}{(ax-1)^5} + \frac{15\,(ax+1)^4}{(ax-1)^4} + \frac{20\,(ax}{(ax-}\right.}\right.$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="giac")
```

output
```
4/45*(2*a^5*log(-(a*x + 1)/(a*x - 1) - 1) - 2*a^5*log(-(a*x + 1)/(a*x - 1)
) + 30*(a*x + 1)^3*a^5*log(-(a*x + 1)/(a*x - 1))^2/((a*x - 1)^3*((a*x + 1)
^6/(a*x - 1)^6 + 6*(a*x + 1)^5/(a*x - 1)^5 + 15*(a*x + 1)^4/(a*x - 1)^4 +
20*(a*x + 1)^3/(a*x - 1)^3 + 15*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x
 - 1) + 1)) + 2*(10*(a*x + 1)^2*a^5/(a*x - 1)^2 + 5*(a*x + 1)*a^5/(a*x - 1
) + a^5)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)^5/(a*x - 1)^5 + 5*(a*x + 1)^
4/(a*x - 1)^4 + 10*(a*x + 1)^3/(a*x - 1)^3 + 10*(a*x + 1)^2/(a*x - 1)^2 +
5*(a*x + 1)/(a*x - 1) + 1) - (2*(a*x + 1)^3*a^5/(a*x - 1)^3 + 7*(a*x + 1)^
2*a^5/(a*x - 1)^2 + 2*(a*x + 1)*a^5/(a*x - 1))/((a*x + 1)^4/(a*x - 1)^4 +
4*(a*x + 1)^3/(a*x - 1)^3 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x + 1)/(a*x -
 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.96

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7}\,dx$$
$$= \frac{8\,a^6\,\ln(x)}{45} - \frac{\frac{3\,a^2}{4}-\frac{7\,a^4\,x^2}{2}}{45\,x^4} - \ln(1-a\,x)^2\left(\frac{\frac{a^4\,x^4}{2}-\frac{a^2\,x^2}{2}+\frac{1}{6}}{4\,x^6}-\frac{a^6}{24}\right) - \ln(a\,x+1)^2\left(\frac{\frac{a^4\,x^4}{8}-\frac{a^2\,x^2}{8}+\frac{1}{24}}{x^6}-\frac{a^6}{24}\right) - \ln(1-a\,x)\left(\frac{a\left(\frac{137\,a^5\,x^5}{2}-30\,a^4\,x^4+15\,a^3\,x^3-10\,a^2\,x^2+\frac{15\,a\,x}{2}-6\right)}{360\,x^5} - \ln(a\,x+1)\left(\frac{\frac{a^4\,x^4}{2}-\frac{a^2\,x^2}{2}+\frac{1}{6}}{2\,x^6}-\frac{a^6}{12}\right) - \frac{a\,(137\,a^5\,x^5+60\,a^4\,x^4+30\,a^3\,x^3+20\,a^2\,x^2+15\,a\,x+12)}{720\,x^5} + \frac{5\,a^8\,x^2-\frac{15\,a^9\,x^3}{2}}{60\,a^5\,x^5} + \frac{\frac{15\,a^9\,x^3}{2}+5\,a^8\,x^2}{60\,a^5\,x^5}\right) - \frac{4\,a^6\,\ln(a^2\,x^2-1)}{45} - \frac{a\,\ln(a\,x+1)\,\left(\frac{a^4\,x^4}{6}-\frac{a^2\,x^2}{9}+\frac{1}{30}\right)}{x^5}$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^7,x)
```

output
```
(8*a^6*log(x))/45 - ((3*a^2)/4 - (7*a^4*x^2)/2)/(45*x^4) - log(1 - a*x)^2*
(((a^4*x^4)/2 - (a^2*x^2)/2 + 1/6)/(4*x^6) - a^6/24) - log(a*x + 1)^2*(((a
^4*x^4)/8 - (a^2*x^2)/8 + 1/24)/x^6 - a^6/24) - log(1 - a*x)*((a*((15*a*x)
/2 - 10*a^2*x^2 + 15*a^3*x^3 - 30*a^4*x^4 + (137*a^5*x^5)/2 - 6))/(360*x^5
) - log(a*x + 1)*(((a^4*x^4)/2 - (a^2*x^2)/2 + 1/6)/(2*x^6) - a^6/12) - (a
*(15*a*x + 20*a^2*x^2 + 30*a^3*x^3 + 60*a^4*x^4 + 137*a^5*x^5 + 12))/(720*
x^5) + (5*a^8*x^2 - (15*a^9*x^3)/2)/(60*a^5*x^5) + (5*a^8*x^2 + (15*a^9*x^
3)/2)/(60*a^5*x^5)) - (4*a^6*log(a^2*x^2 - 1))/45 - (a*log(a*x + 1)*((a^4*
x^4)/6 - (a^2*x^2)/9 + 1/30))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^7} dx$$
$$= \frac{30atanh(ax)^2 a^6x^6 - 90atanh(ax)^2 a^4x^4 + 90atanh(ax)^2 a^2x^2 - 30atanh(ax)^2 - 32atanh(ax) a^6x^6 - 60$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^7,x)
```

output
```
(30*atanh(a*x)**2*a**6*x**6 - 90*atanh(a*x)**2*a**4*x**4 + 90*atanh(a*x)**
2*a**2*x**2 - 30*atanh(a*x)**2 - 32*atanh(a*x)*a**6*x**6 - 60*atanh(a*x)*a
**5*x**5 + 40*atanh(a*x)*a**3*x**3 - 12*atanh(a*x)*a*x - 32*log(a**2*x - a
)*a**6*x**6 + 32*log(x)*a**6*x**6 + 14*a**4*x**4 - 3*a**2*x**2)/(180*x**6)
```

3.215 $\int \frac{(1-a^2x^2)^2\text{arctanh}(ax)^2}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 183

$$\begin{aligned}\int \frac{(1-a^2x^2)^2\text{arctanh}(ax)^2}{x^8}\,dx = &-\frac{a^2}{105x^5}+\frac{17a^4}{630x^3}+\frac{a^6}{210x}-\frac{1}{210}a^7\text{arctanh}(ax)\\&-\frac{a\text{arctanh}(ax)}{21x^6}+\frac{9a^3\text{arctanh}(ax)}{70x^4}-\frac{8a^5\text{arctanh}(ax)}{105x^2}\\&+\frac{8}{105}a^7\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)^2}{7x^7}\\&+\frac{2a^2\text{arctanh}(ax)^2}{5x^5}-\frac{a^4\text{arctanh}(ax)^2}{3x^3}\\&+\frac{16}{105}a^7\text{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)\\&-\frac{8}{105}a^7\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-1/105*a^2/x^5+17/630*a^4/x^3+1/210*a^6/x-1/210*a^7*arctanh(a*x)-1/21*a*ar
ctanh(a*x)/x^6+9/70*a^3*arctanh(a*x)/x^4-8/105*a^5*arctanh(a*x)/x^2+8/105*
a^7*arctanh(a*x)^2-1/7*arctanh(a*x)^2/x^7+2/5*a^2*arctanh(a*x)^2/x^5-1/3*a
^4*arctanh(a*x)^2/x^3+16/105*a^7*arctanh(a*x)*ln(2-2/(a*x+1))-8/105*a^7*po
lylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.77

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8} dx$$
$$= \frac{a^2x^2(-6+17a^2x^2+3a^4x^4)+6(-15+42a^2x^2-35a^4x^4+8a^7x^7)\operatorname{arctanh}(ax)^2+3ax\operatorname{arctanh}(ax)\,(-1\ldots}{630x^7}$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

output
```
(a^2*x^2*(-6 + 17*a^2*x^2 + 3*a^4*x^4) + 6*(-15 + 42*a^2*x^2 - 35*a^4*x^4
+ 8*a^7*x^7)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[a*x]*(-10 + 27*a^2*x^2 - 16*a^
4*x^4 - a^6*x^6 + 32*a^6*x^6*Log[1 - E^(-2*ArcTanh[a*x])]) - 48*a^7*x^7*Po
lyLog[2, E^(-2*ArcTanh[a*x])])/(630*x^7)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8} dx$$
$$\Big\downarrow \text{ 6574}$$
$$\int \left(\frac{a^4\operatorname{arctanh}(ax)^2}{x^4}-\frac{2a^2\operatorname{arctanh}(ax)^2}{x^6}+\frac{\operatorname{arctanh}(ax)^2}{x^8}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{8}{105}a^7\text{arctanh}(ax)^2 - \frac{1}{210}a^7\text{arctanh}(ax) + \frac{16}{105}a^7\text{arctanh}(ax)\log\left(2 - \frac{2}{ax+1}\right) - \frac{8}{105}a^7\,\text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + \frac{a^6}{210x} - \frac{8a^5\text{arctanh}(ax)}{105x^2} - \frac{a^4\text{arctanh}(ax)^2}{3x^3} + \frac{17a^4}{630x^3} + \frac{9a^3\text{arctanh}(ax)}{70x^4} + \frac{2a^2\text{arctanh}(ax)^2}{5x^5} - \frac{a^2}{105x^5} - \frac{\text{arctanh}(ax)^2}{7x^7} - \frac{a\text{arctanh}(ax)}{21x^6}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

output
```
-1/105*a^2/x^5 + (17*a^4)/(630*x^3) + a^6/(210*x) - (a^7*ArcTanh[a*x])/210
 - (a*ArcTanh[a*x])/(21*x^6) + (9*a^3*ArcTanh[a*x])/(70*x^4) - (8*a^5*ArcT
anh[a*x])/(105*x^2) + (8*a^7*ArcTanh[a*x]^2)/105 - ArcTanh[a*x]^2/(7*x^7)
+ (2*a^2*ArcTanh[a*x]^2)/(5*x^5) - (a^4*ArcTanh[a*x]^2)/(3*x^3) + (16*a^7*
ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/105 - (8*a^7*PolyLog[2, -1 + 2/(1 + a*x
)])/105
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.38

method	result
derivativedivides	$a^7\left(-\frac{\text{arctanh}(ax)^2}{3a^3x^3} - \frac{\text{arctanh}(ax)^2}{7a^7x^7} + \frac{2\,\text{arctanh}(ax)^2}{5a^5x^5} - \frac{\text{arctanh}(ax)}{21a^6x^6} + \frac{9\,\text{arctanh}(ax)}{70a^4x^4} - \frac{8\,\text{arctanh}(ax)}{105a^2x^2} + \right.$
default	$a^7\left(-\frac{\text{arctanh}(ax)^2}{3a^3x^3} - \frac{\text{arctanh}(ax)^2}{7a^7x^7} + \frac{2\,\text{arctanh}(ax)^2}{5a^5x^5} - \frac{\text{arctanh}(ax)}{21a^6x^6} + \frac{9\,\text{arctanh}(ax)}{70a^4x^4} - \frac{8\,\text{arctanh}(ax)}{105a^2x^2} + \right.$
parts	$-\frac{a^4\,\text{arctanh}(ax)^2}{3x^3} - \frac{\text{arctanh}(ax)^2}{7x^7} + \frac{2a^2\,\text{arctanh}(ax)^2}{5x^5} - \frac{a\,\text{arctanh}(ax)}{21x^6} + \frac{9a^3\,\text{arctanh}(ax)}{70x^4} - \frac{8a^5\,\text{arctanh}(}{105x^2}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
a^7*(-1/3*arctanh(a*x)^2/a^3/x^3-1/7*arctanh(a*x)^2/a^7/x^7+2/5*arctanh(a*
x)^2/a^5/x^5-1/21*arctanh(a*x)/a^6/x^6+9/70*arctanh(a*x)/a^4/x^4-8/105*arc
tanh(a*x)/a^2/x^2+16/105*arctanh(a*x)*ln(a*x)-8/105*arctanh(a*x)*ln(a*x-1)
-8/105*arctanh(a*x)*ln(a*x+1)-8/105*dilog(a*x)-8/105*dilog(a*x+1)-8/105*ln
(a*x)*ln(a*x+1)-2/105*ln(a*x-1)^2+8/105*dilog(1/2*a*x+1/2)+4/105*ln(a*x-1)
*ln(1/2*a*x+1/2)-4/105*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+2/105*
ln(a*x+1)^2+1/210/a/x-1/105/a^5/x^5+17/630/a^3/x^3+1/420*ln(a*x-1)-1/420*l
n(a*x+1))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8}\,dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^8}\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^8, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8}\,dx = \int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^8}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**8,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.39

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8} dx$$
$$= \frac{1}{1260}\left(96\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^5 - 96(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax))\right.$$
$$- \frac{1}{210}\left(16a^6\log(a^2x^2-1) - 16a^6\log(x^2) + \frac{16a^4x^4-27a^2x^2+10}{x^6}\right)a \operatorname{artanh}(ax)$$
$$- \frac{(35a^4x^4-42a^2x^2+15)\operatorname{artanh}(ax)^2}{105x^7}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="maxima")
```

output
```
1/1260*(96*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^5 -
 96*(log(a*x + 1)*log(x) + dilog(-a*x))*a^5 + 96*(log(-a*x + 1)*log(x) + d
ilog(a*x))*a^5 - 3*a^5*log(a*x + 1) + 3*a^5*log(a*x - 1) + 2*(12*a^5*x^5*l
og(a*x + 1)^2 - 24*a^5*x^5*log(a*x + 1)*log(a*x - 1) - 12*a^5*x^5*log(a*x
- 1)^2 + 3*a^4*x^4 + 17*a^2*x^2 - 6)/x^5)*a^2 - 1/210*(16*a^6*log(a^2*x^2
- 1) - 16*a^6*log(x^2) + (16*a^4*x^4 - 27*a^2*x^2 + 10)/x^6)*a*arctanh(a*x
) - 1/105*(35*a^4*x^4 - 42*a^2*x^2 + 15)*arctanh(a*x)^2/x^7
```

Giac [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8} dx = \int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^8} dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2}{x^8}\,dx$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^8,x)
```

output
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^8, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^8}\,dx = \frac{-210atanh(ax)^2\,a^4x^4+252atanh(ax)^2\,a^2x^2-90atanh(ax)^2-3atanh(ax)\,a^7x^7-48atanh(ax)\,a^5x^5+8}{630x^7}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^8,x)
```

output
```
( - 210*atanh(a*x)**2*a**4*x**4 + 252*atanh(a*x)**2*a**2*x**2 - 90*atanh(a
*x)**2 - 3*atanh(a*x)*a**7*x**7 - 48*atanh(a*x)*a**5*x**5 + 81*atanh(a*x)*
a**3*x**3 - 30*atanh(a*x)*a*x - 96*int(atanh(a*x)/(a**2*x**3 - x),x)*a**7*
x**7 + 3*a**6*x**6 + 17*a**4*x**4 - 6*a**2*x**2)/(630*x**7)
```

3.216 $\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 170

$$\begin{aligned}\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} \, dx = &-\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a\operatorname{arctanh}(ax)}{28x^7} \\ &+ \frac{a^3\operatorname{arctanh}(ax)}{12x^5} - \frac{a^5\operatorname{arctanh}(ax)}{36x^3} - \frac{a^7\operatorname{arctanh}(ax)}{12x} \\ &+ \frac{1}{24}a^8\operatorname{arctanh}(ax)^2 - \frac{\operatorname{arctanh}(ax)^2}{8x^8} \\ &+ \frac{a^2\operatorname{arctanh}(ax)^2}{3x^6} - \frac{a^4\operatorname{arctanh}(ax)^2}{4x^4} \\ &+ \frac{4}{63}a^8\log(x) - \frac{2}{63}a^8\log\left(1-a^2x^2\right)\end{aligned}$$

output
```
-1/168*a^2/x^6+1/84*a^4/x^4+5/504*a^6/x^2-1/28*a*arctanh(a*x)/x^7+1/12*a^3
*arctanh(a*x)/x^5-1/36*a^5*arctanh(a*x)/x^3-1/12*a^7*arctanh(a*x)/x+1/24*a
^8*arctanh(a*x)^2-1/8*arctanh(a*x)^2/x^8+1/3*a^2*arctanh(a*x)^2/x^6-1/4*a^
4*arctanh(a*x)^2/x^4+4/63*a^8*ln(x)-2/63*a^8*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} dx$$
$$= \frac{-2ax(9-21a^2x^2+7a^4x^4+21a^6x^6)\operatorname{arctanh}(ax)+21(-1+a^2x^2)^3(3+a^2x^2)\operatorname{arctanh}(ax)^2+a^2x^2(-3}{504x^8}$$

input
```
Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

output
```
(-2*a*x*(9 - 21*a^2*x^2 + 7*a^4*x^4 + 21*a^6*x^6)*ArcTanh[a*x] + 21*(-1 +
a^2*x^2)^3*(3 + a^2*x^2)*ArcTanh[a*x]^2 + a^2*x^2*(-3 + 6*a^2*x^2 + 5*a^4*
x^4 + 32*a^6*x^6*Log[x] - 16*a^6*x^6*Log[1 - a^2*x^2]))/(504*x^8)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6574, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} dx$$
$$\Big\downarrow \text{6574}$$
$$\int \left(\frac{a^4\operatorname{arctanh}(ax)^2}{x^5}-\frac{2a^2\operatorname{arctanh}(ax)^2}{x^7}+\frac{\operatorname{arctanh}(ax)^2}{x^9}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{24}a^8\operatorname{arctanh}(ax)^2+\frac{4}{63}a^8\log(x)-\frac{a^7\operatorname{arctanh}(ax)}{12x}+\frac{5a^6}{504x^2}-\frac{a^5\operatorname{arctanh}(ax)}{36x^3}-\frac{a^4\operatorname{arctanh}(ax)^2}{4x^4}+\frac{a^4}{84x^4}+\frac{a^3\operatorname{arctanh}(ax)}{12x^5}+\frac{a^2\operatorname{arctanh}(ax)^2}{3x^6}-\frac{a^2}{168x^6}-\frac{2}{63}a^8\log\left(1-a^2x^2\right)-\frac{\operatorname{arctanh}(ax)^2}{8x^8}-\frac{a\operatorname{arctanh}(ax)}{28x^7}$$

input
```
Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

output
```
-1/168*a^2/x^6 + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a*ArcTanh[a*x])/(28*x
^7) + (a^3*ArcTanh[a*x])/(12*x^5) - (a^5*ArcTanh[a*x])/(36*x^3) - (a^7*Arc
Tanh[a*x])/(12*x) + (a^8*ArcTanh[a*x]^2)/24 - ArcTanh[a*x]^2/(8*x^8) + (a^
2*ArcTanh[a*x]^2)/(3*x^6) - (a^4*ArcTanh[a*x]^2)/(4*x^4) + (4*a^8*Log[x])/
63 - (2*a^8*Log[1 - a^2*x^2])/63
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6574
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{21 \operatorname{arctanh}(ax)^2 a^8 x^8+32 \ln(x) a^8 x^8-32 \ln(ax-1) x^8 a^8-32 \operatorname{arctanh}(ax) a^8 x^8+5 a^8 x^8-42 \operatorname{arctanh}(ax) a^7 x^7+5 a^6 x^6-1}{}$
derivativedivides	$a^8 \left(\frac{\operatorname{arctanh}(ax)^2}{3a^6x^6}-\frac{\operatorname{arctanh}(ax)^2}{8a^8x^8}-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}-\frac{\operatorname{arctanh}(ax)}{28a^7x^7}+\frac{\operatorname{arctanh}(ax)}{12a^5x^5}-\frac{\operatorname{arctanh}(ax)}{36a^3x^3}-\frac{\operatorname{arctan}}{12a}\right.$
default	$a^8 \left(\frac{\operatorname{arctanh}(ax)^2}{3a^6x^6}-\frac{\operatorname{arctanh}(ax)^2}{8a^8x^8}-\frac{\operatorname{arctanh}(ax)^2}{4a^4x^4}-\frac{\operatorname{arctanh}(ax)}{28a^7x^7}+\frac{\operatorname{arctanh}(ax)}{12a^5x^5}-\frac{\operatorname{arctanh}(ax)}{36a^3x^3}-\frac{\operatorname{arctan}}{12a}\right.$
parts	$-\frac{a^4 \operatorname{arctanh}(ax)^2}{4x^4}+\frac{a^2 \operatorname{arctanh}(ax)^2}{3x^6}-\frac{\operatorname{arctanh}(ax)^2}{8x^8}-\frac{a \operatorname{arctanh}(ax)}{28x^7}+\frac{a^3 \operatorname{arctanh}(ax)}{12x^5}-\frac{a^5 \operatorname{arctanh}(ax)}{36x^3}$
risch	$\frac{(a^8x^8-6a^4x^4+8a^2x^2-3)\ln(ax+1)^2}{96x^8}-\frac{(21a^8x^8\ln(-ax+1)+42a^7x^7+14a^5x^5-126x^4\ln(-ax+1)a^4-42a^3x^3+168x}{1008x^8}$

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/504*(21*arctanh(a*x)^2*a^8*x^8+32*ln(x)*a^8*x^8-32*ln(a*x-1)*x^8*a^8-32*
arctanh(a*x)*a^8*x^8+5*a^8*x^8-42*arctanh(a*x)*a^7*x^7+5*a^6*x^6-14*arctan
h(a*x)*a^5*x^5-126*a^4*x^4*arctanh(a*x)^2+6*a^4*x^4+42*a^3*x^3*arctanh(a*x
)+168*a^2*x^2*arctanh(a*x)^2-3*a^2*x^2-18*a*x*arctanh(a*x)-63*arctanh(a*x)
^2)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} \, dx = -\frac{64 \, a^8x^8 \log(a^2x^2-1) - 128 \, a^8x^8 \log(x) - 20 \, a^6x^6 - 24 \, a^4x^4 + 12 \, a^2x^2 - 21 \, (a^8x^8 - 6 \, a^4x^4 + 8 \, a^2x^2 -}{2016 \, x^8}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="fricas")
```

output
```
-1/2016*(64*a^8*x^8*log(a^2*x^2 - 1) - 128*a^8*x^8*log(x) - 20*a^6*x^6 - 2
4*a^4*x^4 + 12*a^2*x^2 - 21*(a^8*x^8 - 6*a^4*x^4 + 8*a^2*x^2 - 3)*log(-(a*
x + 1)/(a*x - 1))^2 + 4*(21*a^7*x^7 + 7*a^5*x^5 - 21*a^3*x^3 + 9*a*x)*log(
-(a*x + 1)/(a*x - 1)))/x^8
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} \, dx = \begin{cases} \frac{4a^8 \log(x)}{63} - \frac{4a^8 \log\left(x-\frac{1}{a}\right)}{63} + \frac{a^8 \operatorname{atanh}^2(ax)}{24} - \frac{4a^8 \operatorname{atanh}(ax)}{63} - \frac{a^7 \operatorname{atanh}(ax)}{12x} + \frac{5a^6}{504x^2} - \frac{a^5 \operatorname{atanh}(ax)}{36x^3} - \frac{a^4 \operatorname{atanh}^2(ax)}{4x^4} + \\ 0 \end{cases}$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**9,x)
```

output
```
Piecewise((4*a**8*log(x)/63 - 4*a**8*log(x - 1/a)/63 + a**8*atanh(a*x)**2/
24 - 4*a**8*atanh(a*x)/63 - a**7*atanh(a*x)/(12*x) + 5*a**6/(504*x**2) - a
**5*atanh(a*x)/(36*x**3) - a**4*atanh(a*x)**2/(4*x**4) + a**4/(84*x**4) +
a**3*atanh(a*x)/(12*x**5) + a**2*atanh(a*x)**2/(3*x**6) - a**2/(168*x**6)
- a*atanh(a*x)/(28*x**7) - atanh(a*x)**2/(8*x**8), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9}\,dx$$
$$= \frac{1}{2016}\left(128\,a^6\log(x) - \frac{21\,a^6x^6\log(ax+1)^2 + 21\,a^6x^6\log(ax-1)^2 + 64\,a^6x^6\log(ax-1) - 20\,a^4x^4 - \cdots}{x^6}\right.$$
$$+ \frac{1}{504}\left(21\,a^7\log(ax+1) - 21\,a^7\log(ax-1) - \frac{2\,(21\,a^6x^6 + 7\,a^4x^4 - 21\,a^2x^2 + 9)}{x^7}\right)a\,\operatorname{artanh}(ax)$$
$$- \frac{(6\,a^4x^4 - 8\,a^2x^2 + 3)\operatorname{artanh}(ax)^2}{24\,x^8}$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="maxima")
```

output
```
1/2016*(128*a^6*log(x) - (21*a^6*x^6*log(a*x + 1)^2 + 21*a^6*x^6*log(a*x -
 1)^2 + 64*a^6*x^6*log(a*x - 1) - 20*a^4*x^4 - 24*a^2*x^2 - 2*(21*a^6*x^6*
log(a*x - 1) - 32*a^6*x^6)*log(a*x + 1) + 12)/x^6)*a^2 + 1/504*(21*a^7*log
(a*x + 1) - 21*a^7*log(a*x - 1) - 2*(21*a^6*x^6 + 7*a^4*x^4 - 21*a^2*x^2 +
 9)/x^7)*a*arctanh(a*x) - 1/24*(6*a^4*x^4 - 8*a^2*x^2 + 3)*arctanh(a*x)^2/
x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(144) = 288$.

Time = 0.13 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.83

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} \, dx = \text{Too large to display}$$

input

```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="giac")
```

output

```
2/63*(2*a^7*log(-(a*x + 1)/(a*x - 1) - 1) - 2*a^7*log(-(a*x + 1)/(a*x - 1)
) + 84*((a*x + 1)^5*a^7/(a*x - 1)^5 - (a*x + 1)^4*a^7/(a*x - 1)^4 + (a*x +
 1)^3*a^7/(a*x - 1)^3)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)^8/(a*x - 1)^
8 + 8*(a*x + 1)^7/(a*x - 1)^7 + 28*(a*x + 1)^6/(a*x - 1)^6 + 56*(a*x + 1)^
5/(a*x - 1)^5 + 70*(a*x + 1)^4/(a*x - 1)^4 + 56*(a*x + 1)^3/(a*x - 1)^3 +
28*(a*x + 1)^2/(a*x - 1)^2 + 8*(a*x + 1)/(a*x - 1) + 1) + 2*(28*(a*x + 1)^
4*a^7/(a*x - 1)^4 + 7*(a*x + 1)^3*a^7/(a*x - 1)^3 + 21*(a*x + 1)^2*a^7/(a*
x - 1)^2 + 7*(a*x + 1)*a^7/(a*x - 1) + a^7)*log(-(a*x + 1)/(a*x - 1))/((a*
x + 1)^7/(a*x - 1)^7 + 7*(a*x + 1)^6/(a*x - 1)^6 + 21*(a*x + 1)^5/(a*x - 1
)^5 + 35*(a*x + 1)^4/(a*x - 1)^4 + 35*(a*x + 1)^3/(a*x - 1)^3 + 21*(a*x +
1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1) + 1) - (2*(a*x + 1)^5*a^7/(a*x -
1)^5 + 11*(a*x + 1)^4*a^7/(a*x - 1)^4 + 6*(a*x + 1)^3*a^7/(a*x - 1)^3 + 11
*(a*x + 1)^2*a^7/(a*x - 1)^2 + 2*(a*x + 1)*a^7/(a*x - 1))/((a*x + 1)^6/(a*
x - 1)^6 + 6*(a*x + 1)^5/(a*x - 1)^5 + 15*(a*x + 1)^4/(a*x - 1)^4 + 20*(a*
x + 1)^3/(a*x - 1)^3 + 15*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a*x - 1)
+ 1))*a
```

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.10

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} dx = \frac{4a^8 \ln(x)}{63} + \frac{a^8 \ln(ax+1)^2}{96} + \frac{a^8 \ln(1-ax)^2}{96} - \frac{\ln(ax+1)^2}{32x^8} - \frac{\ln(1-ax)^2}{32x^8} - \frac{2a^8 \ln(a^2x^2-1)}{63} - \frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a^8 \ln(ax+1)\ln(1-ax)}{48} + \frac{\ln(ax+1)\ln(1-ax)}{16x^8} + \frac{a^2\ln(ax+1)^2}{12x^6} - \frac{a^4\ln(ax+1)^2}{16x^4} + \frac{a^2\ln(1-ax)^2}{12x^6} - \frac{a^4\ln(1-ax)^2}{16x^4} - \frac{a\ln(ax+1)}{56x^7} + \frac{a\ln(1-ax)}{56x^7} + \frac{a^3\ln(ax+1)}{24x^5} - \frac{a^5\ln(ax+1)}{72x^3} - \frac{a^7\ln(ax+1)}{24x} - \frac{a^3\ln(1-ax)}{24x^5} + \frac{a^5\ln(1-ax)}{72x^3} + \frac{a^7\ln(1-ax)}{24x} - \frac{a^2\ln(ax+1)\ln(1-ax)}{6x^6} + \frac{a^4\ln(ax+1)\ln(1-ax)}{8x^4}$$

input
```
int((atanh(a*x)^2*(a^2*x^2 - 1)^2)/x^9,x)
```

output
```
(4*a^8*log(x))/63 + (a^8*log(a*x + 1)^2)/96 + (a^8*log(1 - a*x)^2)/96 - lo
g(a*x + 1)^2/(32*x^8) - log(1 - a*x)^2/(32*x^8) - (2*a^8*log(a^2*x^2 - 1))
/63 - a^2/(168*x^6) + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a^8*log(a*x + 1)
*log(1 - a*x))/48 + (log(a*x + 1)*log(1 - a*x))/(16*x^8) + (a^2*log(a*x +
1)^2)/(12*x^6) - (a^4*log(a*x + 1)^2)/(16*x^4) + (a^2*log(1 - a*x)^2)/(12*
x^6) - (a^4*log(1 - a*x)^2)/(16*x^4) - (a*log(a*x + 1))/(56*x^7) + (a*log(
1 - a*x))/(56*x^7) + (a^3*log(a*x + 1))/(24*x^5) - (a^5*log(a*x + 1))/(72*
x^3) - (a^7*log(a*x + 1))/(24*x) - (a^3*log(1 - a*x))/(24*x^5) + (a^5*log(
1 - a*x))/(72*x^3) + (a^7*log(1 - a*x))/(24*x) - (a^2*log(a*x + 1)*log(1 -
 a*x))/(6*x^6) + (a^4*log(a*x + 1)*log(1 - a*x))/(8*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96

$$\int \frac{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}{x^9} dx$$
$$= \frac{21atanh(ax)^2 a^8x^8 - 126atanh(ax)^2 a^4x^4 + 168atanh(ax)^2 a^2x^2 - 63atanh(ax)^2 - 32atanh(ax) a^8x^8 -}{}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^2/x^9,x)
```

output
```
(21*atanh(a*x)**2*a**8*x**8 - 126*atanh(a*x)**2*a**4*x**4 + 168*atanh(a*x)
**2*a**2*x**2 - 63*atanh(a*x)**2 - 32*atanh(a*x)*a**8*x**8 - 42*atanh(a*x)
*a**7*x**7 - 14*atanh(a*x)*a**5*x**5 + 42*atanh(a*x)*a**3*x**3 - 18*atanh(
a*x)*a*x - 32*log(a**2*x - a)*a**8*x**8 + 32*log(x)*a**8*x**8 + 5*a**6*x**
6 + 6*a**4*x**4 - 3*a**2*x**2)/(504*x**8)
```

3.217 $\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 248

$$\begin{aligned}\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^3 \, dx = & -\frac{1-a^2x^2}{20a} - x\operatorname{arctanh}(ax) - \frac{1}{10}x(1-a^2x^2)\operatorname{arctanh}(ax) \\ & + \frac{2(1-a^2x^2)\operatorname{arctanh}(ax)^2}{5a} \\ & + \frac{3(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{20a} + \frac{8\operatorname{arctanh}(ax)^3}{15a} \\ & + \frac{8}{15}x\operatorname{arctanh}(ax)^3 + \frac{4}{15}x(1-a^2x^2)\operatorname{arctanh}(ax)^3 \\ & + \frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax)^3 \\ & - \frac{8\operatorname{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{5a} - \frac{\log(1-a^2x^2)}{2a} \\ & - \frac{8\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a} \\ & + \frac{4\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{5a}\end{aligned}$$

output
```
-1/20*(-a^2*x^2+1)/a-x*arctanh(a*x)-1/10*x*(-a^2*x^2+1)*arctanh(a*x)+2/5*(
-a^2*x^2+1)*arctanh(a*x)^2/a+3/20*(-a^2*x^2+1)^2*arctanh(a*x)^2/a+8/15*arc
tanh(a*x)^3/a+8/15*x*arctanh(a*x)^3+4/15*x*(-a^2*x^2+1)*arctanh(a*x)^3+1/5
*x*(-a^2*x^2+1)^2*arctanh(a*x)^3-8/5*arctanh(a*x)^2*ln(2/(-a*x+1))/a-1/2*l
n(-a^2*x^2+1)/a-8/5*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+4/5*polylog(3,1
-2/(-a*x+1))/a
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3 \, dx$$

$$= \frac{-3+3a^2x^2-66ax\operatorname{arctanh}(ax)+6a^3x^3\operatorname{arctanh}(ax)+33\operatorname{arctanh}(ax)^2-42a^2x^2\operatorname{arctanh}(ax)^2+9a^4x^4a}{}$$

input
```
Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

output
```
(-3 + 3*a^2*x^2 - 66*a*x*ArcTanh[a*x] + 6*a^3*x^3*ArcTanh[a*x] + 33*ArcTan
h[a*x]^2 - 42*a^2*x^2*ArcTanh[a*x]^2 + 9*a^4*x^4*ArcTanh[a*x]^2 - 32*ArcTa
nh[a*x]^3 + 60*a*x*ArcTanh[a*x]^3 - 40*a^3*x^3*ArcTanh[a*x]^3 + 12*a^5*x^5
*ArcTanh[a*x]^3 - 96*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 30*Log[
1 - a^2*x^2] + 96*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 48*PolyL
og[3, -E^(-2*ArcTanh[a*x])])/(60*a)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {6506, 6504, 6436, 240, 6506, 6436, 240, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3 \, dx$$

$$\Big\downarrow \text{6506}$$

$$-\frac{3}{10}\int \left(1-a^2x^2\right)\operatorname{arctanh}(ax)dx+\frac{4}{5}\int \left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2}{20a}$$

$$\Big\downarrow \text{6504}$$

$$-\frac{3}{10}\left(\frac{2}{3}\int \operatorname{arctanh}(ax)dx+\frac{1}{3}x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{4}{5}\int \left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2}{20a}$$

$$\Big\downarrow \text{6436}$$

$$-\frac{3}{10}\left(\frac{2}{3}\left(x\operatorname{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{4}{5}\int \left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2}{20a}$$

$$\Big\downarrow \text{240}$$

$$\frac{4}{5}\int \left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2}{20a}-\frac{3}{10}\left(\frac{1}{3}x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\operatorname{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$$\Big\downarrow \text{6506}$$

$$\frac{4}{5}\left(-\int \operatorname{arctanh}(ax)dx+\frac{2}{3}\int \operatorname{arctanh}(ax)^3dx+\frac{1}{3}x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}{2a}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^2}{20a}-\frac{3}{10}\left(\frac{1}{3}x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\operatorname{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$$\Big\downarrow \text{6436}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx\right)+a\int\frac{x}{1-a^2x^2}dx+\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)^3+\frac{(1-a^2x^2)}{}\right.$$
$$\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)^3+\frac{3(1-a^2x^2)^2\text{arctanh}(ax)^2}{20a}-$$
$$\frac{3}{10}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$\downarrow$ 240

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx\right)+\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\text{arctanh}(ax)^2}{2a}-\frac{\text{lo}}{}\right.$$
$$\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)^3+\frac{3(1-a^2x^2)^2\text{arctanh}(ax)^2}{20a}-$$
$$\frac{3}{10}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$\downarrow$ 6546

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\int\frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)^3+\frac{(1-a^2x^2)}{}\right.$$
$$\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)^3+\frac{3(1-a^2x^2)^2\text{arctanh}(ax)^2}{20a}-$$
$$\frac{3}{10}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$\downarrow$ 6470

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(1-\right.$$
$$\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)^3+\frac{3(1-a^2x^2)^2\text{arctanh}(ax)^2}{20a}-$$
$$\frac{3}{10}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$\downarrow$ 6620

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right.}{a}\right.\right.\right.$$

$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}{20a}-$$
$$\frac{3}{10}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

$$\Big\downarrow \; 7164$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\right.\right.\right.$$

$$\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3+\frac{3\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}{20a}-$$
$$\frac{3}{10}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-a^2x^2}{6a}\right)$$

input
```
Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

output
```
(3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2)/(20*a) + (x*(1 - a^2*x^2)^2*ArcTanh[a*x
]^3)/5 - (3*((1 - a^2*x^2)/(6*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x])/3 + (2*(
x*ArcTanh[a*x] + Log[1 - a^2*x^2]/(2*a)))/3))/10 + (4*(-(x*ArcTanh[a*x]) +
 ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(2*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x]^3)/3
 - Log[1 - a^2*x^2]/(2*a) + (2*(x*ArcTanh[a*x]^3 - 3*a*(-1/3*ArcTanh[a*x]^
3/a^2 + ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(ArcTanh[a*x]*PolyL
og[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a))/3))/5
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.51 (sec) , antiderivative size = 858, normalized size of antiderivative = 3.46

method	result	size
derivativedivides	Expression too large to display	858
default	Expression too large to display	858
parts	Expression too large to display	865

input
```
int((-a^2*x^2+1)^2*arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(2/5*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*x)
^2+1/5*arctanh(a*x)^3*a^5*x^5+4/5*arctanh(a*x)^2*ln(a*x+1)-8/5*arctanh(a*x
)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/10-2/3*arctanh(a*x)^3*a^3*x^3-4/5*I*P
i*arctanh(a*x)^2+1/10*a*x+arctanh(a*x)^3*a*x+4/5*arctanh(a*x)^2*ln(a*x-1)-
8/5*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+8/15*arctanh(a*x)^3+11
/20*arctanh(a*x)^2+2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-2/5*I*Pi*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2
+3/20*a^4*x^4*arctanh(a*x)^2-7/10*a^2*x^2*arctanh(a*x)^2-8/5*arctanh(a*x)^
2*ln(2)-4/5*(a*x+1)*arctanh(a*x)+1/20*(a*x-1)^2-2/5*I*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2+4/5*I*Pi*csgn(I/(
-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-2/5*I*Pi*csgn(I*(a*x+1)^2/(a^2
*x^2-1))^3*arctanh(a*x)^2-4/5*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*ar
ctanh(a*x)^2+1/10*(a^2*x^2-4*a*x+7)*(a*x+1)*arctanh(a*x)+3/10*(a*x-3)*(a*x
+1)*arctanh(a*x)+ln((a*x+1)^2/(-a^2*x^2+1)+1)-4/5*I*Pi*csgn(I*(a*x+1)/(-a^
2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-2/5*I*Pi*cs
gn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/
(a^2*x^2-1)+1))^2*arctanh(a*x)^2+4/5*polylog(3,-(a*x+1)^2/(-a^2*x^2+1)))
```

Fricas [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3\,dx = \int \left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3\,dx=\int (ax-1)^2\,(ax+1)^2\operatorname{atanh}^3{(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)**2*atanh(a*x)**3,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3, x)
```

Maxima [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3\,dx=\int \left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2400*(36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 + 480*a*x - 6
0*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1)^2/a -
1/8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/
a - 1/1440000*(288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x
 + 1) - 6)*(a*x - 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 1
2*log(-a*x + 1) - 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x +
 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*lo
g(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3
 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/432*(4*(9*log
(-a*x + 1)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(
4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 +
 108*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)
)/a - 1/8*integrate(-1/150*(150*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2
 + a*x - 1)*log(a*x + 1)^3 + (36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*
a^2*x^2 - 450*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*
x + 1)^2 + 480*a*x - 60*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1)
)*log(-a*x + 1))/(a*x - 1), x)
```

Giac [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3 \, dx = \int \left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^3 \, dx$$

input
```
integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*arctanh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3 \, dx = \int \operatorname{atanh}(a\,x)^3 \left(a^2\,x^2-1\right)^2 dx$$

input
```
int(atanh(a*x)^3*(a^2*x^2 - 1)^2,x)
```

output
```
int(atanh(a*x)^3*(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3 \, dx$$
$$= \frac{12atanh(ax)^3 a^5x^5 - 40atanh(ax)^3 a^3x^3 + 60atanh(ax)^3 ax + 9atanh(ax)^2 a^4x^4 - 42atanh(ax)^2 a^2x^2 +}{}$$

input
```
int((-a^2*x^2+1)^2*atanh(a*x)^3,x)
```

output
```
(12*atanh(a*x)**3*a**5*x**5 - 40*atanh(a*x)**3*a**3*x**3 + 60*atanh(a*x)**
3*a*x + 9*atanh(a*x)**2*a**4*x**4 - 42*atanh(a*x)**2*a**2*x**2 + 33*atanh(
a*x)**2 + 6*atanh(a*x)*a**3*x**3 - 66*atanh(a*x)*a*x - 60*atanh(a*x) + 96*
int((atanh(a*x)**2*x)/(a**2*x**2 - 1),x)*a**2 - 60*log(a**2*x - a) + 3*a**
2*x**2)/(60*a)
```

3.218 $\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)} dx = \operatorname{Int}\left(\frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}, x\right)$$

output
```
Defer(Int)(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)} dx = \int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)} dx$$

input
```
Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
```

output
```
Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-a^2x^2)^2}{\text{arctanh}(ax)} \, dx$$
$$\Big\downarrow \text{6651}$$
$$\int \frac{x(1-a^2x^2)^2}{\text{arctanh}(ax)}dx$$

input
```
Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(-a^2x^2+1)^2}{\text{arctanh}\,(ax)}dx$$

input
```
int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

output
```
int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2x}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \int \frac{x(ax-1)^2(ax+1)^2}{\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x*(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2x}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2x}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \int \frac{x\,(a^2\,x^2-1)^2}{\operatorname{atanh}(a\,x)}\,dx$$

input
```
int((x*(a^2*x^2 - 1)^2)/atanh(a*x),x)
```

output
```
int((x*(a^2*x^2 - 1)^2)/atanh(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \left(\int \frac{x^5}{atanh\,(ax)}dx\right)a^4 - 2\left(\int \frac{x^3}{atanh\,(ax)}dx\right)a^2 + \int \frac{x}{atanh\,(ax)}dx$$

input
```
int(x*(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
int(x**5/atanh(a*x),x)*a**4 - 2*int(x**3/atanh(a*x),x)*a**2 + int(x/atanh(
a*x),x)
```

3.219 $\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \operatorname{Int}\left(\frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^2/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx = \int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

output
```
Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)}dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)}dx$$

input
```
Int[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(-a^2x^2+1)^2}{\text{arctanh}\,(ax)}dx$$

input
```
int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

output
```
int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2}{\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)}\,dx = \int \frac{(ax-1)^2\,(ax+1)^2}{\text{atanh}\,(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2}{\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx=\int \frac{(a^2x^2-1)^2}{\operatorname{artanh}(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx=\int \frac{(a^2\,x^2-1)^2}{\operatorname{atanh}(a\,x)}\,dx$$

input
```
int((a^2*x^2 - 1)^2/atanh(a*x),x)
```

output
```
int((a^2*x^2 - 1)^2/atanh(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.37

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)}\,dx$$
$$= \frac{\left(\int \frac{x^6}{atanh(ax)a^2x^2-atanh(ax)}dx\right)a^7 - 3\left(\int \frac{x^4}{atanh(ax)a^2x^2-atanh(ax)}dx\right)a^5 + 3\left(\int \frac{x^2}{atanh(ax)a^2x^2-atanh(ax)}dx\right)a^3 + \mathrm{lo}}{a}$$

input
```
int((-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
(int(x**6/(atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**7 - 3*int(x**4/(atanh(
a*x)*a**2*x**2 - atanh(a*x)),x)*a**5 + 3*int(x**2/(atanh(a*x)*a**2*x**2 -
atanh(a*x)),x)*a**3 + log(atanh(a*x)))/a
```

3.220 $\int \frac{(1-a^2x^2)^2}{x\textbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx=\text{Int}\left(\frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx=\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx$$

input
```
Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
```

output
```
Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}dx$$

input
```
Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(-a^2x^2+1)^2}{x \operatorname{arctanh}(ax)}dx$$

input
```
int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

output
```
int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2}{x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx = \int \frac{(ax-1)^2\,(ax+1)^2}{x\,\text{atanh}\,(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)**2/x/atanh(a*x),x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2}{x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx = \int \frac{(a^2x^2-1)^2}{x\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx = \int \frac{(a^2\,x^2-1)^2}{x\,\text{atanh}\,(a\,x)}\,dx$$

input
```
int((a^2*x^2 - 1)^2/(x*atanh(a*x)),x)
```

output
```
int((a^2*x^2 - 1)^2/(x*atanh(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)}\,dx = \left(\int \frac{x^3}{atanh\,(ax)}dx\right)a^4 - 2\left(\int \frac{x}{atanh\,(ax)}dx\right)a^2 + \int \frac{1}{atanh\,(ax)\,x}dx$$

input

```
int((-a^2*x^2+1)^2/x/atanh(a*x),x)
```

output

```
int(x**3/atanh(a*x),x)*a**4 - 2*int(x/atanh(a*x),x)*a**2 + int(1/(atanh(a*
x)*x),x)
```

3.221 $\int \frac{x(1-a^2x^2)^2}{\textbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \operatorname{Int}\left(\frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx$$

input
```
Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

output
```
Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(1-a^2x^2\right)^2}{\text{arctanh}(ax)^2}\,dx$$
$$\Big\downarrow \ 6651$$
$$\int \frac{x\left(1-a^2x^2\right)^2}{\text{arctanh}(ax)^2}dx$$

input
```
Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(-a^2x^2+1)^2}{\text{arctanh}\left(ax\right)^2}dx$$

input
```
int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

output
```
int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2x}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x(ax-1)^2\,(ax+1)^2}{\operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(x*(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2x}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)/(a*log(a*x + 1) - a*log(-a*x + 1))
 + integrate(-2*(7*a^6*x^6 - 15*a^4*x^4 + 9*a^2*x^2 - 1)/(a*log(a*x + 1) -
 a*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x(1-a^2x^2)^2}{\text{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2x}{\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2*x/arctanh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x(1-a^2x^2)^2}{\text{arctanh}(ax)^2}\,dx = \int \frac{x\,(a^2\,x^2-1)^2}{\text{atanh}\,(a\,x)^2}\,dx$$

input
```
int((x*(a^2*x^2 - 1)^2)/atanh(a*x)^2,x)
```

output
```
int((x*(a^2*x^2 - 1)^2)/atanh(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{x(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \left(\int \frac{x^5}{atanh\,(ax)^2}dx\right)a^4 - 2\left(\int \frac{x^3}{atanh\,(ax)^2}dx\right)a^2 + \int \frac{x}{atanh\,(ax)^2}dx$$

input
```
int(x*(-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
int(x**5/atanh(a*x)**2,x)*a**4 - 2*int(x**3/atanh(a*x)**2,x)*a**2 + int(x/
atanh(a*x)**2,x)
```

3.222 $\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2} \, dx = \operatorname{Int}\left(\frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}, x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2} \, dx = \int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2} \, dx$$

input
```
Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]
```

output
```
Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \ 6651$$

$$\int \frac{(1-a^2x^2)^2}{\text{arctanh}(ax)^2}dx$$

input
```
Int[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(-a^2x^2+1)^2}{\text{arctanh}\left(ax\right)^2}dx$$

input
```
int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2}{\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(ax-1)^2\,(ax+1)^2}{\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2}{\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*log(a*x + 1) - a*log(-a*x + 1))
 + integrate(-12*(a^5*x^5 - 2*a^3*x^3 + a*x)/(log(a*x + 1) - log(-a*x + 1)
), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2}{\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2/arctanh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{(a^2\,x^2-1)^2}{\operatorname{atanh}(a\,x)^2}\,dx$$

input
```
int((a^2*x^2 - 1)^2/atanh(a*x)^2,x)
```

output
```
int((a^2*x^2 - 1)^2/atanh(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{(1-a^2x^2)^2}{\operatorname{arctanh}(ax)^2}\,dx = \left(\int \frac{x^4}{atanh\,(ax)^2}dx\right)a^4 - 2\left(\int \frac{x^2}{atanh\,(ax)^2}dx\right)a^2 + \int \frac{1}{atanh\,(ax)^2}dx$$

input
```
int((-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
int(x**4/atanh(a*x)**2,x)*a**4 - 2*int(x**2/atanh(a*x)**2,x)*a**2 + int(1/
atanh(a*x)**2,x)
```

3.223 $\int \frac{(1-a^2x^2)^2}{x\textbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \text{Int}\left(\frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
```

output
```
Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}dx$$

input
```
Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(-a^2x^2+1)^2}{x \operatorname{arctanh}(ax)^2}dx$$

input
```
int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2}{x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{(ax-1)^2\,(ax+1)^2}{x\,\text{atanh}^2\,(ax)}\,dx$$

input
```
integrate((-a**2*x**2+1)**2/x/atanh(a*x)**2,x)
```

output
```
Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2}{x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*x*log(a*x + 1) - a*x*log(-a*x +
 1)) + integrate(-2*(5*a^6*x^6 - 9*a^4*x^4 + 3*a^2*x^2 + 1)/(a*x^2*log(a*x
 + 1) - a*x^2*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{(a^2x^2-1)^2}{x\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \int \frac{(a^2\,x^2-1)^2}{x\,\text{atanh}\,(a\,x)^2}\,dx$$

input
```
int((a^2*x^2 - 1)^2/(x*atanh(a*x)^2),x)
```

output
```
int((a^2*x^2 - 1)^2/(x*atanh(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{(1-a^2x^2)^2}{x\text{arctanh}(ax)^2}\,dx = \left(\int \frac{x^3}{atanh\left(ax\right)^2}dx\right)a^4 - 2\left(\int \frac{x}{atanh\left(ax\right)^2}dx\right)a^2 + \int \frac{1}{atanh\left(ax\right)^2 x}dx$$

input
```
int((-a^2*x^2+1)^2/x/atanh(a*x)^2,x)
```

output
```
int(x**3/atanh(a*x)**2,x)*a**4 - 2*int(x/atanh(a*x)**2,x)*a**2 + int(1/(at
anh(a*x)**2*x),x)
```

3.224 $\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)\,dx = \frac{4(1-a^2x^2)}{35a} + \frac{3(1-a^2x^2)^2}{70a} + \frac{(1-a^2x^2)^3}{42a} + \frac{16}{35}x\operatorname{arctanh}(ax) + \frac{8}{35}x(1-a^2x^2)\operatorname{arctanh}(ax) + \frac{6}{35}x(1-a^2x^2)^2\operatorname{arctanh}(ax) + \frac{1}{7}x(1-a^2x^2)^3\operatorname{arctanh}(ax) + \frac{8\log(1-a^2x^2)}{35a}$$

output
```
4/35*(-a^2*x^2+1)/a+3/70*(-a^2*x^2+1)^2/a+1/42*(-a^2*x^2+1)^3/a+16/35*x*ar
ctanh(a*x)+8/35*x*(-a^2*x^2+1)*arctanh(a*x)+6/35*x*(-a^2*x^2+1)^2*arctanh(
a*x)+1/7*x*(-a^2*x^2+1)^3*arctanh(a*x)+8/35*ln(-a^2*x^2+1)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \left(1-a^2x^2\right)^3 \text{arctanh}(ax)\,dx = \frac{-57a^2x^2+24a^4x^4-5a^6x^6-6ax(-35+35a^2x^2-21a^4x^4+5a^6x^6)\,\text{arctanh}(ax)+48\log\left(1-a^2x^2\right)}{210a}$$

input
```
Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

output
```
(-57*a^2*x^2 + 24*a^4*x^4 - 5*a^6*x^6 - 6*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x
^4 + 5*a^6*x^6)*ArcTanh[a*x] + 48*Log[1 - a^2*x^2])/(210*a)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6504, 6504, 6504, 6436, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^3 \text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6504}$$

$$\frac{6}{7}\int \left(1-a^2x^2\right)^2 \text{arctanh}(ax)dx+\frac{1}{7}x\left(1-a^2x^2\right)^3 \text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^3}{42a}$$

$$\Big\downarrow \text{6504}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \left(1-a^2x^2\right) \text{arctanh}(ax)dx+\frac{1}{5}x\left(1-a^2x^2\right)^2 \text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^2}{20a}\right)+\frac{1}{7}x\left(1-a^2x^2\right)^3 \text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^3}{42a}$$

$$\Big\downarrow \text{6504}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \operatorname{arctanh}(ax)dx+\frac{1}{3}x(1-a^2x^2)\operatorname{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax)+\frac{(1-a^2x}{20a}\right.$$
$$\frac{1}{7}x(1-a^2x^2)^3\operatorname{arctanh}(ax)+\frac{(1-a^2x^2)^3}{42a}$$

$\big\downarrow$ 6436

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\operatorname{arctanh}(ax)-a\int\frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x(1-a^2x^2)\operatorname{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)+\frac{1}{5}x(1-a^2x^2)^2\operatorname{arctan}\right.$$
$$\frac{1}{7}x(1-a^2x^2)^3\operatorname{arctanh}(ax)+\frac{(1-a^2x^2)^3}{42a}$$

$\big\downarrow$ 240

$$\frac{1}{7}x(1-a^2x^2)^3\operatorname{arctanh}(ax)+$$
$$\frac{6}{7}\left(\frac{1}{5}x(1-a^2x^2)^2\operatorname{arctanh}(ax)+\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\operatorname{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\operatorname{arctanh}(ax)\right)+\frac{1-a}{6a}\right.\right.$$
$$\frac{(1-a^2x^2)^3}{42a}$$

input
```
Int[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

output
```
(1 - a^2*x^2)^3/(42*a) + (x*(1 - a^2*x^2)^3*ArcTanh[a*x])/7 + (6*((1 - a^2
*x^2)^2/(20*a) + (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 + (4*((1 - a^2*x^2)/(6
*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*x] + Log[1 - a^2*
x^2]/(2*a)))/3))/5))/7
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.58

method	result
parts	$-\frac{\operatorname{arctanh}(ax)a^6x^7}{7}+\frac{3\operatorname{arctanh}(ax)a^4x^5}{5}-x^3a^2\operatorname{arctanh}(ax)+x\operatorname{arctanh}(ax)-\frac{a\left(\frac{5a^4x^6}{6}-4a^2x\right.}{}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{3\operatorname{arctanh}(ax)a^5x^5}{5}-a^3x^3\operatorname{arctanh}(ax)+ax\operatorname{arctanh}(ax)-\frac{a^6x^6}{42}+\frac{4a^4x^4}{35}-\frac{19a^2x^2}{70}+\frac{8\ln(ax-1)}{35}+\frac{8\ln(}{3}}{a}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)a^7x^7}{7}+\frac{3\operatorname{arctanh}(ax)a^5x^5}{5}-a^3x^3\operatorname{arctanh}(ax)+ax\operatorname{arctanh}(ax)-\frac{a^6x^6}{42}+\frac{4a^4x^4}{35}-\frac{19a^2x^2}{70}+\frac{8\ln(ax-1)}{35}+\frac{8\ln(}{3}}{a}$
parallelrisch	$-\frac{30\operatorname{arctanh}(ax)a^7x^7+5a^6x^6-126\operatorname{arctanh}(ax)a^5x^5-24a^4x^4+210a^3x^3\operatorname{arctanh}(ax)+57a^2x^2-210ax\operatorname{arctanh}(ax)-}{210a}$
risch	$\left(-\frac{1}{14}a^6x^7+\frac{3}{10}a^4x^5-\frac{1}{2}a^2x^3+\frac{1}{2}x\right)\ln(ax+1)+\frac{a^6x^7\ln(-ax+1)}{14}-\frac{a^5x^6}{42}-\frac{3a^4x^5\ln(-ax+1)}{10}$
meijerg	$-\frac{\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{\sqrt{a^2x^2}}-2\ln(-a^2x^2+1)}{4a}-\frac{\frac{x^2a^2(4a^4x^4+6a^2x^2+12)}{42}-\frac{2x^8a^8\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a\right.}{7\sqrt{a^2x^2}}}{4a}$

input
```
int((-a^2*x^2+1)^3*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/7*arctanh(a*x)*a^6*x^7+3/5*arctanh(a*x)*a^4*x^5-x^3*a^2*arctanh(a*x)+x*
arctanh(a*x)-1/35*a*(5/6*a^4*x^6-4*a^2*x^4+19/2*x^2-8/a^2*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)\,dx = -\frac{5\,a^6x^6-24\,a^4x^4+57\,a^2x^2+3\,(5\,a^7x^7-21\,a^5x^5+35\,a^3x^3-35\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-48\,\log\left(a^2x^2-1\right)}{210\,a}$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/210*(5*a^6*x^6 - 24*a^4*x^4 + 57*a^2*x^2 + 3*(5*a^7*x^7 - 21*a^5*x^5 +
35*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 48*log(a^2*x^2 - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)\,dx = \begin{cases} -\frac{a^6x^7\operatorname{atanh}(ax)}{7}-\frac{a^5x^6}{42}+\frac{3a^4x^5\operatorname{atanh}(ax)}{5}+\frac{4a^3x^4}{35}-a^2x^3\operatorname{atanh}(ax)-\frac{19ax^2}{70}+x\operatorname{atanh}(ax)+\frac{16\log\left(x-\frac{1}{a}\right)}{35a}+ \\ 0 \end{cases}$$

input
```
integrate((-a**2*x**2+1)**3*atanh(a*x),x)
```

output
```
Piecewise((-a**6*x**7*atanh(a*x)/7 - a**5*x**6/42 + 3*a**4*x**5*atanh(a*x)
/5 + 4*a**3*x**4/35 - a**2*x**3*atanh(a*x) - 19*a*x**2/70 + x*atanh(a*x) +
 16*log(x - 1/a)/(35*a) + 16*atanh(a*x)/(35*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)\,dx$$
$$= -\frac{1}{210}\left(5\,a^4x^6 - 24\,a^2x^4 + 57\,x^2 - \frac{48\,\log\left(ax+1\right)}{a^2} - \frac{48\,\log\left(ax-1\right)}{a^2}\right)a - \frac{1}{35}\left(5\,a^6x^7 - 21\,a^4x^5 + 35\,a^2x^3 - 35\,x\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/210*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*log(a*x
 - 1)/a^2)*a - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3 - 35*x)*arctanh(a
*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(122) = 244.

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.10

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{8}{105}\,a\left(\frac{6\,\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^2} - \frac{6\,\log\left(\left|-\frac{ax+1}{ax-1}+1\right|\right)}{a^2} - \frac{\frac{6\,(ax+1)^5}{(ax-1)^5} - \frac{33\,(ax+1)^4}{(ax-1)^4} + \frac{74\,(ax+1)^3}{(ax-1)^3} - \frac{33\,(ax+1)^2}{(ax-1)^2} + \frac{6\,(ax+1)}{ax-1}}{a^2\left(\frac{ax+1}{ax-1}-1\right)^6}\right.$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="giac")
```

output
```
8/105*a*(6*log(abs(-a*x - 1)/abs(a*x - 1))/a^2 - 6*log(abs(-(a*x + 1)/(a*x
 - 1) + 1))/a^2 - (6*(a*x + 1)^5/(a*x - 1)^5 - 33*(a*x + 1)^4/(a*x - 1)^4
+ 74*(a*x + 1)^3/(a*x - 1)^3 - 33*(a*x + 1)^2/(a*x - 1)^2 + 6*(a*x + 1)/(a
*x - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^6) - 6*(35*(a*x + 1)^3/(a*x - 1)^3
 - 21*(a*x + 1)^2/(a*x - 1)^2 + 7*(a*x + 1)/(a*x - 1) - 1)*log(-(a*((a*x +
 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1
) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^2*((a*x + 1)/(a*x - 1) - 1)^7)
)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)\,dx = x\,\mathrm{atanh}(a\,x)-\frac{19\,a\,x^2}{70}+\frac{8\,\ln\left(a^2\,x^2-1\right)}{35\,a}+\frac{4\,a^3\,x^4}{35}-\frac{a^5\,x^6}{42}-a^2\,x^3\,\mathrm{atanh}(a\,x)+\frac{3\,a^4\,x^5\,\mathrm{atanh}(a\,x)}{5}-\frac{a^6\,x^7\,\mathrm{atanh}(a\,x)}{7}$$

input
```
int(-atanh(a*x)*(a^2*x^2 - 1)^3,x)
```

output
```
x*atanh(a*x) - (19*a*x^2)/70 + (8*log(a^2*x^2 - 1))/(35*a) + (4*a^3*x^4)/3
5 - (a^5*x^6)/42 - a^2*x^3*atanh(a*x) + (3*a^4*x^5*atanh(a*x))/5 - (a^6*x^
7*atanh(a*x))/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)\,dx = \frac{-30 atanh(ax)\,a^7x^7+126 atanh(ax)\,a^5x^5-210 atanh(ax)\,a^3x^3+210 atanh(ax)\,ax+96 atanh(ax)+96}{210a}$$

input
```
int((-a^2*x^2+1)^3*atanh(a*x),x)
```

output
```
( - 30*atanh(a*x)*a**7*x**7 + 126*atanh(a*x)*a**5*x**5 - 210*atanh(a*x)*a*
*3*x**3 + 210*atanh(a*x)*a*x + 96*atanh(a*x) + 96*log(a**2*x - a) - 5*a**6
*x**6 + 24*a**4*x**4 - 57*a**2*x**2)/(210*a)
```

3.225 $\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$\begin{aligned}\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2 \, dx = & -\frac{38x}{105}+\frac{19a^2x^3}{315}-\frac{a^4x^5}{105}+\frac{8(1-a^2x^2)\operatorname{arctanh}(ax)}{35a} \\ & +\frac{3(1-a^2x^2)^2\operatorname{arctanh}(ax)}{35a} \\ & +\frac{(1-a^2x^2)^3\operatorname{arctanh}(ax)}{21a}+\frac{16\operatorname{arctanh}(ax)^2}{35a} \\ & +\frac{16}{35}x\operatorname{arctanh}(ax)^2+\frac{8}{35}x(1-a^2x^2)\operatorname{arctanh}(ax)^2 \\ & +\frac{6}{35}x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2 \\ & +\frac{1}{7}x(1-a^2x^2)^3\operatorname{arctanh}(ax)^2 \\ & -\frac{32\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{35a} \\ & -\frac{16\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}\end{aligned}$$

output
```
-38/105*x+19/315*a^2*x^3-1/105*a^4*x^5+8/35*(-a^2*x^2+1)*arctanh(a*x)/a+3/
35*(-a^2*x^2+1)^2*arctanh(a*x)/a+1/21*(-a^2*x^2+1)^3*arctanh(a*x)/a+16/35*
arctanh(a*x)^2/a+16/35*x*arctanh(a*x)^2+8/35*x*(-a^2*x^2+1)*arctanh(a*x)^2
+6/35*x*(-a^2*x^2+1)^2*arctanh(a*x)^2+1/7*x*(-a^2*x^2+1)^3*arctanh(a*x)^2-
32/35*arctanh(a*x)*ln(2/(-a*x+1))/a-16/35*polylog(2,1-2/(-a*x+1))/a
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.55

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2 \, dx = -\frac{114ax - 19a^3x^3 + 3a^5x^5 + 9(-1+ax)^4\left(16+29ax+20a^2x^2+5a^3x^3\right)\operatorname{arctanh}(ax)^2 + 3\operatorname{arctanh}(ax)}{315a}$$

input
```
Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

output
```
-1/315*(114*a*x - 19*a^3*x^3 + 3*a^5*x^5 + 9*(-1 + a*x)^4*(16 + 29*a*x + 2
0*a^2*x^2 + 5*a^3*x^3)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-38 + 57*a^2*x^2 -
 24*a^4*x^4 + 5*a^6*x^6 + 96*Log[1 + E^(-2*ArcTanh[a*x])]) - 144*PolyLog[2
, -E^(-2*ArcTanh[a*x])])/a
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6506, 210, 2009, 6506, 2009, 6506, 24, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2 \, dx$$

$$\Big\downarrow \; 6506$$

$$\frac{6}{7}\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2dx - \frac{1}{21}\int \left(1-a^2x^2\right)^2 dx + \frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2 + \frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}$$

$$\Big\downarrow \; 210$$

$$\frac{6}{7}\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2dx - \frac{1}{21}\int \left(a^4x^4-2a^2x^2+1\right) dx + \frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2 + \frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{7}\int \left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2dx+\frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6506}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2dx-\frac{1}{10}\int \left(1-a^2x^2\right)dx+\frac{1}{5}x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)}{10a}\right)+\frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2dx+\frac{1}{5}x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}-x\right)\right)+\frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6506}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \operatorname{arctanh}(ax)^2dx-\frac{\int 1dx}{3}+\frac{1}{3}x\left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right) \operatorname{arctanh}(ax)}{3a}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2 \ldots\right)+\frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \operatorname{arctanh}(ax)^2dx+\frac{1}{3}x\left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right) \operatorname{arctanh}(ax)}{3a}-\frac{x}{3}\right)+\frac{1}{5}x\left(1-a^2x^2\right)^2 \operatorname{ar}\ldots\right)+\frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \text{6436}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\operatorname{arctanh}(ax)^2-2a\int \frac{x\operatorname{arctanh}(ax)}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right) \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right) \operatorname{arctanh}(ax)}{3a}-\ldots\right)\right)+\frac{1}{7}x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \; 6546$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^2+\frac{(1-a^2\right.\right.$$
$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \; 6470$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{arc}\right.\right.$$
$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \; 2849$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\,\text{a}\right.\right.$$
$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

$$\Big\downarrow \; 2752$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\right.\right.$$
$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}{21a}+\frac{1}{21}\left(-\frac{1}{5}a^4x^5+\frac{2a^2x^3}{3}-x\right)$$

input
```
Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

output
```
(-x + (2*a^2*x^3)/3 - (a^4*x^5)/5)/21 + ((1 - a^2*x^2)^3*ArcTanh[a*x])/(21
*a) + (x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2)/7 + (6*((-x + (a^2*x^3)/3)/10 + (
(1 - a^2*x^2)^2*ArcTanh[a*x])/(10*a) + (x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2)/
5 + (4*(-1/3*x + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*a) + (x*(1 - a^2*x^2)*Arc
Tanh[a*x]^2)/3 + (2*(x*ArcTanh[a*x]^2 - 2*a*(-1/2*ArcTanh[a*x]^2/a^2 + ((A
rcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)))/
3))/5))/7
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{3\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\operatorname{arctanh}(ax)^2 a^3 x^3+\operatorname{arctanh}(ax)^2 ax-\frac{\operatorname{arctanh}(ax)a^6x^6}{21}+\frac{8a^4x^4\operatorname{arctanh}(ax)}{35}-$
default	$-\frac{\operatorname{arctanh}(ax)^2 a^7 x^7}{7}+\frac{3\operatorname{arctanh}(ax)^2 a^5 x^5}{5}-\operatorname{arctanh}(ax)^2 a^3 x^3+\operatorname{arctanh}(ax)^2 ax-\frac{\operatorname{arctanh}(ax)a^6x^6}{21}+\frac{8a^4x^4\operatorname{arctanh}(ax)}{35}-$
parts	$-\frac{\operatorname{arctanh}(ax)^2 a^6 x^7}{7}+\frac{3\operatorname{arctanh}(ax)^2 a^4 x^5}{5}-x^3a^2\operatorname{arctanh}(ax)^2+x\operatorname{arctanh}(ax)^2-\frac{a^5\operatorname{arctanh}(}{21}$
risch	$-\frac{38x}{105}+\frac{4a^3\ln(ax+1)x^4}{35}-\frac{4a^3\ln(-ax+1)x^4}{35}+\frac{a^5\ln(-ax+1)x^6}{42}-\frac{a^5\ln(ax+1)x^6}{42}+\frac{19a^2x^3}{315}-\frac{a^4x^5}{105}+$

input
```
int((-a^2*x^2+1)^3*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/7*arctanh(a*x)^2*a^7*x^7+3/5*arctanh(a*x)^2*a^5*x^5-arctanh(a*x)^2
*a^3*x^3+arctanh(a*x)^2*a*x-1/21*arctanh(a*x)*a^6*x^6+8/35*a^4*x^4*arctanh
(a*x)-19/35*a^2*x^2*arctanh(a*x)+16/35*arctanh(a*x)*ln(a*x-1)+16/35*arctan
h(a*x)*ln(a*x+1)-16/35*dilog(1/2*a*x+1/2)-8/35*ln(a*x-1)*ln(1/2*a*x+1/2)+4
/35*ln(a*x-1)^2+8/35*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-4/35*ln(
a*x+1)^2-1/105*a^5*x^5+19/315*a^3*x^3-38/105*a*x-19/105*ln(a*x-1)+19/105*l
n(a*x+1))
```

Fricas [F]

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2 \, dx = \int -\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^2 \, dx$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2 \, dx = -\int 3a^2x^2 \operatorname{atanh}^2(ax) \, dx - \int \left(-3a^4x^4 \operatorname{atanh}^2(ax)\right) dx - \int a^6x^6 \operatorname{atanh}^2(ax) \, dx - \int \left(-\operatorname{atanh}^2(ax)\right) dx$$

input
```
integrate((-a**2*x**2+1)**3*atanh(a*x)**2,x)
```

output
```
-Integral(3*a**2*x**2*atanh(a*x)**2, x) - Integral(-3*a**4*x**4*atanh(a*x)
**2, x) - Integral(a**6*x**6*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2,
x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2\,dx =$$
$$-\frac{1}{315}a^2\left(\frac{3a^5x^5-19a^3x^3+114ax+36\log(ax+1)^2-72\log(ax+1)\log(ax-1)-36\log(ax-1)\right.$$
$$-\frac{1}{105}\left(5a^4x^6-24a^2x^4+57x^2-\frac{48\log(ax+1)}{a^2}-\frac{48\log(ax-1)}{a^2}\right)a\,\operatorname{artanh}(ax)$$
$$-\frac{1}{35}\left(5a^6x^7-21a^4x^5+35a^2x^3-35x\right)\operatorname{artanh}(ax)^2$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-1/315*a^2*((3*a^5*x^5 - 19*a^3*x^3 + 114*a*x + 36*log(a*x + 1)^2 - 72*log
(a*x + 1)*log(a*x - 1) - 36*log(a*x - 1)^2 + 57*log(a*x - 1))/a^3 + 144*(l
og(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 - 57*log(a*x +
 1)/a^3) - 1/105*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 -
48*log(a*x - 1)/a^2)*a*arctanh(a*x) - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^
2*x^3 - 35*x)*arctanh(a*x)^2
```

Giac [F]

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2\,dx = \int -(a^2x^2-1)^3\operatorname{artanh}(ax)^2\,dx$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2\,dx = -\int \operatorname{atanh}(a\,x)^2\,\left(a^2\,x^2-1\right)^3\,dx$$

input
```
int(-atanh(a*x)^2*(a^2*x^2 - 1)^3,x)
```

output
```
-int(atanh(a*x)^2*(a^2*x^2 - 1)^3, x)
```

Reduce [F]

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2\,dx = \frac{-45atanh(ax)^2\,a^7x^7+189atanh(ax)^2\,a^5x^5-315atanh(ax)^2\,a^3x^3+315atanh(ax)^2\,ax-15atanh(ax)\,a^{6}\cdots}{}$$

input
```
int((-a^2*x^2+1)^3*atanh(a*x)^2,x)
```

output
```
( - 45*atanh(a*x)**2*a**7*x**7 + 189*atanh(a*x)**2*a**5*x**5 - 315*atanh(a
*x)**2*a**3*x**3 + 315*atanh(a*x)**2*a*x - 15*atanh(a*x)*a**6*x**6 + 72*at
anh(a*x)*a**4*x**4 - 171*atanh(a*x)*a**2*x**2 + 114*atanh(a*x) + 288*int((
atanh(a*x)*x)/(a**2*x**2 - 1),x)*a**2 - 3*a**5*x**5 + 19*a**3*x**3 - 114*a
*x)/(315*a)
```

3.226 $\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 338

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3\,dx = -\frac{13(1-a^2x^2)}{210a}-\frac{(1-a^2x^2)^2}{140a}-\frac{14}{15}x\operatorname{arctanh}(ax) -\frac{13}{105}x(1-a^2x^2)\operatorname{arctanh}(ax) -\frac{1}{35}x(1-a^2x^2)^2\operatorname{arctanh}(ax) +\frac{12(1-a^2x^2)\operatorname{arctanh}(ax)^2}{35a} +\frac{9(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}{70a} +\frac{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}{14a}+\frac{16\operatorname{arctanh}(ax)^3}{35a} +\frac{16}{35}x\operatorname{arctanh}(ax)^3+\frac{8}{35}x(1-a^2x^2)\operatorname{arctanh}(ax)^3 +\frac{6}{35}x(1-a^2x^2)^2\operatorname{arctanh}(ax)^3 +\frac{1}{7}x(1-a^2x^2)^3\operatorname{arctanh}(ax)^3 -\frac{48\operatorname{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{35a}-\frac{7\log(1-a^2x^2)}{15a} -\frac{48\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a} +\frac{24\operatorname{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{35a}$$

output
```
1/210*(13*a^2*x^2-13)/a-1/140*(-a^2*x^2+1)^2/a-14/15*x*arctanh(a*x)-13/105
*x*(-a^2*x^2+1)*arctanh(a*x)-1/35*x*(-a^2*x^2+1)^2*arctanh(a*x)+12/35*(-a^
2*x^2+1)*arctanh(a*x)^2/a+9/70*(-a^2*x^2+1)^2*arctanh(a*x)^2/a+1/14*(-a^2*
x^2+1)^3*arctanh(a*x)^2/a+16/35*arctanh(a*x)^3/a+16/35*x*arctanh(a*x)^3+8/
35*x*(-a^2*x^2+1)*arctanh(a*x)^3+6/35*x*(-a^2*x^2+1)^2*arctanh(a*x)^3+1/7*
x*(-a^2*x^2+1)^3*arctanh(a*x)^3-48/35*arctanh(a*x)^2*ln(2/(-a*x+1))/a-7/15
*ln(-a^2*x^2+1)/a-48/35*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a+24/35*polyl
og(3,1-2/(-a*x+1))/a
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.68

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^3 \, dx =$$
$$-\frac{29 - 32a^2x^2 + 3a^4x^4 + 456ax\operatorname{arctanh}(ax) - 76a^3x^3\operatorname{arctanh}(ax) + 12a^5x^5\operatorname{arctanh}(ax) - 228\operatorname{arctanh}(}{}$$

input
```
Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

output
```
-1/420*(29 - 32*a^2*x^2 + 3*a^4*x^4 + 456*a*x*ArcTanh[a*x] - 76*a^3*x^3*Ar
cTanh[a*x] + 12*a^5*x^5*ArcTanh[a*x] - 228*ArcTanh[a*x]^2 + 342*a^2*x^2*Ar
cTanh[a*x]^2 - 144*a^4*x^4*ArcTanh[a*x]^2 + 30*a^6*x^6*ArcTanh[a*x]^2 + 19
2*ArcTanh[a*x]^3 - 420*a*x*ArcTanh[a*x]^3 + 420*a^3*x^3*ArcTanh[a*x]^3 - 2
52*a^5*x^5*ArcTanh[a*x]^3 + 60*a^7*x^7*ArcTanh[a*x]^3 + 576*ArcTanh[a*x]^2
*Log[1 + E^(-2*ArcTanh[a*x])] + 196*Log[1 - a^2*x^2] - 576*ArcTanh[a*x]*Po
lyLog[2, -E^(-2*ArcTanh[a*x])] - 288*PolyLog[3, -E^(-2*ArcTanh[a*x])])/a
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.38, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {6506, 6504, 6504, 6436, 240, 6506, 6504, 6436, 240, 6506, 6436, 240, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^2x^2)^3 \operatorname{arctanh}(ax)^3 \, dx$$

$$\Big\downarrow \text{6506}$$

$$-\frac{1}{7}\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)dx + \frac{6}{7}\int (1-a^2x^2)^2 \operatorname{arctanh}(ax)^3dx + \frac{1}{7}x(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3 + \frac{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}{14a}$$

$$\Big\downarrow \text{6504}$$

$$\frac{1}{7}\left(-\frac{4}{5}\int \left(1-a^2x^2\right)\text{arctanh}(ax)dx-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)-\frac{\left(1-a^2x^2\right)^2}{20a}\right)+$$
$$\frac{6}{7}\int \left(1-a^2x^2\right)^2\text{arctanh}(ax)^3dx+\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}$$

$\downarrow$ 6504

$$\frac{1}{7}\left(-\frac{4}{5}\left(\frac{2}{3}\int \text{arctanh}(ax)dx+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)-\frac{\left(1-a^2\right.}{20a}\right.$$
$$\frac{6}{7}\int \left(1-a^2x^2\right)^2\text{arctanh}(ax)^3dx+\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}$$

$\downarrow$ 6436

$$\frac{1}{7}\left(-\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\right)+\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\right)-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arct}\right.$$
$$\frac{6}{7}\int \left(1-a^2x^2\right)^2\text{arctanh}(ax)^3dx+\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}$$

$\downarrow$ 240

$$\frac{6}{7}\int \left(1-a^2x^2\right)^2\text{arctanh}(ax)^3dx+\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}+$$
$$\frac{1}{7}\left(-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\downarrow$ 6506

$$\frac{6}{7}\left(-\frac{3}{10}\int \left(1-a^2x^2\right)\text{arctanh}(ax)dx+\frac{4}{5}\int \left(1-a^2x^2\right)\text{arctanh}(ax)^3dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3+\frac{3(1-}{}\right.$$
$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}+$$
$$\frac{1}{7}\left(-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\downarrow$ 6504

$$\frac{6}{7}\Bigg(-\frac{3}{10}\Bigg(\frac{2}{3}\int \text{arctanh}(ax)dx+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\Bigg)+\frac{4}{5}\int (1-a^2x^2)\,\text{arctanh}(ax)^3dx+\frac{1}{5}$$
$$\frac{1}{7}x(1-a^2x^2)^3\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{14a}+$$
$$\frac{1}{7}\Bigg(-\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)-\frac{4}{5}\Bigg(\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{2}{3}\Bigg(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\Bigg)+\frac{1-}{6}$$

$$\Big\downarrow \; 6436$$

$$\frac{6}{7}\Bigg(-\frac{3}{10}\Bigg(\frac{2}{3}\Bigg(x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx\Bigg)+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{1-a^2x^2}{6a}\Bigg)+\frac{4}{5}\int (1-a^2x^2)\,\text{ar}$$
$$\frac{1}{7}x(1-a^2x^2)^3\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{14a}+$$
$$\frac{1}{7}\Bigg(-\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)-\frac{4}{5}\Bigg(\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{2}{3}\Bigg(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\Bigg)+\frac{1-}{6}$$

$$\Big\downarrow \; 240$$

$$\frac{6}{7}\Bigg(\frac{4}{5}\int (1-a^2x^2)\,\text{arctanh}(ax)^3dx+\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)^3+\frac{3(1-a^2x^2)^2\,\text{arctanh}(ax)^2}{20a}-\frac{3}{10}\Bigg(\frac{1}{3}x(1-$$
$$\frac{1}{7}x(1-a^2x^2)^3\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{14a}+$$
$$\frac{1}{7}\Bigg(-\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)-\frac{4}{5}\Bigg(\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{2}{3}\Bigg(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\Bigg)+\frac{1-}{6}$$

$$\Big\downarrow \; 6506$$

$$\frac{6}{7}\Bigg(\frac{4}{5}\Bigg(-\int \text{arctanh}(ax)dx+\frac{2}{3}\int \text{arctanh}(ax)^3dx+\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\,\text{arctanh}(ax)^2}{2a}\Bigg)+$$
$$\frac{1}{7}x(1-a^2x^2)^3\,\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}{14a}+$$
$$\frac{1}{7}\Bigg(-\frac{1}{5}x(1-a^2x^2)^2\,\text{arctanh}(ax)-\frac{4}{5}\Bigg(\frac{1}{3}x(1-a^2x^2)\,\text{arctanh}(ax)+\frac{2}{3}\Bigg(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\Bigg)+\frac{1-}{6}$$

$$\Big\downarrow \; 6436$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx\right)+a\int\frac{x}{1-a^2x^2}dx+\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)^3+\frac{(1-a\right.\right.$$

$$\frac{1}{7}x(1-a^2x^2)^3\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\text{arctanh}(ax)^2}{14a}+$$

$$\frac{1}{7}\left(-\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\downarrow$ 240

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\int\frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx\right)+\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)^3+\frac{(1-a^2x^2)\text{arctanh}(ax)^2}{2a}\right.\right.$$

$$\frac{1}{7}x(1-a^2x^2)^3\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\text{arctanh}(ax)^2}{14a}+$$

$$\frac{1}{7}\left(-\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\downarrow$ 6546

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\int\frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)^3+\frac{(1-a^2\right.\right.$$

$$\frac{1}{7}x(1-a^2x^2)^3\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\text{arctanh}(ax)^2}{14a}+$$

$$\frac{1}{7}\left(-\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\downarrow$ 6470

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\right.\right.$$

$$\frac{1}{7}x(1-a^2x^2)^3\text{arctanh}(ax)^3+\frac{(1-a^2x^2)^3\text{arctanh}(ax)^2}{14a}+$$

$$\frac{1}{7}\left(-\frac{1}{5}x(1-a^2x^2)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\downarrow$ 6620

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right.}{a}\right.\right.\right.\right.$$

$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}+$$

$$\frac{1}{7}\left(-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

$\Big\downarrow$ 7164

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}\right.\right.\right.\right.$$

$$\frac{1}{7}x\left(1-a^2x^2\right)^3\text{arctanh}(ax)^3+\frac{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}{14a}+$$

$$\frac{1}{7}\left(-\frac{1}{5}x\left(1-a^2x^2\right)^2\text{arctanh}(ax)-\frac{4}{5}\left(\frac{1}{3}x\left(1-a^2x^2\right)\text{arctanh}(ax)+\frac{2}{3}\left(\frac{\log\left(1-a^2x^2\right)}{2a}+x\text{arctanh}(ax)\right)+\frac{1-}{6}\right.\right.$$

input
```
Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

output
```
((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(14*a) + (x*(1 - a^2*x^2)^3*ArcTanh[a*x]^
3)/7 + (-1/20*(1 - a^2*x^2)^2/a - (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 - (4*
((1 - a^2*x^2)/(6*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*
x] + Log[1 - a^2*x^2]/(2*a)))/3))/5)/7 + (6*((3*(1 - a^2*x^2)^2*ArcTanh[a*
x]^2)/(20*a) + (x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/5 - (3*((1 - a^2*x^2)/(6
*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x])/3 + (2*(x*ArcTanh[a*x] + Log[1 - a^2*
x^2]/(2*a)))/3))/10 + (4*(-(x*ArcTanh[a*x]) + ((1 - a^2*x^2)*ArcTanh[a*x]^
2)/(2*a) + (x*(1 - a^2*x^2)*ArcTanh[a*x]^3)/3 - Log[1 - a^2*x^2]/(2*a) + (
2*(x*ArcTanh[a*x]^3 - 3*a*(-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[
2/(1 - a*x)])/a - 2*(-1/2*(ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + P
olyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a)))/3))/5))/7
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.28 (sec) , antiderivative size = 978, normalized size of antiderivative = 2.89

method	result	size
derivativedivides	Expression too large to display	978
default	Expression too large to display	978
parts	Expression too large to display	983

input
```
int((-a^2*x^2+1)^3*arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(12/35*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*
x)^2+3/5*arctanh(a*x)^3*a^5*x^5+24/35*arctanh(a*x)^2*ln(a*x+1)-48/35*arcta
nh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/14*arctanh(a*x)^2*a^6*x^6-arcta
nh(a*x)^3*a^3*x^3-24/35*I*Pi*arctanh(a*x)^2+13/105*a*x+arctanh(a*x)^3*a*x+
24/35*arctanh(a*x)^2*ln(a*x-1)-48/35*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a
^2*x^2+1))+16/35*arctanh(a*x)^3+19/35*arctanh(a*x)^2+12/35*a^4*x^4*arctanh
(a*x)^2-57/70*a^2*x^2*arctanh(a*x)^2-13/105-48/35*arctanh(a*x)^2*ln(2)-24/
35*(a*x+1)*arctanh(a*x)-1/140*(a*x-1)^4-1/35*(a*x-1)^3-1/35*(a^4*x^4-6*a^3
*x^3+16*a^2*x^2-26*a*x+31)*(a*x+1)*arctanh(a*x)-1/7*(a^3*x^3-5*a^2*x^2+11*
a*x-15)*(a*x+1)*arctanh(a*x)-1/7*arctanh(a*x)^3*a^7*x^7-12/35*I*Pi*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2-12/35*I
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2+24/35*I*Pi*csgn(I/(-(a*
x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-24/35*I*Pi*csgn(I/(-(a*x+1)^2/(a^2
*x^2-1)+1))^3*arctanh(a*x)^2-12/35*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2
-12/35*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*arctanh(a*x)^2+12/35*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-24/35*I*Pi*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctan...
```

Fricas [F]

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3\,dx = \int -\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3\,dx = -\int 3a^2x^2 \operatorname{atanh}^3(ax)\,dx - \int \left(-3a^4x^4 \operatorname{atanh}^3(ax)\right)\,dx - \int a^6x^6 \operatorname{atanh}^3(ax)\,dx - \int \left(-\operatorname{atanh}^3(ax)\right)\,dx$$

input
```
integrate((-a**2*x**2+1)**3*atanh(a*x)**3,x)
```

output
```
-Integral(3*a**2*x**2*atanh(a*x)**3, x) - Integral(-3*a**4*x**4*atanh(a*x)
**3, x) - Integral(a**6*x**6*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3,
x)
```

Maxima [F]

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3\,dx = \int -\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^3\,dx$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
1/19600*(150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 840*a^4*x^4 + 1330*a^3*
x^3 - 1995*a^2*x^2 - 3360*a*x - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 -
 35*a*x - 16)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*l
og(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a + 1/691488000*(36000*(34
3*log(-a*x + 1)^3 - 147*log(-a*x + 1)^2 + 42*log(-a*x + 1) - 6)*(a*x - 1)^
7 + 2401000*(36*log(-a*x + 1)^3 - 18*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 1
)*(a*x - 1)^6 + 2074464*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log
(-a*x + 1) - 6)*(a*x - 1)^5 + 13505625*(32*log(-a*x + 1)^3 - 24*log(-a*x +
 1)^2 + 12*log(-a*x + 1) - 3)*(a*x - 1)^4 + 48020000*(9*log(-a*x + 1)^3 -
9*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 64827000*(4*log(-a*
x + 1)^3 - 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 86436000
*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a
- 1/480000*(288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x +
1) - 6)*(a*x - 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*l
og(-a*x + 1) - 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)
^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-
a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 -
3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/288*(4*(9*log(-a
*x + 1)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*l
og(-a*x + 1)^3 - 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 +...
```

Giac [F]

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3 \, dx = \int -\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^3 \, dx$$

input
```
integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3 \, dx = -\int \operatorname{atanh}(a\,x)^3 \left(a^2\,x^2-1\right)^3 \, dx$$

input
```
int(-atanh(a*x)^3*(a^2*x^2 - 1)^3,x)
```

output
```
-int(atanh(a*x)^3*(a^2*x^2 - 1)^3, x)
```

Reduce [F]

$$\int \left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3 \, dx = \frac{-60atanh(ax)^3 a^7x^7 + 252atanh(ax)^3 a^5x^5 - 420atanh(ax)^3 a^3x^3 + 420atanh(ax)^3 ax - 30atanh(ax)^2 a}{}$$

input
```
int((-a^2*x^2+1)^3*atanh(a*x)^3,x)
```

output
```
( - 60*atanh(a*x)**3*a**7*x**7 + 252*atanh(a*x)**3*a**5*x**5 - 420*atanh(a
*x)**3*a**3*x**3 + 420*atanh(a*x)**3*a*x - 30*atanh(a*x)**2*a**6*x**6 + 14
4*atanh(a*x)**2*a**4*x**4 - 342*atanh(a*x)**2*a**2*x**2 + 228*atanh(a*x)**
2 - 12*atanh(a*x)*a**5*x**5 + 76*atanh(a*x)*a**3*x**3 - 456*atanh(a*x)*a*x
 - 392*atanh(a*x) + 576*int((atanh(a*x)**2*x)/(a**2*x**2 - 1),x)*a**2 - 39
2*log(a**2*x - a) - 3*a**4*x**4 + 32*a**2*x**2)/(420*a)
```

3.227 $\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}\,dx = -\frac{x}{2a^3}+\frac{\text{arctanh}(ax)}{2a^4}-\frac{x^2\text{arctanh}(ax)}{2a^2}-\frac{\text{arctanh}(ax)^2}{2a^4}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}$$

output
```
-1/2*x/a^3+1/2*arctanh(a*x)/a^4-1/2*x^2*arctanh(a*x)/a^2-1/2*arctanh(a*x)^
2/a^4+arctanh(a*x)*ln(2/(-a*x+1))/a^4+1/2*polylog(2,1-2/(-a*x+1))/a^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{-ax+\text{arctanh}(ax)^2+\text{arctanh}(ax)\left(1-a^2x^2+2\log\left(1+e^{-2\text{arctanh}(ax)}\right)\right)-\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right)}{2a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

output
```
(-(a*x) + ArcTanh[a*x]^2 + ArcTanh[a*x]*(1 - a^2*x^2 + 2*Log[1 + E^(-2*Arc
Tanh[a*x])]) - PolyLog[2, -E^(-2*ArcTanh[a*x])])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\text{arctanh}(ax)dx}{a^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\int \frac{x^2}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{6546}$$

$$\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

output
```
-(((x^2*ArcTanh[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2) + (-1/
2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 -
 2/(1 - a*x)]/(2*a))/a)/a^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{-\frac{a^2x^2\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\ln(ax-1)^2}{8}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\cdots}{a^4}$
default	$\frac{-\frac{a^2x^2\operatorname{arctanh}(ax)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\ln(ax-1)^2}{8}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\cdots}{a^4}$
risch	$\frac{\ln(-ax+1)x^2}{4a^2}-\frac{\ln(-ax+1)}{4a^4}-\frac{x}{2a^3}+\frac{\ln(-ax+1)^2}{8a^4}+\frac{\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\ln(-ax+1)}{4a^4}-\frac{\operatorname{dilog}\left(-\frac{ax}{2}+\frac{1}{2}\right)}{4a^4}-\frac{\ln(ax+1)x\cdots}{4a^2}$
parts	$-\frac{x^2\operatorname{arctanh}(ax)}{2a^2}-\frac{\operatorname{arctanh}(ax)\ln\left(a^2x^2-1\right)}{2a^4}-\frac{a\left(-\frac{\ln(ax+1)\ln\left(a^2x^2-1\right)}{2a^5}+\frac{\ln(ax+1)^2}{4a^5}+\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{2a^5}-\frac{\ln(\cdots}{}\cdots\right)}{\cdots}$

input
```
int(x^3*arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/2*a^2*x^2*arctanh(a*x)-1/2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*
x)*ln(a*x+1)-1/2*a*x-1/4*ln(a*x-1)+1/4*ln(a*x+1)-1/8*ln(a*x-1)^2+1/2*dilog
(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)
-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2))
```

Fricas [F]

$$\int \frac{x^3\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx=\int -\frac{x^3\operatorname{artanh}(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3 \text{arctanh}(ax)}{1-a^2x^2}\,dx = -\int \frac{x^3\,\text{atanh}\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x**3*atanh(a*x)/(-a**2*x**2+1),x)
```

output
```
-Integral(x**3*atanh(a*x)/(a**2*x**2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

$$\int \frac{x^3 \text{arctanh}(ax)}{1-a^2x^2}\,dx = -\frac{1}{8}a\left(\frac{4\,ax-\log\,(ax+1)^2+2\,\log\,(ax+1)\log\,(ax-1)+\log\,(ax-1)^2+2\,\log\,(ax-1)}{a^5} - \frac{4\,(\log\,(ax\right.$$
$$-\frac{1}{2}\left(\frac{x^2}{a^2}+\frac{\log\,(a^2x^2-1)}{a^4}\right)\text{artanh}\,(ax)$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/8*a*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) + log(a*x -
1)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1
/2*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) - 1/2*(x^2/a^2 + log(a^2*x^2 - 1)
/a^4)*arctanh(a*x)
```

Giac [F]

$$\int \frac{x^3 \text{arctanh}(ax)}{1-a^2x^2}\,dx = \int -\frac{x^3 \,\text{artanh}\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arctanh}(ax)}{1-a^2x^2}\,dx = -\int \frac{x^3 \,\text{atanh}(a\,x)}{a^2\,x^2-1}\,dx$$

input
```
int(-(x^3*atanh(a*x))/(a^2*x^2 - 1),x)
```

output
```
-int((x^3*atanh(a*x))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x^3 \text{arctanh}(ax)}{1-a^2x^2}\,dx = -\left(\int \frac{atanh(ax)\,x^3}{a^2x^2-1}dx\right)$$

input
```
int(x^3*atanh(a*x)/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)*x**3)/(a**2*x**2 - 1),x)
```

3.228 $\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx = -\frac{x\text{arctanh}(ax)}{a^2}+\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\log(1-a^2x^2)}{2a^3}$$

output
```
-x*arctanh(a*x)/a^2+1/2*arctanh(a*x)^2/a^3-1/2*ln(-a^2*x^2+1)/a^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx = -\frac{x\text{arctanh}(ax)}{a^2}+\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\log(1-a^2x^2)}{2a^3}$$

input
```
Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

output
```
-((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3
)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6542, 6436, 240, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)dx}{a^2}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{6510}$$

$$\frac{\text{arctanh}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}$$

input
```
Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^2/(2*a^3) - (x*ArcTanh[a*x] + Log[1 - a^2*x^2]/(2*a))/a^2
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{2ax\,\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2+2\ln(ax-1)+2\,\operatorname{arctanh}(ax)}{2a^3}$
risch	$\frac{\ln(ax+1)^2}{8a^3}-\frac{(2ax+\ln(-ax+1))\ln(ax+1)}{4a^3}+\frac{\ln(-ax+1)x}{2a^2}+\frac{\ln(-ax+1)^2}{8a^3}-\frac{\ln(a^2x^2-1)}{2a^3}$
derivativedivides	$\frac{-ax\,\operatorname{arctanh}(ax)-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(ax-1)}{2}-\frac{\ln(ax+1)}{2}-}{a^3}$
default	$\frac{-ax\,\operatorname{arctanh}(ax)-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\frac{\ln(ax-1)}{2}-\frac{\ln(ax+1)}{2}-}{a^3}$
parts	$-\frac{x\,\operatorname{arctanh}(ax)}{a^2}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2a^3}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2a^3}-\frac{a\left(\frac{\frac{\ln(ax-1)^2}{4}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\ln(ax-1)\ln(}{2}}{a^4}\right.}{}$

input
```
int(x^2*arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*x*arctanh(a*x)-arctanh(a*x)^2+2*ln(a*x-1)+2*arctanh(a*x))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx = -\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right) - \log\left(-\frac{ax+1}{ax-1}\right)^2 + 4\,\log\left(a^2x^2-1\right)}{8\,a^3}$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
-1/8*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a*x - 1))^2 + 4*lo
g(a^2*x^2 - 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx = \begin{cases} -\frac{x\,\text{atanh}\,(ax)}{a^2} - \frac{\log\left(x-\frac{1}{a}\right)}{a^3} + \frac{\text{atanh}^2\,(ax)}{2a^3} - \frac{\text{atanh}\,(ax)}{a^3} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*atanh(a*x)/(-a**2*x**2+1),x)
```

output
```
Piecewise((-x*atanh(a*x)/a**2 - log(x - 1/a)/a**3 + atanh(a*x)**2/(2*a**3)
 - atanh(a*x)/a**3, Ne(a, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{x^2 \operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = -\frac{1}{2}\left(\frac{2\,x}{a^2} - \frac{\log\left(ax+1\right)}{a^3} + \frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right) + \frac{2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right) - \log\left(ax+1\right)^2 - \log\left(ax-1\right)^2 - 4\,\log\left(ax-1\right)}{8\,a^3}$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x) + 1/8*(2
*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log
(a*x - 1))/a^3
```

Giac [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \int -\frac{x^2\operatorname{artanh}\left(ax\right)}{a^2x^2-1}\,dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{\ln(a\,x+1)^2}{8\,a^3} - \ln(1-a\,x)\left(\frac{\ln(a\,x+1)}{4\,a^3} - \frac{x}{2\,a^2}\right) + \frac{\ln(1-a\,x)^2}{8\,a^3} - \frac{\ln(a^2\,x^2-1)}{2\,a^3} - \frac{x\,\ln(a\,x+1)}{2\,a^2}$$

input
```
int(-(x^2*atanh(a*x))/(a^2*x^2 - 1),x)
```

output
```
log(a*x + 1)^2/(8*a^3) - log(1 - a*x)*(log(a*x + 1)/(4*a^3) - x/(2*a^2)) +
 log(1 - a*x)^2/(8*a^3) - log(a^2*x^2 - 1)/(2*a^3) - (x*log(a*x + 1))/(2*a
^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{atanh(ax)^2 - 2atanh(ax)\,ax - 2atanh(ax) - 2\log(a^2x-a)}{2a^3}$$

input
```
int(x^2*atanh(a*x)/(-a^2*x^2+1),x)
```

output
```
(atanh(a*x)**2 - 2*atanh(a*x)*a*x - 2*atanh(a*x) - 2*log(a**2*x - a))/(2*a
**3)
```

3.229 $\int \frac{x\text{arctanh}(ax)}{1-a^2x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2} dx = -\frac{\text{arctanh}(ax)^2}{2a^2} + \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{\text{PolyLog}\left(2, 1-\frac{2}{1-ax}\right)}{2a^2}$$

output
```
-1/2*arctanh(a*x)^2/a^2+arctanh(a*x)*ln(2/(-a*x+1))/a^2+1/2*polylog(2,1-2/
(-a*x+1))/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2} dx = -\frac{-\text{arctanh}(ax)\left(\text{arctanh}(ax) + 2\log\left(1+e^{-2\text{arctanh}(ax)}\right)\right) + \text{PolyLog}\left(2, -e^{-2\text{arctanh}(ax)}\right)}{2a^2}$$

input
```
Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

output
```
-1/2*(-(ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 + E^(-2*ArcTanh[a*x])])) + Po
lyLog[2, -E^(-2*ArcTanh[a*x])])/a^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6546}$$

$$\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}$$

input
```
Int[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

output
```
-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2,
1 - 2/(1 - a*x)]/(2*a))/a
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

method	result
risch	$\frac{\ln(-ax+1)^2}{8a^2}+\frac{\ln(\frac{ax}{2}+\frac{1}{2})\ln(-ax+1)}{4a^2}-\frac{\operatorname{dilog}(-\frac{ax}{2}+\frac{1}{2})}{4a^2}-\frac{\ln(ax+1)^2}{8a^2}-\frac{\ln(-\frac{ax}{2}+\frac{1}{2})\ln(ax+1)}{4a^2}+\frac{\operatorname{dilog}(\frac{ax}{2}+}{4a^2}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)^2}{8}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(ax+1)^2}{8}-\frac{(\ln(ax+1)-}{}}{a^2}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)^2}{8}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(ax+1)^2}{8}-\frac{(\ln(ax+1)-}{}}{a^2}$
parts	$-\frac{\ln(a^2x^2-1)\operatorname{arctanh}(ax)}{2a^2}+\frac{\frac{\ln(ax+1)\ln(a^2x^2-1)}{2a}-\frac{\frac{\ln(ax+1)^2}{4}+\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}}{a}}{2a}$

input
```
int(x*arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*ln(-a*x+1)^2+1/4/a^2*ln(1/2*a*x+1/2)*ln(-a*x+1)-1/4/a^2*dilog(-1/2
*a*x+1/2)-1/8/a^2*ln(a*x+1)^2-1/4/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/4/a^2*d
ilog(1/2*a*x+1/2)
```

Fricas [F]

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}\,dx=\int -\frac{x\,\text{artanh}\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}\,dx=-\int \frac{x\,\text{atanh}\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x*atanh(a*x)/(-a**2*x**2+1),x)
```

output
```
-Integral(x*atanh(a*x)/(a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(47) = 94$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.31

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}\,dx=-\frac{1}{8}\,a\left(\frac{\log\left(ax+1\right)^2+2\,\log\left(ax+1\right)\log\left(ax-1\right)-\log\left(ax-1\right)^2}{a^3}-\frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\text{Li}\right.}{a^3}\right.$$
$$+\frac{\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\log\left(a^2x^2-1\right)}{4\,a}-\frac{\text{artanh}\,(ax)\log\left(a^2x^2-1\right)}{2\,a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/8*a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a^
3 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3) + 1/4
*(log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a - 1/2*arctanh(a*x)*l
og(a^2*x^2 - 1)/a^2
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}\,dx = \int -\frac{x\,\text{artanh}\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}\,dx = -\int \frac{x\,\text{atanh}(a\,x)}{a^2\,x^2-1}\,dx$$

input
```
int(-(x*atanh(a*x))/(a^2*x^2 - 1),x)
```

output
```
-int((x*atanh(a*x))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}\,dx = -\left(\int \frac{atanh(ax)\,x}{a^2x^2-1}dx\right)$$

input
```
int(x*atanh(a*x)/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)*x)/(a**2*x**2 - 1),x)
```

3.230 $\int \frac{\text{arctanh}(ax)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{\text{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{\text{arctanh}(ax)^2}{2a}$$

output
```
1/2*arctanh(a*x)^2/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{\text{arctanh}(ax)^2}{2a}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^2/(2*a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\operatorname{arctanh}(ax)^2}{2a}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^2/(2*a)
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^2}{2a}$
default	$\frac{\operatorname{arctanh}(ax)^2}{2a}$
parallelrisch	$\frac{\operatorname{arctanh}(ax)^2}{2a}$
risch	$\frac{\ln(ax+1)^2}{8a} - \frac{\ln(-ax+1)\ln(ax+1)}{4a} + \frac{\ln(-ax+1)^2}{8a}$
parts	$\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2a} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2a} - \frac{a\left(\frac{\frac{\ln(ax-1)^2}{4} - \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2} - \frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}}{a^2} - \frac{-\frac{\ln(ax+1)^2}{4}}{\cdots}\right.}{2}$

input
```
int(arctanh(a*x)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(a*x)^2/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8\,a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \begin{cases} \frac{\operatorname{atanh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1),x)
```

output
```
Piecewise((atanh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(11) = 22.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 5.00

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax) - \frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) + \log(ax-1)^2}{8\,a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x) - 1/8*(log(a*x + 1)^2 -
 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8\,a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{\left(\ln\left(a\,x+1\right)-\ln\left(1-a\,x\right)\right)^2}{8\,a}$$

input
```
int(-atanh(a*x)/(a^2*x^2 - 1),x)
```

output
```
(log(a*x + 1) - log(1 - a*x))^2/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}\,dx = \frac{atanh(ax)^2}{2a}$$

input
```
int(atanh(a*x)/(-a^2*x^2+1),x)
```

output
```
atanh(a*x)**2/(2*a)
```

3.231 $\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{\operatorname{arctanh}(ax)}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{2}\operatorname{arctanh}(ax)^2 + \operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
1/2*arctanh(a*x)^2+arctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1)
)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}(ax)}{x\left(1-a^2x^2\right)}\,dx = -\frac{1}{2}\operatorname{arctanh}(ax)^2 + \operatorname{arctanh}(ax)\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

output
```
-1/2*ArcTanh[a*x]^2 + ArcTanh[a*x]*Log[1 - E^(2*ArcTanh[a*x])] + PolyLog[2
, E^(2*ArcTanh[a*x])]/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx$$

$$\Big\downarrow \text{6550}$$

$$\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6494}$$

$$-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

input
```
Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

output
```
ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(
1 + a*x)]/2
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{\ln(ax+1)^2}{8}-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(-ax+1)^2}{8}+\operatorname{dilog}$
derivativedivides	$-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{\ln(ax-1)^2}{8}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}$
default	$-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{\ln(ax-1)^2}{8}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}$
parts	$\operatorname{arctanh}(ax)\ln(x)-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{a\left(\frac{\frac{\ln(ax-1)^2}{4}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\ln(}{}}{a}\right.}{}$

input
```
int(arctanh(a*x)/x/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(a*x+1)^2-1/2*dilog(a*x+1)-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*
a*x+1/2)+1/4*dilog(1/2*a*x+1/2)+1/8*ln(-a*x+1)^2+1/2*dilog(-a*x+1)+1/4*(ln
(-a*x+1)-ln(-1/2*a*x+1/2))*ln(1/2*a*x+1/2)-1/4*dilog(-1/2*a*x+1/2)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(ax)}{a^2x^3-x}\,dx$$

input
```
integrate(atanh(a*x)/x/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)/(a**2*x**3 - x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(40) = 80.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.93

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}\,dx$$
$$= \frac{1}{8}a\left(\frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right) + \operatorname{Li}_2(\cdot\right.}{a}\right.$$
$$-\frac{1}{2}\left(\log\left(a^2x^2-1\right) - \log\left(x^2\right)\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/8*a*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a +
 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*
x + 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a)
- 1/2*(log(a^2*x^2 - 1) - log(x^2))*arctanh(a*x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}\,dx = \int -\frac{\text{artanh}\,(ax)}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}\,dx = -\int \frac{\text{atanh}(a\,x)}{x\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)/(x*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)/(x*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)}\,dx = -\left(\int \frac{atanh(ax)}{a^2x^3-x}dx\right)$$

input
```
int(atanh(a*x)/x/(-a^2*x^2+1),x)
```

output
```
- int(atanh(a*x)/(a**2*x**3 - x),x)
```

3.232 $\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx = -\frac{\operatorname{arctanh}(ax)}{x} + \frac{1}{2}a\operatorname{arctanh}(ax)^2 + a\log(x) - \frac{1}{2}a\log\left(1-a^2x^2\right)$$

output
```
-arctanh(a*x)/x+1/2*a*arctanh(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx = -\frac{\operatorname{arctanh}(ax)}{x} + \frac{1}{2}a\operatorname{arctanh}(ax)^2 + a\log(x) - \frac{1}{2}a\log\left(1-a^2x^2\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

output
```
-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])
/2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)} \, dx$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)}{x^2}dx$$

$$\Big\downarrow \text{6452}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+a\int \frac{1}{x (1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\int \frac{1}{x^2 (1-a^2x^2)}dx^2-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}$$

input
```
Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

output
```
-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + (a*(Log[x^2] - Log[1 - a^2*x^2]
))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{\operatorname{arctanh}(ax)^2 ax+2a\ln(x)x-2\ln(ax-1)ax-2ax\operatorname{arctanh}(ax)-2\operatorname{arctanh}(ax)}{2x}$
risch	$\frac{a\ln(ax+1)^2}{8}-\frac{(ax\ln(-ax+1)+2)\ln(ax+1)}{4x}+\frac{a\ln(-ax+1)^2x+8a\ln(x)x-4a\ln(a^2x^2-1)x+4\ln(-ax+1)}{8x}$
parts	$\frac{a\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)}{x}-\frac{a\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{a\left(\frac{\ln(ax-1)^2}{4}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}-2\ln(x\right.}{}$
derivativedivides	$a\left(\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\ln(\right.$
default	$a\left(\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)}{ax}-\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}-\ln(\right.$

input
```
int(arctanh(a*x)/x^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctanh(a*x)^2*a*x+2*a*ln(x)*x-2*ln(a*x-1)*a*x-2*a*x*arctanh(a*x)-2*a
rctanh(a*x))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx = \frac{ax\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,ax\log\left(a^2x^2-1\right)+8\,ax\log(x)-4\,\log\left(-\frac{ax+1}{ax-1}\right)}{8\,x}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^2 - 4*a*x*log(a^2*x^2 - 1) + 8*a*x*log(
x) - 4*log(-(a*x + 1)/(a*x - 1)))/x
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx$$
$$= \begin{cases} a\log(x) - a\log\left(x-\frac{1}{a}\right) + \frac{a\,\operatorname{atanh}^2(ax)}{2} - a\,\operatorname{atanh}(ax) - \frac{\operatorname{atanh}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(a*x)/x**2/(-a**2*x**2+1),x)
```

output
```
Piecewise((a*log(x) - a*log(x - 1/a) + a*atanh(a*x)**2/2 - a*atanh(a*x) -
atanh(a*x)/x, Ne(a, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx$$
$$= \frac{1}{8}\left(2\left(\log(ax-1)-2\right)\log(ax+1) - \log(ax+1)^2 - \log(ax-1)^2 - 4\log(ax-1) + 8\log(x)\right)a$$
$$+ \frac{1}{2}\left(a\log(ax+1) - a\log(ax-1) - \frac{2}{x}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
 4*log(a*x - 1) + 8*log(x))*a + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x
)*arctanh(a*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)x^2}\,dx$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx = \frac{a\ln(a\,x+1)^2}{8} + \frac{a\ln(1-a\,x)^2}{8} - \frac{\ln(a\,x+1)}{2\,x} + \frac{\ln(1-a\,x)}{2\,x} - \frac{a\,\ln(a^2\,x^2-1)}{2} + a\,\ln(x) - \frac{a\,\ln(a\,x+1)\,\ln(1-a\,x)}{4}$$

input
```
int(-atanh(a*x)/(x^2*(a^2*x^2 - 1)),x)
```

output
```
(a*log(a*x + 1)^2)/8 + (a*log(1 - a*x)^2)/8 - log(a*x + 1)/(2*x) + log(1 -
 a*x)/(2*x) - (a*log(a^2*x^2 - 1))/2 + a*log(x) - (a*log(a*x + 1)*log(1 -
a*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)}\,dx = \frac{atanh(ax)^2\,ax - 2atanh(ax)\,ax - 2atanh(ax) - 2\log(a^2x-a)\,ax + 2\log(x)\,ax}{2x}$$

input
```
int(atanh(a*x)/x^2/(-a^2*x^2+1),x)
```

output

```
(atanh(a*x)**2*a*x - 2*atanh(a*x)*a*x - 2*atanh(a*x) - 2*log(a**2*x - a)*a
*x + 2*log(x)*a*x)/(2*x)
```

3.233 $\int \frac{\text{arctanh}(ax)}{x^3(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{\text{arctanh}(ax)}{x^3(1-a^2x^2)}\,dx = -\frac{a}{2x}+\frac{1}{2}a^2\text{arctanh}(ax)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a^2\text{arctanh}(ax)^2+a^2\text{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{2}a^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
-1/2*a/x+1/2*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+1/2*a^2*arctanh(a*x)^2+
a^2*arctanh(a*x)*ln(2-2/(a*x+1))-1/2*a^2*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\text{arctanh}(ax)}{x^3(1-a^2x^2)}\,dx = -\frac{1}{2}a^2\left(\frac{1}{ax}-\text{arctanh}(ax)\left(1-\frac{1}{a^2x^2}+\text{arctanh}(ax)+2\log\left(1-e^{-2\text{arctanh}(ax)}\right)\right)+\text{PolyLog}\left(2,e^{-2\text{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

output
```
-1/2*(a^2*(1/(a*x) - ArcTanh[a*x]*(1 - 1/(a^2*x^2) + ArcTanh[a*x] + 2*Log[
1 - E^(-2*ArcTanh[a*x])]) + PolyLog[2, E^(-2*ArcTanh[a*x])]))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^3(1-a^2x^2)}dx$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+\int \frac{\text{arctanh}(ax)}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \ 2897$$

$$a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

input
```
Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

output
```
-1/2*ArcTanh[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 + a^2*(ArcTanh[a*
x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/
2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(74) = 148.

Time = 0.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.02

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{1}{2ax}-\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)}{2a^2x^2}+\operatorname{arctanh}(ax)\ln(ax)-\frac{1}{2ax}-\right.$
risch	$-\frac{a^2\ln(ax+1)^2}{8}-\frac{a^2\operatorname{dilog}(ax+1)}{2}-\frac{a^2\ln(ax)}{4}-\frac{a}{2x}+\frac{a^2\ln(ax+1)}{4}-\frac{\ln(ax+1)}{4x^2}-\frac{a^2\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{4}$
parts	$-\frac{\operatorname{arctanh}(ax)}{2x^2}+\operatorname{arctanh}(ax)a^2\ln(x)-\frac{\operatorname{arctanh}(ax)a^2\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)a^2\ln(ax-1)}{2}-\frac{a\left(-\right.}{}$

input
```
int(arctanh(a*x)/x^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*arctanh(a*
x)/a^2/x^2+arctanh(a*x)*ln(a*x)-1/2/a/x-1/4*ln(a*x-1)+1/4*ln(a*x+1)-1/8*ln
(a*x-1)^2+1/2*dilog(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)+1/8*ln(a*x+
1)^2-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-1/2*dilog(a*x)-1/2*d
ilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1 - a^2x^2)}\, dx = \int -\frac{\operatorname{artanh}(ax)}{(a^2x^2 - 1)x^3}\, dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1 - a^2x^2)}\, dx = -\int \frac{\operatorname{atanh}(ax)}{a^2x^5 - x^3}\, dx$$

input
```
integrate(atanh(a*x)/x**3/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)/(a**2*x**5 - x**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(73) = 146$.

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}\,dx$$
$$= \frac{1}{8}\left(4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a-4\left(\log\left(ax+1\right)\log\left(x\right)+\operatorname{Li}_2(-ax)\right)a+4\left(\right.$$
$$-\frac{1}{2}\left(a^2\log\left(a^2x^2-1\right)-a^2\log\left(x^2\right)+\frac{1}{x^2}\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/8*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))
*a + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log
(a*x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a - 1/2*(a^2*log(a^2*x
^2 - 1) - a^2*log(x^2) + 1/x^2)*arctanh(a*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\left(1-a^2x^2\right)}\,dx = \int -\frac{\operatorname{artanh}\left(ax\right)}{(a^2x^2-1)x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)} \, dx = -\int \frac{\operatorname{atanh}(a\,x)}{x^3\,(a^2\,x^2-1)} \, dx$$

input
```
int(-atanh(a*x)/(x^3*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)/(x^3*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)} \, dx = -\left(\int \frac{atanh(ax)}{a^2x^5-x^3}dx\right)$$

input
```
int(atanh(a*x)/x^3/(-a^2*x^2+1),x)
```

output
```
- int(atanh(a*x)/(a**2*x**5 - x**3),x)
```

3.234 $\int \frac{x^3\textbf{arctanh}(ax)^2}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{x^3\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{x\text{arctanh}(ax)}{a^3}+\frac{\text{arctanh}(ax)^2}{2a^4}-\frac{x^2\text{arctanh}(ax)^2}{2a^2} -\frac{\text{arctanh}(ax)^3}{3a^4}+\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4}-\frac{\log\left(1-a^2x^2\right)}{2a^4} +\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}$$

output
```
-x*arctanh(a*x)/a^3+1/2*arctanh(a*x)^2/a^4-1/2*x^2*arctanh(a*x)^2/a^2-1/3*
arctanh(a*x)^3/a^4+arctanh(a*x)^2*ln(2/(-a*x+1))/a^4-1/2*ln(-a^2*x^2+1)/a^
4+arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^4-1/2*polylog(3,1-2/(-a*x+1))/a^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int \frac{x^3\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{ax\text{arctanh}(ax)-\frac{1}{2}(1-a^2x^2)\,\text{arctanh}(ax)^2-\frac{1}{3}\text{arctanh}(ax)^3-\text{arctanh}(ax)^2\log\left(1+e^{-2\text{arctanh}(ax)}\right)-}{a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

output
```
-((a*x*ArcTanh[a*x] - ((1 - a^2*x^2)*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^3/3
- ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - Log[1/Sqrt[1 - a^2*x^2]] +
 ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh
[a*x])]/2)/a^4)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6542, 6452, 6542, 6436, 240, 6510, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int x\text{arctanh}(ax)^2dx}{a^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^2-a\int \frac{x^2\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{6542}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^2-a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)dx}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^2-a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)-a\int \frac{x}{1-a^2x^2}dx}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\text{arctanh}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{\frac{\int \frac{\text{arctanh}(ax)^2}{1-ax}dx}{a} - \frac{\text{arctanh}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\text{arctanh}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} - 2\int \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a} - \frac{\text{arctanh}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\text{arctanh}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 6620}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} - 2\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx - \frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a} - \frac{\text{arctanh}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\text{arctanh}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} - 2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a} - \frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a} - \frac{\text{arctanh}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\text{arctanh}(ax)^2 - a\left(\frac{\text{arctanh}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

output
```
-(((x^2*ArcTanh[a*x]^2)/2 - a*(ArcTanh[a*x]^2/(2*a^3) - (x*ArcTanh[a*x] +
Log[1 - a^2*x^2]/(2*a))/a^2))/a^2) + (-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[
a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a
*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a)/a^2
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.62 (sec) , antiderivative size = 750, normalized size of antiderivative = 5.56

method	result
derivativedivides	$\frac{-\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}+\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)-\frac{\operatorname{polylog}\left(3,-\right.}{2}}{}$
default	$\frac{-\frac{a^2x^2\operatorname{arctanh}(ax)^2}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}+\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,-\frac{(ax+1)^2}{-a^2x^2+1}\right)-\frac{\operatorname{polylog}\left(3,-\right.}{2}}{}$
parts	Expression too large to display

input
```
int(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/2*a^2*x^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctan
h(a*x)^2*ln(a*x+1)+arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*pol
ylog(3,-(a*x+1)^2/(-a^2*x^2+1))+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/
2))-1/3*arctanh(a*x)^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*
x)^2+arctanh(a*x)^2*ln(2)+1/2*I*Pi*arctanh(a*x)^2-1/2*I*Pi*csgn(I/(-(a*x+1
)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2-1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-
1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1
)^2/(a^2*x^2-1)+1))*arctanh(a*x)^2+1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)
+1))^3*arctanh(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a
^2*x^2-1)+1))^3*arctanh(a*x)^2-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+ln((
a*x+1)^2/(-a^2*x^2+1)+1)+1/2*arctanh(a*x)^2-(a*x+1)*arctanh(a*x)+1/2*I*Pi*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh
(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*arctanh(a*x)^2+1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2)
```

Fricas [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{1-a^2x^2}\,dx=\int -\frac{x^3\,\text{artanh}\,(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\int \frac{x^3 \operatorname{atanh}^2(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1),x)
```

output
```
-Integral(x**3*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \int -\frac{x^3 \operatorname{artanh}(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/24*(3*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^4 +
 1/4*integrate(-(a^3*x^3*log(a*x + 1)^2 - (a^3*x^3 + a^2*x^2 + (2*a^3*x^3
+ a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^5*x^2 - a^3), x)
```

Giac [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \int -\frac{x^3 \operatorname{artanh}(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\int \frac{x^3\,\text{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-(x^3*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

output
```
-int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\left(\int \frac{atanh(ax)^2\,x^3}{a^2x^2-1}dx\right)$$

input
```
int(x^3*atanh(a*x)^2/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)**2*x**3)/(a**2*x**2 - 1),x)
```

3.235 $\int \frac{x^2 \mathbf{arctanh}(ax)^2}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{\text{arctanh}(ax)^2}{a^3} - \frac{x\text{arctanh}(ax)^2}{a^2} + \frac{\text{arctanh}(ax)^3}{3a^3} + \frac{2\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}$$

output
```
-arctanh(a*x)^2/a^3-x*arctanh(a*x)^2/a^2+1/3*arctanh(a*x)^3/a^3+2*arctanh(
a*x)*ln(2/(-a*x+1))/a^3+polylog(2,1-2/(-a*x+1))/a^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{-\frac{1}{3}\text{arctanh}(ax)\left(-3ax\text{arctanh}(ax)+\text{arctanh}(ax)(3+\text{arctanh}(ax))+6\log\left(1+e^{-2\text{arctanh}(ax)}\right)\right)+\text{Po}}{a^3}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

output
```
-((-1/3*(ArcTanh[a*x]*(-3*a*x*ArcTanh[a*x] + ArcTanh[a*x]*(3 + ArcTanh[a*x
]) + 6*Log[1 + E^(-2*ArcTanh[a*x])])) + PolyLog[2, -E^(-2*ArcTanh[a*x])])/
a^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)^2dx}{a^2}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)^2-2a\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}$$

$\Big\downarrow$ 2752

$$\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}$$

input
```
Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^3/(3*a^3) - (x*ArcTanh[a*x]^2 - 2*a*(-1/2*ArcTanh[a*x]^2/a^2
+ ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/
a))/a^2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.53 (sec) , antiderivative size = 5330, normalized size of antiderivative = 71.07

method	result	size
derivativedivides	Expression too large to display	5330
default	Expression too large to display	5330
parts	Expression too large to display	5580

input
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = \int -\frac{x^2\,\text{artanh}\,(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\int \frac{x^2\,\text{atanh}^2\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1),x)
```

output
```
-Integral(x**2*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(70) = 140.

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.67

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{1}{2}\left(\frac{2\,x}{a^2}-\frac{\log\,(ax+1)}{a^3}+\frac{\log\,(ax-1)}{a^3}\right)\text{artanh}\,(ax)^2 - \frac{\frac{3\,(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\,\left(\log(ax-1)^2-4\,\log(ax-1)\right)\log(ax+1)+6\,\log(ax-1)^2}{a} - \frac{24\,(\log(ax-1)\log(\frac{1}{2}}{}}{24\,a^2} + \frac{(2\,(\log\,(ax-1)-2)\log\,(ax+1)-\log\,(ax+1)^2-\log\,(ax-1)^2-4\,\log\,(ax-1))\,\text{artanh}\,(ax)}{4\,a^3}$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x)^2 - 1/24
*((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 -
 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a -
24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a)/a^2 + 1/4*
(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*l
og(a*x - 1))*arctanh(a*x)/a^3
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = \int -\frac{x^2\,\text{artanh}\left(ax\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\int \frac{x^2\,\text{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-(x^2*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

output
```
-int((x^2*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\left(\int \frac{atanh(ax)^2\,x^2}{a^2x^2-1}dx\right)$$

input
```
int(x^2*atanh(a*x)^2/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)**2*x**2)/(a**2*x**2 - 1),x)
```

3.236 $\int \frac{x\textbf{arctanh}(ax)^2}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{\text{arctanh}(ax)^3}{3a^2} + \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2} - \frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}$$

output
```
-1/3*arctanh(a*x)^3/a^2+arctanh(a*x)^2*ln(2/(-a*x+1))/a^2+arctanh(a*x)*pol
ylog(2,1-2/(-a*x+1))/a^2-1/2*polylog(3,1-2/(-a*x+1))/a^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\frac{-\frac{1}{3}\text{arctanh}(ax)^3 - \text{arctanh}(ax)^2\log\left(1+e^{-2\text{arctanh}(ax)}\right) + \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right) +}{a^2}$$

input
```
Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

output
```
-((-1/3*ArcTanh[a*x]^3 - ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + Arc
Tanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x
])]/2)/a^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 6620}$$

$$\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}$$

input
```
Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

output
```
-1/3*ArcTanh[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(
ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/
(4*a)))/a
```

Defintions of rubi rules used

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.96 (sec) , antiderivative size = 638, normalized size of antiderivative = 8.18

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^3}{3}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right.\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^3}{3}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right.\right.}{}}{}$
parts	$-\frac{\ln(a^2x^2-1)\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{\operatorname{arctanh}(ax)^3}{3a}+\frac{\left(i\pi\operatorname{csgn}\left(\frac{i}{\left(-\frac{(ax+1)^2}{a^2x^2-1}+1\right)^2}\right)\operatorname{csgn}\left(\frac{}{(a^2\right.\right.}{}}{}$

input
```
int(x*arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/2*arctanh(a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(
a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3+1/4*(I*Pi*csgn(I/
(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*
x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi-I*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))
^2+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+I*Pi*csgn(I*(a*x+1)/(-a^2*x
^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1
))^2+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+4*ln(2))*arctanh
(a*x)^2+arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*polylog(3,-(a*
x+1)^2/(-a^2*x^2+1)))
```

Fricas [F]

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = \int -\frac{x\,\text{artanh}\,(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = -\int \frac{x\,\text{atanh}^2\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x*atanh(a*x)**2/(-a**2*x**2+1),x)
```

output
```
-Integral(x*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx = \int -\frac{x\,\text{artanh}\,(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/24*(3*log(a*x + 1)*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^2 + 1/4*integra
te(-(a*x*log(a*x + 1)^2 - (3*a*x + 1)*log(a*x + 1)*log(-a*x + 1))/(a^3*x^2
 - a), x)
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx=\int -\frac{x\,\text{artanh}\,(ax)^2}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx=-\int \frac{x\,\text{atanh}(a\,x)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-(x*atanh(a*x)^2)/(a^2*x^2 - 1),x)
```

output
```
-int((x*atanh(a*x)^2)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}\,dx=-\left(\int \frac{atanh(ax)^2\,x}{a^2x^2-1}dx\right)$$

input
```
int(x*atanh(a*x)^2/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)**2*x)/(a**2*x**2 - 1),x)
```

3.237 $\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2} dx = \frac{\text{arctanh}(ax)^3}{3a}$$

output
```
1/3*arctanh(a*x)^3/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2} dx = \frac{\text{arctanh}(ax)^3}{3a}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^3/(3*a)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\text{arctanh}(ax)^3}{3a}$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^3/(3*a)
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^3}{3a}$
default	$\frac{\operatorname{arctanh}(ax)^3}{3a}$
parallelrisch	$\frac{\operatorname{arctanh}(ax)^3}{3a}$
risch	$\frac{\ln(ax+1)^3}{24a} - \frac{\ln(-ax+1)\ln(ax+1)^2}{8a} + \frac{\ln(-ax+1)^2\ln(ax+1)}{8a} - \frac{\ln(-ax+1)^3}{24a}$
parts	$\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2a} - \frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2a} - a\left(\frac{\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a^2} - \frac{\operatorname{arctanh}(ax)^3}{3a^2} + \frac{i\pi}{\ldots}\right.$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctanh(a*x)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \begin{cases} \frac{\operatorname{atanh}^3(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1),x)
```

output
```
Piecewise((atanh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(11) = 22.

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 9.77

$$\int \frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2 - \frac{\left(\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) + \log(ax-1)^2\right)\operatorname{artanh}(ax)}{4a} + \frac{\log(ax+1)^3 - 3\log(ax+1)^2\log(ax-1) + 3\log(ax+1)\log(ax-1)^2 - \log(ax-1)^3}{24a}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^2 - 1/4*(log(a*x + 1)^2
 - 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)*arctanh(a*x)/a + 1/24*(lo
g(a*x + 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)
^2 - log(a*x - 1)^3)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 5.23

$$\int \frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \frac{\ln\left(a\,x+1\right)^3}{24\,a} - \frac{\ln\left(1-a\,x\right)^3}{24\,a} + \frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^2}{8\,a} - \frac{\ln\left(a\,x+1\right)^2\,\ln\left(1-a\,x\right)}{8\,a}$$

input
```
int(-atanh(a*x)^2/(a^2*x^2 - 1),x)
```

output
```
log(a*x + 1)^3/(24*a) - log(1 - a*x)^3/(24*a) + (log(a*x + 1)*log(1 - a*x)
^2)/(8*a) - (log(a*x + 1)^2*log(1 - a*x))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}\,dx = \frac{atanh(ax)^3}{3a}$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1),x)
```

output
```
atanh(a*x)**3/(3*a)
```

3.238 $\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = \frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)$$

output
```
1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2-2/(a*x+1))-arctanh(a*x)*polylog(2,-
1+2/(a*x+1))-1/2*polylog(3,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = -\frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

output
```
-1/3*ArcTanh[a*x]^3 + ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + ArcTanh
[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - PolyLog[3, E^(2*ArcTanh[a*x])]/2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}dx$$

$$\Big\downarrow \text{ 6550}$$

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\operatorname{arctanh}(ax)^3$$

$$\Big\downarrow \text{ 6494}$$

$$-2a\int \frac{\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6618}$$

$$-2a\left(\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 7164}$$

$$-2a\left(\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

input
```
Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

output
```
ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x
]*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)
)
```

Defintions of rubi rules used

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.30 (sec) , antiderivative size = 1108, normalized size of antiderivative = 16.79

method	result	size
derivativedivides	Expression too large to display	1108
default	Expression too large to display	1108
parts	Expression too large to display	1494

input
```
int(arctanh(a*x)^2/x/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^2*ln
(a*x-1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3+1
/4*(I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I
*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2
*x^2-1)+1))^2-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I*(a
*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-2*I*Pi*csgn(I*(-(a*x+1)^2/
(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+
1))^2-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^
2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1
)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)
/(-(a*x+1)^2/(a^2*x^2-1)+1))+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I
*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+4*ln(2))*arctanh(a*x)^
2-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1-(a*x+1)/(-...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}^2(ax)}{a^2x^3-x}\,dx$$

input
```
integrate(atanh(a*x)**2/x/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)**2/(a**2*x**3 - x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/8*log(a*x + 1)*log(-a*x + 1)^2 - 1/24*log(-a*x + 1)^3 + 1/4*integrate((
(a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1) - log(a*x + 1)^2)/(a^2*x^3
- x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^2}{x\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)^2/(x*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)^2/(x*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}\,dx = -\left(\int \frac{atanh(ax)^2}{a^2x^3-x}dx\right)$$

input
```
int(atanh(a*x)^2/x/(-a^2*x^2+1),x)
```

output
```
- int(atanh(a*x)**2/(a**2*x**3 - x),x)
```

3.239 $\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = a\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{x}+\frac{1}{3}a\operatorname{arctanh}(ax)^3+2a\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)-a\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
a*arctanh(a*x)^2-arctanh(a*x)^2/x+1/3*a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln
(2-2/(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = -a\left(-\frac{1}{3}\operatorname{arctanh}(ax)\left(-\frac{3\operatorname{arctanh}(ax)}{ax}+\operatorname{arctanh}(ax)(3+\operatorname{arctanh}(ax))+6\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)\right)+\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

output
```
-(a*(-1/3*(ArcTanh[a*x]*((-3*ArcTanh[a*x])/(a*x) + ArcTanh[a*x]*(3 + ArcTa
nh[a*x]) + 6*Log[1 - E^(-2*ArcTanh[a*x])])) + PolyLog[2, E^(-2*ArcTanh[a*x
])]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)} \, dx$$

$$\Big\downarrow \text{6544}$$

$$a^2 \int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{6452}$$

$$a^2 \int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+2a\int \frac{\text{arctanh}(ax)}{x (1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6510}$$

$$2a\int \frac{\text{arctanh}(ax)}{x (1-a^2x^2)}dx+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6550}$$

$$2a\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6494}$$

$$2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{2897}$$

$$2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

input
```
Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

output
```
-(ArcTanh[a*x]^2/x) + (a*ArcTanh[a*x]^3)/3 + 2*a*(ArcTanh[a*x]^2/2 + ArcTa
nh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.47 (sec) , antiderivative size = 4380, normalized size of antiderivative = 66.36

method	result	size
derivativedivides	Expression too large to display	4380
default	Expression too large to display	4380
parts	Expression too large to display	4383

input
```
int(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/2*arctanh(a*x)^2*ln(a*x+1)-arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1
/2))-arctanh(a*x)^2/a/x+1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-
1)+1))*arctanh(a*x)^2-1/2*arctanh(a*x)^2*ln(a*x-1)+1/2*I*Pi*csgn(I/(-(a*x+
1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2-1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2
-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+
1)^2/(a^2*x^2-1)+1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog
(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*
arctanh(a*x)^3-arctanh(a*x)^2-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)*ln(1-(a
*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^
2-1)+1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*Pi*csgn(I/(-(a*x+1)^
2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I
*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*dilog(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))-1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx=\int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = -\int \frac{\text{atanh}^2(ax)}{a^2x^4-x^2}\,dx$$

input
```
integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)**2/(a**2*x**4 - x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(63) = 126.

Time = 0.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.59

$$\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = -\frac{1}{24}a^2\left(\frac{3(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\left(\log(ax-1)^2-4\log(a\right.}{a}\right. \\ +\frac{1}{4}(2(\log(ax-1)-2)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\log(ax-1)+8\log(x))a\,\text{arta} \\ +\frac{1}{2}\left(a\log(ax+1)-a\log(ax-1)-\frac{2}{x}\right)\text{artanh}(ax)^2$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/24*a^2*((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x
 - 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1
)^2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a +
24*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilo
g(a*x))/a) + 1/4*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log
(a*x - 1)^2 - 4*log(a*x - 1) + 8*log(x))*a*arctanh(a*x) + 1/2*(a*log(a*x +
 1) - a*log(a*x - 1) - 2/x)*arctanh(a*x)^2
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)^2/(x^2*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)}\,dx = \frac{atanh(ax)^3\,ax - 3atanh(ax)^2 - 6\left(\int \frac{atanh(ax)}{a^2x^3-x}dx\right)ax}{3x}$$

input
```
int(atanh(a*x)^2/x^2/(-a^2*x^2+1),x)
```

output
```
(atanh(a*x)**3*a*x - 3*atanh(a*x)**2 - 6*int(atanh(a*x)/(a**2*x**3 - x),x)
*a*x)/(3*x)
```

3.240 $\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = &-\frac{a\operatorname{arctanh}(ax)}{x}+\frac{1}{2}a^2\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{2x^2}\\&+\frac{1}{3}a^2\operatorname{arctanh}(ax)^3+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)\\&+a^2\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right)\\&-a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\\&-\frac{1}{2}a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-a*arctanh(a*x)/x+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+1/3*a^2*ar
ctanh(a*x)^3+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+a^2*arctanh(a*x)^2*ln(2-2/(a
*x+1))-a^2*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*a^2*polylog(3,-1+2/(a*
x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = -a^2\left(-\frac{i\pi^3}{24}+\frac{\operatorname{arctanh}(ax)}{ax}+\frac{(1-a^2x^2)\operatorname{arctanh}(ax)^2}{2a^2x^2}+\frac{1}{3}\operatorname{arctanh}(ax)^3-\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)-\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

output
```
-(a^2*((-1/24*I)*Pi^3 + ArcTanh[a*x]/(a*x) + ((1 - a^2*x^2)*ArcTanh[a*x]^2
)/(2*a^2*x^2) + ArcTanh[a*x]^3/3 - ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x
])] - Log[(a*x)/Sqrt[1 - a^2*x^2]] - ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[
a*x])] + PolyLog[3, E^(2*ArcTanh[a*x])]/2))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6544, 6452, 6544, 6452, 243, 47, 14, 16, 6510, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}dx+\int \frac{\operatorname{arctanh}(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)}dx - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + \int \frac{\text{arctanh}(ax)}{x^2}dx\right) - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6452}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + a\int \frac{1}{x(1-a^2x^2)}dx - \frac{\text{arctanh}(ax)}{x}\right) - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + \frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2 - \frac{\text{arctanh}(ax)}{x}\right) - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + \frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2 + \int \frac{1}{x^2}dx^2\right) - \frac{\text{arctanh}(ax)}{x}\right) - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + \frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2 + \log\left(x^2\right)\right) - \frac{\text{arctanh}(ax)}{x}\right) - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx + a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + \frac{1}{2}a\left(\log\left(x^2\right) - \log\left(1-a^2x^2\right)\right) - \frac{\text{arctanh}(ax)}{x}\right) - \frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)}dx+$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 6550}$$

$$a^2\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 6494}$$

$$a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 6618}$$

$$a^2\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 7164}$$

$$a^2\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

input
```
Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

output
```
-1/2*ArcTanh[a*x]^2/x^2 + a*(-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + (a
*(Log[x^2] - Log[1 - a^2*x^2]))/2) + a^2*(ArcTanh[a*x]^3/3 + ArcTanh[a*x]^
2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/
(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.34 (sec) , antiderivative size = 1316, normalized size of antiderivative = 9.54

method	result	size
derivativedivides	Expression too large to display	1316
default	Expression too large to display	1316
parts	Expression too large to display	1735

input
```
int(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^
(1/2))-1/2*arctanh(a*x)^2/a^2/x^2-1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2
/(a^2*x^2-1)+1))*arctanh(a*x)^2+ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*arcta
nh(a*x)^2*ln(a*x-1)+1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(
a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^
2*x^2-1)+1))*arctanh(a*x)^2+1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+1/
2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2-2*polylog(3,-(a
*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arct
anh(a*x)^3+1/2*arctanh(a*x)^2+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+
1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x
)^2*ln(2)-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+1/2*I*Pi*csgn(I*(a*x+1)/(
-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2+1/4*I*Pi
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctan
h(a*x)^2-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(
a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
+1/2*I*Pi*arctanh(a*x)^2+1/2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x
+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)...
```

Fricas [F]

$$\int \frac{\text{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = \int -\frac{\text{artanh}\,(ax)^2}{(a^2x^2-1)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}^2(ax)}{a^2x^5-x^3}\,dx$$

input
```
integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)**2/(a**2*x**5 - x**3), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/24*(a^2*x^2*log(-a*x + 1)^3 + 3*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1
)^2)/x^2 + 1/4*integrate(-(log(a*x + 1)^2 - (a^2*x^2 + a*x + (a^4*x^4 + a^
3*x^3 + 2)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^5 - x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)^2/(x^3*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)}\,dx = -\left(\int \frac{atanh(ax)^2}{a^2x^5-x^3}dx\right)$$

input
```
int(atanh(a*x)^2/x^3/(-a^2*x^2+1),x)
```

output
```
- int(atanh(a*x)**2/(a**2*x**5 - x**3),x)
```

3.241 $\int \frac{x^3\text{arctanh}(ax)^3}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int \frac{x^3\text{arctanh}(ax)^3}{1-a^2x^2}\,dx =& -\frac{3\text{arctanh}(ax)^2}{2a^4}-\frac{3x\text{arctanh}(ax)^2}{2a^3}+\frac{\text{arctanh}(ax)^3}{2a^4}\\ &-\frac{x^2\text{arctanh}(ax)^3}{2a^2}-\frac{\text{arctanh}(ax)^4}{4a^4}+\frac{3\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}\\ &+\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^4}+\frac{3\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}\\ &+\frac{3\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}\\ &-\frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}\\ &+\frac{3\,\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}\end{aligned}$$

output
```
-3/2*arctanh(a*x)^2/a^4-3/2*x*arctanh(a*x)^2/a^3+1/2*arctanh(a*x)^3/a^4-1/
2*x^2*arctanh(a*x)^3/a^2-1/4*arctanh(a*x)^4/a^4+3*arctanh(a*x)*ln(2/(-a*x+
1))/a^4+arctanh(a*x)^3*ln(2/(-a*x+1))/a^4+3/2*polylog(2,1-2/(-a*x+1))/a^4+
3/2*arctanh(a*x)^2*polylog(2,1-2/(-a*x+1))/a^4-3/2*arctanh(a*x)*polylog(3,
1-2/(-a*x+1))/a^4+3/4*polylog(4,1-2/(-a*x+1))/a^4
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.69

$$\int \frac{x^3\text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\frac{-6\text{arctanh}(ax)^2+6ax\text{arctanh}(ax)^2-2(1-a^2x^2)\,\text{arctanh}(ax)^3-\text{arctanh}(ax)^4-12\text{arctanh}(ax)\log}{}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

output
```
-1/4*(-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 - 2*(1 - a^2*x^2)*ArcTanh[a
*x]^3 - ArcTanh[a*x]^4 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] - 4*
ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(1 + ArcTanh[a*x]^2)*PolyL
og[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x
])] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/a^4
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6542, 6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^3}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\int x\text{arctanh}(ax)^3dx}{a^2}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\int \frac{x^2\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)^2dx}{a^2}\right)}{a^2}$$

$\Big\downarrow$ 6436

$$\frac{\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)^2-2a\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}\right)}{a^2}$$

$\Big\downarrow$ 6510

$$\frac{\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}\right)}{a^2}$$

$\Big\downarrow$ 6546

$$\frac{\frac{\int \frac{\text{arctanh}(ax)^3}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2}$$

$\Big\downarrow$ 6470

$$\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}-\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2}$$

$\Big\downarrow$ 2849

$$\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}-$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{a^2}$$

$\downarrow$ 2752

$$\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}-$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{a^2}$$

$\downarrow$ 6620

$$\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(\int\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}-$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{a^2}$$

$\downarrow$ 6624

$$\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\text{arc}}{a^2}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{a^2}$$

$$\Big\downarrow \; 7164$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a}\right)}{a}-\frac{\text{arctanh}(}{4a^2}}{a^2}$$

$$\frac{\frac{1}{2}x^2\text{arctanh}(ax)^3-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^3}{3a^3}-\frac{x\text{arctanh}(ax)^2-2a\left(\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}\right)}{a^2}\right.}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

output
```
-((((x^2*ArcTanh[a*x]^3)/2 - (3*a*(ArcTanh[a*x]^3/(3*a^3) - (x*ArcTanh[a*x]
^2 - 2*a*(-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a*x)])/a + P
olyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a))/a^2))/2)/a^2) + (-1/4*ArcTanh[a*x]^4
/a^2 + ((ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a - 3*(-1/2*(ArcTanh[a*x]^2*Poly
Log[2, 1 - 2/(1 - a*x)])/a + (ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/(2
*a) - PolyLog[4, 1 - 2/(1 - a*x)]/(4*a)))/a)/a^2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [A] (verified)

Time = 22.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^4}{4}-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3)(ax-1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)+\frac{3\operatorname{arctanh}(ax)^2\operatorname{pol}}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^4}{4}-\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3)(ax-1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)+\frac{3\operatorname{arctanh}(ax)^2\operatorname{pol}}{}}{}$

input
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*arctanh(a*x)^4-1/2*arctanh(a*x)^2*(a*x*arctanh(a*x)+arctanh(a*
x)+3)*(a*x-1)+arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/2*arctanh(a*x)
^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2*arctanh(a*x)*polylog(3,-(a*x+1)^
2/(-a^2*x^2+1))+3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3*arctanh(a*x)^2+3*
arctanh(a*x)*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/2*polylog(2,-(a*x+1)^2/(-a^2*x
^2+1)))
```

Fricas [F]

$$\int \frac{x^3\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x^3\operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3 \text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\int \frac{x^3 \operatorname{atanh}^3(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1),x)
```

output
```
-Integral(x**3*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x^3 \text{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x^3 \operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/64*(4*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^4 -
1/8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)^3 - 6*a^3*x^3*log(a*x + 1)^2*log
(-a*x + 1) + 3*(a^3*x^3 + a^2*x^2 + (2*a^3*x^3 + a*x + 1)*log(a*x + 1))*lo
g(-a*x + 1)^2)/(a^5*x^2 - a^3), x)
```

Giac [F]

$$\int \frac{x^3 \text{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x^3 \operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\int \frac{x^3\,\text{atanh}(a\,x)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-(x^3*atanh(a*x)^3)/(a^2*x^2 - 1),x)
```

output
```
-int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\left(\int \frac{atanh(ax)^3\,x^3}{a^2x^2-1}dx\right)$$

input
```
int(x^3*atanh(a*x)^3/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)**3*x**3)/(a**2*x**2 - 1),x)
```

3.242 $\int \frac{x^2 \text{arctanh}(ax)^3}{1-a^2x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2} dx = -\frac{\text{arctanh}(ax)^3}{a^3} - \frac{x\text{arctanh}(ax)^3}{a^2} + \frac{\text{arctanh}(ax)^4}{4a^3} + \frac{3\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3} - \frac{3\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3}$$

output
```
-arctanh(a*x)^3/a^3-x*arctanh(a*x)^3/a^2+1/4*arctanh(a*x)^4/a^3+3*arctanh(
a*x)^2*ln(2/(-a*x+1))/a^3+3*arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^3-3/2*p
olylog(3,1-2/(-a*x+1))/a^3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}\,dx = \frac{\text{arctanh}(ax)^2\left((4-4ax)\text{arctanh}(ax)+\text{arctanh}(ax)^2+12\log\left(1+e^{-2\text{arctanh}(ax)}\right)\right)-12\text{arctanh}(ax)\,\text{Poly}}{4a^3}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

output
```
(ArcTanh[a*x]^2*((4 - 4*a*x)*ArcTanh[a*x] + ArcTanh[a*x]^2 + 12*Log[1 + E^
(-2*ArcTanh[a*x])]) - 12*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6
*PolyLog[3, -E^(-2*ArcTanh[a*x])])/(4*a^3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6542, 6436, 6510, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{\int \text{arctanh}(ax)^3dx}{a^2}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}-\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\int \frac{x\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{6546}$$

$$\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\int \frac{\text{arctanh}(ax)^2}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a^2}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int \frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a^2}$$

$$\Big\downarrow \text{6620}$$

$$\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a^2}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\text{arctanh}(ax)^4}{4a^3}-\frac{x\text{arctanh}(ax)^3-3a\left(\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}\right)}{a^2}$$

input

```
Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

output

```
ArcTanh[a*x]^4/(4*a^3) - (x*ArcTanh[a*x]^3 - 3*a*(-1/3*ArcTanh[a*x]^3/a^2
+ ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(ArcTanh[a*x]*PolyLog[2,
1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a))/a^2
```

Defintions of rubi rules used

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.36 (sec) , antiderivative size = 736, normalized size of antiderivative = 7.15

method	result
derivativedivides	$\frac{-\operatorname{arctanh}(ax)^3 ax-\frac{\operatorname{arctanh}(ax)^3 \ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{2}-\operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{i\pi \operatorname{cs}}{}}{}$
default	$\frac{-\operatorname{arctanh}(ax)^3 ax-\frac{\operatorname{arctanh}(ax)^3 \ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)^3 \ln(ax+1)}{2}-\operatorname{arctanh}(ax)^3 \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{i\pi \operatorname{cs}}{}}{}$
parts	$-\frac{x\operatorname{arctanh}(ax)^3}{a^2}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2a^3}-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2a^3}-\frac{3a\left(\frac{2\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3a^4}-\frac{\operatorname{arc}}{}\right.}{}$

input
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-arctanh(a*x)^3*a*x-1/2*arctanh(a*x)^3*ln(a*x-1)+1/2*arctanh(a*x)^3
*ln(a*x+1)-arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*arctanh(a*x)^
4+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a
*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^3-1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-1/4*I*Pi*csgn(I/
(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*
x^2-1)+1))^2*arctanh(a*x)^3-1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3+1/4*I*Pi*csgn(I/(-(a*x+1)^2/(
a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
(-(a*x+1)^2/(a^2*x^2-1)+1))*arctanh(a*x)^3+1/2*I*Pi*arctanh(a*x)^3-1/2*I*P
i*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^3+1/2*I*Pi*csgn(I/(-(a
*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^3-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^3-arctanh(a*x)^3-3/2*polyl
og(3,-(a*x+1)^2/(-a^2*x^2+1))-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arc
tanh(a*x)^3+3*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3*arctanh(a*x)*p
olylog(2,-(a*x+1)^2/(-a^2*x^2+1)))
```

Fricas [F]

$$\int \frac{x^2 \text{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x^2\,\text{artanh}\,(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2 \text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\int \frac{x^2\,\text{atanh}^3\,(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1),x)
```

output
```
-Integral(x**2*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x^2 \operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/64*(4*(2*a*x - log(a*x + 1) - 2)*log(-a*x + 1)^3 + log(-a*x + 1)^4 - 6*(
4*(a*x + 1)*log(a*x + 1) - log(a*x + 1)^2)*log(-a*x + 1)^2)/a^3 + 1/8*inte
grate(-1/2*(2*a^2*x^2*log(a*x + 1)^3 - 3*((2*a^2*x^2 - a*x - 1)*log(a*x +
1)^2 + 4*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^4*x^2 - a^2
), x)
```

Giac [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x^2 \operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\int \frac{x^2 \operatorname{atanh}(a\,x)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-(x^2*atanh(a*x)^3)/(a^2*x^2 - 1),x)
```

output
```
-int((x^2*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\left(\int \frac{atanh(ax)^3\,x^2}{a^2x^2-1}dx\right)$$

input
```
int(x^2*atanh(a*x)^3/(-a^2*x^2+1),x)
```

output
```
 - int((atanh(a*x)**3*x**2)/(a**2*x**2 - 1),x)
```

3.243 $\int \frac{x\textbf{arctanh}(ax)^3}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\frac{\text{arctanh}(ax)^4}{4a^2} + \frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{3\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2} - \frac{3\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2} + \frac{3\,\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^2}$$

output
```
-1/4*arctanh(a*x)^4/a^2+arctanh(a*x)^3*ln(2/(-a*x+1))/a^2+3/2*arctanh(a*x)
^2*polylog(2,1-2/(-a*x+1))/a^2-3/2*arctanh(a*x)*polylog(3,1-2/(-a*x+1))/a^
2+3/4*polylog(4,1-2/(-a*x+1))/a^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}\,dx =$$
$$-\frac{-\text{arctanh}(ax)^4-4\text{arctanh}(ax)^3\log\left(1+e^{-2\text{arctanh}(ax)}\right)+6\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right)\cdots}{4a^2}$$

input
```
Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

output
```
-1/4*(-ArcTanh[a*x]^4 - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*
ArcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3
, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/a^2
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6546, 6470, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx$$
$$\Big\downarrow \text{6546}$$
$$\frac{\int \frac{\text{arctanh}(ax)^3}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}$$
$$\Big\downarrow \text{6470}$$
$$\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}$$
$$\Big\downarrow \text{6620}$$

$$\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}$$

$\downarrow$ 6624

$$\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}$$

$\downarrow$ 7164

$$\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a}\right)}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}$$

input
```
Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

output
```
-1/4*ArcTanh[a*x]^4/a^2 + ((ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a - 3*(-1/2*(
ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/a + (ArcTanh[a*x]*PolyLog[3, 1
 - 2/(1 - a*x)])/(2*a) - PolyLog[4, 1 - 2/(1 - a*x)]/(4*a)))/a
```

Defintions of rubi rules used

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.60 (sec) , antiderivative size = 670, normalized size of antiderivative = 6.20

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\left(i\pi\ \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right.\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}+\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\left(i\pi\ \operatorname{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}+1}\right.\right.}{}}{}$
parts	$-\frac{\ln(a^2x^2-1)\operatorname{arctanh}(ax)^3}{2a^2}+\frac{\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{\operatorname{arctanh}(ax)^4}{4a}+\frac{\left(-i\pi\ \operatorname{csgn}\left(\frac{i}{\left(-\frac{(ax+1)^2}{a^2x^2-1}+1\right)^2}\right)\operatorname{csgn}\left(\frac{i(a}{a^2}\right.\right.}{}}{}$

input
```
int(x*arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/2*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(
a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4+1/4*(I*Pi*csgn(I/
(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*
x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi-I*Pi*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))
^2+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+I*Pi*csgn(I*(a*x+1)/(-a^2*x
^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1
))^2+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^2-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+4*ln(2))*arctanh
(a*x)^3+3/2*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2*arctanh(
a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2
+1)))
```

Fricas [F]

$$\int \frac{x\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx=\int -\frac{x\operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = -\int \frac{x\operatorname{atanh}^3(ax)}{a^2x^2-1}\,dx$$

input
```
integrate(x*atanh(a*x)**3/(-a**2*x**2+1),x)
```

output
```
-Integral(x*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x\operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/64*(4*log(a*x + 1)*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^2 - 1/8*integrat
e(1/2*(2*a*x*log(a*x + 1)^3 - 6*a*x*log(a*x + 1)^2*log(-a*x + 1) + 3*(3*a*
x + 1)*log(a*x + 1)*log(-a*x + 1)^2)/(a^3*x^2 - a), x)
```

Giac [F]

$$\int \frac{x\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \int -\frac{x\operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}\,dx=-\int \frac{x\,\text{atanh}(a\,x)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-(x*atanh(a*x)^3)/(a^2*x^2 - 1),x)
```

output
```
-int((x*atanh(a*x)^3)/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}\,dx=-\left(\int \frac{atanh(ax)^3\,x}{a^2x^2-1}dx\right)$$

input
```
int(x*atanh(a*x)^3/(-a^2*x^2+1),x)
```

output
```
- int((atanh(a*x)**3*x)/(a**2*x**2 - 1),x)
```

3.244 $\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \frac{\operatorname{arctanh}(ax)^4}{4a}$$

output
```
1/4*arctanh(a*x)^4/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \frac{\operatorname{arctanh}(ax)^4}{4a}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^4/(4*a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2} \, dx$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\operatorname{arctanh}(ax)^4}{4a}$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^4/(4*a)
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)^4}{4a}$
default	$\frac{\operatorname{arctanh}(ax)^4}{4a}$
parallelrisch	$\frac{\operatorname{arctanh}(ax)^4}{4a}$
risch	$\frac{\ln(ax+1)^4}{64a} - \frac{\ln(-ax+1)\ln(ax+1)^3}{16a} + \frac{3\ln(-ax+1)^2\ln(ax+1)^2}{32a} - \frac{\ln(-ax+1)^3\ln(ax+1)}{16a} + \frac{\ln(-ax+1)^4}{64a}$
parts	$\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2a} - \frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2a} - \frac{3a\left(\frac{2\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3a^2} - \frac{\operatorname{arctanh}(ax)^4}{6a^2} + \frac{i\pi \operatorname{arctanh}(\cdots}{\cdots}\right.}{\cdots}$

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(a*x)^4/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \begin{cases} \frac{\operatorname{atanh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1),x)
```

output
```
Piecewise((atanh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(11) = 22.

Time = 0.04 (sec) , antiderivative size = 209, normalized size of antiderivative = 16.08

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx = \frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3$$
$$+ \frac{1}{64}a\left(\frac{8\left(\log(ax+1)^3 - 3\log(ax+1)^2\log(ax-1) + 3\log(ax+1)\log(ax-1)^2 - \log(ax-1)^3\right)\mathrm{a}}{a^2}\right.$$
$$- \frac{3\left(\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) + \log(ax-1)^2\right)\operatorname{artanh}(ax)^2}{8a}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^3 + 1/64*a*(8*(log(a*x
+ 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2 - l
og(a*x - 1)^3)*arctanh(a*x)/a^2 - (log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(a
*x - 1) + 6*log(a*x + 1)^2*log(a*x - 1)^2 - 4*log(a*x + 1)*log(a*x - 1)^3
+ log(a*x - 1)^4)/a^2) - 3/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1)
 + log(a*x - 1)^2)*arctanh(a*x)^2/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx=\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 6.92

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx=\frac{\ln\left(a\,x+1\right)^4}{64\,a}+\frac{\ln\left(1-a\,x\right)^4}{64\,a}-\frac{\ln\left(a\,x+1\right)\,\ln\left(1-a\,x\right)^3}{16\,a}-\frac{\ln\left(a\,x+1\right)^3\,\ln\left(1-a\,x\right)}{16\,a}+\frac{3\ln\left(a\,x+1\right)^2\ln\left(1-a\,x\right)^2}{32\,a}$$

input
```
int(-atanh(a*x)^3/(a^2*x^2 - 1),x)
```

output
```
log(a*x + 1)^4/(64*a) + log(1 - a*x)^4/(64*a) - (log(a*x + 1)*log(1 - a*x)
^3)/(16*a) - (log(a*x + 1)^3*log(1 - a*x))/(16*a) + (3*log(a*x + 1)^2*log(
1 - a*x)^2)/(32*a)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}\,dx=\frac{atanh(ax)^4}{4a}$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1),x)
```

output
```
atanh(a*x)**4/(4*a)
```

3.245 $\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = \frac{1}{4}\operatorname{arctanh}(ax)^4 + \operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) - \frac{3}{2}\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right) - \frac{3}{4}\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)$$

output
```
1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2*polyl
og(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*polylog(4,
-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = -\frac{1}{4}\operatorname{arctanh}(ax)^4 + \operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + \frac{3}{2}\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - \frac{3}{2}\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right) + \frac{3}{4}\operatorname{PolyLog}\left(4,e^{2\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

output
```
-1/4*ArcTanh[a*x]^4 + ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] + (3*ArcT
anh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*PolyLog[3,
E^(2*ArcTanh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6550, 6494, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx$$

$$\Big\downarrow \text{6550}$$

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(ax+1)}dx + \frac{1}{4}\operatorname{arctanh}(ax)^4$$

$$\Big\downarrow \text{6494}$$

$$-3a\int \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx + \frac{1}{4}\operatorname{arctanh}(ax)^4 + \operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{6618}$$

$$-3a\left(\frac{\operatorname{arctanh}(ax)^2 \operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int \frac{\operatorname{arctanh}(ax) \operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{6622}$$

$$-3a\left(-\frac{1}{2}\int \frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\operatorname{arctanh}(ax)^2 \operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax) \operatorname{PolyLog}\left(3,\frac{2}{ax+1}\right.}{2a}\right.$$
$$\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{7164}$$

$$-3a\left(\frac{\operatorname{arctanh}(ax)^2 \operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax) \operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)+$$
$$\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

input
```
Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

output
```
ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - 3*a*((ArcTanh[a*x
]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/
(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a))
```

Defintions of rubi rules used

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.55 (sec) , antiderivative size = 1165, normalized size of antiderivative = 12.80

method	result	size
derivativedivides	Expression too large to display	1165
default	Expression too large to display	1165
parts	Expression too large to display	1558

input
```
int(arctanh(a*x)^3/x/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*
ln(a*x)+arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4+1
/4*(I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I
*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2
*x^2-1)+1))^2-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I*(a
*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-2*I*Pi*csgn(I*(-(a*x+1)^2/
(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+
1))^2-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^
2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1
)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)
/(-(a*x+1)^2/(a^2*x^2-1)+1))+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I
*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+4*ln(2))*arctanh(a*x)^
3-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^3*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,-(a...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}^3(ax)}{a^2x^3-x}\,dx$$

input
```
integrate(atanh(a*x)**3/x/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)**3/(a**2*x**3 - x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/16*log(a*x + 1)*log(-a*x + 1)^3 + 1/64*log(-a*x + 1)^4 - 1/8*integrate(1
/2*(3*(a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1)^2 + 2*log(a*x + 1)^3
- 6*log(a*x + 1)^2*log(-a*x + 1))/(a^2*x^3 - x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^3}{x\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)^3/(x*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)^3/(x*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}\,dx = -\left(\int \frac{atanh(ax)^3}{a^2x^3-x}dx\right)$$

input
```
int(atanh(a*x)^3/x/(-a^2*x^2+1),x)
```

output
```
- int(atanh(a*x)**3/(a**2*x**3 - x),x)
```

3.246 $\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx = a\operatorname{arctanh}(ax)^3 - \frac{\operatorname{arctanh}(ax)^3}{x} + \frac{1}{4}a\operatorname{arctanh}(ax)^4 + 3a\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - 3a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) - \frac{3}{2}a\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)$$

output
```
a*arctanh(a*x)^3-arctanh(a*x)^3/x+1/4*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*
ln(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-
1+2/(a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx = -a\left(-\frac{i\pi^3}{8}+\operatorname{arctanh}(ax)^3+\frac{\operatorname{arctanh}(ax)^3}{ax}-\frac{1}{4}\operatorname{arctanh}(ax)^4 - 3\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) - 3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) + \frac{3}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

output
```
-(a*((-1/8*I)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - ArcTanh[a*x]^
4/4 - 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLo
g[2, E^(2*ArcTanh[a*x])] + (3*PolyLog[3, E^(2*ArcTanh[a*x])])/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6544, 6452, 6510, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}dx+\int \frac{\operatorname{arctanh}(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{6452}$$

$$3a\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)}dx+a^2\int \frac{\operatorname{arctanh}(ax)^3}{1-a^2x^2}dx-\frac{\operatorname{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \; 6510$$

$$3a\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)}dx+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \; 6550$$

$$3a\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \; 6494$$

$$3a\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \; 6618$$

$$3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}($$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \; 7164$$

$$3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

input
```
Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

output
```
-(ArcTanh[a*x]^3/x) + (a*ArcTanh[a*x]^4)/4 + 3*a*(ArcTanh[a*x]^3/3 + ArcTa
nh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 +
 a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
 + 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.44 (sec) , antiderivative size = 810, normalized size of antiderivative = 9.00

method	result
derivativedivides	$a\left(-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^3}{xa}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}-\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right.\right.$
default	$a\left(-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\frac{\operatorname{arctanh}(ax)^3}{xa}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}-\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right.\right.$
parts	Expression too large to display

input
```
int(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*arctanh(a*x)^3*ln(a*x-1)-arctanh(a*x)^3/x/a+1/2*arctanh(a*x)^3*ln(
a*x+1)-arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(3,-(a*x+1)/
(-a^2*x^2+1)^(1/2))-6*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(
I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^3-1/4*I*Pi*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^3+1/4*arctanh(a*x)^4
-arctanh(a*x)^3+3*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctan
h(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*ln(1-(a*x+1
)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3-1/4*I*Pi*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-
1/2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^3+1/2*I*Pi*arct
anh(a*x)^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^3-1/4*I*Pi*csgn(I/(-(a*x+1
)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1
))^2*arctanh(a*x)^3-1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3+1/4*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-
1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1
)^2/(a^2*x^2-1)+1))*arctanh(a*x)^3)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1 - a^2x^2)} \, dx = -\int \frac{\operatorname{atanh}^3(ax)}{a^2x^4 - x^2} \, dx$$

input
```
integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)**3/(a**2*x**4 - x**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1 - a^2x^2)} \, dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2 - 1)x^2} \, dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/64*(a*x*log(-a*x + 1)^4 - 4*(a*x*log(a*x + 1) + 2*a*x - 2)*log(-a*x + 1)
^3 + 6*(a*x*log(a*x + 1)^2 - 4*(a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/x
- 1/8*integrate(1/2*(2*log(a*x + 1)^3 + 3*((a^3*x^3 + a^2*x^2 - 2)*log(a*x
 + 1)^2 - 4*(a^3*x^3 + 2*a^2*x^2 + a*x)*log(a*x + 1))*log(-a*x + 1))/(a^2*
x^4 - x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1 - a^2x^2)} \, dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2 - 1)x^2} \, dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)^3/(x^2*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)}\,dx = \frac{atanh(ax)^4\,ax-4atanh(ax)^3-12\left(\int \frac{atanh(ax)^2}{a^2x^3-x}dx\right)ax}{4x}$$

input
```
int(atanh(a*x)^3/x^2/(-a^2*x^2+1),x)
```

output
```
(atanh(a*x)**4*a*x - 4*atanh(a*x)**3 - 12*int(atanh(a*x)**2/(a**2*x**3 - x
),x)*a*x)/(4*x)
```

3.247 $\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 200

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)} \, dx &= \frac{3}{2}a^2\operatorname{arctanh}(ax)^2 - \frac{3a\operatorname{arctanh}(ax)^2}{2x} \\ &+ \frac{1}{2}a^2\operatorname{arctanh}(ax)^3 - \frac{\operatorname{arctanh}(ax)^3}{2x^2} + \frac{1}{4}a^2\operatorname{arctanh}(ax)^4 \\ &+ 3a^2\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) \\ &+ a^2\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right) \\ &- \frac{3}{2}a^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ &- \frac{3}{2}a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ &- \frac{3}{2}a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right) \\ &- \frac{3}{4}a^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x+1/2*a^2*arctanh(a*x)^3-1/2*a
rctanh(a*x)^3/x^2+1/4*a^2*arctanh(a*x)^4+3*a^2*arctanh(a*x)*ln(2-2/(a*x+1)
)+a^2*arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/(a*x+1))-3/2*a
^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))-3/2*a^2*arctanh(a*x)*polylog(3,-
1+2/(a*x+1))-3/4*a^2*polylog(4,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)}\,dx = -\frac{1}{64}a^2\left(-\pi^4-96\operatorname{arctanh}(ax)^2+\frac{96\operatorname{arctanh}(ax)^2}{ax}+\frac{32(1-a^2x^2)\operatorname{arctanh}(ax)^3}{a^2x^2}+16\operatorname{arctanh}(ax)^4-192\operatorname{arctanh}(ax)\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)-64\operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)+96\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right)-96\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)+96\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)-48\operatorname{PolyLog}\left(4,e^{2\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

output
```
-1/64*(a^2*(-Pi^4 - 96*ArcTanh[a*x]^2 + (96*ArcTanh[a*x]^2)/(a*x) + (32*(1
 - a^2*x^2)*ArcTanh[a*x]^3)/(a^2*x^2) + 16*ArcTanh[a*x]^4 - 192*ArcTanh[a*
x]*Log[1 - E^(-2*ArcTanh[a*x])] - 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a
*x])] + 96*PolyLog[2, E^(-2*ArcTanh[a*x])] - 96*ArcTanh[a*x]^2*PolyLog[2,
E^(2*ArcTanh[a*x])] + 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] - 48*
PolyLog[4, E^(2*ArcTanh[a*x])]))
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6544, 6452, 6544, 6452, 6510, 6550, 6494, 2897, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x^3(1-a^2x^2)}dx$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx+\int \frac{\text{arctanh}(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}a\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)}dx+a^2\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6544}$$

$$\frac{3}{2}a\left(a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^2}{x^2}dx\right)+a^2\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}a\left(a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+2a\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^2}{x}\right)+a^2\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$\frac{3}{2}a\left(2a\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}\right)+a^2\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx+\frac{1}{4}\text{arctanh}(ax)^4\right)+$$
$$\frac{3}{2}a\left(2a\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}\right)-$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 6494

$$\frac{3}{2}a\left(2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\text{arc}\right.$$
$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)-$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 2897

$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 6618

$$a^2\left(-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int \frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{4}\text{arctanh}(ax)^4\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 6622

$$a^2\left(-3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{a\ldots}\right.}{2a}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\Big\downarrow$ 7164

$$a^2\left(-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

input
```
Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

output
```
-1/2*ArcTanh[a*x]^3/x^2 + (3*a*(-(ArcTanh[a*x]^2/x) + (a*ArcTanh[a*x]^3)/3
 + 2*a*(ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2,
-1 + 2/(1 + a*x)]/2)))/2 + a^2*(ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 -
2/(1 + a*x)] - 3*a*((ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) +
(ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 +
 a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(182) = 364$.

Time = 35.40 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.84

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3ax)(ax-1)}{2a^2x^2}+\operatorname{arctanh}(ax)^3\ln\left(1-\right.\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{4}+\frac{\operatorname{arctanh}(ax)^2(ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)+3ax)(ax-1)}{2a^2x^2}+\operatorname{arctanh}(ax)^3\ln\left(1-\right.\right.$

input
```
int(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/4*arctanh(a*x)^4+1/2*arctanh(a*x)^2*(a*x*arctanh(a*x)+arctanh(a*x)
+3*a*x)*(a*x-1)/a^2/x^2+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*
arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylo
g(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+ar
ctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,-
(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)
^(1/2))+6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2+3*arctan
h(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(
1/2))+3*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1 - a^2x^2)} dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2 - 1)x^3} dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1 - a^2x^2)} dx = -\int \frac{\operatorname{atanh}^3(ax)}{a^2x^5 - x^3} dx$$

input
```
integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1),x)
```

output
```
-Integral(atanh(a*x)**3/(a**2*x**5 - x**3), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1 - a^2x^2)} dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2 - 1)x^3} dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/64*(a^2*x^2*log(-a*x + 1)^4 + 4*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)
^3)/x^2 - 1/8*integrate(1/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x
+ 1) + 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 + 2)*log(a*x + 1))*log(-a*x +
 1)^2)/(a^2*x^5 - x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,(a^2\,x^2-1)}\,dx$$

input
```
int(-atanh(a*x)^3/(x^3*(a^2*x^2 - 1)),x)
```

output
```
-int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)}\,dx = -\left(\int \frac{atanh(ax)^3}{a^2x^5-x^3}dx\right)$$

input
```
int(atanh(a*x)^3/x^3/(-a^2*x^2+1),x)
```

output
```
- int(atanh(a*x)**3/(a**2*x**5 - x**3),x)
```

3.248 $\int \frac{\sqrt{\text{arctanh}(ax)}}{1-a^2x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{1-a^2x^2} dx = \frac{2\text{arctanh}(ax)^{3/2}}{3a}$$

output
```
2/3*arctanh(a*x)^(3/2)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{1-a^2x^2} dx = \frac{2\text{arctanh}(ax)^{3/2}}{3a}$$

input
```
Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
```

output
```
(2*ArcTanh[a*x]^(3/2))/(3*a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2} \, dx$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{2\operatorname{arctanh}(ax)^{3/2}}{3a}$$

input
```
Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
```

output
```
(2*ArcTanh[a*x]^(3/2))/(3*a)
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}(ax)^{\frac{3}{2}}}{3a}$	12
default	$\frac{2 \operatorname{arctanh}(ax)^{\frac{3}{2}}}{3a}$	12

input
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctanh(a*x)^(3/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2}\,dx = \frac{\sqrt{\frac{1}{2}}\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{3\,a}$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2}\,dx = \begin{cases} \frac{2\operatorname{atanh}^{\frac{3}{2}}(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1),x)
```

output
```
Piecewise((2*atanh(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2}\,dx = \int -\frac{\sqrt{\operatorname{artanh}(ax)}}{a^2x^2-1}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2}\,dx = \frac{\sqrt{2}\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6\,a}$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2}\,dx = \frac{2\operatorname{atanh}(a\,x)^{3/2}}{3\,a}$$

input
```
int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1),x)
```

output
```
(2*atanh(a*x)^(3/2))/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{1-a^2x^2}\,dx = \frac{2\sqrt{atanh\left(ax\right)}\,atanh(ax)}{3a}$$

input
```
int(atanh(a*x)^(1/2)/(-a^2*x^2+1),x)
```

output
```
(2*sqrt(atanh(a*x))*atanh(a*x))/(3*a)
```

3.249 $\int \frac{x}{(1-a^2x^2)\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx=\text{Int}\left(\frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx=\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx$$

input
```
Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

output
```
Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx$$

$$\Big\downarrow \; 6651$$

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}dx$$

input
```
Int[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-a^2x^2+1)\,\text{arctanh}\,(ax)}dx$$

input
```
int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

output
```
int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx = \int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx = -\int \frac{x}{a^2x^2\,\text{atanh}\,(ax)-\text{atanh}\,(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)/atanh(a*x),x)
```

output
```
-Integral(x/(a**2*x**2*atanh(a*x) - atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx = \int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = \int -\frac{x}{(a^2x^2-1)\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = -\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-x/(atanh(a*x)*(a^2*x^2 - 1)),x)
```

output
```
-int(x/(atanh(a*x)*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx = -\left(\int \frac{x}{atanh\,(ax)\,a^2x^2 - atanh\,(ax)}dx\right)$$

input
```
int(x/(-a^2*x^2+1)/atanh(a*x),x)
```

output
```
- int(x/(atanh(a*x)*a**2*x**2 - atanh(a*x)),x)
```

3.250 $\int \frac{1}{(1-a^2x^2)\,\mathbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx = \frac{\log(\text{arctanh}(ax))}{a}$$

output
```
ln(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)}\,dx = \frac{\log(\text{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

output
```
Log[ArcTanh[a*x]]/a
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6508}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6508}$$

$$\frac{\log(\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

output
```
Log[ArcTanh[a*x]]/a
```

Defintions of rubi rules used

rule 6508
```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\operatorname{arctanh}(ax))}{a}$	10
default	$\frac{\ln(\operatorname{arctanh}(ax))}{a}$	10
parallelrisch	$\frac{\ln(\operatorname{arctanh}(ax))}{a}$	10
risch	$\frac{\ln(\ln(ax+1)-\ln(-ax+1))}{a}$	22

input
```
int(1/(-a^2*x^2+1)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(arctanh(a*x))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = \frac{\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)}{a}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

output
```
log(log(-(a*x + 1)/(a*x - 1)))/a
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = \frac{\log\left(\operatorname{atanh}\left(ax\right)\right)}{a}$$

input
```
integrate(1/(-a**2*x**2+1)/atanh(a*x),x)
```

output
```
log(atanh(a*x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = \frac{\log\left(-\log\left(ax+1\right)+\log\left(-ax+1\right)\right)}{a}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

output
```
log(-log(a*x + 1) + log(-a*x + 1))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = \frac{\log\left(\left|\log\left(-\frac{ax+1}{ax-1}\right)\right|\right)}{a}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

output
```
log(abs(log(-(a*x + 1)/(a*x - 1))))/a
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = \frac{\ln\left(\operatorname{atanh}(a\,x)\right)}{a}$$

input
```
int(-1/(atanh(a*x)*(a^2*x^2 - 1)),x)
```

output
```
log(atanh(a*x))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx=\frac{\log(atanh(ax))}{a}$$

input
```
int(1/(-a^2*x^2+1)/atanh(a*x),x)
```

output
```
log(atanh(a*x))/a
```

3.251 $\int \frac{1}{x(1-a^2x^2)\mathbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-a^2x^2)\,\text{arctanh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{x(1-a^2x^2)\,\text{arctanh}(ax)}dx$$

input
```
Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)\text{arctanh}\left(ax\right)}dx$$

input
```
int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^{2}x^{2}\right)\operatorname{arctanh}(ax)}\,dx=\int -\frac{1}{(a^{2}x^{2}-1)x\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-1/((a^2*x^3 - x)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^{2}x^{2}\right)\operatorname{arctanh}(ax)}\,dx=-\int \frac{1}{a^{2}x^{3}\operatorname{atanh}\left(ax\right)-x\operatorname{atanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)/atanh(a*x),x)
```

output
```
-Integral(1/(a**2*x**3*atanh(a*x) - x*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(1-a^{2}x^{2}\right)\operatorname{arctanh}(ax)}\,dx=\int -\frac{1}{(a^{2}x^{2}-1)x\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)\,\mathrm{arctanh}(ax)}\,dx = \int -\frac{1}{(a^2x^2-1)x\,\mathrm{artanh}\,(ax)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(1-a^2x^2)\,\mathrm{arctanh}(ax)}\,dx = -\int \frac{1}{x\,\mathrm{atanh}\,(a\,x)\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(x*atanh(a*x)*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(x*atanh(a*x)*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{1}{atanh\,(ax)\,a^2x^3 - atanh\,(ax)\,x}dx\right)$$

input

```
int(1/x/(-a^2*x^2+1)/atanh(a*x),x)
```

output

```
- int(1/(atanh(a*x)*a**2*x**3 - atanh(a*x)*x),x)
```

3.252 $\int \frac{x}{(1-a^2x^2)\mathbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = -\frac{x}{a\operatorname{arctanh}(ax)} + \frac{\operatorname{Int}\left(\frac{1}{\operatorname{arctanh}(ax)},x\right)}{a}$$

output
```
-x/a/arctanh(a*x)+Defer(Int)(1/arctanh(a*x),x)/a
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx$$

input
```
Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6548}$$

$$\frac{\int \frac{1}{\operatorname{arctanh}(ax)}dx}{a}-\frac{x}{a\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6444}$$

$$\frac{\int \frac{1}{\operatorname{arctanh}(ax)}dx}{a}-\frac{x}{a\operatorname{arctanh}(ax)}$$

input
```
Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-a^2x^2+1)\operatorname{arctanh}(ax)^2}dx$$

input
```
int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

output
```
int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = \int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\int \frac{x}{a^2x^2\,\text{atanh}^2\,(ax)-\text{atanh}^2\,(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

output
```
-Integral(x/(a**2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = \int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*x/(a*log(a*x + 1) - a*log(-a*x + 1)) - 2*integrate(-1/(a*log(a*x + 1) -
 a*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = \int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\int \frac{x}{\text{atanh}\,(a\,x)^2\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-x/(atanh(a*x)^2*(a^2*x^2 - 1)),x)
```

output
```
-int(x/(atanh(a*x)^2*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\left(\int \frac{x}{atanh\,(ax)^2\,a^2x^2 - atanh\,(ax)^2}dx\right)$$

input
```
int(x/(-a^2*x^2+1)/atanh(a*x)^2,x)
```

output
```
 - int(x/(atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)
```

3.253 $\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{a\text{arctanh}(ax)}$$

output
```
-1/a/arctanh(a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{a\text{arctanh}(ax)}$$

input
```
Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*ArcTanh[a*x]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx$$

$$\Big\downarrow \text{6510}$$

$$-\frac{1}{a\text{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*ArcTanh[a*x]))
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{1}{a\,\text{arctanh}(ax)}$	12
default	$-\frac{1}{a\,\text{arctanh}(ax)}$	12
parallelrisch	$-\frac{1}{a\,\text{arctanh}(ax)}$	12
risch	$\frac{2}{a(-\ln(ax+1)+\ln(-ax+1))}$	24

input
```
int(1/(-a^2*x^2+1)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/arctanh(a*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-2/(a*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{a\,\text{atanh}\,(ax)}$$

input
```
integrate(1/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

output
```
-1/(a*atanh(a*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = -\frac{2}{a\log(ax+1) - a\log(-ax+1)}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2/(a*log(a*x + 1) - a*log(-a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = -\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
-2/(a*log(-(a*x + 1)/(a*x - 1)))
```

Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = -\frac{2}{a\,\left(\ln(a\,x+1) - \ln(1-a\,x)\right)}$$

input
```
int(-1/(atanh(a*x)^2*(a^2*x^2 - 1)),x)
```

output
```
-2/(a*(log(a*x + 1) - log(1 - a*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{atanh\,(ax)\,a}$$

input
```
int(1/(-a^2*x^2+1)/atanh(a*x)^2,x)
```

output
```
( - 1)/(atanh(a*x)*a)
```

3.254 $\int \frac{1}{x(1-a^2x^2)\mathbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\,dx = -\frac{1}{ax\operatorname{arctanh}(ax)} - \frac{\operatorname{Int}\left(\frac{1}{x^2\operatorname{arctanh}(ax)},x\right)}{a}$$

output
```
-1/a/x/arctanh(a*x)-Defer(Int)(1/x^2/arctanh(a*x),x)/a
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-a^2x^2)\,\text{arctanh}(ax)^2}\,dx$$

$$\Big\downarrow \text{ 6552}$$

$$-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6468}$$

$$-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

input
```
Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(-a^2x^2+1)\,\text{arctanh}\,(ax)^2}dx$$

input
```
int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

output
```
int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{1}{(a^2x^2-1)x\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-1/((a^2*x^3 - x)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = -\int \frac{1}{a^2x^3\operatorname{atanh}^2(ax) - x\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

output
```
-Integral(1/(a**2*x**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{1}{(a^2x^2-1)x\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) + 2*integrate(-1/(a*x^2*log(a*x
+ 1) - a*x^2*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\,dx = \int -\frac{1}{(a^2x^2-1)x\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\,dx = -\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(x*atanh(a*x)^2*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{1}{atanh\left(ax\right)^2 a^2x^3 - atanh\left(ax\right)^2 x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)/atanh(a*x)^2,x)
```

output
```
- int(1/(atanh(a*x)**2*a**2*x**3 - atanh(a*x)**2*x),x)
```

3.255 $\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx = -\frac{x}{2a\operatorname{arctanh}(ax)^2} + \frac{\operatorname{Int}\left(\frac{1}{\operatorname{arctanh}(ax)^2}, x\right)}{2a}$$

output
```
-1/2*x/a/arctanh(a*x)^2+1/2*Defer(Int)(1/arctanh(a*x)^2,x)/a
```

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx = \int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx$$

input
```
Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

output
```
Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6548}$$

$$\frac{\int \frac{1}{\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6444}$$

$$\frac{\int \frac{1}{\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\operatorname{arctanh}(ax)^2}$$

input
```
Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-a^2x^2+1)\operatorname{arctanh}(ax)^3}dx$$

input
```
int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

output
```
int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x}{(a^2x^2-1)\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = -\int \frac{x}{a^2x^2\operatorname{atanh}^3(ax)-\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

output
```
-Integral(x/(a**2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 5.65

$$\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x}{(a^2x^2-1)\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*l
og(a*x + 1)^2 - 2*a^2*log(a*x + 1)*log(-a*x + 1) + a^2*log(-a*x + 1)^2) +
2*integrate(-x/(log(a*x + 1) - log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^3}\,dx = \int -\frac{x}{(a^2x^2-1)\,\text{artanh}\,(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^3}\,dx = -\int \frac{x}{\text{atanh}\,(a\,x)^3\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-x/(atanh(a*x)^3*(a^2*x^2 - 1)),x)
```

output
```
-int(x/(atanh(a*x)^3*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{x}{(1-a^2x^2)\,\text{arctanh}(ax)^3}\,dx = -\left(\int \frac{x}{atanh\,(ax)^3\,a^2x^2 - atanh\,(ax)^3}dx\right)$$

input
```
int(x/(-a^2*x^2+1)/atanh(a*x)^3,x)
```

output
```
- int(x/(atanh(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)
```

3.256 $\int \frac{1}{(1-a^2x^2)\textbf{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arctanh}(ax)^2}$$

output
```
-1/2/a/arctanh(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{arctanh}(ax)^2}$$

input
```
Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*ArcTanh[a*x]^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{6510\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \text{6510}$$

$$-\frac{1}{2a\operatorname{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*ArcTanh[a*x]^2)
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2a \operatorname{arctanh}(ax)^2}$	12
default	$-\frac{1}{2a \operatorname{arctanh}(ax)^2}$	12
parallelrisch	$-\frac{1}{2a \operatorname{arctanh}(ax)^2}$	12
risch	$-\frac{2}{a(-\ln(ax+1)+\ln(-ax+1))^2}$	24

input
```
int(1/(-a^2*x^2+1)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/arctanh(a*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = -\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
-2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2a\operatorname{atanh}^2(ax)}$$

input
```
integrate(1/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

output
```
-1/(2*a*atanh(a*x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(11) = 22$.

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^3}\,dx = -\frac{2}{a\log\left(ax+1\right)^2 - 2\,a\log\left(ax+1\right)\log\left(-ax+1\right) + a\log\left(-ax+1\right)^2}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-2/(a*log(a*x + 1)^2 - 2*a*log(a*x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{(1-a^2x^2)\,\text{arctanh}(ax)^3}\,dx = -\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

input
```
integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
-2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1}{(1-a^2x^2)\,\mathrm{arctanh}(ax)^3}\,dx = -\frac{2}{a\,(\ln(a\,x+1)-\ln(1-a\,x))^2}$$

input
```
int(-1/(atanh(a*x)^3*(a^2*x^2 - 1)),x)
```

output
```
-2/(a*(log(a*x + 1) - log(1 - a*x))^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-a^2x^2)\,\mathrm{arctanh}(ax)^3}\,dx = -\frac{1}{2atanh\,(ax)^2\,a}$$

input
```
int(1/(-a^2*x^2+1)/atanh(a*x)^3,x)
```

output
```
( - 1)/(2*atanh(a*x)**2*a)
```

3.257 $\int \frac{1}{x(1-a^2x^2)\mathbf{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx = -\frac{1}{2ax\operatorname{arctanh}(ax)^2} - \frac{\operatorname{Int}\left(\frac{1}{x^2\operatorname{arctanh}(ax)^2}, x\right)}{2a}$$

output
```
-1/2/a/x/arctanh(a*x)^2-1/2*Defer(Int)(1/x^2/arctanh(a*x)^2,x)/a
```

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx = \int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^3} \, dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (1-a^2x^2) \operatorname{arctanh}(ax)^3} \, dx$$

$$\Big\downarrow \text{ 6552}$$

$$-\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 6468}$$

$$-\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\operatorname{arctanh}(ax)^2}$$

input
```
Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(-a^2x^2+1\right) \operatorname{arctanh}\left(ax\right)^3}dx$$

input
```
int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

output
```
int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{1}{(a^2x^2-1)x\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/((a^2*x^3 - x)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3}\,dx = -\int \frac{1}{a^2x^3\operatorname{atanh}^3\left(ax\right)-x\operatorname{atanh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

output
```
-Integral(1/(a**2*x**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 6.09

$$\int \frac{1}{x\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{1}{(a^2x^2-1)x\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-(2*a*x + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*x
^2*log(a*x + 1)^2 - 2*a^2*x^2*log(a*x + 1)*log(-a*x + 1) + a^2*x^2*log(-a*
x + 1)^2) - 2*integrate(-1/(a^2*x^3*log(a*x + 1) - a^2*x^3*log(-a*x + 1)),
 x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{1}{(a^2x^2-1)x\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(1-a^2x^2)\operatorname{arctanh}(ax)^3}\,dx = -\int \frac{1}{x\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(x*atanh(a*x)^3*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{x(1-a^2x^2)\,\text{arctanh}(ax)^3}\,dx = -\left(\int \frac{1}{atanh\,(ax)^3\,a^2x^3 - atanh\,(ax)^3\,x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)/atanh(a*x)^3,x)
```

output
```
- int(1/(atanh(a*x)**3*a**2*x**3 - atanh(a*x)**3*x),x)
```

3.258 $\int \frac{\text{arctanh}(ax)^p}{1-a^2x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{\text{arctanh}(ax)^p}{1-a^2x^2} dx = \frac{\text{arctanh}(ax)^{1+p}}{a(1+p)}$$

output
```
arctanh(a*x)^(p+1)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}(ax)^p}{1-a^2x^2} dx = \frac{\text{arctanh}(ax)^{1+p}}{a(1+p)}$$

input
```
Integrate[ArcTanh[a*x]^p/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{\operatorname{arctanh}(ax)^{p+1}}{a(p+1)}$$

input
```
Int[ArcTanh[a*x]^p/(1 - a^2*x^2),x]
```

output
```
ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(ax)^{p+1}}{a(p+1)}$	18
default	$\frac{\operatorname{arctanh}(ax)^{p+1}}{a(p+1)}$	18
parallelrisch	$\frac{\operatorname{arctanh}(ax)^{p}\operatorname{arctanh}(ax)}{a(p+1)}$	20

input
```
int(arctanh(a*x)^p/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctanh(a*x)^(p+1)/a/(p+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.94

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx = \frac{\cosh\left(p\log\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+\log\left(-\frac{ax+1}{ax-1}\right)\sinh\left(p\log\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)}{2\left(ap+a\right)}$$

input
```
integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*(cosh(p*log(1/2*log(-(a*x + 1)/(a*x - 1))))*log(-(a*x + 1)/(a*x - 1))
+ log(-(a*x + 1)/(a*x - 1))*sinh(p*log(1/2*log(-(a*x + 1)/(a*x - 1)))))/(a
*p + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx = \begin{cases} \frac{\begin{cases} \frac{\operatorname{atanh}^{p+1}(ax)}{p+1} & \text{for}\ p \neq -1 \\ \log\left(\operatorname{atanh}(ax)\right) & \text{otherwise} \end{cases}}{a} & \text{for}\ a \neq 0 \\ 0^p x & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(a*x)**p/(-a**2*x**2+1),x)
```

output
```
Piecewise((Piecewise((atanh(a*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(atanh(
a*x)), True))/a, Ne(a, 0)), (0**p*x, True))
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx = \int -\frac{\operatorname{artanh}\left(ax\right)^p}{a^2x^2-1}\,dx$$

input
```
integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="maxima")
```

output
```
-integrate(arctanh(a*x)^p/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx = \frac{\left(\frac{1}{2}\log\left(-\frac{ax+1}{ax-1}\right)\right)^{p+1}}{a(p+1)}$$

input
```
integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="giac")
```

output
```
(1/2*log(-(a*x + 1)/(a*x - 1)))^(p + 1)/(a*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx = \begin{cases} \frac{\ln(\operatorname{atanh}(a\,x))}{a} & \text{ if } p=-1\\ \frac{\operatorname{atanh}(a\,x)^{p+1}}{a\,(p+1)} & \text{ if } p\neq -1 \end{cases}$$

input
```
int(-atanh(a*x)^p/(a^2*x^2 - 1),x)
```

output
```
piecewise(p == -1, log(atanh(a*x))/a, p ~= -1, atanh(a*x)^(p + 1)/(a*(p +
1)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(ax)^p}{1-a^2x^2}\,dx = \frac{atanh(ax)^p\,atanh(ax)}{a\,(p+1)}$$

input
```
int(atanh(a*x)^p/(-a^2*x^2+1),x)
```

output
```
(atanh(a*x)**p*atanh(a*x))/(a*(p + 1))
```

3.259 $\int \frac{x^3\textbf{arctanh}(ax)}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{x}{4a^3(1-a^2x^2)}-\frac{\text{arctanh}(ax)}{4a^4}+\frac{\text{arctanh}(ax)}{2a^4(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{2a^4} -\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}$$

output
```
-1/4*x/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)/a^4+1/2*arctanh(a*x)/a^4/(-a^2*x^
2+1)+1/2*arctanh(a*x)^2/a^4-arctanh(a*x)*ln(2/(-a*x+1))/a^4-1/2*polylog(2,
1-2/(-a*x+1))/a^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{4\text{arctanh}(ax)^2-2\text{arctanh}(ax)\left(\cosh(2\text{arctanh}(ax))-4\log\left(1+e^{-2\text{arctanh}(ax)}\right)\right)-4\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right)-\ldots}{8a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

output
```
-1/8*(4*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 4*Log[1 +
E^(-2*ArcTanh[a*x])]) - 4*PolyLog[2, -E^(-2*ArcTanh[a*x])] + Sinh[2*ArcTan
h[a*x]])/a^4
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6590, 6546, 6470, 2849, 2752, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a^2}-\frac{\int \frac{x\text{arctanh}(ax)}{1-a^2x^2}dx}{a^2}$$

$$\Big\downarrow \text{6546}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a^2}-\frac{\frac{\int \frac{\text{arctanh}(ax)}{1-ax}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}$$

$$\Big\downarrow \text{6470}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a^2}-\frac{\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\text{arctanh}(ax)^2}{2a^2}}{a^2}$$

$$\Big\downarrow \text{6556}$$

$$\frac{\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}}{a^2}-\frac{\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\operatorname{arctanh}(ax)^2}{2a^2}}{a^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}}{a^2}-\frac{\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\operatorname{arctanh}(ax)^2}{2a^2}}{a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}}{2a}}{a^2}-\frac{\frac{\frac{\operatorname{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}}{a}-\frac{\operatorname{arctanh}(ax)^2}{2a^2}}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

output
```
(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/
(2*a))/(2*a))/a^2 - (-1/2*ArcTanh[a*x]^2/a^2 + ((ArcTanh[a*x]*Log[2/(1 - a
*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)}{4ax+4}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\frac{\ln(ax-1)^2}{8}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}}{a^4}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)}{4ax+4}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\frac{\ln(ax-1)^2}{8}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}}{a^4}$
parts	$\frac{\operatorname{arctanh}(ax)}{4a^4(ax+1)}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2a^4}-\frac{\operatorname{arctanh}(ax)}{4a^4(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2a^4}-\frac{a\left(-\frac{\ln(ax-1)^2}{2a^5}+\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{a^5}\right.}{}$
risch	$\frac{\ln(ax+1)^2}{8a^4}+\frac{\ln(ax-1)}{16a^4}-\frac{\ln(ax+1)x}{16a^3(ax-1)}-\frac{\ln(ax+1)}{16a^4(ax-1)}+\frac{\ln(ax+1)}{8a^4(ax+1)}+\frac{1}{8a^4(ax+1)}+\frac{\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln(ax+1)}{4a^4}$

input
```
int(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*arctanh(a*x)/(a*x-1)+1/2*arctanh(a*x)*ln(a*x-1)+1/4*arctanh(a*
x)/(a*x+1)+1/2*arctanh(a*x)*ln(a*x+1)+1/8*ln(a*x-1)^2-1/2*dilog(1/2*a*x+1/
2)-1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x+1)^2+1/4*(ln(a*x+1)-ln(1/2*a*x
+1/2))*ln(-1/2*a*x+1/2)+1/8/(a*x-1)+1/8*ln(a*x-1)+1/8/(a*x+1)-1/8*ln(a*x+1
))
```

Fricas [F]

$$\int \frac{x^3\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx=\int \frac{x^3\operatorname{artanh}(ax)}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \int \frac{x^3\,\text{atanh}\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x**3*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.62

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{1}{8}\,a\left(\frac{(a^2x^2-1)\log(ax+1)^2 - 2\,(a^2x^2-1)\log(ax+1)\log(ax-1) - (a^2x^2-1)\log(ax-1)^2 - 2\,a}{a^7x^2-a^5}\right. - \frac{1}{2}\left(\frac{1}{a^6x^2-a^4} - \frac{\log(a^2x^2-1)}{a^4}\right)\text{artanh}\,(ax)$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a
*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2*a*x - (a^2*x^2 - 1)*log(a*x - 1
))/(a^7*x^2 - a^5) + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x +
 1/2))/a^5 + log(a*x + 1)/a^5) - 1/2*(1/(a^6*x^2 - a^4) - log(a^2*x^2 - 1)
/a^4)*arctanh(a*x)
```

Giac [F]

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx=\int \frac{x^3\,\text{artanh}\,(ax)}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx=\int \frac{x^3\,\text{atanh}(a\,x)}{(a^2\,x^2-1)^2}\,dx$$

input
```
int((x^3*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

output
```
int((x^3*atanh(a*x))/(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx=\int \frac{atanh(ax)\,x^3}{a^4x^4-2a^2x^2+1}dx$$

input
```
int(x^3*atanh(a*x)/(-a^2*x^2+1)^2,x)
```

output
```
int((atanh(a*x)*x**3)/(a**4*x**4 - 2*a**2*x**2 + 1),x)
```

3.260 $\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{1}{4a^3(1-a^2x^2)} + \frac{x\text{arctanh}(ax)}{2a^2(1-a^2x^2)} - \frac{\text{arctanh}(ax)^2}{4a^3}$$

output
```
-1/4/a^3/(-a^2*x^2+1)+1/2*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-1/4*arctanh(a*x)
^2/a^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{1-2ax\text{arctanh}(ax)+(1-a^2x^2)\,\text{arctanh}(ax)^2}{4a^3(-1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

output
```
(1 - 2*a*x*ArcTanh[a*x] + (1 - a^2*x^2)*ArcTanh[a*x]^2)/(4*a^3*(-1 + a^2*x
^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6560, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{ 6560}$$

$$-\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{2a^2}+\frac{x\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{1}{4a^3(1-a^2x^2)}$$

$$\Big\downarrow \text{ 6510}$$

$$-\frac{\text{arctanh}(ax)^2}{4a^3}+\frac{x\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{1}{4a^3(1-a^2x^2)}$$

input
```
Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

output
```
-1/4*1/(a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*a^2*(1 - a^2*x^2)) - ArcT
anh[a*x]^2/(4*a^3)
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6560
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x] + Sim
p[1/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q
, -5/2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{a^2x^2\operatorname{arctanh}(ax)^2-a^2x^2+2ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2}{4(a^2x^2-1)a^3}$
risch	$-\frac{\ln(ax+1)^2}{16a^3}+\frac{(x^2\ln(-ax+1)a^2-2ax-\ln(-ax+1))\ln(ax+1)}{8a^3(a^2x^2-1)}-\frac{a^2x^2\ln(-ax+1)^2-4ax\ln(-ax+1)-\ln(-ax+1)\cdots}{16a^3(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{8}+\frac{\ln(ax+1)\cdots}{16}}{a^3}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{8}+\frac{\ln(ax+1)\cdots}{16}}{a^3}$
parts	$-\frac{\operatorname{arctanh}(ax)}{4a^3(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4a^3}-\frac{\operatorname{arctanh}(ax)}{4a^3(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4a^3}-\frac{a\left(-\frac{\ln(ax+1)^2}{4}+\frac{(\ln(ax+1)\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a^2*x^2*arctanh(a*x)^2-a^2*x^2+2*a*x*arctanh(a*x)-arctanh(a*x)^2)/(a
^2*x^2-1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\,(a^5x^2-a^3)}$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
 - 1))^2 - 4)/(a^5*x^2 - a^3)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \int \frac{x^2\,\text{atanh}\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x**2*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(49) = 98.

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.21

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2}+\frac{\log\left(ax+1\right)}{a^3}-\frac{\log\left(ax-1\right)}{a^3}\right)\text{artanh}\left(ax\right) + \frac{\left((a^2x^2-1)\log\left(ax+1\right)^2-2\,(a^2x^2-1)\log\left(ax+1\right)\log\left(ax-1\right)+(a^2x^2-1)\log\left(ax-1\right)^2+4\right)a}{16\,(a^6x^2-a^4)}$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a
*x) + 1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*lo
g(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a/(a^6*x^2 - a^4)
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)}{(1-a^2x^2)^2} \, dx = \int \frac{x^2 \operatorname{artanh}(ax)}{(a^2x^2-1)^2} \, dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{x^2 \text{arctanh}(ax)}{(1-a^2x^2)^2} \, dx = \ln(1-a\,x) \left(\frac{\ln(a\,x+1)}{8\,a^3} + \frac{x}{2\,a^2\,(2\,a^2\,x^2-2)}\right) - \frac{\ln(a\,x+1)^2}{16\,a^3} - \frac{\ln(1-a\,x)^2}{16\,a^3} - \frac{1}{2\,a^2\,(2\,a-2\,a^3\,x^2)} - \frac{x\,\ln(a\,x+1)}{4\,a^3\,\left(a\,x^2-\frac{1}{a}\right)}$$

input
```
int((x^2*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

output
```
log(1 - a*x)*(log(a*x + 1)/(8*a^3) + x/(2*a^2*(2*a^2*x^2 - 2))) - log(a*x
+ 1)^2/(16*a^3) - log(1 - a*x)^2/(16*a^3) - 1/(2*a^2*(2*a - 2*a^3*x^2)) -
(x*log(a*x + 1))/(4*a^3*(a*x^2 - 1/a))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{-atanh(ax)^2\,a^2x^2 + atanh(ax)^2 - 2atanh(ax)\,ax + a^2x^2}{4a^3\,(a^2x^2-1)}$$

input
```
int(x^2*atanh(a*x)/(-a^2*x^2+1)^2,x)
```

output
```
( - atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2 - 2*atanh(a*x)*a*x + a**2*x**2
)/(4*a**3*(a**2*x**2 - 1))
```

3.261 $\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2} dx = -\frac{x}{4a(1-a^2x^2)} - \frac{\text{arctanh}(ax)}{4a^2} + \frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}$$

output
```
-1/4*x/a/(-a^2*x^2+1)-1/4*arctanh(a*x)/a^2+1/2*arctanh(a*x)/a^2/(-a^2*x^2+
1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2} dx = \frac{2ax - 4\text{arctanh}(ax) + (-1+a^2x^2)\log(1-ax) + \log(1+ax) - a^2x^2\log(1+ax)}{8a^2(-1+a^2x^2)}$$

input
```
Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

output
```
(2*a*x - 4*ArcTanh[a*x] + (-1 + a^2*x^2)*Log[1 - a*x] + Log[1 + a*x] - a^2
*x^2*Log[1 + a*x])/(8*a^2*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{ 6556}$$

$$\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}$$

input
```
Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

output
```
ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(
2*a))/(2*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result
parallelrisch	$-\frac{a^{2} x^{2} \operatorname{arctanh}\left(a x\right)-a x+\operatorname{arctanh}\left(a x\right)}{4\left(a^{2} x^{2}-1\right) a^{2}}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}\left(a x\right)}{2\left(a^{2} x^{2}-1\right)}+\frac{1}{8 a x-8}+\frac{\ln\left(a x-1\right)}{8}+\frac{1}{8 a x+8}-\frac{\ln\left(a x+1\right)}{8}}{a^{2}}$
default	$\frac{-\frac{\operatorname{arctanh}\left(a x\right)}{2\left(a^{2} x^{2}-1\right)}+\frac{1}{8 a x-8}+\frac{\ln\left(a x-1\right)}{8}+\frac{1}{8 a x+8}-\frac{\ln\left(a x+1\right)}{8}}{a^{2}}$
parts	$-\frac{\operatorname{arctanh}\left(a x\right)}{2 a^{2}\left(a^{2} x^{2}-1\right)}+\frac{\frac{1}{4\left(a x+1\right) a}-\frac{\ln\left(a x+1\right)}{4 a}+\frac{1}{4 a\left(a x-1\right)}+\frac{\ln\left(a x-1\right)}{4 a}}{2 a}$
risch	$-\frac{\ln\left(a x+1\right)}{4 a^{2}\left(a^{2} x^{2}-1\right)}-\frac{\ln\left(a x+1\right) a^{2} x^{2}-x^{2} \ln\left(-a x+1\right) a^{2}-2 a x-\ln\left(a x+1\right)-\ln\left(-a x+1\right)}{8 a^{2}\left(a x-1\right)\left(a x+1\right)}$
orering	$-\frac{\left(a x-1\right)\left(a x+1\right)\left(2 a^{2} x^{2}+1\right) \operatorname{arctanh}\left(a x\right)}{2 a^{2}\left(-a^{2} x^{2}+1\right)^{2}}-\frac{\left(a x+1\right)^{2}\left(a x-1\right)^{2}\left(\frac{\operatorname{arctanh}\left(a x\right)}{\left(-a^{2} x^{2}+1\right)^{2}}+\frac{x a}{\left(-a^{2} x^{2}+1\right)^{3}}+\frac{4 x^{2} \operatorname{arctanh}\left(a x\right) a^{2}}{\left(-a^{2} x^{2}+1\right)^{3}}\right)}{4 a^{2}}$

input
```
int(x*arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a^2*x^2*arctanh(a*x)-a*x+arctanh(a*x))/(a^2*x^2-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{2\,ax - (a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{8\,(a^4x^2-a^2)}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
1/8*(2*a*x - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \begin{cases} -\frac{a^2x^2\,\text{atanh}\,(ax)}{4a^4x^2-4a^2} + \frac{ax}{4a^4x^2-4a^2} - \frac{\text{atanh}\,(ax)}{4a^4x^2-4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

output
```
Piecewise((-a**2*x**2*atanh(a*x)/(4*a**4*x**2 - 4*a**2) + a*x/(4*a**4*x**2
 - 4*a**2) - atanh(a*x)/(4*a**4*x**2 - 4*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}}{8a} - \frac{\text{artanh}\,(ax)}{2\,(a^2x^2-1)a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)/a - 1/2*arctanh(
a*x)/((a^2*x^2 - 1)*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(47) = 94.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.80

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{1}{16}\left(\left(\frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right) - \frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)a$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
-1/16*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*((a
*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x
 - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) - (a*x + 1)/((a*x - 1)*a^3) +
 (a*x - 1)/((a*x + 1)*a^3))*a
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{\text{atanh}(a\,x)}{4\,a^2} - \frac{\frac{\text{atanh}(a\,x)}{2} - \frac{a\,x}{4}}{a^2\,\left(a^2\,x^2-1\right)}$$

input
```
int((x*atanh(a*x))/(a^2*x^2 - 1)^2,x)
```

output
```
- atanh(a*x)/(4*a^2) - (atanh(a*x)/2 - (a*x)/4)/(a^2*(a^2*x^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{-4atanh(ax)\,a^2x^2 - \log(a^2x-a)\,a^2x^2 + \log(a^2x-a) + \log(a^2x+a)\,a^2x^2 - \log(a^2x+a) + 2ax}{8a^2\,(a^2x^2-1)}$$

input
```
int(x*atanh(a*x)/(-a^2*x^2+1)^2,x)
```

output
```
( - 4*atanh(a*x)*a**2*x**2 - log(a**2*x - a)*a**2*x**2 + log(a**2*x - a) +
 log(a**2*x + a)*a**2*x**2 - log(a**2*x + a) + 2*a*x)/(8*a**2*(a**2*x**2 -
 1))
```

3.262 $\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2} dx = -\frac{1}{4a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)} + \frac{\operatorname{arctanh}(ax)^2}{4a}$$

output
```
-1/4/a/(-a^2*x^2+1)+x*arctanh(a*x)/(-2*a^2*x^2+2)+1/4*arctanh(a*x)^2/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2} dx = \frac{1 - 2ax\operatorname{arctanh}(ax) + (-1 + a^2x^2)\operatorname{arctanh}(ax)^2}{4a(-1 + a^2x^2)}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

output
```
(1 - 2*a*x*ArcTanh[a*x] + (-1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a*(-1 + a^2*x^
2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{ 6518}$$

$$-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

output
```
-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*
x]^2/(4*a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{-a^2x^2\operatorname{arctanh}(ax)^2-a^2x^2+2ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)^2}{4(a^2x^2-1)a}$
risch	$\frac{\ln(ax+1)^2}{16a}-\frac{(x^2\ln(-ax+1)a^2+2ax-\ln(-ax+1))\ln(ax+1)}{8(a^2x^2-1)a}+\frac{a^2x^2\ln(-ax+1)^2+4ax\ln(-ax+1)-\ln(-ax+1)^2+}{16a(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax+1)}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\ln(ax-1)^2}{16}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{8}-\frac{\ln(ax+1)^2}{16}+}{a}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{4(ax+1)}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4}-\frac{\ln(ax-1)^2}{16}+\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{8}-\frac{\ln(ax+1)^2}{16}+}{a}$
parts	$-\frac{\operatorname{arctanh}(ax)}{4(ax+1)a}+\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{4a}-\frac{\operatorname{arctanh}(ax)}{4a(ax-1)}-\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{4a}-\frac{a\left(\frac{\frac{\ln(ax-1)^2}{4}-\frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}}{a}\right.}{}$

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-a^2*x^2*arctanh(a*x)^2-a^2*x^2+2*a*x*arctanh(a*x)+arctanh(a*x)^2)/(
a^2*x^2-1)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int\frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx=-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\,(a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
 - 1))^2 - 4)/(a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(46) = 92$.

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2\,x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax) - \frac{\left((a^2x^2-1)\log(ax+1)^2 - 2\,(a^2x^2-1)\log(ax+1)\log(ax-1) + (a^2x^2-1)\log(ax-1)^2 - 4\right)a}{16\,(a^4x^2-a^2)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x) -
1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
- 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a/(a^4*x^2 - a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(46) = 92$.

Time = 1.11 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.72

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{1}{8}a^2\left(\frac{(ax-1)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}-1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{a-\frac{a\left(\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1\right)}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}}+1}\right)}{(ax+1)a^4}+\frac{ax-1}{(ax+1)a^4}\right)$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
1/8*a^2*((a*x - 1)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/(a - a*(a*((a*x + 1)/
(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) +
1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) - 1)/(a*((a*x + 1)/(a*x - 1) + 1)/(a
- a*(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x
 + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + 1))/((a*x + 1)*a^
4) + (a*x - 1)/((a*x + 1)*a^4))
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.96

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{\ln\left(a\,x+1\right)^2}{16\,a}-\ln\left(1-a\,x\right)\,\left(\frac{\ln\left(a\,x+1\right)}{8\,a}-\frac{x}{2\,\left(2\,a^2\,x^2-2\right)}\right)+\frac{\ln\left(1-a\,x\right)^2}{16\,a}+\frac{1}{2\,a\,\left(2\,a^2\,x^2-2\right)}-\frac{x\,\ln\left(a\,x+1\right)}{4\,a\,\left(a\,x^2-\frac{1}{a}\right)}$$

input
```
int(atanh(a*x)/(a^2*x^2 - 1)^2,x)
```

output
```
log(a*x + 1)^2/(16*a) - log(1 - a*x)*(log(a*x + 1)/(8*a) - x/(2*(2*a^2*x^2
 - 2))) + log(1 - a*x)^2/(16*a) + 1/(2*a*(2*a^2*x^2 - 2)) - (x*log(a*x + 1
))/(4*a*(a*x^2 - 1/a))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}\,dx = \frac{atanh(ax)^2\,a^2x^2 - atanh(ax)^2 - 2atanh(ax)\,ax + a^2x^2}{4a\,(a^2x^2-1)}$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^2,x)
```

output
```
(atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2 - 2*atanh(a*x)*a*x + a**2*x**2)/(
4*a*(a**2*x**2 - 1))
```

3.263 $\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^2}\,dx = -\frac{ax}{4(1-a^2x^2)} - \frac{1}{4}\operatorname{arctanh}(ax) + \frac{\operatorname{arctanh}(ax)}{2(1-a^2x^2)} + \frac{1}{2}\operatorname{arctanh}(ax)^2 + \operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
-1/4*a*x/(-a^2*x^2+1)-1/4*arctanh(a*x)+arctanh(a*x)/(-2*a^2*x^2+2)+1/2*arc
tanh(a*x)^2+arctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^2}\,dx = \frac{1}{8}\big(4\operatorname{arctanh}(ax)^2 + 2\operatorname{arctanh}(ax)\left(\cosh(2\operatorname{arctanh}(ax)) + 4\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)\right) - 4\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right) - \sinh(2\operatorname{arctanh}(ax))\big)$$

input
```
Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

output
```
(4*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] + 4*Log[1 - E^(-2
*ArcTanh[a*x])]) - 4*PolyLog[2, E^(-2*ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]
])/8
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6592, 6550, 6494, 2897, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx$$

$$\Big\downarrow \text{6550}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{6494}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{2897}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

$$\Big\downarrow \; 215$$

$$a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

$$\Big\downarrow \; 219$$

$$a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

input
```
Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

output
```
ArcTanh[a*x]^2/2 + a^2*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^
2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a)) + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] -
 PolyLog[2, -1 + 2/(1 + a*x)]/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(80) = 160$.

Time = 0.46 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{\operatorname{arctanh}(ax)}{4ax+4} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} + \operatorname{arctanh}(ax)\ln(ax) -$
default	$\frac{\operatorname{arctanh}(ax)}{4ax+4} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} + \operatorname{arctanh}(ax)\ln(ax) -$
risch	$-\frac{\ln(ax+1)^2}{8} - \frac{\operatorname{dilog}(ax+1)}{2} + \frac{\ln(ax+1)}{8ax+8} + \frac{1}{8ax+8} - \frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\frac{1}{2}\right)\right)\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)}{4} + \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}$
parts	$\operatorname{arctanh}(ax)\ln(x) + \frac{\operatorname{arctanh}(ax)}{4ax+4} - \frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arctanh}(ax)}{4(ax-1)} - \frac{\operatorname{arctanh}(ax)\ln(ax-1)}{2} -$

input
```
int(arctanh(a*x)/x/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(a*x)/(a*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/4*arctanh(a*x)/(a*x-
1)-1/2*arctanh(a*x)*ln(a*x-1)+arctanh(a*x)*ln(a*x)-1/2*dilog(a*x)-1/2*dilo
g(a*x+1)-1/2*ln(a*x)*ln(a*x+1)-1/8*ln(a*x-1)^2+1/2*dilog(1/2*a*x+1/2)+1/4*
ln(a*x-1)*ln(1/2*a*x+1/2)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*
ln(-1/2*a*x+1/2)+1/8/(a*x-1)+1/8*ln(a*x-1)+1/8/(a*x+1)-1/8*ln(a*x+1)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^2 x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}\,dx = \int \frac{\operatorname{atanh}\left(ax\right)}{x\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

input
```
integrate(atanh(a*x)/x/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(78) = 156.

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}\,dx$$
$$= \frac{1}{8}a\left(\frac{\left(a^2x^2-1\right)\log\left(ax+1\right)^2-2\left(a^2x^2-1\right)\log\left(ax+1\right)\log\left(ax-1\right)-\left(a^2x^2-1\right)\log\left(ax-1\right)^2+2\,ax}{a^3x^2-a}\right.$$
$$-\frac{1}{2}\left(\frac{1}{a^2x^2-1}+\log\left(a^2x^2-1\right)-\log\left(x^2\right)\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 2*a*x)/(a^3*x^2 - a) + 4*(log(a*x
- 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x + 1)*log(x
) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a - log(a*x + 1
)/a + log(a*x - 1)/a) - 1/2*(1/(a^2*x^2 - 1) + log(a^2*x^2 - 1) - log(x^2)
)*arctanh(a*x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}\,dx = \int \frac{\text{artanh}\,(ax)}{(a^2x^2-1)^2x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}\,dx = \int \frac{\text{atanh}(a\,x)}{x\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(atanh(a*x)/(x*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)/(x*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}\,dx = \int \frac{atanh(ax)}{a^4x^5-2a^2x^3+x}dx$$

input
```
int(atanh(a*x)/x/(-a^2*x^2+1)^2,x)
```

output
```
int(atanh(a*x)/(a**4*x**5 - 2*a**2*x**3 + x),x)
```

3.264 $\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^2}\,dx = -\frac{a}{4(1-a^2x^2)} - \frac{\text{arctanh}(ax)}{x} + \frac{a^2x\text{arctanh}(ax)}{2(1-a^2x^2)} + \frac{3}{4}a\text{arctanh}(ax)^2 + a\log(x) - \frac{1}{2}a\log(1-a^2x^2)$$

output
```
-1/4*a/(-a^2*x^2+1)-arctanh(a*x)/x+a^2*x*arctanh(a*x)/(-2*a^2*x^2+2)+3/4*a
*arctanh(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^2}\,dx = \frac{1}{4}\left(-\frac{2(-2+3a^2x^2)\text{arctanh}(ax)}{x(-1+a^2x^2)} + 3a\text{arctanh}(ax)^2 + a\left(\frac{1}{-1+a^2x^2} + 4\log(ax) - 2\log(1-a^2x^2)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

output
```
((-2*(-2 + 3*a^2*x^2)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)) + 3*a*ArcTanh[a*x]^
2 + a*((-1 + a^2*x^2)^(-1) + 4*Log[a*x] - 2*Log[1 - a^2*x^2]))/4
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6592, 6518, 241, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^2} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx + \int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)}dx$$

$$\Big\downarrow \text{6518}$$

$$a^2 \left(-\frac{1}{2}a \int \frac{x}{(1-a^2x^2)^2}dx + \frac{x\text{arctanh}(ax)}{2(1-a^2x^2)} + \frac{\text{arctanh}(ax)^2}{4a}\right) + \int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)}dx$$

$$\Big\downarrow \text{241}$$

$$\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)}dx + a^2 \left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)} - \frac{1}{4a(1-a^2x^2)} + \frac{\text{arctanh}(ax)^2}{4a}\right)$$

$$\Big\downarrow \text{6544}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + \int \frac{\text{arctanh}(ax)}{x^2}dx + a^2 \left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)} - \frac{1}{4a(1-a^2x^2)} + \frac{\text{arctanh}(ax)^2}{4a}\right)$$

$$\Big\downarrow \text{6452}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx + a \int \frac{1}{x(1-a^2x^2)}dx + a^2 \left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)} - \frac{1}{4a(1-a^2x^2)} + \frac{\text{arctanh}(ax)^2}{4a}\right) - \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2+ a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log(x^2)\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ \frac{1}{2}a\left(\log(x^2)-\log(1-a^2x^2)\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{6510}$$

$$a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{1}{2}a\left(\log(x^2)-\log(1-a^2x^2)\right)+ \frac{1}{2}a\,\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}$$

input
```
Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

output
```
-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a^2*(-1/4*1/(a*(1 - a^2*x^2)) +
 (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)) + (a*(Log[x^2]
 - Log[1 - a^2*x^2]))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

method	result
parallelrisch	$\frac{3\,\mathrm{arctanh}(ax)^2 a^3x^3+4\ln(x)a^3x^3-4\ln(ax-1)x^3a^3-4a^3x^3\,\mathrm{arctanh}(ax)+a^3x^3-6a^2x^2\,\mathrm{arctanh}(ax)-3\,\mathrm{arctanh}(ax)^2 a}{4(a^2x^2-1)x}$
derivativedivides	$a\left(-\frac{\mathrm{arctanh}(ax)}{4(ax+1)}+\frac{3\,\mathrm{arctanh}(ax)\ln(ax+1)}{4}-\frac{\mathrm{arctanh}(ax)}{4(ax-1)}-\frac{3\,\mathrm{arctanh}(ax)\ln(ax-1)}{4}-\frac{\mathrm{arctanh}(ax)}{ax}-3\mathrm{l}\right.$
default	$a\left(-\frac{\mathrm{arctanh}(ax)}{4(ax+1)}+\frac{3\,\mathrm{arctanh}(ax)\ln(ax+1)}{4}-\frac{\mathrm{arctanh}(ax)}{4(ax-1)}-\frac{3\,\mathrm{arctanh}(ax)\ln(ax-1)}{4}-\frac{\mathrm{arctanh}(ax)}{ax}-3\mathrm{l}\right.$
parts	$-\frac{\mathrm{arctanh}(ax)}{x}-\frac{\mathrm{arctanh}(ax)a}{4(ax+1)}+\frac{3a\,\mathrm{arctanh}(ax)\ln(ax+1)}{4}-\frac{\mathrm{arctanh}(ax)a}{4(ax-1)}-\frac{3a\,\mathrm{arctanh}(ax)\ln(ax-1)}{4}-$
risch	$\frac{3a\ln(ax+1)^2}{16}-\frac{(3a^3x^3\ln(-ax+1)+6a^2x^2-3ax\ln(-ax+1)-4)\ln(ax+1)}{8(a^2x^2-1)x}+\frac{3a^3x^3\ln(-ax+1)^2+16\ln(x)a^3x^3-8}{}$

input
```
int(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(3*arctanh(a*x)^2*a^3*x^3+4*ln(x)*a^3*x^3-4*ln(a*x-1)*x^3*a^3-4*a^3*x^
3*arctanh(a*x)+a^3*x^3-6*a^2*x^2*arctanh(a*x)-3*arctanh(a*x)^2*a*x-4*a*ln(
x)*x+4*ln(a*x-1)*a*x+4*a*x*arctanh(a*x)+4*arctanh(a*x))/(a^2*x^2-1)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)^2}\,dx$$
$$= \frac{3(a^3x^3-ax)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 4\,ax - 8(a^3x^3-ax)\log(a^2x^2-1) + 16(a^3x^3-ax)\log(x) - 4(3\,a^2x^2 -}{16(a^2x^3-x)}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*(3*(a^3*x^3 - a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x - 8*(a^3*x^3 -
 a*x)*log(a^2*x^2 - 1) + 16*(a^3*x^3 - a*x)*log(x) - 4*(3*a^2*x^2 - 2)*log
(-(a*x + 1)/(a*x - 1)))/(a^2*x^3 - x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(68) = 136.

Time = 0.89 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.09

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)^2}\,dx$$
$$= \begin{cases} \frac{4a^3x^3\log(x)}{4a^2x^3-4x} - \frac{4a^3x^3\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x} + \frac{3a^3x^3\operatorname{atanh}^2(ax)}{4a^2x^3-4x} - \frac{4a^3x^3\operatorname{atanh}(ax)}{4a^2x^3-4x} - \frac{6a^2x^2\operatorname{atanh}(ax)}{4a^2x^3-4x} - \frac{4ax\log(x)}{4a^2x^3-4x} + \frac{4ax\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x} - \\ 0 \end{cases}$$

input
```
integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**2,x)
```

output
```
Piecewise((4*a**3*x**3*log(x)/(4*a**2*x**3 - 4*x) - 4*a**3*x**3*log(x - 1/
a)/(4*a**2*x**3 - 4*x) + 3*a**3*x**3*atanh(a*x)**2/(4*a**2*x**3 - 4*x) - 4
*a**3*x**3*atanh(a*x)/(4*a**2*x**3 - 4*x) - 6*a**2*x**2*atanh(a*x)/(4*a**2
*x**3 - 4*x) - 4*a*x*log(x)/(4*a**2*x**3 - 4*x) + 4*a*x*log(x - 1/a)/(4*a*
*2*x**3 - 4*x) - 3*a*x*atanh(a*x)**2/(4*a**2*x**3 - 4*x) + 4*a*x*atanh(a*x
)/(4*a**2*x**3 - 4*x) + a*x/(4*a**2*x**3 - 4*x) + 4*atanh(a*x)/(4*a**2*x**
3 - 4*x), Ne(a, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(72) = 144$.

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^2}\,dx = -\frac{1}{16}a\left(\frac{3(a^2x^2-1)\log(ax+1)^2 - 6(a^2x^2-1)\log(ax+1)\log(ax-1) + 3(a^2x^2-1)\log(ax-1)^2 -}{a^2x^2-1}\right. + \frac{1}{4}\left(3a\log(ax+1) - 3a\log(ax-1) - \frac{2(3a^2x^2-2)}{a^2x^3-x}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/16*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*x + 1)*lo
g(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8*log(a*x
 + 1) + 8*log(a*x - 1) - 16*log(x)) + 1/4*(3*a*log(a*x + 1) - 3*a*log(a*x
- 1) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^2 x^2}\,dx$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.61

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^2} \, dx = \frac{3 a \ln (a x+1)^2}{16}+\frac{3 a \ln (1-a x)^2}{16} +\frac{a}{2 (2 a^2 x^2-2)}-\frac{a \ln (a^2 x^2-1)}{2}+a \ln (x) -\ln (1-a x) \left(\frac{\frac{3 a^2 x^2}{2}-1}{2 x-2 a^2 x^3}+\frac{3 a \ln (a x+1)}{8}\right) +\frac{\ln (a x+1) \left(\frac{3 a x^2}{4}-\frac{1}{2 a}\right)}{\frac{x}{a}-a x^3}$$

input
```
int(atanh(a*x)/(x^2*(a^2*x^2 - 1)^2),x)
```

output
```
(3*a*log(a*x + 1)^2)/16 + (3*a*log(1 - a*x)^2)/16 + a/(2*(2*a^2*x^2 - 2))
- (a*log(a^2*x^2 - 1))/2 + a*log(x) - log(1 - a*x)*(((3*a^2*x^2)/2 - 1)/(2
*x - 2*a^2*x^3) + (3*a*log(a*x + 1))/8) + (log(a*x + 1)*((3*a*x^2)/4 - 1/(
2*a)))/(x/a - a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^2} \, dx = \frac{3atanh(ax)^2 a^3x^3-3atanh(ax)^2 ax-4atanh(ax) a^3x^3-6atanh(ax) a^2x^2+4atanh(ax) ax+4atanh(a}{4x (a^2x^2-1)}$$

input
```
int(atanh(a*x)/x^2/(-a^2*x^2+1)^2,x)
```

output
```
(3*atanh(a*x)**2*a**3*x**3 - 3*atanh(a*x)**2*a*x - 4*atanh(a*x)*a**3*x**3
- 6*atanh(a*x)*a**2*x**2 + 4*atanh(a*x)*a*x + 4*atanh(a*x) - 4*log(a**2*x
- a)*a**3*x**3 + 4*log(a**2*x - a)*a*x + 4*log(x)*a**3*x**3 - 4*log(x)*a*x
 + a**3*x**3)/(4*x*(a**2*x**2 - 1))
```

3.265 $\int \frac{\text{arctanh}(ax)}{x^3(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\begin{aligned}\int \frac{\text{arctanh}(ax)}{x^3(1-a^2x^2)^2}\,dx = & -\frac{a}{2x}-\frac{a^3x}{4(1-a^2x^2)}+\frac{1}{4}a^2\text{arctanh}(ax)\\ & -\frac{\text{arctanh}(ax)}{2x^2}+\frac{a^2\text{arctanh}(ax)}{2(1-a^2x^2)}+a^2\text{arctanh}(ax)^2\\ & +2a^2\text{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)\\ & -a^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-1/2*a/x-a^3*x/(-4*a^2*x^2+4)+1/4*a^2*arctanh(a*x)-1/2*arctanh(a*x)/x^2+a^
2*arctanh(a*x)/(-2*a^2*x^2+2)+a^2*arctanh(a*x)^2+2*a^2*arctanh(a*x)*ln(2-2
/(a*x+1))-a^2*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \frac{\text{arctanh}(ax)}{x^3 (1-a^2x^2)^2} \, dx = \frac{1}{8}a^2 \left(-\frac{4}{ax} + 8\text{arctanh}(ax)^2 + 2\text{arctanh}(ax) \left(2 - \frac{2}{a^2x^2} + \cosh(2\text{arctanh}(ax)) + 8\log\left(1-e^{-2\text{arctanh}(ax)}\right) \right) - 8\,\text{PolyLog}\left(2, e^{-2\text{arctanh}(ax)}\right) - \sinh(2\text{arctanh}(ax)) \right)$$

input
```
Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

output
```
(a^2*(-4/(a*x) + 8*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(2 - 2/(a^2*x^2) + Cosh
[2*ArcTanh[a*x]] + 8*Log[1 - E^(-2*ArcTanh[a*x])]) - 8*PolyLog[2, E^(-2*Ar
cTanh[a*x])] - Sinh[2*ArcTanh[a*x]]))/8
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.55, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6592, 6544, 6452, 264, 219, 6550, 6494, 2897, 6592, 6550, 6494, 2897, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^3 (1-a^2x^2)^2} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{x (1-a^2x^2)^2} dx + \int \frac{\text{arctanh}(ax)}{x^3 (1-a^2x^2)} dx$$

$$\Big\downarrow \text{6544}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{x (1-a^2x^2)^2} dx + a^2 \int \frac{\text{arctanh}(ax)}{x (1-a^2x^2)} dx + \int \frac{\text{arctanh}(ax)}{x^3} dx$$

$$\Big\downarrow \text{6452}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6550}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6494}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{2897}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6592}$$

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx\right)+a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+ a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)- \frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+ a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)- \frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{2897}$$

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+ a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)- \frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\right.\right. a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)- \frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{215}$$

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{Po}\right.$$
$$a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-$$
$$\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

$$\Big\downarrow \text{219}$$

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{Pol}\right.$$
$$a^2\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)-$$
$$\frac{\text{arctanh}(ax)}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)$$

input
```
Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

output
```
-1/2*ArcTanh[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 + a^2*(ArcTanh[a*
x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/
2) + a^2*(ArcTanh[a*x]^2/2 + a^2*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/
(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a)) + ArcTanh[a*x]*Log[2 - 2/(1
 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.73

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}-\operatorname{arctanh}(ax)\ln(ax-1)+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\operatorname{arctanh}(ax)\ln(ax+1)-\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)}{4(ax-1)}-\operatorname{arctanh}(ax)\ln(ax-1)+\frac{\operatorname{arctanh}(ax)}{4ax+4}-\operatorname{arctanh}(ax)\ln(ax+1)-\right.$
parts	$-\frac{\operatorname{arctanh}(ax)}{2x^2}+2\operatorname{arctanh}(ax)\,a^2\ln(x)+\frac{a^2\operatorname{arctanh}(ax)}{4ax+4}-\operatorname{arctanh}(ax)\,a^2\ln(ax+1)-$
risch	$-\frac{a^3\ln(-ax+1)x}{16(-ax-1)}-\frac{a^2}{8(-ax+1)}-\frac{a^2\ln(ax+1)^2}{4}+\frac{a^2\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{2}-a^2\operatorname{dilog}(ax+1)-\frac{a^2\ln(ax)}{4}+$

input
```
int(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/4*arctanh(a*x)/(a*x-1)-arctanh(a*x)*ln(a*x-1)+1/4*arctanh(a*x)/(a*
x+1)-arctanh(a*x)*ln(a*x+1)-1/2*arctanh(a*x)/a^2/x^2+2*arctanh(a*x)*ln(a*x
)-dilog(a*x)-dilog(a*x+1)-ln(a*x)*ln(a*x+1)-1/4*ln(a*x-1)^2+dilog(1/2*a*x+
1/2)+1/2*ln(a*x-1)*ln(1/2*a*x+1/2)-1/2*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2
*a*x+1/2)+1/4*ln(a*x+1)^2+1/8/(a*x-1)-1/8*ln(a*x-1)+1/8/(a*x+1)+1/8*ln(a*x
+1)-1/2/a/x)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^2 x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}(ax)}{x^3 (ax-1)^2 (ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(110) = 220$.

Time = 0.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^2}\,dx$$
$$= \frac{1}{8}\left(8\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a - 8(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax))a + 8(\right.$$
$$-\frac{1}{2}\left(2a^2\log\left(a^2x^2-1\right) - 2a^2\log\left(x^2\right) + \frac{2a^2x^2-1}{a^2x^4-x^2}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/8*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 8*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 8*(log(-a*x + 1)*log(x) + dilog(a*x))
*a + a*log(a*x + 1) - a*log(a*x - 1) - 2*(a^2*x^2 - (a^3*x^3 - a*x)*log(a*
x + 1)^2 + 2*(a^3*x^3 - a*x)*log(a*x + 1)*log(a*x - 1) + (a^3*x^3 - a*x)*l
og(a*x - 1)^2 - 2)/(a^2*x^3 - x))*a - 1/2*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*
log(x^2) + (2*a^2*x^2 - 1)/(a^2*x^4 - x^2))*arctanh(a*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^2x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{x^3\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(atanh(a*x)/(x^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)/(x^3*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\left(1-a^2x^2\right)^2}\,dx=\int \frac{atanh(ax)}{a^4x^7-2a^2x^5+x^3}dx$$

input
```
int(atanh(a*x)/x^3/(-a^2*x^2+1)^2,x)
```

output
```
int(atanh(a*x)/(a**4*x**7 - 2*a**2*x**5 + x**3),x)
```

3.266 $\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{1}{4a^4(1-a^2x^2)} - \frac{x\text{arctanh}(ax)}{2a^3(1-a^2x^2)} - \frac{\text{arctanh}(ax)^2}{4a^4} + \frac{\text{arctanh}(ax)^2}{2a^4(1-a^2x^2)} + \frac{\text{arctanh}(ax)^3}{3a^4} - \frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4} + \frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}$$

output
```
1/4/a^4/(-a^2*x^2+1)-1/2*x*arctanh(a*x)/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)^
2/a^4+1/2*arctanh(a*x)^2/a^4/(-a^2*x^2+1)+1/3*arctanh(a*x)^3/a^4-arctanh(a
*x)^2*ln(2/(-a*x+1))/a^4-arctanh(a*x)*polylog(2,1-2/(-a*x+1))/a^4+1/2*poly
log(3,1-2/(-a*x+1))/a^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.64

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$$
$$= \frac{-\frac{1}{3}\text{arctanh}(ax)^3 + \frac{1}{8}(1+2\text{arctanh}(ax)^2)\cosh(2\text{arctanh}(ax)) - \text{arctanh}(ax)^2\log\left(1+e^{-2\text{arctanh}(ax)}\right) + \text{a}}{}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

output
```
(-1/3*ArcTanh[a*x]^3 + ((1 + 2*ArcTanh[a*x]^2)*Cosh[2*ArcTanh[a*x]])/8 - A
rcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]*PolyLog[2, -E^(-
2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])]/2 - (ArcTanh[a*x]*Sinh
[2*ArcTanh[a*x]])/4)/a^4
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6590, 6546, 6470, 6556, 6518, 241, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$$
$$\Big\downarrow \text{6590}$$
$$\frac{\int \frac{x\,\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{a^2} - \frac{\int \frac{x\,\text{arctanh}(ax)^2}{1-a^2x^2}dx}{a^2}$$
$$\Big\downarrow \text{6546}$$
$$\frac{\int \frac{x\,\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{a^2} - \frac{\frac{\int \frac{\text{arctanh}(ax)^2}{1-ax}dx}{a} - \frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$
$$\Big\downarrow \text{6470}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$

$$\Big\downarrow \; 6556$$

$$\frac{\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int\frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx}{a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$

$$\Big\downarrow \; 6518$$

$$\frac{\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int\frac{x}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$

$$\Big\downarrow \; 241$$

$$\frac{\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\int\frac{\text{arctanh}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$

$$\Big\downarrow \; 6620$$

$$\frac{\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$

$$\Big\downarrow \; 7164$$

$$\frac{\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^3}{3a^2}}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

output
```
(ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*Arc
Tanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a)/a^2 - (-1/3*ArcTan
h[a*x]^3/a^2 + ((ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(ArcTanh[a*x
]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a)/
a^2
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.04 (sec) , antiderivative size = 735, normalized size of antiderivative = 4.57

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)^2}{4ax+4}+\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}-\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{\operatorname{arctanh}(}{3}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2}+\frac{\operatorname{arctanh}(ax)^2}{4ax+4}+\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2}-\operatorname{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{\operatorname{arctanh}(}{3}}{}$
parts	$\frac{\operatorname{arctanh}(ax)^2}{4a^4(ax+1)}+\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{2a^4}-\frac{\operatorname{arctanh}(ax)^2}{4a^4(ax-1)}+\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{2a^4}-\frac{a\left(\frac{2\operatorname{arctanh}(ax)^2\ln\left(\frac{a}{\sqrt{-a}}\right)}{a^5}\right.}{}$

input
```
int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*arctanh(a*x)^2/(a*x-1)+1/2*arctanh(a*x)^2*ln(a*x-1)+1/4*arctan
h(a*x)^2/(a*x+1)+1/2*arctanh(a*x)^2*ln(a*x+1)-arctanh(a*x)^2*ln((a*x+1)/(-
a^2*x^2+1)^(1/2))+1/3*arctanh(a*x)^3-1/8*arctanh(a*x)*(a*x-1)/(a*x+1)-1/16
*(a*x-1)/(a*x+1)+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)-1/16*(a*x+1)/(a*x-1)-arc
tanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a*x+1)^2/(-a^
2*x^2+1))-1/4*(I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(a*x+1)/
(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-I*Pi*csgn(I/(-(a*x+1)^
2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/(-(a*x+1)^2/(a^2*x^2-1)+1))-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+2*I*Pi*csgn(I/(-(a*x+1)^2
/(a^2*x^2-1)+1))^3+I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^
2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1)
)^2+2*I*Pi+4*ln(2)+1)*arctanh(a*x)^2)
```

Fricas [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \int \frac{x^3 \, \text{artanh}\,(ax)^2}{(a^2x^2-1)^2} \, dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \int \frac{x^3 \, \text{atanh}^2\,(ax)}{(ax-1)^2\,(ax+1)^2} \, dx$$

input
```
integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x**3*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \int \frac{x^3 \, \text{artanh}\,(ax)^2}{(a^2x^2-1)^2} \, dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-3/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a
^3), x) - 1/4*a^2*integrate(x^2*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^
5*x^2 + a^3), x) - 1/32*(a*(2/(a^7*x - a^6) - log(a*x + 1)/a^6 + log(a*x -
 1)/a^6) + 4*log(-a*x + 1)/(a^7*x^2 - a^5))*a + 1/4*a*integrate(x*log(a*x
+ 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x) + 1/24*((a^2*x^2 - 1)*l
og(-a*x + 1)^3 + 3*((a^2*x^2 - 1)*log(a*x + 1) - 1)*log(-a*x + 1)^2)/(a^6*
x^2 - a^4) + 1/4*integrate(a^3*x^3*log(a*x + 1)^2/(a^7*x^4 - 2*a^5*x^2 + a
^3), x) + 1/4*integrate(log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 +
a^3), x) + 1/4*integrate(log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
```

Giac [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx=\int \frac{x^3\,\text{artanh}\,(ax)^2}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx=\int \frac{x^3\,\text{atanh}(a\,x)^2}{(a^2\,x^2-1)^2}\,dx$$

input
```
int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

output
```
int((x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \int \frac{atanh(ax)^2 x^3}{a^4x^4 - 2a^2x^2 + 1} dx$$

input
```
int(x^3*atanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

output
```
int((atanh(a*x)**2*x**3)/(a**4*x**4 - 2*a**2*x**2 + 1),x)
```

3.267 $\int \frac{x^2\textbf{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{x}{4a^2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{4a^3}-\frac{\text{arctanh}(ax)}{2a^3(1-a^2x^2)}+\frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)^3}{6a^3}$$

output
```
1/4*x/a^2/(-a^2*x^2+1)+1/4*arctanh(a*x)/a^3-1/2*arctanh(a*x)/a^3/(-a^2*x^2
+1)+1/2*x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)-1/6*arctanh(a*x)^3/a^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{12\text{arctanh}(ax)-12ax\text{arctanh}(ax)^2+(4-4a^2x^2)\,\text{arctanh}(ax)^3-3(2ax+(-1+a^2x^2)\log(1-ax)+(}{24a^3(-1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

output
```
(12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + (4 - 4*a^2*x^2)*ArcTanh[a*x]^3
- 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(2
4*a^3*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6562, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx$$

$$\Big\downarrow \text{6562}$$

$$-\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a} - \frac{\text{arctanh}(ax)^3}{6a^3} + \frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}$$

$$\Big\downarrow \text{6556}$$

$$-\frac{\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)} - \frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}}{a} - \frac{\text{arctanh}(ax)^3}{6a^3} + \frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}$$

$$\Big\downarrow \text{215}$$

$$-\frac{\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)} - \frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx + \frac{x}{2(1-a^2x^2)}}{2a}}{a} - \frac{\text{arctanh}(ax)^3}{6a^3} + \frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}(ax)^3}{6a^3} + \frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)} - \frac{\frac{x}{2(1-a^2x^2)} + \frac{\text{arctanh}(ax)}{2a}}{2a}}{a}$$

input
```
Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

output
```
(x*ArcTanh[a*x]^2)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(6*a^3) - (ArcTa
nh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))
/(2*a))/a
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6562
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> Simp[-(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(2*c^2*d*(d + e*x^2))), x] - Simp[b*(
p/(2*c))   Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 22.65 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

method	result
parallelrisch	$-\frac{2\,\mathrm{arctanh}(ax)^3 a^2x^2-3a^2x^2\,\mathrm{arctanh}(ax)+6\,\mathrm{arctanh}(ax)^2 ax-2\,\mathrm{arctanh}(ax)^3+3ax-3\,\mathrm{arctanh}(ax)}{12(a^2x^2-1)a^3}$
risch	$-\frac{\ln(ax+1)^3}{48a^3}+\frac{(x^2\ln(-ax+1)a^2-2ax-\ln(-ax+1))\ln(ax+1)^2}{16a^3(a^2x^2-1)}-\frac{\left(a^2x^2\ln(-ax+1)^2-4ax\ln(-ax+1)-\ln(-ax\right.}{16a^3(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\mathrm{arctanh}(ax)^2}{4(ax-1)}+\frac{\mathrm{arctanh}(ax)^2\ln(ax-1)}{4}-\frac{\mathrm{arctanh}(ax)^2}{4(ax+1)}-\frac{\mathrm{arctanh}(ax)^2\ln(ax+1)}{4}+\frac{\mathrm{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{i\pi\mathrm{csgn}\left(\right.}{}}{}$
default	$\frac{-\frac{\mathrm{arctanh}(ax)^2}{4(ax-1)}+\frac{\mathrm{arctanh}(ax)^2\ln(ax-1)}{4}-\frac{\mathrm{arctanh}(ax)^2}{4(ax+1)}-\frac{\mathrm{arctanh}(ax)^2\ln(ax+1)}{4}+\frac{\mathrm{arctanh}(ax)^2\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{i\pi\mathrm{csgn}\left(\right.}{}}{}$
parts	$-\frac{\mathrm{arctanh}(ax)^2}{4a^3(ax+1)}-\frac{\mathrm{arctanh}(ax)^2\ln(ax+1)}{4a^3}-\frac{\mathrm{arctanh}(ax)^2}{4a^3(ax-1)}+\frac{\mathrm{arctanh}(ax)^2\ln(ax-1)}{4a^3}-\frac{a\left(-\frac{i\pi\mathrm{csgn}\left(\frac{i}{-\frac{(ax+1)^2}{a^2x^2-1}}\right.}{2}\right.}{}$

input
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(2*arctanh(a*x)^3*a^2*x^2-3*a^2*x^2*arctanh(a*x)+6*arctanh(a*x)^2*a*
x-2*arctanh(a*x)^3+3*a*x-3*arctanh(a*x))/(a^2*x^2-1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{x^2\mathrm{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$$
$$= -\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2+(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\,(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{48\,(a^5x^2-a^3)}$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^2
- a^3)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \int \frac{x^2\operatorname{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x**2*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(81) = 162$.

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.90

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\text{artanh}(ax)^2 - \frac{\left((a^2x^2-1)\log(ax+1)^3-3\,(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3+12\,ax\right.}{48\,(a^7x^2-a^5)} + \frac{\left((a^2x^2-1)\log(ax+1)^2-2\,(a^2x^2-1)\log(ax+1)\log(ax-1)+(a^2x^2-1)\log(ax-1)^2+4\right)a\,\text{art}}{8\,(a^6x^2-a^4)}$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a
*x)^2 - 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^
2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*(2*a^2*x^2 - (a
^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 - 1)*log(a*x - 1
))*a^2/(a^7*x^2 - a^5) + 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 -
1)*log(a*x + 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh
(a*x)/(a^6*x^2 - a^4)
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \int \frac{x^2 \,\text{artanh}\,(ax)^2}{(a^2x^2-1)^2} \, dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.46

$$\begin{aligned}\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx &= \frac{\ln(1-a\,x)}{4\,a^3-4\,a^5\,x^2} - \frac{\ln(a\,x+1)^3}{48\,a^3} + \frac{\ln(1-a\,x)^3}{48\,a^3} \\ &\quad + \frac{x}{4\,a^2-4\,a^4\,x^2} - \frac{\ln(a\,x+1)}{4\,(a^3-a^5\,x^2)} + \frac{x\ln(1-a\,x)^2}{8\,a^2-8\,a^4\,x^2} \\ &\quad - \frac{\ln(a\,x+1)\,\ln(1-a\,x)^2}{16\,a^3} + \frac{\ln(a\,x+1)^2\,\ln(1-a\,x)}{16\,a^3} \\ &\quad + \frac{x\ln(a\,x+1)^2}{8\,(a^2-a^4\,x^2)} - \frac{x\,\ln(a\,x+1)\,\ln(1-a\,x)}{4\,a^2-4\,a^4\,x^2} - \frac{\text{atan}(a\,x\,1\text{i})\,1\text{i}}{4\,a^3}\end{aligned}$$

input
```
int((x^2*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

output
```
log(1 - a*x)/(4*a^3 - 4*a^5*x^2) - log(a*x + 1)^3/(48*a^3) + log(1 - a*x)^
3/(48*a^3) + x/(4*a^2 - 4*a^4*x^2) - (atan(a*x*1i)*1i)/(4*a^3) - log(a*x +
 1)/(4*(a^3 - a^5*x^2)) + (x*log(1 - a*x)^2)/(8*a^2 - 8*a^4*x^2) - (log(a*
x + 1)*log(1 - a*x)^2)/(16*a^3) + (log(a*x + 1)^2*log(1 - a*x))/(16*a^3) +
 (x*log(a*x + 1)^2)/(8*(a^2 - a^4*x^2)) - (x*log(a*x + 1)*log(1 - a*x))/(4
*a^2 - 4*a^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$$
$$= \frac{-4atanh(ax)^3\,a^2x^2+4atanh(ax)^3-12atanh(ax)^2\,ax+12atanh(ax)\,a^2x^2+3\log(a^2x-a)\,a^2x^2-3\log}{24a^3\,(a^2x^2-1)}$$

input
```
int(x^2*atanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

output
```
( - 4*atanh(a*x)**3*a**2*x**2 + 4*atanh(a*x)**3 - 12*atanh(a*x)**2*a*x + 1
2*atanh(a*x)*a**2*x**2 + 3*log(a**2*x - a)*a**2*x**2 - 3*log(a**2*x - a) -
 3*log(a**2*x + a)*a**2*x**2 + 3*log(a**2*x + a) - 6*a*x)/(24*a**3*(a**2*x
**2 - 1))
```

3.268 $\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{1}{4a^2(1-a^2x^2)} - \frac{x\text{arctanh}(ax)}{2a(1-a^2x^2)} - \frac{\text{arctanh}(ax)^2}{4a^2} + \frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}$$

output
```
1/4/a^2/(-a^2*x^2+1)-1/2*x*arctanh(a*x)/a/(-a^2*x^2+1)-1/4*arctanh(a*x)^2/
a^2+1/2*arctanh(a*x)^2/a^2/(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.52

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{1-2ax\text{arctanh}(ax)+(1+a^2x^2)\,\text{arctanh}(ax)^2}{4a^2-4a^4x^2}$$

input
```
Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

output
```
(1 - 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a^2 - 4*a^4*x^2
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6556, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{ 6556}$$

$$\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}$$

$$\Big\downarrow \text{ 6518}$$

$$\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}$$

input
```
Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

output
```
ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcT
anh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

method	result
parallelrisch	$-\frac{a^{2} x^{2} \operatorname{arctanh}\left(a x\right)^{2}+a^{2} x^{2}-2 a x \,\operatorname{arctanh}\left(a x\right)+\operatorname{arctanh}\left(a x\right)^{2}}{4 \left(a^{2} x^{2}-1\right) a^{2}}$
risch	$-\frac{\left(a^{2} x^{2}+1\right) \ln \left(a x +1\right)^{2}}{16 a^{2} \left(a x -1\right) \left(a x +1\right)}+\frac{\left(x^{2} \ln \left(-a x +1\right) a^{2}+2 a x +\ln \left(-a x +1\right)\right) \ln \left(a x +1\right)}{8 a^{2} \left(a x -1\right) \left(a x +1\right)}-\frac{a^{2} x^{2} \ln \left(-a x +1\right)^{2}+4 a x \ln \left(-a x +1\right)+\cdots}{16 a^{2} \left(a x -1\right) \left(a x +1\right)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}\left(a x \right)^{2}}{2 \left(a^{2} x^{2}-1\right)}+\frac{\operatorname{arctanh}\left(a x \right)}{4 a x +4}-\frac{\operatorname{arctanh}\left(a x \right) \ln \left(a x +1\right)}{4}+\frac{\operatorname{arctanh}\left(a x \right)}{4 a x -4}+\frac{\operatorname{arctanh}\left(a x \right) \ln \left(a x -1\right)}{4}+\frac{\ln \left(a x +1\right)^{2}}{16}-\frac{\left(\ln \left(a x +1\right)-\ln \left(\frac{a x}{2}+\cdots\right)\cdots\right)}{8}\cdots}{a^{2}}$
default	$\frac{-\frac{\operatorname{arctanh}\left(a x \right)^{2}}{2 \left(a^{2} x^{2}-1\right)}+\frac{\operatorname{arctanh}\left(a x \right)}{4 a x +4}-\frac{\operatorname{arctanh}\left(a x \right) \ln \left(a x +1\right)}{4}+\frac{\operatorname{arctanh}\left(a x \right)}{4 a x -4}+\frac{\operatorname{arctanh}\left(a x \right) \ln \left(a x -1\right)}{4}+\frac{\ln \left(a x +1\right)^{2}}{16}-\frac{\left(\ln \left(a x +1\right)-\ln \left(\frac{a x}{2}+\cdots\right)\cdots\right)}{8}\cdots}{a^{2}}$
parts	$-\frac{\operatorname{arctanh}\left(a x \right)^{2}}{2 a^{2} \left(a^{2} x^{2}-1\right)}+\frac{\frac{\operatorname{arctanh}\left(a x \right)}{4 a x +4}-\frac{\operatorname{arctanh}\left(a x \right) \ln \left(a x +1\right)}{4}+\frac{\operatorname{arctanh}\left(a x \right)}{4 a x -4}+\frac{\operatorname{arctanh}\left(a x \right) \ln \left(a x -1\right)}{4}+\frac{\ln \left(a x +1\right)^{2}}{16}-\frac{\left(\ln \left(a x +1\right)-\ln \left(\cdots\right)\cdots\right)}{\cdots}\cdots}{a^{2}}$
orering	$-\frac{\left(a x -1\right) \left(a x +1\right) \left(10 a^{4} x^{4}+3 a^{2} x^{2}+1\right) \operatorname{arctanh}\left(a x \right)^{2}}{4 a^{4} x^{2} \left(-a^{2} x^{2}+1\right)^{2}}-\frac{\left(5 a^{2} x^{2}+1\right) \left(a x +1\right)^{2} \left(a x -1\right)^{2} \left(\frac{\operatorname{arctanh}\left(a x \right)^{2}}{\left(-a^{2} x^{2}+1\right)^{2}}+\frac{2 x \,\operatorname{arctanh}\left(a x \right) a}{\left(-a^{2} x^{2}+1\right)^{3}}\cdots\right)}{4 a^{4} x^{2}}$

input
```
int(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a^2*x^2*arctanh(a*x)^2+a^2*x^2-2*a*x*arctanh(a*x)+arctanh(a*x)^2)/(a
^2*x^2-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\,(a^4x^2-a^2)}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \int \frac{x\,\text{atanh}^2\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(71) = 142$.

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.78

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\text{artanh}\,(ax)}{4\,a} + \frac{(a^2x^2-1)\log\,(ax+1)^2 - 2\,(a^2x^2-1)\log\,(ax+1)\log\,(ax-1) + (a^2x^2-1)\log\,(ax-1)^2 - 4}{16\,(a^4x^2-a^2)} - \frac{\text{artanh}\,(ax)^2}{2\,(a^2x^2-1)a^2}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)/a +
 1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
 - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^4*x^2 - a^2) - 1/2*arctanh(a*
x)^2/((a^2*x^2 - 1)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.71

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = -\frac{1}{32}\left(\left(\frac{ax+1}{(ax-1)a^3} + \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 2\left(\frac{ax+1}{(ax-1)a^3} - \frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)\right.$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
-1/32*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*x +
 1)/(a*x - 1))^2 - 2*((a*x + 1)/((a*x - 1)*a^3) - (a*x - 1)/((a*x + 1)*a^3
))*log(-(a*x + 1)/(a*x - 1)) + 2*(a*x + 1)/((a*x - 1)*a^3) + 2*(a*x - 1)/(
(a*x + 1)*a^3))*a
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.41

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \ln(1-a\,x)\,\left(\frac{\frac{x}{2}-\frac{1}{2\,a}}{4\,a-4\,a^3\,x^2}+\frac{\frac{x}{2}+\frac{1}{2\,a}}{4\,a-4\,a^3\,x^2}+\ln(a\,x+1)\,\left(\frac{1}{8\,a^2}+\frac{1}{2\,a^2\,(2\,a^2\,x^2-2)}\right)\right) - \ln(1-a\,x)^2\left(\frac{1}{16\,a^2}+\frac{1}{2\,a^2\,(4\,a^2\,x^2-4)}\right)-\frac{1}{2\,a^2\,(2\,a^2\,x^2-2)} - \ln(a\,x+1)^2\left(\frac{1}{8\,a^3\,\left(a\,x^2-\frac{1}{a}\right)}+\frac{1}{16\,a^2}\right)+\frac{x\,\ln(a\,x+1)}{4\,a^2\,\left(a\,x^2-\frac{1}{a}\right)}$$

input
```
int((x*atanh(a*x)^2)/(a^2*x^2 - 1)^2,x)
```

output
```
log(1 - a*x)*((x/2 - 1/(2*a))/(4*a - 4*a^3*x^2) + (x/2 + 1/(2*a))/(4*a - 4
*a^3*x^2) + log(a*x + 1)*(1/(8*a^2) + 1/(2*a^2*(2*a^2*x^2 - 2)))) - log(1
- a*x)^2*(1/(16*a^2) + 1/(2*a^2*(4*a^2*x^2 - 4))) - 1/(2*a^2*(2*a^2*x^2 -
2)) - log(a*x + 1)^2*(1/(8*a^3*(a*x^2 - 1/a)) + 1/(16*a^2)) + (x*log(a*x +
 1))/(4*a^2*(a*x^2 - 1/a))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{-atanh(ax)^2\,a^2x^2-atanh(ax)^2+2atanh(ax)\,ax-a^2x^2}{4a^2\,(a^2x^2-1)}$$

input
```
int(x*atanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

output
```
( - atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2 + 2*atanh(a*x)*a*x - a**2*x**2
)/(4*a**2*(a**2*x**2 - 1))
```

3.269 $\int \frac{\mathbf{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \frac{x}{4(1-a^2x^2)} + \frac{\operatorname{arctanh}(ax)}{4a} - \frac{\operatorname{arctanh}(ax)}{2a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)} + \frac{\operatorname{arctanh}(ax)^3}{6a}$$

output
```
x/(-4*a^2*x^2+4)+1/4*arctanh(a*x)/a-1/2*arctanh(a*x)/a/(-a^2*x^2+1)+x*arct
anh(a*x)^2/(-2*a^2*x^2+2)+1/6*arctanh(a*x)^3/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2} \, dx = \frac{12\operatorname{arctanh}(ax) - 12ax\operatorname{arctanh}(ax)^2 + 4(-1+a^2x^2)\operatorname{arctanh}(ax)^3 - 3(2ax + (-1+a^2x^2)\log(1-ax) +}{24a(-1+a^2x^2)}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

output
```
(12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^3
 - 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(
24*a*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6518, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx$$

$$\Big\downarrow \text{ 6518}$$

$$-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}$$

$$\Big\downarrow \text{ 6556}$$

$$-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}$$

$$\Big\downarrow \text{ 215}$$

$$-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

output
```
(x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a
*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*
a))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 23.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

method	result
parallelrisch	$-\frac{-2\operatorname{arctanh}(ax)^{3}a^{2}x^{2}-3a^{2}x^{2}\operatorname{arctanh}(ax)+6\operatorname{arctanh}(ax)^{2}ax+2\operatorname{arctanh}(ax)^{3}+3ax-3\operatorname{arctanh}(ax)}{12(a^{2}x^{2}-1)a}$
risch	$\frac{\ln(ax+1)^{3}}{48a}-\frac{(x^{2}\ln(-ax+1)a^{2}+2ax-\ln(-ax+1))\ln(ax+1)^{2}}{16(a^{2}x^{2}-1)a}+\frac{\left(a^{2}x^{2}\ln(-ax+1)^{2}+4ax\ln(-ax+1)-\ln(-ax+1)\right.}{16a(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^{2}}{4(ax+1)}+\frac{\operatorname{arctanh}(ax)^{2}\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)^{2}}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)^{2}\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)^{2}\ln\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}+\frac{i\pi\operatorname{arctanh}(}{4}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^{2}}{4(ax+1)}+\frac{\operatorname{arctanh}(ax)^{2}\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)^{2}}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)^{2}\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)^{2}\ln\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}+\frac{i\pi\operatorname{arctanh}(}{4}}{}$
parts	$-\frac{\operatorname{arctanh}(ax)^{2}}{4(ax+1)a}+\frac{\operatorname{arctanh}(ax)^{2}\ln(ax+1)}{4a}-\frac{\operatorname{arctanh}(ax)^{2}}{4a(ax-1)}-\frac{\operatorname{arctanh}(ax)^{2}\ln(ax-1)}{4a}-\frac{a\left(\frac{i\pi\operatorname{csgn}\left(\frac{i(a}{(a^{2}x^{2}-1)(}\right.}{}\right.}{}$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-2*arctanh(a*x)^3*a^2*x^2-3*a^2*x^2*arctanh(a*x)+6*arctanh(a*x)^2*a
*x+2*arctanh(a*x)^3+3*a*x-3*arctanh(a*x))/(a^2*x^2-1)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = -\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\,(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{48\,(a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^3*x^2
- a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.05

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2 + \frac{((a^2x^2-1)\log(ax+1)^3 - 3(a^2x^2-1)\log(ax+1)^2\log(ax-1) - (a^2x^2-1)\log(ax-1)^3 - 12ax}{48(a^5x^2-a^3)} - \frac{((a^2x^2-1)\log(ax+1)^2 - 2(a^2x^2-1)\log(ax+1)\log(ax-1) + (a^2x^2-1)\log(ax-1)^2 - 4)a\operatorname{art}}{8(a^4x^2-a^2)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2
+ 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(
a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2
 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2
/(a^5*x^2 - a^3) - 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log
(a*x + 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/
(a^4*x^2 - a^2)
```

Giac [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{1}{16}a^2\left(\frac{(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax+1)a^4} + \frac{2(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)}{(ax+1)a^4} + \frac{2(ax-1)}{(ax+1)a^4}\right)$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
1/16*a^2*((a*x - 1)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)*a^4) + 2*(a*x -
 1)*log(-(a*x + 1)/(a*x - 1))/((a*x + 1)*a^4) + 2*(a*x - 1)/((a*x + 1)*a^4
))
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx &= \frac{\ln(a\,x+1)^3}{48\,a} - \frac{\ln(a\,x+1)}{4\,(a-a^3\,x^2)} - \frac{\ln(1-a\,x)^3}{48\,a} \\ &\quad - \frac{x}{4\,a^2\,x^2-4} + \frac{\ln(1-a\,x)}{4\,a-4\,a^3\,x^2} + \frac{\ln(a\,x+1)\,\ln(1-a\,x)^2}{16\,a} \\ &\quad - \frac{\ln(a\,x+1)^2\,\ln(1-a\,x)}{16\,a} - \frac{x\ln(a\,x+1)^2}{8\,(a^2\,x^2-1)} - \frac{x\ln(1-a\,x)^2}{2\,(4\,a^2\,x^2-4)} \\ &\quad + \frac{x\,\ln(a\,x+1)\,\ln(1-a\,x)}{4\,a^2\,x^2-4} - \frac{\operatorname{atan}(a\,x\,1\mathrm{i})\,1\mathrm{i}}{4\,a}\end{aligned}$$

input
```
int(atanh(a*x)^2/(a^2*x^2 - 1)^2,x)
```

output
```
log(a*x + 1)^3/(48*a) - log(a*x + 1)/(4*(a - a^3*x^2)) - log(1 - a*x)^3/(4
8*a) - x/(4*a^2*x^2 - 4) - (atan(a*x*1i)*1i)/(4*a) + log(1 - a*x)/(4*a - 4
*a^3*x^2) + (log(a*x + 1)*log(1 - a*x)^2)/(16*a) - (log(a*x + 1)^2*log(1 -
 a*x))/(16*a) - (x*log(a*x + 1)^2)/(8*(a^2*x^2 - 1)) - (x*log(1 - a*x)^2)/
(2*(4*a^2*x^2 - 4)) + (x*log(a*x + 1)*log(1 - a*x))/(4*a^2*x^2 - 4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}\,dx = \frac{4atanh(ax)^3\,a^2x^2-4atanh(ax)^3-12atanh(ax)^2\,ax+12atanh(ax)\,a^2x^2+3\log(a^2x-a)\,a^2x^2-3\log(a}{24a\,(a^2x^2-1)}$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

output
```
(4*atanh(a*x)**3*a**2*x**2 - 4*atanh(a*x)**3 - 12*atanh(a*x)**2*a*x + 12*a
tanh(a*x)*a**2*x**2 + 3*log(a**2*x - a)*a**2*x**2 - 3*log(a**2*x - a) - 3*
log(a**2*x + a)*a**2*x**2 + 3*log(a**2*x + a) - 6*a*x)/(24*a*(a**2*x**2 -
1))
```

3.270 $\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^2} dx = \frac{1}{4(1-a^2x^2)} - \frac{ax\operatorname{arctanh}(ax)}{2(1-a^2x^2)} - \frac{1}{4}\operatorname{arctanh}(ax)^2 + \frac{\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)} + \frac{1}{3}\operatorname{arctanh}(ax)^3 + \operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)$$

output

```
1/(-4*a^2*x^2+4)-a*x*arctanh(a*x)/(-2*a^2*x^2+2)-1/4*arctanh(a*x)^2+arctan
h(a*x)^2/(-2*a^2*x^2+2)+1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2-2/(a*x+1))-
arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*polylog(3,-1+2/(a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx = \frac{1}{24}\left(i\pi^3 - 8\text{arctanh}(ax)^3 + 3\cosh(2\text{arctanh}(ax)) + 6\text{arctanh}(ax)^2\cosh(2\text{arctanh}(ax)) + 24\text{arctanh}(ax)^2\log\left(1-e^{2\text{arctanh}(ax)}\right) + 24\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(ax)}\right) - 12\,\text{PolyLog}\left(3,e^{2\text{arctanh}(ax)}\right) - 6\text{arctanh}(ax)\sinh(2\text{arctanh}(ax))\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

output
```
(I*Pi^3 - 8*ArcTanh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*Cos
h[2*ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 24*Arc
Tanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - 12*PolyLog[3, E^(2*ArcTanh[a*x]
)] - 6*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/24
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6592, 6550, 6494, 6556, 6518, 241, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx + \int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx$$

$$\Big\downarrow \text{ 6550}$$

$$a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3$$

$$\Big\downarrow \text{ 6494}$$

$$a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6518}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 241}$$

$$-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6618}$$

$$-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2\,(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{7164}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2\,(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

input
```
Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

output
```
ArcTanh[a*x]^3/3 + a^2*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*
(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)
)/a) + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2,
 -1 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 42.65 (sec) , antiderivative size = 1203, normalized size of antiderivative = 8.85

method	result	size
derivativedivides	Expression too large to display	1203
default	Expression too large to display	1203
parts	Expression too large to display	1602

input
```
int(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctanh(a*x)^2/(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x-1)+1/4*arctanh(a*x)^
2/(a*x+1)-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln(a*x)+arctanh(a*x)
^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3-1/8*arctanh(a*x)*(a*x
-1)/(a*x+1)-1/16*(a*x-1)/(a*x+1)+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)-1/16*(a*
x+1)/(a*x-1)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln
(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+
1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x
+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*(-2*I*Pi*csgn(I/(-(a*x+1)
^2/(a^2*x^2-1)+1))^2-2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a
*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi-2*I*Pi*csgn(I/
(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/
(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)
^2/(a^2*x^2-1))-I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+I*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*
csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*
x+1)^2/(a^2*x^2-1)+1))+2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+2*I*Pi*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+I*Pi*cs
gn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{x(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**2/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/4*a^4*integrate(x^4*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x)
, x) + 1/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x
^3 + x), x) - 1/32*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/a
^3) + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^2 - 1/4*a^2*integrate(x^2*log(a*x
 + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/4*a*integrate(x*log(
a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) + 1/4*a*integrate(x*l
og(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/24*((a^2*x^2 - 1)*log(-a*x
+ 1)^3 + 3*((a^2*x^2 - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^2*x^2 - 1)
 + 1/4*integrate(log(a*x + 1)^2/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/2*integr
ate(log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx = \int \frac{\text{artanh}\,(ax)^2}{(a^2x^2-1)^2x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^2}\,dx = \int \frac{\text{atanh}(a\,x)^2}{x\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx = \int \frac{atanh(ax)^2}{a^4x^5-2a^2x^3+x}dx$$

input
```
int(atanh(a*x)^2/x/(-a^2*x^2+1)^2,x)
```

output
```
int(atanh(a**x)**2/(a**4*x**5 - 2*a**2*x**3 + x),x)
```

3.271 $\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 142

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}\,dx = {} & \frac{a^2x}{4(1-a^2x^2)}+\frac{1}{4}a\operatorname{arctanh}(ax)-\frac{a\operatorname{arctanh}(ax)}{2(1-a^2x^2)} \\ & +a\operatorname{arctanh}(ax)^2-\frac{\operatorname{arctanh}(ax)^2}{x}+\frac{a^2x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)} \\ & +\frac{1}{2}a\operatorname{arctanh}(ax)^3+2a\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) \\ & -a\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
a^2*x/(-4*a^2*x^2+4)+1/4*a*arctanh(a*x)-a*arctanh(a*x)/(-2*a^2*x^2+2)+a*ar
ctanh(a*x)^2-arctanh(a*x)^2/x+a^2*x*arctanh(a*x)^2/(-2*a^2*x^2+2)+1/2*a*ar
ctanh(a*x)^3+2*a*arctanh(a*x)*ln(2-2/(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)^2} dx$$
$$= \frac{4ax\text{arctanh}(ax)^3 - 2ax\text{arctanh}(ax) \left(\cosh(2\text{arctanh}(ax)) - 8\log\left(1-e^{-2\text{arctanh}(ax)}\right)\right) - 8ax \,\text{PolyLog}\,(2}{8x}$$

input
```
Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

output
```
(4*a*x*ArcTanh[a*x]^3 - 2*a*x*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 8*Log[1
 - E^(-2*ArcTanh[a*x])]) - 8*a*x*PolyLog[2, E^(-2*ArcTanh[a*x])] + a*x*Sin
h[2*ArcTanh[a*x]] + 2*ArcTanh[a*x]^2*(-4 + 4*a*x + a*x*Sinh[2*ArcTanh[a*x]
]))/(8*x)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6592, 6518, 6544, 6452, 6510, 6550, 6494, 2897, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)^2} dx$$
$$\Big\downarrow \text{ 6592}$$
$$a^2 \int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2} dx + \int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)} dx$$
$$\Big\downarrow \text{ 6518}$$
$$a^2 \left(-a \int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2} dx + \frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)} + \frac{\text{arctanh}(ax)^3}{6a}\right) + \int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)} dx$$
$$\Big\downarrow \text{ 6544}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{ 6452}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+2a\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{ 6510}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+2a\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{ 6550}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+2a\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{ 6494}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{ 2897}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{ 6556}$$

$$a^2\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+ 2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+ \frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{215}$$

$$a^2\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+ 2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+ \frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{219}$$

$$a^2\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)+ 2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+ \frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

input
```
Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

output
```
-(ArcTanh[a*x]^2/x) + (a*ArcTanh[a*x]^3)/3 + a^2*((x*ArcTanh[a*x]^2)/(2*(1
 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)
) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))) + 2*a*(ArcTanh[a*x]
^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 20.66 (sec) , antiderivative size = 3005, normalized size of antiderivative = 21.16

method	result	size
default	Expression too large to display	3005
parts	Expression too large to display	3015
derivativedivides	Expression too large to display	3050

input
```
int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(3/4*arctanh(a*x)^2*ln(a*x+1)-3/2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)
^(1/2))-arctanh(a*x)^2/a/x+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)+1/8*arctanh(a*
x)*(a*x-1)/(a*x+1)-3/4*arctanh(a*x)^2*ln(a*x-1)+polylog(2,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*arctanh(a*x)^3-arct
anh(a*x)^2+3/8*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*(arcta
nh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))
+dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2)))+3/4*I*Pi*arctanh(a*x)^2+1/16*(a*x-1)
/(a*x+1)-1/16*(a*x+1)/(a*x-1)+3/8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*(arctanh(a*x)*ln(1+(
a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+dilog(1+(a*x+
1)/(-a^2*x^2+1)^(1/2)))-3/8*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-
1)+1))*(-arctanh(a*x)^2+arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arct
anh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,-(a*x+1)/(-a^2*x^2+1)^
(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2)))+3/8*I*Pi*csgn(I/(-(a*x+1)^2/
(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2
*(-arctanh(a*x)^2+arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*
x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2)))-3/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{x^2(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**2/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(128) = 256.

Time = 0.04 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}\,dx$$
$$= \frac{1}{16}a^2\left(\frac{(a^2x^2-1)\log(ax+1)^3-(a^2x^2-1)\log(ax-1)^3+(4a^2x^2-3(a^2x^2-1)\log(ax-1)-4)\log\right.}{}$$
$$-\frac{1}{8}a\left(\frac{3(a^2x^2-1)\log(ax+1)^2-6(a^2x^2-1)\log(ax+1)\log(ax-1)+3(a^2x^2-1)\log(ax-1)^2-}{a^2x^2-1}\right.$$
$$+\frac{1}{4}\left(3a\log(ax+1)-3a\log(ax-1)-\frac{2(3a^2x^2-2)}{a^2x^3-x}\right)\operatorname{artanh}(ax)^2$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/16*a^2*(((a^2*x^2 - 1)*log(a*x + 1)^3 - (a^2*x^2 - 1)*log(a*x - 1)^3 + (
4*a^2*x^2 - 3*(a^2*x^2 - 1)*log(a*x - 1) - 4)*log(a*x + 1)^2 - 4*(a^2*x^2
- 1)*log(a*x - 1)^2 - 4*a*x + (3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 8*(a^2*x^2
 - 1)*log(a*x - 1))*log(a*x + 1))/(a^3*x^2 - a) + 16*(log(a*x - 1)*log(1/2
*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 16*(log(a*x + 1)*log(x) + dilog(-
a*x))/a + 16*(log(-a*x + 1)*log(x) + dilog(a*x))/a + 2*log(a*x + 1)/a - 2*
log(a*x - 1)/a) - 1/8*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)
*log(a*x + 1)*log(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2
- 1) + 8*log(a*x + 1) + 8*log(a*x - 1) - 16*log(x))*arctanh(a*x) + 1/4*(3*
a*log(a*x + 1) - 3*a*log(a*x - 1) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arcta
nh(a*x)^2
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)^2x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx=\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,\left(a^2\,x^2-1\right)^2}\,dx$$

input
```
int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^2}\,dx$$
$$= \frac{4atanh(ax)^3 a^3x^3 - 4atanh(ax)^3 ax - 12atanh(ax)^2 a^2x^2 + 8atanh(ax)^2 + 12atanh(ax)\,a^3x^3 + 16\left(\int \frac{}{a^4 \cdots}\right.}{}$$

input
```
int(atanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x)
```

output
```
(4*atanh(a*x)**3*a**3*x**3 - 4*atanh(a*x)**3*a*x - 12*atanh(a*x)**2*a**2*x
**2 + 8*atanh(a*x)**2 + 12*atanh(a*x)*a**3*x**3 + 16*int(atanh(a*x)/(a**4*
x**5 - 2*a**2*x**3 + x),x)*a**3*x**3 - 16*int(atanh(a*x)/(a**4*x**5 - 2*a*
*2*x**3 + x),x)*a*x + 3*log(a**2*x - a)*a**3*x**3 - 3*log(a**2*x - a)*a*x
- 3*log(a**2*x + a)*a**3*x**3 + 3*log(a**2*x + a)*a*x - 6*a**2*x**2)/(8*x*
(a**2*x**2 - 1))
```

3.272 $\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)^2}\,dx &= \frac{a^2}{4(1-a^2x^2)} - \frac{a\operatorname{arctanh}(ax)}{x} - \frac{a^3x\operatorname{arctanh}(ax)}{2(1-a^2x^2)} \\ &\quad + \frac{1}{4}a^2\operatorname{arctanh}(ax)^2 - \frac{\operatorname{arctanh}(ax)^2}{2x^2} + \frac{a^2\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)} \\ &\quad + \frac{2}{3}a^2\operatorname{arctanh}(ax)^3 + a^2\log(x) - \frac{1}{2}a^2\log\left(1-a^2x^2\right) \\ &\quad + 2a^2\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right) \\ &\quad - 2a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ &\quad - a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
a^2/(-4*a^2*x^2+4)-a*arctanh(a*x)/x-a^3*x*arctanh(a*x)/(-2*a^2*x^2+2)+1/4*
a^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2+a^2*arctanh(a*x)^2/(-2*a^2*x^2+2
)+2/3*a^2*arctanh(a*x)^3+a^2*ln(x)-1/2*a^2*ln(-a^2*x^2+1)+2*a^2*arctanh(a*
x)^2*ln(2-2/(a*x+1))-2*a^2*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-a^2*polylo
g(3,-1+2/(a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)^2}\,dx = a^2\left(2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) + \frac{1}{24}\left(2i\pi^3-16\operatorname{arctanh}(ax)^3+3\cosh(2\operatorname{arctanh}(ax)) + 6\operatorname{arctanh}(ax)^2\left(2-\frac{2}{a^2x^2}+\cosh(2\operatorname{arctanh}(ax))+8\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)\right) + 24\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right)-24\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right) - \frac{6\operatorname{arctanh}(ax)(4+ax\sinh(2\operatorname{arctanh}(ax)))}{ax}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

output
```
a^2*(2*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((2*I)*Pi^3 - 16*ArcT
anh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*(2 - 2/(a^2*x^2) +
Cosh[2*ArcTanh[a*x]] + 8*Log[1 - E^(2*ArcTanh[a*x])]) + 24*Log[(a*x)/Sqrt[
1 - a^2*x^2]] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] - (6*ArcTanh[a*x]*(4 + a
*x*Sinh[2*ArcTanh[a*x]]))/(a*x))/24)
```

Rubi [A] (verified)

Time = 3.69 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.53, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {6592, 6544, 6452, 6544, 6452, 243, 47, 14, 16, 6510, 6550, 6494, 6592, 6550, 6494, 6556, 6518, 241, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^2}\,dx$$

$\Big\downarrow$ 6592

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + \int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)}dx$$

$\Big\downarrow$ 6544

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)}dx + \int \frac{\operatorname{arctanh}(ax)^2}{x^3}dx$$

$\Big\downarrow$ 6452

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)}dx + a \int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)}dx - \frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6544

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)}dx + a\left(a^2 \int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx + \int \frac{\operatorname{arctanh}(ax)}{x^2}dx\right) - \frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6452

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)}dx + a\left(a^2 \int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx + a \int \frac{1}{x (1-a^2x^2)}dx - \frac{\operatorname{arctanh}(ax)}{x}\right) - \frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 243

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)}dx + a\left(a^2 \int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx + \frac{1}{2}a \int \frac{1}{x^2 (1-a^2x^2)}dx^2 - \frac{\operatorname{arctanh}(ax)}{x}\right) - \frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 47

$$a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)^2}dx + a^2 \int \frac{\operatorname{arctanh}(ax)^2}{x (1-a^2x^2)}dx + a\left(a^2 \int \frac{\operatorname{arctanh}(ax)}{1-a^2x^2}dx + \frac{1}{2}a\left(a^2 \int \frac{1}{1-a^2x^2}dx^2 + \int \frac{1}{x^2}dx^2\right) - \frac{\operatorname{arctanh}(ax)}{x}\right) - \frac{\operatorname{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 14

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 16

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6510

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6550

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^2}dx+a^2\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6494

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^2}dx+a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6592

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)}dx\right)+a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+ a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+ a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+ a^2\left(a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{}{a}\right.\right. a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+ a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctan}\right. a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6518}$$

$$a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+ a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+}\right.}{1-a^2x^2}\right. a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\big\downarrow$ 241

$$a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\big\downarrow$ 6618

$$a^2\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(\right.$$
$$a^2\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arc}}{2(1}}{}\right.\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

$\big\downarrow$ 7164

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right.}{2a}\right.\right.$$
$$a^2\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\right.$$
$$a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}\right)-\frac{\text{arctanh}(ax)^2}{2x^2}$$

input
```
Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

output
```
-1/2*ArcTanh[a*x]^2/x^2 + a*(-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + (a
*(Log[x^2] - Log[1 - a^2*x^2]))/2) + a^2*(ArcTanh[a*x]^3/3 + ArcTanh[a*x]^
2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/
(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a))) + a^2*(ArcTanh[a*x]^3/3 + a^2
*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*Ar
cTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a) + ArcTanh[a*x]^2*
Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/(2
*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 43.70 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.90

method	result	size
derivativedivides	Expression too large to display	1415
default	Expression too large to display	1415
parts	Expression too large to display	1839

input
```
int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1
)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*arctan
h(a*x)^2-arctanh(a*x)^2*ln(a*x+1)+2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*arctanh(a*x)^2/a^2/x^2+1/8*arctanh(a*x)*(a*x+1)/(a*x-1)-1/8*ar
ctanh(a*x)*(a*x-1)/(a*x+1)+I*Pi*arctanh(a*x)^2+ln(1+(a*x+1)/(-a^2*x^2+1)^(
1/2))-arctanh(a*x)^2*ln(a*x-1)+I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a
*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a*x)^2-4*polylog(3,-(a*x+1)/(-a^2*x^2+1)
^(1/2))-4*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+I*Pi*csgn(I/(-(a*x+1)^2/(a
^2*x^2-1)+1))^3*arctanh(a*x)^2-2/3*arctanh(a*x)^3+1/4*arctanh(a*x)^2+4*arc
tanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*arctanh(a*x)*polylog(2,
(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)^2*ln(2)-1/16*(a*x-1)/(a*x+1)-1/
16*(a*x+1)/(a*x-1)-2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+2*arctanh
(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a
^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^2/(a*x-1)+1/4*arctanh(a*x)^2/(a*x+1)+1/2
*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*arctanh(a
*x)^2+1/2*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2-I*Pi*csgn(I/
(-(a*x+1)^2/(a^2*x^2-1)+1))^2*arctanh(a*x)^2+ln((a*x+1)/(-a^2*x^2+1)^(1/2)
-1)+2*arctanh(a*x)^2*ln(a*x)+1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+1/2*I*Pi*csgn(I/(-(a*x+1)^2/
(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1)...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\left(1-a^2x^2\right)^2}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)^2x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^2} dx = \int \frac{\operatorname{atanh}^2(ax)}{x^3 (ax-1)^2 (ax+1)^2} dx$$

input
```
integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**2/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^2} dx = \int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2 x^3} dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*a^6*integrate(x^6*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^
3), x) + 1/2*a^5*integrate(x^5*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2
*x^5 + x^3), x) - 1/16*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x -
1)/a^3) + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^4 - 1/2*a^4*integrate(x^4*log
(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/2*a^3*integrat
e(x^3*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) + 1/2*a^3
*integrate(x^3*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a^2*int
egrate(x^2*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a*integrate
(x*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/24*(2*(a^4*x^4 - a^2*
x^2)*log(-a*x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1) -
 1)*log(-a*x + 1)^2)/(a^2*x^4 - x^2) + 1/4*integrate(log(a*x + 1)^2/(a^4*x
^7 - 2*a^2*x^5 + x^3), x) - 1/2*integrate(log(a*x + 1)*log(-a*x + 1)/(a^4*
x^7 - 2*a^2*x^5 + x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^2} \, dx = \int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2 x^3} \, dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^2} \, dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)^2/(x^3*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^2} \, dx = \int \frac{atanh(ax)^2}{a^4x^7-2a^2x^5+x^3} dx$$

input
```
int(atanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x)
```

output
```
int(atanh(a*x)**2/(a**4*x**7 - 2*a**2*x**5 + x**3),x)
```

3.273 $\int \frac{x^3 \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 227

$$\begin{aligned}\int \frac{x^3 \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2} dx &= -\frac{3x}{8a^3(1-a^2x^2)} - \frac{3\operatorname{arctanh}(ax)}{8a^4} + \frac{3\operatorname{arctanh}(ax)}{4a^4(1-a^2x^2)} \\ &\quad - \frac{3x\operatorname{arctanh}(ax)^2}{4a^3(1-a^2x^2)} - \frac{\operatorname{arctanh}(ax)^3}{4a^4} + \frac{\operatorname{arctanh}(ax)^3}{2a^4(1-a^2x^2)} \\ &\quad + \frac{\operatorname{arctanh}(ax)^4}{4a^4} - \frac{\operatorname{arctanh}(ax)^3 \log\left(\frac{2}{1-ax}\right)}{a^4} \\ &\quad - \frac{3\operatorname{arctanh}(ax)^2 \operatorname{PolyLog}\left(2, 1-\frac{2}{1-ax}\right)}{2a^4} \\ &\quad + \frac{3\operatorname{arctanh}(ax) \operatorname{PolyLog}\left(3, 1-\frac{2}{1-ax}\right)}{2a^4} \\ &\quad - \frac{3\operatorname{PolyLog}\left(4, 1-\frac{2}{1-ax}\right)}{4a^4}\end{aligned}$$

output
```
-3/8*x/a^3/(-a^2*x^2+1)-3/8*arctanh(a*x)/a^4+3/4*arctanh(a*x)/a^4/(-a^2*x^
2+1)-3/4*x*arctanh(a*x)^2/a^3/(-a^2*x^2+1)-1/4*arctanh(a*x)^3/a^4+1/2*arct
anh(a*x)^3/a^4/(-a^2*x^2+1)+1/4*arctanh(a*x)^4/a^4-arctanh(a*x)^3*ln(2/(-a
*x+1))/a^4-3/2*arctanh(a*x)^2*polylog(2,1-2/(-a*x+1))/a^4+3/2*arctanh(a*x)
*polylog(3,1-2/(-a*x+1))/a^4-3/4*polylog(4,1-2/(-a*x+1))/a^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^2} \, dx$$
$$= \frac{-4\text{arctanh}(ax)^4 + 6\text{arctanh}(ax)\cosh(2\text{arctanh}(ax)) + 4\text{arctanh}(ax)^3\cosh(2\text{arctanh}(ax)) - 16\text{arctanh}}{}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

output
```
(-4*ArcTanh[a*x]^4 + 6*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^
3*Cosh[2*ArcTanh[a*x]] - 16*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] +
24*ArcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 24*ArcTanh[a*x]*PolyL
og[3, -E^(-2*ArcTanh[a*x])] + 12*PolyLog[4, -E^(-2*ArcTanh[a*x])] - 3*Sinh
[2*ArcTanh[a*x]] - 6*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/(16*a^4)
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6590, 6546, 6470, 6556, 6518, 6556, 215, 219, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^2} \, dx$$
$$\Big\downarrow \text{6590}$$
$$\frac{\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx}{a^2} - \frac{\int \frac{x\text{arctanh}(ax)^3}{1-a^2x^2}dx}{a^2}$$
$$\Big\downarrow \text{6546}$$
$$\frac{\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx}{a^2} - \frac{\frac{\int \frac{\text{arctanh}(ax)^3}{1-ax}dx}{a} - \frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$$\Big\downarrow \; 6470$$

$$\frac{\int \frac{x\,\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$$\Big\downarrow \; 6556$$

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$$\Big\downarrow \; 6518$$

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\int\frac{x\,\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\,\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$$\Big\downarrow \; 6556$$

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\,\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\,\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\int\frac{\text{arctanh}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$\Big\downarrow$ 6620

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(\int\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\frac{\text{arctanh}(ax)^4}{4a^2}}{a^2}$$

$\Big\downarrow$ 6624

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}\right)}{a}-\text{arc}}{a^2}$$

$\Big\downarrow$ 7164

$$\frac{\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}}{a^2}-\frac{\frac{\frac{\text{arctanh}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a}-3\left(-\frac{\text{arctanh}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\text{arctanh}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a}\right)}{a}-\frac{\text{arctanh(}}{4a^2}}{a^2}$$

input

```
Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

output
```
(ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2)) - (3*((x*ArcTanh[a*x]^2)/(2*(1 - a^2
*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x
/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/(2*a))/a^2 - (-1/4*ArcTa
nh[a*x]^4/a^2 + ((ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a - 3*(-1/2*(ArcTanh[a*
x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/a + (ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 -
a*x)])/(2*a) - PolyLog[4, 1 - 2/(1 - a*x)]/(4*a)))/a)/a^2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.72 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.55

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^3}{4ax+4}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)^3}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{\operatorname{arctanh}(ax}{4}}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^3}{4ax+4}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2}-\frac{\operatorname{arctanh}(ax)^3}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2}-\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{\operatorname{arctanh}(ax}{4}}{}$
parts	$\frac{\operatorname{arctanh}(ax)^3}{4a^4(ax+1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{2a^4}-\frac{\operatorname{arctanh}(ax)^3}{4a^4(ax-1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{2a^4}-\frac{3a\left(\frac{4\operatorname{arctanh}(ax)^3\ln\left(\frac{}{\sqrt{-}}\right.}{3a^5}\right.}{}$

input
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*arctanh(a*x)^3/(a*x+1)+1/2*arctanh(a*x)^3*ln(a*x+1)-1/4*arctanh
(a*x)^3/(a*x-1)+1/2*arctanh(a*x)^3*ln(a*x-1)-arctanh(a*x)^3*ln((a*x+1)/(-a
^2*x^2+1)^(1/2))+1/4*arctanh(a*x)^4-3/16*arctanh(a*x)^2*(a*x-1)/(a*x+1)-3/
16*arctanh(a*x)*(a*x-1)/(a*x+1)-3/32*(a*x-1)/(a*x+1)+3/16*(a*x+1)*arctanh(
a*x)^2/(a*x-1)-3/16*arctanh(a*x)*(a*x+1)/(a*x-1)+3/32*(a*x+1)/(a*x-1)-3/2*
arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2*arctanh(a*x)*polylog
(3,-(a*x+1)^2/(-a^2*x^2+1))-3/4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-1/4*(2*
I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2
-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2-I*Pi*cs
gn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^3-I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1))^3+2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))^2+I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1
)^2/(a^2*x^2-1))-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+2*I*Pi+4*ln(2
)+1)*arctanh(a*x)^3)
```

Fricas [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \int \frac{x^3\,\text{artanh}\,(ax)^3}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \int \frac{x^3\,\text{atanh}^3\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x**3*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \int \frac{x^3\,\text{artanh}\,(ax)^3}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) - 1)*
log(-a*x + 1)^3)/(a^6*x^2 - a^4) + 1/8*integrate(1/2*(2*a^3*x^3*log(a*x +
1)^3 - 6*a^3*x^3*log(a*x + 1)^2*log(-a*x + 1) - 3*(a*x - (3*a^3*x^3 + a^2*
x^2 - a*x - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^7*x^4 - 2*a^5*x^2 + a
^3), x)
```

Giac [F]

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^2} \, dx = \int \frac{x^3 \operatorname{artanh}(ax)^3}{(a^2x^2-1)^2} \, dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^2} \, dx = \int \frac{x^3 \operatorname{atanh}(a\,x)^3}{(a^2\,x^2-1)^2} \, dx$$

input
```
int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

output
```
int((x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^2} \, dx = \int \frac{atanh(ax)^3 x^3}{a^4x^4-2a^2x^2+1} dx$$

input
```
int(x^3*atanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

output
```
int((atanh(a*x)**3*x**3)/(a**4*x**4 - 2*a**2*x**2 + 1),x)
```

3.274 $\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = -\frac{3}{8a^3(1-a^2x^2)}+\frac{3x\text{arctanh}(ax)}{4a^2(1-a^2x^2)}+\frac{3\text{arctanh}(ax)^2}{8a^3} - \frac{3\text{arctanh}(ax)^2}{4a^3(1-a^2x^2)}+\frac{x\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)^4}{8a^3}$$

output
```
-3/8/a^3/(-a^2*x^2+1)+3/4*x*arctanh(a*x)/a^2/(-a^2*x^2+1)+3/8*arctanh(a*x)
^2/a^3-3/4*arctanh(a*x)^2/a^3/(-a^2*x^2+1)+1/2*x*arctanh(a*x)^3/a^2/(-a^2*
x^2+1)-1/8*arctanh(a*x)^4/a^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{3-6ax\text{arctanh}(ax)+3(1+a^2x^2)\,\text{arctanh}(ax)^2-4ax\text{arctanh}(ax)^3+(1-a^2x^2)\,\text{arctanh}(ax)^4}{8a^3(-1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

output
```
(3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a
*x]^3 + (1 - a^2*x^2)*ArcTanh[a*x]^4)/(8*a^3*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6562, 6556, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{ 6562}$$

$$-\frac{3\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{2a}-\frac{\text{arctanh}(ax)^4}{8a^3}+\frac{x\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}$$

$$\Big\downarrow \text{ 6556}$$

$$-\frac{3\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)}{2a}-\frac{\text{arctanh}(ax)^4}{8a^3}+\frac{x\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}$$

$$\Big\downarrow \text{ 6518}$$

$$-\frac{3\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)}{2a}-\frac{\text{arctanh}(ax)^4}{8a^3}+\frac{x\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{\text{arctanh}(ax)^4}{8a^3}+\frac{x\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)}{2a}$$

input
```
Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

output
```
(x*ArcTanh[a*x]^3)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(8*a^3) - (3*(Ar
cTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTan
h[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a))/(2*a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6562
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> Simp[-(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(2*c^2*d*(d + e*x^2))), x] - Simp[b*(
p/(2*c))   Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 22.72 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

method	result
parallelrisch	$-\frac{\operatorname{arctanh}(ax)^4 a^2x^2-3a^2x^2 \operatorname{arctanh}(ax)^2+4\operatorname{arctanh}(ax)^3 ax-3a^2x^2-\operatorname{arctanh}(ax)^4+6ax \operatorname{arctanh}(ax)-3\operatorname{arctanh}(a}{8(a^2x^2-1)a^3}$
risch	$-\frac{\ln(ax+1)^4}{128a^3}+\frac{(x^2\ln(-ax+1)a^2-2ax-\ln(-ax+1))\ln(ax+1)^3}{32a^3(a^2x^2-1)}-\frac{3\left(a^2x^2\ln(-ax+1)^2-2a^2x^2-4ax\ln(-ax+1)-\right.}{64a^3(ax-1)(ax+1}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^3}{4(ax+1)}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)^3}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{4}+\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{i\pi\operatorname{csgn}\left(\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^3}{4(ax+1)}-\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)^3}{4(ax-1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{4}+\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{i\pi\operatorname{csgn}\left(\right.}{}}{}$
parts	Expression too large to display

input
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(arctanh(a*x)^4*a^2*x^2-3*a^2*x^2*arctanh(a*x)^2+4*arctanh(a*x)^3*a*x
-3*a^2*x^2-arctanh(a*x)^4+6*a*x*arctanh(a*x)-3*arctanh(a*x)^2)/(a^2*x^2-1)
/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{x^2\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx =$$
$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^4+48\,ax\log\left(-\frac{ax+1}{ax-1}\right)-12\,(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2-48}{128\,(a^5x^2-a^3)}$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + (a^2*x^2 - 1)*log(-(a*x + 1)/(
a*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*
x + 1)/(a*x - 1))^2 - 48)/(a^5*x^2 - a^3)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \int \frac{x^2\,\text{atanh}^3\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x**2*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(105) = 210.

Time = 0.05 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.84

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2} + \frac{\log\,(ax+1)}{a^3} - \frac{\log\,(ax-1)}{a^3}\right)\text{artanh}\,(ax)^3 + \frac{3\,((a^2x^2-1)\log\,(ax+1)^2 - 2\,(a^2x^2-1)\log\,(ax+1)\log\,(ax-1) + (a^2x^2-1)\log\,(ax-1)^2 + 4)a\,\text{ar}}{16\,(a^6x^2-a^4)} + \frac{1}{128}\left(\frac{((a^2x^2-1)\log\,(ax+1)^4 - 4\,(a^2x^2-1)\log\,(ax+1)^3\log\,(ax-1) + (a^2x^2-1)\log\,(ax-1)^4 -}{}\right.$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a
*x)^3 + 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*
log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)^2/(a^6*x^2
 - a^4) + 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x +
 1)^3*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 - 6*(2*a^2*x^2 - (a^2*x^
2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1)^2 - 12*(a^2*x^2 - 1)*log(a*x - 1)^
2 - 4*((a^2*x^2 - 1)*log(a*x - 1)^3 - 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*
x + 1) + 48)*a^2/(a^8*x^2 - a^6) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^
2*x^2 - 1)*log(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12
*a*x - 3*(2*a^2*x^2 - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(
a^2*x^2 - 1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^2 - a^5))*a
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^3}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.39

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{3\ln(a\,x+1)^2}{32\,a^3} - \frac{3\ln(a\,x+1)^2}{16\,(a^3-a^5\,x^2)} + \frac{3\ln(1-a\,x)^2}{32\,a^3} - \frac{\ln(a\,x+1)^4}{128\,a^3} - \frac{\ln(1-a\,x)^4}{128\,a^3} - \frac{3\ln(1-a\,x)^2}{16\,a^3-16\,a^5\,x^2} - \frac{3}{2\,(4\,a^3-4\,a^5\,x^2)} - \frac{x\ln(1-a\,x)^3}{2\,(8\,a^2-8\,a^4\,x^2)} - \frac{3\,\ln(a\,x+1)\,\ln(1-a\,x)}{16\,a^3} + \frac{3\,\ln(a\,x+1)\,\ln(1-a\,x)}{8\,a^3-8\,a^5\,x^2} + \frac{3\,x\,\ln(a\,x+1)}{8\,(a^2-a^4\,x^2)} + \frac{\ln(a\,x+1)\,\ln(1-a\,x)^3}{32\,a^3} + \frac{\ln(a\,x+1)^3\,\ln(1-a\,x)}{32\,a^3} - \frac{6\,x\,\ln(1-a\,x)}{16\,a^2-16\,a^4\,x^2} + \frac{x\ln(a\,x+1)^3}{16\,(a^2-a^4\,x^2)} - \frac{3\ln(a\,x+1)^2\ln(1-a\,x)^2}{64\,a^3} + \frac{6\,x\,\ln(a\,x+1)\,\ln(1-a\,x)^2}{32\,a^2-32\,a^4\,x^2} - \frac{6\,x\ln(a\,x+1)^2\,\ln(1-a\,x)}{32\,a^2-32\,a^4\,x^2}$$

input
```
int((x^2*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

output
```
(3*log(a*x + 1)^2)/(32*a^3) - (3*log(a*x + 1)^2)/(16*(a^3 - a^5*x^2)) + (3
*log(1 - a*x)^2)/(32*a^3) - log(a*x + 1)^4/(128*a^3) - log(1 - a*x)^4/(128
*a^3) - (3*log(1 - a*x)^2)/(16*a^3 - 16*a^5*x^2) - 3/(2*(4*a^3 - 4*a^5*x^2
)) - (x*log(1 - a*x)^3)/(2*(8*a^2 - 8*a^4*x^2)) - (3*log(a*x + 1)*log(1 -
a*x))/(16*a^3) + (3*log(a*x + 1)*log(1 - a*x))/(8*a^3 - 8*a^5*x^2) + (3*x*
log(a*x + 1))/(8*(a^2 - a^4*x^2)) + (log(a*x + 1)*log(1 - a*x)^3)/(32*a^3)
 + (log(a*x + 1)^3*log(1 - a*x))/(32*a^3) - (6*x*log(1 - a*x))/(16*a^2 - 1
6*a^4*x^2) + (x*log(a*x + 1)^3)/(16*(a^2 - a^4*x^2)) - (3*log(a*x + 1)^2*l
og(1 - a*x)^2)/(64*a^3) + (6*x*log(a*x + 1)*log(1 - a*x)^2)/(32*a^2 - 32*a
^4*x^2) - (6*x*log(a*x + 1)^2*log(1 - a*x))/(32*a^2 - 32*a^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^2} \, dx$$
$$= \frac{-atanh(ax)^4 a^2x^2 + atanh(ax)^4 - 4atanh(ax)^3 ax + 3atanh(ax)^2 a^2x^2 + 3atanh(ax)^2 - 6atanh(ax)\, ax}{8a^3(a^2x^2-1)}$$

input
```
int(x^2*atanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

output
```
( - atanh(a*x)**4*a**2*x**2 + atanh(a*x)**4 - 4*atanh(a*x)**3*a*x + 3*atan
h(a*x)**2*a**2*x**2 + 3*atanh(a*x)**2 - 6*atanh(a*x)*a*x + 3*a**2*x**2)/(8
*a**3*(a**2*x**2 - 1))
```

3.275 $\int \frac{x\textbf{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 119

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = -\frac{3x}{8a(1-a^2x^2)} - \frac{3\text{arctanh}(ax)}{8a^2} + \frac{3\text{arctanh}(ax)}{4a^2(1-a^2x^2)} - \frac{3x\text{arctanh}(ax)^2}{4a(1-a^2x^2)} - \frac{\text{arctanh}(ax)^3}{4a^2} + \frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}$$

output
```
-3/8*x/a/(-a^2*x^2+1)-3/8*arctanh(a*x)/a^2+3/4*arctanh(a*x)/a^2/(-a^2*x^2+
1)-3/4*x*arctanh(a*x)^2/a/(-a^2*x^2+1)-1/4*arctanh(a*x)^3/a^2+1/2*arctanh(
a*x)^3/a^2/(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{6ax - 12\text{arctanh}(ax) + 12ax\text{arctanh}(ax)^2 - 4(1+a^2x^2)\,\text{arctanh}(ax)^3 + 3(-1+a^2x^2)\log(1-ax) - 3(}{16a^2(-1+a^2x^2)}$$

input
```
Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

output
```
(6*a*x - 12*ArcTanh[a*x] + 12*a*x*ArcTanh[a*x]^2 - 4*(1 + a^2*x^2)*ArcTanh
[a*x]^3 + 3*(-1 + a^2*x^2)*Log[1 - a*x] - 3*(-1 + a^2*x^2)*Log[1 + a*x])/(
16*a^2*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6556, 6518, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{2a}$$

$$\Big\downarrow \text{6518}$$

$$\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}$$

$$\Big\downarrow \text{6556}$$

$$\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}$$

$$\Big\downarrow \text{215}$$

$$\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)} - a\left(\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)} - \frac{\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}}{2a}\right) + \frac{\operatorname{arctanh}(ax)^3}{6a}\right)}{2a}$$

input

```
Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

output

```
ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2)) - (3*((x*ArcTanh[a*x]^2)/(2*(1 - a^2*
x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/
(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/(2*a)
```

Defintions of rubi rules used

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6518

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 32.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.61

method	result
parallelrisch	$-\frac{2\operatorname{arctanh}(ax)^{3}a^{2}x^{2}+3a^{2}x^{2}\operatorname{arctanh}(ax)-6\operatorname{arctanh}(ax)^{2}ax+2\operatorname{arctanh}(ax)^{3}-3ax+3\operatorname{arctanh}(ax)}{8(a^{2}x^{2}-1)a^{2}}$
risch	$-\frac{(a^{2}x^{2}+1)\ln(ax+1)^{3}}{32a^{2}(ax-1)(ax+1)}+\frac{3(x^{2}\ln(-ax+1)a^{2}+2ax+\ln(-ax+1))\ln(ax+1)^{2}}{32a^{2}(ax-1)(ax+1)}-\frac{3\left(a^{2}x^{2}\ln(-ax+1)^{2}+4ax\ln(-ax+\right.}{32a^{2}(ax-1)}$
orering	$-\frac{(ax-1)(ax+1)(9a^{6}x^{6}+3a^{4}x^{4}+2a^{2}x^{2}+1)\operatorname{arctanh}(ax)^{3}}{2a^{4}x^{2}(-a^{2}x^{2}+1)^{2}}-\frac{(ax+1)^{2}(ax-1)^{2}(17a^{4}x^{4}+6a^{2}x^{2}+2)\left(\frac{\operatorname{arctanh}(ax)^{3}}{(-a^{2}x^{2}+1)^{2}}+\right.}{4a^{4}x^{2}}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^{3}}{2\left(a^{2}x^{2}-1\right)}+\frac{3\operatorname{arctanh}(ax)^{2}}{8(ax+1)}-\frac{3\operatorname{arctanh}(ax)^{2}\ln(ax+1)}{8}+\frac{3\operatorname{arctanh}(ax)^{2}}{8(ax-1)}+\frac{3\operatorname{arctanh}(ax)^{2}\ln(ax-1)}{8}+\frac{3\operatorname{arctanh}(ax)^{2}\ln\left(\frac{a}{\sqrt{-}}\right.}{4}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^{3}}{2\left(a^{2}x^{2}-1\right)}+\frac{3\operatorname{arctanh}(ax)^{2}}{8(ax+1)}-\frac{3\operatorname{arctanh}(ax)^{2}\ln(ax+1)}{8}+\frac{3\operatorname{arctanh}(ax)^{2}}{8(ax-1)}+\frac{3\operatorname{arctanh}(ax)^{2}\ln(ax-1)}{8}+\frac{3\operatorname{arctanh}(ax)^{2}\ln\left(\frac{a}{\sqrt{-}}\right.}{4}}{}$
parts	$-\frac{\operatorname{arctanh}(ax)^{3}}{2a^{2}(a^{2}x^{2}-1)}+\frac{\frac{3\operatorname{arctanh}(ax)^{2}}{2(4ax-4)}+\frac{3\operatorname{arctanh}(ax)^{2}\ln(ax-1)}{8}+\frac{3\operatorname{arctanh}(ax)^{2}}{2(4ax+4)}-\frac{3\operatorname{arctanh}(ax)^{2}\ln(ax+1)}{8}+\frac{3\operatorname{arctanh}(ax)^{2}\ln(}{4}}{}$

input
```
int(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*arctanh(a*x)^3*a^2*x^2+3*a^2*x^2*arctanh(a*x)-6*arctanh(a*x)^2*a*x
+2*arctanh(a*x)^3-3*a*x+3*arctanh(a*x))/(a^2*x^2-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int \frac{x\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\,(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)}{32\,(a^4x^2-a^2)}$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
1/32*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 -
 a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx=\int \frac{x\,\text{atanh}^3\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(x*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.50

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx=\frac{3\left(\frac{2\,x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\text{artanh}\,(ax)^2}{8\,a}-\frac{\frac{\left((a^2x^2-1)\log(ax+1)^3-3\,(a^2x^2-1)\log(ax+1)^2\log(ax-1)-(a^2x^2-1)\log(ax-1)^3-12\,ax+3\left(2\,a^2x^2+(a^2x^2-1)\log(ax-1)^2-2\right)\log(ax+}{a^5x^2-a^3}}{32\,a}-\frac{\text{artanh}\,(ax)^3}{2\,(a^2x^2-1)a^2}$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
3/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2/a
 - 1/32*(((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*lo
g(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x
^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a
^2/(a^5*x^2 - a^3) - 6*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log
(a*x + 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/
(a^4*x^2 - a^2))/a - 1/2*arctanh(a*x)^3/((a^2*x^2 - 1)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx =$$
$$-\frac{1}{64}\left(\left(\frac{ax+1}{(ax-1)a^3}+\frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3-3\left(\frac{ax+1}{(ax-1)a^3}-\frac{ax-1}{(ax+1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)\right.$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
-1/64*(((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/((a*x + 1)*a^3))*log(-(a*x +
 1)/(a*x - 1))^3 - 3*((a*x + 1)/((a*x - 1)*a^3) - (a*x - 1)/((a*x + 1)*a^3
))*log(-(a*x + 1)/(a*x - 1))^2 + 6*((a*x + 1)/((a*x - 1)*a^3) + (a*x - 1)/
((a*x + 1)*a^3))*log(-(a*x + 1)/(a*x - 1)) - 6*(a*x + 1)/((a*x - 1)*a^3) +
 6*(a*x - 1)/((a*x + 1)*a^3))*a
```

Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.01

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx =$$
$$-\frac{6\,\ln(1-a\,x)-6\,\ln(a\,x+1)+12\,a\,x-\ln(a\,x+1)^3+\ln(1-a\,x)^3-3\,\ln(a\,x+1)\,\ln(1-a\,x)^2+3}{}$$

input
```
int((x*atanh(a*x)^3)/(a^2*x^2 - 1)^2,x)
```

output
```
-(6*log(1 - a*x) - 6*log(a*x + 1) + 12*a*x - log(a*x + 1)^3 + log(1 - a*x)
^3 - 3*log(a*x + 1)*log(1 - a*x)^2 + 3*log(a*x + 1)^2*log(1 - a*x) - a^2*x
^2*(6*log(a*x + 1) - 6*log(1 - a*x)) - a^2*x^2*log(a*x + 1)^3 + a^2*x^2*lo
g(1 - a*x)^3 + 6*a*x*log(a*x + 1)^2 + 6*a*x*log(1 - a*x)^2 - 12*a*x*log(a*
x + 1)*log(1 - a*x) - 3*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 + 3*a^2*x^2*lo
g(a*x + 1)^2*log(1 - a*x))/(32*a^2 - 32*a^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx$$
$$= \frac{-4atanh(ax)^3\,a^2x^2 - 4atanh(ax)^3 + 12atanh(ax)^2\,ax - 12atanh(ax)\,a^2x^2 - 3\log(a^2x-a)\,a^2x^2 + 3\log}{16a^2\,(a^2x^2-1)}$$

input
```
int(x*atanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

output
```
( - 4*atanh(a*x)**3*a**2*x**2 - 4*atanh(a*x)**3 + 12*atanh(a*x)**2*a*x - 1
2*atanh(a*x)*a**2*x**2 - 3*log(a**2*x - a)*a**2*x**2 + 3*log(a**2*x - a) +
 3*log(a**2*x + a)*a**2*x**2 - 3*log(a**2*x + a) + 6*a*x)/(16*a**2*(a**2*x
**2 - 1))
```

3.276 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2} dx = -\frac{3}{8a(1-a^2x^2)} + \frac{3x\operatorname{arctanh}(ax)}{4(1-a^2x^2)} + \frac{3\operatorname{arctanh}(ax)^2}{8a} - \frac{3\operatorname{arctanh}(ax)^2}{4a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)} + \frac{\operatorname{arctanh}(ax)^4}{8a}$$

output
```
-3/8/a/(-a^2*x^2+1)+3*x*arctanh(a*x)/(-4*a^2*x^2+4)+3/8*arctanh(a*x)^2/a-3
/4*arctanh(a*x)^2/a/(-a^2*x^2+1)+x*arctanh(a*x)^3/(-2*a^2*x^2+2)+1/8*arcta
nh(a*x)^4/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2} dx = \frac{3 - 6ax\operatorname{arctanh}(ax) + 3(1+a^2x^2)\operatorname{arctanh}(ax)^2 - 4ax\operatorname{arctanh}(ax)^3 + (-1+a^2x^2)\operatorname{arctanh}(ax)^4}{8a(-1+a^2x^2)}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

output
```
(3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a
*x]^3 + (-1 + a^2*x^2)*ArcTanh[a*x]^4)/(8*a*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6518, 6556, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx$$

$$\Big\downarrow \text{6518}$$

$$-\frac{3}{2}a\int \frac{x\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^4}{8a}$$

$$\Big\downarrow \text{6556}$$

$$-\frac{3}{2}a\left(\frac{\operatorname{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^4}{8a}$$

$$\Big\downarrow \text{6518}$$

$$-\frac{3}{2}a\left(\frac{\operatorname{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^4}{8a}$$

$$\Big\downarrow \text{241}$$

$$\frac{x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\operatorname{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\operatorname{arctanh}(ax)^4}{8a}$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

output
```
(x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a) - (3*a*(ArcTan
h[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*
x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a))/2
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 24.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

method	result
parallelrisch	$-\frac{-\operatorname{arctanh}(ax)^4 a^2x^2-3a^2x^2\operatorname{arctanh}(ax)^2+4\operatorname{arctanh}(ax)^3ax-3a^2x^2+\operatorname{arctanh}(ax)^4+6ax\operatorname{arctanh}(ax)-3\operatorname{arctanh}}{8(a^2x^2-1)a}$
risch	$\frac{\ln(ax+1)^4}{128a}-\frac{(x^2\ln(-ax+1)a^2+2ax-\ln(-ax+1))\ln(ax+1)^3}{32(a^2x^2-1)a}+\frac{3\left(a^2x^2\ln(-ax+1)^2+2a^2x^2+4ax\ln(-ax+1)-\ln\right.}{64a(ax-1)(ax+1)}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^3}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)^3}{4(ax+1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{i\pi\operatorname{csgn}\Big(}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^3}{4(ax-1)}-\frac{\operatorname{arctanh}(ax)^3\ln(ax-1)}{4}-\frac{\operatorname{arctanh}(ax)^3}{4(ax+1)}+\frac{\operatorname{arctanh}(ax)^3\ln(ax+1)}{4}-\frac{\operatorname{arctanh}(ax)^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{i\pi\operatorname{csgn}\Big(}{}}{}$
parts	Expression too large to display

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-arctanh(a*x)^4*a^2*x^2-3*a^2*x^2*arctanh(a*x)^2+4*arctanh(a*x)^3*a*
x-3*a^2*x^2+arctanh(a*x)^4+6*a*x*arctanh(a*x)-3*arctanh(a*x)^2)/(a^2*x^2-1
)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx =$$
$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3-(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^4+48\,ax\log\left(-\frac{ax+1}{ax-1}\right)-12\,(a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2-48}{128\,(a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 - (a^2*x^2 - 1)*log(-(a*x + 1)/(
a*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*
x + 1)/(a*x - 1))^2 - 48)/(a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(99) = 198$.

Time = 0.04 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.99

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3$$
$$-\frac{3\left((a^2x^2-1)\log(ax+1)^2 - 2(a^2x^2-1)\log(ax+1)\log(ax-1) + (a^2x^2-1)\log(ax-1)^2 - 4\right)a \text{ ar}}{16(a^4x^2-a^2)}$$
$$-\frac{1}{128}\left(\frac{\left((a^2x^2-1)\log(ax+1)^4 - 4(a^2x^2-1)\log(ax+1)^3\log(ax-1) + (a^2x^2-1)\log(ax-1)^4 + \right.}{}\right.$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^3
- 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*
x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)^2/(a^4*x^2 - a^2
) - 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^3*
log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 + 6*(2*a^2*x^2 + (a^2*x^2 - 1)
*log(a*x - 1)^2 - 2)*log(a*x + 1)^2 + 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*
((a^2*x^2 - 1)*log(a*x - 1)^3 + 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1)
 - 48)*a^2/(a^6*x^2 - a^4) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2
- 1)*log(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x +
 3*(2*a^2*x^2 + (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^
2 - 1)*log(a*x - 1))*a*arctanh(a*x)/(a^5*x^2 - a^3))*a
```

Giac [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx$$
$$=\frac{1}{32}\,a^2\left(\frac{(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^3}{(ax+1)a^4}+\frac{3\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)^2}{(ax+1)a^4}+\frac{6\,(ax-1)\log\left(-\frac{ax+1}{ax-1}\right)}{(ax+1)a^4}+\frac{6\,(ax-1)}{(ax+1)a^4}\right)$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
1/32*a^2*((a*x - 1)*log(-(a*x + 1)/(a*x - 1))^3/((a*x + 1)*a^4) + 3*(a*x -
 1)*log(-(a*x + 1)/(a*x - 1))^2/((a*x + 1)*a^4) + 6*(a*x - 1)*log(-(a*x +
1)/(a*x - 1))/((a*x + 1)*a^4) + 6*(a*x - 1)/((a*x + 1)*a^4))
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.29

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx = \frac{3\ln(a\,x+1)^2}{32\,a} - \frac{3}{2\,(4\,a-4\,a^3\,x^2)} - \frac{3\ln(1-a\,x)^2}{16\,a-16\,a^3\,x^2} + \frac{3\ln(1-a\,x)^2}{32\,a} + \frac{\ln(a\,x+1)^4}{128\,a} + \frac{\ln(1-a\,x)^4}{128\,a} - \frac{3\ln(a\,x+1)^2}{16\,(a-a^3\,x^2)} - \frac{3\,\ln(a\,x+1)\,\ln(1-a\,x)}{16\,a} - \frac{\ln(a\,x+1)\,\ln(1-a\,x)^3}{32\,a} - \frac{\ln(a\,x+1)^3\,\ln(1-a\,x)}{32\,a} - \frac{3\,x\,\ln(a\,x+1)}{8\,(a^2\,x^2-1)} + \frac{6\,x\,\ln(1-a\,x)}{16\,a^2\,x^2-16} + \frac{3\,\ln(a\,x+1)\,\ln(1-a\,x)}{8\,a-8\,a^3\,x^2} + \frac{3\ln(a\,x+1)^2\ln(1-a\,x)^2}{64\,a} - \frac{x\ln(a\,x+1)^3}{16\,(a^2\,x^2-1)} + \frac{x\ln(1-a\,x)^3}{2\,(8\,a^2\,x^2-8)} - \frac{6\,x\,\ln(a\,x+1)\,\ln(1-a\,x)^2}{32\,a^2\,x^2-32} + \frac{6\,x\ln(a\,x+1)^2\,\ln(1-a\,x)}{32\,a^2\,x^2-32}$$

input
```
int(atanh(a*x)^3/(a^2*x^2 - 1)^2,x)
```

output
```
(3*log(a*x + 1)^2)/(32*a) - 3/(2*(4*a - 4*a^3*x^2)) - (3*log(1 - a*x)^2)/(
16*a - 16*a^3*x^2) + (3*log(1 - a*x)^2)/(32*a) + log(a*x + 1)^4/(128*a) +
log(1 - a*x)^4/(128*a) - (3*log(a*x + 1)^2)/(16*(a - a^3*x^2)) - (3*log(a*
x + 1)*log(1 - a*x))/(16*a) - (log(a*x + 1)*log(1 - a*x)^3)/(32*a) - (log(
a*x + 1)^3*log(1 - a*x))/(32*a) - (3*x*log(a*x + 1))/(8*(a^2*x^2 - 1)) + (
6*x*log(1 - a*x))/(16*a^2*x^2 - 16) + (3*log(a*x + 1)*log(1 - a*x))/(8*a -
 8*a^3*x^2) + (3*log(a*x + 1)^2*log(1 - a*x)^2)/(64*a) - (x*log(a*x + 1)^3
)/(16*(a^2*x^2 - 1)) + (x*log(1 - a*x)^3)/(2*(8*a^2*x^2 - 8)) - (6*x*log(a
*x + 1)*log(1 - a*x)^2)/(32*a^2*x^2 - 32) + (6*x*log(a*x + 1)^2*log(1 - a*
x))/(32*a^2*x^2 - 32)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}\,dx$$
$$= \frac{atanh(ax)^4 a^2x^2 - atanh(ax)^4 - 4atanh(ax)^3 ax + 3atanh(ax)^2 a^2x^2 + 3atanh(ax)^2 - 6atanh(ax) ax +}{8a(a^2x^2-1)}$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

output
```
(atanh(a*x)**4*a**2*x**2 - atanh(a*x)**4 - 4*atanh(a*x)**3*a*x + 3*atanh(a
*x)**2*a**2*x**2 + 3*atanh(a*x)**2 - 6*atanh(a*x)*a*x + 3*a**2*x**2)/(8*a*
(a**2*x**2 - 1))
```

3.277 $\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 193

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx &= -\frac{3ax}{8(1-a^2x^2)} - \frac{3}{8}\operatorname{arctanh}(ax) + \frac{3\operatorname{arctanh}(ax)}{4(1-a^2x^2)} \\ &\quad - \frac{3ax\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)} - \frac{1}{4}\operatorname{arctanh}(ax)^3 + \frac{\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)} \\ &\quad + \frac{1}{4}\operatorname{arctanh}(ax)^4 + \operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right) \\ &\quad - \frac{3}{2}\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ &\quad - \frac{3}{2}\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right) \\ &\quad - \frac{3}{4}\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-3*a*x/(-8*a^2*x^2+8)-3/8*arctanh(a*x)+3*arctanh(a*x)/(-4*a^2*x^2+4)-3*a*x
*arctanh(a*x)^2/(-4*a^2*x^2+4)-1/4*arctanh(a*x)^3+arctanh(a*x)^3/(-2*a^2*x
^2+2)+1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^2
*polylog(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*poly
log(4,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx = \frac{1}{64}\left(\pi^4 - 16\operatorname{arctanh}(ax)^4 + 24\operatorname{arctanh}(ax)\cosh(2\operatorname{arctanh}(ax)) + 16\operatorname{arctanh}(ax)^3\cosh(2\operatorname{arctanh}(ax)) + 64\operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + 96\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - 96\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{2\operatorname{arctanh}(ax)}\right) - 12\sinh(2\operatorname{arctanh}(ax)) - 24\operatorname{arctanh}(ax)^2\sinh(2\operatorname{arctanh}(ax))\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

output
```
(Pi^4 - 16*ArcTanh[a*x]^4 + 24*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 16*ArcT
anh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a
*x])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]
*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])] - 12*S
inh[2*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/64
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6592, 6550, 6494, 6556, 6518, 6556, 215, 219, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{x\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}dx + \int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)}dx$$

$$\Big\downarrow \; 6550$$

$$a^2 \int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx + \int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx + \frac{1}{4}\text{arctanh}(ax)^4$$

$$\Big\downarrow \; 6494$$

$$a^2 \int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx - 3a \int \frac{\text{arctanh}(ax)^2 \log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx + \frac{1}{4}\text{arctanh}(ax)^4 + \text{arctanh}(ax)^3 \log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 6556$$

$$a^2 \left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{2a}\right) - 3a \int \frac{\text{arctanh}(ax)^2 \log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx + \frac{1}{4}\text{arctanh}(ax)^4 + \text{arctanh}(ax)^3 \log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 6518$$

$$a^2 \left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx + \frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)} + \frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right) - 3a \int \frac{\text{arctanh}(ax)^2 \log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx + \frac{1}{4}\text{arctanh}(ax)^4 + \text{arctanh}(ax)^3 \log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 6556$$

$$a^2 \left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)} - \frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)} - \frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right) + \frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)} + \frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right) - 3a \int \frac{\text{arctanh}(ax)^2 \log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx + \frac{1}{4}\text{arctanh}(ax)^4 + \text{arctanh}(ax)^3 \log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 215$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$\big\downarrow$ 219

$$-3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$\big\downarrow$ 6618

$$-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$\big\downarrow$ 6622

$$-3a\left(-\frac{1}{2}\int \frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}\right.}{2a}\right.$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\left(\frac{x\,\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)+$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 7164$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\left(\frac{x\,\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-$$

$$3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)+$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

input
```
Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

output
```
ArcTanh[a*x]^4/4 + a^2*(ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2)) - (3*((x*ArcT
anh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*
a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/(
2*a)) + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - 3*a*((ArcTanh[a*x]^2*PolyLog
[2, -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])
/(2*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 65.90 (sec) , antiderivative size = 1300, normalized size of antiderivative = 6.74

method	result	size
derivativedivides	Expression too large to display	1300
default	Expression too large to display	1300
parts	Expression too large to display	1711

input
```
int(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(a*x)^3/(a*x+1)-1/2*arctanh(a*x)^3*ln(a*x+1)-1/4*arctanh(a*x)^3
/(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x-1)+arctanh(a*x)^3*ln(a*x)+arctanh(a*x)^
3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4-3/16*arctanh(a*x)^2*(a
*x-1)/(a*x+1)-3/16*arctanh(a*x)*(a*x-1)/(a*x+1)-3/32*(a*x-1)/(a*x+1)+3/16*
(a*x+1)*arctanh(a*x)^2/(a*x-1)-3/16*arctanh(a*x)*(a*x+1)/(a*x-1)+3/32*(a*x
+1)/(a*x-1)-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^3*ln(
1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylo
g(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*
x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^
(1/2))+1/4*(-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2-2*I*Pi*csgn(I*(-(
a*x+1)^2/(a^2*x^2-1)-1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^
2*x^2-1)+1))^2+2*I*Pi-2*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(
a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+I*Pi*csgn(I*(a*x+1)/
(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-I*Pi*csgn(I/(-(a*x+1)^
2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+2*I*Pi
*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csg
n(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+2*I*Pi*csgn(...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)^3}{\left(a^2x^2-1\right)^2x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{x(ax-1)^2(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**3/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) + 1)*l
og(-a*x + 1)^3)/(a^2*x^2 - 1) - 1/8*integrate(-1/2*(2*log(a*x + 1)^3 - 6*l
og(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 - a^2*
x^2 - a*x - 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*x^5 - 2*a^2*x^3 + x), x
)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^2}\,dx = \int \frac{atanh(ax)^3}{a^4x^5-2a^2x^3+x}dx$$

input
```
int(atanh(a*x)^3/x/(-a^2*x^2+1)^2,x)
```

output
```
int(atanh(a*x)**3/(a**4*x**5 - 2*a**2*x**3 + x),x)
```

3.278 $\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 191

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx = & -\frac{3a}{8(1-a^2x^2)}+\frac{3a^2x\operatorname{arctanh}(ax)}{4(1-a^2x^2)}+\frac{3}{8}a\operatorname{arctanh}(ax)^2 \\ & -\frac{3a\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)}+a\operatorname{arctanh}(ax)^3-\frac{\operatorname{arctanh}(ax)^3}{x} \\ & +\frac{a^2x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{3}{8}a\operatorname{arctanh}(ax)^4 \\ & +3a\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right) \\ & -3a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ & -\frac{3}{2}a\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output

```
-3*a/(-8*a^2*x^2+8)+3*a^2*x*arctanh(a*x)/(-4*a^2*x^2+4)+3/8*a*arctanh(a*x)
^2-3*a*arctanh(a*x)^2/(-4*a^2*x^2+4)+a*arctanh(a*x)^3-arctanh(a*x)^3/x+a^2
*x*arctanh(a*x)^3/(-2*a^2*x^2+2)+3/8*a*arctanh(a*x)^4+3*a*arctanh(a*x)^2*l
n(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1
+2/(a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx = \frac{1}{16}a\Bigg(2i\pi^3 - 16\operatorname{arctanh}(ax)^3 - \frac{16\operatorname{arctanh}(ax)^3}{ax} + 6\operatorname{arctanh}(ax)^4 - 3\cosh(2\operatorname{arctanh}(ax)) - 6\operatorname{arctanh}(ax)^2\cosh(2\operatorname{arctanh}(ax)) + 48\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) + 48\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - 24\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right) + 6\operatorname{arctanh}(ax)\sinh(2\operatorname{arctanh}(ax)) + 4\operatorname{arctanh}(ax)^3\sinh(2\operatorname{arctanh}(ax))\Bigg)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

output
```
(a*((2*I)*Pi^3 - 16*ArcTanh[a*x]^3 - (16*ArcTanh[a*x]^3)/(a*x) + 6*ArcTanh
[a*x]^4 - 3*Cosh[2*ArcTanh[a*x]] - 6*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]] +
 48*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 48*ArcTanh[a*x]*PolyLog[2
, E^(2*ArcTanh[a*x])] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] + 6*ArcTanh[a*x]
*Sinh[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3*Sinh[2*ArcTanh[a*x]]))/16
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6592, 6518, 6544, 6452, 6510, 6550, 6494, 6556, 6518, 241, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx$$

$$\Big\downarrow \text{6518}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx$$

$$\Big\downarrow \text{6544}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+a^2\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{6452}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx+a^2\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6510}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+3a\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+3a\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\downarrow$ 6518

$$a^2\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int\frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(a}{8a}\right.$$
$$3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\downarrow$ 241

$$3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\downarrow$ 6618

$$3a\left(-2a\left(\frac{\text{arctanh}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(\right.$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\downarrow$ 7164

$$a^2\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

input
```
Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

output
```
-(ArcTanh[a*x]^3/x) + (a*ArcTanh[a*x]^4)/4 + a^2*((x*ArcTanh[a*x]^3)/(2*(1
 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a) - (3*a*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2
*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) +
ArcTanh[a*x]^2/(4*a))/a))/2) + 3*a*(ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[
2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a)
+ PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [A] (verified)

Time = 36.93 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.42

method	result
derivativedivides	$a\left(\frac{3\operatorname{arctanh}(ax)^4}{8}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax)^3-6\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-3\right)}{32(ax-1)}+\frac{\left(4\operatorname{arctanh}(ax)^3+6\operatorname{arctanh}\right.}{3}\right.$
default	$a\left(\frac{3\operatorname{arctanh}(ax)^4}{8}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax)^3-6\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-3\right)}{32(ax-1)}+\frac{\left(4\operatorname{arctanh}(ax)^3+6\operatorname{arctanh}\right.}{3}\right.$

input
```
int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(3/8*arctanh(a*x)^4-1/32*(a*x+1)*(4*arctanh(a*x)^3-6*arctanh(a*x)^2+6*ar
ctanh(a*x)-3)/(a*x-1)+1/32*(4*arctanh(a*x)^3+6*arctanh(a*x)^2+6*arctanh(a*
x)+3)*(a*x-1)/(a*x+1)+arctanh(a*x)^3/a/x*(a*x-1)-2*arctanh(a*x)^3+3*arctan
h(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,-(a*x+1
)/(-a^2*x^2+1)^(1/2))-6*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a
*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(2,(a*x+1)/(-
a^2*x^2+1)^(1/2))-6*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{x^2 (ax-1)^2 (ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**3/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}\,dx$$
$$= \frac{3atanh(ax)^4\,a^3x^3 - 3atanh(ax)^4\,ax - 12atanh(ax)^3\,a^2x^2 + 8atanh(ax)^3 + 9atanh(ax)^2\,a^3x^3 + 9atanh(}{8x\,(a^2x^2-}$$

input
```
int(atanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x)
```

output
```
(3*atanh(a*x)**4*a**3*x**3 - 3*atanh(a*x)**4*a*x - 12*atanh(a*x)**3*a**2*x
**2 + 8*atanh(a*x)**3 + 9*atanh(a*x)**2*a**3*x**3 + 9*atanh(a*x)**2*a*x -
18*atanh(a*x)*a**2*x**2 + 24*int(atanh(a*x)**2/(a**4*x**5 - 2*a**2*x**3 +
x),x)*a**3*x**3 - 24*int(atanh(a*x)**2/(a**4*x**5 - 2*a**2*x**3 + x),x)*a*
x + 9*a**3*x**3)/(8*x*(a**2*x**2 - 1))
```

3.279 $\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 302

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^2}\,dx &= -\frac{3a^3x}{8(1-a^2x^2)}-\frac{3}{8}a^2\operatorname{arctanh}(ax)+\frac{3a^2\operatorname{arctanh}(ax)}{4(1-a^2x^2)}\\&\quad+\frac{3}{2}a^2\operatorname{arctanh}(ax)^2-\frac{3a\operatorname{arctanh}(ax)^2}{2x}-\frac{3a^3x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)}\\&\quad+\frac{1}{4}a^2\operatorname{arctanh}(ax)^3-\frac{\operatorname{arctanh}(ax)^3}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}\\&\quad+\frac{1}{2}a^2\operatorname{arctanh}(ax)^4+3a^2\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right)\\&\quad+2a^2\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right)\\&\quad-\frac{3}{2}a^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\\&\quad-3a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\\&\quad-3a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\\&\quad-\frac{3}{2}a^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-3*a^3*x/(-8*a^2*x^2+8)-3/8*a^2*arctanh(a*x)+3*a^2*arctanh(a*x)/(-4*a^2*x^
2+4)+3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x-3*a^3*x*arctanh(a*x)^2/
(-4*a^2*x^2+4)+1/4*a^2*arctanh(a*x)^3-1/2*arctanh(a*x)^3/x^2+a^2*arctanh(a
*x)^3/(-2*a^2*x^2+2)+1/2*a^2*arctanh(a*x)^4+3*a^2*arctanh(a*x)*ln(2-2/(a*x
+1))+2*a^2*arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/(a*x+1))-
3*a^2*arctanh(a*x)^2*polylog(2,-1+2/(a*x+1))-3*a^2*arctanh(a*x)*polylog(3,
-1+2/(a*x+1))-3/2*a^2*polylog(4,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^2}\,dx = \frac{1}{32}a^2\left(\pi^4 + 48\operatorname{arctanh}(ax)^2 - \frac{48\operatorname{arctanh}(ax)^2}{ax} - \frac{16(1-a^2x^2)\operatorname{arctanh}(ax)^3}{a^2x^2} - 16\operatorname{arctanh}(ax)^4 + 12\operatorname{arctanh}(ax)\cosh(2\operatorname{arctanh}(ax)) + 8\operatorname{arctanh}(ax)^3\cosh(2\operatorname{arctanh}(ax)) + 96\operatorname{arctanh}(ax)\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right) + 64\operatorname{arctanh}(ax)^3\log\left(1-e^{2\operatorname{arctanh}(ax)}\right) - 48\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right) + 96\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) - 96\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{2\operatorname{arctanh}(ax)}\right) - 6\sinh(2\operatorname{arctanh}(ax)) - 12\operatorname{arctanh}(ax)^2\sinh(2\operatorname{arctanh}(ax))\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

output
```
(a^2*(Pi^4 + 48*ArcTanh[a*x]^2 - (48*ArcTanh[a*x]^2)/(a*x) - (16*(1 - a^2*
x^2)*ArcTanh[a*x]^3)/(a^2*x^2) - 16*ArcTanh[a*x]^4 + 12*ArcTanh[a*x]*Cosh[
2*ArcTanh[a*x]] + 8*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 96*ArcTanh[a*x]*
Log[1 - E^(-2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x]
)] - 48*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(
2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*Pol
yLog[4, E^(2*ArcTanh[a*x])] - 6*Sinh[2*ArcTanh[a*x]] - 12*ArcTanh[a*x]^2*S
inh[2*ArcTanh[a*x]]))/32
```

Rubi [A] (verified)

Time = 4.82 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.47, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {6592, 6544, 6452, 6544, 6452, 6510, 6550, 6494, 2897, 6592, 6550, 6494, 6556, 6518, 6556, 215, 219, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x^3 (1-a^2x^2)^2} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)^3}{x^3 (1-a^2x^2)}dx$$

$$\Big\downarrow \text{6544}$$

$$a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)^2}dx+a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)}dx+\int \frac{\text{arctanh}(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}a \int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)}dx+a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)^2}dx+a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6544}$$

$$\frac{3}{2}a \left(a^2 \int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)^2}{x^2}dx\right)+a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)^2}dx+a^2 \int \frac{\text{arctanh}(ax)^3}{x (1-a^2x^2)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}a\left(a^2\int \frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+2a\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)^2}{x}\right)+ a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx+a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$\frac{3}{2}a\left(2a\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}\right)+a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx+ a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)}dx-\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6550}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx+a^2\left(\int \frac{\text{arctanh}(ax)^3}{x(ax+1)}dx+\frac{1}{4}\text{arctanh}(ax)^4\right)+ \frac{3}{2}a\left(2a\left(\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}\right)- \frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6494}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx+ \frac{3}{2}a\left(2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\text{ar} \right. a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)- \frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{2897}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx+ a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+ \frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right. \frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6592}$$

$$a^2\left(a^2\int\frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\int\frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)}dx\right)+$$
$$a^2\left(-3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(a^2\int\frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\int\frac{\text{arctanh}(ax)^3}{x(ax+1)}dx+\frac{1}{4}\text{arctanh}(ax)^4\right)+$$
$$a^2\left(-3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(-3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(a^2\int\frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx-3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{a\cdots}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{2a}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arc}\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\Big\downarrow$ 6518

$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-3a\int \frac{\text{arctanh}(ax)^2\log}{1-a^2x}\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\Big\downarrow$ 6556

$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-3a\int \frac{\text{arc}}{}\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$\Big\downarrow$ 215

$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)\right)-$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow 219$$

$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2\left(1-a^2\right.}}{}\right.\right.}{2a}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$
$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow 6618$$

$$a^2\left(-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{4}\text{arctanh}(ax)^4\right.$$

$$a^2\left(-3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+a^2\left(\frac{\text{arctanh}(a}{2a^2\,(1-a^2}\right.\right.$$

$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$

$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \; 6622$$

$$a^2\left(-3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{}{}\right.}{2a}\right.\right.$$

$$a^2\left(-3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{}{}\right.}{2a}\right.\right.$$

$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\text{arctanh}(ax)^3-\right.$$

$$\frac{\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \; 7164$$

$$a^2\left(a^2\left(\frac{\operatorname{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}}{2a}\right)+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)}{2a}\right)-3\right.$$

$$a^2\left(-3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right.\right.$$

$$\frac{3}{2}a\left(2a\left(\frac{1}{2}\operatorname{arctanh}(ax)^2+\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+\frac{1}{3}a\operatorname{arctanh}(ax)^3-\right.$$

$$\frac{\operatorname{arctanh}(ax)^3}{2x^2}$$

input
```
Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

output
```
-1/2*ArcTanh[a*x]^3/x^2 + (3*a*(-(ArcTanh[a*x]^2/x) + (a*ArcTanh[a*x]^3)/3
 + 2*a*(ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2,
-1 + 2/(1 + a*x)]/2)))/2 + a^2*(ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 -
2/(1 + a*x)] - 3*a*((ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) +
(ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 +
 a*x)]/(4*a))) + a^2*(ArcTanh[a*x]^4/4 + a^2*(ArcTanh[a*x]^3/(2*a^2*(1 - a
^2*x^2)) - (3*((x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a)
 - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[
a*x]/(2*a))/(2*a))))/(2*a)) + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - 3*a*((
ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog
[3, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [A] (verified)

Time = 90.92 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.48

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{2}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax)^3-6\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-3\right)}{32(ax-1)}-\frac{\left(4\operatorname{arctanh}(ax)^3+6\operatorname{arcta}\right.}{}\right.$
default	$a^2\left(-\frac{\operatorname{arctanh}(ax)^4}{2}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax)^3-6\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-3\right)}{32(ax-1)}-\frac{\left(4\operatorname{arctanh}(ax)^3+6\operatorname{arcta}\right.}{}\right.$

input
```
int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arctanh(a*x)^4-1/32*(a*x+1)*(4*arctanh(a*x)^3-6*arctanh(a*x)^2+6
*arctanh(a*x)-3)/(a*x-1)-1/32*(4*arctanh(a*x)^3+6*arctanh(a*x)^2+6*arctanh
(a*x)+3)*(a*x-1)/(a*x+1)+1/2*arctanh(a*x)^2*(a*x*arctanh(a*x)+arctanh(a*x)
+3*a*x)*(a*x-1)/a^2/x^2-3*arctanh(a*x)^2+3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)*ln(1
+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*ar
ctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)^2*polylog(2,(
a*x+1)/(-a^2*x^2+1)^(1/2))-12*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^
(1/2))+12*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)^3*ln(1+(a*x
+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(
1/2))-12*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*polylog(4,
-(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^2}\,dx=\int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^2}\, dx = \int \frac{\operatorname{atanh}^3(ax)}{x^3 (ax-1)^2 (ax+1)^2}\, dx$$

input
```
integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**2,x)
```

output
```
Integral(atanh(a*x)**3/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^2}\, dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2 x^3}\, dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/32*((a^4*x^4 - a^2*x^2)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4 - a^
2*x^2)*log(a*x + 1) - 1)*log(-a*x + 1)^3)/(a^2*x^4 - x^2) - 1/8*integrate(
-1/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x + 1) - 3*(2*a^4*x^4 + 2
*a^3*x^3 - a^2*x^2 - a*x + 2*(a^6*x^6 + a^5*x^5 - a^4*x^4 - a^3*x^3 - 1)*l
og(a*x + 1))*log(-a*x + 1)^2)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(atanh(a*x)^3/(x^3*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^2}\,dx = \int \frac{atanh(ax)^3}{a^4x^7-2a^2x^5+x^3}dx$$

input
```
int(atanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x)
```

output
```
int(atanh(a*x)**3/(a**4*x**7 - 2*a**2*x**5 + x**3),x)
```

3.280 $\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx = \frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)} + \frac{\text{arctanh}(ax)^{3/2}}{3a} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)}{16a} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)}{16a}$$

output
```
x*arctanh(a*x)^(1/2)/(-2*a^2*x^2+2)+1/3*arctanh(a*x)^(3/2)/a+1/32*2^(1/2)*
Pi^(1/2)*erf(2^(1/2)*arctanh(a*x)^(1/2))/a-1/32*2^(1/2)*Pi^(1/2)*erfi(2^(1
/2)*arctanh(a*x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx = \sqrt{\text{arctanh}(ax)}\left(-\frac{x}{2(-1+a^2x^2)} + \frac{\text{arctanh}(ax)}{3a}\right) - \frac{\sqrt{\frac{\pi}{2}}\left(-\text{erf}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right) + \text{erfi}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)\right)}{16a}$$

input
```
Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

output
```
Sqrt[ArcTanh[a*x]]*(-1/2*x/(-1 + a^2*x^2) + ArcTanh[a*x]/(3*a)) - (Sqrt[Pi
/2]*(-Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]]))
/(16*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6518, 6596, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6518}$$

$$-\frac{1}{4}a\int \frac{x}{(1-a^2x^2)^2\sqrt{\text{arctanh}(ax)}}dx+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{6596}$$

$$-\frac{\int \frac{ax}{(1-a^2x^2)\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)}{4a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{5971}$$

$$-\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{2\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)}{4a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)}{8a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)}{8a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)}{8a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{3789}$$

$$\frac{i\left(\frac{1}{2}i\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)-\frac{1}{2}i\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{\text{arctanh}(ax)}}d\text{arctanh}(ax)\right)}{8a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{2611}$$

$$\frac{i\left(i\int e^{2\text{arctanh}(ax)}d\sqrt{\text{arctanh}(ax)}-i\int e^{-2\text{arctanh}(ax)}d\sqrt{\text{arctanh}(ax)}\right)}{8a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{2633}$$

$$\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)-i\int e^{-2\text{arctanh}(ax)}d\sqrt{\text{arctanh}(ax)}\right)}{8a}+\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{2634}$$

$$\frac{x\sqrt{\text{arctanh}(ax)}}{2(1-a^2x^2)}+\frac{i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)\right)}{8a}+\frac{\text{arctanh}(ax)^{3/2}}{3a}$$

input
```
Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

output
```
(x*Sqrt[ArcTanh[a*x]])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^(3/2)/(3*a) + ((I/
8)*((-1/2*I)*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]] + (I/2)*Sqrt[Pi/2]
*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]]))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{arctanh}\left(ax\right)}}{\left(-a^{2}x^{2}+1\right)^{2}}dx$$

input
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

output
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx = \int \frac{\sqrt{\operatorname{atanh}(ax)}}{(ax-1)^2\,(ax+1)^2}\,dx$$

input
```
integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**2,x)
```

output
```
Integral(sqrt(atanh(a*x))/((a*x - 1)**2*(a*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx = \int \frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2}\,dx = \int \frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^2}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2} \, dx = \int \frac{\sqrt{\mathrm{atanh}\left(a\,x\right)}}{{\left(a^2\,x^2-1\right)}^2} \,d x$$

input
```
int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^2,x)
```

output
```
int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2} \, dx = \int \frac{\sqrt{atanh\left(a x \right)}}{a^{4} x^{4}-2 a^{2} x^{2}+1}d x$$

input
```
int(atanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

output
```
int(sqrt(atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1),x)
```

3.281 $\int \frac{x^4}{(1-a^2x^2)^2\text{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a^5} - \frac{3\log(\operatorname{arctanh}(ax))}{2a^5} + \frac{\operatorname{Int}\left(\frac{1}{\operatorname{arctanh}(ax)}, x\right)}{a^4}$$

output
```
1/2*Chi(2*arctanh(a*x))/a^5-3/2*ln(arctanh(a*x))/a^5+Defer(Int)(1/arctanh(
a*x),x)/a^4
```

Mathematica [N/A]

Not integrable

Time = 3.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx$$

input
```
Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx$$
$$\Big\downarrow \; 6651$$
$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx$$

input
```
Int[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(-a^2x^2+1)^2 \operatorname{arctanh}(ax)} dx$$

input
```
int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

output
```
int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(x^4/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(x**4/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

input
```
int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{atanh\,(ax)\,a^4x^4 - 2atanh\,(ax)\,a^2x^2 + atanh\,(ax)}dx$$

input
```
int(x^4/(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
int(x**4/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.282 $\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx=\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2a^4}-\frac{\operatorname{Int}\left(\frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)},x\right)}{a^2}$$

output
```
1/2*Shi(2*arctanh(a*x))/a^4-Defer(Int)(x/(-a^2*x^2+1)/arctanh(a*x),x)/a^2
```

Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx=\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx$$

input
```
Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(-a^2x^2+1)^2 \operatorname{arctanh}(ax)}dx$$

input
```
int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

output
```
int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

input
```
int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{x^3}{atanh\,(ax)\,a^4x^4 - 2atanh\,(ax)\,a^2x^2 + atanh\,(ax)} dx$$

input
```
int(x^3/(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
int(x**3/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.283 $\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}\,dx=\frac{\text{Chi}(2\text{arctanh}(ax))}{2a^3}-\frac{\log(\text{arctanh}(ax))}{2a^3}$$

output
```
1/2*Chi(2*arctanh(a*x))/a^3-1/2*ln(arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}\,dx=\frac{\text{Chi}(2\text{arctanh}(ax))}{2a^3}-\frac{\log(\text{arctanh}(ax))}{2a^3}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \; 6596$$

$$\frac{\int \frac{a^2x^2}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -\frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\int \left(\frac{1}{2\operatorname{arctanh}(ax)} - \frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax)) - \frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^3}$$

input
```
Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcTanh[a*x]]/2 - Log[ArcTanh[a*x]]/2)/a^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\ln(\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}}{a^3}$	22
default	$\frac{-\frac{\ln(\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}}{a^3}$	22

input
```
int(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/2*ln(arctanh(a*x))+1/2*Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(23) = 46$.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx = -\frac{2 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4 a^3}$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a*x + 1)/(a*x - 1)
) - log_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx = \int \frac{x^2}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)} dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx = \int \frac{x^2}{(a^2x^2-1)^2 \operatorname{artanh}(ax)} dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(a^2x^2-1)^2 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \frac{\left(\int \frac{1}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+atanh(ax)}dx\right)a-\log(atanh(ax))}{a^3}$$

input
```
int(x^2/(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
(int(1/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a -
 log(atanh(a*x)))/a**3
```

3.284 $\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 14

$$\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx=\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2a^2}$$

output
```
1/2*Shi(2*arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx=\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{\int \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{2a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\frac{i\sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{2a^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int \frac{\sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{2a^2}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2a^2}$$

input
```
Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\text{Shi}(2\,\text{arctanh}(ax))}{2a^2}$	13
default	$\frac{\text{Shi}(2\,\text{arctanh}(ax))}{2a^2}$	13

input
```
int(x/(-a^2*x^2+1)^2/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*Shi(2*arctanh(a*x))/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{x}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)}\,dx = \frac{\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a^2}$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/4*(log_integral(-(a*x + 1)/(a*x - 1)) - log_integral(-(a*x - 1)/(a*x + 1
)))/a^2
```

Sympy [F]

$$\int \frac{x}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x}{\left(ax-1\right)^2\left(ax+1\right)^2 \operatorname{atanh}\left(ax\right)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(a^2x^2-1)^2\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(a^2x^2-1)^2\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2}\,dx$$

input
```
int(x/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

output
```
int(x/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \int \frac{x}{atanh\,(ax)\,a^4x^4 - 2atanh\,(ax)\,a^2x^2 + atanh\,(ax)}dx$$

input
```
int(x/(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.285 $\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a} + \frac{\log(\operatorname{arctanh}(ax))}{2a}$$

output
```
1/2*Chi(2*arctanh(a*x))/a+1/2*ln(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\,dx = \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a} + \frac{\log(\operatorname{arctanh}(ax))}{2a}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{6530}$$

$$\frac{\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\frac{1}{2\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x]]/2)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{\ln(\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}}{a}$	22
default	$\frac{\frac{\ln(\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}}{a}$	22

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*ln(arctanh(a*x))+1/2*Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx = \frac{2 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4a}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a*x + 1)/(a*x - 1))
 + log_integral(-(a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx = \int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)} dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)} dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^2} \, dx$$

input
```
int(1/(atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{atanh(ax)\,a^4x^4 - 2atanh(ax)\,a^2x^2 + atanh(ax)} dx$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
int(1/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.286 $\int \frac{1}{x(1-a^2x^2)^2\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(1-a^2x^2)^2 \text{arctanh}(ax)}\,dx = \frac{1}{2}\text{Shi}(2\text{arctanh}(ax)) + \text{Int}\left(\frac{1}{x(1-a^2x^2)\,\text{arctanh}(ax)},x\right)$$

output
```
1/2*Shi(2*arctanh(a*x))+Defer(Int)(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^2 \text{arctanh}(ax)}\,dx = \int \frac{1}{x(1-a^2x^2)^2 \text{arctanh}(ax)}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx$$

input
```
Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(-a^2x^2+1\right)^2 \operatorname{arctanh}\left(ax\right)}dx$$

input
```
int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{(a^2x^2-1)^2 x \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{x (ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x),x)
```

output
```
Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{(a^2x^2-1)^2 x \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\left(a^2x^2-1\right)^2x\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,\left(a^2\,x^2-1\right)^2}\,dx$$

input
```
int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{atanh\left(ax\right)a^4x^5-2atanh\left(ax\right)a^2x^3+atanh\left(ax\right)x}dx$$

input
```
int(1/x/(-a^2*x^2+1)^2/atanh(a*x),x)
```

output
```
int(1/(atanh(a*x)*a**4*x**5 - 2*atanh(a*x)*a**2*x**3 + atanh(a*x)*x),x)
```

3.287 $\int \frac{x^3}{(1-a^2x^2)^2\textbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}\,dx = \frac{x}{a^3\operatorname{arctanh}(ax)} - \frac{x}{a^3(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{a^4} - \frac{\operatorname{Int}\left(\frac{1}{\operatorname{arctanh}(ax)},x\right)}{a^3}$$

output
```
x/a^3/arctanh(a*x)-x/a^3/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))/a^4
-Defer(Int)(1/arctanh(a*x),x)/a^3
```

Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}\,dx$$

input
```
Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6548}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\frac{\int \frac{1}{\operatorname{arctanh}(ax)}dx}{a} - \frac{x}{a\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 6444}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\frac{\int \frac{1}{\operatorname{arctanh}(ax)}dx}{a} - \frac{x}{a\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 6594}$$

$$\frac{\frac{\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx}{a} + a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2} - \frac{\frac{\int \frac{1}{\operatorname{arctanh}(ax)}dx}{a} - \frac{x}{a\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 6530}$$

$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}+\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{6596}$$

$$\frac{\frac{\int \frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}+\frac{\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}+\frac{-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}$$

input
```
Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(-a^2x^2+1\right)^2 \operatorname{arctanh}\left(ax\right)^2}dx$$

input
```
int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

output
```
int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^3}{\left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^3}{\left(ax-1\right)^2\left(ax+1\right)^2 \operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.32

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*x^3/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) + integra
te(-2*(a^2*x^4 - 3*x^2)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4
 - 2*a^3*x^2 + a)*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^3}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(a^2\,x^2-1\right)}^2}\,dx$$

input
```
int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx$$
$$= \int \frac{x^3}{atanh\left(ax\right)^2 a^4x^4 - 2atanh\left(ax\right)^2 a^2x^2 + atanh\left(ax\right)^2}dx$$

input
```
int(x^3/(-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
int(x**3/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)
**2),x)
```

3.288 $\int \frac{x^2}{(1-a^2x^2)^2 \text{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx = -\frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a^3}$$

output
```
-x^2/a/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx = \frac{x^2}{a(-1+a^2x^2)\operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a^3}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
x^2/(a*(-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6568, 6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{ 6568}$$

$$\frac{2\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx}{a} - \frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{2\int \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3} - \frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{2\int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3} - \frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3} - \frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\int -\frac{i\sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)} - \frac{i\int \frac{\sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a^3} - \frac{x^2}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

input
```
Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
-(x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{a^3}$	36
default	$\frac{\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{a^3}$	36

input
```
int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctan
h(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

$$\int \frac{x^2}{(1-a^2x^2)^2\,\mathrm{arctanh}(ax)^2}\,dx = \frac{4\,a^2x^2+\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,(a^5x^2-a^3)\log\left(-\frac{ax+1}{ax-1}\right)}$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*x^2 + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*
x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((
a^5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x^2}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)} \, dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x^2}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*x^2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - 4*integ
rate(-x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a
)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx$$
$$= \frac{2atanh(ax)\left(\int \frac{x}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+atanh(ax)}dx\right)a^2x^2 - 2atanh(ax)\left(\int \frac{x}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+ata}\right.}{atanh(ax)\,a\,(a^2x^2-1)}$$

input
```
int(x^2/(-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
(2*atanh(a*x)*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh
(a*x)),x)*a**2*x**2 - 2*atanh(a*x)*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a
*x)*a**2*x**2 + atanh(a*x)),x) + x**2)/(atanh(a*x)*a*(a**2*x**2 - 1))
```

3.289 $\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx = -\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{a^2}$$

output
```
-x/a/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx = \frac{\frac{ax}{(-1+a^2x^2)\operatorname{arctanh}(ax)} + \operatorname{Chi}(2\operatorname{arctanh}(ax))}{a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
((a*x)/((-1 + a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]])/a^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(36) = 72.

Time = 0.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx$$

$$\Big\downarrow \text{6594}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx}{a} + a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6530}$$

$$a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx + \frac{\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx + \frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3793}$$

$$a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx + \frac{\int \left(\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)} + \frac{1}{2\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx + \frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax)) + \frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2} - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\,\text{arctanh}(ax)}$$

input
```
Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
-(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + (CoshIntegral[2*ArcTanh[a*x]]/2 - Lo
g[ArcTanh[a*x]]/2)/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x
]]/2)/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Chi}(2\,\mathrm{arctanh}(ax))}{a^2}$	28
default	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Chi}(2\,\mathrm{arctanh}(ax))}{a^2}$	28

input
```
int(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/2/arctanh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.94

$$\int \frac{x}{(1-a^2x^2)^2\,\mathrm{arctanh}(ax)^2}\,dx = \frac{4\,ax+\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,(a^4x^2-a^2)\log\left(-\frac{ax+1}{ax-1}\right)}$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*(4*a*x + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2
- 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^4*
x^2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*x/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - integrate
(-2*(a^2*x^2 + 1)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a
^3*x^2 + a)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x}{atanh(ax)^2 a^4x^4 - 2atanh(ax)^2 a^2x^2 + atanh(ax)^2} dx$$

input
```
int(x/(-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
int(x/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2
),x)
```

3.290 $\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}\,dx=-\frac{1}{a(1-a^2x^2)\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a}$$

output
```
-1/a/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}\,dx=\frac{\frac{1}{(-1+a^2x^2)\operatorname{arctanh}(ax)}+\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
(1/((-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6528, 6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{ 6528}$$

$$2a \int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} dx - \frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{2 \int \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a} - \frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{2 \int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a} - \frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a} - \frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)} + \frac{\int -\frac{i \sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)} - \frac{i \int \frac{\sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a} - \frac{1}{a(1-a^2x^2) \operatorname{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{a}$	36
default	$\frac{-\frac{1}{2\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{a}$	36

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh
(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91

$$\int \frac{1}{(1-a^2x^2)^2\,\mathrm{arctanh}(ax)^2}\,dx = \frac{\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+4}{2\,(a^3x^2-a)\log\left(-\frac{ax+1}{ax-1}\right)}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log
_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) + 4)/((a^3*x^2
- a)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(ax-1\right)^2\left(ax+1\right)^2 \operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-4*a*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a
^2*x^2 + 1)*log(-a*x + 1)), x) + 2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2
- a)*log(-a*x + 1))
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{1}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx = \frac{2atanh(ax)\left(\int \frac{x}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+atanh(ax)}dx\right)a^4x^2 - 2atanh(ax)\left(\int \frac{x}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+ata}\right.}{atanh(ax)\,a\,(a^2x^2-1)}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
(2*atanh(a*x)*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh
(a*x)),x)*a**4*x**2 - 2*atanh(a*x)*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a
*x)*a**2*x**2 + atanh(a*x)),x)*a**2 + 1)/(atanh(a*x)*a*(a**2*x**2 - 1))
```

3.291 $\int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2} dx = -\frac{1}{ax\operatorname{arctanh}(ax)} - \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)} + \operatorname{Chi}(2\operatorname{arctanh}(ax)) - \frac{\operatorname{Int}\left(\frac{1}{x^2\operatorname{arctanh}(ax)}, x\right)}{a}$$

output
```
-1/a/x/arctanh(a*x)-a*x/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arctanh(a*x))-Defe
r(Int)(1/x^2/arctanh(a*x),x)/a
```

Mathematica [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2} dx = \int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^2} dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (1-a^2x^2)^2 \text{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^2 \text{arctanh}(ax)^2}dx + \int \frac{1}{x (1-a^2x^2) \text{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6552}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^2 \text{arctanh}(ax)^2}dx - \frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a} - \frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6468}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^2 \text{arctanh}(ax)^2}dx - \frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a} - \frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6594}$$

$$a^2 \left(\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}dx}{a} + a \int \frac{x^2}{(1-a^2x^2)^2 \text{arctanh}(ax)}dx - \frac{x}{a (1-a^2x^2) \text{arctanh}(ax)}\right) - \frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a} - \frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6530}$$

$$a^2\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 3042

$$a^2\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 3793

$$a^2\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arcta}}\right.$$
$$-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 2009

$$a^2\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 6596

$$a^2\left(\frac{\int \frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax}\right.$$
$$-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 3042

$$a^2\left(\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}\right.$$

$$\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{25}$$

$$a^2\left(-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}\right.$$

$$\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{3793}$$

$$a^2\left(-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a\left(1-\right.}\right.$$

$$\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}+$$

$$a^2\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arct}}\right.$$

$$\frac{1}{ax\text{arctanh}(ax)}$$

input
```
Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(-a^{2} x^{2}+1\right)^{2} \operatorname{arctanh}\left(a x\right)^{2}}d x$$

input
```
int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

output
```
int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx=\int \frac{1}{{\left(a^{2} x^{2} - 1\right)}^{2} x \operatorname{artanh}\left(a x\right)^{2}} \,d x$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \, dx=\int \frac{1}{x \left(a x - 1\right)^{2} \left(a x + 1\right)^{2} \operatorname{atanh}^{2}{\left(a x \right)}}\, dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

output
```
Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 5.59

$$\int \frac{1}{x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx = \int \frac{1}{(a^2x^2-1)^2 x \operatorname{artanh}(ax)^2} dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2/((a^3*x^3 - a*x)*log(a*x + 1) - (a^3*x^3 - a*x)*log(-a*x + 1)) - integra
te(-2*(3*a^2*x^2 - 1)/((a^5*x^6 - 2*a^3*x^4 + a*x^2)*log(a*x + 1) - (a^5*x
^6 - 2*a^3*x^4 + a*x^2)*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} dx = \int \frac{1}{(a^2x^2-1)^2 x \operatorname{artanh}(ax)^2} dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}\, dx = \int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^2}\, dx$$

input
```
int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}\, dx = \int \frac{1}{atanh\left(ax\right)^2 a^4x^5-2atanh\left(ax\right)^2 a^2x^3+atanh\left(ax\right)^2 x}dx$$

input
```
int(1/x/(-a^2*x^2+1)^2/atanh(a*x)^2,x)
```

output
```
int(1/(atanh(a*x)**2*a**4*x**5 - 2*atanh(a*x)**2*a**2*x***3 + atanh(a*x)**2
*x),x)
```

3.292 $\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}\,dx = \frac{x}{2a^3\operatorname{arctanh}(ax)^2} - \frac{x}{2a^3(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{1+a^2x^2}{2a^4(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a^4} - \frac{\operatorname{Int}\left(\frac{1}{\operatorname{arctanh}(ax)^2}, x\right)}{2a^3}$$

output
```
1/2*x/a^3/arctanh(a*x)^2-1/2*x/a^3/(-a^2*x^2+1)/arctanh(a*x)^2-1/2*(a^2*x^
2+1)/a^4/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))/a^4-1/2*Defer(Int)(
1/arctanh(a*x)^2,x)/a^3
```

Mathematica [N/A]

Not integrable

Time = 5.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx$$

input
```
Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{6548}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx}{a^2} - \frac{\frac{\int \frac{1}{\operatorname{arctanh}(ax)^2}dx}{2a} - \frac{x}{2a\operatorname{arctanh}(ax)^2}}{a^2}$$

$$\Big\downarrow \text{6444}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)^3}dx}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}$$

$\Big\downarrow$ 6558

$$\frac{2\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}$$

$\Big\downarrow$ 6596

$$\frac{\frac{2\int \frac{ax}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}$$

$\Big\downarrow$ 5971

$$\frac{\frac{2\int \frac{\sinh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}+\frac{\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\Big\downarrow$ 26

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}+$$
$$\frac{-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-$$
$$\frac{\frac{\int \frac{1}{\text{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\text{arctanh}(ax)^2}}{a^2}$$

input
```
Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(-a^2x^2+1)^2 \operatorname{arctanh}(ax)^3}dx$$

input
```
int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

output
```
int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^3(ax)} \, dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

output
```
Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 9.50

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
(2*a*x^3 - (a^2*x^4 - 3*x^2)*log(a*x + 1) + (a^2*x^4 - 3*x^2)*log(-a*x + 1
))/((a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a
*x + 1) + (a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-2*(a^4*x^5 - 2*a^2
*x^3 + 3*x)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1) - (a^6*x^4 - 2*a^4*x
^2 + a^2)*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} dx = \int \frac{x^3}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} dx = \int \frac{x^3}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^2} dx$$

input
```
int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} dx = \int \frac{x^3}{atanh(ax)^3 a^4x^4 - 2atanh(ax)^3 a^2x^2 + atanh(ax)^3} dx$$

input
```
int(x^3/(-a^2*x^2+1)^2/atanh(a*x)^3,x)
```

output
```
int(x**3/(atanh(a*x)**3*a**4*x**4 - 2*atanh(a*x)**3*a**2*x**2 + atanh(a*x)
**3),x)
```

3.293 $\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)^3}\,dx = -\frac{x^2}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2} - \frac{x}{a^2(1-a^2x^2)\,\text{arctanh}(ax)} + \frac{\text{Chi}(2\text{arctanh}(ax))}{a^3}$$

output
```
-1/2*x^2/a/(-a^2*x^2+1)/arctanh(a*x)^2-x/a^2/(-a^2*x^2+1)/arctanh(a*x)+Chi
(2*arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)^3}\,dx = \frac{\frac{ax(ax+2\text{arctanh}(ax))}{(-1+a^2x^2)\text{arctanh}(ax)^2} + 2\text{Chi}(2\text{arctanh}(ax))}{2a^3}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
((a*x*(a*x + 2*ArcTanh[a*x]))/((-1 + a^2*x^2)*ArcTanh[a*x]^2) + 2*CoshInte
gral[2*ArcTanh[a*x]])/(2*a^3)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6568, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^2 \text{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6568}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)^2}dx}{a}-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$\frac{\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx}{a}+a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}+\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}$$

$$\Big\downarrow \; 3793$$

$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 6596$$

$$\frac{\frac{\int \frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}+\frac{\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}+\frac{-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}$$

$$\Big\downarrow \; 3793$$

$$\frac{-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}$$

$$\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a}-$$

$$\frac{x^2}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}$$

input
```
Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
-1/2*x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) + (-(x/(a*(1 - a^2*x^2)*ArcTanh[
a*x])) + (CoshIntegral[2*ArcTanh[a*x]]/2 - Log[ArcTanh[a*x]]/2)/a^2 + (Cos
hIntegral[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x]]/2)/a^2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{1}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Chi}(2\,\mathrm{arctanh}(ax))}{a^3}$	51
default	$\frac{\frac{1}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Chi}(2\,\mathrm{arctanh}(ax))}{a^3}$	51

input
```
int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2/arct
anh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.05

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\, dx = \frac{4\,a^2x^2 + 4\,ax\log\left(-\frac{ax+1}{ax-1}\right) + \left((a^2x^2-1)\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + (a^2x^2-1)\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(\right.}{2\,(a^5x^2-a^3)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*x^2 + 4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_inte
gral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x +
1)))*log(-(a*x + 1)/(a*x - 1))^2)/((a^5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1
))^2)
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\, dx = \int \frac{x^2}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}^3(ax)}\, dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

output
```
Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^2}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
2*(a*x^2 + x*log(a*x + 1) - x*log(-a*x + 1))/((a^4*x^2 - a^2)*log(a*x + 1)
^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^4*x^2 - a^2)*log(-a
*x + 1)^2) - integrate(-2*(a^2*x^2 + 1)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a
*x + 1) - (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^2}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x^2}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx = \frac{2atanh(ax)^2 \left(\int \frac{x}{atanh(ax)^2a^4x^4-2atanh(ax)^2a^2x^2+atanh(ax)^2}dx\right)a^2x^2 - 2atanh(ax)^2 \left(\int \frac{x}{atanh(ax)^2a^4x^4-2atanh(ax)^2a}\right.}{2atanh\,(ax)^2\,a\,(a^2x^2-1)}$$

input
```
int(x^2/(-a^2*x^2+1)^2/atanh(a*x)^3,x)
```

output
```
(2*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**
2 + atanh(a*x)**2),x)*a**2*x**2 - 2*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**
4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x) + x**2)/(2*atanh(a*
x)**2*a*(a**2*x**2 - 1))
```

3.294 $\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{x}{(1-a^2x^2)^2\,\text{arctanh}(ax)^3}\,dx = -\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2} - \frac{1+a^2x^2}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)} + \frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}$$

output
```
-1/2*x/a/(-a^2*x^2+1)/arctanh(a*x)^2-1/2*(a^2*x^2+1)/a^2/(-a^2*x^2+1)/arct
anh(a*x)+Shi(2*arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x}{(1-a^2x^2)^2\,\text{arctanh}(ax)^3}\,dx = \frac{ax+(1+a^2x^2)\,\text{arctanh}(ax)+2(-1+a^2x^2)\,\text{arctanh}(ax)^2\text{Shi}(2\text{arctanh}(ax))}{2a^2(-1+a^2x^2)\,\text{arctanh}(ax)^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
(a*x + (1 + a^2*x^2)*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*SinhIn
tegral[2*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6558, 6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6558}$$

$$2\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{2\int \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{2\int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i\sin(2i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}$$

input
```
Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
-1/2*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^
2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{a^2}$	43
default	$\frac{-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{a^2}$	43

input
```
int(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/2/arctanh(a*x)*cosh(2*ar
ctanh(a*x))+Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.88

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx$$
$$= \frac{\left((a^2x^2-1)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,ax+2\,(a^2x^2+}{2\,(a^4x^2-a^2)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log
_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x + 2*(
a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^2 - a^2)*log(-(a*x + 1)/(a
*x - 1))^2)
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{x}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

output
```
Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
(2*a*x + (a^2*x^2 + 1)*log(a*x + 1) - (a^2*x^2 + 1)*log(-a*x + 1))/((a^4*x
^2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) +
(a^4*x^2 - a^2)*log(-a*x + 1)^2) - 4*integrate(-x/((a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{x}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx$$
$$= \frac{2atanh(ax)^2 \left(\int \frac{x}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+atanh(ax)}dx\right) a^3x^2 - 2atanh(ax)^2 \left(\int \frac{x}{atanh(ax)a^4x^4-2atanh(ax)a^2x^2+}\right.}{}$$

input
```
int(x/(-a^2*x^2+1)^2/atanh(a*x)^3,x)
```

output
```
(2*atanh(a*x)**2*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + at
anh(a*x)),x)*a**3*x**2 - 2*atanh(a*x)**2*int(x/(atanh(a*x)*a**4*x**4 - 2*a
tanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a + atanh(a*x)**2*int(1/(atanh(a*x)**
2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x)*a**2*x**2 - at
anh(a*x)**2*int(1/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + a
tanh(a*x)**2),x) + atanh(a*x)*a*x**2 + x)/(2*atanh(a*x)**2*a*(a**2*x**2 -
1))
```

3.295 $\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = -\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{x}{(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\text{Chi}(2\text{arctanh}(ax))}{a}$$

output
```
-1/2/a/(-a^2*x^2+1)/arctanh(a*x)^2-x/(-a^2*x^2+1)/arctanh(a*x)+Chi(2*arcta
nh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \frac{1+2ax\text{arctanh}(ax)+2(-1+a^2x^2)\operatorname{arctanh}(ax)^2\text{Chi}(2\text{arctanh}(ax))}{2a(-1+a^2x^2)\operatorname{arctanh}(ax)^2}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
(1 + 2*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*CoshIntegral[2*A
rcTanh[a*x]])/(2*a*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.78, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {6528, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6528}$$

$$a\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$a\left(\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx}{a}+a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$a\left(a\int \frac{x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}+a\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\int\frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)$$

$$\Big\downarrow \; 3793$$

$$a\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\int\left(\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\frac{1}{2\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctan}\right.$$
$$\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$a\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-$$
$$\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \; 6596$$

$$a\left(\frac{\int\frac{a^2x^2}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right.$$
$$\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}+a\left(\frac{\int-\frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}+a\left(-\frac{\int\frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)$$

$$\Big\downarrow \; 3793$$

$$a\left(-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a\,(1-}\right.$$
$$\frac{1}{2a\,(1-a^2x^2)\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$a\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arcta}}\right.$$
$$\frac{1}{2a\,(1-a^2x^2)\,\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) + a*(-(x/(a*(1 - a^2*x^2)*ArcTanh[
a*x])) + (CoshIntegral[2*ArcTanh[a*x]]/2 - Log[ArcTanh[a*x]]/2)/a^2 + (Cos
hIntegral[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x]]/2)/a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{4 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{2 \operatorname{arctanh}(ax)}+\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{a}$	51
default	$\frac{-\frac{1}{4 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{2 \operatorname{arctanh}(ax)}+\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{a}$	51

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2/arcta
nh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.10

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx = \frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right) + \left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + (a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{2\,(a^3x^2-a)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
1/2*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integral(-(a*x +
 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a
*x + 1)/(a*x - 1))^2 + 4)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
2*(a*x*log(a*x + 1) - a*x*log(-a*x + 1) + 1)/((a^3*x^2 - a)*log(a*x + 1)^2
 - 2*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1) + (a^3*x^2 - a)*log(-a*x + 1
)^2) - integrate(-2*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)
- (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{1}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}\,dx$$
$$= \frac{2atanh(ax)^2 \left(\int \frac{x}{atanh(ax)^2 a^4x^4 - 2atanh(ax)^2 a^2x^2 + atanh(ax)^2}dx\right) a^4x^2 - 2atanh(ax)^2 \left(\int \frac{x}{atanh(ax)^2 a^4x^4 - 2atanh(ax)^2 a^2x^2 + atanh(ax)^2}dx\right) a^2 + 1}{2atanh\,(ax)^2\, a\,(a^2x^2-1)}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^3,x)
```

output
```
(2*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**
2 + atanh(a*x)**2),x)*a**4*x**2 - 2*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**
4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x)*a**2 + 1)/(2*atanh(
a*x)**2*a*(a**2*x**2 - 1))
```

3.296 $\int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2ax\operatorname{arctanh}(ax)^2} - \frac{ax}{2(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\operatorname{arctanh}(ax)} + \operatorname{Shi}(2\operatorname{arctanh}(ax)) - \frac{\operatorname{Int}\left(\frac{1}{x^2\operatorname{arctanh}(ax)^2},x\right)}{2a}$$

output
```
-1/2/a/x/arctanh(a*x)^2-1/2*a*x/(-a^2*x^2+1)/arctanh(a*x)^2-1/2*(a^2*x^2+1
)/(-a^2*x^2+1)/arctanh(a*x)+Shi(2*arctanh(a*x))-1/2*Defer(Int)(1/x^2/arcta
nh(a*x)^2,x)/a
```

Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{x(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3}dx+\int \frac{1}{x\left(1-a^2x^2\right)\text{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6552}$$

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3}dx-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6468}$$

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3}dx-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6558}$$

$$a^2\left(2\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6596}$$

$$a^2\left(\frac{2\int \frac{ax}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\big\downarrow$ 5971

$$a^2\left(\frac{2\int \frac{\sinh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\big\downarrow$ 27

$$a^2\left(\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\big\downarrow$ 3042

$$a^2\left(\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\big\downarrow$ 26

$$a^2\left(-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\big\downarrow$ 3779

$$a^2\left(\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}+\frac{x}{2a\,(1-a^2x^2)\,\text{arctanh}(ax)^2}}{\frac{1}{2ax\text{arctanh}(ax)^2}}-\frac{a^2x^2+1}{2a^2\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-$$

input
```
Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(-a^2x^2+1)^2\,\text{arctanh}\,(ax)^3}dx$$

input
```
int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

output
```
int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}\,dx=\int \frac{1}{(a^2x^2-1)^2x\,\text{artanh}\,(ax)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{x\left(ax-1\right)^2\left(ax+1\right)^2 \operatorname{atanh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

output
```
Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 9.86

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\left(a^2x^2-1\right)^2 x \operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
(2*a*x + (3*a^2*x^2 - 1)*log(a*x + 1) - (3*a^2*x^2 - 1)*log(-a*x + 1))/((a
^4*x^4 - a^2*x^2)*log(a*x + 1)^2 - 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1)*log(
-a*x + 1) + (a^4*x^4 - a^2*x^2)*log(-a*x + 1)^2) - integrate(-2*(3*a^4*x^4
 - 2*a^2*x^2 + 1)/((a^6*x^7 - 2*a^4*x^5 + a^2*x^3)*log(a*x + 1) - (a^6*x^7
 - 2*a^4*x^5 + a^2*x^3)*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\left(a^2x^2-1\right)^2 x \operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{x \operatorname{atanh}\left(a\,x\right)^3 \left(a^2\,x^2-1\right)^2}\,dx$$

input
```
int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^3}\,dx$$
$$= \int \frac{1}{atanh\left(ax\right)^3 a^4x^5-2atanh\left(ax\right)^3 a^2x^3+atanh\left(ax\right)^3 x}dx$$

input
```
int(1/x/(-a^2*x^2+1)^2/atanh(a*x)^3,x)
```

output
```
int(1/(atanh(a*x)**3*a**4*x**5 - 2*atanh(a*x)**3*a**2*x**3 + atanh(a*x)**3
*x),x)
```

3.297 $\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} \, dx = -\frac{1}{3a(1-a^2x^2)\operatorname{arctanh}(ax)^3} - \frac{x}{3(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{1+a^2x^2}{3a(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3a}$$

output
```
-1/3/a/(-a^2*x^2+1)/arctanh(a*x)^3-1/3*x/(-a^2*x^2+1)/arctanh(a*x)^2-1/3*(
a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/3*Shi(2*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} \, dx = \frac{1 + ax\operatorname{arctanh}(ax) + (1+a^2x^2)\operatorname{arctanh}(ax)^2 + 2(-1+a^2x^2)\operatorname{arctanh}(ax)^3\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3a(-1+a^2x^2)\operatorname{arctanh}(ax)^3}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

output
```
(1 + a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*Ar
cTanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]])/(3*a*(-1 + a^2*x^2)*ArcTanh[a*x
]^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6528, 6558, 6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} dx$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{2}{3}a\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx - \frac{1}{3a(1-a^2x^2)\operatorname{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 6558}$$

$$\frac{2}{3}a\left(2\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}dx - \frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right) - \frac{1}{3a(1-a^2x^2)\operatorname{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right) - \frac{1}{3a(1-a^2x^2)\operatorname{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{\sinh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{1}{3a(1-a^2x^2)\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}a\left(\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{1}{3a(1-a^2x^2)\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{3a(1-a^2x^2)\,\text{arctanh}(ax)^3}+\frac{2}{3}a\left(\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{3a(1-a^2x^2)\,\text{arctanh}(ax)^3}+\frac{2}{3}a\left(-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)$$

$$\Big\downarrow \; 3779$$

$$\frac{2}{3}a\left(\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{1}{3a(1-a^2x^2)\,\text{arctanh}(ax)^3}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

output
```
-1/3*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^3) + (2*a*(-1/2*x/(a*(1 - a^2*x^2)*Ar
cTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhInt
egral[2*ArcTanh[a*x]]/a^2))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{-\frac{1}{6\operatorname{arctanh}(ax)^{3}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^{3}}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^{2}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{3\operatorname{arctanh}(ax)}+\frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3}}{a}$	68
default	$\frac{-\frac{1}{6\operatorname{arctanh}(ax)^{3}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^{3}}-\frac{\sinh(2\operatorname{arctanh}(ax))}{6\operatorname{arctanh}(ax)^{2}}-\frac{\cosh(2\operatorname{arctanh}(ax))}{3\operatorname{arctanh}(ax)}+\frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3}}{a}$	68

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/6/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6/arcta
nh(a*x)^2*sinh(2*arctanh(a*x))-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*S
hi(2*arctanh(a*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.56

$$\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^4}\,dx = \frac{\left((a^2x^2-1)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)}{3\,(a^3x^2-a)\log\left(-\frac{ax+1}{ax-1}\right)^3}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="fricas")
```

output
```
1/3*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log
_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^3 + 4*a*x*log(-
(a*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 8)/((
a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^3)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} dx = \int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^4(ax)} dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**4,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^4} dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="maxima")
```

output
```
-8*a*integrate(-1/3*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/3*(2*a*x*log(a*x + 1) + (a^2*x^2 +
1)*log(a*x + 1)^2 + (a^2*x^2 + 1)*log(-a*x + 1)^2 - 2*(a*x + (a^2*x^2 + 1)
*log(a*x + 1))*log(-a*x + 1) + 4)/((a^3*x^2 - a)*log(a*x + 1)^3 - 3*(a^3*x
^2 - a)*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^2 - a)*log(a*x + 1)*log(-a
*x + 1)^2 - (a^3*x^2 - a)*log(-a*x + 1)^3)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4}\,dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^4}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^4\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4}\,dx$$
$$= \frac{atanh(ax)^3\left(\int \frac{x^2}{atanh(ax)^2a^4x^4-2atanh(ax)^2a^2x^2+atanh(ax)^2}dx\right)a^5x^2 - atanh(ax)^3\left(\int \frac{x^2}{atanh(ax)^2a^4x^4-2atanh(ax)^2a^2x^2}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^4,x)
```

output
```
(atanh(a*x)**3*int(x**2/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x*
*2 + atanh(a*x)**2),x)*a**5*x**2 - atanh(a*x)**3*int(x**2/(atanh(a*x)**2*a
**4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x)*a**3 + 2*atanh(a*
x)**3*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a**2*x**2 + atanh(a*x)),x
)*a**4*x**2 - 2*atanh(a*x)**3*int(x/(atanh(a*x)*a**4*x**4 - 2*atanh(a*x)*a
**2*x**2 + atanh(a*x)),x)*a**2 + atanh(a*x)**2 + atanh(a*x)*a*x + 1)/(3*at
anh(a*x)**3*a*(a**2*x**2 - 1))
```

3.298 $\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5} dx = -\frac{1}{4a(1-a^2x^2)\operatorname{arctanh}(ax)^4} - \frac{x}{6(1-a^2x^2)\operatorname{arctanh}(ax)^3} - \frac{1+a^2x^2}{12a(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{x}{3(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{3a}$$

output
```
-1/4/a/(-a^2*x^2+1)/arctanh(a*x)^4-1/6*x/(-a^2*x^2+1)/arctanh(a*x)^3-1/12*
(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)^2-1/3*x/(-a^2*x^2+1)/arctanh(a*x)+
1/3*Chi(2*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^5}\,dx = \frac{3+2ax\operatorname{arctanh}(ax)+\left(1+a^2x^2\right)\operatorname{arctanh}(ax)^2+4ax\operatorname{arctanh}(ax)^3+4(-1+a^2x^2)\operatorname{arctanh}(ax)^4\operatorname{Chi}(2a}{12a\left(-1+a^2x^2\right)\operatorname{arctanh}(ax)^4}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

output
```
(3 + 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[a*x
]^3 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]])/(12*a*
(-1 + a^2*x^2)*ArcTanh[a*x]^4)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6528, 6558, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^5}\,dx$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^4}dx-\frac{1}{4a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^4}$$

$$\Big\downarrow \text{ 6558}$$

$$\frac{1}{2}a\left(\frac{2}{3}\int \frac{x}{\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^2}dx-\frac{x}{3a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^3}-\frac{a^2x^2+1}{6a^2\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^2}\right)-\frac{1}{4a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^4}$$

$$\Big\downarrow \text{ 6594}$$

$$\frac{1}{2}a\left(\frac{2}{3}\left(\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}dx}{a}+a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{}{3a\,(1-a^2\ldots}\right.$$
$$\frac{1}{4a\,(1-a^2x^2)\,\text{arctanh}(ax)^4}$$

$\big\downarrow$ 6530

$$\frac{1}{2}a\left(\frac{2}{3}\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\ldots\right.$$
$$\frac{1}{4a\,(1-a^2x^2)\,\text{arctanh}(ax)^4}$$

$\big\downarrow$ 3042

$$-\frac{1}{4a\,(1-a^2x^2)\,\text{arctanh}(ax)^4}+$$
$$\frac{1}{2}a\left(\frac{2}{3}\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\ldots\right.$$

$\big\downarrow$ 3793

$$\frac{1}{2}a\left(\frac{2}{3}\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{a}\ldots}\right.\right.$$
$$\frac{1}{4a\,(1-a^2x^2)\,\text{arctanh}(ax)^4}$$

$\big\downarrow$ 2009

$$\frac{1}{2}a\left(\frac{2}{3}\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(a\ldots}\right.\right.$$
$$\frac{1}{4a\,(1-a^2x^2)\,\text{arctanh}(ax)^4}$$

$\big\downarrow$ 6596

$$\frac{1}{2}a\left(\frac{2}{3}\left(\frac{\int \frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctan}\ldots}\right.\right.$$
$$\frac{1}{4a\,(1-a^2x^2)\,\text{arctanh}(ax)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4a(1-a^2x^2)\operatorname{arctanh}(ax)^4}+$$

$$\frac{1}{2}a\left(\frac{2}{3}\left(\frac{\int -\frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4a(1-a^2x^2)\operatorname{arctanh}(ax)^4}+$$

$$\frac{1}{2}a\left(\frac{2}{3}\left(-\frac{\int \frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}\right.\right.$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{2}a\left(\frac{2}{3}\left(-\frac{\int\left(\frac{1}{2\operatorname{arctanh}(ax)}-\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\right.\right.$$

$$\frac{1}{4a(1-a^2x^2)\operatorname{arctanh}(ax)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\left(\frac{2}{3}\left(\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))-\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)}\right.\right.$$

$$\frac{1}{4a(1-a^2x^2)\operatorname{arctanh}(ax)^4}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

output
```
-1/4*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^4) + (a*(-1/3*x/(a*(1 - a^2*x^2)*ArcT
anh[a*x]^3) - (1 + a^2*x^2)/(6*a^2*(1 - a^2*x^2)*ArcTanh[a*x]^2) + (2*(-(x
/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + (CoshIntegral[2*ArcTanh[a*x]]/2 - Log[A
rcTanh[a*x]]/2)/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x]]/
2)/a^2))/3))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{-\frac{1}{8 \operatorname{arctanh}(ax)^4}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)^4}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^3}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{3}}{a}$
default	$\frac{-\frac{1}{8 \operatorname{arctanh}(ax)^4}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)^4}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^3}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{3}}{a}$

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/8/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12*sinh
(2*arctanh(a*x))/arctanh(a*x)^3-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1
/6/arctanh(a*x)*sinh(2*arctanh(a*x))+1/3*Chi(2*arctanh(a*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5} \, dx$$
$$= \frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3 + \left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + (a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^4}{6\,(a^3x^2-a)\log\left(-\frac{ax+1}{ax-1}\right)^4}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="fricas")
```

output
```
1/6*(4*a*x*log(-(a*x + 1)/(a*x - 1))^3 + ((a^2*x^2 - 1)*log_integral(-(a*x
 + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-
(a*x + 1)/(a*x - 1))^4 + 8*a*x*log(-(a*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)
*log(-(a*x + 1)/(a*x - 1))^2 + 24)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1)
)^4)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5} \, dx = \int \frac{1}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}^5(ax)} \, dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**5,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**5), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}\,dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^5}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="maxima")
```

output
```
1/3*(2*a*x*log(a*x + 1)^3 - 2*a*x*log(-a*x + 1)^3 + 4*a*x*log(a*x + 1) + (
a^2*x^2 + 1)*log(a*x + 1)^2 + (a^2*x^2 + 6*a*x*log(a*x + 1) + 1)*log(-a*x
+ 1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*a*x + (a^2*x^2 + 1)*log(a*x + 1))*log
(-a*x + 1) + 12)/((a^3*x^2 - a)*log(a*x + 1)^4 - 4*(a^3*x^2 - a)*log(a*x +
 1)^3*log(-a*x + 1) + 6*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*(
a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^3 + (a^3*x^2 - a)*log(-a*x + 1)^4)
 - integrate(-2/3*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) -
(a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}\,dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^5}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^5\,(a^2\,x^2-1)^2}\,dx$$

input
```
int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}\,dx$$
$$= \frac{2atanh(ax)^4 \left(\int \frac{x^2}{atanh(ax)^3a^4x^4-2atanh(ax)^3a^2x^2+atanh(ax)^3}dx\right)a^5x^2 - 2atanh(ax)^4\left(\int \frac{x^2}{atanh(ax)^3a^4x^4-2atanh(ax)^3a}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^5,x)
```

output
```
(2*atanh(a*x)**4*int(x**2/(atanh(a*x)**3*a**4*x**4 - 2*atanh(a*x)**3*a**2*
x**2 + atanh(a*x)**3),x)*a**5*x**2 - 2*atanh(a*x)**4*int(x**2/(atanh(a*x)*
*3*a**4*x**4 - 2*atanh(a*x)**3*a**2*x**2 + atanh(a*x)**3),x)*a**3 + 2*atan
h(a*x)**4*int(x/(atanh(a*x)**2*a**4*x**4 - 2*atanh(a*x)**2*a**2*x**2 + ata
nh(a*x)**2),x)*a**4*x**2 - 2*atanh(a*x)**4*int(x/(atanh(a*x)**2*a**4*x**4
- 2*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x)*a**2 + atanh(a*x)**2 + 2*a
tanh(a*x)*a*x + 3)/(12*atanh(a*x)**4*a*(a**2*x**2 - 1))
```

3.299 $\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx = -\frac{1}{5a(1-a^2x^2)\operatorname{arctanh}(ax)^5} - \frac{x}{10(1-a^2x^2)\operatorname{arctanh}(ax)^4} - \frac{1+a^2x^2}{30a(1-a^2x^2)\operatorname{arctanh}(ax)^3} - \frac{x}{15(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{1+a^2x^2}{15a(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{15a}$$

output
```
-1/5/a/(-a^2*x^2+1)/arctanh(a*x)^5-1/10*x/(-a^2*x^2+1)/arctanh(a*x)^4-1/30
*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)^3-1/15*x/(-a^2*x^2+1)/arctanh(a*x
)^2-1/15*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/15*Shi(2*arctanh(a*x))/
a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx = \frac{6+3ax\operatorname{arctanh}(ax)+(1+a^2x^2)\operatorname{arctanh}(ax)^2+2ax\operatorname{arctanh}(ax)^3+2(1+a^2x^2)\operatorname{arctanh}(ax)^4+4(-1}{30a\,(-1+a^2x^2)\operatorname{arctanh}(ax)^5}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

output
```
(6 + 3*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*a*x*ArcTanh[a*x
]^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^5*Sin
hIntegral[2*ArcTanh[a*x]])/(30*a*(-1 + a^2*x^2)*ArcTanh[a*x]^5)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6528, 6558, 6558, 6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx$$

$$\Big\downarrow \; 6528$$

$$\frac{2}{5}a\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}dx-\frac{1}{5a\,(1-a^2x^2)\operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \; 6558$$

$$\frac{2}{5}a\left(\frac{1}{3}\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx-\frac{x}{4a\,(1-a^2x^2)\operatorname{arctanh}(ax)^4}-\frac{a^2x^2+1}{12a^2\,(1-a^2x^2)\operatorname{arctanh}(ax)^3}\right)-\frac{1}{5a\,(1-a^2x^2)\operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \; 6558$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(2\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{}{4a\left(1-\right.}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6596}$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(\frac{2\int \frac{ax}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{}{4}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{5971}$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(\frac{2\int \frac{\sinh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{}{4a}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{}{4a\,(}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5a\left(1-a^2x^2\right)\text{arctanh}(ax)^5}+$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{}{4}\right.$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{5a\left(1-a^2x^2\right)\text{arctanh}(ax)^5}+$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{}{4}\right.$$

$$\Big\downarrow \text{3779}$$

$$\frac{2}{5}a\left(\frac{1}{3}\left(\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a\,(1-a^2x^2)\,\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{x}{4a\,(1-a^2x^2)\,\text{arctan}}\right.$$
$$\frac{1}{5a\,(1-a^2x^2)\,\text{arctanh}(ax)^5}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

output
```
-1/5*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^5) + (2*a*(-1/4*x/(a*(1 - a^2*x^2)*Ar
cTanh[a*x]^4) - (1 + a^2*x^2)/(12*a^2*(1 - a^2*x^2)*ArcTanh[a*x]^3) + (-1/
2*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^2)*
ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2)/3))/5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{1}{10 \operatorname{arctanh}(ax)^5}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{10 \operatorname{arctanh}(ax)^5}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{20 \operatorname{arctanh}(ax)^4}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{30 \operatorname{arctanh}(ax)^3}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{30 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax)}{15 \operatorname{arctanh}(ax)}}{a}$
default	$\frac{-\frac{1}{10 \operatorname{arctanh}(ax)^5}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{10 \operatorname{arctanh}(ax)^5}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{20 \operatorname{arctanh}(ax)^4}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{30 \operatorname{arctanh}(ax)^3}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{30 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax)}{15 \operatorname{arctanh}(ax)}}{a}$

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/10/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/ar
ctanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))
-1/30/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/15/arctanh(a*x)*cosh(2*arctanh
(a*x))+2/15*Shi(2*arctanh(a*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.30

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6}\,dx = \frac{\left((a^2x^2-1)\log\_\text{integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\log\_\text{integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^5+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3}{15\,(a^3x^2-a)\log\left(-\frac{ax+}{ax-}\right.}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="fricas")
```

output
```
1/15*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*lo
g_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^5 + 4*a*x*log(
-(a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 24
*a*x*log(-(a*x + 1)/(a*x - 1)) + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))
^2 + 96)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^5)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6}\,dx = \int \frac{1}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^6(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**6,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**6), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^6} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="maxima")
```

output
```
-8*a*integrate(-1/15*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4
- 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/15*(2*a*x*log(a*x + 1)^3 + (a^2*x^
2 + 1)*log(a*x + 1)^4 + (a^2*x^2 + 1)*log(-a*x + 1)^4 - 2*(a*x + 2*(a^2*x^
2 + 1)*log(a*x + 1))*log(-a*x + 1)^3 + 12*a*x*log(a*x + 1) + 2*(a^2*x^2 +
1)*log(a*x + 1)^2 + 2*(a^2*x^2 + 3*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(
a*x + 1)^2 + 1)*log(-a*x + 1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*(a^2*x^2 + 1
)*log(a*x + 1)^3 + 6*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 4
8)/((a^3*x^2 - a)*log(a*x + 1)^5 - 5*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x
 + 1) + 10*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^3*x^2 - a)
*log(a*x + 1)^2*log(-a*x + 1)^3 + 5*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x +
1)^4 - (a^3*x^2 - a)*log(-a*x + 1)^5)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^6} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx = \int \frac{1}{\operatorname{atanh}(a\,x)^6 \,(a^2\,x^2-1)^2} \, dx$$

input
```
int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6} \, dx = \frac{3atanh(ax)^5 \left(\int \frac{x^2}{atanh(ax)^4 a^4x^4-2atanh(ax)^4 a^2x^2+atanh(ax)^4} dx\right) a^5x^2 - 3atanh(ax)^5 \left(\int \frac{x^2}{atanh(ax)^4 a^4x^4-2atanh(ax)^4 a}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^6,x)
```

output
```
(3*atanh(a*x)**5*int(x**2/(atanh(a*x)**4*a**4*x**4 - 2*atanh(a*x)**4*a**2*
x**2 + atanh(a*x)**4),x)*a**5*x**2 - 3*atanh(a*x)**5*int(x**2/(atanh(a*x)*
*4*a**4*x**4 - 2*atanh(a*x)**4*a**2*x**2 + atanh(a*x)**4),x)*a**3 + 2*atan
h(a*x)**5*int(x/(atanh(a*x)**3*a**4*x**4 - 2*atanh(a*x)**3*a**2*x**2 + ata
nh(a*x)**3),x)*a**4*x**2 - 2*atanh(a*x)**5*int(x/(atanh(a*x)**3*a**4*x**4
- 2*atanh(a*x)**3*a**2*x**2 + atanh(a*x)**3),x)*a**2 + atanh(a*x)**2 + 3*a
tanh(a*x)*a*x + 6)/(30*atanh(a*x)**5*a*(a**2*x**2 - 1))
```

3.300 $\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^7}\,dx = -\frac{1}{6a(1-a^2x^2)\operatorname{arctanh}(ax)^6} - \frac{x}{15(1-a^2x^2)\operatorname{arctanh}(ax)^5} - \frac{1+a^2x^2}{60a(1-a^2x^2)\operatorname{arctanh}(ax)^4} - \frac{x}{45(1-a^2x^2)\operatorname{arctanh}(ax)^3} - \frac{1+a^2x^2}{90a(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{2x}{45(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{2\operatorname{Chi}(2\operatorname{arctanh}(ax))}{45a}$$

output
```
-1/6/a/(-a^2*x^2+1)/arctanh(a*x)^6-1/15*x/(-a^2*x^2+1)/arctanh(a*x)^5-1/60
*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)^4-1/45*x/(-a^2*x^2+1)/arctanh(a*x
)^3-1/90*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)^2-2/45*x/(-a^2*x^2+1)/arc
tanh(a*x)+2/45*Chi(2*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} \, dx$$
$$= \frac{30 + 12ax\operatorname{arctanh}(ax) + 3(1+a^2x^2)\operatorname{arctanh}(ax)^2 + 4ax\operatorname{arctanh}(ax)^3 + 2(1+a^2x^2)\operatorname{arctanh}(ax)^4 + 8a}{180a\,(-1+a^2x^2)\operatorname{arctanh}(ax)^6}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

output
```
(30 + 12*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh
[a*x]^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 8*a*x*ArcTanh[a*x]^5 + 8*(-1 +
a^2*x^2)*ArcTanh[a*x]^6*CoshIntegral[2*ArcTanh[a*x]])/(180*a*(-1 + a^2*x^2
)*ArcTanh[a*x]^6)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {6528, 6558, 6558, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} \, dx$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{1}{3}a\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^6}dx - \frac{1}{6a\,(1-a^2x^2)\operatorname{arctanh}(ax)^6}$$

$$\Big\downarrow \text{ 6558}$$

$$\frac{1}{3}a\left(\frac{1}{5}\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4}dx - \frac{x}{5a\,(1-a^2x^2)\operatorname{arctanh}(ax)^5} - \frac{a^2x^2+1}{20a^2\,(1-a^2x^2)\operatorname{arctanh}(ax)^4}\right) - \frac{1}{6a\,(1-a^2x^2)\operatorname{arctanh}(ax)^6}$$

$$\Big\downarrow \text{6558}$$

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\int\frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}dx-\frac{x}{3a(1-a^2x^2)\operatorname{arctanh}(ax)^3}-\frac{a^2x^2+1}{6a^2(1-a^2x^2)\operatorname{arctanh}(ax)^2}\right)-\frac{}{5a(1}\right.$$
$$\frac{1}{6a(1-a^2x^2)\operatorname{arctanh}(ax)^6}$$

$$\Big\downarrow \text{6594}$$

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(\frac{\int\frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx}{a}+a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-\frac{}{3a(1-}\right.\right.$$
$$\frac{1}{6a(1-a^2x^2)\operatorname{arctanh}(ax)^6}$$

$$\Big\downarrow \text{6530}$$

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\int\frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)\right.\right.$$
$$\frac{1}{6a(1-a^2x^2)\operatorname{arctanh}(ax)^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6a(1-a^2x^2)\operatorname{arctanh}(ax)^6}+$$
$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\int\frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right.\right.\right.$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\int\left(\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\frac{1}{2\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}-\frac{}{a(1-a^2x}\right.\right.\right.$$
$$\frac{1}{6a(1-a^2x^2)\operatorname{arctanh}(ax)^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(a\int\frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\operatorname{arctanh}\right.}\right.\right.\right.$$
$$\frac{1}{6a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^6}$$

$\Big\downarrow$ 6596

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(\frac{\int\frac{a^2x^2}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\operatorname{arc}\right.}\right.\right.\right.$$
$$\frac{1}{6a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^6}$$

$\Big\downarrow$ 3042

$$-\frac{1}{6a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^6}+$$
$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(\frac{\int-\frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\operatorname{arct}\right.}\right.\right.\right.$$

$\Big\downarrow$ 25

$$-\frac{1}{6a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^6}+$$
$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(-\frac{\int\frac{\sin(i\operatorname{arctanh}(ax))^2}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\operatorname{arct}\right.}\right.\right.\right.$$

$\Big\downarrow$ 3793

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(-\frac{\int\left(\frac{1}{2\operatorname{arctanh}(ax)}-\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}\right.\right.\right.$$
$$\frac{1}{6a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^6}$$

$\Big\downarrow$ 2009

$$\frac{1}{3}a\left(\frac{1}{5}\left(\frac{2}{3}\left(\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))-\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{2}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{}{a\left(1-a^2\right.}\right.\right.\right.$$
$$\frac{1}{6a\left(1-a^2x^2\right)\operatorname{arctanh}(ax)^6}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

output
```
-1/6*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^6) + (a*(-1/5*x/(a*(1 - a^2*x^2)*ArcT
anh[a*x]^5) - (1 + a^2*x^2)/(20*a^2*(1 - a^2*x^2)*ArcTanh[a*x]^4) + (-1/3*
x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - (1 + a^2*x^2)/(6*a^2*(1 - a^2*x^2)*Ar
cTanh[a*x]^2) + (2*(-(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + (CoshIntegral[2*
ArcTanh[a*x]]/2 - Log[ArcTanh[a*x]]/2)/a^2 + (CoshIntegral[2*ArcTanh[a*x]]
/2 + Log[ArcTanh[a*x]]/2)/a^2))/3)/5))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{1}{12\operatorname{arctanh}(ax)^6}-\frac{\cosh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^6}-\frac{\sinh(2\operatorname{arctanh}(ax))}{30\operatorname{arctanh}(ax)^5}-\frac{\cosh(2\operatorname{arctanh}(ax))}{60\operatorname{arctanh}(ax)^4}-\frac{\sinh(2\operatorname{arctanh}(ax))}{90\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax)}{90\operatorname{arctanh}(ax)^2}}{a}$
default	$\frac{-\frac{1}{12\operatorname{arctanh}(ax)^6}-\frac{\cosh(2\operatorname{arctanh}(ax))}{12\operatorname{arctanh}(ax)^6}-\frac{\sinh(2\operatorname{arctanh}(ax))}{30\operatorname{arctanh}(ax)^5}-\frac{\cosh(2\operatorname{arctanh}(ax))}{60\operatorname{arctanh}(ax)^4}-\frac{\sinh(2\operatorname{arctanh}(ax))}{90\operatorname{arctanh}(ax)^3}-\frac{\cosh(2\operatorname{arctanh}(ax)}{90\operatorname{arctanh}(ax)^2}}{a}$

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/12/arctanh(a*x)^6-1/12/arctanh(a*x)^6*cosh(2*arctanh(a*x))-1/30/ar
ctanh(a*x)^5*sinh(2*arctanh(a*x))-1/60/arctanh(a*x)^4*cosh(2*arctanh(a*x))
-1/90*sinh(2*arctanh(a*x))/arctanh(a*x)^3-1/90/arctanh(a*x)^2*cosh(2*arcta
nh(a*x))-1/45/arctanh(a*x)*sinh(2*arctanh(a*x))+2/45*Chi(2*arctanh(a*x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} dx = \frac{4\,ax \log\left(-\frac{ax+1}{ax-1}\right)^5 + \left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + (a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^6}{45\,(a^3x^2-a}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="fricas")
```

output
```
1/45*(4*a*x*log(-(a*x + 1)/(a*x - 1))^5 + ((a^2*x^2 - 1)*log_integral(-(a*
x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(
-(a*x + 1)/(a*x - 1))^6 + 8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 +
 1)*log(-(a*x + 1)/(a*x - 1))^4 + 96*a*x*log(-(a*x + 1)/(a*x - 1)) + 12*(a
^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 480)/((a^3*x^2 - a)*log(-(a*x +
1)/(a*x - 1))^6)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} dx = \int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^7(ax)} dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**7,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**7), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^7} dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="maxima")
```

output
```
2/45*(2*a*x*log(a*x + 1)^5 - 2*a*x*log(-a*x + 1)^5 + 4*a*x*log(a*x + 1)^3
+ (a^2*x^2 + 1)*log(a*x + 1)^4 + (a^2*x^2 + 10*a*x*log(a*x + 1) + 1)*log(-
a*x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + a*x + (a^2*x^2 + 1)*log(a*x + 1))*l
og(-a*x + 1)^3 + 48*a*x*log(a*x + 1) + 6*(a^2*x^2 + 1)*log(a*x + 1)^2 + 2*
(10*a*x*log(a*x + 1)^3 + 3*a^2*x^2 + 6*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*
log(a*x + 1)^2 + 3)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x + 1)^4 + 6*a*x*log(
a*x + 1)^2 + 2*(a^2*x^2 + 1)*log(a*x + 1)^3 + 24*a*x + 6*(a^2*x^2 + 1)*log
(a*x + 1))*log(-a*x + 1) + 240)/((a^3*x^2 - a)*log(a*x + 1)^6 - 6*(a^3*x^2
 - a)*log(a*x + 1)^5*log(-a*x + 1) + 15*(a^3*x^2 - a)*log(a*x + 1)^4*log(-
a*x + 1)^2 - 20*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^3 + 15*(a^3*x^2
 - a)*log(a*x + 1)^2*log(-a*x + 1)^4 - 6*(a^3*x^2 - a)*log(a*x + 1)*log(-a
*x + 1)^5 + (a^3*x^2 - a)*log(-a*x + 1)^6) - integrate(-4/45*(a^2*x^2 + 1)
/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-
a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^7} dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} \, dx = \int \frac{1}{\operatorname{atanh}(a\,x)^7\,(a^2\,x^2-1)^2} \, dx$$

input
```
int(1/(atanh(a*x)^7*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^7*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7} \, dx$$
$$= \frac{4atanh(ax)^6 \left(\int \frac{x^2}{atanh(ax)^5 a^4x^4-2atanh(ax)^5 a^2x^2+atanh(ax)^5} dx\right) a^5x^2 - 4atanh(ax)^6 \left(\int \frac{x^2}{atanh(ax)^5 a^4x^4-2atanh(ax)^5 a\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^7,x)
```

output
```
(4*atanh(a*x)**6*int(x**2/(atanh(a*x)**5*a**4*x**4 - 2*atanh(a*x)**5*a**2*
x**2 + atanh(a*x)**5),x)*a**5*x**2 - 4*atanh(a*x)**6*int(x**2/(atanh(a*x)*
*5*a**4*x**4 - 2*atanh(a*x)**5*a**2*x**2 + atanh(a*x)**5),x)*a**3 + 2*atan
h(a*x)**6*int(x/(atanh(a*x)**4*a**4*x**4 - 2*atanh(a*x)**4*a**2*x**2 + ata
nh(a*x)**4),x)*a**4*x**2 - 2*atanh(a*x)**6*int(x/(atanh(a*x)**4*a**4*x**4
- 2*atanh(a*x)**4*a**2*x**2 + atanh(a*x)**4),x)*a**2 + atanh(a*x)**2 + 4*a
tanh(a*x)*a*x + 10)/(60*atanh(a*x)**6*a*(a**2*x**2 - 1))
```

3.301 $\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 211

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8} \, dx = -\frac{1}{7a(1-a^2x^2)\operatorname{arctanh}(ax)^7} - \frac{x}{21(1-a^2x^2)\operatorname{arctanh}(ax)^6} - \frac{1+a^2x^2}{105a(1-a^2x^2)\operatorname{arctanh}(ax)^5} - \frac{x}{105(1-a^2x^2)\operatorname{arctanh}(ax)^4} - \frac{1+a^2x^2}{315a(1-a^2x^2)\operatorname{arctanh}(ax)^3} - \frac{2x}{315(1-a^2x^2)\operatorname{arctanh}(ax)^2} - \frac{2(1+a^2x^2)}{315a(1-a^2x^2)\operatorname{arctanh}(ax)} + \frac{4\operatorname{Shi}(2\operatorname{arctanh}(ax))}{315a}$$

output
```
-1/7/a/(-a^2*x^2+1)/arctanh(a*x)^7-1/21*x/(-a^2*x^2+1)/arctanh(a*x)^6-1/10
5*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)^5-1/105*x/(-a^2*x^2+1)/arctanh(a
*x)^4-1/315*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)^3-2/315*x/(-a^2*x^2+1)
/arctanh(a*x)^2-2/315*(a^2*x^2+1)/a/(-a^2*x^2+1)/arctanh(a*x)+4/315*Shi(2*
arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8} \, dx = \frac{45 + 15ax\operatorname{arctanh}(ax) + 3(1+a^2x^2) \operatorname{arctanh}(ax)^2 + 3ax\operatorname{arctanh}(ax)^3 + (1+a^2x^2) \operatorname{arctanh}(ax)^4 + 2ax}{315a\,(-1+a^2x^2) \operatorname{arct}}$$

input
```
Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

output
```
(45 + 15*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh
[a*x]^3 + (1 + a^2*x^2)*ArcTanh[a*x]^4 + 2*a*x*ArcTanh[a*x]^5 + 2*(1 + a^2
*x^2)*ArcTanh[a*x]^6 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^7*SinhIntegral[2*ArcT
anh[a*x]])/(315*a*(-1 + a^2*x^2)*ArcTanh[a*x]^7)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6528, 6558, 6558, 6558, 6596, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8} \, dx$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{2}{7}a\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^7}dx - \frac{1}{7a\,(1-a^2x^2) \operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{ 6558}$$

$$\frac{2}{7}a\left(\frac{2}{15}\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}dx - \frac{x}{6a\,(1-a^2x^2) \operatorname{arctanh}(ax)^6} - \frac{a^2x^2+1}{30a^2\,(1-a^2x^2) \operatorname{arctanh}(ax)^5}\right) - \frac{1}{7a\,(1-a^2x^2) \operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{6558}$$

$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\int\frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}dx-\frac{x}{4a(1-a^2x^2)\operatorname{arctanh}(ax)^4}-\frac{a^2x^2+1}{12a^2(1-a^2x^2)\operatorname{arctanh}(ax)^3}\right)-\frac{}{6a}\right.$$
$$\frac{1}{7a(1-a^2x^2)\operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{6558}$$

$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(2\int\frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-\frac{}{4a}\right.\right.$$
$$\frac{1}{7a(1-a^2x^2)\operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{6596}$$

$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(\frac{2\int\frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)\right.\right.$$
$$\frac{1}{7a(1-a^2x^2)\operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{5971}$$

$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(\frac{2\int\frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)\right.\right.$$
$$\frac{1}{7a(1-a^2x^2)\operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(\frac{\int\frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-\right.\right.$$
$$\frac{1}{7a(1-a^2x^2)\operatorname{arctanh}(ax)^7}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7a(1-a^2x^2)\text{arctanh}(ax)^7}+$$
$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}\right)\right.\right.$$

$\downarrow$ 26

$$-\frac{1}{7a(1-a^2x^2)\text{arctanh}(ax)^7}+$$
$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}\right)\right.\right.$$

$\downarrow$ 3779

$$\frac{2}{7}a\left(\frac{2}{15}\left(\frac{1}{3}\left(\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{x}{4a(1-a^2x^2)\text{a}}\right.\right.$$
$$\frac{1}{7a(1-a^2x^2)\text{arctanh}(ax)^7}$$

input
```
Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

output
```
-1/7*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^7) + (2*a*(-1/6*x/(a*(1 - a^2*x^2)*Ar
cTanh[a*x]^6) - (1 + a^2*x^2)/(30*a^2*(1 - a^2*x^2)*ArcTanh[a*x]^5) + (2*(
-1/4*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - (1 + a^2*x^2)/(12*a^2*(1 - a^2*x
^2)*ArcTanh[a*x]^3) + (-1/2*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*
x^2)/(2*a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2
)/3))/15))/7
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{14\,\mathrm{arctanh}(ax)^7}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{14\,\mathrm{arctanh}(ax)^7}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{42\,\mathrm{arctanh}(ax)^6}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{105\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{210\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{315\,\mathrm{arctanh}(ax)^3}}{a}$
default	$\frac{-\frac{1}{14\,\mathrm{arctanh}(ax)^7}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{14\,\mathrm{arctanh}(ax)^7}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{42\,\mathrm{arctanh}(ax)^6}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{105\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{210\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{315\,\mathrm{arctanh}(ax)^3}}{a}$

input
```
int(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/14/arctanh(a*x)^7-1/14/arctanh(a*x)^7*cosh(2*arctanh(a*x))-1/42/ar
ctanh(a*x)^6*sinh(2*arctanh(a*x))-1/105/arctanh(a*x)^5*cosh(2*arctanh(a*x)
)-1/210/arctanh(a*x)^4*sinh(2*arctanh(a*x))-1/315/arctanh(a*x)^3*cosh(2*ar
ctanh(a*x))-1/315/arctanh(a*x)^2*sinh(2*arctanh(a*x))-2/315/arctanh(a*x)*c
osh(2*arctanh(a*x))+4/315*Shi(2*arctanh(a*x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.18

$$\int \frac{1}{(1-a^2x^2)^2\,\mathrm{arctanh}(ax)^8}\,dx = \frac{2\left(\left((a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-(a^2x^2-1)\,\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^7+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)\right.}{}$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="fricas")
```

output
```
2/315*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*l
og_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^7 + 4*a*x*log
(-(a*x + 1)/(a*x - 1))^5 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^6 + 2
4*a*x*log(-(a*x + 1)/(a*x - 1))^3 + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
1))^4 + 480*a*x*log(-(a*x + 1)/(a*x - 1)) + 48*(a^2*x^2 + 1)*log(-(a*x + 1
)/(a*x - 1))^2 + 2880)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^7)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8} \, dx = \int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^8(ax)} \, dx$$

input
```
integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**8,x)
```

output
```
Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**8), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8} \, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^8} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="maxima")
```

output
```
-16*a*integrate(-1/315*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^
4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 4/315*(2*a*x*log(a*x + 1)^5 + (a^2
*x^2 + 1)*log(a*x + 1)^6 + (a^2*x^2 + 1)*log(-a*x + 1)^6 - 2*(a*x + 3*(a^2
*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)^5 + 12*a*x*log(a*x + 1)^3 + 2*(a^2*x
^2 + 1)*log(a*x + 1)^4 + (2*a^2*x^2 + 10*a*x*log(a*x + 1) + 15*(a^2*x^2 +
1)*log(a*x + 1)^2 + 2)*log(-a*x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + 5*(a^2*
x^2 + 1)*log(a*x + 1)^3 + 3*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x +
 1)^3 + 240*a*x*log(a*x + 1) + 24*(a^2*x^2 + 1)*log(a*x + 1)^2 + (20*a*x*l
og(a*x + 1)^3 + 15*(a^2*x^2 + 1)*log(a*x + 1)^4 + 24*a^2*x^2 + 36*a*x*log(
a*x + 1) + 12*(a^2*x^2 + 1)*log(a*x + 1)^2 + 24)*log(-a*x + 1)^2 - 2*(5*a*
x*log(a*x + 1)^4 + 3*(a^2*x^2 + 1)*log(a*x + 1)^5 + 18*a*x*log(a*x + 1)^2
+ 4*(a^2*x^2 + 1)*log(a*x + 1)^3 + 120*a*x + 24*(a^2*x^2 + 1)*log(a*x + 1)
)*log(-a*x + 1) + 1440)/((a^3*x^2 - a)*log(a*x + 1)^7 - 7*(a^3*x^2 - a)*lo
g(a*x + 1)^6*log(-a*x + 1) + 21*(a^3*x^2 - a)*log(a*x + 1)^5*log(-a*x + 1)
^2 - 35*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^3 + 35*(a^3*x^2 - a)*lo
g(a*x + 1)^3*log(-a*x + 1)^4 - 21*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x +
1)^5 + 7*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^6 - (a^3*x^2 - a)*log(-a
*x + 1)^7)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8}\, dx = \int \frac{1}{(a^2x^2-1)^2 \operatorname{artanh}(ax)^8}\, dx$$

input
```
integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8}\, dx = \int \frac{1}{\operatorname{atanh}(a\,x)^8\,(a^2\,x^2-1)^2}\, dx$$

input
```
int(1/(atanh(a*x)^8*(a^2*x^2 - 1)^2),x)
```

output
```
int(1/(atanh(a*x)^8*(a^2*x^2 - 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^8}\, dx = \frac{5atanh(ax)^7 \left(\int \frac{x^2}{atanh(ax)^6 a^4x^4-2atanh(ax)^6 a^2x^2+atanh(ax)^6}dx\right) a^5x^2 - 5atanh(ax)^7 \left(\int \frac{x^2}{atanh(ax)^6 a^4x^4-2atanh(ax)^6 a}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^2/atanh(a*x)^8,x)
```

output
```
(5*atanh(a*x)**7*int(x**2/(atanh(a*x)**6*a**4*x**4 - 2*atanh(a*x)**6*a**2*
x**2 + atanh(a*x)**6),x)*a**5*x**2 - 5*atanh(a*x)**7*int(x**2/(atanh(a*x)*
*6*a**4*x**4 - 2*atanh(a*x)**6*a**2*x**2 + atanh(a*x)**6),x)*a**3 + 2*atan
h(a*x)**7*int(x/(atanh(a*x)**5*a**4*x**4 - 2*atanh(a*x)**5*a**2*x**2 + ata
nh(a*x)**5),x)*a**4*x**2 - 2*atanh(a*x)**7*int(x/(atanh(a*x)**5*a**4*x**4
- 2*atanh(a*x)**5*a**2*x**2 + atanh(a*x)**5),x)*a**2 + atanh(a*x)**2 + 5*a
tanh(a*x)*a*x + 15)/(105*atanh(a*x)**7*a*(a**2*x**2 - 1))
```

3.302 $\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3} dx = -\frac{x^3}{16a(1-a^2x^2)^2} + \frac{3x}{32a^3(1-a^2x^2)} - \frac{3\text{arctanh}(ax)}{32a^4} + \frac{x^4\text{arctanh}(ax)}{4(1-a^2x^2)^2}$$

output
```
-1/16*x^3/a/(-a^2*x^2+1)^2+3/32*x/a^3/(-a^2*x^2+1)-3/32*arctanh(a*x)/a^4+1
/4*x^4*arctanh(a*x)/(-a^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3} dx = -\frac{x}{16a^3(-1+a^2x^2)^2} - \frac{5x}{32a^3(-1+a^2x^2)} + \frac{(-1+2a^2x^2)\,\text{arctanh}(ax)}{4a^4(-1+a^2x^2)^2} - \frac{5\log(1-ax)}{64a^4} + \frac{5\log(1+ax)}{64a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

output
```
-1/16*x/(a^3*(-1 + a^2*x^2)^2) - (5*x)/(32*a^3*(-1 + a^2*x^2)) + ((-1 + 2*
a^2*x^2)*ArcTanh[a*x])/(4*a^4*(-1 + a^2*x^2)^2) - (5*Log[1 - a*x])/(64*a^4
) + (5*Log[1 + a*x])/(64*a^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6570, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6570}$$

$$\frac{x^4\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{4}a\int \frac{x^4}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{4}a\left(\frac{x^3}{4a^2(1-a^2x^2)^2}-\frac{3\int \frac{x^2}{(1-a^2x^2)^2}dx}{4a^2}\right)$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{4}a\left(\frac{x^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{1-a^2x^2}dx}{2a^2}\right)}{4a^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x^4\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{4}a\left(\frac{x^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x}{2a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)}{2a^3}\right)}{4a^2}\right)$$

input
```
Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

output
```
(x^4*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) - (a*(x^3/(4*a^2*(1 - a^2*x^2)^2) -
 (3*(x/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]/(2*a^3)))/(4*a^2)))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result
parallelrisch	$-\frac{-5a^4x^4\,\text{arctanh}(ax)+5a^3x^3-6a^2x^2\,\text{arctanh}(ax)-3ax+3\,\text{arctanh}(ax)}{32(a^2x^2-1)^2a^4}$
derivativedivides	$\frac{\frac{\text{arctanh}(ax)}{16(ax+1)^2}-\frac{3\,\text{arctanh}(ax)}{16(ax+1)}+\frac{\text{arctanh}(ax)}{16(ax-1)^2}+\frac{3\,\text{arctanh}(ax)}{16(ax-1)}+\frac{1}{64(ax+1)^2}-\frac{5}{64(ax+1)}+\frac{5\ln(ax+1)}{64}-\frac{1}{64(ax-1)^2}-\frac{5}{64(ax-1)}-\frac{5\,\text{l}}{}}{a^4}$
default	$\frac{\frac{\text{arctanh}(ax)}{16(ax+1)^2}-\frac{3\,\text{arctanh}(ax)}{16(ax+1)}+\frac{\text{arctanh}(ax)}{16(ax-1)^2}+\frac{3\,\text{arctanh}(ax)}{16(ax-1)}+\frac{1}{64(ax+1)^2}-\frac{5}{64(ax+1)}+\frac{5\ln(ax+1)}{64}-\frac{1}{64(ax-1)^2}-\frac{5}{64(ax-1)}-\frac{5\,\text{l}}{}}{a^4}$
parts	$\frac{\text{arctanh}(ax)}{16a^4(ax+1)^2}-\frac{3\,\text{arctanh}(ax)}{16a^4(ax+1)}+\frac{\text{arctanh}(ax)}{16a^4(ax-1)^2}+\frac{3\,\text{arctanh}(ax)}{16a^4(ax-1)}-\frac{-\frac{1}{16a(ax+1)^2}+\frac{5}{16a(ax+1)}-\frac{5\ln(ax+1)}{16a}+\frac{}{16a(}}{4a^3}$
orering	$-\frac{(ax-1)(ax+1)(5a^4x^4+3a^2x^2-3)\,\text{arctanh}(ax)}{8a^4(-a^2x^2+1)^3}-\frac{(5a^2x^2-3)(ax+1)^2(ax-1)^2\left(\frac{3x^2\,\text{arctanh}(ax)}{(-a^2x^2+1)^3}+\frac{x^3a}{(-a^2x^2+1)^4}+\frac{6}{}\right.}{32x^2a^4}$
risch	$\frac{(2a^2x^2-1)\ln(ax+1)}{8a^4(a^2x^2-1)^2}+\frac{5\ln(-ax-1)a^4x^4-5\ln(ax-1)x^4a^4-10a^3x^3-10\ln(-ax-1)a^2x^2+10\ln(ax-1)a^2x^2-16x^2\,\text{l}}{64a^4(ax-1)(ax+1)(a^2x^2-1)}$

input
```
int(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-5*a^4*x^4*arctanh(a*x)+5*a^3*x^3-6*a^2*x^2*arctanh(a*x)-3*a*x+3*ar
ctanh(a*x))/(a^2*x^2-1)^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{10\,a^3x^3-6\,ax-(5\,a^4x^4+6\,a^2x^2-3)\log\left(-\frac{ax+1}{ax-1}\right)}{64\,(a^8x^4-2\,a^6x^2+a^4)}$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/64*(10*a^3*x^3 - 6*a*x - (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*
x - 1)))/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65) = 130$.

Time = 0.78 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \begin{cases} \frac{5a^4x^4\,\text{atanh}\,(ax)}{32a^8x^4-64a^6x^2+32a^4} - \frac{5a^3x^3}{32a^8x^4-64a^6x^2+32a^4} + \frac{6a^2x^2\,\text{atanh}\,(ax)}{32a^8x^4-64a^6x^2+32a^4} + \frac{3ax}{32a^8x^4-64a^6x^2+32a^4} - \frac{3\,\text{atanh}\,(ax)}{32a^8x^4-64a^6x^2+32a^4} & \text{fo} \\ 0 & \text{ot} \end{cases}$$

input
```
integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

output
```
Piecewise((5*a**4*x**4*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4)
- 5*a**3*x**3/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 6*a**2*x**2*atanh(
a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 3*a*x/(32*a**8*x**4 - 64*a*
*6*x**2 + 32*a**4) - 3*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4),
 Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{1}{64}\,a\left(\frac{2\,(5\,a^2x^3-3\,x)}{a^8x^4-2\,a^6x^2+a^4} - \frac{5\,\log\,(ax+1)}{a^5} + \frac{5\,\log\,(ax-1)}{a^5}\right) + \frac{(2\,a^2x^2-1)\,\text{artanh}\,(ax)}{4\,(a^8x^4-2\,a^6x^2+a^4)}$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/
a^5 + 5*log(a*x - 1)/a^5) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/(a^8*x^4 - 2*
a^6*x^2 + a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(66) = 132$.

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.10

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= \frac{1}{256}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5} + \frac{\frac{(ax+1)^2a^5}{(ax-1)^2}+\frac{4(ax+1)a^5}{ax-1}}{a^{10}}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right) + \frac{(ax-1)^2\left(\frac{8(ax+1}{ax-1}\right.}{(ax+1)^2a}\right.$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
1/256*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + ((a*
x + 1)^2*a^5/(a*x - 1)^2 + 4*(a*x + 1)*a^5/(a*x - 1))/a^10)*log(-(a*((a*x
+ 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x -
1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + (a*x - 1)^2*(8*(a*x + 1)/(a*x
- 1) + 1)/((a*x + 1)^2*a^5) - ((a*x + 1)^2*a^5/(a*x - 1)^2 + 8*(a*x + 1)*a
^5/(a*x - 1))/a^10)*a
```

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= \frac{5\,\text{atanh}(a\,x)}{32\,a^4} + \frac{\frac{\ln(1-a\,x)}{8} - \frac{\ln(a\,x+1)}{8} + \frac{3\,a\,x}{32} + x^2\left(\frac{a^2\,\ln(a\,x+1)}{4} - \frac{a^2\,\ln(1-a\,x)}{4}\right) - \frac{5\,a^3\,x^3}{32}}{a^4\,(a^2\,x^2-1)^2}$$

input
```
int(-(x^3*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

output
```
(5*atanh(a*x))/(32*a^4) + (log(1 - a*x)/8 - log(a*x + 1)/8 + (3*a*x)/32 +
x^2*((a^2*log(a*x + 1))/4 - (a^2*log(1 - a*x))/4) - (5*a^3*x^3)/32)/(a^4*(
a^2*x^2 - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

$$\int \frac{x^3 \text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= \frac{16atanh(ax)\,a^4x^4 + 3\log(a^2x-a)\,a^4x^4 - 6\log(a^2x-a)\,a^2x^2 + 3\log(a^2x-a) - 3\log(a^2x+a)\,a^4x^4 +}{64a^4\,(a^4x^4-2a^2x^2+1)}$$

input
```
int(x^3*atanh(a*x)/(-a^2*x^2+1)^3,x)
```

output
```
(16*atanh(a*x)*a**4*x**4 + 3*log(a**2*x - a)*a**4*x**4 - 6*log(a**2*x - a)
*a**2*x**2 + 3*log(a**2*x - a) - 3*log(a**2*x + a)*a**4*x**4 + 6*log(a**2*
x + a)*a**2*x**2 - 3*log(a**2*x + a) - 10*a**3*x**3 + 6*a*x)/(64*a**4*(a**
4*x**4 - 2*a**2*x**2 + 1))
```

3.303 $\int \frac{x^2 \mathbf{arctanh}(ax)}{(1-a^2x^2)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{1}{16a^3(1-a^2x^2)^2}+\frac{1}{16a^3(1-a^2x^2)}+\frac{x\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{x\text{arctanh}(ax)}{8a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)^2}{16a^3}$$

output
```
-1/16/a^3/(-a^2*x^2+1)^2+1/16/a^3/(-a^2*x^2+1)+1/4*x*arctanh(a*x)/a^2/(-a^
2*x^2+1)^2-1/8*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-1/16*arctanh(a*x)^2/a^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{a^2x^2-2(ax+a^3x^3)\,\text{arctanh}(ax)+(-1+a^2x^2)^2\,\text{arctanh}(ax)^2}{16a^3(-1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

output
```
-1/16*(a^2*x^2 - 2*(a*x + a^3*x^3)*ArcTanh[a*x] + (-1 + a^2*x^2)^2*ArcTanh
[a*x]^2)/(a^3*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6560, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3} \, dx$$

$$\Big\downarrow \text{ 6560}$$

$$-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{4a^2}+\frac{x\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{1}{16a^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 6518}$$

$$-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{4a^2}+\frac{x\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{1}{16a^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{x\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{4a^2}-\frac{1}{16a^3(1-a^2x^2)^2}$$

input
```
Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

output
```
-1/16*1/(a^3*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*a^2*(1 - a^2*x^2)^2) -
 (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[
a*x]^2/(4*a))/(4*a^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6560
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x] + Sim
p[1/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q
, -5/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result
parallelrisch	$-\frac{a^4x^4\operatorname{arctanh}(ax)^2-2a^3x^3\operatorname{arctanh}(ax)-2a^2x^2\operatorname{arctanh}(ax)^2+a^2x^2-2ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)^2}{16(a^2x^2-1)^2a^3}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax-16}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax+16}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{16}+\frac{\ln(ax+1)^2}{64}-\frac{(\ln(ax+1}{}}{a^3}$
default	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax-16}+\frac{\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{\operatorname{arctanh}(ax)}{16ax+16}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{16}+\frac{\ln(ax+1)^2}{64}-\frac{(\ln(ax+1}{}}{a^3}$
risch	$-\frac{\ln(ax+1)^2}{64a^3}+\frac{(x^4\ln(-ax+1)a^4+2a^3x^3-2x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1))\ln(ax+1)}{32a^3(a^2x^2-1)^2}-\frac{a^4x^4\ln(-ax+1)^2+}{}$
parts	$-\frac{\operatorname{arctanh}(ax)}{16a^3(ax+1)^2}+\frac{\operatorname{arctanh}(ax)}{16a^3(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{16a^3}+\frac{\operatorname{arctanh}(ax)}{16a^3(ax-1)^2}+\frac{\operatorname{arctanh}(ax)}{16a^3(ax-1)}+\frac{\operatorname{arctanh}(ax)\ln(a}{16a^3}$

input
```
int(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(a^4*x^4*arctanh(a*x)^2-2*a^3*x^3*arctanh(a*x)-2*a^2*x^2*arctanh(a*x
)^2+a^2*x^2-2*a*x*arctanh(a*x)+arctanh(a*x)^2)/(a^2*x^2-1)^2/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{4\,a^2x^2+(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,(a^3x^3+ax)\log\left(-\frac{ax+1}{ax-1}\right)}{64\,(a^7x^4-2\,a^5x^2+a^3)}$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/64*(4*a^2*x^2 + (a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 -
 4*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\int \frac{x^2\,\text{atanh}\,(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x**2*atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(86) = 172$.

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.79

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= \frac{1}{16}\left(\frac{2\,(a^2x^3+x)}{a^6x^4-2\,a^4x^2+a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3}\right)\text{artanh}\,(ax)$$
$$- \frac{\left(4\,a^2x^2 - (a^4x^4-2\,a^2x^2+1)\log(ax+1)^2 + 2\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1) - (a^4x^4 - \right.}{64\,(a^8x^4-2\,a^6x^2+a^4)}$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log
(a*x - 1)/a^3)*arctanh(a*x) - 1/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (
a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \int -\frac{x^2\,\text{artanh}\,(ax)}{(a^2x^2-1)^3}\,dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.50

$$\int \frac{x^2 \text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \ln(1-a\,x)\left(\frac{\ln(a\,x+1)}{32\,a^3} - \frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{2\,a^4\,x^4-4\,a^2\,x^2+2}\right) - \frac{\ln(a\,x+1)^2}{64\,a^3} - \frac{\ln(1-a\,x)^2}{64\,a^3} - \frac{x^2}{2\,(8\,a^5\,x^4-16\,a^3\,x^2+8\,a)} + \frac{\ln(a\,x+1)\,\left(\frac{x}{16\,a^3}+\frac{x^3}{16\,a}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

input
```
int(-(x^2*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

output
```
log(1 - a*x)*(log(a*x + 1)/(32*a^3) - (x/(8*a^2) + x^3/8)/(2*a^4*x^4 - 4*a
^2*x^2 + 2)) - log(a*x + 1)^2/(64*a^3) - log(1 - a*x)^2/(64*a^3) - x^2/(2*
(8*a - 16*a^3*x^2 + 8*a^5*x^4)) + (log(a*x + 1)*(x/(16*a^3) + x^3/(16*a)))
/(1/a - 2*a*x^2 + a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{-2atanh(ax)^2\,a^4x^4+4atanh(ax)^2\,a^2x^2-2atanh(ax)^2+4atanh(ax)\,a^3x^3+4atanh(ax)\,ax-a^4x^4-1}{32a^3\,(a^4x^4-2a^2x^2+1)}$$

input
```
int(x^2*atanh(a*x)/(-a^2*x^2+1)^3,x)
```

output
```
( - 2*atanh(a*x)**2*a**4*x**4 + 4*atanh(a*x)**2*a**2*x**2 - 2*atanh(a*x)**
2 + 4*atanh(a*x)*a**3*x**3 + 4*atanh(a*x)*a*x - a**4*x**4 - 1)/(32*a**3*(a
**4*x**4 - 2*a**2*x**2 + 1))
```

3.304 $\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 75

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{x}{16a(1-a^2x^2)^2} - \frac{3x}{32a(1-a^2x^2)} - \frac{3\text{arctanh}(ax)}{32a^2} + \frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}$$

output
```
-1/16*x/a/(-a^2*x^2+1)^2-3/32*x/a/(-a^2*x^2+1)-3/32*arctanh(a*x)/a^2+1/4*a
rctanh(a*x)/a^2/(-a^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{x}{16a(-1+a^2x^2)^2} + \frac{3x}{32a(-1+a^2x^2)} + \frac{\text{arctanh}(ax)}{4a^2(-1+a^2x^2)^2} + \frac{3\log(1-ax)}{64a^2} - \frac{3\log(1+ax)}{64a^2}$$

input
```
Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

output
```
-1/16*x/(a*(-1 + a^2*x^2)^2) + (3*x)/(32*a*(-1 + a^2*x^2)) + ArcTanh[a*x]/
(4*a^2*(-1 + a^2*x^2)^2) + (3*Log[1 - a*x])/(64*a^2) - (3*Log[1 + a*x])/(6
4*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6556, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\int \frac{1}{(1-a^2x^2)^3}dx}{4a}$$

$$\Big\downarrow \text{215}$$

$$\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\int \frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}}{4a}$$

$$\Big\downarrow \text{215}$$

$$\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}$$

input
```
Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

output
```
ArcTanh[a*x]/(4*a^2*(1 - a^2*x^2)^2) - (x/(4*(1 - a^2*x^2)^2) + (3*(x/(2*(
1 - a^2*x^2)) + ArcTanh[a*x]/(2*a)))/4)/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
parallelrisch	$-\frac{3a^4x^4\,\mathrm{arctanh}(ax)-3a^3x^3-6a^2x^2\,\mathrm{arctanh}(ax)+5ax-5\,\mathrm{arctanh}(ax)}{32(a^2x^2-1)^2a^2}$
derivativedivides	$\frac{\frac{\mathrm{arctanh}(ax)}{4\left(a^2x^2-1\right)^2}-\frac{1}{64(ax-1)^2}+\frac{3}{64(ax-1)}+\frac{3\ln(ax-1)}{64}+\frac{1}{64(ax+1)^2}+\frac{3}{64(ax+1)}-\frac{3\ln(ax+1)}{64}}{a^2}$
default	$\frac{\frac{\mathrm{arctanh}(ax)}{4\left(a^2x^2-1\right)^2}-\frac{1}{64(ax-1)^2}+\frac{3}{64(ax-1)}+\frac{3\ln(ax-1)}{64}+\frac{1}{64(ax+1)^2}+\frac{3}{64(ax+1)}-\frac{3\ln(ax+1)}{64}}{a^2}$
parts	$\frac{\mathrm{arctanh}(ax)}{4a^2(a^2x^2-1)^2}-\frac{-\frac{1}{16a(ax+1)^2}-\frac{3}{16a(ax+1)}+\frac{3\ln(ax+1)}{16a}+\frac{1}{16a(ax-1)^2}-\frac{3}{16a(ax-1)}-\frac{3\ln(ax-1)}{16a}}{4a}$
orering	$\frac{(ax-1)(ax+1)(9a^4x^4-14a^2x^2-5)\,\mathrm{arctanh}(ax)}{16a^2(-a^2x^2+1)^3}+\frac{(3a^2x^2-5)(ax+1)^2(ax-1)^2\left(\frac{\mathrm{arctanh}(ax)}{\left(-a^2x^2+1\right)^3}+\frac{xa}{\left(-a^2x^2+1\right)^4}+\frac{6x^2a}{(-}\right.}{32a^2}$
risch	$\frac{\ln(ax+1)}{8a^2(a^2x^2-1)^2}+\frac{3x^4\ln(-ax+1)a^4-3\ln(ax+1)a^4x^4+6a^3x^3-6x^2\ln(-ax+1)a^2+6\ln(ax+1)a^2x^2-10ax-5\ln(-ax}{64a^2(ax-1)(ax+1)(a^2x^2-1)}$

input
```
int(x*arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(3*a^4*x^4*arctanh(a*x)-3*a^3*x^3-6*a^2*x^2*arctanh(a*x)+5*a*x-5*arc
tanh(a*x))/(a^2*x^2-1)^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{x\mathrm{arctanh}(ax)}{(1-a^2x^2)^3}\,dx=\frac{6\,a^3x^3-10\,ax-(3\,a^4x^4-6\,a^2x^2-5)\log\left(-\frac{ax+1}{ax-1}\right)}{64\,(a^6x^4-2\,a^4x^2+a^2)}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/64*(6*a^3*x^3 - 10*a*x - (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x
 - 1)))/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(61) = 122$.

Time = 0.80 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.11

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= \begin{cases} -\frac{3a^4x^4\,\text{atanh}\,(ax)}{32a^6x^4-64a^4x^2+32a^2} + \frac{3a^3x^3}{32a^6x^4-64a^4x^2+32a^2} + \frac{6a^2x^2\,\text{atanh}\,(ax)}{32a^6x^4-64a^4x^2+32a^2} - \frac{5ax}{32a^6x^4-64a^4x^2+32a^2} + \frac{5\,\text{atanh}\,(ax)}{32a^6x^4-64a^4x^2+32a^2} & \\ 0 & \end{cases}$$

input
```
integrate(x*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

output
```
Piecewise((-3*a**4*x**4*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2)
 + 3*a**3*x**3/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) + 6*a**2*x**2*atanh
(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) - 5*a*x/(32*a**6*x**4 - 64*a
**4*x**2 + 32*a**2) + 5*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2)
, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1} - \frac{3\,\log(ax+1)}{a} + \frac{3\,\log(ax-1)}{a}}{64\,a} + \frac{\text{artanh}\,(ax)}{4\,(a^2x^2-1)^2a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)/a + 1/4*arctanh(a*x)/((a^2*x^2 - 1)^2*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(64) = 128$.

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.19

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx =$$

$$-\frac{1}{256}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{\frac{(ax+1)^2a^3}{(ax-1)^2}-\frac{4(ax+1)a^3}{ax-1}}{a^6}\right)\log\left(-\frac{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}+1}{\frac{a\left(\frac{ax+1}{ax-1}+1\right)}{\frac{(ax+1)a}{ax-1}-a}-1}\right)+\frac{(ax-1)^2\left(\frac{8(a}{ax}\right.}{(ax+1)}\right.$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
-1/256*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - ((a
*x + 1)^2*a^3/(a*x - 1)^2 - 4*(a*x + 1)*a^3/(a*x - 1))/a^6)*log(-(a*((a*x
+ 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x -
1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1)) + (a*x - 1)^2*(8*(a*x + 1)/(a*x
- 1) - 1)/((a*x + 1)^2*a^3) + ((a*x + 1)^2*a^3/(a*x - 1)^2 - 8*(a*x + 1)*a
^3/(a*x - 1))/a^6)*a
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{\frac{3\ln(a\,x-1)}{64}-\frac{3\ln(a\,x+1)}{64}}{a^2}$$
$$+\frac{\frac{\text{atanh}(a\,x)}{4}-x^2\left(a^2\left(\frac{3\ln(a\,x-1)}{32}-\frac{3\ln(a\,x+1)}{32}\right)-2\,a^2\left(\frac{3\ln(a\,x-1)}{64}-\frac{3\ln(a\,x+1)}{64}\right)\right)-\frac{5\,a\,x}{32}+\frac{3\,a^3\,x^3}{32}}{a^2\,(a^2\,x^2-1)^2}$$

input
```
int(-(x*atanh(a*x))/(a^2*x^2 - 1)^3,x)
```

output
```
((3*log(a*x - 1))/64 - (3*log(a*x + 1))/64)/a^2 + (atanh(a*x)/4 - x^2*(a^2
*((3*log(a*x - 1))/32 - (3*log(a*x + 1))/32) - 2*a^2*((3*log(a*x - 1))/64
- (3*log(a*x + 1))/64)) - (5*a*x)/32 + (3*a^3*x^3)/32)/(a^2*(a^2*x^2 - 1)^
2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= \frac{-16atanh(ax)\,a^4x^4+32atanh(ax)\,a^2x^2-5\log(a^2x-a)\,a^4x^4+10\log(a^2x-a)\,a^2x^2-5\log(a^2x-a)+}{64a^2\,(a^4x^4-2a^2x^2+1)}$$

input
```
int(x*atanh(a*x)/(-a^2*x^2+1)^3,x)
```

output
```
( - 16*atanh(a*x)*a**4*x**4 + 32*atanh(a*x)*a**2*x**2 - 5*log(a**2*x - a)*
a**4*x**4 + 10*log(a**2*x - a)*a**2*x**2 - 5*log(a**2*x - a) + 5*log(a**2*
x + a)*a**4*x**4 - 10*log(a**2*x + a)*a**2*x**2 + 5*log(a**2*x + a) + 6*a*
*3*x**3 - 10*a*x)/(64*a**2*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.305 $\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\frac{1}{16a(1-a^2x^2)^2}-\frac{3}{16a(1-a^2x^2)}+\frac{x\operatorname{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3x\operatorname{arctanh}(ax)}{8(1-a^2x^2)}+\frac{3\operatorname{arctanh}(ax)^2}{16a}$$

output
```
-1/16/a/(-a^2*x^2+1)^2-3/16/a/(-a^2*x^2+1)+1/4*x*arctanh(a*x)/(-a^2*x^2+1)
^2+3*x*arctanh(a*x)/(-8*a^2*x^2+8)+3/16*arctanh(a*x)^2/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{-4+3a^2x^2+(10ax-6a^3x^3)\operatorname{arctanh}(ax)+3(-1+a^2x^2)^2\operatorname{arctanh}(ax)^2}{16a(-1+a^2x^2)^2}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

output
```
(-4 + 3*a^2*x^2 + (10*a*x - 6*a^3*x^3)*ArcTanh[a*x] + 3*(-1 + a^2*x^2)^2*A
rcTanh[a*x]^2)/(16*a*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6522, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{ 6522}$$

$$\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 6518}$$

$$\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

output
```
-1/16*1/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-
1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x
]^2/(4*a)))/4
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{-3a^4x^4\operatorname{arctanh}(ax)^2-4a^4x^4+6a^3x^3\operatorname{arctanh}(ax)+6a^2x^2\operatorname{arctanh}(ax)^2+5a^2x^2-10ax\operatorname{arctanh}(ax)-3\operatorname{arctanh}(ax}{16(a^2x^2-1)^2a}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{16}-\frac{3\ln(ax-1)^2}{64}+}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{16}-\frac{3\ln(ax-1)^2}{64}+}{}$
risch	$\frac{3\ln(ax+1)^2}{64a}-\frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax+3\ln(-ax+1))\ln(ax+1)}{32(a^2x^2-1)^2a}+\frac{3a^4x^4\ln(-ax+1)}{}$
parts	$-\frac{\operatorname{arctanh}(ax)}{16a(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16a(ax+1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{16a}+\frac{\operatorname{arctanh}(ax)}{16a(ax-1)^2}-\frac{3\operatorname{arctanh}(ax)}{16a(ax-1)}-\frac{3\operatorname{arctanh}(ax)}{16a}$

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-3*a^4*x^4*arctanh(a*x)^2-4*a^4*x^4+6*a^3*x^3*arctanh(a*x)+6*a^2*x^
2*arctanh(a*x)^2+5*a^2*x^2-10*a*x*arctanh(a*x)-3*arctanh(a*x)^2)/(a^2*x^2-
1)^2/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{12\,a^2x^2+3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-16}{64\,(a^5x^4-2\,a^3x^2+a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/64*(12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
 - 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^5*x^4 - 2*a^3*
x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(80) = 160$.

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= -\frac{1}{16}\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)$$
$$+\frac{\left(12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)^2+6\,(a^4x^4-2\,a^2x^2+1)\log\left(ax+1\right)\log\left(ax-1\right)-3\,(a\right.}{64\,(a^6x^4-2\,a^4x^2+a^2)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a +
3*log(a*x - 1)/a)*arctanh(a*x) + 1/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x -
1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a/(a^6*x^4 - 2*a^4*x
^2 + a^2)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}\left(ax\right)}{\left(a^2x^2-1\right)^3}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{\frac{3\,a\,x^2}{2}-\frac{2}{a}}{8\,a^4\,x^4-16\,a^2\,x^2+8} - \ln\left(1-a\,x\right)\left(\frac{3\,\ln\left(a\,x+1\right)}{32\,a}+\frac{\frac{5\,x}{8}-\frac{3\,a^2\,x^3}{8}}{2\,a^4\,x^4-4\,a^2\,x^2+2}\right) + \frac{3\ln\left(a\,x+1\right)^2}{64\,a}+\frac{3\ln\left(1-a\,x\right)^2}{64\,a}+\frac{\ln\left(a\,x+1\right)\left(\frac{5\,x}{16\,a}-\frac{3\,a\,x^3}{16}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

input
```
int(-atanh(a*x)/(a^2*x^2 - 1)^3,x)
```

output
```
((3*a*x^2)/2 - 2/a)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 - a*x)*((3*log(a*
x + 1))/(32*a) + ((5*x)/8 - (3*a^2*x^3)/8)/(2*a^4*x^4 - 4*a^2*x^2 + 2)) +
(3*log(a*x + 1)^2)/(64*a) + (3*log(1 - a*x)^2)/(64*a) + (log(a*x + 1)*((5*
x)/(16*a) - (3*a*x^3)/16))/(1/a - 2*a*x^2 + a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}\,dx = \frac{6atanh(ax)^2\,a^4x^4-12atanh(ax)^2\,a^2x^2+6atanh(ax)^2-12atanh(ax)\,a^3x^3+20atanh(ax)\,ax+3a^4x^4-}{32a\left(a^4x^4-2a^2x^2+1\right)}$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^3,x)
```

output
```
(6*atanh(a*x)**2*a**4*x**4 - 12*atanh(a*x)**2*a**2*x**2 + 6*atanh(a*x)**2
- 12*atanh(a*x)*a**3*x**3 + 20*atanh(a*x)*a*x + 3*a**4*x**4 - 5)/(32*a*(a*
*4*x**4 - 2*a**2*x**2 + 1))
```

3.306 $\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 129

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3} dx = -\frac{ax}{16(1-a^2x^2)^2} - \frac{11ax}{32(1-a^2x^2)} - \frac{11}{32}\operatorname{arctanh}(ax) + \frac{\operatorname{arctanh}(ax)}{4(1-a^2x^2)^2} + \frac{\operatorname{arctanh}(ax)}{2(1-a^2x^2)} + \frac{1}{2}\operatorname{arctanh}(ax)^2 + \operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
-1/16*a*x/(-a^2*x^2+1)^2-11*a*x/(-32*a^2*x^2+32)-11/32*arctanh(a*x)+1/4*ar
ctanh(a*x)/(-a^2*x^2+1)^2+arctanh(a*x)/(-2*a^2*x^2+2)+1/2*arctanh(a*x)^2+a
rctanh(a*x)*ln(2-2/(a*x+1))-1/2*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3} dx = \frac{1}{128}\left(64\operatorname{arctanh}(ax)^2 + 4\operatorname{arctanh}(ax)\left(12\cosh(2\operatorname{arctanh}(ax)) + \cosh(4\operatorname{arctanh}(ax)) + 32\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)\right) - 64\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right) - 24\sinh(2\operatorname{arctanh}(ax)) - \sinh(4\operatorname{arctanh}(ax))\right)$$

input
```
Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

output
```
(64*ArcTanh[a*x]^2 + 4*ArcTanh[a*x]*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcT
anh[a*x]] + 32*Log[1 - E^(-2*ArcTanh[a*x])]) - 64*PolyLog[2, E^(-2*ArcTanh
[a*x])] - 24*Sinh[2*ArcTanh[a*x]] - Sinh[4*ArcTanh[a*x]])/128
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.51, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6592, 6556, 215, 215, 219, 6592, 6550, 6494, 2897, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\int \frac{1}{(1-a^2x^2)^3}dx}{4a}\right)+\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$a^2\left(\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\int \frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$a^2\left(\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\int \frac{\text{arctanh}(ax)}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{219}$$

$$\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^2}dx+a^2\left(\frac{\text{arctanh}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)}dx+ a^2\left(\frac{\text{arctanh}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)$$

$$\Big\downarrow \text{ 6550}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\int \frac{\text{arctanh}(ax)}{x(ax+1)}dx+ a^2\left(\frac{\text{arctanh}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\frac{1}{2}\text{arctanh}(ax)^2$$

$$\Big\downarrow \text{ 6494}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+ a^2\left(\frac{\text{arctanh}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+ \text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 2897}$$

$$a^2\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+a^2\left(\frac{\text{arctanh}(ax)}{4a^2\left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+ \frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

$$\Big\downarrow \text{ 6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+$$
$$a^2\left(\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+$$
$$\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

$$\Big\downarrow \ 215$$

$$a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+$$
$$a^2\left(\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+$$
$$\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

$$\Big\downarrow \ 219$$

$$a^2\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+$$
$$a^2\left(\frac{\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}}{4a}\right)+\frac{1}{2}\text{arctanh}(ax)^2+$$
$$\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)$$

input
```
Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

output
```
ArcTanh[a*x]^2/2 + a^2*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^
2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a)) + a^2*(ArcTanh[a*x]/(4*a^2*(1 - a^2*x
^2)^2) - (x/(4*(1 - a^2*x^2)^2) + (3*(x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(
2*a)))/4)/(4*a)) + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(
1 + a*x)]/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(114) = 228.

Time = 0.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{5\operatorname{arctanh}(ax)}{16(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5\operatorname{arct}}{16(}$
default	$\operatorname{arctanh}(ax)\ln(ax)+\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}+\frac{5\operatorname{arctanh}(ax)}{16(ax+1)}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5\operatorname{arct}}{16(}$
parts	$\operatorname{arctanh}(ax)\ln(x)+\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{\operatorname{arctanh}(ax)\ln(ax+1)}{2}+\frac{5\operatorname{arctanh}(ax)}{16(ax+1)}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\operatorname{arctanh}$
risch	$-\frac{\ln(ax+1)^2}{8}+\frac{11\ln(ax-1)}{128}-\frac{5\ln(ax+1)(ax+1)}{64(ax-1)}+\frac{1}{64ax-64}-\frac{\ln(ax+1)(ax+1)(ax-3)}{128(ax-1)^2}-\frac{\operatorname{dilog}(ax+1)}{2}+$

input
```
int(arctanh(a*x)/x/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
arctanh(a*x)*ln(a*x)+1/16*arctanh(a*x)/(a*x+1)^2+5/16*arctanh(a*x)/(a*x+1)
-1/2*arctanh(a*x)*ln(a*x+1)+1/16*arctanh(a*x)/(a*x-1)^2-5/16*arctanh(a*x)/
(a*x-1)-1/2*arctanh(a*x)*ln(a*x-1)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)-
1/2*dilog(a*x)-1/8*ln(a*x-1)^2+1/2*dilog(1/2*a*x+1/2)+1/4*ln(a*x-1)*ln(1/2
*a*x+1/2)-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/8*ln(a*x+1)^2
+1/64/(a*x+1)^2+11/64/(a*x+1)-11/64*ln(a*x+1)-1/64/(a*x-1)^2+11/64/(a*x-1)
+11/64*ln(a*x-1)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^3x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(ax)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

input
```
integrate(atanh(a*x)/x/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(110) = 220$.

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.08

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3}\,dx$$
$$= \frac{1}{64}a\left(\frac{2\left(11\,a^3x^3+4\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)^2-8\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)\log\left(ax-1\right)\right.}{a^5x^4-2\,a^3x^2+a}\right.$$
$$-\frac{1}{4}\left(\frac{2\,a^2x^2-3}{a^4x^4-2\,a^2x^2+1}+2\,\log\left(a^2x^2-1\right)-2\,\log\left(x^2\right)\right)\operatorname{artanh}\left(ax\right)$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/64*a*(2*(11*a^3*x^3 + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 - 8*(a^
4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 4*(a^4*x^4 - 2*a^2*x^2
+ 1)*log(a*x - 1)^2 - 13*a*x)/(a^5*x^4 - 2*a^3*x^2 + a) + 32*(log(a*x - 1)
*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 32*(log(a*x + 1)*log(x) +
 dilog(-a*x))/a + 32*(log(-a*x + 1)*log(x) + dilog(a*x))/a - 11*log(a*x +
1)/a + 11*log(a*x - 1)/a) - 1/4*((2*a^2*x^2 - 3)/(a^4*x^4 - 2*a^2*x^2 + 1)
 + 2*log(a^2*x^2 - 1) - 2*log(x^2))*arctanh(a*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^3x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(a\,x)}{x\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-atanh(a*x)/(x*(a^2*x^2 - 1)^3),x)
```

output
```
-int(atanh(a*x)/(x*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^3} \, dx = -\left(\int \frac{atanh(ax)}{a^6x^7 - 3a^4x^5 + 3a^2x^3 - x} dx\right)$$

input
```
int(atanh(a*x)/x/(-a^2*x^2+1)^3,x)
```

output
```
- int(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),x)
```

3.307 $\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^3}\,dx = -\frac{a}{16(1-a^2x^2)^2} - \frac{7a}{16(1-a^2x^2)} - \frac{\text{arctanh}(ax)}{x} + \frac{a^2x\text{arctanh}(ax)}{4(1-a^2x^2)^2} + \frac{7a^2x\text{arctanh}(ax)}{8(1-a^2x^2)} + \frac{15}{16}a\text{arctanh}(ax)^2 + a\log(x) - \frac{1}{2}a\log(1-a^2x^2)$$

output
```
-1/16*a/(-a^2*x^2+1)^2-7*a/(-16*a^2*x^2+16)-arctanh(a*x)/x+1/4*a^2*x*arcta
nh(a*x)/(-a^2*x^2+1)^2+7*a^2*x*arctanh(a*x)/(-8*a^2*x^2+8)+15/16*a*arctanh
(a*x)^2+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^3}\,dx = \frac{1}{16}\left(-\frac{2(8-25a^2x^2+15a^4x^4)\,\text{arctanh}(ax)}{x(-1+a^2x^2)^2} + 15a\text{arctanh}(ax)^2 + a\left(\frac{-8+7a^2x^2}{(-1+a^2x^2)^2} + 16\log(x) - 8\log(1-a^2x^2)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

output
```
((-2*(8 - 25*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)^2) + 15
*a*ArcTanh[a*x]^2 + a*((-8 + 7*a^2*x^2)/(-1 + a^2*x^2)^2 + 16*Log[x] - 8*L
og[1 - a^2*x^2]))/16
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.67, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6592, 6522, 6518, 241, 6592, 6518, 241, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^3} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6522}$$

$$a^2 \left(\frac{3}{4} \int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4 (1-a^2x^2)^2}-\frac{1}{16a (1-a^2x^2)^2}\right)+\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6518}$$

$$a^2 \left(\frac{3}{4} \left(-\frac{1}{2}a \int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2 (1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4 (1-a^2x^2)^2}-\frac{1}{16a (1-a^2x^2)^2}\right)+ \int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{241}$$

$$\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^2}dx+ a^2 \left(\frac{x\text{arctanh}(ax)}{4 (1-a^2x^2)^2}+\frac{3}{4} \left(\frac{x\text{arctanh}(ax)}{2 (1-a^2x^2)}-\frac{1}{4a (1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a (1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)}dx+ a^2\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{6518}$$

$$a^2\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)}dx+ a^2\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{241}$$

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)}dx+a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{6544}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\int \frac{\text{arctanh}(ax)}{x^2}dx+ a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+a\int \frac{1}{x(1-a^2x^2)}dx+ a^2\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)- \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}dx^2+ a^2\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}\right)- \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}\right)- \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}\right)- \frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$a^2\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx+a^2\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}\right)+ \frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{6510}$$

$$a^2\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+ a^2\left(\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}\right)+ \frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\text{arctanh}(ax)^2-\frac{\text{arctanh}(ax)}{x}$$

input
```
Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

output
```
-(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a^2*(-1/4*1/(a*(1 - a^2*x^2)) +
 (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)) + a^2*(-1/16*1
/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-1/4*1/(
a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*
a)))/4) + (a*(Log[x^2] - Log[1 - a^2*x^2]))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.61

method	result
parallelrisch	$\frac{15\,\mathrm{arctanh}(ax)^2 ax-30\,\mathrm{arctanh}(ax)^2 a^3x^3+15\,\mathrm{arctanh}(ax)^2 a^5x^5-16ax\,\mathrm{arctanh}(ax)-16\,\mathrm{arctanh}(ax)a^5x^5+32\ln(ax-}{}$
derivativedivides	$a\left(\frac{\mathrm{arctanh}(ax)}{16(ax-1)^2}-\frac{7\,\mathrm{arctanh}(ax)}{16(ax-1)}-\frac{15\,\mathrm{arctanh}(ax)\ln(ax-1)}{16}-\frac{\mathrm{arctanh}(ax)}{16(ax+1)^2}-\frac{7\,\mathrm{arctanh}(ax)}{16(ax+1)}+\frac{15\,\mathrm{arctanh}(}{}\right.$
default	$a\left(\frac{\mathrm{arctanh}(ax)}{16(ax-1)^2}-\frac{7\,\mathrm{arctanh}(ax)}{16(ax-1)}-\frac{15\,\mathrm{arctanh}(ax)\ln(ax-1)}{16}-\frac{\mathrm{arctanh}(ax)}{16(ax+1)^2}-\frac{7\,\mathrm{arctanh}(ax)}{16(ax+1)}+\frac{15\,\mathrm{arctanh}(}{}\right.$
parts	$-\frac{\mathrm{arctanh}(ax)}{x}-\frac{\mathrm{arctanh}(ax)a}{16(ax+1)^2}-\frac{7\,\mathrm{arctanh}(ax)a}{16(ax+1)}+\frac{15a\,\mathrm{arctanh}(ax)\ln(ax+1)}{16}+\frac{\mathrm{arctanh}(ax)a}{16(ax-1)^2}-\frac{7\,\mathrm{arctanh}}{16(ax-}$
risch	$\frac{15a\ln(ax+1)^2}{64}-\frac{(15x^5\ln(-ax+1)a^5+30a^4x^4-30a^3x^3\ln(-ax+1)-50a^2x^2+15ax\ln(-ax+1)+16)\ln(ax+1)}{32x(a^2x^2-1)^2}+\frac{1}{}$

input

```
int(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output

```
1/16*(15*arctanh(a*x)^2*a*x-30*arctanh(a*x)^2*a^3*x^3+15*arctanh(a*x)^2*a^
5*x^5-16*a*x*arctanh(a*x)-16*arctanh(a*x)*a^5*x^5+32*ln(a*x-1)*x^3*a^3+8*a
^5*x^5-9*a^3*x^3+16*ln(x)*a^5*x^5+16*a*ln(x)*x-32*ln(x)*a^3*x^3+32*a^3*x^3
*arctanh(a*x)-30*a^4*x^4*arctanh(a*x)+50*a^2*x^2*arctanh(a*x)-16*arctanh(a
*x)-16*ln(a*x-1)*x^5*a^5-16*ln(a*x-1)*a*x)/(a^2*x^2-1)^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.31

$$\int \frac{\mathrm{arctanh}(ax)}{x^2(1-a^2x^2)^3}\,dx = \frac{28\,a^3x^3+15\,(a^5x^5-2\,a^3x^3+ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-32\,ax-32\,(a^5x^5-2\,a^3x^3+ax)\log(a^2x^2-1)+64\,(a}{64\,(a^4x^5-2\,a^2x^3+x)}$$

input

```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/64*(28*a^3*x^3 + 15*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)
)^2 - 32*a*x - 32*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a^2*x^2 - 1) + 64*(a^5*x
^5 - 2*a^3*x^3 + a*x)*log(x) - 4*(15*a^4*x^4 - 25*a^2*x^2 + 8)*log(-(a*x +
 1)/(a*x - 1)))/(a^4*x^5 - 2*a^2*x^3 + x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(107) = 214$.

Time = 1.61 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.46

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^3} dx$$

$$= \begin{cases} \frac{16a^5x^5 \log(x)}{16a^4x^5-32a^2x^3+16x} - \frac{16a^5x^5 \log\left(x-\frac{1}{a}\right)}{16a^4x^5-32a^2x^3+16x} + \frac{15a^5x^5 \operatorname{atanh}^2(ax)}{16a^4x^5-32a^2x^3+16x} - \frac{16a^5x^5 \operatorname{atanh}(ax)}{16a^4x^5-32a^2x^3+16x} - \frac{30a^4x^4 \operatorname{atanh}(ax)}{16a^4x^5-32a^2x^3+16x} - \frac{32a}{16a^4x^5} \\ 0 \end{cases}$$

input
```
integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**3,x)
```

output
```
Piecewise((16*a**5*x**5*log(x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a
**5*x**5*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 15*a**5*x**5*
atanh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a**5*x**5*atanh(a*
x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 30*a**4*x**4*atanh(a*x)/(16*a**4
*x**5 - 32*a**2*x**3 + 16*x) - 32*a**3*x**3*log(x)/(16*a**4*x**5 - 32*a**2
*x**3 + 16*x) + 32*a**3*x**3*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 1
6*x) - 30*a**3*x**3*atanh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 3
2*a**3*x**3*atanh(a*x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 7*a**3*x**3/
(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 50*a**2*x**2*atanh(a*x)/(16*a**4*x*
*5 - 32*a**2*x**3 + 16*x) + 16*a*x*log(x)/(16*a**4*x**5 - 32*a**2*x**3 + 1
6*x) - 16*a*x*log(x - 1/a)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) + 15*a*x*a
tanh(a*x)**2/(16*a**4*x**5 - 32*a**2*x**3 + 16*x) - 16*a*x*atanh(a*x)/(16*
a**4*x**5 - 32*a**2*x**3 + 16*x) - 8*a*x/(16*a**4*x**5 - 32*a**2*x**3 + 16
*x) - 16*atanh(a*x)/(16*a**4*x**5 - 32*a**2*x**3 + 16*x), Ne(a, 0)), (0, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^3}\,dx$$
$$= \frac{1}{64}a\left(\frac{28\,a^2x^2 - 15\,(a^4x^4 - 2\,a^2x^2 + 1)\log{(ax+1)}^2 + 30\,(a^4x^4 - 2\,a^2x^2 + 1)\log{(ax+1)}\log{(ax-1)} -}{a^4x^4 - 2\,a^2x^2 + 1}\right.$$
$$+ \frac{1}{16}\left(15\,a\log{(ax+1)} - 15\,a\log{(ax-1)} - \frac{2\,(15\,a^4x^4 - 25\,a^2x^2 + 8)}{a^4x^5 - 2\,a^2x^3 + x}\right)\operatorname{artanh}{(ax)}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/64*a*((28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^
4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 3
2*log(a*x - 1) + 64*log(x)) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1)
- 2*(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}{(ax)}}{(a^2x^2-1)^3x^2}\,dx$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^3}\,dx = \frac{15\,a\ln\left(a\,x+1\right)^2}{64}-\frac{4\,a-\frac{7\,a^3\,x^2}{2}}{8\,a^4\,x^4-16\,a^2\,x^2+8}+\frac{15\,a\ln\left(1-a\,x\right)^2}{64}-\frac{a\,\ln\left(a^2\,x^2-1\right)}{2}+a\,\ln\left(x\right)+\ln\left(1-a\,x\right)\left(\frac{\frac{15\,a^4\,x^4}{8}-\frac{25\,a^2\,x^2}{8}+1}{2\,a^4\,x^5-4\,a^2\,x^3+2\,x}-\frac{15\,a\,\ln\left(a\,x+1\right)}{32}\right)-\frac{\ln\left(a\,x+1\right)\left(\frac{1}{2\,a}-\frac{25\,a\,x^2}{16}+\frac{15\,a^3\,x^4}{16}\right)}{\frac{x}{a}-2\,a\,x^3+a^3\,x^5}$$

input
```
int(-atanh(a*x)/(x^2*(a^2*x^2 - 1)^3),x)
```

output
```
(15*a*log(a*x + 1)^2)/64 - (4*a - (7*a^3*x^2)/2)/(8*a^4*x^4 - 16*a^2*x^2 +
 8) + (15*a*log(1 - a*x)^2)/64 - (a*log(a^2*x^2 - 1))/2 + a*log(x) + log(1
 - a*x)*(((15*a^4*x^4)/8 - (25*a^2*x^2)/8 + 1)/(2*x - 4*a^2*x^3 + 2*a^4*x^
5) - (15*a*log(a*x + 1))/32) - (log(a*x + 1)*(1/(2*a) - (25*a*x^2)/16 + (1
5*a^3*x^4)/16))/(x/a - 2*a*x^3 + a^3*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^3}\,dx = \frac{30 atanh(ax)^2 a^5x^5-60 atanh(ax)^2 a^3x^3+30 atanh(ax)^2 ax-32 atanh(ax)\,a^5x^5-60 atanh(ax)\,a^4x^4+}{}$$

input
```
int(atanh(a*x)/x^2/(-a^2*x^2+1)^3,x)
```

output
```
(30*atanh(a*x)**2*a**5*x**5 - 60*atanh(a*x)**2*a**3*x**3 + 30*atanh(a*x)**
2*a*x - 32*atanh(a*x)*a**5*x**5 - 60*atanh(a*x)*a**4*x**4 + 64*atanh(a*x)*
a**3*x**3 + 100*atanh(a*x)*a**2*x**2 - 32*atanh(a*x)*a*x - 32*atanh(a*x) -
 32*log(a**2*x - a)*a**5*x**5 + 64*log(a**2*x - a)*a**3*x**3 - 32*log(a**2
*x - a)*a*x + 32*log(x)*a**5*x**5 - 64*log(x)*a**3*x**3 + 32*log(x)*a*x +
7*a**5*x**5 - 9*a*x)/(32*x*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.308 $\int \frac{x^3\textbf{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{x^4}{32(1-a^2x^2)^2} - \frac{3}{32a^4(1-a^2x^2)} - \frac{x^3\text{arctanh}(ax)}{8a(1-a^2x^2)^2} + \frac{3x\text{arctanh}(ax)}{16a^3(1-a^2x^2)} - \frac{3\text{arctanh}(ax)^2}{32a^4} + \frac{x^4\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}$$

output
```
1/32*x^4/(-a^2*x^2+1)^2-3/32/a^4/(-a^2*x^2+1)-1/8*x^3*arctanh(a*x)/a/(-a^2
*x^2+1)^2+3/16*x*arctanh(a*x)/a^3/(-a^2*x^2+1)-3/32*arctanh(a*x)^2/a^4+1/4
*x^4*arctanh(a*x)^2/(-a^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{-4+5a^2x^2+(6ax-10a^3x^3)\,\text{arctanh}(ax)+(-3+6a^2x^2+5a^4x^4)\,\text{arctanh}(ax)^2}{32a^4(-1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

output
```
(-4 + 5*a^2*x^2 + (6*a*x - 10*a^3*x^3)*ArcTanh[a*x] + (-3 + 6*a^2*x^2 + 5*
a^4*x^4)*ArcTanh[a*x]^2)/(32*a^4*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6570, 6564, 6560, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6570}$$

$$\frac{x^4\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{1}{2}a\int \frac{x^4\text{arctanh}(ax)}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6564}$$

$$\frac{x^4\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{1}{2}a\left(-\frac{3\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{4a^2}+\frac{x^3\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{x^4}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{6560}$$

$$\frac{x^4\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{1}{2}a\left(-\frac{3\left(-\frac{\int \frac{\text{arctanh}(ax)}{1-a^2x^2}dx}{2a^2}+\frac{x\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{1}{4a^3(1-a^2x^2)}\right)}{4a^2}+\frac{x^3\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{x^4}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{x^4\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{1}{2}a\left(\frac{x^3\text{arctanh}(ax)}{4a^2(1-a^2x^2)^2}-\frac{x^4}{16a(1-a^2x^2)^2}-\frac{3\left(-\frac{\text{arctanh}(ax)^2}{4a^3}+\frac{x\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{1}{4a^3(1-a^2x^2)}\right)}{4a^2}\right)$$

input
```
Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

output
```
(x^4*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) - (a*(-1/16*x^4/(a*(1 - a^2*x^2)^
2) + (x^3*ArcTanh[a*x])/(4*a^2*(1 - a^2*x^2)^2) - (3*(-1/4*1/(a^3*(1 - a^2
*x^2)) + (x*ArcTanh[a*x])/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^2/(4*a^3)))
/(4*a^2)))/2
```

Defintions of rubi rules used

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6560
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x] + Sim
p[1/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q
, -5/2]
```

rule 6564
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(q_), x_Symbol] :> Simp[(-b)*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)),
x] + (Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(c^2*d
*m)), x] - Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c
^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
parallelrisch	$-\frac{-5a^4x^4 \operatorname{arctanh}(ax)^2-4a^4x^4+10a^3x^3 \operatorname{arctanh}(ax)-6a^2x^2 \operatorname{arctanh}(ax)^2+3a^2x^2-6ax \operatorname{arctanh}(ax)+3 \operatorname{arctanh}(ax}{32(a^2x^2-1)^2 a^4}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2}{16(ax-1)^2}+\frac{3 \operatorname{arctanh}(ax)^2}{16(ax-1)}+\frac{\operatorname{arctanh}(ax)^2}{16(ax+1)^2}-\frac{3 \operatorname{arctanh}(ax)^2}{16(ax+1)}-\frac{\operatorname{arctanh}(ax)}{32(ax-1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32(ax-1)}-\frac{5 \operatorname{arctanh}(ax) \ln(ax-1)}{32}+\frac{\operatorname{arctan}}{32(a}}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2}{16(ax-1)^2}+\frac{3 \operatorname{arctanh}(ax)^2}{16(ax-1)}+\frac{\operatorname{arctanh}(ax)^2}{16(ax+1)^2}-\frac{3 \operatorname{arctanh}(ax)^2}{16(ax+1)}-\frac{\operatorname{arctanh}(ax)}{32(ax-1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32(ax-1)}-\frac{5 \operatorname{arctanh}(ax) \ln(ax-1)}{32}+\frac{\operatorname{arctan}}{32(a}}{}$
risch	$\frac{(5a^4x^4+6a^2x^2-3) \ln(ax+1)^2}{128a^4(a^2x^2-1)^2}-\frac{(5x^4 \ln(-ax+1)a^4+10a^3x^3+6x^2 \ln(-ax+1)a^2-6ax-3 \ln(-ax+1)) \ln(ax+1)}{64a^4(ax-1)(ax+1)(a^2x^2-1)}+$
parts	$\frac{\operatorname{arctanh}(ax)^2}{16a^4(ax+1)^2}-\frac{3 \operatorname{arctanh}(ax)^2}{16a^4(ax+1)}+\frac{\operatorname{arctanh}(ax)^2}{16a^4(ax-1)^2}+\frac{3 \operatorname{arctanh}(ax)^2}{16a^4(ax-1)}-\frac{\operatorname{arctanh}(ax)}{32a^4(ax-1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32a^4(ax-1)}-5 \operatorname{ar}$
orering	$-\frac{(ax-1)(ax+1)(12a^6x^6+13a^4x^4+9a^2x^2-15) \operatorname{arctanh}(ax)^2}{16a^4(-a^2x^2+1)^3}-\frac{(ax+1)^2(ax-1)^2(16a^4x^4+5a^2x^2-12)\left(\frac{3x^2 \operatorname{arctan}}{(-a^2x^2}\right.}{32a^4x}$

input
```
int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-5*a^4*x^4*arctanh(a*x)^2-4*a^4*x^4+10*a^3*x^3*arctanh(a*x)-6*a^2*x
^2*arctanh(a*x)^2+3*a^2*x^2-6*a*x*arctanh(a*x)+3*arctanh(a*x)^2)/(a^2*x^2-
1)^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.78

$$\int \frac{x^3 \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3} \, dx = \frac{20 \, a^2x^2 + (5 \, a^4x^4 + 6 \, a^2x^2 - 3) \log\left(-\frac{ax+1}{ax-1}\right)^2 - 4 \, (5 \, a^3x^3 - 3 \, ax) \log\left(-\frac{ax+1}{ax-1}\right) - 16}{128 \, (a^8x^4 - 2 \, a^6x^2 + a^4)}$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(20*a^2*x^2 + (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1))^
2 - 4*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^8*x^4 - 2*a^6
*x^2 + a^4)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = -\int \frac{x^3\,\text{atanh}^2\,(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x**3*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(110) = 220$.

Time = 0.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.78

$$\begin{aligned}
&\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx \\
&= -\frac{1}{32}\,a\left(\frac{2\,(5\,a^2x^3-3\,x)}{a^8x^4-2\,a^6x^2+a^4}-\frac{5\,\log(ax+1)}{a^5}+\frac{5\,\log(ax-1)}{a^5}\right)\text{artanh}\,(ax) \\
&\quad+\frac{(20\,a^2x^2-5\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2+10\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1)-5\,(a}{128\,(a^{10}x^4-2\,a^8x^2+a^6)} \\
&\quad+\frac{(2\,a^2x^2-1)\,\text{artanh}\,(ax)^2}{4\,(a^8x^4-2\,a^6x^2+a^4)}
\end{aligned}$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/32*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/
a^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x) + 1/128*(20*a^2*x^2 - 5*(a^4*x^4 -
2*a^2*x^2 + 1)*log(a*x + 1)^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*
log(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a^2/(a^10*
x^4 - 2*a^8*x^2 + a^6) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/(a^8*x^4 - 2*a
^6*x^2 + a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.97

$$\int \frac{x^3 \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= \frac{1}{512}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5} + \frac{(ax+1)^2}{(ax-1)^2a^5} + \frac{4(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 2\left(\frac{(ax-1)^2\left(\frac{8(ax}{ax}\right.}{(ax+1)}\right.\right.$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
1/512*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (a*x
 + 1)^2/((a*x - 1)^2*a^5) + 4*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/(a
*x - 1))^2 + 2*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5)
- (a*x + 1)^2/((a*x - 1)^2*a^5) - 8*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x +
 1)/(a*x - 1)) + (a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5
) + (a*x + 1)^2/((a*x - 1)^2*a^5) + 16*(a*x + 1)/((a*x - 1)*a^5))*a
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.93

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \ln(a\,x+1)^2\left(\frac{5}{128\,a^4}-\frac{\frac{1}{16\,a^5}-\frac{x^2}{8\,a^3}}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}\right) - \ln(1-a\,x)\left(\frac{\frac{3\,x}{8}+a\,x^2-\frac{3}{4\,a}-\frac{5\,a^2\,x^3}{8}}{8\,a^7\,x^4-16\,a^5\,x^2+8\,a^3} + \frac{\frac{3\,x}{8}-a\,x^2+\frac{3}{4\,a}-\frac{5\,a^2\,x^3}{8}}{8\,a^7\,x^4-16\,a^5\,x^2+8\,a^3} - \ln(a\,x+1)\left(\frac{\frac{1}{4\,a^4}-\frac{x^2}{2\,a^2}}{2\,a^4\,x^4-4\,a^2\,x^2+2}-\frac{5\,(a^4\,x^4-2\,a^2\,x^2+1)}{32\,a^4\,(2\,a^4\,x^4-4\,a^2\,x^2+2)}\right)\right) - \ln(1-a\,x)^2\left(\frac{\frac{1}{4\,a^4}-\frac{x^2}{2\,a^2}}{4\,a^4\,x^4-8\,a^2\,x^2+4}-\frac{5}{128\,a^4}\right) - \frac{\frac{2}{a^2}-\frac{5\,x^2}{2}}{16\,a^6\,x^4-32\,a^4\,x^2+16\,a^2} + \frac{\ln(a\,x+1)\left(\frac{3\,x}{32\,a^4}-\frac{5\,x^3}{32\,a^2}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

input

```
int(-(x^3*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

output

```
log(a*x + 1)^2*(5/(128*a^4) - (1/(16*a^5) - x^2/(8*a^3))/(1/a - 2*a*x^2 +
a^3*x^4)) - log(1 - a*x)*(((3*x)/8 + a*x^2 - 3/(4*a) - (5*a^2*x^3)/8)/(8*a
^3 - 16*a^5*x^2 + 8*a^7*x^4) + ((3*x)/8 - a*x^2 + 3/(4*a) - (5*a^2*x^3)/8)
/(8*a^3 - 16*a^5*x^2 + 8*a^7*x^4) - log(a*x + 1)*((1/(4*a^4) - x^2/(2*a^2)
)/(2*a^4*x^4 - 4*a^2*x^2 + 2) - (5*(a^4*x^4 - 2*a^2*x^2 + 1))/(32*a^4*(2*a
^4*x^4 - 4*a^2*x^2 + 2)))) - log(1 - a*x)^2*((1/(4*a^4) - x^2/(2*a^2))/(4*
a^4*x^4 - 8*a^2*x^2 + 4) - 5/(128*a^4)) - (2/a^2 - (5*x^2)/2)/(16*a^2 - 32
*a^4*x^2 + 16*a^6*x^4) + (log(a*x + 1)*((3*x)/(32*a^4) - (5*x^3)/(32*a^2))
)/(1/a - 2*a*x^2 + a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{x^3 \text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= \frac{10atanh(ax)^2\,a^4x^4 + 12atanh(ax)^2\,a^2x^2 - 6atanh(ax)^2 - 20atanh(ax)\,a^3x^3 + 12atanh(ax)\,ax + 5a^4x^4}{64a^4\,(a^4x^4 - 2a^2x^2 + 1)}$$

input
```
int(x^3*atanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

output
```
(10*atanh(a*x)**2*a**4*x**4 + 12*atanh(a*x)**2*a**2*x**2 - 6*atanh(a*x)**2
 - 20*atanh(a*x)*a**3*x**3 + 12*atanh(a*x)*a*x + 5*a**4*x**4 - 3)/(64*a**4
*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.309 $\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\begin{aligned}\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx =& \frac{x}{32a^2(1-a^2x^2)^2}-\frac{x}{64a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)}{64a^3}\\&-\frac{\text{arctanh}(ax)}{8a^3(1-a^2x^2)^2}+\frac{\text{arctanh}(ax)}{8a^3(1-a^2x^2)}\\&+\frac{x\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{x\text{arctanh}(ax)^2}{8a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)^3}{24a^3}\end{aligned}$$

output
```
1/32*x/a^2/(-a^2*x^2+1)^2-1/64*x/a^2/(-a^2*x^2+1)-1/64*arctanh(a*x)/a^3-1/
8*arctanh(a*x)/a^3/(-a^2*x^2+1)^2+1/8*arctanh(a*x)/a^3/(-a^2*x^2+1)+1/4*x*
arctanh(a*x)^2/a^2/(-a^2*x^2+1)^2-1/8*x*arctanh(a*x)^2/a^2/(-a^2*x^2+1)-1/
24*arctanh(a*x)^3/a^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$=\frac{6ax(1+a^2x^2)-48a^2x^2\text{arctanh}(ax)+48(ax+a^3x^3)\,\text{arctanh}(ax)^2-16(-1+a^2x^2)^2\,\text{arctanh}(ax)^3+3}{384a^3\,(-1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

output
```
(6*a*x*(1 + a^2*x^2) - 48*a^2*x^2*ArcTanh[a*x] + 48*(a*x + a^3*x^3)*ArcTan
h[a*x]^2 - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3 + 3*(-1 + a^2*x^2)^2*Log[1 -
 a*x] - 3*(-1 + a^2*x^2)^2*Log[1 + a*x])/(384*a^3*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {6590, 6518, 6526, 215, 215, 219, 6518, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx}{a^2}-\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{a^2}$$

$$\Big\downarrow \text{6518}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx}{a^2}-\frac{-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \text{6526}$$

$$\frac{\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\int \frac{1}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}}{a^2}-\frac{-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\int \frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}}{a^2}-\frac{-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}}{a^2}-\frac{-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)}{a^2}-\frac{-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \; 6518$$

$$\frac{\frac{3}{4}\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\Big(\frac{3}{4}\Big(\frac{x}{2(1-a^2x^2)}+\text{arc}}{a^2}-\frac{-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \; 6556$$

$$\frac{\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\right.\right.}{a^2}$$

$$\frac{-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}}{a^2}$$

$$\frac{-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(a}{2a^2(1-a^2x}\right.\right.}{a^2}$$

$$\frac{\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}}{a^2}$$

input

```
Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

output

```
-(((x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTan
h[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/
(2*a)))/a^2) + (-1/8*ArcTanh[a*x]/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x]^2)
/(4*(1 - a^2*x^2)^2) + (x/(4*(1 - a^2*x^2)^2) + (3*(x/(2*(1 - a^2*x^2)) +
ArcTanh[a*x]/(2*a)))/4)/8 + (3*((x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + Arc
Tanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2
*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/4)/a^2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 50.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{8\operatorname{arctanh}(ax)^3a^4x^4+3a^4x^4\operatorname{arctanh}(ax)-24\operatorname{arctanh}(ax)^2a^3x^3-16\operatorname{arctanh}(ax)^3a^2x^2-3a^3x^3+18a^2x^2\operatorname{arctanh}(a\ldots}{192(a^2x^2-1)^2a^3}$
risch	$-\frac{\ln(ax+1)^3}{192a^3}+\frac{(x^4\ln(-ax+1)a^4+2a^3x^3-2x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1))\ln(ax+1)^2}{64a^3(a^2x^2-1)^2}-\frac{(a^4x^4\ln(-ax+1)^2\ldots}{\ldots}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)^2}{16(ax+1)^2}+\frac{\operatorname{arctanh}(ax)^2}{16ax+16}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{16}+\frac{\operatorname{arctanh}(ax)^2}{16(ax-1)^2}+\frac{\operatorname{arctanh}(ax)^2}{16ax-16}+\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{16}+\frac{\operatorname{arctanh}(ax)^2\ldots}{\ldots}}{\ldots}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)^2}{16(ax+1)^2}+\frac{\operatorname{arctanh}(ax)^2}{16ax+16}-\frac{\operatorname{arctanh}(ax)^2\ln(ax+1)}{16}+\frac{\operatorname{arctanh}(ax)^2}{16(ax-1)^2}+\frac{\operatorname{arctanh}(ax)^2}{16ax-16}+\frac{\operatorname{arctanh}(ax)^2\ln(ax-1)}{16}+\frac{\operatorname{arctanh}(ax)^2\ldots}{\ldots}}{\ldots}$
parts	Expression too large to display

input
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(8*arctanh(a*x)^3*a^4*x^4+3*a^4*x^4*arctanh(a*x)-24*arctanh(a*x)^2*
a^3*x^3-16*arctanh(a*x)^3*a^2*x^2-3*a^3*x^3+18*a^2*x^2*arctanh(a*x)-24*arc
tanh(a*x)^2*a*x+8*arctanh(a*x)^3-3*a*x+3*arctanh(a*x))/(a^2*x^2-1)^2/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= \frac{6\,a^3x^3 - 2\,(a^4x^4 - 2\,a^2x^2 + 1)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 12\,(a^3x^3 + ax)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 6\,ax - 3\,(a^4x^4 + 6\,a^2x^2 +}{384\,(a^7x^4 - 2\,a^5x^2 + a^3)}$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^3
 + 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 6*a*x - 3*(a^4*x^4 + 6
*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F]

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = -\int \frac{x^2 \operatorname{atanh}^2(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

input
```
integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x**2*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(141) = 282.

Time = 0.04 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.38

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= \frac{1}{16}\left(\frac{2\,(a^2x^3+x)}{a^6x^4-2\,a^4x^2+a^2}-\frac{\log{(ax+1)}}{a^3}+\frac{\log{(ax-1)}}{a^3}\right)\text{artanh}\,(ax)^2$$
$$+\frac{(6\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log{(ax+1)}^3+6\,(a^4x^4-2\,a^2x^2+1)\log{(ax+1)}^2\log{(ax-1)}+2\,(a^4$$
$$-\frac{(4\,a^2x^2-(a^4x^4-2\,a^2x^2+1)\log{(ax+1)}^2+2\,(a^4x^4-2\,a^2x^2+1)\log{(ax+1)}\log{(ax-1)}-(a^4x^4}{32\,(a^8x^4-2\,a^6x^2+a^4)}$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log
(a*x - 1)/a^3)*arctanh(a*x)^2 + 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2
+ 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x -
 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*
a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 1)*log(a*x + 1) + 3
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^9*x^4 - 2*a^7*x^2 + a^5) -
 1/32*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(
a*x - 1)^2)*a*arctanh(a*x)/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \int -\frac{x^2\,\text{artanh}\,(ax)^2}{(a^2x^2-1)^3}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.15

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \ln(1-a\,x)\left(\frac{\frac{3\,a\,x^3}{2}-\frac{x}{2\,a}+x^2}{32\,a^5\,x^4-64\,a^3\,x^2+32\,a}+\frac{\frac{x}{2\,a}-\frac{3\,a\,x^3}{2}+x^2}{32\,a^5\,x^4-64\,a^3\,x^2+32\,a}+\frac{\ln(a\,x+1)^2}{64\,a^3}-\frac{\ln(a\,x+1)\,\left(2\,a^2\,x^3+2\,x\right)}{32\,a^6\,x^4-64\,a^4\,x^2+32\,a^2}\right)+\frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{8\,a^4\,x^4-16\,a^2\,x^2+8}-\ln(1-a\,x)^2\left(\frac{\ln(a\,x+1)}{64\,a^3}-\frac{\frac{x}{8\,a^2}+\frac{x^3}{8}}{4\,a^4\,x^4-8\,a^2\,x^2+4}\right)-\frac{\ln(a\,x+1)^3}{192\,a^3}+\frac{\ln(1-a\,x)^3}{192\,a^3}+\frac{\ln(a\,x+1)^2\left(\frac{x}{32\,a^3}+\frac{x^3}{32\,a}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}-\frac{x^2\,\ln(a\,x+1)}{16\,a^2\,\left(\frac{1}{a}-2\,a\,x^2+a^3\,x^4\right)}+\frac{\text{atan}(a\,x\,1\text{i})\,1\text{i}}{64\,a^3}$$

input
```
int(-(x^2*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

output
```
log(1 - a*x)*(((3*a*x^3)/2 - x/(2*a) + x^2)/(32*a - 64*a^3*x^2 + 32*a^5*x^
4) + (x/(2*a) - (3*a*x^3)/2 + x^2)/(32*a - 64*a^3*x^2 + 32*a^5*x^4) + log(
a*x + 1)^2/(64*a^3) - (log(a*x + 1)*(2*x + 2*a^2*x^3))/(32*a^2 - 64*a^4*x^
2 + 32*a^6*x^4)) + (x/(8*a^2) + x^3/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(
1 - a*x)^2*(log(a*x + 1)/(64*a^3) - (x/(8*a^2) + x^3/8)/(4*a^4*x^4 - 8*a^2
*x^2 + 4)) - log(a*x + 1)^3/(192*a^3) + log(1 - a*x)^3/(192*a^3) + (atan(a
*x*1i)*1i)/(64*a^3) + (log(a*x + 1)^2*(x/(32*a^3) + x^3/(32*a)))/(1/a - 2*
a*x^2 + a^3*x^4) - (x^2*log(a*x + 1))/(16*a^2*(1/a - 2*a*x^2 + a^3*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.26

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^3} \, dx$$
$$= \frac{-16atanh(ax)^3 a^4x^4 + 32atanh(ax)^3 a^2x^2 - 16atanh(ax)^3 + 48atanh(ax)^2 a^3x^3 + 48atanh(ax)^2 ax - 2\cdots}{}$$

input
```
int(x^2*atanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

output
```
( - 16*atanh(a*x)**3*a**4*x**4 + 32*atanh(a*x)**3*a**2*x**2 - 16*atanh(a*x
)**3 + 48*atanh(a*x)**2*a**3*x**3 + 48*atanh(a*x)**2*a*x - 24*atanh(a*x)*a
**4*x**4 - 24*atanh(a*x) - 9*log(a**2*x - a)*a**4*x**4 + 18*log(a**2*x - a
)*a**2*x**2 - 9*log(a**2*x - a) + 9*log(a**2*x + a)*a**4*x**4 - 18*log(a**
2*x + a)*a**2*x**2 + 9*log(a**2*x + a) + 6*a**3*x**3 + 6*a*x)/(384*a**3*(a
**4*x**4 - 2*a**2*x**2 + 1))
```

3.310 $\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{1}{32a^2(1-a^2x^2)^2}+\frac{3}{32a^2(1-a^2x^2)}-\frac{x\text{arctanh}(ax)}{8a(1-a^2x^2)^2}-\frac{3x\text{arctanh}(ax)}{16a(1-a^2x^2)}-\frac{3\text{arctanh}(ax)^2}{32a^2}+\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}$$

output
```
1/32/a^2/(-a^2*x^2+1)^2+3/32/a^2/(-a^2*x^2+1)-1/8*x*arctanh(a*x)/a/(-a^2*x
^2+1)^2-3/16*x*arctanh(a*x)/a/(-a^2*x^2+1)-3/32*arctanh(a*x)^2/a^2+1/4*arc
tanh(a*x)^2/a^2/(-a^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{4-3a^2x^2+2ax(-5+3a^2x^2)\,\text{arctanh}(ax)+(5+6a^2x^2-3a^4x^4)\,\text{arctanh}(ax)^2}{32a^2(-1+a^2x^2)^2}$$

input
```
Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

output
```
(4 - 3*a^2*x^2 + 2*a*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x] + (5 + 6*a^2*x^2 - 3*
a^4*x^4)*ArcTanh[a*x]^2)/(32*a^2*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6556, 6522, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx}{2a}$$

$$\Big\downarrow \text{6522}$$

$$\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}}{2a}$$

$$\Big\downarrow \text{6518}$$

$$\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}}{2a}$$

$$\Big\downarrow \text{241}$$

$$\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}$$

input
```
Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

output
```
ArcTanh[a*x]^2/(4*a^2*(1 - a^2*x^2)^2) - (-1/16*1/(a*(1 - a^2*x^2)^2) + (x
*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-1/4*1/(a*(1 - a^2*x^2)) + (x*Arc
Tanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)))/4)/(2*a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

method	result
parallelrisch	$-\frac{3a^4x^4\operatorname{arctanh}(ax)^2+4a^4x^4-6a^3x^3\operatorname{arctanh}(ax)-6a^2x^2\operatorname{arctanh}(ax)^2-5a^2x^2+10ax\operatorname{arctanh}(ax)-5\operatorname{arctanh}(ax)^2}{32(a^2x^2-1)^2a^2}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(ax)^2}{4(a^2x^2-1)^2}+\frac{\operatorname{arctanh}(ax)}{32(ax+1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax+1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{32}-\frac{\operatorname{arctanh}(ax)}{32(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax-1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{32}+}{}$
default	$\frac{\frac{\operatorname{arctanh}(ax)^2}{4(a^2x^2-1)^2}+\frac{\operatorname{arctanh}(ax)}{32(ax+1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax+1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax+1)}{32}-\frac{\operatorname{arctanh}(ax)}{32(ax-1)^2}+\frac{3\operatorname{arctanh}(ax)}{32(ax-1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{32}+}{}$
parts	$\frac{\operatorname{arctanh}(ax)^2}{4a^2(a^2x^2-1)^2}-\frac{\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax-1)}-\frac{3\operatorname{arctanh}(ax)\ln(ax-1)}{16}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{3\operatorname{arctanh}(ax)}{16(ax+1)}+\frac{3\operatorname{arctanh}(ax)\ln(ax-}{16}}{}$
risch	$-\frac{(3a^4x^4-6a^2x^2-5)\ln(ax+1)^2}{128a^2(ax-1)(ax+1)(a^2x^2-1)}+\frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax-5\ln(-ax+1))\ln(ax+1)}{64a^2(ax-1)(ax+1)(a^2x^2-1)}$
orering	$\frac{(ax-1)(ax+1)(60a^6x^6-49a^4x^4-34a^2x^2-15)\operatorname{arctanh}(ax)^2}{32a^2(-a^2x^2+1)^3}+\frac{(ax+1)^2(ax-1)^2(24a^4x^4-23a^2x^2-10)\left(\frac{\operatorname{arctanh}(ax}{(-a^2x^2+1}\right.}{32a^2}$

input
```
int(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(3*a^4*x^4*arctanh(a*x)^2+4*a^4*x^4-6*a^3*x^3*arctanh(a*x)-6*a^2*x^2
*arctanh(a*x)^2-5*a^2*x^2+10*a*x*arctanh(a*x)-5*arctanh(a*x)^2)/(a^2*x^2-1
)^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \frac{x\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$=-\frac{12\,a^2x^2+(3\,a^4x^4-6\,a^2x^2-5)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)-16}{128\,(a^6x^4-2\,a^4x^2+a^2)}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/128*(12*a^2*x^2 + (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))
^2 - 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^6*x^4 - 2*a^
4*x^2 + a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = -\int \frac{x\,\text{atanh}^2\,(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.65

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}\right)\text{artanh}\,(ax)}{32\,a} - \frac{12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log\,(ax+1)^2+6\,(a^4x^4-2\,a^2x^2+1)\log\,(ax+1)\log\,(ax-1)-3\,(a^4}{128\,(a^6x^4-2\,a^4x^2+a^2)} + \frac{\text{artanh}\,(ax)^2}{4\,(a^2x^2-1)^2a^2}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/32*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)/a - 1/128*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x
- 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)/(a^6*x^4 - 2*a^4*x
^2 + a^2) + 1/4*arctanh(a*x)^2/((a^2*x^2 - 1)^2*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.01

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = -\frac{1}{512}\left(2\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{4(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+2\left(\frac{(ax-1)^2\left(\right.}{(ax+\right.}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
-1/512*(2*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - (a*
x + 1)^2/((a*x - 1)^2*a^3) + 4*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)/(
a*x - 1))^2 + 2*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3)
 + (a*x + 1)^2/((a*x - 1)^2*a^3) - 8*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x
+ 1)/(a*x - 1)) + (a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^
3) - (a*x + 1)^2/((a*x - 1)^2*a^3) + 16*(a*x + 1)/((a*x - 1)*a^3))*a
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.55

$$\begin{aligned}\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx &= \ln(a\,x+1)^2\left(\frac{1}{16\,a^3\,\left(\frac{1}{a}-2\,a\,x^2+a^3\,x^4\right)}-\frac{3}{128\,a^2}\right)\\ &\quad-\ln(1-a\,x)^2\left(\frac{3}{128\,a^2}-\frac{1}{4\,a^2\,\left(4\,a^4\,x^4-8\,a^2\,x^2+4\right)}\right)\\ &\quad-\ln(1-a\,x)\left(\frac{\frac{1}{4\,a}-\frac{5\,x}{8}+\frac{3\,a^2\,x^3}{8}}{8\,a^5\,x^4-16\,a^3\,x^2+8\,a}-\frac{\frac{5\,x}{8}+\frac{1}{4\,a}-\frac{3\,a^2\,x^3}{8}}{8\,a^5\,x^4-16\,a^3\,x^2+8\,a}\right.\\ &\quad+\ln(a\,x+1)\left(\frac{1}{4\,a^2\,\left(2\,a^4\,x^4-4\,a^2\,x^2+2\right)}\right.\\ &\quad\left.\left.-\frac{3\,\left(a^4\,x^4-2\,a^2\,x^2+1\right)}{32\,a^2\,\left(2\,a^4\,x^4-4\,a^2\,x^2+2\right)}\right)\right)\\ &\quad+\frac{\frac{2}{a^2}-\frac{3\,x^2}{2}}{16\,a^4\,x^4-32\,a^2\,x^2+16}-\frac{\ln(a\,x+1)\,\left(\frac{5\,x}{32\,a^2}-\frac{3\,x^3}{32}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}\end{aligned}$$

input
```
int(-(x*atanh(a*x)^2)/(a^2*x^2 - 1)^3,x)
```

output
```
log(a*x + 1)^2*(1/(16*a^3*(1/a - 2*a*x^2 + a^3*x^4)) - 3/(128*a^2)) - log(
1 - a*x)^2*(3/(128*a^2) - 1/(4*a^2*(4*a^4*x^4 - 8*a^2*x^2 + 4))) - log(1 -
 a*x)*((1/(4*a) - (5*x)/8 + (3*a^2*x^3)/8)/(8*a - 16*a^3*x^2 + 8*a^5*x^4)
- ((5*x)/8 + 1/(4*a) - (3*a^2*x^3)/8)/(8*a - 16*a^3*x^2 + 8*a^5*x^4) + log
(a*x + 1)*(1/(4*a^2*(2*a^4*x^4 - 4*a^2*x^2 + 2)) - (3*(a^4*x^4 - 2*a^2*x^2
 + 1))/(32*a^2*(2*a^4*x^4 - 4*a^2*x^2 + 2)))) + (2/a^2 - (3*x^2)/2)/(16*a^
4*x^4 - 32*a^2*x^2 + 16) - (log(a*x + 1)*((5*x)/(32*a^2) - (3*x^3)/32))/(1
/a - 2*a*x^2 + a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= \frac{-6atanh(ax)^2\,a^4x^4+12atanh(ax)^2\,a^2x^2+10atanh(ax)^2+12atanh(ax)\,a^3x^3-20atanh(ax)\,ax-3a^4x}{64a^2\,(a^4x^4-2a^2x^2+1)}$$

input
```
int(x*atanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

output
```
( - 6*atanh(a*x)**2*a**4*x**4 + 12*atanh(a*x)**2*a**2*x**2 + 10*atanh(a*x)
**2 + 12*atanh(a*x)*a**3*x**3 - 20*atanh(a*x)*a*x - 3*a**4*x**4 + 5)/(64*a
**2*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.311 $\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3} dx = \frac{x}{32(1-a^2x^2)^2} + \frac{15x}{64(1-a^2x^2)} + \frac{15\operatorname{arctanh}(ax)}{64a} - \frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2} - \frac{3\operatorname{arctanh}(ax)}{8a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2} + \frac{3x\operatorname{arctanh}(ax)^2}{8(1-a^2x^2)} + \frac{\operatorname{arctanh}(ax)^3}{8a}$$

output
```
1/32*x/(-a^2*x^2+1)^2+15*x/(-64*a^2*x^2+64)+15/64*arctanh(a*x)/a-1/8*arcta
nh(a*x)/a/(-a^2*x^2+1)^2-3/8*arctanh(a*x)/a/(-a^2*x^2+1)+1/4*x*arctanh(a*x
)^2/(-a^2*x^2+1)^2+3*x*arctanh(a*x)^2/(-8*a^2*x^2+8)+1/8*arctanh(a*x)^3/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{1}{128}\left(\frac{4x}{(-1+a^2x^2)^2} - \frac{30x}{-1+a^2x^2} + \frac{16(-4+3a^2x^2)\operatorname{arctanh}(ax)}{a(-1+a^2x^2)^2} - \frac{16x(-5+3a^2x^2)\operatorname{arctanh}(ax)^2}{(-1+a^2x^2)^2} + \frac{16\operatorname{arctanh}(ax)^3}{a} - \frac{15\log(1-ax)}{a} + \frac{15\log(1+ax)}{a}\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

output
```
((4*x)/(-1 + a^2*x^2)^2 - (30*x)/(-1 + a^2*x^2) + (16*(-4 + 3*a^2*x^2)*Arc
Tanh[a*x])/(a*(-1 + a^2*x^2)^2) - (16*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x]^2)/(
-1 + a^2*x^2)^2 + (16*ArcTanh[a*x]^3)/a - (15*Log[1 - a*x])/a + (15*Log[1
+ a*x])/a)/128
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6526, 215, 215, 219, 6518, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{3}{4}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}dx + \frac{1}{8}\int \frac{1}{(1-a^2x^2)^3}dx + \frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2} - \frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{1}{8}\left(\frac{3}{4}\int \frac{1}{\left(1-a^2x^2\right)^2}dx+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{1}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)$$

$$\Big\downarrow \text{6518}$$

$$\frac{3}{4}\left(-a\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)$$

$$\Big\downarrow \text{6556}$$

$$\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\int \frac{1}{\left(1-a^2x^2\right)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}-a\left(\frac{\text{arctanh}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\frac{x}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

output
```
-1/8*ArcTanh[a*x]/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2
)^2) + (x/(4*(1 - a^2*x^2)^2) + (3*(x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*
a)))/4)/8 + (3*((x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a
) - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh
[a*x]/(2*a))/(2*a))))/4
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 76.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.79

method	result
parallelrisch	$-\frac{-8\operatorname{arctanh}(ax)^3a^4x^4-15a^4x^4\operatorname{arctanh}(ax)+24\operatorname{arctanh}(ax)^2a^3x^3+16\operatorname{arctanh}(ax)^3a^2x^2+15a^3x^3+6a^2x^2\operatorname{arctanh}}{64(a^2x^2-1)^2a}$
risch	$\frac{\ln(ax+1)^3}{64a}-\frac{(3x^4\ln(-ax+1)a^4+6a^3x^3-6x^2\ln(-ax+1)a^2-10ax+3\ln(-ax+1))\ln(ax+1)^2}{64(a^2x^2-1)^2a}+\frac{(3a^4x^4\ln(-ax+}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-8*arctanh(a*x)^3*a^4*x^4-15*a^4*x^4*arctanh(a*x)+24*arctanh(a*x)^2
*a^3*x^3+16*arctanh(a*x)^3*a^2*x^2+15*a^3*x^3+6*a^2*x^2*arctanh(a*x)-40*ar
ctanh(a*x)^2*a*x-8*arctanh(a*x)^3-17*a*x+17*arctanh(a*x))/(a^2*x^2-1)^2/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx =$$
$$-\frac{30\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^3+4\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-34\,ax-(15\,a^4x^4-6}{128\,(a^5x^4-2\,a^3x^2+a)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))
^3 + 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 34*a*x - (15*a^4*
x^4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(129) = 258$.

Time = 0.04 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= -\frac{1}{16}\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2$$
$$-\frac{\left(30\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^3+6\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2\log(ax-1)+2\,(a\right.}{}$$
$$+\frac{\left(12\,a^2x^2-3\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2+6\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)\log(ax-1)-3\,(a\right.}{32\,(a^6x^4-2\,a^4x^2+a^2)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a +
3*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a
*x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x
^4 - 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x
+ 1) + 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2
 + a^3) + 1/32*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 +
6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2
*x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.37

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx = \frac{\frac{17\,x}{8}-\frac{15\,a^2\,x^3}{8}}{8\,a^4\,x^4-16\,a^2\,x^2+8} - \ln(1-a\,x)\left(\frac{3\ln(a\,x+1)^2}{64\,a}-\frac{\frac{7\,x}{2}-3\,a\,x^2+\frac{4}{a}-\frac{5\,a^2\,x^3}{2}}{32\,a^4\,x^4-64\,a^2\,x^2+32} + \frac{\frac{7\,x}{2}+3\,a\,x^2-\frac{4}{a}-\frac{5\,a^2\,x^3}{2}}{32\,a^4\,x^4-64\,a^2\,x^2+32}+\frac{\ln(a\,x+1)\,\left(10\,x-6\,a^2\,x^3\right)}{32\,a^4\,x^4-64\,a^2\,x^2+32}\right) + \ln(1-a\,x)^2\left(\frac{3\,\ln(a\,x+1)}{64\,a}+\frac{\frac{5\,x}{8}-\frac{3\,a^2\,x^3}{8}}{4\,a^4\,x^4-8\,a^2\,x^2+4}\right) + \frac{\ln(a\,x+1)^3}{64\,a}-\frac{\ln(1-a\,x)^3}{64\,a}-\frac{\ln(a\,x+1)\,\left(\frac{1}{4\,a^2}-\frac{3\,x^2}{16}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4} + \frac{\ln(a\,x+1)^2\,\left(\frac{5\,x}{32\,a}-\frac{3\,a\,x^3}{32}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}-\frac{\operatorname{atan}(a\,x\,1\mathrm{i})\,15\mathrm{i}}{64\,a}$$

input

```
int(-atanh(a*x)^2/(a^2*x^2 - 1)^3,x)
```

output

```
((17*x)/8 - (15*a^2*x^3)/8)/(8*a^4*x^4 - 16*a^2*x^2 + 8) - log(1 - a*x)*((
3*log(a*x + 1)^2)/(64*a) - ((7*x)/2 - 3*a*x^2 + 4/a - (5*a^2*x^3)/2)/(32*a
^4*x^4 - 64*a^2*x^2 + 32) + ((7*x)/2 + 3*a*x^2 - 4/a - (5*a^2*x^3)/2)/(32*
a^4*x^4 - 64*a^2*x^2 + 32) + (log(a*x + 1)*(10*x - 6*a^2*x^3))/(32*a^4*x^4
 - 64*a^2*x^2 + 32)) + log(1 - a*x)^2*((3*log(a*x + 1))/(64*a) + ((5*x)/8
- (3*a^2*x^3)/8)/(4*a^4*x^4 - 8*a^2*x^2 + 4)) + log(a*x + 1)^3/(64*a) - lo
g(1 - a*x)^3/(64*a) - (atan(a*x*1i)*15i)/(64*a) - (log(a*x + 1)*(1/(4*a^2)
 - (3*x^2)/16))/(1/a - 2*a*x^2 + a^3*x^4) + (log(a*x + 1)^2*((5*x)/(32*a)
- (3*a*x^3)/32))/(1/a - 2*a*x^2 + a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}\,dx$$
$$= \frac{16atanh(ax)^3\,a^4x^4 - 32atanh(ax)^3\,a^2x^2 + 16atanh(ax)^3 - 48atanh(ax)^2\,a^3x^3 + 80atanh(ax)^2\,ax + 24a}{}$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

output
```
(16*atanh(a*x)**3*a**4*x**4 - 32*atanh(a*x)**3*a**2*x**2 + 16*atanh(a*x)**
3 - 48*atanh(a*x)**2*a**3*x**3 + 80*atanh(a*x)**2*a*x + 24*atanh(a*x)*a**4
*x**4 - 40*atanh(a*x) - 3*log(a**2*x - a)*a**4*x**4 + 6*log(a**2*x - a)*a*
*2*x**2 - 3*log(a**2*x - a) + 3*log(a**2*x + a)*a**4*x**4 - 6*log(a**2*x +
 a)*a**2*x**2 + 3*log(a**2*x + a) - 30*a**3*x**3 + 34*a*x)/(128*a*(a**4*x*
*4 - 2*a**2*x**2 + 1))
```

3.312 $\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 196

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx &= \frac{1}{32(1-a^2x^2)^2}+\frac{11}{32(1-a^2x^2)}-\frac{ax\operatorname{arctanh}(ax)}{8(1-a^2x^2)^2}\\&\quad-\frac{11ax\operatorname{arctanh}(ax)}{16(1-a^2x^2)}-\frac{11}{32}\operatorname{arctanh}(ax)^2\\&\quad+\frac{\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}+\frac{\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{1}{3}\operatorname{arctanh}(ax)^3\\&\quad+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right)\\&\quad-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\\&\quad-\frac{1}{2}\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output

```
1/32/(-a^2*x^2+1)^2+11/(-32*a^2*x^2+32)-1/8*a*x*arctanh(a*x)/(-a^2*x^2+1)^
2-11*a*x*arctanh(a*x)/(-16*a^2*x^2+16)-11/32*arctanh(a*x)^2+1/4*arctanh(a*
x)^2/(-a^2*x^2+1)^2+arctanh(a*x)^2/(-2*a^2*x^2+2)+1/3*arctanh(a*x)^3+arcta
nh(a*x)^2*ln(2-2/(a*x+1))-arctanh(a*x)*polylog(2,-1+2/(a*x+1))-1/2*polylog
(3,-1+2/(a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx = \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right) + \frac{1}{768}\Big(32i\pi^3 - 256\operatorname{arctanh}(ax)^3 + 144\cosh(2\operatorname{arctanh}(ax)) + 3\cosh(4\operatorname{arctanh}(ax)) + 24\operatorname{arctanh}(ax)^2\left(12\cosh(2\operatorname{arctanh}(ax)) + \cosh(4\operatorname{arctanh}(ax)) + 32\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)\right) - 384\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right) - 12\operatorname{arctanh}(ax)(24\sinh(2\operatorname{arctanh}(ax)) + \sinh(4\operatorname{arctanh}(ax)))\Big)$$

input
```
Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

output
```
ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((32*I)*Pi^3 - 256*ArcTanh[a
*x]^3 + 144*Cosh[2*ArcTanh[a*x]] + 3*Cosh[4*ArcTanh[a*x]] + 24*ArcTanh[a*x
]^2*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTanh[a*x]] + 32*Log[1 - E^(2*ArcT
anh[a*x])]) - 384*PolyLog[3, E^(2*ArcTanh[a*x])] - 12*ArcTanh[a*x]*(24*Sin
h[2*ArcTanh[a*x]] + Sinh[4*ArcTanh[a*x]]))/768
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6592, 6556, 6522, 6518, 241, 6592, 6550, 6494, 6556, 6518, 241, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx$$

$$\Big\downarrow \; 6592$$

$$a^2\int \frac{x\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}dx + \int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6556}$$

$$a^2 \left(\frac{\text{arctanh}(ax)^2}{4a^2 \left(1-a^2x^2\right)^2}-\frac{\int \frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^3}dx}{2a}\right)+\int \frac{\text{arctanh}(ax)^2}{x \left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \text{6522}$$

$$a^2 \left(\frac{\text{arctanh}(ax)^2}{4a^2 \left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\int \frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{1}{16a\left(1-a^2x^2\right)^2}}{2a}\right)+\int \frac{\text{arctanh}(ax)^2}{x \left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \text{6518}$$

$$a^2 \left(\frac{\text{arctanh}(ax)^2}{4a^2 \left(1-a^2x^2\right)^2}-\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{1}{16a\left(1-a^2x^2\right)^2}}{2a}\right)+\int \frac{\text{arctanh}(ax)^2}{x \left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \text{241}$$

$$\int \frac{\text{arctanh}(ax)^2}{x \left(1-a^2x^2\right)^2}dx+a^2 \left(\frac{\text{arctanh}(ax)^2}{4a^2 \left(1-a^2x^2\right)^2}-\frac{\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}}{2a}\right)$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\int \frac{\text{arctanh}(ax)^2}{x \left(1-a^2x^2\right)}dx+a^2 \left(\frac{\text{arctanh}(ax)^2}{4a^2 \left(1-a^2x^2\right)^2}-\frac{\frac{x\text{arctanh}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2\left(1-a^2x^2\right)}-\frac{1}{4a\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a\left(1-a^2x^2\right)^2}}{2a}\right)$$

$$\Big\downarrow \text{6550}$$

$$a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+$$
$$a^2\left(\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)+$$
$$\frac{1}{3}\text{arctanh}(ax)^3$$

$\Big\downarrow$ 6494

$$a^2\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$
$$a^2\left(\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)+$$
$$\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$\Big\downarrow$ 6556

$$a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$
$$a^2\left(\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)+$$
$$\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$\Big\downarrow$ 6518

$$a^2\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)-$$
$$2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$
$$a^2\left(\frac{\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)+$$
$$\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$\Big\downarrow$ 241

$$-2a\int \frac{\operatorname{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+ a^2\left(\frac{\operatorname{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}}{a}\right)+ a^2\left(\frac{\operatorname{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\operatorname{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)+ \frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 6618$$

$$-2a\left(\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+ a^2\left(\frac{\operatorname{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}}{a}\right)+ a^2\left(\frac{\operatorname{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\operatorname{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)+ \frac{1}{3}\operatorname{arctanh}(ax)^3+\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 7164$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}}{a}\right)+ a^2\left(\frac{\operatorname{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{\frac{x\operatorname{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}}{2a}\right)- 2a\left(\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\operatorname{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\operatorname{arctanh}(ax)^3+ \operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)$$

input

```
Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

output
```
ArcTanh[a*x]^3/3 + a^2*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*
(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)
)/a) + a^2*(ArcTanh[a*x]^2/(4*a^2*(1 - a^2*x^2)^2) - (-1/16*1/(a*(1 - a^2*
x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-1/4*1/(a*(1 - a^2*x^
2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)))/4)/(2*a)
) + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLog[2, -1
 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 261.64 (sec) , antiderivative size = 1305, normalized size of antiderivative = 6.66

method	result	size
derivativedivides	Expression too large to display	1305
default	Expression too large to display	1305
parts	Expression too large to display	1716

input
```
int(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)
)+3/16*arctanh(a*x)*(a*x+1)/(a*x-1)-3/16*arctanh(a*x)*(a*x-1)/(a*x+1)-1/2*
arctanh(a*x)^2*ln(a*x-1)-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylo
g(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3+1/512*(a*x+1)^2/(a*x-1)
^2+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*arctanh(a*x)^2/(a*x-1)^2+1/16*arc
tanh(a*x)^2/(a*x+1)^2-3/32*(a*x-1)/(a*x+1)-3/32*(a*x+1)/(a*x-1)-arctanh(a*
x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)
^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/512*(a*x-1)^2/(a
*x+1)^2-5/16*arctanh(a*x)^2/(a*x-1)+5/16*arctanh(a*x)^2/(a*x+1)+arctanh(a*
x)^2*ln(a*x)+1/32*(16*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+8*I*Pi*csg
n(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-8*I*Pi*csg
n(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+
1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+16*I*Pi*csgn(I*(-(a*x+1)^2/(a
^2*x^2-1)-1))*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x
^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))+16*I*Pi+16*I*Pi*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-16*I*Pi*csgn(I/(-(a*x+1)^2/(
a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(a^2*x^2-1)+1
))^2+8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-8*I*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2+8*I*P...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^3x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^2/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^3}\,dx = -\int \frac{\operatorname{atanh}^2(ax)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

input
```
integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a**x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)^3x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/2*a^6*integrate(1/2*x^6*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5
+ 3*a^2*x^3 - x), x) + 1/2*a^5*integrate(1/2*x^5*log(a*x + 1)*log(-a*x + 1
)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/256*(a*(2*(5*a^2*x^2 + 3*a
*x - 6)/(a^8*x^3 - a^7*x^2 - a^6*x + a^5) - 5*log(a*x + 1)/a^5 + 5*log(a*x
 - 1)/a^5) + 16*(2*a^2*x^2 - 1)*log(-a*x + 1)/(a^8*x^4 - 2*a^6*x^2 + a^4))
*a^4 - a^4*integrate(1/2*x^4*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x
^5 + 3*a^2*x^3 - x), x) - a^3*integrate(1/2*x^3*log(a*x + 1)*log(-a*x + 1)
/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + 1/2*a^3*integrate(1/2*x^3*log
(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 3/512*(a*(2*(3*a^2*
x^2 - 3*a*x - 2)/(a^6*x^3 - a^5*x^2 - a^4*x + a^3) - 3*log(a*x + 1)/a^3 +
3*log(a*x - 1)/a^3) - 16*log(-a*x + 1)/(a^6*x^4 - 2*a^4*x^2 + a^2))*a^2 +
1/2*a^2*integrate(1/2*x^2*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5
+ 3*a^2*x^3 - x), x) + 1/2*a*integrate(1/2*x*log(a*x + 1)*log(-a*x + 1)/(a
^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 3/4*a*integrate(1/2*x*log(-a*x +
 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/48*(2*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(-a*x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a
*x + 1) - 3)*log(-a*x + 1)^2)/(a^4*x^4 - 2*a^2*x^2 + 1) - 1/2*integrate(1/
2*log(a*x + 1)^2/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + integrate(1/2
*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^3}\,dx=\int -\frac{\operatorname{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)^3x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^2}{x\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-atanh(a*x)^2/(x*(a^2*x^2 - 1)^3),x)
```

output
```
-int(atanh(a*x)^2/(x*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^3}\,dx = -\left(\int \frac{atanh(ax)^2}{a^6x^7-3a^4x^5+3a^2x^3-x}dx\right)$$

input
```
int(atanh(a*x)^2/x/(-a^2*x^2+1)^3,x)
```

output
```
- int(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),x)
```

3.313 $\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 209

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx = \frac{a^2x}{32(1-a^2x^2)^2}+\frac{31a^2x}{64(1-a^2x^2)}+\frac{31}{64}a\operatorname{arctanh}(ax) - \frac{a\operatorname{arctanh}(ax)}{8(1-a^2x^2)^2}-\frac{7a\operatorname{arctanh}(ax)}{8(1-a^2x^2)}+a\operatorname{arctanh}(ax)^2 - \frac{\operatorname{arctanh}(ax)^2}{x}+\frac{a^2x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}+\frac{7a^2x\operatorname{arctanh}(ax)^2}{8(1-a^2x^2)} + \frac{5}{8}a\operatorname{arctanh}(ax)^3+2a\operatorname{arctanh}(ax)\log\left(2-\frac{2}{1+ax}\right) - a\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output

```
1/32*a^2*x/(-a^2*x^2+1)^2+31*a^2*x/(-64*a^2*x^2+64)+31/64*a*arctanh(a*x)-1
/8*a*arctanh(a*x)/(-a^2*x^2+1)^2-7*a*arctanh(a*x)/(-8*a^2*x^2+8)+a*arctanh
(a*x)^2-arctanh(a*x)^2/x+1/4*a^2*x*arctanh(a*x)^2/(-a^2*x^2+1)^2+7*a^2*x*a
rctanh(a*x)^2/(-8*a^2*x^2+8)+5/8*a*arctanh(a*x)^3+2*a*arctanh(a*x)*ln(2-2/
(a*x+1))-a*polylog(2,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx = -a\left(-\frac{5}{8}\operatorname{arctanh}(ax)^3+\frac{1}{64}\operatorname{arctanh}(ax)\left(32\cosh(2\operatorname{arctanh}(ax))+\cosh(4\operatorname{arctanh}(ax))-128\log\left(1-e^{-2\operatorname{arctanh}(ax)}\right)\right)+\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(ax)}\right)-\frac{1}{4}\sinh(2\operatorname{arctanh}(ax))+\operatorname{arctanh}(ax)^2\left(-1+\frac{1}{ax}+\frac{ax}{-1+a^2x^2}-\frac{1}{32}\sinh(4\operatorname{arctanh}(ax))\right)-\frac{1}{256}\sinh(4\operatorname{arctanh}(ax))\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

output
```
-(a*((-5*ArcTanh[a*x]^3)/8 + (ArcTanh[a*x]*(32*Cosh[2*ArcTanh[a*x]] + Cosh
[4*ArcTanh[a*x]] - 128*Log[1 - E^(-2*ArcTanh[a*x])]))/64 + PolyLog[2, E^(-
2*ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]]/4 + ArcTanh[a*x]^2*(-1 + 1/(a*x) +
 (a*x)/(-1 + a^2*x^2) - Sinh[4*ArcTanh[a*x]]/32) - Sinh[4*ArcTanh[a*x]]/25
6))
```

Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.84, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {6592, 6526, 215, 215, 219, 6518, 6556, 215, 219, 6592, 6518, 6544, 6452, 6510, 6550, 6494, 2897, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx+\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6526}$$

$$a^2\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\int \frac{1}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)+\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$a^2\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\int \frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)+\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$a^2\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arc}}{8a(}\right.$$
$$\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{219}$$

$$a^2\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2}\right.\right.$$
$$\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6518}$$

$$a^2\left(\frac{3}{4}\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\right.\right.$$
$$\int \frac{\text{arctanh}(ax)^2}{x^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(\frac{3}{4}\left(-a\left(\frac{\operatorname{arctanh}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\int\frac{1}{\left(1-a^2x^2\right)^2}dx}{2a}\right)+\frac{x\operatorname{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)+\frac{x\operatorname{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\operatorname{arctan}}{8a(1-}\right.$$
$$\int\frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)^2}dx$$

$\big\downarrow$ 215

$$a^2\left(\frac{3}{4}\left(-a\left(\frac{\operatorname{arctanh}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}}{2a}\right)+\frac{x\operatorname{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)+\frac{x\operatorname{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}\right.$$
$$\int\frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)^2}dx$$

$\big\downarrow$ 219

$$\int\frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)^2}dx+$$
$$a^2\left(\frac{x\operatorname{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\operatorname{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(}{2\left(1-a^2\right.}\right.\right.$$

$\big\downarrow$ 6592

$$a^2\int\frac{\operatorname{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\int\frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)}dx+$$
$$a^2\left(\frac{x\operatorname{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\operatorname{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(}{2\left(1-a^2\right.}\right.\right.$$

$\big\downarrow$ 6518

$$a^2\left(-a\int\frac{x\operatorname{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\frac{x\operatorname{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)+\int\frac{\operatorname{arctanh}(ax)^2}{x^2\left(1-a^2x^2\right)}dx+$$
$$a^2\left(\frac{x\operatorname{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\operatorname{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(}{2\left(1-a^2\right.}\right.\right.$$

$\big\downarrow$ 6544

$$a^2\left(-a\int\frac{x\operatorname{arctanh}(ax)}{\left(1-a^2x^2\right)^2}dx+\frac{x\operatorname{arctanh}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)+a^2\int\frac{\operatorname{arctanh}(ax)^2}{1-a^2x^2}dx+$$
$$\int\frac{\operatorname{arctanh}(ax)^2}{x^2}dx+$$
$$a^2\left(\frac{x\operatorname{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\operatorname{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2\left(1-a^2x^2\right)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(}{2\left(1-a^2\right.}\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$a^2\left(-a\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+a^2\int\frac{\text{arctanh}(ax)^2}{1-a^2x^2}dx+ 2a\int\frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+ a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right. \frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6510}$$

$$a^2\left(-a\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+2a\int\frac{\text{arctanh}(ax)}{x(1-a^2x^2)}dx+ a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right. \frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(-a\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+ 2a\left(\int\frac{\text{arctanh}(ax)}{x(ax+1)}dx+\frac{1}{2}\text{arctanh}(ax)^2\right)+ a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right. \frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(-a\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+ 2a\left(-a\int\frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)\right)+ a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right. \frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{2897}$$

$$a^2\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right.$$
$$2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+$$
$$\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right.$$
$$2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+$$
$$\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{215}$$

$$a^2\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right.$$
$$2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+$$
$$\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{219}$$

$$a^2\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(}{2(1-a^2}\right.\right.$$
$$2a\left(\frac{1}{2}\text{arctanh}(ax)^2+\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\right)+$$
$$\frac{1}{3}a\text{arctanh}(ax)^3-\frac{\text{arctanh}(ax)^2}{x}$$

input
```
Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

output
```
-(ArcTanh[a*x]^2/x) + (a*ArcTanh[a*x]^3)/3 + a^2*((x*ArcTanh[a*x]^2)/(2*(1
 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)
) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))) + a^2*(-1/8*ArcTanh
[a*x]/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (x/(4
*(1 - a^2*x^2)^2) + (3*(x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a)))/4)/8 +
(3*((x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTa
nh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))
/(2*a))))/4) + 2*a*(ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] -
 PolyLog[2, -1 + 2/(1 + a*x)]/2)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 174.77 (sec) , antiderivative size = 3116, normalized size of antiderivative = 14.91

method	result	size
default	Expression too large to display	3116
parts	Expression too large to display	3127
derivativedivides	Expression too large to display	3152

input
```
int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(15/16*arctanh(a*x)^2*ln(a*x+1)-15/8*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2
+1)^(1/2))-arctanh(a*x)^2/a/x+1/4*arctanh(a*x)*(a*x+1)/(a*x-1)+1/4*arctanh
(a*x)*(a*x-1)/(a*x+1)-15/16*arctanh(a*x)^2*ln(a*x-1)+polylog(2,-(a*x+1)/(-
a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*Pi*csgn(I/
(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*(-arctanh(a*x)^2+arctanh(a*x)*ln(
1+(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2)
)+polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1
/2)))+5/8*arctanh(a*x)^3-arctanh(a*x)^2+1/512*(a*x+1)^2/(a*x-1)^2-15/32*I*
Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*(-arctanh(a*x)*ln(1+(a*
x+1)/(-a^2*x^2+1)^(1/2))+dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)
/(-a^2*x^2+1)^(1/2)))+1/16*arctanh(a*x)^2/(a*x-1)^2-1/16*arctanh(a*x)^2/(a
*x+1)^2+1/8*(a*x-1)/(a*x+1)-1/8*(a*x+1)/(a*x-1)+15/32*I*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^3*(-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+dilog((a*
x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2)))+15/16*I*Pi*c
sgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*(-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))+dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)/(-a^2*x^2+1)^(
1/2)))-15/16*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*(-arctanh(a*x)*ln(1
+(a*x+1)/(-a^2*x^2+1)^(1/2))+dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^3 x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^2/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)^3} \, dx = -\int \frac{\text{atanh}^2(ax)}{a^6x^8 - 3a^4x^6 + 3a^2x^4 - x^2} \, dx$$

input
```
integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)**2/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(186) = 372.

Time = 0.06 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.56

$$\int \frac{\text{arctanh}(ax)^2}{x^2 (1-a^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/128*a^2*(2*(31*a^3*x^3 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 5
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - (16*a^4*x^4 - 32*a^2*x^2 - 15*
(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1) + 16)*log(a*x + 1)^2 + 16*(a^4*x^4
- 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 33*a*x - (15*(a^4*x^4 - 2*a^2*x^2 + 1)*l
og(a*x - 1)^2 - 32*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))/(
a^5*x^4 - 2*a^3*x^2 + a) - 128*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1
/2*a*x + 1/2))/a + 128*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 128*(log(-a
*x + 1)*log(x) + dilog(a*x))/a - 31*log(a*x + 1)/a + 31*log(a*x - 1)/a) +
1/32*a*((28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^
4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 3
2*log(a*x - 1) + 64*log(x))*arctanh(a*x) + 1/16*(15*a*log(a*x + 1) - 15*a*
log(a*x - 1) - 2*(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*
arctanh(a*x)^2
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^3x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(atanh(a*x)^2/(x^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^3}\,dx$$
$$= \frac{80atanh(ax)^3a^5x^5-160atanh(ax)^3a^3x^3+80atanh(ax)^3ax-240atanh(ax)^2a^4x^4+400atanh(ax)^2a^2}{}$$

input
```
int(atanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x)
```

output

```
(80*atanh(a*x)**3*a**5*x**5 - 160*atanh(a*x)**3*a**3*x**3 + 80*atanh(a*x)*
*3*a*x - 240*atanh(a*x)**2*a**4*x**4 + 400*atanh(a*x)**2*a**2*x**2 - 128*a
tanh(a*x)**2 + 120*atanh(a*x)*a**5*x**5 - 200*atanh(a*x)*a*x - 256*int(ata
nh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),x)*a**5*x**5 + 512*int
(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),x)*a**3*x**3 - 256
*int(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),x)*a*x - 15*lo
g(a**2*x - a)*a**5*x**5 + 30*log(a**2*x - a)*a**3*x**3 - 15*log(a**2*x - a
)*a*x + 15*log(a**2*x + a)*a**5*x**5 - 30*log(a**2*x + a)*a**3*x**3 + 15*l
og(a**2*x + a)*a*x - 150*a**4*x**4 + 170*a**2*x**2)/(128*x*(a**4*x**4 - 2*
a**2*x**2 + 1))
```

3.314 $\int \frac{x^3\textbf{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\frac{3x^3}{128a(1-a^2x^2)^2}+\frac{45x}{256a^3(1-a^2x^2)}+\frac{27\text{arctanh}(ax)}{256a^4}+\frac{3x^4\text{arctanh}(ax)}{32(1-a^2x^2)^2}-\frac{9\text{arctanh}(ax)}{32a^4(1-a^2x^2)}-\frac{3x^3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{9x\text{arctanh}(ax)^2}{32a^3(1-a^2x^2)}-\frac{3\text{arctanh}(ax)^3}{32a^4}+\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}$$

output
```
-3/128*x^3/a/(-a^2*x^2+1)^2+45/256*x/a^3/(-a^2*x^2+1)+27/256*arctanh(a*x)/
a^4+3/32*x^4*arctanh(a*x)/(-a^2*x^2+1)^2-9/32*arctanh(a*x)/a^4/(-a^2*x^2+1
)-3/16*x^3*arctanh(a*x)^2/a/(-a^2*x^2+1)^2+9/32*x*arctanh(a*x)^2/a^3/(-a^2
*x^2+1)-3/32*arctanh(a*x)^3/a^4+1/4*x^4*arctanh(a*x)^3/(-a^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.70

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^3} \, dx = \frac{48(-4+5a^2x^2)\text{arctanh}(ax)-48ax(-3+5a^2x^2)\text{arctanh}(ax)^2+16(-3+6a^2x^2+5a^4x^4)\text{arctanh}(ax)}{512a^4(-1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

output
```
(48*(-4 + 5*a^2*x^2)*ArcTanh[a*x] - 48*a*x*(-3 + 5*a^2*x^2)*ArcTanh[a*x]^2
 + 16*(-3 + 6*a^2*x^2 + 5*a^4*x^4)*ArcTanh[a*x]^3 + 3*(30*a*x - 34*a^3*x^3
 - 17*(-1 + a^2*x^2)^2*Log[1 - a*x] + 17*(-1 + a^2*x^2)^2*Log[1 + a*x]))/(
512*a^4*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6570, 6566, 252, 252, 219, 6562, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^3} \, dx$$

$$\Big\downarrow \text{ 6570}$$

$$\frac{x^4 \text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3}{4}a\int \frac{x^4 \text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{ 6566}$$

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-$$
$$\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{4a^2}+\frac{1}{8}\int\frac{x^4}{(1-a^2x^2)^3}dx-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}\right)$$

$\downarrow$ 252

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-$$
$$\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(1-a^2x^2)^2}-\frac{3\int\frac{x^2}{(1-a^2x^2)^2}dx}{4a^2}\right)-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}\right)$$

$\downarrow$ 252

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-$$
$$\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{1-a^2x^2}dx}{2a^2}\right)}{4a^2}\right)-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arct}}{4a^2(1}\right.$$

$\downarrow$ 219

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-$$
$$\frac{3}{4}a\left(-\frac{3\int\frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{4a^2}-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x}{2a^2(1-a^2x^2)}-\frac{\text{arcta}}{2}\right.}{4a^2}\right.\right.$$

$\downarrow$ 6562

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-$$
$$\frac{3}{4}a\left(-\frac{3\left(-\frac{\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}-\frac{\text{arctanh}(ax)^3}{6a^3}+\frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}\right)}{4a^2}-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{}{4a^2(}\right.\right.$$

$\downarrow$ 6556

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3}{4}a\left(-\frac{3\left(-\frac{\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{(1-a^2x^2)^2}dx}{2a}}{a}-\frac{\text{arctanh}(ax)^3}{6a^3}+\frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}\right)}{4a^2}-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}+\right.$$

$$\downarrow 215$$

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3}{4}a\left(-\frac{3\left(-\frac{\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}}{a}-\frac{\text{arctanh}(ax)^3}{6a^3}+\frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}\right)}{4a^2}-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}}{4a^2(1-a}\right.$$

$$\downarrow 219$$

$$\frac{x^4\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3}{4}a\left(-\frac{x^4\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{x^3\text{arctanh}(ax)^2}{4a^2(1-a^2x^2)^2}-\frac{3\left(-\frac{\text{arctanh}(ax)^3}{6a^3}+\frac{x\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}}{2a}}{2a}}{a}\right.}{4a^2}\right.$$

input
```
Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

output
```
(x^4*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) - (3*a*(-1/8*(x^4*ArcTanh[a*x])/(
a*(1 - a^2*x^2)^2) + (x^3*ArcTanh[a*x]^2)/(4*a^2*(1 - a^2*x^2)^2) + (x^3/(
4*a^2*(1 - a^2*x^2)^2) - (3*(x/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]/(2*a^3
)))/(4*a^2))/8 - (3*((x*ArcTanh[a*x]^2)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*
x]^3/(6*a^3) - (ArcTanh[a*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2))
+ ArcTanh[a*x]/(2*a))/(2*a))/a))/(4*a^2)))/4
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6562
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> Simp[-(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(2*c^2*d*(d + e*x^2))), x] - Simp[b*(
p/(2*c))   Int[x*((a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6566
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Simp[(-b)*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a +
b*ArcTanh[c*x])^(p - 1)/(c*d*m^2)), x] + (Simp[f*(f*x)^(m - 1)*(d + e*x^2)^
(q + 1)*((a + b*ArcTanh[c*x])^p/(c^2*d*m)), x] - Simp[f^2*((m - 1)/(c^2*d*m
))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p, x], x] +
 Simp[b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^
(p - 2), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] &&
 EqQ[m + 2*q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 342.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

method	result
parallelrisch	$-\frac{-40\operatorname{arctanh}(ax)^3a^4x^4-51a^4x^4\operatorname{arctanh}(ax)+120\operatorname{arctanh}(ax)^2a^3x^3-48\operatorname{arctanh}(ax)^3a^2x^2+51a^3x^3-18a^2x^2\operatorname{arcta}}{256(a^2x^2-1)^2a^4}$
risch	$\frac{(5a^4x^4+6a^2x^2-3)\ln(ax+1)^3}{256a^4(a^2x^2-1)^2}-\frac{3(5x^4\ln(-ax+1)a^4+10a^3x^3+6x^2\ln(-ax+1)a^2-6ax-3\ln(-ax+1))\ln(ax+1)^2}{256a^4(ax-1)(ax+1)(a^2x^2-1)}$
orering	$-\frac{(ax-1)(ax+1)(153a^8x^8+64a^6x^6+48a^4x^4-135)\operatorname{arctanh}(ax)^3}{64a^6x^2(-a^2x^2+1)^3}-\frac{(ax+1)^2(ax-1)^2(578a^6x^6+107a^4x^4-150a^2x^2}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/256*(-40*arctanh(a*x)^3*a^4*x^4-51*a^4*x^4*arctanh(a*x)+120*arctanh(a*x
)^2*a^3*x^3-48*arctanh(a*x)^3*a^2*x^2+51*a^3*x^3-18*a^2*x^2*arctanh(a*x)-7
2*arctanh(a*x)^2*a*x+24*arctanh(a*x)^3-45*a*x+45*arctanh(a*x))/(a^2*x^2-1)
^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.73

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\frac{102\,a^3x^3 - 2\,(5\,a^4x^4 + 6\,a^2x^2 - 3)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 12\,(5\,a^3x^3 - 3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 90\,ax - 3\,(17\,a^4x}{512\,(a^8x^4 - 2\,a^6x^2 + a^4)}$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/512*(102*a^3*x^3 - 2*(5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x -
1))^3 + 12*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 90*a*x - 3*(1
7*a^4*x^4 + 6*a^2*x^2 - 15)*log(-(a*x + 1)/(a*x - 1)))/(a^8*x^4 - 2*a^6*x^
2 + a^4)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\int \frac{x^3\,\text{atanh}^3\,(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

input
```
integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x**3*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(167) = 334$.

Time = 0.04 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.28

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$= -\frac{3}{64}\,a\left(\frac{2\,(5\,a^2x^3-3\,x)}{a^8x^4-2\,a^6x^2+a^4}-\frac{5\,\log(ax+1)}{a^5}+\frac{5\,\log(ax-1)}{a^5}\right)\text{artanh}\,(ax)^2$$
$$+\frac{(2\,a^2x^2-1)\,\text{artanh}\,(ax)^3}{4\,(a^8x^4-2\,a^6x^2+a^4)}$$
$$-\frac{1}{512}\left(\frac{(102\,a^3x^3-10\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^3+30\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2\log(ax-\right.$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-3/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/
a^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x)^2 + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x
)^3/(a^8*x^4 - 2*a^6*x^2 + a^4) - 1/512*((102*a^3*x^3 - 10*(a^4*x^4 - 2*a^
2*x^2 + 1)*log(a*x + 1)^3 + 30*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*lo
g(a*x - 1) + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 90*a*x - 3*(17*
a^4*x^4 - 34*a^2*x^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 17)*l
og(a*x + 1) + 51*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^11*x^4 - 2
*a^9*x^2 + a^7) - 12*(20*a^2*x^2 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1
)^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 5*(a^4*x^4
- 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^10*x^4 - 2*a^8*x^2
 + a^6))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(167) = 334$.

Time = 0.13 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.78

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$=\frac{1}{2048}\left(4\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}+1\right)}{(ax+1)^2a^5}+\frac{(ax+1)^2}{(ax-1)^2a^5}+\frac{4(ax+1)}{(ax-1)a^5}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+6\left(\frac{(ax-1)^2\left(\frac{8(}{a}\right.}{(ax+1}\right.\right.$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
1/2048*(4*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5) + (a*
x + 1)^2/((a*x - 1)^2*a^5) + 4*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x + 1)/(
a*x - 1))^3 + 6*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)^2*a^5)
 - (a*x + 1)^2/((a*x - 1)^2*a^5) - 8*(a*x + 1)/((a*x - 1)*a^5))*log(-(a*x
+ 1)/(a*x - 1))^2 + 6*((a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) + 1)/((a*x + 1)
^2*a^5) + (a*x + 1)^2/((a*x - 1)^2*a^5) + 16*(a*x + 1)/((a*x - 1)*a^5))*lo
g(-(a*x + 1)/(a*x - 1)) + 3*(a*x - 1)^2*(32*(a*x + 1)/(a*x - 1) + 1)/((a*x
 + 1)^2*a^5) - 3*(a*x + 1)^2/((a*x - 1)^2*a^5) - 96*(a*x + 1)/((a*x - 1)*a
^5))*a
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.16

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$=\frac{48\,\ln(1-a\,x)-48\,\ln(a\,x+1)+51\,\text{atanh}(a\,x)+45\,a\,x-3\ln(a\,x+1)^3+3\ln(1-a\,x)^3-9\,\ln(a\,x+}{}$$

input
```
int(-(x^3*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

output
```
(48*log(1 - a*x) - 48*log(a*x + 1) + 51*atanh(a*x) + 45*a*x - 3*log(a*x +
1)^3 + 3*log(1 - a*x)^3 - 9*log(a*x + 1)*log(1 - a*x)^2 + 9*log(a*x + 1)^2
*log(1 - a*x) - 51*a^3*x^3 + 6*a^2*x^2*log(a*x + 1)^3 - 6*a^2*x^2*log(1 -
a*x)^3 - 30*a^3*x^3*log(a*x + 1)^2 - 30*a^3*x^3*log(1 - a*x)^2 + 5*a^4*x^4
*log(a*x + 1)^3 - 5*a^4*x^4*log(1 - a*x)^3 - 102*a^2*x^2*atanh(a*x) + 51*a
^4*x^4*atanh(a*x) + 18*a*x*log(a*x + 1)^2 + 18*a*x*log(1 - a*x)^2 + 60*a^2
*x^2*log(a*x + 1) - 60*a^2*x^2*log(1 - a*x) - 36*a*x*log(a*x + 1)*log(1 -
a*x) + 18*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 - 18*a^2*x^2*log(a*x + 1)^2*
log(1 - a*x) + 15*a^4*x^4*log(a*x + 1)*log(1 - a*x)^2 - 15*a^4*x^4*log(a*x
 + 1)^2*log(1 - a*x) + 60*a^3*x^3*log(a*x + 1)*log(1 - a*x))/(256*a^4*(a^2
*x^2 - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^3} \, dx$$
$$= \frac{80atanh(ax)^3 a^4x^4 + 96atanh(ax)^3 a^2x^2 - 48atanh(ax)^3 - 240atanh(ax)^2 a^3x^3 + 144atanh(ax)^2 ax + 1$$

input
```
int(x^3*atanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

output
```
(80*atanh(a*x)**3*a**4*x**4 + 96*atanh(a*x)**3*a**2*x**2 - 48*atanh(a*x)**
3 - 240*atanh(a*x)**2*a**3*x**3 + 144*atanh(a*x)**2*a*x + 120*atanh(a*x)*a
**4*x**4 - 72*atanh(a*x) + 9*log(a**2*x - a)*a**4*x**4 - 18*log(a**2*x - a
)*a**2*x**2 + 9*log(a**2*x - a) - 9*log(a**2*x + a)*a**4*x**4 + 18*log(a**
2*x + a)*a**2*x**2 - 9*log(a**2*x + a) - 102*a**3*x**3 + 90*a*x)/(512*a**4
*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.315 $\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 215

$$\begin{aligned}\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx =& -\frac{3}{128a^3(1-a^2x^2)^2}+\frac{3}{128a^3(1-a^2x^2)}\\ &+\frac{3x\text{arctanh}(ax)}{32a^2(1-a^2x^2)^2}-\frac{3x\text{arctanh}(ax)}{64a^2(1-a^2x^2)}\\ &-\frac{3\text{arctanh}(ax)^2}{128a^3}-\frac{3\text{arctanh}(ax)^2}{16a^3(1-a^2x^2)^2}+\frac{3\text{arctanh}(ax)^2}{16a^3(1-a^2x^2)}\\ &+\frac{x\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{x\text{arctanh}(ax)^3}{8a^2(1-a^2x^2)}-\frac{\text{arctanh}(ax)^4}{32a^3}\end{aligned}$$

output
```
-3/128/a^3/(-a^2*x^2+1)^2+3/128/a^3/(-a^2*x^2+1)+3/32*x*arctanh(a*x)/a^2/(
-a^2*x^2+1)^2-3/64*x*arctanh(a*x)/a^2/(-a^2*x^2+1)-3/128*arctanh(a*x)^2/a^
3-3/16*arctanh(a*x)^2/a^3/(-a^2*x^2+1)^2+3/16*arctanh(a*x)^2/a^3/(-a^2*x^2
+1)+1/4*x*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^2-1/8*x*arctanh(a*x)^3/a^2/(-a^2
*x^2+1)-1/32*arctanh(a*x)^4/a^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$= \frac{-3a^2x^2+6(ax+a^3x^3)\,\text{arctanh}(ax)-3(1+6a^2x^2+a^4x^4)\,\text{arctanh}(ax)^2+16(ax+a^3x^3)\,\text{arctanh}(ax)^3}{128a^3\left(-1+a^2x^2\right)^2}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

output
```
(-3*a^2*x^2 + 6*(a*x + a^3*x^3)*ArcTanh[a*x] - 3*(1 + 6*a^2*x^2 + a^4*x^4)
*ArcTanh[a*x]^2 + 16*(a*x + a^3*x^3)*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*A
rcTanh[a*x]^4)/(128*a^3*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6590, 6518, 6526, 6518, 6522, 6518, 241, 6556, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$

$$\Big\downarrow \; 6590$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^3}dx}{a^2}-\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx}{a^2}$$

$$\Big\downarrow \; 6518$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^3}dx}{a^2}-\frac{-\frac{3}{2}a\int \frac{x\,\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\,\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$$\Big\downarrow \; 6526$$

$$\frac{\frac{3}{8}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{3}{4}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}}{a^2}-\frac{-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$\downarrow$ 6518

$$\frac{\frac{3}{8}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax}{16a(1-a^2x^2)^2}}{a^2}$$
$$\frac{-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$\downarrow$ 6522

$$\frac{\frac{3}{8}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax}{8a}\right.}{a^2}$$
$$\frac{-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$\downarrow$ 6518

$$\frac{\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(a}{(1-a^2x^2)^2}\right.}{a^2}$$
$$\frac{-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$\downarrow$ 241

$$\frac{\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}\right.}{a^2}$$
$$\frac{-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$\downarrow$ 6556

$$\frac{\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+}{a^2}$$

$$\frac{-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$$\Big\downarrow \text{6518}$$

$$\frac{\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{ar}}{4}}{a^2}$$

$$\frac{-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}}{a^2}$$

input
```
Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

output
```
-(((x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a) - (3*a*(Arc
Tanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh
[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a))/2)/a^2) + ((-3*ArcTan
h[a*x]^2)/(16*a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2)
+ (3*(-1/16*1/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) +
 (3*(-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTa
nh[a*x]^2/(4*a)))/4))/8 + (3*((x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + ArcTa
nh[a*x]^4/(8*a) - (3*a*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*
(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)
)/a))/2))/4)/a^2
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 349.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

method	result
parallelrisch	$-\frac{4 \operatorname{arctanh}(ax)^4 a^4 x^4+3a^4x^4 \operatorname{arctanh}(ax)^2-16 \operatorname{arctanh}(ax)^3 a^3 x^3-8 \operatorname{arctanh}(ax)^4 a^2 x^2-6a^3x^3 \operatorname{arctanh}(ax)+18a^2x}{128(a^2x^2-1)^2 a^3}$
risch	$-\frac{\ln(ax+1)^4}{512a^3}+\frac{(x^4\ln(-ax+1)a^4+2a^3x^3-2x^2\ln(-ax+1)a^2+2ax+\ln(-ax+1))\ln(ax+1)^3}{128a^3(a^2x^2-1)^2}-\frac{3\left(2a^4x^4\ln(-ax+1\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(4*arctanh(a*x)^4*a^4*x^4+3*a^4*x^4*arctanh(a*x)^2-16*arctanh(a*x)^
3*a^3*x^3-8*arctanh(a*x)^4*a^2*x^2-6*a^3*x^3*arctanh(a*x)+18*a^2*x^2*arcta
nh(a*x)^2-16*arctanh(a*x)^3*a*x+3*a^2*x^2+4*arctanh(a*x)^4-6*a*x*arctanh(a
*x)+3*arctanh(a*x)^2)/(a^2*x^2-1)^2/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\frac{(a^4x^4 - 2\,a^2x^2 + 1)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 12\,a^2x^2 - 8\,(a^3x^3 + ax)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 3\,(a^4x^4 + 6\,a^2x^2 + 1)\log\left(\right.}{512\,(a^7x^4 - 2\,a^5x^2 + a^3)}$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/512*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 12*a^2*x^2
 - 8*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(a^4*x^4 + 6*a^2*x^2
+ 1)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x
- 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F]

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\int \frac{x^2\,\text{atanh}^3\,(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

input
```
integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x**2*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(187) = 374$.

Time = 0.05 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.06

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log
(a*x - 1)/a^3)*arctanh(a*x)^3 - 3/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) -
 (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a*arctanh(a*x)^2/(a^8*x^4 - 2*a
^6*x^2 + a^4) + 1/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 - 4*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x - 1)^4 - 12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a
^2*x^2 + 1)*log(a*x - 1)^2 + 1)*log(a*x + 1)^2 + 3*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 3*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))*a^2/(a^10*x^4 - 2*a^8*x^
2 + a^6) + 4*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 6*(
a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x - 1))*a*arctanh(a*x)/(a^9*x^4 - 2*a^7*x^2 + a^5))*a
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)^3}{(1-a^2x^2)^3} \, dx = \int -\frac{x^2 \operatorname{artanh}(ax)^3}{(a^2x^2-1)^3} \, dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 831, normalized size of antiderivative = 3.87

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \text{Too large to display}$$

input
```
int(-(x^2*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

output
```
(3*log(a*x + 1)*log(1 - a*x))/(4*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4)) - (3
*log(1 - a*x)^2)/(512*a^3) - log(a*x + 1)^4/(512*a^3) - log(1 - a*x)^4/(51
2*a^3) - (3*x^2)/(2*(64*a - 128*a^3*x^2 + 64*a^5*x^4)) - (x*log(1 - a*x)^3
)/(8*(8*a^2 - 16*a^4*x^2 + 8*a^6*x^4)) - (6*x^2*log(1 - a*x)^2)/(128*a - 2
56*a^3*x^2 + 128*a^5*x^4) - (3*log(a*x + 1)^2)/(512*a^3) + (x^3*log(a*x +
1)^3)/(64*(a^4*x^4 - 2*a^2*x^2 + 1)) - (x^3*log(1 - a*x)^3)/(8*(8*a^4*x^4
- 16*a^2*x^2 + 8)) + (3*x*log(a*x + 1))/(128*(a^2 - 2*a^4*x^2 + a^6*x^4))
+ (log(a*x + 1)*log(1 - a*x)^3)/(128*a^3) + (log(a*x + 1)^3*log(1 - a*x))/
(128*a^3) - (3*x*log(1 - a*x))/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (3*
x^2*log(a*x + 1)^2)/(64*(a - 2*a^3*x^2 + a^5*x^4)) + (x*log(a*x + 1)^3)/(6
4*(a^2 - 2*a^4*x^2 + a^6*x^4)) - (3*log(a*x + 1)^2*log(1 - a*x)^2)/(256*a^
3) + (3*x^3*log(a*x + 1))/(128*(a^4*x^4 - 2*a^2*x^2 + 1)) - (3*a*x^3*log(1
 - a*x))/(128*a - 256*a^3*x^2 + 128*a^5*x^4) + (6*x*log(a*x + 1)*log(1 - a
*x)^2)/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (6*x*log(a*x + 1)^2*log(1 -
 a*x))/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) + (6*x^2*log(a*x + 1)*log(1 -
 a*x))/(64*a - 128*a^3*x^2 + 64*a^5*x^4) + (6*a^2*x^3*log(a*x + 1)*log(1 -
 a*x)^2)/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (6*a^2*x^3*log(a*x + 1)^2
*log(1 - a*x))/(128*a^2 - 256*a^4*x^2 + 128*a^6*x^4) - (3*a^2*x^2*log(a*x
+ 1)*log(1 - a*x))/(2*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4)) + (3*a^4*x^4*lo
g(a*x + 1)*log(1 - a*x))/(4*(64*a^3 - 128*a^5*x^2 + 64*a^7*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$= \frac{-8atanh(ax)^4 a^4x^4 + 16atanh(ax)^4 a^2x^2 - 8atanh(ax)^4 + 32atanh(ax)^3 a^3x^3 + 32atanh(ax)^3 ax - 6ata}{256a^3 (a^4x^4 -}$$

input
```
int(x^2*atanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

output
```
( - 8*atanh(a*x)**4*a**4*x**4 + 16*atanh(a*x)**4*a**2*x**2 - 8*atanh(a*x)*
*4 + 32*atanh(a*x)**3*a**3*x**3 + 32*atanh(a*x)**3*a*x - 6*atanh(a*x)**2*a
**4*x**4 - 36*atanh(a*x)**2*a**2*x**2 - 6*atanh(a*x)**2 + 12*atanh(a*x)*a*
*3*x**3 + 12*atanh(a*x)*a*x - 3*a**4*x**4 - 3)/(256*a**3*(a**4*x**4 - 2*a*
*2*x**2 + 1))
```

3.316 $\int \frac{x\textbf{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\begin{aligned}\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx =& -\frac{3x}{128a(1-a^2x^2)^2}-\frac{45x}{256a(1-a^2x^2)}-\frac{45\text{arctanh}(ax)}{256a^2}\\&+\frac{3\text{arctanh}(ax)}{32a^2(1-a^2x^2)^2}+\frac{9\text{arctanh}(ax)}{32a^2(1-a^2x^2)}-\frac{3x\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}\\&-\frac{9x\text{arctanh}(ax)^2}{32a(1-a^2x^2)}-\frac{3\text{arctanh}(ax)^3}{32a^2}+\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}\end{aligned}$$

output

```
-3/128*x/a/(-a^2*x^2+1)^2-45/256*x/a/(-a^2*x^2+1)-45/256*arctanh(a*x)/a^2+
3/32*arctanh(a*x)/a^2/(-a^2*x^2+1)^2+9/32*arctanh(a*x)/a^2/(-a^2*x^2+1)-3/
16*x*arctanh(a*x)^2/a/(-a^2*x^2+1)^2-9/32*x*arctanh(a*x)^2/a/(-a^2*x^2+1)-
3/32*arctanh(a*x)^3/a^2+1/4*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.79

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$= \frac{-102ax+90a^3x^3-48(-4+3a^2x^2)\,\text{arctanh}(ax)+48ax(-5+3a^2x^2)\,\text{arctanh}(ax)^2+(80+96a^2x^2-}{512a^2}$$

input
```
Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

output
```
(-102*a*x + 90*a^3*x^3 - 48*(-4 + 3*a^2*x^2)*ArcTanh[a*x] + 48*a*x*(-5 + 3
*a^2*x^2)*ArcTanh[a*x]^2 + (80 + 96*a^2*x^2 - 48*a^4*x^4)*ArcTanh[a*x]^3 +
 45*(-1 + a^2*x^2)^2*Log[1 - a*x] - 45*Log[1 + a*x] + 90*a^2*x^2*Log[1 + a
*x] - 45*a^4*x^4*Log[1 + a*x])/(512*a^2*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6556, 6526, 215, 215, 219, 6518, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}dx$$
$$\Big\downarrow \text{ 6556}$$
$$\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx}{4a}$$
$$\Big\downarrow \text{ 6526}$$
$$\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\int \frac{1}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)}{4a}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\int\frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)}{4a}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)}{4a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)\right)}{4a}$$

$$\Big\downarrow \text{ 6518}$$

$$\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\left(-a\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\cdots\right.\right.\right.}{4a}$$

$$\Big\downarrow \text{ 6556}$$

$$\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{\left(1-a^2x^2\right)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\cdots\right.\right.}{4a}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(\cdots}{8a(1-a^2x^{\cdots}}\right.}{4a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-$$

$$\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\operatorname{arctan}}{2a^2(1-}\right.\right.\right.}{4a}$$

input
```
Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

output
```
ArcTanh[a*x]^3/(4*a^2*(1 - a^2*x^2)^2) - (3*(-1/8*ArcTanh[a*x]/(a*(1 - a^2
*x^2)^2) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (x/(4*(1 - a^2*x^2)^2)
 + (3*(x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a)))/4)/8 + (3*((x*ArcTanh[a*
x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a^2*(1
 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/4))/(4*
a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 848, normalized size of antiderivative = 4.51

Expression too large to display

input
```
int(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

output
```
1/a^2*(-9/128*I*arctanh(a*x)^2*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*Pi-9/64*arctanh(a*x)^2*ln(a*x+1)+9/32*arctanh(a*x)^2*ln((a*x+1)/(-a^2
*x^2+1)^(1/2))-3/32*arctanh(a*x)*(a*x+1)/(a*x-1)-3/32*arctanh(a*x)*(a*x-1)
/(a*x+1)+9/64*arctanh(a*x)^2*ln(a*x-1)+9/128*I*arctanh(a*x)^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1))^3*Pi+9/64*I*arctanh(a*x)^2*csgn(I/(-(a*x+1)^2/(a^2*x^2-1
)+1))^2*Pi-9/64*I*arctanh(a*x)^2*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3*Pi+9
/128*I*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)
+1))^3*Pi-9/64*I*arctanh(a*x)^2*Pi-3/32*arctanh(a*x)^3-3/2048*(a*x+1)^2/(a
*x-1)^2-3/64*arctanh(a*x)^2/(a*x-1)^2+3/64*arctanh(a*x)^2/(a*x+1)^2-3/64*(
a*x-1)/(a*x+1)+3/64*(a*x+1)/(a*x-1)+3/2048*(a*x-1)^2/(a*x+1)^2+9/64*arctan
h(a*x)^2/(a*x-1)+9/64*arctanh(a*x)^2/(a*x+1)+1/4/(a^2*x^2-1)^2*arctanh(a*x
)^3-9/128*I*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x
^2-1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi+9/64*I*arctanh(a*x)^2*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi+9/128*I*arc
tanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*Pi+9/128*I*arctanh(a*x)^2*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/(-(a*x+1)^2/(a^2*x^2-1)+1))^2*Pi+3/512*arctanh(a*x)*(
a*x+1)^2/(a*x-1)^2+3/512*(a*x-1)^2*arctanh(a*x)/(a*x+1)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

$$\int \frac{x\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \frac{90\,a^3x^3 - 2\,(3\,a^4x^4 - 6\,a^2x^2 - 5)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 12\,(3\,a^3x^3 - 5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 102\,ax - 3\,(15\,a^4x^4 \cdots}{512\,(a^6x^4 - 2\,a^4x^2 + a^2)}$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/512*(90*a^3*x^3 - 2*(3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1)
)^3 + 12*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 102*a*x - 3*(15
*a^4*x^4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^6*x^4 - 2*a^4*x^2
 + a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\int \frac{x\,\text{atanh}^3\,(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(x*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(163) = 326.

Time = 0.06 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.24

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \frac{3\left(\frac{2\,(3\,a^2x^3-5\,x)}{a^4x^4-2\,a^2x^2+1}-\frac{3\,\log(ax+1)}{a}+\frac{3\,\log(ax-1)}{a}\right)\text{artanh}\,(ax)^2}{64\,a}$$
$$+\frac{3\left(\frac{\left(30\,a^3x^3-2\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^3+6\,(a^4x^4-2\,a^2x^2+1)\log(ax+1)^2\log(ax-1)+2\,(a^4x^4-2\,a^2x^2+1)\log(ax-1)^3-34\,ax-3\,\left(5\right.\right.}{a^7x^4-2\,a^5x^2+a^3}\right.}{}$$
$$+\frac{\text{artanh}\,(ax)^3}{4\,(a^2x^2-1)^2a^2}$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
3/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^2/a + 3/512*((30*a^3*x^3 - 2*(a^4*x^4 - 2*a^
2*x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log
(a*x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4
*x^4 - 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*
x + 1) + 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x
^2 + a^3) - 4*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2))/
a + 1/4*arctanh(a*x)^3/((a^2*x^2 - 1)^2*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(163) = 326$.

Time = 0.13 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.82

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\frac{1}{2048}\left(4\left(\frac{(ax-1)^2\left(\frac{4(ax+1)}{ax-1}-1\right)}{(ax+1)^2a^3}-\frac{(ax+1)^2}{(ax-1)^2a^3}+\frac{4(ax+1)}{(ax-1)a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+6\left(\frac{(ax-1)^2(}{(ax+}\right.\right.$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
-1/2048*(4*((a*x - 1)^2*(4*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3) - (a
*x + 1)^2/((a*x - 1)^2*a^3) + 4*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x + 1)/
(a*x - 1))^3 + 6*((a*x - 1)^2*(8*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1)^2*a^3
) + (a*x + 1)^2/((a*x - 1)^2*a^3) - 8*(a*x + 1)/((a*x - 1)*a^3))*log(-(a*x
 + 1)/(a*x - 1))^2 + 6*((a*x - 1)^2*(16*(a*x + 1)/(a*x - 1) - 1)/((a*x + 1
)^2*a^3) - (a*x + 1)^2/((a*x - 1)^2*a^3) + 16*(a*x + 1)/((a*x - 1)*a^3))*l
og(-(a*x + 1)/(a*x - 1)) + 3*(a*x - 1)^2*(32*(a*x + 1)/(a*x - 1) - 1)/((a*
x + 1)^2*a^3) + 3*(a*x + 1)^2/((a*x - 1)^2*a^3) - 96*(a*x + 1)/((a*x - 1)*
a^3))*a
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.20

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\frac{48\ln(1-a\,x)-48\ln(a\,x+1)+45\,\text{atanh}(a\,x)+51\,a\,x-5\ln(a\,x+1)^3+5\ln(1-a\,x)^3-15\ln(a}{}$$

input
```
int(-(x*atanh(a*x)^3)/(a^2*x^2 - 1)^3,x)
```

output
```
-(48*log(1 - a*x) - 48*log(a*x + 1) + 45*atanh(a*x) + 51*a*x - 5*log(a*x +
 1)^3 + 5*log(1 - a*x)^3 - 15*log(a*x + 1)*log(1 - a*x)^2 + 15*log(a*x + 1
)^2*log(1 - a*x) - 45*a^3*x^3 - 6*a^2*x^2*log(a*x + 1)^3 + 6*a^2*x^2*log(1
 - a*x)^3 - 18*a^3*x^3*log(a*x + 1)^2 - 18*a^3*x^3*log(1 - a*x)^2 + 3*a^4*
x^4*log(a*x + 1)^3 - 3*a^4*x^4*log(1 - a*x)^3 - 90*a^2*x^2*atanh(a*x) + 45
*a^4*x^4*atanh(a*x) + 30*a*x*log(a*x + 1)^2 + 30*a*x*log(1 - a*x)^2 + 36*a
^2*x^2*log(a*x + 1) - 36*a^2*x^2*log(1 - a*x) - 60*a*x*log(a*x + 1)*log(1
- a*x) - 18*a^2*x^2*log(a*x + 1)*log(1 - a*x)^2 + 18*a^2*x^2*log(a*x + 1)^
2*log(1 - a*x) + 9*a^4*x^4*log(a*x + 1)*log(1 - a*x)^2 - 9*a^4*x^4*log(a*x
 + 1)^2*log(1 - a*x) + 36*a^3*x^3*log(a*x + 1)*log(1 - a*x))/(256*a^2*(a^2
*x^2 - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^3} dx = \frac{-48atanh(ax)^3 a^4x^4 + 96atanh(ax)^3 a^2x^2 + 80atanh(ax)^3 + 144atanh(ax)^2 a^3x^3 - 240atanh(ax)^2 ax -$$

input
```
int(x*atanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

output
```
( - 48*atanh(a*x)**3*a**4*x**4 + 96*atanh(a*x)**3*a**2*x**2 + 80*atanh(a*x
)**3 + 144*atanh(a*x)**2*a**3*x**3 - 240*atanh(a*x)**2*a*x - 72*atanh(a*x)
*a**4*x**4 + 120*atanh(a*x) + 9*log(a**2*x - a)*a**4*x**4 - 18*log(a**2*x
- a)*a**2*x**2 + 9*log(a**2*x - a) - 9*log(a**2*x + a)*a**4*x**4 + 18*log(
a**2*x + a)*a**2*x**2 - 9*log(a**2*x + a) + 90*a**3*x**3 - 102*a*x)/(512*a
**2*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.317 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3} dx = -\frac{3}{128a(1-a^2x^2)^2} - \frac{45}{128a(1-a^2x^2)} + \frac{3x\operatorname{arctanh}(ax)}{32(1-a^2x^2)^2} + \frac{45x\operatorname{arctanh}(ax)}{64(1-a^2x^2)} + \frac{45\operatorname{arctanh}(ax)^2}{128a} - \frac{3\operatorname{arctanh}(ax)^2}{16a(1-a^2x^2)^2} - \frac{9\operatorname{arctanh}(ax)^2}{16a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)^3}{4(1-a^2x^2)^2} + \frac{3x\operatorname{arctanh}(ax)^3}{8(1-a^2x^2)} + \frac{3\operatorname{arctanh}(ax)^4}{32a}$$

output
```
-3/128/a/(-a^2*x^2+1)^2-45/128/a/(-a^2*x^2+1)+3/32*x*arctanh(a*x)/(-a^2*x^
2+1)^2+45*x*arctanh(a*x)/(-64*a^2*x^2+64)+45/128*arctanh(a*x)^2/a-3/16*arc
tanh(a*x)^2/a/(-a^2*x^2+1)^2-9/16*arctanh(a*x)^2/a/(-a^2*x^2+1)+1/4*x*arct
anh(a*x)^3/(-a^2*x^2+1)^2+3*x*arctanh(a*x)^3/(-8*a^2*x^2+8)+3/32*arctanh(a
*x)^4/a
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.55

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \frac{-48+45a^2x^2+(102ax-90a^3x^3)\operatorname{arctanh}(ax)+3(-17-6a^2x^2+15a^4x^4)\operatorname{arctanh}(ax)^2+(80ax-48}{128a\,(-1+a^2x^2)^2}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

output
```
(-48 + 45*a^2*x^2 + (102*a*x - 90*a^3*x^3)*ArcTanh[a*x] + 3*(-17 - 6*a^2*x
^2 + 15*a^4*x^4)*ArcTanh[a*x]^2 + (80*a*x - 48*a^3*x^3)*ArcTanh[a*x]^3 + 1
2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)/(128*a*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.39, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6526, 6518, 6522, 6518, 241, 6556, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{3}{8}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{3}{4}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\frac{x\operatorname{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\operatorname{arctanh}(ax)^2}{16a(1-a^2x^2)^2}$$

$$\Big\downarrow \text{6518}$$

$$\frac{3}{8}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)^4}{8a}\right)+\frac{x\operatorname{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\operatorname{arctanh}(ax)^2}{16a(1-a^2x^2)^2}$$

$$\Big\downarrow \text{6522}$$

$$\frac{3}{8}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}$$

$$\Big\downarrow \; 6518$$

$$\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}$$

$$\Big\downarrow \; 241$$

$$\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \; 6556$$

$$\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \; 6518$$

$$\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(a}{8a}\right.$$

$$\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)$$

$$\Big\downarrow \text{241}$$

$$\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

output
```
(-3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x]^3)/(4*(1 - a^
2*x^2)^2) + (3*(-1/16*1/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2
*x^2)^2) + (3*(-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2
)) + ArcTanh[a*x]^2/(4*a)))/4))/8 + (3*((x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2
)) + ArcTanh[a*x]^4/(8*a) - (3*a*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (
-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*
x]^2/(4*a))/a))/2))/4
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 12.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{-12 \operatorname{arctanh}(ax)^4 a^4 x^4-45 a^4 x^4 \operatorname{arctanh}(ax)^2+48 \operatorname{arctanh}(ax)^3 a^3 x^3-48 a^4 x^4+24 \operatorname{arctanh}(ax)^4 a^2 x^2+90 a^3 x^3 \operatorname{arct}}{128(a^2}$
risch	$\frac{3 \ln(ax+1)^4}{512a}-\frac{(3x^4 \ln(-ax+1)a^4+6a^3x^3-6x^2 \ln(-ax+1)a^2-10ax+3 \ln(-ax+1)) \ln(ax+1)^3}{128(a^2x^2-1)^2 a}+\frac{3\left(6a^4x^4 \ln(-ax\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(-12*arctanh(a*x)^4*a^4*x^4-45*a^4*x^4*arctanh(a*x)^2+48*arctanh(a*
x)^3*a^3*x^3-48*a^4*x^4+24*arctanh(a*x)^4*a^2*x^2+90*a^3*x^3*arctanh(a*x)+
18*a^2*x^2*arctanh(a*x)^2-80*arctanh(a*x)^3*a*x*x+51*a^2*x^2-12*arctanh(a*x)
^4-102*a*x*arctanh(a*x)+51*arctanh(a*x)^2)/(a^2*x^2-1)^2/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \frac{3\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)^4+180\,a^2x^2-8\,(3\,a^3x^3-5\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3+3\,(15\,a^4x^4-6\,a^2x^2-}{512\,(a^5x^4-2\,a^3x^2+a)}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/512*(3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 180*a^2*x
^2 - 8*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(15*a^4*x^4 - 6
*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^3*x^3 - 17*a*x)*log(
-(a*x + 1)/(a*x - 1)) - 192)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}^3(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(175) = 350.

Time = 0.08 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.27

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a +
3*log(a*x - 1)/a)*arctanh(a*x)^3 + 3/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x
- 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)^2/(
a^6*x^4 - 2*a^4*x^2 + a^2) - 3/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1
)^4 - 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x - 1)^4 - 60*a^2*x^2 + 3*(5*a^4*x^4 - 10*a^2*x^2 +
2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)^2 + 15*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*
x - 1)^3 + 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1) + 64)*a
^2/(a^8*x^4 - 2*a^6*x^2 + a^4) + 4*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1
) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 - 1
0*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1) +
 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^4 - 2*a^
5*x^2 + a^3))*a
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \int -\frac{\text{artanh}\,(ax)^3}{(a^2x^2-1)^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.63

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx = \frac{\frac{45\,a\,x^2}{2}-\frac{24}{a}}{64\,a^4\,x^4-128\,a^2\,x^2+64} - \ln\left(a\,x+1\right)^2\left(\frac{\frac{3}{16\,a^2}-\frac{9\,x^2}{64}}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}-\frac{45}{512\,a}\right) - \ln\left(1-a\,x\right)^3\left(\frac{3\,\ln\left(a\,x+1\right)}{128\,a}+\frac{\frac{5\,x}{8}-\frac{3\,a^2\,x^3}{8}}{8\,a^4\,x^4-16\,a^2\,x^2+8}\right) - \ln\left(1-a\,x\right)\left(\frac{3\ln\left(a\,x+1\right)^3}{128\,a}+\ln\left(a\,x+1\right)\left(\frac{\frac{21\,x}{2}+9\,a\,x^2-\frac{12}{a}-\frac{15\,a^2\,x^3}{2}}{64\,a^4\,x^4-128\,a^2\,x^2+64} - \frac{\frac{21\,x}{2}-9\,a\,x^2+\frac{12}{a}-\frac{15\,a^2\,x^3}{2}}{64\,a^4\,x^4-128\,a^2\,x^2+64}+\frac{45\left(a^4\,x^4-2\,a^2\,x^2+1\right)}{4\,a\,\left(64\,a^4\,x^4-128\,a^2\,x^2+64\right)}\right) + \frac{\frac{9\,x}{2}+\frac{3\,a\,x^2}{2}-\frac{3}{2\,a}-\frac{9\,a^2\,x^3}{2}}{128\,a^4\,x^4-256\,a^2\,x^2+128}+\frac{21\,x+\frac{33\,a\,x^2}{2}-\frac{39}{2\,a}-18\,a^2\,x^3}{128\,a^4\,x^4-256\,a^2\,x^2+128} + \frac{\frac{51\,x}{2}-18\,a\,x^2+\frac{21}{a}-\frac{45\,a^2\,x^3}{2}}{128\,a^4\,x^4-256\,a^2\,x^2+128}+\frac{\ln\left(a\,x+1\right)^2\left(30\,x-18\,a^2\,x^3\right)}{128\,a^4\,x^4-256\,a^2\,x^2+128}\right) + \frac{3\ln\left(a\,x+1\right)^4}{512\,a}+\frac{3\ln\left(1-a\,x\right)^4}{512\,a}+\ln\left(1-a\,x\right)^2\left(\frac{9\ln\left(a\,x+1\right)^2}{256\,a} + \frac{45}{512\,a}-\frac{\frac{21\,x}{2}-9\,a\,x^2+\frac{12}{a}-\frac{15\,a^2\,x^3}{2}}{128\,a^4\,x^4-256\,a^2\,x^2+128} + \frac{\frac{21\,x}{2}+9\,a\,x^2-\frac{12}{a}-\frac{15\,a^2\,x^3}{2}}{128\,a^4\,x^4-256\,a^2\,x^2+128}+\frac{\ln\left(a\,x+1\right)\,\left(30\,x-18\,a^2\,x^3\right)}{128\,a^4\,x^4-256\,a^2\,x^2+128}\right) + \frac{\ln\left(a\,x+1\right)\,\left(\frac{51\,x}{128\,a}-\frac{45\,a\,x^3}{128}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}+\frac{\ln\left(a\,x+1\right)^3\,\left(\frac{5\,x}{64\,a}-\frac{3\,a\,x^3}{64}\right)}{\frac{1}{a}-2\,a\,x^2+a^3\,x^4}$$

input
```
int(-atanh(a*x)^3/(a^2*x^2 - 1)^3,x)
```

output
```
((45*a*x^2)/2 - 24/a)/(64*a^4*x^4 - 128*a^2*x^2 + 64) - log(a*x + 1)^2*((3
/(16*a^2) - (9*x^2)/64)/(1/a - 2*a*x^2 + a^3*x^4) - 45/(512*a)) - log(1 -
a*x)^3*((3*log(a*x + 1))/(128*a) + ((5*x)/8 - (3*a^2*x^3)/8)/(8*a^4*x^4 -
16*a^2*x^2 + 8)) - log(1 - a*x)*((3*log(a*x + 1)^3)/(128*a) + log(a*x + 1)
*(((21*x)/2 + 9*a*x^2 - 12/a - (15*a^2*x^3)/2)/(64*a^4*x^4 - 128*a^2*x^2 +
 64) - ((21*x)/2 - 9*a*x^2 + 12/a - (15*a^2*x^3)/2)/(64*a^4*x^4 - 128*a^2*
x^2 + 64) + (45*(a^4*x^4 - 2*a^2*x^2 + 1))/(4*a*(64*a^4*x^4 - 128*a^2*x^2
+ 64))) + ((9*x)/2 + (3*a*x^2)/2 - 3/(2*a) - (9*a^2*x^3)/2)/(128*a^4*x^4 -
 256*a^2*x^2 + 128) + (21*x + (33*a*x^2)/2 - 39/(2*a) - 18*a^2*x^3)/(128*a
^4*x^4 - 256*a^2*x^2 + 128) + ((51*x)/2 - 18*a*x^2 + 21/a - (45*a^2*x^3)/2
)/(128*a^4*x^4 - 256*a^2*x^2 + 128) + (log(a*x + 1)^2*(30*x - 18*a^2*x^3))
/(128*a^4*x^4 - 256*a^2*x^2 + 128)) + (3*log(a*x + 1)^4)/(512*a) + (3*log(
1 - a*x)^4)/(512*a) + log(1 - a*x)^2*((9*log(a*x + 1)^2)/(256*a) + 45/(512
*a) - ((21*x)/2 - 9*a*x^2 + 12/a - (15*a^2*x^3)/2)/(128*a^4*x^4 - 256*a^2*
x^2 + 128) + ((21*x)/2 + 9*a*x^2 - 12/a - (15*a^2*x^3)/2)/(128*a^4*x^4 - 2
56*a^2*x^2 + 128) + (log(a*x + 1)*(30*x - 18*a^2*x^3))/(128*a^4*x^4 - 256*
a^2*x^2 + 128)) + (log(a*x + 1)*((51*x)/(128*a) - (45*a*x^3)/128))/(1/a -
2*a*x^2 + a^3*x^4) + (log(a*x + 1)^3*((5*x)/(64*a) - (3*a*x^3)/64))/(1/a -
 2*a*x^2 + a^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}\,dx$$
$$= \frac{24atanh(ax)^4 a^4x^4 - 48atanh(ax)^4 a^2x^2 + 24atanh(ax)^4 - 96atanh(ax)^3 a^3x^3 + 160atanh(ax)^3 ax + 90}{256a(a^4}$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

output
```
(24*atanh(a*x)**4*a**4*x**4 - 48*atanh(a*x)**4*a**2*x**2 + 24*atanh(a*x)**
4 - 96*atanh(a*x)**3*a**3*x**3 + 160*atanh(a*x)**3*a*x + 90*atanh(a*x)**2*
a**4*x**4 - 36*atanh(a*x)**2*a**2*x**2 - 102*atanh(a*x)**2 - 180*atanh(a*x
)*a**3*x**3 + 204*atanh(a*x)*a*x + 45*a**4*x**4 - 51)/(256*a*(a**4*x**4 -
2*a**2*x**2 + 1))
```

3.318 $\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 277

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx &= -\frac{3ax}{128(1-a^2x^2)^2}-\frac{141ax}{256(1-a^2x^2)} \\ &\quad -\frac{141}{256}\operatorname{arctanh}(ax)+\frac{3\operatorname{arctanh}(ax)}{32(1-a^2x^2)^2}+\frac{33\operatorname{arctanh}(ax)}{32(1-a^2x^2)} \\ &\quad -\frac{3ax\operatorname{arctanh}(ax)^2}{16(1-a^2x^2)^2}-\frac{33ax\operatorname{arctanh}(ax)^2}{32(1-a^2x^2)} \\ &\quad -\frac{11}{32}\operatorname{arctanh}(ax)^3+\frac{\operatorname{arctanh}(ax)^3}{4(1-a^2x^2)^2}+\frac{\operatorname{arctanh}(ax)^3}{2(1-a^2x^2)} \\ &\quad +\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{1+ax}\right) \\ &\quad -\frac{3}{2}\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ &\quad -\frac{3}{2}\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right) \\ &\quad -\frac{3}{4}\operatorname{PolyLog}\left(4,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-3/128*a*x/(-a^2*x^2+1)^2-141*a*x/(-256*a^2*x^2+256)-141/256*arctanh(a*x)+
3/32*arctanh(a*x)/(-a^2*x^2+1)^2+33*arctanh(a*x)/(-32*a^2*x^2+32)-3/16*a*x
*arctanh(a*x)^2/(-a^2*x^2+1)^2-33*a*x*arctanh(a*x)^2/(-32*a^2*x^2+32)-11/3
2*arctanh(a*x)^3+1/4*arctanh(a*x)^3/(-a^2*x^2+1)^2+arctanh(a*x)^3/(-2*a^2*
x^2+2)+1/4*arctanh(a*x)^4+arctanh(a*x)^3*ln(2-2/(a*x+1))-3/2*arctanh(a*x)^
2*polylog(2,-1+2/(a*x+1))-3/2*arctanh(a*x)*polylog(3,-1+2/(a*x+1))-3/4*pol
ylog(4,-1+2/(a*x+1))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx = \frac{16\pi^4 - 256\operatorname{arctanh}(ax)^4 + 576\operatorname{arctanh}(ax)\cosh(2\operatorname{arctanh}(ax)) + 384\operatorname{arctanh}(ax)^3\cosh(2\operatorname{arctanh}(ax))}{}$$

input
```
Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

output
```
(16*Pi^4 - 256*ArcTanh[a*x]^4 + 576*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 38
4*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 12*ArcTanh[a*x]*Cosh[4*ArcTanh[a*x
]] + 32*ArcTanh[a*x]^3*Cosh[4*ArcTanh[a*x]] + 1024*ArcTanh[a*x]^3*Log[1 -
E^(2*ArcTanh[a*x])] + 1536*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] -
 1536*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 768*PolyLog[4, E^(2*Ar
cTanh[a*x])] - 288*Sinh[2*ArcTanh[a*x]] - 576*ArcTanh[a*x]^2*Sinh[2*ArcTan
h[a*x]] - 3*Sinh[4*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[4*ArcTanh[a*x]])
/1024
```

Rubi [A] (verified)

Time = 3.76 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.75, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {6592, 6556, 6526, 215, 215, 219, 6518, 6556, 215, 219, 6592, 6550, 6494, 6556, 6518, 6556, 215, 219, 6618, 6622, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^3}dx$$

$$\Big\downarrow \; 6592$$

$$a^2\int \frac{x\text{arctanh}(ax)^3}{\left(1-a^2x^2\right)^3}dx+\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \; 6556$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^3}dx}{4a}\right)+\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \; 6526$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{1}{8}\int \frac{1}{\left(1-a^2x^2\right)^3}dx+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}\right)}{4a}\right)+\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \; 215$$

$$a^2\Bigg(\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{1}{8}\left(\frac{3}{4}\int \frac{1}{\left(1-a^2x^2\right)^2}dx+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{\text{arctanh}(ax)}{8a\left(1-a^2x^2\right)^2}\Bigg)}{4a}$$

$$\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \; 215$$

$$a^2\Bigg(\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\Big(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{1}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}\right)+\frac{x}{4\left(1-a^2x^2\right)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4\left(1-a^2x^2\right)^2}-}{4a}$$

$$\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \; 219$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{3}{4}\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\right.\right.}{4a}\right.$$

$$\int\frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \; 6518$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{3}{4}\left(-a\int\frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(a}{8a(1-a^2x^2}\right.}{4a}\right.$$

$$\int\frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \; 6556$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(a}{4(1-a^2x^2)}\right.}{4a}\right.$$

$$\int\frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \; 215$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{xa}{}\right.}{4a}\right.$$

$$\int\frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)^2}dx$$

$$\Big\downarrow \; 219$$

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^2}dx+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{ar}}{2}\right.\right.}{4a}\right.$$

$\downarrow$ 6592

$$a^2\int \frac{x\operatorname{arctanh}(ax)^3}{\left(1-a^2x^2\right)^2}dx+\int \frac{\operatorname{arctanh}(ax)^3}{x\left(1-a^2x^2\right)}dx+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{ar}}{2}\right.\right.}{4a}\right.$$

$\downarrow$ 6550

$$a^2\int \frac{x\operatorname{arctanh}(ax)^3}{\left(1-a^2x^2\right)^2}dx+\int \frac{\operatorname{arctanh}(ax)^3}{x(ax+1)}dx+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\operatorname{arctanh}(ax)^4$$

$\downarrow$ 6494

$$a^2\int \frac{x\operatorname{arctanh}(ax)^3}{\left(1-a^2x^2\right)^2}dx-3a\int \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx}{2a}\right)-3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6518}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-$$

$$3a\int \frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{ 6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int\frac{1}{\left(1-a^2x^2\right)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-$$

$$3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{215}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int\frac{1}{1-a^2x^2}dx+\frac{x}{2\left(1-a^2x^2\right)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)-$$

$$3a\int\frac{\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \text{219}$$

$$-3a\int \frac{\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}}{2a}\right)+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)}{2a}\right)+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\downarrow \text{6618}$$

$$-3a\left(\frac{\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\int\frac{\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\operatorname{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}}{2a}\right)+\frac{\operatorname{arctanh}(ax)^3}{6a}\right)}{2a}\right)+$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\operatorname{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\operatorname{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\operatorname{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\operatorname{arctanh}(ax)^4+\operatorname{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\downarrow \text{6622}$$

$$-3a\left(-\frac{1}{2}\int\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}\right.}{2a}\right.$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)+$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{ar}}{2}\right.\right.}{4a}\right.$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

$$\Big\downarrow \; 7164$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{2a^2(1-a^2x^2)}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}}{2a}\right)+\frac{\text{arctanh}(ax)^3}{6a}\right)}{2a}\right)+$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{ar}}{2}\right.\right.}{4a}\right.$$

$$3a\left(\frac{\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4a}\right)+$$

$$\frac{1}{4}\text{arctanh}(ax)^4+\text{arctanh}(ax)^3\log\left(2-\frac{2}{ax+1}\right)$$

input
```
Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

output
```
ArcTanh[a*x]^4/4 + a^2*(ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2)) - (3*((x*ArcT
anh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*
a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/(
2*a)) + a^2*(ArcTanh[a*x]^3/(4*a^2*(1 - a^2*x^2)^2) - (3*(-1/8*ArcTanh[a*x
]/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (x/(4*(1
- a^2*x^2)^2) + (3*(x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a)))/4)/8 + (3*(
(x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a
*x]/(2*a^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*
a))))/4))/(4*a)) + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - 3*a*((ArcTanh[a*x
]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + (ArcTanh[a*x]*PolyLog[3, -1 + 2/
(1 + a*x)])/(2*a) + PolyLog[4, -1 + 2/(1 + a*x)]/(4*a))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6622
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.92 (sec) , antiderivative size = 1446, normalized size of antiderivative = 5.22

method	result	size
derivativedivides	Expression too large to display	1446
default	Expression too large to display	1446
parts	Expression too large to display	1875

input
```
int(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/256*arctanh(a*x)^2*(a*x+1)^2/(a*x-1)^2+3/256*(a*x-1)^2*arctanh(a*x)^2/(
a*x+1)^2-9/32*arctanh(a*x)*(a*x+1)/(a*x-1)-9/32*arctanh(a*x)*(a*x-1)/(a*x+
1)+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,-(a*x+1)/(-a^2*x^2+
1)^(1/2))+arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(
a*x)-1/4*arctanh(a*x)^4+1/16*arctanh(a*x)^3/(a*x-1)^2+1/16*arctanh(a*x)^3/
(a*x+1)^2-3/2048*(a*x+1)^2/(a*x-1)^2-1/2*arctanh(a*x)^3*ln(a*x+1)+3*arctan
h(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1
)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)
^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(
1/2))+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-9/64*(a*x-1)/(a*x+1)
+9/64*(a*x+1)/(a*x-1)+3/2048*(a*x-1)^2/(a*x+1)^2+5/16*arctanh(a*x)^3/(a*x+
1)-5/16*arctanh(a*x)^3/(a*x-1)+3/512*arctanh(a*x)*(a*x+1)^2/(a*x-1)^2+3/51
2*(a*x-1)^2*arctanh(a*x)/(a*x+1)^2-1/2*arctanh(a*x)^3*ln(a*x-1)+9/32*(a*x+
1)*arctanh(a*x)^2/(a*x-1)-9/32*arctanh(a*x)^2*(a*x-1)/(a*x+1)+1/32*(16*I*P
i*csgn(I/(-(a*x+1)^2/(a^2*x^2-1)+1))^3+8*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-8*I*Pi*csgn(I/(-(a*x+1)^2/(a^2*x^2-
1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(-(a*x+1
)^2/(a^2*x^2-1)+1))+16*I*Pi*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1))*csgn(I/(-(a
*x+1)^2/(a^2*x^2-1)+1))*csgn(I*(-(a*x+1)^2/(a^2*x^2-1)-1)/(-(a*x+1)^2/(...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^3}\,dx=\int -\frac{\operatorname{artanh}\left(ax\right)^3}{\left(a^2x^2-1\right)^3x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^3/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}^3(ax)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

input
```
integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)**3/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^3x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/64*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^
4 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log(-a*x + 1)^3)/(a^4*x^4 - 2*a^2*x^2
 + 1) - 1/8*integrate(1/4*(4*log(a*x + 1)^3 - 12*log(a*x + 1)^2*log(-a*x +
 1) + 3*(2*a^4*x^4 + 2*a^3*x^3 - 3*a^2*x^2 - 3*a*x + 2*(a^6*x^6 + a^5*x^5
- 2*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 + a*x + 2)*log(a*x + 1))*log(-a*x + 1)^2
)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^3x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^3}{x\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-atanh(a*x)^3/(x*(a^2*x^2 - 1)^3),x)
```

output
```
-int(atanh(a*x)^3/(x*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^3}\,dx = -\left(\int \frac{atanh(ax)^3}{a^6x^7-3a^4x^5+3a^2x^3-x}dx\right)$$

input
```
int(atanh(a*x)^3/x/(-a^2*x^2+1)^3,x)
```

output
```
- int(atanh(a*x)**3/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),x)
```

3.319 $\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 281

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^3}\,dx &= -\frac{3a}{128(1-a^2x^2)^2}-\frac{93a}{128(1-a^2x^2)}+\frac{3a^2x\operatorname{arctanh}(ax)}{32(1-a^2x^2)^2}\\&\quad+\frac{93a^2x\operatorname{arctanh}(ax)}{64(1-a^2x^2)}+\frac{93}{128}a\operatorname{arctanh}(ax)^2\\&\quad-\frac{3a\operatorname{arctanh}(ax)^2}{16(1-a^2x^2)^2}-\frac{21a\operatorname{arctanh}(ax)^2}{16(1-a^2x^2)}+a\operatorname{arctanh}(ax)^3\\&\quad-\frac{\operatorname{arctanh}(ax)^3}{x}+\frac{a^2x\operatorname{arctanh}(ax)^3}{4(1-a^2x^2)^2}+\frac{7a^2x\operatorname{arctanh}(ax)^3}{8(1-a^2x^2)}\\&\quad+\frac{15}{32}a\operatorname{arctanh}(ax)^4+3a\operatorname{arctanh}(ax)^2\log\left(2-\frac{2}{1+ax}\right)\\&\quad-3a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)\\&\quad-\frac{3}{2}a\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-3/128*a/(-a^2*x^2+1)^2-93*a/(-128*a^2*x^2+128)+3/32*a^2*x*arctanh(a*x)/(-
a^2*x^2+1)^2+93*a^2*x*arctanh(a*x)/(-64*a^2*x^2+64)+93/128*a*arctanh(a*x)^
2-3/16*a*arctanh(a*x)^2/(-a^2*x^2+1)^2-21*a*arctanh(a*x)^2/(-16*a^2*x^2+16
)+a*arctanh(a*x)^3-arctanh(a*x)^3/x+1/4*a^2*x*arctanh(a*x)^3/(-a^2*x^2+1)^
2+7*a^2*x*arctanh(a*x)^3/(-8*a^2*x^2+8)+15/32*a*arctanh(a*x)^4+3*a*arctanh
(a*x)^2*ln(2-2/(a*x+1))-3*a*arctanh(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*pol
ylog(3,-1+2/(a*x+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.78

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^3}\,dx = -a\left(-\frac{i\pi^3}{8}+\operatorname{arctanh}(ax)^3+\frac{\operatorname{arctanh}(ax)^3}{ax}-\frac{ax\operatorname{arctanh}(ax)^3}{1-a^2x^2}-\frac{15}{32}\operatorname{arctanh}(ax)^4+\frac{3}{8}\cosh(2\operatorname{arctanh}(ax))+\frac{3}{4}\operatorname{arctanh}(ax)^2\cosh(2\operatorname{arctanh}(ax))+\frac{3\cosh(4\operatorname{arctanh}(ax))}{1024}+\frac{3}{128}\operatorname{arctanh}(ax)^2\cosh(4\operatorname{arctanh}(ax))-3\operatorname{arctanh}(ax)^2\log\left(1-e^{2\operatorname{arctanh}(ax)}\right)-3\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(ax)}\right)+\frac{3}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(ax)}\right)-\frac{3}{4}\operatorname{arctanh}(ax)\sinh(2\operatorname{arctanh}(ax))-\frac{3}{256}\operatorname{arctanh}(ax)\sinh(4\operatorname{arctanh}(ax))-\frac{1}{32}\operatorname{arctanh}(ax)^3\sinh(4\operatorname{arctanh}(ax))\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

output
```
-(a*((-1/8*I)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - (a*x*ArcTanh[
a*x]^3)/(1 - a^2*x^2) - (15*ArcTanh[a*x]^4)/32 + (3*Cosh[2*ArcTanh[a*x]])/
8 + (3*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]])/4 + (3*Cosh[4*ArcTanh[a*x]])/1
024 + (3*ArcTanh[a*x]^2*Cosh[4*ArcTanh[a*x]])/128 - 3*ArcTanh[a*x]^2*Log[1
 - E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + (
3*PolyLog[3, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]]
)/4 - (3*ArcTanh[a*x]*Sinh[4*ArcTanh[a*x]])/256 - (ArcTanh[a*x]^3*Sinh[4*A
rcTanh[a*x]])/32))
```

Rubi [A] (verified)

Time = 5.28 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.86, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {6592, 6526, 6518, 6522, 6518, 241, 6556, 6518, 241, 6592, 6518, 6544, 6452, 6510, 6550, 6494, 6556, 6518, 241, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x^2\left(1-a^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{\left(1-a^2x^2\right)^3}dx+\int \frac{\text{arctanh}(ax)^3}{x^2\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 6526}$$

$$a^2\left(\frac{3}{8}\int \frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^3}dx+\frac{3}{4}\int \frac{\text{arctanh}(ax)^3}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{3\text{arctanh}(ax)^2}{16a\left(1-a^2x^2\right)^2}\right)+\int \frac{\text{arctanh}(ax)^3}{x^2\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 6518}$$

$$a^2\left(\frac{3}{8}\int \frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^2}dx+\frac{x\text{arctanh}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4\left(1-a^2x^2\right)^2}-\right.$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2\left(1-a^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 6522}$$

$$a^2\left(\frac{3}{8}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(a}{2(1-a^2x^2}\right.\right.$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}dx$$

$\downarrow$ 6518

$$a^2\left(\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\right.\right.$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}dx$$

$\downarrow$ 241

$$a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\right.\right.$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}dx$$

$\downarrow$ 6556

$$a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3a}{16a}\right.$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}dx$$

$\downarrow$ 6518

$$a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctan}}{8}\right.\right.$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}dx$$

$\downarrow$ 241

$$\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)^2}dx+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$

$\downarrow$ 6592

$$a^2\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$

$\downarrow$ 6518

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\int \frac{\text{arctanh}(ax)^3}{x^2(1-a^2x^2)}dx+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$

$\downarrow$ 6544

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+a^2\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx+$$
$$\int \frac{\text{arctanh}(ax)^3}{x^2}dx+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$

$\downarrow$ 6452

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx+$$
$$a^2\int \frac{\text{arctanh}(ax)^3}{1-a^2x^2}dx+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{\text{arctanh}(ax)^3}{x}$$

$\downarrow$ 6510

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+3a\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)}dx+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6550}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$3a\left(\int \frac{\text{arctanh}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\text{arctanh}(ax)^3\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6494}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$3a\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$3a\left(-2a\int \frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{6518}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int\frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(a}{8a}\right.$$
$$3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\Big\downarrow$ 241

$$3a\left(-2a\int\frac{\text{arctanh}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\left(2-\frac{2}{ax+1}\right)\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\Big\downarrow$ 6618

$$3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}-\frac{1}{2}\int\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(\right.$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

$\Big\downarrow$ 7164

$$a^2\left(\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{\text{arctanh}(ax)^4}{8a}\right)+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$
$$3a\left(-2a\left(\frac{\text{arctanh}(ax)\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2a}+\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}\right)+\frac{1}{3}\text{arctanh}(ax)^3+\text{arctanh}(ax)^2\log\right.$$
$$\frac{1}{4}a\text{arctanh}(ax)^4-\frac{\text{arctanh}(ax)^3}{x}$$

input
```
Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

output
```
-(ArcTanh[a*x]^3/x) + (a*ArcTanh[a*x]^4)/4 + a^2*((x*ArcTanh[a*x]^3)/(2*(1
 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a) - (3*a*(ArcTanh[a*x]^2/(2*a^2*(1 - a^2
*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) +
ArcTanh[a*x]^2/(4*a))/a))/2) + a^2*((-3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2
)^2) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (3*(-1/16*1/(a*(1 - a^2*x^
2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-1/4*1/(a*(1 - a^2*x^2)
) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)))/4))/8 + (3
*((x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a) - (3*a*(ArcT
anh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[
a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a))/2))/4) + 3*a*(ArcTanh[
a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcTanh[a*x]*PolyLo
g[2, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [A] (verified)

Time = 7.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.25

method	result
derivativedivides	$a\left(\frac{15\operatorname{arctanh}(ax)^4}{32}+\frac{\left(32\operatorname{arctanh}(ax)^3-24\operatorname{arctanh}(ax)^2+12\operatorname{arctanh}(ax)-3\right)(ax+1)^2}{2048(ax-1)^2}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax\right.}{}\right.$
default	$a\left(\frac{15\operatorname{arctanh}(ax)^4}{32}+\frac{\left(32\operatorname{arctanh}(ax)^3-24\operatorname{arctanh}(ax)^2+12\operatorname{arctanh}(ax)-3\right)(ax+1)^2}{2048(ax-1)^2}-\frac{(ax+1)\left(4\operatorname{arctanh}(ax\right.}{}\right.$

input
```
int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(15/32*arctanh(a*x)^4+1/2048*(32*arctanh(a*x)^3-24*arctanh(a*x)^2+12*arc
tanh(a*x)-3)*(a*x+1)^2/(a*x-1)^2-1/16*(a*x+1)*(4*arctanh(a*x)^3-6*arctanh(
a*x)^2+6*arctanh(a*x)-3)/(a*x-1)+1/16*(4*arctanh(a*x)^3+6*arctanh(a*x)^2+6
*arctanh(a*x)+3)*(a*x-1)/(a*x+1)-1/2048*(32*arctanh(a*x)^3+24*arctanh(a*x)
^2+12*arctanh(a*x)+3)*(a*x-1)^2/(a*x+1)^2+arctanh(a*x)^3/a/x*(a*x-1)-2*arc
tanh(a*x)^3+3*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*
x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(3,(a*x+1)/(-a^2*x^2+1)^
(1/2))+3*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
2)))
```

Fricas [F]

$$\int\frac{\operatorname{arctanh}(ax)^3}{x^2\left(1-a^2x^2\right)^3}\,dx=\int-\frac{\operatorname{artanh}\left(ax\right)^3}{\left(a^2x^2-1\right)^3x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-arctanh(a*x)^3/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}^3(ax)}{a^6x^8-3a^4x^6+3a^2x^4-x^2}\,dx$$

input
```
integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(atanh(a*x)**3/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^3}\,dx = \int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^3x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^3}\,dx = -\int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(atanh(a*x)^3/(x^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2 (1-a^2x^2)^3}\,dx = \frac{120atanh(ax)^4\,a^5x^5 - 240atanh(ax)^4\,a^3x^3 + 120atanh(ax)^4\,ax - 480atanh(ax)^3\,a^4x^4 + 800atanh(ax)^3}{}$$

input
```
int(atanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x)
```

output
```
(120*atanh(a*x)**4*a**5*x**5 - 240*atanh(a*x)**4*a**3*x**3 + 120*atanh(a*x
)**4*a*x - 480*atanh(a*x)**3*a**4*x**4 + 800*atanh(a*x)**3*a**2*x**2 - 256
*atanh(a*x)**3 + 450*atanh(a*x)**2*a**5*x**5 - 180*atanh(a*x)**2*a**3*x**3
 - 510*atanh(a*x)**2*a*x - 900*atanh(a*x)*a**4*x**4 + 1020*atanh(a*x)*a**2
*x**2 - 768*int(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x),
x)*a**5*x**5 + 1536*int(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x*
*3 - x),x)*a**3*x**3 - 768*int(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*
a**2*x**3 - x),x)*a*x + 225*a**5*x**5 - 255*a*x)/(256*x*(a**4*x**4 - 2*a**
2*x**2 + 1))
```

3.320 $\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = \frac{\operatorname{arctanh}(ax)^{3/2}}{4a} + \frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arctanh}(ax)}\right)}{256a} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arctanh}(ax)}\right)}{16a} - \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arctanh}(ax)}\right)}{256a} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arctanh}(ax)}\right)}{16a} + \frac{\sqrt{\operatorname{arctanh}(ax)}\sinh(2\operatorname{arctanh}(ax))}{4a} + \frac{\sqrt{\operatorname{arctanh}(ax)}\sinh(4\operatorname{arctanh}(ax))}{32a}$$

output
```
1/4*arctanh(a*x)^(3/2)/a+1/256*Pi^(1/2)*erf(2*arctanh(a*x)^(1/2))/a+1/32*2
^(1/2)*Pi^(1/2)*erf(2^(1/2)*arctanh(a*x)^(1/2))/a-1/256*Pi^(1/2)*erfi(2*ar
ctanh(a*x)^(1/2))/a-1/32*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arctanh(a*x)^(1/2))
/a+1/4*arctanh(a*x)^(1/2)*sinh(2*arctanh(a*x))/a+1/32*arctanh(a*x)^(1/2)*s
inh(4*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = \frac{\frac{32\sqrt{\operatorname{arctanh}(ax)}\left(5ax-3a^3x^3+2(-1+a^2x^2)^2\operatorname{arctanh}(ax)\right)}{(-1+a^2x^2)^2}+\frac{\sqrt{\operatorname{arctanh}(ax)}\Gamma\left(\frac{1}{2},-4\operatorname{arctanh}(ax)\right)}{\sqrt{-\operatorname{arctanh}(ax)}}+\frac{8\sqrt{2}\sqrt{\operatorname{arctanh}(ax)}\Gamma\left(\right.}{\sqrt{-\operatorname{arcta}}}}{256a}$$

input
```
Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

output
```
((32*Sqrt[ArcTanh[a*x]]*(5*a*x - 3*a^3*x^3 + 2*(-1 + a^2*x^2)^2*ArcTanh[a*
x]))/(-1 + a^2*x^2)^2 + (Sqrt[ArcTanh[a*x]]*Gamma[1/2, -4*ArcTanh[a*x]])/S
qrt[-ArcTanh[a*x]] + (8*Sqrt[2]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -2*ArcTanh[a
*x]])/Sqrt[-ArcTanh[a*x]] - 8*Sqrt[2]*Gamma[1/2, 2*ArcTanh[a*x]] - Gamma[1
/2, 4*ArcTanh[a*x]])/(256*a)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx$$

$$\Big\downarrow \text{ 6530}$$

$$\frac{\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^2}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\operatorname{arctanh}(ax)}\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^4d\operatorname{arctanh}(ax)}{a}$$

$\downarrow$ 3793

$$\frac{\int \left(\frac{1}{2}\sqrt{\text{arctanh}(ax)}\cosh(2\text{arctanh}(ax))+\frac{1}{8}\sqrt{\text{arctanh}(ax)}\cosh(4\text{arctanh}(ax))+\frac{3}{8}\sqrt{\text{arctanh}(ax)}\right)d\text{arctanh}(ax}{a}$$

$\downarrow$ 2009

$$\frac{\frac{1}{256}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arctanh}(ax)}\right)+\frac{1}{16}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)-\frac{1}{256}\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arctanh}(ax)}\right)-\frac{1}{16}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{a}\right.}{a}$$

input
```
Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

output
```
(ArcTanh[a*x]^(3/2)/4 + (Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/256 + (Sqrt[P
i/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/16 - (Sqrt[Pi]*Erfi[2*Sqrt[ArcTanh[a
*x]]])/256 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/16 + (Sqrt[ArcT
anh[a*x]]*Sinh[2*ArcTanh[a*x]])/4 + (Sqrt[ArcTanh[a*x]]*Sinh[4*ArcTanh[a*x
]])/32)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(-a^2x^2+1)^3}dx$$

input
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

output
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = -\int \frac{\sqrt{\operatorname{atanh}(ax)}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

input
```
integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**3,x)
```

output
```
-Integral(sqrt(atanh(a*x))/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = \int -\frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^3}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = \int -\frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^3}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = \int -\frac{\sqrt{\operatorname{atanh}(a\,x)}}{(a^2\,x^2-1)^3}\,dx$$

input
```
int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1)^3,x)
```

output
```
int(-atanh(a*x)^(1/2)/(a^2*x^2 - 1)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^3}\,dx = -\left(\int \frac{\sqrt{atanh\,(ax)}}{a^6x^6-3a^4x^4+3a^2x^2-1}dx\right)$$

input
```
int(atanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

output
```
- int(sqrt(atanh(a*x))/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1),x)
```

3.321 $\int \frac{x^6}{(1-a^2x^2)^3\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \text{Int}\left(\frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 7.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx$$

input
```
Int[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(-a^2x^2+1)^3 \operatorname{arctanh}(ax)}dx$$

input
```
int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

output
```
int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^6}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-x^6/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^6}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**6/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(x**6/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**
2*atanh(a*x) - atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^6}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^6}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^6}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

input
```
int(-x^6/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{x^6}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx$$
$$= -\left(\int \frac{x^6}{atanh\,(ax)\,a^6x^6 - 3atanh\,(ax)\,a^4x^4 + 3atanh\,(ax)\,a^2x^2 - atanh\,(ax)}dx\right)$$

input
```
int(x^6/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
- int(x**6/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*
a**2*x**2 - atanh(a*x)),x)
```

3.322 $\int \frac{x^5}{(1-a^2x^2)^3\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^5}{(1-a^2x^2)^3\,\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{x^5}{(1-a^2x^2)^3\,\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(1-a^2x^2)^3\,\text{arctanh}(ax)}\,dx = \int \frac{x^5}{(1-a^2x^2)^3\,\text{arctanh}(ax)}\,dx$$

input
```
Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$$
$$\Big\downarrow \; 6651$$
$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx$$

input
```
Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(-a^2x^2+1)^3 \operatorname{arctanh}(ax)} dx$$

input
```
int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

output
```
int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^5}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^5}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(x**5/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**
2*atanh(a*x) - atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^5}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^5}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^5}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^5/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$$
$$= -\left(\int \frac{x^5}{atanh(ax)\, a^6x^6 - 3atanh(ax)\, a^4x^4 + 3atanh(ax)\, a^2x^2 - atanh(ax)} dx\right)$$

input
```
int(x^5/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
- int(x**5/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*
a**2*x**2 - atanh(a*x)),x)
```

3.323 $\int \frac{x^4}{(1-a^2x^2)^3 \text{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = -\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a^5} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8a^5} + \frac{3\log(\operatorname{arctanh}(ax))}{8a^5}$$

output
```
-1/2*Chi(2*arctanh(a*x))/a^5+1/8*Chi(4*arctanh(a*x))/a^5+3/8*ln(arctanh(a*
x))/a^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \frac{-4\operatorname{Chi}(2\operatorname{arctanh}(ax)) + \operatorname{Chi}(4\operatorname{arctanh}(ax)) + 3\log(\operatorname{arctanh}(ax))}{8a^5}$$

input
```
Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(-4*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*ArcTanh[a*x]] + 3*Log[Ar
cTanh[a*x]])/(8*a^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6596, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{a^4x^4}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(i\operatorname{arctanh}(ax))^4}{\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^5}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(-\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)} + \frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} + \frac{3}{8\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\operatorname{Chi}(4\operatorname{arctanh}(ax)) + \frac{3}{8}\log(\operatorname{arctanh}(ax))}{a^5}$$

input
```
Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(-1/2*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*ArcTanh[a*x]]/8 + (3*L
og[ArcTanh[a*x]])/8)/a^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{3\ln(\operatorname{arctanh}(ax))}{8}-\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8}}{a^5}$	31
default	$\frac{\frac{3\ln(\operatorname{arctanh}(ax))}{8}-\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8}}{a^5}$	31

input
```
int(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(3/8*ln(arctanh(a*x))-1/2*Chi(2*arctanh(a*x))+1/8*Chi(4*arctanh(a*x)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.88

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx = \frac{6 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) + \text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right) - 4 \text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)}{16a^5}$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1
)/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2
*a*x + 1)) - 4*log_integral(-(a*x + 1)/(a*x - 1)) - 4*log_integral(-(a*x -
 1)/(a*x + 1)))/a^5
```

Sympy [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx = -\int \frac{x^4}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)} dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(x**4/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**
2*atanh(a*x) - atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^4}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^4/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}\,dx$$
$$=\frac{-2\left(\int \frac{x^2}{atanh(ax)a^6x^6-3atanh(ax)a^4x^4+3atanh(ax)a^2x^2-atanh(ax)}dx\right)a^3+\left(\int \frac{1}{atanh(ax)a^6x^6-3atanh(ax)a^4x^4+3atanh(ax)a^2x^2}\right.}{a^5}$$

input
```
int(x^4/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
( - 2*int(x**2/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*
x)*a**2*x**2 - atanh(a*x)),x)*a**3 + int(1/(atanh(a*x)*a**6*x**6 - 3*atanh
(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a + log(atanh(a*
x)))/a**5
```

3.324 $\int \frac{x^3}{(1-a^2x^2)^3\textbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}\,dx = -\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{4a^4}+\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{8a^4}$$

output
```
-1/4*Shi(2*arctanh(a*x))/a^4+1/8*Shi(4*arctanh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}\,dx = \frac{-2\operatorname{Shi}(2\operatorname{arctanh}(ax))+\operatorname{Shi}(4\operatorname{arctanh}(ax))}{8a^4}$$

input
```
Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(-2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^3\,\text{arctanh}(ax)}\,dx$$

$$\Big\downarrow \ 6596$$

$$\frac{\int \frac{a^3x^3}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^4}$$

$$\Big\downarrow \ 5971$$

$$\frac{\int \left(\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))-\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))}{a^4}$$

input
```
Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(-1/4*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]]/8)/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{8}-\frac{\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{4}}{a^4}$	24
default	$\frac{\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{8}-\frac{\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{4}}{a^4}$	24

input
```
int(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*Shi(4*arctanh(a*x))-1/4*Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(25) = 50.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.52

$$\int \frac{x^3}{(1-a^2x^2)^3\,\mathrm{arctanh}(ax)}\,dx = \frac{\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-2\,\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+2\,\mathrm{log\_integral}\left(-\right.}{16\,a^4}$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_inte
gral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*log_integral(-(a*x +
 1)/(a*x - 1)) + 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^4
```

Sympy [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = -\int \frac{x^3}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)} \, dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(x**3/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**
2*atanh(a*x) - atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \int -\frac{x^3}{(a^2x^2-1)^3 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \int -\frac{x^3}{(a^2x^2-1)^3 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^3}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

input
```
int(-x^3/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x^3}{atanh(ax)\,a^6x^6-3atanh(ax)\,a^4x^4+3atanh(ax)\,a^2x^2-atanh(ax)}dx\right)$$

input
```
int(x^3/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
- int(x**3/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*
a**2*x**2 - atanh(a*x)),x)
```

3.325 $\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx = \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8a^3} - \frac{\log(\operatorname{arctanh}(ax))}{8a^3}$$

output
```
1/8*Chi(4*arctanh(a*x))/a^3-1/8*ln(arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} dx = \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax)) - \log(\operatorname{arctanh}(ax))}{8a^3}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(CoshIntegral[4*ArcTanh[a*x]] - Log[ArcTanh[a*x]])/(8*a^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{\int \frac{a^2x^2}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} - \frac{1}{8\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{8}\text{Chi}(4\operatorname{arctanh}(ax)) - \frac{1}{8}\log(\operatorname{arctanh}(ax))}{a^3}$$

input
```
Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(CoshIntegral[4*ArcTanh[a*x]]/8 - Log[ArcTanh[a*x]]/8)/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\mathrm{Chi}(4\,\mathrm{arctanh}(ax))}{8}-\frac{\ln(4\,\mathrm{arctanh}(ax))}{8}}{a^3}$	24
default	$\frac{\frac{\mathrm{Chi}(4\,\mathrm{arctanh}(ax))}{8}-\frac{\ln(4\,\mathrm{arctanh}(ax))}{8}}{a^3}$	24

input
```
int(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/8*Chi(4*arctanh(a*x))-1/8*ln(4*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(23) = 46$.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{x^2}{(1-a^2x^2)^3\,\mathrm{arctanh}(ax)}\,dx = -\frac{2\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)}{16\,a^3}$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/16*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral((a^2*x^2 + 2*a*x +
1)/(a^2*x^2 - 2*a*x + 1)) - log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 +
2*a*x + 1)))/a^3
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^2}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(x**2/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**
2*atanh(a*x) - atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^2}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x^2}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x^2}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^2/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x^2}{atanh(ax)\,a^6x^6-3atanh(ax)\,a^4x^4+3atanh(ax)\,a^2x^2-atanh(ax)}dx\right)$$

input
```
int(x^2/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
 - int(x**2/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*
a**2*x**2 - atanh(a*x)),x)
```

3.326 $\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{x}{(1-a^2x^2)^3\,\text{arctanh}(ax)}\,dx = \frac{\text{Shi}(2\text{arctanh}(ax))}{4a^2} + \frac{\text{Shi}(4\text{arctanh}(ax))}{8a^2}$$

output
```
1/4*Shi(2*arctanh(a*x))/a^2+1/8*Shi(4*arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1-a^2x^2)^3\,\text{arctanh}(ax)}\,dx = \frac{2\text{Shi}(2\text{arctanh}(ax)) + \text{Shi}(4\text{arctanh}(ax))}{8a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$$
$$\Big\downarrow \text{6596}$$
$$\frac{\int \frac{ax}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^2}$$
$$\Big\downarrow \text{5971}$$
$$\frac{\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{4}\text{Shi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\operatorname{arctanh}(ax))}{a^2}$$

input
```
Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(SinhIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8)/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{8}+\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{4}}{a^2}$	24
default	$\frac{\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{8}+\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{4}}{a^2}$	24

input
```
int(x/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/8*Shi(4*arctanh(a*x))+1/4*Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.52

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}\,dx = \frac{\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+2\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-2\,\text{log\_integral}\left(-\right.}{16\,a^2}$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_inte
gral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 2*log_integral(-(a*x +
 1)/(a*x - 1)) - 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^2
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{x}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(x/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*a
tanh(a*x) - atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{x}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}\,dx = -\int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3}\,dx$$

input
```
int(-x/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x}{atanh\,(ax)\,a^6x^6-3atanh\,(ax)\,a^4x^4+3atanh\,(ax)\,a^2x^2-atanh\,(ax)}dx\right)$$

input
```
int(x/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
- int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**
2*x**2 - atanh(a*x)),x)
```

3.327 $\int \frac{1}{(1-a^2x^2)^3 \text{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8a} + \frac{3\log(\operatorname{arctanh}(ax))}{8a}$$

output
```
1/2*Chi(2*arctanh(a*x))/a+1/8*Chi(4*arctanh(a*x))/a+3/8*ln(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = -\frac{-4\operatorname{Chi}(2\operatorname{arctanh}(ax)) - \operatorname{Chi}(4\operatorname{arctanh}(ax)) - 3\log(\operatorname{arctanh}(ax))}{8a}$$

input
```
Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
-1/8*(-4*CoshIntegral[2*ArcTanh[a*x]] - CoshIntegral[4*ArcTanh[a*x]] - 3*L
og[ArcTanh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6530}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}+\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{3}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{3}{8}\log(\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
(CoshIntegral[2*ArcTanh[a*x]]/2 + CoshIntegral[4*ArcTanh[a*x]]/8 + (3*Log[
ArcTanh[a*x]])/8)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{3\ln(\operatorname{arctanh}(ax))}{8}+\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8}}{a}$	31
default	$\frac{\frac{3\ln(\operatorname{arctanh}(ax))}{8}+\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2}+\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{8}}{a}$	31

input
```
int(1/(-a^2*x^2+1)^3/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(3/8*ln(arctanh(a*x))+1/2*Chi(2*arctanh(a*x))+1/8*Chi(4*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.88

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx$$
$$= \frac{6\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \log\_integral\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + \log\_integral\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) + 4\log\_integral\left(-\frac{ax+1}{ax-1}\right.}{16\,a}$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1
)/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2
*a*x + 1)) + 4*log_integral(-(a*x + 1)/(a*x - 1)) + 4*log_integral(-(a*x -
 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx$$
$$= -\int \frac{1}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(1/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*a
tanh(a*x) - atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\int \frac{1}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-1/(atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{1}{atanh\,(ax)\,a^6x^6 - 3atanh\,(ax)\,a^4x^4 + 3atanh\,(ax)\,a^2x^2 - atanh\,(ax)}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
 - int(1/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**
2*x**2 - atanh(a*x)),x)
```

3.328 $\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \frac{3}{4}\operatorname{Shi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\operatorname{Shi}(4\operatorname{arctanh}(ax)) - \operatorname{Int}\left(\frac{1}{x(-1+a^2x^2)\operatorname{arctanh}(ax)}, x\right)$$

output
```
3/4*Shi(2*arctanh(a*x))+1/8*Shi(4*arctanh(a*x))-Defer(Int)(1/x/(a^2*x^2-1)
/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx$$

input
```
Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3\text{arctanh}\left(ax\right)}dx$$

input
```
int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)} \, dx = \int -\frac{1}{\left(a^2x^2-1\right)^3 x \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)} \, dx$$
$$= -\int \frac{1}{a^6x^7 \operatorname{atanh}(ax) - 3a^4x^5 \operatorname{atanh}(ax) + 3a^2x^3 \operatorname{atanh}(ax) - x \operatorname{atanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

output
```
-Integral(1/(a**6*x**7*atanh(a*x) - 3*a**4*x**5*atanh(a*x) + 3*a**2*x**3*a
tanh(a*x) - x*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \int -\frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

output
```
-integrate(1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = \int -\frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \, dx = -\int \frac{1}{x \operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^3} \, dx$$

input
```
int(-1/(x*atanh(a*x)*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{1}{atanh\left(ax\right)a^6x^7-3atanh\left(ax\right)a^4x^5+3atanh\left(ax\right)a^2x^3-atanh\left(ax\right)x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)^3/atanh(a*x),x)
```

output
```
 - int(1/(atanh(a*x)*a**6*x**7 - 3*atanh(a*x)*a**4*x**5 + 3*atanh(a*x)*a**
2*x**3 - atanh(a*x)*x),x)
```

3.329 $\int \frac{x^5}{(1-a^2x^2)^3\textbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^5}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}\,dx = -\frac{x}{a^5\operatorname{arctanh}(ax)} - \frac{x}{a^5(1-a^2x^2)^2\operatorname{arctanh}(ax)} + \frac{2x}{a^5(1-a^2x^2)\operatorname{arctanh}(ax)} - \frac{3\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a^6} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a^6} + \frac{\operatorname{Int}\left(\frac{1}{\operatorname{arctanh}(ax)},x\right)}{a^5}$$

output
```
-x/a^5/arctanh(a*x)-x/a^5/(-a^2*x^2+1)^2/arctanh(a*x)+2*x/a^5/(-a^2*x^2+1)
/arctanh(a*x)-3/2*Chi(2*arctanh(a*x))/a^6+1/2*Chi(4*arctanh(a*x))/a^6+Defe
r(Int)(1/arctanh(a*x),x)/a^5
```

Mathematica [N/A]

Not integrable

Time = 7.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = \int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$$

input
```
Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\int \frac{x^3}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\frac{\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx}{a^2}}{a^2} - \frac{\frac{\int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2) \operatorname{arctanh}(ax)^2}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{ 6548}$$

$$\frac{\frac{\int \frac{x}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}dx}{a^2}}{a^2}-\frac{\frac{\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}dx}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}}{a^2}$$

$\Big\downarrow$ 6444

$$\frac{\frac{\int \frac{x}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}dx}{a^2}}{a^2}-\frac{\frac{\int \frac{x}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}dx}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}}{a^2}$$

$\Big\downarrow$ 6594

$$\frac{\frac{\frac{\int \frac{1}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx}{a}+a\int \frac{\cdots}{\left(1-a^2x^2\right)^2 \cdots}}{a^2}}{a^2}-\frac{\frac{\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx}{a}+a\int \frac{x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx-\frac{x}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}}{a^2}$$

$\Big\downarrow$ 6530

$$\frac{\frac{3a\int \frac{x^2}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx+\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{a\int \frac{x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx+\cdots}{\cdots}}{a^2}-\frac{\frac{a\int \frac{x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}dx+\frac{\int \frac{1}{\left(1-a^2x^2\right)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+}{}}{a^2}$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}+\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(}}{a^2}$$

$$\big\downarrow \; 3793$$

$$\frac{\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}}{a^2}$$

$$\frac{\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-}{a^2}}{a^2}$$

$$\big\downarrow \; 2009$$

$$\frac{\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{a\int}{}}{a^2}$$

$$\frac{\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(}}{a^2}}{a^2}$$

$$\big\downarrow \; 6596$$

$$\frac{\frac{\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}}{a^2}$$

$$\frac{\frac{\frac{\int \frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-}{a^2}}{a^2}$$

$$\big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}}{a^2}$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}+\frac{\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a\left(1-a^2x^2\right)}}{a^2}}{a^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}}{a^2}$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}-\frac{x}{a\text{arctanh}(ax)}}{a^2}+\frac{\frac{-\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a\left(1-a^2x^2\right)}}{a^2}}{a^2}$$

$\Big\downarrow$ 3793

$$\frac{\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}}{a^2}$$

$$\frac{\frac{-\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}}{a^2}-\frac{\int}{}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}}{a^2}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{\text{arctanh}(ax)}dx}{a}}{a^2}$$

$\Big\downarrow$ 5971

$$\frac{\frac{\frac{3\int\left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)}}{a^2}}{a^2}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{1}{\text{arctanh}(ax)}dx}{a}}{a^2}$$

$$a^2$$

$\downarrow$ 2009

$$\frac{\frac{\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctan}}}{a^2}}{a^2}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{1}{\text{arctanh}(ax)}dx}{a}}{a^2}$$

$$a^2$$

input
```
Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(-a^2x^2+1)^3 \operatorname{arctanh}(ax)^2} dx$$

input
```
int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

output
```
int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{x^5}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = -\int \frac{x^5}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)} \, dx$$

input
```
integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(x**5/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a*
*2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.86

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{x^5}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*x^5/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)
*log(-a*x + 1)) - integrate(-2*(a^2*x^6 - 5*x^4)/((a^7*x^6 - 3*a^5*x^4 + 3
*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*
x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{x^5}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\int \frac{x^5}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^5/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^5/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{x^5}{atanh(ax)^2 a^6x^6-3atanh(ax)^2 a^4x^4+3atanh(ax)^2 a^2x^2-atanh(ax)^2}dx\right)$$

input
```
int(x^5/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
- int(x**5/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh
(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)
```

3.330 $\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = -\frac{x^4}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)} - \frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a^5} + \frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2a^5}$$

output
```
-x^4/a/(-a^2*x^2+1)^2/arctanh(a*x)-Shi(2*arctanh(a*x))/a^5+1/2*Shi(4*arcta
nh(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = \frac{-\frac{2a^4x^4}{(-1+a^2x^2)^2 \operatorname{arctanh}(ax)} - 2\operatorname{Shi}(2\operatorname{arctanh}(ax)) + \operatorname{Shi}(4\operatorname{arctanh}(ax))}{2a^5}$$

input
```
Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
((-2*a^4*x^4)/((-1 + a^2*x^2)^2*ArcTanh[a*x]) - 2*SinhIntegral[2*ArcTanh[a
*x]] + SinhIntegral[4*ArcTanh[a*x]])/(2*a^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6568, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{6568}$$

$$\frac{4\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx}{a} - \frac{x^4}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{4\int \frac{a^3x^3}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^5} - \frac{x^4}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{4\int \left(\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} - \frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^5} - \frac{x^4}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{1}{8}\operatorname{Shi}(4\operatorname{arctanh}(ax)) - \frac{1}{4}\operatorname{Shi}(2\operatorname{arctanh}(ax))\right)}{a^5} - \frac{x^4}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}$$

input
```
Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
-(x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(-1/4*SinhIntegral[2*ArcTanh[
a*x]] + SinhIntegral[4*ArcTanh[a*x]]/8))/a^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}-\mathrm{Shi}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{2}}{a^5}$	62
default	$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}-\mathrm{Shi}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{2}}{a^5}$	62

input
```
int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-3/8/arctanh(a*x)+1/2/arctanh(a*x)*cosh(2*arctanh(a*x))-Shi(2*arcta
nh(a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(50) = 100$.

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.38

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\frac{8\,a^4x^4 - \left((a^4x^4 - 2\,a^2x^2 + 1)\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - (a^4x^4 - 2\,a^2x^2 + 1)\,\text{log\_integral}\left(\frac{a^2x^2-2\,a}{a^2x^2+2\,a}\right.\right.}{4\,(a^9x^4 - 2\,a}$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a^4*x^4 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
 + 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2
*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log
_integral(-(a*x + 1)/(a*x - 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral
(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^9*x^4 - 2*a^7*x^2 +
 a^5)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\int \frac{x^4}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(x**4/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a*
*2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*x^4/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)
*log(-a*x + 1)) + 8*integrate(-x^3/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*
log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\int \frac{x^4}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^4/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^4/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx$$
$$= \frac{-4atanh(ax)\left(\int \frac{x^3}{atanh(ax)a^6x^6-3atanh(ax)a^4x^4+3atanh(ax)a^2x^2-atanh(ax)}dx\right)a^6x^2+4atanh(ax)\left(\int \frac{}{atanh(ax)a^6x^6-}\right.}{}$$

input
```
int(x^4/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
( - 4*atanh(a*x)*int(x**3/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 +
 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**6*x**2 + 4*atanh(a*x)*int(x**3
/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 -
 atanh(a*x)),x)*a**4 + 4*atanh(a*x)*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(
a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**4*x**2 - 4*ata
nh(a*x)*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x
)*a**2*x**2 - atanh(a*x)),x)*a**2 - atanh(a*x)*int(1/(atanh(a*x)**2*a**6*x
**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**
2),x)*a**3*x**2 + atanh(a*x)*int(1/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)
**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a - a**2*x**
2 - 1)/(atanh(a*x)*a**5*(a**2*x**2 - 1))
```

3.331 $\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = -\frac{x^3}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)} - \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a^4} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a^4}$$

output
```
-x^3/a/(-a^2*x^2+1)^2/arctanh(a*x)-1/2*Chi(2*arctanh(a*x))/a^4+1/2*Chi(4*a
rctanh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = \frac{-2a^3x^3 - (-1+a^2x^2)^2 \operatorname{arctanh}(ax)\operatorname{Chi}(2\operatorname{arctanh}(ax)) + (-1+a^2x^2)^2 \operatorname{arctanh}(ax)\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a^4(-1+a^2x^2)^2 \operatorname{arctanh}(ax)}$$

input
```
Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
(-2*a^3*x^3 - (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] +
 (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^4*(-1 +
a^2*x^2)^2*ArcTanh[a*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.

Time = 1.79 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6590, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{6594}$$

$$\frac{\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx}{a} + 3a\int \frac{x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx - \frac{x}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{a^2} - \frac{\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx}{a} + a\int \frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{6530}$$

$$\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx + \frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{a^2} - \frac{a\int \frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx + \frac{\int \frac{1}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arcta}}}{a^2}$$
$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}$$
$$\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{6596}$$

$$\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}}{a^2}$$
$$\frac{\frac{\int \frac{a^2x^2}{\left(1-a^2x^2\right)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}}{a^2}$$
$$\frac{\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}}{a^2}$$

$$\frac{-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\downarrow$ 3793

$$\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}}{a^2}$$

$$\frac{-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\downarrow$ 2009

$$\frac{\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}}{a^2}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\downarrow$ 5971

$$\frac{\frac{3\int \left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a}}{a^2}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\downarrow$ 2009

$$\frac{\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)}}{a^2}$$

$$\frac{\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

input
```
Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
(-(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (3*(CoshIntegral[4*ArcTanh[a*x]]/
8 - Log[ArcTanh[a*x]]/8))/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + CoshInte
gral[4*ArcTanh[a*x]]/8 + (3*Log[ArcTanh[a*x]])/8)/a^2)/a^2 - (-(x/(a*(1 -
a^2*x^2)*ArcTanh[a*x])) + (CoshIntegral[2*ArcTanh[a*x]]/2 - Log[ArcTanh[a*
x]]/2)/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x]]/2)/a^2)/a
^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(4 \operatorname{arctanh}(ax))}{2}+\frac{\sinh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)}-\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{2}}{a^4}$	54
default	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(4 \operatorname{arctanh}(ax))}{2}+\frac{\sinh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)}-\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{2}}{a^4}$	54

input
```
int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/8*sinh(4*arctanh(a*x))/arctanh(a*x)+1/2*Chi(4*arctanh(a*x))+1/4/
arctanh(a*x)*sinh(2*arctanh(a*x))-1/2*Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 4.20

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx =$$
$$-\frac{8\,a^3x^3 - \left((a^4x^4 - 2\,a^2x^2 + 1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + (a^4x^4 - 2\,a^2x^2 + 1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,a}{a^2x^2+2\,a}\right.\right.}{4\,(a^8x^4 - 2\,a^6}$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a^3*x^3 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
 + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2
*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_i
ntegral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4
)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx$$
$$= -\int \frac{x^3}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(x**3/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a*
*2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{x^3}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*x^3/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)
*log(-a*x + 1)) + integrate(-2*(a^2*x^4 + 3*x^2)/((a^7*x^6 - 3*a^5*x^4 + 3
*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*
x + 1)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{x^3}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = -\int \frac{x^3}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3} \, dx$$

input
```
int(-x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^3/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^3}{\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2}\,dx$$
$$= -\left(\int \frac{x^3}{atanh\left(ax\right)^2 a^6x^6 - 3atanh\left(ax\right)^2 a^4x^4 + 3atanh\left(ax\right)^2 a^2x^2 - atanh\left(ax\right)^2}dx\right)$$

input
```
int(x^3/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
 - int(x**3/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh
(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)
```

3.332 $\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = -\frac{x^2}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2a^3}$$

output
```
-x^2/a/(-a^2*x^2+1)^2/arctanh(a*x)+1/2*Shi(4*arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \frac{-2a^2x^2 + (-1+a^2x^2)^2 \operatorname{arctanh}(ax)\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2a^3(-1+a^2x^2)^2 \operatorname{arctanh}(ax)}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
(-2*a^2*x^2 + (-1 + a^2*x^2)^2*ArcTanh[a*x]*SinhIntegral[4*ArcTanh[a*x]])/
(2*a^3*(-1 + a^2*x^2)^2*ArcTanh[a*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. $2(41) = 82$.

Time = 0.81 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6590, 6528, 6596, 5971, 27, 2009, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{4a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{a^2}-\frac{2a\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{\frac{4\int \frac{ax}{\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{a^2}-\frac{\frac{2\int \frac{ax}{\left(1-a^2x^2\right)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\frac{4\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{a^2}-\frac{\frac{2\int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{4\int \left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}+\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}}{a^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}}{a^2}$$

$$\Big\downarrow \text{ 3779}$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a}-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

input
```
Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
-((-(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a)/a
^2) + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTanh[
a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/a^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2}+\frac{1}{8\operatorname{arctanh}(ax)}}{a^3}$	38
default	$\frac{-\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2}+\frac{1}{8\operatorname{arctanh}(ax)}}{a^3}$	38

input
```
int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x))+1/8/
arctanh(a*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(38) = 76.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.00

$$\int \frac{x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}\,dx =$$
$$-\frac{8\,a^2x^2-\left((a^4x^4-2\,a^2x^2+1)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-(a^4x^4-2\,a^2x^2+1)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,a}{a^2x^2+2\,a}\right.\right.}{4\,(a^7x^4-2\,a^5x^2+a^3)\log\left(-\frac{ax+1}{ax-1}\right)}$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
 + 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2
*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^7
*x^4 - 2*a^5*x^2 + a^3)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$$
$$= -\int \frac{x^2}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)} dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(x**2/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a*
*2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = \int -\frac{x^2}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*x^2/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)
*log(-a*x + 1)) + integrate(-4*(a^2*x^3 + x)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3
*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{x^2}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = -\int \frac{x^2}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3} \, dx$$

input
```
int(-x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^2/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx$$
$$= \frac{-2atanh(ax)\left(\int \frac{x^3}{atanh(ax)a^6x^6-3atanh(ax)a^4x^4+3atanh(ax)a^2x^2-atanh(ax)}dx\right)a^6x^4+4atanh(ax)\left(\int \frac{}{atanh(ax)a^6x^6-}\right.}{}$$

input
```
int(x^2/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
( - 2*atanh(a*x)*int(x**3/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 +
 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**6*x**4 + 4*atanh(a*x)*int(x**3
/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 -
 atanh(a*x)),x)*a**4*x**2 - 2*atanh(a*x)*int(x**3/(atanh(a*x)*a**6*x**6 -
3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**2 - 2*
atanh(a*x)*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(
a*x)*a**2*x**2 - atanh(a*x)),x)*a**4*x**4 + 4*atanh(a*x)*int(x/(atanh(a*x)
*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x))
,x)*a**2*x**2 - 2*atanh(a*x)*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a*
*4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x) - x**2)/(atanh(a*x)*a*(a
**4*x**4 - 2*a**2*x**2 + 1))
```

3.333 $\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = -\frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{2a^2} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a^2}$$

output
```
-x/a/(-a^2*x^2+1)^2/arctanh(a*x)+1/2*Chi(2*arctanh(a*x))/a^2+1/2*Chi(4*arc
tanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx = \frac{-2ax + (-1+a^2x^2)^2 \operatorname{arctanh}(ax)\operatorname{Chi}(2\operatorname{arctanh}(ax)) + (-1+a^2x^2)^2 \operatorname{arctanh}(ax)\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a^2(-1+a^2x^2)^2 \operatorname{arctanh}(ax)}$$

input
```
Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
(-2*a*x + (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] + (-1
 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*
x^2)^2*ArcTanh[a*x])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.64, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6594}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6530}$$

$$3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3793}$$

$$3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx + \frac{\int \left(\frac{\cosh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)} + \frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} + \frac{3}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx + \frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\text{Chi}(4\operatorname{arctanh}(ax)) + \frac{3}{8}\log(\operatorname{arctanh}(ax))}{a^2} - \frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2} + \frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\text{Chi}(4\operatorname{arctanh}(ax)) + \frac{3}{8}\log(\operatorname{arctanh}(ax))}{a^2} - \frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{3\int \left(\frac{\cosh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} - \frac{1}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2} + \frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\text{Chi}(4\operatorname{arctanh}(ax)) + \frac{3}{8}\log(\operatorname{arctanh}(ax))}{a^2} - \frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\left(\frac{1}{8}\text{Chi}(4\operatorname{arctanh}(ax)) - \frac{1}{8}\log(\operatorname{arctanh}(ax))\right)}{a^2} + \frac{\frac{1}{2}\text{Chi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\text{Chi}(4\operatorname{arctanh}(ax)) + \frac{3}{8}\log(\operatorname{arctanh}(ax))}{a^2} - \frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

input
```
Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
-(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (3*(CoshIntegral[4*ArcTanh[a*x]]/8
 - Log[ArcTanh[a*x]]/8))/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + CoshInteg
ral[4*ArcTanh[a*x]]/8 + (3*Log[ArcTanh[a*x]])/8)/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(4 \operatorname{arctanh}(ax))}{2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{2}}{a^2}$	54
default	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(4 \operatorname{arctanh}(ax))}{2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{2}}{a^2}$	54

input
```
int(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/8*sinh(4*arctanh(a*x))/arctanh(a*x)+1/2*Chi(4*arctanh(a*x))-1/4/
arctanh(a*x)*sinh(2*arctanh(a*x))+1/2*Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(48) = 96$.

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 4.25

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx =$$
$$-\frac{8\,ax-\left(\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,ax+}{a^2x^2+2\,ax+}\right.\right.}{4\left(a^6x^4-2\,a^4x\right.}$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a*x - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1
)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2
 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integ
ral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x -
 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*lo
g(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx$$
$$= -\int \frac{x}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(x/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*
x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \int -\frac{x}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2*x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*l
og(-a*x + 1)) + integrate(-2*(3*a^2*x^2 + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3
*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
1)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \int -\frac{x}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}\,dx = -\int \frac{x}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}\,dx$$
$$= -\left(\int \frac{x}{atanh(ax)^2\,a^6x^6-3atanh(ax)^2\,a^4x^4+3atanh(ax)^2\,a^2x^2-atanh(ax)^2}dx\right)$$

input
```
int(x/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
- int(x/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*
x)**2*a**2*x**2 - atanh(a*x)**2),x)
```

3.334 $\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{1}{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{a(1-a^2x^2)^2\,\text{arctanh}(ax)} + \frac{\text{Shi}(2\text{arctanh}(ax))}{a} + \frac{\text{Shi}(4\text{arctanh}(ax))}{2a}$$

output
```
-1/a/(-a^2*x^2+1)^2/arctanh(a*x)+Shi(2*arctanh(a*x))/a+1/2*Shi(4*arctanh(a
*x))/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}\,dx = \frac{-\frac{2}{(-1+a^2x^2)^2\text{arctanh}(ax)} + 2\text{Shi}(2\text{arctanh}(ax)) + \text{Shi}(4\text{arctanh}(ax))}{2a}$$

input
```
Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
(-2/((-1 + a^2*x^2)^2*ArcTanh[a*x]) + 2*SinhIntegral[2*ArcTanh[a*x]] + Sin
hIntegral[4*ArcTanh[a*x]])/(2*a)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6528, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6528}$$

$$4a\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{4\int \frac{ax}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{4\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{1}{4}\text{Shi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\operatorname{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTanh[a*x]]/4
 + SinhIntegral[4*ArcTanh[a*x]]/8))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{2}}{a}$	60
default	$\frac{-\frac{3}{8\,\mathrm{arctanh}(ax)}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{2\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(2\,\mathrm{arctanh}(ax))-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)}+\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{2}}{a}$	60

input
```
int(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3/8/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh
(a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(46) = 92$.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 4.53

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \frac{\left((a^4x^4-2a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-(a^4x^4-2a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+2\,(a\right.}{4\,(a^5x^4-2\,a^3x^2}$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/4*(((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^
2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x
+ 1)/(a^2*x^2 + 2*a*x + 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x + 1)/(a*x - 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(
a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-
(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\int \frac{1}{a^6x^6 \operatorname{atanh}^2(ax) - 3a^4x^4 \operatorname{atanh}^2(ax) + 3a^2x^2 \operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(1/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*
x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
8*a*integrate(-x/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x) - 2/((a^5*x^4 - 2*a^
3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1))
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}\,dx = -\int \frac{1}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-1/(atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} dx$$
$$= \frac{-4atanh(ax) \left(\int \frac{x}{atanh(ax)a^6x^6-3atanh(ax)a^4x^4+3atanh(ax)a^2x^2-atanh(ax)} dx\right) a^6x^4 + 8atanh(ax) \left(\int \frac{}{atanh(ax)a^6x^6-} \right.}{atanh\,(a}$$

input
```
int(1/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
( - 4*atanh(a*x)*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*
atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**6*x**4 + 8*atanh(a*x)*int(x/(atan
h(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh
(a*x)),x)*a**4*x**2 - 4*atanh(a*x)*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a
*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**2 - 1)/(atanh(a
*x)*a*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.335 $\int \frac{1}{x(1-a^2x^2)^3\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x(1-a^2x^2)^3\,\text{arctanh}(ax)^2}\,dx = &-\frac{1}{ax\text{arctanh}(ax)} - \frac{ax}{(1-a^2x^2)^2\,\text{arctanh}(ax)} \\ &- \frac{ax}{(1-a^2x^2)\,\text{arctanh}(ax)} + \frac{3}{2}\text{Chi}(2\text{arctanh}(ax)) \\ &+ \frac{1}{2}\text{Chi}(4\text{arctanh}(ax)) - \frac{\text{Int}\left(\frac{1}{x^2\text{arctanh}(ax)},x\right)}{a}\end{aligned}$$

output
```
-1/a/x/arctanh(a*x)-a*x/(-a^2*x^2+1)^2/arctanh(a*x)-a*x/(-a^2*x^2+1)/arcta
nh(a*x)+3/2*Chi(2*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-Defer(Int)(1/x^2/a
rctanh(a*x),x)/a
```

Mathematica [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^3\,\text{arctanh}(ax)^2}\,dx = \int \frac{1}{x(1-a^2x^2)^3\,\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx+\int \frac{1}{x (1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx+a^2 \int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx+\int \frac{1}{x (1-a^2x^2) \operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6552}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx+a^2 \int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx-\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)}dx}{a}-\frac{1}{ax\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6468}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx+a^2 \int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}dx-\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)}dx}{a}-\frac{1}{ax\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \textcolor{blue}{6594}$$

$$a^2\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)}dx-\frac{x}{a\,(1-a^2x^2)^2\,\text{arctanh}(ax)}\right)+a^2\left(\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}dx}{a}+a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \textcolor{blue}{6530}$$

$$a^2\left(3a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)^2\,\text{arctanh}(ax)}\right)+a^2\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$a^2\left(a\int \frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)+a^2\left(3a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)^2\,\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \textcolor{blue}{3793}$$

$$a^2\left(a\int\frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\int\left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arcta}}\right.$$
$$a^2\left(3a\int\frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)}dx+\frac{\int\left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax}{a^2}\right.$$
$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow\ 2009$$

$$a^2\left(3a\int\frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a\,(1}\right.$$
$$a^2\left(a\int\frac{x^2}{(1-a^2x^2)^2\,\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right)$$
$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow\ 6596$$

$$a^2\left(\frac{3\int\frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.$$
$$a^2\left(\frac{\int\frac{a^2x^2}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax}\right.$$
$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow\ 3042$$

$$a^2\left(\frac{3\int\frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.$$
$$a^2\left(\frac{\int-\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\,(1-a^2x^2)\,\text{arctanh}(ax)}\right.$$
$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 25

$$a^2\left(\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.$$
$$a^2\left(-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right.$$
$$\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 3793

$$a^2\left(\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.$$
$$a^2\left(-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1\right.$$
$$\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}-\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 2009

$$a^2\left(\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.$$
$$\frac{\int \frac{1}{x^2\text{arctanh}(ax)}dx}{a}+$$
$$a^2\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arct}}\right.$$
$$\frac{1}{ax\text{arctanh}(ax)}$$

$\Big\downarrow$ 5971

$$a^2\left(\frac{3\int\left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log\right.$$

$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)}dx}{a}+$$

$$a^2\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arct}\right.$$

$$\frac{1}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int\frac{1}{x^2\text{arctanh}(ax)}dx}{a}+$$

$$a^2\left(\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctan}\right.$$

$$a^2\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)\text{arct}\right.$$

$$\frac{1}{ax\text{arctanh}(ax)}$$

input
```
Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int\frac{1}{x\left(-a^2x^2+1\right)^3\text{arctanh}\left(ax\right)^2}dx$$

input
```
int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

output
```
int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}\,dx=\int -\frac{1}{\left(a^2x^2-1\right)^3x\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}\,dx$$
$$=-\int \frac{1}{a^6x^7\operatorname{atanh}^2\left(ax\right)-3a^4x^5\operatorname{atanh}^2\left(ax\right)+3a^2x^3\operatorname{atanh}^2\left(ax\right)-x\operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

output
```
-Integral(1/(a**6*x**7*atanh(a*x)**2 - 3*a**4*x**5*atanh(a*x)**2 + 3*a**2*
x**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.95

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-2/((a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) - (a^5*x^5 - 2*a^3*x^3 + a*x)
*log(-a*x + 1)) + integrate(-2*(5*a^2*x^2 - 1)/((a^7*x^8 - 3*a^5*x^6 + 3*a
^3*x^4 - a*x^2)*log(a*x + 1) - (a^7*x^8 - 3*a^5*x^6 + 3*a^3*x^4 - a*x^2)*l
og(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} \, dx = \int -\frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2}\,dx = -\int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^2\,\left(a^2\,x^2-1\right)^3}\,dx$$

input
```
int(-1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(x*atanh(a*x)^2*(a^2*x^2 - 1)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{1}{atanh\left(ax\right)^2 a^6x^7-3atanh\left(ax\right)^2 a^4x^5+3atanh\left(ax\right)^2 a^2x^3-atanh\left(ax\right)^2 x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)^3/atanh(a*x)^2,x)
```

output
```
 - int(1/(atanh(a*x)**2*a**6*x**7 - 3*atanh(a*x)**2*a**4*x**5 + 3*atanh(a*
x)**2*a**2*x**3 - atanh(a*x)**2*x),x)
```

3.336 $\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx = -\frac{x^4}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} - \frac{2x}{a^4(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{2x}{a^4(1-a^2x^2) \operatorname{arctanh}(ax)} - \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{a^5} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{a^5}$$

output
```
-1/2*x^4/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2*x/a^4/(-a^2*x^2+1)^2/arctanh(a*
x)+2*x/a^4/(-a^2*x^2+1)/arctanh(a*x)-Chi(2*arctanh(a*x))/a^5+Chi(4*arctanh
(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\frac{\frac{a^3x^3(ax+4\operatorname{arctanh}(ax))}{(-1+a^2x^2)^2\operatorname{arctanh}(ax)^2}+2\operatorname{Chi}(2\operatorname{arctanh}(ax))-2\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a^5}$$

input
```
Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*((a^3*x^3*(a*x + 4*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTanh[a*x]^2) +
 2*CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]])/a^5
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(100) = 200.

Time = 2.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6568, 6590, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{ 6568}$$

$$\frac{2\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx}{a}-\frac{x^4}{2a(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{2\left(\frac{\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}dx}{a^2}\right)}{a}-\frac{x^4}{2a(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$\frac{2\left(\frac{\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}dx}{a}+a\int \frac{}{(1-a^2x}}{}\right.}{a}$$

$$\frac{x^4}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$\frac{2\left(\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d}{}\right.}{a}$$

$$\frac{x^4}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^4}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}+$$

$$\frac{2\left(\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d}{}\right.}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{2\left(\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}}{a^2}\right.}{a}$$

$$\frac{x^4}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\frac{3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}-\frac{a\int}{}\right.$$

$$a$$

$$\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$\downarrow$ 6596

$$2\left(\frac{\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}\right.$$

$$a$$

$$\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$\downarrow$ 3042

$$-\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}+$$

$$2\left(\frac{\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}\right.$$

$$a$$

$\downarrow$ 25

$$-\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}+$$

$$2\left(\frac{\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}\right.$$

$$a$$

$\downarrow$ 3793

$$2\left(\frac{\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}}{a^2}\right.$$

$$\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$2\left(\frac{\frac{3\int \frac{a^2x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}}{a^2}\right.$$

a

$$\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$\downarrow$ 5971

$$2\left(\frac{\frac{3\int\left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a}}{a^2}\right.$$

$$\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$2\left(\frac{\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}}{a^2}\right.$$

a

$$\frac{x^4}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

input
```
Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + (2*((-(x/(a*(1 - a^2*x^2)^2*
ArcTanh[a*x])) + (3*(CoshIntegral[4*ArcTanh[a*x]]/8 - Log[ArcTanh[a*x]]/8)
)/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + CoshIntegral[4*ArcTanh[a*x]]/8 +
 (3*Log[ArcTanh[a*x]])/8)/a^2)/a^2 - (-(x/(a*(1 - a^2*x^2)*ArcTanh[a*x]))
+ (CoshIntegral[2*ArcTanh[a*x]]/2 - Log[ArcTanh[a*x]]/2)/a^2 + (CoshIntegr
al[2*ArcTanh[a*x]]/2 + Log[ArcTanh[a*x]]/2)/a^2)/a^2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6568

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

rule 6590

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{3}{16 \operatorname{arctanh}(ax)^2}+\frac{\cosh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)^2}+\frac{\sinh(2 \operatorname{arctanh}(ax))}{2 \operatorname{arctanh}(ax)}-\operatorname{Chi}(2 \operatorname{arctanh}(ax))-\frac{\cosh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)^2}-\frac{\sinh(4 \operatorname{arctanh}(ax}{4 \operatorname{arctanh}(ax)}}{a^5}$
default	$\frac{-\frac{3}{16 \operatorname{arctanh}(ax)^2}+\frac{\cosh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)^2}+\frac{\sinh(2 \operatorname{arctanh}(ax))}{2 \operatorname{arctanh}(ax)}-\operatorname{Chi}(2 \operatorname{arctanh}(ax))-\frac{\cosh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)^2}-\frac{\sinh(4 \operatorname{arctanh}(ax}{4 \operatorname{arctanh}(ax)}}{a^5}$

input
```
int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-3/16/arctanh(a*x)^2+1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))+1/2/ar
ctanh(a*x)*sinh(2*arctanh(a*x))-Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*co
sh(4*arctanh(a*x))-1/4*sinh(4*arctanh(a*x))/arctanh(a*x)+Chi(4*arctanh(a*x
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.56

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\frac{4\,a^4x^4 + 8\,a^3x^3 \log\left(-\frac{ax+1}{ax-1}\right) - \left((a^4x^4 - 2\,a^2x^2 + 1)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + (a^4x^4 - 2\,a^2x^2 + 1)\right.}{2\,(a}$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^4*x^4 + 8*a^3*x^3*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*
x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*
x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x +
 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^4
*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(
a*x - 1))^2)/((a^9*x^4 - 2*a^7*x^2 + a^5)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\int \frac{x^4}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)} \, dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

output
```
-Integral(x**4/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a*
*2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-2*(a*x^4 + 2*x^3*log(a*x + 1) - 2*x^3*log(-a*x + 1))/((a^6*x^4 - 2*a^4*x^
2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)*log(-
a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + integrate(-4*(a^
2*x^4 + 3*x^2)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1) - (a^
8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x^4}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\int \frac{x^4}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^4/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^4/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \frac{-4atanh(ax)^2\left(\int \frac{x^3}{atanh(ax)^2a^6x^6-3atanh(ax)^2a^4x^4+3atanh(ax)^2a^2x^2-atanh(ax)^2}dx\right)a^6x^2+4atanh(ax)^2\left(\int \frac{}{atanh(ax}\right.}{}$$

input
```
int(x^4/(-a^2*x^2+1)^3/atanh(a*x)^3,x)
```

output
```
( - 4*atanh(a*x)**2*int(x**3/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a*
*4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**6*x**2 + 4*atan
h(a*x)**2*int(x**3/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 +
3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**4 + 4*atanh(a*x)**2*int(x
/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a*
*2*x**2 - atanh(a*x)**2),x)*a**4*x**2 - 4*atanh(a*x)**2*int(x/(atanh(a*x)*
*2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - ata
nh(a*x)**2),x)*a**2 - 2*atanh(a*x)**2*int(1/(atanh(a*x)**3*a**6*x**6 - 3*a
tanh(a*x)**3*a**4*x**4 + 3*atanh(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)*a**
3*x**2 + 2*atanh(a*x)**2*int(1/(atanh(a*x)**3*a**6*x**6 - 3*atanh(a*x)**3*
a**4*x**4 + 3*atanh(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)*a - a**2*x**2 -
1)/(2*atanh(a*x)**2*a**5*(a**2*x**2 - 1))
```

3.337 $\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\begin{aligned}\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx = & -\frac{x^3}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \\ & -\frac{3x^2}{2a^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \\ & -\frac{x^4}{2(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \\ & -\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2a^4} + \frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{a^4}\end{aligned}$$

output
```
-1/2*x^3/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-3/2*x^2/a^2/(-a^2*x^2+1)^2/arctan
h(a*x)-1/2*x^4/(-a^2*x^2+1)^2/arctanh(a*x)-1/2*Shi(2*arctanh(a*x))/a^4+Shi
(4*arctanh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\frac{\frac{a^2x^2\left(ax+(3+a^2x^2)\operatorname{arctanh}(ax)\right)}{(-1+a^2x^2)^2\operatorname{arctanh}(ax)^2} + \operatorname{Shi}(2\operatorname{arctanh}(ax)) - 2\operatorname{Shi}(4\operatorname{arctanh}(ax))}{2a^4}$$

input
```
Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*((a^2*x^2*(a*x + (3 + a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTa
nh[a*x]^2) + SinhIntegral[2*ArcTanh[a*x]] - 2*SinhIntegral[4*ArcTanh[a*x]]
)/a^4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 270 vs. 2(107) = 214.

Time = 2.38 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.52, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6590, 6558, 6594, 6528, 6590, 6528, 6596, 5971, 27, 2009, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}dx}{a^2} - \frac{\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{6558}$$

$$\frac{\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^3}dx}{a^2} - \frac{2\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx - \frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\big\downarrow$ 6594

$$\frac{\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{2a} + \frac{3}{2}a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx - \frac{x}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}}{a^2} - \frac{2\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx - \frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\big\downarrow$ 6528

$$\frac{\frac{3}{2}a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx + \frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx - \frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a} - \frac{x}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}}{a^2} - \frac{2\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx - \frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\big\downarrow$ 6590

$$\frac{\frac{3}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{a^2} - \frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)^2}dx}{a^2}\right) + \frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx - \frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a} - \cdots}{a^2} - \frac{2\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx - \frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\big\downarrow$ 6528

$$\frac{\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx - \frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a} + \frac{3}{2}a\left(\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx - \frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2} - \frac{2a\int \cdots}{\cdots}\right.\cdots}{a^2} - \frac{2\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx - \frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2} - \frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\big\downarrow$ 6596

$$\frac{\frac{\frac{4\int \frac{ax}{\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{\frac{4\int \frac{ax}{\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{ar}}}{a^2}\right.}{a^2}$$

$$\frac{\frac{2\int \frac{ax}{\left(1-a^2x^2\right)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\frac{4\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctan}}{8\operatorname{arctanh}}\right.}{a}\right.$$

$$\frac{\frac{2\int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{4\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4\int \left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctan}}{8\operatorname{arctanh}}\right.}{a}\right.$$

$$\frac{\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\operatorname{Shi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\operatorname{Shi}(4\operatorname{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}}{a^2}-\frac{\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)\operatorname{ar}}}{a^2}\right.}{a^2}$$

$$\frac{\frac{\int \frac{\sinh(2\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}+\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}}{a}}{a^2}\right.}{a^2}$$

$$\frac{\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\Big\downarrow$ 26

$$\frac{\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d}{a}}{a^2}\right.}{a^2}$$

$$\frac{-\frac{i\int \frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

$\Big\downarrow$ 3779

$$\frac{\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a}-\frac{1}{a\left(1-a^2x^2\right)\text{arctanh}(ax)}}{a^2}\right)+}{a^2}$$

$$\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a(1-a^2x^2)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\text{arctanh}(ax)}}{a^2}$$

input
```
Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-((-1/2*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2
*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2)/a^2) + (-1/2*x/(a*
(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + (3*a*(-((-(1/(a*(1 - a^2*x^2)*ArcTanh[a*
x])) + SinhIntegral[2*ArcTanh[a*x]]/a)/a^2) + (-(1/(a*(1 - a^2*x^2)^2*ArcT
anh[a*x])) + (4*(SinhIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a
*x]]/8))/a)/a^2))/2 + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhInt
egral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/(2*a))/a^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(4\,\mathrm{arctanh}(ax))+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^2}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}-\frac{\mathrm{Shi}(2\,\mathrm{arctanh}(}{2}}{a^4}$
default	$\frac{-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{Shi}(4\,\mathrm{arctanh}(ax))+\frac{\sinh(2\,\mathrm{arctanh}(ax))}{8\,\mathrm{arctanh}(ax)^2}+\frac{\cosh(2\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}-\frac{\mathrm{Shi}(2\,\mathrm{arctanh}(}{2}}{a^4}$

input
```
int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/16*sinh(4*arctanh(a*x))/arctanh(a*x)^2-1/4/arctanh(a*x)*cosh(4*a
rctanh(a*x))+Shi(4*arctanh(a*x))+1/8/arctanh(a*x)^2*sinh(2*arctanh(a*x))+1
/4/arctanh(a*x)*cosh(2*arctanh(a*x))-1/2*Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(96) = 192$.

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.50

$$\int \frac{x^3}{(1-a^2x^2)^3\,\mathrm{arctanh}(ax)^3}\,dx =$$
$$-\frac{8\,a^3x^3-\left(2\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-2\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2}{a^2x^2}\right.\right.}{}$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(8*a^3*x^3 - (2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a
*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(
(a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*l
og_integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral
(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^4*x^4 + 3*a^2*x
^2)*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log(-(a*x + 1)
/(a*x - 1))^2)
```

Sympy [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\int \frac{x^3}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

output
```
-Integral(x**3/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a*
*2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x^3}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x^3}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-(2*a*x^3 + (a^2*x^4 + 3*x^2)*log(a*x + 1) - (a^2*x^4 + 3*x^2)*log(-a*x +
1))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 +
 a^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x +
1)^2) + integrate(-2*(5*a^2*x^3 + 3*x)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 -
 a^2)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)
), x)
```

Giac [F]

$$\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x^3}{(a^2x^2-1)^3\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}\,dx = -\int \frac{x^3}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^3/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}\,dx$$
$$= \frac{-5atanh(ax)^2\left(\int \frac{x^2}{atanh(ax)^2a^6x^6-3atanh(ax)^2a^4x^4+3atanh(ax)^2a^2x^2-atanh(ax)^2}dx\right)a^7x^4+10atanh(ax)^2\left(\int \frac{}{atanh(a}\right.}{}$$

input
```
int(x^3/(-a^2*x^2+1)^3/atanh(a*x)^3,x)
```

output

```
( - 5*atanh(a*x)**2*int(x**2/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a*
*4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**7*x**4 + 10*ata
nh(a*x)**2*int(x**2/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 +
 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**5*x**2 - 5*atanh(a*x)**2
*int(x**2/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a
*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**3 + atanh(a*x)**2*int(1/(atanh(a*x
)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - a
tanh(a*x)**2),x)*a**5*x**4 - 2*atanh(a*x)**2*int(1/(atanh(a*x)**2*a**6*x**
6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2)
,x)*a**3*x**2 + atanh(a*x)**2*int(1/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x
)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a - atanh(a
*x)*a**4*x**4 + 2*atanh(a*x)*a**2*x**2 - atanh(a*x) - a**3*x**3)/(2*atanh(
a*x)**2*a**4*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.338 $\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx = -\frac{x^2}{2a(-1+a^2x^2)^2 \operatorname{arctanh}(ax)^2} - \frac{2x}{a^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{x}{a^2(1-a^2x^2) \operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{a^3}$$

output
```
-1/2*x^2/a/(a^2*x^2-1)^2/arctanh(a*x)^2-2*x/a^2/(-a^2*x^2+1)^2/arctanh(a*x
)+x/a^2/(-a^2*x^2+1)/arctanh(a*x)+Chi(4*arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} dx = -\frac{x(ax+2(1+a^2x^2)\operatorname{arctanh}(ax))}{2a^2(-1+a^2x^2)^2 \operatorname{arctanh}(ax)^2} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{a^3}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*(x*(a*x + 2*(1 + a^2*x^2)*ArcTanh[a*x]))/(a^2*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^2) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 229 vs. $2(86) = 172$.

Time = 2.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.66, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6590, 6528, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{6528}$$

$$\frac{2a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}}{a^2}-\frac{a\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}}{a^2}$$

$$\Big\downarrow \text{6594}$$

$$\frac{2a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}}{a^2}-\frac{a\left(\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx}{a}+a\int \frac{x^2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}}{a^2}$$

$\downarrow$ 6530

$$\frac{2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)^2\text{arct}}}{a^2}$$

$$\frac{a\left(a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)\text{arctanh}(}}{a^2}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}+2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arc}}\right.}{a^2}$$

$$\frac{-\frac{1}{2a(1-a^2x^2)\text{arctanh}(ax)^2}+a\left(a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}}\right.}{a^2}$$

$\downarrow$ 3793

$$\frac{2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{a\left(a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{1}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{}{2a(1-}}{a^2}$$

$\downarrow$ 2009

$$\frac{2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}}\right.}{a^2}$$

$$\frac{a\left(a\int \frac{x^2}{(1-a^2x^2)^2\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)\text{arcta}}}{a^2}$$

$\downarrow$ 6596

$$\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{a\left(\frac{\int \frac{a^2x^2}{\left(1-a^2x^2\right)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{}{2a(1-a^2}}{a^2}$$

$\downarrow$ 3042

$$\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{-\frac{1}{2a(1-a^2x^2)\text{arctanh}(ax)^2}+a\left(\frac{\int -\frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x}\right.}{a^2}$$

$\downarrow$ 25

$$\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{-\frac{1}{2a(1-a^2x^2)\text{arctanh}(ax)^2}+a\left(-\frac{\int \frac{\sin(i\text{arctanh}(ax))^2}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x}\right.}{a^2}$$

$\downarrow$ 3793

$$\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{a\left(-\frac{\int \left(\frac{1}{2\text{arctanh}(ax)}-\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(}\right.}{a^2}$$

$\downarrow$ 2009

$$\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{a^2}$$

$$\frac{a\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{}{2a(1}}{a^2}$$

$\Big\downarrow$ 5971

$$\frac{2a\left(\frac{3\int \left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.}{a^2}$$

$$\frac{a\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{}{2a(1}}{a^2}$$

$\Big\downarrow$ 2009

$$\frac{2a\left(\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-}\right.}{a^2}$$

$$\frac{a\left(\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))-\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{2}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)\text{arctanh}(ax)}\right)-\frac{}{2a(1}}{a^2}$$

input
```
Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
(-1/2*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 2*a*(-(x/(a*(1 - a^2*x^2)^2*A
rcTanh[a*x])) + (3*(CoshIntegral[4*ArcTanh[a*x]]/8 - Log[ArcTanh[a*x]]/8))
/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + CoshIntegral[4*ArcTanh[a*x]]/8 +
(3*Log[ArcTanh[a*x]])/8)/a^2))/a^2 - (-1/2*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]
^2) + a*(-(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + (CoshIntegral[2*ArcTanh[a*x
]]/2 - Log[ArcTanh[a*x]]/2)/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + Log[Ar
cTanh[a*x]]/2)/a^2))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

method	result	size
derivativedivides	$\frac{-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{Chi}(4\,\mathrm{arctanh}(ax))+\frac{1}{16\,\mathrm{arctanh}(ax)^2}}{a^3}$	51
default	$\frac{-\frac{\cosh(4\,\mathrm{arctanh}(ax))}{16\,\mathrm{arctanh}(ax)^2}-\frac{\sinh(4\,\mathrm{arctanh}(ax))}{4\,\mathrm{arctanh}(ax)}+\mathrm{Chi}(4\,\mathrm{arctanh}(ax))+\frac{1}{16\,\mathrm{arctanh}(ax)^2}}{a^3}$	51

input
```
int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/16/arctanh(a*x)^2*cosh(4*arctanh(a*x))-1/4*sinh(4*arctanh(a*x))/
arctanh(a*x)+Chi(4*arctanh(a*x))+1/16/arctanh(a*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.24

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\frac{4\,a^2x^2 - \left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + \left(a^4x^4 - 2\,a^2x^2 + 1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,a}{a^2x^2+2\,a}\right.\right.}{2\left(a^7x^4 - 2\,a^5x^2 + a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
 + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2
*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*
(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^4 - 2*a^5*x^2 + a^3)*lo
g(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\int \frac{x^2}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)} \, dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

output
```
-Integral(x**2/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a*
*2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \int -\frac{x^2}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-2*(a*x^2 + (a^2*x^3 + x)*log(a*x + 1) - (a^2*x^3 + x)*log(-a*x + 1))/((a^
6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*lo
g(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) +
integrate(-2*(a^4*x^4 + 6*a^2*x^2 + 1)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 -
 a^2)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)
), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \int -\frac{x^2}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3} \, dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\int \frac{x^2}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^3} \, dx$$

input
```
int(-x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x^2/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \frac{-2atanh(ax)^2 \left(\int \frac{x^3}{atanh(ax)^2a^6x^6-3atanh(ax)^2a^4x^4+3atanh(ax)^2a^2x^2-atanh(ax)^2}dx\right)a^6x^4+4atanh(ax)^2 \left(\int \frac{}{atanh(ax)}\right.}{}$$

input
```
int(x^2/(-a^2*x^2+1)^3/atanh(a*x)^3,x)
```

output
```
( - 2*atanh(a*x)**2*int(x**3/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a*
*4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**6*x**4 + 4*atan
h(a*x)**2*int(x**3/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 +
3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**4*x**2 - 2*atanh(a*x)**2*
int(x**3/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*
x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**2 - 2*atanh(a*x)**2*int(x/(atanh(a*
x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 -
atanh(a*x)**2),x)*a**4*x**4 + 4*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**6*x*
*6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2
),x)*a**2*x**2 - 2*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**6*x**6 - 3*atanh(
a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x) - x**2)/
(2*atanh(a*x)**2*a*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.339 $\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\frac{x}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} - \frac{2}{a^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{3}{2a^2(1-a^2x^2) \operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{2a^2} + \frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{a^2}$$

output
```
-1/2*x/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2/a^2/(-a^2*x^2+1)^2/arctanh(a*x)+3
/2/a^2/(-a^2*x^2+1)/arctanh(a*x)+1/2*Shi(2*arctanh(a*x))/a^2+Shi(4*arctanh
(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}\,dx = -\frac{ax+\operatorname{arctanh}(ax)+3a^2x^2\operatorname{arctanh}(ax)-(-1+a^2x^2)^2\operatorname{arctanh}(ax)^2\operatorname{Shi}(2\operatorname{arctanh}(ax))-2(-1+a^2x}{2a^2(-1+a^2x^2)^2\operatorname{arctanh}(ax)^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*(a*x + ArcTanh[a*x] + 3*a^2*x^2*ArcTanh[a*x] - (-1 + a^2*x^2)^2*ArcTa
nh[a*x]^2*SinhIntegral[2*ArcTanh[a*x]] - 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2
*SinhIntegral[4*ArcTanh[a*x]])/(a^2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6594, 6528, 6590, 6528, 6596, 5971, 27, 2009, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \text{6594}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx}{2a}+\frac{3}{2}a\int \frac{x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx-\frac{x}{2a(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6528}$$

$$\frac{3}{2}a\int \frac{x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx+\frac{4a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{2a}-\frac{x}{2a(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \; 6590$$

$$\frac{3}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)^2}dx}{a^2}\right)+\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 6528$$

$$\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{2a\int \frac{x}{(1-a^2x^2)^2\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}}{a^2}\right)-\frac{x}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 6596$$

$$\frac{\frac{4\int \frac{ax}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{\frac{4\int \frac{ax}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{2\int \frac{ax}{(1-a^2x^2)\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x}}{a^2}\right.$$

$$\frac{x}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 5971$$

$$\frac{\frac{4\int \left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{\frac{4\int \left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{2\int \frac{\sinh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}d\text{arctanh}}{a}}{a}\right.$$

$$\frac{x}{2a(1-a^2x^2)^2\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+$$

$$\frac{3}{2}a\left(\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(\ldots}{a}}{a^2}\right.$$

$$\frac{x}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{\ldots}{a(1-\ldots}}{a^2}\right.$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 3042

$$\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}+\frac{\int-\frac{i\sin(2i\text{arcta}\ldots}{\text{arctan}\ldots}}{}}{a^2}\right.$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 26

$$\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}-\frac{i\int\frac{\sin(2i\text{arctan}}{\text{arctanh}}}{}}{a^2}\right.$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 3779

$$\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a}-\frac{1}{a(1-a^2x^2)\text{arctanh}(}}{a^2}\right.$$

$$\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}$$

input
```
Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + (3*a*(-((-(1/(a*(1 - a^2*x^2)*
ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a)/a^2) + (-(1/(a*(1 - a^2*x
^2)^2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4
*ArcTanh[a*x]]/8))/a)/a^2))/2 + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (
4*(SinhIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/(2
*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{\sinh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\operatorname{Shi}(4\operatorname{arctanh}(ax))-\frac{\sinh(2\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(2\operatorname{arctanh}(}{2}}{a^2}$
default	$\frac{-\frac{\sinh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)^2}-\frac{\cosh(4\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\operatorname{Shi}(4\operatorname{arctanh}(ax))-\frac{\sinh(2\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)^2}-\frac{\cosh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\operatorname{Shi}(2\operatorname{arctanh}(}{2}}{a^2}$

input
```
int(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/16*sinh(4*arctanh(a*x))/arctanh(a*x)^2-1/4/arctanh(a*x)*cosh(4*a
rctanh(a*x))+Shi(4*arctanh(a*x))-1/8/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1
/4/arctanh(a*x)*cosh(2*arctanh(a*x))+1/2*Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.56

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \frac{\left(2\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-2\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+(}{4}$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
1/4*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*
a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-
(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(
a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 - 8*a*x - 4*(3*a^2*x^2 + 1)*log(-(a
*x + 1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a*x + 1)/(a*x - 1))
^2)
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\int \frac{x}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)} \, dx$$

input
```
integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

output
```
-Integral(x/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*
x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-(2*a*x + (3*a^2*x^2 + 1)*log(a*x + 1) - (3*a^2*x^2 + 1)*log(-a*x + 1))/((
a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*
log(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2)
+ integrate(-2*(3*a^2*x^3 + 5*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*lo
g(a*x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{x}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\int \frac{x}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-x/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

output
```
-int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \text{Too large to display}$$

input
```
int(x/(-a^2*x^2+1)^3/atanh(a*x)^3,x)
```

output
```
( - 12*atanh(a*x)**2*int(x**3/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x*
*4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**8*x**4 + 24*atanh(a*x)**2*
int(x**3/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**
2*x**2 - atanh(a*x)),x)*a**6*x**2 - 12*atanh(a*x)**2*int(x**3/(atanh(a*x)*
a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),
x)*a**4 - 8*atanh(a*x)**2*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*
x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**6*x**4 + 16*atanh(a*x)**
2*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2
*x**2 - atanh(a*x)),x)*a**4*x**2 - 8*atanh(a*x)**2*int(x/(atanh(a*x)*a**6*
x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a*
*2 - 3*atanh(a*x)**2*int(1/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4
*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**5*x**4 + 6*atanh(
a*x)**2*int(1/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*ata
nh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**3*x**2 - 3*atanh(a*x)**2*int(1
/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a*
*2*x**2 - atanh(a*x)**2),x)*a - 6*atanh(a*x)*a**2*x**2 + atanh(a*x) - 2*a*
x)/(4*atanh(a*x)**2*a**2*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.340 $\int \frac{1}{(1-a^2x^2)^3 \mathbf{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\frac{1}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} - \frac{2x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{a} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{a}$$

output
```
-1/2/a/(-a^2*x^2+1)^2/arctanh(a*x)^2-2*x/(-a^2*x^2+1)^2/arctanh(a*x)+Chi(2
*arctanh(a*x))/a+Chi(4*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \frac{-1-4ax\operatorname{arctanh}(ax)+2(-1+a^2x^2)^2 \operatorname{arctanh}(ax)^2\operatorname{Chi}(2\operatorname{arctanh}(ax))+2(-1+a^2x^2)^2 \operatorname{arctanh}(ax)^2\mathrm{C}}{2a(-1+a^2x^2)^2 \operatorname{arctanh}(ax)^2}$$

input
```
Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
(-1 - 4*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[
2*ArcTanh[a*x]] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[4*ArcTanh
[a*x]])/(2*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6528, 6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6528}$$

$$2a\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$2a\left(\frac{\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2 \operatorname{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}+$$
$$2a\left(3a\int\frac{x^2}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx+\frac{\int\frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}\right)$$

$\downarrow$ 3793

$$2a\left(3a\int\frac{x^2}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx+\frac{\int\left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$2a\left(3a\int\frac{x^2}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a\left(1\right.}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 6596

$$2a\left(\frac{3\int\frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 5971

$$2a\left(\frac{3\int\left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log}{a^2}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$2a\left(\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctan}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 2*a*(-(x/(a*(1 - a^2*x^2)^2*Ar
cTanh[a*x])) + (3*(CoshIntegral[4*ArcTanh[a*x]]/8 - Log[ArcTanh[a*x]]/8))/
a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + CoshIntegral[4*ArcTanh[a*x]]/8 + (
3*Log[ArcTanh[a*x]])/8)/a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{3}{16 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{2 \operatorname{arctanh}(ax)}+\operatorname{Chi}(2 \operatorname{arctanh}(ax))-\frac{\cosh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)^2}-\frac{\sinh(4 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)} \cdots}{a}$
default	$\frac{-\frac{3}{16 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{2 \operatorname{arctanh}(ax)}+\operatorname{Chi}(2 \operatorname{arctanh}(ax))-\frac{\cosh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)^2}-\frac{\sinh(4 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)} \cdots}{a}$

input
```
int(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3/16/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2/arct
anh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh
(4*arctanh(a*x))-1/4*sinh(4*arctanh(a*x))/arctanh(a*x)+Chi(4*arctanh(a*x))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(65) = 130$.

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.49

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right) - \left((a^4x^4 - 2\,a^2x^2 + 1)\log\_integral\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + (a^4x^4 - 2\,a^2x^2 + 1)\log\_integra}{2\,(a^5x}$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(8*a*x*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_int
egral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2
+ 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4
- 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2
 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4)
/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\int \frac{1}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

output
```
-Integral(1/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*
x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-2*(2*a*x*log(a*x + 1) - 2*a*x*log(-a*x + 1) + 1)/((a^5*x^4 - 2*a^3*x^2 +
a)*log(a*x + 1)^2 - 2*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1)
 + (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^2) + integrate(-4*(3*a^2*x^2 +
1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6*x^6 - 3*a^4*
x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = -\int \frac{1}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-1/(atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}\,dx = \frac{-4atanh(ax)^2\left(\int \frac{x}{atanh(ax)^2a^6x^6-3atanh(ax)^2a^4x^4+3atanh(ax)^2a^2x^2-atanh(ax)^2}dx\right)a^6x^4+8atanh(ax)^2\left(\int \frac{}{atanh(ax)}\right.}{2at}$$

input
```
int(1/(-a^2*x^2+1)^3/atanh(a*x)^3,x)
```

output
```
( - 4*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*
x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**6*x**4 + 8*atanh(a
*x)**2*int(x/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atan
h(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**4*x**2 - 4*atanh(a*x)**2*int(x/
(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**
2*x**2 - atanh(a*x)**2),x)*a**2 - 1)/(2*atanh(a*x)**2*a*(a**4*x**4 - 2*a**
2*x**2 + 1))
```

3.341 $\int \frac{1}{x(1-a^2x^2)^3\mathbf{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}\,dx = & -\frac{1}{2ax\operatorname{arctanh}(ax)^2}-\frac{ax}{2(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}\\ & -\frac{ax}{2(1-a^2x^2)\operatorname{arctanh}(ax)^2}\\ & -\frac{2}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}\\ & +\frac{3}{2(1-a^2x^2)\operatorname{arctanh}(ax)}\\ & -\frac{1+a^2x^2}{2(1-a^2x^2)\operatorname{arctanh}(ax)}+\frac{3}{2}\operatorname{Shi}(2\operatorname{arctanh}(ax))\\ & +\operatorname{Shi}(4\operatorname{arctanh}(ax))-\frac{\operatorname{Int}\left(\frac{1}{x^2\operatorname{arctanh}(ax)^2},x\right)}{2a}\end{aligned}$$

output
```
-1/2/a/x/arctanh(a*x)^2-1/2*a*x/(-a^2*x^2+1)^2/arctanh(a*x)^2-1/2*a*x/(-a^
2*x^2+1)/arctanh(a*x)^2-2/(-a^2*x^2+1)^2/arctanh(a*x)+3/2/(-a^2*x^2+1)/arc
tanh(a*x)-1/2*(a^2*x^2+1)/(-a^2*x^2+1)/arctanh(a*x)+3/2*Shi(2*arctanh(a*x)
)+Shi(4*arctanh(a*x))-1/2*Defer(Int)(1/x^2/arctanh(a*x)^2,x)/a
```

Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = \int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}dx + \int \frac{1}{x(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}dx + a^2 \int \frac{x}{(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3}dx + \int \frac{1}{x(1-a^2x^2) \operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6552}$$

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^3 \text{arctanh}(ax)^3}dx+a^2\int \frac{x}{\left(1-a^2x^2\right)^2 \text{arctanh}(ax)^3}dx-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6468

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^3 \text{arctanh}(ax)^3}dx+a^2\int \frac{x}{\left(1-a^2x^2\right)^2 \text{arctanh}(ax)^3}dx-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6558

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^3 \text{arctanh}(ax)^3}dx+a^2\left(2\int \frac{x}{\left(1-a^2x^2\right)^2 \text{arctanh}(ax)}dx-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6594

$$a^2\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{2a}+\frac{3}{2}a\int \frac{x^2}{\left(1-a^2x^2\right)^3 \text{arctanh}(ax)^2}dx-\frac{x}{2a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}\right)+a^2\left(2\int \frac{x}{\left(1-a^2x^2\right)^2 \text{arctanh}(ax)}dx-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6528

$$a^2\left(\frac{3}{2}a\int \frac{x^2}{\left(1-a^2x^2\right)^3 \text{arctanh}(ax)^2}dx+\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a\left(1-a^2x^2\right)^2 \text{a}\right.$$

$$a^2\left(2\int \frac{x}{\left(1-a^2x^2\right)^2 \text{arctanh}(ax)}dx-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6590

$$a^2\left(\frac{3}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^2}dx}{a^2}\right)+\frac{4a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\operatorname{arc}}}{2a}\right.$$
$$a^2\left(2\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)-$$
$$\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 6528

$$a^2\left(2\int \frac{x}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}dx-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)+$$
$$a^2\left(\frac{4a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}}}{a^2}\right.\right.$$
$$\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 6596

$$a^2\left(\frac{2\int \frac{ax}{(1-a^2x^2)\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)+$$
$$a^2\left(\frac{\frac{4\int \frac{ax}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{\frac{4\int \frac{ax}{(1-a^2x^2)^2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{}{a(}}{a^2}\right.\right.$$
$$\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 5971

$$a^2\left(\frac{2\int \frac{\sinh(2\operatorname{arctanh}(ax))}{2\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{2a(1-a^2x^2)\operatorname{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2(1-a^2x^2)\operatorname{arctanh}(ax)}\right)+$$
$$a^2\left(\frac{\frac{4\int\left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\operatorname{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4\int\left(\frac{\sinh(2\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\operatorname{si}\right.}{}\right.\right.$$
$$\frac{\int \frac{1}{x^2\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \; 27$$

$$a^2\left(\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)+a^2\left(\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\text{si}\right.}{}\right.\right.$$

$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$a^2\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-}{a^2}\right.\right.$$

$$a^2\left(\frac{\int \frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-$$

$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 3042$$

$$a^2\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}+\frac{\int-\frac{i\sin(2i}{\text{arc}}}{}}{a^2}\right.\right.$$

$$a^2\left(\frac{\int -\frac{i\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-$$

$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \; 26$$

$$a^2\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}-\frac{i\int\frac{\sin(2i\text{ar}}{\text{arc}}}{}}{a^2}\right.\right.$$

$$a^2\left(-\frac{i\int\frac{\sin(2i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)-$$

$$\frac{\int\frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}-\frac{1}{2ax\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{\int\frac{1}{x^2\text{arctanh}(ax)^2}dx}{2a}+$$

$$a^2\left(\frac{\text{Shi}(2\text{arctanh}(ax))}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\text{arctanh}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\text{arctanh}(ax)}\right)+$$

$$a^2\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a}-\frac{1}{a(1-a^2x^2)\text{arcta}}}{a^2}\right.\right.$$

$$\frac{1}{2ax\text{arctanh}(ax)^2}$$

input
```
Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int\frac{1}{x\left(-a^2x^2+1\right)^3\operatorname{arctanh}\left(ax\right)^3}dx$$

input
```
int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

output
```
int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3}\,dx = \int -\frac{1}{\left(a^2x^2-1\right)^3 x \operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3}\,dx = -\int \frac{1}{a^6x^7 \operatorname{atanh}^3\left(ax\right) - 3a^4x^5 \operatorname{atanh}^3\left(ax\right) + 3a^2x^3 \operatorname{atanh}^3\left(ax\right) - x \operatorname{atanh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

output
```
-Integral(1/(a**6*x**7*atanh(a*x)**3 - 3*a**4*x**5*atanh(a*x)**3 + 3*a**2*
x**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 11.59

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3}\,dx=\int -\frac{1}{\left(a^2x^2-1\right)^3 x \operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
-(2*a*x + (5*a^2*x^2 - 1)*log(a*x + 1) - (5*a^2*x^2 - 1)*log(-a*x + 1))/((
a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(a*x + 1)^2 - 2*(a^6*x^6 - 2*a^4*x^4 + a
^2*x^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(-
a*x + 1)^2) + integrate(-2*(10*a^4*x^4 - 3*a^2*x^2 + 1)/((a^8*x^9 - 3*a^6*
x^7 + 3*a^4*x^5 - a^2*x^3)*log(a*x + 1) - (a^8*x^9 - 3*a^6*x^7 + 3*a^4*x^5
 - a^2*x^3)*log(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \operatorname{arctanh}(ax)^3}\,dx=\int -\frac{1}{\left(a^2x^2-1\right)^3 x \operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x (1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\int \frac{1}{x \operatorname{atanh}(a\,x)^3 \, (a^2\,x^2-1)^3} \, dx$$

input
```
int(-1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(x*atanh(a*x)^3*(a^2*x^2 - 1)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{1}{x (1-a^2x^2)^3 \operatorname{arctanh}(ax)^3} \, dx = -\left(\int \frac{1}{atanh(ax)^3 a^6x^7 - 3atanh(ax)^3 a^4x^5 + 3atanh(ax)^3 a^2x^3 - atanh(ax)^3 x} dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)^3/atanh(a*x)^3,x)
```

output
```
- int(1/(atanh(a*x)**3*a**6*x**7 - 3*atanh(a*x)**3*a**4*x**5 + 3*atanh(a*
x)**3*a**2*x**3 - atanh(a*x)**3*x),x)
```

3.342 $\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx = -\frac{1}{3a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} - \frac{2x}{3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} - \frac{8}{3a(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{2}{a(1-a^2x^2) \operatorname{arctanh}(ax)} + \frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{3a} + \frac{4\operatorname{Shi}(4\operatorname{arctanh}(ax))}{3a}$$

output
```
-1/3/a/(-a^2*x^2+1)^2/arctanh(a*x)^3-2/3*x/(-a^2*x^2+1)^2/arctanh(a*x)^2-8
/3/a/(-a^2*x^2+1)^2/arctanh(a*x)+2/a/(-a^2*x^2+1)/arctanh(a*x)+2/3*Shi(2*a
rctanh(a*x))/a+4/3*Shi(4*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx = -\frac{1+2ax\operatorname{arctanh}(ax)+2\operatorname{arctanh}(ax)^2+6a^2x^2\operatorname{arctanh}(ax)^2-2(-1+a^2x^2)^2\operatorname{arctanh}(ax)^3\operatorname{Shi}(2\operatorname{arcta}}{3a(-1+a^2x^2)^2\operatorname{arctanh}(ax)^3}$$

input
```
Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

output
```
-1/3*(1 + 2*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2
 - 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]] - 4*(-1
+ a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2
)^2*ArcTanh[a*x]^3)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.75, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6528, 6594, 6528, 6590, 6528, 6596, 5971, 27, 2009, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx$$

$$\Big\downarrow \text{6528}$$

$$\frac{4}{3}a\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^3}dx-\frac{1}{3a(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}$$

$$\Big\downarrow \text{6594}$$

$$\frac{4}{3}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{2a}+\frac{3}{2}a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}dx-\frac{x}{2a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^2}\right)-\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{4}{3}a\left(\frac{3}{2}a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)^2}dx+\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a\,(1-a^2x^2)^2}\right.\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)^2}dx}{a^2}\right)+\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{ar}}}{2a}\right.\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{4}{3}a\left(\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctan}}}{a^2}\right.\right.\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{4}{3}a\left(\frac{\frac{4\int \frac{ax}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{\frac{4\int \frac{ax}{(1-a^2x^2)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{}{a}}{a^2}\right.\right.\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{4}{3}a\left(\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\cdots\right.\right.$$

$$\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{4}{3}a\left(\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\cdots\right.\right.$$

$$\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\cdots}{a^2}\right.\right.$$

$$\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}+$$

$$\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}+\frac{\int-\frac{i\sin(2\cdots}{\cdots}}{\cdots}}{a^2}\right.\right.$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}+$$

$$\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{-\frac{1}{a(1-a^2x^2)\text{arctanh}(ax)}-\frac{i\int\frac{\sin(2i\cdots}{\cdots}}{\cdots}}{a^2}\right.\right.$$

$$\Big\downarrow 3779$$

$$\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\text{Shi}(2\text{arctanh}(ax))}{a}-\frac{1}{a(1-a^2x^2)\text{arct}}}{a^2}\right.\right.$$

$$\frac{1}{3a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^3}$$

input
```
Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

output
```
-1/3*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + (4*a*(-1/2*x/(a*(1 - a^2*x^2)^
2*ArcTanh[a*x]^2) + (3*a*(-((-(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhInt
egral[2*ArcTanh[a*x]]/a)/a^2) + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (
4*(SinhIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/a^
2))/2 + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTan
h[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/(2*a)))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{8 \operatorname{arctanh}(ax)^3}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)^3}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{3 \operatorname{arctanh}(ax)}+\frac{2 \operatorname{Shi}(2 \operatorname{arctanh}(ax))}{3}-\frac{\cosh(4 \operatorname{arctanh}(ax))}{24 \operatorname{arctanh}(ax)^3}}{a}$
default	$\frac{-\frac{1}{8 \operatorname{arctanh}(ax)^3}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)^3}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)^2}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{3 \operatorname{arctanh}(ax)}+\frac{2 \operatorname{Shi}(2 \operatorname{arctanh}(ax))}{3}-\frac{\cosh(4 \operatorname{arctanh}(ax))}{24 \operatorname{arctanh}(ax)^3}}{a}$

input
```
int(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/8/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6/arcta
nh(a*x)^2*sinh(2*arctanh(a*x))-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*S
hi(2*arctanh(a*x))-1/24/arctanh(a*x)^3*cosh(4*arctanh(a*x))-1/12*sinh(4*ar
ctanh(a*x))/arctanh(a*x)^2-1/3/arctanh(a*x)*cosh(4*arctanh(a*x))+4/3*Shi(4
*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(111) = 222$.

Time = 0.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.18

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4}\,dx$$
$$= \frac{\left(2\left(a^4x^4-2a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-2\left(a^4x^4-2a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+(\right.}{}$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="fricas")
```

output
```
1/3*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*
a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-
(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(
a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^3 - 8*a*x*log(-(a*x + 1)/(a*x - 1)) -
 4*(3*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 - 8)/((a^5*x^4 - 2*a^3*x^2
+ a)*log(-(a*x + 1)/(a*x - 1))^3)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx$$
$$= -\int \frac{1}{a^6x^6 \operatorname{atanh}^4(ax) - 3a^4x^4 \operatorname{atanh}^4(ax) + 3a^2x^2 \operatorname{atanh}^4(ax) - \operatorname{atanh}^4(ax)} dx$$

input
```
integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**4,x)
```

output
```
-Integral(1/(a**6*x**6*atanh(a*x)**4 - 3*a**4*x**4*atanh(a*x)**4 + 3*a**2*
x**2*atanh(a*x)**4 - atanh(a*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^4} dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="maxima")
```

output
```
-4/3*(2*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 1
)*log(-a*x + 1)^2 - 2*(a*x + (3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) +
 2)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^3 - 3*(a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*
log(-a*x + 1)^2 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^3) + integrate(-
8/3*(3*a^3*x^3 + 5*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1
) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} \, dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^4} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} \, dx = -\int \frac{1}{\operatorname{atanh}(a\,x)^4\,(a^2\,x^2-1)^3} \, dx$$

input
```
int(-1/(atanh(a*x)^4*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(atanh(a*x)^4*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4} dx$$
$$= \frac{-6atanh(ax)^3 \left(\int \frac{x^2}{atanh(ax)^2 a^6x^6-3atanh(ax)^2 a^4x^4+3atanh(ax)^2 a^2x^2-atanh(ax)^2} dx\right) a^7x^4 + 12atanh(ax)^3 \left(\int \frac{}{atanh(a}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^3/atanh(a*x)^4,x)
```

output
```
( - 6*atanh(a*x)**3*int(x**2/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a*
*4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**7*x**4 + 12*ata
nh(a*x)**3*int(x**2/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 +
 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**5*x**2 - 6*atanh(a*x)**3
*int(x**2/(atanh(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a
*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**3 - 8*atanh(a*x)**3*int(x/(atanh(a
*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*
x)),x)*a**6*x**4 + 16*atanh(a*x)**3*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(
a*x)*a**4*x**4 + 3*atanh(a*x)*a**2*x**2 - atanh(a*x)),x)*a**4*x**2 - 8*ata
nh(a*x)**3*int(x/(atanh(a*x)*a**6*x**6 - 3*atanh(a*x)*a**4*x**4 + 3*atanh(
a*x)*a**2*x**2 - atanh(a*x)),x)*a**2 - 2*atanh(a*x)**2 - 2*atanh(a*x)*a*x
- 1)/(3*atanh(a*x)**3*a*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.343 $\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 170

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5} dx = -\frac{1}{4a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} + \frac{1}{2a(1-a^2x^2) \operatorname{arctanh}(ax)^2} - \frac{8x}{3(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{x}{(1-a^2x^2) \operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{3a} + \frac{4\operatorname{Chi}(4\operatorname{arctanh}(ax))}{3a}$$

output
```
-1/4/a/(-a^2*x^2+1)^2/arctanh(a*x)^4-1/3*x/(-a^2*x^2+1)^2/arctanh(a*x)^3-2
/3/a/(-a^2*x^2+1)^2/arctanh(a*x)^2+1/2/a/(-a^2*x^2+1)/arctanh(a*x)^2-8/3*x
/(-a^2*x^2+1)^2/arctanh(a*x)+x/(-a^2*x^2+1)/arctanh(a*x)+1/3*Chi(2*arctanh
(a*x))/a+4/3*Chi(4*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5} \, dx =$$
$$-\frac{3+4ax\operatorname{arctanh}(ax)+2\operatorname{arctanh}(ax)^2+6a^2x^2\operatorname{arctanh}(ax)^2+20ax\operatorname{arctanh}(ax)^3+12a^3x^3\operatorname{arctanh}(ax}{12a\left(-1+a^2x\right.}$$

input
```
Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

output
```
-1/12*(3 + 4*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^
2 + 20*a*x*ArcTanh[a*x]^3 + 12*a^3*x^3*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2
*ArcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh
[a*x]^4*CoshIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. 2(170) = 340.

Time = 3.77 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.895, Rules used = {6528, 6594, 6528, 6590, 6528, 6594, 6530, 3042, 3793, 2009, 6596, 3042, 25, 3793, 2009, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5} \, dx$$

$$\Big\downarrow \text{6528}$$

$$a\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4}dx - \frac{1}{4a\left(1-a^2x^2\right)^2 \operatorname{arctanh}(ax)^4}$$

$$\Big\downarrow \text{6594}$$

$$a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^3}dx}{3a}+a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)^3}dx-\frac{x}{3a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^3}\right)-\frac{1}{4a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^4}$$

$\Big\downarrow$ 6528

$$a\left(a\int \frac{x^2}{(1-a^2x^2)^3\,\text{arctanh}(ax)^3}dx+\frac{2a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}}{3a}-\frac{x}{3a\,(1-a^2x^2)^2\,a\right.$$

$$\frac{1}{4a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^4}$$

$\Big\downarrow$ 6590

$$a\left(a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^3}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^2\text{arctanh}(ax)^3}dx}{a^2}\right)+\frac{2a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)^2\text{arc}}}{3a}\right.$$

$$\frac{1}{4a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^4}$$

$\Big\downarrow$ 6528

$$a\left(\frac{2a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}}{3a}+a\left(\frac{2a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx-\frac{1}{2a(1-a^2x^2)^2\text{arctan}}}{a^2}\right.\right.$$

$$\frac{1}{4a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^4}$$

$\Big\downarrow$ 6594

$$a\left(\frac{2a\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx}{a}+3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}\right)-\frac{1}{2a(1-a^2x^2)^2\text{arctanh}(ax}}{3a}\right.$$

$$\frac{1}{4a\,(1-a^2x^2)^2\,\text{arctanh}(ax)^4}$$

$\Big\downarrow$ 6530

$$a\left(\frac{2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arctanh}(ax)}\right)-\frac{}{2a(1-a^2x^2)^2}}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}+$$

$$a\left(\frac{-\frac{1}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}+2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^4}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{}{a(1-a^2x^2)^2}\right.}{3a}\right.$$

$$\Big\downarrow \text{3793}$$

$$a\left(\frac{2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\int \left(\frac{\cosh(2\text{arctanh}(ax))}{2\text{arctanh}(ax)}+\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{3}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}-\frac{}{a(1-a^2}\right.}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \text{2009}$$

$$a\left(\frac{2a\left(3a\int \frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)}dx+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^2\text{arct}}\right.}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \text{6596}$$

$$a\left(\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2}\right.}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}+$$

$$a\left(\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2}\right.}{3a}\right.$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}+$$

$$a\left(\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2}\right.}{3a}\right.$$

$$\Big\downarrow \text{3793}$$

$$a\left(\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2}\right.}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \text{2009}$$

$$a\left(\frac{2a\left(\frac{3\int \frac{a^2x^2}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}-\frac{}{a(1-a^2}\right.}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \ 5971$$

$$a\left(\frac{2a\left(\frac{3\int \left(\frac{\cosh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}-\frac{1}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(}{a^2}\right.}{3a}\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

$$\Big\downarrow \ 2009$$

$$a\left(a\left(\frac{2a\left(\frac{3\left(\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))-\frac{1}{8}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Chi}(4\text{arctanh}(ax))+\frac{3}{8}\log(\text{arctanh}(ax))}{a^2}\right.}{a^2}\right.\right.$$

$$\frac{1}{4a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^4}$$

input
```
Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

output
```
-1/4*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^4) + a*(-1/3*x/(a*(1 - a^2*x^2)^2*A
rcTanh[a*x]^3) + a*((-1/2*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 2*a*(-(x/
(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (3*(CoshIntegral[4*ArcTanh[a*x]]/8 - L
og[ArcTanh[a*x]]/8))/a^2 + (CoshIntegral[2*ArcTanh[a*x]]/2 + CoshIntegral[
4*ArcTanh[a*x]]/8 + (3*Log[ArcTanh[a*x]])/8)/a^2))/a^2 - (-1/2*1/(a*(1 - a
^2*x^2)*ArcTanh[a*x]^2) + a*(-(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + (CoshIn
tegral[2*ArcTanh[a*x]]/2 - Log[ArcTanh[a*x]]/2)/a^2 + (CoshIntegral[2*ArcT
anh[a*x]]/2 + Log[ArcTanh[a*x]]/2)/a^2))/a^2) + (-1/2*1/(a*(1 - a^2*x^2)^2
*ArcTanh[a*x]^2) + 2*a*(-(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (3*(CoshIn
tegral[4*ArcTanh[a*x]]/8 - Log[ArcTanh[a*x]]/8))/a^2 + (CoshIntegral[2*Arc
Tanh[a*x]]/2 + CoshIntegral[4*ArcTanh[a*x]]/8 + (3*Log[ArcTanh[a*x]])/8)/a
^2))/(3*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{3}{32 \operatorname{arctanh}(ax)^4}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)^4}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^3}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{3}}{a}$
default	$\frac{-\frac{3}{32 \operatorname{arctanh}(ax)^4}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)^4}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^3}-\frac{\cosh(2 \operatorname{arctanh}(ax))}{12 \operatorname{arctanh}(ax)^2}-\frac{\sinh(2 \operatorname{arctanh}(ax))}{6 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(2 \operatorname{arctanh}(ax))}{3}}{a}$

input
```
int(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3/32/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12*sin
h(2*arctanh(a*x))/arctanh(a*x)^3-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-
1/6/arctanh(a*x)*sinh(2*arctanh(a*x))+1/3*Chi(2*arctanh(a*x))-1/32/arctanh
(a*x)^4*cosh(4*arctanh(a*x))-1/24*sinh(4*arctanh(a*x))/arctanh(a*x)^3-1/12
/arctanh(a*x)^2*cosh(4*arctanh(a*x))-1/3*sinh(4*arctanh(a*x))/arctanh(a*x)
+4/3*Chi(4*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(151) = 302.

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.78

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}\, dx$$
$$= \frac{\left(4\,(a^4x^4-2\,a^2x^2+1)\log\_integral\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+4\,(a^4x^4-2\,a^2x^2+1)\log\_integral\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+(\right.}{}$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="fricas")
```

output
```
1/6*((4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*
a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-
(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(
a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x +
 1)/(a*x - 1))^3 - 16*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3*a^2*x^2 + 1)*lo
g(-(a*x + 1)/(a*x - 1))^2 - 24)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/
(a*x - 1))^4)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}\,dx$$
$$= -\int \frac{1}{a^6x^6 \operatorname{atanh}^5(ax) - 3a^4x^4 \operatorname{atanh}^5(ax) + 3a^2x^2 \operatorname{atanh}^5(ax) - \operatorname{atanh}^5(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**5,x)
```

output
```
-Integral(1/(a**6*x**6*atanh(a*x)**5 - 3*a**4*x**4*atanh(a*x)**5 + 3*a**2*
x**2*atanh(a*x)**5 - atanh(a*x)**5), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^5}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="maxima")
```

output
```
-2/3*((3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - (3*a^3*x^3 + 5*a*x)*log(-a*x +
1)^3 + 4*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 +
3*(3*a^3*x^3 + 5*a*x)*log(a*x + 1) + 1)*log(-a*x + 1)^2 - (3*(3*a^3*x^3 +
5*a*x)*log(a*x + 1)^2 + 4*a*x + 2*(3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x +
 1) + 6)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4 - 4*(a^5*x^4 - 2*a^3*x^
2 + a)*log(a*x + 1)^3*log(-a*x + 1) + 6*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x
+ 1)^2*log(-a*x + 1)^2 - 4*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x
 + 1)^3 + (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^4) + integrate(-2/3*(3*a
^4*x^4 + 24*a^2*x^2 + 5)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x +
1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^5}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}\,dx = -\int \frac{1}{\operatorname{atanh}(a\,x)^5\,(a^2\,x^2-1)^3}\,dx$$

input
```
int(-1/(atanh(a*x)^5*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(atanh(a*x)^5*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}\,dx$$
$$= \frac{-12atanh(ax)^4 \left(\int \frac{x^2}{atanh(ax)^3a^6x^6-3atanh(ax)^3a^4x^4+3atanh(ax)^3a^2x^2-atanh(ax)^3}dx\right)a^7x^4+24atanh(ax)^4\left(\int \frac{}{atanh(}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^3/atanh(a*x)^5,x)
```

output
```
( - 12*atanh(a*x)**4*int(x**2/(atanh(a*x)**3*a**6*x**6 - 3*atanh(a*x)**3*a
**4*x**4 + 3*atanh(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)*a**7*x**4 + 24*at
anh(a*x)**4*int(x**2/(atanh(a*x)**3*a**6*x**6 - 3*atanh(a*x)**3*a**4*x**4
+ 3*atanh(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)*a**5*x**2 - 12*atanh(a*x)*
*4*int(x**2/(atanh(a*x)**3*a**6*x**6 - 3*atanh(a*x)**3*a**4*x**4 + 3*atanh
(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)*a**3 - 8*atanh(a*x)**4*int(x/(atanh
(a*x)**2*a**6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2
 - atanh(a*x)**2),x)*a**6*x**4 + 16*atanh(a*x)**4*int(x/(atanh(a*x)**2*a**
6*x**6 - 3*atanh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x
)**2),x)*a**4*x**2 - 8*atanh(a*x)**4*int(x/(atanh(a*x)**2*a**6*x**6 - 3*at
anh(a*x)**2*a**4*x**4 + 3*atanh(a*x)**2*a**2*x**2 - atanh(a*x)**2),x)*a**2
 - 2*atanh(a*x)**2 - 4*atanh(a*x)*a*x - 3)/(12*atanh(a*x)**4*a*(a**4*x**4
- 2*a**2*x**2 + 1))
```

3.344 $\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 257

$$\begin{aligned}\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6}\,dx = &-\frac{1}{5a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5} \\ &-\frac{x}{5(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4} \\ &-\frac{4}{15a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^3} \\ &+\frac{1}{5a(1-a^2x^2) \operatorname{arctanh}(ax)^3} \\ &-\frac{8x}{15(1-a^2x^2)^2 \operatorname{arctanh}(ax)^2} \\ &+\frac{x}{5(1-a^2x^2) \operatorname{arctanh}(ax)^2} \\ &-\frac{32}{15a(1-a^2x^2)^2 \operatorname{arctanh}(ax)} \\ &+\frac{8}{5a(1-a^2x^2) \operatorname{arctanh}(ax)} \\ &+\frac{1+a^2x^2}{5a(1-a^2x^2) \operatorname{arctanh}(ax)} \\ &+\frac{2\operatorname{Shi}(2\operatorname{arctanh}(ax))}{15a}+\frac{16\operatorname{Shi}(4\operatorname{arctanh}(ax))}{15a}\end{aligned}$$

output
```
-1/5/a/(-a^2*x^2+1)^2/arctanh(a*x)^5-1/5*x/(-a^2*x^2+1)^2/arctanh(a*x)^4-4
/15/a/(-a^2*x^2+1)^2/arctanh(a*x)^3+1/5/a/(-a^2*x^2+1)/arctanh(a*x)^3-8/15
*x/(-a^2*x^2+1)^2/arctanh(a*x)^2+1/5*x/(-a^2*x^2+1)/arctanh(a*x)^2-32/15/a
/(-a^2*x^2+1)^2/arctanh(a*x)+8/5/a/(-a^2*x^2+1)/arctanh(a*x)+1/5*(a^2*x^2+
1)/a/(-a^2*x^2+1)/arctanh(a*x)+2/15*Shi(2*arctanh(a*x))/a+16/15*Shi(4*arct
anh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6} dx = -\frac{3+3ax\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)^2+3a^2x^2\operatorname{arctanh}(ax)^2+5ax\operatorname{arctanh}(ax)^3+3a^3x^3\operatorname{arctanh}(ax)^3 -}{}$$

input
```
Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

output
```
-1/15*(3 + 3*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2
+ 5*a*x*ArcTanh[a*x]^3 + 3*a^3*x^3*ArcTanh[a*x]^3 + 5*ArcTanh[a*x]^4 + 24*
a^2*x^2*ArcTanh[a*x]^4 + 3*a^4*x^4*ArcTanh[a*x]^4 - 2*(-1 + a^2*x^2)^2*Arc
Tanh[a*x]^5*SinhIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x
]^5*SinhIntegral[4*ArcTanh[a*x]])/(a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 621 vs. 2(257) = 514.

Time = 4.80 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.42, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {6528, 6594, 6528, 6590, 6528, 6558, 6594, 6528, 6590, 6528, 6596, 5971, 27, 2009, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6}\,dx$$

$$\Big\downarrow \text{6528}$$

$$\frac{4}{5}a\int \frac{x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^5}dx - \frac{1}{5a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6594}$$

$$\frac{4}{5}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^4}dx}{4a} + \frac{3}{4}a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4}dx - \frac{x}{4a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^4}\right) - \frac{1}{5a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6528}$$

$$\frac{4}{5}a\left(\frac{3}{4}a\int \frac{x^2}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^4}dx + \frac{\frac{4}{3}a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}dx - \frac{1}{3a(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}}{4a} - \frac{}{4a(1-a^2x^{}}\right. \frac{1}{5a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6590}$$

$$\frac{4}{5}a\left(\frac{3}{4}a\left(\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^4}dx}{a^2} - \frac{\int \frac{1}{(1-a^2x^2)^2\operatorname{arctanh}(ax)^4}dx}{a^2}\right) + \frac{\frac{4}{3}a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}dx - \frac{}{3a(1-a^2x^2)^{}}}{4a}\right. \frac{1}{5a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6528}$$

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}dx - \frac{1}{3a(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}}{4a} + \frac{3}{4}a\left(\frac{\frac{4}{3}a\int \frac{x}{(1-a^2x^2)^3\operatorname{arctanh}(ax)^3}dx - \frac{1}{3a(1-a^2x^2)^2\text{ar}}}{a^2}\right.\right. \frac{1}{5a(1-a^2x^2)^2 \operatorname{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6558}$$

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\int\frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^3}dx-\frac{1}{3a(1-a^2x^2)^2\text{arctanh}(ax)^3}}{4a}+\frac{3}{4}a\left(\frac{\frac{4}{3}a\int\frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)^3}dx-\frac{1}{3a(1-a^2x^2)^2\text{ar}}}{a^2}\right.\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6594}$$

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{\int\frac{1}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}dx}{2a}+\frac{3}{2}a\int\frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx-\frac{x}{2a(1-a^2x^2)^2\text{arctanh}(ax)^2}\right)-\frac{1}{3a(1-a^2x^2)^2\text{arcta}}}{4a}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6528}$$

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{3}{2}a\int\frac{x^2}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx+\frac{4a\int\frac{x}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{2a}-\frac{x}{2a(1-a^2x^2)^2\text{arctanh}(a}\right.}{4a}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6590}$$

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\int\frac{1}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}dx}{a^2}-\frac{\int\frac{1}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)^2}dx}{a^2}\right)+\frac{4a\int\frac{x}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx-\frac{1}{a\left(1-a^2x^2\right)^2\text{arct}}}{2a}\right.}{4a}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$$\Big\downarrow \text{6528}$$

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{4a\int\frac{x}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4a\int\frac{x}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}dx-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax}}{a^2}\right.\right.}{4}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$\downarrow$ 6596

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{\frac{4\int\frac{ax}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{\frac{4\int\frac{ax}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{}{a(}}{a^2}\right.\right.}{}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$\downarrow$ 5971

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\right.}{}\right.\right.}{}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$\downarrow$ 27

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{2a}+\frac{3}{2}a\left(\frac{4\int\left(\frac{\sinh(2\text{arctanh}(ax))}{4\text{arctanh}(ax)}+\ldots\right.}{\ldots}\right.\right.}{\ldots}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$\downarrow$ 2009

$$\frac{4}{5}a\left(\frac{\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{\frac{\int\frac{\sinh(2\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\ldots}{a^2}\ldots\right.\right.}{\ldots}\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}$$

$\downarrow$ 3042

$$-\frac{1}{5a\left(1-a^2x^2\right)^2\text{arctanh}(ax)^5}+$$

$$\frac{4}{5}a\left(\frac{-\frac{1}{3a(1-a^2x^2)^2\text{arctanh}(ax)^3}+\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\text{arctanh}(ax)}}{a^2}-\frac{-\ldots}{a\left(1-\ldots\right.}\ldots\right.\right.}{\ldots}\right.$$

$\downarrow$ 26

$$-\frac{1}{5a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^5}+$$

$$\frac{4}{5}a\left(\frac{-\frac{1}{3a(1-a^2x^2)^2\operatorname{arctanh}(ax)^3}+\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\operatorname{Shi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\operatorname{Shi}(4\operatorname{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}}{a^2}-\frac{-\frac{}{a(1\cdots}}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3779}$$

$$\frac{4}{5}a\left(\frac{3}{4}a\left(\frac{\frac{4}{3}a\left(\frac{3}{2}a\left(\frac{\frac{4\left(\frac{1}{4}\operatorname{Shi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\operatorname{Shi}(4\operatorname{arctanh}(ax))\right)}{a}-\frac{1}{a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)}}{a^2}-\frac{\frac{\operatorname{Shi}(2\operatorname{arctanh}(ax))}{a}-\frac{}{a\left(1-a^2x^2\right)\cdots}}{a^2}\right.\right.}{}\right.\right.$$

$$\frac{1}{5a\left(1-a^2x^2\right)^2\operatorname{arctanh}(ax)^5}$$

input
```
Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

output
```
-1/5*1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^5) + (4*a*(-1/4*x/(a*(1 - a^2*x^2)^
2*ArcTanh[a*x]^4) + (3*a*(-((-1/3*1/(a*(1 - a^2*x^2)*ArcTanh[a*x]^3) + (2*
a*(-1/2*x/(a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2
*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2))/3)/a^2) + (-1/3*1
/(a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + (4*a*(-1/2*x/(a*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2) + (3*a*(-((-(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[
2*ArcTanh[a*x]]/a)/a^2) + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(Sin
hIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/a^2))/2
+ (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTanh[a*x]
]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/(2*a)))/3)/a^2))/4 + (-1/3*1/(a*
(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + (4*a*(-1/2*x/(a*(1 - a^2*x^2)^2*ArcTanh[
a*x]^2) + (3*a*(-((-(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*Ar
cTanh[a*x]]/a)/a^2) + (-(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhInt
egral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*ArcTanh[a*x]]/8))/a)/a^2))/2 + (-
(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTanh[a*x]]/4
+ SinhIntegral[4*ArcTanh[a*x]]/8))/a)/(2*a)))/3)/(4*a)))/5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6558
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[x*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x
^2))), x] + (Simp[(1 + c^2*x^2)*((a + b*ArcTanh[c*x])^(p + 2)/(b^2*e*(p + 1
)*(p + 2)*(d + e*x^2))), x] + Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*
ArcTanh[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{3}{40\,\mathrm{arctanh}(ax)^5}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{10\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{20\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^3}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax)}{15\,\mathrm{arctanh}(ax)}$
default	$-\frac{3}{40\,\mathrm{arctanh}(ax)^5}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{10\,\mathrm{arctanh}(ax)^5}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{20\,\mathrm{arctanh}(ax)^4}-\frac{\cosh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^3}-\frac{\sinh(2\,\mathrm{arctanh}(ax))}{30\,\mathrm{arctanh}(ax)^2}-\frac{\cosh(2\,\mathrm{arctanh}(ax)}{15\,\mathrm{arctanh}(ax)}$

input
```
int(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3/40/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/ar
ctanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))
-1/30/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/15/arctanh(a*x)*cosh(2*arctanh
(a*x))+2/15*Shi(2*arctanh(a*x))-1/40/arctanh(a*x)^5*cosh(4*arctanh(a*x))-1
/40/arctanh(a*x)^4*sinh(4*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(4*arctanh
(a*x))-1/15*sinh(4*arctanh(a*x))/arctanh(a*x)^2-4/15/arctanh(a*x)*cosh(4*a
rctanh(a*x))+16/15*Shi(4*arctanh(a*x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.33

$$\int \frac{1}{(1-a^2x^2)^3\,\mathrm{arctanh}(ax)^6}\,dx = \frac{\left(8\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-8\,(a^4x^4-2\,a^2x^2+1)\,\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\cdots\right.}{}$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="fricas")
```

output
```
1/15*((8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2
*x^2 - 2*a*x + 1)) - 8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2
*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(
-(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^5 - 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*l
og(-(a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(a*x - 1
))^3 - 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 8*(3*a^2*x^2 + 1)*log(-(a*x + 1)
/(a*x - 1))^2 - 96)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^5
)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6}\,dx = -\int \frac{1}{a^6x^6 \operatorname{atanh}^6(ax) - 3a^4x^4 \operatorname{atanh}^6(ax) + 3a^2x^2 \operatorname{atanh}^6(ax) - \operatorname{atanh}^6(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**6,x)
```

output
```
-Integral(1/(a**6*x**6*atanh(a*x)**6 - 3*a**4*x**4*atanh(a*x)**6 + 3*a**2*
x**2*atanh(a*x)**6 - atanh(a*x)**6), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6}\,dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^6}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="maxima")
```

output
```
-2/15*((3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^4 + (3*a^4*x^4 + 24*a^2*x
^2 + 5)*log(-a*x + 1)^4 + 2*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - 2*(3*a^3*
x^3 + 5*a*x + 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1))*log(-a*x + 1)^3
 + 24*a*x*log(a*x + 1) + 4*(3*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(6*a^2*x^2 +
 3*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^2 + 3*(3*a^3*x^3 + 5*a*x)*log
(a*x + 1) + 2)*log(-a*x + 1)^2 - 2*(2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x
 + 1)^3 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^2 + 12*a*x + 4*(3*a^2*x^2 + 1
)*log(a*x + 1))*log(-a*x + 1) + 48)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1
)^5 - 5*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4*log(-a*x + 1) + 10*(a^5*x
^4 - 2*a^3*x^2 + a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^5*x^4 - 2*a^3*x
^2 + a)*log(a*x + 1)^2*log(-a*x + 1)^3 + 5*(a^5*x^4 - 2*a^3*x^2 + a)*log(a
*x + 1)*log(-a*x + 1)^4 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^5) + int
egrate(-8/15*(15*a^3*x^3 + 17*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*
log(a*x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6} \, dx = \int -\frac{1}{(a^2x^2-1)^3 \operatorname{artanh}(ax)^6} \, dx$$

input
```
integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6} \, dx = -\int \frac{1}{\operatorname{atanh}(a\,x)^6\,(a^2\,x^2-1)^3} \, dx$$

input
```
int(-1/(atanh(a*x)^6*(a^2*x^2 - 1)^3),x)
```

output
```
-int(1/(atanh(a*x)^6*(a^2*x^2 - 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)^6}\,dx$$
$$= \frac{-9atanh(ax)^5 \left(\int \frac{x^2}{atanh(ax)^4a^6x^6-3atanh(ax)^4a^4x^4+3atanh(ax)^4a^2x^2-atanh(ax)^4}dx\right)a^7x^4+18atanh(ax)^5\left(\int \frac{}{atanh(a}\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^3/atanh(a*x)^6,x)
```

output
```
( - 9*atanh(a*x)**5*int(x**2/(atanh(a*x)**4*a**6*x**6 - 3*atanh(a*x)**4*a*
*4*x**4 + 3*atanh(a*x)**4*a**2*x**2 - atanh(a*x)**4),x)*a**7*x**4 + 18*ata
nh(a*x)**5*int(x**2/(atanh(a*x)**4*a**6*x**6 - 3*atanh(a*x)**4*a**4*x**4 +
 3*atanh(a*x)**4*a**2*x**2 - atanh(a*x)**4),x)*a**5*x**2 - 9*atanh(a*x)**5
*int(x**2/(atanh(a*x)**4*a**6*x**6 - 3*atanh(a*x)**4*a**4*x**4 + 3*atanh(a
*x)**4*a**2*x**2 - atanh(a*x)**4),x)*a**3 - 4*atanh(a*x)**5*int(x/(atanh(a
*x)**3*a**6*x**6 - 3*atanh(a*x)**3*a**4*x**4 + 3*atanh(a*x)**3*a**2*x**2 -
 atanh(a*x)**3),x)*a**6*x**4 + 8*atanh(a*x)**5*int(x/(atanh(a*x)**3*a**6*x
**6 - 3*atanh(a*x)**3*a**4*x**4 + 3*atanh(a*x)**3*a**2*x**2 - atanh(a*x)**
3),x)*a**4*x**2 - 4*atanh(a*x)**5*int(x/(atanh(a*x)**3*a**6*x**6 - 3*atanh
(a*x)**3*a**4*x**4 + 3*atanh(a*x)**3*a**2*x**2 - atanh(a*x)**3),x)*a**2 -
atanh(a*x)**2 - 3*atanh(a*x)*a*x - 3)/(15*atanh(a*x)**5*a*(a**4*x**4 - 2*a
**2*x**2 + 1))
```

3.345 $\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx = -\frac{1}{36a(1-a^2x^2)^3}-\frac{5}{96a(1-a^2x^2)^2}-\frac{5}{32a(1-a^2x^2)}+\frac{x\operatorname{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5x\operatorname{arctanh}(ax)}{24(1-a^2x^2)^2}+\frac{5x\operatorname{arctanh}(ax)}{16(1-a^2x^2)}+\frac{5\operatorname{arctanh}(ax)^2}{32a}$$

output
```
-1/36/a/(-a^2*x^2+1)^3-5/96/a/(-a^2*x^2+1)^2-5/32/a/(-a^2*x^2+1)+1/6*x*x*arc
tanh(a*x)/(-a^2*x^2+1)^3+5/24*x*x*arctanh(a*x)/(-a^2*x^2+1)^2+5*x*x*arctanh(a*
x)/(-16*a^2*x^2+16)+5/32*arctanh(a*x)^2/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx = \frac{68-105a^2x^2+45a^4x^4-6ax(33-40a^2x^2+15a^4x^4)\operatorname{arctanh}(ax)+45(-1+a^2x^2)^3\operatorname{arctanh}(ax)^2}{288a(-1+a^2x^2)^3}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
```

output
```
(68 - 105*a^2*x^2 + 45*a^4*x^4 - 6*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcT
anh[a*x] + 45*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2)/(288*a*(-1 + a^2*x^2)^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6522, 6522, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^4}dx$$

$$\Big\downarrow \text{6522}$$

$$\frac{5}{6}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}-\frac{1}{36a(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6522}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}-\frac{1}{36a(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6518}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}-\frac{1}{36a(1-a^2x^2)^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right)-\frac{1}{36a(1-a^2x^2)^3}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
```

output
```
-1/36*1/(a*(1 - a^2*x^2)^3) + (x*ArcTanh[a*x])/(6*(1 - a^2*x^2)^3) + (5*(-
1/16*1/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-1
/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]
^2/(4*a)))/4))/6
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
parallelrisch	$-\frac{-45 \operatorname{arctanh}(ax)^2 a^6 x^6+198 a x \operatorname{arctanh}(ax)+90 \operatorname{arctanh}(ax) a^5 x^5-99 a^2 x^2-68 a^6 x^6+159 a^4 x^4+45 \operatorname{arctanh}(ax)^2-}{288(a^2x^2-1)^3 a}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(ax)}{48(ax-1)^3}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32(ax-1)}-\frac{5 \operatorname{arctanh}(ax)\ln(ax-1)}{32}-\frac{\operatorname{arctanh}(ax)}{48(ax+1)^3}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32(ax+1)}+\frac{5 \operatorname{arctanh}(}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(ax)}{48(ax-1)^3}+\frac{\operatorname{arctanh}(ax)}{16(ax-1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32(ax-1)}-\frac{5 \operatorname{arctanh}(ax)\ln(ax-1)}{32}-\frac{\operatorname{arctanh}(ax)}{48(ax+1)^3}-\frac{\operatorname{arctanh}(ax)}{16(ax+1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32(ax+1)}+\frac{5 \operatorname{arctanh}(}{}}{}$
risch	$\frac{5\ln(ax+1)^2}{128a}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-80a^3x^3+45x^2\ln(-ax+1)a^2+66ax-15\ln(-ax+}{192(a^2x^2-1)^3 a}$
parts	$-\frac{\operatorname{arctanh}(ax)}{48a(ax+1)^3}-\frac{\operatorname{arctanh}(ax)}{16a(ax+1)^2}-\frac{5 \operatorname{arctanh}(ax)}{32a(ax+1)}+\frac{5 \operatorname{arctanh}(ax)\ln(ax+1)}{32a}-\frac{\operatorname{arctanh}(ax)}{48a(ax-1)^3}+\frac{\operatorname{arctanh}(ax)}{16a(ax-1)^2}-$

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/288*(-45*arctanh(a*x)^2*a^6*x^6+198*a*x*arctanh(a*x)+90*arctanh(a*x)*a^
5*x^5-99*a^2*x^2-68*a^6*x^6+159*a^4*x^4+45*arctanh(a*x)^2-240*a^3*x^3*arct
anh(a*x)+135*a^4*x^4*arctanh(a*x)^2-135*a^2*x^2*arctanh(a*x)^2)/(a^2*x^2-1
)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx$$
$$=\frac{180\,a^4x^4-420\,a^2x^2+45\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)^2-12\,(15\,a^5x^5-40\,a^3x^3+33\,ax)\log}{1152\,(a^7x^6-3\,a^5x^4+3\,a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

output
```
1/1152*(180*a^4*x^4 - 420*a^2*x^2 + 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 -
1)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*log
(-(a*x + 1)/(a*x - 1)) + 272)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx = \int \frac{\operatorname{atanh}(ax)}{(ax-1)^4\,(ax+1)^4}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**4,x)
```

output
```
Integral(atanh(a*x)/((a*x - 1)**4*(a*x + 1)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(114) = 228.

Time = 0.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx$$
$$= -\frac{1}{96}\left(\frac{2\,(15\,a^4x^5 - 40\,a^2x^3 + 33\,x)}{a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1} - \frac{15\,\log(ax+1)}{a} + \frac{15\,\log(ax-1)}{a}\right)\operatorname{artanh}(ax)$$
$$+ \frac{(180\,a^4x^4 - 420\,a^2x^2 - 45\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log(ax+1)^2 + 90\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1}{1152\,(a^8x^6 - 3\,a^6x^4 + 3\,a^4x}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

output
```
-1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x) + 1/1152*(180*
a^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x +
 1)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1)
 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a/(a^8*x
^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^4}\,dx = \int \frac{\text{artanh}\,(ax)}{(a^2x^2-1)^4}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(a^2*x^2 - 1)^4, x)
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.54

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^4}\,dx = \frac{\frac{34}{3\,a}-\frac{35\,a\,x^2}{2}+\frac{15\,a^3\,x^4}{2}}{48\,a^6\,x^6-144\,a^4\,x^4+144\,a^2\,x^2-48} - \ln(1-a\,x)\left(\frac{5\,\ln(a\,x+1)}{64\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{2\,a^6\,x^6-6\,a^4\,x^4+6\,a^2\,x^2-2}\right) + \frac{5\ln(a\,x+1)^2}{128\,a}+\frac{5\ln(1-a\,x)^2}{128\,a} - \frac{\ln(a\,x+1)\left(\frac{11\,x}{32\,a}-\frac{5\,a\,x^3}{12}+\frac{5\,a^3\,x^5}{32}\right)}{3\,a\,x^2-\frac{1}{a}-3\,a^3\,x^4+a^5\,x^6}$$

input
```
int(atanh(a*x)/(a^2*x^2 - 1)^4,x)
```

output
```
(34/(3*a) - (35*a*x^2)/2 + (15*a^3*x^4)/2)/(144*a^2*x^2 - 144*a^4*x^4 + 48
*a^6*x^6 - 48) - log(1 - a*x)*((5*log(a*x + 1))/(64*a) - ((11*x)/16 - (5*a
^2*x^3)/6 + (5*a^4*x^5)/16)/(6*a^2*x^2 - 6*a^4*x^4 + 2*a^6*x^6 - 2)) + (5*
log(a*x + 1)^2)/(128*a) + (5*log(1 - a*x)^2)/(128*a) - (log(a*x + 1)*((11*
x)/(32*a) - (5*a*x^3)/12 + (5*a^3*x^5)/32))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a
^5*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}\,dx$$
$$= \frac{45atanh(ax)^2 a^6x^6 - 135atanh(ax)^2 a^4x^4 + 135atanh(ax)^2 a^2x^2 - 45atanh(ax)^2 - 90atanh(ax)\, a^5x^5 +}{288a\,(a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1)}$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^4,x)
```

output
```
(45*atanh(a*x)**2*a**6*x**6 - 135*atanh(a*x)**2*a**4*x**4 + 135*atanh(a*x)
**2*a**2*x**2 - 45*atanh(a*x)**2 - 90*atanh(a*x)*a**5*x**5 + 240*atanh(a*x
)*a**3*x**3 - 198*atanh(a*x)*a*x + 15*a**6*x**6 - 60*a**2*x**2 + 53)/(288*
a*(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1))
```

3.346 $\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 214

$$\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx = \frac{x}{108(1-a^2x^2)^3}+\frac{65x}{1728(1-a^2x^2)^2}+\frac{245x}{1152(1-a^2x^2)} +\frac{245\text{arctanh}(ax)}{1152a}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3} -\frac{5\text{arctanh}(ax)}{48a(1-a^2x^2)^2}-\frac{5\text{arctanh}(ax)}{16a(1-a^2x^2)}+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3} +\frac{5x\text{arctanh}(ax)^2}{24(1-a^2x^2)^2}+\frac{5x\text{arctanh}(ax)^2}{16(1-a^2x^2)}+\frac{5\text{arctanh}(ax)^3}{48a}$$

output
```
1/108*x/(-a^2*x^2+1)^3+65/1728*x/(-a^2*x^2+1)^2+245*x/(-1152*a^2*x^2+1152)
+245/1152*arctanh(a*x)/a-1/18*arctanh(a*x)/a/(-a^2*x^2+1)^3-5/48*arctanh(a
*x)/a/(-a^2*x^2+1)^2-5/16*arctanh(a*x)/a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)^2
/(-a^2*x^2+1)^3+5/24*x*arctanh(a*x)^2/(-a^2*x^2+1)^2+5*x*arctanh(a*x)^2/(-
16*a^2*x^2+16)+5/48*arctanh(a*x)^3/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx$$
$$= \frac{-\frac{64x}{(-1+a^2x^2)^3}+\frac{260x}{(-1+a^2x^2)^2}-\frac{1470x}{-1+a^2x^2}+\frac{48(68-105a^2x^2+45a^4x^4)\operatorname{arctanh}(ax)}{a(-1+a^2x^2)^3}-\frac{144x(33-40a^2x^2+15a^4x^4)\operatorname{arctanh}(ax)^2}{(-1+a^2x^2)^3}}{6912}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

output
```
((-64*x)/(-1 + a^2*x^2)^3 + (260*x)/(-1 + a^2*x^2)^2 - (1470*x)/(-1 + a^2*
x^2) + (48*(68 - 105*a^2*x^2 + 45*a^4*x^4)*ArcTanh[a*x])/(a*(-1 + a^2*x^2)
^3) - (144*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^2)/(-1 + a^2*x^2)
^3 + (720*ArcTanh[a*x]^3)/a - (735*Log[1 - a*x])/a + (735*Log[1 + a*x])/a)
/6912
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.56, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {6526, 215, 215, 215, 219, 6526, 215, 215, 219, 6518, 6556, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx$$
$$\Big\downarrow \; 6526$$
$$\frac{5}{6}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^3}dx+\frac{1}{18}\int \frac{1}{(1-a^2x^2)^4}dx+\frac{x\operatorname{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\operatorname{arctanh}(ax)}{18a(1-a^2x^2)^3}$$
$$\Big\downarrow \; 215$$

$$\frac{5}{6}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx+\frac{1}{18}\left(\frac{5}{6}\int \frac{1}{(1-a^2x^2)^3}dx+\frac{x}{6(1-a^2x^2)^3}\right)+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}$$

$\Big\downarrow$ 215

$$\frac{5}{6}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx+\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}$$

$\Big\downarrow$ 215

$$\frac{5}{6}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx+\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}$$

$\Big\downarrow$ 219

$$\frac{5}{6}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$\Big\downarrow$ 6526

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\int \frac{1}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$\Big\downarrow$ 215

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\int \frac{1}{(1-a^2x^2)^2}dx+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$$\Big\downarrow 215$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{1}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arct}}{8a(1}\right.$$
$$\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+$$
$$\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$$\Big\downarrow 219$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2}\right.\right.$$
$$\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+$$
$$\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$$\Big\downarrow 6518$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-a\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\right.\right.\right.$$
$$\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+$$
$$\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$$\Big\downarrow 6556$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\int \frac{1}{(1-a^2x^2)^2}dx}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctan}}{8a(1-}\right.$$
$$\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+$$
$$\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$$\Big\downarrow 215$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-a\left(\frac{\text{arctanh}(ax)}{2a^2(1-a^2x^2)}-\frac{\frac{1}{2}\int \frac{1}{1-a^2x^2}dx+\frac{x}{2(1-a^2x^2)}}{2a}\right)+\frac{x\text{arctanh}(ax)^2}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^3}{6a}\right)+\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}\right.$$

$$\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+$$

$$\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x\text{arctanh}(ax)^2}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)}{18a(1-a^2x^2)^3}+$$

$$\frac{1}{18}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{x}{6(1-a^2x^2)^3}\right)+$$

$$\frac{5}{6}\left(\frac{x\text{arctanh}(ax)^2}{4(1-a^2x^2)^2}-\frac{\text{arctanh}(ax)}{8a(1-a^2x^2)^2}+\frac{1}{8}\left(\frac{3}{4}\left(\frac{x}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)}{2a}\right)+\frac{x}{4(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\frac{x\text{arctanh}(a}{2(1-a^2x}\right.\right.$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

output
```
-1/18*ArcTanh[a*x]/(a*(1 - a^2*x^2)^3) + (x*ArcTanh[a*x]^2)/(6*(1 - a^2*x^
2)^3) + (x/(6*(1 - a^2*x^2)^3) + (5*(x/(4*(1 - a^2*x^2)^2) + (3*(x/(2*(1 -
 a^2*x^2)) + ArcTanh[a*x]/(2*a)))/4))/6)/18 + (5*(-1/8*ArcTanh[a*x]/(a*(1
- a^2*x^2)^2) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (x/(4*(1 - a^2*x^
2)^2) + (3*(x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a)))/4)/8 + (3*((x*ArcTa
nh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a) - a*(ArcTanh[a*x]/(2*a
^2*(1 - a^2*x^2)) - (x/(2*(1 - a^2*x^2)) + ArcTanh[a*x]/(2*a))/(2*a))))/4)
)/6
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 14.47 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.79

method	result
parallelrisch	$-\frac{2376 \operatorname{arctanh}(ax)^2 ax-2880 \operatorname{arctanh}(ax)^2 a^3x^3+1080 \operatorname{arctanh}(ax)^2 a^5x^5-1080 \operatorname{arctanh}(ax)^3 a^2x^2-735 \operatorname{arctanh}(ax}{}$
risch	$\frac{5\ln(ax+1)^3}{384a}-\frac{(15a^6x^6\ln(-ax+1)+30a^5x^5-45x^4\ln(-ax+1)a^4-80a^3x^3+45x^2\ln(-ax+1)a^2+66ax-15\ln(-ax+}{384(a^2x^2-1)^3a}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3456*(2376*arctanh(a*x)^2*a*x-2880*arctanh(a*x)^2*a^3*x^3+1080*arctanh(
a*x)^2*a^5*x^5-1080*arctanh(a*x)^3*a^2*x^2-735*arctanh(a*x)*a^6*x^6+897*a*
x+735*a^5*x^5-1600*a^3*x^3+360*arctanh(a*x)^3+1125*a^4*x^4*arctanh(a*x)+31
5*a^2*x^2*arctanh(a*x)-897*arctanh(a*x)+1080*arctanh(a*x)^3*a^4*x^4-360*x^
6*arctanh(a*x)^3*a^6)/(a^2*x^2-1)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx =$$
$$-\frac{1470\,a^5x^5 - 3200\,a^3x^3 - 90\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 36\left(15\,a^5x^5 - 40\,a^3x^3 + 33\,a\right.}{6912\left(a^7x^6 - 3\,a^5x^4 + 3\,a^3\right.}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

output
```
-1/6912*(1470*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1)*log(-(a*x + 1)/(a*x - 1))^3 + 36*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*
log(-(a*x + 1)/(a*x - 1))^2 + 1794*a*x - 3*(245*a^6*x^6 - 375*a^4*x^4 - 10
5*a^2*x^2 + 299)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{(ax-1)^4\,(ax+1)^4}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**4,x)
```

output
```
Integral(atanh(a*x)**2/((a*x - 1)**4*(a*x + 1)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(183) = 366$.

Time = 0.04 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.41

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx = -\frac{1}{96}\left(\frac{2\,(15\,a^4x^5-40\,a^2x^3+33\,x)}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}-\frac{15\,\log\left(ax+1\right)}{a}+\frac{15\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^2 - \frac{(1470\,a^5x^5-3200\,a^3x^3-90\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(ax+1\right)^3+270\,(a^6x^6-3\,a^4x^4+3\,a^2x^2 \cdots}{\cdots} + \frac{(180\,a^4x^4-420\,a^2x^2-45\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(ax+1\right)^2+90\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1 \cdots}{576\,(a^8x^6-3\,a^6x^4+\cdots}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

output
```
-1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/6912*(14
70*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a
*x + 1)^3 + 270*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a
*x - 1) + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a
*x - 15*(49*a^6*x^6 - 147*a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4
+ 3*a^2*x^2 - 1)*log(a*x - 1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*
x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 -
a^3) + 1/576*(180*a^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*
x^2 - 1)*log(a*x + 1)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
 + 1)*log(a*x - 1) - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)
^2 + 272)*a*arctanh(a*x)/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^4}\,dx$$

input

```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="giac")
```

output

```
integrate(arctanh(a*x)^2/(a^2*x^2 - 1)^4, x)
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx &= \ln(1-a\,x)^2\left(\frac{5\,\ln(a\,x+1)}{128\,a}-\frac{\frac{5\,a^4\,x^5}{16}-\frac{5\,a^2\,x^3}{6}+\frac{11\,x}{16}}{4\,a^6\,x^6-12\,a^4\,x^4+12\,a^2\,x^2-4}\right)\\&\quad-\frac{\frac{245\,a^4\,x^5}{8}-\frac{200\,a^2\,x^3}{3}+\frac{299\,x}{8}}{144\,a^6\,x^6-432\,a^4\,x^4+432\,a^2\,x^2-144}\\&\quad-\ln(1-a\,x)\left(\frac{5\ln(a\,x+1)^2}{128\,a}\right.\\&\qquad+\frac{\frac{37\,x}{2}-35\,a\,x^2+\frac{68}{3\,a}-\frac{82\,a^2\,x^3}{3}+15\,a^3\,x^4+\frac{23\,a^4\,x^5}{2}}{192\,a^6\,x^6-576\,a^4\,x^4+576\,a^2\,x^2-192}\\&\qquad-\frac{\frac{37\,x}{2}+35\,a\,x^2-\frac{68}{3\,a}-\frac{82\,a^2\,x^3}{3}-15\,a^3\,x^4+\frac{23\,a^4\,x^5}{2}}{192\,a^6\,x^6-576\,a^4\,x^4+576\,a^2\,x^2-192}\\&\qquad\left.-\frac{\ln(a\,x+1)\left(10\,a^4\,x^5-\frac{80\,a^2\,x^3}{3}+22\,x\right)}{64\,a^6\,x^6-192\,a^4\,x^4+192\,a^2\,x^2-64}\right)+\frac{5\ln(a\,x+1)^3}{384\,a}\\&\quad-\frac{5\ln(1-a\,x)^3}{384\,a}+\frac{\ln(a\,x+1)\left(\frac{17}{72\,a^2}-\frac{35\,x^2}{96}+\frac{5\,a^2\,x^4}{32}\right)}{3\,a\,x^2-\frac{1}{a}-3\,a^3\,x^4+a^5\,x^6}\\&\quad-\frac{\ln(a\,x+1)^2\left(\frac{11\,x}{64\,a}-\frac{5\,a\,x^3}{24}+\frac{5\,a^3\,x^5}{64}\right)}{3\,a\,x^2-\frac{1}{a}-3\,a^3\,x^4+a^5\,x^6}-\frac{\operatorname{atan}(a\,x\,1\mathrm{i})\,245\mathrm{i}}{1152\,a}\end{aligned}$$

input

```
int(atanh(a*x)^2/(a^2*x^2 - 1)^4,x)
```

output
```
log(1 - a*x)^2*((5*log(a*x + 1))/(128*a) - ((11*x)/16 - (5*a^2*x^3)/6 + (5
*a^4*x^5)/16)/(12*a^2*x^2 - 12*a^4*x^4 + 4*a^6*x^6 - 4)) - ((299*x)/8 - (2
00*a^2*x^3)/3 + (245*a^4*x^5)/8)/(432*a^2*x^2 - 432*a^4*x^4 + 144*a^6*x^6
- 144) - log(1 - a*x)*((5*log(a*x + 1)^2)/(128*a) + ((37*x)/2 - 35*a*x^2 +
 68/(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(576*a^2*x^2 - 5
76*a^4*x^4 + 192*a^6*x^6 - 192) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*a^
2*x^3)/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(576*a^2*x^2 - 576*a^4*x^4 + 192*a
^6*x^6 - 192) - (log(a*x + 1)*(22*x - (80*a^2*x^3)/3 + 10*a^4*x^5))/(192*a
^2*x^2 - 192*a^4*x^4 + 64*a^6*x^6 - 64)) + (5*log(a*x + 1)^3)/(384*a) - (5
*log(1 - a*x)^3)/(384*a) - (atan(a*x*1i)*245i)/(1152*a) + (log(a*x + 1)*(1
7/(72*a^2) - (35*x^2)/96 + (5*a^2*x^4)/32))/(3*a*x^2 - 1/a - 3*a^3*x^4 + a
^5*x^6) - (log(a*x + 1)^2*((11*x)/(64*a) - (5*a*x^3)/24 + (5*a^3*x^5)/64))
/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^4}\,dx$$
$$= \frac{720atanh(ax)^3\,a^6x^6 - 2160atanh(ax)^3\,a^4x^4 + 2160atanh(ax)^3\,a^2x^2 - 720atanh(ax)^3 - 2160atanh(ax)^2}{}$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^4,x)
```

output
```
(720*atanh(a*x)**3*a**6*x**6 - 2160*atanh(a*x)**3*a**4*x**4 + 2160*atanh(a
*x)**3*a**2*x**2 - 720*atanh(a*x)**3 - 2160*atanh(a*x)**2*a**5*x**5 + 5760
*atanh(a*x)**2*a**3*x**3 - 4752*atanh(a*x)**2*a*x + 720*atanh(a*x)*a**6*x*
*6 - 2880*atanh(a*x)*a**2*x**2 + 2544*atanh(a*x) - 375*log(a**2*x - a)*a**
6*x**6 + 1125*log(a**2*x - a)*a**4*x**4 - 1125*log(a**2*x - a)*a**2*x**2 +
 375*log(a**2*x - a) + 375*log(a**2*x + a)*a**6*x**6 - 1125*log(a**2*x + a
)*a**4*x**4 + 1125*log(a**2*x + a)*a**2*x**2 - 375*log(a**2*x + a) - 1470*
a**5*x**5 + 3200*a**3*x**3 - 1794*a*x)/(6912*a*(a**6*x**6 - 3*a**4*x**4 +
3*a**2*x**2 - 1))
```

3.347 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 291

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4} dx = -\frac{1}{216a(1-a^2x^2)^3} - \frac{65}{2304a(1-a^2x^2)^2} - \frac{245}{768a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)}{36(1-a^2x^2)^3} + \frac{65x\operatorname{arctanh}(ax)}{576(1-a^2x^2)^2} + \frac{245x\operatorname{arctanh}(ax)}{384(1-a^2x^2)} + \frac{245\operatorname{arctanh}(ax)^2}{768a} - \frac{\operatorname{arctanh}(ax)^2}{12a(1-a^2x^2)^3} - \frac{5\operatorname{arctanh}(ax)^2}{32a(1-a^2x^2)^2} - \frac{15\operatorname{arctanh}(ax)^2}{32a(1-a^2x^2)} + \frac{x\operatorname{arctanh}(ax)^3}{6(1-a^2x^2)^3} + \frac{5x\operatorname{arctanh}(ax)^3}{24(1-a^2x^2)^2} + \frac{5x\operatorname{arctanh}(ax)^3}{16(1-a^2x^2)} + \frac{5\operatorname{arctanh}(ax)^4}{64a}$$

output
```
-1/216/a/(-a^2*x^2+1)^3-65/2304/a/(-a^2*x^2+1)^2-245/768/a/(-a^2*x^2+1)+1/
36*x*arctanh(a*x)/(-a^2*x^2+1)^3+65/576*x*arctanh(a*x)/(-a^2*x^2+1)^2+245*
x*arctanh(a*x)/(-384*a^2*x^2+384)+245/768*arctanh(a*x)^2/a-1/12*arctanh(a*
x)^2/a/(-a^2*x^2+1)^3-5/32*arctanh(a*x)^2/a/(-a^2*x^2+1)^2-15/32*arctanh(a
*x)^2/a/(-a^2*x^2+1)+1/6*x*arctanh(a*x)^3/(-a^2*x^2+1)^3+5/24*x*arctanh(a*
x)^3/(-a^2*x^2+1)^2+5*x*arctanh(a*x)^3/(-16*a^2*x^2+16)+5/64*arctanh(a*x)^
4/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx = \frac{2432-4605a^2x^2+2205a^4x^4-6ax(897-1600a^2x^2+735a^4x^4)\operatorname{arctanh}(ax)+9(299-105a^2x^2-375a}{6912a\,(}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

output
```
(2432 - 4605*a^2*x^2 + 2205*a^4*x^4 - 6*a*x*(897 - 1600*a^2*x^2 + 735*a^4*
x^4)*ArcTanh[a*x] + 9*(299 - 105*a^2*x^2 - 375*a^4*x^4 + 245*a^6*x^6)*ArcT
anh[a*x]^2 - 144*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^3 + 540*(
-1 + a^2*x^2)^3*ArcTanh[a*x]^4)/(6912*a*(-1 + a^2*x^2)^3)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.66, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {6526, 6522, 6522, 6518, 241, 6526, 6518, 6522, 6518, 241, 6556, 6518, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{1}{6}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^4}dx+\frac{5}{6}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}dx+\frac{x\operatorname{arctanh}(ax)^3}{6\,(1-a^2x^2)^3}-\frac{\operatorname{arctanh}(ax)^2}{12a\,(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6522}$$

$$\frac{1}{6}\left(\frac{5}{6}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{x\operatorname{arctanh}(ax)}{6\,(1-a^2x^2)^3}-\frac{1}{36a\,(1-a^2x^2)^3}\right)+\frac{5}{6}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^3}dx+\frac{x\operatorname{arctanh}(ax)^3}{6\,(1-a^2x^2)^3}-\frac{\operatorname{arctanh}(ax)^2}{12a\,(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6522}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}-\frac{1}{36a(1-a^2x^2)^3}\right)+ \frac{5}{6}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6518}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{x\text{ar}}{6(}\right. \frac{5}{6}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{5}{6}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^3}dx+\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+ \frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$$\Big\downarrow \text{6526}$$

$$\frac{5}{6}\left(\frac{3}{8}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{3}{4}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}\right)+ \frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+ \frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$$\Big\downarrow \text{6518}$$

$$\frac{5}{6}\left(\frac{3}{8}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\right. \frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+ \frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$$\Big\downarrow \text{6522}$$

$$\frac{5}{6}\left(\frac{3}{8}\left(\frac{3}{4}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax}{2(1-a^2x^2}\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+$$

$$\frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$\downarrow$ 6518

$$\frac{5}{6}\left(\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)+\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}-\frac{1}{16a(1-a^2x^2)^2}\right)+\frac{3}{4}\left(\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+$$

$$\frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$\downarrow$ 241

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+$$

$$\frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$\downarrow$ 6556

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^2}dx}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^4}{8a}\right)+\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{ar}}{16a}\right.$$

$$\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+$$

$$\frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$\downarrow$ 6518

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\text{arctanh}(ax)^2}{2a^2(1-a^2x^2)}-\frac{-\frac{1}{2}a\int\frac{x}{(1-a^2x^2)^2}dx+\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}}{a}\right)+\frac{x\text{arctanh}(ax)^3}{2(1-a^2x^2)}+\frac{\text{arctan}}{8}\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+$$

$$\frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$$\Big\downarrow \text{241}$$

$$\frac{x\text{arctanh}(ax)^3}{6(1-a^2x^2)^3}-\frac{\text{arctanh}(ax)^2}{12a(1-a^2x^2)^3}+$$

$$\frac{1}{6}\left(\frac{x\text{arctanh}(ax)}{6(1-a^2x^2)^3}+\frac{5}{6}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right)-\frac{1}{16a(1-a^2x^2)^2}\right.\right.$$

$$\frac{5}{6}\left(\frac{x\text{arctanh}(ax)^3}{4(1-a^2x^2)^2}-\frac{3\text{arctanh}(ax)^2}{16a(1-a^2x^2)^2}+\frac{3}{8}\left(\frac{x\text{arctanh}(ax)}{4(1-a^2x^2)^2}+\frac{3}{4}\left(\frac{x\text{arctanh}(ax)}{2(1-a^2x^2)}-\frac{1}{4a(1-a^2x^2)}+\frac{\text{arctanh}(ax)^2}{4a}\right.\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

output
```
-1/12*ArcTanh[a*x]^2/(a*(1 - a^2*x^2)^3) + (x*ArcTanh[a*x]^3)/(6*(1 - a^2*
x^2)^3) + (-1/36*1/(a*(1 - a^2*x^2)^3) + (x*ArcTanh[a*x])/(6*(1 - a^2*x^2)
^3) + (5*(-1/16*1/(a*(1 - a^2*x^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^
2) + (3*(-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + A
rcTanh[a*x]^2/(4*a)))/4))/6)/6 + (5*((-3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^
2)^2) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (3*(-1/16*1/(a*(1 - a^2*x
^2)^2) + (x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (3*(-1/4*1/(a*(1 - a^2*x^2
)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a)))/4))/8 + (
3*((x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a) - (3*a*(Arc
Tanh[a*x]^2/(2*a^2*(1 - a^2*x^2)) - (-1/4*1/(a*(1 - a^2*x^2)) + (x*ArcTanh
[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^2/(4*a))/a))/2))/4))/6
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6518
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcTanh[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcTanh[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 7.47 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.73

method	result
parallelrisch	$-\frac{-2205 \operatorname{arctanh}(ax)^2 a^6 x^6+5382 a x \operatorname{arctanh}(ax)+4410 \operatorname{arctanh}(ax) a^5 x^5-5760 \operatorname{arctanh}(ax)^3 a^3 x^3+4752 \operatorname{arctanh}(a}{}$
risch	$\frac{5 \ln(ax+1)^4}{1024 a}-\frac{(15 a^6 x^6 \ln(-ax+1)+30 a^5 x^5-45 x^4 \ln(-ax+1) a^4-80 a^3 x^3+45 x^2 \ln(-ax+1) a^2+66 a x-15 \ln(-ax+}{768 (a^2 x^2-1)^3 a}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6912*(-2205*arctanh(a*x)^2*a^6*x^6+5382*a*x*arctanh(a*x)+4410*arctanh(a
*x)*a^5*x^5-5760*arctanh(a*x)^3*a^3*x^3+4752*arctanh(a*x)^3*a*x+1620*arcta
nh(a*x)^4*a^4*x^4-1620*arctanh(a*x)^4*a^2*x^2-2691*a^2*x^2-2432*a^6*x^6+50
91*a^4*x^4+540*arctanh(a*x)^4-2691*arctanh(a*x)^2-9600*a^3*x^3*arctanh(a*x
)+3375*a^4*x^4*arctanh(a*x)^2+945*a^2*x^2*arctanh(a*x)^2+2160*arctanh(a*x)
^3*a^5*x^5-540*a^6*arctanh(a*x)^4*x^6)/(a^2*x^2-1)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx = \frac{8820\, a^4x^4 + 135\,(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1)\log\left(-\frac{ax+1}{ax-1}\right)^4 - 18420\,a^2x^2 - 72\,(15\,a^5x^5 - 40\,a^3x^3 + 33\,a}{276}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

output
```
1/27648*(8820*a^4*x^4 + 135*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*
x + 1)/(a*x - 1))^4 - 18420*a^2*x^2 - 72*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x
)*log(-(a*x + 1)/(a*x - 1))^3 + 9*(245*a^6*x^6 - 375*a^4*x^4 - 105*a^2*x^2
 + 299)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(735*a^5*x^5 - 1600*a^3*x^3 + 897
*a*x)*log(-(a*x + 1)/(a*x - 1)) + 9728)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{(ax-1)^4(ax+1)^4}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**4,x)
```

output
```
Integral(atanh(a*x)**3/((a*x - 1)**4*(a*x + 1)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(251) = 502$.

Time = 0.06 (sec) , antiderivative size = 871, normalized size of antiderivative = 2.99

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

output
```
-1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^3 + 1/384*(180
*a^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
+ 1)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1
) - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a*arcta
nh(a*x)^2/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2) + 1/27648*((8820*a^4*x^4
 - 135*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^4 + 540*(a^6*x^6
 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3*log(a*x - 1) - 135*(a^6*x^6 -
 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^4 - 18420*a^2*x^2 - 45*(49*a^6*x^
6 - 147*a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*l
og(a*x - 1)^2 - 49)*log(a*x + 1)^2 - 2205*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1)*log(a*x - 1)^2 + 90*(6*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
 - 1)^3 + 49*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x +
 1) + 9728)*a^2/(a^10*x^6 - 3*a^8*x^4 + 3*a^6*x^2 - a^4) - 12*(1470*a^5*x^
5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3
 + 270*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x - 1) +
 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x - 15*(
49*a^6*x^6 - 147*a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x
^2 - 1)*log(a*x - 1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3*a
^2*x^2 - 1)*log(a*x - 1))*a*arctanh(a*x)/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x...
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4} dx = \int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^4} dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(a^2*x^2 - 1)^4, x)
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 1041, normalized size of antiderivative = 3.58

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx = \text{Too large to display}$$

input
```
int(atanh(a*x)^3/(a^2*x^2 - 1)^4,x)
```

output
```
(1216/(3*a) - (1535*a*x^2)/2 + (735*a^3*x^4)/2)/(3456*a^2*x^2 - 3456*a^4*x
^4 + 1152*a^6*x^6 - 1152) - log(1 - a*x)^3*((5*log(a*x + 1))/(256*a) - ((1
1*x)/16 - (5*a^2*x^3)/6 + (5*a^4*x^5)/16)/(24*a^2*x^2 - 24*a^4*x^4 + 8*a^6
*x^6 - 8)) + (5*log(a*x + 1)^4)/(1024*a) + (5*log(1 - a*x)^4)/(1024*a) + l
og(1 - a*x)^2*((15*log(a*x + 1)^2)/(512*a) + 245/(3072*a) + ((37*x)/2 - 35
*a*x^2 + 68/(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(768*a^2
*x^2 - 768*a^4*x^4 + 256*a^6*x^6 - 256) - ((37*x)/2 + 35*a*x^2 - 68/(3*a)
- (82*a^2*x^3)/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(768*a^2*x^2 - 768*a^4*x^4
 + 256*a^6*x^6 - 256) - (log(a*x + 1)*(66*x - 80*a^2*x^3 + 30*a^4*x^5))/(7
68*a^2*x^2 - 768*a^4*x^4 + 256*a^6*x^6 - 256)) + log(a*x + 1)^2*((17/(96*a
^2) - (35*x^2)/128 + (15*a^2*x^4)/128)/(3*a*x^2 - 1/a - 3*a^3*x^4 + a^5*x^
6) + 245/(3072*a)) + log(1 - a*x)*((36*x + 22*a*x^2 - 23/(2*a) - 67*a^2*x^
3 - (21*a^3*x^4)/2 + 31*a^4*x^5)/(2304*a^2*x^2 - 2304*a^4*x^4 + 768*a^6*x^
6 - 768) - (5*log(a*x + 1)^3)/(256*a) - log(a*x + 1)*(((37*x)/2 - 35*a*x^2
 + 68/(3*a) - (82*a^2*x^3)/3 + 15*a^3*x^4 + (23*a^4*x^5)/2)/(384*a^2*x^2 -
 384*a^4*x^4 + 128*a^6*x^6 - 128) - ((37*x)/2 + 35*a*x^2 - 68/(3*a) - (82*
a^2*x^3)/3 - 15*a^3*x^4 + (23*a^4*x^5)/2)/(384*a^2*x^2 - 384*a^4*x^4 + 128
*a^6*x^6 - 128) + (245*(3*a^2*x^2 - 3*a^4*x^4 + a^6*x^6 - 1))/(12*a*(384*a
^2*x^2 - 384*a^4*x^4 + 128*a^6*x^6 - 128))) + ((227*x)/2 + 173*a*x^2 - 593
/(6*a) - (599*a^2*x^3)/3 - (159*a^3*x^4)/2 + (183*a^4*x^5)/2)/(2304*a^2...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^4}\,dx$$
$$= \frac{540 atanh(ax)^4 a^6x^6 - 1620 atanh(ax)^4 a^4x^4 + 1620 atanh(ax)^4 a^2x^2 - 540 atanh(ax)^4 - 2160 atanh(ax)^3}{}$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^4,x)
```

output
```
(540*atanh(a*x)**4*a**6*x**6 - 1620*atanh(a*x)**4*a**4*x**4 + 1620*atanh(a
*x)**4*a**2*x**2 - 540*atanh(a*x)**4 - 2160*atanh(a*x)**3*a**5*x**5 + 5760
*atanh(a*x)**3*a**3*x**3 - 4752*atanh(a*x)**3*a*x + 2205*atanh(a*x)**2*a**
6*x**6 - 3375*atanh(a*x)**2*a**4*x**4 - 945*atanh(a*x)**2*a**2*x**2 + 2691
*atanh(a*x)**2 - 4410*atanh(a*x)*a**5*x**5 + 9600*atanh(a*x)*a**3*x**3 - 5
382*atanh(a*x)*a*x + 735*a**6*x**6 - 2400*a**2*x**2 + 1697)/(6912*a*(a**6*
x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1))
```

3.348 $\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 252

$$\begin{aligned}\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx &= \frac{5\operatorname{arctanh}(ax)^{3/2}}{24a} + \frac{3\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arctanh}(ax)}\right)}{512a} \\ &\quad + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arctanh}(ax)}\right)}{256a} + \frac{\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\sqrt{6}\sqrt{\operatorname{arctanh}(ax)}\right)}{768a} \\ &\quad - \frac{3\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arctanh}(ax)}\right)}{512a} - \frac{15\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arctanh}(ax)}\right)}{256a} \\ &\quad - \frac{\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\sqrt{6}\sqrt{\operatorname{arctanh}(ax)}\right)}{768a} \\ &\quad + \frac{15\sqrt{\operatorname{arctanh}(ax)}\sinh(2\operatorname{arctanh}(ax))}{64a} \\ &\quad + \frac{3\sqrt{\operatorname{arctanh}(ax)}\sinh(4\operatorname{arctanh}(ax))}{64a} \\ &\quad + \frac{\sqrt{\operatorname{arctanh}(ax)}\sinh(6\operatorname{arctanh}(ax))}{192a}\end{aligned}$$

output
```
5/24*arctanh(a*x)^(3/2)/a+3/512*Pi^(1/2)*erf(2*arctanh(a*x)^(1/2))/a+15/51
2*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*arctanh(a*x)^(1/2))/a+1/4608*6^(1/2)*Pi^(1/
2)*erf(6^(1/2)*arctanh(a*x)^(1/2))/a-3/512*Pi^(1/2)*erfi(2*arctanh(a*x)^(1
/2))/a-15/512*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*arctanh(a*x)^(1/2))/a-1/4608*6
^(1/2)*Pi^(1/2)*erfi(6^(1/2)*arctanh(a*x)^(1/2))/a+15/64*arctanh(a*x)^(1/2
)*sinh(2*arctanh(a*x))/a+3/64*arctanh(a*x)^(1/2)*sinh(4*arctanh(a*x))/a+1/
192*arctanh(a*x)^(1/2)*sinh(6*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \frac{-\frac{3168x\sqrt{\text{arctanh}(ax)}}{(-1+a^2x^2)^3} + \frac{3840a^2x^3\sqrt{\text{arctanh}(ax)}}{(-1+a^2x^2)^3} - \frac{1440a^4x^5\sqrt{\text{arctanh}(ax)}}{(-1+a^2x^2)^3} + \frac{960\text{arctanh}(ax)^{3/2}}{a} + \frac{\sqrt{6}\sqrt{\text{arctanh}(ax)}\Gamma}{a\sqrt{-\text{ar}}}}{}$$

input
```
Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

output
```
((-3168*x*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)^3 + (3840*a^2*x^3*Sqrt[ArcTan
h[a*x]])/(-1 + a^2*x^2)^3 - (1440*a^4*x^5*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^
2)^3 + (960*ArcTanh[a*x]^(3/2))/a + (Sqrt[6]*Sqrt[ArcTanh[a*x]]*Gamma[1/2,
 -6*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (27*Sqrt[ArcTanh[a*x]]*Gamma[
1/2, -4*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (135*Sqrt[2]*Sqrt[ArcTanh
[a*x]]*Gamma[1/2, -2*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) - (135*Sqrt[2]
*Gamma[1/2, 2*ArcTanh[a*x]])/a - (27*Gamma[1/2, 4*ArcTanh[a*x]])/a - (Sqrt
[6]*Gamma[1/2, 6*ArcTanh[a*x]])/a)/4608
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^4}dx$$

$$\Big\downarrow \text{6530}$$

$$\frac{\int \frac{\sqrt{\text{arctanh}(ax)}}{(1-a^2x^2)^3}d\text{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\text{arctanh}(ax)}\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^6d\text{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{15}{32}\sqrt{\text{arctanh}(ax)}\cosh(2\text{arctanh}(ax))+\frac{3}{16}\sqrt{\text{arctanh}(ax)}\cosh(4\text{arctanh}(ax))+\frac{1}{32}\sqrt{\text{arctanh}(ax)}\cosh(6\text{arcta}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{512}\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arctanh}(ax)}\right)+\frac{15}{256}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arctanh}(ax)}\right)+\frac{1}{768}\sqrt{\frac{\pi}{6}}\text{erf}\left(\sqrt{6}\sqrt{\text{arctanh}(ax)}\right)-\frac{3}{512}\sqrt{\pi}\text{erfi}\left(2\sqrt{\right.$$

input
```
Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

output
```
((5*ArcTanh[a*x]^(3/2))/24 + (3*Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/512 +
(15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/256 + (Sqrt[Pi/6]*Erf[Sqrt
[6]*Sqrt[ArcTanh[a*x]]])/768 - (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcTanh[a*x]]])/512
 - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/256 - (Sqrt[Pi/6]*Erfi
[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/768 + (15*Sqrt[ArcTanh[a*x]]*Sinh[2*ArcTanh[
a*x]])/64 + (3*Sqrt[ArcTanh[a*x]]*Sinh[4*ArcTanh[a*x]])/64 + (Sqrt[ArcTanh
[a*x]]*Sinh[6*ArcTanh[a*x]])/192)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(-a^2x^2+1)^4}dx$$

input
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

output
```
int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \int \frac{\sqrt{\operatorname{atanh}(ax)}}{(ax-1)^4\,(ax+1)^4}\,dx$$

input
```
integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**4,x)
```

output
```
Integral(sqrt(atanh(a*x))/((a*x - 1)**4*(a*x + 1)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \int \frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^4}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

Giac [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \int \frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^4}\,dx$$

input
```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \int \frac{\sqrt{\operatorname{atanh}(a\,x)}}{(a^2\,x^2-1)^4}\,dx$$

input
```
int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^4,x)
```

output
```
int(atanh(a*x)^(1/2)/(a^2*x^2 - 1)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(ax)}}{(1-a^2x^2)^4}\,dx = \int \frac{\sqrt{atanh(ax)}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}dx$$

input
```
int(atanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

output
```
int(sqrt(atanh(a*x))/(a**8*x**8 - 4*a**6*x**6 + 6*a**4*x**4 - 4*a**2*x**2
+ 1),x)
```

3.349 $\int \frac{x^8}{(1-a^2x^2)^4\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^8}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}\,dx = \operatorname{Int}\left(\frac{x^8}{(1-a^2x^2)^4\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 7.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}\,dx = \int \frac{x^8}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}dx$$

input
```
Int[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(-a^2x^2+1)^4 \operatorname{arctanh}(ax)}dx$$

input
```
int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

output
```
int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{x^8}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(x^8/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*
x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{x^8}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} \, dx$$

input
```
integrate(x**8/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**8/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{x^8}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^8}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^8}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

input
```
int(x^8/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^8/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{x^8}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$
$$= \int \frac{x^8}{atanh\left(ax\right)a^8x^8-4atanh\left(ax\right)a^6x^6+6atanh\left(ax\right)a^4x^4-4atanh\left(ax\right)a^2x^2+atanh\left(ax\right)}dx$$

input
```
int(x^8/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x**8/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**
4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.350 $\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \operatorname{Int}\left(\frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}, x\right)$$

output
```
Defer(Int)(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 16.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

input
```
Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$
$$\Big\downarrow \; 6651$$
$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}dx$$

input
```
Int[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(-a^2x^2+1)^4 \operatorname{arctanh}(ax)}dx$$

input
```
int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

output
```
int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^7}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(x^7/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*
x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^7}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**7/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**7/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^7}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^7}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 4.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^7}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

input
```
int(x^7/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^7/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{x^7}{\left(1-a^2x^2\right)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \int \frac{x^7}{atanh\left(ax\right)a^8x^8 - 4atanh\left(ax\right)a^6x^6 + 6atanh\left(ax\right)a^4x^4 - 4atanh\left(ax\right)a^2x^2 + atanh\left(ax\right)}dx$$

input
```
int(x^7/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x**7/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**
4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.351 $\int \frac{x^6}{(1-a^2x^2)^4\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x^6}{(1-a^2x^2)^4\,\text{arctanh}(ax)}\,dx = \frac{15\text{Chi}(2\text{arctanh}(ax))}{32a^7} - \frac{3\text{Chi}(4\text{arctanh}(ax))}{16a^7} + \frac{\text{Chi}(6\text{arctanh}(ax))}{32a^7} - \frac{5\log(\text{arctanh}(ax))}{16a^7}$$

output
```
15/32*Chi(2*arctanh(a*x))/a^7-3/16*Chi(4*arctanh(a*x))/a^7+1/32*Chi(6*arct
anh(a*x))/a^7-5/16*ln(arctanh(a*x))/a^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{(1-a^2x^2)^4\,\text{arctanh}(ax)}\,dx = \frac{15\text{Chi}(2\text{arctanh}(ax)) - 6\text{Chi}(4\text{arctanh}(ax)) + \text{Chi}(6\text{arctanh}(ax)) - 10\log(\text{arctanh}(ax))}{32a^7}$$

input
```
Integrate[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(15*CoshIntegral[2*ArcTanh[a*x]] - 6*CoshIntegral[4*ArcTanh[a*x]] + CoshIn
tegral[6*ArcTanh[a*x]] - 10*Log[ArcTanh[a*x]])/(32*a^7)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6596, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{\int \frac{a^6x^6}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\frac{\sin(i\operatorname{arctanh}(ax))^6}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^7}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sin(i\operatorname{arctanh}(ax))^6}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^7}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{\int \left(-\frac{15\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{3\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}-\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{5}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^7}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{15}{32}\text{Chi}(2\operatorname{arctanh}(ax))-\frac{3}{16}\text{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\text{Chi}(6\operatorname{arctanh}(ax))-\frac{5}{16}\log(\operatorname{arctanh}(ax))}{a^7}$$

input
```
Int[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
((15*CoshIntegral[2*ArcTanh[a*x]])/32 - (3*CoshIntegral[4*ArcTanh[a*x]])/1
6 + CoshIntegral[6*ArcTanh[a*x]]/32 - (5*Log[ArcTanh[a*x]])/16)/a^7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{5 \ln(\operatorname{arctanh}(ax))}{16}+\frac{15 \operatorname{Chi}(2 \operatorname{arctanh}(ax))}{32}-\frac{3 \operatorname{Chi}(4 \operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6 \operatorname{arctanh}(ax))}{32}}{a^7}$	40
default	$\frac{-\frac{5 \ln(\operatorname{arctanh}(ax))}{16}+\frac{15 \operatorname{Chi}(2 \operatorname{arctanh}(ax))}{32}-\frac{3 \operatorname{Chi}(4 \operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6 \operatorname{arctanh}(ax))}{32}}{a^7}$	40

input
```
int(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(-5/16*ln(arctanh(a*x))+15/32*Chi(2*arctanh(a*x))-3/16*Chi(4*arctanh
(a*x))+1/32*Chi(6*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 4.00

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = -\frac{20 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right) + 6\ \text{l}}{}$$

input
```
integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*
x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x
^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_int
egral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 15*log_integral(-(a*x + 1)/(a*x -
 1)) - 15*log_integral(-(a*x - 1)/(a*x + 1)))/a^7
```

Sympy [F]

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^6}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**6/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**6/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^6}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^6}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^6}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4}\,dx$$

input
```
int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^6/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x^6}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \frac{3\left(\int \frac{x^4}{atanh(ax)a^8x^8-4atanh(ax)a^6x^6+6atanh(ax)a^4x^4-4atanh(ax)a^2x^2+atanh(ax)}dx\right)a^5 - 3\left(\int \frac{}{atanh(ax)a^8x^8-4atanh(ax)a^6x^6-}\right.}{}$$

input
```
int(x^6/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
(3*int(x**4/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*
a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**5 - 3*int(x**2/(ata
nh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*at
anh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**3 + int(1/(atanh(a*x)*a**8*x**8 - 4
*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 +
atanh(a*x)),x)*a - log(atanh(a*x)))/a**7
```

3.352 $\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \frac{5\operatorname{Shi}(2\operatorname{arctanh}(ax))}{32a^6} - \frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{8a^6} + \frac{\operatorname{Shi}(6\operatorname{arctanh}(ax))}{32a^6}$$

output
```
5/32*Shi(2*arctanh(a*x))/a^6-1/8*Shi(4*arctanh(a*x))/a^6+1/32*Shi(6*arctan
h(a*x))/a^6
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \frac{5\operatorname{Shi}(2\operatorname{arctanh}(ax)) - 4\operatorname{Shi}(4\operatorname{arctanh}(ax)) + \operatorname{Shi}(6\operatorname{arctanh}(ax))}{32a^6}$$

input
```
Integrate[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(5*SinhIntegral[2*ArcTanh[a*x]] - 4*SinhIntegral[4*ArcTanh[a*x]] + SinhInt
egral[6*ArcTanh[a*x]])/(32*a^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{a^5x^5}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^6}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{5\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)} - \frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} + \frac{\sinh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{5}{32}\text{Shi}(2\operatorname{arctanh}(ax)) - \frac{1}{8}\text{Shi}(4\operatorname{arctanh}(ax)) + \frac{1}{32}\text{Shi}(6\operatorname{arctanh}(ax))}{a^6}$$

input
```
Int[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
((5*SinhIntegral[2*ArcTanh[a*x]])/32 - SinhIntegral[4*ArcTanh[a*x]]/8 + Si
nhIntegral[6*ArcTanh[a*x]]/32)/a^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arctanh}(ax))}{32}+\frac{5\,\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{32}}{a^6}$	33
default	$\frac{-\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arctanh}(ax))}{32}+\frac{5\,\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{32}}{a^6}$	33

input
```
int(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(-1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(a*x))+5/32*Shi(2*arctan
h(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.65

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right) - 4\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)}{64\,a^6}$$

input
```
integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3
+ 3*a^2*x^2 + 3*a*x + 1)) - 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2
- 2*a*x + 1)) + 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)
) + 5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x
+ 1)))/a^6
```

Sympy [F]

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{x^5}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} dx$$

input
```
integrate(x**5/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**5/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^5}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^5}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^5}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4}\,dx$$

input
```
int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^5/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x^5}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \int \frac{x^5}{atanh\,(ax)\,a^8x^8 - 4atanh\,(ax)\,a^6x^6 + 6atanh\,(ax)\,a^4x^4 - 4atanh\,(ax)\,a^2x^2 + atanh\,(ax)}dx$$

input
```
int(x^5/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x**5/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**
4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.353 $\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = -\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32a^5} - \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16a^5} + \frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32a^5} + \frac{\log(\operatorname{arctanh}(ax))}{16a^5}$$

output
```
-1/32*Chi(2*arctanh(a*x))/a^5-1/16*Chi(4*arctanh(a*x))/a^5+1/32*Chi(6*arct
anh(a*x))/a^5+1/16*ln(arctanh(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \frac{-\operatorname{Chi}(2\operatorname{arctanh}(ax)) - 2\operatorname{Chi}(4\operatorname{arctanh}(ax)) + \operatorname{Chi}(6\operatorname{arctanh}(ax)) + 2\log(\operatorname{arctanh}(ax))}{32a^5}$$

input
```
Integrate[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(-CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]] + CoshInte
gral[6*ArcTanh[a*x]] + 2*Log[ArcTanh[a*x]])/(32*a^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{a^4x^4}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^5}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(-\frac{\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}-\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{1}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{32}\text{Chi}(2\operatorname{arctanh}(ax))-\frac{1}{16}\text{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\text{Chi}(6\operatorname{arctanh}(ax))+\frac{1}{16}\log(\operatorname{arctanh}(ax))}{a^5}$$

input
```
Int[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(-1/32*CoshIntegral[2*ArcTanh[a*x]] - CoshIntegral[4*ArcTanh[a*x]]/16 + Co
shIntegral[6*ArcTanh[a*x]]/32 + Log[ArcTanh[a*x]]/16)/a^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{\ln(\operatorname{arctanh}(ax))}{16}-\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32}-\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32}}{a^5}$	40
default	$\frac{\frac{\ln(\operatorname{arctanh}(ax))}{16}-\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32}-\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32}}{a^5}$	40

input
```
int(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/16*ln(arctanh(a*x))-1/32*Chi(2*arctanh(a*x))-1/16*Chi(4*arctanh(a
*x))+1/32*Chi(6*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \frac{4\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) + \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) - 2\,\text{log\_}}{64}$$

input
```
integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integ
ral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2
 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - log_integral(-(a*x + 1)/(a*x - 1))
- log_integral(-(a*x - 1)/(a*x + 1)))/a^5
```

Sympy [F]

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**4/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^4}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4}\,dx$$

input
```
int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^4/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \int \frac{x^4}{atanh(ax)\,a^8x^8 - 4atanh(ax)\,a^6x^6 + 6atanh(ax)\,a^4x^4 - 4atanh(ax)\,a^2x^2 + atanh(ax)}dx$$

input
```
int(x^4/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x**4/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**
4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.354 $\int \frac{x^3}{(1-a^2x^2)^4\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{x^3}{(1-a^2x^2)^4\,\text{arctanh}(ax)}\,dx=-\frac{3\text{Shi}(2\text{arctanh}(ax))}{32a^4}+\frac{\text{Shi}(6\text{arctanh}(ax))}{32a^4}$$

output
```
-3/32*Shi(2*arctanh(a*x))/a^4+1/32*Shi(6*arctanh(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(1-a^2x^2)^4\,\text{arctanh}(ax)}\,dx=\frac{-3\text{Shi}(2\text{arctanh}(ax))+\text{Shi}(6\text{arctanh}(ax))}{32a^4}$$

input
```
Integrate[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(-3*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[6*ArcTanh[a*x]])/(32*a^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \ 6596$$

$$\frac{\int \frac{a^3x^3}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^4}$$

$$\Big\downarrow \ 5971$$

$$\frac{\int \left(\frac{\sinh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}-\frac{3\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{32}\text{Shi}(6\operatorname{arctanh}(ax))-\frac{3}{32}\text{Shi}(2\operatorname{arctanh}(ax))}{a^4}$$

input
```
Int[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
((-3*SinhIntegral[2*ArcTanh[a*x]])/32 + SinhIntegral[6*ArcTanh[a*x]]/32)/a
^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{\operatorname{Shi}(6 \operatorname{arctanh}(ax))}{32}-\frac{3 \operatorname{Shi}(2 \operatorname{arctanh}(ax))}{32}}{a^4}$	24
default	$\frac{\frac{\operatorname{Shi}(6 \operatorname{arctanh}(ax))}{32}-\frac{3 \operatorname{Shi}(2 \operatorname{arctanh}(ax))}{32}}{a^4}$	24

input
```
int(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/32*Shi(6*arctanh(a*x))-3/32*Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.69

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \frac{\text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right)-3\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+3\,\text{l}}{64\,a^4}$$

input
```
integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3
+ 3*a^2*x^2 + 3*a*x + 1)) - 3*log_integral(-(a*x + 1)/(a*x - 1)) + 3*log_i
ntegral(-(a*x - 1)/(a*x + 1)))/a^4
```

Sympy [F]

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**3/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^3}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{x^3}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{x^3}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4} dx$$

input
```
int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^3/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x^3}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{x^3}{atanh(ax)\,a^8x^8 - 4atanh(ax)\,a^6x^6 + 6atanh(ax)\,a^4x^4 - 4atanh(ax)\,a^2x^2 + atanh(ax)} dx$$

input
```
int(x^3/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x**3/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**
4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.355 $\int \frac{x^2}{(1-a^2x^2)^4 \mathbf{arctanh}(ax)} dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = -\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32a^3} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16a^3} + \frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32a^3} - \frac{\log(\operatorname{arctanh}(ax))}{16a^3}$$

output
```
-1/32*Chi(2*arctanh(a*x))/a^3+1/16*Chi(4*arctanh(a*x))/a^3+1/32*Chi(6*arct
anh(a*x))/a^3-1/16*ln(arctanh(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = -\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32a^3} + \frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16a^3} + \frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32a^3} - \frac{\log(\operatorname{arctanh}(ax))}{16a^3}$$

input
```
Integrate[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
-1/32*CoshIntegral[2*ArcTanh[a*x]]/a^3 + CoshIntegral[4*ArcTanh[a*x]]/(16*
a^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{a^2x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(-\frac{\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}-\frac{1}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))-\frac{1}{16}\log(\operatorname{arctanh}(ax))}{a^3}$$

input
```
Int[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(-1/32*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*ArcTanh[a*x]]/16 + Co
shIntegral[6*ArcTanh[a*x]]/32 - Log[ArcTanh[a*x]]/16)/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{\ln(\operatorname{arctanh}(ax))}{16}-\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32}+\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32}}{a^3}$	40
default	$\frac{-\frac{\ln(\operatorname{arctanh}(ax))}{16}-\frac{\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32}+\frac{\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32}}{a^3}$	40

input
```
int(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/16*ln(arctanh(a*x))-1/32*Chi(2*arctanh(a*x))+1/16*Chi(4*arctanh(
a*x))+1/32*Chi(6*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx =$$
$$-\frac{4\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right) - 2\,\text{lo}}{6}$$

input
```
integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x
^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^
3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_inte
gral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^
2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + log_integral(-(a*x + 1)/(a*x - 1))
 + log_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

Sympy [F]

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(ax-1)^4\,(ax+1)^4\,\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x**2/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4}\,dx$$

input
```
int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x^2/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \int \frac{x^2}{atanh\left(ax\right)a^8x^8-4atanh\left(ax\right)a^6x^6+6atanh\left(ax\right)a^4x^4-4atanh\left(ax\right)a^2x^2+atanh\left(ax\right)}dx$$

input
```
int(x^2/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x**2/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**
4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.356 $\int \frac{x}{(1-a^2x^2)^4\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x}{(1-a^2x^2)^4\,\text{arctanh}(ax)}\,dx = \frac{5\text{Shi}(2\text{arctanh}(ax))}{32a^2} + \frac{\text{Shi}(4\text{arctanh}(ax))}{8a^2} + \frac{\text{Shi}(6\text{arctanh}(ax))}{32a^2}$$

output
```
5/32*Shi(2*arctanh(a*x))/a^2+1/8*Shi(4*arctanh(a*x))/a^2+1/32*Shi(6*arctan
h(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-a^2x^2)^4\,\text{arctanh}(ax)}\,dx = \frac{5\text{Shi}(2\text{arctanh}(ax))}{32a^2} + \frac{\text{Shi}(4\text{arctanh}(ax))}{8a^2} + \frac{\text{Shi}(6\text{arctanh}(ax))}{32a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(
8*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \; 6596$$

$$\frac{\int \frac{ax}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \; 5971$$

$$\frac{\int \left(\frac{5\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)} + \frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)} + \frac{\sinh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{5}{32}\operatorname{Shi}(2\operatorname{arctanh}(ax)) + \frac{1}{8}\operatorname{Shi}(4\operatorname{arctanh}(ax)) + \frac{1}{32}\operatorname{Shi}(6\operatorname{arctanh}(ax))}{a^2}$$

input
```
Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
((5*SinhIntegral[2*ArcTanh[a*x]])/32 + SinhIntegral[4*ArcTanh[a*x]]/8 + Si
nhIntegral[6*ArcTanh[a*x]]/32)/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arctanh}(ax))}{32}+\frac{5\,\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{32}}{a^2}$	33
default	$\frac{\frac{\mathrm{Shi}(4\,\mathrm{arctanh}(ax))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arctanh}(ax))}{32}+\frac{5\,\mathrm{Shi}(2\,\mathrm{arctanh}(ax))}{32}}{a^2}$	33

input
```
int(x/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(a*x))+5/32*Shi(2*arctanh
(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.65

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right) + 4\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)}{64\,a^2}$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3
+ 3*a^2*x^2 + 3*a*x + 1)) + 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2
- 2*a*x + 1)) - 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)
) + 5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x
+ 1)))/a^2
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{x}{\operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4}\,dx$$

input
```
int(x/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(x/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \int \frac{x}{atanh\,(ax)\,a^8x^8 - 4atanh\,(ax)\,a^6x^6 + 6atanh\,(ax)\,a^4x^4 - 4atanh\,(ax)\,a^2x^2 + atanh\,(ax)}dx$$

input
```
int(x/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**4*x
**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.357 $\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \frac{15\text{Chi}(2\operatorname{arctanh}(ax))}{32a} + \frac{3\text{Chi}(4\operatorname{arctanh}(ax))}{16a} + \frac{\text{Chi}(6\operatorname{arctanh}(ax))}{32a} + \frac{5\log(\operatorname{arctanh}(ax))}{16a}$$

output
```
15/32*Chi(2*arctanh(a*x))/a+3/16*Chi(4*arctanh(a*x))/a+1/32*Chi(6*arctanh(
a*x))/a+5/16*ln(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \frac{15\text{Chi}(2\operatorname{arctanh}(ax)) + 6\text{Chi}(4\operatorname{arctanh}(ax)) + \text{Chi}(6\operatorname{arctanh}(ax)) + 10\log(\operatorname{arctanh}(ax))}{32a}$$

input
```
Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
(15*CoshIntegral[2*ArcTanh[a*x]] + 6*CoshIntegral[4*ArcTanh[a*x]] + CoshIn
tegral[6*ArcTanh[a*x]] + 10*Log[ArcTanh[a*x]])/(32*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx$$

$$\Big\downarrow \text{ 6530}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^6}{\operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\int \left(\frac{15\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)} + \frac{3\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)} + \frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)} + \frac{5}{16\operatorname{arctanh}(ax)}\right) d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{15}{32}\text{Chi}(2\operatorname{arctanh}(ax)) + \frac{3}{16}\text{Chi}(4\operatorname{arctanh}(ax)) + \frac{1}{32}\text{Chi}(6\operatorname{arctanh}(ax)) + \frac{5}{16}\log(\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
((15*CoshIntegral[2*ArcTanh[a*x]])/32 + (3*CoshIntegral[4*ArcTanh[a*x]])/1
6 + CoshIntegral[6*ArcTanh[a*x]]/32 + (5*Log[ArcTanh[a*x]])/16)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{5\ln(\operatorname{arctanh}(ax))}{16}+\frac{15\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32}+\frac{3\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32}}{a}$	40
default	$\frac{\frac{5\ln(\operatorname{arctanh}(ax))}{16}+\frac{15\operatorname{Chi}(2\operatorname{arctanh}(ax))}{32}+\frac{3\operatorname{Chi}(4\operatorname{arctanh}(ax))}{16}+\frac{\operatorname{Chi}(6\operatorname{arctanh}(ax))}{32}}{a}$	40

input
```
int(1/(-a^2*x^2+1)^4/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(5/16*ln(arctanh(a*x))+15/32*Chi(2*arctanh(a*x))+3/16*Chi(4*arctanh(a*
x))+1/32*Chi(6*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \frac{20 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) + \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) + 6 \log}{}$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x
^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^
3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_inte
gral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^
2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 15*log_integral(-(a*x + 1)/(a*x -
1)) + 15*log_integral(-(a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(ax-1)^4\,(ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)\,(a^2\,x^2-1)^4}\,dx$$

input
```
int(1/(atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(1/(atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$
$$= \int \frac{1}{atanh\,(ax)\,a^8x^8 - 4atanh\,(ax)\,a^6x^6 + 6atanh\,(ax)\,a^4x^4 - 4atanh\,(ax)\,a^2x^2 + atanh\,(ax)}dx$$

input
```
int(1/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(1/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)*a**4*x
**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)
```

3.358 $\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \operatorname{Int}\left(\frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)}, x\right)$$

output
```
Defer(Int)(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx$$

input
```
Int[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(-a^2x^2+1)^4 \operatorname{arctanh}(ax)} dx$$

input
```
int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{1}{(a^2x^2-1)^4 x \operatorname{artanh}(ax)} dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^8*x^9 - 4*a^6*x^7 + 6*a^4*x^5 - 4*a^2*x^3 + x)*arctanh(a*x)
), x)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{1}{x(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(1/(x*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx = \int \frac{1}{(a^2x^2-1)^4 x \operatorname{artanh}(ax)} dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\left(a^2x^2-1\right)^4x\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{x\operatorname{atanh}\left(a\,x\right)\,\left(a^2\,x^2-1\right)^4}\,dx$$

input
```
int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(1/(x*atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx$$
$$= \int \frac{1}{atanh(ax)\, a^8x^9 - 4atanh(ax)\, a^6x^7 + 6atanh(ax)\, a^4x^5 - 4atanh(ax)\, a^2x^3 + atanh(ax)\, x} dx$$

input
```
int(1/x/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(1/(atanh(a*x)*a**8*x**9 - 4*atanh(a*x)*a**6*x**7 + 6*atanh(a*x)*a**4*x
**5 - 4*atanh(a*x)*a**2*x**3 + atanh(a*x)*x),x)
```

3.359 $\int \frac{1}{x^2(1-a^2x^2)^4\textbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{x^2\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{6651}$$

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)}dx$$

input
```
Int[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (-a^2x^2+1)^4 \operatorname{arctanh}(ax)}dx$$

input
```
int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

output
```
int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{(a^2x^2-1)^4 x^2 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^8*x^10 - 4*a^6*x^8 + 6*a^4*x^6 - 4*a^2*x^4 + x^2)*arctanh(a
*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{x^2 (ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} \, dx$$

input
```
integrate(1/x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

output
```
Integral(1/(x**2*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{(a^2x^2-1)^4 x^2 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{(a^2x^2-1)^4 x^2 \operatorname{artanh}(ax)} \, dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atanh}(a\,x)\,\left(a^2\,x^2-1\right)^4} \, dx$$

input
```
int(1/(x^2*atanh(a*x)*(a^2*x^2 - 1)^4),x)
```

output
```
int(1/(x^2*atanh(a*x)*(a^2*x^2 - 1)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (1-a^2x^2)^4 \operatorname{arctanh}(ax)}\,dx$$
$$= \int \frac{1}{atanh(ax)\, a^8x^{10} - 4atanh(ax)\, a^6x^8 + 6atanh(ax)\, a^4x^6 - 4atanh(ax)\, a^2x^4 + atanh(ax)\, x^2}dx$$

input
```
int(1/x^2/(-a^2*x^2+1)^4/atanh(a*x),x)
```

output
```
int(1/(atanh(a*x)*a**8*x**10 - 4*atanh(a*x)*a**6*x**8 + 6*atanh(a*x)*a**4*
x**6 - 4*atanh(a*x)*a**2*x**4 + atanh(a*x)*x**2),x)
```

3.360 $\int \frac{x}{(1-a^2x^2)^4\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{x}{(1-a^2x^2)^4\,\text{arctanh}(ax)^2}\,dx = -\frac{x}{a(1-a^2x^2)^3\,\text{arctanh}(ax)} + \frac{5\text{Chi}(2\text{arctanh}(ax))}{16a^2} + \frac{\text{Chi}(4\text{arctanh}(ax))}{2a^2} + \frac{3\text{Chi}(6\text{arctanh}(ax))}{16a^2}$$

output
```
-x/a/(-a^2*x^2+1)^3/arctanh(a*x)+5/16*Chi(2*arctanh(a*x))/a^2+1/2*Chi(4*ar
ctanh(a*x))/a^2+3/16*Chi(6*arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{x}{(1-a^2x^2)^4\,\text{arctanh}(ax)^2}\,dx = \frac{\frac{16ax}{(-1+a^2x^2)^3\text{arctanh}(ax)} + 5\text{Chi}(2\text{arctanh}(ax)) + 8\text{Chi}(4\text{arctanh}(ax)) + 3\text{Chi}(6\text{arctanh}(ax))}{16a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

output
```
((16*a*x)/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + 5*CoshIntegral[2*ArcTanh[a*x]]
 + 8*CoshIntegral[4*ArcTanh[a*x]] + 3*CoshIntegral[6*ArcTanh[a*x]])/(16*a^
2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.79, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^4\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6594}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx}{a}+5a\int \frac{x^2}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6530}$$

$$5a\int \frac{x^2}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$5a\int \frac{x^2}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^6}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5a\int \frac{x^2}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx+\int \left(\frac{15\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{3\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{5}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \; 2009$$

$$\frac{5a\int \frac{x^2}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx+\frac{15}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{3}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))+\frac{5}{16}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \; 6596$$

$$\frac{\frac{5\int \frac{a^2x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{15}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{3}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))+\frac{5}{16}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \; 5971$$

$$\frac{\frac{5\int \left(-\frac{\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}-\frac{1}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}+\frac{15}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{3}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))+\frac{5}{16}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{5\left(-\frac{1}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{1}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))-\frac{1}{16}\log(\operatorname{arctanh}(ax))\right)}{a^2}+\frac{15}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{3}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))+\frac{5}{16}\log(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}$$

input
```
Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

output
```
-(x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (5*(-1/32*CoshIntegral[2*ArcTanh[a
*x]] + CoshIntegral[4*ArcTanh[a*x]]/16 + CoshIntegral[6*ArcTanh[a*x]]/32 -
 Log[ArcTanh[a*x]]/16))/a^2 + ((15*CoshIntegral[2*ArcTanh[a*x]])/32 + (3*C
oshIntegral[4*ArcTanh[a*x]])/16 + CoshIntegral[6*ArcTanh[a*x]]/32 + (5*Log
[ArcTanh[a*x]])/16)/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(4 \operatorname{arctanh}(ax))}{2}-\frac{\sinh(6 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{3 \operatorname{Chi}(6 \operatorname{arctanh}(ax))}{16}-\frac{5 \sinh(2 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{5 \operatorname{Chi}(2 \operatorname{arcta}}{16}}{a^2}$
default	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{8 \operatorname{arctanh}(ax)}+\frac{\operatorname{Chi}(4 \operatorname{arctanh}(ax))}{2}-\frac{\sinh(6 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{3 \operatorname{Chi}(6 \operatorname{arctanh}(ax))}{16}-\frac{5 \sinh(2 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{5 \operatorname{Chi}(2 \operatorname{arcta}}{16}}{a^2}$

input
```
int(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/8*sinh(4*arctanh(a*x))/arctanh(a*x)+1/2*Chi(4*arctanh(a*x))-1/32
/arctanh(a*x)*sinh(6*arctanh(a*x))+3/16*Chi(6*arctanh(a*x))-5/32/arctanh(a
*x)*sinh(2*arctanh(a*x))+5/16*Chi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 6.24

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}\,dx = \frac{64\,ax + \left(3\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) + 3\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 -\right.\right.}{}$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/32*(64*a*x + (3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3
*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 3*(a^6*
x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 8*(a
^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^
2*x^2 + 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral
(-(a*x + 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integ
ral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^
4 + 3*a^4*x^2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(ax-1)^4\,(ax+1)^4\,\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
```

output
```
Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
2*x/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5
*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)) - integrate(-2*(5*a^2*x^2 + 1)/((a^9*
x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(a*x + 1) - (a^9*x^8 - 4*a
^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^2} \, dx$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{x}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^4} \, dx$$

input
```
int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^4),x)
```

output
```
int(x/(atanh(a*x)^2*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx$$
$$= \int \frac{x}{atanh(ax)^2 a^8x^8 - 4atanh(ax)^2 a^6x^6 + 6atanh(ax)^2 a^4x^4 - 4atanh(ax)^2 a^2x^2 + atanh(ax)^2} dx$$

input
```
int(x/(-a^2*x^2+1)^4/atanh(a*x)^2,x)
```

output
```
int(x/(atanh(a*x)**2*a**8*x**8 - 4*atanh(a*x)**2*a**6*x**6 + 6*atanh(a*x)*
*2*a**4*x**4 - 4*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x)
```

3.361 $\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = -\frac{1}{a(1-a^2x^2)^3 \operatorname{arctanh}(ax)} + \frac{15\operatorname{Shi}(2\operatorname{arctanh}(ax))}{16a} + \frac{3\operatorname{Shi}(4\operatorname{arctanh}(ax))}{4a} + \frac{3\operatorname{Shi}(6\operatorname{arctanh}(ax))}{16a}$$

output
```
-1/a/(-a^2*x^2+1)^3/arctanh(a*x)+15/16*Shi(2*arctanh(a*x))/a+3/4*Shi(4*arc
tanh(a*x))/a+3/16*Shi(6*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \frac{\frac{1}{(-1+a^2x^2)^3 \operatorname{arctanh}(ax)} + \frac{15}{16}\operatorname{Shi}(2\operatorname{arctanh}(ax)) + \frac{3}{4}\operatorname{Shi}(4\operatorname{arctanh}(ax)) + \frac{3}{16}\operatorname{Shi}(6\operatorname{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

output
```
(1/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + (15*SinhIntegral[2*ArcTanh[a*x]])/16
+ (3*SinhIntegral[4*ArcTanh[a*x]])/4 + (3*SinhIntegral[6*ArcTanh[a*x]])/16
)/a
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6528, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6528}$$

$$6a\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^3 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{6\int \frac{ax}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^3 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{6\int \left(\frac{5\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{\sinh(4\operatorname{arctanh}(ax))}{8\operatorname{arctanh}(ax)}+\frac{\sinh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^3 \operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6\left(\frac{5}{32}\text{Shi}(2\operatorname{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\text{Shi}(6\operatorname{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^3 \operatorname{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (6*((5*SinhIntegral[2*ArcTanh[a*x]
])/32 + SinhIntegral[4*ArcTanh[a*x]]/8 + SinhIntegral[6*ArcTanh[a*x]]/32))
/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-\frac{5}{16 \operatorname{arctanh}(ax)}-\frac{15 \cosh(2 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{15 \operatorname{Shi}(2 \operatorname{arctanh}(ax))}{16}-\frac{3 \cosh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)}+\frac{3 \operatorname{Shi}(4 \operatorname{arctanh}(ax))}{4}-\frac{\cosh(6 \operatorname{arctan}}{32 \operatorname{arctanh}}}{a}$
default	$\frac{-\frac{5}{16 \operatorname{arctanh}(ax)}-\frac{15 \cosh(2 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{15 \operatorname{Shi}(2 \operatorname{arctanh}(ax))}{16}-\frac{3 \cosh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)}+\frac{3 \operatorname{Shi}(4 \operatorname{arctanh}(ax))}{4}-\frac{\cosh(6 \operatorname{arctan}}{32 \operatorname{arctanh}}}{a}$

input
```
int(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-5/16/arctanh(a*x)-15/32/arctanh(a*x)*cosh(2*arctanh(a*x))+15/16*Shi(
2*arctanh(a*x))-3/16/arctanh(a*x)*cosh(4*arctanh(a*x))+3/4*Shi(4*arctanh(a
*x))-1/32/arctanh(a*x)*cosh(6*arctanh(a*x))+3/16*Shi(6*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 6.26

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \frac{3 \left((a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1) \log\_integral \left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1} \right) - (a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1) \log\_i\right.}{}$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
1/32*(3*((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*
a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - (a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3
*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 4*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*l
og_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^6*x^6 - 3*
a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a
*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)
/(a*x - 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x -
 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*
a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
```

output
```
Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
-12*a*integrate(-x/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(
a*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)
), x) + 2/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 -
 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \int \frac{1}{\operatorname{atanh}(a\,x)^2\,(a^2\,x^2-1)^4} \, dx$$

input
```
int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^4),x)
```

output
```
int(1/(atanh(a*x)^2*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} \, dx = \frac{6atanh(ax)\left(\int \frac{x}{atanh(ax)a^8x^8-4atanh(ax)a^6x^6+6atanh(ax)a^4x^4-4atanh(ax)a^2x^2+atanh(ax)}dx\right)a^8x^6-18atanh(ax)\left(\int\right.}{}$$

input
```
int(1/(-a^2*x^2+1)^4/atanh(a*x)^2,x)
```

output
```
(6*atanh(a*x)*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*ata
nh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**8*x**6 - 18
*atanh(a*x)*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh
(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**6*x**4 + 18*a
tanh(a*x)*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a
*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**4*x**2 - 6*atan
h(a*x)*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6 + 6*atanh(a*x)
*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**2 + 1)/(atanh(a*x)
*a*(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1))
```

3.362 $\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx = -\frac{x}{2a(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} - \frac{3}{a^2(1-a^2x^2)^3 \operatorname{arctanh}(ax)} + \frac{5}{2a^2(1-a^2x^2)^2 \operatorname{arctanh}(ax)} + \frac{5\operatorname{Shi}(2\operatorname{arctanh}(ax))}{16a^2} + \frac{\operatorname{Shi}(4\operatorname{arctanh}(ax))}{a^2} + \frac{9\operatorname{Shi}(6\operatorname{arctanh}(ax))}{16a^2}$$

output
```
-1/2*x/a/(-a^2*x^2+1)^3/arctanh(a*x)^2-3/a^2/(-a^2*x^2+1)^3/arctanh(a*x)+5
/2/a^2/(-a^2*x^2+1)^2/arctanh(a*x)+5/16*Shi(2*arctanh(a*x))/a^2+Shi(4*arct
anh(a*x))/a^2+9/16*Shi(6*arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.64

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx = \frac{\frac{8\left(ax+(1+5a^2x^2)\operatorname{arctanh}(ax)\right)}{(-1+a^2x^2)^3\operatorname{arctanh}(ax)^2} + 5\mathrm{Shi}(2\operatorname{arctanh}(ax)) + 16\mathrm{Shi}(4\operatorname{arctanh}(ax)) + 9\mathrm{Shi}(6\operatorname{arctanh}(ax))}{16a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

output
```
((8*(a*x + (1 + 5*a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^3*ArcTanh[a*x]^2
) + 5*SinhIntegral[2*ArcTanh[a*x]] + 16*SinhIntegral[4*ArcTanh[a*x]] + 9*S
inhIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6594, 6528, 6590, 6528, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx$$

$$\Big\downarrow \text{6594}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^4\operatorname{arctanh}(ax)^2}dx}{2a} + \frac{5}{2}a\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}dx - \frac{x}{2a(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6528}$$

$$\frac{5}{2}a\int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2}dx + \frac{6a\int \frac{x}{(1-a^2x^2)^4\operatorname{arctanh}(ax)}dx - \frac{1}{a(1-a^2x^2)^3\operatorname{arctanh}(ax)}}{2a} - \frac{x}{2a(1-a^2x^2)^3\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6590}$$

$$\frac{5}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^4\text{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{1}{(1-a^2x^2)^3\text{arctanh}(ax)^2}dx}{a^2}\right)+\frac{6a\int \frac{x}{(1-a^2x^2)^4\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{2a}-\frac{x}{2a\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6528}$$

$$\frac{6a\int \frac{x}{(1-a^2x^2)^4\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{2a}+\frac{5}{2}a\left(\frac{6a\int \frac{x}{(1-a^2x^2)^4\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{a^2}-\frac{4a\int \frac{x}{(1-a^2x^2)^3\text{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^2\text{arctanh}(ax)}}{a^2}\right.$$

$$\frac{x}{2a\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6596}$$

$$\frac{\frac{6\int \frac{ax}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{2a}+\frac{5}{2}a\left(\frac{\frac{6\int \frac{ax}{\left(1-a^2x^2\right)^3\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{a^2}-\frac{\frac{4\int \frac{ax}{\left(1-a^2x^2\right)^2\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{}{a(1-a^2}}{a^2}\right.$$

$$\frac{x}{2a\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\frac{6\int \left(\frac{5\sinh(2\text{arctanh}(ax))}{32\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{\sinh(6\text{arctanh}(ax))}{32\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{2a}+\frac{5}{2}a\left(\frac{\frac{6\int \left(\frac{5\sinh(2\text{arctanh}(ax))}{32\text{arctanh}(ax)}+\frac{\sinh(4\text{arctanh}(ax))}{8\text{arctanh}(ax)}+\frac{\sinh(6\text{arctanh}(ax))}{32\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{a^2}-\frac{\frac{4\int \left(\frac{\sinh(2}{4\text{ar}}\right.}{}}{}\right.$$

$$\frac{x}{2a\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{2}a\left(\frac{\frac{6\left(\frac{5}{32}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))+\frac{1}{32}\text{Shi}(6\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{a^2}-\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arctanh}(\right.}{}\right.$$

$$\frac{\frac{6\left(\frac{5}{32}\text{Shi}(2\text{arctanh}(ax))+\frac{1}{8}\text{Shi}(4\text{arctanh}(ax))+\frac{1}{32}\text{Shi}(6\text{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^3\text{arctanh}(ax)}}{2a}-\frac{x}{2a\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}$$

input
```
Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

output
```
-1/2*x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) + (5*a*(-((-(1/(a*(1 - a^2*x^2)^
2*ArcTanh[a*x])) + (4*(SinhIntegral[2*ArcTanh[a*x]]/4 + SinhIntegral[4*Arc
Tanh[a*x]]/8))/a)/a^2) + (-(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (6*((5*S
inhIntegral[2*ArcTanh[a*x]])/32 + SinhIntegral[4*ArcTanh[a*x]]/8 + SinhInt
egral[6*ArcTanh[a*x]]/32))/a)/a^2))/2 + (-(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*
x])) + (6*((5*SinhIntegral[2*ArcTanh[a*x]])/32 + SinhIntegral[4*ArcTanh[a*
x]]/8 + SinhIntegral[6*ArcTanh[a*x]]/32))/a)/(2*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)^2}-\frac{\cosh(4 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)}+\operatorname{Shi}(4 \operatorname{arctanh}(ax))-\frac{\sinh(6 \operatorname{arctanh}(ax))}{64 \operatorname{arctanh}(ax)^2}-\frac{3\cosh(6 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{9 \operatorname{Shi}(6 \operatorname{arcta}}{16}}{a^2}$
default	$\frac{-\frac{\sinh(4 \operatorname{arctanh}(ax))}{16 \operatorname{arctanh}(ax)^2}-\frac{\cosh(4 \operatorname{arctanh}(ax))}{4 \operatorname{arctanh}(ax)}+\operatorname{Shi}(4 \operatorname{arctanh}(ax))-\frac{\sinh(6 \operatorname{arctanh}(ax))}{64 \operatorname{arctanh}(ax)^2}-\frac{3\cosh(6 \operatorname{arctanh}(ax))}{32 \operatorname{arctanh}(ax)}+\frac{9 \operatorname{Shi}(6 \operatorname{arcta}}{16}}{a^2}$

input
```
int(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/16*sinh(4*arctanh(a*x))/arctanh(a*x)^2-1/4/arctanh(a*x)*cosh(4*a
rctanh(a*x))+Shi(4*arctanh(a*x))-1/64/arctanh(a*x)^2*sinh(6*arctanh(a*x))-
3/32/arctanh(a*x)*cosh(6*arctanh(a*x))+9/16*Shi(6*arctanh(a*x))-5/64/arcta
nh(a*x)^2*sinh(2*arctanh(a*x))-5/32/arctanh(a*x)*cosh(2*arctanh(a*x))+5/16
*Shi(2*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(103) = 206$.

Time = 0.08 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.92

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \frac{\left(9\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\operatorname{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-9\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\operatorname{log\_}\right.}{}$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
1/32*((9*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*
a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - 9*(a^6*x^6 - 3*a
^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a
^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 16*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1
)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 16*(a^6*x^6
- 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 +
 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 64*a*x + 32*(5*a^2*x^2 +
 1)*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*lo
g(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{x}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

output
```
Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{x}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
(2*a*x + (5*a^2*x^2 + 1)*log(a*x + 1) - (5*a^2*x^2 + 1)*log(-a*x + 1))/((a
^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^8*x^6 - 3*a^6*
x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^8*x^6 - 3*a^6*x^4 +
 3*a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-4*(5*a^2*x^3 + 4*x)/((a^8*
x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a
^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{x}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1-a^2x^2\right)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,\left(a^2\,x^2-1\right)^4}\,dx$$

input
```
int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^4),x)
```

output
```
int(x/(atanh(a*x)^3*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{x}{\left(1-a^2x^2\right)^4 \operatorname{arctanh}(ax)^3}\,dx = \text{Too large to display}$$

input
```
int(x/(-a^2*x^2+1)^4/atanh(a*x)^3,x)
```

output

```
(60*atanh(a*x)**2*int(x**3/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6*x**6
+ 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a**10*x
**6 - 180*atanh(a*x)**2*int(x**3/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)*a**6
*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)),x)*a
**8*x**4 + 180*atanh(a*x)**2*int(x**3/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)
*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x))
,x)*a**6*x**2 - 60*atanh(a*x)**2*int(x**3/(atanh(a*x)*a**8*x**8 - 4*atanh(
a*x)*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a
*x)),x)*a**4 + 18*atanh(a*x)**2*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x)
*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x))
,x)*a**8*x**6 - 54*atanh(a*x)**2*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*x
)*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x)
),x)*a**6*x**4 + 54*atanh(a*x)**2*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a*
x)*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*x
)),x)*a**4*x**2 - 18*atanh(a*x)**2*int(x/(atanh(a*x)*a**8*x**8 - 4*atanh(a
*x)*a**6*x**6 + 6*atanh(a*x)*a**4*x**4 - 4*atanh(a*x)*a**2*x**2 + atanh(a*
x)),x)*a**2 + 5*atanh(a*x)**2*int(1/(atanh(a*x)**2*a**8*x**8 - 4*atanh(a*x
)**2*a**6*x**6 + 6*atanh(a*x)**2*a**4*x**4 - 4*atanh(a*x)**2*a**2*x**2 + a
tanh(a*x)**2),x)*a**7*x**6 - 15*atanh(a*x)**2*int(1/(atanh(a*x)**2*a**8*x*
*8 - 4*atanh(a*x)**2*a**6*x**6 + 6*atanh(a*x)**2*a**4*x**4 - 4*atanh(a*...
```

3.363 $\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx = -\frac{1}{2a(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2} - \frac{3x}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} + \frac{15\operatorname{Chi}(2\operatorname{arctanh}(ax))}{16a} + \frac{3\operatorname{Chi}(4\operatorname{arctanh}(ax))}{2a} + \frac{9\operatorname{Chi}(6\operatorname{arctanh}(ax))}{16a}$$

output
```
-1/2/a/(-a^2*x^2+1)^3/arctanh(a*x)^2-3*x/(-a^2*x^2+1)^3/arctanh(a*x)+15/16
*Chi(2*arctanh(a*x))/a+3/2*Chi(4*arctanh(a*x))/a+9/16*Chi(6*arctanh(a*x))/
a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} dx = \frac{1}{16}\left(\frac{8}{a(-1+a^2x^2)^3 \operatorname{arctanh}(ax)^2} + \frac{48x}{(-1+a^2x^2)^3 \operatorname{arctanh}(ax)} + \frac{15\operatorname{Chi}(2\operatorname{arctanh}(ax))}{a} + \frac{24\operatorname{Chi}(4\operatorname{arctanh}(ax))}{a} + \frac{9\operatorname{Chi}(6\operatorname{arctanh}(ax))}{a}\right)$$

input
```
Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

output
```
(8/(a*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2) + (48*x)/((-1 + a^2*x^2)^3*ArcTanh[
a*x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/a + (24*CoshIntegral[4*ArcTanh[a
*x]])/a + (9*CoshIntegral[6*ArcTanh[a*x]])/a)/16
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6528, 6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3} \, dx$$

$$\Big\downarrow \text{6528}$$

$$3a \int \frac{x}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^2} dx - \frac{1}{2a\,(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$3a \left(\frac{\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx}{a} + 5a \int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx - \frac{x}{a\,(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \right) - \frac{1}{2a\,(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$3a \left(5a \int \frac{x^2}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)} dx + \frac{\int \frac{1}{(1-a^2x^2)^3 \operatorname{arctanh}(ax)} d\operatorname{arctanh}(ax)}{a^2} - \frac{x}{a\,(1-a^2x^2)^3 \operatorname{arctanh}(ax)} \right) - \frac{1}{2a\,(1-a^2x^2)^3 \operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2a\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}+$$

$$3a\left(5a\int\frac{x^2}{\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}dx+\frac{\int\frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^6}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)}\right)$$

$\Big\downarrow$ 3793

$$3a\left(5a\int\frac{x^2}{\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}dx+\frac{\int\left(\frac{15\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{3\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{}{16\text{ar}}\right.}{a^2}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}$$

$\Big\downarrow$ 2009

$$3a\left(5a\int\frac{x^2}{\left(1-a^2x^2\right)^4\operatorname{arctanh}(ax)}dx+\frac{\frac{15}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{3}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(ax))+}{a^2}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}$$

$\Big\downarrow$ 6596

$$3a\left(\frac{5\int\frac{a^2x^2}{(1-a^2x^2)^3\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{15}{32}\operatorname{Chi}(2\operatorname{arctanh}(ax))+\frac{3}{16}\operatorname{Chi}(4\operatorname{arctanh}(ax))+\frac{1}{32}\operatorname{Chi}(6\operatorname{arctanh}(a}{a^2}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}$$

$\Big\downarrow$ 5971

$$3a\left(\frac{5\int\left(-\frac{\cosh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{\cosh(4\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(6\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}-\frac{1}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{15}{32}\operatorname{Chi}(}{}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^3\operatorname{arctanh}(ax)^2}$$

$\Big\downarrow$ 2009

$$3a\left(\frac{5\left(-\frac{1}{32}\text{Chi}(2\text{arctanh}(ax))+\frac{1}{16}\text{Chi}(4\text{arctanh}(ax))+\frac{1}{32}\text{Chi}(6\text{arctanh}(ax))-\frac{1}{16}\log(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{15}{32}\text{Chi}(\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^3\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) + 3*a*(-(x/(a*(1 - a^2*x^2)^3*Ar
cTanh[a*x])) + (5*(-1/32*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*Arc
Tanh[a*x]]/16 + CoshIntegral[6*ArcTanh[a*x]]/32 - Log[ArcTanh[a*x]]/16))/a
^2 + ((15*CoshIntegral[2*ArcTanh[a*x]])/32 + (3*CoshIntegral[4*ArcTanh[a*x
]])/16 + CoshIntegral[6*ArcTanh[a*x]]/32 + (5*Log[ArcTanh[a*x]])/16)/a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{-\frac{5}{32\operatorname{arctanh}(ax)^2}-\frac{15\cosh(2\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)^2}-\frac{15\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{15\operatorname{Chi}(2\operatorname{arctanh}(ax))}{16}-\frac{3\cosh(4\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)^2}-\frac{3\sinh(4\ \mathrm{a}}{8\ \mathrm{arct}}}{a}$
default	$\frac{-\frac{5}{32\operatorname{arctanh}(ax)^2}-\frac{15\cosh(2\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)^2}-\frac{15\sinh(2\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)}+\frac{15\operatorname{Chi}(2\operatorname{arctanh}(ax))}{16}-\frac{3\cosh(4\operatorname{arctanh}(ax))}{32\operatorname{arctanh}(ax)^2}-\frac{3\sinh(4\ \mathrm{a}}{8\ \mathrm{arct}}}{a}$

input
```
int(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-5/32/arctanh(a*x)^2-15/64/arctanh(a*x)^2*cosh(2*arctanh(a*x))-15/32/
arctanh(a*x)*sinh(2*arctanh(a*x))+15/16*Chi(2*arctanh(a*x))-3/32/arctanh(a
*x)^2*cosh(4*arctanh(a*x))-3/8*sinh(4*arctanh(a*x))/arctanh(a*x)+3/2*Chi(4
*arctanh(a*x))-1/64/arctanh(a*x)^2*cosh(6*arctanh(a*x))-3/32/arctanh(a*x)*
sinh(6*arctanh(a*x))+9/16*Chi(6*arctanh(a*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(79) = 158$.

Time = 0.09 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.89

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \frac{192\,ax\log\left(-\frac{ax+1}{ax-1}\right) + 3\left(3\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\_integral\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) + 3\left(a^6x^6 - 3\right.\right.}{}$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
1/32*(192*a*x*log(-(a*x + 1)/(a*x - 1)) + 3*(3*(a^6*x^6 - 3*a^4*x^4 + 3*a^
2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^
2*x^2 + 3*a*x - 1)) + 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral
(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/
(a^2*x^2 - 2*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4
+ 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*
x - 1))^2 + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-(a*x + 1)/(a*x
 - 1))^2)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

output
```
Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
2*(3*a*x*log(a*x + 1) - 3*a*x*log(-a*x + 1) + 1)/((a^7*x^6 - 3*a^5*x^4 + 3
*a^3*x^2 - a)*log(a*x + 1)^2 - 2*(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log
(a*x + 1)*log(-a*x + 1) + (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)^2) - integrate(-6*(5*a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 -
4*a^2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2
 + 1)*log(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^3\,(a^2\,x^2-1)^4}\,dx$$

input
```
int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^4),x)
```

output
```
int(1/(atanh(a*x)^3*(a^2*x^2 - 1)^4), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^4 \operatorname{arctanh}(ax)^3}\,dx = \frac{6atanh(ax)^2 \left(\int \frac{x}{atanh(ax)^2a^8x^8-4atanh(ax)^2a^6x^6+6atanh(ax)^2a^4x^4-4atanh(ax)^2a^2x^2+atanh(ax)^2}dx\right)a^8x^6-18atanh(ax}{}$$

input
```
int(1/(-a^2*x^2+1)^4/atanh(a*x)^3,x)
```

output
```
(6*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**8*x**8 - 4*atanh(a*x)**2*a**6*x**
6 + 6*atanh(a*x)**2*a**4*x**4 - 4*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2)
,x)*a**8*x**6 - 18*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**8*x**8 - 4*atanh(
a*x)**2*a**6*x**6 + 6*atanh(a*x)**2*a**4*x**4 - 4*atanh(a*x)**2*a**2*x**2
+ atanh(a*x)**2),x)*a**6*x**4 + 18*atanh(a*x)**2*int(x/(atanh(a*x)**2*a**8
*x**8 - 4*atanh(a*x)**2*a**6*x**6 + 6*atanh(a*x)**2*a**4*x**4 - 4*atanh(a*
x)**2*a**2*x**2 + atanh(a*x)**2),x)*a**4*x**2 - 6*atanh(a*x)**2*int(x/(ata
nh(a*x)**2*a**8*x**8 - 4*atanh(a*x)**2*a**6*x**6 + 6*atanh(a*x)**2*a**4*x*
*4 - 4*atanh(a*x)**2*a**2*x**2 + atanh(a*x)**2),x)*a**2 + 1)/(2*atanh(a*x)
**2*a*(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1))
```

3.364 $\int \frac{x^5 \operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{x^5 \operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{5x\sqrt{1-a^2x^2}}{24a^5}-\frac{x^3\sqrt{1-a^2x^2}}{20a^3}+\frac{89\arcsin(ax)}{120a^6}-\frac{8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{15a^6}-\frac{4x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{15a^4}-\frac{x^4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{5a^2}$$

output
```
-5/24*x*(-a^2*x^2+1)^(1/2)/a^5-1/20*x^3*(-a^2*x^2+1)^(1/2)/a^3+89/120*arcs
in(a*x)/a^6-8/15*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^6-4/15*x^2*(-a^2*x^2+1)
^(1/2)*arctanh(a*x)/a^4-1/5*x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int \frac{x^5 \operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{ax\sqrt{1-a^2x^2}(25+6a^2x^2)-89\arcsin(ax)+8\sqrt{1-a^2x^2}(8+4a^2x^2+3a^4x^4)\operatorname{arctanh}(ax)}{120a^6}$$

input
```
Integrate[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/120*(a*x*Sqrt[1 - a^2*x^2]*(25 + 6*a^2*x^2) - 89*ArcSin[a*x] + 8*Sqrt[1
 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/a^6
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.57, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6578, 262, 262, 223, 6578, 262, 223, 6556, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6578}$$

$$\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}dx}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}$$

$$\Big\downarrow \text{6578}$$

$$\frac{4\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}$$

$$\Big\downarrow \text{262}$$

$$\frac{4\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}$$

$$\Big\downarrow \text{223}$$

$$\frac{4\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}$$

$$\Big\downarrow \text{6556}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{4\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}$$

input
```
Int[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + A
rcSin[a*x]/(2*a^3)))/(4*a^2))/(5*a) - (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/(5*a^2) + (4*((-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a
) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2))/(3*a^2)))/(5*a^2)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

method	result
default	$-\frac{(24a^4x^4 \operatorname{arctanh}(ax)+6a^3x^3+32a^2x^2 \operatorname{arctanh}(ax)+25ax+64 \operatorname{arctanh}(ax))\sqrt{-a^2x^2+1}}{120a^6}+\frac{89i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{120a^6}-\frac{89i\ln\left(\right.}{}$

input
```
int(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(24*a^4*x^4*arctanh(a*x)+6*a^3*x^3+32*a^2*x^2*arctanh(a*x)+25*a*x+6
4*arctanh(a*x))*(-a^2*x^2+1)^(1/2)/a^6+89/120*I/a^6*ln((a*x+1)/(-a^2*x^2+1
)^(1/2)+I)-89/120*I/a^6*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{x^5\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\left(6\,a^3x^3+25\,ax+4\,(3\,a^4x^4+4\,a^2x^2+8)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+178\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^6}$$

input
```
integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*((6*a^3*x^3 + 25*a*x + 4*(3*a^4*x^4 + 4*a^2*x^2 + 8)*log(-(a*x + 1)
/(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 178*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a^6
```

Sympy [F]

$$\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^5\,\text{atanh}\,(ax)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(x**5*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**5*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.17

$$\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{120}\,a\left(\frac{3\left(\frac{2\sqrt{-a^2x^2+1}x^3}{a^2}+\frac{3\sqrt{-a^2x^2+1}x}{a^4}-\frac{3\arcsin(ax)}{a^5}\right)}{a^2}+\frac{16\left(\frac{\sqrt{-a^2x^2+1}x}{a^2}-\frac{\arcsin(ax)}{a^3}\right)}{a^4}-\frac{64\arcsin\,(ax)}{a^7}\right) - \frac{1}{15}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2}+\frac{4\sqrt{-a^2x^2+1}x^2}{a^4}+\frac{8\sqrt{-a^2x^2+1}}{a^6}\right)\text{artanh}\,(ax)$$

input
```
integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/120*a*(3*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2 + 1)*x/a^4 - 3
*arcsin(a*x)/a^5)/a^2 + 16*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)/a^
4 - 64*arcsin(a*x)/a^7) - 1/15*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2
*x^2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*arctanh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^5\,\text{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^5*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^5*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)\,x^5}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^5*atanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)*x**5)/sqrt( - a**2*x**2 + 1),x)
```

3.365 $\int \frac{x^4\textbf{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{5\sqrt{1-a^2x^2}}{8a^5}+\frac{(1-a^2x^2)^{3/2}}{12a^5}-\frac{3x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}-\frac{3\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{4a^5} - \frac{3i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{8a^5}+\frac{3i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{8a^5}$$

output
```
-5/8*(-a^2*x^2+1)^(1/2)/a^5+1/12*(-a^2*x^2+1)^(3/2)/a^5-3/8*x*(-a^2*x^2+1)
^(1/2)*arctanh(a*x)/a^4-1/4*x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2-3/4*ar
ctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-3/8*I*polylog(2,-I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a^5
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

$$\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\left(-13-2a^2x^2-15ax\text{arctanh}(ax)-6ax(-1+a^2x^2)\,\text{arctanh}(ax)-\frac{9i\text{arctanh}(ax)\left(\log\left(1-ie^{-\text{arc}\ldots}\right.\right.}{\sqrt{\ldots}}\right.}{24a^5}$$

input
```
Integrate[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-13 - 2*a^2*x^2 - 15*a*x*ArcTanh[a*x] - 6*a*x*(-1 + a^
2*x^2)*ArcTanh[a*x] - ((9*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log
[1 + I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((9*I)*(PolyLog[2, (-I)/E^Arc
Tanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^5)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6578, 243, 53, 2009, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \; 6578$$
$$\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}$$
$$\Big\downarrow \; 243$$
$$\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}$$
$$\Big\downarrow \; 53$$

$$\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}$$

$$\Big\downarrow \text{6578}$$

$$\frac{3\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}$$

$$\Big\downarrow \text{6512}$$

$$-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}$$

input
```
Int[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))/(8*a) - (x^
3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4*a^2) + (3*(-1/2*Sqrt[1 - a^2*x^2]/a^3
 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*x]/
Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1
 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2)))
/(4*a^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

method	result
default	$-\frac{(6a^3x^3 \operatorname{arctanh}(ax)+2a^2x^2+9ax \operatorname{arctanh}(ax)+13)\sqrt{-a^2x^2+1}}{24a^5}-\frac{3i \operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a^5}+\frac{3i \operatorname{arctanh}(ax)\ln\left(1\right.}{8a^5}$

input
```
int(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2+9*a*x*arctanh(a*x)+13)*(-a^2*x^2+1
)^(1/2)/a^5-3/8*I/a^5*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*
I/a^5*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a^5*dilog(1+I*
(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a^5*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)
```

Fricas [F]

$$\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{atanh}\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**4*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4 \text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^4 \text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\text{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)\,x^4}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^4*atanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.366 $\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}}{6a^3}+\frac{5\arcsin(ax)}{6a^4} -\frac{2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}$$

output
```
-1/6*x*(-a^2*x^2+1)^(1/2)/a^3+5/6*arcsin(a*x)/a^4-2/3*(-a^2*x^2+1)^(1/2)*a
rctanh(a*x)/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{ax\sqrt{1-a^2x^2}-5\arcsin(ax)+2\sqrt{1-a^2x^2}(2+a^2x^2)\,\text{arctanh}(ax)}{6a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/6*(a*x*Sqrt[1 - a^2*x^2] - 5*ArcSin[a*x] + 2*Sqrt[1 - a^2*x^2]*(2 + a^2
*x^2)*ArcTanh[a*x])/a^4
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6578, 262, 223, 6556, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6578}$$

$$\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}$$

$$\Big\downarrow \text{6556}$$

$$\frac{2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}$$

input
```
Int[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a) - (x^2*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^
2]*ArcTanh[a*x])/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{(2a^2x^2 \operatorname{arctanh}(ax)+ax+4 \operatorname{arctanh}(ax))\sqrt{-a^2x^2+1}}{6a^4}+\frac{5i \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{6a^4}-\frac{5i \ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-i\right)}{6a^4}$	97

input
```
int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*x^2*arctanh(a*x)+a*x+4*arctanh(a*x))*(-a^2*x^2+1)^(1/2)/a^4+5/
6*I/a^4*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)-5/6*I/a^4*ln((a*x+1)/(-a^2*x^2+1)
^(1/2)-I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{x^3\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\left(ax+(a^2x^2+2)\log\left(-\frac{ax+1}{ax-1}\right)\right)+10\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^4}$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1))) +
 10*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^3\,\text{atanh}\,(ax)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=-\frac{1}{6}\,a\left(\frac{\frac{\sqrt{-a^2x^2+1}x}{a^2}-\frac{\arcsin(ax)}{a^3}}{a^2}-\frac{4\,\arcsin\,(ax)}{a^5}\right)-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\text{artanh}\,(ax)$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*a*((sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)/a^2 - 4*arcsin(a*x)/a
^5) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arctanh(
a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^3\,\text{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{atanh(ax)\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*atanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.367 $\int \frac{x^2\textbf{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{2a^3}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2} -\frac{\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a^3} -\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a^3}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a^3}$$

output
```
-1/2*(-a^2*x^2+1)^(1/2)/a^3-1/2*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2-arct
an((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/2*I*polylog(2,-I*(-a*x
+1)^(1/2)/(a*x+1)^(1/2))/a^3+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}+ax\sqrt{1-a^2x^2}\text{arctanh}(ax)+i\text{arctanh}(ax)\log\left(1-ie^{-\text{arctanh}(ax)}\right)-i\text{arctanh}(ax)\log\left(1+\right.}{2a^3}$$

input
```
Integrate[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2] + a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + I*ArcTanh[a
*x]*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] +
 I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x]])/a^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6578}$$

$$\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}$$

$$\Big\downarrow \text{6512}$$

$$\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}$$

input
```
Int[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) +
((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, (
(-I)*Sqrt[1 - a*x]])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sq
rt[1 + a*x]])/a)/(2*a^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

method	result
default	$-\frac{(ax\ \operatorname{arctanh}(ax)+1)\sqrt{-a^2x^2+1}}{2a^3}-\frac{i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a^3}+\frac{i\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a^3}-\frac{i\operatorname{dilog}\left(1+\frac{i}{\sqrt{-}}\right.}{2a^3}$

input
```
int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a^3-1/2*I/a^3*arctanh(a*x)*ln
(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a^3*arctanh(a*x)*ln(1-I*(a*x+1)/(-a
^2*x^2+1)^(1/2))-1/2*I/a^3*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a^3
*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2 \text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\, dx = \int \frac{x^2 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\, dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\, dx = \int \frac{x^2 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\, dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\, dx = \int \frac{x^2 \operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\, dx$$

input
```
int((x^2*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{atanh(ax)\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*atanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.368 $\int \frac{x\textbf{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}$$

output
```
arcsin(a*x)/a^2-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin(ax) - \sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}$$

input
```
Integrate[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(ArcSin[a*x] - Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6556, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}$$

input
```
Int[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)}{a^2}+\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{a^2}-\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-i\right)}{a^2}$	79

input
```
int(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2+I/a^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I
)-I/a^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.81

$$\int \frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=-\frac{\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)+4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)) + 4*arctan((sqrt(-a^2*x
^2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F]

$$\int \frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{a^2} + \frac{\arcsin\,(ax)}{a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/a^2 + arcsin(a*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin\,(ax)\,\text{sgn}(a)}{a|a|} - \frac{\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)}{2\,a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/2*sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*atanh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*atanh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{atanh(ax)\,x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*atanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)*x)/sqrt( - a**2*x**2 + 1),x)
```

3.369 $\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
-2*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-I*polylog(2,-I*(-a*
x+1)^(1/2)/(a*x+1)^(1/2))/a+I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

$$\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{i\left(\text{arctanh}(ax)\left(\log\left(1-ie^{-\text{arctanh}(ax)}\right)-\log\left(1+ie^{-\text{arctanh}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\text{arctanh}(ax)}\right)-\text{Poly}}{a}$$

input
```
Integrate[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

output
```
((-I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]
) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{6512\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6512}$$

$$-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((
-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a
```

Defintions of rubi rules used

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

method	result	siz
default	$-\frac{i\left(\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)-\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)-\operatorname{dilog}\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)+\operatorname{dilog}\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\right)}{a}$	11

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I/a*(arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1-I*
(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+dilog(1+
I*(a*x+1)/(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{atanh}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)/sqrt( - a**2*x**2 + 1),x)
```

3.370 $\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = -2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-2*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^
(1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \text{arctanh}(ax)\left(\log\left(1-e^{-\text{arctanh}(ax)}\right) - \log\left(1+e^{-\text{arctanh}(ax)}\right)\right) + \text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) - \text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
ArcTanh[a*x]*(Log[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])]) + P
olyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6580}$$

$$-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

input
```
Int[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1
 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Defintions of rubi rules used

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

method	result
default	$\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{arctanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-$ p

input
```
int(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+
1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{atanh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\text{atanh}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(atanh(a*x)/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.371 $\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)}{x} - a\,\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x-a*arctanh((-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)}{x} + a\log(x) - a\log\left(1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + a*Log[x] - a*Log[1 + Sqrt[1 - a^2*
x^2]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6570, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6570}$$

$$a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

input
```
Int[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, \operatorname{arctanh}\left(a x \right)}{x}-a \ln \left(1+\frac{a x +1}{\sqrt{-a^{2} x^{2}+1}}\right)+a \ln \left(\frac{a x +1}{\sqrt{-a^{2} x^{2}+1}}-1\right)$	70

input
```
int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*ln
((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{2\,ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*log(-(a*x
+ 1)/(a*x - 1)))/x
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)}{x}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*arctan
h(a*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(38) = 76$.

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.64

$$\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{1}{2}a\left(\log\left(\sqrt{-a^2x^2+1}+1\right)-\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)$$
$$+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\log\left(-\frac{ax+1}{ax-1}\right)$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*(log(sqrt(-a^2*x^2 + 1) + 1) - log(-sqrt(-a^2*x^2 + 1) + 1)) + 1/4*
(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*abs(
a) + a)/(x*abs(a)))*log(-(a*x + 1)/(a*x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(atanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.372 $\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2x^2} - a^2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \frac{1}{2}a^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\frac{1}{2}a^2\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2-a^2*ar
ctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/2*a^2*polylog(2,-(-a*x+
1)^(1/2)/(a*x+1)^(1/2))-1/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\text{arctanh}(ax)\right) - \text{arctanh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) + 4\text{arctanh}(ax)\log\left(1-e^{-\text{arctanh}(ax)}\right) - 4\text{arctanh}(ax)\log\left(1+e^{-\text{arctanh}(ax)}\right) + 4\,\text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) - 4\,\text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right) - \text{arctanh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) + 2\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 + 4*Ar
cTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh
[a*x])] + 4*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*PolyLog[2, E^(-ArcTanh[a*x]
)] - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{6588}$$

$$\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx + \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}$$

$$\downarrow \text{242}$$

$$\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\downarrow \text{6580}$$

$$\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

input

```
Int[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output

```
-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) +
(a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(
Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/
2
```

Defintions of rubi rules used

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6580

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

method	result
default	$-\frac{(ax+\operatorname{arctanh}(ax))\sqrt{-a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a^2\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{a^2\operatorname{arctanh}(ax)\ln\left(1-\right.}{2}$

input
```
int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+arctanh(a*x))*(-a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(a*x)*ln(1-(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1
/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,-(a
*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(atanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.373 $\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{3a^4} - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^3} - \frac{10\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{3a^4} - \frac{2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2} - \frac{5i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{3a^4} + \frac{5i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{3a^4}$$

output
```
-1/3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^3-10/3
*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4-2/3*(-a^2*x^2+1)^(1
/2)*arctanh(a*x)^2/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/a^2-5/3*I
*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4+5/3*I*polylog(2,I*(-a*x+1)
^(1/2)/(a*x+1)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.78

$$\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\left(-1-ax\text{arctanh}(ax)-3\text{arctanh}(ax)^2+(1-a^2x^2)\,\text{arctanh}(ax)^2-\frac{5i\text{arctanh}(ax)\left(\log\left(1-ie^{-\text{ar}}\right.\right.}{\sqrt{}}\right.}{3a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-1 - a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + (1 - a^2*x^
2)*ArcTanh[a*x]^2 - ((5*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1
 + I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((5*I)*(PolyLog[2, (-I)/E^ArcTa
nh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(3*a^4)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6578, 6556, 6512, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \; 6578$$
$$\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$
$$\Big\downarrow \; 6556$$

$$\frac{2\left(\frac{2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}\right)}{3a^2}+\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6512}$$

$$\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6578}$$

$$\frac{2\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right)}{3a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\downarrow \text{6512}$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}+\frac{2\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}$$

input
```
Int[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (2*(-1/2*Sqrt[1 - a^2*x^
2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*
a^2)))/(3*a) + (2*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2) + (2*((-2*Arc
Tan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a))/a))/(3*a^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\left(a^{2}x^{2}\operatorname{arctanh}(ax)^{2}+ax\operatorname{arctanh}(ax)+2\operatorname{arctanh}(ax)^{2}+1\right)\sqrt{-a^{2}x^{2}+1}}{3a^{4}}-\frac{5i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^{2}x^{2}+1}}\right)}{3a^{4}}+\frac{5i\operatorname{arctanh}(a$

input
```
int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)+2*arctanh(a*x)^2+1)*(-a^2*x^
2+1)^(1/2)/a^4-5/3*I/a^4*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5
/3*I/a^4*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-5/3*I/a^4*dilog(1
+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/3*I/a^4*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1
/2))
```

Fricas [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \,\text{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \,\text{atanh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \,\text{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\,\text{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2 x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*atanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)**2*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.374 $\int \frac{x^2\textbf{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\begin{aligned}\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx =& \frac{\arcsin(ax)}{a^3}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^3}\\&-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}+\frac{\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2}{a^3}\\&-\frac{i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{a^3}\\&+\frac{i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{a^3}\\&+\frac{i\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^3}-\frac{i\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^3}\end{aligned}$$

output
```
arcsin(a*x)/a^3-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^3-1/2*x*(-a^2*x^2+1)^(1/
2)*arctanh(a*x)^2/a^2+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^
3-I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+I*arctanh(a*
x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+I*polylog(3,-I*(a*x+1)/(-a^
2*x^2+1)^(1/2))/a^3-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\left(-2\text{arctanh}(ax) - ax\text{arctanh}(ax)^2 - \frac{i\left(4i\arctan\left(\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)+\text{arctanh}(ax)^2\log\left(1-ie^{-\text{arctan}\right.\right.}{}\right.}{}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-2*ArcTanh[a*x] - a*x*ArcTanh[a*x]^2 - (I*((4*I)*ArcTa
n[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTa
nh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^Arc
Tanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-
I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/
(2*a^3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6578, 6514, 3042, 4668, 3011, 2720, 6556, 223, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{6578}$$
$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a} + \frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$
$$\Big\downarrow \text{6514}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a^3}+\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\Big\downarrow$ 4668

$$\frac{-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+}{2a^3}$$
$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\Big\downarrow$ 3011

$$\frac{2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,ie^{\text{ar}}\right.\right.}{2a^3}$$
$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\Big\downarrow$ 2720

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$
$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\Big\downarrow$ 6556

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$
$$\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\Big\downarrow$ 223

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-}{2a^3}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

input
```
Int[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (ArcSin[a*x]/a^2 - (Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/a^2)/a + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^
2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-
I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])
+ PolyLog[3, I*E^ArcTanh[a*x]]))/(2*a^3)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x^2\,\text{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*atanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)**2*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.375 $\int \frac{x\textbf{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{4\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2} - \frac{2i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^2} + \frac{2i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^2}$$

output
```
-4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^2-(-a^2*x^2+1)^(1/2
)*arctanh(a*x)^2/a^2-2*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2+2*
I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\text{arctanh}(ax)\left(\sqrt{1-a^2x^2}\text{arctanh}(ax) + 2i\left(\log\left(1-ie^{-\text{arctanh}(ax)}\right) - \log\left(1+ie^{-\text{arctanh}(ax)}\right)\right)\right) + 2i\,\text{Poly}}{a^2}$$

input
```
Integrate[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + (2*I)*(Log[1 - I/E^ArcTa
nh[a*x]] - Log[1 + I/E^ArcTanh[a*x]])) + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a
*x]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]])/a^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6556, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}$$

$$\Big\downarrow \text{6512}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}$$

input
```
Int[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2) + (2*((-2*ArcTan[Sqrt[1 - a*x]/S
qrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1
+ a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))/a
```

Defintions of rubi rules used

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)^2}{a^2}-\frac{2i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{a^2}+\frac{2i\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{a^2}-\frac{2i\operatorname{dilog}\left(1+\frac{i(ax}{\sqrt{-a^2}}\right.}{a^2}$

input
```
int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/a^2-2*I/a^2*arctanh(a*x)*ln(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+2*I/a^2*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))-2*I/a^2*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I/a^2*dilog(1-I*(a*x+1)
/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x\,\text{atanh}^2\,(ax)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=\int \frac{x\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2\,x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*atanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)**2*x)/sqrt( - a**2*x**2 + 1),x)
```

3.376 $\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2}{a} - \frac{2i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)}{a} + \frac{2i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{arctanh}(ax)}\right)}{a} + \frac{2i\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)}{a} - \frac{2i\operatorname{PolyLog}\left(3,ie^{\operatorname{arctanh}(ax)}\right)}{a}$$

output
```
2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-2*I*arctanh(a*x)*pol
ylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+2*I*arctanh(a*x)*polylog(2,I*(a*x+
1)/(-a^2*x^2+1)^(1/2))/a+2*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-2*
I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{i\left(-\operatorname{arctanh}(ax)^2\left(\log\left(1-ie^{-\operatorname{arctanh}(ax)}\right)-\log\left(1+ie^{-\operatorname{arctanh}(ax)}\right)\right)-2\operatorname{arctanh}(ax)\left(\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arc}}\right.\right.\right.}{a}$$

input
```
Integrate[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

output
```
(I*(-(ArcTanh[a*x]^2*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]
])) - 2*ArcTanh[a*x]*(PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^Arc
Tanh[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcTanh[a*x]] - PolyLog[3, I/E^ArcTanh[
a*x]])))/a
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6514, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \; 6514$$

$$\frac{\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2 d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \operatorname{arctanh}(ax)^2 \csc\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right) d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \; 4668$$

$$\frac{-2i\int \operatorname{arctanh}(ax)\log\left(1-ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)+2i\int \operatorname{arctanh}(ax)\log\left(1+ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)+}{a}$$

$\downarrow$ 3011

$$\frac{2i\left(\int \operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)\right)-2i\left(\int \operatorname{PolyLog}\left(2,ie^{\operatorname{a}}\right.\right.}{a}$$

$\downarrow$ 2720

$$\frac{2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)de^{\operatorname{arctanh}(ax)}-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\operatorname{arc}}\right.}{}$$

$\downarrow$ 7143

$$\frac{2\operatorname{arctanh}(ax)^2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)+2i\left(\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)\right)-}{a}$$

input
```
Int[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

output
```
(2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2
, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcT
anh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]]) + PolyLog[3, I*E^ArcTanh[a*x]]))/a
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\operatorname{arctanh}\left(ax\right)^{2}}{\sqrt{-a^{2}x^{2}+1}}dx$$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**2/sqrt( - a**2*x**2 + 1),x)
```

3.377 $\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\begin{aligned}\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = & -2\text{arctanh}\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2 \\ & - 2\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right) \\ & + 2\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{\text{arctanh}(ax)}\right) \\ & + 2\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right) - 2\,\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\end{aligned}$$

output
```
-2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh(a*x)*polyl
og(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/
(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - \operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 2\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 2\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Log[1 + E^(-Arc
Tanh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 2*ArcTanh[a*
x]*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*Po
lyLog[3, E^(-ArcTanh[a*x])]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \; 6582$$

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{ax}\,d\operatorname{arctanh}(ax)$$

$$\Big\downarrow \; 3042$$

$$\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$\downarrow$ 26

$$i\int \text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$\downarrow$ 4670

$$i\left(2i\int \text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int \text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\right.$$

$\downarrow$ 3011

$$i\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{PolyLog}\right.\right.$$

$\downarrow$ 2720

$$i\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\right.\right.$$

$\downarrow$ 7143

$$i\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)-\text{a}\right.\right.$$

input
```
Int[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
I*((2*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*Po
lyLog[2, -E^ArcTanh[a*x]]) + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTa
nh[a*x]*PolyLog[2, E^ArcTanh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.32

method	result
default	$\operatorname{arctanh}(ax)^2 \ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+2\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-2\operatorname{polylog}\left(3,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right.$

input
```
int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(
a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh
(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(atanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.378 $\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x} - 4a\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + 2a\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - 2a\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x-4*a*arctanh(a*x)*arctanh((-a*x+1)^(1/
2)/(a*x+1)^(1/2))+2*a*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*a*polylog
(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\operatorname{arctanh}(ax)\left(\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) + 2ax\left(-\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) + \log\left(1+e^{-\operatorname{arctanh}(ax)}\right)\right)\right)}{x} + 2a\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 2a\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2*a*x*(-Log[1 - E^(-ArcT
anh[a*x])] + Log[1 + E^(-ArcTanh[a*x])])))/x) + 2*a*PolyLog[2, -E^(-ArcTan
h[a*x])] - 2*a*PolyLog[2, E^(-ArcTanh[a*x])]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6570, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{ 6570}$$

$$2a\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{ 6580}$$

$$2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}$$

input
```
Int[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) + 2*a*(-2*ArcTanh[a*x]*ArcTanh[Sqr
t[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - P
olyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])
```

Defintions of rubi rules used

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)^2}{x}+2a\,\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+2a\,\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-2a\,\mathrm{ar}$

input
```
int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x+2*a*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x
^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*arctanh(a*x)*ln
(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(atanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.379 $\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\begin{aligned}\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx =& -\frac{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}\\ &-a^2\text{arctanh}\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2-a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\\ &-a^2\text{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\\ &+a^2\text{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{\text{arctanh}(ax)}\right)\\ &+a^2\operatorname{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-a^2\operatorname{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\end{aligned}$$

output
```
-a*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2
/x^2-a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-
a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x
+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{1}{8}a^2\left(-4\operatorname{arctanh}(ax)\coth\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right.$$
$$-\operatorname{arctanh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$+4\operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right)$$
$$-4\operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right)$$
$$+8\log\left(\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$
$$+8\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right)$$
$$-8\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)$$
$$+8\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right)-8\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right)$$
$$-\operatorname{arctanh}(ax)^2\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$\left.+4\operatorname{arctanh}(ax)\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a
*x]/2]^2 + 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]^2*
Log[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 8*ArcTanh[a*x]*
PolyLog[2, -E^(-ArcTanh[a*x])] - 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x
])] + 8*PolyLog[3, -E^(-ArcTanh[a*x])] - 8*PolyLog[3, E^(-ArcTanh[a*x])] -
 ArcTanh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/
2]))/8
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6588, 6570, 243, 73, 221, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6588}$$

$$\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\int \frac{\operatorname{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6570}$$

$$\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6582}$$

$$\frac{1}{2}a^2\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{ax}d\operatorname{arctanh}(ax)+a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a^2\int i\operatorname{arctanh}(ax)^2\csc(i\operatorname{arctanh}(ax))d\operatorname{arctanh}(ax)+a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ia^2\int \operatorname{arctanh}(ax)^2\csc(i\operatorname{arctanh}(ax))d\operatorname{arctanh}(ax)+a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{2}ia^2\left(2i\int \operatorname{arctanh}(ax)\log\left(1-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-2i\int \operatorname{arctanh}(ax)\log\left(1+e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(a\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ia^2\left(-2i\left(\int \operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right)+2i\left(\int \operatorname{Poly}\right.\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}-a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 7143

$$\frac{1}{2}ia^2\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

input
```
Int[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2 + a*(-((Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]) + (I/2)*a^2*((2*I)*ArcTanh[E^
ArcTanh[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^ArcTanh
[a*x]]) + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog[2,
E^ArcTanh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.51

method	result
default	$-\frac{(2ax+\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)\sqrt{-a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+a^2\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,\right.$

input
```
int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*x+arctanh(a*x))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arct
anh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2,(a*
x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^2\left(ax\right)}{x^3\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(atanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.380 $\int \frac{x^3\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\begin{aligned}\int \frac{x^3\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx =& \frac{\arcsin(ax)}{a^4}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^4}\\&-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^3}\\&+\frac{5\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2}{a^4}\\&-\frac{2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}\\&-\frac{5i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{a^4}\\&+\frac{5i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{a^4}\\&+\frac{5i\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^4}-\frac{5i\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^4}\end{aligned}$$

output
```
arcsin(a*x)/a^4-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^4-1/2*x*(-a^2*x^2+1)^(1/
2)*arctanh(a*x)^2/a^3+5*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/
a^4-2/3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^3/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*a
rctanh(a*x)^3/a^2-5*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)/a^4+5*I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5*I*pol
ylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-5*I*polylog(3,I*(a*x+1)/(-a^2*x^
2+1)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

$$\int \frac{x^3\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}\left(-3ax\text{arctanh}(ax)^2+2(1-a^2x^2)\,\text{arctanh}(ax)^3-6\text{arctanh}(ax)\,(1+\text{arctanh}(ax)^2)-\frac{3i\left(4i\,a\right.}{}\right.}{}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-3*a*x*ArcTanh[a*x]^2 + 2*(1 - a^2*x^2)*ArcTanh[a*x]^3
 - 6*ArcTanh[a*x]*(1 + ArcTanh[a*x]^2) - ((3*I)*((4*I)*ArcTan[Tanh[ArcTanh
[a*x]/2]] + 5*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 5*ArcTanh[a*x]^2*
Log[1 + I/E^ArcTanh[a*x]] + 10*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]
] - 10*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 10*PolyLog[3, (-I)/E^Ar
cTanh[a*x]] - 10*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(6*a^4
)
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.45, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6578, 6556, 6514, 3042, 4668, 3011, 2720, 6578, 6514, 3042, 4668, 3011, 2720, 6556, 223, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6578}$$

$$\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}+\frac{2\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6556}$$

$$\frac{2\left(\frac{3\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}\right)}{3a^2}+\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{6514}$$

$$\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}+\frac{2\left(\frac{3\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{4668}$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\cdots\right.}{a^2}\cdots\right.}{3a^2}\cdots$$

$$\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{P}\right.}{3a^2}\right.}{\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}}$$

$\downarrow$ 2720

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.}{\frac{\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}}$$

$\downarrow$ 6578

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.}{\frac{\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}}$$

$\downarrow$ 6514

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.}{\frac{\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}}$$

$\downarrow$ 3042

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.$$

$$\frac{\frac{\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a^3}+\frac{\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$\downarrow$ 4668

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.$$

$$\frac{\frac{-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2\text{arctanh}(ax)^2\text{arctan}}{2a^3}}{a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$\downarrow$ 3011

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.$$

$$\frac{2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(a\right.}{2a^3}$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$\downarrow$ 2720

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a}\right.\right.\right.}{}\right.$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,\right.\right.}{2a^3}$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$\downarrow$ 6556

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a\right.\right.\right.}}{}\right.$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,\right.\right.}{2a^3}$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$\downarrow$ 223

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(a\right.\right.\right.}}{}\right.$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,\right.\right.}{2a^3}$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}$$

$\downarrow$ 7143

$$2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie\right.\right.\right.}{a^2}\right.$$

$$3a^2$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{3a^2}+$$

$$\frac{\frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\text{PolyLog}\left(3,ie^{\text{arcta}}\right.\right.}{2a^3}}{a}$$

input
```
Int[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/a^2 + (-1/2*(x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/a^2 + (ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x
])/a^2)/a + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-(ArcTanh[a*
x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-I)*E^ArcTanh[a*x]]) - (
2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]]) + PolyLog[3, I*E^ArcTan
h[a*x]]))/(2*a^3))/a + (2*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/a^2) + (3*
(2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2
, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcT
anh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]]) + PolyLog[3, I*E^ArcTanh[a*x]])))/a
^2))/(3*a^2)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*atanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)**3*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.381 $\int \frac{x^2 \text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 305

$$\begin{aligned}
\int \frac{x^2\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = & -\frac{6\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a^3}-\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^3} \\
& -\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}+\frac{\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^3}{a^3} \\
& -\frac{3i\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{2a^3} \\
& +\frac{3i\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{2a^3} \\
& -\frac{3i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3}+\frac{3i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3} \\
& +\frac{3i\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^3} \\
& -\frac{3i\text{arctanh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^3} \\
& -\frac{3i\,\text{PolyLog}\left(4,-ie^{\text{arctanh}(ax)}\right)}{a^3}+\frac{3i\,\text{PolyLog}\left(4,ie^{\text{arctanh}(ax)}\right)}{a^3}
\end{aligned}$$

output
```
-6*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-3/2*(-a^2*x^2+1)^
(1/2)*arctanh(a*x)^2/a^3-1/2*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^3/a^2+arcta
n((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a^3-3/2*I*arctanh(a*x)^2*poly
log(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+3/2*I*arctanh(a*x)^2*polylog(2,I*
(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a^3+3*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3+3*I*arctanh(a*x
)*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*arctanh(a*x)*polylog(3,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1
/2))/a^3+3*I*polylog(4,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.87

$$\int \frac{x^2\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx =$$
$$-\frac{i\left(7\pi^4+8i\pi^3\text{arctanh}(ax)+24\pi^2\text{arctanh}(ax)^2-192i\sqrt{1-a^2x^2}\text{arctanh}(ax)^2-32i\pi\text{arctanh}(ax)^3-\cdots\right.}{}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (
192*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 - (64*I
)*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + 384*ArcTanh[a
*x]*Log[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] - 384
*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E
^ArcTanh[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*A
rcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E
^ArcTanh[a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I
)*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a
*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(Pi^2 - (4*I)
*Pi*ArcTanh[a*x] - 4*(2 + ArcTanh[a*x]^2))*PolyLog[2, (-I)/E^ArcTanh[a*x]]
 - 384*PolyLog[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E
^ArcTanh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh
[a*x]*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[
a*x]] + 384*ArcTanh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x
]*PolyLog[3, (-I)*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]
] + 384*PolyLog[4, (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*
x]]))/a^3
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6578, 6514, 3042, 4668, 3011, 6556, 6512, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6578}$$

$$\frac{3\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a}+\frac{\int \frac{\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\Big\downarrow \text{ 6514}$$

$$\frac{3\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a}+\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3d\text{arctanh}(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\downarrow \text{3042}$$

$$\frac{\int \text{arctanh}(ax)^3 \csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right) d\text{arctanh}(ax)}{2a^3}+\frac{3\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\downarrow \text{4668}$$

$$\frac{-3i\int \text{arctanh}(ax)^2 \log\left(1-ie^{\text{arctanh}(ax)}\right) d\text{arctanh}(ax)+3i\int \text{arctanh}(ax)^2 \log\left(1+ie^{\text{arctanh}(ax)}\right) d\text{arctanh}(ax)\cdots}{2a^3}$$
$$\frac{3\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\downarrow \text{3011}$$

$$\frac{3i\left(2\int \text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right) d\text{arctanh}(ax)-\text{arctanh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-3i(2\cdots}{}$$
$$\frac{3\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\downarrow \text{6556}$$

$$\frac{3i\left(2\int \text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right) d\text{arctanh}(ax)-\text{arctanh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-3i(2\cdots}{}$$
$$\frac{3\left(\frac{2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}\right)}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\downarrow \text{6512}$$

$$\frac{3i\left(2\int \text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right) d\text{arctanh}(ax)-\text{arctanh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-3i(2\cdots}{}$$
$$\frac{3\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\downarrow \text{7163}$$

$$\frac{3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\int\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right)-\text{arctanh}(ax)^2\,\text{Poly}\right.}{}$$

$$\frac{3\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\Big\downarrow \; 2720$$

$$\frac{3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right)-\text{arctanh}\right.}{}$$

$$\frac{3\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

$$\Big\downarrow \; 7143$$

$$\frac{2\text{arctanh}(ax)^3\arctan\left(e^{\text{arctanh}(ax)}\right)+3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{PolyLog}\left(4,-ie^{\text{arctanh}(ax)}\right)\right.\right.}{}$$

$$\frac{3\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2a^2}$$

input
```
Int[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/a^2 + (3*(-((Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^2)/a^2) + (2*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[
a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLo
g[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))/a))/(2*a) + (2*ArcTan[E^ArcTanh
[a*x]]*ArcTanh[a*x]^3 + (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[
a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]] - PolyLog[4, (-I)
*E^ArcTanh[a*x]])) - (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[a*x]])
 + 2*(ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]] - PolyLog[4, I*E^ArcTanh[a
*x]])))/(2*a^3)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2 \text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\text{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*atanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)**3*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.382 $\int \frac{x\textbf{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{6\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2}{a^2} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2} - \frac{6i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{a^2} + \frac{6i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{a^2} + \frac{6i\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^2} - \frac{6i\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^2}$$

output

```
6*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^2-(-a^2*x^2+1)^(1/2)
*arctanh(a*x)^3/a^2-6*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/
2))/a^2+6*I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+6*I*p
olylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2-6*I*polylog(3,I*(a*x+1)/(-a^2*
x^2+1)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx =$$
$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+3i\text{arctanh}(ax)^2\log\left(1-ie^{-\text{arctanh}(ax)}\right)-3i\text{arctanh}(ax)^2\log\left(1+ie^{-\text{arctanh}(ax)}\right)\ldots}{}$$

input
```
Integrate[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 + (3*I)*ArcTanh[a*x]^2*Log[1 - I/E^Arc
Tanh[a*x]] - (3*I)*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*ArcTan
h[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*PolyLog[2, I/E
^ArcTanh[a*x]] + (6*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (6*I)*PolyLog[3,
I/E^ArcTanh[a*x]])/a^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6556, 6514, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \text{6556}$$
$$\frac{3\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}$$
$$\Big\downarrow \text{6514}$$
$$\frac{3\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{a^2}$$

$\Big\downarrow$ 4668

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right.}{a^2}$$

$\Big\downarrow$ 3011

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,i\right.\right.\right.}{a^2}$$

$\Big\downarrow$ 2720

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-}\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right.\right.}{a^2}$$

input
```
Int[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/a^2) + (3*(2*ArcTan[E^ArcTanh[a*x]]*A
rcTanh[a*x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + P
olyLog[3, (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^Arc
Tanh[a*x]]) + PolyLog[3, I*E^ArcTanh[a*x]])))/a^2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x \operatorname{arctanh}(ax)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\text{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3\,x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*atanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((atanh(a*x)**3*x)/sqrt( - a**2*x**2 + 1),x)
```

3.383 $\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^3}{a} - \frac{3i\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)}{a} + \frac{3i\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,ie^{\operatorname{arctanh}(ax)}\right)}{a} + \frac{6i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)}{a} - \frac{6i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,ie^{\operatorname{arctanh}(ax)}\right)}{a} - \frac{6i\operatorname{PolyLog}\left(4,-ie^{\operatorname{arctanh}(ax)}\right)}{a} + \frac{6i\operatorname{PolyLog}\left(4,ie^{\operatorname{arctanh}(ax)}\right)}{a}$$

output

```
2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a-3*I*arctanh(a*x)^2*p
olylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+3*I*arctanh(a*x)^2*polylog(2,I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a+6*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a^2*x
^2+1)^(1/2))/a-6*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a-
6*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+6*I*polylog(4,I*(a*x+1)/(-a
^2*x^2+1)^(1/2))/a
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 451 vs. 2(153) = 306.

Time = 0.30 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{i(7\pi^4 + 8i\pi^3\operatorname{arctanh}(ax) + 24\pi^2\operatorname{arctanh}(ax)^2 - 32i\pi\operatorname{arctanh}(ax)^3 - 16\operatorname{arctanh}(ax)^4 + 8i\pi^3\log(1+}{}$$

input
```
Integrate[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

output
```
((-1/64*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (3
2*I)*Pi*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcTan
h[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - (96*I)*Pi*ArcTa
nh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh[a*x]^3*Log[1 + I/E^ArcTan
h[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] + (96*I)*Pi*ArcTa
nh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcTanh[a*x]]
 + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi +
(2*I)*ArcTanh[a*x])/4]] - 48*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E
^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - 48*P
i^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcTanh[a
*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)*E^A
rcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4, (-
I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6514, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx$$

$\downarrow$ 6514

$$\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3d\text{arctanh}(ax)}{a}$$

$\downarrow$ 3042

$$\frac{\int \text{arctanh}(ax)^3\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{a}$$

$\downarrow$ 4668

$$\frac{-3i\int \text{arctanh}(ax)^2\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+3i\int \text{arctanh}(ax)^2\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\ -}{a}$$

$\downarrow$ 3011

$$3i\left(2\int \text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-3i(2$$

$\downarrow$ 7163

$$3i\left(2\left(\text{arctanh}(ax)\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\int \text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right)-\text{arctanh}(ax)^2\text{Poly}$$

$\downarrow$ 2720

$$3i\left(2\left(\text{arctanh}(ax)\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right)-\text{arctanh}$$

$\downarrow$ 7143

$$2\text{arctanh}(ax)^3\arctan\left(e^{\text{arctanh}(ax)}\right)+3i\left(2\left(\text{arctanh}(ax)\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{PolyLog}\left(4,-ie^{\text{arctanh}(ax)}\right)\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

output
```
(2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 + (3*I)*(-(ArcTanh[a*x]^2*PolyLog
[2, (-I)*E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]
] - PolyLog[4, (-I)*E^ArcTanh[a*x]])) - (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2,
 I*E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]] - PolyL
og[4, I*E^ArcTanh[a*x]])))/a
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\operatorname{arctanh}\left(ax\right)^{3}}{\sqrt{-a^{2}x^{2}+1}}dx$$

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^{3}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**3/sqrt( - a**2*x**2 + 1),x)
```

3.384 $\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = &-2\operatorname{arctanh}\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^3\\ &-3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\\ &+3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right)\\ &+6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right)\\ &-6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)\\ &-6\operatorname{PolyLog}\left(4,-e^{\operatorname{arctanh}(ax)}\right)+6\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)\end{aligned}$$

output
```
-2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-3*arctanh(a*x)^2*pol
ylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a
rctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a
^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \frac{1}{8}\left(\pi^4 - 2\operatorname{arctanh}(ax)^4 - 8\operatorname{arctanh}(ax)^3\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 8\operatorname{arctanh}(ax)^3\log\left(1-e^{\operatorname{arctanh}(ax)}\right) + 24\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) + 24\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right) + 48\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 48\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,-e^{-\operatorname{arctanh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(Pi^4 - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8
*ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^
(-ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTa
nh[a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcT
anh[a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*
x]])/8
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6582, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \; 6582$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{ax}d\text{arctanh}(ax)$$

$\Big\downarrow$ 3042

$$\int i\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$\Big\downarrow$ 26

$$i\int \text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$\Big\downarrow$ 4670

$$i\left(3i\int \text{arctanh}(ax)^2\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-3i\int \text{arctanh}(ax)^2\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax\right.$$

$\Big\downarrow$ 3011

$$i\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$

$\Big\downarrow$ 7163

$$i\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right)-\text{arctanh}(ax\right.\right.$$

$\Big\downarrow$ 2720

$$i\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right)-\right.\right.$$

$\Big\downarrow$ 7143

$$i\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{PolyLog}\left(4,-e^{\text{arctanh}(ax)}\right)\right)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e\right.\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
I*((2*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - (3*I)*(-(ArcTanh[a*x]^2*
PolyLog[2, -E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]]
 - PolyLog[4, -E^ArcTanh[a*x]])) + (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, E^Ar
cTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - PolyLog[4, E^A
rcTanh[a*x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.11

method	result
default	$\operatorname{arctanh}(ax)^3 \ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+3\operatorname{arctanh}(ax)^2 \operatorname{polylog}\left(2, \frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-6\operatorname{arctanh}(ax)\operatorname{pol}$

input
```
int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)
^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^3*ln(1+(a*x+1
)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^3\left(ax\right)}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(atanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.385 $\int \frac{\text{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{\text{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = -6a\,\text{arctanh}\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2 - \frac{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)^3}{x} - 6a\,\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right) + 6a\,\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{\text{arctanh}(ax)}\right) + 6a\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right) - 6a\,\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)$$

output
```
-6*a*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-(-a^2*x^2+1)^(1/2)
*arctanh(a*x)^3/x-6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x
+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = a\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{ax} + 3\operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - 3\operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 6\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 6\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(a*x)) + 3*ArcTanh[a*x]^2*Log[1 -
E^(-ArcTanh[a*x])] - 3*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 6*ArcTa
nh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*PolyLog[2, E^(-Arc
Tanh[a*x])] + 6*PolyLog[3, -E^(-ArcTanh[a*x])] - 6*PolyLog[3, E^(-ArcTanh[
a*x])])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6570, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{ 6570}$$

$$3a\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{ 6582}$$

$$3a\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+3a\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+3ia\int \text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$3ia\left(2i\int \text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int \text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$3ia\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{PolyL}\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$3ia\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2\right.$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$3ia\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)-\right.\right.$$

input

```
Int[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/x) + (3*I)*a*((2*I)*ArcTanh[E^ArcTanh
[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]])
 + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, E^ArcTa
nh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.92

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)^3}{x}+3a\operatorname{arctanh}(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+6a\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,\frac{ax}{\sqrt{-a^2}}\right.$

input
```
int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)^3/x+3*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*
polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*
a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^3\left(ax\right)}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(atanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.386 $\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx =& -\frac{3a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{2x^2}\\
&-a^2\operatorname{arctanh}\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^3\\
&-6a^2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&-\frac{3}{2}a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\\
&+\frac{3}{2}a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right)\\
&+3a^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-3a^2\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&+3a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right)\\
&-3a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)\\
&-3a^2\operatorname{PolyLog}\left(4,-e^{\operatorname{arctanh}(ax)}\right)+3a^2\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)\end{aligned}$$

output
```
-3/2*a*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x-1/2*(-a^2*x^2+1)^(1/2)*arctanh(
a*x)^3/x^2-a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-6*a^2*ar
ctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*polylog(2,(a*x
+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3*a
^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(
a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+
1)^(1/2))-3*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(4,(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 6.35 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{1}{16}a\Bigg(a\pi^4 - 2a\operatorname{arctanh}(ax)^4 - 12a\operatorname{arctanh}(ax)^2\coth\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) - 2a\operatorname{arctanh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) + 48a\operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - 48a\operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) - 8a\operatorname{arctanh}(ax)^3\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 8a\operatorname{arctanh}(ax)^3\log\left(1-e^{\operatorname{arctanh}(ax)}\right) + 24a\left(2+\operatorname{arctanh}(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 48a\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 24a\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right) + 48a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 48a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right) + 48a\operatorname{PolyLog}\left(4,-e^{-\operatorname{arctanh}(ax)}\right) + 48a\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right) + 12a\operatorname{arctanh}(ax)^2\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) - \frac{4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{x}\Bigg)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a*(a*Pi^4 - 2*a*ArcTanh[a*x]^4 - 12*a*ArcTanh[a*x]^2*Coth[ArcTanh[a*x]/2]
 - 2*a*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2 + 48*a*ArcTanh[a*x]*Log[1 - E
^(-ArcTanh[a*x])] - 48*a*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] - 8*a*Arc
Tanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*a*ArcTanh[a*x]^3*Log[1 - E^ArcT
anh[a*x]] + 24*a*(2 + ArcTanh[a*x]^2)*PolyLog[2, -E^(-ArcTanh[a*x])] - 48*
a*PolyLog[2, E^(-ArcTanh[a*x])] + 24*a*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh
[a*x]] + 48*a*ArcTanh[a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*a*ArcTanh[a
*x]*PolyLog[3, E^ArcTanh[a*x]] + 48*a*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*
a*PolyLog[4, E^ArcTanh[a*x]] + 12*a*ArcTanh[a*x]^2*Tanh[ArcTanh[a*x]/2] -
(4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3*Tanh[ArcTanh[a*x]/2])/x))/16
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6588, 6570, 6580, 6582, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6588}$$

$$\frac{3}{2}a\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6570}$$

$$\frac{3}{2}a\left(2a\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\right)+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6580}$$

$$\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 6582

$$\frac{1}{2}a^2\int\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{ax}d\text{arctanh}(ax)+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 3042

$$\frac{1}{2}a^2\int i\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 26

$$\frac{1}{2}ia^2\int\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 4670

$$\frac{1}{2}ia^2\left(3i\int\text{arctanh}(ax)^2\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-3i\int\text{arctanh}(ax)^2\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 3011

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 7163

$$\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right)-\text{arctanh}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 2720

$$\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right)\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 7143

$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{PolyLog}\left(4,-e^{\text{arctanh}(ax)}\right)\right)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

input

```
Int[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/x^2 + (3*a*(-((Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^2)/x) + 2*a*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*
x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x
]/Sqrt[1 + a*x]])))/2 + (I/2)*a^2*((2*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a
*x]^3 - (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, -E^ArcTanh[a*x]]) + 2*(ArcTanh[
a*x]*PolyLog[3, -E^ArcTanh[a*x]] - PolyLog[4, -E^ArcTanh[a*x]])) + (3*I)*(
-(ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3,
E^ArcTanh[a*x]] - PolyLog[4, E^ArcTanh[a*x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.44

method	result
default	$-\frac{(3ax+\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)^2\sqrt{-a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}(ax)^3\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{3a^2\operatorname{arctanh}(ax)^2\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}$

input
```
int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*a*x+arctanh(a*x))*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2+1/2*a^2*ar
ctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^
2*x^2+1)^(1/2))+3*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*arctan
h(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*
x^2+1)^(1/2))-3*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a
*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^
(1/2))-3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(atanh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(atanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.387 $\int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}} \, dx = \operatorname{Int}\left(\frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}} \, dx = \int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}} \, dx$$

input
```
Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arctanh}(ax)}{(1-a^2x^2)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{x^m \text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx$$

input
```
Int[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

output
```
int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{x^m \text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [N/A]

Not integrable

Time = 46.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**m*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atanh}(a\,x)}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^m*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^m*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{x^m \operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{x^m\, atanh(ax)}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^m*atanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

output
```
 - int((x**m*atanh(a*x))/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*
x**2 + 1)),x)
```

3.388 $\int \frac{x^3 \mathbf{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{x}{a^3\sqrt{1-a^2x^2}} - \frac{\arcsin(ax)}{a^4} + \frac{\text{arctanh}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^4}$$

output
```
-x/a^3/(-a^2*x^2+1)^(1/2)-arcsin(a*x)/a^4+arctanh(a*x)/a^4/(-a^2*x^2+1)^(1
/2)+(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{ax\sqrt{1-a^2x^2} + (1-a^2x^2)\arcsin(ax) + \sqrt{1-a^2x^2}(-2+a^2x^2)\,\text{arctanh}(ax)}{a^4\,(-1+a^2x^2)}$$

input
```
Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
(a*x*Sqrt[1 - a^2*x^2] + (1 - a^2*x^2)*ArcSin[a*x] + Sqrt[1 - a^2*x^2]*(-2
 + a^2*x^2)*ArcTanh[a*x])/(a^4*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6590, 6556, 208, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a^2}$$

$$\Big\downarrow \text{6556}$$

$$\frac{\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}}{a^2}-\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}}{a^2}-\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
(-(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2]))/a^2 -
(ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2)/a^2
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

method	result	siz
default	$\frac{\sqrt{-a^{2} x^{2}+1}\, \left(a^{2} x^{2} \operatorname{arctanh}\left(a x\right)-i \sqrt{-a^{2} x^{2}+1}\, \ln \left(\frac{a x+1}{\sqrt{-a^{2} x^{2}+1}}-i\right)+i \sqrt{-a^{2} x^{2}+1}\, \ln \left(\frac{a x+1}{\sqrt{-a^{2} x^{2}+1}}+i\right)+a x-2 \operatorname{arctanh}\left(a x\right)\right)}{a^{4} \left(a^{2} x^{2}-1\right)}$	123

input
```
int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-a^2*x^2+1)^(1/2)*(a^2*x^2*arctanh(a*x)-I*(-a^2*x^2+1)^(1/2)*ln((a*x+1)/(
-a^2*x^2+1)^(1/2)-I)+I*(-a^2*x^2+1)^(1/2)*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)
+a*x-2*arctanh(a*x))/a^4/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{4\,(a^2x^2-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\left(2\,ax+(a^2x^2-2)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\,(a^6x^2-a^4)}$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(4*(a^2*x^2 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(2*a*x + (a^2*x^2 - 2)*log(-(a*x + 1)/(a*x - 1))))/(a^6*x^2 - a^4)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3\,\text{atanh}\,(ax)}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**3*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = a\left(\frac{\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin(ax)}{a^3}}{a^2}-\frac{2\,x}{\sqrt{-a^2x^2+1}a^4}\right) - \left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2}-\frac{2}{\sqrt{-a^2x^2+1}a^4}\right)\text{artanh}\,(ax)$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
a*((x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3)/a^2 - 2*x/(sqrt(-a^2*x^2
 + 1)*a^4)) - (x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4))*
arctanh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3\,\text{atanh}(a\,x)}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^3*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)\,x^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^3*atanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)*x**3)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*
x**2 + 1)),x)
```

3.389 $\int \frac{x^2\textbf{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{1}{a^3\sqrt{1-a^2x^2}} + \frac{x\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}} + \frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a^3} + \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3} - \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3}$$

output
```
-1/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)+2*arctan((
-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3+I*polylog(2,-I*(-a*x+1)^(1/2
)/(a*x+1)^(1/2))/a^3-I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{i\left(\frac{i}{\sqrt{1-a^2x^2}} - \frac{iax\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} + \text{arctanh}(ax)\log\left(1-ie^{-\text{arctanh}(ax)}\right) - \text{arctanh}(ax)\log\cdots}{a^3}$$

input
```
Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
(I*(I/Sqrt[1 - a^2*x^2] - (I*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] +
 PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6560, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{6560}$$

$$-\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a^2}+\frac{x\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{1}{a^3\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{6512}$$

$$-\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{a^2}+\frac{x\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{1}{a^3\sqrt{1-a^2x^2}}$$

input
```
Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
-(1/(a^3*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) -
((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sq
rt[1 + a*x]])/a)/a^2
```

Defintions of rubi rules used

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6560
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*c^2*d*(q + 1))), x] + Sim
p[1/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q
, -5/2]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.39

method	result
default	$-\frac{(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2a^3(ax-1)}-\frac{(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2a^3(ax+1)}+\frac{i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{a^3}-\frac{i\operatorname{arctan}}{}$

input
```
int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x-1)-1/2*(arctanh(a*
x)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x+1)+I/a^3*arctanh(a*x)*ln(1+I*(a*x+
1)/(-a^2*x^2+1)^(1/2))-I/a^3*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))+I/a^3*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I/a^3*dilog(1-I*(a*x+1)/(-a
^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{atanh}\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**2*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{atanh}(a\,x)}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^2*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)\,x^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^2*atanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)*x**2)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*
x**2 + 1)),x)
```

3.390 $\int \frac{x\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{x}{a\sqrt{1-a^2x^2}} + \frac{\operatorname{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}$$

output
```
-x/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{x\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{-ax+\operatorname{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}$$

input
```
Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
(-(a*x) + ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6556, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{(1-a^2x^2)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{208}$$

$$\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}$$

input
```
Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

output
```
-(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,(-ax+\operatorname{arctanh}(ax))}{a^{2}(a^{2}x^{2}-1)}$	38
orering	$-\frac{2(ax-1)(ax+1)(a^{2}x^{2}+1)\operatorname{arctanh}(ax)}{a^{2}(-a^{2}x^{2}+1)^{\frac{3}{2}}}-\frac{(ax+1)^{2}(ax-1)^{2}\left(\frac{\operatorname{arctanh}(ax)}{(-a^{2}x^{2}+1)^{\frac{3}{2}}}+\frac{xa}{(-a^{2}x^{2}+1)^{\frac{5}{2}}}+\frac{3x^{2}\operatorname{arctanh}(ax)a^{2}}{(-a^{2}x^{2}+1)^{\frac{5}{2}}}\right)}{a^{2}}$	118

input
```
int(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-a^2*x^2+1)^(1/2)*(-a*x+arctanh(a*x))/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{\sqrt{-a^2x^2+1}\left(2\,ax-\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\,(a^4x^2-a^2)}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-a^2*x^2 + 1)*(2*a*x - log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{atanh}\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{-a^2x^2+1}a}+\frac{\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-x/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{\sqrt{-a^2x^2+1}x}{(a^2x^2-1)a}+\frac{\log\left(-\frac{ax+1}{ax-1}\right)}{2\sqrt{-a^2x^2+1}a^2}$$

input
```
integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*a) + 1/2*log(-(a*x + 1)/(a*x - 1))/(sq
rt(-a^2*x^2 + 1)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{atanh}(a\,x)}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x*atanh(a*x))/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x*atanh(a*x))/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)\,x}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x*atanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

output
```
 - int((atanh(a*x)*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**
2 + 1)),x)
```

3.391 $\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}} + \frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}$$

output
```
-1/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{-1+ax\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

output
```
(-1 + a*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6520}$$

$$\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

output
```
-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,(ax\,\text{arctanh}(ax)-1)}{a(a^2x^2-1)}$	38
orering	$-\frac{4(ax+1)x(ax-1)\,\text{arctanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}-\frac{(ax+1)^2(ax-1)^2\left(\frac{a}{(-a^2x^2+1)^{\frac{5}{2}}}+\frac{3\,\text{arctanh}(ax)a^2x}{(-a^2x^2+1)^{\frac{5}{2}}}\right)}{a^2}$	87

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-a^2*x^2+1)^(1/2)*(a*x*arctanh(a*x)-1)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\,(a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
x*arctanh(a*x)/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2x^2+1}x\log\left(-\frac{ax+1}{ax-1}\right)}{2\,(a^2x^2-1)} - \frac{1}{\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1) - 1/(sqr
t(-a^2*x^2 + 1)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2 +
1)),x)
```

3.392 $\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx = -\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} - 2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-a*x/(-a^2*x^2+1)^(1/2)+arctanh(a*x)/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)*arc
tanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)-polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx = -\frac{ax}{\sqrt{1-a^2x^2}} + \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} + \operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - \operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + \operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - \operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
-((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + Po
lyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6592, 6556, 208, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx + \int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(\frac{\operatorname{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{1}{(1-a^2x^2)^{3/2}}dx}{a}\right) + \int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{208}$$

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx + a^2\left(\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{x}{a\sqrt{1-a^2x^2}}\right)$$

$$\Big\downarrow \text{ 6580}$$

$$a^2\left(\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{x}{a\sqrt{1-a^2x^2}}\right) - 2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{PolyLog}\left(2, -\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{PolyLog}\left(2, \frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

input
```
Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
a^2*(-(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2])) -
2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1
- a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.40

method	result
default	$-\frac{(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)(ax+1)}}{2ax+2}+\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+$

input
```
int(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)+1
)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a
^2*x^2+1)^(1/2))-polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{x(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/x/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x\,(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{x\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)/(x*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)/(x*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x\,(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^2x^3-\sqrt{-a^2x^2+1}\,x}dx\right)$$

input
```
int(atanh(a*x)/x/(-a^2*x^2+1)^(3/2),x)
```

output
```
 - int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**3 - sqrt( - a**2*x**2 +
1)*x),x)
```

3.393 $\int \frac{\textbf{arctanh}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx = -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x} - a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-a/(-a^2*x^2+1)^(1/2)+a^2*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(
1/2)*arctanh(a*x)/x-a*arctanh((-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\text{arctanh}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx = \frac{(-1+2a^2x^2)\,\text{arctanh}(ax)+ax\left(-1+\sqrt{1-a^2x^2}\log(x)-\sqrt{1-a^2x^2}\log\left(1+\sqrt{1\cdots}\right)\right)}{x\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

output
```
((-1 + 2*a^2*x^2)*ArcTanh[a*x] + a*x*(-1 + Sqrt[1 - a^2*x^2]*Log[x] - Sqrt
[1 - a^2*x^2]*Log[1 + Sqrt[1 - a^2*x^2]]))/(x*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6592, 6520, 6570, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{x^2 (1-a^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx + \int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6520}$$

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx + a^2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right)$$

$$\Big\downarrow \text{6570}$$

$$a\int \frac{1}{x\sqrt{1-a^2x^2}}dx + a^2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2 + a^2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a} + a^2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{221}$$

$$a^2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right) - a\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

input
```
Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + a^2*(-(1/(a*Sqrt[1 - a^2*x^2])) +
(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(74) = 148$.

Time = 1.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.22

method	result
default	$-\frac{2\,\mathrm{arctanh}(ax)a^2x^2\sqrt{-a^2x^2+1}+\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^3x^3-a\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)x-\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)a^3x^3+\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-\right.}{(a^2x^2-1)x}$

input
```
int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*arctanh(a*x)*a^2*x^2*(-a^2*x^2+1)^(1/2)+ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2
))*a^3*x^3-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))*x-ln((a*x+1)/(-a^2*x^2+1)^(1
/2)-1)*a^3*x^3+ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)*a*x-a*x*(-a^2*x^2+1)^(1/2)
-(-a^2*x^2+1)^(1/2)*arctanh(a*x))/(a^2*x^2-1)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{\mathrm{arctanh}(ax)}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = -\frac{2\,a^3x^3-2\,ax-2\,(a^3x^3-ax)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\left(2\,ax-(2\,a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\,(a^2x^3-x)}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*x^3 - 2*a*x - 2*(a^3*x^3 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x
) - sqrt(-a^2*x^2 + 1)*(2*a*x - (2*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1)))
)/(a^2*x^3 - x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{x^2 \left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^{3/2}}\,dx = -a\left(\frac{1}{\sqrt{-a^2x^2+1}} + \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)\right) + \left(\frac{2\,a^2x}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}x}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-a*(1/sqrt(-a^2*x^2 + 1) + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))) +
(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(74) = 148$.

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx = -\frac{1}{2}\,a\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{1}{2}\,a\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{2\sqrt{-a^2x^2+1}a^2x}{a^2x^2-1}-\frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\log\left(-\frac{ax+1}{ax-1}\right)-\frac{a}{\sqrt{-a^2x^2+1}}$$

input
```
integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*a*log(sqrt(-a^2*x^2 + 1) + 1) + 1/2*a*log(-sqrt(-a^2*x^2 + 1) + 1) +
1/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 2*sqrt(-a^2*x^2 + 1)
*a^2*x/(a^2*x^2 - 1) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a)))*log(-(a
*x + 1)/(a*x - 1)) - a/sqrt(-a^2*x^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{x^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^2 (1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^2x^4-\sqrt{-a^2x^2+1}\,x^2}dx\right)$$

input
```
int(atanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
 - int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**4 - sqrt( - a**2*x**2 +
1)*x**2),x)
```

3.394 $\int \frac{\operatorname{arctanh}(ax)}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)}{x^3(1-a^2x^2)^{3/2}}\,dx &= -\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\\ &\quad-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2x^2}-3a^2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\ &\quad+\frac{3}{2}a^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\frac{3}{2}a^2\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\end{aligned}$$

output

```
-a^3*x/(-a^2*x^2+1)^(1/2)-1/2*a*(-a^2*x^2+1)^(1/2)/x+a^2*arctanh(a*x)/(-a^
2*x^2+1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2-3*a^2*arctanh(a*x)*
arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/2*a^2*polylog(2,-(-a*x+1)^(1/2)/(a
*x+1)^(1/2))-3/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}(ax)}{x^3(1-a^2x^2)^{3/2}}\,dx = \frac{1}{8}a^2\left(-\frac{8ax}{\sqrt{1-a^2x^2}}+\frac{8\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{ax\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{\sqrt{1-a^2x^2}} - \operatorname{arctanh}(ax)\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) + 12\operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - 12\operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 12\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 12\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) - \operatorname{arctanh}(ax)\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) + 2\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(a^2*((-8*a*x)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a
*x*Csch[ArcTanh[a*x]/2]^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]*Csch[ArcTanh[a
*x]/2]^2 + 12*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]*Lo
g[1 + E^(-ArcTanh[a*x])] + 12*PolyLog[2, -E^(-ArcTanh[a*x])] - 12*PolyLog[
2, E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTa
nh[a*x]/2]))/8
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6592, 6588, 242, 6580, 6592, 6556, 208, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{x^3(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6592$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^{3/2}}dx+\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6588}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}$$

$$\Big\downarrow \text{242}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\Big\downarrow \text{6580}$$

$$a^2\int \frac{\text{arctanh}(ax)}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\Big\downarrow \text{6592}$$

$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)}{\left(1-a^2x^2\right)^{3/2}}dx+\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx\right)+\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}dx}{a}\right)+\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx\right)+\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\Big\downarrow \text{208}$$

$$a^2\left(\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx + a^2\left(\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{x}{a\sqrt{1-a^2x^2}}\right)\right) + \frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{PolyLog}\left(2, -\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{PolyLog}\left(2, \frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\Big\downarrow \text{ 6580}$$

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{x}{a\sqrt{1-a^2x^2}}\right) - 2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{PolyLog}\left(2, -\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{PolyL}\right. \frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{PolyLog}\left(2, -\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{PolyLog}\left(2, \frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{2x}$$

input
```
Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

output
```
-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) +
(a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(
Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/
2 + a^2*(a^2*(-(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*
x^2])) - 2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2,
-(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(151) = 302$.

Time = 1.41 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.94

method	result
default	$-\frac{3\operatorname{arctanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^4x^4-3\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^4x^4+3\operatorname{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^4x^4-3\operatorname{polylog}\left(\right.}{}$

input
```
int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4*x^4-3*arctanh(a*
x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4*x^4+3*polylog(2,-(a*x+1)/(-a^2*x^2
+1)^(1/2))*a^4*x^4-3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4*x^4-(-a^2*x
^2+1)^(1/2)*a^3*x^3-3*arctanh(a*x)*a^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))*x^
2+3*arctanh(a*x)*a^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2+3*arctanh(a*x)*a
^2*x^2*(-a^2*x^2+1)^(1/2)-3*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2
+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2-a*x*(-a^2*x^2+1)^(1/2)-(-
a^2*x^2+1)^(1/2)*arctanh(a*x))/x^2/(a*x+1)/(a*x-1)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{x^3 (-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{x^3\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{x^3 (1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^2x^5 - \sqrt{-a^2x^2+1}\,x^3}dx\right)$$

input
```
int(atanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**5 - sqrt( - a**2*x**2 +
1)*x**3),x)
```

3.395 $\int \frac{x^m \textbf{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}} \, dx$$
$$\Big\downarrow \text{6651}$$
$$\int \frac{x^m \text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx$$

input
```
Int[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \operatorname{arctanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1),
x)
```

Sympy [N/A]

Not integrable

Time = 43.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**m*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}\left(ax\right)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 3.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atanh}(a\,x)^2}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^m*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^m*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{x^m atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^m*atanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
 - int((x**m*atanh(a*x)**2)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a*
*2*x**2 + 1)),x)
```

3.396 $\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{2}{a^4\sqrt{1-a^2x^2}} - \frac{2x\text{arctanh}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{4\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a^4} + \frac{\text{arctanh}(ax)^2}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^4} + \frac{2i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^4} - \frac{2i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^4}$$

output
```
2/a^4/(-a^2*x^2+1)^(1/2)-2*x*arctanh(a*x)/a^3/(-a^2*x^2+1)^(1/2)+4*arctan(
(-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4+arctanh(a*x)^2/a^4/(-a^2*x^
2+1)^(1/2)+(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/a^4+2*I*polylog(2,-I*(-a*x+1)
^(1/2)/(a*x+1)^(1/2))/a^4-2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^
4
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.89

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{2i\,\text{PolyLog}\left(2,-ie^{-\text{arctanh}(ax)}\right)+\frac{2+(2-a^2x^2)\text{arctanh}(ax)^2-2\text{arctanh}(ax)\left(ax-i\sqrt{1-a^2x^2}\log\right.}{}}{a^4}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
((2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (2 + (2 - a^2*x^2)*ArcTanh[a*x]^2
 - 2*ArcTanh[a*x]*(a*x - I*Sqrt[1 - a^2*x^2]*Log[1 - I/E^ArcTanh[a*x]] + I
*Sqrt[1 - a^2*x^2]*Log[1 + I/E^ArcTanh[a*x]]) - (2*I)*Sqrt[1 - a^2*x^2]*Po
lyLog[2, I/E^ArcTanh[a*x]])/Sqrt[1 - a^2*x^2])/a^4
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6590, 6556, 6512, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{x\,\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \frac{x\,\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a^2}$$

$$\Big\downarrow \text{6556}$$

$$\frac{\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{\frac{2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)^2}{a^2}}{a^2}$$

$$\Big\downarrow \text{6512}$$

$$\frac{\frac{\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^{3/2}}dx}{a}}{a^2}-\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}}{a^2}}{}$$

$\Big\downarrow$ 6520

$$\frac{\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\,\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}}{a^2}-\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
(ArcTanh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2]) - (2*(-(1/(a*Sqrt[1 - a^2*x^2])) +
 (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/a)/a^2 - (-((Sqrt[1 - a^2*x^2]*ArcTa
nh[a*x]^2)/a^2) + (2*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x]
)/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2,
 (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))/a)/a^2
```

Defintions of rubi rules used

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.18

method	result
default	$\frac{\sqrt{-a^2x^2+1}\left(a^2x^2\operatorname{arctanh}(ax)^2-2i\sqrt{-a^2x^2+1}\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)+2i\sqrt{-a^2x^2+1}\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)\right.}{a^4(a^2x^2-1)}$

input
```
int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-a^2*x^2+1)^(1/2)*(a^2*x^2*arctanh(a*x)^2-2*I*(-a^2*x^2+1)^(1/2)*arctanh(
a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I*(-a^2*x^2+1)^(1/2)*arctanh(a*x
)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*x*arctanh(a*x)+2*I*dilog(1-I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*(-a^2*x^2+1)^(1/2)-2*I*(-a^2*x^2+1)^(1/2)*dilog(1+
I*(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)^2-2)/a^4/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx=\int \frac{x^3\,\text{artanh}\,(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1),
x)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx=\int \frac{x^3\,\text{atanh}^2\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**3*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx=\int \frac{x^3\,\text{artanh}\,(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3\,\text{atanh}(a\,x)^2}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^3*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2\,x^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^3*atanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)**2*x**3)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a*
*2*x**2 + 1)),x)
```

3.397 $\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\text{arctanh}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2}{a^3} + \frac{2i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{a^3} - \frac{2i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{a^3} - \frac{2i\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^3} + \frac{2i\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^3}$$

output
```
2*x/a^2/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh
(a*x)^2/a^2/(-a^2*x^2+1)^(1/2)-2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctan
h(a*x)^2/a^3+2*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
-2*I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-2*I*polylog(
3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+2*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^
(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.13

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{i\left(-\frac{2iax}{\sqrt{1-a^2x^2}}+\frac{2i\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{iax\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}+\text{arctanh}(ax)^2\log\left(1-ie^{-\text{arctanh}(a\cdots}\right.\right.}{}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
(I*(((-2*I)*a*x)/Sqrt[1 - a^2*x^2] + ((2*I)*ArcTanh[a*x])/Sqrt[1 - a^2*x^2
] - (I*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - I/E^
ArcTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*
PolyLog[2, (-I)/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*
x]] + 2*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))
/a^3
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6590, 6514, 3042, 4668, 3011, 2720, 6524, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a^2}$$

$$\Big\downarrow \text{ 6514}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2} - \frac{\int \text{arctanh}(ax)^2 \csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{a^3}$$

$\downarrow$ 4668

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2} - \frac{-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+}{a^3}$$

$\downarrow$ 3011

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2} - \frac{2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,ie^{\text{a}}\right.\right.}{a^3}$$

$\downarrow$ 2720

$$\frac{\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a^2} - \frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$

$\downarrow$ 6524

$$\frac{2\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}}{a^2} - \frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$

$\downarrow$ 208

$$\frac{\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}}{a^2} - \frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$

$\downarrow$ 7143

$$\frac{\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}}{a^2} - \frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-}{a^3}$$

input
```
Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
((2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*Arc
Tanh[a*x]^2)/Sqrt[1 - a^2*x^2])/a^2 - (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*
x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3,
 (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]
]) + PolyLog[3, I*E^ArcTanh[a*x]]))/a^3
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1),
x)
```

Sympy [F]

$$\int \frac{x^2\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**2*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{atanh}(a\,x)^2}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^2*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2 \text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2\,x^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^2*atanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
 - int((atanh(a*x)**2*x**2)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a*
*2*x**2 + 1)),x)
```

3.398 $\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{2}{a^2\sqrt{1-a^2x^2}} - \frac{2x\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}} + \frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}$$

output
```
2/a^2/(-a^2*x^2+1)^(1/2)-2*x*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x
)^2/a^2/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{2-2ax\text{arctanh}(ax)+\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}$$

input
```
Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
(2 - 2*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6556, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6556}$$

$$\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{ 6520}$$

$$\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}$$

input
```
Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

output
```
ArcTanh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2]) - (2*(-(1/(a*Sqrt[1 - a^2*x^2])) +
(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/a
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(-2ax\,\operatorname{arctanh}(ax)+\operatorname{arctanh}(ax)^2+2\right)}{a^2(a^2x^2-1)}$
orering	$-\frac{(ax-1)(ax+1)(8a^4x^4+3a^2x^2+2)\operatorname{arctanh}(ax)^2}{x^2a^4(-a^2x^2+1)^{\frac{3}{2}}}-\frac{(ax+1)^2(ax-1)^2(7a^2x^2+2)\left(\frac{\operatorname{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{2x\,\operatorname{arctanh}(ax)a}{(-a^2x^2+1)^{\frac{5}{2}}}+\frac{3x^2\,\operatorname{arctanh}(}{(-a^2x^2+}\right.}{a^4x^2}$

input
```
int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-a^2*x^2+1)^(1/2)*(-2*a*x*arctanh(a*x)+arctanh(a*x)^2+2)/(a^2*x^2-
1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{x\mathrm{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx=\frac{\sqrt{-a^2x^2+1}\left(4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\log\left(-\frac{ax+1}{ax-1}\right)^2-8\right)}{4\,(a^4x^2-a^2)}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-a^2*x^2 + 1)*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(
a*x - 1))^2 - 8)/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{atanh}^2\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = -\frac{2\,x\,\text{artanh}\,(ax)}{\sqrt{-a^2x^2+1}a} + \frac{\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}a^2} + \frac{2}{\sqrt{-a^2x^2+1}a^2}$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-2*x*arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^2/(sqrt(-a^2*x^2 +
 1)*a^2) + 2/(sqrt(-a^2*x^2 + 1)*a^2)
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{artanh}\,(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{atanh}(a\,x)^2}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x*atanh(a*x)^2)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2\,x}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x*atanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)**2*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*
x**2 + 1)),x)
```

3.399 $\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx=\frac{2x}{\sqrt{1-a^2x^2}}-\frac{2\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}$$

output
```
2*x/(-a^2*x^2+1)^(1/2)-2*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^
2/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx=\frac{2ax-2\operatorname{arctanh}(ax)+ax\operatorname{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

output
```
(2*a*x - 2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6524, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6524}$$

$$2\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

output
```
(2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcT
anh[a*x]^2)/Sqrt[1 - a^2*x^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\left(\operatorname{arctanh}\left(ax\right)^{2}ax+2ax-2\operatorname{arctanh}\left(ax\right)\right)}{a\left(a^{2}x^{2}-1\right)}$
orering	$\frac{\left(-12a^{4}x^{5}+11a^{2}x^{3}+x\right)\operatorname{arctanh}\left(ax\right)^{2}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}-\frac{\left(ax+1\right)^{2}\left(ax-1\right)^{2}\left(8a^{2}x^{2}+1\right)\left(\frac{2\operatorname{arctanh}\left(ax\right)a}{\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}+\frac{3\operatorname{arctanh}\left(ax\right)^{2}a^{2}x}{\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}\right)}{a^{2}}-\frac{x\left(ax+1\right)^{3}\left(ax-1\right)}{}$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^2*a*x+2*a*x-2*arctanh(a*x))/(a^2*x^2
-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^2+8\,ax-4\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{4\left(a^3x^2-a\right)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1))^2 + 8*a*x - 4*log(-
(a*x + 1)/(a*x - 1)))/(a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \frac{x\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}} + \frac{2x}{\sqrt{-a^2x^2+1}} - \frac{2\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1) + 2*x/sqrt(-a^2*x^2 + 1) - 2*arctanh(a
*x)/(sqrt(-a^2*x^2 + 1)*a)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
+ 1)),x)
```

3.400 $\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx = \frac{2}{\sqrt{1-a^2x^2}} - \frac{2ax\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} + \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} - 2\operatorname{arctanh}\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2 - 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right) + 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right) + 2\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right) - 2\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)$$

output
```
2/(-a^2*x^2+1)^(1/2)-2*a*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^2/
(-a^2*x^2+1)^(1/2)-2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*
arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog
(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*
polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx = \frac{2}{\sqrt{1-a^2x^2}} - \frac{2ax\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} + \frac{\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} + \operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - \operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 2\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 2\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x
]^2/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTan
h[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 2*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E
^(-ArcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a*x])]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6592, 6556, 6520, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{x\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx + \int \frac{\operatorname{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx}{a}\right)+\int\frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6520}$$

$$\int\frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \text{6582}$$

$$\int\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)+a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{26}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+i\int\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{4670}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(2i\int\text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int\text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{PolyLog}\right.\right.$$

$\Big\downarrow$ 2720

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\right.\right.$$

$\Big\downarrow$ 7143

$$a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)-\text{a}\right.\right.$$

input
```
Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
a^2*(ArcTanh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2]) - (2*(-(1/(a*Sqrt[1 - a^2*x^2]
)) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/a) + I*((2*I)*ArcTanh[E^ArcTanh[
a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]])
+ PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, E^ArcTan
h[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^2-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^2+2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2ax+2}+\operatorname{arctanl}$

input
```
int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/
2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)+arcta
nh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1
)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)
^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^
2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{atanh}^2\left(ax\right)}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**2/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)^2/(x*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x\,(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^2x^3-\sqrt{-a^2x^2+1}\,x}dx\right)$$

input
```
int(atanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**2*x**3 - sqrt( - a**2*x**2
+ 1)*x),x)
```

3.401 $\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx = \frac{2a^2x}{\sqrt{1-a^2x^2}} - \frac{2a\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} + \frac{a^2x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x} - 4a\operatorname{arctanh}(ax)\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + 2a\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - 2a\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
2*a^2*x/(-a^2*x^2+1)^(1/2)-2*a*arctanh(a*x)/(-a^2*x^2+1)^(1/2)+a^2*x*arcta
nh(a*x)^2/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x-4*a*arcta
nh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*a*polylog(2,-(-a*x+1)^(1/2
)/(a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^{3/2}}\,dx = \frac{a\left(4ax - 4\operatorname{arctanh}(ax) + 2ax\operatorname{arctanh}(ax)^2 - \frac{1}{2}ax\operatorname{arctanh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(a\right.\right.}{}$$

input
```
Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(a*(4*a*x - 4*ArcTanh[a*x] + 2*a*x*ArcTanh[a*x]^2 - (a*x*ArcTanh[a*x]^2*Cs
ch[ArcTanh[a*x]/2]^2)/2 + 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-Arc
Tanh[a*x])] - 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
+ 4*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*Sqrt[1 - a^2*x^2]
*PolyLog[2, E^(-ArcTanh[a*x])] - (2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*Sinh[Arc
Tanh[a*x]/2]^2)/(a*x)))/(2*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6592, 6524, 208, 6570, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx + \int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6524}$$

$$a^2\left(2\int \frac{1}{(1-a^2x^2)^{3/2}}dx + \frac{x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{2\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right) + \int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{208}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx+a^2\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)$$

$$\Big\downarrow \text{6570}$$

$$2a\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+a^2\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6580}$$

$$a^2\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}+2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)$$

input

```
Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

output

```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) + a^2*((2*x)/Sqrt[1 - a^2*x^2] - (
2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^
2]) + 2*a*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[
2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])
```

Defintions of rubi rules used

rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 6524

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(151) = 302$.

Time = 0.51 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2\,\mathrm{arctanh}(ax)^2\sqrt{-a^2x^2+1}\,a^2x^2+2\,\mathrm{arctanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^3x^3-2\,\mathrm{arctanh}(ax)a\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)x-2\,\mathrm{arctanh}(ax)}{}$

input
```
int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)*a^2*x^2+2*arctanh(a*x)*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))*a^3*x^3-2*arctanh(a*x)*a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/
2))*x-2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^3+2*arctanh(a*
x)*a*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*x+2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^
(1/2))*a^3*x^3-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))*x-2*polylog(2,(a
*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^3+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2)
)*x+2*(-a^2*x^2+1)^(1/2)*a^2*x^2-2*x*arctanh(a*x)*a*(-a^2*x^2+1)^(1/2)-(-a
^2*x^2+1)^(1/2)*arctanh(a*x)^2)/(a^2*x^2-1)/x
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}^2\left(ax\right)}{x^2\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**2/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)^2/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^2 (1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^2x^4-\sqrt{-a^2x^2+1}\,x^2}dx\right)$$

input
```
int(atanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**2*x**4 - sqrt( - a**2*x**2
+ 1)*x**2),x)
```

3.402 $\int \frac{\text{arctanh}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\begin{aligned}\int \frac{\text{arctanh}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx &= \frac{2a^2}{\sqrt{1-a^2x^2}}-\frac{2a^3x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}\\ &-\frac{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}+\frac{a^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}\\ &-3a^2\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2-a^2\arctan\left(\sqrt{1-a^2x^2}\right)\\ &-3a^2\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\\ &+3a^2\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{\text{arctanh}(ax)}\right)\\ &+3a^2\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-3a^2\,\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\end{aligned}$$

output
```
2*a^2/(-a^2*x^2+1)^(1/2)-2*a^3*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-a*(-a^2*x
^2+1)^(1/2)*arctanh(a*x)/x+a^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-1/2*(-a^2
*x^2+1)^(1/2)*arctanh(a*x)^2/x^2-3*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
*arctanh(a*x)^2-a^2*arctanh((-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog
(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2
*x^2+1)^(1/2))+3*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(
3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.90 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx = \frac{1}{8}a^2\left(\frac{16}{\sqrt{1-a^2x^2}} - \frac{16ax\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} + \frac{8\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{2ax\operatorname{arctanh}(ax)\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{\sqrt{1-a^2x^2}} - \operatorname{arctanh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) + 12\operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - 12\operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 8\log\left(\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right) + 24\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 24\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 24\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 24\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right) - \operatorname{arctanh}(ax)^2\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right) + 4\operatorname{arctanh}(ax)\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(a^2*(16/Sqrt[1 - a^2*x^2] - (16*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (8*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x]*Csch[ArcTanh[a*x]/
2]^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*x]/2]^2 + 12*ArcTa
nh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]^2*Log[1 + E^(-ArcTa
nh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 24*ArcTanh[a*x]*PolyLog[2, -E^(-
ArcTanh[a*x])] - 24*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 24*PolyLo
g[3, -E^(-ArcTanh[a*x])] - 24*PolyLog[3, E^(-ArcTanh[a*x])] - ArcTanh[a*x]
^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.43, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6592, 6588, 6570, 243, 73, 221, 6582, 3042, 26, 4670, 3011, 2720, 6592, 6556, 6520, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}dx+\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6588}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6570}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x(1-a^2x^2)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+ a\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+ a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6582}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)+ a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+ a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{26}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}ia^2\int \text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+ a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{4670}$$

$$a^2\int \frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+ \frac{1}{2}ia^2\left(2i\int \text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int \text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(a\right. a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ia^2\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{Poly}\right.\right.$$
$$a^2\int\frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 2720

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a^2\int\frac{\text{arctanh}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}dx+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6592

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a^2\left(a^2\int\frac{x\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx+\int\frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx\right)+$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6556

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int\frac{\text{arctanh}(ax)}{\left(1-a^2x^2\right)^{3/2}}dx}{a}\right)+\int\frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx\right)+$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6520

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+a^2\left(\int\frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)\right)\right)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\downarrow$ 6582

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+a^2\left(\int\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)+a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)\right)\right)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\downarrow$ 3042

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)\right)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}\right.$$

$\downarrow$ 26

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+i\int\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)\right)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}\right.$$

$\downarrow$ 4670

$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(2i\int\text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(a\right.\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 3011

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-2i\left(\int\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\right.\right.\right.$$
$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 2720

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de\right.\right.\right.$$
$$\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 7143

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{Pol}\right.\right.\right.$$
$$\frac{1}{2}ia^2\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

input
```
Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2 + a*(-((Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]) + (I/2)*a^2*((2*I)*ArcTanh[E^
ArcTanh[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^ArcTanh
[a*x]]) + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog[2,
E^ArcTanh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]])) + a^2*(a^2*(ArcTanh[a*x]^2
/(a^2*Sqrt[1 - a^2*x^2]) - (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x
])/Sqrt[1 - a^2*x^2]))/a) + I*((2*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^
2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]]) + PolyLog[3, -E^Arc
Tanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]]) + PolyLog[
3, E^ArcTanh[a*x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.42

method	result
default	$-\frac{a^{2}\left(\operatorname{arctanh}(ax)^{2}-2\operatorname{arctanh}(ax)+2\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^{2}+2\operatorname{arctanh}(ax)+2\right)a^{2}\sqrt{-(ax-1)(ax+1)}}{2ax+2}-\frac{(2ax}{}$

input
```
int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1
)+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+
1)-1/2*(2*a*x+arctanh(a*x))*arctanh(a*x)*(-(a*x-1)*(a*x+1))^(1/2)/x^2-2*a^
2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))-3*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*ln(1
+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x
^2+1)^(1/2))+3*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^{3/2}} \, dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3} \, dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^{3/2}} \, dx = \int \frac{\operatorname{atanh}^2\left(ax\right)}{x^3\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**2/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{x^3\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)^2/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{x^3 (1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^2x^5 - \sqrt{-a^2x^2+1}\,x^3}dx\right)$$

input
```
int(atanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**2*x**5 - sqrt( - a**2*x**2
+ 1)*x**3),x)
```

3.403 $\int \frac{x^m \textbf{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx$$

input
```
Int[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1),
x)
```

Sympy [N/A]

Not integrable

Time = 97.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \,\text{atanh}^3\,(ax)}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**m*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 4.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atanh}(a\,x)^3}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^m*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^m*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m \operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{x^m\, atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^m*atanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((x**m*atanh(a*x)**3)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a*
*2*x**2 + 1)),x)
```

3.404 $\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\begin{aligned}\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx &= -\frac{6x}{a^3\sqrt{1-a^2x^2}}+\frac{6\text{arctanh}(ax)}{a^4\sqrt{1-a^2x^2}}\\ &\quad -\frac{3x\text{arctanh}(ax)^2}{a^3\sqrt{1-a^2x^2}}-\frac{6\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2}{a^4}+\frac{\text{arctanh}(ax)^3}{a^4\sqrt{1-a^2x^2}}\\ &\quad +\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^4}+\frac{6i\text{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{a^4}\\ &\quad -\frac{6i\text{arctanh}(ax)\operatorname{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{a^4}\\ &\quad -\frac{6i\operatorname{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^4}+\frac{6i\operatorname{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^4}\end{aligned}$$

output
```
-6*x/a^3/(-a^2*x^2+1)^(1/2)+6*arctanh(a*x)/a^4/(-a^2*x^2+1)^(1/2)-3*x*arct
anh(a*x)^2/a^3/(-a^2*x^2+1)^(1/2)-6*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arc
tanh(a*x)^2/a^4+arctanh(a*x)^3/a^4/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(1/2)*a
rctanh(a*x)^3/a^4+6*I*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)/a^4-6*I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-6*I*pol
ylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+6*I*polylog(3,I*(a*x+1)/(-a^2*x^
2+1)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.13

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{6i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{-\text{arctanh}(ax)}\right)-6i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{-\text{arctanh}(}\right.}{}$$

input
```
Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*P
olyLog[2, I/E^ArcTanh[a*x]] + (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x
]^2 + 2*ArcTanh[a*x]^3 - a^2*x^2*ArcTanh[a*x]^3 + (3*I)*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - (3*I)*Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, (-I
)/E^ArcTanh[a*x]] - (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, I/E^ArcTanh[a*x]])/
Sqrt[1 - a^2*x^2])/a^4
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6590, 6556, 6514, 3042, 4668, 3011, 2720, 6524, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{a^2}$$

$$\Big\downarrow \text{ 6556}$$

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{\frac{3\int\frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}}{a^2}$$

$\downarrow$ 6514

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{\frac{3\int\sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}}{a^2}$$

$\downarrow$ 3042

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\int\text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{a^2}}{a^2}$$

$\downarrow$ 4668

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(-2i\int\text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int\text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arct}\right.}{a^2}}{a^2}$$

$\downarrow$ 3011

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int\text{PolyL}\right.\right.}{}}{a^2}$$

$\downarrow$ 2720

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int\frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}}{a^2}-\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)\right.}{}}{}$$

$\downarrow$ 6524

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(2\int\frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)}{a}}{a^2}-$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right.\right.}{}$$

$\downarrow$ 208

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}}{a^2}-$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right.\right.}{}$$

$\downarrow$ 7143

$$\frac{\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}}{a^2}-$$

$$\frac{-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{a^2}+\frac{3\left(2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arc}\ldots}\right.\right.\right.}{a^2}}{a^2}$$

input
```
Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
(ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^2*x^2]) - (3*((2*x)/Sqrt[1 - a^2*x^2] - (2
*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2
]))/a)/a^2 - (-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/a^2) + (3*(2*ArcTan[E^A
rcTanh[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcT
anh[a*x]]) + PolyLog[3, (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*Poly
Log[2, I*E^ArcTanh[a*x]]) + PolyLog[3, I*E^ArcTanh[a*x]])))/a^2)/a^2
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^3 \operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}dx$$

input
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1),
x)
```

Sympy [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3\,\text{atanh}^3\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**3*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3\,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^3\,\text{atanh}(a\,x)^3}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^3*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3 \text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3\,x^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^3*atanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)**3*x**3)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a*
*2*x**2 + 1)),x)
```

3.405 $\int \frac{x^2\textbf{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 246

$$\begin{aligned}\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx &= -\frac{6}{a^3\sqrt{1-a^2x^2}}+\frac{6x\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}\\ &-\frac{3\text{arctanh}(ax)^2}{a^3\sqrt{1-a^2x^2}}+\frac{x\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{2\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^3}{a^3}\\ &+\frac{3i\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{a^3}\\ &-\frac{3i\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{a^3}\\ &-\frac{6i\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{a^3}\\ &+\frac{6i\text{arctanh}(ax)\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{a^3}\\ &+\frac{6i\,\text{PolyLog}\left(4,-ie^{\text{arctanh}(ax)}\right)}{a^3}-\frac{6i\,\text{PolyLog}\left(4,ie^{\text{arctanh}(ax)}\right)}{a^3}\end{aligned}$$

output
```
-6/a^3/(-a^2*x^2+1)^(1/2)+6*x*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)-3*arctan
h(a*x)^2/a^3/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^3/a^2/(-a^2*x^2+1)^(1/2)-2*
arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a^3+3*I*arctanh(a*x)^2*p
olylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-3*I*arctanh(a*x)^2*polylog(2,I
*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-6*I*arctanh(a*x)*polylog(3,-I*(a*x+1)/(-a
^2*x^2+1)^(1/2))/a^3+6*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/
2))/a^3+6*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-6*I*polylog(4,I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 541 vs. 2(246) = 492.

Time = 0.66 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.20

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{7i\pi^4 - \frac{384}{\sqrt{1-a^2x^2}} - 8\pi^3\text{arctanh}(ax) + \frac{384ax\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} + 24i\pi^2\text{arctanh}(ax)^2 - \frac{192a\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
((7*I)*Pi^4 - 384/Sqrt[1 - a^2*x^2] - 8*Pi^3*ArcTanh[a*x] + (384*a*x*ArcTa
nh[a*x])/Sqrt[1 - a^2*x^2] + (24*I)*Pi^2*ArcTanh[a*x]^2 - (192*ArcTanh[a*x
]^2)/Sqrt[1 - a^2*x^2] + 32*Pi*ArcTanh[a*x]^3 + (64*a*x*ArcTanh[a*x]^3)/Sq
rt[1 - a^2*x^2] - (16*I)*ArcTanh[a*x]^4 - 8*Pi^3*Log[1 + I/E^ArcTanh[a*x]]
 + (48*I)*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 96*Pi*ArcTanh[a*x]
^2*Log[1 + I/E^ArcTanh[a*x]] - (64*I)*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a
*x]] - (48*I)*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] - 96*Pi*ArcTanh[
a*x]^2*Log[1 - I*E^ArcTanh[a*x]] + 8*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + (64*
I)*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] - 8*Pi^3*Log[Tan[(Pi + (2*I)*A
rcTanh[a*x])/4]] - (48*I)*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^Ar
cTanh[a*x]] + (192*I)*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (48
*I)*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] - 192*Pi*ArcTanh[a*x]*PolyLog[2, I*E
^ArcTanh[a*x]] - 192*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + (384*I)*ArcTanh[
a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (384*I)*ArcTanh[a*x]*PolyLog[3, (-I
)*E^ArcTanh[a*x]] + 192*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + (384*I)*PolyLog[
4, (-I)/E^ArcTanh[a*x]] + (384*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/(64*a^3
)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6590, 6514, 3042, 4668, 3011, 6524, 6520, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6590}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \frac{\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx}{a^2}$$

$$\Big\downarrow \text{6514}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3d\text{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{\int \text{arctanh}(ax)^3\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{a^3}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{-3i\int \text{arctanh}(ax)^2\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+3i\int \text{arctanh}(ax)^2\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\cdot}{a^3}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx}{a^2}-\frac{3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-3i\left(2\right.}{}$$

$$\Big\downarrow \text{6524}$$

$$\frac{6\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx + \frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}}{a^2} - \frac{}{3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax) - \text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right) - 3i\left(2\right.}$$

$\downarrow$ 6520

$$\frac{\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}} + 6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right)}{a^2} - \frac{}{3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax) - \text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right) - 3i\left(2\right.}$$

$\downarrow$ 7163

$$\frac{\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}} + 6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right)}{a^2} - \frac{}{3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right) - \int \text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right) - \text{arctanh}(ax)^2\,\text{Poly}\right.}$$

$\downarrow$ 2720

$$\frac{\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}} + 6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right)}{a^2} - \frac{}{3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right) - \int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right) - \text{arctanh}\right.}$$

$\downarrow$ 7143

$$\frac{\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}} + 6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{1}{a\sqrt{1-a^2x^2}}\right)}{a^2} - \frac{}{2\text{arctanh}(ax)^3\arctan\left(e^{\text{arctanh}(ax)}\right) + 3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right) - \text{PolyLog}\left(4,-ie^{\text{arctanh}(ax)}\right)\right)\right.}$$

input

```
Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output

```
((-3*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a
^2*x^2] + 6*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^
2]))/a^2 - (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 + (3*I)*(-(ArcTanh[a*x
]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, (-I)*E^A
rcTanh[a*x]] - PolyLog[4, (-I)*E^ArcTanh[a*x]])) - (3*I)*(-(ArcTanh[a*x]^2
*PolyLog[2, I*E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*
x]] - PolyLog[4, I*E^ArcTanh[a*x]])))/a^3
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6590
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*A
rcTanh[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcT
anh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && In
tegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2 \operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1),
x)
```

Sympy [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{atanh}^3\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x**2*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x^2\,\text{atanh}(a\,x)^3}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x^2*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3\,x^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x^2*atanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)**3*x**2)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a*
*2*x**2 + 1)),x)
```

3.406 $\int \frac{x\textbf{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx=-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{6\text{arctanh}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{3x\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

output
```
-6*x/a/(-a^2*x^2+1)^(1/2)+6*arctanh(a*x)/a^2/(-a^2*x^2+1)^(1/2)-3*x*arctan
h(a*x)^2/a/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^3/a^2/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx=\frac{-6ax+6\text{arctanh}(ax)-3ax\text{arctanh}(ax)^2+\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

input
```
Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
(-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^2 + ArcTanh[a*x]^3)/(a^2*Sqr
t[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6556, 6524, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx$$
$$\Big\downarrow \text{6556}$$
$$\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}$$
$$\Big\downarrow \text{6524}$$
$$\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(2\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)}{a}$$
$$\Big\downarrow \text{208}$$
$$\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}$$

input
```
Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

output
```
ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^2*x^2]) - (3*((2*x)/Sqrt[1 - a^2*x^2] - (2*
ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2]
))/a
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(-3\operatorname{arctanh}(ax)^2ax+\operatorname{arctanh}(ax)^3-6ax+6\operatorname{arctanh}(ax)\right)}{a^2(a^2x^2-1)}$
orering	$-\frac{4(ax-1)(ax+1)(6a^6x^6+a^4x^4+a^2x^2+2)\operatorname{arctanh}(ax)^3}{x^2a^4(-a^2x^2+1)^{\frac{3}{2}}}-\frac{2(ax+1)^2(ax-1)^2(18a^4x^4+7a^2x^2+4)\left(\frac{\operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{3x\operatorname{arctanh}(a}{(-a^2x^2+1)}\right.}{a^4x^2}$

input
```
int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-a^2*x^2+1)^(1/2)*(-3*arctanh(a*x)^2*a*x+arctanh(a*x)^3-6*a*x+6*ar
ctanh(a*x))/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\sqrt{-a^2x^2+1}\left(6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2 - \log\left(-\frac{ax+1}{ax-1}\right)^3 + 48\,ax - 24\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{8\,(a^4x^2-a^2)}$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(-a^2*x^2 + 1)*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - log(-(a*x + 1)
/(a*x - 1))^3 + 48*a*x - 24*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [F]

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{atanh}^3\,(ax)}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(x*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\frac{3\,x\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}a} + \frac{\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}a^2} - \frac{6\left(\frac{x}{\sqrt{-a^2x^2+1}} - \frac{\text{artanh}(ax)}{\sqrt{-a^2x^2+1}a}\right)}{a}$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-3*x*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^3/(sqrt(-a^2*x^2
 + 1)*a^2) - 6*(x/sqrt(-a^2*x^2 + 1) - arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a)
)/a
```

Giac [F]

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{artanh}\,(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{x\,\text{atanh}(a\,x)^3}{(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2),x)
```

output
```
int((x*atanh(a*x)^3)/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3\,x}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(x*atanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int((atanh(a*x)**3*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*
x**2 + 1)),x)
```

3.407 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx=-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{6x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{3\operatorname{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}$$

output
```
-6/a/(-a^2*x^2+1)^(1/2)+6*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)
^2/a/(-a^2*x^2+1)^(1/2)+x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx=\frac{-6+6ax\operatorname{arctanh}(ax)-3\operatorname{arctanh}(ax)^2+ax\operatorname{arctanh}(ax)^3}{a\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

output
```
(-6 + 6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + a*x*ArcTanh[a*x]^3)/(a*Sqrt[
1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6524, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6524$$

$$6\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\operatorname{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 6520$$

$$\frac{x\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\operatorname{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

output
```
(-3*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^
2*x^2] + 6*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2
])
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\left(\operatorname{arctanh}(ax)^{3}ax+6ax\operatorname{arctanh}(ax)-3\operatorname{arctanh}(ax)^{2}-6\right)}{a(a^{2}x^{2}-1)}$
orering	$-\frac{8(ax-1)(ax+1)x(9a^{2}x^{2}-4)\operatorname{arctanh}(ax)^{3}}{(-a^{2}x^{2}+1)^{\frac{3}{2}}}-\frac{2(ax+1)^{2}(ax-1)^{2}(34a^{2}x^{2}-5)\left(\frac{3\operatorname{arctanh}(ax)^{2}a}{(-a^{2}x^{2}+1)^{\frac{5}{2}}}+\frac{3\operatorname{arctanh}(ax)^{3}a^{2}x}{(-a^{2}x^{2}+1)^{\frac{5}{2}}}\right)}{a^{2}}-\frac{16x(ax}{}$

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^3*a*x+6*a*x*arctanh(a*x)-3*arctanh(a
*x)^2-6)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\frac{\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^3+24\,ax\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(-\frac{ax+1}{ax-1}\right)^2-48\right)\sqrt{-a^2x^2+1}}{8\,(a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^3 + 24*a*x*log(-(a*x + 1)/(a*x - 1)) -
 6*log(-(a*x + 1)/(a*x - 1))^2 - 48)*sqrt(-a^2*x^2 + 1)/(a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{x\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}} + 6\,a\left(\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}a} - \frac{1}{\sqrt{-a^2x^2+1}a^2}\right) - \frac{3\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1) + 6*a*(x*arctanh(a*x)/(sqrt(-a^2*x^2 +
 1)*a) - 1/(sqrt(-a^2*x^2 + 1)*a^2)) - 3*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1
)*a)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x)
```

3.408 $\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = &-\frac{6ax}{\sqrt{1-a^2x^2}}+\frac{6\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\\ &-\frac{3ax\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}+\frac{\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^3\\ &-3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\\ &+3\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right)\\ &+6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right)\\ &-6\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)\\ &-6\operatorname{PolyLog}\left(4,-e^{\operatorname{arctanh}(ax)}\right)+6\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)\end{aligned}$$

output
```
-6*a*x/(-a^2*x^2+1)^(1/2)+6*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*a*x*arctanh(
a*x)^2/(-a^2*x^2+1)^(1/2)+arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)-2*arctanh((a*x
+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-3*arctanh(a*x)^2*polylog(2,-(a*x+1)
/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2)
)+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*pol
ylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \frac{1}{8}\Bigg(\pi^4 - \frac{48ax}{\sqrt{1-a^2x^2}} + \frac{48\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{24ax\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{8\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - 2\operatorname{arctanh}(ax)^4 - 8\operatorname{arctanh}(ax)^3\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 8\operatorname{arctanh}(ax)^3\log\left(1-e^{\operatorname{arctanh}(ax)}\right) + 24\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) + 24\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right) + 48\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 48\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,-e^{-\operatorname{arctanh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)\Bigg)$$

input
```
Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(Pi^4 - (48*a*x)/Sqrt[1 - a^2*x^2] + (48*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] -
 (24*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x]^3)/Sqrt[1 - a
^2*x^2] - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] +
 8*ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -
E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*Arc
Tanh[a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^Ar
cTanh[a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[
a*x]])/8
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6592, 6556, 6524, 208, 6582, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6556}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx}{a}\right)+\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6524}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(2\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{208}$$

$$\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx+a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \text{6582}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{ax}d\text{arctanh}(ax)+a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+\int i\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{26}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\int \text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 4670}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(3i\int \text{arctanh}(ax)^2\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-3i\int \text{arctanh}(ax)^2\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax\right.$$

$$\Big\downarrow \text{ 3011}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-3i\left(2\int \text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right.$$

$$\Big\downarrow \text{ 7163}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-3i\left(2\left(\text{arctanh}(ax)\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right)-\text{arctanh}(ax\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-3i\left(2\left(\text{arctanh}(ax)\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right)-\right.\right.$$

$$\Big\downarrow \text{ 7143}$$

$$a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+$$
$$i\left(-3i\left(2\left(\text{arctanh}(ax)\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{PolyLog}\left(4,-e^{\text{arctanh}(ax)}\right)\right)-\text{arctanh}(ax)^2\text{PolyLog}\left(2,-e^{\text{a}}\right.\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
a^2*(ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^2*x^2]) - (3*((2*x)/Sqrt[1 - a^2*x^2]
- (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/Sqrt[1 - a^2
*x^2]))/a) + I*((2*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - (3*I)*(-(Ar
cTanh[a*x]^2*PolyLog[2, -E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, -E^
ArcTanh[a*x]] - PolyLog[4, -E^ArcTanh[a*x]])) + (3*I)*(-(ArcTanh[a*x]^2*Po
lyLog[2, E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - P
olyLog[4, E^ArcTanh[a*x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\left(\operatorname{arctanh}(ax)^3-3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^3+3\operatorname{arctanh}(ax)^2+6\operatorname{arctanh}(ax)+6\right)}{2ax+2}$

input
```
int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))
^(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-(a
*x-1)*(a*x+1))^(1/2)/(a*x+1)+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2
))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*p
olylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2
))-arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylo
g(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x
^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx=\int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\,(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)^3/(x*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x\,(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^3-\sqrt{-a^2x^2+1}\,x}dx\right)$$

input
```
int(atanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**2*x**3 - sqrt( - a**2*x**2
+ 1)*x),x)
```

3.409 $\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx &= -\frac{6a}{\sqrt{1-a^2x^2}}+\frac{6a^2x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}\\ &\quad-\frac{3a\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-6a\operatorname{arctanh}\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+\frac{a^2x\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}\\ &\quad-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{x}-6a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\\ &\quad+6a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right)\\ &\quad+6a\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right)-6a\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)\end{aligned}$$

output
```
-6*a/(-a^2*x^2+1)^(1/2)+6*a^2*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*a*arctan
h(a*x)^2/(-a^2*x^2+1)^(1/2)-6*a*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctan
h(a*x)^2+a^2*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)*arctan
h(a*x)^3/x-6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arc
tanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a
^2*x^2+1)^(1/2))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx = -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{a^2x\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{a^2x\operatorname{arctanh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{4\sqrt{1-a^2x^2}} + 3a\operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - 3a\operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 6a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 6a\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 6a\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 6a\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right) + \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3\sinh^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{x}$$

input
```
Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a
*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x
^2] - (a^2*x*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2)/(4*Sqrt[1 - a^2*x^2])
+ 3*a*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 3*a*ArcTanh[a*x]^2*Log[1
 + E^(-ArcTanh[a*x])] + 6*a*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] -
6*a*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 6*a*PolyLog[3, -E^(-ArcTa
nh[a*x])] - 6*a*PolyLog[3, E^(-ArcTanh[a*x])] + (Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^3*Sinh[ArcTanh[a*x]/2]^2)/x
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6592, 6524, 6520, 6570, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{x^2 (1-a^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\int \frac{\text{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6524}$$

$$a^2\left(6\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}\right)+\int \frac{\text{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6520}$$

$$\int \frac{\text{arctanh}(ax)^3}{x^2\sqrt{1-a^2x^2}}dx+ a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$$\Big\downarrow \text{ 6570}$$

$$3a\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+ a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$$\Big\downarrow \text{ 6582}$$

$$3a\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+ a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$3a\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+ a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$$\Big\downarrow \text{ 26}$$

$$3ia\int \text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+ a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$$\Big\downarrow \text{ 4670}$$

$$3ia\left(2i\int \text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int \text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$\big\downarrow$ 3011

$$3ia\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{PolyL}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$\big\downarrow$ 2720

$$3ia\left(-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

$\big\downarrow$ 7143

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{x}+$$
$$a^2\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)+$$
$$3ia\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)-\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/x) + a^2*((-3*ArcTanh[a*x]^2)/(a*Sqrt
[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] + 6*(-(1/(a*Sqrt[1 -
 a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2])) + (3*I)*a*((2*I)*ArcTan
h[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^Arc
Tanh[a*x]]) + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog
[2, E^ArcTanh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(235) = 470$.

Time = 0.70 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.61

method	result
default	$-\frac{3\operatorname{arctanh}(ax)^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^3x^3-3\operatorname{arctanh}(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^3x^3+2\operatorname{arctanh}(ax)^3\sqrt{-a^2x^2+1}\,a^2x^2+6\operatorname{arctanh}}{}$

input
```
int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(3*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^3-3*arctanh(a*x)
^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^3+2*arctanh(a*x)^3*(-a^2*x^2+1)^
(1/2)*a^2*x^2+6*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^
3-6*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^3-6*polylog(3
,-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^3*x^3+6*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/
2))*a^3*x^3+6*arctanh(a*x)*a^2*x^2*(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)^2*a*x
*(-a^2*x^2+1)^(1/2)-3*arctanh(a*x)^2*a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))*x+
3*arctanh(a*x)^2*a*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*x-(-a^2*x^2+1)^(1/2)*a
rctanh(a*x)^3-6*arctanh(a*x)*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))*x+6*
arctanh(a*x)*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))*x-6*a*x*(-a^2*x^2+1)^
(1/2)+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))*x-6*a*polylog(3,(a*x+1)/(
-a^2*x^2+1)^(1/2))*x)/(a^2*x^2-1)/x
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{artanh}\left(ax\right)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{x^2(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)^3/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^4-\sqrt{-a^2x^2+1}\,x^2}dx\right)$$

input
```
int(atanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**2*x**4 - sqrt( - a**2*x**2
+ 1)*x**2),x)
```

3.410 $\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 360

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^{3/2}}\,dx &= -\frac{6a^3x}{\sqrt{1-a^2x^2}}+\frac{6a^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}} \\
&- \frac{3a^3x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2x}+\frac{a^2\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}} \\
&- \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{2x^2}-3a^2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^3 \\
&- 6a^2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\
&- \frac{9}{2}a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right) \\
&+ \frac{9}{2}a^2\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right)+3a^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\
&- 3a^2\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+9a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right) \\
&- 9a^2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right) \\
&- 9a^2\operatorname{PolyLog}\left(4,-e^{\operatorname{arctanh}(ax)}\right)+9a^2\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)\end{aligned}$$

output
```
-6*a^3*x/(-a^2*x^2+1)^(1/2)+6*a^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3*a^3*x*
arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)-3/2*a*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/
x+a^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x
)^3/x^2-3*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3-6*a^2*arc
tanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-9/2*a^2*arctanh(a*x)^2*pol
ylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+
1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3*a^
2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))+9*a^2*arctanh(a*x)*polylog(3,-(a
*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1
)^(1/2))-9*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,(a*x
+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{1}{16}a^2\left(3\pi^4-\frac{96ax}{\sqrt{1-a^2x^2}}+\frac{96\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{48ax\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\right.$$
$$+\frac{16\operatorname{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-6\operatorname{arctanh}(ax)^4-\frac{6ax\operatorname{arctanh}(ax)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{\sqrt{1-a^2x^2}}$$
$$-2\operatorname{arctanh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)+48\operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right)$$
$$-48\operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right)-24\operatorname{arctanh}(ax)^3\log\left(1+e^{-\operatorname{arctanh}(ax)}\right)$$
$$+24\operatorname{arctanh}(ax)^3\log\left(1-e^{\operatorname{arctanh}(ax)}\right)+24\left(2+3\operatorname{arctanh}(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right)$$
$$-48\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)+72\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arctanh}(ax)}\right)$$
$$+144\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right)-144\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)$$
$$+144\operatorname{PolyLog}\left(4,-e^{-\operatorname{arctanh}(ax)}\right)+144\operatorname{PolyLog}\left(4,e^{\operatorname{arctanh}(ax)}\right)$$
$$\left.-2\operatorname{arctanh}(ax)^3\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)+12\operatorname{arctanh}(ax)^2\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$

input
```
Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(a^2*(3*Pi^4 - (96*a*x)/Sqrt[1 - a^2*x^2] + (96*ArcTanh[a*x])/Sqrt[1 - a^2
*x^2] - (48*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (16*ArcTanh[a*x]^3)/Sq
rt[1 - a^2*x^2] - 6*ArcTanh[a*x]^4 - (6*a*x*ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2)/Sqrt[1 - a^2*x^2] - 2*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2 + 48*
ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*Log[1 + E^(-ArcT
anh[a*x])] - 24*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x
]^3*Log[1 - E^ArcTanh[a*x]] + 24*(2 + 3*ArcTanh[a*x]^2)*PolyLog[2, -E^(-Ar
cTanh[a*x])] - 48*PolyLog[2, E^(-ArcTanh[a*x])] + 72*ArcTanh[a*x]^2*PolyLo
g[2, E^ArcTanh[a*x]] + 144*ArcTanh[a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 1
44*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] + 144*PolyLog[4, -E^(-ArcTanh[a
*x])] + 144*PolyLog[4, E^ArcTanh[a*x]] - 2*ArcTanh[a*x]^3*Sech[ArcTanh[a*x
]/2]^2 + 12*ArcTanh[a*x]^2*Tanh[ArcTanh[a*x]/2]))/16
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.16 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.36, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6592, 6588, 6570, 6580, 6582, 3042, 26, 4670, 3011, 6592, 6556, 6524, 208, 6582, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{6592}$$

$$a^2 \int \frac{\operatorname{arctanh}(ax)^3}{x (1-a^2x^2)^{3/2}}dx+\int \frac{\operatorname{arctanh}(ax)^3}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6588}$$

$$\frac{3}{2}a \int \frac{\operatorname{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx+a^2 \int \frac{\operatorname{arctanh}(ax)^3}{x (1-a^2x^2)^{3/2}}dx+\frac{1}{2}a^2 \int \frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6570}$$

$$\frac{3}{2}a\left(2a\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\right)+a^2\int\frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 6580

$$a^2\int\frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx+\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 6582

$$a^2\int\frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{ax}d\text{arctanh}(ax)+\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 3042

$$a^2\int\frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}a^2\int i\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 26

$$a^2\int\frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}ia^2\int\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 4670

$$a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+$$
$$\frac{1}{2}ia^2\left(3i\int \text{arctanh}(ax)^2\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-3i\int \text{arctanh}(ax)^2\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 3011

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a^2\int \frac{\text{arctanh}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}dx+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 6592

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(a^2\int \frac{x\text{arctanh}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}dx+\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx\right)+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 6556

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\int \frac{\text{arctanh}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx}{a}\right)+\int \frac{\text{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx\right)+$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6524}$$

$$\frac{1}{2}ia^2\left(-3i\left(2\int \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(a^2\left(\frac{\operatorname{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(2\int\frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)}{a}\right)+\int\frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx\right)+$$
$$\frac{3}{2}a\left(2a\left(-2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{1}{2}ia^2\left(-3i\left(2\int \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(\int\frac{\operatorname{arctanh}(ax)^3}{x\sqrt{1-a^2x^2}}dx+a^2\left(\frac{\operatorname{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)\right)+$$
$$\frac{3}{2}a\left(2a\left(-2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6582}$$

$$\frac{1}{2}ia^2\left(-3i\left(2\int \operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(\int\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{ax}d\operatorname{arctanh}(ax)+a^2\left(\frac{\operatorname{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right.}{a}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+\int i\text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))da\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 26

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+i\int \text{arctanh}(ax)^3\csc(i\text{arctanh}(ax))da\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 4670

$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(3i\int \text{arctanh}(ax)^2\log\left(1-e^{\text{arctanh}}\right.\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\big\downarrow$ 3011

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,\right.\right.\right.\right.$$
$$\frac{1}{2}ia^2\left(-3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 7163

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-\right.\right.\right.\right.\right.$$
$$\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int \text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\right)-\text{arctanh}\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 2720

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-\right.\right.\right.\right.\right.$$
$$\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}\right)\right.\right.$$
$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

$\downarrow$ 7143

$$\frac{3}{2}a\left(2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-a^2}}{}\right.$$

$$a^2\left(a^2\left(\frac{\text{arctanh}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)}{a}\right)+i\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-\right.\right.\right.\right.\right.$$

$$\frac{1}{2}ia^2\left(-3i\left(2\left(\text{arctanh}(ax)\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{PolyLog}\left(4,-e^{\text{arctanh}(ax)}\right)\right)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2\right.\right.\right.$$

$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}{2x^2}$$

input
```
Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/x^2 + (3*a*(-((Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^2)/x) + 2*a*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*
x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x
]/Sqrt[1 + a*x]])))/2 + a^2*(a^2*(ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^2*x^2]) -
 (3*((2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x
*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2]))/a) + I*((2*I)*ArcTanh[E^ArcTanh[a*x]]
*ArcTanh[a*x]^3 - (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, -E^ArcTanh[a*x]]) + 2
*(ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]] - PolyLog[4, -E^ArcTanh[a*x]]))
 + (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*P
olyLog[3, E^ArcTanh[a*x]] - PolyLog[4, E^ArcTanh[a*x]])))) + (I/2)*a^2*((2
*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - (3*I)*(-(ArcTanh[a*x]^2*PolyL
og[2, -E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]] - Po
lyLog[4, -E^ArcTanh[a*x]])) + (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh
[a*x]]) + 2*(ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - PolyLog[4, E^ArcTan
h[a*x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

rule 6592
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh
[c*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Integers
Q[p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.34

method	result
default	$-\frac{a^{2}\left(\operatorname{arctanh}(ax)^{3}-3\operatorname{arctanh}(ax)^{2}+6\ \operatorname{arctanh}(ax)-6\right)\sqrt{-(ax-1)(ax+1)}}{2(ax-1)}+\frac{\left(\operatorname{arctanh}(ax)^{3}+3\operatorname{arctanh}(ax)^{2}+6\ \operatorname{arctanh}(ax)+\right.}{2ax+2}$

input
```
int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x
+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*
a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(3*a*x+arctanh(a*x))*arctanh(a*x)
^2*(-(a*x-1)*(a*x+1))^(1/2)/x^2+3/2*a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*
x^2+1)^(1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
-9*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,
(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))-9/2*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*
a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*polylog(4,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1
/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*ln(1+(
a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{x^3(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral(atanh(a*x)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3(1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{x^3\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(atanh(a*x)^3/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{x^3 (1-a^2x^2)^{3/2}}\,dx = -\left(\int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^5-\sqrt{-a^2x^2+1}\,x^3}dx\right)$$

input
```
int(atanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x)
```

output
```
- int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**2*x**5 - sqrt( - a**2*x**2
+ 1)*x**3),x)
```

3.411 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx = \text{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx$$

input
```
Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx$$

input
```
Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)}dx$$

input
```
int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

output
```
int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)),
x)
```

Sympy [N/A]

Not integrable

Time = 65.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^m}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

output
```
Integral(x**m/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\,\mathrm{artanh}\,(ax)}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 4.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}\,dx = \int \frac{x^m}{\mathrm{atanh}\,(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^m/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x^m/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x^m}{\sqrt{-a^2x^2+1}\,atanh\,(ax)\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh\,(ax)}dx\right)$$

input
```
int(x^m/(-a^2*x^2+1)^(3/2)/atanh(a*x),x)
```

output
```
- int(x**m/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**2 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)),x)
```

3.412 $\int \frac{x^2}{(1-a^2x^2)^{3/2}\mathbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \operatorname{Int}\left(\frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}dx$$

input
```
Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\,\text{arctanh}\,(ax)}dx$$

input
```
int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

output
```
int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)),
x)
```

Sympy [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

output
```
Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 4.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x^2}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^2/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x^2/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x^2}{\sqrt{-a^2x^2+1}\,atanh\,(ax)\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh\,(ax)}dx\right)$$

input
```
int(x^2/(-a^2*x^2+1)^(3/2)/atanh(a*x),x)
```

output
```
 - int(x**2/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**2 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)),x)
```

3.413 $\int \frac{x}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 9

$$\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx = \frac{\text{Shi}(\text{arctanh}(ax))}{a^2}$$

output
```
Shi(arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx = \frac{\text{Shi}(\text{arctanh}(ax))}{a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
SinhIntegral[ArcTanh[a*x]]/a^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6596, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{ax}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i\sin(i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sin(i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(\text{arctanh}(ax))}{a^2}$$

input
```
Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
SinhIntegral[ArcTanh[a*x]]/a^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\operatorname{Shi}(\operatorname{arctanh}(ax))}{a^2}$	10

input
```
int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
Shi(arctanh(a*x))/a^2
```

Fricas [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

output
```
Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{x}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{x}{\sqrt{-a^2x^2+1}\,atanh(ax)\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)}dx\right)$$

input
```
int(x/(-a^2*x^2+1)^(3/2)/atanh(a*x),x)
```

output

```
 - int(x/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**2 - sqrt( - a**2*x**2
+ 1)*atanh(a*x)),x)
```

3.414 $\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{a}$$

output
```
Chi(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
CoshIntegral[ArcTanh[a*x]]/a
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6530, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)} \, dx$$
$$\Big\downarrow \text{ 6530}$$
$$\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$
$$\Big\downarrow \text{ 3782}$$
$$\frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
CoshIntegral[ArcTanh[a*x]]/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{a}$	10

input
```
int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
Chi(arctanh(a*x))/a
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}\,dx = \int \frac{1}{\mathrm{atanh}\,(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh\,(ax)}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^(3/2)/atanh(a*x),x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**2 - sqrt( - a**2*x**2
+ 1)*atanh(a*x)),x)
```

3.415 $\int \frac{1}{x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \operatorname{Int}\left(\frac{1}{x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (1-a^2x^2)^{3/2} \text{arctanh}(ax)} \, dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{x (1-a^2x^2)^{3/2} \text{arctanh}(ax)}dx$$

input
```
Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x (-a^2x^2+1)^{\frac{3}{2}} \operatorname{arctanh}(ax)}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 5.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

output
```
Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-a^{2} x^{2}\right)^{3/2} \operatorname{arctanh}(a x)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^{2} x^{2}\right)^{3/2} \operatorname{arctanh}(a x)}\, dx = \int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

input
```
int(1/(x*atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{1}{x\left(1-a^{2} x^{2}\right)^{3/2} \operatorname{arctanh}(a x)} \, dx = -\left(\int \frac{1}{\sqrt{-a^{2} x^{2}+1}\, atanh\left(a x\right) a^{2} x^{3}-\sqrt{-a^{2} x^{2}+1}\, atanh\left(a x\right) x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)^(3/2)/atanh(a*x),x)
```

output
```
 - int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**3 - sqrt( - a**2*x**2
+ 1)*atanh(a*x)*x),x)
```

3.416 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = \text{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}dx$$

input
```
Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)^2}dx$$

input
```
int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

output
```
int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{atanh}(a\,x)^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^m/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x^m/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{x^m}{\sqrt{-a^2x^2+1}\,atanh(ax)^2\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)^2}dx\right)$$

input
```
int(x^m/(-a^2*x^2+1)^(3/2)/atanh(a*x)^2,x)
```

output
```
 - int(x**m/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**2 - sqrt( - a**2
*x**2 + 1)*atanh(a*x)**2),x)
```

3.417 $\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = -\frac{1}{a^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(\operatorname{arctanh}(ax))}{a^3} - \frac{\operatorname{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2},x\right)}{a^2}$$

output
```
-1/a^3/(-a^2*x^2+1)^(1/2)/arctanh(a*x)+Shi(arctanh(a*x))/a^3-Defer(Int)(1/
(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)/a^2
```

Mathematica [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx$$

input
```
Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \ 6590$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \ 6528$$

$$\frac{a\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \ 6596$$

$$\frac{\frac{\int \frac{ax}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{a^2}+\frac{-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}+\frac{\int -\frac{i\sin(i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}}{a^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{a^2}+\frac{-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}-\frac{i\int \frac{\sin(i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}}{a^2}$$

$$\Big\downarrow \ 3779$$

$$\frac{\frac{\text{Shi}(\text{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{6651}$$

$$\frac{\frac{\text{Shi}(\text{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{a^2}$$

input
```
Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{arctanh}\left(ax\right)^2}dx$$

input
```
int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

output
```
int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2}\,dx=\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2)
, x)
```

Sympy [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

output
```
Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{atanh}(a\,x)^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x^2/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx =$$
$$-\left(\int \frac{x^2}{\sqrt{-a^2x^2+1}\,atanh(ax)^2\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)^2}dx\right)$$

input
```
int(x^2/(-a^2*x^2+1)^(3/2)/atanh(a*x)^2,x)
```

output
```
- int(x**2/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**2 - sqrt( - a**2
*x**2 + 1)*atanh(a*x)**2),x)
```

3.418 $\int \frac{x}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = -\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)} + \frac{\text{Chi}(\text{arctanh}(ax))}{a^2}$$

output
```
-x/a/(-a^2*x^2+1)^(1/2)/arctanh(a*x)+Chi(arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = \frac{-\frac{ax}{\sqrt{1-a^2x^2}\text{arctanh}(ax)} + \text{Chi}(\text{arctanh}(ax))}{a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
(-((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]])/a
^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6568, 6530, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6568}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx}{a}-\frac{x}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6530}$$

$$\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}+\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}$$

input
```
Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
-(x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

method	result	size
default	$\frac{\operatorname{arctanh}(ax)\operatorname{Chi}(\operatorname{arctanh}(ax))a^2x^2+ax\sqrt{-a^2x^2+1}-\operatorname{Chi}(\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)}{a^2\operatorname{arctanh}(ax)(a^2x^2-1)}$	65

input
```
int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(arctanh(a*x)*Chi(arctanh(a*x))*a^2*x^2+a*x*(-a^2*x^2+1)^(1/2)-Chi(a
rctanh(a*x))*arctanh(a*x))/arctanh(a*x)/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
x)
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

output
```
Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{x}{\operatorname{atanh}(a\,x)^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(x/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{x}{\sqrt{-a^2x^2+1}\,atanh(ax)^2\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)^2}dx\right)$$

input
```
int(x/(-a^2*x^2+1)^(3/2)/atanh(a*x)^2,x)
```

output

```
 - int(x/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**2 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)**2),x)
```

3.419 $\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}\,\text{arctanh}(ax)} + \frac{\text{Shi}(\text{arctanh}(ax))}{a}$$

output
```
-1/a/(-a^2*x^2+1)^(1/2)/arctanh(a*x)+Shi(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = \frac{-\frac{1}{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)} + \text{Shi}(\text{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
(-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6528, 6596, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6528}$$

$$a\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}dx-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{ax}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\int -\frac{i\sin(i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}-\frac{i\int \frac{\sin(i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(\text{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.77

method	result	size
default	$\frac{\operatorname{arctanh}(ax)\operatorname{Shi}(\operatorname{arctanh}(ax))a^2x^2-\operatorname{Shi}(\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)+\sqrt{-a^2x^2+1}}{a\operatorname{arctanh}(ax)(a^2x^2-1)}$	62

input
```
int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2-Shi(arctanh(a*x))*arctanh(a*x)
+(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)^2\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)^2}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^(3/2)/atanh(a*x)^2,x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**2 - sqrt( - a**2*x**
*2 + 1)*atanh(a*x)**2),x)
```

3.420 $\int \frac{1}{x(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = -\frac{ax}{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)} - \frac{\sqrt{1-a^2x^2}}{ax\,\text{arctanh}(ax)} + \text{Chi}(\text{arctanh}(ax)) - \frac{\text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\,\text{arctanh}(ax)},x\right)}{a}$$

output
```
-a*x/(-a^2*x^2+1)^(1/2)/arctanh(a*x)-(-a^2*x^2+1)^(1/2)/a/x/arctanh(a*x)+C
hi(arctanh(a*x))-Defer(Int)(1/x^2/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)/a
```

Mathematica [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = \int \frac{1}{x(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6592}$$

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}dx+\int \frac{1}{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{ 6568}$$

$$a^2\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}dx}{a}-\frac{x}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}dx}{a}-\frac{\sqrt{1-a^2x^2}}{ax\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 6530}$$

$$a^2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}dx}{a}-\frac{\sqrt{1-a^2x^2}}{ax\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\left(-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}dx}{a}+\frac{\sqrt{1-a^2x^2}}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}dx}{a}+a^2\left(\frac{\text{Chi}(\text{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{\sqrt{1-a^2x^2}}{ax\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6651}$$

$$-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}dx}{a}+a^2\left(\frac{\text{Chi}(\text{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{\sqrt{1-a^2x^2}}{ax\text{arctanh}(ax)}$$

input
```
Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)^2}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

output
```
int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 7.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{x\,\operatorname{atanh}\left(a\,x\right)^2\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(x*atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,a^2x^3-\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)^(3/2)/atanh(a*x)^2,x)
```

output
```
 - int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**3 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)**2*x),x)
```

3.421 $\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \operatorname{Int}\left(\frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3},x\right)$$

output
```
Defer(Int)(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx$$

input
```
Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \; 6651$$

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}dx$$

input
```
Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)^3}dx$$

input
```
int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

output
```
int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^m}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^m}{\operatorname{atanh}(a\,x)^3\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^m/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x^m/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^m}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{x^m}{\sqrt{-a^2x^2+1}\,atanh(ax)^3\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)^3}dx\right)$$

input
```
int(x^m/(-a^2*x^2+1)^(3/2)/atanh(a*x)^3,x)
```

output
```
- int(x**m/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**2 - sqrt( - a**2
*x**2 + 1)*atanh(a*x)**3),x)
```

3.422 $\int \frac{x^2}{(1-a^2x^2)^{3/2}\textbf{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2a^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2} - \frac{x}{2a^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{2a^3} - \frac{\operatorname{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3},x\right)}{a^2}$$

output
```
-1/2/a^3/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2-1/2*x/a^2/(-a^2*x^2+1)^(1/2)/ar
ctanh(a*x)+1/2*Chi(arctanh(a*x))/a^3-Defer(Int)(1/(-a^2*x^2+1)^(1/2)/arcta
nh(a*x)^3,x)/a^2
```

Mathematica [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx$$

input
```
Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \text{ 6590}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^3}dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2}dx-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{ 6568}$$

$$\frac{\frac{1}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}dx}{a}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{ 6530}$$

$$\frac{\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}+ -\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}+\frac{1}{2}a\left(-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}\right)}{a^2}$$

$\Big\downarrow$ 3782

$$\frac{\frac{1}{2}a\left(\frac{\text{Chi}(\text{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}$$

$\Big\downarrow$ 6651

$$\frac{\frac{1}{2}a\left(\frac{\text{Chi}(\text{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx}{a^2}$$

input
```
Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{arctanh}(ax)^3}dx$$

input
```
int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

output
```
int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3)
, x)
```

Sympy [N/A]

Not integrable

Time = 5.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^2}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

output
```
Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^2}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x^2/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{x^2}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^3\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh\,(ax)^3}dx\right)$$

input
```
int(x^2/(-a^2*x^2+1)^(3/2)/atanh(a*x)^3,x)
```

output
```
 - int(x**2/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**2 - sqrt( - a**2
*x**2 + 1)*atanh(a*x)**3),x)
```

3.423 $\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(\operatorname{arctanh}(ax))}{2a^2}$$

output
```
-1/2*x/a/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2-1/2/a^2/(-a^2*x^2+1)^(1/2)/arct
anh(a*x)+1/2*Shi(arctanh(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \frac{-\frac{ax+\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}+\operatorname{Shi}(\operatorname{arctanh}(ax))}{2a^2}$$

input
```
Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
(-((a*x + ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + SinhIntegral
[ArcTanh[a*x]])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6568, 6528, 6596, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{ 6568}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{a\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 6596}$$

$$\frac{\frac{\int \frac{ax}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}+\frac{-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}+\frac{\int -\frac{i\sin(i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}+\frac{-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}-\frac{i\int \frac{\sin(i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}}{2a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\frac{\text{Shi}(\text{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

input
```
Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) + (-(1/(a*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\operatorname{arctanh}(ax)^2 \operatorname{Shi}(\operatorname{arctanh}(ax))a^2x^2-\operatorname{Shi}(\operatorname{arctanh}(ax)) \operatorname{arctanh}(ax)^2+ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}\, \operatorname{arctanh}(ax)}{2a^2 \operatorname{arctanh}(ax)^2(a^2x^2-1)}$	87

input
```
int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(arctanh(a*x)^2*Shi(arctanh(a*x))*a^2*x^2-Shi(arctanh(a*x))*arctan
h(a*x)^2+a*x*(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(1/2)*arctanh(a*x))/arctanh(a
*x)^2/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3),
x)
```

Sympy [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\int \frac{x}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

output
```
Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\int \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\int \frac{x}{\operatorname{atanh}\left(a\,x\right)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(x/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{x}{\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3 a^2x^2-\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3}dx\right)$$

input
```
int(x/(-a^2*x^2+1)^(3/2)/atanh(a*x)^3,x)
```

output
```
 - int(x/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**2 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)**3),x)
```

3.424 $\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)} + \frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{2a}$$

output
```
-1/2/a/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2-1/2*x/(-a^2*x^2+1)^(1/2)/arctanh(
a*x)+1/2*Chi(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \frac{-\frac{1+ax\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2} + \operatorname{Chi}(\operatorname{arctanh}(ax))}{2a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
(-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegr
al[ArcTanh[a*x]])/(2*a)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6528, 6568, 6530, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6528}$$

$$\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}dx-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6568}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}dx}{a}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}+\frac{1}{2}a\left(-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}\right)$$

$$\Big\downarrow \text{3782}$$

$$\frac{1}{2}a\left(\frac{\text{Chi}(\text{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) + (a*(-(x/(a*Sqrt[1 - a^2*x^2]
*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\operatorname{arctanh}(ax)^2 \operatorname{Chi}(\operatorname{arctanh}(ax)) a^2 x^2+x \operatorname{arctanh}(ax) a \sqrt{-a^2x^2+1}-\operatorname{Chi}(\operatorname{arctanh}(ax)) \operatorname{arctanh}(ax)^2+\sqrt{-a^2x^2+1}}{2a \operatorname{arctanh}(ax)^2 (a^2x^2-1)}$	86

input
```
int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+x*arctanh(a*x)*a*(-a^2*x^2
+1)^(1/2)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x
)^2/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)^3} dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}} \operatorname{artanh}(ax)^3} dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3 a^2x^2-\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^(3/2)/atanh(a*x)^3,x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**2 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)**3),x)
```

3.425 $\int \frac{1}{x(1-a^2x^2)^{3/2}\text{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\text{arctanh}(ax)^3}\,dx = -\frac{ax}{2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\text{arctanh}(ax)^2} - \frac{1}{2\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{1}{2}\text{Shi}(\text{arctanh}(ax))-\frac{\text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2},x\right)}{2a}$$

output
```
-1/2*a*x/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2-1/2*(-a^2*x^2+1)^(1/2)/a/x/arct
anh(a*x)^2-1/2/(-a^2*x^2+1)^(1/2)/arctanh(a*x)+1/2*Shi(arctanh(a*x))-1/2*D
efer(Int)(1/x^2/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)/a
```

Mathematica [N/A]

Not integrable

Time = 11.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\text{arctanh}(ax)^3}\,dx = \int \frac{1}{x(1-a^2x^2)^{3/2}\text{arctanh}(ax)^3}\,dx$$

input
```
Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6592}$$

$$a^2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}dx+\int \frac{1}{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{6568}$$

$$a^2\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{\sqrt{1-a^2x^2}}{2ax\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6528}$$

$$a^2\left(\frac{a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}dx-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}dx}{2a}-\frac{\sqrt{1-a^2x^2}}{2ax\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6596}$$

$$a^2\left(\frac{\frac{\int \frac{ax}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{2a}-\frac{\sqrt{1-a^2x^2}}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{2a}+a^2\left(-\frac{x}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}+\frac{-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\int -\frac{i\sin(i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}}{2a}\right)-\frac{\sqrt{1-a^2x^2}}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 26

$$-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{2a}+a^2\left(-\frac{x}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}+\frac{-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}-\frac{i\int \frac{\sin(i\text{arctanh}(ax))}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}}{2a}\right)-\frac{\sqrt{1-a^2x^2}}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 3779

$$-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{2a}+a^2\left(\frac{\frac{\text{Shi}(\text{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\right)-\frac{\sqrt{1-a^2x^2}}{2ax\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6651

$$a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx}{2a}+\frac{\text{Shi}(\text{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}}{2a}-\frac{x}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\right)-\frac{\sqrt{1-a^2x^2}}{2ax\text{arctanh}(ax)^2}$$

input
```
Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{arctanh}\left(ax\right)^3}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

output
```
int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)^3}\,dx=\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\,\text{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 9.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{3}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)^{3}}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{atanh}\left(a\,x\right)^3\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3 a^2x^3-\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3 x}dx\right)$$

input
```
int(1/x/(-a^2*x^2+1)^(3/2)/atanh(a*x)^3,x)
```

output
```
 - int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**3 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)**3*x),x)
```

3.426 $\int x^4\sqrt{1-a^2x^2}\,\textbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\int x^4\sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx=\frac{\sqrt{1-a^2x^2}}{16a^5}-\frac{7(1-a^2x^2)^{3/2}}{72a^5}+\frac{(1-a^2x^2)^{5/2}}{30a^5}-\frac{x\sqrt{1-a^2x^2}\,\text{arctanh}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\,\text{arctanh}(ax)}{24a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\,\text{arctanh}(ax)-\frac{\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{8a^5}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{16a^5}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{16a^5}$$

output
```
1/16*(-a^2*x^2+1)^(1/2)/a^5-7/72*(-a^2*x^2+1)^(3/2)/a^5+1/30*(-a^2*x^2+1)^
(5/2)/a^5-1/16*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^4-1/24*x^3*(-a^2*x^2+1)
^(1/2)*arctanh(a*x)/a^2+1/6*x^5*(-a^2*x^2+1)^(1/2)*arctanh(a*x)-1/8*arctan
((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-1/16*I*polylog(2,-I*(-a*x+
1)^(1/2)/(a*x+1)^(1/2))/a^5+1/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a^5
```

Mathematica [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.73

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$
$$=\frac{\sqrt{1-a^2x^2}\left(45+70(-1+a^2x^2)+24(-1+a^2x^2)^2+45ax\text{arctanh}(ax)+210ax(-1+a^2x^2)\,\text{arctanh}(ax\right.}{}$$

input
```
Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(45 + 70*(-1 + a^2*x^2) + 24*(-1 + a^2*x^2)^2 + 45*a*x*
ArcTanh[a*x] + 210*a*x*(-1 + a^2*x^2)*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2
)^2*ArcTanh[a*x] - ((45*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[
1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^
ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6572, 243, 53, 2009, 6578, 243, 53, 2009, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$
$$\Big\downarrow \text{ 6572}$$
$$\frac{1}{6}\int\frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\int\frac{x^5}{\sqrt{1-a^2x^2}}dx+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{6}\int\frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{12}a\int\frac{x^4}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)$$
$$\Big\downarrow \text{ 53}$$

$$\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{12}a\int \left(\frac{\left(1-a^2x^2\right)^{3/2}}{a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}+\frac{1}{a^4\sqrt{1-a^2x^2}}\right)dx^2+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{6578}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{6578}$$

$$\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1}}{}}{8a}\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$\downarrow$ 241

$$\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2}}{a^4}}{8a}\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$\downarrow$ 6512

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+$$

$$\frac{1}{6}\left(-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+}{2a^2}\right.}{4}\right.$$

input
```
Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
-1/12*(a*((-2*Sqrt[1 - a^2*x^2])/a^6 + (4*(1 - a^2*x^2)^(3/2))/(3*a^6) - (
2*(1 - a^2*x^2)^(5/2))/(5*a^6))) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/6
+ (((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))/(8*a) -
(x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4*a^2) + (3*(-1/2*Sqrt[1 - a^2*x^2]/
a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*
x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2
)))/(4*a^2))/6
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(120 \operatorname{arctanh}(ax)a^5x^5+24a^4x^4-30a^3x^3 \operatorname{arctanh}(ax)+22a^2x^2-45ax \operatorname{arctanh}(ax)-1)}{720a^5} - \frac{i \operatorname{arctanh}(ax)\ln\left(1-\right.}{16a^5}$

input
```
int(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/720/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*a^5*x^5+24*a^4*x^4-30
*a^3*x^3*arctanh(a*x)+22*a^2*x^2-45*a*x*arctanh(a*x)-1)-1/16*I*ln(1+I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^5-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5
+1/16*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5
```

Fricas [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}x^4\,\text{artanh}\,(ax)\;dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Sympy [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int x^4\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)\,dx$$

input
```
integrate(x**4*(-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

output
```
Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}x^4\,\text{artanh}\,(ax)\;dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Giac [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}x^4\,\text{artanh}\,(ax)\,\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int x^4\,\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x^4*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x^4*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^4dx$$

input
```
int(x^4*(-a^2*x^2+1)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**4,x)
```

3.427 $\int x^3\sqrt{1-a^2x^2}\,\mathbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx=\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{x^3\sqrt{1-a^2x^2}}{20a}+\frac{11\arcsin(ax)}{120a^4}-\frac{2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{15a^4}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

output
```
1/24*x*(-a^2*x^2+1)^(1/2)/a^3+1/20*x^3*(-a^2*x^2+1)^(1/2)/a+11/120*arcsin(
a*x)/a^4-2/15*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^4-1/15*x^2*(-a^2*x^2+1)^(1
/2)*arctanh(a*x)/a^2+1/5*x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \frac{ax\sqrt{1-a^2x^2}(5+6a^2x^2)+11\arcsin(ax)+8\sqrt{1-a^2x^2}(-2-a^2x^2+3a^4x^4)\,\text{arctanh}(ax)}{120a^4}$$

input
```
Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
(a*x*Sqrt[1 - a^2*x^2]*(5 + 6*a^2*x^2) + 11*ArcSin[a*x] + 8*Sqrt[1 - a^2*x
^2]*(-2 - a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/(120*a^4)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.54, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6572, 262, 262, 223, 6578, 262, 223, 6556, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6572}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{5}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{5}a\left(\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{5}a\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\downarrow$ 223

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 6578

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 262

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 223

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)+\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 6556

$$\frac{1}{5}\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)+$$
$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$
$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$
$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

input
```
Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
-1/5*(a*(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/2*(x*Sqrt[1 - a^2*x^2])
/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2))) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x])/5 + ((-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a) - (x
^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/a^2))/(3*a^2))/5
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(24a^4x^4\operatorname{arctanh}(ax)+6a^3x^3-8a^2x^2\operatorname{arctanh}(ax)+5ax-16\operatorname{arctanh}(ax))}{120a^4}+\frac{11i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{120a^4}-\frac{11i\ln}{}$

input
```
int(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/120/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*a^3*x^3-8*a^
2*x^2*arctanh(a*x)+5*a*x-16*arctanh(a*x))+11/120*I*ln((a*x+1)/(-a^2*x^2+1)
^(1/2)+I)/a^4-11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \frac{\left(6\,a^3x^3+5\,ax+4\,(3\,a^4x^4-a^2x^2-2)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}-22\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/120*((6*a^3*x^3 + 5*a*x + 4*(3*a^4*x^4 - a^2*x^2 - 2)*log(-(a*x + 1)/(a*
x - 1)))*sqrt(-a^2*x^2 + 1) - 22*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
^4
```

Sympy [F]

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int x^{3}\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{atanh}\,(ax)\,dx$$

input
```
integrate(x**3*(-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

output
```
Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = -\frac{1}{120}\,a\left(\frac{3\left(\frac{2\,(-a^2x^2+1)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{-a^2x^2+1}x}{a^2}-\frac{\arcsin(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{-a^2x^2+1}x+\frac{\arcsin(ax)}{a}\right)}{a^4}\right) - \frac{1}{15}\left(\frac{3\,(-a^2x^2+1)^{\frac{3}{2}}x^2}{a^2}+\frac{2\,(-a^2x^2+1)^{\frac{3}{2}}}{a^4}\right)\text{artanh}\,(ax)$$

input
```
integrate(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/120*a*(3*(2*(-a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(-a^2*x^2 + 1)*x/a^2 - arc
sin(a*x)/a^3)/a^2 - 8*(sqrt(-a^2*x^2 + 1)*x + arcsin(a*x)/a)/a^4) - 1/15*(
3*(-a^2*x^2 + 1)^(3/2)*x^2/a^2 + 2*(-a^2*x^2 + 1)^(3/2)/a^4)*arctanh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int x^3\,\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x^3*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x^3*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^3dx$$

input
```
int(x^3*(-a^2*x^2+1)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**3,x)
```

3.428 $\int x^2\sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int x^2\sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx=\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{(1-a^2x^2)^{3/2}}{12a^3}-\frac{x\sqrt{1-a^2x^2}\,\text{arctanh}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\text{arctanh}(ax)-\frac{\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{4a^3}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{8a^3}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{8a^3}$$

output
```
1/8*(-a^2*x^2+1)^(1/2)/a^3-1/12*(-a^2*x^2+1)^(3/2)/a^3-1/8*x*(-a^2*x^2+1)^
(1/2)*arctanh(a*x)/a^2+1/4*x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)-1/4*arctan(
(-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/8*I*polylog(2,-I*(-a*x+1)
^(1/2)/(a*x+1)^(1/2))/a^3+1/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/
a^3
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$
$$=\frac{\sqrt{1-a^2x^2}\left(1+2a^2x^2+3ax\text{arctanh}(ax)+6ax(-1+a^2x^2)\,\text{arctanh}(ax)-\frac{3i\text{arctanh}(ax)\left(\log\left(1-ie^{-\text{arctanh}(}\right.\right.}{\sqrt{1-a^2}}\right.}{24a^3}$$

input
```
Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2 + 3*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x
^2)*ArcTanh[a*x] - ((3*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1
+ I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((3*I)*(PolyLog[2, (-I)/E^ArcTan
h[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6572, 243, 53, 2009, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6572}$$

$$\frac{1}{4}\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\int\frac{x^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{8}a\int\frac{x^2}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{8}a\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2+\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\Big\downarrow$ 2009

$$\frac{1}{4}\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$\Big\downarrow$ 6578

$$\frac{1}{4}\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right)+\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$\Big\downarrow$ 241

$$\frac{1}{4}\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)+\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$\Big\downarrow$ 6512

$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{4}\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)$$

input
```
Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
-1/8*(a*((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))) +
(x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/4 + (-1/2*Sqrt[1 - a^2*x^2]/a^3 - (x*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1
 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x
]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.90

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(6a^3x^3\,\mathrm{arctanh}(ax)+2a^2x^2-3ax\,\mathrm{arctanh}(ax)+1)}{24a^3}-\frac{i\,\mathrm{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a^3}+\frac{i\,\mathrm{arctanh}(ax)\ln\left(\right.}{8a}$

input
```
int(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-3*a*x*
arctanh(a*x)+1)-1/8*I/a^3*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+
1/8*I/a^3*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*I*dilog(1+I*
(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)/a^3
```

Fricas [F]

$$\int x^2\sqrt{1-a^2x^2}\mathrm{arctanh}(ax)\,dx=\int \sqrt{-a^2x^2+1}x^2\,\mathrm{artanh}\left(ax\right)\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Sympy [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int x^2\sqrt{-(ax-1)(ax+1)}\,\text{atanh}(ax)\,dx$$

input
```
integrate(x**2*(-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

output
```
Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}x^2\,\text{artanh}(ax)\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Giac [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}x^2\,\text{artanh}(ax)\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx=\int x^2\,\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x^2*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x^2*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx=\int\sqrt{-a^2x^2+1}\,atanh(ax)\,x^2dx$$

input
```
int(x^2*(-a^2*x^2+1)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**2,x)
```

3.429 $\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)\,dx=\frac{x\sqrt{1-a^2x^2}}{6a}+\frac{\arcsin(ax)}{6a^2}-\frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{3a^2}$$

output
```
1/6*x*(-a^2*x^2+1)^(1/2)/a+1/6*arcsin(a*x)/a^2-1/3*(-a^2*x^2+1)^(3/2)*arct
anh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)\,dx=\frac{ax\sqrt{1-a^2x^2}+\arcsin(ax)-2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{6a^2}$$

input
```
Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
(a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x] - 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])
/(6*a^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6556, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int \sqrt{1-a^2x^2}dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3a^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{2}\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}x\sqrt{1-a^2x^2}+\frac{\arcsin(ax)}{2a}}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3a^2}$$

input
```
Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
((x*Sqrt[1 - a^2*x^2])/2 + ArcSin[a*x]/(2*a))/(3*a) - ((1 - a^2*x^2)^(3/2)
*ArcTanh[a*x])/(3*a^2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

method	result	size
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\,\operatorname{arctanh}(ax)+ax-2\,\operatorname{arctanh}(ax))}{6a^2}+\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{6a^2}-\frac{i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-i\right)}{6a^2}$	99

input
```
int(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-2*arctanh(a*x
))+1/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^2-1/6*I*ln((a*x+1)/(-a^2*x^2+1
)^(1/2)-I)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)\,dx = \frac{\sqrt{-a^2x^2+1}\left(ax+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)-2\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^2}$$

input
```
integrate(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))) -
2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F]

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx=\int x\sqrt{-(ax-1)(ax+1)}\,\text{atanh}(ax)\,dx$$

input
```
integrate(x*(-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

output
```
Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx=-\frac{(-a^2x^2+1)^{\frac{3}{2}}\,\text{artanh}(ax)}{3\,a^2}+\frac{\sqrt{-a^2x^2+1}x+\frac{\arcsin(ax)}{a}}{6\,a}$$

input
```
integrate(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/a^2 + 1/6*(sqrt(-a^2*x^2 + 1)*x + a
rcsin(a*x)/a)/a
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int x\,\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x*atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x*atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)\,xdx$$

input
```
int(x*(-a^2*x^2+1)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x,x)
```

3.430 $\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)\,dx = \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax) - \frac{\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\mathrm{arctanh}(ax)}{a} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
1/2*(-a^2*x^2+1)^(1/2)/a+1/2*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)-arctan((-a*
x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-1/2*I*polylog(2,-I*(-a*x+1)^(1/2)
/(a*x+1)^(1/2))/a+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)\,dx$$

$$= \frac{\sqrt{1-a^2x^2}\left(1+ax\,\mathrm{arctanh}(ax) - \frac{i\left(\mathrm{arctanh}(ax)\left(\log\left(1-ie^{-\mathrm{arctanh}(ax)}\right)-\log\left(1+ie^{-\mathrm{arctanh}(ax)}\right)\right)+\mathrm{PolyLog}\left(2,-ie^{-\mathrm{arctan}}\right.\right.}{\sqrt{1-a^2x^2}}\right.}{2a}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^A
rcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]
] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6504, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{ 6504}$$

$$\frac{1}{2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}$$

$$\Big\downarrow \text{ 6512}$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

input
```
Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 + ((-2*ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*
x]])/a)/2
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

method	result
default	$\frac{(ax\ \mathrm{arctanh}(ax)+1)\sqrt{-a^2x^2+1}}{2a}-\frac{i\,\mathrm{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{i\,\mathrm{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{i\,\mathrm{dilog}\left(1+\frac{i(ax}{\sqrt{-a^2}}\right.}{2a}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a-1/2*I/a*arctanh(a*x)*ln(1+I*
(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+
1)^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*
(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{atanh}\left(ax\right)dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x),x)
```

3.431 $\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}\,dx = -\arcsin(ax)+\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - 2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-arcsin(a*x)+(-a^2*x^2+1)^(1/2)*arctanh(a*x)-2*arctanh(a*x)*arctanh((-a*x+
1)^(1/2)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-polylog(2
,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx = -2\arctan\left(\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right) + \sqrt{1-a^2x^2}\text{arctanh}(ax) + \text{arctanh}(ax)\log\left(1-e^{-\text{arctanh}(ax)}\right) - \text{arctanh}(ax)\log\left(1+e^{-\text{arctanh}(ax)}\right) + \text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) - \text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

output
```
-2*ArcTan[Tanh[ArcTanh[a*x]/2]] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + ArcTanh
[a*x]*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 + PolyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6572, 223, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx$$

$$\Big\downarrow \; 6572$$

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx - a\int \frac{1}{\sqrt{1-a^2x^2}}dx + \sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \; 223$$

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx + \sqrt{1-a^2x^2}\text{arctanh}(ax) - \arcsin(ax)$$

$$\Big\downarrow \text{6580}$$

$$\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

output
```
-ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 2*ArcTanh[a*x]*ArcTanh[Sqr
t[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - P
olyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

method	result
default	$\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)-2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{dilog}\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{arctanh}(ax)\ln\left(1\right.$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
(-a^2*x^2+1)^(1/2)*arctanh(a*x)-2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog
(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2
))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx=\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx=\int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)}{x}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x}dx$$

input

```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x,x)
```

output

```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x,x)
```

3.432 $\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = -\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x} + 2a\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax) - a\text{arctanh}\left(\sqrt{1-a^2x^2}\right) + ia\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right) - ia\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x+2*a*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2)
)*arctanh(a*x)-a*arctanh((-a^2*x^2+1)^(1/2))+I*a*polylog(2,-I*(-a*x+1)^(1/
2)/(a*x+1)^(1/2))-I*a*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{ax} + i\text{arctanh}(ax)\log\left(1-ie^{-\text{arctanh}(ax)}\right) - i\text{arctanh}(ax)\log\left(1+ie^{-\text{arctanh}(ax)}\right) - \log\left(\cosh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right) + \log\left(\sinh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right) + i\,\text{PolyLog}\left(2,-ie^{-\text{arctanh}(ax)}\right) - i\,\text{PolyLog}\left(2,ie^{-\text{arctanh}(ax)}\right)\right)$$

input

```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

output

```
a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I/E^
ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - Log[Cosh[ArcTan
h[a*x]/2]] + Log[Sinh[ArcTanh[a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]]
 - I*PolyLog[2, I/E^ArcTanh[a*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6576, 6512, 6570, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-a^2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6512}$$

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{ 6570}$$

$$a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

$$\Big\downarrow \text{ 221}$$

$$-\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]] - a^2
*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2,
((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/S
qrt[1 + a*x]])/a)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)}{x}+ia\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)+ia\operatorname{dilog}\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)-ia\operatorname{dilog}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x+I*a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x
^2+1)^(1/2))+I*a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I*a*dilog(1-I*(a*x+
1)/(-a^2*x^2+1)^(1/2))-I*a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = \int \frac{\sqrt{-(ax-1)\,(ax+1)}\,\text{atanh}\,(ax)}{x^2}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x**2,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^2}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^2,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^2}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x^2,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**2,x)
```

3.433 $\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = -\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} + a^2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{1}{2}a^2\,\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \frac{1}{2}a^2\,\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output

```
-1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2+a^2*ar
ctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/2*a^2*polylog(2,-(-a*x+
1)^(1/2)/(a*x+1)^(1/2))+1/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \frac{1}{8}a^2\left(-2\coth\left(\frac{1}{2}\text{arctanh}(ax)\right) - \text{arctanh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) - 4\text{arctanh}(ax)\log\left(1-e^{-\text{arctanh}(ax)}\right) + 4\text{arctanh}(ax)\log\left(1+e^{-\text{arctanh}(ax)}\right) - 4\,\text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) + 4\,\text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right) - \text{arctanh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) + 2\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

output
```
(a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 4*Ar
cTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh
[a*x])] - 4*PolyLog[2, -E^(-ArcTanh[a*x])] + 4*PolyLog[2, E^(-ArcTanh[a*x]
)] - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6572, 242, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx$$

$$\Big\downarrow \text{6572}$$

$$-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx + a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}$$

$$\Big\downarrow \text{242}$$

$$-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2} - \frac{a\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{6588}$$

$$-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx - \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{2x}$$

$$\Big\downarrow \text{6580}$$

$$-\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{2x}$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

output
```
-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) -
(a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(
Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/
2
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

method	result
default	$-\frac{(ax+\operatorname{arctanh}(ax))\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{a^2\operatorname{polylog}\left(2,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a^2\operatorname{arctanh}(ax)\ln\left(1-\right.}{2}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+arctanh(a*x))*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(a*x)*ln(1-(
a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1
/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,-(a
*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)}{x^3}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x**3,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^3}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^3,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^3}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x^3,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**3,x)
```

3.434 $\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4} dx = -\frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3x^3} + \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-1/6*a*(-a^2*x^2+1)^(1/2)/x^2-1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3+1/6*
a^3*arctanh((-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4} dx = -\frac{ax\sqrt{1-a^2x^2} + 2(1-a^2x^2)^{3/2}\text{arctanh}(ax) + a^3x^3\log(x) - a^3x^3\log\left(1+\sqrt{1-a^2x^2}\right)}{6x^3}$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

output
```
-1/6*(a*x*Sqrt[1 - a^2*x^2] + 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x] + a^3*x^3
*Log[x] - a^3*x^3*Log[1 + Sqrt[1 - a^2*x^2]])/x^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6570, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4} \, dx$$

$$\Big\downarrow \text{6570}$$

$$\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}dx-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}a\int \frac{\sqrt{1-a^2x^2}}{x^4}dx^2-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{6}a\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}a\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

output
```
-1/3*((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3 + (a*(-(Sqrt[1 - a^2*x^2]/x^2)
 + a^2*ArcTanh[Sqrt[1 - a^2*x^2]]))/6
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(2a^2x^2\,\mathrm{arctanh}(ax)-ax-2\,\mathrm{arctanh}(ax))}{6x^3}-\frac{a^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)}{6}+\frac{a^3\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{6}$	96

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x
^3-1/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+1/6*a^3*ln(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-a^2x^2}\mathrm{arctanh}(ax)}{x^4}\,dx = -\frac{a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\left(ax-(a^2x^2-1)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{6\,x^3}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(a*x -
(a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4}\,dx=\int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)}{x^4}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x**4,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}a^2-\frac{(-a^2x^2+1)^{\frac{3}{2}}}{x^2}\right)a - \frac{(-a^2x^2+1)^{\frac{3}{2}}\,\text{artanh}\,(ax)}{3\,x^3}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

output
```
1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*
a^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a - 1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/
x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^4}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^4,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^4}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x^4,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**4,x)
```

3.435 $\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 191

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx = -\frac{a\sqrt{1-a^2x^2}}{12x^3}-\frac{a^3\sqrt{1-a^2x^2}}{24x} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{8x^2} + \frac{1}{4}a^4\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{1}{8}a^4\,\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \frac{1}{8}a^4\,\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/12*a*(-a^2*x^2+1)^(1/2)/x^3-1/24*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*(-a^2*x^2
+1)^(1/2)*arctanh(a*x)/x^4+1/8*a^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2+1/4
*a^4*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/8*a^4*polylog(2,
-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/8*a^4*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1
/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx = \frac{1}{192}a^4\left(-8\coth\left(\frac{1}{2}\text{arctanh}(ax)\right) - 6\text{arctanh}(ax)\text{csch}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) - \frac{ax\text{csch}^4\left(\frac{1}{2}\text{arctanh}(ax)\right)}{\sqrt{1-a^2x^2}} - 3\text{arctanh}(ax)\text{csch}^4\left(\frac{1}{2}\text{arctanh}(ax)\right) - 24\text{arctanh}(ax)\log\left(1-e^{-\text{arctanh}(ax)}\right) + 24\text{arctanh}(ax)\log\left(1+e^{-\text{arctanh}(ax)}\right) - 24\,\text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) + 24\,\text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right) - 6\text{arctanh}(ax)\text{sech}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) + 3\text{arctanh}(ax)\text{sech}^4\left(\frac{1}{2}\text{arctanh}(ax)\right) - \frac{16(1-a^2x^2)^{3/2}\sinh^4\left(\frac{1}{2}\text{arctanh}(ax)\right)}{a^3x^3} + 8\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

output
```
(a^4*(-8*Coth[ArcTanh[a*x]/2] - 6*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (a
*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*ArcTanh[a*x]*Csch[ArcTanh
[a*x]/2]^4 - 24*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]*
Log[1 + E^(-ArcTanh[a*x])] - 24*PolyLog[2, -E^(-ArcTanh[a*x])] + 24*PolyLo
g[2, E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 3*ArcTan
h[a*x]*Sech[ArcTanh[a*x]/2]^4 - (16*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/
2]^4)/(a^3*x^3) + 8*Tanh[ArcTanh[a*x]/2]))/192
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.54, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6572, 245, 242, 6588, 245, 242, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}dx$$

$$\Big\downarrow \text{6572}$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx+\frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx+\frac{1}{3}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \text{6588}$$

$$\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{3}\left(-\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\Big\downarrow$ 6588

$$\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax}{4x^4}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\Big\downarrow$ 242

$$\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\Big\downarrow$ 6580

$$\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

output
```
(a*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x)))/3 - (Sq
rt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^4) + (-1/4*(a*(-1/3*Sqrt[1 - a^2*x^2]/x
^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x))) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/
(4*x^4) - (3*a^2*(-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(2*x^2) + (a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x
]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]
/Sqrt[1 + a*x]]))/2))/4)/3
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.86

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(-a^3x^3+3a^2x^2\,\text{arctanh}(ax)-2ax-6\,\text{arctanh}(ax))}{24x^4}-\frac{\text{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)a^4}{8}-\frac{\text{polylog}\left(2,\frac{a}{\sqrt{-}}\right.}{8}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(-(a*x-1)*(a*x+1))^(1/2)*(-a^3*x^3+3*a^2*x^2*arctanh(a*x)-2*a*x-6*arc
tanh(a*x))/x^4-1/8*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4-1/8*p
olylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4+1/8*arctanh(a*x)*ln(1+(a*x+1)/(-a
^2*x^2+1)^(1/2))*a^4+1/8*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx=\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)}{x^5}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx=\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{atanh}\left(ax\right)}{x^5}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x**5,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^5}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^5}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^5,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^5}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x^5,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**5,x)
```

3.436 $\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^6}\,dx = -\frac{a\sqrt{1-a^2x^2}}{20x^4}-\frac{a^3\sqrt{1-a^2x^2}}{24x^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{5x^5}+\frac{a^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{15x^3}+\frac{2a^4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{15x}+\frac{11}{120}a^5\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output

```
-1/20*a*(-a^2*x^2+1)^(1/2)/x^4-1/24*a^3*(-a^2*x^2+1)^(1/2)/x^2-1/5*(-a^2*x
^2+1)^(1/2)*arctanh(a*x)/x^5+1/15*a^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^3+
2/15*a^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x+11/120*a^5*arctanh((-a^2*x^2+1)
^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx = \frac{1}{120}\left(-\frac{a\sqrt{1-a^2x^2}(6+5a^2x^2)}{x^4} + \frac{8\sqrt{1-a^2x^2}(-3+a^2x^2+2a^4x^4)\,\text{arctanh}(ax)}{x^5} - 11a^5\log(x) + 11a^5\log\left(1+\sqrt{1-a^2x^2}\right)\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

output
```
(-((a*Sqrt[1 - a^2*x^2]*(6 + 5*a^2*x^2))/x^4) + (8*Sqrt[1 - a^2*x^2]*(-3 +
 a^2*x^2 + 2*a^4*x^4)*ArcTanh[a*x])/x^5 - 11*a^5*Log[x] + 11*a^5*Log[1 + S
qrt[1 - a^2*x^2]])/120
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 331 vs. 2(150) = 300.

Time = 1.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.21, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {6572, 243, 52, 52, 73, 221, 6588, 243, 52, 52, 73, 221, 6588, 243, 52, 73, 221, 6570, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx$$

$$\Big\downarrow \text{6572}$$

$$-\frac{1}{4}\int \frac{\text{arctanh}(ax)}{x^6\sqrt{1-a^2x^2}}dx + \frac{1}{4}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{4}\int \frac{\text{arctanh}(ax)}{x^6\sqrt{1-a^2x^2}}dx+\frac{1}{8}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}$$

$$\Big\downarrow \textbf{52}$$

$$-\frac{1}{4}\int \frac{\text{arctanh}(ax)}{x^6\sqrt{1-a^2x^2}}dx+\frac{1}{8}a\left(\frac{3}{4}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}$$

$$\Big\downarrow \textbf{52}$$

$$-\frac{1}{4}\int \frac{\text{arctanh}(ax)}{x^6\sqrt{1-a^2x^2}}dx+\frac{1}{8}a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{1}{4}\int \frac{\text{arctanh}(ax)}{x^6\sqrt{1-a^2x^2}}dx+\frac{1}{8}a\left(\frac{3}{4}a^2\left(-\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{1}{4}\int \frac{\text{arctanh}(ax)}{x^6\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \textbf{6588}$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\int \frac{\text{arctanh}(ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{1}{5}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \textbf{243}$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\int \frac{\text{arctanh}(ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{1}{10}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}dx^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \; 52$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\int \frac{\text{arctanh}(ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{1}{10}a\left(\frac{3}{4}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \; 52$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\int \frac{\text{arctanh}(ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{1}{10}a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)+\frac{\sqrt{1-a^2x}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\int \frac{\text{arctanh}(ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{1}{10}a\left(\frac{3}{4}a^2\left(-\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)+\frac{\sqrt{1-a^2}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\int \frac{\text{arctanh}(ax)}{x^4\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^5}-\frac{1}{10}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \; 6588$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx+\frac{1}{3}a\int\frac{1}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^3}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax}{5x^5}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \ 243$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx+\frac{1}{6}a\int\frac{1}{x^4\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^3}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(a}{5x^5}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx+\frac{1}{6}a\left(\frac{1}{2}a^2\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^3}\right)+\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx+\frac{1}{6}a\left(-\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^3}\right)\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx+\frac{1}{6}a\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^3}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{6570}$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\left(a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2}}{x^2}\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-}}{}\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{4}{5}a^2\left(\frac{2}{3}a^2\left(-\frac{\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-}}{}\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^5}+$$
$$\frac{1}{8}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)+$$
$$\frac{1}{4}\left(\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^5}-\frac{1}{10}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{4}{5}a^2\left(\frac{2}{3}\right.\right.$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

output
```
-1/4*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5 + (a*(-1/2*Sqrt[1 - a^2*x^2]/x^4
 + (3*a^2*(-(Sqrt[1 - a^2*x^2]/x^2) - a^2*ArcTanh[Sqrt[1 - a^2*x^2]]))/4))
/8 + ((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(5*x^5) - (4*a^2*(-1/3*(Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/x^3 + (2*a^2*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) -
 a*ArcTanh[Sqrt[1 - a^2*x^2]]))/3 + (a*(-(Sqrt[1 - a^2*x^2]/x^2) - a^2*Arc
Tanh[Sqrt[1 - a^2*x^2]]))/6))/5 - (a*(-1/2*Sqrt[1 - a^2*x^2]/x^4 + (3*a^2*
(-(Sqrt[1 - a^2*x^2]/x^2) - a^2*ArcTanh[Sqrt[1 - a^2*x^2]]))/4))/10)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(16a^4x^4\operatorname{arctanh}(ax)-5a^3x^3+8a^2x^2\operatorname{arctanh}(ax)-6ax-24\operatorname{arctanh}(ax)\right)}{120x^5}-\frac{11a^5\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)}{120}+\frac{11a}{}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/120*(-(a*x-1)*(a*x+1))^(1/2)*(16*a^4*x^4*arctanh(a*x)-5*a^3*x^3+8*a^2*x^
2*arctanh(a*x)-6*a*x-24*arctanh(a*x))/x^5-11/120*a^5*ln((a*x+1)/(-a^2*x^2+
1)^(1/2)-1)+11/120*a^5*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx = -\frac{11\,a^5x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(5\,a^3x^3+6\,ax-4\,(2\,a^4x^4+a^2x^2-3)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

output
```
-1/120*(11*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 + 6*a*x -
4*(2*a^4*x^4 + a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))
/x^5
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}(ax)}{x^6}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x**6,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx$$
$$= \frac{1}{120}\left(3\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-3\sqrt{-a^2x^2+1}a^4+8\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}\right.\right.$$
$$-\frac{1}{15}\left(\frac{2\,(-a^2x^2+1)^{\frac{3}{2}}a^2}{x^3}+\frac{3\,(-a^2x^2+1)^{\frac{3}{2}}}{x^5}\right)\text{artanh}\,(ax)$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

output
```
1/120*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*sqrt(-a^2*x^2
 + 1)*a^4 + 8*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2
*x^2 + 1)*a^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a^2 - 3*(-a^2*x^2 + 1)^(3/2)*a^2
/x^2 - 6*(-a^2*x^2 + 1)^(3/2)/x^4)*a - 1/15*(2*(-a^2*x^2 + 1)^(3/2)*a^2/x^
3 + 3*(-a^2*x^2 + 1)^(3/2)/x^5)*arctanh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^6}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^6,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^6}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^6}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x^6,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**6,x)
```

3.437 $\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^7}\,dx =& -\frac{a\sqrt{1-a^2x^2}}{30x^5}-\frac{11a^3\sqrt{1-a^2x^2}}{360x^3}\\&+\frac{a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{6x^6}\\&+\frac{a^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{24x^4}+\frac{a^4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{16x^2}\\&+\frac{1}{8}a^6\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\&-\frac{1}{16}a^6\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\&+\frac{1}{16}a^6\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\end{aligned}$$

output

```
-1/30*a*(-a^2*x^2+1)^(1/2)/x^5-11/360*a^3*(-a^2*x^2+1)^(1/2)/x^3+1/720*a^5
*(-a^2*x^2+1)^(1/2)/x-1/6*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^6+1/24*a^2*(-a
^2*x^2+1)^(1/2)*arctanh(a*x)/x^4+1/16*a^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/
x^2+1/8*a^6*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/16*a^6*po
lylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/16*a^6*polylog(2,(-a*x+1)^(1/2)/(
a*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^7}\,dx = \frac{a^6\left(-76\coth\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)-90\operatorname{arctanh}(ax)\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)-\frac{26ax\operatorname{csch}^4\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{\sqrt{1-a^2x^2}}-90\operatorname{arct}\right.}{}$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```

output
```
(a^6*(-76*Coth[ArcTanh[a*x]/2] - 90*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 -
(26*a*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 90*ArcTanh[a*x]*Csch[A
rcTanh[a*x]/2]^4 - (3*a*x*Csch[ArcTanh[a*x]/2]^6)/Sqrt[1 - a^2*x^2] - 15*A
rcTanh[a*x]*Csch[ArcTanh[a*x]/2]^6 - 360*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[
a*x])] + 360*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] - 360*PolyLog[2, -E^(
-ArcTanh[a*x])] + 360*PolyLog[2, E^(-ArcTanh[a*x])] - 90*ArcTanh[a*x]*Sech
[ArcTanh[a*x]/2]^2 + 90*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - 15*ArcTanh[a
*x]*Sech[ArcTanh[a*x]/2]^6 - (416*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]
^4)/(a^3*x^3) + 76*Tanh[ArcTanh[a*x]/2] + 6*Sech[ArcTanh[a*x]/2]^4*Tanh[Ar
cTanh[a*x]/2]))/5760
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.80, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6572, 245, 245, 242, 6588, 245, 245, 242, 6588, 245, 242, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^7}\,dx$$

$$\Big\downarrow \text{6572}$$

$$-\frac{1}{5}\int \frac{\operatorname{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx+\frac{1}{5}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx+\frac{1}{5}a\left(\frac{4}{5}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx+\frac{1}{5}a\left(\frac{4}{5}a^2\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{6588}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\left(\frac{4}{5}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\left(\frac{4}{5}a^2\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)+\frac{\sqrt{1-a^2x^2}\text{a}}{6x}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}-\frac{1}{6}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2}}{5x^5}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\big\downarrow$ 6588

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax}{6x^6}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\big\downarrow$ 245

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{1}{4}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)+\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\big\downarrow$ 242

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}+\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)+\frac{\sqrt{1-}}{}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\big\downarrow$ 6588

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctan}}{4x^4}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\big\downarrow$ 242

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}+\right.\right.$$
$$\left.\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\right)$$

$$\Big\downarrow \text{6580}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right.\right.\right.\right.$$
$$\left.\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\right)$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```

output
```
(a*(-1/5*Sqrt[1 - a^2*x^2]/x^5 + (4*a^2*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a
^2*Sqrt[1 - a^2*x^2])/(3*x)))/5))/5 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(5*
x^6) + (-1/6*(a*(-1/5*Sqrt[1 - a^2*x^2]/x^5 + (4*a^2*(-1/3*Sqrt[1 - a^2*x^
2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x)))/5)) + (Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(6*x^6) - (5*a^2*((a*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 -
 a^2*x^2])/(3*x)))/4 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (3*a^2*(
-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) +
(a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(
Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/
2))/4))/6)/5
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.75

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,(a^5x^5+45a^4x^4\operatorname{arctanh}(ax)-22a^3x^3+30a^2x^2\operatorname{arctanh}(ax)-24ax-120\operatorname{arctanh}(ax))}{720x^6}-\frac{a^6\operatorname{arctanh}(ax)\ln\left(1-\right.}{16}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/720*(-(a*x-1)*(a*x+1))^(1/2)*(a^5*x^5+45*a^4*x^4*arctanh(a*x)-22*a^3*x^3
+30*a^2*x^2*arctanh(a*x)-24*a*x-120*arctanh(a*x))/x^6-1/16*a^6*arctanh(a*x
)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polylog(2,(a*x+1)/(-a^2*x^2+1)
^(1/2))+1/16*a^6*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^7}\,dx=\int \frac{\sqrt{-a^2x^2+1}\operatorname{artanh}\left(ax\right)}{x^7}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^7,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^7}\,dx=\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\operatorname{atanh}\left(ax\right)}{x^7}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)/x**7,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**7, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^7}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^7}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^7,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^7}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^7,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^7}\,dx = \int \frac{\text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}}{x^7}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^7,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(1/2))/x^7, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^7}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^7}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)/x^7,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**7,x)
```

3.438 $\int x^4\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 336

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2} - \frac{19\arcsin(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{360a^5} + \frac{11x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{180a^3} + \frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{15a} - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{16a^4} - \frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{24a^2} + \frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)^2 + \frac{\arctan\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2}{8a^5} - \frac{i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)}{8a^5} + \frac{i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)}{8a^5} + \frac{i\,\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)}{8a^5} - \frac{i\,\text{PolyLog}\left(3,ie^{\text{arctanh}(ax)}\right)}{8a^5}$$

output

```
1/18*x*(-a^2*x^2+1)^(1/2)/a^4+1/60*x^3*(-a^2*x^2+1)^(1/2)/a^2-19/360*arcsi
n(a*x)/a^5-1/360*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^5+11/180*x^2*(-a^2*x^2+
1)^(1/2)*arctanh(a*x)/a^3+1/15*x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a-1/16*
x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/a^4-1/24*x^3*(-a^2*x^2+1)^(1/2)*arctan
h(a*x)^2/a^2+1/6*x^5*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2+1/8*arctan((a*x+1)/
(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^5-1/8*I*arctanh(a*x)*polylog(2,-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/8*I*arctanh(a*x)*polylog(2,I*(a*x+1)/(-a^2*
x^2+1)^(1/2))/a^5+1/8*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5-1/8*I
*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.80

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$=\frac{\sqrt{1-a^2x^2}\Big(90\text{arctanh}(ax)+140(-1+a^2x^2)\,\text{arctanh}(ax)+48(-1+a^2x^2)^2\,\text{arctanh}(ax)+120ax(-1+}{}$$

input
```
Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(90*ArcTanh[a*x] + 140*(-1 + a^2*x^2)*ArcTanh[a*x] + 48
*(-1 + a^2*x^2)^2*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 +
 6*a*x*(-1 + a^2*x^2)*(2 + 35*ArcTanh[a*x]^2) + a*x*(52 + 45*ArcTanh[a*x]^
2) - (I*((-76*I)*ArcTan[Tanh[ArcTanh[a*x]/2]] + 45*ArcTanh[a*x]^2*Log[1 -
I/E^ArcTanh[a*x]] - 45*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 90*ArcTa
nh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 90*ArcTanh[a*x]*PolyLog[2, I/E^A
rcTanh[a*x]] + 90*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 90*PolyLog[3, I/E^ArcT
anh[a*x]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 892 vs. 2(336) = 672.

Time = 7.14 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.65, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.042, Rules used = {6576, 6578, 6578, 262, 223, 262, 223, 6514, 3042, 4668, 3011, 2720, 6556, 223, 6578, 262, 223, 6514, 3042, 4668, 3011, 2720, 6556, 223, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{x^4\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx - a^2\int \frac{x^6\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6578}$$

$$\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-a^2\left(\frac{\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{5\int \frac{x^4\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{6a^2}\right)-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{6578}$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}}{2a}+\frac{3\left(\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}-a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}dx}{5a}-}{3a}\right.$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}}{2a}+\frac{3\left(\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}-a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-}{5a}}{3}\right.$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3\left(\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\dots}{5a}}{3\dots}\right.$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\left(\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}\dots\right)}{4a\dots}}{\dots}\right.$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3\left(\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}+$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}\right.$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$\downarrow$ 6514

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}\right.$$

$$\frac{3\left(\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}-$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}}{\ldots}\right.$$

$$\frac{3\left(\frac{\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a^3}+\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}-$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow 4668$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}}{\ldots}\right.$$

$$\frac{3\left(\frac{-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2\text{arctanh}(ax)^2\text{ ar}}{2a^3}\right.}{4a^2}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow 3011$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}\right.$$

$$\frac{3\left(\frac{2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,ie^{\text{arctanh}(ax)}\right)d\text{arctan}\right.}{2a^3}\right.}{}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \; 2720$$

$$\frac{\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}\right.$$

$$\frac{3\left(\frac{2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\right.}{2a^3}\right.}{}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \; 6556$$

$$\frac{\frac{2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}-$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}\right.$$

$$\frac{3\left(\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\right.}{2a^3}\right.}{}$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}$$

$$\Big\downarrow \; 223$$

$$\frac{3\left(\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\right.}{2a^3}\right.}{}$$

$$a^2\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)}{6a^2}+\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arc}}{5a^2}\right.$$

$$\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}+$$

$$\frac{\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}$$

$$\Big\downarrow \; 6578$$

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}\cdots}{3a\cdots}\right.}{\cdots}}{3a}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 262

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}\cdots}{3a\cdots}\right.}{\cdots}}{3a}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 223

$$\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}}{}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}}{3a}\right.}{3a}}{}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\right.}{}\right.$$

$\downarrow$ 6514

$$\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}}{}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}}{3a}\right.}{3a}}{}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\right.}{}\right.$$

$\downarrow$ 3042

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}}{3a^2}\right.}{}}{3a}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\right.}{}\right.$$

$\downarrow$ 4668

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}}{3a^2}\right.}{}}{3a}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\right.}{}\right.$$

$\downarrow$ 3011

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}-$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}}{3a^2}\right.}{}}{3a}\right.$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\right.}{}\right.$$

$\downarrow$ 2720

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}+$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\right.}{}\right.$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}}{3a^2}\right.}{}}{3a}\right.$$

$\downarrow$ 6556

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}+$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\ldots\right.}{\ldots}\right.$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}\ldots}{3a^{\ldots}}\ldots\right.}{\ldots}}{3a}\ldots\right.$$

$\downarrow$ 223

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}+$$

$$3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{Poly}\ldots\right.}{\ldots}\right.$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}\ldots}{3a^{\ldots}}\ldots\right.}{\ldots}}{3a}\ldots\right.$$

$\downarrow$ 7143

$$-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^3}{4a^2}+$$

$$\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^2}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}+\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}\right)}{3a^2}}{2a}+$$

$$\frac{3\left(-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}+\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a^2}}{a}+\frac{2\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2+2i\left(\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctan}\cdots}\right.\right.}{}\right.}{4a^2}$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2x^5}{6a^2}+\frac{-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)x^4}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}+\frac{4\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arct}\cdots}{3a^2}\right.}{}}{3a}\right.$$

input
```
Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + ((-1/2*(x*Sqrt[1 - a^2*x
^2])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
])/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2))/
(3*a^2))/(2*a) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (ArcS
in[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2)/a + (2*ArcTan[E^ArcTan
h[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a
*x]]) + PolyLog[3, (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2
, I*E^ArcTanh[a*x]]) + PolyLog[3, I*E^ArcTanh[a*x]]))/(2*a^3)))/(4*a^2) -
a^2*(-1/6*(x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + ((-1/4*(x^3*Sqrt[1
- a^2*x^2])/a^2 + (3*(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3)
))/(4*a^2))/(5*a) - (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(5*a^2) + (4*((-1
/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a) - (x^2*Sqrt[1 -
a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*
ArcTanh[a*x])/a^2))/(3*a^2)))/(5*a^2))/(3*a) + (5*(-1/4*(x^3*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x]^2)/a^2 + ((-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/
(2*a^3))/(3*a) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin
[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2))/(3*a^2))/(2*a) + (3*(-1
/2*(x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (ArcSin[a*x]/a^2 - (Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/a^2)/a + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2
+ (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (...
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int x^4\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)^2\,dx$$

input
```
int(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

output
```
int(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Fricas [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx=\int\sqrt{-a^2x^2+1}x^4\,\operatorname{artanh}(ax)^2\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx=\int x^4\sqrt{-(ax-1)\,(ax+1)}\,\text{atanh}^2\,(ax)\,dx$$

input
```
integrate(x**4*(-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

output
```
Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Maxima [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx=\int \sqrt{-a^2x^2+1}x^4\,\text{artanh}\,(ax)^2\,\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Giac [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx=\int \sqrt{-a^2x^2+1}x^4\,\text{artanh}\,(ax)^2\,\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx=\int x^4\,\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x^4*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x^4*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx=\int\sqrt{-a^2x^2+1}\,atanh(ax)^2\,x^4dx$$

input
```
int(x^4*(-a^2*x^2+1)^(1/2)*atanh(a*x)^2,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*x**4,x)
```

3.439 $\int x^3\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 281

$$\begin{aligned}\int x^3\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\,dx &= \frac{11\sqrt{1-a^2x^2}}{60a^4}-\frac{(1-a^2x^2)^{3/2}}{30a^4}\\&+\frac{x\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)}{12a^3}\\&+\frac{x^3\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)}{10a}\\&-\frac{11\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\mathrm{arctanh}(ax)}{30a^4}\\&-\frac{2\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2}{15a^4}\\&-\frac{x^2\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2}{15a^2}\\&+\frac{1}{5}x^4\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\\&-\frac{11i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{60a^4}\\&+\frac{11i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{60a^4}\end{aligned}$$

output
```
11/60*(-a^2*x^2+1)^(1/2)/a^4-1/30*(-a^2*x^2+1)^(3/2)/a^4+1/12*x*(-a^2*x^2+
1)^(1/2)*arctanh(a*x)/a^3+1/10*x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a-11/30
*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^4-2/15*(-a^2*x^2+1)^(
1/2)*arctanh(a*x)^2/a^4-1/15*x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/a^2+1/5
*x^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2-11/60*I*polylog(2,-I*(-a*x+1)^(1/2)
/(a*x+1)^(1/2))/a^4+11/60*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.62

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\Big(11+11ax\text{arctanh}(ax)+6ax(-1+a^2x^2)\,\text{arctanh}(ax)+12(-1+a^2x^2)^2\,\text{arctanh}(ax)^2+2(}{}$$

input
```
Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(11 + 11*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^2)*ArcTan
h[a*x] + 12*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*(1 + 10*Arc
Tanh[a*x]^2) - ((11*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcT
anh[a*x]]))/Sqrt[1 - a^2*x^2]))/(60*a^4)
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 926 vs. 2(281) = 562.

Time = 4.34 (sec) , antiderivative size = 926, normalized size of antiderivative = 3.30, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6576, 6578, 6556, 6512, 6578, 241, 243, 53, 2009, 6512, 6556, 6512, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx-a^2\int \frac{x^5\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6578}$$

$$\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\left(a^2\left(\frac{2\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a}+\frac{4\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{5a^2}\right)\right)-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6556}$$

$$\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\left(\frac{2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}\right)}{3a^2}-\left(a^2\left(\frac{2\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a}+\frac{4\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{5a^2}\right)\right)-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6512}$$

$$\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}-\left(a^2\left(\frac{2\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a}+\frac{4\int \frac{x^3\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{5a^2}\right)\right)+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6578}$$

$$\frac{2\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right)}{3a}-$$

$$\left(a^2\left(\frac{4\left(\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3}{\sqrt{1-a^2\cdots}}\cdots}{4a}\cdots\right.}{5a\cdots}\right.\right.$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \; 241$$

$$-\left(a^2\left(\frac{4\left(\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x\cdots}{\sqrt{1-\cdots}}\cdots}{4\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$\frac{2\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}+$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \; 243$$

$$-\left(a^2\left(\frac{4\left(\frac{2\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{x}{\sqrt{1-}}}{8}\right.}{}\right.\right.$$

$$\frac{2\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}+$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$\downarrow$ 53

$$-\left(a^2\left(\frac{4\left(\frac{2\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\left(\frac{}{a^2\sqrt{}}\right.}{}\right.}{}\right.\right.$$

$$\frac{2\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}+$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$\downarrow$ 2009

$$-\left(a^2\left(\frac{4\left(\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-\right.}{}\right.\right.$$

$$\frac{2\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}+$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}$$

$\big\downarrow$ 6512

$$-\left(a^2\left(\frac{4\left(\frac{2\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-\right.}{}\right.\right.$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-$$

$$\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}+$$

$$\frac{2\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}$$

$\big\downarrow$ 6556

$$-\left(a^2\left(\frac{4\left(\frac{2\left(\frac{2\int\frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}\right)}{3a^2}+\frac{2\int\frac{x^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{3a^2}\right)}{5a^2}+\frac{2\left(\frac{3\int\frac{x}{}}{}\right.}{}\right.\right.$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{3a^2}+$$

$$\frac{2\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}$$

$\downarrow$ 6512

$$-\left(a^2\left(\frac{4\left(\frac{2\int\frac{x^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLo}}{}\right.}{a}\right.}{3a^2}\right.}{5a^2}\right.\right.$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-$$

$$\frac{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{3a^2}+$$

$$\frac{2\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}$$

$\downarrow$ 6578

$$-\left(a^2\left(\frac{4\left(\frac{2\left(\frac{\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}\right)}{3a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arc}\cdots}{a}\cdots\right.}{\cdots}\right.}{\cdots}\right)}{5a^2}\right.\right.$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{3a^2}+$$

$$\frac{2\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}$$

$$\Big\downarrow \; 241$$

$$-\left(a^2\left(\frac{4\left(\frac{2\left(\frac{\int\frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctan}\right.}{a}}{}\right.}{3a^2}\right.}{5a^2}\right.\right.$$

$$\frac{2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)}{3a^2}-$$

$$\frac{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{3a^2}+$$

$$\frac{2\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}$$

$$\Big\downarrow \text{6512}$$

$$-\left(\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2x^4}{5a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)x^3}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(}{2a^2}\right.}{}\right.}{}\right.\right.$$

$$\frac{2\left(-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}+\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{3a}-$$

$$\frac{\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{3a^2}+}{}$$

$$\frac{2\left(\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}\right)}{3a^2}$$

input
```
Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (2*(-1/2*Sqrt[1 - a^2*x^
2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*
a^2)))/(3*a) + (2*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2) + (2*((-2*Arc
Tan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a))/a))/(3*a^2) - a^2*(-1/5*(x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a
^2 + (2*(((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))/(8
*a) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4*a^2) + (3*(-1/2*Sqrt[1 - a^2
*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[
1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/
(2*a^2)))/(4*a^2)))/(5*a) + (4*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/a^2 + (2*(-1/2*Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]
)/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*
PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[
1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2)))/(3*a) + (2*(-((Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]^2)/a^2) + (2*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a
*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*Poly...
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.75

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\,\left(12a^4x^4\operatorname{arctanh}(ax)^2+6a^3x^3\operatorname{arctanh}(ax)-4a^2x^2\operatorname{arctanh}(ax)^2+2a^2x^2+5ax\operatorname{arctanh}(ax)-8\operatorname{arctanh}(ax)^2+9\right.}{60a^4}$

input
```
int(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(12*a^4*x^4*arctanh(a*x)^2+6*a^3*x^3*arc
tanh(a*x)-4*a^2*x^2*arctanh(a*x)^2+2*a^2*x^2+5*a*x*arctanh(a*x)-8*arctanh(
a*x)^2+9)-11/60*I/a^4*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+11/6
0*I/a^4*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-11/60*I*dilog(1+I*
(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+11/60*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/
2))/a^4
```

Fricas [F]

$$\int x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx=\int\sqrt{-a^2x^2+1}x^3\operatorname{artanh}\left(ax\right)^2\,dx$$

input
```
integrate(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx=\int x^3\sqrt{-(ax-1)(ax+1)}\operatorname{atanh}^2(ax)\,dx$$

input
```
integrate(x**3*(-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

output
```
Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Maxima [F]

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}x^3\,\text{artanh}\,(ax)^2\,dx$$

input
```
integrate(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int x^3\,\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x^3*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x^3*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx=\int\sqrt{-a^2x^2+1}\,atanh(ax)^2\,x^3dx$$

input
```
int(x^3*(-a^2*x^2+1)^(1/2)*atanh(a*x)^2,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*x**3,x)
```

3.440 $\int x^2\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\begin{aligned}\int x^2\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\,dx &= \frac{x\sqrt{1-a^2x^2}}{12a^2}-\frac{\arcsin(ax)}{6a^3}+\frac{\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)}{12a^3}\\
&+\frac{x^2\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)}{6a}\\
&-\frac{x\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2}{8a^2}\\
&+\frac{1}{4}x^3\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2\\
&+\frac{\arctan\left(e^{\mathrm{arctanh}(ax)}\right)\mathrm{arctanh}(ax)^2}{4a^3}\\
&-\frac{i\,\mathrm{arctanh}(ax)\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arctanh}(ax)}\right)}{4a^3}\\
&+\frac{i\,\mathrm{arctanh}(ax)\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arctanh}(ax)}\right)}{4a^3}\\
&+\frac{i\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{arctanh}(ax)}\right)}{4a^3}\\
&-\frac{i\,\mathrm{PolyLog}\left(3,ie^{\mathrm{arctanh}(ax)}\right)}{4a^3}\end{aligned}$$

output
```
1/12*x*(-a^2*x^2+1)^(1/2)/a^2-1/6*arcsin(a*x)/a^3+1/12*(-a^2*x^2+1)^(1/2)*
arctanh(a*x)/a^3+1/6*x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a-1/8*x*(-a^2*x^2
+1)^(1/2)*arctanh(a*x)^2/a^2+1/4*x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2+1/4
*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a^3-1/4*I*arctanh(a*x)*
polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/4*I*arctanh(a*x)*polylog(2,
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/4*I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^
(1/2))/a^3-1/4*I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.90

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$=\frac{\sqrt{1-a^2x^2}\Big(6\text{arctanh}(ax)-4(1-a^2x^2)\,\text{arctanh}(ax)-6ax(1-a^2x^2)\,\text{arctanh}(ax)^2+ax(2+3\text{arctanh}}{}$$

input
```
Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(6*ArcTanh[a*x] - 4*(1 - a^2*x^2)*ArcTanh[a*x] - 6*a*x*
(1 - a^2*x^2)*ArcTanh[a*x]^2 + a*x*(2 + 3*ArcTanh[a*x]^2) - (I*((-8*I)*Arc
Tan[Tanh[ArcTanh[a*x]/2]] + 3*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 3
*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyLog[2, (-I)
/E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 6*PolyLog
[3, (-I)/E^ArcTanh[a*x]] - 6*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x
^2]))/(24*a^3)
```

Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.87, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6576, 6578, 6514, 3042, 4668, 3011, 2720, 6556, 223, 6578, 262, 223, 6514, 3042, 4668, 3011, 2720, 6556, 223, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx - a^2\int \frac{x^4\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6578}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a} + \frac{\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2} - \left(a^2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2} + \frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a} - \frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)\right) - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{ 6514}$$

$$\frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a} - \left(a^2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2} + \frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a} - \frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)\right) + \frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a^3} + \frac{\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a} - \left(a^2\left(\frac{3\int \frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2} + \frac{\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a} - \frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)\right) - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{-2i\int \operatorname{arctanh}(ax)\log\left(1-ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)+2i\int \operatorname{arctanh}(ax)\log\left(1+ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)+}{2a^3}$$

$$\frac{\int \frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-$$

$$\left(a^2\left(\frac{3\int \frac{x^2\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{4a^2}\right)\right)-$$

$$\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2i\left(\int \operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)\right)-2i\left(\int \operatorname{PolyLog}\left(2,ie^{\operatorname{ar}}\right.\right.}{2a^3}$$

$$\frac{\int \frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-$$

$$\left(a^2\left(\frac{3\int \frac{x^2\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{4a^2}\right)\right)-$$

$$\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{2720}$$

$$2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)de^{\operatorname{arctanh}(ax)}-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\operatorname{arc}}\right.$$

$$\frac{\int \frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}-$$

$$\left(a^2\left(\frac{3\int \frac{x^2\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{4a^2}\right)\right)-$$

$$\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{6556}$$

$$2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.$$

$$\frac{\frac{\int\frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-$$

$$\left(a^2\left(\frac{3\int\frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)\right)-$$

$$\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{223}$$

$$2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.$$

$$\left(a^2\left(\frac{3\int\frac{x^2\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}\right)\right)+$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{6578}$$

$$2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.$$

$$\left(a^2\left(\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int\frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}}{2a}+\frac{3\left(\frac{\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int\frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-}}{}\right.}{4a^2}\right.\right.$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arc}\cdots}\right.}{\left(a^2\left(\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int\frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}}{2a}+\frac{3\left(\frac{\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int\frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}\cdots}{2a^2}\cdots\right.}{4a^2}\cdots\right.\right.}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\downarrow$ 223

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arc}\cdots}\right.}{\left(a^2\left(\frac{3\left(\frac{\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int\frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}\right)}{4a^2}+\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctan}\cdots}{3a^2}}{2a}\cdots\right.\right.}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\downarrow$ 6514

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arc}\cdots}\right.}{\left(a^2\left(\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}+\frac{3\left(\frac{\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a}+\frac{\int\sqrt{1-a^2x^2}\text{arctanh}\cdots}{2a\cdots}\cdots\right.}{\cdots}\cdots\right.\right.}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$\downarrow$ 3042

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}\cdots\right.}{\left(a^2\left(\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}+\frac{3\left(\frac{\int\text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{ar}\cdots}{2a^3}\right.}{\cdots}\right.\right.\qquad\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}}$$

$\Big\downarrow$ 4668

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}\cdots\right.}{\left(a^2\left(\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}+\frac{3\left(\frac{-2i\int\text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{a}\cdots}{\cdots}\right.}{\cdots}\right.\right.\qquad\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}}$$

$\Big\downarrow$ 3011

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}\cdots\right.}{\left(a^2\left(\frac{\frac{2\int\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}+\frac{3\left(\frac{2i\left(\int\operatorname{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.\qquad\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}}$$

$\Big\downarrow$ 2720

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}\cdots}\right.}{\left(a^2\left(\frac{\frac{2\int\frac{x\,\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}+\frac{3\left(\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(a\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.\right.}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{6556}$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}\cdots}\right.}{\left(a^2\left(\frac{\frac{2\left(\frac{\int\frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}}{2a}+\frac{3\left(\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{Pol}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}\cdots}\right.}{\left(a^2\left(\frac{3\left(\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{arctanh}(ax)}\cdots\right.}{2a^3}\right.}{\cdots}\right.\right.}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-}{2a^3}$$

$$\frac{\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}}{a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a^2}-$$

$$\left(a^2\left(-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{4a^2}+\frac{3\left(\frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\,\text{P}\right.}{}\right.}{}\right.\right.$$

input
```
Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (ArcSin[a*x]/a^2 - (Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/a^2)/a + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^
2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-
I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])
+ PolyLog[3, I*E^ArcTanh[a*x]]))/(2*a^3) - a^2*(-1/4*(x^3*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/a^2 + ((-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*
a^3))/(3*a) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin[a*
x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2))/(3*a^2))/(2*a) + (3*(-1/2*
(x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2 + (ArcSin[a*x]/a^2 - (Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/a^2)/a + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 + (
2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-I)*E^
ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]]) + Pol
yLog[3, I*E^ArcTanh[a*x]]))/(2*a^3)))/(4*a^2))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int x^2\sqrt{-a^2x^2+1}\, \operatorname{arctanh}\left(ax\right)^2dx$$

input
```
int(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

output
```
int(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Fricas [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}x^2\,\text{artanh}\,(ax)^2\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int x^2\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2\,(ax)\,dx$$

input
```
integrate(x**2*(-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

output
```
Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Maxima [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}x^2\,\text{artanh}\,(ax)^2\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Giac [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}x^2\,\text{artanh}\,(ax)^2\;dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int x^2\,\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x^2*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x^2*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)^2\,x^2dx$$

input
```
int(x^2*(-a^2*x^2+1)^(1/2)*atanh(a*x)^2,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*x**2,x)
```

3.441 $\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \frac{\sqrt{1-a^2x^2}}{3a^2}+\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a} - \frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{3a^2} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2}{3a^2} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{3a^2}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{3a^2}$$

output
```
1/3*(-a^2*x^2+1)^(1/2)/a^2+1/3*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a-2/3*arc
tan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^2-1/3*(-a^2*x^2+1)^(3/2)*
arctanh(a*x)^2/a^2-1/3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2+1/
3*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.77

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\left(1+ax\text{arctanh}(ax)-(1-a^2x^2)\,\text{arctanh}(ax)^2-\frac{i\left(\text{arctanh}(ax)\left(\log\left(1-ie^{-\text{arctanh}(ax)}\right)-\log\left(1+ie^{-\text{ar}}\right.\right.\right.}{\cdots}\right.}{3a^2}$$

input
```
Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (1 - a^2*x^2)*ArcTanh[a*x]^2 -
(I*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) +
 PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 -
 a^2*x^2]))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6556, 6504, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{2\int \sqrt{1-a^2x^2}\text{arctanh}(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6504}$$

$$\frac{2\left(\frac{1}{2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)^2}{3a^2}$$

$$\Big\downarrow \text{6512}$$

$$-\frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}{3a^2}+$$
$$\frac{2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)}{3a}$$

input
```
Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
-1/3*((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2)/a^2 + (2*(Sqrt[1 - a^2*x^2]/(2*a
) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 + ((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[
1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*
x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/2))/(3*a)
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\left(a^2x^2\operatorname{arctanh}(ax)^2+ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^2+1\right)}{3a^2}-\frac{i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{3a^2}+\frac{i\operatorname{arctanh}(}{}$

input
```
int(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)-
arctanh(a*x)^2+1)-1/3*I/a^2*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/3*I/a^2*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*I*dilog(1+
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+1/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/
2))/a^2
```

Fricas [F]

$$\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx=\int\sqrt{-a^2x^2+1}x\operatorname{artanh}(ax)^2\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx=\int x\sqrt{-(ax-1)(ax+1)}\operatorname{atanh}^2(ax)\,dx$$

input
```
integrate(x*(-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

output
```
Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Maxima [F]

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}x\,\text{artanh}\,(ax)^2\,dx$$

input
```
integrate(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int x\,\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(x*atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(x*atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)^2\,xdx$$

input
```
int(x*(-a^2*x^2+1)^(1/2)*atanh(a*x)^2,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*x,x)
```

3.442 $\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = -\frac{\arcsin(ax)}{a}+\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a}+\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2+\frac{\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2}{a}-\frac{i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)}{a}+\frac{i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{arctanh}(ax)}\right)}{a}+\frac{i\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)}{a}-\frac{i\operatorname{PolyLog}\left(3,ie^{\operatorname{arctanh}(ax)}\right)}{a}$$

output
```
-arcsin(a*x)/a+(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a+1/2*x*(-a^2*x^2+1)^(1/2)*
arctanh(a*x)^2+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-I*arcta
nh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*arctanh(a*x)*polylog(
2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\left(2\text{arctanh}(ax)+ax\text{arctanh}(ax)^2-\frac{i\left(-4i\arctan\left(\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)+\text{arctanh}(ax)^2\log\left(1-ie^{-\text{arctanh}}\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTa
n[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTa
nh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^Arc
Tanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-
I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/
(2*a)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6506, 223, 6514, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$\Big\downarrow \text{ 6506}$$
$$\frac{1}{2}\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx-\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}$$
$$\Big\downarrow \text{ 223}$$
$$\frac{1}{2}\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \; 6514$$

$$\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \; 4668$$

$$\frac{-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+}{2a}$$
$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \; 3011$$

$$\frac{2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,ie^{a}\right.\right.}{2a}$$
$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \; 2720$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$
$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}+$$
$$\frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-}{2a}$$

input
```
Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
-(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/2 + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-
(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-I)*E^ArcTanh
[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]]) + PolyLog[3,
 I*E^ArcTanh[a*x]]))/(2*a)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \sqrt{-a^2x^2+1}\, \operatorname{arctanh}(ax)^2\, dx$$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Fricas [F]

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\, dx = \int \sqrt{-a^2x^2+1}\, \operatorname{artanh}(ax)^2\, dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2(ax)\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Maxima [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}\,\text{artanh}(ax)^2\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = \int \operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)^2\,dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^2,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2,x)
```

3.443 $\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\,dx &= 4\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax) \\ &\quad+\sqrt{1-a^2x^2}\text{arctanh}(ax)^2 \\ &\quad-2\text{arctanh}\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2 \\ &\quad-2\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right) \\ &\quad+2\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{\text{arctanh}(ax)}\right) \\ &\quad+2i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\ &\quad-2i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\ &\quad+2\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right) \\ &\quad-2\,\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\end{aligned}$$

output
```
4*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)+(-a^2*x^2+1)^(1/2)*arc
tanh(a*x)^2-2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-2*arctanh
(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x
+1)/(-a^2*x^2+1)^(1/2))+2*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*I
*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+
1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x}\,dx = \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2 + \operatorname{arctanh}(ax)^2\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) + 2i\operatorname{arctanh}(ax)\log\left(1-ie^{-\operatorname{arctanh}(ax)}\right) - 2i\operatorname{arctanh}(ax)\log\left(1+ie^{-\operatorname{arctanh}(ax)}\right) - \operatorname{arctanh}(ax)^2\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) + 2i\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arctanh}(ax)}\right) - 2i\operatorname{PolyLog}\left(2,ie^{-\operatorname{arctanh}(ax)}\right) - 2\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right) + 2\operatorname{PolyLog}\left(3,-e^{-\operatorname{arctanh}(ax)}\right) - 2\operatorname{PolyLog}\left(3,e^{-\operatorname{arctanh}(ax)}\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```

output
```
Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x]
)] + (2*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (2*I)*ArcTanh[a*x]*Log
[1 + I/E^ArcTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*Arc
Tanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*PolyLog[2, (-I)/E^ArcTanh
[a*x]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, E^
(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyLog[3, E^(-Arc
Tanh[a*x])]
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6576, 6556, 6512, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx - a^2\int \frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6556}$$

$$\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx - a^2\left(\frac{2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{a} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2}\right)$$

$$\Big\downarrow \text{ 6512}$$

$$\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx - a^2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2} + \frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$$\Big\downarrow \text{ 6582}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax) - a^2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2} + \frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax) - a^2\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{a^2} + \frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\int \operatorname{arctanh}(ax)^2\csc(i\operatorname{arctanh}(ax))d\operatorname{arctanh}(ax)-$$
$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$\downarrow$ 4670

$$i\left(2i\int \operatorname{arctanh}(ax)\log\left(1-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-2i\int \operatorname{arctanh}(ax)\log\left(1+e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\right.$$
$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$\downarrow$ 3011

$$i\left(-2i\left(\int \operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right)+2i\left(\int \operatorname{PolyLog}\right.\right.$$
$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$\downarrow$ 2720

$$i\left(-2i\left(\int e^{-\operatorname{arctanh}(ax)}\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)de^{\operatorname{arctanh}(ax)}-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right)+2i\left(\right.\right.$$
$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

$\downarrow$ 7143

$$i\left(-2i\left(\operatorname{PolyLog}\left(3,-e^{\operatorname{arctanh}(ax)}\right)-\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arctanh}(ax)}\right)\right)+2i\left(\operatorname{PolyLog}\left(3,e^{\operatorname{arctanh}(ax)}\right)-\operatorname{a}\right.\right.$$

$$a^2\left(-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{a^2}+\frac{2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{a}\right)$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```

output
```
-(a^2*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^2) + (2*((-2*ArcTan[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))/a)
) + I*((2*I)*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x
]*PolyLog[2, -E^ArcTanh[a*x]]) + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(A
rcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)^2}{x}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x,x)
```

output
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x,x)
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2/x,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}\,dx = \int \frac{\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x}\,dx$$

input
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x,x)
```

output
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2}{x}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)^2}{x}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^2/x,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x)**2)/x,x)
```

3.444 $\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 197

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^2}\,dx = &-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x} \\ &- 2a\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2 \\ &- 4a\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\ &+ 2ia\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right) \\ &- 2ia\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{arctanh}(ax)}\right) \\ &+ 2a\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\ &- 2a\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\ &- 2ia\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right) \\ &+ 2ia\operatorname{PolyLog}\left(3,ie^{\operatorname{arctanh}(ax)}\right)\end{aligned}$$

output
```
-(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x-2*a*arctan((a*x+1)/(-a^2*x^2+1)^(1/2)
)*arctanh(a*x)^2-4*a*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+2*
I*a*arctanh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-2*I*a*arctanh(a*
x)*polylog(2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*polylog(2,-(-a*x+1)^(1/2)/(
a*x+1)^(1/2))-2*a*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))-2*I*a*polylog(3,
-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I*a*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2
))
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^2}\,dx = a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax} + 2\text{arctanh}(ax)\log\left(1-e^{-\text{arctanh}(ax)}\right) + i\text{arctanh}(ax)^2\log\left(1-ie^{-\text{arctanh}(ax)}\right) - i\text{arctanh}(ax)^2\log\left(1+ie^{-\text{arctanh}(ax)}\right) - 2\text{arctanh}(ax)\log\left(1+e^{-\text{arctanh}(ax)}\right) + 2\,\text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) + 2i\text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{-\text{arctanh}(ax)}\right) - 2i\text{arctanh}(ax)\,\text{PolyLog}\left(2,ie^{-\text{arctanh}(ax)}\right) - 2\,\text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right) + 2i\,\text{PolyLog}\left(3,-ie^{-\text{arctanh}(ax)}\right) - 2i\,\text{PolyLog}\left(3,ie^{-\text{arctanh}(ax)}\right)\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

output
```
a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(a*x)) + 2*ArcTanh[a*x]*Log[1 - E^
(-ArcTanh[a*x])] + I*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[
a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x]
)] + 2*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*ArcTanh[a*x]*PolyLog[2, (-I)
/E^ArcTanh[a*x]] - (2*I)*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] - 2*Pol
yLog[2, E^(-ArcTanh[a*x])] + (2*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (2*I)
*PolyLog[3, I/E^ArcTanh[a*x]])
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6576, 6514, 3042, 4668, 3011, 2720, 6570, 6580, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-a^2\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 6514}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-a\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-a\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 4668}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-$$
$$a\left(-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(a\right.$$

$$\Big\downarrow \text{ 3011}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-$$
$$a\left(2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-$$
$$a\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\right.\right.$$

$$\Big\downarrow \text{6570}$$

$$2a\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-a\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}$$

$$\Big\downarrow \text{6580}$$

$$-a\left(2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}+$$
$$2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x}-$$
$$a\left(2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax}\right.\right.\right.$$
$$2a\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) + 2*a*(-2*ArcTanh[a*x]*ArcTanh[Sqr
t[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - P
olyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]) - a*(2*ArcTan[E^ArcTanh[a*x]]*ArcT
anh[a*x]^2 + (2*I)*(-(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + Poly
Log[3, (-I)*E^ArcTanh[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTan
h[a*x]]) + PolyLog[3, I*E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)^2}{x^2}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^2,x)
```

output
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^2}\,dx=\int \frac{\sqrt{-(ax-1)\,(ax+1)}\,\text{atanh}^2\,(ax)}{x^2}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2/x**2,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^2}\,dx=\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^2}\,dx$$

input
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^2,x)
```

output
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)^2}{x^2}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^2/x^2,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x)**2)/x**2,x)
```

3.445 $\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}\,dx &= -\frac{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}\\ &\quad+a^2\text{arctanh}\left(e^{\text{arctanh}(ax)}\right)\text{arctanh}(ax)^2\\ &\quad-a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\\ &\quad+a^2\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\\ &\quad-a^2\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{\text{arctanh}(ax)}\right)\\ &\quad-a^2\,\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)\\ &\quad+a^2\,\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right)\end{aligned}$$

output

```
-a*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^2
/x^2+a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2-a^2*arctanh((-
a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x
+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}\,dx = \frac{1}{8}a^2\left(-4\text{arctanh}(ax)\coth\left(\frac{1}{2}\text{arctanh}(ax)\right) - \text{arctanh}(ax)^2\text{csch}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) - 4\text{arctanh}(ax)^2\log\left(1-e^{-\text{arctanh}(ax)}\right) + 4\text{arctanh}(ax)^2\log\left(1+e^{-\text{arctanh}(ax)}\right) + 8\log\left(\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right) - 8\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right) + 8\text{arctanh}(ax)\,\text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right) - 8\,\text{PolyLog}\left(3,-e^{-\text{arctanh}(ax)}\right) + 8\,\text{PolyLog}\left(3,e^{-\text{arctanh}(ax)}\right) - \text{arctanh}(ax)^2\text{sech}^2\left(\frac{1}{2}\text{arctanh}(ax)\right) + 4\text{arctanh}(ax)\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

output
```
(a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a
*x]/2]^2 - 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]^2*
Log[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] - 8*ArcTanh[a*x]*
PolyLog[2, -E^(-ArcTanh[a*x])] + 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x
])] - 8*PolyLog[3, -E^(-ArcTanh[a*x])] + 8*PolyLog[3, E^(-ArcTanh[a*x])] -
 ArcTanh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/
2]))/8
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {6576, 6582, 3042, 26, 4670, 3011, 2720, 6588, 6570, 243, 73, 221, 6582, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx-a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{6582}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx-a^2\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx-a^2\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx-ia^2\int \text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)$$

$$\Big\downarrow \text{4670}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx- ia^2\left(2i\int \text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int \text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax$$

$$\Big\downarrow \text{3011}$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx- ia^2\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{PolyI}\right.\right.$$

$$\Big\downarrow \ 2720$$

$$\int \frac{\text{arctanh}(ax)^2}{x^3\sqrt{1-a^2x^2}}dx - ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)} - \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2\right.$$

$$\Big\downarrow \ 6588$$

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)} - \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right. \frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx + a\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 6570$$

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)} - \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right. \frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx + a\left(a\int \frac{1}{x\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 243$$

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)} - \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right. \frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx + a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2 - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 73$$

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)} - \text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\right. \frac{1}{2}a^2\int \frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx + a\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a} - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 221$$

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)^2}{x\sqrt{1-a^2x^2}}dx+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 6582

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\frac{1}{2}a^2\int\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{ax}d\text{arctanh}(ax)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 3042

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\frac{1}{2}a^2\int i\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 26

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\frac{1}{2}ia^2\int\text{arctanh}(ax)^2\csc(i\text{arctanh}(ax))d\text{arctanh}(ax)+a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$\Big\downarrow$ 4670

$$-ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+\frac{1}{2}ia^2\left(2i\int\text{arctanh}(ax)\log\left(1-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-2i\int\text{arctanh}(ax)\log\left(1+e^{\text{arctanh}(ax)}\right)d\text{arctanh}(a\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{2}ia^2\left(-2i\left(\int \text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\int \text{Poly}\right.\right.$$
$$ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 2720$$

$$-\frac{1}{2}ia^2\left(-2i\left(\int e^{-\text{arctanh}(ax)}\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

$$\Big\downarrow \ 7143$$

$$-\frac{1}{2}ia^2\left(-2i\left(\text{PolyLog}\left(3,-e^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arctanh}(ax)}\right)\right)+2i\left(\text{PolyLog}\left(3,e^{\text{arctanh}(ax)}\right.\right.\right.$$
$$a\left(-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2x^2}$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2 + a*(-((Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]) - (I/2)*a^2*((2*I)*ArcTanh[E^
ArcTanh[a*x]]*ArcTanh[a*x]^2 - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, -E^ArcTanh
[a*x]]) + PolyLog[3, -E^ArcTanh[a*x]]) + (2*I)*(-(ArcTanh[a*x]*PolyLog[2,
E^ArcTanh[a*x]]) + PolyLog[3, E^ArcTanh[a*x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6582
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csch[x], x], x, Arc
Tanh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.52

method	result
default	$-\frac{(2ax+\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\operatorname{arctanh}(ax)^2\ln\left(1-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}-a^2\operatorname{arctanh}(ax)\operatorname{polylog}\left(2,\right.$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*x+arctanh(a*x))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arct
anh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2,(a*
x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^3}\,dx=\int \frac{\sqrt{-a^2x^2+1}\operatorname{artanh}\left(ax\right)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2(ax)}{x^3}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2/x**3,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^2}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^3}\,dx$$

input
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^3,x)
```

output
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)^2}{x^3}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^2/x^3,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x)**2)/x**3,x)
```

3.446 $\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^4}\,dx = -\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}{3x^3}+\frac{2}{3}a^3\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\frac{1}{3}a^3\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\frac{1}{3}a^3\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/3*a^2*(-a^2*x^2+1)^(1/2)/x-1/3*a*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2-1/
3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)^2/x^3+2/3*a^3*arctanh(a*x)*arctanh((-a*x
+1)^(1/2)/(a*x+1)^(1/2))-1/3*a^3*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))+
1/3*a^3*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^4}\,dx = -\frac{1}{3}a^3\,\text{PolyLog}\left(2,-e^{-\text{arctanh}(ax)}\right)$$
$$-\frac{(1-a^2x^2)^{3/2}\left(4\text{arctanh}(ax)^2+2(-1+\cosh(2\text{arctanh}(ax)))-\frac{4a^3x^3\,\text{PolyLog}\left(2,e^{-\text{arctanh}(ax)}\right)}{(1-a^2x^2)^{3/2}}+\text{arctanh}(\right.}{12}$$

input
```
Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

output
```
-1/3*(a^3*PolyLog[2, -E^(-ArcTanh[a*x])]) - ((1 - a^2*x^2)^(3/2)*(4*ArcTan
h[a*x]^2 + 2*(-1 + Cosh[2*ArcTanh[a*x]]) - (4*a^3*x^3*PolyLog[2, E^(-ArcTa
nh[a*x])])/(1 - a^2*x^2)^(3/2) + ArcTanh[a*x]*(2*Sinh[2*ArcTanh[a*x]] + ((
Log[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])])*(-3*a*x + Sqrt[1
- a^2*x^2]*Sinh[3*ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2])))/(12*x^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6570, 6572, 242, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^4}\,dx$$

$$\Big\downarrow \text{6570}$$

$$\frac{2}{3}a\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}dx-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6572}$$

$$\frac{2}{3}a\left(-\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{2}{3}a\left(-\int \frac{\operatorname{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x^2}-\frac{a\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6588}$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\operatorname{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6580}$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\left(-2\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1-}}{}\right.$$
$$-\frac{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}{3x^3}$$

input
```
Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

output
```
-1/3*((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2)/x^3 + (2*a*(-1/2*(a*Sqrt[1 - a^2
*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - (a^2*(-2*ArcTanh[a*x
]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/2))/3
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01

method	result
default	$\frac{\sqrt{-(ax-1)(ax+1)}\left(a^{2}x^{2}\operatorname{arctanh}(ax)^{2}-a^{2}x^{2}-ax\operatorname{arctanh}(ax)-\operatorname{arctanh}(ax)^{2}\right)}{3x^{3}}-\frac{a^{3}\operatorname{arctanh}(ax)\ln\left(1-\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)}{3}-\frac{a^{3}\operatorname{po}}{}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2-a^2*x^2-a*x*arctanh(a
*x)-arctanh(a*x)^2)/x^3-1/3*a^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/
2))-1/3*a^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*a^3*arctanh(a*x)*ln(
1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*a^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2
))
```

Fricas [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^4}\,dx=\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^4}\,dx=\int \frac{\sqrt{-(ax-1)(ax+1)}\operatorname{atanh}^2(ax)}{x^4}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2/x**4,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^4}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{x^4}\,dx = \int \frac{\text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}}{x^4}\,dx$$

input
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^4,x)
```

output
```
int((atanh(a*x)^2*(1 - a^2*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{x^4}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)^2}{x^4}dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^2/x^4,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x)**2)/x**4,x)
```

3.447 $\int x^4(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\int x^4\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\frac{3\sqrt{1-a^2x^2}}{128a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{192a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}$$
$$+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{128a^4}-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{64a^2}$$
$$+\frac{3}{16}x^5\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{3\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax)}{64a^5}-\frac{3i\operatorname{PolyLog}}{1}$$

output
```
3/128*(-a^2*x^2+1)^(1/2)/a^5+1/192*(-a^2*x^2+1)^(3/2)/a^5-3/80*(-a^2*x^2+1
)^(5/2)/a^5+1/56*(-a^2*x^2+1)^(7/2)/a^5-3/128*x*(-a^2*x^2+1)^(1/2)*arctanh
(a*x)/a^4-1/64*x^3*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2+3/16*x^5*(-a^2*x^2+
1)^(1/2)*arctanh(a*x)-1/8*a^2*x^7*(-a^2*x^2+1)^(1/2)*arctanh(a*x)-3/64*arc
tan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^5-3/128*I*polylog(2,-I*(-
a*x+1)^(1/2)/(a*x+1)^(1/2))/a^5+3/128*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)
^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.93

$$\int x^4(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \frac{121\sqrt{1-a^2x^2}+218a^2x^2\sqrt{1-a^2x^2}+216a^4x^4\sqrt{1-a^2x^2}-240a^6x^6\sqrt{1-a^2x^2}}{}$$

input
```
Integrate[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(121*Sqrt[1 - a^2*x^2] + 218*a^2*x^2*Sqrt[1 - a^2*x^2] + 216*a^4*x^4*Sqrt[
1 - a^2*x^2] - 240*a^6*x^6*Sqrt[1 - a^2*x^2] - 315*a*x*Sqrt[1 - a^2*x^2]*A
rcTanh[a*x] - 210*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2520*a^5*x^5*Sq
rt[1 - a^2*x^2]*ArcTanh[a*x] - 1680*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]
 - (315*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (315*I)*ArcTanh[a*x]*L
og[1 + I/E^ArcTanh[a*x]] - (315*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (315*
I)*PolyLog[2, I/E^ArcTanh[a*x]])/(13440*a^5)
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 791 vs. 2(292) = 584.

Time = 2.80 (sec) , antiderivative size = 791, normalized size of antiderivative = 2.71, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {6576, 6572, 243, 53, 2009, 6578, 243, 53, 2009, 6578, 241, 243, 53, 2009, 6512, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx - a^2\int x^6\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx$$

$\Big\downarrow$ 6572

$$\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-a^2\left(\frac{1}{8}\int \frac{x^6\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{8}a\int \frac{x^7}{\sqrt{1-a^2x^2}}dx+\frac{1}{8}x^7\sqrt{1-a^2x^2}\text{arctanh}(ax)\right)-\frac{1}{6}a\int \frac{x^5}{\sqrt{1-a^2x^2}}dx+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\Big\downarrow$ 243

$$\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-a^2\left(\frac{1}{8}\int \frac{x^6\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{16}a\int \frac{x^6}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{8}x^7\sqrt{1-a^2x^2}\text{arctanh}(ax)\right)-\frac{1}{12}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\Big\downarrow$ 53

$$\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-a^2\left(\frac{1}{8}\int \frac{x^6\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{16}a\int \left(-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6}+\frac{3\left(1-a^2x^2\right)^{3/2}}{a^6}-\frac{3\sqrt{1-a^2x^2}}{a^6}+\frac{1}{a^6\sqrt{1-a^2x^2}}\right)dx^2+\right.$$
$$\frac{1}{12}a\int \left(\frac{\left(1-a^2x^2\right)^{3/2}}{a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}+\frac{1}{a^4\sqrt{1-a^2x^2}}\right)dx^2+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\Big\downarrow$ 2009

$$\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-a^2\left(\frac{1}{8}\int \frac{x^6\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{8}x^7\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{16}a\left(\frac{2\left(1-a^2x^2\right)^{7/2}}{7a^8}-\frac{6\left(1-a^2x^2\right)^{5/2}}{5a^8}+\frac{2\left(1-a^2x\right.}{a^8}\right.\right.$$
$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$\Big\downarrow$ 6578

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)-$$
$$a^2\left(\frac{1}{8}\left(\frac{5\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}+\frac{\int \frac{x^5}{\sqrt{1-a^2x^2}}dx}{6a}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}\right)+\frac{1}{8}x^7\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{16}a\right.$$
$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)-$$
$$a^2\left(\frac{1}{8}\left(\frac{5\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}+\frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}dx^2}{12a}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}\right)+\frac{1}{8}x^7\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{16}\right.$$
$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)-$$
$$a^2\left(\frac{1}{8}\left(\frac{5\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}+\frac{\int \left(\frac{\left(1-a^2x^2\right)^{3/2}}{a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}+\frac{1}{a^4\sqrt{1-a^2x^2}}\right)dx^2}{12a}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}\right)+\frac{1}{8}\right.$$
$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right)-$$
$$a^2\left(\frac{1}{8}\left(\frac{5\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}+\frac{-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}}{12a}\right)+\frac{1}{8}x^7\sqrt{}\right.$$
$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{6578}$$

$$\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int\frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{}}{}}{8a}\right.$$

$$a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}+\frac{-\frac{2\left(1-a^2x^2\right.}{5a^6}}{}\right.\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \; 241$$

$$-a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}+\frac{-\frac{2\left(1-a\right.}{5a}}{}\right.\right.$$

$$\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a}}{a^4}}{8a}\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \; 243$$

$$-a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{x^2}{\sqrt{1-a^2x^2}}dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}+\frac{-\frac{2(1-a}{5}}{}\right.\right.$$

$$\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2}}{a^4}}{8a}\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{ 53}$$

$$-a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\int\frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(a}{6a^2}\right.\right.$$

$$\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2}}{a^4}}{8a}\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right.$$

$$a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}+\right.\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)$$

$$\Big\downarrow \text{6512}$$

$$-a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6a^2}\right.\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+$$

$$\frac{1}{6}\left(-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+}{2a^2}\right.}{4\ldots}\right.$$

$$\Big\downarrow \text{6578}$$

$$-a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\left(\frac{\int\frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int\frac{x}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2}}{a^4}}{8a}\right.}{6a^2}\right.\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+$$

$$\frac{1}{6}\left(-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+}{2a^2}\right.}{4}\right.$$

$$\Big\downarrow \; 241$$

$$-a^2\left(\frac{1}{8}\left(\frac{5\left(\frac{3\left(\frac{\int\frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2}+\frac{\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right)}{6a^2}\right.\right.$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+$$

$$\frac{1}{6}\left(-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+}{2a^2}\right.}{4}\right.$$

$$\Big\downarrow \; 6512$$

$$\frac{1}{6}x^5\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+$$

$$\frac{1}{6}\left(-\frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}+\frac{3\left(-\frac{\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+}{2a^2}\right.}{4}\right.$$

$$a^2\left(\frac{1}{8}x^7\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{16}a\left(\frac{2(1-a^2x^2)^{7/2}}{7a^8}-\frac{6(1-a^2x^2)^{5/2}}{5a^8}+\frac{2(1-a^2x^2)^{3/2}}{a^8}-\frac{2\sqrt{1-a^2x^2}}{a^8}\right)+\right.$$

input
```
Int[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
-1/12*(a*((-2*Sqrt[1 - a^2*x^2])/a^6 + (4*(1 - a^2*x^2)^(3/2))/(3*a^6) - (
2*(1 - a^2*x^2)^(5/2))/(5*a^6))) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/6
+ (((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))/(8*a) -
(x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4*a^2) + (3*(-1/2*Sqrt[1 - a^2*x^2]/
a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*
x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2
)))/(4*a^2))/6 - a^2*(-1/16*(a*((-2*Sqrt[1 - a^2*x^2])/a^8 + (2*(1 - a^2*x
^2)^(3/2))/a^8 - (6*(1 - a^2*x^2)^(5/2))/(5*a^8) + (2*(1 - a^2*x^2)^(7/2))
/(7*a^8))) + (x^7*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/8 + (((-2*Sqrt[1 - a^2*x
^2])/a^6 + (4*(1 - a^2*x^2)^(3/2))/(3*a^6) - (2*(1 - a^2*x^2)^(5/2))/(5*a^
6))/(12*a) - (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*a^2) + (5*(((-2*Sqrt[
1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))/(8*a) - (x^3*Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/(4*a^2) + (3*(-1/2*Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a
*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/
a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2)))/(4*a^2)))
/(6*a^2))/8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.74

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(1680 \operatorname{arctanh}(ax)a^7x^7+240a^6x^6-2520 \operatorname{arctanh}(ax)a^5x^5-216a^4x^4+210a^3x^3 \operatorname{arctanh}(ax)-218a^2x^2+315a}{13440a^5}$

input
```
int(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/13440/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(1680*arctanh(a*x)*a^7*x^7+240*a^6*x
^6-2520*arctanh(a*x)*a^5*x^5-216*a^4*x^4+210*a^3*x^3*arctanh(a*x)-218*a^2*
x^2+315*a*x*arctanh(a*x)-121)-3/128*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*a
rctanh(a*x)/a^5+3/128*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^
5-3/128*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/128*I*dilog(1-I*(a*x
+1)/(-a^2*x^2+1)^(1/2))/a^5
```

Fricas [F]

$$\int x^4 (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \int (-a^2x^2+1)^{\frac{3}{2}} x^4 \operatorname{artanh}(ax)\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^6 - x^4)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int x^4 (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \int x^4 (-(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)\,dx$$

input
```
integrate(x**4*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

output
```
Integral(x**4*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [F]

$$\int x^4(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \int(-a^2x^2+1)^{\frac{3}{2}}x^4\operatorname{artanh}(ax)\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

Giac [F]

$$\int x^4(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \int(-a^2x^2+1)^{\frac{3}{2}}x^4\operatorname{artanh}(ax)\,dx$$

input
```
integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \int x^4\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x^4*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

output
```
int(x^4*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^4 (1 - a^2x^2)^{3/2} \operatorname{arctanh}(ax)\, dx = -\left(\int \sqrt{-a^2x^2 + 1}\, atanh(ax)\, x^6 dx\right) a^2 + \int \sqrt{-a^2x^2 + 1}\, atanh(ax)\, x^4 dx$$

input
```
int(x^4*(-a^2*x^2+1)^(3/2)*atanh(a*x),x)
```

output
```
 - int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**6,x)*a**2 + int(sqrt( - a**2*x
**2 + 1)*atanh(a*x)*x**4,x)
```

3.448 $\int x^3(1-a^2x^2)^{3/2}\,\textbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 186

$$\int x^3\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\frac{3x\sqrt{1-a^2x^2}}{112a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{17\arcsin(ax)}{560a^4}-\frac{2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{35a^4}-\frac{x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{35a^2}+\frac{8}{35}x^4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)$$

output
```
3/112*x*(-a^2*x^2+1)^(1/2)/a^3+23/840*x^3*(-a^2*x^2+1)^(1/2)/a-1/42*a*x^5*
(-a^2*x^2+1)^(1/2)+17/560*arcsin(a*x)/a^4-2/35*(-a^2*x^2+1)^(1/2)*arctanh(
a*x)/a^4-1/35*x^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2+8/35*x^4*(-a^2*x^2+1
)^(1/2)*arctanh(a*x)-1/7*a^2*x^6*(-a^2*x^2+1)^(1/2)*arctanh(a*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42

$$\int x^3(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \frac{ax\sqrt{1-a^2x^2}(45+46a^2x^2-40a^4x^4)+51\arcsin(ax)-48(1-a^2x^2)^{5/2}(2+5$$

input
```
Integrate[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(a*x*Sqrt[1 - a^2*x^2]*(45 + 46*a^2*x^2 - 40*a^4*x^4) + 51*ArcSin[a*x] - 4
8*(1 - a^2*x^2)^(5/2)*(2 + 5*a^2*x^2)*ArcTanh[a*x])/(1680*a^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 566 vs. 2(186) = 372.

Time = 2.14 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.04, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {6576, 6572, 262, 262, 223, 262, 223, 6578, 262, 223, 262, 223, 6556, 223, 6578, 262, 223, 6556, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx-a^2\int x^5\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx$$

$$\Big\downarrow \text{6572}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{7}\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{7}a\int \frac{x^6}{\sqrt{1-a^2x^2}}dx + \frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(ax)\right) - \frac{1}{5}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx + \frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{7}\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{7}a\left(\frac{5\int \frac{x^4}{\sqrt{1-a^2x^2}}dx}{6a^2} - \frac{x^5\sqrt{1-a^2x^2}}{6a^2}\right) + \frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(ax)\right) - \frac{1}{5}a\left(\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right) + \frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{7}\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{7}a\left(\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)}{6a^2} - \frac{x^5\sqrt{1-a^2x^2}}{6a^2}\right) + \frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(a\right. \frac{1}{5}a\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right) + \frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{7}\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{7}a\left(\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)}{6a^2} - \frac{x^5\sqrt{1-a^2x^2}}{6a^2}\right) + \frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(a\right. \frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax) - \frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-$$

$$a^2\left(\frac{1}{7}\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}}{6a^2}\right)+\frac{1}{7}x^6\sqrt{1-a\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 223

$$\frac{1}{5}\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx-$$

$$a^2\left(\frac{1}{7}\int \frac{x^5\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{7}a\left(\frac{5\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)}{6a^2}-\frac{x^5\sqrt{\right.\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 6578

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)-$$

$$a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}dx}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}\right)+\frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{7}a\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$\downarrow$ 262

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)-$$

$$a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}\right)+\frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctan}\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\downarrow 223$$

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}\right)+\frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctan}\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\downarrow 262$$

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}\right)+\frac{1}{7}x^6\sqrt{1-}\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\downarrow 223$$

$$\frac{1}{5}\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}\right)+\frac{1}{7}x^6\sqrt{1-a^2\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{6556}$$

$$\frac{1}{5}\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}\right)+\frac{1}{7}x^6\sqrt{1-a^2\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$-a^2\left(\frac{1}{7}\left(\frac{4\int \frac{x^3\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}}{5a}\right)+\frac{1}{7}x^6\sqrt{1-\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$

$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{6578}$$

$$-a^2\left(\frac{1}{7}\left(\frac{4\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{3\left(\frac{\arcsin(a}{2a^3}\right.}{}\right.\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$

$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{262}$$

$$-a^2\left(\frac{1}{7}\left(\frac{4\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\right.\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$

$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$-a^2\left(\frac{1}{7}\left(\frac{4\left(\frac{2\int \frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{3\left(\right.}{}\right.\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$

$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{ 6556}$$

$$-a^2\left(\frac{1}{7}\left(\frac{4\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{a}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}\right.\right.$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$

$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$

$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{5}x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)+$$
$$\frac{1}{5}\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3a^2}+\frac{\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}}{3a}\right)-$$
$$a^2\left(\frac{1}{7}x^6\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{1}{7}\left(-\frac{x^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5a^2}+\frac{4\left(\frac{2\left(\frac{\arcsin(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-}}{}\right.}{5a^2}\right.\right.$$
$$\frac{1}{5}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)$$

input
```
Int[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
-1/5*(a*(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/2*(x*Sqrt[1 - a^2*x^2])
/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2))) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x])/5 + ((-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a) - (x
^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/a^2))/(3*a^2))/5 - a^2*(-1/7*(a*(-1/6*(x^5*Sqrt[
1 - a^2*x^2])/a^2 + (5*(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/2*(x*Sqr
t[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2)))/(6*a^2))) + (x^6*Sqr
t[1 - a^2*x^2]*ArcTanh[a*x])/7 + ((-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(
-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2))/(5*a) - (x
^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(5*a^2) + (4*((-1/2*(x*Sqrt[1 - a^2*x^2
])/a^2 + ArcSin[a*x]/(2*a^3))/(3*a) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/(3*a^2) + (2*(ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2))/(3
*a^2)))/(5*a^2))/7)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.75

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(240 \operatorname{arctanh}(ax)a^6x^6+40a^5x^5-384a^4x^4 \operatorname{arctanh}(ax)-46a^3x^3+48a^2x^2 \operatorname{arctanh}(ax)-45ax+96 \operatorname{arctanh}(a}{1680a^4}$

input
```
int(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/1680/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(240*arctanh(a*x)*a^6*x^6+40*a^5*x^5-
384*a^4*x^4*arctanh(a*x)-46*a^3*x^3+48*a^2*x^2*arctanh(a*x)-45*a*x+96*arct
anh(a*x))+17/560*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-17/560*I*ln((a*x+1
)/(-a^2*x^2+1)^(1/2)-I)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.57

$$\int x^3 \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\,dx =$$

$$-\frac{\left(40\,a^5x^5-46\,a^3x^3-45\,ax+24\,(5\,a^6x^6-8\,a^4x^4+a^2x^2+2)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+102\arctan\left(\right.}{1680\,a^4}$$

input
```
integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/1680*((40*a^5*x^5 - 46*a^3*x^3 - 45*a*x + 24*(5*a^6*x^6 - 8*a^4*x^4 + a
^2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 102*arctan((sq
rt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F]

$$\int x^3 \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\,dx = \int x^3 (-(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)\,dx$$

input
```
integrate(x**3*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

output
```
Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int x^3 \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\,dx =$$
$$-\frac{1}{1680}a\left(\frac{5\left(\frac{8(-a^2x^2+1)^{\frac{5}{2}}x}{a^2}-\frac{2(-a^2x^2+1)^{\frac{3}{2}}x}{a^2}-\frac{3\sqrt{-a^2x^2+1}x}{a^2}-\frac{3\arcsin(ax)}{a^3}\right)}{a^2}-\frac{12\left(2(-a^2x^2+1)^{\frac{3}{2}}x+3\sqrt{-a^2x^2\right.}}{a^4}\right.$$
$$-\frac{1}{35}\left(\frac{5(-a^2x^2+1)^{\frac{5}{2}}x^2}{a^2}+\frac{2(-a^2x^2+1)^{\frac{5}{2}}}{a^4}\right)\operatorname{artanh}(ax)$$

input
```
integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/1680*a*(5*(8*(-a^2*x^2 + 1)^(5/2)*x/a^2 - 2*(-a^2*x^2 + 1)^(3/2)*x/a^2
- 3*sqrt(-a^2*x^2 + 1)*x/a^2 - 3*arcsin(a*x)/a^3)/a^2 - 12*(2*(-a^2*x^2 +
1)^(3/2)*x + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a*x)/a)/a^4) - 1/35*(5*(-a^
2*x^2 + 1)^(5/2)*x^2/a^2 + 2*(-a^2*x^2 + 1)^(5/2)/a^4)*arctanh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \int x^3 \operatorname{atanh}(a\,x)\, \left(1-a^2\,x^2\right)^{3/2} \,dx$$

input
```
int(x^3*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

output
```
int(x^3*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3 (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = -\left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^5dx\right)a^2 + \int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^3dx$$

input
```
int(x^3*(-a^2*x^2+1)^(3/2)*atanh(a*x),x)
```

output
```
 - int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**5,x)*a**2 + int(sqrt( - a**2*x
**2 + 1)*atanh(a*x)*x**3,x)
```

3.449 $\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\int x^2\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\frac{\sqrt{1-a^2x^2}}{16a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}-\frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{16a^2}+\frac{7}{24}x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-\frac{\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax)}{8a^3}-\frac{i\,\mathrm{PolyLog}\left(2\right.}{16}$$

output

```
1/16*(-a^2*x^2+1)^(1/2)/a^3+1/72*(-a^2*x^2+1)^(3/2)/a^3-1/30*(-a^2*x^2+1)^
(5/2)/a^3-1/16*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a^2+7/24*x^3*(-a^2*x^2+1)
^(1/2)*arctanh(a*x)-1/6*a^2*x^5*(-a^2*x^2+1)^(1/2)*arctanh(a*x)-1/8*arctan
((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a^3-1/16*I*polylog(2,-I*(-a*x+
1)^(1/2)/(a*x+1)^(1/2))/a^3+1/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a^3
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.92

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \frac{31\sqrt{1-a^2x^2}+38a^2x^2\sqrt{1-a^2x^2}-24a^4x^4\sqrt{1-a^2x^2}-45ax\sqrt{1-a^2x^2}\operatorname{arcta}}{}$$

input
```
Integrate[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(31*Sqrt[1 - a^2*x^2] + 38*a^2*x^2*Sqrt[1 - a^2*x^2] - 24*a^4*x^4*Sqrt[1 -
 a^2*x^2] - 45*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 210*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[a*x] - 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (45*I)
*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (45*I)*ArcTanh[a*x]*Log[1 + I/E^
ArcTanh[a*x]] - (45*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (45*I)*PolyLog[2,
 I/E^ArcTanh[a*x]])/(720*a^3)
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 562 vs. 2(243) = 486.

Time = 1.89 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6576, 6572, 243, 53, 2009, 6578, 241, 243, 53, 2009, 6512, 6578, 241, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{ 6576}$$

$$\int x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx-a^2\int x^4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx$$

$$\Big\downarrow \text{ 6572}$$

$$\frac{1}{4}\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{6}a\int \frac{x^5}{\sqrt{1-a^2x^2}}dx + \frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)\right) - \frac{1}{4}a\int \frac{x^3}{\sqrt{1-a^2x^2}}dx + \frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\downarrow$ 243

$$\frac{1}{4}\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{12}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx^2 + \frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)\right) - \frac{1}{8}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2 + \frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\downarrow$ 53

$$\frac{1}{4}\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{8}a\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2 - a^2\left(\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - \frac{1}{12}a\int \left(\frac{\left(1-a^2x^2\right)^{3/2}}{a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4} + \frac{1}{a^4\sqrt{1-a^2x^2}}\right)dx^2 + \frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arct}\right. \frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$\downarrow$ 2009

$$\frac{1}{4}\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx - a^2\left(\frac{1}{6}\int \frac{x^4\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx + \frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax) - \frac{1}{12}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6} + \frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6} - \frac{2\sqrt{1-a}}{a^6}\right.\right. \frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax) - \frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$\downarrow$ 6578

$$\frac{1}{4}\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a} - \frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}\right) - a^2\left(\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2} + \frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a} - \frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right) + \frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax) - \frac{1}{12}a\right. \frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax) - \frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$$\Big\downarrow \text{241}$$

$$-a^2\left(\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}\right.$$
$$\frac{1}{4}\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)+$$
$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$$\Big\downarrow \text{243}$$

$$-a^2\left(\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{1}\right.$$
$$\frac{1}{4}\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)+$$
$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$$\Big\downarrow \text{53}$$

$$-a^2\left(\frac{1}{6}\left(\frac{3\int \frac{x^2\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2}{8a}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}\right)+\frac{1}{6}x^5\sqrt{1-a^2x^2}\right.$$
$$\frac{1}{4}\left(\frac{\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)+$$
$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2} - \frac{\sqrt{1-a^2x^2}}{2a^3}\right) -$$

$$a^2\left(\frac{1}{6}\left(\frac{3\int \frac{x^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2} + \frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right) + \frac{1}{6}x^5\sqrt{1-a^2x^2}\operatorname{arctanh}\right.$$

$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - \frac{1}{8}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right)$$

$\downarrow$ 6512

$$-a^2\left(\frac{1}{6}\left(\frac{3\int \frac{x^2\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2} + \frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}}{8a}\right) + \frac{1}{6}x^5\sqrt{1-a^2x^2}\operatorname{arctan}\right.$$

$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - \frac{1}{8}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right) +$$

$$\frac{1}{4}\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a} - \frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2} - \frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2} - \frac{\sqrt{1-a}}{2a^3}\right.$$

$\downarrow$ 6578

$$-a^2\left(\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}}dx}{2a} - \frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2}\right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4a^2} + \frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}}{8}\right.\right.$$

$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - \frac{1}{8}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right) +$$

$$\frac{1}{4}\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a} - \frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2} - \frac{x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{2a^2} - \frac{\sqrt{1-a}}{2a^3}\right.$$

$\downarrow$ 241

$$-a^2\left(\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}}{2a^3}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4a^2}+\frac{\frac{2(1-a^2x^2)^{3/2}}{3a^4}-}{8a}\right.\right.$$

$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+$$

$$\frac{1}{4}\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a}}{2a^3}\right.$$

$$\Big\downarrow \text{6512}$$

$$\frac{1}{4}x^3\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{8}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+$$

$$\frac{1}{4}\left(\frac{-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}}{2a^2}-\frac{x\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2a^2}-\frac{\sqrt{1-a}}{2a^3}\right.$$

$$a^2\left(\frac{1}{6}x^5\sqrt{1-a^2x^2}\text{arctanh}(ax)-\frac{1}{12}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+\frac{1}{6}\left(-\frac{x^3\sqrt{1-a^2}}{}\right.\right.$$

input
```
Int[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
-1/8*(a*((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))) +
(x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/4 + (-1/2*Sqrt[1 - a^2*x^2]/a^3 - (x*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1
 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x
]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*a^2))/4 - a^
2*(-1/12*(a*((-2*Sqrt[1 - a^2*x^2])/a^6 + (4*(1 - a^2*x^2)^(3/2))/(3*a^6)
- (2*(1 - a^2*x^2)^(5/2))/(5*a^6))) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/6 + (((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))/(8*a)
 - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4*a^2) + (3*(-1/2*Sqrt[1 - a^2*x^
2]/a^3 - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) + ((-2*ArcTan[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a)/(2*
a^2)))/(4*a^2))/6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6578
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(-f)*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a
+ b*ArcTanh[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1
)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[f^2*((m - 1
)/(c^2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d + e*x^2]), x]
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(120\operatorname{arctanh}(ax)a^5x^5+24a^4x^4-210a^3x^3\operatorname{arctanh}(ax)-38a^2x^2+45ax\operatorname{arctanh}(ax)-31)}{720a^3}-\frac{i\operatorname{arctanh}(ax)\,\mathrm{l}}{}$

input
```
int(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/720/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*a^5*x^5+24*a^4*x^4-2
10*a^3*x^3*arctanh(a*x)-38*a^2*x^2+45*a*x*arctanh(a*x)-31)-1/16*I/a^3*arct
anh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*I/a^3*arctanh(a*x)*ln(1-I
*(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^3+1/16*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Fricas [F]

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx=\int(-a^2x^2+1)^{\frac{3}{2}}x^2\operatorname{artanh}(ax)\;dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^4 - x^2)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx=\int x^2(-(ax-1)\,(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)\,dx$$

input
```
integrate(x**2*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

output
```
Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [F]

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx=\int(-a^2x^2+1)^{\frac{3}{2}}x^2\operatorname{artanh}(ax)\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

Giac [F]

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx=\int(-a^2x^2+1)^{\frac{3}{2}}x^2\operatorname{artanh}(ax)\,dx$$

input
```
integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx=\int x^2\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x^2*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

output
```
int(x^2*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2 (1 - a^2 x^2)^{3/2} \operatorname{arctanh}(ax)\, dx = -\left(\int \sqrt{-a^2x^2+1}\, atanh(ax)\, x^4 dx\right) a^2 + \int \sqrt{-a^2x^2+1}\, atanh(ax)\, x^2 dx$$

input
```
int(x^2*(-a^2*x^2+1)^(3/2)*atanh(a*x),x)
```

output
```
 - int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**4,x)*a**2 + int(sqrt( - a**2*x
**2 + 1)*atanh(a*x)*x**2,x)
```

3.450 $\int x(1-a^2x^2)^{3/2}\,\textbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\frac{3x\sqrt{1-a^2x^2}}{40a}+\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3\arcsin(ax)}{40a^2}-\frac{\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)}{5a^2}$$

output
```
3/40*x*(-a^2*x^2+1)^(1/2)/a+1/20*x*(-a^2*x^2+1)^(3/2)/a+3/40*arcsin(a*x)/a
^2-1/5*(-a^2*x^2+1)^(5/2)*arctanh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\frac{ax(5-2a^2x^2)\sqrt{1-a^2x^2}+3\arcsin(ax)-8(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{40a^2}$$

input
```
Integrate[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(a*x*(5 - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] + 3*ArcSin[a*x] - 8*(1 - a^2*x^2)^(
5/2)*ArcTanh[a*x])/(40*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6556, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6556}$$

$$\frac{\int \left(1-a^2x^2\right)^{3/2}dx}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)}{5a^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}\int \sqrt{1-a^2x^2}dx+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)}{5a^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)}{5a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}+\frac{\arcsin(ax)}{2a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)}{5a^2}$$

input
```
Int[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
((x*(1 - a^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - a^2*x^2])/2 + ArcSin[a*x]/(2*
a)))/4)/(5*a) - ((1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*a^2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(8a^4x^4\operatorname{arctanh}(ax)+2a^3x^3-16a^2x^2\operatorname{arctanh}(ax)-5ax+8\operatorname{arctanh}(ax))}{40a^2}+\frac{3i\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}+i\right)}{40a^2}-\frac{3i\ln}{}$

input
```
int(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/40/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)+2*a^3*x^3-16*a^
2*x^2*arctanh(a*x)-5*a*x+8*arctanh(a*x))+3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1
/2)+I)/a^2-3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = -\frac{\left(2\,a^3x^3-5\,ax+4\,(a^4x^4-2\,a^2x^2+1)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{40\,a^2}$$

input
```
integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
-1/40*((2*a^3*x^3 - 5*a*x + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*
x - 1)))*sqrt(-a^2*x^2 + 1) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^
2
```

Sympy [F]

$$\int x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \int x(-(ax-1)\,(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)\,dx$$

input
```
integrate(x*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

output
```
Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = -\frac{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)}{5\,a^2} + \frac{2\,(-a^2x^2+1)^{\frac{3}{2}}x+3\,\sqrt{-a^2x^2+1}x+\frac{3\,\arcsin(ax)}{a}}{40\,a}$$

input
```
integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
-1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/a^2 + 1/40*(2*(-a^2*x^2 + 1)^(3/2)*
x + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a*x)/a)/a
```

Giac [F(-2)]

Exception generated.

$$\int x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \int x\,\mathrm{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x*atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

output
```
int(x*atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = -\left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^3dx\right)a^2 + \int \sqrt{-a^2x^2+1}\,atanh(ax)\,xdx$$

input
```
int(x*(-a^2*x^2+1)^(3/2)*atanh(a*x),x)
```

output
```
 - int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**3,x)*a**2 + int(sqrt( - a**2*x
**2 + 1)*atanh(a*x)*x,x)
```

3.451 $\int (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{(1-a^2x^2)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) + \frac{1}{4}x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax) - \frac{3\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax)}{4a} - \frac{3i\,\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{8a} + \frac{3i\,\operatorname{PolyLog}\left(2\right.}{8a}$$

output
```
3/8*(-a^2*x^2+1)^(1/2)/a+1/12*(-a^2*x^2+1)^(3/2)/a+3/8*x*(-a^2*x^2+1)^(1/2
)*arctanh(a*x)+1/4*x*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-3/4*arctan((-a*x+1)^(
1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-3/8*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+
1)^(1/2))/a+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \frac{11\sqrt{1-a^2x^2} - 2a^2x^2\sqrt{1-a^2x^2} + 15ax\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - 6a^3x^3\sqrt{1-}{}$$

input
```
Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTa
nh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh
[a*x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcT
anh[a*x]])/(24*a)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6504, 6504, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \ 6504$$

$$\frac{3}{4}\int \sqrt{1-a^2x^2}\text{arctanh}(ax)dx+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \ 6504$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}\right)+ \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \ 6512$$

$$\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} \right.\right. \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}$$

input
```
Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(1 - a^2*x^2)^(3/2)/(12*a) + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 + (3*(
Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 + ((-2*ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*
x]])/a)/2))/4
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\left(6 a^{3} x^{3} \operatorname{arctanh}\left(a x\right)+2 a^{2} x^{2}-15 a x \operatorname{arctanh}\left(a x\right)-11\right) \sqrt{-a^{2} x^{2}+1}}{24 a}-\frac{3 i \operatorname{arctanh}\left(a x\right) \ln \left(1+\frac{i \left(a x+1\right)}{\sqrt{-a^{2} x^{2}+1}}\right)}{8 a}+\frac{3 i \operatorname{arctanh}\left(a x\right) \ln \left(\right.}{8 a}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+
1)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a
*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\,dx = \int \left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\,dx = \int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right)dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [F]

$$\int \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\,dx = \int \left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int (1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = \int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

input
```
int(atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int (1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)\,dx = -\left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^2dx\right)a^2 + \int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x),x)
```

output
```
- int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**2,x)*a**2 + int(sqrt( - a**2*x
**2 + 1)*atanh(a*x),x)
```

3.452 $\int \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x}\,dx = -\frac{1}{6}ax\sqrt{1-a^2x^2} - \frac{7}{6}\arcsin(ax) + \sqrt{1-a^2x^2}\text{arctanh}(ax) + \frac{1}{3}(1-a^2x^2)^{3/2}\,\text{arctanh}(ax) - 2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/6*a*x*(-a^2*x^2+1)^(1/2)-7/6*arcsin(a*x)+(-a^2*x^2+1)^(1/2)*arctanh(a*x
)+1/3*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-2*arctanh(a*x)*arctanh((-a*x+1)^(1/2
)/(a*x+1)^(1/2))+polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-polylog(2,(-a*x+
1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x}\,dx = \frac{1}{6}\Big(-ax\sqrt{1-a^2x^2} - 14\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right) + 8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - 2a^2x^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) + 6\operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right) - 6\operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right) + 6\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right) - 6\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)\Big)$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

output
```
(-(a*x*Sqrt[1 - a^2*x^2]) - 14*ArcTan[Tanh[ArcTanh[a*x]/2]] + 8*Sqrt[1 - a
^2*x^2]*ArcTanh[a*x] - 2*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*ArcTan
h[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x
])] + 6*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*PolyLog[2, E^(-ArcTanh[a*x])])/
6
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6576, 6556, 211, 223, 6572, 223, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x}dx - a^2\int x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx$$

$$\Big\downarrow \text{6556}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}dx - a^2\left(\frac{\int \sqrt{1-a^2x^2}dx}{3a} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3a^2}\right)$$

$$\Big\downarrow \text{211}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}dx - a^2\left(\frac{\frac{1}{2}\int \frac{1}{\sqrt{1-a^2x^2}}dx + \frac{1}{2}x\sqrt{1-a^2x^2}}{3a} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}dx - a^2\left(\frac{\frac{1}{2}x\sqrt{1-a^2x^2} + \frac{\arcsin(ax)}{2a}}{3a} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3a^2}\right)$$

$$\Big\downarrow \text{6572}$$

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx - a\int \frac{1}{\sqrt{1-a^2x^2}}dx - \left(a^2\left(\frac{\frac{1}{2}x\sqrt{1-a^2x^2} + \frac{\arcsin(ax)}{2a}}{3a} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3a^2}\right)\right) + \sqrt{1-a^2x^2}\text{arctanh}(ax)$$

$$\Big\downarrow \text{223}$$

$$\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx - \left(a^2\left(\frac{\frac{1}{2}x\sqrt{1-a^2x^2} + \frac{\arcsin(ax)}{2a}}{3a} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3a^2}\right)\right) + \sqrt{1-a^2x^2}\text{arctanh}(ax) - \arcsin(ax)$$

$$\Big\downarrow \text{6580}$$

$$-\left(a^2\left(\frac{\frac{1}{2}x\sqrt{1-a^2x^2} + \frac{\arcsin(ax)}{2a}}{3a} - \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{3a^2}\right)\right) + \sqrt{1-a^2x^2}\text{arctanh}(ax) - \arcsin(ax) - 2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{PolyLog}\left(2, -\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{PolyLog}\left(2, \frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

output
```
-ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - a^2*(((x*Sqrt[1 - a^2*x^2]
)/2 + ArcSin[a*x]/(2*a))/(3*a) - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*a^2
)) - 2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6556
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTanh[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

method	result
default	$-\frac{(2a^2x^2\operatorname{arctanh}(ax)+ax-8\operatorname{arctanh}(ax))\sqrt{-a^2x^2+1}}{6}-\frac{7\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{3}-\operatorname{dilog}\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\operatorname{arctan}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*x^2*arctanh(a*x)+a*x-8*arctanh(a*x))*(-a^2*x^2+1)^(1/2)-7/3*ar
ctan((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arcta
nh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x}\,dx=\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x}\,dx = \int \frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\text{atanh}\,(ax)}{x}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\,\text{artanh}\,(ax)}{x}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x}\,dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x}dx - \left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,xdx\right)a^2$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x,x) - int(sqrt( - a**2*x**2 + 1)*
atanh(a*x)*x,x)*a**2
```

3.453 $\int \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\begin{aligned}\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx &= -\frac{1}{2}a\sqrt{1-a^2x^2} \\ &- \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x} - \frac{1}{2}a^2x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) \\ &+ 3a\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax) - a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) \\ &+ \frac{3}{2}ia\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{3}{2}ia\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)\end{aligned}$$

output
```
-1/2*a*(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x-1/2*a^2*x*(-a^
2*x^2+1)^(1/2)*arctanh(a*x)+3*a*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arcta
nh(a*x)-a*arctanh((-a^2*x^2+1)^(1/2))+3/2*I*a*polylog(2,-I*(-a*x+1)^(1/2)/
(a*x+1)^(1/2))-3/2*I*a*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \frac{1}{2}\left(-a\sqrt{1-a^2x^2} - \frac{2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x} - a^2x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) + 3ia\operatorname{arctanh}(ax)\log\left(1-ie^{-\operatorname{arctanh}(ax)}\right) - 3ia\operatorname{arctanh}(ax)\log\left(1+ie^{-\operatorname{arctanh}(ax)}\right) - 2a\log\left(\cosh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right) + 2a\log\left(\sinh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right) + 3ia\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arctanh}(ax)}\right) - 3ia\operatorname{PolyLog}\left(2,ie^{-\operatorname{arctanh}(ax)}\right)\right)$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

output
```
(-(a*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - a^2*x*Sqr
t[1 - a^2*x^2]*ArcTanh[a*x] + (3*I)*a*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x
]] - (3*I)*a*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - 2*a*Log[Cosh[ArcTanh
[a*x]/2]] + 2*a*Log[Sinh[ArcTanh[a*x]/2]] + (3*I)*a*PolyLog[2, (-I)/E^ArcT
anh[a*x]] - (3*I)*a*PolyLog[2, I/E^ArcTanh[a*x]])/2
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6576, 6504, 6512, 6576, 6512, 6570, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx$$

$$\Big\downarrow \; 6576$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx-a^2\int \sqrt{1-a^2x^2}\text{arctanh}(ax)dx$$

$$\Big\downarrow \text{6504}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx- a^2\left(\frac{1}{2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}\right)$$

$$\Big\downarrow \text{6512}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx- a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.$$

$$\Big\downarrow \text{6576}$$

$$-a^2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx- \left(a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.\right.$$

$$\Big\downarrow \text{6512}$$

$$\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx- a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.$$
$$\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)$$

$$\Big\downarrow \text{6570}$$

$$a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-$$
$$a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.$$
$$\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-$$
$$a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.$$
$$\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$
$$\Big\downarrow \text{73}$$
$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-$$
$$a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.$$
$$\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$
$$\Big\downarrow \text{221}$$

$$-a^2\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right.\right.$$

$$\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right)-$$

$$a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]] - a^2
*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2,
((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/S
qrt[1 + a*x]])/a) - a^2*(Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])/2 + ((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a -
 (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*S
qrt[1 - a*x])/Sqrt[1 + a*x]])/a)/2)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\left(a^{2} x^{2} \operatorname{arctanh}\left(a x \right)+a x +2 \operatorname{arctanh}\left(a x \right)\right) \sqrt{-a^{2} x^{2}+1}}{2 x}-a \ln \left(1+\frac{a x +1}{\sqrt{-a^{2} x^{2}+1}}\right)+a \ln \left(\frac{a x +1}{\sqrt{-a^{2} x^{2}+1}}-1\right)+\frac{3 i a \operatorname{arct}}{}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a^2*x^2*arctanh(a*x)+a*x+2*arctanh(a*x))*(-a^2*x^2+1)^(1/2)/x-a*ln(1
+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+3/2*I*a*ar
ctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*I*a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))-3/2*I*a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*I*a*ar
ctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^2}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**2,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**2, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^2}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^2,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^2}dx - \left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx\right)a^2$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x^2,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**2,x) - int(sqrt( - a**2*x**2 +
1)*atanh(a*x),x)*a**2
```

3.454 $\int \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^3}\,dx = -\frac{a\sqrt{1-a^2x^2}}{2x} + a^2\arcsin(ax) - a^2\sqrt{1-a^2x^2}\text{arctanh}(ax) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2} + 3a^2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{3}{2}a^2\,\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \frac{3}{2}a^2\,\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/2*a*(-a^2*x^2+1)^(1/2)/x+a^2*arcsin(a*x)-a^2*(-a^2*x^2+1)^(1/2)*arctanh
(a*x)-1/2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2+3*a^2*arctanh(a*x)*arctanh((
-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/2*a^2*polylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/
2))+3/2*a^2*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.94

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^3}\,dx = \frac{1}{8}a^2\left(16\arctan\left(\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)\right.$$
$$-8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)-2\coth\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$-\operatorname{arctanh}(ax)\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)-12\operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right)$$
$$+12\operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right)$$
$$-12\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right)+12\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)$$
$$\left.-\operatorname{arctanh}(ax)\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)+2\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

output
```
(a^2*(16*ArcTan[Tanh[ArcTanh[a*x]/2]] - 8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] -
 2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 12*ArcTanh
[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 12*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x
])] - 12*PolyLog[2, -E^(-ArcTanh[a*x])] + 12*PolyLog[2, E^(-ArcTanh[a*x])]
 - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.40, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6576, 6572, 223, 242, 6580, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^3}\,dx$$
$$\Big\downarrow \; 6576$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}dx-a^2\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x}dx$$

$$\Big\downarrow \text{6572}$$

$$-\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\sqrt{1-a^2x^2}\text{arctanh}(ax)\right)\right)-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}$$

$$\Big\downarrow \text{223}$$

$$-\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)\right)\right)-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}$$

$$\Big\downarrow \text{242}$$

$$-\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)\right)\right)-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}-\frac{a\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{6580}$$

$$-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\left(a^2\left(\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Po}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}-\frac{a\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{6588}$$

$$-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\left(a^2\left(\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Po}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\left(a^2\left(\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Po}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

$\downarrow$ 6580

$$-a^2\left(\sqrt{1-a^2x^2}\text{arctanh}(ax)-\arcsin(ax)-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{P}\right.$$
$$\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

output
```
-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) -
(a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(
Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/
2 - a^2*(-ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 2*ArcTanh[a*x]*Ar
cTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a
*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\left(2a^2x^2\,\mathrm{arctanh}(ax)+ax+\mathrm{arctanh}(ax)\right)\sqrt{-a^2x^2+1}}{2x^2}+2a^2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{3a^2\,\mathrm{dilog}\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{3a^2\,\mathrm{a}}{}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a^2*x^2*arctanh(a*x)+a*x+arctanh(a*x))*(-a^2*x^2+1)^(1/2)/x^2+2*a^
2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^
(1/2))+3/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*dilog
((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^3}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^3}\,dx = \int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^3}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**3,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**3, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^3}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\text{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^3}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^3,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^3}dx - \left(\int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x}dx\right)a^2$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x^3,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**3,x) - int((sqrt( - a**2*x**2 +
 1)*atanh(a*x))/x,x)*a**2
```

3.455 $\int \frac{(1-a^2x^2)^{3/2}\textbf{arctanh}(ax)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^4}\,dx = -\frac{a\sqrt{1-a^2x^2}}{6x^2} + \frac{a^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{x} - \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{3x^3} - 2a^3\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax) + \frac{7}{6}a^3\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) - ia^3\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right) + ia^3\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/6*a*(-a^2*x^2+1)^(1/2)/x^2+a^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x-1/3*(-
a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3-2*a^3*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2
))*arctanh(a*x)+7/6*a^3*arctanh((-a^2*x^2+1)^(1/2))-I*a^3*polylog(2,-I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))+I*a^3*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.27

$$\int \frac{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}{x^4}\,dx = -a^3\left(-\frac{\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)}{ax} + i\,\mathrm{arctanh}(ax)\log\left(1-ie^{-\mathrm{arctanh}(ax)}\right) - i\,\mathrm{arctanh}(ax)\log\left(1+ie^{-\mathrm{arctanh}(ax)}\right) - \log\left(\cosh\left(\frac{1}{2}\mathrm{arctanh}(ax)\right)\right) + \log\left(\sinh\left(\frac{1}{2}\mathrm{arctanh}(ax)\right)\right) + i\,\mathrm{PolyLog}\left(2,-ie^{-\mathrm{arctanh}(ax)}\right) - i\,\mathrm{PolyLog}\left(2,ie^{-\mathrm{arctanh}(ax)}\right)\right) - \frac{(1-a^2x^2)^{3/2}\left(8\,\mathrm{arctanh}(ax) + 2\sinh(2\,\mathrm{arctanh}(ax)) + \frac{\left(\log\left(\cosh\left(\frac{1}{2}\mathrm{arctanh}(ax)\right)\right)-\log\left(\sinh\left(\frac{1}{2}\mathrm{arctanh}(ax)\right)\right)\right)(}{\sqrt{1-a^2x^2}}\right.}{24x^3}$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

output
```
-(a^3*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 -
I/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - Log[Cosh[Ar
cTanh[a*x]/2]] + Log[Sinh[ArcTanh[a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a
*x]] - I*PolyLog[2, I/E^ArcTanh[a*x]])) - ((1 - a^2*x^2)^(3/2)*(8*ArcTanh[
a*x] + 2*Sinh[2*ArcTanh[a*x]] + ((Log[Cosh[ArcTanh[a*x]/2]] - Log[Sinh[Arc
Tanh[a*x]/2]])*(3*a*x - Sqrt[1 - a^2*x^2]*Sinh[3*ArcTanh[a*x]]))/Sqrt[1 -
a^2*x^2]))/(24*x^3)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6576, 6570, 243, 51, 73, 221, 6576, 6512, 6570, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}{x^4}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^4}dx-a^2\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx$$

$$\Big\downarrow \text{6570}$$

$$a^2\left(-\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx\right)+\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}dx-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$a^2\left(-\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx\right)+\frac{1}{6}a\int \frac{\sqrt{1-a^2x^2}}{x^4}dx^2-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{51}$$

$$a^2\left(-\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx\right)+\frac{1}{6}a\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$a^2\left(-\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx\right)+\frac{1}{6}a\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$a^2\left(-\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}dx\right)+\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{6576}$$

$$-\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx-a^2\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx\right)\right)+\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{6512}$$

$$-\left(a^2\left(\int \frac{\text{arctanh}(ax)}{x^2\sqrt{1-a^2x^2}}dx - a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,\frac{i}{\cdot}\right.}{a}\right.\right.\right.$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}}{x^2}\right) - \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{6570}$$

$$-\left(a^2\left(a\int \frac{1}{x\sqrt{1-a^2x^2}}dx - \left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}\left(\right.}{a}\right.\right.\right.\right.$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}}{x^2}\right) - \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\left(a^2\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2 - \left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLo}}{}\right.\right.\right.\right.$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}}{x^2}\right) - \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$-\left(a^2\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a} - \left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}}{}\right.\right.\right.\right.$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}}{x^2}\right) - \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$-\left(a^2\left(-\left(a^2\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)\right) - a\,\text{ar}\right.\right.$$

$$\frac{1}{6}a\left(a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}}{x^2}\right) - \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{3x^3}$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

output
```
-1/3*((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3 + (a*(-(Sqrt[1 - a^2*x^2]/x^2)
 + a^2*ArcTanh[Sqrt[1 - a^2*x^2]]))/6 - a^2*(-((Sqrt[1 - a^2*x^2]*ArcTanh[
a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]] - a^2*((-2*ArcTan[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 +
 a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.15

method	result
default	$\frac{(8a^2x^2 \operatorname{arctanh}(ax)-ax-2 \operatorname{arctanh}(ax))\sqrt{-a^2x^2+1}}{6x^3}+\frac{7a^3\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{6}-\frac{7a^3\ln\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}-1\right)}{6}-ia^3 \operatorname{arctanh}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(8*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))*(-a^2*x^2+1)^(1/2)/x^3+7/6
*a^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-7/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)-1)-I*a^3*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I*a^3*dilog(1+I
*(a*x+1)/(-a^2*x^2+1)^(1/2))+I*a^3*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+I
*a^3*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^4}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\,\text{artanh}\,(ax)}{x^4}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^4, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^4}\,dx = \int \frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\text{atanh}\,(ax)}{x^4}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**4,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**4, x)
```

Maxima [F]

$$\int \frac{(1 - a^2x^2)^{3/2} \operatorname{arctanh}(ax)}{x^4} \, dx = \int \frac{(-a^2x^2 + 1)^{\frac{3}{2}} \operatorname{artanh}(ax)}{x^4} \, dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - a^2x^2)^{3/2} \operatorname{arctanh}(ax)}{x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1 - a^2x^2)^{3/2} \operatorname{arctanh}(ax)}{x^4} \, dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(1 - a^2\,x^2\right)^{3/2}}{x^4} \, dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^4,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}{x^4}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^4}dx - \left(\int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^2}dx\right)a^2$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x^4,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**4,x) - int((sqrt( - a**2*x**2 +
 1)*atanh(a*x))/x**2,x)*a**2
```

3.456 $\int \frac{(1-a^2x^2)^{3/2}\mathbf{arctanh}(ax)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 191

$$\begin{aligned}\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^5}\,dx &= -\frac{a\sqrt{1-a^2x^2}}{12x^3} \\ &+ \frac{11a^3\sqrt{1-a^2x^2}}{24x} - \frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{4x^4} \\ &+ \frac{5a^2\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{8x^2} - \frac{3}{4}a^4\operatorname{arctanh}(ax)\operatorname{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) \\ &+ \frac{3}{8}a^4\operatorname{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{3}{8}a^4\operatorname{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\end{aligned}$$

output
```
-1/12*a*(-a^2*x^2+1)^(1/2)/x^3+11/24*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*(-a^2*x^
2+1)^(1/2)*arctanh(a*x)/x^4+5/8*a^2*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^2-3/
4*a^4*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+3/8*a^4*polylog(2
,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-3/8*a^4*polylog(2,(-a*x+1)^(1/2)/(a*x+1)^(
1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.48

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^5}\,dx = \frac{1}{192}a\left(40a^3\coth\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right.$$
$$+18a^3\operatorname{arctanh}(ax)\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$-\frac{a^4x\operatorname{csch}^4\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{\sqrt{1-a^2x^2}}-3a^3\operatorname{arctanh}(ax)\operatorname{csch}^4\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$+72a^3\operatorname{arctanh}(ax)\log\left(1-e^{-\operatorname{arctanh}(ax)}\right)$$
$$-72a^3\operatorname{arctanh}(ax)\log\left(1+e^{-\operatorname{arctanh}(ax)}\right)+72a^3\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(ax)}\right)$$
$$-72a^3\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(ax)}\right)+18a^3\operatorname{arctanh}(ax)\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$+3a^3\operatorname{arctanh}(ax)\operatorname{sech}^4\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)$$
$$-\frac{16\sqrt{1-a^2x^2}\sinh^4\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{x^3}$$
$$\left.+\frac{16a^2\sqrt{1-a^2x^2}\sinh^4\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)}{x}-40a^3\tanh\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)\right)$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

output
```
(a*(40*a^3*Coth[ArcTanh[a*x]/2] + 18*a^3*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]
^2 - (a^4*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*a^3*ArcTanh[a*x]
*Csch[ArcTanh[a*x]/2]^4 + 72*a^3*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] -
 72*a^3*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 72*a^3*PolyLog[2, -E^(-A
rcTanh[a*x])] - 72*a^3*PolyLog[2, E^(-ArcTanh[a*x])] + 18*a^3*ArcTanh[a*x]
*Sech[ArcTanh[a*x]/2]^2 + 3*a^3*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - (16*
Sqrt[1 - a^2*x^2]*Sinh[ArcTanh[a*x]/2]^4)/x^3 + (16*a^2*Sqrt[1 - a^2*x^2]*
Sinh[ArcTanh[a*x]/2]^4)/x - 40*a^3*Tanh[ArcTanh[a*x]/2]))/192
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 430 vs. 2(191) = 382.

Time = 1.66 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6576, 6572, 242, 245, 242, 6588, 242, 245, 242, 6580, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{x^5}dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}dx-a^2\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^3}dx$$

$$\Big\downarrow \text{6572}$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}\right)\right)+\frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}-\frac{a\sqrt{1-a^2x^2}}{x}\right)\right)+\frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}-\frac{a\sqrt{1-a^2x^2}}{x}\right)\right)+\frac{1}{3}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}$$

$$\Big\downarrow \; 242$$

$$-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^2}-\frac{a\sqrt{1-a^2x^2}}{x}\right)\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \; 6588$$

$$-\left(a^2\left(-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{x}\right)\right)+\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \; 242$$

$$-\left(a^2\left(-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)\right)+\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \; 245$$

$$-\left(a^2\left(-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)\right)+\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$$\Big\downarrow \; 242$$

$$-\left(a^2\left(-\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)\right)+$$
$$\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\big\downarrow$ 6580

$$\frac{1}{3}\left(-\frac{3}{4}a^2\int \frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)-$$
$$\left(a^2\left(-\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1}}{}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\big\downarrow$ 6588

$$\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax}{4x^4}\right.$$
$$\left(a^2\left(-\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1}}{}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\big\downarrow$ 242

$$\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\right.\right.$$
$$\left(a^2\left(-\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\frac{\sqrt{1}}{}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

$\big\downarrow$ 6580

$$-\left(a^2\left(-\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\right.\right.$$
$$\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)-\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

output
```
(a*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x)))/3 - (Sq
rt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^4) + (-1/4*(a*(-1/3*Sqrt[1 - a^2*x^2]/x
^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x))) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/
(4*x^4) - (3*a^2*(-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(2*x^2) + (a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x
]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]
/Sqrt[1 + a*x]]))/2))/4)/3 - a^2*(-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/(2*x^2) - (a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a
*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[
2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(11 a^{3} x^{3}+15 a^{2} x^{2} \operatorname{arctanh}\left(a x\right)-2 a x-6 \operatorname{arctanh}\left(a x\right)\right) \sqrt{-a^{2} x^{2}+1}}{24 x^{4}}+\frac{3 \operatorname{arctanh}\left(a x\right) \ln\left(1-\frac{a x+1}{\sqrt{-a^{2} x^{2}+1}}\right) a^{4}}{8}+\frac{3 \operatorname{polylog}\left(2, \frac{a x+}{\sqrt{-a^{2} x}}\right.}{8}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(11*a^3*x^3+15*a^2*x^2*arctanh(a*x)-2*a*x-6*arctanh(a*x))*(-a^2*x^2+1
)^(1/2)/x^4+3/8*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4+3/8*poly
log(2,(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4-3/8*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))*a^4-3/8*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))*a^4
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)}{x^5} dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}} \operatorname{artanh}(ax)}{x^5} dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)}{x^5} dx = \int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}} \operatorname{atanh}(ax)}{x^5} dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**5,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**5, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^5}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^5}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^5}\,dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^5}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^5,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}{x^5}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^5}dx - \left(\int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^3}dx\right)a^2$$

input

```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x^5,x)
```

output

```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**5,x) - int((sqrt( - a**2*x**2 +
 1)*atanh(a*x))/x**3,x)*a**2
```

3.457 $\int \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{x^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{a(1-a^2x^2)^{3/2}}{20x^4} - \frac{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)}{5x^5} - \frac{3}{40}a^5\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
3/40*a^3*(-a^2*x^2+1)^(1/2)/x^2-1/20*a*(-a^2*x^2+1)^(3/2)/x^4-1/5*(-a^2*x^
2+1)^(5/2)*arctanh(a*x)/x^5-3/40*a^5*arctanh((-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \left(-\frac{a}{20x^4}+\frac{a^3}{8x^2}\right)\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}(-1+a^2x^2)^2\,\text{arctanh}(ax)}{5x^5} + \frac{3}{40}a^5\log(x) - \frac{3}{40}a^5\log\left(1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

output
```
(-1/20*a/x^4 + a^3/(8*x^2))*Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*(-1 + a
^2*x^2)^2*ArcTanh[a*x])/(5*x^5) + (3*a^5*Log[x])/40 - (3*a^5*Log[1 + Sqrt[
1 - a^2*x^2]])/40
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6570, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^6}\,dx$$

$$\Big\downarrow \text{6570}$$

$$\frac{1}{5}a\int \frac{(1-a^2x^2)^{3/2}}{x^5}dx-\frac{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}a\int \frac{(1-a^2x^2)^{3/2}}{x^6}dx^2-\frac{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\int \frac{\sqrt{1-a^2x^2}}{x^4}dx^2-\frac{(1-a^2x^2)^{3/2}}{2x^4}\right)-\frac{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{(1-a^2x^2)^{3/2}}{2x^4}\right)-\frac{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{(1-a^2x^2)^{3/2}}{2x^4}\right)-\frac{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{5x^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{10}a\left(-\frac{3}{4}a^2\left(a^2\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{(1-a^2x^2)^{3/2}}{2x^4}\right)-\frac{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}{5x^5}$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

output
```
-1/5*((1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/x^5 + (a*(-1/2*(1 - a^2*x^2)^(3/2)
/x^4 - (3*a^2*(-(Sqrt[1 - a^2*x^2]/x^2) + a^2*ArcTanh[Sqrt[1 - a^2*x^2]]))
/4))/10
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6570
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a
+ b*ArcTanh[c*x])^p/(d*(m + 1))), x] - Simp[b*c*(p/(m + 1))   Int[(f*x)^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
 && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(8a^4x^4\,\operatorname{arctanh}(ax)-5a^3x^3-16a^2x^2\,\operatorname{arctanh}(ax)+2ax+8\,\operatorname{arctanh}(ax))}{40x^5}-\frac{3a^5\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)}{40}+\frac{3a^5}{}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)-5*a^3*x^3-16*a^2*x^
2*arctanh(a*x)+2*a*x+8*arctanh(a*x))/x^5-3/40*a^5*ln(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))+3/40*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^6}\,dx = \frac{3\,a^5x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(5\,a^3x^3-2\,ax-4\,(a^4x^4-2\,a^2x^2+1)\log\left(-\right.\right.}{40\,x^5}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

output
```
1/40*(3*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 - 2*a*x - 4*(
a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x^
5
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \int \frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\text{atanh}\,(ax)}{x^6}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**6,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \frac{1}{40}\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4 - 3\,a^4\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+3\,\sqrt{-a^2x^2+\right.$$
$$-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}\,\text{artanh}\,(ax)}{5\,x^5}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

output
```
1/40*((-a^2*x^2 + 1)^(3/2)*a^4 - 3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2
/abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^4 + (-a^2*x^2 + 1)^(5/2)*a^2/x^2 - 2*(-a
^2*x^2 + 1)^(5/2)/x^4)*a - 1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/x^5
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \int \frac{\text{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^6}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^6,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^6}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^6}dx - \left(\int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^4}dx\right)a^2$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x^6,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**6,x) - int((sqrt( - a**2*x**2 +
 1)*atanh(a*x))/x**4,x)*a**2
```

3.458 $\int \frac{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\int \frac{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}{x^7}\,dx = -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3} + \frac{31a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}+\frac{7a^2\sqrt{1-a^2x^2}\text{arctanh}(ax)}{24x^4} - \frac{a^4\sqrt{1-a^2x^2}\text{arctanh}(ax)}{16x^2}-\frac{1}{8}a^6\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \frac{1}{16}a^6\,\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\frac{1}{16}a^6\,\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

output
```
-1/30*a*(-a^2*x^2+1)^(1/2)/x^5+19/360*a^3*(-a^2*x^2+1)^(1/2)/x^3+31/720*a^
5*(-a^2*x^2+1)^(1/2)/x-1/6*(-a^2*x^2+1)^(1/2)*arctanh(a*x)/x^6+7/24*a^2*(-
a^2*x^2+1)^(1/2)*arctanh(a*x)/x^4-1/16*a^4*(-a^2*x^2+1)^(1/2)*arctanh(a*x)
/x^2-1/8*a^6*arctanh(a*x)*arctanh((-a*x+1)^(1/2)/(a*x+1)^(1/2))+1/16*a^6*p
olylog(2,-(-a*x+1)^(1/2)/(a*x+1)^(1/2))-1/16*a^6*polylog(2,(-a*x+1)^(1/2)/
(a*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 4.15 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.95

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx = \frac{82a^7x^4\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arctanh}(ax)\right)+90a^6x^3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arc}\right.}{}$$

input
```
Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

output
```
(82*a^7*x^4*Csch[ArcTanh[a*x]/2]^2 + 90*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]*Csch[ArcTanh[a*x]/2]^2 + 4*a^7*x^4*Csch[ArcTanh[a*x]/2]^4 - 3*a^7*x^4
*Csch[ArcTanh[a*x]/2]^6 - 15*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Csch[A
rcTanh[a*x]/2]^6 + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-
ArcTanh[a*x])] - 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-Ar
cTanh[a*x])] + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])
] - 360*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, E^(-ArcTanh[a*x])] + 328*a^5*
x^2*(-1 + a^2*x^2)*Sinh[ArcTanh[a*x]/2]^2 + 360*a^4*x*(1 - a^2*x^2)^(3/2)*
ArcTanh[a*x]*Sinh[ArcTanh[a*x]/2]^2 + 64*a^3*Sinh[ArcTanh[a*x]/2]^4 - 128*
a^5*x^2*Sinh[ArcTanh[a*x]/2]^4 + 64*a^7*x^4*Sinh[ArcTanh[a*x]/2]^4 - (192*
a*(-1 + a^2*x^2)^3*Sinh[ArcTanh[a*x]/2]^6)/x^2 - (960*(1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]*Sinh[ArcTanh[a*x]/2]^6)/x^3)/(5760*x^3*Sqrt[1 - a^2*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 737 vs. 2(243) = 486.

Time = 2.71 (sec) , antiderivative size = 737, normalized size of antiderivative = 3.03, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {6576, 6572, 245, 242, 245, 242, 6588, 245, 242, 245, 242, 6588, 242, 245, 242, 6580, 6588, 242, 6580}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx$$

$$\Big\downarrow \text{6576}$$

$$\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^7}dx-a^2\int \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{x^5}dx$$

$$\Big\downarrow \text{6572}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx+\frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}\right)\right)+\frac{1}{5}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx+\frac{1}{3}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}\right)\right)+\frac{1}{5}a\left(\frac{4}{5}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)\right)+\frac{1}{5}a\left(\frac{4}{5}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx-\left(a^2\left(-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)\right)+\frac{1}{5}a\left(\frac{4}{5}a^2\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{5}\int \frac{\text{arctanh}(ax)}{x^7\sqrt{1-a^2x^2}}dx - \left(a^2\left(-\frac{1}{3}\int \frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{3x^4}+\frac{1}{3}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)\right) - \frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\downarrow$ 6588

$$\frac{1}{5}\left(-\frac{5}{6}a^2\int\frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\int\frac{1}{x^6\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}\right) - \left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\int\frac{1}{x^4\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(a}{3x^4}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\downarrow$ 245

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{1}{4}a\left(\frac{2}{3}a^2\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\int\frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\left(\frac{4}{5}a^2\int\frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\downarrow$ 242

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)-\frac{\sqrt{1-}}{}\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\int\frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\left(\frac{4}{5}a^2\int\frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}\right)-$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$\downarrow$ 245

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)-\frac{\sqrt{1-}}{}\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\int\frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{1}{6}a\left(\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)+\frac{\sqrt{1-a^2x^2}\text{a}}{6x}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{242}$$

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)-\frac{\sqrt{1-}}{}\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\int\frac{\text{arctanh}(ax)}{x^5\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{6x^6}-\frac{1}{6}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2}}{5x^5}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{6588}$$

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}\right)+\frac{\sqrt{1-a^2x^2}\text{arct}}{4x^4}\right.\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{1}{4}a\int\frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax}{6x^6}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{242}$$

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right.\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{1}{4}a\int\frac{1}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax}{6x^6}\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{245}$$

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right.\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx+\frac{1}{4}a\left(\frac{2}{3}a^2\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right)+\right.$$
$$\left.\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\right.$$

$$\Big\downarrow \text{ 242}$$

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}\right.\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}+\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)+\frac{\sqrt{1-\ldots}}{}\right.$$
$$\left.\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\right.$$

$$\Big\downarrow \text{ 6580}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{\text{arctanh}(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}+\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\right)+\frac{\sqrt{1-\ldots}}{}\right.$$
$$\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.\right.\right.\right.\right.\right.$$
$$\left.\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\right.$$

$$\Big\downarrow \text{ 6588}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}\right)-\frac{\sqrt{1-a^2x^2}\text{arctan}\ldots}{4x^4}\right.\right.$$
$$\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.\right.\right.\right.\right.\right.$$
$$\left.\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\right.$$

$$\Big\downarrow \text{ 242}$$

$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{\text{arctanh}(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{2x^2}-\frac{a\sqrt{1-a^2x^2}}{2x}\right)-\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{4x^4}+\right.\right.\right.$$
$$\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right.\right.\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

$$\Big\downarrow \text{6580}$$

$$-\left(a^2\left(\frac{1}{3}\left(-\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-a}}{\sqrt{ax+}}\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{5}\left(-\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\left(-2\text{arctanh}(ax)\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right.\right.\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{5x^6}+\frac{1}{5}a\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)$$

input
```
Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

output
```
(a*(-1/5*Sqrt[1 - a^2*x^2]/x^5 + (4*a^2*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a
^2*Sqrt[1 - a^2*x^2])/(3*x)))/5))/5 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(5*
x^6) - a^2*((a*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*
x)))/3 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^4) + (-1/4*(a*(-1/3*Sqrt[1
- a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x))) + (Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])/(4*x^4) - (3*a^2*(-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1 - a^2
*x^2]*ArcTanh[a*x])/(2*x^2) + (a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/
Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog[2, S
qrt[1 - a*x]/Sqrt[1 + a*x]]))/2))/4)/3) + (-1/6*(a*(-1/5*Sqrt[1 - a^2*x^2]
/x^5 + (4*a^2*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x
)))/5)) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6) - (5*a^2*((a*(-1/3*Sqrt
[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x)))/4 - (Sqrt[1 - a^2*x^
2]*ArcTanh[a*x])/(4*x^4) + (3*a^2*(-1/2*(a*Sqrt[1 - a^2*x^2])/x - (Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(2*x^2) + (a^2*(-2*ArcTanh[a*x]*ArcTanh[Sqrt[1 -
a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyLog
[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]))/2))/4))/6)/5
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6572
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTanh[c
*x])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTanh[c*x])
/Sqrt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sq
rt[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e,
 0] && NeQ[m, -2]
```

rule 6576
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a
+ b*ArcTanh[c*x])^p, x], x] - Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x
^2)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m},
x] && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ
[p, 1] && IntegerQ[q]))
```

rule 6580
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]], x] + (Simp[(b/Sqrt[d])*PolyLog[2, -Sqrt[1 - c*x]/Sqrt[1 + c*x
]], x] - Simp[(b/Sqrt[d])*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]], x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

rule 6588
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*A
rcTanh[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^
(m + 1)*((a + b*ArcTanh[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] + Simp[c^2*(
(m + 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/Sqrt[d +
 e*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && G
tQ[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{-(ax-1)(ax+1)}\,(-31a^5x^5+45a^4x^4\operatorname{arctanh}(ax)-38a^3x^3-210a^2x^2\operatorname{arctanh}(ax)+24ax+120\operatorname{arctanh}(ax))}{720x^6}+\frac{a^6\operatorname{arctanh}(a}{}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/720*(-(a*x-1)*(a*x+1))^(1/2)*(-31*a^5*x^5+45*a^4*x^4*arctanh(a*x)-38*a^
3*x^3-210*a^2*x^2*arctanh(a*x)+24*a*x+120*arctanh(a*x))/x^6+1/16*a^6*arcta
nh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*polylog(2,(a*x+1)/(-a^2*
x^2+1)^(1/2))-1/16*a^6*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*
a^6*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx=\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^7}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx = \int \frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^7}\,dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**7,x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**7, x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^7}\,dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^7, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx = \int \frac{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}{x^7}\,dx$$

input
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^7,x)
```

output
```
int((atanh(a*x)*(1 - a^2*x^2)^(3/2))/x^7, x)
```

Reduce [F]

$$\int \frac{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}{x^7}\,dx = \int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^7}dx - \left(\int \frac{\sqrt{-a^2x^2+1}\,atanh(ax)}{x^5}dx\right)a^2$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x)/x^7,x)
```

output
```
int((sqrt( - a**2*x**2 + 1)*atanh(a*x))/x**7,x) - int((sqrt( - a**2*x**2 +
 1)*atanh(a*x))/x**5,x)*a**2
```

3.459 $\int (1-a^2x^2)^{5/2} \operatorname{arctanh}(ax) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 233

$$\int (1-a^2x^2)^{5/2} \operatorname{arctanh}(ax) \, dx = \frac{5\sqrt{1-a^2x^2}}{16a} + \frac{5(1-a^2x^2)^{3/2}}{72a} + \frac{(1-a^2x^2)^{5/2}}{30a}$$
$$+\frac{5}{16}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)+\frac{5}{24}x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)+\frac{1}{6}x(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)-\frac{5\arctan\left(\frac{\sqrt{1-}}{\sqrt{1-}}\right.}{}$$

output
```
5/16*(-a^2*x^2+1)^(1/2)/a+5/72*(-a^2*x^2+1)^(3/2)/a+1/30*(-a^2*x^2+1)^(5/2
)/a+5/16*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)+5/24*x*(-a^2*x^2+1)^(3/2)*arcta
nh(a*x)+1/6*x*(-a^2*x^2+1)^(5/2)*arctanh(a*x)-5/8*arctan((-a*x+1)^(1/2)/(a
*x+1)^(1/2))*arctanh(a*x)/a-5/16*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a+5/16*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96

$$\int (1 -a^2x^2)^{5/2} \operatorname{arctanh}(ax) \, dx = \frac{299\sqrt{1-a^2x^2} - 98a^2x^2\sqrt{1-a^2x^2} + 24a^4x^4\sqrt{1-a^2x^2} + 495ax\sqrt{1-a^2x^2}\operatorname{arc}}{}$$

input
```
Integrate[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

output
```
(299*Sqrt[1 - a^2*x^2] - 98*a^2*x^2*Sqrt[1 - a^2*x^2] + 24*a^4*x^4*Sqrt[1
- a^2*x^2] + 495*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 390*a^3*x^3*Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x] + 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (225
*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (225*I)*ArcTanh[a*x]*Log[1 +
I/E^ArcTanh[a*x]] - (225*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (225*I)*Poly
Log[2, I/E^ArcTanh[a*x]])/(720*a)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6504, 6504, 6504, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^{5/2} \text{arctanh}(ax) \, dx$$

$$\Big\downarrow \text{6504}$$

$$\frac{5}{6}\int \left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)dx+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{6504}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-a^2x^2}\text{arctanh}(ax)dx+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}\right)+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{6504}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)+\frac{(1-}{}\right.$$
$$\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{6512}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right.}{a}\right.\right.\right.$$

$$\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}$$

input
```
Int[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

output
```
(1 - a^2*x^2)^(5/2)/(30*a) + (x*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/6 + (5*(
(1 - a^2*x^2)^(3/2)/(12*a) + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 + (3*(
Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 + ((-2*ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*
x]])/a)/2))/4))/6
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.83

method	result
default	$\frac{(120 \operatorname{arctanh}(ax)a^5x^5+24a^4x^4-390a^3x^3 \operatorname{arctanh}(ax)-98a^2x^2+495ax \operatorname{arctanh}(ax)+299)\sqrt{-a^2x^2+1}}{720a} - \frac{5i \operatorname{arctanh}(ax) \ln\left(1+\right.}{16a}$

input
```
int((-a^2*x^2+1)^(5/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/720*(120*arctanh(a*x)*a^5*x^5+24*a^4*x^4-390*a^3*x^3*arctanh(a*x)-98*a^2
*x^2+495*a*x*arctanh(a*x)+299)*(-a^2*x^2+1)^(1/2)/a-5/16*I/a*arctanh(a*x)*
ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-
a^2*x^2+1)^(1/2))-5/16*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*
dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \left(1-a^2x^2\right)^{5/2} \operatorname{arctanh}(ax)\,dx = \int \left(-a^2x^2+1\right)^{\frac{5}{2}} \operatorname{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right)^{5/2} \operatorname{arctanh}(ax)\,dx = \int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}} \operatorname{atanh}\left(ax\right)dx$$

input
```
integrate((-a**2*x**2+1)**(5/2)*atanh(a*x),x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x), x)
```

Maxima [F]

$$\int \left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)\,dx=\int \left(-a^2x^2+1\right)^{\frac{5}{2}}\operatorname{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(5/2)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)\,dx=\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{5/2}\,dx$$

input
```
int(atanh(a*x)*(1 - a^2*x^2)^(5/2),x)
```

output
```
int(atanh(a*x)*(1 - a^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(1-a^2x^2\right)^{5/2} \operatorname{arctanh}(ax)\, dx = \left(\int \sqrt{-a^2x^2+1}\, atanh(ax)\, x^4 dx\right) a^4 - 2\left(\int \sqrt{-a^2x^2+1}\, atanh(ax)\, x^2 dx\right) a^2 + \int \sqrt{-a^2x^2+1}\, atanh(ax)\, dx$$

input
```
int((-a^2*x^2+1)^(5/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**4,x)*a**4 - 2*int(sqrt( - a**2*x*
*2 + 1)*atanh(a*x)*x**2,x)*a**2 + int(sqrt( - a**2*x**2 + 1)*atanh(a*x),x)
```

3.460 $\int (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{(1-a^2x^2)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) + \frac{1}{4}x(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax) - \frac{3\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax)}{4a} - \frac{3i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{8a} + \frac{3i\operatorname{PolyLog}\left(2\right.}{8a}$$

output
```
3/8*(-a^2*x^2+1)^(1/2)/a+1/12*(-a^2*x^2+1)^(3/2)/a+3/8*x*(-a^2*x^2+1)^(1/2
)*arctanh(a*x)+1/4*x*(-a^2*x^2+1)^(3/2)*arctanh(a*x)-3/4*arctan((-a*x+1)^(
1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-3/8*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+
1)^(1/2))/a+3/8*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int (1-a^2x^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \frac{11\sqrt{1-a^2x^2} - 2a^2x^2\sqrt{1-a^2x^2} + 15ax\sqrt{1-a^2x^2}\operatorname{arctanh}(ax) - 6a^3x^3\sqrt{1-}}{}$$

input
```
Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTa
nh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh
[a*x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcT
anh[a*x]])/(24*a)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6504, 6504, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx$$

$$\Big\downarrow \ 6504$$

$$\frac{3}{4}\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)dx+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \ 6504$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \ 6512$$

$$\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\right.\right.$$
$$\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}$$

input
```
Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(1 - a^2*x^2)^(3/2)/(12*a) + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 + (3*(
Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 + ((-2*ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*
x]])/a)/2))/4
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

method	result
default	$-\frac{(6a^3x^3 \operatorname{arctanh}(ax)+2a^2x^2-15ax \operatorname{arctanh}(ax)-11)\sqrt{-a^2x^2+1}}{24a} - \frac{3i \operatorname{arctanh}(ax) \ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{8a} + \frac{3i \operatorname{arctanh}(ax) \ln\left(\right.}{8a}$

input
```
int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+
1)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a
*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\, dx = \int \left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)\, dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\, dx = \int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right) dx$$

input
```
integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

output
```
Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [F]

$$\int \left(1-a^2x^2\right)^{3/2} \operatorname{arctanh}(ax)\, dx = \int \left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)\, dx$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx=\int \operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}\,dx$$

input
```
int(atanh(a*x)*(1 - a^2*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)*(1 - a^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx= -\left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^2dx\right)a^2+\int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx$$

input
```
int((-a^2*x^2+1)^(3/2)*atanh(a*x),x)
```

output
```
- int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**2,x)*a**2 + int(sqrt( - a**2*x
**2 + 1)*atanh(a*x),x)
```

3.461 $\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)\,dx = \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax) - \frac{\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\mathrm{arctanh}(ax)}{a} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
1/2*(-a^2*x^2+1)^(1/2)/a+1/2*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)-arctan((-a*
x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a-1/2*I*polylog(2,-I*(-a*x+1)^(1/2)
/(a*x+1)^(1/2))/a+1/2*I*polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)\,dx = \frac{\sqrt{1-a^2x^2}\left(1+ax\,\mathrm{arctanh}(ax)-\frac{i\left(\mathrm{arctanh}(ax)\left(\log\left(1-ie^{-\mathrm{arctanh}(ax)}\right)-\log\left(1+ie^{-\mathrm{arctanh}(ax)}\right)\right)+\mathrm{PolyLog}\left(2,-ie^{-\mathrm{arctan}\ldots}\right.\right.}{\sqrt{1-a^2x^2}}\right.}{2a}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
(Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^A
rcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]
] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6504, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx$$

$$\Big\downarrow \text{6504}$$

$$\frac{1}{2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}$$

$$\Big\downarrow \text{6512}$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

input
```
Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

output
```
Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 + ((-2*ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-I)*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*
x]])/a)/2
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

method	result
default	$\frac{(ax\,\operatorname{arctanh}(ax)+1)\sqrt{-a^2x^2+1}}{2a}-\frac{i\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{i\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{i\,\mathrm{dilog}\left(1+\frac{i(ax}{\sqrt{-a^2}}\right.}{2a}$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a-1/2*I/a*arctanh(a*x)*ln(1+I*
(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+
1)^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*
(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx=\int \sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F]

$$\int \sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx=\int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{atanh}\left(ax\right)dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [F]

$$\int \sqrt{1-a^2x^2}\,\text{arctanh}(ax)\,dx=\int \sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \text{atanh}(a\,x)\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(atanh(a*x)*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x),x)
```

3.462 $\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = -\frac{1}{9a\,(1-a^2x^2)^{3/2}} - \frac{2}{3a\sqrt{1-a^2x^2}} + \frac{x\operatorname{arctanh}(ax)}{3\,(1-a^2x^2)^{3/2}} + \frac{2x\operatorname{arctanh}(ax)}{3\sqrt{1-a^2x^2}}$$

output
```
-1/9/a/(-a^2*x^2+1)^(3/2)-2/3/a/(-a^2*x^2+1)^(1/2)+1/3*x*arctanh(a*x)/(-a^
2*x^2+1)^(3/2)+2/3*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.55

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = -\frac{7-6a^2x^2+(-9ax+6a^3x^3)\operatorname{arctanh}(ax)}{9a\,(1-a^2x^2)^{3/2}}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

output
```
-1/9*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x])/(a*(1 - a^2*x^2)^
(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6522}$$

$$\frac{2}{3}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\operatorname{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{6520}$$

$$\frac{x\operatorname{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

output
```
-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*(1 - a^2*x^2)^(3/2))
+ (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/3
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\,\left(6 a^{3} x^{3} \operatorname{arctanh}\left(a x\right)-6 a^{2} x^{2}-9 a x \operatorname{arctanh}\left(a x\right)+7\right)}{9 a \left(a^{2} x^{2}-1\right)^{2}}$	59
orering	$\frac{\left(4 a^{4} x^{5}-\frac{80}{9} a^{2} x^{3}+\frac{44}{9} x\right) \operatorname{arctanh}\left(a x\right)}{\left(-a^{2} x^{2}+1\right)^{\frac{5}{2}}}+\frac{\left(6 a^{2} x^{2}-7\right) \left(a x+1\right)^{2} \left(a x-1\right)^{2} \left(\frac{a}{\left(-a^{2} x^{2}+1\right)^{\frac{7}{2}}}+\frac{5 x \operatorname{arctanh}\left(a x\right) a^{2}}{\left(-a^{2} x^{2}+1\right)^{\frac{7}{2}}}\right)}{9 a^{2}}$	105

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9/a*(-a^2*x^2+1)^(1/2)*(6*a^3*x^3*arctanh(a*x)-6*a^2*x^2-9*a*x*arctanh(
a*x)+7)/(a^2*x^2-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = \frac{\left(12\,a^2x^2 - 3\,(2\,a^3x^3 - 3\,ax)\log\left(-\frac{ax+1}{ax-1}\right) - 14\right)\sqrt{-a^2x^2+1}}{18\,(a^5x^4 - 2\,a^3x^2 + a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 14)*s
qrt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**(5/2),x)
```

output
```
Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = -\frac{1}{9}\,a\left(\frac{6}{\sqrt{-a^2x^2+1}a^2} + \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}a^2}\right) + \frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2x^2+1}} + \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
-1/9*a*(6/(sqrt(-a^2*x^2 + 1)*a^2) + 1/((-a^2*x^2 + 1)^(3/2)*a^2)) + 1/3*(
2*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = -\frac{(2\,a^2x^2-3)\sqrt{-a^2x^2+1}x\log\left(-\frac{ax+1}{ax-1}\right)}{6\,(a^2x^2-1)^2} - \frac{6\,a^2x^2-7}{9\,(a^2x^2-1)\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(2*a^2*x^2 - 3)*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x
^2 - 1)^2 - 1/9*(6*a^2*x^2 - 7)/((a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\mathrm{atanh}(a\,x)}{{(1-a^2\,x^2)}^{5/2}}\,dx$$

input
```
int(atanh(a*x)/(1 - a^2*x^2)^(5/2),x)
```

output
```
int(atanh(a*x)/(1 - a^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^(5/2),x)
```

output
```
int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2 + 1
)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)
```

3.463 $\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = -\frac{1}{25a(1-a^2x^2)^{5/2}} - \frac{4}{45a(1-a^2x^2)^{3/2}} - \frac{8}{15a\sqrt{1-a^2x^2}} + \frac{x\operatorname{arctanh}(ax)}{5(1-a^2x^2)^{5/2}} + \frac{4x\operatorname{arctanh}(ax)}{15(1-a^2x^2)^{3/2}} + \frac{8x\operatorname{arctanh}(ax)}{15\sqrt{1-a^2x^2}}$$

output
```
-1/25/a/(-a^2*x^2+1)^(5/2)-4/45/a/(-a^2*x^2+1)^(3/2)-8/15/a/(-a^2*x^2+1)^(
1/2)+1/5*x*arctanh(a*x)/(-a^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)/(-a^2*x^2+1
)^(3/2)+8/15*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = \frac{-149+260a^2x^2-120a^4x^4+15ax(15-20a^2x^2+8a^4x^4)\operatorname{arctanh}(ax)}{225a(1-a^2x^2)^{5/2}}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

output
```
(-149 + 260*a^2*x^2 - 120*a^4*x^4 + 15*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*A
rcTanh[a*x])/(225*a*(1 - a^2*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6522, 6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{6522}$$

$$\frac{4}{5}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{6520}$$

$$\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{1}{25a(1-a^2x^2)^{5/2}}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

output
```
-1/25*1/(a*(1 - a^2*x^2)^(5/2)) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)^(5/2))
 + (4*(-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*(1 - a^2*x^2)^
(3/2)) + (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^
2]))/3))/5
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.59

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\left(120 \operatorname{arctanh}\left(a x\right) a^{5} x^{5}-120 a^{4} x^{4}-300 a^{3} x^{3} \operatorname{arctanh}\left(a x\right)+260 a^{2} x^{2}+225 a x \operatorname{arctanh}\left(a x\right)-149\right)}{225 a \left(a^{2} x^{2}-1\right)^{3}}$
orering	$\frac{\left(-\frac{64}{15} a^{6} x^{7}+\frac{616}{45} a^{4} x^{5}-\frac{3388}{225} a^{2} x^{3}+\frac{1268}{225} x\right) \operatorname{arctanh}\left(a x\right)}{\left(-a^{2} x^{2}+1\right)^{\frac{7}{2}}}-\frac{\left(120 a^{4} x^{4}-260 a^{2} x^{2}+149\right) \left(a x+1\right)^{2} \left(a x-1\right)^{2} \left(\frac{a}{\left(-a^{2} x^{2}+1\right)^{\frac{9}{2}}}+\frac{7 \operatorname{arctanh}(a}{\left(-a^{2}x^{2}+\right.}\right.}{225 a^{2}}$

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/225/a*(-a^2*x^2+1)^(1/2)*(120*arctanh(a*x)*a^5*x^5-120*a^4*x^4-300*a^3*
x^3*arctanh(a*x)+260*a^2*x^2+225*a*x*arctanh(a*x)-149)/(a^2*x^2-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = \frac{\left(240\,a^4x^4-520\,a^2x^2-15\,(8\,a^5x^5-20\,a^3x^3+15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)+298\right)\sqrt{-a^2x^2+}}{450\,(a^7x^6-3\,a^5x^4+3\,a^3x^2-a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

output
```
1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*lo
g(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4 + 3
*a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**(7/2),x)
```

output
```
Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = -\frac{1}{225}\,a\left(\frac{120}{\sqrt{-a^2x^2+1}a^2}+\frac{20}{(-a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{9}{(-a^2x^2+1)^{\frac{5}{2}}a^2}\right) + \frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{3\,x}{(-a^2x^2+1)^{\frac{5}{2}}}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

output
```
-1/225*a*(120/(sqrt(-a^2*x^2 + 1)*a^2) + 20/((-a^2*x^2 + 1)^(3/2)*a^2) + 9
/((-a^2*x^2 + 1)^(5/2)*a^2)) + 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^
2 + 1)^(3/2) + 3*x/(-a^2*x^2 + 1)^(5/2))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = -\frac{\sqrt{-a^2x^2+1}(4\,(2\,a^4x^2-5\,a^2)x^2+15)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\,(a^2x^2-1)^3} + \frac{20\,a^2x^2-120\,(a^2x^2-1)^2-29}{225\,(a^2x^2-1)^2\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

output
```
-1/30*sqrt(-a^2*x^2 + 1)*(4*(2*a^4*x^2 - 5*a^2)*x^2 + 15)*x*log(-(a*x + 1)
/(a*x - 1))/(a^2*x^2 - 1)^3 + 1/225*(20*a^2*x^2 - 120*(a^2*x^2 - 1)^2 - 29
)/((a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(1-a^2\,x^2)^{7/2}}\,dx$$

input
```
int(atanh(a*x)/(1 - a^2*x^2)^(7/2),x)
```

output
```
int(atanh(a*x)/(1 - a^2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}\,dx = -\left(\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^(7/2),x)
```

output
```
 - int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x**2
+ 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2 +
1)),x)
```

3.464 $\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = -\frac{1}{49a(1-a^2x^2)^{7/2}}-\frac{6}{175a(1-a^2x^2)^{5/2}}-\frac{8}{105a(1-a^2x^2)^{3/2}}-\frac{16}{35a\sqrt{1-a^2x^2}}+\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6x\text{arctanh}(ax)}{35(1-a^2x^2)^{5/2}}+\frac{8x\text{arctanh}(ax)}{35(1-a^2x^2)^{3/2}}+\frac{16x\text{arctanh}(ax)}{35\sqrt{1-a^2x^2}}$$

output
```
-1/49/a/(-a^2*x^2+1)^(7/2)-6/175/a/(-a^2*x^2+1)^(5/2)-8/105/a/(-a^2*x^2+1)
^(3/2)-16/35/a/(-a^2*x^2+1)^(1/2)+1/7*x*arctanh(a*x)/(-a^2*x^2+1)^(7/2)+6/
35*x*arctanh(a*x)/(-a^2*x^2+1)^(5/2)+8/35*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2
)+16/35*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = \frac{-2161+5726a^2x^2-5320a^4x^4+1680a^6x^6-105ax(-35+70a^2x^2-56a^4x^4+16a^6}{3675a\,(1-a^2x^2)^{7/2}}$$

input
```
Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

output
```
(-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6 - 105*a*x*(-35 + 70*a^
2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x])/(3675*a*(1 - a^2*x^2)^(7/2)
)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6522, 6522, 6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)}{7\,(1-a^2x^2)^{7/2}}-\frac{1}{49a\,(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)}{5\,(1-a^2x^2)^{5/2}}-\frac{1}{25a\,(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(ax)}{7\,(1-a^2x^2)^{7/2}}-\frac{1}{49a\,(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}-\frac{1}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \; 6520$$

$$\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{1}{25a(1-a^2x^2)^{5/2}}\right)-\frac{1}{49a(1-a^2x^2)^{7/2}}$$

input
```
Int[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

output
```
-1/49*1/(a*(1 - a^2*x^2)^(7/2)) + (x*ArcTanh[a*x])/(7*(1 - a^2*x^2)^(7/2))
 + (6*(-1/25*1/(a*(1 - a^2*x^2)^(5/2)) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)
^(5/2)) + (4*(-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*(1 - a^
2*x^2)^(3/2)) + (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 -
 a^2*x^2]))/3))/5))/7
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.56

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,(1680\,\operatorname{arctanh}(ax)a^7x^7-1680a^6x^6-5880\,\operatorname{arctanh}(ax)a^5x^5+5320a^4x^4+7350a^3x^3\,\operatorname{arctanh}(ax)-5726a^2x^2-3675ax}{3675a(a^2x^2-1)^4}$
orering	$\frac{\left(\frac{32}{7}a^8x^9-\frac{96}{5}a^6x^7+\frac{5364}{175}a^4x^5-\frac{27336}{1225}a^2x^3+\frac{7708}{1225}x\right)\operatorname{arctanh}(ax)}{(-a^2x^2+1)^{\frac{9}{2}}}+\frac{(1680a^6x^6-5320a^4x^4+5726a^2x^2-2161)(ax+1)^2(ax-1)^2\left(\right.}{3675a^2}$

input
```
int(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3675/a*(-a^2*x^2+1)^(1/2)*(1680*arctanh(a*x)*a^7*x^7-1680*a^6*x^6-5880*
arctanh(a*x)*a^5*x^5+5320*a^4*x^4+7350*a^3*x^3*arctanh(a*x)-5726*a^2*x^2-3
675*a*x*arctanh(a*x)+2161)/(a^2*x^2-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx=\frac{(3360\,a^6x^6-10640\,a^4x^4+11452\,a^2x^2-105\,(16\,a^7x^7-56\,a^5x^5+70\,a^3x^3-35\,ax)\,\mathrm{l}}{7350\,(a^9x^8-4\,a^7x^6+6\,a^5x^4-4\,a^3x^2+a)}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

output
```
1/7350*(3360*a^6*x^6 - 10640*a^4*x^4 + 11452*a^2*x^2 - 105*(16*a^7*x^7 - 5
6*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 4322)*sqrt(-a
^2*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{9}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*x**2+1)**(9/2),x)
```

output
```
Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = -\frac{1}{3675}\,a\left(\frac{1680}{\sqrt{-a^2x^2+1}a^2} + \frac{280}{(-a^2x^2+1)^{\frac{3}{2}}a^2} + \frac{126}{(-a^2x^2+1)^{\frac{5}{2}}a^2} + \frac{75}{(-a^2x^2+1)^{\frac{7}{2}}a^2}\right) + \frac{1}{35}\left(\frac{16\,x}{\sqrt{-a^2x^2+1}} + \frac{8\,x}{(-a^2x^2+1)^{\frac{3}{2}}} + \frac{6\,x}{(-a^2x^2+1)^{\frac{5}{2}}} + \frac{5\,x}{(-a^2x^2+1)^{\frac{7}{2}}}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

output
```
-1/3675*a*(1680/(sqrt(-a^2*x^2 + 1)*a^2) + 280/((-a^2*x^2 + 1)^(3/2)*a^2)
+ 126/((-a^2*x^2 + 1)^(5/2)*a^2) + 75/((-a^2*x^2 + 1)^(7/2)*a^2)) + 1/35*(
16*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 + 1)^(5
/2) + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = -\frac{\sqrt{-a^2x^2+1}(2\,(4\,(2\,a^6x^2-7\,a^4)x^2+35\,a^2)x^2-35)x\log\left(-\frac{ax+1}{ax-1}\right)}{70\,(a^2x^2-1)^4} - \frac{126\,a^2x^2+1680\,(a^2x^2-1)^3-280\,(a^2x^2-1)^2-201}{3675\,(a^2x^2-1)^3\sqrt{-a^2x^2+1}a}$$

input
```
integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

output
```
-1/70*sqrt(-a^2*x^2 + 1)*(2*(4*(2*a^6*x^2 - 7*a^4)*x^2 + 35*a^2)*x^2 - 35)
*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1)^4 - 1/3675*(126*a^2*x^2 + 1680*
(a^2*x^2 - 1)^3 - 280*(a^2*x^2 - 1)^2 - 201)/((a^2*x^2 - 1)^3*sqrt(-a^2*x^
2 + 1)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(1-a^2\,x^2)^{9/2}}\,dx$$

input
```
int(atanh(a*x)/(1 - a^2*x^2)^(9/2),x)
```

output
```
int(atanh(a*x)/(1 - a^2*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^8x^8 - 4\sqrt{-a^2x^2+1}\,a^6x^6 + 6\sqrt{-a^2x^2+1}\,a^4x^4 - 4\sqrt{-a^2x^2+1}\,a^2x^2}$$

input
```
int(atanh(a*x)/(-a^2*x^2+1)^(9/2),x)
```

output
```
int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**8*x**8 - 4*sqrt( - a**2*x**2 + 1
)*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 4*sqrt( - a**2*x**2 + 1
)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)
```

3.465 $\int (c-a^2cx^2)^{3/2} \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 291

$$\int (c-a^2cx^2)^{3/2} \operatorname{arctanh}(ax)\,dx = \frac{3c\sqrt{c-a^2cx^2}}{8a} + \frac{(c-a^2cx^2)^{3/2}}{12a} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\operatorname{arctanh}(ax) + \frac{1}{4}x(c-a^2cx^2)^{3/2}\operatorname{arctanh}(ax) - \frac{3c^2\sqrt{1-a^2x^2}\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\operatorname{arctanh}(ax)}{4a\sqrt{c-a^2cx^2}} - \frac{3ic^2\sqrt{1-a^2x^2}\operatorname{PolyLog}\left(2,\right.}{8a\sqrt{c-a^2cx^2}}$$

output
```
3/8*c*(-a^2*c*x^2+c)^(1/2)/a+1/12*(-a^2*c*x^2+c)^(3/2)/a+3/8*c*x*(-a^2*c*x
^2+c)^(1/2)*arctanh(a*x)+1/4*x*(-a^2*c*x^2+c)^(3/2)*arctanh(a*x)-3/4*c^2*(
-a^2*x^2+1)^(1/2)*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a/(-a^
2*c*x^2+c)^(1/2)-3/8*I*c^2*(-a^2*x^2+1)^(1/2)*polylog(2,-I*(-a*x+1)^(1/2)/
(a*x+1)^(1/2))/a/(-a^2*c*x^2+c)^(1/2)+3/8*I*c^2*(-a^2*x^2+1)^(1/2)*polylog
(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71

$$\int \left(c-a^2cx^2\right)^{3/2}\text{arctanh}(ax)\,dx =$$
$$-\frac{c\sqrt{c-a^2cx^2}\left(-11\sqrt{1-a^2x^2}+2a^2x^2\sqrt{1-a^2x^2}-15ax\sqrt{1-a^2x^2}\text{arctanh}(ax)+6a^3x^3\sqrt{1-a^2x^2}\text{arctan}\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
-1/24*(c*Sqrt[c - a^2*c*x^2]*(-11*Sqrt[1 - a^2*x^2] + 2*a^2*x^2*Sqrt[1 - a
^2*x^2] - 15*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*a^3*x^3*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] + (9*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (9*I)*Ar
cTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (9*I)*PolyLog[2, I/E^ArcTanh[a*x]]))/(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6504, 6504, 6516, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(ax)\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 6504}$$

$$\frac{3}{4}c\int \sqrt{c-a^2cx^2}\text{arctanh}(ax)dx+\frac{1}{4}x\text{arctanh}(ax)\left(c-a^2cx^2\right)^{3/2}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{ 6504}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\text{arctanh}(ax)\sqrt{c-a^2cx^2}+\frac{\sqrt{c-a^2cx^2}}{2a}\right)+\frac{1}{4}x\text{arctanh}(ax)\left(c-a^2cx^2\right)^{3/2}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{6516}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{1-a^2x^2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2\sqrt{c-a^2cx^2}}+\frac{1}{2}x\text{arctanh}(ax)\sqrt{c-a^2cx^2}+\frac{\sqrt{c-a^2cx^2}}{2a}\right)+$$
$$\frac{1}{4}x\text{arctanh}(ax)\left(c-a^2cx^2\right)^{3/2}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{6512}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{1-a^2x^2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{2\sqrt{c-a^2cx^2}}+\frac{1}{2}x\text{arctanh}(ax)\sqrt{c}\right.$$
$$\frac{1}{4}x\text{arctanh}(ax)\left(c-a^2cx^2\right)^{3/2}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

output
```
(c - a^2*c*x^2)^(3/2)/(12*a) + (x*(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x])/4 +
(3*c*(Sqrt[c - a^2*c*x^2]/(2*a) + (x*Sqrt[c - a^2*c*x^2]*ArcTanh[a*x])/2 +
 (c*Sqrt[1 - a^2*x^2]*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x
])/a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2
, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))/(2*Sqrt[c - a^2*c*x^2])))/4
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6516
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTanh[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.19

method	result
default	$-\frac{c\sqrt{-(ax-1)c(ax+1)}\,(6a^3x^3\,\mathrm{arctanh}(ax)+2a^2x^2-15ax\,\mathrm{arctanh}(ax)-11)}{24a}+\frac{3ic\sqrt{-(ax-1)c(ax+1)}\,\sqrt{-a^2x^2+1}\,\mathrm{arctanh}(ax)}{8a(ax+1)(ax-1)}$

input
```
int((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/24*c/a*(-(a*x-1)*c*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*
a*x*arctanh(a*x)-11)+3/8*I*c/a*(-(a*x-1)*c*(a*x+1))^(1/2)/(a*x+1)*(-a^2*x^
2+1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*c
/a*(-(a*x-1)*c*(a*x+1))^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*arctanh(a
*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*c/a*(-(a*x-1)*c*(a*x+1))^(1/2
)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))
-3/8*I*c/a*(-(a*x-1)*c*(a*x+1))^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*d
ilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \left(c - a^2 c x^2\right)^{3/2} \operatorname{arctanh}(ax)\, dx = \int \left(-a^2 c x^2 + c\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)\, dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Sympy [F]

$$\int \left(c - a^2 c x^2\right)^{3/2} \operatorname{arctanh}(ax)\, dx = \int \left(-c(ax - 1)\left(ax + 1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right) dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*atanh(a*x),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Maxima [F]

$$\int \left(c - a^2 c x^2\right)^{3/2} \operatorname{arctanh}(ax)\, dx = \int \left(-a^2 c x^2 + c\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)\, dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx = \int \operatorname{atanh}(a\,x)\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(atanh(a*x)*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\operatorname{arctanh}(ax)\,dx = \sqrt{c}\,c\left(-\left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,x^2dx\right)a^2 + \int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)*atanh(a*x),x)
```

output
```
sqrt(c)*c*( - int(sqrt( - a**2*x**2 + 1)*atanh(a*x)*x**2,x)*a**2 + int(sqr
t( - a**2*x**2 + 1)*atanh(a*x),x))
```

3.466 $\int \sqrt{c-a^2cx^2}\,\text{arctanh}(ax)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 235

$$\int \sqrt{c-a^2cx^2}\,\text{arctanh}(ax)\,dx=\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\,\text{arctanh}(ax)-\frac{c\sqrt{1-a^2x^2}\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{ic\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{ic\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a\sqrt{c-a^2cx^2}}$$

output
```
1/2*(-a^2*c*x^2+c)^(1/2)/a+1/2*x*(-a^2*c*x^2+c)^(1/2)*arctanh(a*x)-c*(-a^2
*x^2+1)^(1/2)*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a/(-a^2*c*
x^2+c)^(1/2)-1/2*I*c*(-a^2*x^2+1)^(1/2)*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1
)^(1/2))/a/(-a^2*c*x^2+c)^(1/2)+1/2*I*c*(-a^2*x^2+1)^(1/2)*polylog(2,I*(-a
*x+1)^(1/2)/(a*x+1)^(1/2))/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.51

$$\int \sqrt{c-a^2cx^2}\text{arctanh}(ax)\,dx = \frac{\sqrt{c(1-a^2x^2)}\left(1+ax\text{arctanh}(ax)-\frac{i\left(\text{arctanh}(ax)\left(\log\left(1-ie^{-\text{arctanh}(ax)}\right)-\log\left(1+ie^{-\text{arctanh}(ax)}\right)\right)+\text{PolyLog}\left(2,-ie^{-a}\right.\right.}{\sqrt{1-a^2x^2}}\right)}{2a}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

output
```
(Sqrt[c*(1 - a^2*x^2)]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I
/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6504, 6516, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(ax)\sqrt{c-a^2cx^2}\,dx$$

$$\Big\downarrow \text{6504}$$

$$\frac{1}{2}c\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\text{arctanh}(ax)\sqrt{c-a^2cx^2}+\frac{\sqrt{c-a^2cx^2}}{2a}$$

$$\Big\downarrow \text{6516}$$

$$\frac{c\sqrt{1-a^2x^2}\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{2\sqrt{c-a^2cx^2}}+\frac{1}{2}x\text{arctanh}(ax)\sqrt{c-a^2cx^2}+\frac{\sqrt{c-a^2cx^2}}{2a}$$

$$\Big\downarrow \text{6512}$$

$$\frac{c\sqrt{1-a^2x^2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{2\sqrt{c-a^2cx^2}}+\frac{1}{2}x\operatorname{arctanh}(ax)\sqrt{c-a^2cx^2}+\frac{\sqrt{c-a^2cx^2}}{2a}$$

input
```
Int[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

output
```
Sqrt[c - a^2*c*x^2]/(2*a) + (x*Sqrt[c - a^2*c*x^2]*ArcTanh[a*x])/2 + (c*Sq
rt[1 - a^2*x^2]*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a -
 (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*S
qrt[1 - a*x])/Sqrt[1 + a*x]])/a))/(2*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6516
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTanh[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.36

method	result
default	$\frac{(ax\ \mathrm{arctanh}(ax)+1)\sqrt{-(ax-1)c(ax+1)}}{2a}+\frac{i\sqrt{-(ax-1)c(ax+1)}\,\sqrt{-a^2x^2+1}\ \mathrm{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{2a(ax+1)(ax-1)}-\frac{i\sqrt{-(ax-1)c(}}{}$

input
```
int((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*c*(a*x+1))^(1/2)/a+1/2*I/a*(-(a*x-1)*c*
(a*x+1))^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1+I*(a*x
+1)/(-a^2*x^2+1)^(1/2))-1/2*I/a*(-(a*x-1)*c*(a*x+1))^(1/2)/(a*x+1)*(-a^2*x
^2+1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/
a*(-(a*x-1)*c*(a*x+1))^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*dilog(1+I*
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I/a*(-(a*x-1)*c*(a*x+1))^(1/2)/(a*x+1)*(-a
^2*x^2+1)^(1/2)/(a*x-1)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \sqrt{c-a^2cx^2}\mathrm{arctanh}(ax)\,dx=\int \sqrt{-a^2cx^2+c}\,\mathrm{artanh}\left(ax\right)\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Sympy [F]

$$\int \sqrt{c-a^2cx^2}\text{arctanh}(ax)\,dx = \int \sqrt{-c\,(ax-1)\,(ax+1)}\,\text{atanh}\,(ax)\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*atanh(a*x),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Maxima [F]

$$\int \sqrt{c-a^2cx^2}\text{arctanh}(ax)\,dx = \int \sqrt{-a^2cx^2+c}\,\text{artanh}\,(ax)\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\text{arctanh}(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c-a^2cx^2}\operatorname{arctanh}(ax)\,dx=\int \operatorname{atanh}(a\,x)\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(atanh(a*x)*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c-a^2cx^2}\operatorname{arctanh}(ax)\,dx=\sqrt{c}\left(\int \sqrt{-a^2x^2+1}\,atanh(ax)\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*atanh(a*x),x)
```

output
```
sqrt(c)*int(sqrt( - a**2*x**2 + 1)*atanh(a*x),x)
```

3.467 $\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 182

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{arctanh}(ax)}{a\sqrt{c-a^2cx^2}} - \frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} + \frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}$$

output
```
-2*(-a^2*x^2+1)^(1/2)*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a/
(-a^2*c*x^2+c)^(1/2)-I*(-a^2*x^2+1)^(1/2)*polylog(2,-I*(-a*x+1)^(1/2)/(a*x
+1)^(1/2))/a/(-a^2*c*x^2+c)^(1/2)+I*(-a^2*x^2+1)^(1/2)*polylog(2,I*(-a*x+1
)^(1/2)/(a*x+1)^(1/2))/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = -\frac{i\sqrt{c(1-a^2x^2)}\left(\operatorname{arctanh}(ax)\left(\log\left(1-ie^{-\operatorname{arctanh}(ax)}\right)-\log\left(1+ie^{-\operatorname{arctanh}(ax)}\right)\right)+\operatorname{PolyLog}\left(2,-ie^{-\operatorname{arct}\right.\right.}{ac\sqrt{1-a^2x^2}}$$

input
```
Integrate[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

output
```
((-I)*Sqrt[c*(1 - a^2*x^2)]*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log
[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E
^ArcTanh[a*x]]))/(a*c*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6516, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx$$

$$\Big\downarrow \text{ 6516}$$

$$\frac{\sqrt{1-a^2x^2}\int \frac{\operatorname{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 6512}$$

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

output

```
(Sqrt[1 - a^2*x^2]*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/
a - (I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (
I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a))/Sqrt[c - a^2*c*x^2]
```

Defintions of rubi rules used

rule 6512

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6516

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTanh[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.66

method	result
default	$\frac{i\sqrt{-(ax-1)c(ax+1)}\,\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{(ax-1)(ax+1)ca}-\frac{i\sqrt{-(ax-1)c(ax+1)}\,\sqrt{-a^2x^2+1}\,\operatorname{arctanh}(ax)\ln\left(1-\frac{i(ax}{\sqrt{-a^2}}\right.}{(ax-1)(ax+1)ca}$

input

```
int(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
I*(-(a*x-1)*c*(a*x+1))^(1/2)*(-a^2*x^2+1)^(1/2)*arctanh(a*x)*ln(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+1)/c/a-I*(-(a*x-1)*c*(a*x+1))^(1/2)*(-a
^2*x^2+1)^(1/2)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a
*x+1)/c/a+I*(-(a*x-1)*c*(a*x+1))^(1/2)*(-a^2*x^2+1)^(1/2)*dilog(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+1)/c/a-I*(-(a*x-1)*c*(a*x+1))^(1/2)*(-a
^2*x^2+1)^(1/2)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/(a*x-1)/(a*x+1)/c/a
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arctanh(a*x)/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atanh(a*x)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\text{artanh}\,(ax)}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\text{atanh}(a\,x)}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(atanh(a*x)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c-a^2cx^2}}\,dx = \frac{\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(atanh(a*x)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atanh(a*x)/sqrt( - a**2*x**2 + 1),x)/sqrt(c)
```

3.468 $\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{1}{ac\sqrt{c-a^2cx^2}}+\frac{x\operatorname{arctanh}(ax)}{c\sqrt{c-a^2cx^2}}$$

output
```
-1/a/c/(-a^2*c*x^2+c)^(1/2)+x*arctanh(a*x)/c/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2cx^2}(1-ax\operatorname{arctanh}(ax))}{ac^2\,(-1+a^2x^2)}$$

input
```
Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1 - a*x*ArcTanh[a*x]))/(a*c^2*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6520}$$

$$\frac{x\text{arctanh}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a*x])/(c*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

method	result	size
default	$-\frac{(\text{arctanh}(ax)-1)\sqrt{-(ax-1)c(ax+1)}}{2a(ax-1)c^2}-\frac{(\text{arctanh}(ax)+1)\sqrt{-(ax-1)c(ax+1)}}{2a(ax+1)c^2}$	74
orering	$-\frac{4(ax+1)x(ax-1)\,\text{arctanh}(ax)}{(-a^2c\,x^2+c)^{\frac{3}{2}}}-\frac{(ax+1)^2(ax-1)^2\left(\frac{a}{(-a^2x^2+1)(-a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{3\,\text{arctanh}(ax)a^2cx}{(-a^2c\,x^2+c)^{\frac{5}{2}}}\right)}{a^2}$	103

input
```
int(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctanh(a*x)-1)*(-(a*x-1)*c*(a*x+1))^(1/2)/a/(a*x-1)/c^2-1/2*(arctan
h(a*x)+1)*(-(a*x-1)*c*(a*x+1))^(1/2)/a/(a*x+1)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\left(a^3c^2x^2-ac^2\right)}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-a^2*c*x^2 + c)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*c^2*x^2
 - a*c^2)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(44) = 88.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2\left(\frac{\sqrt{-a^2cx^2+c}}{a^4cx+a^3c} - \frac{\sqrt{-a^2cx^2+c}}{a^4cx-a^3c}\right)}{2\,c} + \frac{x\,\operatorname{artanh}\left(ax\right)}{\sqrt{-a^2cx^2+c}c}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*a^2*(sqrt(-a^2*c*x^2 + c)/(a^4*c*x + a^3*c) - sqrt(-a^2*c*x^2 + c)/(a
^4*c*x - a^3*c))/c + x*arctanh(a*x)/(sqrt(-a^2*c*x^2 + c)*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}x\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^2cx^2-c\right)c} - \frac{1}{\sqrt{-a^2cx^2+c}ac}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-a^2*c*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1))/((a^2*c*x^2 - c)*c)
- 1/(sqrt(-a^2*c*x^2 + c)*a*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(atanh(a*x)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(atanh(a*x)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2 +
 1)),x))/(sqrt(c)*c)
```

3.469 $\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}} - \frac{2}{3ac^2\sqrt{c-a^2cx^2}} + \frac{x\operatorname{arctanh}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}} + \frac{2x\operatorname{arctanh}(ax)}{3c^2\sqrt{c-a^2cx^2}}$$

output
```
-1/9/a/c/(-a^2*c*x^2+c)^(3/2)-2/3/a/c^2/(-a^2*c*x^2+c)^(1/2)+1/3*x*arctanh
(a*x)/c/(-a^2*c*x^2+c)^(3/2)+2/3*x*arctanh(a*x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{c-a^2cx^2}\left(7-6a^2x^2+\left(-9ax+6a^3x^3\right)\operatorname{arctanh}(ax)\right)}{9ac^3\left(-1+a^2x^2\right)^2}$$

input
```
Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/9*(Sqrt[c - a^2*c*x^2]*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*
x]))/(a*c^3*(-1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6522}$$

$$\frac{2\int \frac{\text{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x\text{arctanh}(ax)}{3c\,(c-a^2cx^2)^{3/2}}-\frac{1}{9ac\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 6520}$$

$$\frac{x\text{arctanh}(ax)}{3c\,(c-a^2cx^2)^{3/2}}+\frac{2\left(\frac{x\text{arctanh}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}\right)}{3c}-\frac{1}{9ac\,(c-a^2cx^2)^{3/2}}$$

input
```
Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/9*1/(a*c*(c - a^2*c*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*c*(c - a^2*c*x^2)
^(3/2)) + (2*(-(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a*x])/(c*Sqrt[c
- a^2*c*x^2])))/(3*c)
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result
orering	$\frac{\left(4a^4x^5-\frac{80}{9}a^2x^3+\frac{44}{9}x\right)\operatorname{arctanh}(ax)}{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}+\frac{\left(6a^2x^2-7\right)\left(ax+1\right)^2\left(ax-1\right)^2\left(\frac{a}{\left(-a^2x^2+1\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}+\frac{5\operatorname{arctanh}(ax)a^2cx}{\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}\right)}{9a^2}$
default	$\frac{(ax+1)(3\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)c(ax+1)}}{72a(ax-1)^2c^3}-\frac{3(\operatorname{arctanh}(ax)-1)\sqrt{-(ax-1)c(ax+1)}}{8a\,c^3(ax-1)}-\frac{3(\operatorname{arctanh}(ax)+1)\sqrt{-(ax-1)c(ax+}}{8a(ax+1)c^3}$

input
```
int(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(4*a^4*x^5-80/9*a^2*x^3+44/9*x)*arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)+1/9*(6*a
^2*x^2-7)/a^2*(a*x+1)^2*(a*x-1)^2*(a/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2)+5*a
rctanh(a*x)/(-a^2*c*x^2+c)^(7/2)*a^2*c*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}\left(12\,a^2x^2-3\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)}{18\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/18*sqrt(-a^2*c*x^2 + c)*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x +
1)/(a*x - 1)) - 14)/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{1}{9}\,a\left(\frac{6}{\sqrt{-a^2cx^2+c}a^2c^2}+\frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}a^2c}\right) + \frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2cx^2+c}c^2}+\frac{x}{(-a^2cx^2+c)^{\frac{3}{2}}c}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/9*a*(6/(sqrt(-a^2*c*x^2 + c)*a^2*c^2) + 1/((-a^2*c*x^2 + c)^(3/2)*a^2*c
)) + 1/3*(2*x/(sqrt(-a^2*c*x^2 + c)*c^2) + x/((-a^2*c*x^2 + c)^(3/2)*c))*a
rctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}\left(\frac{2\,a^2x^2}{c}-\frac{3}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{6\,(a^2cx^2-c)^2} - \frac{6\,a^2cx^2-7\,c}{9\,(a^2cx^2-c)\sqrt{-a^2cx^2+c}ac^2}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(-a^2*c*x^2 + c)*(2*a^2*x^2/c - 3/c)*x*log(-(a*x + 1)/(a*x - 1))/
(a^2*c*x^2 - c)^2 - 1/9*(6*a^2*c*x^2 - 7*c)/((a^2*c*x^2 - c)*sqrt(-a^2*c*x
^2 + c)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(atanh(a*x)/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(atanh(a*x)/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input

```
int(atanh(a*x)/(-a^2*c*x^2+c)^(5/2),x)
```

output

```
int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2 + 1
)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.470 $\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}+\frac{x\operatorname{arctanh}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x\operatorname{arctanh}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x\operatorname{arctanh}(ax)}{15c^3\sqrt{c-a^2cx^2}}$$

output
```
-1/25/a/c/(-a^2*c*x^2+c)^(5/2)-4/45/a/c^2/(-a^2*c*x^2+c)^(3/2)-8/15/a/c^3/
(-a^2*c*x^2+c)^(1/2)+1/5*x*arctanh(a*x)/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arct
anh(a*x)/c^2/(-a^2*c*x^2+c)^(3/2)+8/15*x*arctanh(a*x)/c^3/(-a^2*c*x^2+c)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\left(149-260a^2x^2+120a^4x^4-15ax(15-20a^2x^2+8a^4x^4)\operatorname{arctanh}(ax)\right)}{225ac^4\left(-1+a^2x^2\right)^3}$$

input
```
Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(149 - 260*a^2*x^2 + 120*a^4*x^4 - 15*a*x*(15 - 20*a^
2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]))/(225*a*c^4*(-1 + a^2*x^2)^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6522, 6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{6522}$$

$$\frac{4\int \frac{\text{arctanh}(ax)}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{x\text{arctanh}(ax)}{5c\,(c-a^2cx^2)^{5/2}}-\frac{1}{25ac\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{4\left(\frac{2\int \frac{\text{arctanh}(ax)}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x\text{arctanh}(ax)}{3c(c-a^2cx^2)^{3/2}}-\frac{1}{9ac(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\text{arctanh}(ax)}{5c\,(c-a^2cx^2)^{5/2}}-\frac{1}{25ac\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6520}$$

$$\frac{x\text{arctanh}(ax)}{5c\,(c-a^2cx^2)^{5/2}}+\frac{4\left(\frac{x\text{arctanh}(ax)}{3c(c-a^2cx^2)^{3/2}}+\frac{2\left(\frac{x\text{arctanh}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}\right)}{3c}-\frac{1}{9ac(c-a^2cx^2)^{3/2}}\right)}{5c}-\frac{1}{25ac\,(c-a^2cx^2)^{5/2}}$$

input
```
Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/25*1/(a*c*(c - a^2*c*x^2)^(5/2)) + (x*ArcTanh[a*x])/(5*c*(c - a^2*c*x^2
)^(5/2)) + (4*(-1/9*1/(a*c*(c - a^2*c*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*c*
(c - a^2*c*x^2)^(3/2)) + (2*(-(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a
*x])/(c*Sqrt[c - a^2*c*x^2])))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

method	result
orering	$\frac{\left(-\frac{64}{15} a^{6} x^{7}+\frac{616}{45} a^{4} x^{5}-\frac{3388}{225} a^{2} x^{3}+\frac{1268}{225} x\right) \operatorname{arctanh}(a x)}{\left(-a^{2} c \,x^{2}+c\right)^{\frac{7}{2}}}-\frac{\left(120 a^{4} x^{4}-260 a^{2} x^{2}+149\right) \left(a x+1\right)^{2} \left(a x-1\right)^{2} \left(\frac{a}{\left(-a^{2} x^{2}+1\right) \left(-a^{2} c \,x^{2}+c\right)^{\frac{7}{2}}}\right.}{225 a^{2}}$
default	$-\frac{\left(a x+1\right)^{2} \left(-1+5 \,\operatorname{arctanh}(a x)\right) \sqrt{-\left(a x-1\right) c \left(a x+1\right)}}{800 a \left(a x-1\right)^{3} c^{4}}+\frac{5 \left(a x+1\right) \left(3 \,\operatorname{arctanh}(a x)-1\right) \sqrt{-\left(a x-1\right) c \left(a x+1\right)}}{288 a \,c^{4} \left(a x-1\right)^{2}}-\frac{5 \left(\operatorname{arctanh}(a x)-1\right) \sqrt{}}{16 a \,c^{4} (a}$

input
```
int(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-64/15*a^6*x^7+616/45*a^4*x^5-3388/225*a^2*x^3+1268/225*x)*arctanh(a*x)/(
-a^2*c*x^2+c)^(7/2)-1/225*(120*a^4*x^4-260*a^2*x^2+149)/a^2*(a*x+1)^2*(a*x
-1)^2*(a/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2)+7*arctanh(a*x)/(-a^2*c*x^2+c)^(
9/2)*a^2*c*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\left(240\,a^4x^4 - 520\,a^2x^2 - 15\left(8\,a^5x^5 - 20\,a^3x^3 + 15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) + 298\right)\sqrt{-a^2cx^2}}{450\left(a^7c^4x^6 - 3\,a^5c^4x^4 + 3\,a^3c^4x^2 - ac^4\right)}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*lo
g(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*c*x^2 + c)/(a^7*c^4*x^6 - 3*a^5*c
^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx =$$
$$-\frac{1}{225}\,a\left(\frac{120}{\sqrt{-a^2cx^2+c}a^2c^3} + \frac{20}{(-a^2cx^2+c)^{\frac{3}{2}}a^2c^2} + \frac{9}{(-a^2cx^2+c)^{\frac{5}{2}}a^2c}\right)$$
$$+\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2cx^2+c}c^3} + \frac{4\,x}{(-a^2cx^2+c)^{\frac{3}{2}}c^2} + \frac{3\,x}{(-a^2cx^2+c)^{\frac{5}{2}}c}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/225*a*(120/(sqrt(-a^2*c*x^2 + c)*a^2*c^3) + 20/((-a^2*c*x^2 + c)^(3/2)*
a^2*c^2) + 9/((-a^2*c*x^2 + c)^(5/2)*a^2*c)) + 1/15*(8*x/(sqrt(-a^2*c*x^2
+ c)*c^3) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)
*c))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2a^4x^2}{c}-\frac{5a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^2cx^2-c\right)^3} - \frac{120\left(a^2cx^2-c\right)^2-20\left(a^2cx^2-c\right)c+9c^2}{225\left(a^2cx^2-c\right)^2\sqrt{-a^2cx^2+c}ac^3}$$

input
```
integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-1/30*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c - 5*a^2/c)*x^2 + 15/c)*x*log(-(
a*x + 1)/(a*x - 1))/(a^2*c*x^2 - c)^3 - 1/225*(120*(a^2*c*x^2 - c)^2 - 20*
(a^2*c*x^2 - c)*c + 9*c^2)/((a^2*c*x^2 - c)^2*sqrt(-a^2*c*x^2 + c)*a*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\mathrm{atanh}(a\,x)}{{\left(c-a^2\,c\,x^2\right)}^{7/2}} \,d x$$

input
```
int(atanh(a*x)/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(atanh(a*x)/(c - a^2*c*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\int \frac{atanh(ax)}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(atanh(a*x)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
( - int(atanh(a*x)/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x**2
 + 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2 +
 1)),x))/(sqrt(c)*c**3)
```

3.471 $\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\begin{aligned}\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = & -\frac{\arcsin(ax)}{a}+\frac{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}{a} \\ & +\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2 \\ & +\frac{\arctan\left(e^{\operatorname{arctanh}(ax)}\right)\operatorname{arctanh}(ax)^2}{a} \\ & -\frac{i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{arctanh}(ax)}\right)}{a} \\ & +\frac{i\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{arctanh}(ax)}\right)}{a} \\ & +\frac{i\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)}{a} \\ & -\frac{i\operatorname{PolyLog}\left(3,ie^{\operatorname{arctanh}(ax)}\right)}{a}\end{aligned}$$

output
```
-arcsin(a*x)/a+(-a^2*x^2+1)^(1/2)*arctanh(a*x)/a+1/2*x*(-a^2*x^2+1)^(1/2)*
arctanh(a*x)^2+arctan((a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2/a-I*arcta
nh(a*x)*polylog(2,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*arctanh(a*x)*polylog(
2,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+I*polylog(3,-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a-I*polylog(3,I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$= \frac{\sqrt{1-a^2x^2}\left(2\text{arctanh}(ax)+ax\text{arctanh}(ax)^2-\frac{i\left(-4i\arctan\left(\tanh\left(\frac{1}{2}\text{arctanh}(ax)\right)\right)+\text{arctanh}(ax)^2\log\left(1-ie^{-\text{arctanh}}\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
(Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTa
n[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTa
nh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^Arc
Tanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-
I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/
(2*a)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6506, 223, 6514, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx$$
$$\Big\downarrow \text{6506}$$
$$\frac{1}{2}\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx-\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}$$
$$\Big\downarrow \text{223}$$
$$\frac{1}{2}\int \frac{\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \text{6514}$$

$$\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2d\text{arctanh}(ax)}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \text{arctanh}(ax)^2\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-2i\int \text{arctanh}(ax)\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+2i\int \text{arctanh}(ax)\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+}{2a}$$
$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2i\left(\int \text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int \text{PolyLog}\left(2,ie^{a}\right.\right.}{2a}$$
$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2i\left(\int e^{-\text{arctanh}(ax)}\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)de^{\text{arctanh}(ax)}-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-2i\left(\int e^{-\text{ar}}\right.}{}$$
$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^2+\frac{\sqrt{1-a^2x^2}\text{arctanh}(ax)}{a}-\frac{\arcsin(ax)}{a}+$$
$$\frac{2\text{arctanh}(ax)^2\arctan\left(e^{\text{arctanh}(ax)}\right)+2i\left(\text{PolyLog}\left(3,-ie^{\text{arctanh}(ax)}\right)-\text{arctanh}(ax)\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-}{2a}$$

input
```
Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

output
```
-(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/2 + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 + (2*I)*(-
(ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + PolyLog[3, (-I)*E^ArcTanh
[a*x]]) - (2*I)*(-(ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]]) + PolyLog[3,
 I*E^ArcTanh[a*x]]))/(2*a)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \sqrt{-a^2x^2+1}\, \operatorname{arctanh}(ax)^2\, dx$$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Fricas [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\, dx = \int \sqrt{-a^2x^2+1}\, \operatorname{artanh}(ax)^2\, dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F]

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = \int \sqrt{-(ax-1)(ax+1)}\operatorname{atanh}^2(ax)\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Maxima [F]

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^2\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \text{atanh}(a\,x)^2\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)^2*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^2\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)^2\,dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^2,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2,x)
```

3.472 $\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \frac{2x}{27(1-a^2x^2)^{3/2}} + \frac{40x}{27\sqrt{1-a^2x^2}} - \frac{2\operatorname{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}} - \frac{4\operatorname{arctanh}(ax)}{3a\sqrt{1-a^2x^2}} + \frac{x\operatorname{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}} + \frac{2x\operatorname{arctanh}(ax)^2}{3\sqrt{1-a^2x^2}}$$

output
```
2/27*x/(-a^2*x^2+1)^(3/2)+40/27*x/(-a^2*x^2+1)^(1/2)-2/9*arctanh(a*x)/a/(-
a^2*x^2+1)^(3/2)-4/3*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+1/3*x*arctanh(a*x)^
2/(-a^2*x^2+1)^(3/2)+2/3*x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \frac{42ax - 40a^3x^3 + 6(-7+6a^2x^2)\operatorname{arctanh}(ax) - 9ax(-3+2a^2x^2)\operatorname{arctanh}(ax)^2}{27a(1-a^2x^2)^{3/2}}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

output
```
(42*a*x - 40*a^3*x^3 + 6*(-7 + 6*a^2*x^2)*ArcTanh[a*x] - 9*a*x*(-3 + 2*a^2
*x^2)*ArcTanh[a*x]^2)/(27*a*(1 - a^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6526, 209, 208, 6524, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{2}{3}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{2}{9}\int \frac{1}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2}{3}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{2}{9}\left(\frac{2}{3}\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x}{3(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2}{3}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{9}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)$$

$$\Big\downarrow \text{6524}$$

$$\frac{2}{3}\left(2\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{9}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{x\operatorname{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\operatorname{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\operatorname{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\operatorname{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)+\frac{2}{9}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

output
```
(2*(x/(3*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - a^2*x^2])))/9 - (2*ArcTa
nh[a*x])/(9*a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(
3/2)) + (2*((2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2
]) + (x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2]))/3
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

method	result
default	$-\frac{\sqrt{-a^{2}x^{2}+1}\,\left(18\operatorname{arctanh}(ax)^{2}a^{3}x^{3}+40a^{3}x^{3}-36a^{2}x^{2}\operatorname{arctanh}(ax)-27\operatorname{arctanh}(ax)^{2}ax-42ax+42\operatorname{arctanh}(ax)\right)}{27a\left(a^{2}x^{2}-1\right)^{2}}$
orering	$\frac{\left(\frac{200}{9}a^{6}x^{7}-\frac{1222}{27}a^{4}x^{5}+\frac{595}{27}a^{2}x^{3}+x\right)\operatorname{arctanh}(ax)^{2}}{\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}+\frac{(ax+1)^{2}(ax-1)^{2}\left(80a^{4}x^{4}-78a^{2}x^{2}-7\right)\left(\frac{2\operatorname{arctanh}(ax)a}{\left(-a^{2}x^{2}+1\right)^{\frac{7}{2}}}+\frac{5\operatorname{arctanh}(ax)^{2}a^{2}x}{\left(-a^{2}x^{2}+1\right)^{\frac{7}{2}}}\right)}{9a^{2}}$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^2*a^3*x^3+40*a^3*x^3-36*a^2*x^
2*arctanh(a*x)-27*arctanh(a*x)^2*a*x-42*a*x+42*arctanh(a*x))/(a^2*x^2-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = -\frac{\left(160\,a^3x^3+9\,(2\,a^3x^3-3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2-168\,ax-12\,(6\,a^2x^2-7)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{108\,(a^5x^4-2\,a^3x^2+a)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/108*(160*a^3*x^3 + 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 -
168*a*x - 12*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)
/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**(5/2),x)
```

output
```
Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(115) = 230.

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.19

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2x^2+1}} + \frac{x}{(-a^2x^2+1)^{\frac{3}{2}}}\right)\operatorname{artanh}(ax)^2 + \frac{1}{27}\,a\left(\frac{\frac{2\,x}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}a^2x+\sqrt{-a^2x^2+1}a}}{a} + \frac{\frac{2\,x}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}a^2x-\sqrt{-a^2x^2+1}a}}{a} - \frac{18\sqrt{-a^2x^2+1}}{(a^2x+a)a} - \frac{18\,\sqrt{}}{(a}\right.$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)^2 + 1/2
7*a*((2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2
 + 1)*a))/a + (2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt
(-a^2*x^2 + 1)*a))/a - 18*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 18*sqrt(-a^
2*x^2 + 1)/((a^2*x - a)*a) - 18*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 18
*log(-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 3*log(a*x + 1)/((-a^2*x^2 + 1)^(
3/2)*a^2) + 3*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2))
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{(1-a^2\,x^2)^{5/2}}\,dx$$

input
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(5/2),x)
```

output
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^(5/2),x)
```

output
```
int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2
+ 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)
```

3.473 $\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\begin{aligned}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx &= \frac{2x}{125\,(1-a^2x^2)^{5/2}}+\frac{272x}{3375\,(1-a^2x^2)^{3/2}}\\&+\frac{4144x}{3375\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{25a\,(1-a^2x^2)^{5/2}}-\frac{8\text{arctanh}(ax)}{45a\,(1-a^2x^2)^{3/2}}\\&-\frac{16\text{arctanh}(ax)}{15a\sqrt{1-a^2x^2}}+\frac{x\text{arctanh}(ax)^2}{5\,(1-a^2x^2)^{5/2}}+\frac{4x\text{arctanh}(ax)^2}{15\,(1-a^2x^2)^{3/2}}+\frac{8x\text{arctanh}(ax)^2}{15\sqrt{1-a^2x^2}}\end{aligned}$$

output

```
2/125*x/(-a^2*x^2+1)^(5/2)+272/3375*x/(-a^2*x^2+1)^(3/2)+4144/3375*x/(-a^2
*x^2+1)^(1/2)-2/25*arctanh(a*x)/a/(-a^2*x^2+1)^(5/2)-8/45*arctanh(a*x)/a/(
-a^2*x^2+1)^(3/2)-16/15*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+1/5*x*arctanh(a*
x)^2/(-a^2*x^2+1)^(5/2)+4/15*x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2)+8/15*x*ar
ctanh(a*x)^2/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = \frac{4470ax - 8560a^3x^3 + 4144a^5x^5 - 30(149 - 260a^2x^2 + 120a^4x^4)\operatorname{arctanh}(ax) + 225a}{3375a\,(1-a^2x^2)^{5/2}}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

output
```
(4470*a*x - 8560*a^3*x^3 + 4144*a^5*x^5 - 30*(149 - 260*a^2*x^2 + 120*a^4*
x^4)*ArcTanh[a*x] + 225*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^2)/
(3375*a*(1 - a^2*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6526, 209, 209, 208, 6526, 209, 208, 6524, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 6526}$$

$$\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx + \frac{2}{25}\int \frac{1}{(1-a^2x^2)^{7/2}}dx + \frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}} - \frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx + \frac{2}{25}\left(\frac{4}{5}\int \frac{1}{(1-a^2x^2)^{5/2}}dx + \frac{x}{5(1-a^2x^2)^{5/2}}\right) + \frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}} - \frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx+\frac{2}{25}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x}{3(1-a^2x^2)^{3/2}}\right)+\frac{x}{5(1-a^2x^2)^{5/2}}\right)+\frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx+\frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 6526$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{2}{9}\int \frac{1}{(1-a^2x^2)^{5/2}}dx+\frac{x\operatorname{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\operatorname{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 209$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{2}{9}\left(\frac{2}{3}\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x}{3(1-a^2x^2)^{3/2}}\right)+\frac{x\operatorname{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\operatorname{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 208$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{x\operatorname{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\operatorname{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{9}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)+\frac{x\operatorname{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\operatorname{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 6524$$

$$\frac{4}{5}\left(\frac{2}{3}\left(2\int\frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{9}\left(\frac{}{3\sqrt{}}\right.\right.$$

$$\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\text{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+$$

$$\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\text{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+$$

$$\frac{4}{5}\left(\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}+\frac{2x}{\sqrt{1-a^2x^2}}\right)+\frac{2}{9}\left(\frac{2x}{3\sqrt{1-a^2x^2}}\right.\right.$$

$$\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

output
```
(2*(x/(5*(1 - a^2*x^2)^(5/2)) + (4*(x/(3*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*S
qrt[1 - a^2*x^2])))/5))/25 - (2*ArcTanh[a*x])/(25*a*(1 - a^2*x^2)^(5/2)) +
 (x*ArcTanh[a*x]^2)/(5*(1 - a^2*x^2)^(5/2)) + (4*((2*(x/(3*(1 - a^2*x^2)^(
3/2)) + (2*x)/(3*Sqrt[1 - a^2*x^2])))/9 - (2*ArcTanh[a*x])/(9*a*(1 - a^2*x
^2)^(3/2)) + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(3/2)) + (2*((2*x)/Sqrt[1
 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/
Sqrt[1 - a^2*x^2]))/3))/5
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.57

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(1800\,\mathrm{arctanh}(ax)^2a^5x^5+4144a^5x^5-3600a^4x^4\,\mathrm{arctanh}(ax)-4500\,\mathrm{arctanh}(ax)^2a^3x^3-8560a^3x^3+7800a^2x^2\,\mathrm{arctan}\right.}{3375a(a^2x^2-1)^3}$
orering	$\frac{\left(-\frac{116032}{3375}a^8x^9+\frac{355816}{3375}a^6x^7-\frac{363104}{3375}a^4x^5+\frac{23989}{675}a^2x^3+x\right)\mathrm{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{7}{2}}}-\frac{(ax+1)^2(ax-1)^2\left(33152a^6x^6-66680a^4x^4+31860a^2x\right.}{3375a^2}$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3375/a*(-a^2*x^2+1)^(1/2)*(1800*arctanh(a*x)^2*a^5*x^5+4144*a^5*x^5-360
0*a^4*x^4*arctanh(a*x)-4500*arctanh(a*x)^2*a^3*x^3-8560*a^3*x^3+7800*a^2*x
^2*arctanh(a*x)+3375*arctanh(a*x)^2*a*x+4470*a*x-4470*arctanh(a*x))/(a^2*x
^2-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = -\frac{\left(16576\,a^5x^5 - 34240\,a^3x^3 + 225\,(8\,a^5x^5 - 20\,a^3x^3 + 15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 17880\,ax - 60\,(120\,a^4x^4 - 2\cdots\right.}{13500\,(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a)}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

output
```
-1/13500*(16576*a^5*x^5 - 34240*a^3*x^3 + 225*(8*a^5*x^5 - 20*a^3*x^3 + 15
*a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 17880*a*x - 60*(120*a^4*x^4 - 260*a^2*
x^2 + 149)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*
x^4 + 3*a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**(7/2),x)
```

output
```
Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(172) = 344$.

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.47

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = \frac{1}{15}\left(\frac{8x}{\sqrt{-a^2x^2+1}}+\frac{4x}{(-a^2x^2+1)^{\frac{3}{2}}}+\frac{3x}{(-a^2x^2+1)^{\frac{5}{2}}}\right)\operatorname{artanh}(ax)^2 + \frac{1}{3375}a\left(\frac{9\left(\frac{8x}{\sqrt{-a^2x^2+1}}+\frac{4x}{(-a^2x^2+1)^{\frac{3}{2}}}-\frac{3}{(-a^2x^2+1)^{\frac{3}{2}}a^2x+(-a^2x^2+1)^{\frac{3}{2}}a}\right)}{a}+\frac{9\left(\frac{8x}{\sqrt{-a^2x^2+1}}+\frac{4x}{(-a^2x^2+1)^{\frac{3}{2}}}-\frac{}{(-a^2x^2+1)}\right.}{a}\right.$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

output
```
1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) + 3*x/(-a^2*x^2 +
1)^(5/2))*arctanh(a*x)^2 + 1/3375*a*(9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2
*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a^2*x^2 + 1)^(3/2)*a))
/a + 9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 +
 1)^(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 100*(2*x/sqrt(-a^2*x^2 + 1)
 - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a))/a + 100*(2*x/sqrt(
-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a^2*x^2 + 1)*a))/a - 1
800*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 1800*sqrt(-a^2*x^2 + 1)/((a^2*x -
 a)*a) - 1800*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 1800*log(-a*x + 1)/(
sqrt(-a^2*x^2 + 1)*a^2) - 300*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) + 30
0*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) - 135*log(a*x + 1)/((-a^2*x^2 +
 1)^(5/2)*a^2) + 135*log(-a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2))
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{{(1-a^2\,x^2)}^{7/2}}\,dx$$

input
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(7/2),x)
```

output
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx = -\left(\int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^(7/2),x)
```

output
```
- int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x*
*2 + 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x)
```

3.474 $\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 277

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \frac{2x}{343(1-a^2x^2)^{7/2}} + \frac{888x}{42875(1-a^2x^2)^{5/2}} + \frac{30256x}{385875(1-a^2x^2)^{3/2}} + \frac{413312x}{385875\sqrt{1-a^2x^2}} - \frac{2\operatorname{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}} - \frac{12\operatorname{arctanh}(ax)}{175a(1-a^2x^2)^{5/2}} - \frac{16\operatorname{arctanh}(ax)}{105a(1-a^2x^2)^{3/2}} - \frac{32\operatorname{arctanh}(ax)}{35a\sqrt{1-a^2x^2}} + \frac{x\operatorname{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}} + \frac{6x\operatorname{arctanh}(ax)^2}{35(1-a^2x^2)^{5/2}} + \frac{8x\operatorname{arctanh}(ax)^2}{35(1-a^2x^2)^{3/2}} + \frac{16x\operatorname{arctanh}(ax)^2}{35\sqrt{1-a^2x^2}}$$

output
```
2/343*x/(-a^2*x^2+1)^(7/2)+888/42875*x/(-a^2*x^2+1)^(5/2)+30256/385875*x/(
-a^2*x^2+1)^(3/2)+413312/385875*x/(-a^2*x^2+1)^(1/2)-2/49*arctanh(a*x)/a/(
-a^2*x^2+1)^(7/2)-12/175*arctanh(a*x)/a/(-a^2*x^2+1)^(5/2)-16/105*arctanh(
a*x)/a/(-a^2*x^2+1)^(3/2)-32/35*arctanh(a*x)/a/(-a^2*x^2+1)^(1/2)+1/7*x*ar
ctanh(a*x)^2/(-a^2*x^2+1)^(7/2)+6/35*x*arctanh(a*x)^2/(-a^2*x^2+1)^(5/2)+8
/35*x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2)+16/35*x*arctanh(a*x)^2/(-a^2*x^2+1
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.43

$$\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \frac{2ax(226905-654220a^2x^2+635096a^4x^4-206656a^6x^6)+210(-2161+5726a^2x^2 -}{3858}$$

input
```
Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

output
```
(2*a*x*(226905 - 654220*a^2*x^2 + 635096*a^4*x^4 - 206656*a^6*x^6) + 210*(
-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*x] - 11025*a
*x*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]^2)/(385875*a*
(1 - a^2*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.55, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6526, 209, 209, 209, 208, 6526, 209, 209, 208, 6526, 209, 208, 6524, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}dx$$

$$\Big\downarrow \; 6526$$

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}dx+\frac{2}{49}\int \frac{1}{(1-a^2x^2)^{9/2}}dx+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}dx+\frac{2}{49}\left(\frac{6}{7}\int \frac{1}{(1-a^2x^2)^{7/2}}dx+\frac{x}{7(1-a^2x^2)^{7/2}}\right)+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}dx+\frac{2}{49}\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{(1-a^2x^2)^{5/2}}dx+\frac{x}{5(1-a^2x^2)^{5/2}}\right)+\frac{x}{7(1-a^2x^2)^{7/2}}\right)+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}$$

$\Big\downarrow$ 209

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}dx+\frac{2}{49}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x}{3(1-a^2x^2)^{3/2}}\right)+\frac{x}{5(1-a^2x^2)^{5/2}}\right)+\frac{x}{7(1-a^2x^2)^{7/2}}\right)+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}$$

$\Big\downarrow$ 208

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$\Big\downarrow$ 6526

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx+\frac{2}{25}\int \frac{1}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\text{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$\Big\downarrow$ 209

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx+\frac{2}{25}\left(\frac{4}{5}\int \frac{1}{(1-a^2x^2)^{5/2}}dx+\frac{x}{5(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\text{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$\Big\downarrow$ 209

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx+\frac{2}{25}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x}{3(1-a^2x^2)^{3/2}}\right)+\frac{x}{5(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(a\right.$$
$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\text{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{2}{25}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{}{3(1}\right.\right.\right.$$
$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$$\Big\downarrow \text{6526}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{2}{9}\int \frac{1}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\right.$$
$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{2}{9}\left(\frac{2}{3}\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x}{3(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax}{9a(1-a^2x^2)^{3}}\right.\right.$$
$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)^2}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{9}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)+\frac{xa}{5(\cdots}\right.$$
$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$\downarrow$ 6524

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(2\int \frac{1}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arctanh}(ax)}{a\sqrt{1-a^2x^2}}\right)+\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{9}\left(\cdots\right.\right.\right.$$
$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

$\downarrow$ 208

$$\frac{x\text{arctanh}(ax)^2}{7(1-a^2x^2)^{7/2}}-\frac{2\text{arctanh}(ax)}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{7}\left(\frac{x\text{arctanh}(ax)^2}{5(1-a^2x^2)^{5/2}}-\frac{2\text{arctanh}(ax)}{25a(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)^2}{3(1-a^2x^2)^{3/2}}-\frac{2\text{arctanh}(ax)}{9a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\text{arcta}\cdots}{a\sqrt{1-\cdots}}\right.\right.\right.$$
$$\frac{2}{49}\left(\frac{x}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3(1-a^2x^2)^{3/2}}\right)\right)\right)$$

input
```
Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

output
```
(2*(x/(7*(1 - a^2*x^2)^(7/2)) + (6*(x/(5*(1 - a^2*x^2)^(5/2)) + (4*(x/(3*(
1 - a^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - a^2*x^2])))/5))/7))/49 - (2*ArcTan
h[a*x])/(49*a*(1 - a^2*x^2)^(7/2)) + (x*ArcTanh[a*x]^2)/(7*(1 - a^2*x^2)^(
7/2)) + (6*((2*(x/(5*(1 - a^2*x^2)^(5/2)) + (4*(x/(3*(1 - a^2*x^2)^(3/2))
+ (2*x)/(3*Sqrt[1 - a^2*x^2])))/5))/25 - (2*ArcTanh[a*x])/(25*a*(1 - a^2*x
^2)^(5/2)) + (x*ArcTanh[a*x]^2)/(5*(1 - a^2*x^2)^(5/2)) + (4*((2*(x/(3*(1
- a^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - a^2*x^2])))/9 - (2*ArcTanh[a*x])/(9*
a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(3/2)) + (2*(
(2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcT
anh[a*x]^2)/Sqrt[1 - a^2*x^2]))/3))/5))/7
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]), 
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.55

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(176400\operatorname{arctanh}(ax)^2a^7x^7+413312a^7x^7-352800\operatorname{arctanh}(ax)a^6x^6-617400\operatorname{arctanh}(ax)^2a^5x^5-1270192a^5x^5+111\right.}{}$
orering	$\frac{\left(\frac{413312}{8575}a^{10}x^{11}-\frac{8428176}{42875}a^8x^9+\frac{12911712}{42875}a^6x^7-\frac{179314}{875}a^4x^5+\frac{62669}{1225}a^2x^3+x\right)\operatorname{arctanh}(ax)^2}{(-a^2x^2+1)^{\frac{9}{2}}}+\frac{(ax+1)^2(ax-1)^2\left(826624a^8x^8-2505\right.}{}$

input
```
int(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/385875/a*(-a^2*x^2+1)^(1/2)*(176400*arctanh(a*x)^2*a^7*x^7+413312*a^7*x
^7-352800*arctanh(a*x)*a^6*x^6-617400*arctanh(a*x)^2*a^5*x^5-1270192*a^5*x
^5+1117200*a^4*x^4*arctanh(a*x)+771750*arctanh(a*x)^2*a^3*x^3+1308440*a^3*
x^3-1202460*a^2*x^2*arctanh(a*x)-385875*arctanh(a*x)^2*a*x-453810*a*x+4538
10*arctanh(a*x))/(a^2*x^2-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx =$$
$$-\frac{\left(1653248\,a^7x^7 - 5080768\,a^5x^5 + 5233760\,a^3x^3 + 11025\left(16\,a^7x^7 - 56\,a^5x^5 + 70\,a^3x^3 - 35\,ax\right)\log\left(-\frac{ax+}{ax-}\right.\right.}{1543500\left(a^9x^8 - 4\,a^7x^6 + 6\right.}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

output
```
-1/1543500*(1653248*a^7*x^7 - 5080768*a^5*x^5 + 5233760*a^3*x^3 + 11025*(1
6*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1))^2
- 1815240*a*x - 420*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*lo
g(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x
^4 - 4*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**2/(-a**2*x**2+1)**(9/2),x)
```

output
```
Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(229) = 458$.

Time = 0.19 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

output
```
1/35*(16*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 +
 1)^(5/2) + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)^2 + 1/385875*a*(225*(16
*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)
*a^2*x + (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 225*(16*x
/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a
^2*x - (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 882*(8*x/sq
rt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*
x + (-a^2*x^2 + 1)^(3/2)*a))/a + 882*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x
^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a
 + 9800*(2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*
x^2 + 1)*a))/a + 9800*(2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*
x - sqrt(-a^2*x^2 + 1)*a))/a - 176400*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) -
 176400*sqrt(-a^2*x^2 + 1)/((a^2*x - a)*a) - 176400*log(a*x + 1)/(sqrt(-a^
2*x^2 + 1)*a^2) + 176400*log(-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 29400*lo
g(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) + 29400*log(-a*x + 1)/((-a^2*x^2 + 1
)^(3/2)*a^2) - 13230*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2) + 13230*log(-
a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2) - 7875*log(a*x + 1)/((-a^2*x^2 + 1)^(7
/2)*a^2) + 7875*log(-a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2))
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)^2}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^2}{(1-a^2\,x^2)^{9/2}}\,dx$$

input
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(9/2),x)
```

output
```
int(atanh(a*x)^2/(1 - a^2*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^2}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{atanh(ax)^2}{\sqrt{-a^2x^2+1}\,a^8x^8 - 4\sqrt{-a^2x^2+1}\,a^6x^6 + 6\sqrt{-a^2x^2+1}\,a^4x^4 - 4\sqrt{-a^2x^2+1}\,a^2x}$$

input
```
int(atanh(a*x)^2/(-a^2*x^2+1)^(9/2),x)
```

output
```
int(atanh(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**8*x**8 - 4*sqrt( - a**2*x**2
+ 1)*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 4*sqrt( - a**2*x**2
+ 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)
```

3.475 $\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 302

$$\begin{aligned}\int \sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^3\,dx &= \frac{6\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\mathrm{arctanh}(ax)}{a} \\ &+ \frac{3\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^2}{2a} \\ &+ \frac{1}{2}x\sqrt{1-a^2x^2}\,\mathrm{arctanh}(ax)^3 \\ &+ \frac{\arctan\left(e^{\mathrm{arctanh}(ax)}\right)\mathrm{arctanh}(ax)^3}{a} \\ &- \frac{3i\,\mathrm{arctanh}(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{\mathrm{arctanh}(ax)}\right)}{2a} \\ &+ \frac{3i\,\mathrm{arctanh}(ax)^2\,\mathrm{PolyLog}\left(2,ie^{\mathrm{arctanh}(ax)}\right)}{2a} \\ &+ \frac{3i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} - \frac{3i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &+ \frac{3i\,\mathrm{arctanh}(ax)\,\mathrm{PolyLog}\left(3,-ie^{\mathrm{arctanh}(ax)}\right)}{a} \\ &- \frac{3i\,\mathrm{arctanh}(ax)\,\mathrm{PolyLog}\left(3,ie^{\mathrm{arctanh}(ax)}\right)}{a} \\ &- \frac{3i\,\mathrm{PolyLog}\left(4,-ie^{\mathrm{arctanh}(ax)}\right)}{a} \\ &+ \frac{3i\,\mathrm{PolyLog}\left(4,ie^{\mathrm{arctanh}(ax)}\right)}{a}\end{aligned}$$

output
```
6*arctan((-a*x+1)^(1/2)/(a*x+1)^(1/2))*arctanh(a*x)/a+3/2*(-a^2*x^2+1)^(1/
2)*arctanh(a*x)^2/a+1/2*x*(-a^2*x^2+1)^(1/2)*arctanh(a*x)^3+arctan((a*x+1)
/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3/a-3/2*I*arctanh(a*x)^2*polylog(2,-I*(a
*x+1)/(-a^2*x^2+1)^(1/2))/a+3/2*I*arctanh(a*x)^2*polylog(2,I*(a*x+1)/(-a^2
*x^2+1)^(1/2))/a+3*I*polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-3*I*poly
log(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+3*I*arctanh(a*x)*polylog(3,-I*(a*x
+1)/(-a^2*x^2+1)^(1/2))/a-3*I*arctanh(a*x)*polylog(3,I*(a*x+1)/(-a^2*x^2+1
)^(1/2))/a-3*I*polylog(4,-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a+3*I*polylog(4,I*
(a*x+1)/(-a^2*x^2+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.88

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx =$$

$$-\frac{i\left(7\pi^4+8i\pi^3\text{arctanh}(ax)+24\pi^2\text{arctanh}(ax)^2+192i\sqrt{1-a^2x^2}\text{arctanh}(ax)^2-32i\pi\text{arctanh}(ax)^3+\cdots\right.}{}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

output
```
((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 + (
192*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 + (64*I
)*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 - 384*ArcTanh[a
*x]*Log[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + 384
*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E
^ArcTanh[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*A
rcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E
^ArcTanh[a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I
)*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a
*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(8 + Pi^2 - (
4*I)*Pi*ArcTanh[a*x] - 4*ArcTanh[a*x]^2)*PolyLog[2, (-I)/E^ArcTanh[a*x]] +
 384*PolyLog[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^A
rcTanh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a
*x]*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*
x]] + 384*ArcTanh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*
PolyLog[3, (-I)*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]]
+ 384*PolyLog[4, (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]
]))/a
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6506, 6512, 6514, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx$$

$$\Big\downarrow \text{ 6506}$$

$$-3\int \frac{\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}\int \frac{\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a}$$

$$\Big\downarrow \text{ 6512}$$

$$\frac{1}{2}\int \frac{\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{ 6514}$$

$$\frac{\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3d\text{arctanh}(ax)}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \text{arctanh}(ax)^3\csc\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)d\text{arctanh}(ax)}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{-3i\int \text{arctanh}(ax)^2\log\left(1-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)+3i\int \text{arctanh}(ax)^2\log\left(1+ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)\ \cdots}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{3i\left(2\int \text{arctanh}(ax)\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)d\text{arctanh}(ax)-\text{arctanh}(ax)^2\,\text{PolyLog}\left(2,-ie^{\text{arctanh}(ax)}\right)\right)-3i(2\ \cdots}{\cdots}+\frac{1}{2}x\sqrt{1-a^2x^2}\text{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\text{arctanh}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{3i\left(2\left(\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)-\int \operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)\right)-\operatorname{arctanh}(ax)^2\operatorname{Poly}}{}$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{3i\left(2\left(\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)-\int e^{-\operatorname{arctanh}(ax)}\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)de^{\operatorname{arctanh}(ax)}\right)-\operatorname{arctanh}}{}$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^3+\frac{3\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}{2a}-3\left(-\frac{2\arctan\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\operatorname{arctanh}(ax)}{a}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}\right)+\frac{2\operatorname{arctanh}(ax)^3\arctan\left(e^{\operatorname{arctanh}(ax)}\right)+3i\left(2\left(\operatorname{arctanh}(ax)\operatorname{PolyLog}\left(3,-ie^{\operatorname{arctanh}(ax)}\right)-\operatorname{PolyLog}\left(4,-ie^{\operatorname{arctanh}(ax)}\right)\right.\right.}{}$$

input
```
Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

output
```
(3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/2 - 3*((-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (
I*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqr
t[1 - a*x])/Sqrt[1 + a*x]])/a) + (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3
+ (3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]]) + 2*(ArcTanh[a*
x]*PolyLog[3, (-I)*E^ArcTanh[a*x]] - PolyLog[4, (-I)*E^ArcTanh[a*x]])) - (
3*I)*(-(ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[a*x]]) + 2*(ArcTanh[a*x]*Pol
yLog[3, I*E^ArcTanh[a*x]] - PolyLog[4, I*E^ArcTanh[a*x]])))/(2*a)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6506
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[b*p*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^(p - 1)/(2*c*q*(2*
q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTanh[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x])^p,
 x], x] - Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*
(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6514
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTa
nh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0
] && GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \sqrt{-a^2x^2+1}\, \operatorname{arctanh}(ax)^3\, dx$$

input
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

output
```
int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

Fricas [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx=\int \sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Sympy [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx=\int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\text{atanh}^3\left(ax\right)dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**3,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3, x)
```

Maxima [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx=\int \sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx = \int \text{atanh}(a\,x)^3\,\sqrt{1-a^2\,x^2}\,dx$$

input
```
int(atanh(a*x)^3*(1 - a^2*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)^3*(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-a^2x^2}\text{arctanh}(ax)^3\,dx = \int \sqrt{-a^2x^2+1}\,atanh(ax)^3\,dx$$

input
```
int((-a^2*x^2+1)^(1/2)*atanh(a*x)^3,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3,x)
```

3.476 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 191

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = -\frac{2}{27a\,(1-a^2x^2)^{3/2}} - \frac{40}{9a\sqrt{1-a^2x^2}} + \frac{2x\operatorname{arctanh}(ax)}{9\,(1-a^2x^2)^{3/2}} + \frac{40x\operatorname{arctanh}(ax)}{9\sqrt{1-a^2x^2}} - \frac{\operatorname{arctanh}(ax)^2}{3a\,(1-a^2x^2)^{3/2}} - \frac{2\operatorname{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{x\operatorname{arctanh}(ax)^3}{3\,(1-a^2x^2)^{3/2}} + \frac{2x\operatorname{arctanh}(ax)^3}{3\sqrt{1-a^2x^2}}$$

output
```
-2/27/a/(-a^2*x^2+1)^(3/2)-40/9/a/(-a^2*x^2+1)^(1/2)+2/9*x*arctanh(a*x)/(-
a^2*x^2+1)^(3/2)+40/9*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-1/3*arctanh(a*x)^2
/a/(-a^2*x^2+1)^(3/2)-2*arctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+1/3*x*arctanh(
a*x)^3/(-a^2*x^2+1)^(3/2)+2/3*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \frac{-122+120a^2x^2-6ax(-21+20a^2x^2)\,\text{arctanh}(ax)+9(-7+6a^2x^2)\,\text{arctanh}(ax)^2 \cdot}{27a\,(1-a^2x^2)^{3/2}}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

output
```
(-122 + 120*a^2*x^2 - 6*a*x*(-21 + 20*a^2*x^2)*ArcTanh[a*x] + 9*(-7 + 6*a^
2*x^2)*ArcTanh[a*x]^2 - 9*a*x*(-3 + 2*a^2*x^2)*ArcTanh[a*x]^3)/(27*a*(1 -
a^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6526, 6522, 6520, 6524, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{2}{3}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{2}{3}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{2}{3}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{6520}$$

$$\frac{2}{3}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)$$

$$\Big\downarrow \ 6524$$

$$\frac{2}{3}\left(6\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}\right)+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)$$

$$\Big\downarrow \ 6520$$

$$\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}+6\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)\right)$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

output
```
-1/3*ArcTanh[a*x]^2/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x]^3)/(3*(1 - a
^2*x^2)^(3/2)) + (2*(-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*
(1 - a^2*x^2)^(3/2)) + (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/S
qrt[1 - a^2*x^2]))/3))/3 + (2*((-3*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) +
 (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] + 6*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x
*ArcTanh[a*x])/Sqrt[1 - a^2*x^2])))/3
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(18\operatorname{arctanh}(ax)^3a^3x^3+120a^3x^3\operatorname{arctanh}(ax)-54a^2x^2\operatorname{arctanh}(ax)^2-27\operatorname{arctanh}(ax)^3ax-120a^2x^2-126ax\operatorname{arctanh}\right.}{27a(a^2x^2-1)^2}$
orering	$\frac{\left(\frac{1600}{9}a^6x^7-\frac{11392}{27}a^4x^5+\frac{24880}{81}a^2x^3-\frac{5104}{81}x\right)\operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{5}{2}}}+\frac{2(ax+1)^2(ax-1)^2(4140a^4x^4-4704a^2x^2+487)\left(\frac{3\operatorname{arctanh}(ax)^2a}{(-a^2x^2+1)^{\frac{7}{2}}}+\right.}{81a^2}$

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^3*a^3*x^3+120*a^3*x^3*arctanh(
a*x)-54*a^2*x^2*arctanh(a*x)^2-27*arctanh(a*x)^3*a*x-120*a^2*x^2-126*a*x*a
rctanh(a*x)+63*arctanh(a*x)^2+122)/(a^2*x^2-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \frac{\left(960\,a^2x^2 - 9\,(2\,a^3x^3 - 3\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 18\,(6\,a^2x^2 - 7)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 24\,(20\right.}{216\,(a^5x^4 - 2\,a^3x^2 + a)}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
1/216*(960*a^2*x^2 - 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 1
8*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1))^2 - 24*(20*a^3*x^3 - 21*a*x)*l
og(-(a*x + 1)/(a*x - 1)) - 976)*sqrt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 +
a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{(-(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**(5/2),x)
```

output
```
Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{(1-a^2\,x^2)^{5/2}}\,dx$$

input
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(5/2),x)
```

output
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}\,dx = \int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^(5/2),x)
```

output
```
int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2
+ 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)
```

3.477 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 289

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx =& -\frac{6}{625a(1-a^2x^2)^{5/2}}-\frac{272}{3375a(1-a^2x^2)^{3/2}}\\&-\frac{4144}{1125a\sqrt{1-a^2x^2}}+\frac{6x\operatorname{arctanh}(ax)}{125(1-a^2x^2)^{5/2}}+\frac{272x\operatorname{arctanh}(ax)}{1125(1-a^2x^2)^{3/2}}\\&+\frac{4144x\operatorname{arctanh}(ax)}{1125\sqrt{1-a^2x^2}}-\frac{3\operatorname{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}-\frac{4\operatorname{arctanh}(ax)^2}{15a(1-a^2x^2)^{3/2}}\\&-\frac{8\operatorname{arctanh}(ax)^2}{5a\sqrt{1-a^2x^2}}+\frac{x\operatorname{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}+\frac{4x\operatorname{arctanh}(ax)^3}{15(1-a^2x^2)^{3/2}}+\frac{8x\operatorname{arctanh}(ax)^3}{15\sqrt{1-a^2x^2}}\end{aligned}$$

output
```
-6/625/a/(-a^2*x^2+1)^(5/2)-272/3375/a/(-a^2*x^2+1)^(3/2)-4144/1125/a/(-a^
2*x^2+1)^(1/2)+6/125*x*arctanh(a*x)/(-a^2*x^2+1)^(5/2)+272/1125*x*arctanh(
a*x)/(-a^2*x^2+1)^(3/2)+4144/1125*x*arctanh(a*x)/(-a^2*x^2+1)^(1/2)-3/25*a
rctanh(a*x)^2/a/(-a^2*x^2+1)^(5/2)-4/15*arctanh(a*x)^2/a/(-a^2*x^2+1)^(3/2
)-8/5*arctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+1/5*x*arctanh(a*x)^3/(-a^2*x^2+1
)^(5/2)+4/15*x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2)+8/15*x*arctanh(a*x)^3/(-a
^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = \frac{-63682+125680a^2x^2-62160a^4x^4+30ax(2235-4280a^2x^2+2072a^4x^4)\operatorname{arctanh}(\cdots}{168\cdots}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

output
```
(-63682 + 125680*a^2*x^2 - 62160*a^4*x^4 + 30*a*x*(2235 - 4280*a^2*x^2 + 2
072*a^4*x^4)*ArcTanh[a*x] - 225*(149 - 260*a^2*x^2 + 120*a^4*x^4)*ArcTanh[
a*x]^2 + 1125*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^3)/(16875*a*(
1 - a^2*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.52, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6526, 6522, 6522, 6520, 6526, 6522, 6520, 6524, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{6}{25}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{7/2}}dx+\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\operatorname{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\operatorname{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{25}\left(\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{x\operatorname{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}\right)+\frac{4}{5}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\operatorname{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\operatorname{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{25}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}\right)+ \frac{4}{5}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}$$

$\downarrow$ 6520

$$\frac{4}{5}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+ \frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{}{25a(1-}\right.$$

$\downarrow$ 6526

$$\frac{4}{5}\left(\frac{2}{3}\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{2}{3}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}\right)+ \frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+ \frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{}{25a(1-}\right.$$

$\downarrow$ 6522

$$\frac{4}{5}\left(\frac{2}{3}\left(\frac{2}{3}\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{2}{3}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\right. \frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+ \frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{}{25a(1-}\right.$$

$\downarrow$ 6520

$$\frac{4}{5}\left(\frac{2}{3}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{}{a\sqrt{1}}\right.\right.\right. \frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+ \frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{}{25a(1-}\right.$$

$\downarrow$ 6524

$$\frac{4}{5}\left(\frac{2}{3}\left(6\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}\right)+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x}{3(\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+$$

$$\frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{}{25a(1-\right.$$

$$\downarrow \text{6520}$$

$$\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+$$

$$\frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)-\frac{}{25a(1-\right.$$

$$\frac{4}{5}\left(\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{1}{9a(1-a^2x\right.\right.$$

input
```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

output
```
(-3*ArcTanh[a*x]^2)/(25*a*(1 - a^2*x^2)^(5/2)) + (x*ArcTanh[a*x]^3)/(5*(1
- a^2*x^2)^(5/2)) + (6*(-1/25*1/(a*(1 - a^2*x^2)^(5/2)) + (x*ArcTanh[a*x])
/(5*(1 - a^2*x^2)^(5/2)) + (4*(-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh
[a*x])/(3*(1 - a^2*x^2)^(3/2)) + (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTa
nh[a*x])/Sqrt[1 - a^2*x^2]))/3))/5))/25 + (4*(-1/3*ArcTanh[a*x]^2/(a*(1 -
a^2*x^2)^(3/2)) + (x*ArcTanh[a*x]^3)/(3*(1 - a^2*x^2)^(3/2)) + (2*(-1/9*1/
(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*(1 - a^2*x^2)^(3/2)) + (2*(-
(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/3))/3 + (
2*((-3*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 -
 a^2*x^2] + 6*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*
x^2])))/3))/5
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(9000\operatorname{arctanh}(ax)^3a^5x^5+62160\operatorname{arctanh}(ax)a^5x^5-27000a^4x^4\operatorname{arctanh}(ax)^2-22500\operatorname{arctanh}(ax)^3a^3x^3-62160a^4x^4\right.}{}$
orering	$\frac{\left(-\frac{232064}{675}a^8x^9+\frac{769024}{675}a^6x^7-\frac{68286904}{50625}a^4x^5+\frac{6618136}{10125}a^2x^3-\frac{5075776}{50625}x\right)\operatorname{arctanh}(ax)^3}{(-a^2x^2+1)^{\frac{7}{2}}}-\frac{2(ax+1)^2(ax-1)^2\left(3605280a^6x^6-764424\right.}{}$

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16875/a*(-a^2*x^2+1)^(1/2)*(9000*arctanh(a*x)^3*a^5*x^5+62160*arctanh(a
*x)*a^5*x^5-27000*a^4*x^4*arctanh(a*x)^2-22500*arctanh(a*x)^3*a^3*x^3-6216
0*a^4*x^4-128400*a^3*x^3*arctanh(a*x)+58500*a^2*x^2*arctanh(a*x)^2+16875*a
rctanh(a*x)^3*a*x+125680*a^2*x^2+67050*a*x*arctanh(a*x)-33525*arctanh(a*x)
^2-63682)/(a^2*x^2-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = \frac{\left(497280\,a^4x^4 - 1005440\,a^2x^2 - 1125\,(8\,a^5x^5 - 20\,a^3x^3 + 15\,ax)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 450\right.}{}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

output
```
1/135000*(497280*a^4*x^4 - 1005440*a^2*x^2 - 1125*(8*a^5*x^5 - 20*a^3*x^3
+ 15*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 450*(120*a^4*x^4 - 260*a^2*x^2 + 1
49)*log(-(a*x + 1)/(a*x - 1))^2 - 120*(2072*a^5*x^5 - 4280*a^3*x^3 + 2235*
a*x)*log(-(a*x + 1)/(a*x - 1)) + 509456)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a
^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**(7/2),x)
```

output
```
Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{(1-a^2\,x^2)^{7/2}}\,dx$$

input
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(7/2),x)
```

output
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}\,dx = -\left(\int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx\right)$$

input

```
int(atanh(a*x)^3/(-a^2*x^2+1)^(7/2),x)
```

output

```
 - int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x*
*2 + 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x)
```

3.478 $\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 385

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx =& -\frac{6}{2401a(1-a^2x^2)^{7/2}}-\frac{2664}{214375a(1-a^2x^2)^{5/2}}\\&-\frac{30256}{385875a(1-a^2x^2)^{3/2}}-\frac{413312}{128625a\sqrt{1-a^2x^2}}+\frac{6x\operatorname{arctanh}(ax)}{343(1-a^2x^2)^{7/2}}\\&+\frac{2664x\operatorname{arctanh}(ax)}{42875(1-a^2x^2)^{5/2}}+\frac{30256x\operatorname{arctanh}(ax)}{128625(1-a^2x^2)^{3/2}}+\frac{413312x\operatorname{arctanh}(ax)}{128625\sqrt{1-a^2x^2}}\\&-\frac{3\operatorname{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}-\frac{18\operatorname{arctanh}(ax)^2}{175a(1-a^2x^2)^{5/2}}-\frac{8\operatorname{arctanh}(ax)^2}{35a(1-a^2x^2)^{3/2}}-\frac{48\operatorname{arctanh}(ax)^2}{35a\sqrt{1-a^2x^2}}\\&+\frac{x\operatorname{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}+\frac{6x\operatorname{arctanh}(ax)^3}{35(1-a^2x^2)^{5/2}}+\frac{8x\operatorname{arctanh}(ax)^3}{35(1-a^2x^2)^{3/2}}+\frac{16x\operatorname{arctanh}(ax)^3}{35\sqrt{1-a^2x^2}}\end{aligned}$$

output
```
-6/2401/a/(-a^2*x^2+1)^(7/2)-2664/214375/a/(-a^2*x^2+1)^(5/2)-30256/385875
/a/(-a^2*x^2+1)^(3/2)-413312/128625/a/(-a^2*x^2+1)^(1/2)+6/343*x*arctanh(a
*x)/(-a^2*x^2+1)^(7/2)+2664/42875*x*arctanh(a*x)/(-a^2*x^2+1)^(5/2)+30256/
128625*x*arctanh(a*x)/(-a^2*x^2+1)^(3/2)+413312/128625*x*arctanh(a*x)/(-a^
2*x^2+1)^(1/2)-3/49*arctanh(a*x)^2/a/(-a^2*x^2+1)^(7/2)-18/175*arctanh(a*x
)^2/a/(-a^2*x^2+1)^(5/2)-8/35*arctanh(a*x)^2/a/(-a^2*x^2+1)^(3/2)-48/35*ar
ctanh(a*x)^2/a/(-a^2*x^2+1)^(1/2)+1/7*x*arctanh(a*x)^3/(-a^2*x^2+1)^(7/2)+
6/35*x*arctanh(a*x)^3/(-a^2*x^2+1)^(5/2)+8/35*x*arctanh(a*x)^3/(-a^2*x^2+1
)^(3/2)+16/35*x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.39

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \frac{-44658302 + 132479032a^2x^2 - 131252240a^4x^4 + 43397760a^6x^6 - 210ax(-226905}{}$$

input
```
Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

output
```
(-44658302 + 132479032*a^2*x^2 - 131252240*a^4*x^4 + 43397760*a^6*x^6 - 21
0*a*x*(-226905 + 654220*a^2*x^2 - 635096*a^4*x^4 + 206656*a^6*x^6)*ArcTanh
[a*x] + 11025*(-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh
[a*x]^2 - 385875*a*x*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[
a*x]^3)/(13505625*a*(1 - a^2*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 3.80 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.78, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6526, 6522, 6522, 6522, 6520, 6526, 6522, 6522, 6520, 6526, 6522, 6520, 6524, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{6526}$$

$$\frac{6}{49}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{9/2}}dx+\frac{6}{7}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{49}\left(\frac{6}{7}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}-\frac{1}{49a(1-a^2x^2)^{7/2}}\right)+\frac{6}{7}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{49}\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}\right)+\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}-\frac{1}{49a(1-a^2x^2)^{7/2}}\right)+\frac{6}{7}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{49}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}} \cdots\right.\right.$$
$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{6520}$$

$$\frac{6}{7}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{7/2}}dx+\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{\cdots}{9a(1-\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{6526}$$

$$\frac{6}{7}\left(\frac{6}{25}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{7/2}}dx+\frac{4}{5}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}\right)+$$
$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{\cdots}{9a(1-\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{7}\left(\frac{6}{25}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}\right)+\frac{4}{5}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}\right.$$
$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{7}\left(\frac{6}{25}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}-\frac{1}{25a(1-a^2x^2)^{5/2}}\right.\right.$$
$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$$\Big\downarrow \text{6520}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{5/2}}dx+\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+\frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\right.\right.\right.$$
$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$$\Big\downarrow \text{6526}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\text{arctanh}(ax)}{(1-a^2x^2)^{5/2}}dx+\frac{2}{3}\int \frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}\right)+\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\right.$$
$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$
$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$$\Big\downarrow \text{6522}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2}{3}\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}-\frac{1}{9a(1-a^2x^2)^{3/2}}\right)+\frac{2}{3}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3}}\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$

$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$\Big\downarrow$ 6520

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\text{arctanh}(ax)^3}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\right.\right.\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$

$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$\Big\downarrow$ 6524

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(6\int\frac{\text{arctanh}(ax)}{(1-a^2x^2)^{3/2}}dx+\frac{x\text{arctanh}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\text{arctanh}(ax)^2}{a\sqrt{1-a^2x^2}}\right)+\frac{x\text{arctanh}(ax)^3}{3(1-a^2x^2)^{3/2}}-\frac{\text{arctanh}(ax)^2}{3a(1-a^2x^2)^{3/2}}+\frac{2}{3}\right.\right.$$

$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$

$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$\Big\downarrow$ 6520

$$\frac{x\text{arctanh}(ax)^3}{7(1-a^2x^2)^{7/2}}-\frac{3\text{arctanh}(ax)^2}{49a(1-a^2x^2)^{7/2}}+$$

$$\frac{6}{49}\left(\frac{x\text{arctanh}(ax)}{7(1-a^2x^2)^{7/2}}+\frac{6}{7}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}\right)-\frac{}{9a(1-}\right.\right.\right.$$

$$\frac{6}{7}\left(\frac{x\text{arctanh}(ax)^3}{5(1-a^2x^2)^{5/2}}-\frac{3\text{arctanh}(ax)^2}{25a(1-a^2x^2)^{5/2}}+\frac{6}{25}\left(\frac{x\text{arctanh}(ax)}{5(1-a^2x^2)^{5/2}}+\frac{4}{5}\left(\frac{x\text{arctanh}(ax)}{3(1-a^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\text{arctanh}(ax)}{\sqrt{1-a^2x^2}}-\frac{}{a\sqrt{1}}\right.\right.\right.\right.$$

input

```
Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

output
```
(-3*ArcTanh[a*x]^2)/(49*a*(1 - a^2*x^2)^(7/2)) + (x*ArcTanh[a*x]^3)/(7*(1
- a^2*x^2)^(7/2)) + (6*(-1/49*1/(a*(1 - a^2*x^2)^(7/2)) + (x*ArcTanh[a*x])
/(7*(1 - a^2*x^2)^(7/2)) + (6*(-1/25*1/(a*(1 - a^2*x^2)^(5/2)) + (x*ArcTan
h[a*x])/(5*(1 - a^2*x^2)^(5/2)) + (4*(-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*
ArcTanh[a*x])/(3*(1 - a^2*x^2)^(3/2)) + (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (
x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/3))/5))/7))/49 + (6*((-3*ArcTanh[a*x]^
2)/(25*a*(1 - a^2*x^2)^(5/2)) + (x*ArcTanh[a*x]^3)/(5*(1 - a^2*x^2)^(5/2))
 + (6*(-1/25*1/(a*(1 - a^2*x^2)^(5/2)) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)
^(5/2)) + (4*(-1/9*1/(a*(1 - a^2*x^2)^(3/2)) + (x*ArcTanh[a*x])/(3*(1 - a^
2*x^2)^(3/2)) + (2*(-(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 -
 a^2*x^2]))/3))/5))/25 + (4*(-1/3*ArcTanh[a*x]^2/(a*(1 - a^2*x^2)^(3/2)) +
 (x*ArcTanh[a*x]^3)/(3*(1 - a^2*x^2)^(3/2)) + (2*(-1/9*1/(a*(1 - a^2*x^2)^
(3/2)) + (x*ArcTanh[a*x])/(3*(1 - a^2*x^2)^(3/2)) + (2*(-(1/(a*Sqrt[1 - a^
2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]))/3))/3 + (2*((-3*ArcTanh[a*
x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] + 6*(-(
1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2])))/3))/5))/7
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 6524
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(-b)*p*((a + b*ArcTanh[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])
), x] + (Simp[x*((a + b*ArcTanh[c*x])^p/(d*Sqrt[d + e*x^2])), x] + Simp[b^2
*p*(p - 1)   Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

rule 6526
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p - 1)/(4
*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^p
/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*(a + b*ArcTanh[c*x])^p, x], x] + Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int
[(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.52

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(6174000 \operatorname{arctanh}(ax)^3 a^7 x^7+43397760 \operatorname{arctanh}(ax) a^7 x^7-18522000 \operatorname{arctanh}(ax)^2 a^6 x^6-21609000 \operatorname{arctanh}(ax)^3 a^5\right.}{}$
orering	$\frac{\left(\frac{4959744}{8575} a^{10} x^{11}-\frac{12708544}{5145} a^8 x^9+\frac{525524992}{128625} a^6 x^7-\frac{4829742448}{1500625} a^4 x^5+\frac{5255006528}{4501875} a^2 x^3-\frac{643043504}{4501875} x\right) \operatorname{arctanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}+\frac{2(ax+1)^2(ax-}{}$

input
```
int(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/13505625/a*(-a^2*x^2+1)^(1/2)*(6174000*arctanh(a*x)^3*a^7*x^7+43397760*
arctanh(a*x)*a^7*x^7-18522000*arctanh(a*x)^2*a^6*x^6-21609000*arctanh(a*x)
^3*a^5*x^5-43397760*a^6*x^6-133370160*arctanh(a*x)*a^5*x^5+58653000*a^4*x^
4*arctanh(a*x)^2+27011250*arctanh(a*x)^3*a^3*x^3+131252240*a^4*x^4+1373862
00*a^3*x^3*arctanh(a*x)-63129150*a^2*x^2*arctanh(a*x)^2-13505625*arctanh(a
*x)^3*a*x-132479032*a^2*x^2-47650050*a*x*arctanh(a*x)+23825025*arctanh(a*x
)^2+44658302)/(a^2*x^2-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.56

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \frac{\left(347182080\,a^6x^6 - 1050017920\,a^4x^4 + 1059832256\,a^2x^2 - 385875\,(16\,a^7x^7 - 56\,a^5x\right.}{}$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

output
```
1/108045000*(347182080*a^6*x^6 - 1050017920*a^4*x^4 + 1059832256*a^2*x^2 -
 385875*(16*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*
x - 1))^3 + 22050*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(
-(a*x + 1)/(a*x - 1))^2 - 840*(206656*a^7*x^7 - 635096*a^5*x^5 + 654220*a^
3*x^3 - 226905*a*x)*log(-(a*x + 1)/(a*x - 1)) - 357266416)*sqrt(-a^2*x^2 +
 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}^3{(ax)}}{\left(-(ax-1)(ax+1)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(atanh(a*x)**3/(-a**2*x**2+1)**(9/2),x)
```

output
```
Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(9/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)^3}{(1-a^2\,x^2)^{9/2}}\,dx$$

input
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(9/2),x)
```

output
```
int(atanh(a*x)^3/(1 - a^2*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx = \int \frac{atanh(ax)^3}{\sqrt{-a^2x^2+1}\,a^8x^8-4\sqrt{-a^2x^2+1}\,a^6x^6+6\sqrt{-a^2x^2+1}\,a^4x^4-4\sqrt{-a^2x^2+1}\,a^2x}$$

input
```
int(atanh(a*x)^3/(-a^2*x^2+1)^(9/2),x)
```

output
```
int(atanh(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**8*x**8 - 4*sqrt( - a**2*x**2
+ 1)*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 4*sqrt( - a**2*x**2
+ 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)
```

3.479 $\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)} \, dx = \operatorname{Int}\left(\frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}, x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)} \, dx = \int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)} \, dx$$

input
```
Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]
```

output
```
Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}}{\text{arctanh}(ax)} \, dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{\sqrt{1-a^2x^2}}{\text{arctanh}(ax)}dx$$

input
```
Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-a^2x^2+1}}{\text{arctanh}\,(ax)}dx$$

input
```
int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

output
```
int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}\,dx = \int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\operatorname{atanh}\left(ax\right)}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)/atanh(a*x),x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}\,dx=\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}\,dx=\int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}\left(a\,x\right)}\,dx$$

input
```
int((1 - a^2*x^2)^(1/2)/atanh(a*x),x)
```

output
```
int((1 - a^2*x^2)^(1/2)/atanh(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{atanh\,(ax)}dx$$

input

```
int((-a^2*x^2+1)^(1/2)/atanh(a*x),x)
```

output

```
int(sqrt( - a**2*x**2 + 1)/atanh(a*x),x)
```

3.480 $\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\,dx=\operatorname{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)},x\right)$$

output
```
Defer(Int)(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\,dx$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
```

output
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6651}$$

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}dx$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{arctanh}\left(ax\right)}dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

output
```
int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x),x)
```

output
```
Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx=\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx=\int \frac{1}{\text{atanh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)}dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/atanh(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)),x)
```

3.481 $\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx=\frac{\text{Chi}(\text{arctanh}(ax))}{a}$$

output
```
Chi(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\,dx=\frac{\text{Chi}(\text{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
CoshIntegral[ArcTanh[a*x]]/a
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6530, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx$$
$$\Big\downarrow \text{6530}$$
$$\frac{\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$
$$\Big\downarrow \text{3782}$$
$$\frac{\operatorname{Chi}(\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

output
```
CoshIntegral[ArcTanh[a*x]]/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\mathrm{Chi}(\mathrm{arctanh}(ax))}{a}$	10

input
```
int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
Chi(arctanh(a*x))/a
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\mathrm{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\,\mathrm{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^(3/2)/atanh(a*x),x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**2 - sqrt( - a**2*x**2
+ 1)*atanh(a*x)),x)
```

3.482 $\int \frac{1}{(1-a^2x^2)^{5/2}\textbf{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx = \frac{3\operatorname{Chi}(\operatorname{arctanh}(ax))}{4a} + \frac{\operatorname{Chi}(3\operatorname{arctanh}(ax))}{4a}$$

output
```
3/4*Chi(arctanh(a*x))/a+1/4*Chi(3*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx = -\frac{-3\operatorname{Chi}(\operatorname{arctanh}(ax)) - \operatorname{Chi}(3\operatorname{arctanh}(ax))}{4a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

output
```
-1/4*(-3*CoshIntegral[ArcTanh[a*x]] - CoshIntegral[3*ArcTanh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6530}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^3}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cosh(3\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{3}{4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{4}\text{Chi}(\operatorname{arctanh}(ax))+\frac{1}{4}\text{Chi}(3\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

output
```
((3*CoshIntegral[ArcTanh[a*x]])/4 + CoshIntegral[3*ArcTanh[a*x]]/4)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{3\,\mathrm{Chi}(\operatorname{arctanh}(ax))+\mathrm{Chi}(3\operatorname{arctanh}(ax))}{4a}$	21

input
```
int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*(3*Chi(arctanh(a*x))+Chi(3*arctanh(a*x)))/a
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctan
h(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{5}{2}}\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x),x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\operatorname{atanh}(a\,x)\,\left(1-a^2\,x^2\right)^{5/2}}\,dx$$

input
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(5/2)),x)
```

output
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}\,dx=\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)\,a^4x^4-2\sqrt{-a^2x^2+1}\,atanh(ax)\,a^2x^2+\sqrt{-a}$$

input
```
int(1/(-a^2*x^2+1)^(5/2)/atanh(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**4*x**4 - 2*sqrt( - a**2*x**2 +
 1)*atanh(a*x)*a**2*x**2 + sqrt( - a**2*x**2 + 1)*atanh(a*x)),x)
```

3.483 $\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \frac{5\text{Chi}(\operatorname{arctanh}(ax))}{8a} + \frac{5\text{Chi}(3\operatorname{arctanh}(ax))}{16a} + \frac{\text{Chi}(5\operatorname{arctanh}(ax))}{16a}$$

output
```
5/8*Chi(arctanh(a*x))/a+5/16*Chi(3*arctanh(a*x))/a+1/16*Chi(5*arctanh(a*x)
)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \frac{10\text{Chi}(\operatorname{arctanh}(ax)) + 5\text{Chi}(3\operatorname{arctanh}(ax)) + \text{Chi}(5\operatorname{arctanh}(ax))}{16a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

output
```
(10*CoshIntegral[ArcTanh[a*x]] + 5*CoshIntegral[3*ArcTanh[a*x]] + CoshInte
gral[5*ArcTanh[a*x]])/(16*a)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx$$

$$\Big\downarrow \text{ 6530}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^5}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\int \left(\frac{5\cosh(3\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(5\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{5}{8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{5}{8}\operatorname{Chi}(\operatorname{arctanh}(ax))+\frac{5}{16}\operatorname{Chi}(3\operatorname{arctanh}(ax))+\frac{1}{16}\operatorname{Chi}(5\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

output
```
((5*CoshIntegral[ArcTanh[a*x]])/8 + (5*CoshIntegral[3*ArcTanh[a*x]])/16 +
CoshIntegral[5*ArcTanh[a*x]]/16)/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{10\,\mathrm{Chi}(\operatorname{arctanh}(ax))+5\,\mathrm{Chi}(3\operatorname{arctanh}(ax))+\mathrm{Chi}(5\operatorname{arctanh}(ax))}{16a}$	30

input
```
int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/16*(10*Chi(arctanh(a*x))+5*Chi(3*arctanh(a*x))+Chi(5*arctanh(a*x)))/a
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2
+ 1)*arctanh(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{7}{2}}\operatorname{atanh}(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x),x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)\,(1-a^2\,x^2)^{7/2}}\,dx$$

input
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(7/2)),x)
```

output
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)\,a^6x^6-3\sqrt{-a^2x^2+1}\,atanh(ax)\,a^4x^4+3\sqrt{-a^2x^2+1}\,atanh(ax)\,a^2x^2-\sqrt{-}\right.$$

input
```
int(1/(-a^2*x^2+1)^(7/2)/atanh(a*x),x)
```

output
```
 - int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**6*x**6 - 3*sqrt( - a**2*x**
2 + 1)*atanh(a*x)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**
2 - sqrt( - a**2*x**2 + 1)*atanh(a*x)),x)
```

3.484 $\int \frac{1}{(1-a^2x^2)^{9/2}\text{arctanh}(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)}\,dx=\frac{35\operatorname{Chi}(\operatorname{arctanh}(ax))}{64a}+\frac{21\operatorname{Chi}(3\operatorname{arctanh}(ax))}{64a}+\frac{7\operatorname{Chi}(5\operatorname{arctanh}(ax))}{64a}+\frac{\operatorname{Chi}(7\operatorname{arctanh}(ax))}{64a}$$

output
```
35/64*Chi(arctanh(a*x))/a+21/64*Chi(3*arctanh(a*x))/a+7/64*Chi(5*arctanh(a
*x))/a+1/64*Chi(7*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)}\,dx=-\frac{-35\operatorname{Chi}(\operatorname{arctanh}(ax))-21\operatorname{Chi}(3\operatorname{arctanh}(ax))-7\operatorname{Chi}(5\operatorname{arctanh}(ax))-\operatorname{Chi}(7\operatorname{arctanh}(ax))}{64a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

output
```
-1/64*(-35*CoshIntegral[ArcTanh[a*x]] - 21*CoshIntegral[3*ArcTanh[a*x]] -
7*CoshIntegral[5*ArcTanh[a*x]] - CoshIntegral[7*ArcTanh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6530, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6530}$$

$$\frac{\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^7}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{21\cosh(3\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)}+\frac{7\cosh(5\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)}+\frac{\cosh(7\operatorname{arctanh}(ax))}{64\operatorname{arctanh}(ax)}+\frac{35}{64\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{35}{64}\operatorname{Chi}(\operatorname{arctanh}(ax))+\frac{21}{64}\operatorname{Chi}(3\operatorname{arctanh}(ax))+\frac{7}{64}\operatorname{Chi}(5\operatorname{arctanh}(ax))+\frac{1}{64}\operatorname{Chi}(7\operatorname{arctanh}(ax))}{a}$$

input
```
Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

output
```
((35*CoshIntegral[ArcTanh[a*x]])/64 + (21*CoshIntegral[3*ArcTanh[a*x]])/64
 + (7*CoshIntegral[5*ArcTanh[a*x]])/64 + CoshIntegral[7*ArcTanh[a*x]]/64)/
a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{35\ \mathrm{Chi}(\operatorname{arctanh}(ax))+21\ \mathrm{Chi}(3\ \operatorname{arctanh}(ax))+7\ \mathrm{Chi}(5\ \operatorname{arctanh}(ax))+\mathrm{Chi}(7\ \operatorname{arctanh}(ax))}{64a}$	39

input
```
int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/64*(35*Chi(arctanh(a*x))+21*Chi(3*arctanh(a*x))+7*Chi(5*arctanh(a*x))+Ch
i(7*arctanh(a*x)))/a
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4
*x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{9}{2}}\,\text{atanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x),x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\,\text{artanh}\,(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\operatorname{artanh}(ax)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)\,(1-a^2\,x^2)^{9/2}}\,dx$$

input
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(9/2)),x)
```

output
```
int(1/(atanh(a*x)*(1 - a^2*x^2)^(9/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)\,a^8x^8-4\sqrt{-a^2x^2+1}\,atanh(ax)\,a^6x^6+6\sqrt{-}$$

input
```
int(1/(-a^2*x^2+1)^(9/2)/atanh(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**8*x**8 - 4*sqrt( - a**2*x**2 +
 1)*atanh(a*x)*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**4*x**4 -
 4*sqrt( - a**2*x**2 + 1)*atanh(a*x)*a**2*x**2 + sqrt( - a**2*x**2 + 1)*at
anh(a*x)),x)
```

3.485 $\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx=\operatorname{Int}\left(\frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx=\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx$$

input
```
Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
```

output
```
Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}}{\text{arctanh}(ax)^2}\,dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{\sqrt{1-a^2x^2}}{\text{arctanh}(ax)^2}dx$$

input
```
Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}\left(ax\right)^2}dx$$

input
```
int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

output
```
int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\operatorname{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}\left(a\,x\right)^2}\,dx$$

input
```
int((1 - a^2*x^2)^(1/2)/atanh(a*x)^2,x)
```

output
```
int((1 - a^2*x^2)^(1/2)/atanh(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{atanh\,(ax)^2}dx$$

input

```
int((-a^2*x^2+1)^(1/2)/atanh(a*x)^2,x)
```

output

```
int(sqrt( - a**2*x**2 + 1)/atanh(a*x)**2,x)
```

3.486 $\int \frac{1}{\sqrt{1-a^2x^2}\textbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx = \text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2},x\right)$$

output
```
Defer(Int)(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx = \int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
```

output
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2} \, dx$$
$$\Big\downarrow \, 6651$$
$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}dx$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-a^2x^2+1}\ \text{arctanh}\left(ax\right)^2}dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

output
```
int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2\left(ax\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx=\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}\,dx=\int \frac{1}{\text{atanh}\left(a\,x\right)^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^2}dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/atanh(a*x)^2,x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2),x)
```

3.487 $\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)} + \frac{\operatorname{Shi}(\operatorname{arctanh}(ax))}{a}$$

output
```
-1/a/(-a^2*x^2+1)^(1/2)/arctanh(a*x)+Shi(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \frac{-\frac{1}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)} + \operatorname{Shi}(\operatorname{arctanh}(ax))}{a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
(-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6528, 6596, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6528}$$

$$a\int \frac{x}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)}dx-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{\int \frac{ax}{\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}+\frac{\int -\frac{i\sin(i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}-\frac{i\int \frac{\sin(i\operatorname{arctanh}(ax))}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\operatorname{Shi}(\operatorname{arctanh}(ax))}{a}-\frac{1}{a\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.77

method	result	size
default	$\frac{\operatorname{arctanh}(ax)\operatorname{Shi}(\operatorname{arctanh}(ax))a^2x^2-\operatorname{Shi}(\operatorname{arctanh}(ax))\operatorname{arctanh}(ax)+\sqrt{-a^2x^2+1}}{a\operatorname{arctanh}(ax)(a^2x^2-1)}$	62

input
```
int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2-Shi(arctanh(a*x))*arctanh(a*x)
+(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx=\int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx=\int \frac{1}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx=\int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^2\,(1-a^2\,x^2)^{3/2}}\,dx$$

input
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^2}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)^2\,a^2x^2-\sqrt{-a^2x^2+1}\,atanh(ax)^2}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^(3/2)/atanh(a*x)^2,x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**2 - sqrt( - a**2*x**
*2 + 1)*atanh(a*x)**2),x)
```

3.488 $\int \frac{1}{(1-a^2x^2)^{5/2}\text{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}\,dx = -\frac{1}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)} + \frac{3\text{Shi}(\text{arctanh}(ax))}{4a} + \frac{3\text{Shi}(3\text{arctanh}(ax))}{4a}$$

output
```
-1/a/(-a^2*x^2+1)^(3/2)/arctanh(a*x)+3/4*Shi(arctanh(a*x))/a+3/4*Shi(3*arc
tanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}\,dx = \frac{-\frac{4}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)} + 3\text{Shi}(\text{arctanh}(ax)) + 3\text{Shi}(3\text{arctanh}(ax))}{4a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

output
```
(-4/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]) + 3*SinhIntegral[ArcTanh[a*x]] + 3*
SinhIntegral[3*ArcTanh[a*x]])/(4*a)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6528, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6528}$$

$$3a\int \frac{x}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)}dx-\frac{1}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{3\int \frac{ax}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{3\int \left(\frac{ax}{4\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\sinh(3\text{arctanh}(ax))}{4\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{1}{4}\text{Shi}(\text{arctanh}(ax))+\frac{1}{4}\text{Shi}(3\text{arctanh}(ax))\right)}{a}-\frac{1}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])) + (3*(SinhIntegral[ArcTanh[a*x]]
/4 + SinhIntegral[3*ArcTanh[a*x]]/4))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.

Time = 1.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result
default	$\frac{3 \operatorname{arctanh}(ax) \operatorname{Shi}(\operatorname{arctanh}(ax)) a^2x^2+3 \operatorname{arctanh}(ax) \operatorname{Shi}(3 \operatorname{arctanh}(ax)) a^2x^2-\cosh(3 \operatorname{arctanh}(ax)) a^2x^2-3 \operatorname{Shi}(\operatorname{arctanh}(ax)) \operatorname{arct}}{4a \operatorname{arctanh}(ax)(a^2x^2-1)}$

input
```
int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(3*arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2+3*arctanh(a*x)*Shi(3*arcta
nh(a*x))*a^2*x^2-cosh(3*arctanh(a*x))*a^2*x^2-3*Shi(arctanh(a*x))*arctanh(
a*x)-3*Shi(3*arctanh(a*x))*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctan
h(a*x)))/arctanh(a*x)/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctan
h(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{5}{2}}\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^2\,(1-a^2\,x^2)^{5/2}}\,dx$$

input
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(5/2)),x)
```

output
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\mathrm{arctanh}(ax)^2}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,a^4x^4 - 2\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,a^2x^2 + \sqrt{}$$

input
```
int(1/(-a^2*x^2+1)^(5/2)/atanh(a*x)^2,x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**4*x**4 - 2*sqrt( - a**2*x**
2 + 1)*atanh(a*x)**2*a**2*x**2 + sqrt( - a**2*x**2 + 1)*atanh(a*x)**2),x)
```

3.489 $\int \frac{1}{(1-a^2x^2)^{7/2}\textbf{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^2}\,dx = -\frac{1}{a(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)} + \frac{5\operatorname{Shi}(\operatorname{arctanh}(ax))}{8a} + \frac{15\operatorname{Shi}(3\operatorname{arctanh}(ax))}{16a} + \frac{5\operatorname{Shi}(5\operatorname{arctanh}(ax))}{16a}$$

output
```
-1/a/(-a^2*x^2+1)^(5/2)/arctanh(a*x)+5/8*Shi(arctanh(a*x))/a+15/16*Shi(3*a
rctanh(a*x))/a+5/16*Shi(5*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^2}\,dx = \frac{-\frac{16}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)} + 5(2\operatorname{Shi}(\operatorname{arctanh}(ax)) + 3\operatorname{Shi}(3\operatorname{arctanh}(ax)) -}{16a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

output
```
(-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]) + 5*(2*SinhIntegral[ArcTanh[a*x]]
+ 3*SinhIntegral[3*ArcTanh[a*x]] + SinhIntegral[5*ArcTanh[a*x]]))/(16*a)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6528, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6528}$$

$$5a\int \frac{x}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)}dx-\frac{1}{a(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{5\int \frac{ax}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{5\int \left(\frac{ax}{8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}+\frac{3\sinh(3\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\sinh(5\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a}-\frac{1}{a(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{1}{8}\text{Shi}(\operatorname{arctanh}(ax))+\frac{3}{16}\text{Shi}(3\operatorname{arctanh}(ax))+\frac{1}{16}\text{Shi}(5\operatorname{arctanh}(ax))\right)}{a}-\frac{1}{a(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])) + (5*(SinhIntegral[ArcTanh[a*x]]
/8 + (3*SinhIntegral[3*ArcTanh[a*x]])/16 + SinhIntegral[5*ArcTanh[a*x]]/16
))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(58) = 116$.

Time = 1.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.67

method	result
default	$\frac{10 \operatorname{arctanh}(ax) \operatorname{Shi}(\operatorname{arctanh}(ax)) a^2x^2+15 \operatorname{arctanh}(ax) \operatorname{Shi}(3 \operatorname{arctanh}(ax)) a^2x^2+5 \operatorname{arctanh}(ax) \operatorname{Shi}(5 \operatorname{arctanh}(ax)) a^2x^2-5 \cosh(3}{}$

input
```
int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a*(10*arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2+15*arctanh(a*x)*Shi(3*ar
ctanh(a*x))*a^2*x^2+5*arctanh(a*x)*Shi(5*arctanh(a*x))*a^2*x^2-5*cosh(3*ar
ctanh(a*x))*a^2*x^2-cosh(5*arctanh(a*x))*a^2*x^2-10*Shi(arctanh(a*x))*arct
anh(a*x)-15*Shi(3*arctanh(a*x))*arctanh(a*x)-5*Shi(5*arctanh(a*x))*arctanh
(a*x)+10*(-a^2*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x))+cosh(5*arctanh(a*x)))/a
rctanh(a*x)/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2
+ 1)*arctanh(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{7}{2}}\operatorname{atanh}^2(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\,\text{artanh}\,(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}\,dx = \int \frac{1}{\text{atanh}\,(a\,x)^2\,(1-a^2\,x^2)^{7/2}}\,dx$$

input
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(7/2)),x)
```

output
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,a^6x^6-3\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,a^4x^4+3\sqrt{-a^2x^2+1}\,atanh\,(ax)^2\,a^2x^2-\right.$$

input
```
int(1/(-a^2*x^2+1)^(7/2)/atanh(a*x)^2,x)
```

output
```
 - int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**6*x**6 - 3*sqrt( - a**2*
x**2 + 1)*atanh(a*x)**2*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*atanh(a*x)**2
*a**2*x**2 - sqrt( - a**2*x**2 + 1)*atanh(a*x)**2),x)
```

3.490 $\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = -\frac{1}{a(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)} + \frac{35\operatorname{Shi}(\operatorname{arctanh}(ax))}{64a} + \frac{63\operatorname{Shi}(3\operatorname{arctanh}(ax))}{64a} + \frac{35\operatorname{Shi}(5\operatorname{arctanh}(ax))}{64a} + \frac{7\operatorname{Shi}(7\operatorname{arctanh}(ax))}{64a}$$

output
```
-1/a/(-a^2*x^2+1)^(7/2)/arctanh(a*x)+35/64*Shi(arctanh(a*x))/a+63/64*Shi(3
*arctanh(a*x))/a+35/64*Shi(5*arctanh(a*x))/a+7/64*Shi(7*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = \frac{-\frac{64}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)} + 35\operatorname{Shi}(\operatorname{arctanh}(ax)) + 63\operatorname{Shi}(3\operatorname{arctanh}(ax)) +}{64a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

output
```
(-64/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]) + 35*SinhIntegral[ArcTanh[a*x]] +
63*SinhIntegral[3*ArcTanh[a*x]] + 35*SinhIntegral[5*ArcTanh[a*x]] + 7*Sinh
Integral[7*ArcTanh[a*x]])/(64*a)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6528, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)^2}dx$$

$$\Big\downarrow \text{6528}$$

$$7a\int \frac{x}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}dx-\frac{1}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{6596}$$

$$\frac{7\int \frac{ax}{(1-a^2x^2)^{7/2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a}-\frac{1}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{5971}$$

$$\frac{7\int \left(\frac{5ax}{64\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{9\sinh(3\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{5\sinh(5\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{\sinh(7\text{arctanh}(ax))}{64\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a}-\frac{1}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7\left(\frac{5}{64}\text{Shi}(\text{arctanh}(ax))+\frac{9}{64}\text{Shi}(3\text{arctanh}(ax))+\frac{5}{64}\text{Shi}(5\text{arctanh}(ax))+\frac{1}{64}\text{Shi}(7\text{arctanh}(ax))\right)}{a}-\frac{1}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}$$

input
```
Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

output
```
-(1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x])) + (7*((5*SinhIntegral[ArcTanh[a*
x]])/64 + (9*SinhIntegral[3*ArcTanh[a*x]])/64 + (5*SinhIntegral[5*ArcTanh[
a*x]])/64 + SinhIntegral[7*ArcTanh[a*x]]/64))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(70) = 140.

Time = 1.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.90

method	result
default	$\frac{35 \operatorname{arctanh}(ax) \operatorname{Shi}(\operatorname{arctanh}(ax))a^2x^2+63 \operatorname{arctanh}(ax) \operatorname{Shi}(3 \operatorname{arctanh}(ax))a^2x^2+35 \operatorname{arctanh}(ax) \operatorname{Shi}(5 \operatorname{arctanh}(ax))a^2x^2+7 \operatorname{arcta}}{}$

input
```
int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/a*(35*arctanh(a*x)*Shi(arctanh(a*x))*a^2*x^2+63*arctanh(a*x)*Shi(3*ar
ctanh(a*x))*a^2*x^2+35*arctanh(a*x)*Shi(5*arctanh(a*x))*a^2*x^2+7*arctanh(
a*x)*Shi(7*arctanh(a*x))*a^2*x^2-21*cosh(3*arctanh(a*x))*a^2*x^2-7*cosh(5*
arctanh(a*x))*a^2*x^2-cosh(7*arctanh(a*x))*a^2*x^2-35*Shi(arctanh(a*x))*ar
ctanh(a*x)-63*Shi(3*arctanh(a*x))*arctanh(a*x)-35*Shi(5*arctanh(a*x))*arct
anh(a*x)-7*Shi(7*arctanh(a*x))*arctanh(a*x)+35*(-a^2*x^2+1)^(1/2)+21*cosh(
3*arctanh(a*x))+7*cosh(5*arctanh(a*x))+cosh(7*arctanh(a*x)))/arctanh(a*x)/
(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx=\int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\operatorname{artanh}(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4
*x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**2,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\operatorname{artanh}\left(ax\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{1}{\operatorname{atanh}\left(a\,x\right)^2\,{\left(1-a^2\,x^2\right)}^{9/2}}\,dx$$

input
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(9/2)),x)
```

output
```
int(1/(atanh(a*x)^2*(1 - a^2*x^2)^(9/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^2}\,dx = \int \frac{}{\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^2 a^8x^8-4\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^2 a^6x^6+6}$$

input
```
int(1/(-a^2*x^2+1)^(9/2)/atanh(a*x)^2,x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**8*x**8 - 4*sqrt( - a**2*x**
2 + 1)*atanh(a*x)**2*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a*
*4*x**4 - 4*sqrt( - a**2*x**2 + 1)*atanh(a*x)**2*a**2*x**2 + sqrt( - a**2*
x**2 + 1)*atanh(a*x)**2),x)
```

3.491 $\int \frac{\sqrt{1-a^2x^2}}{\textbf{arctanh}(ax)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3} \, dx = \operatorname{Int}\left(\frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}, x\right)$$

output
```
Defer(Int)((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3} \, dx = \int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3} \, dx$$

input
```
Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
```

output
```
Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3} \, dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}dx$$

input
```
Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{arctanh}\left(ax\right)^3}dx$$

input
```
int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

output
```
int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}\,dx = \int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\operatorname{atanh}^3\left(ax\right)}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
```

output
```
Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}\,dx = \int \frac{\sqrt{1-a^2\,x^2}}{\operatorname{atanh}\left(a\,x\right)^3}\,dx$$

input
```
int((1 - a^2*x^2)^(1/2)/atanh(a*x)^3,x)
```

output
```
int((1 - a^2*x^2)^(1/2)/atanh(a*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1-a^2x^2}}{\operatorname{arctanh}(ax)^3}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{atanh\left(ax\right)^3}dx$$

input
```
int((-a^2*x^2+1)^(1/2)/atanh(a*x)^3,x)
```

output
```
int(sqrt( - a**2*x**2 + 1)/atanh(a*x)**3,x)
```

3.492 $\int \frac{1}{\sqrt{1-a^2x^2}\,\textbf{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)^3}\,dx = \text{Int}\left(\frac{1}{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)^3},x\right)$$

output
```
Defer(Int)(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)^3}\,dx = \int \frac{1}{\sqrt{1-a^2x^2}\,\text{arctanh}(ax)^3}\,dx$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
```

output
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3} \, dx$$
$$\Big\downarrow \text{ 6651}$$
$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}dx$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{arctanh}\left(ax\right)^3}dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

output
```
int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}\,dx=\int \frac{1}{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3}\,dx=\int \frac{1}{\text{atanh}\,(a\,x)^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^3} \, dx = \int \frac{1}{\sqrt{-a^2x^2+1}\, atanh\,(ax)^3} dx$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/atanh(a*x)^3,x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3),x)
```

3.493 $\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2} - \frac{x}{2\sqrt{1-a^2x^2}\text{arctanh}(ax)} + \frac{\text{Chi}(\text{arctanh}(ax))}{2a}$$

output
```
-1/2/a/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2-1/2*x/(-a^2*x^2+1)^(1/2)/arctanh(
a*x)+1/2*Chi(arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^3}\,dx = \frac{-\frac{1+ax\text{arctanh}(ax)}{\sqrt{1-a^2x^2}\text{arctanh}(ax)^2} + \text{Chi}(\text{arctanh}(ax))}{2a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
(-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegr
al[ArcTanh[a*x]])/(2*a)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6528, 6568, 6530, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \ 6528$$

$$\frac{1}{2}a\int \frac{x}{(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}dx-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \ 6568$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}dx}{a}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \ 6530$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}+\frac{1}{2}a\left(-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}\right)$$

$$\Big\downarrow \ 3782$$

$$\frac{1}{2}a\left(\frac{\text{Chi}(\text{arctanh}(ax))}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)-\frac{1}{2a\sqrt{1-a^2x^2}\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) + (a*(-(x/(a*Sqrt[1 - a^2*x^2]
*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6568
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Arc
Tanh[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f
*x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && Lt
Q[p, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\operatorname{arctanh}(ax)^{2} \operatorname{Chi}(\operatorname{arctanh}(ax)) a^{2} x^{2}+x \operatorname{arctanh}(ax) a \sqrt{-a^{2} x^{2}+1}-\operatorname{Chi}(\operatorname{arctanh}(ax)) \operatorname{arctanh}(ax)^{2}+\sqrt{-a^{2} x^{2}+1}}{2 a \operatorname{arctanh}(ax)^{2} (a^{2} x^{2}-1)}$	86

input
```
int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+x*arctanh(a*x)*a*(-a^2*x^2
+1)^(1/2)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x
)^2/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^3\left(ax\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx=\int \frac{1}{\operatorname{atanh}\left(a\,x\right)^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)),x)
```

output
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{3/2}\operatorname{arctanh}(ax)^3}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3 a^2x^2-\sqrt{-a^2x^2+1}\,atanh\left(ax\right)^3}dx\right)$$

input
```
int(1/(-a^2*x^2+1)^(3/2)/atanh(a*x)^3,x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**2 - sqrt( - a**2*x*
*2 + 1)*atanh(a*x)**3),x)
```

3.494 $\int \frac{1}{(1-a^2x^2)^{5/2}\text{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^3}\,dx = -\frac{1}{2a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2} - \frac{3x}{2\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)} + \frac{3\text{Chi}(\text{arctanh}(ax))}{8a} + \frac{9\text{Chi}(3\text{arctanh}(ax))}{8a}$$

output
```
-1/2/a/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2-3/2*x/(-a^2*x^2+1)^(3/2)/arctanh(
a*x)+3/8*Chi(arctanh(a*x))/a+9/8*Chi(3*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^3}\,dx = \frac{-\frac{4(1+3ax\text{arctanh}(ax))}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)^2} + 3\text{Chi}(\text{arctanh}(ax)) + 9\text{Chi}(3\text{arctanh}(ax))}{8a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

output
```
((-4*(1 + 3*a*x*ArcTanh[a*x]))/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) + 3*Co
shIntegral[ArcTanh[a*x]] + 9*CoshIntegral[3*ArcTanh[a*x]])/(8*a)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6528, 6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \text{6528}$$

$$\frac{3}{2}a\int \frac{x}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}dx-\frac{1}{2a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$\frac{3}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^{5/2}\text{arctanh}(ax)}dx}{a}+2a\int \frac{x^2}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)}dx-\frac{x}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\right)-\frac{1}{2a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6530}$$

$$\frac{3}{2}a\left(2a\int \frac{x^2}{(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^{3/2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)}\right)-\frac{1}{2a\,(1-a^2x^2)^{3/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}+$$

$$\frac{3}{2}a\left(2a\int\frac{x^2}{\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)}dx+\frac{\int\frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^3}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}\right)$$

$\downarrow$ 3793

$$\frac{3}{2}a\left(2a\int\frac{x^2}{\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)}dx+\frac{\int\left(\frac{\cosh(3\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}+\frac{3}{4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}-\frac{}{a(1-}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$\frac{3}{2}a\left(2a\int\frac{x^2}{\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)}dx+\frac{\frac{3}{4}\operatorname{Chi}(\operatorname{arctanh}(ax))+\frac{1}{4}\operatorname{Chi}(3\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 6596

$$\frac{3}{2}a\left(\frac{2\int\frac{a^2x^2}{\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{3}{4}\operatorname{Chi}(\operatorname{arctanh}(ax))+\frac{1}{4}\operatorname{Chi}(3\operatorname{arctanh}(ax))}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^{3/2}\operatorname{arc}}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 5971

$$\frac{3}{2}a\left(\frac{2\int\left(\frac{\cosh(3\operatorname{arctanh}(ax))}{4\operatorname{arctanh}(ax)}-\frac{1}{4\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{3}{4}\operatorname{Chi}(\operatorname{arctanh}(ax))+\frac{1}{4}\operatorname{Chi}(3\operatorname{arctanh}(ax))}{a^2}\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\operatorname{arctanh}(ax)^2}$$

$\downarrow$ 2009

$$\frac{3}{2}a\left(\frac{2\left(\frac{1}{4}\text{Chi}(3\text{arctanh}(ax))-\frac{1}{4}\text{Chi}(\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{3}{4}\text{Chi}(\text{arctanh}(ax))+\frac{1}{4}\text{Chi}(3\text{arctanh}(ax))}{a^2}-\frac{}{a\left(1-a^2x^2\right)}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) + (3*a*(-(x/(a*(1 - a^2*x^2)
^(3/2)*ArcTanh[a*x])) + (2*(-1/4*CoshIntegral[ArcTanh[a*x]] + CoshIntegral
[3*ArcTanh[a*x]]/4))/a^2 + ((3*CoshIntegral[ArcTanh[a*x]])/4 + CoshIntegra
l[3*ArcTanh[a*x]]/4)/a^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(67) = 134$.

Time = 1.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

method	result
default	$\frac{3 \operatorname{arctanh}(ax)^2 \operatorname{Chi}(\operatorname{arctanh}(ax)) a^2 x^2 + 9 \operatorname{arctanh}(ax)^2 \operatorname{Chi}(3 \operatorname{arctanh}(ax)) a^2 x^2 - 3 \operatorname{arctanh}(ax) \sinh(3 \operatorname{arctanh}(ax)) a^2 x^2 - \cosh(3}{}$

input
```
int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(3*arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+9*arctanh(a*x)^2*Chi(3*a
rctanh(a*x))*a^2*x^2-3*arctanh(a*x)*sinh(3*arctanh(a*x))*a^2*x^2-cosh(3*ar
ctanh(a*x))*a^2*x^2+3*x*arctanh(a*x)*a*(-a^2*x^2+1)^(1/2)-3*Chi(arctanh(a*
x))*arctanh(a*x)^2-9*Chi(3*arctanh(a*x))*arctanh(a*x)^2+3*sinh(3*arctanh(a
*x))*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a*x)))/arctanh(a*x)^
2/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctan
h(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{5}{2}}\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{5}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^3\,(1-a^2\,x^2)^{5/2}}\,dx$$

input
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(5/2)),x)
```

output
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{5/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^3\,a^4x^4 - 2\sqrt{-a^2x^2+1}\,atanh\,(ax)^3\,a^2x^2 + \sqrt{}}$$

input
```
int(1/(-a^2*x^2+1)^(5/2)/atanh(a*x)^3,x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**4*x**4 - 2*sqrt( - a**2*x**
2 + 1)*atanh(a*x)**3*a**2*x**2 + sqrt( - a**2*x**2 + 1)*atanh(a*x)**3),x)
```

3.495 $\int \frac{1}{(1-a^2x^2)^{7/2}\text{arctanh}(ax)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^3}\,dx = -\frac{1}{2a(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2} - \frac{5x}{2(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)} + \frac{5\text{Chi}(\text{arctanh}(ax))}{16a} + \frac{45\text{Chi}(3\text{arctanh}(ax))}{32a} + \frac{25\text{Chi}(5\text{arctanh}(ax))}{32a}$$

output
```
-1/2/a/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2-5/2*x/(-a^2*x^2+1)^(5/2)/arctanh(
a*x)+5/16*Chi(arctanh(a*x))/a+45/32*Chi(3*arctanh(a*x))/a+25/32*Chi(5*arct
anh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^3}\,dx = \frac{-\frac{16}{(1-a^2x^2)^{5/2}\text{arctanh}(ax)^2} - \frac{80ax}{(1-a^2x^2)^{5/2}\text{arctanh}(ax)} + 10\text{Chi}(\text{arctanh}(ax)) }{32a}$$

input
```
Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

output
```
(-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (80*a*x)/((1 - a^2*x^2)^(5/2)*
ArcTanh[a*x]) + 10*CoshIntegral[ArcTanh[a*x]] + 45*CoshIntegral[3*ArcTanh[
a*x]] + 25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.43, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6528, 6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^3}dx$$

$$\Big\downarrow \text{ 6528}$$

$$\frac{5}{2}a\int \frac{x}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}dx-\frac{1}{2a\,(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 6594}$$

$$\frac{5}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^{7/2}\text{arctanh}(ax)}dx}{a}+4a\int \frac{x^2}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}dx-\frac{x}{a\,(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)}\right)-\frac{1}{2a\,(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 6530}$$

$$\frac{5}{2}a\left(4a\int \frac{x^2}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^{5/2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)}\right)-\frac{1}{2a\,(1-a^2x^2)^{5/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)^2}+\frac{5}{2}a\left(4a\int\frac{x^2}{\left(1-a^2x^2\right)^{7/2}\operatorname{arctanh}(ax)}dx+\frac{\int\frac{\sin\left(i\operatorname{arctanh}(ax)+\frac{\pi}{2}\right)^5}{\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}-\frac{x}{a\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)}\right)$$

$$\Big\downarrow \text{3793}$$

$$\frac{5}{2}a\left(4a\int\frac{x^2}{\left(1-a^2x^2\right)^{7/2}\operatorname{arctanh}(ax)}dx+\frac{\int\left(\frac{5\cosh(3\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(5\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{5}{8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)da}{a^2}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{2}a\left(4a\int\frac{x^2}{\left(1-a^2x^2\right)^{7/2}\operatorname{arctanh}(ax)}dx+\frac{\frac{5}{8}\text{Chi}(\operatorname{arctanh}(ax))+\frac{5}{16}\text{Chi}(3\operatorname{arctanh}(ax))+\frac{1}{16}\text{Chi}(5\operatorname{arctanh}(ax))}{a^2}-\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6596}$$

$$\frac{5}{2}a\left(\frac{4\int\frac{a^2x^2}{\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)}d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{5}{8}\text{Chi}(\operatorname{arctanh}(ax))+\frac{5}{16}\text{Chi}(3\operatorname{arctanh}(ax))+\frac{1}{16}\text{Chi}(5\operatorname{arctanh}(a\right.}{a^2}$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{5}{2}a\left(\frac{4\int\left(\frac{\cosh(3\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}+\frac{\cosh(5\operatorname{arctanh}(ax))}{16\operatorname{arctanh}(ax)}-\frac{1}{8\sqrt{1-a^2x^2}\operatorname{arctanh}(ax)}\right)d\operatorname{arctanh}(ax)}{a^2}+\frac{\frac{5}{8}\text{Chi}(\operatorname{arctanh}(ax))+}{}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\operatorname{arctanh}(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{2}a\left(\frac{4\left(-\frac{1}{8}\text{Chi}(\text{arctanh}(ax))+\frac{1}{16}\text{Chi}(3\text{arctanh}(ax))+\frac{1}{16}\text{Chi}(5\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{5}{8}\text{Chi}(\text{arctanh}(ax))+\frac{5}{16}\text{Chi}(3a\right.$$

$$\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\text{arctanh}(ax)^2}$$

input

```
Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

output

```
-1/2*1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) + (5*a*(-(x/(a*(1 - a^2*x^2)
^(5/2)*ArcTanh[a*x])) + (4*(-1/8*CoshIntegral[ArcTanh[a*x]] + CoshIntegral
[3*ArcTanh[a*x]]/16 + CoshIntegral[5*ArcTanh[a*x]]/16))/a^2 + ((5*CoshInte
gral[ArcTanh[a*x]])/8 + (5*CoshIntegral[3*ArcTanh[a*x]])/16 + CoshIntegral
[5*ArcTanh[a*x]]/16)/a^2))/2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(79) = 158$.

Time = 1.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.92

method	result
default	$\frac{10 \operatorname{arctanh}(ax)^2 \operatorname{Chi}(\operatorname{arctanh}(ax)) a^2 x^2 + 45 \operatorname{arctanh}(ax)^2 \operatorname{Chi}(3 \operatorname{arctanh}(ax)) a^2 x^2 + 25 \operatorname{arctanh}(ax)^2 \operatorname{Chi}(5 \operatorname{arctanh}(ax)) a^2 x^2 - 15}{}$

input
```
int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/a*(10*arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+45*arctanh(a*x)^2*Chi(
3*arctanh(a*x))*a^2*x^2+25*arctanh(a*x)^2*Chi(5*arctanh(a*x))*a^2*x^2-15*a
rctanh(a*x)*sinh(3*arctanh(a*x))*a^2*x^2-5*arctanh(a*x)*sinh(5*arctanh(a*x
))*a^2*x^2-5*cosh(3*arctanh(a*x))*a^2*x^2-cosh(5*arctanh(a*x))*a^2*x^2+10*
x*arctanh(a*x)*a*(-a^2*x^2+1)^(1/2)-10*Chi(arctanh(a*x))*arctanh(a*x)^2-45
*Chi(3*arctanh(a*x))*arctanh(a*x)^2-25*Chi(5*arctanh(a*x))*arctanh(a*x)^2+
15*sinh(3*arctanh(a*x))*arctanh(a*x)+5*sinh(5*arctanh(a*x))*arctanh(a*x)+1
0*(-a^2*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x))+cosh(5*arctanh(a*x)))/arctanh(
a*x)^2/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2
+ 1)*arctanh(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{7}{2}}\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(7/2)*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{7}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^3\,(1-a^2\,x^2)^{7/2}}\,dx$$

input
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(7/2)),x)
```

output
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^3}\,dx = -\left(\int \frac{1}{\sqrt{-a^2x^2+1}\,atanh\,(ax)^3\,a^6x^6-3\sqrt{-a^2x^2+1}\,atanh\,(ax)^3\,a^4x^4+3\sqrt{-a^2x^2+1}\,atanh\,(ax)^3\,a^2x^2-$$

input
```
int(1/(-a^2*x^2+1)^(7/2)/atanh(a*x)^3,x)
```

output
```
- int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**6*x**6 - 3*sqrt( - a**2*
x**2 + 1)*atanh(a*x)**3*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*atanh(a*x)**3
*a**2*x**2 - sqrt( - a**2*x**2 + 1)*atanh(a*x)**3),x)
```

3.496 $\int \frac{1}{(1-a^2x^2)^{9/2}\textbf{arctanh}(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = -\frac{1}{2a(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)^2} - \frac{7x}{2(1-a^2x^2)^{7/2}\operatorname{arctanh}(ax)} + \frac{35\operatorname{Chi}(\operatorname{arctanh}(ax))}{128a} + \frac{189\operatorname{Chi}(3\operatorname{arctanh}(ax))}{128a} + \frac{175\operatorname{Chi}(5\operatorname{arctanh}(ax))}{128a} + \frac{49\operatorname{Chi}(7\operatorname{arctanh}(ax))}{128a}$$

output
```
-1/2/a/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2-7/2*x/(-a^2*x^2+1)^(7/2)/arctanh(
a*x)+35/128*Chi(arctanh(a*x))/a+189/128*Chi(3*arctanh(a*x))/a+175/128*Chi(
5*arctanh(a*x))/a+49/128*Chi(7*arctanh(a*x))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{1}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)^3}\,dx = \frac{1}{128}\left(-\frac{64}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2} - \frac{448x}{(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)} + \frac{35\text{Chi}(\text{arctanh}(ax))}{a} + \frac{189\text{Chi}(3\text{arctanh}(ax))}{a} + \frac{175\text{Chi}(5\text{arctanh}(ax))}{a} + \frac{49\text{Chi}(7\text{arctanh}(ax))}{a}\right)$$

input
```
Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

output
```
(-64/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (448*x)/((1 - a^2*x^2)^(7/2)
*ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/a + (189*CoshIntegral[3*A
rcTanh[a*x]])/a + (175*CoshIntegral[5*ArcTanh[a*x]])/a + (49*CoshIntegral[
7*ArcTanh[a*x]])/a)/128
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6528, 6594, 6530, 3042, 3793, 2009, 6596, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)^3}\,dx$$

$$\Big\downarrow \text{6528}$$

$$\frac{7}{2}a\int \frac{x}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)^2}dx - \frac{1}{2a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{6594}$$

$$\frac{7}{2}a\left(\frac{\int \frac{1}{(1-a^2x^2)^{9/2}\text{arctanh}(ax)}dx}{a}+6a\int \frac{x^2}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}dx-\frac{x}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}\right)-\frac{1}{2a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6530

$$\frac{7}{2}a\left(6a\int \frac{x^2}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}dx+\frac{\int \frac{1}{(1-a^2x^2)^{7/2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}\right)-\frac{1}{2a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 3042

$$-\frac{1}{2a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}+\frac{7}{2}a\left(6a\int \frac{x^2}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}dx+\frac{\int \frac{\sin\left(i\text{arctanh}(ax)+\frac{\pi}{2}\right)^7}{\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}-\frac{x}{a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)}\right)$$

$\Big\downarrow$ 3793

$$\frac{7}{2}a\left(6a\int \frac{x^2}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}dx+\frac{\int \left(\frac{21\cosh(3\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{7\cosh(5\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{\cosh(7\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{}{6}\right.}{a^2}\right.$$
$$\frac{1}{2a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 2009

$$\frac{7}{2}a\left(6a\int \frac{x^2}{(1-a^2x^2)^{9/2}\,\text{arctanh}(ax)}dx+\frac{\frac{35}{64}\text{Chi}(\text{arctanh}(ax))+\frac{21}{64}\text{Chi}(3\text{arctanh}(ax))+\frac{7}{64}\text{Chi}(5\text{arctanh}(ax))}{a^2}\right.$$
$$\frac{1}{2a\,(1-a^2x^2)^{7/2}\,\text{arctanh}(ax)^2}$$

$\Big\downarrow$ 6596

$$\frac{7}{2}a\left(\frac{6\int \frac{a^2x^2}{(1-a^2x^2)^{7/2}\text{arctanh}(ax)}d\text{arctanh}(ax)}{a^2}+\frac{\frac{35}{64}\text{Chi}(\text{arctanh}(ax))+\frac{21}{64}\text{Chi}(3\text{arctanh}(ax))+\frac{7}{64}\text{Chi}(5\text{arctanh}(a\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{5971}$$

$$\frac{7}{2}a\left(\frac{6\int \left(\frac{\cosh(3\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{3\cosh(5\text{arctanh}(ax))}{64\text{arctanh}(ax)}+\frac{\cosh(7\text{arctanh}(ax))}{64\text{arctanh}(ax)}-\frac{5}{64\sqrt{1-a^2x^2}\text{arctanh}(ax)}\right)d\text{arctanh}(ax)}{a^2}+\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\text{arctanh}(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7}{2}a\left(\frac{6\left(-\frac{5}{64}\text{Chi}(\text{arctanh}(ax))+\frac{1}{64}\text{Chi}(3\text{arctanh}(ax))+\frac{3}{64}\text{Chi}(5\text{arctanh}(ax))+\frac{1}{64}\text{Chi}(7\text{arctanh}(ax))\right)}{a^2}+\frac{\frac{35}{64}\text{Chi}\right.$$
$$\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\text{arctanh}(ax)^2}$$

input
```
Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

output
```
-1/2*1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) + (7*a*(-(x/(a*(1 - a^2*x^2)
^(7/2)*ArcTanh[a*x])) + (6*((-5*CoshIntegral[ArcTanh[a*x]])/64 + CoshInteg
ral[3*ArcTanh[a*x]]/64 + (3*CoshIntegral[5*ArcTanh[a*x]])/64 + CoshIntegra
l[7*ArcTanh[a*x]]/64))/a^2 + ((35*CoshIntegral[ArcTanh[a*x]])/64 + (21*Cos
hIntegral[3*ArcTanh[a*x]])/64 + (7*CoshIntegral[5*ArcTanh[a*x]])/64 + Cosh
Integral[7*ArcTanh[a*x]]/64)/a^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6528
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p
 + 1))), x] + Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*A
rcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && LtQ[q, -1] && LtQ[p, -1]
```

rule 6530
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x,
ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && I
LtQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 6594
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])^(
p + 1)/(b*c*d*(p + 1))), x] + (Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p
+ 1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -
1] && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 6596
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sinh[x]^
m/Cosh[x]^(m + 2*(q + 1))), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (In
tegerQ[q] || GtQ[d, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(91) = 182.

Time = 1.46 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.40

method	result
default	$\frac{35\operatorname{arctanh}(ax)^2\operatorname{Chi}(\operatorname{arctanh}(ax))a^2x^2+189\operatorname{arctanh}(ax)^2\operatorname{Chi}(3\operatorname{arctanh}(ax))a^2x^2+175\operatorname{arctanh}(ax)^2\operatorname{Chi}(5\operatorname{arctanh}(ax))a^2x^2+4}{}$

input
```
int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128/a*(35*arctanh(a*x)^2*Chi(arctanh(a*x))*a^2*x^2+189*arctanh(a*x)^2*Ch
i(3*arctanh(a*x))*a^2*x^2+175*arctanh(a*x)^2*Chi(5*arctanh(a*x))*a^2*x^2+4
9*arctanh(a*x)^2*Chi(7*arctanh(a*x))*a^2*x^2-63*arctanh(a*x)*sinh(3*arctan
h(a*x))*a^2*x^2-35*arctanh(a*x)*sinh(5*arctanh(a*x))*a^2*x^2-7*arctanh(a*x
)*sinh(7*arctanh(a*x))*a^2*x^2-21*cosh(3*arctanh(a*x))*a^2*x^2-7*cosh(5*ar
ctanh(a*x))*a^2*x^2-cosh(7*arctanh(a*x))*a^2*x^2+35*x*arctanh(a*x)*a*(-a^2
*x^2+1)^(1/2)-35*Chi(arctanh(a*x))*arctanh(a*x)^2-189*Chi(3*arctanh(a*x))*
arctanh(a*x)^2-175*Chi(5*arctanh(a*x))*arctanh(a*x)^2-49*Chi(7*arctanh(a*x
))*arctanh(a*x)^2+63*sinh(3*arctanh(a*x))*arctanh(a*x)+35*sinh(5*arctanh(a
*x))*arctanh(a*x)+7*sinh(7*arctanh(a*x))*arctanh(a*x)+35*(-a^2*x^2+1)^(1/2
)+21*cosh(3*arctanh(a*x))+7*cosh(5*arctanh(a*x))+cosh(7*arctanh(a*x)))/arc
tanh(a*x)^2/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4
*x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-(ax-1)(ax+1))^{\frac{9}{2}}\operatorname{atanh}^3(ax)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**3,x)
```

output
```
Integral(1/((-(a*x - 1)*(a*x + 1))**(9/2)*atanh(a*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

Giac [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{(-a^2x^2+1)^{\frac{9}{2}}\operatorname{artanh}(ax)^3}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\operatorname{atanh}(a\,x)^3\,(1-a^2\,x^2)^{9/2}}\,dx$$

input
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(9/2)),x)
```

output
```
int(1/(atanh(a*x)^3*(1 - a^2*x^2)^(9/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-a^2x^2)^{9/2}\operatorname{arctanh}(ax)^3}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,atanh(ax)^3\,a^8x^8-4\sqrt{-a^2x^2+1}\,atanh(ax)^3\,a^6x^6+6}$$

input
```
int(1/(-a^2*x^2+1)^(9/2)/atanh(a*x)^3,x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**8*x**8 - 4*sqrt( - a**2*x**
2 + 1)*atanh(a*x)**3*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a*
*4*x**4 - 4*sqrt( - a**2*x**2 + 1)*atanh(a*x)**3*a**2*x**2 + sqrt( - a**2*
x**2 + 1)*atanh(a*x)**3),x)
```

3.497 $\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 122

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx = \frac{d(a+b\text{arctanh}(cx))^3}{3bc} - \frac{e(a+b\text{arctanh}(cx))^3}{3bc^2} + \frac{e(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{c^2} + \frac{be(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^2} - \frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^2}$$

output
```
1/3*d*(a+b*arctanh(c*x))^3/b/c-1/3*e*(a+b*arctanh(c*x))^3/b/c^2+e*(a+b*arc
tanh(c*x))^2*ln(2/(-c*x+1))/c^2+b*e*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x
+1))/c^2-1/2*b^2*e*polylog(3,1-2/(-c*x+1))/c^2
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx$$
$$= \frac{6abcd\text{arctanh}(cx)^2+6abe\text{arctanh}(cx)^2+2b^2cd\text{arctanh}(cx)^3+2b^2e\text{arctanh}(cx)^3+12abe\text{arctanh}(cx)\log}{}$$

input
```
Integrate[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

output
```
(6*a*b*c*d*ArcTanh[c*x]^2 + 6*a*b*e*ArcTanh[c*x]^2 + 2*b^2*c*d*ArcTanh[c*x
]^3 + 2*b^2*e*ArcTanh[c*x]^3 + 12*a*b*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh
[c*x])] + 6*b^2*e*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 3*a^2*c*d*
Log[1 - c*x] - 3*a^2*e*Log[1 - c*x] + 3*a^2*c*d*Log[1 + c*x] - 3*a^2*e*Log
[1 + c*x] - 6*b*e*(a + b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] -
3*b^2*e*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(6*c^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx$$
$$\Big\downarrow \text{6610}$$
$$\int \left(\frac{d(a+b\text{arctanh}(cx))^2}{1-c^2x^2}+\frac{ex(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{be\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{c^2}-\frac{e(a+b\operatorname{arctanh}(cx))^3}{3bc^2}+\frac{e\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))^2}{c^2}+\frac{d(a+b\operatorname{arctanh}(cx))^3}{3bc}-\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^2}$$

input
```
Int[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

output
```
(d*(a + b*ArcTanh[c*x])^3)/(3*b*c) - (e*(a + b*ArcTanh[c*x])^3)/(3*b*c^2)
+ (e*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^2 + (b*e*(a + b*ArcTanh[c*
x])*PolyLog[2, 1 - 2/(1 - c*x)])/c^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])
/(2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6610
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.46 (sec) , antiderivative size = 1435, normalized size of antiderivative = 11.76

method	result	size
derivativedivides	Expression too large to display	1435
default	Expression too large to display	1435
parts	Expression too large to display	1451

input
```
int((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a^2/c*(1/2*(c*d+e)*ln(c*x-1)-1/2*(c*d-e)*ln(c*x+1))-b^2/c*(1/2*arcta
nh(c*x)^2*ln(c*x-1)*c*d+1/2*arctanh(c*x)^2*ln(c*x-1)*e-1/2*arctanh(c*x)^2*
ln(c*x+1)*c*d+1/2*arctanh(c*x)^2*ln(c*x+1)*e+(c*d-e)*arctanh(c*x)^2*ln((c*
x+1)/(-c^2*x^2+1)^(1/2))-1/3*d*c*arctanh(c*x)^3+1/3*e*arctanh(c*x)^3-1/4*(
4*ln(2)*e-I*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+2*I*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))
*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-I*Pi*c*d*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-
(c*x+1)^2/(c^2*x^2-1)))^3+I*Pi*c*d*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*c*d*csgn(I/(1-(c*x+1
)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^
2+2*I*Pi*c*d-I*Pi*e*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))+2*I*Pi*e
*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3-2*I*Pi*e*csgn(I/(1-(c*x+1)^2/(c^2*x^2
-1)))^2-2*I*Pi*c*d*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^
2*x^2-1))^2-I*Pi*c*d*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+I*Pi*e*csgn(I*(c*x+1)
/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+I*Pi*e*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+I*Pi*e*csgn(I/(1-(c*x+1)^2/(c^
2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+2*I*P
i*e+2*I*Pi*c*d*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3-2*I*Pi*c*d*csgn(I/(1-(c
*x+1)^2/(c^2*x^2-1)))^2+I*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-I*Pi*c*d...
```

Fricas [F]

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx=\int -\frac{(ex+d)(b\,\text{artanh}\,(cx)+a)^2}{c^2x^2-1}\,dx$$

input
```
integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*e*x
 + a*b*d)*arctanh(c*x))/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx = -\int \frac{a^2d}{c^2x^2-1}\,dx - \int \frac{a^2ex}{c^2x^2-1}\,dx - \int \frac{b^2d\,\text{atanh}^2(cx)}{c^2x^2-1}\,dx - \int \frac{2abd\,\text{atanh}(cx)}{c^2x^2-1}\,dx - \int \frac{b^2ex\,\text{atanh}^2(cx)}{c^2x^2-1}\,dx - \int \frac{2abex\,\text{atanh}(cx)}{c^2x^2-1}\,dx$$

input
```
integrate((e*x+d)*(a+b*atanh(c*x))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2*d/(c**2*x**2 - 1), x) - Integral(a**2*e*x/(c**2*x**2 - 1),
x) - Integral(b**2*d*atanh(c*x)**2/(c**2*x**2 - 1), x) - Integral(2*a*b*d*
atanh(c*x)/(c**2*x**2 - 1), x) - Integral(b**2*e*x*atanh(c*x)**2/(c**2*x**
2 - 1), x) - Integral(2*a*b*e*x*atanh(c*x)/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx = \int -\frac{(ex+d)(b\,\text{artanh}(cx)+a)^2}{c^2x^2-1}\,dx$$

input
```
integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="maxima")
```

output
```
a*b*d*(log(c*x + 1)/c - log(c*x - 1)/c)*arctanh(c*x) + 1/2*a^2*d*(log(c*x
+ 1)/c - log(c*x - 1)/c) - 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x -
1) + log(c*x - 1)^2)*a*b*d/c - 1/2*a^2*e*log(c^2*x^2 - 1)/c^2 + 1/24*(3*(c
*d - e)*b^2*log(c*x + 1)*log(-c*x + 1)^2 - (c*d + e)*b^2*log(-c*x + 1)^3)/
c^2 - integrate(1/4*(4*a*b*c*e*x*log(c*x + 1) + (b^2*c*e*x + b^2*c*d)*log(
c*x + 1)^2 - (4*a*b*c*e*x - ((c^2*d - 3*c*e)*b^2*x - (c*d + e)*b^2)*log(c*
x + 1))*log(-c*x + 1))/(c^3*x^2 - c), x)
```

Giac [F]

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx=\int -\frac{(ex+d)(b\,\text{artanh}\,(cx)+a)^2}{c^2x^2-1}\,dx$$

input
```
integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="giac")
```

output
```
integrate(-(e*x + d)*(b*arctanh(c*x) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx=\int -\frac{(a+b\,\text{atanh}(c\,x))^2\,(d+e\,x)}{c^2\,x^2-1}\,dx$$

input
```
int(-((a + b*atanh(c*x))^2*(d + e*x))/(c^2*x^2 - 1),x)
```

output
```
int(-((a + b*atanh(c*x))^2*(d + e*x))/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{(d+ex)(a+b\text{arctanh}(cx))^2}{1-c^2x^2}\,dx = \frac{2atanh(cx)^3\,b^2cd+6atanh(cx)^2\,abcd-12\left(\int \frac{atanh(cx)x}{c^2x^2-1}dx\right)ab\,c^2e-6\left(\int \frac{atanh(cx)^2x}{c^2x^2-1}dx\right)b^2c^2e-3\log(c^2x \ldots}{6c^2}$$

input
```
int((e*x+d)*(a+b*atanh(c*x))^2/(-c^2*x^2+1),x)
```

output
```
(2*atanh(c*x)**3*b**2*c*d + 6*atanh(c*x)**2*a*b*c*d - 12*int((atanh(c*x)*x
)/(c**2*x**2 - 1),x)*a*b*c**2*e - 6*int((atanh(c*x)**2*x)/(c**2*x**2 - 1),
x)*b**2*c**2*e - 3*log(c**2*x - c)*a**2*c*d - 3*log(c**2*x - c)*a**2*e + 3
*log(c**2*x + c)*a**2*c*d - 3*log(c**2*x + c)*a**2*e)/(6*c**2)
```

3.498 $\int (c+dx^2)^4 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 245

$$\int (c+dx^2)^4 \operatorname{arctanh}(ax)\,dx = \frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d^2)\,x^4}{1260a^5}+\frac{d^3(36a^2c+7d)\,x^6}{378a^3}+\frac{d^4x^8}{72a}+c^4x\operatorname{arctanh}(ax)+\frac{4}{3}c^3dx^3\operatorname{arctanh}(ax)+\frac{6}{5}c^2d^2x^5\operatorname{arctanh}(ax)+\frac{4}{7}cd^3x^7\operatorname{arctanh}(ax)+\frac{1}{9}d^4x^9\operatorname{arctanh}(ax)+\frac{(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)\log(1-a^2x^2)}{630a^9}$$

output
```
1/630*d*(420*a^6*c^3+378*a^4*c^2*d+180*a^2*c*d^2+35*d^3)*x^2/a^7+1/1260*d^
2*(378*a^4*c^2+180*a^2*c*d+35*d^2)*x^4/a^5+1/378*d^3*(36*a^2*c+7*d)*x^6/a^
3+1/72*d^4*x^8/a+c^4*x*arctanh(a*x)+4/3*c^3*d*x^3*arctanh(a*x)+6/5*c^2*d^2
*x^5*arctanh(a*x)+4/7*c*d^3*x^7*arctanh(a*x)+1/9*d^4*x^9*arctanh(a*x)+1/63
0*(315*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*ln(-a^2
*x^2+1)/a^9
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.87

$$\int (c+dx^2)^4 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{a^2dx^2(420d^3+30a^2d^2(72c+7dx^2)+4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240\ldots}{}$$

input
```
Integrate[(c + d*x^2)^4*ArcTanh[a*x],x]
```

output
```
(a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 27
0*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4
+ 35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180
*c*d^3*x^6 + 35*d^4*x^8)*ArcTanh[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d +
378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6538, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(ax)\left(c+dx^2\right)^4\,dx$$

$$\Big\downarrow \text{6538}$$

$$-a\int \frac{x(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4)}{315(1-a^2x^2)}dx+c^4x\operatorname{arctanh}(ax)+\frac{4}{3}c^3dx^3\operatorname{arctanh}(ax)+\frac{6}{5}c^2d^2x^5\operatorname{arctanh}(ax)+\frac{4}{7}cd^3x^7\operatorname{arctanh}(ax)+\frac{1}{9}d^4x^9\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{315}a\int \frac{x(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4)}{1-a^2x^2}dx+c^4x\text{arctanh}(ax)+\frac{4}{3}c^3dx^3\text{arctanh}(ax)+\frac{6}{5}c^2d^2x^5\text{arctanh}(ax)+\frac{4}{7}cd^3x^7\text{arctanh}(ax)+\frac{1}{9}d^4x^9\text{arctanh}(ax)$$

$\Big\downarrow$ 2331

$$-\frac{1}{630}a\int \frac{35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4}{1-a^2x^2}dx^2+c^4x\text{arctanh}(ax)+\frac{4}{3}c^3dx^3\text{arctanh}(ax)+\frac{6}{5}c^2d^2x^5\text{arctanh}(ax)+\frac{4}{7}cd^3x^7\text{arctanh}(ax)+\frac{1}{9}d^4x^9\text{arctanh}(ax)$$

$\Big\downarrow$ 2389

$$-\frac{1}{630}a\int \left(-\frac{35d^4x^6}{a^2}-\frac{5d^3(36ca^2+7d)x^4}{a^4}-\frac{d^2(378c^2a^4+180cda^2+35d^2)x^2}{a^6}-\frac{d(420c^3a^6+378c^2da^4+180\right.$$
$$c^4x\text{arctanh}(ax)+\frac{4}{3}c^3dx^3\text{arctanh}(ax)+\frac{6}{5}c^2d^2x^5\text{arctanh}(ax)+\frac{4}{7}cd^3x^7\text{arctanh}(ax)+\frac{1}{9}d^4x^9\text{arctanh}(ax)$$

$\Big\downarrow$ 2009

$$-\frac{1}{630}a\left(-\frac{35d^4x^8}{4a^2}-\frac{5d^3x^6(36a^2c+7d)}{3a^4}-\frac{d^2x^4(378a^4c^2+180a^2cd+35d^2)}{2a^6}-\frac{dx^2(420a^6c^3+378a^4c^2d+180a\right.$$
$$c^4x\text{arctanh}(ax)+\frac{4}{3}c^3dx^3\text{arctanh}(ax)+\frac{6}{5}c^2d^2x^5\text{arctanh}(ax)+\frac{4}{7}cd^3x^7\text{arctanh}(ax)+\frac{1}{9}d^4x^9\text{arctanh}(ax)$$

input
```
Int[(c + d*x^2)^4*ArcTanh[a*x],x]
```

output
```
c^4*x*ArcTanh[a*x] + (4*c^3*d*x^3*ArcTanh[a*x])/3 + (6*c^2*d^2*x^5*ArcTanh
[a*x])/5 + (4*c*d^3*x^7*ArcTanh[a*x])/7 + (d^4*x^9*ArcTanh[a*x])/9 - (a*(-
((d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/a^8) - (d^
2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(2*a^6) - (5*d^3*(36*a^2*c + 7
*d)*x^6)/(3*a^4) - (35*d^4*x^8)/(4*a^2) - ((315*a^8*c^4 + 420*a^6*c^3*d +
378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/a^10))/630
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

method	result
parts	$\frac{d^4x^9\operatorname{arctanh}(ax)}{9}+\frac{4c\,d^3x^7\operatorname{arctanh}(ax)}{7}+\frac{6c^2d^2x^5\operatorname{arctanh}(ax)}{5}+\frac{4c^3d\,x^3\operatorname{arctanh}(ax)}{3}+c^4x\operatorname{arctanh}(a$
derivativedivides	$\operatorname{arctanh}(ax)c^4ax+\frac{4a\operatorname{arctanh}(ax)c^3d\,x^3}{3}+\frac{6a\operatorname{arctanh}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arctanh}(ax)c\,d^3x^7}{7}+\frac{a\operatorname{arctanh}(ax)d^4x^9}{9}-\frac{(-315a^8c^4-$
default	$\operatorname{arctanh}(ax)c^4ax+\frac{4a\operatorname{arctanh}(ax)c^3d\,x^3}{3}+\frac{6a\operatorname{arctanh}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arctanh}(ax)c\,d^3x^7}{7}+\frac{a\operatorname{arctanh}(ax)d^4x^9}{9}-\frac{(-315a^8c^4-$
parallelrisch	$-\frac{-210d^4a^4x^4-105d^4a^8x^8-420d^4a^2x^2-140d^4a^6x^6-1080a^6c\,d^3x^4-2160a^4c\,d^3x^2-720c\,a^8d^3x^6-2268c^2a^8d^2x^4-50$
risch	$\left(\frac{1}{18}d^4x^9+\frac{2}{7}c\,d^3x^7+\frac{3}{5}c^2d^2x^5+\frac{2}{3}c^3d\,x^3+\frac{1}{2}c^4x\right)\ln(ax+1)-\frac{d^4x^9\ln(-ax+1)}{18}-\frac{2c\,d^3x^7\ln(}{7}$
meijerg	$-\frac{d^4\left(-\frac{x^2a^2(15a^6x^6+20a^4x^4+30a^2x^2+60)}{270}+\frac{2x^{10}a^{10}\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{9\sqrt{a^2x^2}}-\frac{2\ln(-a^2x^2+1)}{9}\right)}{4a^9}+\frac{d^3c\left(\frac{x^2a^2(}{}\right.}{}$

input
```
int((d*x^2+c)^4*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9*arctanh(a*x)+4/7*c*d^3*x^7*arctanh(a*x)+6/5*c^2*d^2*x^5*arctan
h(a*x)+4/3*c^3*d*x^3*arctanh(a*x)+c^4*x*arctanh(a*x)-1/315*a*(-1/2*d/a^8*(
35/4*a^6*d^3*x^8+60*a^6*c*d^2*x^6+189*a^6*c^2*d*x^4+420*a^6*c^3*x^2+35/3*d
^3*x^6*a^4+90*a^4*c*d^2*x^4+378*a^4*c^2*d*x^2+35/2*d^3*x^4*a^2+180*d^2*c*a
^2*x^2+35*d^3*x^2)+1/2*(-315*a^8*c^4-420*a^6*c^3*d-378*a^4*c^2*d^2-180*a^2
*c*d^3-35*d^4)/a^10*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01

$$\int \left(c+dx^2\right)^4\operatorname{arctanh}(ax)\,dx = \frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3+7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2+180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d+378\,a^6}{}$$

input
```
integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c
^2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2
*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d +
 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d
^4*x^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315
*a^9*c^4*x)*log(-(a*x + 1)/(a*x - 1)))/a^9
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.52

$$\int \left(c+dx^2\right)^4 \operatorname{arctanh}(ax)\,dx$$
$$= \begin{cases} c^4x\operatorname{atanh}(ax) + \frac{4c^3dx^3\operatorname{atanh}(ax)}{3} + \frac{6c^2d^2x^5\operatorname{atanh}(ax)}{5} + \frac{4cd^3x^7\operatorname{atanh}(ax)}{7} + \frac{d^4x^9\operatorname{atanh}(ax)}{9} + \frac{c^4\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^4\operatorname{atanh}}{a} \\ 0 \end{cases}$$

input
```
integrate((d*x**2+c)**4*atanh(a*x),x)
```

output
```
Piecewise((c**4*x*atanh(a*x) + 4*c**3*d*x**3*atanh(a*x)/3 + 6*c**2*d**2*x*
*5*atanh(a*x)/5 + 4*c*d**3*x**7*atanh(a*x)/7 + d**4*x**9*atanh(a*x)/9 + c*
*4*log(x - 1/a)/a + c**4*atanh(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*
x**4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1
/a)/(3*a**3) + 4*c**3*d*atanh(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) +
c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a
**5) + 6*c**2*d**2*atanh(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**
4/(36*a**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*atanh(a*x)/(7*a**7
) + d**4*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*atanh(a*x)/(9*
a**9), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.13

$$\int \left(c+dx^2\right)^4 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3+7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d}{a^8}\right.$$
$$+\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\operatorname{artanh}(ax)$$

input
```
integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^
6*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*
c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3
*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315
*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a
*x - 1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*
c^3*d*x^3 + 315*c^4*x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1471 vs. 2(227) = 454.

Time = 0.18 (sec) , antiderivative size = 1471, normalized size of antiderivative = 6.00

$$\int \left(c+dx^2\right)^4 \operatorname{arctanh}(ax)\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="giac")
```

output
```
1/945*a*(3*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3
+ 35*d^4)*log(abs(-a*x - 1)/abs(a*x - 1))/a^10 - 3*(315*a^8*c^4 + 420*a^6*
c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(abs(-(a*x + 1)/(a*x
- 1) + 1))/a^10 + 8*(3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 +
35*d^4)*(a*x + 1)^7/(a*x - 1)^7 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a
^2*c*d^3 + 7*d^4)*(a*x + 1)^6/(a*x - 1)^6 + (4725*a^6*c^3*d + 6237*a^4*c^2
*d^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^5/(a*x - 1)^5 - 2*(3150*a^6*c^3
*d + 3969*a^4*c^2*d^2 + 2340*a^2*c*d^3 + 455*d^4)*(a*x + 1)^4/(a*x - 1)^4
+ (4725*a^6*c^3*d + 6237*a^4*c^2*d^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)
^3/(a*x - 1)^3 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*d^3 + 7*d^4)
*(a*x + 1)^2/(a*x - 1)^2 + 3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*
d^3 + 35*d^4)*(a*x + 1)/(a*x - 1))/(a^10*((a*x + 1)/(a*x - 1) - 1)^8) + 3*
(315*(a*x + 1)^8*a^8*c^4/(a*x - 1)^8 - 2520*(a*x + 1)^7*a^8*c^4/(a*x - 1)^
7 + 8820*(a*x + 1)^6*a^8*c^4/(a*x - 1)^6 - 17640*(a*x + 1)^5*a^8*c^4/(a*x
- 1)^5 + 22050*(a*x + 1)^4*a^8*c^4/(a*x - 1)^4 - 17640*(a*x + 1)^3*a^8*c^4
/(a*x - 1)^3 + 8820*(a*x + 1)^2*a^8*c^4/(a*x - 1)^2 - 2520*(a*x + 1)*a^8*c
^4/(a*x - 1) + 315*a^8*c^4 + 1260*(a*x + 1)^8*a^6*c^3*d/(a*x - 1)^8 - 7560
*(a*x + 1)^7*a^6*c^3*d/(a*x - 1)^7 + 19320*(a*x + 1)^6*a^6*c^3*d/(a*x - 1)
^6 - 27720*(a*x + 1)^5*a^6*c^3*d/(a*x - 1)^5 + 25200*(a*x + 1)^4*a^6*c^3*d
/(a*x - 1)^4 - 15960*(a*x + 1)^3*a^6*c^3*d/(a*x - 1)^3 + 7560*(a*x + 1)...
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.18

$$\int \left(c+dx^2\right)^4 \operatorname{arctanh}(ax)\,dx$$
$$= x^2\left(\frac{\frac{\frac{d^4}{9a^3}+\frac{4cd^3}{7a}}{a^2}+\frac{6c^2d^2}{5a}}{2a^2}+\frac{2c^3d}{3a}\right)+x^6\left(\frac{d^4}{54a^3}+\frac{2cd^3}{21a}\right)$$
$$+\ln(ax+1)\left(\frac{c^4x}{2}+\frac{2c^3dx^3}{3}+\frac{3c^2d^2x^5}{5}+\frac{2cd^3x^7}{7}+\frac{d^4x^9}{18}\right)$$
$$-\ln(1-ax)\left(\frac{c^4x}{2}+\frac{2c^3dx^3}{3}+\frac{3c^2d^2x^5}{5}+\frac{2cd^3x^7}{7}+\frac{d^4x^9}{18}\right)$$
$$+x^4\left(\frac{\frac{d^4}{9a^3}+\frac{4cd^3}{7a}}{4a^2}+\frac{3c^2d^2}{10a}\right)$$
$$+\frac{\ln(a^2x^2-1)\,(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)}{630a^9}+\frac{d^4x^8}{72a}$$

input
```
int(atanh(a*x)*(c + d*x^2)^4,x)
```

output
```
x^2*(((d^4/(9*a^3) + (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a))/(2*a^2) + (
2*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) + (2*c*d^3)/(21*a)) + log(a*x + 1)*((c
^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^
5)/5) - log(1 - a*x)*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^
3*x^7)/7 + (3*c^2*d^2*x^5)/5) + x^4*((d^4/(9*a^3) + (4*c*d^3)/(7*a))/(4*a^
2) + (3*c^2*d^2)/(10*a)) + (log(a^2*x^2 - 1)*(35*d^4 + 315*a^8*c^4 + 180*a
^2*c*d^3 + 420*a^6*c^3*d + 378*a^4*c^2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.46

$$\int (c+dx^2)^4 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{7560 atanh(ax)\, a^8c^4 + 7560\log(a^2x-a)\, a^8c^4 + 105a^8d^4x^8 + 140a^6d^4x^6 + 210a^4d^4x^4 + 420a^2d^4x^2 + 840}{}$$

input
```
int((d*x^2+c)^4*atanh(a*x),x)
```

output
```
(7560*atanh(a*x)*a**9*c**4*x + 10080*atanh(a*x)*a**9*c**3*d*x**3 + 9072*at
anh(a*x)*a**9*c**2*d**2*x**5 + 4320*atanh(a*x)*a**9*c*d**3*x**7 + 840*atan
h(a*x)*a**9*d**4*x**9 + 7560*atanh(a*x)*a**8*c**4 + 10080*atanh(a*x)*a**6*
c**3*d + 9072*atanh(a*x)*a**4*c**2*d**2 + 4320*atanh(a*x)*a**2*c*d**3 + 84
0*atanh(a*x)*d**4 + 7560*log(a**2*x - a)*a**8*c**4 + 10080*log(a**2*x - a)
*a**6*c**3*d + 9072*log(a**2*x - a)*a**4*c**2*d**2 + 4320*log(a**2*x - a)*
a**2*c*d**3 + 840*log(a**2*x - a)*d**4 + 5040*a**8*c**3*d*x**2 + 2268*a**8
*c**2*d**2*x**4 + 720*a**8*c*d**3*x**6 + 105*a**8*d**4*x**8 + 4536*a**6*c*
*2*d**2*x**2 + 1080*a**6*c*d**3*x**4 + 140*a**6*d**4*x**6 + 2160*a**4*c*d*
*3*x**2 + 210*a**4*d**4*x**4 + 420*a**2*d**4*x**2)/(7560*a**9)
```

3.499 $\int (c+dx^2)^3 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 169

$$\int (c+dx^2)^3 \operatorname{arctanh}(ax)\,dx = \frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5} + \frac{d^2(21a^2c+5d)\,x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\operatorname{arctanh}(ax) + c^2dx^3\operatorname{arctanh}(ax) + \frac{3}{5}cd^2x^5\operatorname{arctanh}(ax) + \frac{1}{7}d^3x^7\operatorname{arctanh}(ax) + \frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log(1-a^2x^2)}{70a^7}$$

output

```
1/70*d*(35*a^4*c^2+21*a^2*c*d+5*d^2)*x^2/a^5+1/140*d^2*(21*a^2*c+5*d)*x^4/
a^3+1/42*d^3*x^6/a+c^3*x*arctanh(a*x)+c^2*d*x^3*arctanh(a*x)+3/5*c*d^2*x^5
*arctanh(a*x)+1/7*d^3*x^7*arctanh(a*x)+1/70*(35*a^6*c^3+35*a^4*c^2*d+21*a^
2*c*d^2+5*d^3)*ln(-a^2*x^2+1)/a^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \left(c+dx^2\right)^3 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{a^2dx^2(30d^2+3a^2d(42c+5dx^2)+a^4(210c^2+63cdx^2+10d^2x^4))+12a^7x(35c^3+35c^2dx^2+21cd^2x^4+5}{420a^7}$$

input
```
Integrate[(c + d*x^2)^3*ArcTanh[a*x],x]
```

output
```
(a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2
+ 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^
6)*ArcTanh[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log
[1 - a^2*x^2])/(420*a^7)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6538, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(ax)\left(c+dx^2\right)^3\,dx$$

$$\Big\downarrow \; 6538$$

$$-a\int \frac{x\left(5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3\right)}{35\left(1-a^2x^2\right)}dx+c^3x\operatorname{arctanh}(ax)+c^2dx^3\operatorname{arctanh}(ax)+\frac{3}{5}cd^2x^5\operatorname{arctanh}(ax)+\frac{1}{7}d^3x^7\operatorname{arctanh}(ax)$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{35}a\int \frac{x\left(5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3\right)}{1-a^2x^2}dx+c^3x\operatorname{arctanh}(ax)+c^2dx^3\operatorname{arctanh}(ax)+\frac{3}{5}cd^2x^5\operatorname{arctanh}(ax)+\frac{1}{7}d^3x^7\operatorname{arctanh}(ax)$$

$\downarrow$ 2331

$$-\frac{1}{70}a\int\frac{5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3}{1-a^2x^2}dx^2+c^3x\text{arctanh}(ax)+c^2dx^3\text{arctanh}(ax)+\frac{3}{5}cd^2x^5\text{arctanh}(ax)+\frac{1}{7}d^3x^7\text{arctanh}(ax)$$

$\downarrow$ 2389

$$-\frac{1}{70}a\int\left(-\frac{5d^3x^4}{a^2}-\frac{d^2(21ca^2+5d)x^2}{a^4}-\frac{d(35c^2a^4+21cda^2+5d^2)}{a^6}+\frac{-35c^3a^6-35c^2da^4-21cd^2a^2-5d^3}{a^6(a^2x^2-1)}\right)$$
$$c^3x\text{arctanh}(ax)+c^2dx^3\text{arctanh}(ax)+\frac{3}{5}cd^2x^5\text{arctanh}(ax)+\frac{1}{7}d^3x^7\text{arctanh}(ax)$$

$\downarrow$ 2009

$$-\frac{1}{70}a\left(-\frac{5d^3x^6}{3a^2}-\frac{d^2x^4(21a^2c+5d)}{2a^4}-\frac{dx^2(35a^4c^2+21a^2cd+5d^2)}{a^6}-\frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log}{a^8}\right.$$
$$c^3x\text{arctanh}(ax)+c^2dx^3\text{arctanh}(ax)+\frac{3}{5}cd^2x^5\text{arctanh}(ax)+\frac{1}{7}d^3x^7\text{arctanh}(ax)$$

input
```
Int[(c + d*x^2)^3*ArcTanh[a*x],x]
```

output
```
c^3*x*ArcTanh[a*x] + c^2*d*x^3*ArcTanh[a*x] + (3*c*d^2*x^5*ArcTanh[a*x])/5
 + (d^3*x^7*ArcTanh[a*x])/7 - (a*(-((d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x
^2)/a^6) - (d^2*(21*a^2*c + 5*d)*x^4)/(2*a^4) - (5*d^3*x^6)/(3*a^2) - ((35
*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/a^8))/70
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
parts	$\frac{d^3x^7\operatorname{arctanh}(ax)}{7}+\frac{3c\,d^2x^5\operatorname{arctanh}(ax)}{5}+c^2d\,x^3\operatorname{arctanh}(ax)+c^3x\operatorname{arctanh}(ax)-\frac{a\left(-\frac{d\left(\frac{5}{3}a^4d\right.}{}\right.}{}$
derivativedivides	$\frac{\operatorname{arctanh}(ax)c^3ax+a\operatorname{arctanh}(ax)c^2d\,x^3+\frac{3a\operatorname{arctanh}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arctanh}(ax)d^3x^7}{7}-\frac{-\frac{5a^6d^3x^6}{6}-\frac{21a^6c\,d^2x^4}{4}-\frac{35a^6c^2d\,x^2}{2}}{}}{}$
default	$\frac{\operatorname{arctanh}(ax)c^3ax+a\operatorname{arctanh}(ax)c^2d\,x^3+\frac{3a\operatorname{arctanh}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arctanh}(ax)d^3x^7}{7}-\frac{-\frac{5a^6d^3x^6}{6}-\frac{21a^6c\,d^2x^4}{4}-\frac{35a^6c^2d\,x^2}{2}}{}}{}$
parallelrisch	$-\frac{-60x^7\operatorname{arctanh}(ax)a^7d^3-252x^5\operatorname{arctanh}(ax)a^7c\,d^2-10a^6d^3x^6-420x^3\operatorname{arctanh}(ax)a^7c^2d-63a^6c\,d^2x^4-420c^3\operatorname{arcta}}{}$
risch	$\left(\frac{1}{14}d^3x^7+\frac{3}{10}c\,d^2x^5+\frac{1}{2}c^2d\,x^3+\frac{1}{2}x\,c^3\right)\ln(ax+1)-\frac{d^3x^7\ln(-ax+1)}{14}-\frac{3c\,d^2x^5\ln(-ax+1)}{10}+$
meijerg	$\frac{d^3\left(\frac{x^2a^2\left(4a^4x^4+6a^2x^2+12\right)}{42}-\frac{2x^8a^8\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(-a^2x^2+1\right)}{7}\right)}{4a^7}-\frac{3d^2c\left(-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\right.}{}$

input
```
int((d*x^2+c)^3*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*d^3*x^7*arctanh(a*x)+3/5*c*d^2*x^5*arctanh(a*x)+c^2*d*x^3*arctanh(a*x)
+c^3*x*arctanh(a*x)-1/35*a*(-1/2*d/a^6*(5/3*a^4*d^2*x^6+21/2*a^4*c*d*x^4+3
5*a^4*c^2*x^2+5/2*d^2*x^4*a^2+21*d*c*a^2*x^2+5*d^2*x^2)+1/2*(-35*a^6*c^3-3
5*a^4*c^2*d-21*a^2*c*d^2-5*d^3)/a^8*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int \left(c+dx^2\right)^3 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{10\,a^6d^3x^6 + 3\,(21\,a^6cd^2 + 5\,a^4d^3)x^4 + 6\,(35\,a^6c^2d + 21\,a^4cd^2 + 5\,a^2d^3)x^2 + 6\,(35\,a^6c^3 + 35\,a^4c^2d + 21\,a^2c}{420\,a^7}$$

input
```
integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
 + 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c
*d^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*
a^7*c^2*d*x^3 + 35*a^7*c^3*x)*log(-(a*x + 1)/(a*x - 1)))/a^7
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.45

$$\int \left(c+dx^2\right)^3 \operatorname{arctanh}(ax)\,dx$$
$$= \begin{cases} c^3x\operatorname{atanh}(ax) + c^2dx^3\operatorname{atanh}(ax) + \frac{3cd^2x^5\operatorname{atanh}(ax)}{5} + \frac{d^3x^7\operatorname{atanh}(ax)}{7} + \frac{c^3\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^3\operatorname{atanh}(ax)}{a} + \frac{c^2dx^2}{2a} + \\ 0 \end{cases}$$

input
```
integrate((d*x**2+c)**3*atanh(a*x),x)
```

output
```
Piecewise((c**3*x*atanh(a*x) + c**2*d*x**3*atanh(a*x) + 3*c*d**2*x**5*atan
h(a*x)/5 + d**3*x**7*atanh(a*x)/7 + c**3*log(x - 1/a)/a + c**3*atanh(a*x)/
a + c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*l
og(x - 1/a)/a**3 + c**2*d*atanh(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3
*x**4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*atanh(a*x)/(5*
a**5) + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*atanh(a*x)
/(7*a**7), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.17

$$\int (c+dx^2)^3 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{1}{420}a\left(\frac{10\,a^4d^3x^6 + 3\,(21\,a^4cd^2 + 5\,a^2d^3)x^4 + 6\,(35\,a^4c^2d + 21\,a^2cd^2 + 5\,d^3)x^2}{a^6} + \frac{6\,(35\,a^6c^3 + 35\,a^4c^2d +\right.$$
$$+ \frac{1}{35}\left(5\,d^3x^7 + 21\,cd^2x^5 + 35\,c^2dx^3 + 35\,c^3x\right)\operatorname{artanh}(ax)$$

input
```
integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^
2*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a
^2*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
*c*d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^
2*d*x^3 + 35*c^3*x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. 2(157) = 314.

Time = 0.16 (sec) , antiderivative size = 930, normalized size of antiderivative = 5.50

$$\int (c+dx^2)^3 \operatorname{arctanh}(ax)\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="giac")
```

output
```
1/105*a*(3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(-a*x
 - 1)/abs(a*x - 1))/a^8 - 3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*
d^3)*log(abs(-(a*x + 1)/(a*x - 1) + 1))/a^8 + 2*(3*(35*a^4*c^2*d + 42*a^2*
c*d^2 + 15*d^3)*(a*x + 1)^5/(a*x - 1)^5 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 +
 15*d^3)*(a*x + 1)^4/(a*x - 1)^4 + 2*(315*a^4*c^2*d + 252*a^2*c*d^2 + 85*d
^3)*(a*x + 1)^3/(a*x - 1)^3 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^3)*(a*
x + 1)^2/(a*x - 1)^2 + 3*(35*a^4*c^2*d + 42*a^2*c*d^2 + 15*d^3)*(a*x + 1)/
(a*x - 1))/(a^8*((a*x + 1)/(a*x - 1) - 1)^6) + 3*(35*(a*x + 1)^6*a^6*c^3/(
a*x - 1)^6 - 210*(a*x + 1)^5*a^6*c^3/(a*x - 1)^5 + 525*(a*x + 1)^4*a^6*c^3
/(a*x - 1)^4 - 700*(a*x + 1)^3*a^6*c^3/(a*x - 1)^3 + 525*(a*x + 1)^2*a^6*c
^3/(a*x - 1)^2 - 210*(a*x + 1)*a^6*c^3/(a*x - 1) + 35*a^6*c^3 + 105*(a*x +
 1)^6*a^4*c^2*d/(a*x - 1)^6 - 420*(a*x + 1)^5*a^4*c^2*d/(a*x - 1)^5 + 665*
(a*x + 1)^4*a^4*c^2*d/(a*x - 1)^4 - 560*(a*x + 1)^3*a^4*c^2*d/(a*x - 1)^3
+ 315*(a*x + 1)^2*a^4*c^2*d/(a*x - 1)^2 - 140*(a*x + 1)*a^4*c^2*d/(a*x - 1
) + 35*a^4*c^2*d + 105*(a*x + 1)^6*a^2*c*d^2/(a*x - 1)^6 - 210*(a*x + 1)^5
*a^2*c*d^2/(a*x - 1)^5 + 315*(a*x + 1)^4*a^2*c*d^2/(a*x - 1)^4 - 420*(a*x
+ 1)^3*a^2*c*d^2/(a*x - 1)^3 + 231*(a*x + 1)^2*a^2*c*d^2/(a*x - 1)^2 - 42*
(a*x + 1)*a^2*c*d^2/(a*x - 1) + 21*a^2*c*d^2 + 35*(a*x + 1)^6*d^3/(a*x - 1
)^6 + 175*(a*x + 1)^4*d^3/(a*x - 1)^4 + 105*(a*x + 1)^2*d^3/(a*x - 1)^2 +
5*d^3)*log(-(a*((a*x + 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + ...
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\int (c+dx^2)^3 \operatorname{arctanh}(ax)\,dx = c^3\,x\,\mathrm{atanh}(a\,x)+\frac{d^3\,x^7\,\mathrm{atanh}(a\,x)}{7}+\frac{c^3\,\ln\left(a^2\,x^2-1\right)}{2\,a}+\frac{d^3\,\ln\left(a^2\,x^2-1\right)}{14\,a^7}+\frac{d^3\,x^6}{42\,a}+\frac{d^3\,x^4}{28\,a^3}+\frac{d^3\,x^2}{14\,a^5}+\frac{c^2\,d\,\ln\left(a^2\,x^2-1\right)}{2\,a^3}+\frac{3\,c\,d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5}+\frac{c^2\,d\,x^2}{2\,a}+\frac{3\,c\,d^2\,x^4}{20\,a}+\frac{3\,c\,d^2\,x^2}{10\,a^3}+c^2\,d\,x^3\,\mathrm{atanh}(a\,x)+\frac{3\,c\,d^2\,x^5\,\mathrm{atanh}(a\,x)}{5}$$

input
```
int(atanh(a*x)*(c + d*x^2)^3,x)
```

output
```
c^3*x*atanh(a*x) + (d^3*x^7*atanh(a*x))/7 + (c^3*log(a^2*x^2 - 1))/(2*a) +
 (d^3*log(a^2*x^2 - 1))/(14*a^7) + (d^3*x^6)/(42*a) + (d^3*x^4)/(28*a^3) +
 (d^3*x^2)/(14*a^5) + (c^2*d*log(a^2*x^2 - 1))/(2*a^3) + (3*c*d^2*log(a^2*
x^2 - 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) + (3*c*d^2*x
^2)/(10*a^3) + c^2*d*x^3*atanh(a*x) + (3*c*d^2*x^5*atanh(a*x))/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.50

$$\int (c+dx^2)^3 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{420atanh(ax)\,a^7c^3x + 420atanh(ax)\,a^7c^2d\,x^3 + 252atanh(ax)\,a^7c\,d^2x^5 + 60atanh(ax)\,a^7d^3x^7 + 420atan}{}$$

input
```
int((d*x^2+c)^3*atanh(a*x),x)
```

output
```
(420*atanh(a*x)*a**7*c**3*x + 420*atanh(a*x)*a**7*c**2*d*x**3 + 252*atanh(
a*x)*a**7*c*d**2*x**5 + 60*atanh(a*x)*a**7*d**3*x**7 + 420*atanh(a*x)*a**6
*c**3 + 420*atanh(a*x)*a**4*c**2*d + 252*atanh(a*x)*a**2*c*d**2 + 60*atanh
(a*x)*d**3 + 420*log(a**2*x - a)*a**6*c**3 + 420*log(a**2*x - a)*a**4*c**2
*d + 252*log(a**2*x - a)*a**2*c*d**2 + 60*log(a**2*x - a)*d**3 + 210*a**6*
c**2*d*x**2 + 63*a**6*c*d**2*x**4 + 10*a**6*d**3*x**6 + 126*a**4*c*d**2*x*
*2 + 15*a**4*d**3*x**4 + 30*a**2*d**3*x**2)/(420*a**7)
```

3.500 $\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{d(10a^2c+3d)\,x^2}{30a^3} + \frac{d^2x^4}{20a} + c^2x\operatorname{arctanh}(ax) + \frac{2}{3}cdx^3\operatorname{arctanh}(ax) + \frac{1}{5}d^2x^5\operatorname{arctanh}(ax) + \frac{(15a^4c^2+10a^2cd+3d^2)\log(1-a^2x^2)}{30a^5}$$

output
```
1/30*d*(10*a^2*c+3*d)*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arctanh(a*x)+2/3*c*d*x^
3*arctanh(a*x)+1/5*d^2*x^5*arctanh(a*x)+1/30*(15*a^4*c^2+10*a^2*c*d+3*d^2)
*ln(-a^2*x^2+1)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{a^2dx^2(6d+a^2(20c+3dx^2)) + 4a^5x(15c^2+10cdx^2+3d^2x^4)\operatorname{arctanh}(ax) + (30a^4c^2+20a^2cd+6d^2)\log}{60a^5}$$

input
```
Integrate[(c + d*x^2)^2*ArcTanh[a*x],x]
```

output
```
(a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3
*d^2*x^4)*ArcTanh[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2
])/(60*a^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6538, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(ax)\left(c+dx^2\right)^2 dx$$

$$\Big\downarrow \text{6538}$$

$$-a\int \frac{x\left(3d^2x^4+10cdx^2+15c^2\right)}{15\left(1-a^2x^2\right)}dx+c^2x\text{arctanh}(ax)+\frac{2}{3}cdx^3\text{arctanh}(ax)+\frac{1}{5}d^2x^5\text{arctanh}(ax)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}a\int \frac{x\left(3d^2x^4+10cdx^2+15c^2\right)}{1-a^2x^2}dx+c^2x\text{arctanh}(ax)+\frac{2}{3}cdx^3\text{arctanh}(ax)+\frac{1}{5}d^2x^5\text{arctanh}(ax)$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}a\int \frac{3d^2x^4+10cdx^2+15c^2}{1-a^2x^2}dx^2+c^2x\text{arctanh}(ax)+\frac{2}{3}cdx^3\text{arctanh}(ax)+\frac{1}{5}d^2x^5\text{arctanh}(ax)$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}a\int \left(-\frac{3d^2x^2}{a^2}-\frac{d\left(10ca^2+3d\right)}{a^4}+\frac{-15c^2a^4-10cda^2-3d^2}{a^4\left(a^2x^2-1\right)}\right)dx^2+c^2x\text{arctanh}(ax)+\frac{2}{3}cdx^3\text{arctanh}(ax)+\frac{1}{5}d^2x^5\text{arctanh}(ax)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{30}a\left(-\frac{3d^2x^4}{2a^2}-\frac{dx^2(10a^2c+3d)}{a^4}-\frac{(15a^4c^2+10a^2cd+3d^2)\log(1-a^2x^2)}{a^6}\right)+ c^2x\text{arctanh}(ax)+\frac{2}{3}cdx^3\text{arctanh}(ax)+\frac{1}{5}d^2x^5\text{arctanh}(ax)$$

input
```
Int[(c + d*x^2)^2*ArcTanh[a*x],x]
```

output
```
c^2*x*ArcTanh[a*x] + (2*c*d*x^3*ArcTanh[a*x])/3 + (d^2*x^5*ArcTanh[a*x])/5
 - (a*(-((d*(10*a^2*c + 3*d)*x^2)/a^4) - (3*d^2*x^4)/(2*a^2) - ((15*a^4*c^
2 + 10*a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/a^6))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
parts	$\frac{d^2x^5\,\mathrm{arctanh}(ax)}{5}+\frac{2cd\,x^3\,\mathrm{arctanh}(ax)}{3}+c^2x\,\mathrm{arctanh}\,(ax)-\frac{a\left(-\frac{d\left(\frac{3}{2}d\,x^4a^2+10a^2c\,x^2+3d\,x^2\right)}{2a^4}+\frac{\left(-15a^4c^2\right.}{}\right.}{15}$
derivativedivides	$\frac{\mathrm{arctanh}(ax)c^2ax+\frac{2a\,\mathrm{arctanh}(ax)cd\,x^3}{3}+\frac{a\,\mathrm{arctanh}(ax)d^2x^5}{5}-\frac{-5a^4cd\,x^2-\frac{3a^4d^2x^4}{4}-\frac{3d^2x^2a^2}{2}-\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\ln(a}{2}}{15a^4}}{a}$
default	$\frac{\mathrm{arctanh}(ax)c^2ax+\frac{2a\,\mathrm{arctanh}(ax)cd\,x^3}{3}+\frac{a\,\mathrm{arctanh}(ax)d^2x^5}{5}-\frac{-5a^4cd\,x^2-\frac{3a^4d^2x^4}{4}-\frac{3d^2x^2a^2}{2}-\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\ln(a}{2}}{15a^4}}{a}$
parallelrisch	$-\frac{-12x^5\,\mathrm{arctanh}(ax)a^5d^2-40x^3\,\mathrm{arctanh}(ax)a^5cd-3a^4d^2x^4-60c^2\,\mathrm{arctanh}(ax)x\,a^5-20a^4cd\,x^2-60\ln(ax-1)a^4c^2-6}{}$
risch	$\left(\frac{1}{10}d^2x^5+\frac{1}{3}cd\,x^3+\frac{1}{2}c^2x\right)\ln\left(ax+1\right)-\frac{d^2x^5\ln(-ax+1)}{10}-\frac{cd\,x^3\ln(-ax+1)}{3}+\frac{d^2x^4}{20a}-\frac{c^2x\ln(-a}{2}$
meijerg	$-\frac{d^2\left(-\frac{a^2x^2\left(3a^2x^2+6\right)}{15}+\frac{2a^6x^6\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{5\sqrt{a^2x^2}}-\frac{2\ln\left(-a^2x^2+1\right)}{5}\right)}{4a^5}+\frac{dc\left(\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right.\right.}{3\sqrt{a^2}}\right.}{}$

input
```
int((d*x^2+c)^2*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^2*x^5*arctanh(a*x)+2/3*c*d*x^3*arctanh(a*x)+c^2*x*arctanh(a*x)-1/15*
a*(-1/2*d/a^4*(3/2*d*x^4*a^2+10*a^2*c*x^2+3*d*x^2)+1/2*(-15*a^4*c^2-10*a^2
*c*d-3*d^2)/a^6*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \left(c+dx^2\right)^2\mathrm{arctanh}(ax)\,dx$$
$$=\frac{3\,a^4d^2x^4+2\,(10\,a^4cd+3\,a^2d^2)x^2+2\,(15\,a^4c^2+10\,a^2cd+3\,d^2)\log\left(a^2x^2-1\right)+2\,(3\,a^5d^2x^5+10\,a^5cdx^3}{60\,a^5}$$

input
```
integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15
*a^5*c^2*x)*log(-(a*x + 1)/(a*x - 1)))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.41

$$\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \begin{cases} c^2x\operatorname{atanh}(ax) + \frac{2cdx^3\operatorname{atanh}(ax)}{3} + \frac{d^2x^5\operatorname{atanh}(ax)}{5} + \frac{c^2\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^2\operatorname{atanh}(ax)}{a} + \frac{cdx^2}{3a} + \frac{d^2x^4}{20a} + \frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3} + \\ 0 \end{cases}$$

input
```
integrate((d*x**2+c)**2*atanh(a*x),x)
```

output
```
Piecewise((c**2*x*atanh(a*x) + 2*c*d*x**3*atanh(a*x)/3 + d**2*x**5*atanh(a
*x)/5 + c**2*log(x - 1/a)/a + c**2*atanh(a*x)/a + c*d*x**2/(3*a) + d**2*x*
*4/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*atanh(a*x)/(3*a**3) + d**2
*x**2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*atanh(a*x)/(5*a**5), N
e(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx$$
$$= \frac{1}{60}a\left(\frac{3a^2d^2x^4 + 2(10a^2cd + 3d^2)x^2}{a^4} + \frac{2(15a^4c^2 + 10a^2cd + 3d^2)\log(ax+1)}{a^6} + \frac{2(15a^4c^2 + 10a^2cd}{a^6}\right.$$
$$+ \frac{1}{15}\left(3d^2x^5 + 10cdx^3 + 15c^2x\right)\operatorname{artanh}(ax)$$

input
```
integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
 10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2
)*log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arctanh(a*x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.79

$$\int (c + dx^2)^2 \operatorname{arctanh}(ax)\, dx$$

$$= \frac{1}{15}a\left(\frac{(15\,a^4c^2 + 10\,a^2cd + 3\,d^2)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^6} - \frac{(15\,a^4c^2 + 10\,a^2cd + 3\,d^2)\log\left(\left|-\frac{ax+1}{ax-1} + 1\right|\right)}{a^6} + \frac{4\left(\frac{(5\,a^2c\right.}{}\right.$$

input
```
integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="giac")
```

output
```
1/15*a*((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(-a*x - 1)/abs(a*x - 1))/
a^6 - (15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(-(a*x + 1)/(a*x - 1) + 1))
/a^6 + 4*((5*a^2*c*d + 3*d^2)*(a*x + 1)^3/(a*x - 1)^3 - (10*a^2*c*d + 3*d^
2)*(a*x + 1)^2/(a*x - 1)^2 + (5*a^2*c*d + 3*d^2)*(a*x + 1)/(a*x - 1))/(a^6
*((a*x + 1)/(a*x - 1) - 1)^4) + (15*(a*x + 1)^4*a^4*c^2/(a*x - 1)^4 - 60*(
a*x + 1)^3*a^4*c^2/(a*x - 1)^3 + 90*(a*x + 1)^2*a^4*c^2/(a*x - 1)^2 - 60*(
a*x + 1)*a^4*c^2/(a*x - 1) + 15*a^4*c^2 + 30*(a*x + 1)^4*a^2*c*d/(a*x - 1)
^4 - 60*(a*x + 1)^3*a^2*c*d/(a*x - 1)^3 + 40*(a*x + 1)^2*a^2*c*d/(a*x - 1)
^2 - 20*(a*x + 1)*a^2*c*d/(a*x - 1) + 10*a^2*c*d + 15*(a*x + 1)^4*d^2/(a*x
 - 1)^4 + 30*(a*x + 1)^2*d^2/(a*x - 1)^2 + 3*d^2)*log(-(a*((a*x + 1)/(a*x
- 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x - 1) + 1)/((
a*x + 1)*a/(a*x - 1) - a) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx = c^2\,x\,\operatorname{atanh}(a\,x) + \frac{d^2\,x^5\,\operatorname{atanh}(a\,x)}{5} + \frac{c^2\,\ln\left(a^2\,x^2-1\right)}{2\,a} + \frac{d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5} + \frac{d^2\,x^4}{20\,a} + \frac{d^2\,x^2}{10\,a^3} + \frac{2\,c\,d\,x^3\,\operatorname{atanh}(a\,x)}{3} + \frac{c\,d\,\ln\left(a^2\,x^2-1\right)}{3\,a^3} + \frac{c\,d\,x^2}{3\,a}$$

input

```
int(atanh(a*x)*(c + d*x^2)^2,x)
```

output

```
c^2*x*atanh(a*x) + (d^2*x^5*atanh(a*x))/5 + (c^2*log(a^2*x^2 - 1))/(2*a) +
 (d^2*log(a^2*x^2 - 1))/(10*a^5) + (d^2*x^4)/(20*a) + (d^2*x^2)/(10*a^3) +
 (2*c*d*x^3*atanh(a*x))/3 + (c*d*log(a^2*x^2 - 1))/(3*a^3) + (c*d*x^2)/(3*
a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int (c+dx^2)^2 \operatorname{arctanh}(ax)\,dx = \frac{60 atanh(ax)\,a^5c^2x + 40 atanh(ax)\,a^5cd\,x^3 + 12 atanh(ax)\,a^5d^2x^5 + 60 atanh(ax)\,a^4c^2 + 40 atanh(ax)\,a^2c$$

input

```
int((d*x^2+c)^2*atanh(a*x),x)
```

output

```
(60*atanh(a*x)*a**5*c**2*x + 40*atanh(a*x)*a**5*c*d*x**3 + 12*atanh(a*x)*a
**5*d**2*x**5 + 60*atanh(a*x)*a**4*c**2 + 40*atanh(a*x)*a**2*c*d + 12*atan
h(a*x)*d**2 + 60*log(a**2*x - a)*a**4*c**2 + 40*log(a**2*x - a)*a**2*c*d +
 12*log(a**2*x - a)*d**2 + 20*a**4*c*d*x**2 + 3*a**4*d**2*x**4 + 6*a**2*d*
*2*x**2)/(60*a**5)
```

3.501 $\int (c+dx^2)\,\text{arctanh}(ax)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (c+dx^2)\,\text{arctanh}(ax)\,dx=\frac{dx^2}{6a}+cx\text{arctanh}(ax)+\frac{1}{3}dx^3\text{arctanh}(ax)+\frac{(3a^2c+d)\log(1-a^2x^2)}{6a^3}$$

output
```
1/6*d*x^2/a+c*x*arctanh(a*x)+1/3*d*x^3*arctanh(a*x)+1/6*(3*a^2*c+d)*ln(-a^
2*x^2+1)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int (c+dx^2)\,\text{arctanh}(ax)\,dx=\frac{dx^2}{6a}+cx\text{arctanh}(ax)+\frac{1}{3}dx^3\text{arctanh}(ax)+\frac{c\log(1-a^2x^2)}{2a}+\frac{d\log(1-a^2x^2)}{6a^3}$$

input
```
Integrate[(c + d*x^2)*ArcTanh[a*x],x]
```

output
```
(d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + (c*Log[1 - a^2
*x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6538, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(ax)\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \text{ 6538}$$

$$-a\int \frac{x\left(dx^2+3c\right)}{3\left(1-a^2x^2\right)}dx+cx\operatorname{arctanh}(ax)+\frac{1}{3}dx^3\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}a\int \frac{x\left(dx^2+3c\right)}{1-a^2x^2}dx+cx\operatorname{arctanh}(ax)+\frac{1}{3}dx^3\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{1}{6}a\int \frac{dx^2+3c}{1-a^2x^2}dx^2+cx\operatorname{arctanh}(ax)+\frac{1}{3}dx^3\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{1}{6}a\int \left(\frac{-3ca^2-d}{a^2\left(a^2x^2-1\right)}-\frac{d}{a^2}\right)dx^2+cx\operatorname{arctanh}(ax)+\frac{1}{3}dx^3\operatorname{arctanh}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{6}a\left(-\frac{dx^2}{a^2}-\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{a^4}\right)+cx\operatorname{arctanh}(ax)+\frac{1}{3}dx^3\operatorname{arctanh}(ax)$$

input
```
Int[(c + d*x^2)*ArcTanh[a*x],x]
```

output
```
c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 - (a*(-((d*x^2)/a^2) - ((3*a^2*c
 + d)*Log[1 - a^2*x^2])/a^4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

method	result
parts	$\frac{d\,x^3\,\mathrm{arctanh}(ax)}{3}+cx\,\mathrm{arctanh}\,(ax)-\frac{a\left(-\frac{d\,x^2}{2a^2}+\frac{\left(-3a^2c-d\right)\ln\left(a^2x^2-1\right)}{2a^4}\right)}{3}$
derivativedivides	$\frac{\mathrm{arctanh}(ax)acx+\frac{a\,\mathrm{arctanh}(ax)d\,x^3}{3}-\frac{-\frac{d\,x^2a^2}{2}-\frac{\left(3a^2c+d\right)\ln(ax-1)}{2}+\frac{\left(-3a^2c-d\right)\ln(ax+1)}{2}}{3a^2}}{a}$
default	$\frac{\mathrm{arctanh}(ax)acx+\frac{a\,\mathrm{arctanh}(ax)d\,x^3}{3}-\frac{-\frac{d\,x^2a^2}{2}-\frac{\left(3a^2c+d\right)\ln(ax-1)}{2}+\frac{\left(-3a^2c-d\right)\ln(ax+1)}{2}}{3a^2}}{a}$
parallelrisch	$-\frac{-2x^3\,\mathrm{arctanh}(ax)a^3d-6c\,\mathrm{arctanh}(ax)x\,a^3-d\,x^2a^2-6\ln(ax-1)a^2c-6\,\mathrm{arctanh}(ax)a^2c-2\ln(ax-1)d-2\,\mathrm{arctanh}(a}{6a^3}$
risch	$\left(\frac{1}{6}d\,x^3+\frac{1}{2}cx\right)\ln\left(ax+1\right)-\frac{d\,x^3\ln(-ax+1)}{6}-\frac{cx\ln(-ax+1)}{2}+\frac{d\,x^2}{6a}+\frac{\ln(a^2x^2-1)c}{2a}+\frac{\ln(a^2x^2-1)c}{6a^3}$
meijerg	$\frac{d\left(\frac{2a^2x^2}{3}-\frac{2a^4x^4\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{3\sqrt{a^2x^2}}+\frac{2\ln\left(-a^2x^2+1\right)}{3}\right)}{4a^3}-\frac{c\left(\frac{2a^2x^2\left(\ln\left(1-\sqrt{a^2x^2}\right)-\ln\left(1+\sqrt{a^2x^2}\right)\right)}{\sqrt{a^2x^2}}-2\ln(-\right.}{4a}$

input
```
int((d*x^2+c)*arctanh(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x^3*arctanh(a*x)+c*x*arctanh(a*x)-1/3*a*(-1/2*d/a^2*x^2+1/2*(-3*a^2*
c-d)/a^4*ln(a^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \left(c+dx^2\right)\mathrm{arctanh}(ax)\,dx = \frac{a^2dx^2+\left(3\,a^2c+d\right)\log\left(a^2x^2-1\right)+\left(a^3dx^3+3\,a^3cx\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,a^3}$$

input
```
integrate((d*x^2+c)*arctanh(a*x),x, algorithm="fricas")
```

output
```
1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*
log(-(a*x + 1)/(a*x - 1)))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int (c + dx^2) \operatorname{arctanh}(ax) \, dx$$
$$= \begin{cases} cx \operatorname{atanh}(ax) + \frac{dx^3 \operatorname{atanh}(ax)}{3} + \frac{c \log\left(x - \frac{1}{a}\right)}{a} + \frac{c \operatorname{atanh}(ax)}{a} + \frac{dx^2}{6a} + \frac{d \log\left(x - \frac{1}{a}\right)}{3a^3} + \frac{d \operatorname{atanh}(ax)}{3a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2+c)*atanh(a*x),x)
```

output
```
Piecewise((c*x*atanh(a*x) + d*x**3*atanh(a*x)/3 + c*log(x - 1/a)/a + c*ata
nh(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*atanh(a*x)/(3*a**3)
, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int (c + dx^2) \operatorname{arctanh}(ax) \, dx$$
$$= \frac{1}{6} a \left(\frac{dx^2}{a^2} + \frac{(3 a^2 c + d) \log(ax + 1)}{a^4} + \frac{(3 a^2 c + d) \log(ax - 1)}{a^4} \right) + \frac{1}{3} (dx^3 + 3 cx) \operatorname{artanh}(ax)$$

input
```
integrate((d*x^2+c)*arctanh(a*x),x, algorithm="maxima")
```

output
```
1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x
- 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arctanh(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.67

$$\int (c + dx^2) \operatorname{arctanh}(ax)\, dx$$
$$= \frac{1}{3}a\left(\frac{(3a^2c + d)\log\left(\frac{|-ax-1|}{|ax-1|}\right)}{a^4} - \frac{(3a^2c + d)\log\left(\left|-\frac{ax+1}{ax-1} + 1\right|\right)}{a^4} + \frac{2(ax+1)d}{(ax-1)a^4\left(\frac{ax+1}{ax-1} - 1\right)^2} + \frac{\left(\frac{3(ax+1)^2a^2c}{(ax-1)^2}\right.}{}\right.$$

input
```
integrate((d*x^2+c)*arctanh(a*x),x, algorithm="giac")
```

output
```
1/3*a*((3*a^2*c + d)*log(abs(-a*x - 1)/abs(a*x - 1))/a^4 - (3*a^2*c + d)*l
og(abs(-(a*x + 1)/(a*x - 1) + 1))/a^4 + 2*(a*x + 1)*d/((a*x - 1)*a^4*((a*x
 + 1)/(a*x - 1) - 1)^2) + (3*(a*x + 1)^2*a^2*c/(a*x - 1)^2 - 6*(a*x + 1)*a
^2*c/(a*x - 1) + 3*a^2*c + 3*(a*x + 1)^2*d/(a*x - 1)^2 + d)*log(-(a*((a*x
+ 1)/(a*x - 1) + 1)/((a*x + 1)*a/(a*x - 1) - a) + 1)/(a*((a*x + 1)/(a*x -
1) + 1)/((a*x + 1)*a/(a*x - 1) - a) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^3
))
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int (c + dx^2) \operatorname{arctanh}(ax)\, dx = \frac{\frac{d\ln(a^2x^2-1)}{6} + a^2\left(\frac{c\ln(a^2x^2-1)}{2} + \frac{dx^2}{6}\right)}{a^3} + \frac{dx^3\,\operatorname{atanh}(ax)}{3} + cx\,\operatorname{atanh}(ax)$$

input
```
int(atanh(a*x)*(c + d*x^2),x)
```

output
```
((d*log(a^2*x^2 - 1))/6 + a^2*((c*log(a^2*x^2 - 1))/2 + (d*x^2)/6))/a^3 +
(d*x^3*atanh(a*x))/3 + c*x*atanh(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int (c + dx^2) \operatorname{arctanh}(ax) \, dx$$
$$= \frac{6atanh(ax)\, a^3cx + 2atanh(ax)\, a^3d\, x^3 + 6atanh(ax)\, a^2c + 2atanh(ax)\, d + 6\log(a^2x - a)\, a^2c + 2\log(a^2x}{6a^3}$$

input
```
int((d*x^2+c)*atanh(a*x),x)
```

output
```
(6*atanh(a*x)*a**3*c*x + 2*atanh(a*x)*a**3*d*x**3 + 6*atanh(a*x)*a**2*c +
2*atanh(a*x)*d + 6*log(a**2*x - a)*a**2*c + 2*log(a**2*x - a)*d + a**2*d*x
**2)/(6*a**3)
```

3.502 $\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 429

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx = &-\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt{d}(1+ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{d}(1+ax)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\end{aligned}$$

output
```
-1/4*ln(-a*x+1)*ln(a*((-c)^(1/2)-d^(1/2)*x)/(a*(-c)^(1/2)-d^(1/2)))/(-c)^(
1/2)/d^(1/2)+1/4*ln(a*x+1)*ln(a*((-c)^(1/2)-d^(1/2)*x)/(a*(-c)^(1/2)+d^(1/
2)))/(-c)^(1/2)/d^(1/2)-1/4*ln(a*x+1)*ln(a*((-c)^(1/2)+d^(1/2)*x)/(a*(-c)^
(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln(-a*x+1)*ln(a*((-c)^(1/2)+d^(1/2)
*x)/(a*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-d^(1/2)*(-a*
x+1)/(a*(-c)^(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,d^(1/2)*(-a*
x+1)/(a*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-d^(1/2)*(a*
x+1)/(a*(-c)^(1/2)-d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,d^(1/2)*(a*x
+1)/(a*(-c)^(1/2)+d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.54

$$\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx =$$

$$-\frac{a\left(-2i\arccos\left(\frac{-a^2c+d}{a^2c+d}\right)\arctan\left(\frac{adx}{\sqrt{a^2cd}}\right)+4\arctan\left(\frac{ac}{\sqrt{a^2cdx}}\right)\operatorname{arctanh}(ax)-\left(\arccos\left(\frac{-a^2c+d}{a^2c+d}\right)+2\operatorname{arct}\right.\right.}{}$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2),x]
```

output
```
-1/4*(a*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2
*c*d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c
) + d)/(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d
 + Sqrt[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (
ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[
(2*a*c*(d + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d
]*x))] + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c
*d]*x)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqr
t[a^2*c + d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*
x]]])] + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c
*d]*x)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^Arc
Tanh[a*x])/(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*
x]]])] + I*(-PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqr
t[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((
-(a^2*c) + d + (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d
)*((-I)*a*c + Sqrt[a^2*c*d]*x))])))/Sqrt[a^2*c*d]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6534, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{c+dx^2} \, dx$$

$$\Big\downarrow \text{6534}$$

$$\frac{1}{2}\int \frac{\log(ax+1)}{dx^2+c}dx-\frac{1}{2}\int \frac{\log(1-ax)}{dx^2+c}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{-c}\log(ax+1)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(ax+1)}{2c\left(\sqrt{d}x+\sqrt{-c}\right)}\right)dx-$$
$$\frac{1}{2}\int \left(\frac{\sqrt{-c}\log(1-ax)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(1-ax)}{2c\left(\sqrt{d}x+\sqrt{-c}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(1-ax)}{\sqrt{-c}a+\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\log(1-ax)\log\left(\right.}{2\sqrt{-c}}\right.$$

$$\frac{1}{2}\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}-\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(ax+1)}{\sqrt{-c}a+\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\log(ax+1)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\log(ax+1)\log\left(\right.}{2\sqrt{-c}}\right.$$

input
```
Int[ArcTanh[a*x]/(c + d*x^2),x]
```

output
```
(-1/2*(Log[1 - a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] - Sqrt[d])]
)/(Sqrt[-c]*Sqrt[d]) + (Log[1 - a*x]*Log[(a*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqr
t[-c] + Sqrt[d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(1 - a*x))
/(a*Sqrt[-c] - Sqrt[d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 -
a*x))/(a*Sqrt[-c] + Sqrt[d])]/(2*Sqrt[-c]*Sqrt[d]))/2 + ((Log[1 + a*x]*Log
[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] + Sqrt[d])])/(2*Sqrt[-c]*Sqrt[d])
- (Log[1 + a*x]*Log[(a*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(2
*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] - Sqrt[d
]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] + S
qrt[d])]/(2*Sqrt[-c]*Sqrt[d]))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6534
```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + c*x]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - c*x]/(d + e*x^2)
, x], x] /; FreeQ[{c, d, e}, x]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\ln(-ax+1)\ln\left(\frac{a\sqrt{-cd}-d(-ax+1)+d}{a\sqrt{-cd}+d}\right)}{4\sqrt{-cd}} - \frac{\ln(-ax+1)\ln\left(\frac{a\sqrt{-cd}+d(-ax+1)-d}{a\sqrt{-cd}-d}\right)}{4\sqrt{-cd}} + \frac{\text{dilog}\left(\frac{a\sqrt{-cd}-d(-ax+1)+d}{a\sqrt{-cd}+d}\right)}{4\sqrt{-cd}} -$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctanh(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-a*x+1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-d*(-a*x+1)+d)/(a*(-c*d)^(1/
2)+d))-1/4*ln(-a*x+1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+d*(-a*x+1)-d)/(a*(-c
*d)^(1/2)-d))+1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-d*(-a*x+1)+d)/(a*(-c*
d)^(1/2)+d))-1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)+d*(-a*x+1)-d)/(a*(-c*d
)^(1/2)-d))+1/4*ln(a*x+1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-d*(a*x+1)+d)/(a*
(-c*d)^(1/2)+d))-1/4*ln(a*x+1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+d*(a*x+1)-d
)/(a*(-c*d)^(1/2)-d))+1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-d*(a*x+1)+d)/
(a*(-c*d)^(1/2)+d))-1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)+d*(a*x+1)-d)/(a
*(-c*d)^(1/2)-d))
```

Fricas [F]

$$\int \frac{\text{arctanh}(ax)}{c+dx^2}\,dx = \int \frac{\text{artanh}\,(ax)}{dx^2+c}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/(d*x^2 + c), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{atanh}(ax)}{c+dx^2}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c),x)
```

output
```
Integral(atanh(a*x)/(c + d*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx = \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)\operatorname{artanh}(ax)}{\sqrt{cd}} + \frac{\left(\arctan\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d}, \frac{adx+d}{a^2c+d}\right) - \arctan\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d}, -\frac{adx-d}{a^2c+d}\right)\right)\log\left(dx^2+c\right) - \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{a^2d}{}\right.}{}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="maxima")
```

output
```
arctan(d*x/sqrt(c*d))*arctanh(a*x)/sqrt(c*d) + 1/4*((arctan2((a^2*x + a)*s
qrt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - arctan2((a^2*x - a)
*sqrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c) -
arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) + arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - I*dilo
g((a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)
*sqrt(d) - d)) - I*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))
/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c + a*d*x + (I*a^2*x
- I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^
2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt
(d) - d)))/sqrt(c*d)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{dx^2+c}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{d\,x^2+c}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2),x)
```

output
```
int(atanh(a*x)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{c+dx^2}\,dx = \frac{-atanh(ax)^2 a-2\left(\int \frac{atanh(ax)}{a^2d\,x^4+a^2c\,x^2-d\,x^2-c}dx\right)a^2c-2\left(\int \frac{atanh(ax)}{a^2d\,x^4+a^2c\,x^2-d\,x^2-c}dx\right)d}{2d}$$

input
```
int(atanh(a*x)/(d*x^2+c),x)
```

output
```
( - atanh(a*x)**2*a - 2*int(atanh(a*x)/(a**2*c*x**2 + a**2*d*x**4 - c - d*
x**2),x)*a**2*c - 2*int(atanh(a*x)/(a**2*c*x**2 + a**2*d*x**4 - c - d*x**2
),x)*d)/(2*d)
```

3.503 $\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 590

$$\begin{aligned}\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = {} & \frac{x\operatorname{arctanh}(ax)}{2c(c+dx^2)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\operatorname{arctanh}(ax)}{2c^{3/2}\sqrt{d}} \\ & +\frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\ & -\frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\ & -\frac{i\log\left(-\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\ & +\frac{i\log\left(\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\ & +\frac{a\log(1-a^2x^2)}{4c(a^2c+d)}-\frac{a\log(c+dx^2)}{4c(a^2c+d)} \\ & +\frac{i\operatorname{PolyLog}\left(2,\frac{a(\sqrt{c}-i\sqrt{d}x)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\operatorname{PolyLog}\left(2,\frac{a(\sqrt{c}-i\sqrt{d}x)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} \\ & +\frac{i\operatorname{PolyLog}\left(2,\frac{a(\sqrt{c}+i\sqrt{d}x)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\operatorname{PolyLog}\left(2,\frac{a(\sqrt{c}+i\sqrt{d}x)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}\end{aligned}$$

output
```
1/2*x*arctanh(a*x)/c/(d*x^2+c)+1/2*arctan(d^(1/2)*x/c^(1/2))*arctanh(a*x)/
c^(3/2)/d^(1/2)+1/8*I*ln(d^(1/2)*(-a*x+1)/(I*a*c^(1/2)+d^(1/2)))*ln(1-I*d^
(1/2)*x/c^(1/2))/c^(3/2)/d^(1/2)-1/8*I*ln(-d^(1/2)*(a*x+1)/(I*a*c^(1/2)-d^
(1/2)))*ln(1-I*d^(1/2)*x/c^(1/2))/c^(3/2)/d^(1/2)-1/8*I*ln(-d^(1/2)*(-a*x+
1)/(I*a*c^(1/2)-d^(1/2)))*ln(1+I*d^(1/2)*x/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*
ln(d^(1/2)*(a*x+1)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*d^(1/2)*x/c^(1/2))/c^(3/2
)/d^(1/2)+1/4*a*ln(-a^2*x^2+1)/c/(a^2*c+d)-1/4*a*ln(d*x^2+c)/c/(a^2*c+d)+1
/8*I*polylog(2,a*(c^(1/2)-I*d^(1/2)*x)/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1
/2)-1/8*I*polylog(2,a*(c^(1/2)-I*d^(1/2)*x)/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)
/d^(1/2)+1/8*I*polylog(2,a*(c^(1/2)+I*d^(1/2)*x)/(a*c^(1/2)-I*d^(1/2)))/c^
(3/2)/d^(1/2)-1/8*I*polylog(2,a*(c^(1/2)+I*d^(1/2)*x)/(a*c^(1/2)+I*d^(1/2)
))/c^(3/2)/d^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.81 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx$$

$$= \frac{a\left(-\frac{2\log\left(1+\frac{(a^2c+d)\cosh(2\operatorname{arctanh}(ax))}{a^2c-d}\right)}{a^2c+d}+\frac{2i\arccos\left(\frac{-a^2c+d}{a^2c+d}\right)\arctan\left(\frac{adx}{\sqrt{a^2cd}}\right)-4\arctan\left(\frac{ac}{\sqrt{a^2cdx}}\right)\operatorname{arctanh}(ax)+\left(\arccos\left(\frac{-a^2}{a^2c}\right.\right.}{}\right.}{}$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

output
```
(a*((-2*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)])/(a^2*c +
d) + ((2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d
]] - 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] + (ArcCos[(-(a^2*c) +
d)/(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + S
qrt[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (ArcC
os[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a
*c*(d + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x)
)] - (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*
x)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^
2*c + d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]]
)] - (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*
x)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh
[a*x])/(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]]
)] + I*(PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2
*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] - PolyLog[2, ((-(a^2
*c) + d + (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-
I)*a*c + Sqrt[a^2*c*d]*x))]))/Sqrt[a^2*c*d] + (4*ArcTanh[a*x]*Sinh[2*ArcTa
nh[a*x]])/(a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]])))/(8*c)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6538, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(c+dx^2)^2} \, dx$$

$$\Big\downarrow \; 6538$$

$$-a\int \frac{\frac{x}{c(dx^2+c)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}}}{2(1-a^2x^2)}dx+\frac{\text{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\text{arctanh}(ax)}{2c(c+dx^2)}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{2}a\int \frac{\frac{x}{c(dx^2+c)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}}}{1-a^2x^2}dx+\frac{\text{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\text{arctanh}(ax)}{2c(c+dx^2)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{1}{2}a\int\left(-\frac{x}{c(ax-1)(ax+1)\left(dx^2+c\right)}-\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}\left(a^2x^2-1\right)}\right)dx+\frac{\operatorname{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\operatorname{arctanh}(ax)}{2c\left(c+dx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a\left(-\frac{\log\left(1-a^2x^2\right)}{2c\left(a^2c+d\right)}+\frac{\log\left(c+dx^2\right)}{2c\left(a^2c+d\right)}-\frac{i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{4ac^{3/2}\sqrt{d}}+\frac{i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{c}a+i\sqrt{d}}\right)}{4ac^{3/2}\sqrt{d}}-\frac{i\operatorname{PolyL}}{}\right.$$

$$\frac{\operatorname{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\operatorname{arctanh}(ax)}{2c\left(c+dx^2\right)}$$

input

```
Int[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

output

```
(x*ArcTanh[a*x])/(2*c*(c + d*x^2)) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTanh[
a*x])/(2*c^(3/2)*Sqrt[d]) - (a*(((-1/4*I)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) + ((I
/4)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]
*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) + ((I/4)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*
Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) -
 ((I/4)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d
]*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) - Log[1 - a^2*x^2]/(2*c*(a^2*c + d)) +
Log[c + d*x^2]/(2*c*(a^2*c + d)) - ((I/4)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[
d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(a*c^(3/2)*Sqrt[d]) + ((I/4)*PolyLog[2, (
a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(a*c^(3/2)*Sqrt[d]) -
 ((I/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(
a*c^(3/2)*Sqrt[d]) + ((I/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt
[c] + I*Sqrt[d])])/(a*c^(3/2)*Sqrt[d])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. 2(430) = 860.

Time = 8.62 (sec) , antiderivative size = 1951, normalized size of antiderivative = 3.31

method	result	size
risch	Expression too large to display	1951
derivativedivides	Expression too large to display	2227
default	Expression too large to display	2227

input
```
int(arctanh(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^2*ln(a*x+1)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^
(1/2)-d*(a*x+1)+d)/(a*(-c*d)^(1/2)+d))*d^2*x^2-1/8*a^2*ln(a*x+1)/c/(a^2*c+
d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+d*(a*x+1)-d)/(a*(-c*d
)^(1/2)-d))*d^2*x^2+1/8*a^2*ln(-a*x+1)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d
)^(1/2)*ln((a*(-c*d)^(1/2)-d*(-a*x+1)+d)/(a*(-c*d)^(1/2)+d))*d^2*x^2-1/8*a
^2*ln(-a*x+1)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2
)+d*(-a*x+1)-d)/(a*(-c*d)^(1/2)-d))*d^2*x^2+1/8*a^4*ln(a*x+1)/(a^2*c+d)/(a
^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-d*(a*x+1)+d)/(a*(-c*d)^(1/
2)+d))*d*x^2-1/8*a^4*ln(a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln
((a*(-c*d)^(1/2)+d*(a*x+1)-d)/(a*(-c*d)^(1/2)-d))*d*x^2+1/8*a^4*ln(-a*x+1)
/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-d*(-a*x+1)+d)
/(a*(-c*d)^(1/2)+d))*d*x^2-1/8*a^4*ln(-a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)/
(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+d*(-a*x+1)-d)/(a*(-c*d)^(1/2)-d))*d*x^2+1/
4*a^4*ln(a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)*x-1/4*a^4*ln(-a*x+1)/(a^2*c+d)
/(a^2*d*x^2+a^2*c)*x-1/8*a/c/(a^2*c+d)*ln((-a*x+1)^2*d+a^2*c-2*d*(-a*x+1)+
d)-1/4*a^2/(a^2*c+d)/(c*d)^(1/2)*arctan(1/2*(2*d*(-a*x+1)-2*d)/a/(c*d)^(1/
2))+1/4*a^3*ln(-a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)-1/8*a/c/(a^2*c+d)*ln((a
*x+1)^2*d+a^2*c-2*d*(a*x+1)+d)-1/4*a^2/(a^2*c+d)/(c*d)^(1/2)*arctan(1/2*(2
*d*(a*x+1)-2*d)/a/(c*d)^(1/2))+1/4*a^3*ln(a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*
c)+1/8/c/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-d*(-a*x+1)+d)/(a*(-c*d)^(1/...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{(dx^2+c)^2}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^2}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**2,x)
```

output
```
Integral(atanh(a*x)/(c + d*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{cdx^2+c^2} + \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c}\right)\operatorname{artanh}(ax) - \frac{\left(2\,acd\log\left(dx^2+c\right) - 2\,acd\log\left(ax+1\right) - 2\,acd\log\left(ax-1\right) + \left(\left(a^2c+d\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2+2\,a\cdots}{a^2c+\cdots}\right)\right.\right.}{}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arctanh(a*x)
 - 1/8*(2*a*c*d*log(d*x^2 + c) - 2*a*c*d*log(a*x + 1) - 2*a*c*d*log(a*x -
1) + ((a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/
(a^2*c + d)) - (a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*
d*x + d)/(a^2*c + d)) + (I*a^2*c + I*d)*dilog((a^2*c + a*d*x - (I*a^2*x -
I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (I*a^2*c + I*
d)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*
sqrt(c)*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilog((a^2*c + a*d*x + (I*a^2*x -
 I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + (-I*a^2*c -
I*d)*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*
a*sqrt(c)*sqrt(d) - d)) - ((a^2*c + d)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)
/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - (a^2*c + d)*arctan2((a^2*x - a)*s
qrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqr
t(c)*sqrt(d))*a/(a^3*c^3*d + a*c^2*d^2)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{artanh}(ax)}{(dx^2+c)^2}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(d\,x^2+c)^2}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^2,x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(atanh(a*x)/(d*x^2+c)^2,x)
```

output

```
(atanh(a*x)**2*a**3*c**2 + atanh(a*x)**2*a**3*c*d*x**2 + atanh(a*x)**2*a*c
*d + atanh(a*x)**2*a*d**2*x**2 - 2*atanh(a*x)*a**2*c*d*x - 2*atanh(a*x)*d*
*2*x + 2*int((atanh(a*x)*x**2)/(a**4*c**3*x**2 + 2*a**4*c**2*d*x**4 + a**4
*c*d**2*x**6 - a**2*c**3 - 3*a**2*c**2*d*x**2 - 3*a**2*c*d**2*x**4 - a**2*
d**3*x**6 + c**2*d + 2*c*d**2*x**2 + d**3*x**4),x)*a**8*c**5 + 2*int((atan
h(a*x)*x**2)/(a**4*c**3*x**2 + 2*a**4*c**2*d*x**4 + a**4*c*d**2*x**6 - a**
2*c**3 - 3*a**2*c**2*d*x**2 - 3*a**2*c*d**2*x**4 - a**2*d**3*x**6 + c**2*d
 + 2*c*d**2*x**2 + d**3*x**4),x)*a**8*c**4*d*x**2 + 4*int((atanh(a*x)*x**2
)/(a**4*c**3*x**2 + 2*a**4*c**2*d*x**4 + a**4*c*d**2*x**6 - a**2*c**3 - 3*
a**2*c**2*d*x**2 - 3*a**2*c*d**2*x**4 - a**2*d**3*x**6 + c**2*d + 2*c*d**2
*x**2 + d**3*x**4),x)*a**6*c**4*d + 4*int((atanh(a*x)*x**2)/(a**4*c**3*x**
2 + 2*a**4*c**2*d*x**4 + a**4*c*d**2*x**6 - a**2*c**3 - 3*a**2*c**2*d*x**2
 - 3*a**2*c*d**2*x**4 - a**2*d**3*x**6 + c**2*d + 2*c*d**2*x**2 + d**3*x**
4),x)*a**6*c**3*d**2*x**2 - 4*int((atanh(a*x)*x**2)/(a**4*c**3*x**2 + 2*a*
*4*c**2*d*x**4 + a**4*c*d**2*x**6 - a**2*c**3 - 3*a**2*c**2*d*x**2 - 3*a**
2*c*d**2*x**4 - a**2*d**3*x**6 + c**2*d + 2*c*d**2*x**2 + d**3*x**4),x)*a*
*2*c**2*d**3 - 4*int((atanh(a*x)*x**2)/(a**4*c**3*x**2 + 2*a**4*c**2*d*x**
4 + a**4*c*d**2*x**6 - a**2*c**3 - 3*a**2*c**2*d*x**2 - 3*a**2*c*d**2*x**4
 - a**2*d**3*x**6 + c**2*d + 2*c*d**2*x**2 + d**3*x**4),x)*a**2*c*d**4*x**
2 - 2*int((atanh(a*x)*x**2)/(a**4*c**3*x**2 + 2*a**4*c**2*d*x**4 + a**4...
```

3.504 $\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 657

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3} \, dx = \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x\operatorname{arctanh}(ax)}{4c(c+dx^2)^2} + \frac{3x\operatorname{arctanh}(ax)}{8c^2(c+dx^2)} + \frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\operatorname{arctanh}(ax)}{8c^{5/2}\sqrt{d}} + \frac{3i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\log\left(-\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}} + \frac{3i\log\left(\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}} + \frac{a(5a^2c+3d)\log(1-a^2x^2)}{16c^2(a^2c+d)^2} - \frac{a(5a^2c+3d)\log(c+dx^2)}{16c^2(a^2c+d)^2} + \frac{3i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}} + \frac{3i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}$$

output
```
1/8*a/c/(a^2*c+d)/(d*x^2+c)+1/4*x*arctanh(a*x)/c/(d*x^2+c)^2+3/8*x*arctanh
(a*x)/c^2/(d*x^2+c)+3/8*arctan(d^(1/2)*x/c^(1/2))*arctanh(a*x)/c^(5/2)/d^(
1/2)+3/32*I*ln(d^(1/2)*(-a*x+1)/(I*a*c^(1/2)+d^(1/2)))*ln(1-I*d^(1/2)*x/c^
(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-d^(1/2)*(a*x+1)/(I*a*c^(1/2)-d^(1/2)))*l
n(1-I*d^(1/2)*x/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-d^(1/2)*(-a*x+1)/(I*a*
c^(1/2)-d^(1/2)))*ln(1+I*d^(1/2)*x/c^(1/2))/c^(5/2)/d^(1/2)+3/32*I*ln(d^(1
/2)*(a*x+1)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*d^(1/2)*x/c^(1/2))/c^(5/2)/d^(1/
2)+1/16*a*(5*a^2*c+3*d)*ln(-a^2*x^2+1)/c^2/(a^2*c+d)^2-1/16*a*(5*a^2*c+3*d
)*ln(d*x^2+c)/c^2/(a^2*c+d)^2+3/32*I*polylog(2,a*(c^(1/2)-I*d^(1/2)*x)/(a*
c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)-I*d^(1/2)*
x)/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)+I*d^
(1/2)*x)/(a*c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2
)+I*d^(1/2)*x)/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1541 vs. 2(657) = 1314.

Time = 9.09 (sec) , antiderivative size = 1541, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

output
```
(a*(-10*a^2*c*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)] - 6*
d*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)] - (3*d*(a^2*c +
d)*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]
] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c) + d
)/(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sq
rt[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (ArcCo
s[(-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*
c*(d + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))
] + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x
)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2
*c + d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])
] + (ArcCos[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x
)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[
a*x])/(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])
] + I*(-PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2
*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((-(a^2
*c) + d + (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-
I)*a*c + Sqrt[a^2*c*d]*x))])))/Sqrt[a^2*c*d] - (3*Sqrt[a^2*c*d]*(a^2*c + d
)*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]
 + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c) +...
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6538, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{6538}$$

$$-a\int \frac{\frac{3dx^3+5cx}{c^2(dx^2+c)^2}+\frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}}{8(1-a^2x^2)}dx+\frac{3\text{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\text{arctanh}(ax)}{8c^2(c+dx^2)}+\frac{x\text{arctanh}(ax)}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{8}a\int \frac{\frac{3dx^3+5cx}{c^2(dx^2+c)^2}+\frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}}{1-a^2x^2}dx+\frac{3\text{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\text{arctanh}(ax)}{8c^2(c+dx^2)}+\frac{x\text{arctanh}(ax)}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 7276$$

$$-\frac{1}{8}a\int \left(-\frac{x(3dx^2+5c)}{c^2(a^2x^2-1)(dx^2+c)^2}-\frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}(a^2x^2-1)}\right)dx+\frac{3\text{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\text{arctanh}(ax)}{8c^2(c+dx^2)}+\frac{x\text{arctanh}(ax)}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{8}a\left(-\frac{(5a^2c+3d)\log(1-a^2x^2)}{2c^2(a^2c+d)^2}+\frac{(5a^2c+3d)\log(c+dx^2)}{2c^2(a^2c+d)^2}-\frac{1}{c(a^2c+d)(c+dx^2)}-\frac{3i\,\text{PolyLog}\left(2,\frac{a(\sqrt{c}-}{a\sqrt{c}}\right.}{4ac^{5/2}\sqrt{d}}\right.$$

$$\frac{3\text{arctanh}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\text{arctanh}(ax)}{8c^2(c+dx^2)}+\frac{x\text{arctanh}(ax)}{4c(c+dx^2)^2}$$

input
```
Int[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

output

```
(x*ArcTanh[a*x])/(4*c*(c + d*x^2)^2) + (3*x*ArcTanh[a*x])/(8*c^2*(c + d*x^
2)) + (3*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTanh[a*x])/(8*c^(5/2)*Sqrt[d]) - (
a*(-(1/(c*(a^2*c + d)*(c + d*x^2))) - (((3*I)/4)*Log[(Sqrt[d]*(1 - a*x))/(
I*a*Sqrt[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(5/2)*Sqrt[d]
) + (((3*I)/4)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 -
 (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(5/2)*Sqrt[d]) + (((3*I)/4)*Log[-((Sqrt[d]*(
1 - a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(
5/2)*Sqrt[d]) - (((3*I)/4)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])
]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(5/2)*Sqrt[d]) - ((5*a^2*c + 3*d)*L
og[1 - a^2*x^2])/(2*c^2*(a^2*c + d)^2) + ((5*a^2*c + 3*d)*Log[c + d*x^2])/
(2*c^2*(a^2*c + d)^2) - (((3*I)/4)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/
(a*Sqrt[c] - I*Sqrt[d])])/(a*c^(5/2)*Sqrt[d]) + (((3*I)/4)*PolyLog[2, (a*(
Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(a*c^(5/2)*Sqrt[d]) - ((
(3*I)/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/
(a*c^(5/2)*Sqrt[d]) + (((3*I)/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a
*Sqrt[c] + I*Sqrt[d])])/(a*c^(5/2)*Sqrt[d])))/8
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6538

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4046 vs. 2(493) = 986.

Time = 2.29 (sec) , antiderivative size = 4047, normalized size of antiderivative = 6.16

method	result	size
derivativedivides	Expression too large to display	4047
default	Expression too large to display	4047
risch	Expression too large to display	4564

input
```
int(arctanh(a*x)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-3/32*((-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d-(-a^2*c*d)^(1/2)*d)/c^3/(a^4*
c^2+2*a^2*c*d+d^2)^2*d^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*
c-2*(-a^2*c*d)^(1/2)+d))-3/16*(-a^2*c*d)^(1/2)/c*a^4/d/(a^4*c^2+2*a^2*c*d+
d^2)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*
d)^(1/2)+d))-3/16*((-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d-(-a^2*c*d)^(1/2)*d)/c^
2/(a^4*c^2+2*a^2*c*d+d^2)^2*a^2*d*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-
a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)+3/16*(c*d)^(1/2)*d^2/c^3*a*arcta
n(1/4*(2*(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(c*d)^(1/2))/(a^2
*c+d)/(a^4*c^2+2*a^2*c*d+d^2)+5/16*(c*d)^(1/2)/c^2*d*a^3*arctan(1/4*(2*(a^
2*c+d)*(a*x+1)^2/(-a^2*x^2+1)+2*a^2*c-2*d)/a/(c*d)^(1/2))/(a^2*c+d)/(a^4*c
^2+2*a^2*c*d+d^2)-3/8*(a^2*c-2*(-a^2*c*d)^(1/2)-d)/c^2/(a^4*c^2+2*a^2*c*d+
d^2)^2*a^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2
)+d))*d^2*arctanh(a*x)-3/4*(a^2*c-2*(-a^2*c*d)^(1/2)-d)/c/(a^4*c^2+2*a^2*c
*d+d^2)^2*a^4*d*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)
^(1/2)+d))*arctanh(a*x)+3/8*(a^2*c-2*(-a^2*c*d)^(1/2)-d)/(a^4*c^2+2*a^2*c*
d+d^2)^2*a^6*arctanh(a*x)^2-5/16/(a^4*c^2+2*a^2*c*d+d^2)*a^6/(a^2*c+d)*ln(
a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*(a*x+1)^2/(-a^2*x^2+1)*a^2*c+d*(a*x+1)^4/
(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)+5/4/(a^4*c^2+2*a^2*c*d+
d^2)*a^6/(a^2*c+d)*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/16*(a^2*c-2*(-a^2*c*d)
^(1/2)-d)/(a^4*c^2+2*a^2*c*d+d^2)^2*a^6*polylog(2,(a^2*c+d)*(a*x+1)^2/(...
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)}{(dx^2+c)^3}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(463) = 926$.

Time $= 0.22$ (sec) , antiderivative size $= 1087$, normalized size of antiderivative $= 1.65$

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*d*x^3 + 5*c*x)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4) + 3*arctan(d*x/sq
rt(c*d))/(sqrt(c*d)*c^2))*arctanh(a*x) + 1/32*(4*a^3*c^3*d + 4*a*c^2*d^2 -
 3*((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*
arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) - (a^
4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(
sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - (-I*a^4*c^
3 - 2*I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d - 2*I*a^2*c*d^2 - I*d^3)*x^2)*
dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqr
t(c)*sqrt(d) - d)) - (-I*a^4*c^3 - 2*I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d
 - 2*I*a^2*c*d^2 - I*d^3)*x^2)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt
(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) - (I*a^4*c^3 + 2*I*a^2*c
^2*d + I*c*d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2 + I*d^3)*x^2)*dilog((a^2*c +
 a*d*x + (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) -
 d)) - (I*a^4*c^3 + 2*I*a^2*c^2*d + I*c*d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2
 + I*d^3)*x^2)*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^
2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) - ((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4
*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*
c + d), (a*d*x + d)/(a^2*c + d)) - (a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c
^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x - a)*sqrt(c)*sqrt(d)/(a^2*c
+ d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d) - 2*(5...
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3}\,dx = \int \frac{\operatorname{artanh}(ax)}{(dx^2+c)^3}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3}\,dx=\int \frac{\mathrm{atanh}(a\,x)}{(d\,x^2+c)^3}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^3,x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^3}\,dx=\text{too large to display}$$

input
```
int(atanh(a*x)/(d*x^2+c)^3,x)
```

output

```
(6*atanh(a*x)**2*a**7*c**5 + 12*atanh(a*x)**2*a**7*c**4*d*x**2 + 6*atanh(a
*x)**2*a**7*c**3*d**2*x**4 + 12*atanh(a*x)**2*a**5*c**4*d + 24*atanh(a*x)*
*2*a**5*c**3*d**2*x**2 + 12*atanh(a*x)**2*a**5*c**2*d**3*x**4 + 6*atanh(a*
x)**2*a**3*c**3*d**2 + 12*atanh(a*x)**2*a**3*c**2*d**3*x**2 + 6*atanh(a*x)
**2*a**3*c*d**4*x**4 - 24*atanh(a*x)*a**6*c**4*d*x - 12*atanh(a*x)*a**6*c*
*3*d**2*x**3 - 52*atanh(a*x)*a**4*c**3*d**2*x - 24*atanh(a*x)*a**4*c**2*d*
*3*x**3 - 32*atanh(a*x)*a**2*c**2*d**3*x - 12*atanh(a*x)*a**2*c*d**4*x**3
- 4*atanh(a*x)*c*d**4*x + 36*int((atanh(a*x)*x**2)/(3*a**6*c**5*x**2 + 9*a
**6*c**4*d*x**4 + 9*a**6*c**3*d**2*x**6 + 3*a**6*c**2*d**3*x**8 - 3*a**4*c
**5 - 15*a**4*c**4*d*x**2 - 27*a**4*c**3*d**2*x**4 - 21*a**4*c**2*d**3*x**
6 - 6*a**4*c*d**4*x**8 + 6*a**2*c**4*d + 17*a**2*c**3*d**2*x**2 + 15*a**2*
c**2*d**3*x**4 + 3*a**2*c*d**4*x**6 - a**2*d**5*x**8 + c**3*d**2 + 3*c**2*
d**3*x**2 + 3*c*d**4*x**4 + d**5*x**6),x)*a**14*c**10 + 72*int((atanh(a*x)
*x**2)/(3*a**6*c**5*x**2 + 9*a**6*c**4*d*x**4 + 9*a**6*c**3*d**2*x**6 + 3*
a**6*c**2*d**3*x**8 - 3*a**4*c**5 - 15*a**4*c**4*d*x**2 - 27*a**4*c**3*d**
2*x**4 - 21*a**4*c**2*d**3*x**6 - 6*a**4*c*d**4*x**8 + 6*a**2*c**4*d + 17*
a**2*c**3*d**2*x**2 + 15*a**2*c**2*d**3*x**4 + 3*a**2*c*d**4*x**6 - a**2*d
**5*x**8 + c**3*d**2 + 3*c**2*d**3*x**2 + 3*c*d**4*x**4 + d**5*x**6),x)*a*
*14*c**9*d*x**2 + 36*int((atanh(a*x)*x**2)/(3*a**6*c**5*x**2 + 9*a**6*c**4
*d*x**4 + 9*a**6*c**3*d**2*x**6 + 3*a**6*c**2*d**3*x**8 - 3*a**4*c**5 -...
```

3.505 $\int \frac{\operatorname{arctanh}(bx)}{1-x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 171

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2} \, dx = \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx) - \frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx) - \frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx) + \frac{1}{4}\log\left(-\frac{b(1+x)}{1-b}\right)\log(1+bx) + \frac{1}{4}\operatorname{PolyLog}\left(2,\frac{1-bx}{1-b}\right) - \frac{1}{4}\operatorname{PolyLog}\left(2,\frac{1-bx}{1+b}\right) + \frac{1}{4}\operatorname{PolyLog}\left(2,\frac{1+bx}{1-b}\right) - \frac{1}{4}\operatorname{PolyLog}\left(2,\frac{1+bx}{1+b}\right)$$

output
```
1/4*ln(-b*(1-x)/(1-b))*ln(-b*x+1)-1/4*ln(b*(1+x)/(1+b))*ln(-b*x+1)-1/4*ln(
b*(1-x)/(1+b))*ln(b*x+1)+1/4*ln(-b*(1+x)/(1-b))*ln(b*x+1)+1/4*polylog(2,(-
b*x+1)/(1-b))-1/4*polylog(2,(-b*x+1)/(1+b))+1/4*polylog(2,(b*x+1)/(1-b))-1
/4*polylog(2,(b*x+1)/(1+b))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.37

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx =$$

$$-\frac{b\left(2i\arccos\left(\frac{1+b^2}{1-b^2}\right)\arctan\left(\frac{bx}{\sqrt{-b^2}}\right)-4\arctan\left(\frac{\sqrt{-b^2}}{bx}\right)\operatorname{arctanh}(bx)-\left(\arccos\left(\frac{1+b^2}{1-b^2}\right)-2\arctan\left(\frac{bx}{\sqrt{-b^2}}\right.\right.\right.}{}$$

input
```
Integrate[ArcTanh[b*x]/(1 - x^2),x]
```

output
```
-1/4*(b*((2*I)*ArcCos[(1 + b^2)/(1 - b^2)]*ArcTan[(b*x)/Sqrt[-b^2]] - 4*Ar
cTan[Sqrt[-b^2]/(b*x)]*ArcTanh[b*x] - (ArcCos[(1 + b^2)/(1 - b^2)] - 2*Arc
Tan[(b*x)/Sqrt[-b^2]])*Log[(2*b*(-I + Sqrt[-b^2])*(-1 + b*x))/((-1 + b^2)*
((-I)*b + Sqrt[-b^2]*x))] - (ArcCos[(1 + b^2)/(1 - b^2)] + 2*ArcTan[(b*x)/
Sqrt[-b^2]])*Log[(2*b*(I + Sqrt[-b^2])*(1 + b*x))/((-1 + b^2)*((-I)*b + Sq
rt[-b^2]*x))] + (ArcCos[(1 + b^2)/(1 - b^2)] - 2*(ArcTan[Sqrt[-b^2]/(b*x)]
 + ArcTan[(b*x)/Sqrt[-b^2]]))*Log[(Sqrt[2]*Sqrt[-b^2])/(Sqrt[-1 + b^2]*E^A
rcTanh[b*x]*Sqrt[1 + b^2 + (-1 + b^2)*Cosh[2*ArcTanh[b*x]]])] + (ArcCos[(1
 + b^2)/(1 - b^2)] + 2*(ArcTan[Sqrt[-b^2]/(b*x)] + ArcTan[(b*x)/Sqrt[-b^2]
]))*Log[(Sqrt[2]*Sqrt[-b^2]*E^ArcTanh[b*x])/(Sqrt[-1 + b^2]*Sqrt[1 + b^2 +
 (-1 + b^2)*Cosh[2*ArcTanh[b*x]]])] + I*(PolyLog[2, ((1 + b^2 - (2*I)*Sqrt
[-b^2])*(b - I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x))] - PolyLog
[2, ((1 + b^2 + (2*I)*Sqrt[-b^2])*(b - I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I
*Sqrt[-b^2]*x))])))/Sqrt[-b^2]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6534, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2} \, dx$$

$$\Big\downarrow \text{6534}$$

$$\frac{1}{2} \int \frac{\log(bx+1)}{1-x^2}dx - \frac{1}{2} \int \frac{\log(1-bx)}{1-x^2}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2} \int \left(\frac{\log(bx+1)}{2(1-x)} + \frac{\log(bx+1)}{2(x+1)}\right) dx - \frac{1}{2} \int \left(\frac{\log(1-bx)}{2(1-x)} + \frac{\log(1-bx)}{2(x+1)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{1-bx}{1-b}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,\frac{1-bx}{b+1}\right) + \frac{1}{2}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx) - \frac{1}{2}\log\left(\frac{b(x+1)}{b+1}\right)\log($$
$$\frac{1}{2}\left(\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{bx+1}{1-b}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,\frac{bx+1}{b+1}\right) - \frac{1}{2}\log\left(\frac{b(1-x)}{b+1}\right)\log(bx+1) + \frac{1}{2}\log\left(-\frac{b(x+1)}{1-b}\right)\log($$

input
```
Int[ArcTanh[b*x]/(1 - x^2),x]
```

output
```
((Log[-((b*(1 - x))/(1 - b))]*Log[1 - b*x])/2 - (Log[(b*(1 + x))/(1 + b)]*
Log[1 - b*x])/2 + PolyLog[2, (1 - b*x)/(1 - b)]/2 - PolyLog[2, (1 - b*x)/(
1 + b)]/2)/2 + (-1/2*(Log[(b*(1 - x))/(1 + b)]*Log[1 + b*x]) + (Log[-((b*(
1 + x))/(1 - b))]*Log[1 + b*x])/2 + PolyLog[2, (1 + b*x)/(1 - b)]/2 - Poly
Log[2, (1 + b*x)/(1 + b)]/2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6534
```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + c*x]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - c*x]/(d + e*x^2)
, x], x] /; FreeQ[{c, d, e}, x]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\ln(-bx+1)\ln\left(\frac{-bx-b}{-b-1}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{-bx-b}{-b-1}\right)}{4}+\frac{\ln(-bx+1)\ln\left(\frac{-bx+b}{-1+b}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{-bx+b}{-1+b}\right)}{4}-\frac{\ln(bx+1)\ln\left(\frac{bx-b}{-b-1}\right)}{4}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(bx)b\ln(bx+b)}{2}-\frac{\operatorname{arctanh}(bx)b\ln(-bx+b)}{2}-\frac{b^2\left(\frac{\frac{\operatorname{dilog}\left(\frac{-bx+1}{1-b}\right)}{2}+\frac{\ln(-bx+b)\ln\left(\frac{-bx+1}{1-b}\right)}{2}}{b}-\frac{\frac{\operatorname{dilog}\left(\frac{-bx-1}{-b-1}\right)}{2}-\frac{\ln(-bx+b)\ln}{2}}{}\right.}{}}{b}$
default	$\frac{\frac{\operatorname{arctanh}(bx)b\ln(bx+b)}{2}-\frac{\operatorname{arctanh}(bx)b\ln(-bx+b)}{2}-\frac{b^2\left(\frac{\frac{\operatorname{dilog}\left(\frac{-bx+1}{1-b}\right)}{2}+\frac{\ln(-bx+b)\ln\left(\frac{-bx+1}{1-b}\right)}{2}}{b}-\frac{\frac{\operatorname{dilog}\left(\frac{-bx-1}{-b-1}\right)}{2}-\frac{\ln(-bx+b)\ln}{2}}{}\right.}{}}{b}$
parts	$\operatorname{arctanh}(x)\operatorname{arctanh}(bx)-b\left(\frac{\operatorname{arctanh}(x)\ln(bx+1)}{2b}-\frac{\operatorname{arctanh}(x)\ln(bx-1)}{2b}+\frac{\ln\left(\frac{bx+b}{1+b}\right)\ln(bx-1)}{4b}+\right.$

input
```
int(arctanh(b*x)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(-b*x+1)*ln((-b*x-b)/(-b-1))-1/4*dilog((-b*x-b)/(-b-1))+1/4*ln(-b*x
+1)*ln((-b*x+b)/(-1+b))+1/4*dilog((-b*x+b)/(-1+b))-1/4*ln(b*x+1)*ln((b*x-b
)/(-b-1))-1/4*dilog((b*x-b)/(-b-1))+1/4*ln(b*x+1)*ln((b*x+b)/(-1+b))+1/4*d
ilog((b*x+b)/(-1+b))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx=\int -\frac{\operatorname{artanh}(bx)}{x^2-1}\,dx$$

input
```
integrate(arctanh(b*x)/(-x^2+1),x, algorithm="fricas")
```

output
```
integral(-arctanh(b*x)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx = -\int \frac{\operatorname{atanh}(bx)}{x^2-1}\,dx$$

input
```
integrate(atanh(b*x)/(-x**2+1),x)
```

output
```
-Integral(atanh(b*x)/(x**2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx$$
$$= \frac{1}{4}b\left(\frac{\log(x+1)\log\left(-\frac{bx+b}{b+1}+1\right)+\operatorname{Li}_2\left(\frac{bx+b}{b+1}\right)}{b} + \frac{\log(x-1)\log\left(\frac{bx-b}{b+1}+1\right)+\operatorname{Li}_2\left(-\frac{bx-b}{b+1}\right)}{b} - \frac{\log(x+1)\operatorname{lo}}{}\right.$$
$$+ \frac{1}{2}(\log(x+1)-\log(x-1))\operatorname{artanh}(bx)$$

input
```
integrate(arctanh(b*x)/(-x^2+1),x, algorithm="maxima")
```

output
```
1/4*b*((log(x + 1)*log(-(b*x + b)/(b + 1) + 1) + dilog((b*x + b)/(b + 1)))
/b + (log(x - 1)*log((b*x - b)/(b + 1) + 1) + dilog(-(b*x - b)/(b + 1)))/b
 - (log(x + 1)*log(-(b*x + b)/(b - 1) + 1) + dilog((b*x + b)/(b - 1)))/b -
 (log(x - 1)*log((b*x - b)/(b - 1) + 1) + dilog(-(b*x - b)/(b - 1)))/b) +
1/2*(log(x + 1) - log(x - 1))*arctanh(b*x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx=\int -\frac{\operatorname{artanh}(bx)}{x^2-1}\,dx$$

input
```
integrate(arctanh(b*x)/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-arctanh(b*x)/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx=-\int \frac{\operatorname{atanh}(b\,x)}{x^2-1}\,dx$$

input
```
int(-atanh(b*x)/(x^2 - 1),x)
```

output
```
-int(atanh(b*x)/(x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(bx)}{1-x^2}\,dx=\frac{atanh(bx)^2\,b}{2}-\left(\int \frac{atanh(bx)}{b^2x^4-b^2x^2-x^2+1}dx\right)b^2+\int \frac{atanh(bx)}{b^2x^4-b^2x^2-x^2+1}dx$$

input
```
int(atanh(b*x)/(-x^2+1),x)
```

output
```
(atanh(b*x)**2*b - 2*int(atanh(b*x)/(b**2*x**4 - b**2*x**2 - x**2 + 1),x)*
b**2 + 2*int(atanh(b*x)/(b**2*x**4 - b**2*x**2 - x**2 + 1),x))/2
```

3.506 $\int \frac{1}{(a-ax^2)(b-2b\,\mathbf{arctanh}(x))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{1}{(a-ax^2)(b-2b\text{arctanh}(x))}\,dx = -\frac{\log(1-2\text{arctanh}(x))}{2ab}$$

output
```
-1/2*ln(1-2*arctanh(x))/a/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-ax^2)(b-2b\text{arctanh}(x))}\,dx = -\frac{\log(-1+2\text{arctanh}(x))}{2ab}$$

input
```
Integrate[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

output
```
-1/2*Log[-1 + 2*ArcTanh[x]]/(a*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6508}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-ax^2)(b-2b\text{arctanh}(x))}dx$$

$$\Big\downarrow \text{ 6508}$$

$$-\frac{\log(1-2\text{arctanh}(x))}{2ab}$$

input
```
Int[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

output
```
-1/2*Log[1 - 2*ArcTanh[x]]/(a*b)
```

Defintions of rubi rules used

rule 6508
```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$-\frac{\ln\left(\operatorname{arctanh}(x)-\frac{1}{2}\right)}{2ba}$	14
default	$-\frac{\ln(2\operatorname{arctanh}(x)b-b)}{2ab}$	19
risch	$-\frac{\ln(-1+\ln(1+x)-\ln(1-x))}{2ba}$	24

input
```
int(1/(-a*x^2+a)/(b-2*arctanh(x)*b),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(arctanh(x)-1/2)/b/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a-ax^2)\,(b-2b\text{arctanh}(x))}\,dx = -\frac{\log\left(\log\left(-\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="fricas")
```

output
```
-1/2*log(log(-(x + 1)/(x - 1)) - 1)/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a-ax^2)\,(b-2b\text{arctanh}(x))}\,dx = -\frac{\log\left(\text{atanh}\,(x)-\frac{1}{2}\right)}{2ab}$$

input
```
integrate(1/(-a*x**2+a)/(b-2*b*atanh(x)),x)
```

output
```
-log(atanh(x) - 1/2)/(2*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a-ax^2)\,(b-2b\text{arctanh}(x))}\,dx = -\frac{\log\left(-\log\left(x+1\right)+\log\left(-x+1\right)+1\right)}{2\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="maxima")
```

output
```
-1/2*log(-log(x + 1) + log(-x + 1) + 1)/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a-ax^2)\,(b-2b\text{arctanh}(x))}\,dx = -\frac{\log\left(\frac{1}{4}\,\pi^2(\text{sgn}(x-1)\,\text{sgn}(-x-1)-1)^2+\left(\log\left(\frac{|-x-1|}{|x-1|}\right)-1\right)^2\right)}{4\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="giac")
```

output
```
-1/4*log(1/4*pi^2*(sgn(x - 1)*sgn(-x - 1) - 1)^2 + (log(abs(-x - 1)/abs(x
- 1)) - 1)^2)/(a*b)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-ax^2)\,(b-2b\mathrm{arctanh}(x))}\,dx = -\frac{\ln\left(2\,\mathrm{atanh}(x)-1\right)}{2\,a\,b}$$

input
```
int(1/((a - a*x^2)*(b - 2*b*atanh(x))),x)
```

output
```
-log(2*atanh(x) - 1)/(2*a*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-ax^2)\,(b-2b\mathrm{arctanh}(x))}\,dx = -\frac{\log(2atanh(x)-1)}{2ab}$$

input
```
int(1/(-a*x^2+a)/(b-2*b*atanh(x)),x)
```

output
```
( - log(2*atanh(x) - 1))/(2*a*b)
```

3.507 $\int \sqrt{c+dx^2}\textbf{arctanh}(ax)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx^2}\operatorname{arctanh}(ax)\,dx = \operatorname{Int}\left(\sqrt{c+dx^2}\operatorname{arctanh}(ax),x\right)$$

output
```
Defer(Int)((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx^2}\operatorname{arctanh}(ax)\,dx = \int \sqrt{c+dx^2}\operatorname{arctanh}(ax)\,dx$$

input
```
Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
```

output
```
Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(ax)\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{6651}$$

$$\int \text{arctanh}(ax)\sqrt{c+dx^2}dx$$

input
```
Int[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{d\,x^2+c}\,\text{arctanh}\left(ax\right)dx$$

input
```
int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

output
```
int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\text{arctanh}(ax)\,dx=\int \sqrt{dx^2+c}\,\text{artanh}\,(ax)\;dx$$

input
```
integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx^2}\text{arctanh}(ax)\,dx=\int \sqrt{c+dx^2}\,\text{atanh}\,(ax)\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*atanh(a*x),x)
```

output
```
Integral(sqrt(c + d*x**2)*atanh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\text{arctanh}(ax)\,dx=\int \sqrt{dx^2+c}\,\text{artanh}\,(ax)\;dx$$

input
```
integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\text{arctanh}(ax)\,dx = \int \sqrt{dx^2+c}\,\text{artanh}\left(ax\right)\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\text{arctanh}(ax)\,dx = \int \text{atanh}(a\,x)\,\sqrt{d\,x^2+c}\,dx$$

input
```
int(atanh(a*x)*(c + d*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)*(c + d*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx^2}\text{arctanh}(ax)\,dx=\int \sqrt{d\,x^2+c}\,atanh(ax)\,dx$$

input
```
int((d*x^2+c)^(1/2)*atanh(a*x),x)
```

output
```
int(sqrt(c + d*x**2)*atanh(a*x),x)
```

3.508 $\int \frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \text{Int}\left(\frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}},x\right)$$

output
```
Defer(Int)(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
```

output
```
Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 6651$$

$$\int \frac{\text{arctanh}(ax)}{\sqrt{c+dx^2}}dx$$

input
```
Int[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{arctanh}\,(ax)}{\sqrt{d\,x^2+c}}dx$$

input
```
int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

output
```
int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{atanh}\left(ax\right)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(atanh(a*x)/sqrt(c + d*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{artanh}\left(ax\right)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{artanh}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^(1/2),x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{atanh(ax)}{\sqrt{d\,x^2+c}}dx$$

input
```
int(atanh(a*x)/(d*x^2+c)^(1/2),x)
```

output
```
int(atanh(a*x)/sqrt(c + d*x**2),x)
```

3.509 $\int \frac{\text{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{\text{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{x\text{arctanh}(ax)}{c\sqrt{c+dx^2}} - \frac{\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

output
```
x*arctanh(a*x)/c/(d*x^2+c)^(1/2)-arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2)
)/c/(a^2*c+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \frac{\text{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{\frac{2x\text{arctanh}(ax)}{\sqrt{c+dx^2}} + \frac{\log(1-ax)+\log(1+ax)-\log\left(ac-dx+\sqrt{a^2c+d}\sqrt{c+dx^2}\right)-\log\left(ac+dx+\sqrt{a^2c+d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c+d}}}{2c}$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

output
```
((2*x*ArcTanh[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a
*c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c +
 d]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6538, 27, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6538}$$

$$\frac{x\operatorname{arctanh}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c(1-a^2x^2)\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x\operatorname{arctanh}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{(1-a^2x^2)\sqrt{dx^2+c}}dx}{c}$$

$$\Big\downarrow \text{353}$$

$$\frac{x\operatorname{arctanh}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{1}{(1-a^2x^2)\sqrt{dx^2+c}}dx^2}{2c}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\operatorname{arctanh}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{1}{-\frac{a^2x^4}{d}+\frac{a^2c}{d}+1}d\sqrt{dx^2+c}}{cd}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\operatorname{arctanh}(ax)}{c\sqrt{c+dx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

input
```
Int[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

output
```
(x*ArcTanh[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^
2*c + d]]/(c*Sqrt[a^2*c + d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Maple [F]

$$\int \frac{\operatorname{arctanh}\left(a x\right)}{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}d x$$

input
```
int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

output
```
int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(54) = 108$.

Time = 0.11 (sec) , antiderivative size = 356, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \left[\frac{2\left(a^2c+d\right)\sqrt{dx^2+c}\,x\log\left(-\frac{ax+1}{ax-1}\right)+\sqrt{a^2c+d}(dx^2+c)\log\left(\frac{a^4d^2x^4+8\,a^4c^2+8\,a^2cd+2\,(4\,a\ldots}{\ldots}\right.}{4\left(a^2c^3+c^2d+(a^2c^2d+cd^2)x^2\right)}\right.$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + sqrt(a^2
*c + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*
d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*
x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d
+ c*d^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)
) + sqrt(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt
(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(
a^2*c^3 + c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(atanh(a*x)/(c + d*x**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(54) = 108.

Time = 0.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.47

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{a^2\left(\frac{\operatorname{arsinh}\left(-\frac{2a^2c}{\sqrt{cd}|2a^2x+2a|}+\frac{2adx}{\sqrt{cd}|2a^2x+2a|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}-\frac{\operatorname{arsinh}\left(\frac{2a^2c}{\sqrt{cd}|2a^2x-2a|}+\frac{2adx}{\sqrt{cd}|2a^2x-2a|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}\right)}{2c} + \frac{x\operatorname{artanh}(ax)}{\sqrt{dx^2+c}c}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*a^2*(arcsinh(-2*a^2*c/(sqrt(c*d)*abs(2*a^2*x + 2*a)) + 2*a*d*x/(sqrt(c
*d)*abs(2*a^2*x + 2*a)))/(a^3*sqrt(c + d/a^2)) - arcsinh(2*a^2*c/(sqrt(c*d
)*abs(2*a^2*x - 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x - 2*a)))/(a^3*sqrt(
c + d/a^2)))/c + x*arctanh(a*x)/(sqrt(d*x^2 + c)*c)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{x\log\left(-\frac{ax+1}{ax-1}\right)}{2\sqrt{dx^2+c}c} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-dc}}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*log(-(a*x + 1)/(a*x - 1))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 +
c)*a/sqrt(-a^2*c - d))/(sqrt(-a^2*c - d)*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{{(d\,x^2+c)}^{3/2}}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^(3/2),x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{atanh(ax)}{\sqrt{d\,x^2+c}\,c+\sqrt{d\,x^2+c}\,d\,x^2}dx$$

input
```
int(atanh(a*x)/(d*x^2+c)^(3/2),x)
```

output
```
int(atanh(a*x)/(sqrt(c + d*x**2)*c + sqrt(c + d*x**2)*d*x**2),x)
```

3.510 $\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x\operatorname{arctanh}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x\operatorname{arctanh}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c+2d)\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}$$

output
```
1/3*a/c/(a^2*c+d)/(d*x^2+c)^(1/2)+1/3*x*arctanh(a*x)/c/(d*x^2+c)^(3/2)+2/3
*x*arctanh(a*x)/c^2/(d*x^2+c)^(1/2)-1/3*(3*a^2*c+2*d)*arctanh(a*(d*x^2+c)^
(1/2)/(a^2*c+d)^(1/2))/c^2/(a^2*c+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{\frac{2ac}{(a^2c+d)\sqrt{c+dx^2}} + \frac{2x(3c+2dx^2)\operatorname{arctanh}(ax)}{(c+dx^2)^{3/2}} + \frac{(3a^2c+2d)\log(1-ax)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d)\log(1+ax)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2}{}}{6c^2}$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

output
```
((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcTanh[a*x]
)/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + (
(3*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c
 - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c +
 2*d)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))
/(6*c^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6538, 27, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6538}$$

$$-a\int \frac{x(2dx^2+3c)}{3c^2(1-a^2x^2)(dx^2+c)^{3/2}}dx+\frac{2x\text{arctanh}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\text{arctanh}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x(2dx^2+3c)}{(1-a^2x^2)(dx^2+c)^{3/2}}dx}{3c^2}+\frac{2x\text{arctanh}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\text{arctanh}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{a\int \frac{2dx^2+3c}{(1-a^2x^2)(dx^2+c)^{3/2}}dx^2}{6c^2}+\frac{2x\text{arctanh}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\text{arctanh}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{a\left(\frac{(3a^2c+2d)\int \frac{1}{(1-a^2x^2)\sqrt{dx^2+c}}dx^2}{a^2c+d}-\frac{2c}{(a^2c+d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\text{arctanh}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\text{arctanh}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(\frac{2(3a^2c+2d)\int \frac{1}{-\frac{a^2x^4}{d}+\frac{a^2c}{d}+1}d\sqrt{dx^2+c}}{d(a^2c+d)}-\frac{2c}{(a^2c+d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\text{arctanh}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\text{arctanh}(ax)}{3c\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(\frac{2(3a^2c+2d)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{a(a^2c+d)^{3/2}}-\frac{2c}{(a^2c+d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\text{arctanh}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\text{arctanh}(ax)}{3c\,(c+dx^2)^{3/2}}$$

input
```
Int[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

output
```
(x*ArcTanh[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[
c + d*x^2]) - (a*((-2*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*(3*a^2*c + 2*d
)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(a*(a^2*c + d)^(3/2))))/(6
*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Maple [F]

$$\int \frac{\operatorname{arctanh}\left(a x\right)}{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}d x$$

input
```
int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

output
```
int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 730, normalized size of antiderivative = 5.70

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \left[\frac{(3\,a^2c^3 + (3\,a^2cd^2 + 2\,d^3)x^4 + 2\,c^2d + 2\,(3\,a^2c^2d + 2\,cd^2)x^2)\sqrt{a^2c+d}\log\left(\frac{a^4d^2x^4+8\,a\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
 2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d +
2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c +
 d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a
*c^2*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*
x^3 + 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt
(d*x^2 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d
^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*
a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*
x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d
)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3
+ 2*a*c^2*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 +
d^3)*x^3 + 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))
*sqrt(d*x^2 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*
c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(atanh(a*x)/(c + d*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{1}{6}\,a\left(\frac{\frac{ad\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^2c^2+cd)\sqrt{a^2c+d}}+\frac{2\,d}{(a^2c^2+cd)\sqrt{dx^2+c}}}{d}+\frac{2\,\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{\sqrt{a^2c+d}ac^2}\right) + \frac{1}{3}\left(\frac{2\,x}{\sqrt{dx^2+c}c^2}+\frac{x}{(dx^2+c)^{\frac{3}{2}}c}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/6*a*((a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)
*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^2 + c*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c
^2 + c*d)*sqrt(d*x^2 + c)))/d + 2*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c +
d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^2))
+ 1/3*(2*x/(sqrt(d*x^2 + c)*c^2) + x/((d*x^2 + c)^(3/2)*c))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{1}{3}\,a\left(\frac{(3\,a^2c+2\,d)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^2c^3+c^2d)\sqrt{-a^2c-d}a}+\frac{1}{(a^2c^2+cd)\sqrt{dx^2+c}}\right) + \frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*a*((3*a^2*c + 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^2*c^
3 + c^2*d)*sqrt(-a^2*c - d)*a) + 1/((a^2*c^2 + c*d)*sqrt(d*x^2 + c))) + 1/
6*x*(2*d*x^2/c^2 + 3/c)*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^(5/2),x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{atanh(ax)}{\sqrt{d\,x^2+c}\,c^2+2\sqrt{d\,x^2+c}\,cd\,x^2+\sqrt{d\,x^2+c}\,d^2x^4}dx$$

input
```
int(atanh(a*x)/(d*x^2+c)^(5/2),x)
```

output
```
int(atanh(a*x)/(sqrt(c + d*x**2)*c**2 + 2*sqrt(c + d*x**2)*c*d*x**2 + sqrt
(c + d*x**2)*d**2*x**4),x)
```

3.511 $\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x\operatorname{arctanh}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\operatorname{arctanh}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\operatorname{arctanh}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{(15a^4c^2+20a^2cd+8d^2)\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c+d)^{5/2}}$$

output
```
1/15*a/c/(a^2*c+d)/(d*x^2+c)^(3/2)+1/15*a*(7*a^2*c+4*d)/c^2/(a^2*c+d)^2/(d
*x^2+c)^(1/2)+1/5*x*arctanh(a*x)/c/(d*x^2+c)^(5/2)+4/15*x*arctanh(a*x)/c^2
/(d*x^2+c)^(3/2)+8/15*x*arctanh(a*x)/c^3/(d*x^2+c)^(1/2)-1/15*(15*a^4*c^2+
20*a^2*c*d+8*d^2)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c^3/(a^2*c+d)
^(5/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{2ac\sqrt{a^2c+d}(c+dx^2)\left(d(5c+4dx^2)+a^2c(8c+7dx^2)\right)+2(a^2c+d)^{5/2}x(15c^2+20c$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x
^2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTanh[a*x
] + (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15
*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^
2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*
Sqrt[c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log
[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(
c + d*x^2)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6538, 27, 7266, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6538}$$

$$-a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{15c^3(1-a^2x^2)(dx^2+c)^{5/2}}dx+\frac{8x\operatorname{arctanh}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\operatorname{arctanh}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\operatorname{arctanh}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{(1-a^2x^2)(dx^2+c)^{5/2}}dx}{15c^3}+\frac{8x\operatorname{arctanh}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\operatorname{arctanh}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\operatorname{arctanh}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{a\int \frac{8d^2x^4+20cdx^2+15c^2}{(1-a^2x^2)(dx^2+c)^{5/2}}dx^2}{30c^3}+\frac{8x\text{arctanh}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\text{arctanh}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\text{arctanh}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{a\int \frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c+d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\text{arctanh}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\text{arctanh}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\text{arctanh}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{a\int \left(\frac{(15c^2a^4+20cda^2+8d^2)d^2}{(ca^2+d)^2(-a^2x^4+a^2c+d)}+\frac{c(7ca^2+4d)d^2}{(ca^2+d)^2x^4}+\frac{3c^2d^2}{(ca^2+d)x^8}\right)d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\text{arctanh}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\text{arctanh}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\text{arctanh}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{c^2d^2}{x^6(a^2c+d)}-\frac{cd^2(7a^2c+4d)}{x^2(a^2c+d)^2}+\frac{d^2(15a^4c^2+20a^2cd+8d^2)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{a(a^2c+d)^{5/2}}\right)}{15c^3d^2}+\frac{8x\text{arctanh}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\text{arctanh}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\text{arctanh}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

input
```
Int[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(x*ArcTanh[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*ArcTanh[a*x])/(15*c^2*(c +
 d*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*c^3*Sqrt[c + d*x^2]) - (a*(-((c^2*
d^2)/((a^2*c + d)*x^6)) - (c*d^2*(7*a^2*c + 4*d))/((a^2*c + d)^2*x^2) + (d
^2*(15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*
c + d]])/(a*(a^2*c + d)^(5/2))))/(15*c^3*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(d\,x^{2}+c)^{\frac{7}{2}}}dx$$

input
```
int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(172) = 344$.

Time = 0.17 (sec) , antiderivative size = 1280, normalized size of antiderivative = 6.40

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5
)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*
d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d
*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*
a^2*x^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3
*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 +
3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^
5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c
^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log(-(a*x + 1)/(a*x - 1)))*
sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c
^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3
*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2
 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a
^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 2
0*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^
3)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c
- d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*
c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*
a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(...
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(atanh(a*x)/(c + d*x**2)**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(172) = 344$.

Time = 0.13 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{1}{30}a\left(\frac{\frac{3a^3d\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^4c^3+2a^2c^2d+cd^2)\sqrt{a^2c+d}}+\frac{2(3(dx^2+c)a^2d+a^2cd+d^2)}{(a^4c^3+2a^2c^2d+cd^2)(dx^2+c)^{\frac{3}{2}}}}{d}+\frac{4\left(\frac{ad\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+da}}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+da}}\right)}{(a^2c^3+c^2d)\sqrt{a^2c+d}}\right.}{d}\right.$$
$$+\frac{1}{15}\left(\frac{8x}{\sqrt{dx^2+c}c^3}+\frac{4x}{(dx^2+c)^{\frac{3}{2}}c^2}+\frac{3x}{(dx^2+c)^{\frac{5}{2}}c}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/30*a*((3*a^3*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^4*c^3 + 2*a^2*c^2*d + c*d^2)*sqrt(a^2*
c + d)) + 2*(3*(d*x^2 + c)*a^2*d + a^2*c*d + d^2)/((a^4*c^3 + 2*a^2*c^2*d
+ c*d^2)*(d*x^2 + c)^(3/2)))/d + 4*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^
2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^3 + c^2*d)*
sqrt(a^2*c + d)) + 2*d/((a^2*c^3 + c^2*d)*sqrt(d*x^2 + c)))/d + 8*log((sqr
t(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c +
d)*a))/(sqrt(a^2*c + d)*a*c^3)) + 1/15*(8*x/(sqrt(d*x^2 + c)*c^3) + 4*x/((
d*x^2 + c)^(3/2)*c^2) + 3*x/((d*x^2 + c)^(5/2)*c))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{1}{15}a\left(\frac{(15a^4c^2+20a^2cd+8d^2)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^4c^5+2a^2c^4d+c^3d^2)\sqrt{-a^2c-d}a}+\frac{7(dx^2+c)a^2c+a^2c^2+4(dx^2}{(a^4c^4+2a^2c^3d+c^2d^2)(dx^2}\right.$$
$$+\frac{\left(4x^2\left(\frac{2d^2x^2}{c^3}+\frac{5d}{c^2}\right)+\frac{15}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30(dx^2+c)^{\frac{5}{2}}}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
1/15*a*((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a
^2*c - d))/((a^4*c^5 + 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c - d)*a) + (7*(d*
x^2 + c)*a^2*c + a^2*c^2 + 4*(d*x^2 + c)*d + c*d)/((a^4*c^4 + 2*a^2*c^3*d
+ c^2*d^2)*(d*x^2 + c)^(3/2))) + 1/30*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 1
5/c)*x*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^(7/2),x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{atanh(ax)}{\sqrt{d\,x^2+c}\,c^3+3\sqrt{d\,x^2+c}\,c^2d\,x^2+3\sqrt{d\,x^2+c}\,c\,d^2x^4+\sqrt{d\,x^2+c}\,d^3x^6}dx$$

input
```
int(atanh(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(atanh(a*x)/(sqrt(c + d*x**2)*c**3 + 3*sqrt(c + d*x**2)*c**2*d*x**2 + 3
*sqrt(c + d*x**2)*c*d**2*x**4 + sqrt(c + d*x**2)*d**3*x**6),x)
```

3.512 $\int \frac{\text{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 283

$$\int \frac{\text{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x\text{arctanh}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\text{arctanh}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\text{arctanh}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\text{arctanh}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4(a^2c+d)^{7/2}}$$

output
```
1/35*a/c/(a^2*c+d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c+6*d)/c^2/(a^2*c+d)^2/
(d*x^2+c)^(3/2)+1/35*a*(19*a^4*c^2+22*a^2*c*d+8*d^2)/c^3/(a^2*c+d)^3/(d*x^
2+c)^(1/2)+1/7*x*arctanh(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arctanh(a*x)/c^2/(d
*x^2+c)^(5/2)+8/35*x*arctanh(a*x)/c^3/(d*x^2+c)^(3/2)+16/35*x*arctanh(a*x)
/c^4/(d*x^2+c)^(1/2)-1/35*(35*a^6*c^3+70*a^4*c^2*d+56*a^2*c*d^2+16*d^3)*ar
ctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c^4/(a^2*c+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{2ac\sqrt{a^2c+d}(c+dx^2)\left(3c^2(a^2c+d)^2+c(a^2c+d)\left(11a^2c+6d\right)(c+dx^2)+3(19a^4c^2\right.}{}$$

input
```
Integrate[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
(2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(1
1*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^
2)^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d
^3*x^6)*ArcTanh[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^
3)*(c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*
c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^
2*d + 56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c
+ d]*Sqrt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d
^3)*(c + d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(2
10*c^4*(a^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6538, 27, 7266, 2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}dx$$

$$\Big\downarrow \ 6538$$

$$-a\int \frac{x\left(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3\right)}{35c^4\left(1-a^2x^2\right)\left(dx^2+c\right)^{7/2}}dx+\frac{16x\operatorname{arctanh}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\operatorname{arctanh}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\operatorname{arctanh}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\operatorname{arctanh}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\int \frac{x(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3)}{(1-a^2x^2)(dx^2+c)^{7/2}}dx}{35c^4}+\frac{16x\text{arctanh}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\text{arctanh}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\text{arctanh}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\text{arctanh}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \; 7266$$

$$-\frac{a\int \frac{16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3}{(1-a^2x^2)(dx^2+c)^{7/2}}dx^2}{70c^4}+\frac{16x\text{arctanh}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\text{arctanh}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\text{arctanh}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\text{arctanh}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \; 2122$$

$$-\frac{a\int \left(\frac{5dc^3}{(ca^2+d)(dx^2+c)^{7/2}}+\frac{d(11ca^2+6d)c^2}{(ca^2+d)^2(dx^2+c)^{5/2}}+\frac{d(19c^2a^4+22cda^2+8d^2)c}{(ca^2+d)^3(dx^2+c)^{3/2}}+\frac{-35c^3a^6-70c^2da^4-56cd^2a^2-16d^3}{(ca^2+d)^3(a^2x^2-1)\sqrt{dx^2+c}}\right)dx^2}{70c^4}+\frac{16x\text{arctanh}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\text{arctanh}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\text{arctanh}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\text{arctanh}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(-\frac{2c^3}{(a^2c+d)(c+dx^2)^{5/2}}-\frac{2c^2(11a^2c+6d)}{3(a^2c+d)^2(c+dx^2)^{3/2}}-\frac{2c(19a^4c^2+22a^2cd+8d^2)}{(a^2c+d)^3\sqrt{c+dx^2}}+\frac{2(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\text{arctanh}\left(\frac{a\sqrt{c+}}{\sqrt{a^2c}}\right.}{a(a^2c+d)^{7/2}}\right.}{70c^4}+\frac{16x\text{arctanh}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\text{arctanh}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\text{arctanh}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\text{arctanh}(ax)}{7c(c+dx^2)^{7/2}}$$

input

```
Int[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

output

```
(x*ArcTanh[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcTanh[a*x])/(35*c^2*(c +
 d*x^2)^(5/2)) + (8*x*ArcTanh[a*x])/(35*c^3*(c + d*x^2)^(3/2)) + (16*x*Arc
Tanh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (a*((-2*c^3)/((a^2*c + d)*(c + d*x^2
)^(5/2)) - (2*c^2*(11*a^2*c + 6*d))/(3*(a^2*c + d)^2*(c + d*x^2)^(3/2)) -
(2*c*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/((a^2*c + d)^3*Sqrt[c + d*x^2]) +
(2*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c +
 d*x^2])/Sqrt[a^2*c + d]])/(a*(a^2*c + d)^(7/2))))/(70*c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

rule 6538
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTanh[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(247) = 494$.

Time = 0.25 (sec) , antiderivative size = 2006, normalized size of antiderivative = 7.09

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 +
70*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d
^3 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 +
 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*
a^4*c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d
^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*
x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a
^2*x^2 + 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4
*d^3 + 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^
6 + 2*(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)
*x^4 + 2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4
)*x^2 + 3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 +
 d^7)*x^7 + 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^
5 + c*d^6)*x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3
*d^4 + c^2*d^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^
4*d^3 + c^3*d^4)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12
+ 4*a^6*c^11*d + 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 +
 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9
*d^3 + 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a
^8*c^10*d^2 + 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*...
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(atanh(a*x)/(d*x**2+c)**(9/2),x)
```

output
```
Integral(atanh(a*x)/(c + d*x**2)**(9/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(247) = 494$.

Time = 0.14 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{1}{210}\,a\left(\frac{\frac{15\,a^5 d\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3)\sqrt{a^2c+d}} + \frac{2\left(15\,(dx^2+c)^2a^4d+3\,a^4c^2d+6\,a^2cd^2+3\,d^3+5\,(a^4cd+a^2d^2)\,(\right.}{(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3)(dx^2+c)^{\frac{5}{2}}}}{d}\right.$$

$$+\frac{1}{35}\left(\frac{16\,x}{\sqrt{dx^2+c}c^4}+\frac{8\,x}{(dx^2+c)^{\frac{3}{2}}c^3}+\frac{6\,x}{(dx^2+c)^{\frac{5}{2}}c^2}+\frac{5\,x}{(dx^2+c)^{\frac{7}{2}}c}\right)\operatorname{artanh}(ax)$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
1/210*a*((15*a^5*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x
^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^6*c^4 + 3*a^4*c^3*d + 3*a^2*c^2*d^2
+ c*d^3)*sqrt(a^2*c + d)) + 2*(15*(d*x^2 + c)^2*a^4*d + 3*a^4*c^2*d + 6*a^
2*c*d^2 + 3*d^3 + 5*(a^4*c*d + a^2*d^2)*(d*x^2 + c))/((a^6*c^4 + 3*a^4*c^3
*d + 3*a^2*c^2*d^2 + c*d^3)*(d*x^2 + c)^(5/2)))/d + 6*(3*a^3*d*log((sqrt(d
*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*
a))/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*sqrt(a^2*c + d)) + 2*(3*(d*x^2 + c)
*a^2*d + a^2*c*d + d^2)/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*(d*x^2 + c)^(3/
2)))/d + 24*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^4 + c^3*d)*sqrt(a^2*c + d)) + 2*d/
((a^2*c^4 + c^3*d)*sqrt(d*x^2 + c)))/d + 48*log((sqrt(d*x^2 + c)*a^2 - sqr
t(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d
)*a*c^4)) + 1/35*(16*x/(sqrt(d*x^2 + c)*c^4) + 8*x/((d*x^2 + c)^(3/2)*c^3)
 + 6*x/((d*x^2 + c)^(5/2)*c^2) + 5*x/((d*x^2 + c)^(7/2)*c))*arctanh(a*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{1}{105}a\left(\frac{3\left(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{\left(a^6c^7+3a^4c^6d+3a^2c^5d^2+c^4d^3\right)\sqrt{-a^2c-d}a}+\frac{57\left(dx^2+c\right)^2a^4}{}\right.$$
$$+\frac{\left(2\left(4x^2\left(\frac{2d^3x^2}{c^4}+\frac{7d^2}{c^3}\right)+\frac{35d}{c^2}\right)x^2+\frac{35}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{70\left(dx^2+c\right)^{\frac{7}{2}}}$$

input
```
integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
1/105*a*(3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*arctan(sqrt
(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 +
c^4*d^3)*sqrt(-a^2*c - d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*
a^4*c^3 + 3*a^4*c^4 + 66*(d*x^2 + c)^2*a^2*c*d + 17*(d*x^2 + c)*a^2*c^2*d
+ 6*a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((
a^6*c^6 + 3*a^4*c^5*d + 3*a^2*c^4*d^2 + c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/7
0*(2*(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*log(-(a*
x + 1)/(a*x - 1))/(d*x^2 + c)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{atanh}(a\,x)}{{(d\,x^2+c)}^{9/2}}\,dx$$

input
```
int(atanh(a*x)/(c + d*x^2)^(9/2),x)
```

output
```
int(atanh(a*x)/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{atanh(ax)}{\sqrt{d\,x^2+c}\,c^4+4\sqrt{d\,x^2+c}\,c^3d\,x^2+6\sqrt{d\,x^2+c}\,c^2d^2x^4+4\sqrt{d\,x^2+c}\,c\,d^3x^6+\sqrt{d\,x^2}}$$

input
```
int(atanh(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(atanh(a*x)/(sqrt(c + d*x**2)*c**4 + 4*sqrt(c + d*x**2)*c**3*d*x**2 + 6
*sqrt(c + d*x**2)*c**2*d**2*x**4 + 4*sqrt(c + d*x**2)*c*d**3*x**6 + sqrt(c
 + d*x**2)*d**4*x**8),x)
```

3.513 $\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 186

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx=\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\text{arctanh}(x)-\frac{a\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\text{arctanh}(x)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{2\sqrt{a-ax^2}}$$

output
```
1/2*(-a*x^2+a)^(1/2)+1/2*x*(-a*x^2+a)^(1/2)*arctanh(x)-a*(-x^2+1)^(1/2)*ar
ctan((1-x)^(1/2)/(1+x)^(1/2))*arctanh(x)/(-a*x^2+a)^(1/2)-1/2*I*a*(-x^2+1)
^(1/2)*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))/(-a*x^2+a)^(1/2)+1/2*I*a*(-x^
2+1)^(1/2)*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))/(-a*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.52

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx = \frac{1}{2}\sqrt{a(1-x^2)}\Bigg(1+x\text{arctanh}(x) - \frac{i\left(\text{arctanh}(x)\left(\log\left(1-ie^{-\text{arctanh}(x)}\right)-\log\left(1+ie^{-\text{arctanh}(x)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\text{arctanh}(x)}\right)-\text{PolyLog}$$
$$\overline{\sqrt{1-x^2}}$$

input
```
Integrate[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

output
```
(Sqrt[a*(1 - x^2)]*(1 + x*ArcTanh[x] - (I*(ArcTanh[x]*(Log[1 - I/E^ArcTanh
[x]] - Log[1 + I/E^ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[
2, I/E^ArcTanh[x]]))/Sqrt[1 - x^2]))/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6504, 6516, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx$$

$$\Big\downarrow \text{6504}$$

$$\frac{1}{2}a\int \frac{\text{arctanh}(x)}{\sqrt{a-ax^2}}dx + \frac{1}{2}x\sqrt{a-ax^2}\text{arctanh}(x) + \frac{1}{2}\sqrt{a-ax^2}$$

$$\Big\downarrow \text{6516}$$

$$\frac{a\sqrt{1-x^2}\int \frac{\text{arctanh}(x)}{\sqrt{1-x^2}}dx}{2\sqrt{a-ax^2}} + \frac{1}{2}x\sqrt{a-ax^2}\text{arctanh}(x) + \frac{1}{2}\sqrt{a-ax^2}$$

$$\Big\downarrow \text{6512}$$

$$\frac{a\sqrt{1-x^2}\left(-2\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\operatorname{arctanh}(x)-i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)+i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}x\sqrt{a-ax^2}\operatorname{arctanh}(x)+\frac{1}{2}\sqrt{a-ax^2}$$

input
```
Int[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

output
```
Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcTanh[x])/2 + (a*Sqrt[1 - x^2]*(-
2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x] - I*PolyLog[2, ((-I)*Sqrt[1 -
 x])/Sqrt[1 + x]] + I*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]]))/(2*Sqrt[a
- a*x^2])
```

Defintions of rubi rules used

rule 6504
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcTanh[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6516
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTanh[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.23

method	result
default	$\frac{(x\,\mathrm{arctanh}(x)+1)\sqrt{-(-1+x)a(1+x)}}{2}+\frac{i\sqrt{-(-1+x)a(1+x)}\,\sqrt{-x^{2}+1}\,\mathrm{arctanh}(x)\ln\left(1+\frac{i(1+x)}{\sqrt{-x^{2}+1}}\right)}{2(1+x)(-1+x)}-\frac{i\sqrt{-(-1+x)a(1+x)}\,\sqrt{\cdots}}{2\cdots}$ [illegible]

input
```
int((-a*x^2+a)^(1/2)*arctanh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x*arctanh(x)+1)*(-(-1+x)*a*(1+x))^(1/2)+1/2*I*(-(-1+x)*a*(1+x))^(1/2)
/(1+x)*(-x^2+1)^(1/2)/(-1+x)*arctanh(x)*ln(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I
*(-(-1+x)*a*(1+x))^(1/2)/(1+x)*(-x^2+1)^(1/2)/(-1+x)*arctanh(x)*ln(1-I*(1+
x)/(-x^2+1)^(1/2))+1/2*I*(-(-1+x)*a*(1+x))^(1/2)/(1+x)*(-x^2+1)^(1/2)/(-1+
x)*dilog(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(-(-1+x)*a*(1+x))^(1/2)/(1+x)*(-x
^2+1)^(1/2)/(-1+x)*dilog(1-I*(1+x)/(-x^2+1)^(1/2))
```

Fricas [F]

$$\int \sqrt{a-ax^2}\operatorname{arctanh}(x)\,dx=\int \sqrt{-ax^2+a}\,\operatorname{artanh}(x)\ dx$$

input
```
integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Sympy [F]

$$\int \sqrt{a-ax^2}\operatorname{arctanh}(x)\,dx=\int \sqrt{-a(x-1)(x+1)}\,\operatorname{atanh}(x)\,dx$$

input
```
integrate((-a*x**2+a)**(1/2)*atanh(x),x)
```

output
```
Integral(sqrt(-a*(x - 1)*(x + 1))*atanh(x), x)
```

Maxima [F]

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx = \int \sqrt{-ax^2+a}\,\text{artanh}\left(x\right)\,dx$$

input
```
integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Giac [F]

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx = \int \sqrt{-ax^2+a}\,\text{artanh}\left(x\right)\,dx$$

input
```
integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx = \int \text{atanh}(x)\,\sqrt{a-a\,x^2}\,dx$$

input
```
int(atanh(x)*(a - a*x^2)^(1/2),x)
```

output
```
int(atanh(x)*(a - a*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a-ax^2}\text{arctanh}(x)\,dx = \sqrt{a}\left(\int \sqrt{-x^2+1}\,atanh(x)\,dx\right)$$

input
```
int((-a*x^2+a)^(1/2)*atanh(x),x)
```

output
```
sqrt(a)*int(sqrt( - x**2 + 1)*atanh(x),x)
```

3.514 $\int \frac{\text{arctanh}(x)}{\sqrt{a-ax^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \frac{\text{arctanh}(x)}{\sqrt{a-ax^2}}\,dx = -\frac{2\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\text{arctanh}(x)}{\sqrt{a-ax^2}} - \frac{i\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}$$

output
```
-2*(-x^2+1)^(1/2)*arctan((1-x)^(1/2)/(1+x)^(1/2))*arctanh(x)/(-a*x^2+a)^(1
/2)-I*(-x^2+1)^(1/2)*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))/(-a*x^2+a)^(1/2
)+I*(-x^2+1)^(1/2)*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))/(-a*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int \frac{\text{arctanh}(x)}{\sqrt{a-ax^2}}\,dx = -\frac{i\sqrt{a(1-x^2)}\left(\text{arctanh}(x)\left(\log\left(1-ie^{-\text{arctanh}(x)}\right)-\log\left(1+ie^{-\text{arctanh}(x)}\right)\right)+\text{PolyLog}\left(2,-ie^{-\text{arctanh}(x)}\right.\right.}{a\sqrt{1-x^2}}$$

input
```
Integrate[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

output
```
((-I)*Sqrt[a*(1 - x^2)]*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[x]] - Log[1 + I/E
^ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2, I/E^ArcTanh[x]]
))/(a*Sqrt[1 - x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6516, 6512}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(x)}{\sqrt{a-ax^2}}dx$$

$$\Big\downarrow \text{6516}$$

$$\frac{\sqrt{1-x^2}\int \frac{\text{arctanh}(x)}{\sqrt{1-x^2}}dx}{\sqrt{a-ax^2}}$$

$$\Big\downarrow \text{6512}$$

$$\frac{\sqrt{1-x^2}\left(-2\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\text{arctanh}(x)-i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)\right)}{\sqrt{a-ax^2}}$$

input
```
Int[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x] - I*PolyLog[
2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]] + I*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 +
 x]]))/Sqrt[a - a*x^2]
```

Defintions of rubi rules used

rule 6512
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcTanh[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6516
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTanh[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.46

method	result
default	$\frac{i \ln\left(1+\frac{i(1+x)}{\sqrt{-x^2+1}}\right)\sqrt{-(-1+x)a(1+x)}\,\sqrt{-x^2+1}\,\operatorname{arctanh}(x)}{a(x^2-1)}-\frac{i \ln\left(1-\frac{i(1+x)}{\sqrt{-x^2+1}}\right)\sqrt{-(-1+x)a(1+x)}\,\sqrt{-x^2+1}\,\operatorname{arctanh}(x)}{a(x^2-1)}+$

input
```
int(arctanh(x)/(-a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*ln(1+I*(1+x)/(-x^2+1)^(1/2))*(-(-1+x)*a*(1+x))^(1/2)*(-x^2+1)^(1/2)*arct
anh(x)/a/(x^2-1)-I*ln(1-I*(1+x)/(-x^2+1)^(1/2))*(-(-1+x)*a*(1+x))^(1/2)*(-
x^2+1)^(1/2)*arctanh(x)/a/(x^2-1)+I*dilog(1+I*(1+x)/(-x^2+1)^(1/2))*(-x^2+
1)^(1/2)*(-(-1+x)*a*(1+x))^(1/2)/a/(x^2-1)-I*dilog(1-I*(1+x)/(-x^2+1)^(1/2
))*(-x^2+1)^(1/2)*(-(-1+x)*a*(1+x))^(1/2)/a/(x^2-1)
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{artanh}(x)}{\sqrt{-ax^2+a}}\,dx$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*x^2 + a)*arctanh(x)/(a*x^2 - a), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{atanh}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

input
```
integrate(atanh(x)/(-a*x**2+a)**(1/2),x)
```

output
```
Integral(atanh(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{arctanh}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{artanh}(x)}{\sqrt{-ax^2+a}}\,dx$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(x)}{\sqrt{a-ax^2}}\,dx=\int \frac{\operatorname{artanh}\left(x\right)}{\sqrt{-ax^2+a}}\,dx$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(x)}{\sqrt{a-ax^2}}\,dx=\int \frac{\operatorname{atanh}(x)}{\sqrt{a-a\,x^2}}\,dx$$

input
```
int(atanh(x)/(a - a*x^2)^(1/2),x)
```

output
```
int(atanh(x)/(a - a*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(x)}{\sqrt{a-ax^2}}\,dx=\frac{\int \frac{atanh(x)}{\sqrt{-x^2+1}}dx}{\sqrt{a}}$$

input
```
int(atanh(x)/(-a*x^2+a)^(1/2),x)
```

output
```
int(atanh(x)/sqrt( - x**2 + 1),x)/sqrt(a)
```

3.515 $\int \frac{\text{arctanh}(x)}{(a-ax^2)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{3/2}} \, dx = -\frac{1}{a\sqrt{a-ax^2}} + \frac{x\text{arctanh}(x)}{a\sqrt{a-ax^2}}$$

output
```
-1/a/(-a*x^2+a)^(1/2)+x*arctanh(x)/a/(-a*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{3/2}} \, dx = \frac{\sqrt{a-ax^2}(1-x\text{arctanh}(x))}{a^2(-1+x^2)}$$

input
```
Integrate[ArcTanh[x]/(a - a*x^2)^(3/2),x]
```

output
```
(Sqrt[a - a*x^2]*(1 - x*ArcTanh[x]))/(a^2*(-1 + x^2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6520$$

$$\frac{x\text{arctanh}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

input
```
Int[ArcTanh[x]/(a - a*x^2)^(3/2),x]
```

output
```
-(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{x\ln(1+x)}{2a\sqrt{-a(x^2-1)}}-\frac{x\ln(1-x)+2}{2a\sqrt{-a(x^2-1)}}$	47
default	$-\frac{(\text{arctanh}(x)-1)\sqrt{-(-1+x)a(1+x)}}{2a^2(-1+x)}-\frac{(1+\text{arctanh}(x))\sqrt{-(-1+x)a(1+x)}}{2a^2(1+x)}$	52
orering	$\frac{(-4x^3+4x)\,\text{arctanh}(x)}{(-a\,x^2+a)^{\frac{3}{2}}}-(-1+x)^2\,(1+x)^2\left(\frac{1}{(-x^2+1)(-a\,x^2+a)^{\frac{3}{2}}}+\frac{3\,\text{arctanh}(x)ax}{(-a\,x^2+a)^{\frac{5}{2}}}\right)$	73

input
```
int(arctanh(x)/(-a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*x/(-a*(x^2-1))^(1/2)*ln(1+x)-1/2/a*(x*ln(1-x)+2)/(-a*(x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx = -\frac{\sqrt{-ax^2+a}\left(x\log\left(-\frac{x+1}{x-1}\right)-2\right)}{2\left(a^2x^2-a^2\right)}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-a*x^2 + a)*(x*log(-(x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(x)}{\left(-a(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(x)/(-a*x**2+a)**(3/2),x)
```

output
```
Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx = \frac{x\,\operatorname{artanh}(x)}{\sqrt{-ax^2+a}a} - \frac{\frac{\sqrt{-ax^2+a}}{ax+a} - \frac{\sqrt{-ax^2+a}}{ax-a}}{2\,a}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x*arctanh(x)/(sqrt(-a*x^2 + a)*a) - 1/2*(sqrt(-a*x^2 + a)/(a*x + a) - sqrt
(-a*x^2 + a)/(a*x - a))/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx = -\frac{\sqrt{-ax^2+a}x\log\left(-\frac{x+1}{x-1}\right)}{2\,(ax^2-a)a} - \frac{1}{\sqrt{-ax^2+a}a}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-a*x^2 + a)*x*log(-(x + 1)/(x - 1))/((a*x^2 - a)*a) - 1/(sqrt(-a
*x^2 + a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx = \int \frac{\operatorname{atanh}(x)}{(a-a\,x^2)^{3/2}}\,dx$$

input
```
int(atanh(x)/(a - a*x^2)^(3/2),x)
```

output
```
int(atanh(x)/(a - a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{3/2}}\,dx = -\frac{\int \frac{atanh(x)}{\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}}dx}{\sqrt{a}\,a}$$

input
```
int(atanh(x)/(-a*x^2+a)^(3/2),x)
```

output
```
( - int(atanh(x)/(sqrt( - x**2 + 1)*x**2 - sqrt( - x**2 + 1)),x))/(sqrt(a)
*a)
```

3.516 $\int \frac{\text{arctanh}(x)}{(a-ax^2)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{5/2}} dx = -\frac{1}{9a(a-ax^2)^{3/2}} - \frac{2}{3a^2\sqrt{a-ax^2}} + \frac{x\text{arctanh}(x)}{3a(a-ax^2)^{3/2}} + \frac{2x\text{arctanh}(x)}{3a^2\sqrt{a-ax^2}}$$

output
```
-1/9/a/(-a*x^2+a)^(3/2)-2/3/a^2/(-a*x^2+a)^(1/2)+1/3*x*arctanh(x)/a/(-a*x^
2+a)^(3/2)+2/3*x*arctanh(x)/a^2/(-a*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{5/2}} dx = -\frac{\sqrt{a-ax^2}(7-6x^2+(-9x+6x^3)\,\text{arctanh}(x))}{9a^3(-1+x^2)^2}$$

input
```
Integrate[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

output
```
-1/9*(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcTanh[x]))/(a^3*(-1 +
x^2)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{6522}$$

$$\frac{2 \int \frac{\text{arctanh}(x)}{(a-ax^2)^{3/2}}dx}{3a}+\frac{x\text{arctanh}(x)}{3a\left(a-ax^2\right)^{3/2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}$$

$$\Big\downarrow \text{6520}$$

$$\frac{x\text{arctanh}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2\left(\frac{x\text{arctanh}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}\right)}{3a}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}$$

input
```
Int[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

output
```
-1/9*1/(a*(a - a*x^2)^(3/2)) + (x*ArcTanh[x])/(3*a*(a - a*x^2)^(3/2)) + (2
*(-(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])))/(3*a)
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x(2x^2-3)\ln(1+x)}{6a^2(x^2-1)\sqrt{-a(x^2-1)}}-\frac{6x^3\ln(1-x)+12x^2-9x\ln(1-x)-14}{18a^2(x^2-1)\sqrt{-a(x^2-1)}}$
orering	$\frac{\left(4x^5-\frac{80}{9}x^3+\frac{44}{9}x\right)\operatorname{arctanh}(x)}{(-a\,x^2+a)^{\frac{5}{2}}}+\frac{(6x^2-7)(-1+x)^2(1+x)^2\left(\frac{1}{(-x^2+1)(-a\,x^2+a)^{\frac{5}{2}}}+\frac{5\operatorname{arctanh}(x)ax}{(-a\,x^2+a)^{\frac{7}{2}}}\right)}{9}$
default	$\frac{(1+x)(-1+3\operatorname{arctanh}(x))\sqrt{-(-1+x)a(1+x)}}{72(-1+x)^2a^3}-\frac{3(\operatorname{arctanh}(x)-1)\sqrt{-(-1+x)a(1+x)}}{8a^3(-1+x)}-\frac{3(1+\operatorname{arctanh}(x))\sqrt{-(-1+x)a(1+x)}}{8(1+x)a^3}-$

input
```
int(arctanh(x)/(-a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*x*(2*x^2-3)/(x^2-1)/(-a*(x^2-1))^(1/2)*ln(1+x)-1/18/a^2*(6*x^3*ln(
1-x)+12*x^2-9*x*ln(1-x)-14)/(x^2-1)/(-a*(x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{5/2}}\,dx=\frac{\sqrt{-ax^2+a}\left(12\,x^2-3\left(2\,x^3-3\,x\right)\log\left(-\frac{x+1}{x-1}\right)-14\right)}{18\left(a^3x^4-2\,a^3x^2+a^3\right)}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log(-(x + 1)/(x - 1)) - 14
)/(a^3*x^4 - 2*a^3*x^2 + a^3)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{5/2}}\,dx = \int \frac{\operatorname{atanh}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(x)/(-a*x**2+a)**(5/2),x)
```

output
```
Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2x}{\sqrt{-ax^2+a}a^2}+\frac{x}{(-ax^2+a)^{\frac{3}{2}}a}\right)\operatorname{artanh}(x) - \frac{2}{3\sqrt{-ax^2+a}a^2} - \frac{1}{9(-ax^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arctanh(x) - 2
/3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{5/2}}\,dx = -\frac{\sqrt{-ax^2+a}x\left(\frac{2x^2}{a}-\frac{3}{a}\right)\log\left(-\frac{x+1}{x-1}\right)}{6(ax^2-a)^2} - \frac{6ax^2-7a}{9(ax^2-a)\sqrt{-ax^2+a}a^2}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(-a*x^2 + a)*x*(2*x^2/a - 3/a)*log(-(x + 1)/(x - 1))/(a*x^2 - a)^
2 - 1/9*(6*a*x^2 - 7*a)/((a*x^2 - a)*sqrt(-a*x^2 + a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{5/2}}\,dx=\int \frac{\operatorname{atanh}(x)}{(a-a\,x^2)^{5/2}}\,dx$$

input
```
int(atanh(x)/(a - a*x^2)^(5/2),x)
```

output
```
int(atanh(x)/(a - a*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{5/2}}\,dx=\frac{\int \frac{atanh(x)}{\sqrt{-x^2+1}\,x^4-2\sqrt{-x^2+1}\,x^2+\sqrt{-x^2+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(atanh(x)/(-a*x^2+a)^(5/2),x)
```

output
```
int(atanh(x)/(sqrt( - x**2 + 1)*x**4 - 2*sqrt( - x**2 + 1)*x**2 + sqrt( -
x**2 + 1)),x)/(sqrt(a)*a**2)
```

3.517 $\int \frac{\text{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{1}{25a\,(a-ax^2)^{5/2}}-\frac{4}{45a^2\,(a-ax^2)^{3/2}} -\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{x\text{arctanh}(x)}{5a\,(a-ax^2)^{5/2}}+\frac{4x\text{arctanh}(x)}{15a^2\,(a-ax^2)^{3/2}}+\frac{8x\text{arctanh}(x)}{15a^3\sqrt{a-ax^2}}$$

output
```
-1/25/a/(-a*x^2+a)^(5/2)-4/45/a^2/(-a*x^2+a)^(3/2)-8/15/a^3/(-a*x^2+a)^(1/
2)+1/5*x*arctanh(x)/a/(-a*x^2+a)^(5/2)+4/15*x*arctanh(x)/a^2/(-a*x^2+a)^(3
/2)+8/15*x*arctanh(x)/a^3/(-a*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = \frac{\sqrt{a-ax^2}(149-260x^2+120x^4-15x(15-20x^2+8x^4)\,\text{arctanh}(x))}{225a^4\,(-1+x^2)^3}$$

input
```
Integrate[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```

output
```
(Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*Arc
Tanh[x]))/(225*a^4*(-1 + x^2)^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6522, 6522, 6520}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 6522}$$

$$\frac{4\int \frac{\text{arctanh}(x)}{(a-ax^2)^{5/2}}dx}{5a}+\frac{x\text{arctanh}(x)}{5a(a-ax^2)^{5/2}}-\frac{1}{25a(a-ax^2)^{5/2}}$$

$$\Big\downarrow \text{ 6522}$$

$$\frac{4\left(\frac{2\int \frac{\text{arctanh}(x)}{(a-ax^2)^{3/2}}dx}{3a}+\frac{x\text{arctanh}(x)}{3a(a-ax^2)^{3/2}}-\frac{1}{9a(a-ax^2)^{3/2}}\right)}{5a}+\frac{x\text{arctanh}(x)}{5a(a-ax^2)^{5/2}}-\frac{1}{25a(a-ax^2)^{5/2}}$$

$$\Big\downarrow \text{ 6520}$$

$$\frac{x\text{arctanh}(x)}{5a(a-ax^2)^{5/2}}+\frac{4\left(\frac{x\text{arctanh}(x)}{3a(a-ax^2)^{3/2}}+\frac{2\left(\frac{x\text{arctanh}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}\right)}{3a}-\frac{1}{9a(a-ax^2)^{3/2}}\right)}{5a}-\frac{1}{25a(a-ax^2)^{5/2}}$$

input
```
Int[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```

output
```
-1/25*1/(a*(a - a*x^2)^(5/2)) + (x*ArcTanh[x])/(5*a*(a - a*x^2)^(5/2)) + (
4*(-1/9*1/(a*(a - a*x^2)^(3/2)) + (x*ArcTanh[x])/(3*a*(a - a*x^2)^(3/2)) +
 (2*(-(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])))/(3*a)
))/(5*a)
```

Defintions of rubi rules used

rule 6520
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTanh[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6522
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcTanh[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

method	result
orering	$\frac{\left(-\frac{64}{15}x^{7}+\frac{616}{45}x^{5}-\frac{3388}{225}x^{3}+\frac{1268}{225}x\right)\operatorname{arctanh}(x)}{\left(-a\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{\left(120x^{4}-260x^{2}+149\right)\left(-1+x\right)^{2}\left(1+x\right)^{2}\left(\frac{1}{\left(-x^{2}+1\right)\left(-a\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{7\operatorname{arctanh}(x)ax}{\left(-a\,x^{2}+a\right)^{\frac{9}{2}}}\right)}{225}$
risch	$\frac{x\left(8x^{4}-20x^{2}+15\right)\ln(1+x)}{30a^{3}\left(x^{2}-1\right)^{2}\sqrt{-a\left(x^{2}-1\right)}}-\frac{120x^{5}\ln(1-x)+240x^{4}-300x^{3}\ln(1-x)-520x^{2}+225x\ln(1-x)+298}{450a^{3}\left(x^{2}-1\right)^{2}\sqrt{-a\left(x^{2}-1\right)}}$
default	$-\frac{\left(1+x\right)^{2}\left(-1+5\operatorname{arctanh}(x)\right)\sqrt{-\left(-1+x\right)a\left(1+x\right)}}{800\left(-1+x\right)^{3}a^{4}}+\frac{5\left(1+x\right)\left(-1+3\operatorname{arctanh}(x)\right)\sqrt{-\left(-1+x\right)a\left(1+x\right)}}{288a^{4}\left(-1+x\right)^{2}}-\frac{5\left(\operatorname{arctanh}(x)-1\right)\sqrt{-(-1}}{16a^{4}(-1+x)}$

input
```
int(arctanh(x)/(-a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-64/15*x^7+616/45*x^5-3388/225*x^3+1268/225*x)*arctanh(x)/(-a*x^2+a)^(7/2
)-1/225*(120*x^4-260*x^2+149)*(-1+x)^2*(1+x)^2*(1/(-x^2+1)/(-a*x^2+a)^(7/2
)+7*arctanh(x)/(-a*x^2+a)^(9/2)*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = \frac{\left(240\,x^4-520\,x^2-15\,(8\,x^5-20\,x^3+15\,x)\log\left(-\frac{x+1}{x-1}\right)+298\right)\sqrt{-ax^2+a}}{450\,(a^4x^6-3\,a^4x^4+3\,a^4x^2-a^4)}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log(-(x + 1)/(x - 1)
) + 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(x)}{(-a\,(x-1)\,(x+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(x)/(-a*x**2+a)**(7/2),x)
```

output
```
Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = \frac{1}{15}\left(\frac{8\,x}{\sqrt{-ax^2+a}a^3}+\frac{4\,x}{(-ax^2+a)^{\frac{3}{2}}a^2}+\frac{3\,x}{(-ax^2+a)^{\frac{5}{2}}a}\right)\operatorname{artanh}(x) - \frac{8}{15\sqrt{-ax^2+a}a^3} - \frac{4}{45\,(-ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25\,(-ax^2+a)^{\frac{5}{2}}a}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a
*x^2 + a)^(5/2)*a))*arctanh(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x
^2 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{\sqrt{-ax^2+a}\left(4\,x^2\left(\frac{2\,x^2}{a}-\frac{5}{a}\right)+\frac{15}{a}\right)x\log\left(-\frac{x+1}{x-1}\right)}{30\,(ax^2-a)^3} - \frac{120\,(ax^2-a)^2-20\,(ax^2-a)a+9\,a^2}{225\,(ax^2-a)^2\sqrt{-ax^2+a}a^3}$$

input
```
integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
-1/30*sqrt(-a*x^2 + a)*(4*x^2*(2*x^2/a - 5/a) + 15/a)*x*log(-(x + 1)/(x -
1))/(a*x^2 - a)^3 - 1/225*(120*(a*x^2 - a)^2 - 20*(a*x^2 - a)*a + 9*a^2)/(
(a*x^2 - a)^2*sqrt(-a*x^2 + a)*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = \int \frac{\operatorname{atanh}(x)}{(a-a\,x^2)^{7/2}}\,dx$$

input
```
int(atanh(x)/(a - a*x^2)^(7/2),x)
```

output
```
int(atanh(x)/(a - a*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\text{arctanh}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{\int \frac{atanh(x)}{\sqrt{-x^2+1}\,x^6-3\sqrt{-x^2+1}\,x^4+3\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}}dx}{\sqrt{a}\,a^3}$$

input
```
int(atanh(x)/(-a*x^2+a)^(7/2),x)
```

output
```
( - int(atanh(x)/(sqrt( - x**2 + 1)*x**6 - 3*sqrt( - x**2 + 1)*x**4 + 3*sq
rt( - x**2 + 1)*x**2 - sqrt( - x**2 + 1)),x))/(sqrt(a)*a**3)
```

3.518 $\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 258

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2} dx = \frac{\operatorname{arctanh}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\operatorname{arctanh}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\operatorname{PolyLog}\left(2,1-\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left(2,1-\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{2\sqrt{b^2-4ac}}$$

output
```
arctanh(x)*ln(2*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(b+2*c-(-4*a*c+b^2)^(1/2))/(1
+x))/(-4*a*c+b^2)^(1/2)-arctanh(x)*ln(2*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(b+2*
c+(-4*a*c+b^2)^(1/2))/(1+x))/(-4*a*c+b^2)^(1/2)-1/2*polylog(2,1-2*(b-(-4*a
*c+b^2)^(1/2)+2*c*x)/(b+2*c-(-4*a*c+b^2)^(1/2))/(1+x))/(-4*a*c+b^2)^(1/2)+
1/2*polylog(2,1-2*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(b+2*c+(-4*a*c+b^2)^(1/2))/
(1+x))/(-4*a*c+b^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx=\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx$$

input
```
Integrate[ArcTanh[x]/(a + b*x + c*x^2),x]
```

output
```
Integrate[ArcTanh[x]/(a + b*x + c*x^2), x]
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{2c\operatorname{arctanh}(x)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c\operatorname{arctanh}(x)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\operatorname{arctanh}(x)\log\left(\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\operatorname{arctanh}(x)\log\left(\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left(2,1-\frac{2\left(b+2cx-\sqrt{b^2-4ac}\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(x+1)}\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,1-\frac{2\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(x+1)}\right)}{2\sqrt{b^2-4ac}}$$

input
```
Int[ArcTanh[x]/(a + b*x + c*x^2),x]
```

output
```
(ArcTanh[x]*Log[(2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2 -
 4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c] - (ArcTanh[x]*Log[(2*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a
*c] - PolyLog[2, 1 - (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[
b^2 - 4*a*c])*(1 + x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 - (2*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))]/(2*Sqrt[
b^2 - 4*a*c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\ln(1-x)\left(\ln\left(\frac{-2c(1-x)+\sqrt{-4ca+b^2}+b+2c}{b+2c+\sqrt{-4ca+b^2}}\right)-\ln\left(\frac{2c(1-x)+\sqrt{-4ca+b^2}-b-2c}{-b-2c+\sqrt{-4ca+b^2}}\right)\right)}{2\sqrt{-4ca+b^2}}+\frac{\operatorname{dilog}\left(\frac{-2c(1-x)+\sqrt{-4ca+b^2}+b+2c}{b+2c+\sqrt{-4ca+b^2}}\right)}{2\sqrt{-4ca+b^2}}-\frac{\operatorname{dilog}\left(\right.}{}$
default	$-\frac{\left(-\sqrt{-4ca+b^2}+a-c\right)\ln\left(1-\frac{(a+b+c)(1+x)^2}{\left(-x^2+1\right)\left(-\sqrt{-4ca+b^2}-a+c\right)}\right)\operatorname{arctanh}(x)}{a^2+2ca-b^2+c^2}+\frac{\left(4ca-b^2+\sqrt{-4ca+b^2}\,a-\sqrt{-4ca+b^2}\,c\right)\ln\left(1-\frac{}{(-x^2}\right.}{(4ca-b^2)(a^2+2ca-}$

input
```
int(arctanh(x)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1-x)*(ln((-2*c*(1-x)+(-4*a*c+b^2)^(1/2)+b+2*c)/(b+2*c+(-4*a*c+b^2)^
(1/2)))-ln((2*c*(1-x)+(-4*a*c+b^2)^(1/2)-b-2*c)/(-b-2*c+(-4*a*c+b^2)^(1/2)
)))/(-4*a*c+b^2)^(1/2)+1/2/(-4*a*c+b^2)^(1/2)*dilog((-2*c*(1-x)+(-4*a*c+b^
2)^(1/2)+b+2*c)/(b+2*c+(-4*a*c+b^2)^(1/2)))-1/2/(-4*a*c+b^2)^(1/2)*dilog((
2*c*(1-x)+(-4*a*c+b^2)^(1/2)-b-2*c)/(-b-2*c+(-4*a*c+b^2)^(1/2)))+1/2*ln(1+
x)*(ln((-2*c*(1+x)+(-4*a*c+b^2)^(1/2)-b+2*c)/(-b+2*c+(-4*a*c+b^2)^(1/2)))-
ln((2*c*(1+x)+(-4*a*c+b^2)^(1/2)+b-2*c)/(b-2*c+(-4*a*c+b^2)^(1/2))))/(-4*a
*c+b^2)^(1/2)+1/2/(-4*a*c+b^2)^(1/2)*dilog((-2*c*(1+x)+(-4*a*c+b^2)^(1/2)-
b+2*c)/(-b+2*c+(-4*a*c+b^2)^(1/2)))-1/2/(-4*a*c+b^2)^(1/2)*dilog((2*c*(1+x
)+(-4*a*c+b^2)^(1/2)+b-2*c)/(b-2*c+(-4*a*c+b^2)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{artanh}(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(arctanh(x)/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{atanh}(x)}{a+bx+cx^2}\,dx$$

input
```
integrate(atanh(x)/(c*x**2+b*x+a),x)
```

output
```
Integral(atanh(x)/(a + b*x + c*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{artanh}(x)}{cx^2+bx+a}\,dx$$

input
```
integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(arctanh(x)/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{atanh}(x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(atanh(x)/(a + b*x + c*x^2),x)
```

output
```
int(atanh(x)/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{arctanh}(x)}{a+bx+cx^2}\,dx = -\left(\int \frac{atanh(x)}{c\,x^4+b\,x^3+a\,x^2-c\,x^2-bx-a}dx\right) + \int \frac{atanh(x)\,x^2}{c\,x^4+b\,x^3+a\,x^2-c\,x^2-bx-a}dx$$

input
```
int(atanh(x)/(c*x^2+b*x+a),x)
```

output
```
 - int(atanh(x)/(a*x**2 - a + b*x**3 - b*x + c*x**4 - c*x**2),x) + int((at
anh(x)*x**2)/(a*x**2 - a + b*x**3 - b*x + c*x**4 - c*x**2),x)
```

3.519 $\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = -\frac{b}{16c^3(1+cx)^2}+\frac{5b}{16c^3(1+cx)} - \frac{5b\,\mathrm{arctanh}(cx)}{16c^3}-\frac{a+b\,\mathrm{arctanh}(cx)}{4c^3(1+cx)^2} + \frac{3(a+b\,\mathrm{arctanh}(cx))}{4c^3(1+cx)}+\frac{(a+b\,\mathrm{arctanh}(cx))^2}{8bc^3}$$

output
```
-1/16*b/c^3/(c*x+1)^2+5/16*b/c^3/(c*x+1)-5/16*b*arctanh(c*x)/c^3-1/4*(a+b*
arctanh(c*x))/c^3/(c*x+1)^2+3/4*(a+b*arctanh(c*x))/c^3/(c*x+1)+1/8*(a+b*ar
ctanh(c*x))^2/b/c^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = \frac{-\frac{2(4a+b)}{(1+cx)^2}+\frac{24a+10b}{1+cx}+\frac{8b(2+3cx)\mathrm{arctanh}(cx)}{(1+cx)^2}+4b\,\mathrm{arctanh}(cx)^2+(-4a+5b)\log(1-cx)+(4a-5b)\log(1+}{32c^3}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x]))/((1 - c*x)*(1 + c*x)^3),x]
```

output
```
((-2*(4*a + b))/(1 + c*x)^2 + (24*a + 10*b)/(1 + c*x) + (8*b*(2 + 3*c*x)*A
rcTanh[c*x])/(1 + c*x)^2 + 4*b*ArcTanh[c*x]^2 + (-4*a + 5*b)*Log[1 - c*x]
+ (4*a - 5*b)*Log[1 + c*x])/(32*c^3)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(cx+1)^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{ax^2}{(cx-1)(cx+1)^3}-\frac{bx^2\text{arctanh}(cx)}{(cx-1)(cx+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\text{arctanh}(cx)}{4c^3}+\frac{3a}{4c^3(cx+1)}-\frac{a}{4c^3(cx+1)^2}+\frac{b\text{arctanh}(cx)^2}{8c^3}+\frac{3b\text{arctanh}(cx)}{4c^3(cx+1)}-\frac{b\text{arctanh}(cx)}{4c^3(cx+1)^2}-\frac{5b\text{arctanh}(cx)}{16c^3}+\frac{5b}{16c^3(cx+1)}-\frac{b}{16c^3(cx+1)^2}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x]))/((1 - c*x)*(1 + c*x)^3),x]
```

output
```
-1/4*a/(c^3*(1 + c*x)^2) - b/(16*c^3*(1 + c*x)^2) + (3*a)/(4*c^3*(1 + c*x)
) + (5*b)/(16*c^3*(1 + c*x)) + (a*ArcTanh[c*x])/(4*c^3) - (5*b*ArcTanh[c*x
])/(16*c^3) - (b*ArcTanh[c*x])/(4*c^3*(1 + c*x)^2) + (3*b*ArcTanh[c*x])/(4
*c^3*(1 + c*x)) + (b*ArcTanh[c*x]^2)/(8*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.26

method	result
parallelrisch	$-\frac{8a c^2x^2+3bcx-2\operatorname{arctanh}(cx)bcx+5\operatorname{arctanh}(cx)bc^2x^2+4bc^2x^2+4acx-3b\operatorname{arctanh}(cx)-4a\operatorname{arctanh}(cx)-2b\operatorname{arcta}\cdots}{16(cx+1)^2c^3}$
derivativedivides	$\frac{-a\left(\frac{1}{4(cx+1)^2}-\frac{3}{4(cx+1)}-\frac{\ln(cx+1)}{8}+\frac{\ln(cx-1)}{8}\right)-b\left(\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{4(cx+1)}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\operatorname{arctanh}(cx)\ln(cx-\cdots}{8}\cdots\right.}{\cdots}$
default	$\frac{-a\left(\frac{1}{4(cx+1)^2}-\frac{3}{4(cx+1)}-\frac{\ln(cx+1)}{8}+\frac{\ln(cx-1)}{8}\right)-b\left(\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{4(cx+1)}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\operatorname{arctanh}(cx)\ln(cx-\cdots}{8}\cdots\right.}{\cdots}$
parts	$-a\left(\frac{1}{4c^3(cx+1)^2}-\frac{3}{4c^3(cx+1)}-\frac{\ln(cx+1)}{8c^3}+\frac{\ln(cx-1)}{8c^3}\right)-\frac{b\left(\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\operatorname{arctanh}(cx)}{4(cx+1)}-\frac{\operatorname{arctanh}(cx)\ln(cx+1\cdots}{8}\cdots\right.}{\cdots}$
risch	$\frac{b\ln(cx+1)^2}{32c^3}-\frac{b(c^2x^2\ln(-cx+1)+2cx\ln(-cx+1)-6cx+\ln(-cx+1)-4)\ln(cx+1)}{16c^3(cx+1)^2}-\frac{-bc^2x^2\ln(-cx+1)^2+4\ln(cx\cdots}{\cdots}$

input
```
int(x^2*(a+b*arctanh(c*x))/(-c*x+1)/(c*x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(8*a*c^2*x^2+3*b*c*x-2*arctanh(c*x)*b*c*x+5*arctanh(c*x)*b*c^2*x^2+4
*b*c^2*x^2+4*a*c*x-3*b*arctanh(c*x)-4*a*arctanh(c*x)-2*b*arctanh(c*x)^2-4*
x^2*arctanh(c*x)*a*c^2-8*x*arctanh(c*x)*a*c-2*b*arctanh(c*x)^2*x^2*c^2-4*b
*arctanh(c*x)^2*x*c)/(c*x+1)^2/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = \frac{2\,(12\,a+5\,b)cx+(bc^2x^2+2\,bcx+b)\log\left(-\frac{cx+1}{cx-1}\right)^2+((4\,a-5\,b)c^2x^2+2\,(4\,a+b)cx+4\,a+3\,b)\log\left(-\frac{c}{c}\right.}{32\,(c^5x^2+2\,c^4x+c^3)}$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(-c*x+1)/(c*x+1)^3,x, algorithm="fricas")
```

output
```
1/32*(2*(12*a + 5*b)*c*x + (b*c^2*x^2 + 2*b*c*x + b)*log(-(c*x + 1)/(c*x -
 1))^2 + ((4*a - 5*b)*c^2*x^2 + 2*(4*a + b)*c*x + 4*a + 3*b)*log(-(c*x + 1
)/(c*x - 1)) + 16*a + 8*b)/(c^5*x^2 + 2*c^4*x + c^3)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = -\int \frac{ax^2}{c^4x^4+2c^3x^3-2cx-1}\,dx - \int \frac{bx^2\,\text{atanh}\,(cx)}{c^4x^4+2c^3x^3-2cx-1}\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x))/(-c*x+1)/(c*x+1)**3,x)
```

output
```
-Integral(a*x**2/(c**4*x**4 + 2*c**3*x**3 - 2*c*x - 1), x) - Integral(b*x*
*2*atanh(c*x)/(c**4*x**4 + 2*c**3*x**3 - 2*c*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(95) = 190$.

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.29

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx$$
$$= \frac{1}{8}\,b\left(\frac{2\,(3\,cx+2)}{c^5x^2+2\,c^4x+c^3}+\frac{\log\left(cx+1\right)}{c^3}-\frac{\log\left(cx-1\right)}{c^3}\right)\text{artanh}\left(cx\right)$$
$$-\frac{\left((c^2x^2+2\,cx+1)\log\left(cx+1\right)^2+(c^2x^2+2\,cx+1)\log\left(cx-1\right)^2-10\,cx+(5\,c^2x^2+10\,cx-2\,(c^2x^2+\right.}{32\,(c^6x^2+2\,c^5x+c^4)}$$
$$+\frac{1}{8}\,a\left(\frac{2\,(3\,cx+2)}{c^5x^2+2\,c^4x+c^3}+\frac{\log\left(cx+1\right)}{c^3}-\frac{\log\left(cx-1\right)}{c^3}\right)$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(-c*x+1)/(c*x+1)^3,x, algorithm="maxima")
```

output
```
1/8*b*(2*(3*c*x + 2)/(c^5*x^2 + 2*c^4*x + c^3) + log(c*x + 1)/c^3 - log(c*
x - 1)/c^3)*arctanh(c*x) - 1/32*((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c
^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 - 10*c*x + (5*c^2*x^2 + 10*c*x - 2*(c^2
*x^2 + 2*c*x + 1)*log(c*x - 1) + 5)*log(c*x + 1) - 5*(c^2*x^2 + 2*c*x + 1)
*log(c*x - 1) - 8)*b*c/(c^6*x^2 + 2*c^5*x + c^4) + 1/8*a*(2*(3*c*x + 2)/(c
^5*x^2 + 2*c^4*x + c^3) + log(c*x + 1)/c^3 - log(c*x - 1)/c^3)
```

Giac [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = \int -\frac{(b\,\text{artanh}\left(cx\right)+a)x^2}{\left(cx+1\right)^3(cx-1)}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(-c*x+1)/(c*x+1)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arctanh(c*x) + a)*x^2/((c*x + 1)^3*(c*x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = \frac{\frac{4\,(2\,a+b)}{c}+x\,(12\,a+5\,b)}{16\,c^4\,x^2+32\,c^3\,x+16\,c^2} - \ln(1-c\,x)\,\left(\frac{\frac{b}{c^3}+\frac{b\,x}{c^2}}{2\,c^2\,x^2+4\,c\,x+2} + \frac{b\,\ln(c\,x+1)}{16\,c^3} - \frac{b\,(3\,c^2\,x^2+10\,c\,x+11)}{16\,c^3\,(2\,c^2\,x^2+4\,c\,x+2)}\right) + \frac{\ln(c\,x+1)\,\left(\frac{5\,b}{32\,c^4}-\frac{3\,b\,x^2}{32\,c^2}+\frac{3\,b\,x}{16\,c^3}\right)}{2\,x+c\,x^2+\frac{1}{c}} + \frac{b\ln(c\,x+1)^2}{32\,c^3} + \frac{b\ln(1-c\,x)^2}{32\,c^3} - \frac{\text{atan}(c\,x\,1\text{i})\,(2\,a-b)\,1\text{i}}{8\,c^3}$$

input
```
int(-(x^2*(a + b*atanh(c*x)))/((c*x - 1)*(c*x + 1)^3),x)
```

output
```
((4*(2*a + b))/c + x*(12*a + 5*b))/(32*c^3*x + 16*c^2 + 16*c^4*x^2) - log(
1 - c*x)*((b/c^3 + (b*x)/c^2)/(4*c*x + 2*c^2*x^2 + 2) + (b*log(c*x + 1))/(
16*c^3) - (b*(10*c*x + 3*c^2*x^2 + 11))/(16*c^3*(4*c*x + 2*c^2*x^2 + 2)))
+ (log(c*x + 1)*((5*b)/(32*c^4) - (3*b*x^2)/(32*c^2) + (3*b*x)/(16*c^3)))/
(2*x + c*x^2 + 1/c) - (atan(c*x*1i)*(2*a - b)*1i)/(8*c^3) + (b*log(c*x + 1
)^2)/(32*c^3) + (b*log(1 - c*x)^2)/(32*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.22

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(1+cx)^3}\,dx = \frac{4atanh(cx)^2\,b\,c^2x^2+8atanh(cx)^2\,bcx+4atanh(cx)^2\,b-12atanh(cx)\,b\,c^2x^2+4atanh(cx)\,b-4\log(cx-$$

input
```
int(x^2*(a+b*atanh(c*x))/(-c*x+1)/(c*x+1)^3,x)
```

output
```
(4*atanh(c*x)**2*b*c**2*x**2 + 8*atanh(c*x)**2*b*c*x + 4*atanh(c*x)**2*b -
 12*atanh(c*x)*b*c**2*x**2 + 4*atanh(c*x)*b - 4*log(c*x - 1)*a*c**2*x**2 -
 8*log(c*x - 1)*a*c*x - 4*log(c*x - 1)*a - log(c*x - 1)*b*c**2*x**2 - 2*lo
g(c*x - 1)*b*c*x - log(c*x - 1)*b + 4*log(c*x + 1)*a*c**2*x**2 + 8*log(c*x
 + 1)*a*c*x + 4*log(c*x + 1)*a + log(c*x + 1)*b*c**2*x**2 + 2*log(c*x + 1)
*b*c*x + log(c*x + 1)*b - 12*a*c**2*x**2 + 4*a - 5*b*c**2*x**2 + 3*b)/(32*
c**3*(c**2*x**2 + 2*c*x + 1))
```

3.520 $\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 107

$$\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx = -\frac{b}{16c^3(1+cx)^2}+\frac{5b}{16c^3(1+cx)} - \frac{5b\,\mathrm{arctanh}(cx)}{16c^3}-\frac{a+b\,\mathrm{arctanh}(cx)}{4c^3(1+cx)^2} + \frac{3(a+b\,\mathrm{arctanh}(cx))}{4c^3(1+cx)}+\frac{(a+b\,\mathrm{arctanh}(cx))^2}{8bc^3}$$

output
```
-1/16*b/c^3/(c*x+1)^2+5/16*b/c^3/(c*x+1)-5/16*b*arctanh(c*x)/c^3-1/4*(a+b*
arctanh(c*x))/c^3/(c*x+1)^2+3/4*(a+b*arctanh(c*x))/c^3/(c*x+1)+1/8*(a+b*ar
ctanh(c*x))^2/b/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+b\,\mathrm{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx = -\frac{\frac{2(4a+b)}{(1+cx)^2}-\frac{2(12a+5b)}{1+cx}-\frac{8b(2+3cx)\mathrm{arctanh}(cx)}{(1+cx)^2}-4b\,\mathrm{arctanh}(cx)^2+(4a-5b)\log(1-cx)+(-4a+5b)\log(1}{32c^3}$$

input
```
Integrate[(x^2*(a + b*ArcTanh[c*x]))/((1 + c*x)^2*(1 - c^2*x^2)),x]
```

output
```
-1/32*((2*(4*a + b))/(1 + c*x)^2 - (2*(12*a + 5*b))/(1 + c*x) - (8*b*(2 +
3*c*x)*ArcTanh[c*x])/(1 + c*x)^2 - 4*b*ArcTanh[c*x]^2 + (4*a - 5*b)*Log[1
- c*x] + (-4*a + 5*b)*Log[1 + c*x])/c^3
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2003, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(cx+1)^2(1-c^2x^2)}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1-cx)(cx+1)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{ax^2}{(cx-1)(cx+1)^3}-\frac{bx^2\text{arctanh}(cx)}{(cx-1)(cx+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\text{arctanh}(cx)}{4c^3}+\frac{3a}{4c^3(cx+1)}-\frac{a}{4c^3(cx+1)^2}+\frac{b\text{arctanh}(cx)^2}{8c^3}+\frac{3b\text{arctanh}(cx)}{4c^3(cx+1)}-\frac{b\text{arctanh}(cx)}{4c^3(cx+1)^2}-\frac{5b\text{arctanh}(cx)}{16c^3}+\frac{5b}{16c^3(cx+1)}-\frac{b}{16c^3(cx+1)^2}$$

input
```
Int[(x^2*(a + b*ArcTanh[c*x]))/((1 + c*x)^2*(1 - c^2*x^2)),x]
```

output
```
-1/4*a/(c^3*(1 + c*x)^2) - b/(16*c^3*(1 + c*x)^2) + (3*a)/(4*c^3*(1 + c*x)
) + (5*b)/(16*c^3*(1 + c*x)) + (a*ArcTanh[c*x])/(4*c^3) - (5*b*ArcTanh[c*x
])/(16*c^3) - (b*ArcTanh[c*x])/(4*c^3*(1 + c*x)^2) + (3*b*ArcTanh[c*x])/(4
*c^3*(1 + c*x)) + (b*ArcTanh[c*x]^2)/(8*c^3)
```

Defintions of rubi rules used

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.26

method	result
parallelrisch	$-\frac{8a\,c^2x^2+3bcx-2\,\operatorname{arctanh}(cx)bcx+5\,\operatorname{arctanh}(cx)b\,c^2x^2+4b\,c^2x^2+4acx-3b\,\operatorname{arctanh}(cx)-4a\,\operatorname{arctanh}(cx)-2b\,\operatorname{arcta}}{16(cx+1)^2c^3}$
derivativedivides	$\frac{-a\left(\frac{1}{4(cx+1)^2}-\frac{3}{4(cx+1)}-\frac{\ln(cx+1)}{8}+\frac{\ln(cx-1)}{8}\right)-b\left(\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\,\operatorname{arctanh}(cx)}{4(cx+1)}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\operatorname{arctanh}(cx)\ln(cx-}{8}\right.}{}$
default	$\frac{-a\left(\frac{1}{4(cx+1)^2}-\frac{3}{4(cx+1)}-\frac{\ln(cx+1)}{8}+\frac{\ln(cx-1)}{8}\right)-b\left(\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\,\operatorname{arctanh}(cx)}{4(cx+1)}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{8}+\frac{\operatorname{arctanh}(cx)\ln(cx-}{8}\right.}{}$
parts	$-a\left(\frac{1}{4c^3(cx+1)^2}-\frac{3}{4c^3(cx+1)}-\frac{\ln(cx+1)}{8c^3}+\frac{\ln(cx-1)}{8c^3}\right)-\frac{b\left(\frac{\operatorname{arctanh}(cx)}{4(cx+1)^2}-\frac{3\,\operatorname{arctanh}(cx)}{4(cx+1)}-\frac{\operatorname{arctanh}(cx)\ln(cx+1}{8}\right.}{}$
risch	$\frac{b\ln(cx+1)^2}{32c^3}-\frac{b\left(c^2x^2\ln(-cx+1)+2cx\ln(-cx+1)-6cx+\ln(-cx+1)-4\right)\ln(cx+1)}{16c^3(cx+1)^2}-\frac{-b\,c^2x^2\ln(-cx+1)^2+4\ln(cx}{}$

input
```
int(x^2*(a+b*arctanh(c*x))/(c*x+1)^2/(-c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(8*a*c^2*x^2+3*b*c*x-2*arctanh(c*x)*b*c*x+5*arctanh(c*x)*b*c^2*x^2+4
*b*c^2*x^2+4*a*c*x-3*b*arctanh(c*x)-4*a*arctanh(c*x)-2*b*arctanh(c*x)^2-4*
x^2*arctanh(c*x)*a*c^2-8*x*arctanh(c*x)*a*c-2*b*arctanh(c*x)^2*x^2*c^2-4*b
*arctanh(c*x)^2*x*c)/(c*x+1)^2/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1+cx)^2\,(1-c^2x^2)}\,dx = \frac{2\,(12\,a+5\,b)cx+(bc^2x^2+2\,bcx+b)\log\left(-\frac{cx+1}{cx-1}\right)^2+((4\,a-5\,b)c^2x^2+2\,(4\,a+b)cx+4\,a+3\,b)\log\left(-\right.}{32\,(c^5x^2+2\,c^4x+c^3)}$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*x+1)^2/(-c^2*x^2+1),x, algorithm="fric
as")
```

output
```
1/32*(2*(12*a + 5*b)*c*x + (b*c^2*x^2 + 2*b*c*x + b)*log(-(c*x + 1)/(c*x -
 1))^2 + ((4*a - 5*b)*c^2*x^2 + 2*(4*a + b)*c*x + 4*a + 3*b)*log(-(c*x + 1
)/(c*x - 1)) + 16*a + 8*b)/(c^5*x^2 + 2*c^4*x + c^3)
```

Sympy [F]

$$\int \frac{x^2(a+b\text{arctanh}(cx))}{(1+cx)^2\,(1-c^2x^2)}\,dx = -\int \frac{ax^2}{c^4x^4+2c^3x^3-2cx-1}\,dx - \int \frac{bx^2\,\text{atanh}\,(cx)}{c^4x^4+2c^3x^3-2cx-1}\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x))/(c*x+1)**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a*x**2/(c**4*x**4 + 2*c**3*x**3 - 2*c*x - 1), x) - Integral(b*x*
*2*atanh(c*x)/(c**4*x**4 + 2*c**3*x**3 - 2*c*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(95) = 190$.

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.29

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx = \frac{1}{8}\,b\left(\frac{2\,(3\,cx+2)}{c^5x^2+2\,c^4x+c^3}+\frac{\log(cx+1)}{c^3}-\frac{\log(cx-1)}{c^3}\right)\operatorname{artanh}(cx) - \frac{\left((c^2x^2+2\,cx+1)\log(cx+1)^2+(c^2x^2+2\,cx+1)\log(cx-1)^2-10\,cx+(5\,c^2x^2+10\,cx-2\,(c^2x^2+\ldots\right.}{32\,(c^6x^2+2\,c^5x+c^4\ldots} + \frac{1}{8}\,a\left(\frac{2\,(3\,cx+2)}{c^5x^2+2\,c^4x+c^3}+\frac{\log(cx+1)}{c^3}-\frac{\log(cx-1)}{c^3}\right)$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*x+1)^2/(-c^2*x^2+1),x, algorithm="maxi
ma")
```

output
```
1/8*b*(2*(3*c*x + 2)/(c^5*x^2 + 2*c^4*x + c^3) + log(c*x + 1)/c^3 - log(c*
x - 1)/c^3)*arctanh(c*x) - 1/32*((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c
^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 - 10*c*x + (5*c^2*x^2 + 10*c*x - 2*(c^2
*x^2 + 2*c*x + 1)*log(c*x - 1) + 5)*log(c*x + 1) - 5*(c^2*x^2 + 2*c*x + 1)
*log(c*x - 1) - 8)*b*c/(c^6*x^2 + 2*c^5*x + c^4) + 1/8*a*(2*(3*c*x + 2)/(c
^5*x^2 + 2*c^4*x + c^3) + log(c*x + 1)/c^3 - log(c*x - 1)/c^3)
```

Giac [F]

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx = \int -\frac{(b\operatorname{artanh}(cx)+a)x^2}{(c^2x^2-1)(cx+1)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))/(c*x+1)^2/(-c^2*x^2+1),x, algorithm="giac
")
```

output
```
integrate(-(b*arctanh(c*x) + a)*x^2/((c^2*x^2 - 1)*(c*x + 1)^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx = \frac{\frac{4\,(2\,a+b)}{c}+x\,(12\,a+5\,b)}{16\,c^4\,x^2+32\,c^3\,x+16\,c^2} - \ln(1-c\,x)\left(\frac{\frac{b}{c^3}+\frac{b\,x}{c^2}}{2\,c^2\,x^2+4\,c\,x+2} + \frac{b\,\ln(c\,x+1)}{16\,c^3} - \frac{b\,(3\,c^2\,x^2+10\,c\,x+11)}{16\,c^3\,(2\,c^2\,x^2+4\,c\,x+2)}\right) + \frac{\ln(c\,x+1)\left(\frac{5\,b}{32\,c^4}-\frac{3\,b\,x^2}{32\,c^2}+\frac{3\,b\,x}{16\,c^3}\right)}{2\,x+c\,x^2+\frac{1}{c}} + \frac{b\ln(c\,x+1)^2}{32\,c^3} + \frac{b\ln(1-c\,x)^2}{32\,c^3} - \frac{\operatorname{atan}(c\,x\,1\mathrm{i})\,(2\,a-b)\,1\mathrm{i}}{8\,c^3}$$

input
```
int(-(x^2*(a + b*atanh(c*x)))/((c^2*x^2 - 1)*(c*x + 1)^2),x)
```

output
```
((4*(2*a + b))/c + x*(12*a + 5*b))/(32*c^3*x + 16*c^2 + 16*c^4*x^2) - log(
1 - c*x)*((b/c^3 + (b*x)/c^2)/(4*c*x + 2*c^2*x^2 + 2) + (b*log(c*x + 1))/(
16*c^3) - (b*(10*c*x + 3*c^2*x^2 + 11))/(16*c^3*(4*c*x + 2*c^2*x^2 + 2)))
+ (log(c*x + 1)*((5*b)/(32*c^4) - (3*b*x^2)/(32*c^2) + (3*b*x)/(16*c^3)))/
(2*x + c*x^2 + 1/c) - (atan(c*x*1i)*(2*a - b)*1i)/(8*c^3) + (b*log(c*x + 1
)^2)/(32*c^3) + (b*log(1 - c*x)^2)/(32*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.22

$$\int \frac{x^2(a+b\operatorname{arctanh}(cx))}{(1+cx)^2(1-c^2x^2)}\,dx$$
$$= \frac{4atanh(cx)^2\,b\,c^2x^2 + 8atanh(cx)^2\,bcx + 4atanh(cx)^2\,b - 12atanh(cx)\,b\,c^2x^2 + 4atanh(cx)\,b - 4\log(cx -}{}$$

input
```
int(x^2*(a+b*atanh(c*x))/(c*x+1)^2/(-c^2*x^2+1),x)
```

output
```
(4*atanh(c*x)**2*b*c**2*x**2 + 8*atanh(c*x)**2*b*c*x + 4*atanh(c*x)**2*b -
 12*atanh(c*x)*b*c**2*x**2 + 4*atanh(c*x)*b - 4*log(c*x - 1)*a*c**2*x**2 -
 8*log(c*x - 1)*a*c*x - 4*log(c*x - 1)*a - log(c*x - 1)*b*c**2*x**2 - 2*lo
g(c*x - 1)*b*c*x - log(c*x - 1)*b + 4*log(c*x + 1)*a*c**2*x**2 + 8*log(c*x
 + 1)*a*c*x + 4*log(c*x + 1)*a + log(c*x + 1)*b*c**2*x**2 + 2*log(c*x + 1)
*b*c*x + log(c*x + 1)*b - 12*a*c**2*x**2 + 4*a - 5*b*c**2*x**2 + 3*b)/(32*
c**3*(c**2*x**2 + 2*c*x + 1))
```

3.521 $\int x^4(a+b\text{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 263

$$\begin{aligned}
&\int x^4(a+b\text{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right) dx \\
&= -\frac{2aex}{5c^4} - \frac{77bex^2}{300c^3} - \frac{9bex^4}{200c} - \frac{2bex\text{arctanh}(cx)}{5c^4} - \frac{2ex^3(a+b\text{arctanh}(cx))}{15c^2} \\
&\quad - \frac{2}{25}ex^5(a+b\text{arctanh}(cx)) + \frac{e(a+b\text{arctanh}(cx))^2}{5bc^5} - \frac{137be\log(1-c^2x^2)}{300c^5} \\
&\quad - \frac{be\log^2(1-c^2x^2)}{20c^5} + \frac{bx^2(d+e\log(1-c^2x^2))}{10c^3} + \frac{bx^4(d+e\log(1-c^2x^2))}{20c} \\
&\quad + \frac{1}{5}x^5(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right) + \frac{b\log(1-c^2x^2)(d+e\log(1-c^2x^2))}{10c^5}
\end{aligned}$$

output

```
-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-9/200*b*e*x^4/c-2/5*b*e*x*arctanh(c*x)/c
^4-2/15*e*x^3*(a+b*arctanh(c*x))/c^2-2/25*e*x^5*(a+b*arctanh(c*x))+1/5*e*(
a+b*arctanh(c*x))^2/b/c^5-137/300*b*e*ln(-c^2*x^2+1)/c^5-1/20*b*e*ln(-c^2*
x^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(-c^2*x^2+1))/c^3+1/20*b*x^4*(d+e*ln(-c^2*x
^2+1))/c+1/5*x^5*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/10*b*ln(-c^2*x^
2+1)*(d+e*ln(-c^2*x^2+1))/c^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.90

$$\int x^4(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-240acex+2bc^2(30d-77e)x^2-80ac^3ex^3+3bc^4(10d-9e)x^4+24ac^5(5d-2e)x^5-8bcx(-15c^4dx^4+\cdots}{}$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
 - 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15
+ 5*c^2*x^2 + 3*c^4*x^4))*ArcTanh[c*x] + 120*b*e*ArcTanh[c*x]^2 + 2*(30*b*
d - 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 +
 c*x] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcTanh[
c*x])*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6647, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6647}$$

$$2c^2e\int\left(\frac{4ac^3x^6+4bc^3\text{arctanh}(cx)x^6+bc^2x^5+2bx^3}{20c^3\left(1-c^2x^2\right)}+\frac{bx\log\left(1-c^2x^2\right)}{10c^5\left(1-c^2x^2\right)}\right)dx+\frac{1}{5}x^5(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+$$
$$2c^2e\left(\frac{a\text{arctanh}(cx)}{5c^7}-\frac{ax}{5c^6}-\frac{ax^3}{15c^4}-\frac{ax^5}{25c^2}+\frac{b\text{arctanh}(cx)^2}{10c^7}-\frac{bx\text{arctanh}(cx)}{5c^6}-\frac{bx^3\text{arctanh}(cx)}{15c^4}-\frac{bx^5\text{arctanh}(c}{25c^2}\right.$$
$$\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}$$

input
```
Int[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*Log[1 - c^2*x^2]
))/(20*c) + (x^5*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/5 + (b*Log
[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5) + 2*c^2*e*(-1/5*(a*x)/c^6
 - (77*b*x^2)/(600*c^5) - (a*x^3)/(15*c^4) - (9*b*x^4)/(400*c^3) - (a*x^5)
/(25*c^2) + (a*ArcTanh[c*x])/(5*c^7) - (b*x*ArcTanh[c*x])/(5*c^6) - (b*x^3
*ArcTanh[c*x])/(15*c^4) - (b*x^5*ArcTanh[c*x])/(25*c^2) + (b*ArcTanh[c*x]^
2)/(10*c^7) - (137*b*Log[1 - c^2*x^2])/(600*c^7) - (b*Log[1 - c^2*x^2]^2)/
(40*c^7))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6647
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 6.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{-154be+60db+60b\,c^2d\,x^2+60bd\ln\left(-c^2x^2+1\right)+120c^5d\,x^5a+30d\,x^4c^4b+120b\,c^5d\,\operatorname{arctanh}(cx)x^5-240acex-80a\,c^3e\,x^3-27b}{}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input

```
int(x^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output

```
1/600*(-154*b*e+60*d*b+60*b*c^2*d*x^2+60*b*d*ln(-c^2*x^2+1)+120*c^5*d*x^5*
a+30*d*x^4*c^4*b+120*b*c^5*d*arctanh(c*x)*x^5-240*a*c*e*x-80*a*c^3*e*x^3-2
7*b*c^4*e*x^4-154*b*c^2*e*x^2-48*a*c^5*e*x^5+120*b*e*ln(-c^2*x^2+1)*arctan
h(c*x)*x^5*c^5-274*ln(-c^2*x^2+1)*b*e+240*arctanh(c*x)*a*e+120*e*b*arctanh
(c*x)^2+30*e*b*ln(-c^2*x^2+1)^2-80*e*b*arctanh(c*x)*x^3*c^3-240*e*b*arctan
h(c*x)*x*c+30*b*e*ln(-c^2*x^2+1)*x^4*c^4+120*a*e*ln(-c^2*x^2+1)*x^5*c^5-48
*x^5*arctanh(c*x)*b*c^5*e+60*x^2*ln(-c^2*x^2+1)*b*c^2*e)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int x^4(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$
$$-\frac{80\,ac^3ex^3-24\,(5\,ac^5d-2\,ac^5e)x^5-3\,(10\,bc^4d-9\,bc^4e)x^4+240\,acex-30\,be\log\left(-c^2x^2+1\right)^2-30\,b}{}$$

input

```
integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d -
9*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*log(-c^2*x^2 + 1)^2 - 30*b*e*log(-(c
*x + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5
 + 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*log(-c^2*x^2 + 1) -
 4*(15*b*c^5*e*x^5*log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b
*c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*log(-(c*x + 1)/(c*x - 1)))/c^5
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.29

$$\int x^4(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^5}{5}+\frac{aex^5\log\left(-c^2x^2+1\right)}{5}-\frac{2aex^5}{25}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\,\text{atanh}\,(cx)}{5c^5}+\frac{bdx^5\,\text{atanh}\,(cx)}{5}+\frac{bex^5\log\left(-c^2x^2+1\right)\text{atanh}\,(cx)}{5}-\\ \frac{adx^5}{5}\end{cases}$$

input
```
integrate(x**4*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2
*a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atanh(c*x)/(5*c**5) + b*d*x
**5*atanh(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*atanh(c*x)/5 - 2*b*e*x**5*
atanh(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*
b*e*x**4/(200*c) - 2*b*e*x**3*atanh(c*x)/(15*c**2) + b*d*x**2/(10*c**3) +
b*e*x**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*
atanh(c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x*
*2 + 1)**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*atanh(
c*x)**2/(5*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.21

$$\int x^4(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{5}\,adx^5$$
$$+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)be$$
$$+\frac{1}{20}\left(4\,x^5\,\text{artanh}\left(cx\right)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bd$$
$$+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)ae$$
$$-\frac{\left(3\left(-10i\,\pi c^4+9\,c^4\right)x^4+2\left(-30i\,\pi c^2+77\,c^2\right)x^2+2\left(-30i\,\pi-15\,c^4x^4-30\,c^2x^2-60\,\log\left(cx-1\right)+1\right.}{600\,c^5}$$

input
```
integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x
^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*e*arctanh(c
*x) + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2
- 1)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^
2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/60
0*(3*(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 + 2*(-30*I
*pi - 15*c^4*x^4 - 30*c^2*x^2 - 60*log(c*x - 1) + 137)*log(c*x + 1) + 2*(-
30*I*pi - 15*c^4*x^4 - 30*c^2*x^2 + 137)*log(c*x - 1))*b*e/c^5
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.19

$$\int x^4(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=-\frac{1}{10}bex^5\log(-cx+1)^2+\frac{1}{25}(5ad-2ae)x^5+\frac{(10bd-9be)x^4}{200c}-\frac{2aex^3}{15c^2}+\frac{1}{10}\left(bex^5+\frac{be}{c^5}\right)\log(cx+1)^2+\frac{1}{300}\left(6(5bd+10ae-2be)x^5+\frac{15bex^4}{c}-\frac{20bex^3}{c^2}+\frac{30bex^2}{c^3}-\frac{60bex}{c^4}\right)\log(cx+1)-\frac{1}{300}\left(6(5bd-10ae-2be)x^5-\frac{15bex^4}{c}-\frac{20bex^3}{c^2}-\frac{30bex^2}{c^3}-\frac{60bex}{c^4}-\frac{60be\log(cx-1)}{c^5}\right)\log(-cx+1)+\frac{(30bd-77be)x^2}{300c^3}-\frac{2aex}{5c^4}-\frac{be\log(cx-1)^2}{10c^5}+\frac{(30bd+60ae-137be)\log(cx+1)}{300c^5}+\frac{(30bd-60ae-137be)\log(cx-1)}{300c^5}$$

input
```
integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
-1/10*b*e*x^5*log(-c*x + 1)^2 + 1/25*(5*a*d - 2*a*e)*x^5 + 1/200*(10*b*d -
 9*b*e)*x^4/c - 2/15*a*e*x^3/c^2 + 1/10*(b*e*x^5 + b*e/c^5)*log(c*x + 1)^2
 + 1/300*(6*(5*b*d + 10*a*e - 2*b*e)*x^5 + 15*b*e*x^4/c - 20*b*e*x^3/c^2 +
 30*b*e*x^2/c^3 - 60*b*e*x/c^4)*log(c*x + 1) - 1/300*(6*(5*b*d - 10*a*e -
2*b*e)*x^5 - 15*b*e*x^4/c - 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 - 60*b*e*x/c^4
 - 60*b*e*log(c*x - 1)/c^5)*log(-c*x + 1) + 1/300*(30*b*d - 77*b*e)*x^2/c^
3 - 2/5*a*e*x/c^4 - 1/10*b*e*log(c*x - 1)^2/c^5 + 1/300*(30*b*d + 60*a*e -
 137*b*e)*log(c*x + 1)/c^5 + 1/300*(30*b*d - 60*a*e - 137*b*e)*log(c*x - 1
)/c^5
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.28

$$\int x^4(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{a\,d\,x^5}{5}-\frac{2\,a\,e\,x^5}{25}+\frac{b\,d\,x^5\,\ln\left(c\,x+1\right)}{10}-\frac{b\,d\,x^5\,\ln\left(1-c\,x\right)}{10}-\frac{b\,e\,x^5\,\ln\left(c\,x+1\right)}{25}+\frac{b\,e\,x^5\,\ln\left(1-c\,x\right)}{25}+\frac{b\,e\ln\left(c\,x+1\right)^2}{10\,c^5}+\frac{b\,e\ln\left(1-c\,x\right)^2}{10\,c^5}-\frac{2\,a\,e\,x}{5\,c^4}-\frac{2\,a\,e\,x^3}{15\,c^2}+\frac{b\,d\,x^4}{20\,c}+\frac{b\,d\,x^2}{10\,c^3}-\frac{9\,b\,e\,x^4}{200\,c}-\frac{77\,b\,e\,x^2}{300\,c^3}+\frac{a\,e\,x^5\,\ln\left(1-c^2\,x^2\right)}{5}-\frac{a\,e\,\ln\left(c\,x-1\right)}{5\,c^5}+\frac{a\,e\,\ln\left(c\,x+1\right)}{5\,c^5}+\frac{b\,d\,\ln\left(c\,x-1\right)}{10\,c^5}+\frac{b\,d\,\ln\left(c\,x+1\right)}{10\,c^5}-\frac{137\,b\,e\,\ln\left(c\,x-1\right)}{300\,c^5}-\frac{137\,b\,e\,\ln\left(c\,x+1\right)}{300\,c^5}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{5\,c^4}\right)}{10\,c^5}-\frac{b\,e\,x\,\ln\left(c\,x+1\right)}{5\,c^4}+\frac{b\,e\,x\,\ln\left(1-c\,x\right)}{5\,c^4}+\frac{b\,e\,x^4\,\ln\left(1-c^2\,x^2\right)}{20\,c}+\frac{b\,e\,x^2\,\ln\left(1-c^2\,x^2\right)}{10\,c^3}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{5\,c^4}\right)\,\ln\left(1-c^2\,x^2\right)}{10\,c^5}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{5\,c^4}\right)\,\ln\left(1-c^2\,x^2\right)}{10\,c^5}-\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)}{15\,c^2}+\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)}{15\,c^2}+\frac{b\,e\,x^5\,\ln\left(c\,x+1\right)\,\ln\left(1-c^2\,x^2\right)}{10}-\frac{b\,e\,x^5\,\ln\left(1-c\,x\right)\,\ln\left(1-c^2\,x^2\right)}{10}$$

input
```
int(x^4*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
(a*d*x^5)/5 - (2*a*e*x^5)/25 + (b*d*x^5*log(c*x + 1))/10 - (b*d*x^5*log(1
- c*x))/10 - (b*e*x^5*log(c*x + 1))/25 + (b*e*x^5*log(1 - c*x))/25 + (b*e*
log(c*x + 1)^2)/(10*c^5) + (b*e*log(1 - c*x)^2)/(10*c^5) - (2*a*e*x)/(5*c^
4) - (2*a*e*x^3)/(15*c^2) + (b*d*x^4)/(20*c) + (b*d*x^2)/(10*c^3) - (9*b*e
*x^4)/(200*c) - (77*b*e*x^2)/(300*c^3) + (a*e*x^5*log(1 - c^2*x^2))/5 - (a
*e*log(c*x - 1))/(5*c^5) + (a*e*log(c*x + 1))/(5*c^5) + (b*d*log(c*x - 1))
/(10*c^5) + (b*d*log(c*x + 1))/(10*c^5) - (137*b*e*log(c*x - 1))/(300*c^5)
 - (137*b*e*log(c*x + 1))/(300*c^5) - (b*e*log(c*x + 1)*log(-(2*a*e - 2*a*
c*e*x)/(5*c^4)))/(10*c^5) - (b*e*log(c*x + 1)*log(-(2*a*e + 2*a*c*e*x)/(5*
c^4)))/(10*c^5) - (b*e*log(1 - c*x)*log(-(2*a*e - 2*a*c*e*x)/(5*c^4)))/(10
*c^5) - (b*e*log(1 - c*x)*log(-(2*a*e + 2*a*c*e*x)/(5*c^4)))/(10*c^5) - (b
*e*x*log(c*x + 1))/(5*c^4) + (b*e*x*log(1 - c*x))/(5*c^4) + (b*e*x^4*log(1
 - c^2*x^2))/(20*c) + (b*e*x^2*log(1 - c^2*x^2))/(10*c^3) + (b*e*log(-(2*a
*e - 2*a*c*e*x)/(5*c^4))*log(1 - c^2*x^2))/(10*c^5) + (b*e*log(-(2*a*e + 2
*a*c*e*x)/(5*c^4))*log(1 - c^2*x^2))/(10*c^5) - (b*e*x^3*log(c*x + 1))/(15
*c^2) + (b*e*x^3*log(1 - c*x))/(15*c^2) + (b*e*x^5*log(c*x + 1)*log(1 - c^
2*x^2))/10 - (b*e*x^5*log(1 - c*x)*log(1 - c^2*x^2))/10
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.17

$$\int x^4(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{120atanh(cx)^2be+120atanh(cx)\log(-c^2x^2+1)\,b\,c^5e\,x^5+120atanh(cx)\,b\,c^5d\,x^5-48atanh(cx)\,b\,c^5e\,x^5}{}$$

input
```
int(x^4*(a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(120*atanh(c*x)**2*b*e + 120*atanh(c*x)*log( - c**2*x**2 + 1)*b*c**5*e*x**
5 + 120*atanh(c*x)*b*c**5*d*x**5 - 48*atanh(c*x)*b*c**5*e*x**5 - 80*atanh(
c*x)*b*c**3*e*x**3 - 240*atanh(c*x)*b*c*e*x + 30*log( - c**2*x**2 + 1)**2*
b*e + 120*log( - c**2*x**2 + 1)*a*c**5*e*x**5 + 120*log( - c**2*x**2 + 1)*
a*e + 30*log( - c**2*x**2 + 1)*b*c**4*e*x**4 + 60*log( - c**2*x**2 + 1)*b*
c**2*e*x**2 + 60*log( - c**2*x**2 + 1)*b*d - 274*log( - c**2*x**2 + 1)*b*e
 - 240*log(c**2*x - c)*a*e + 120*a*c**5*d*x**5 - 48*a*c**5*e*x**5 - 80*a*c
**3*e*x**3 - 240*a*c*e*x + 30*b*c**4*d*x**4 - 27*b*c**4*e*x**4 + 60*b*c**2
*d*x**2 - 154*b*c**2*e*x**2)/(600*c**5)
```

3.522 $\int x^3(a+b\text{arctanh}(cx)) (d+e\log(1-c^2x^2)) dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\begin{aligned}\int x^3(a+b\text{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right) dx &= \frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{b(2d-3e)\text{arctanh}(cx)}{8c^4}\\&+\frac{2be\text{arctanh}(cx)}{3c^4}-\frac{ex^2(a+b\text{arctanh}(cx))}{4c^2}-\frac{1}{8}ex^4(a+b\text{arctanh}(cx))\\&+\frac{bex\log\left(1-c^2x^2\right)}{4c^3}+\frac{bex^3\log\left(1-c^2x^2\right)}{12c}-\frac{e(a+b\text{arctanh}(cx))\log\left(1-c^2x^2\right)}{4c^4}\\&+\frac{1}{4}x^4(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\end{aligned}$$

output
```
1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3+1/24*b*(2*d-e)*x^3/c-1/18*b*e*x^3/c-1/
8*b*(2*d-3*e)*arctanh(c*x)/c^4+2/3*b*e*arctanh(c*x)/c^4-1/4*e*x^2*(a+b*arc
tanh(c*x))/c^2-1/8*e*x^4*(a+b*arctanh(c*x))+1/4*b*e*x*ln(-c^2*x^2+1)/c^3+1
/12*b*e*x^3*ln(-c^2*x^2+1)/c-1/4*e*(a+b*arctanh(c*x))*ln(-c^2*x^2+1)/c^4+1
/4*x^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

$$\int x^3(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{6bc(6d-25e)x-36ac^2ex^2+2bc^3(6d-7e)x^3+18ac^4(2d-e)x^4-18bc^2x^2(-2c^2dx^2+e(2+c^2x^2))\,\text{arc}}{}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^
4*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcTanh[c*
x] + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e
)*Log[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x
^4)*ArcTanh[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6645, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6645}$$

$$-bc\int\left(-\frac{\left(2e-c^2(2d-e)x^2\right)x^2}{8c^2\left(1-c^2x^2\right)}-\frac{e\left(c^2x^2+1\right)\log\left(1-c^2x^2\right)}{4c^4}\right)dx+\frac{1}{4}x^4(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2(a+b\text{arctanh}(cx))}{4c^2}-\frac{e\log\left(1-c^2x^2\right)(a+b\text{arctanh}(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2(a+b\text{arctanh}(cx))}{4c^2}-\frac{e\log\left(1-c^2x^2\right)(a+b\text{arctanh}(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\text{arctanh}(cx))-bc\left(\frac{(2d-3e)\text{arctanh}(cx)}{8c^5}-\frac{2e\text{arctanh}(cx)}{3c^5}-\frac{x(2d-3e)}{8c^4}+\frac{2ex}{3c^4}-\frac{x^3(2d-e)}{24c^2}+\frac{ex^3}{18c^2}-\frac{ex^3\log\left(1-c^2x^2\right)}{12c^2}-\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
-1/4*(e*x^2*(a + b*ArcTanh[c*x]))/c^2 - (e*x^4*(a + b*ArcTanh[c*x]))/8 - (
e*(a + b*ArcTanh[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcTanh[c*x
])*(d + e*Log[1 - c^2*x^2]))/4 - b*c*(-1/8*((2*d - 3*e)*x)/c^4 + (2*e*x)/(
3*c^4) - ((2*d - e)*x^3)/(24*c^2) + (e*x^3)/(18*c^2) + ((2*d - 3*e)*ArcTan
h[c*x])/(8*c^5) - (2*e*ArcTanh[c*x])/(3*c^5) - (e*x*Log[1 - c^2*x^2])/(4*c
^4) - (e*x^3*Log[1 - c^2*x^2])/(12*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6645
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
 + 1)/2, 0]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{75\ \mathrm{arctanh}(cx)be-18a\,c^{2}e\,x^{2}-75bcex+18bcdx-18\ \mathrm{arctanh}(cx)b\,c^{2}e\,x^{2}+18a\,c^{4}d\,x^{4}+6b\,c^{3}d\,x^{3}-18\ \mathrm{arctanh}(cx)bd+18db\ \mathrm{arc}}{}$
orering	$\frac{(9c^{8}x^{8}+76c^{6}x^{6}-497c^{4}x^{4}+1570c^{2}x^{2}-1350)(a+b\ \mathrm{arctanh}(cx))(d+e\ln(-c^{2}x^{2}+1))}{12c^{6}x^{2}(c^{2}x^{2}+3)}-\frac{(19c^{8}x^{8}+274c^{6}x^{6}-2162c^{4}x^{4}+6210}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/72*(75*arctanh(c*x)*b*e-18*a*c^2*e*x^2-75*b*c*e*x+18*b*c*d*x-18*arctanh(
c*x)*b*c^2*e*x^2+18*a*c^4*d*x^4+6*b*c^3*d*x^3-18*arctanh(c*x)*b*d+18*d*b*a
rctanh(c*x)*x^4*c^4-18*a*e-18*arctanh(c*x)*ln(-c^2*x^2+1)*b*e-18*ln(-c^2*x
^2+1)*a*e+18*ln(-c^2*x^2+1)*b*c*e*x-9*a*c^4*e*x^4-7*b*c^3*e*x^3+18*x^4*arc
tanh(c*x)*ln(-c^2*x^2+1)*b*e*c^4+6*b*e*ln(-c^2*x^2+1)*x^3*c^3+18*x^4*ln(-c
^2*x^2+1)*a*e*c^4-9*x^4*arctanh(c*x)*b*c^4*e)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.87

$$\int x^{3}(a+b\mathrm{arctanh}(cx))\left(d+e\log\left(1-c^{2}x^{2}\right)\right)\,dx =$$
$$-\frac{36\,ac^{2}ex^{2}-18\,(2\,ac^{4}d-ac^{4}e)x^{4}-2\,(6\,bc^{3}d-7\,bc^{3}e)x^{3}-6\,(6\,bcd-25\,bce)x-12\,(3\,ac^{4}ex^{4}+bc^{3}ex^{3}}{}$$

input
```
integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b
*c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 +
 3*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d -
b*c^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*l
og(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.24

$$\int x^3(a + b\operatorname{arctanh}(cx)) \left(d + e\log\left(1 - c^2x^2\right)\right) \, dx$$

$$= \begin{cases} \frac{adx^4}{4} + \frac{aex^4\log\left(-c^2x^2+1\right)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae\log\left(-c^2x^2+1\right)}{4c^4} + \frac{bdx^4\operatorname{atanh}\left(cx\right)}{4} + \frac{bex^4\log\left(-c^2x^2+1\right)\operatorname{atanh}\left(cx\right)}{4} - \frac{bex^4\operatorname{ata}}{8} \\ \frac{adx^4}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*
x**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atanh(c*x)/4 +
 b*e*x**4*log(-c**2*x**2 + 1)*atanh(c*x)/4 - b*e*x**4*atanh(c*x)/8 + b*d*x
**3/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e
*x**2*atanh(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*
c**3) - 25*b*e*x/(24*c**3) - b*d*atanh(c*x)/(4*c**4) - b*e*log(-c**2*x**2
+ 1)*atanh(c*x)/(4*c**4) + 25*b*e*atanh(c*x)/(24*c**4), Ne(c, 0)), (a*d*x*
*4/4, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.20

$$\int x^3(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{4}adx^4+\frac{1}{8}\left(2\,x^4\log\left(-c^2x^2+1\right)-c^2\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)be\,\text{artanh}\left(cx\right)+\frac{1}{24}\left(6\,x^4\,\text{artanh}\left(cx\right)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bd+\frac{1}{8}\left(2\,x^4\log\left(-c^2x^2+1\right)-c^2\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)ae-\frac{(2\left(-6i\,\pi c^3+7\,c^3\right)x^3+6\left(-6i\,\pi c+25\,c\right)x+3\left(6i\,\pi-4\,c^3x^3-12\,cx-25\right)\log\left(cx+1\right)+3\left(-6i\,\pi-4\right.}{144\,c^4}$$

input
```
integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 +
2*log(c^2*x^2 - 1)/c^6))*b*e*arctanh(c*x) + 1/24*(6*x^4*arctanh(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/
8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
1)/c^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x
 + 3*(6*I*pi - 4*c^3*x^3 - 12*c*x - 25)*log(c*x + 1) + 3*(-6*I*pi - 4*c^3*
x^3 - 12*c*x + 25)*log(c*x - 1))*b*e/c^4
```

Giac [F(-2)]

Exception generated.

$$\int x^3(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 851, normalized size of antiderivative = 3.78

$$\int x^3(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\text{Too large to display}$$

input
```
int(x^3*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - c*x)^2*((b*e)/(8*c^4) - (b*e*x^4)/8) - log(c*x + 1)^2*((b*e)/(8*c^
4) - (b*e*x^4)/8) + log(1 - c*x)*((x^4*(a*e - (b*d)/2 + (b*e)/4 + (b*e*(lo
g(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/2))/4 - (x^2*((16*a*e - 8*b
*d + 8*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/c - (16*a*e -
 8*b*d + 4*b*e + 8*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/c
))/(32*c) + (b*e*x)/(4*c^3) + (b*e*x^3)/(12*c)) - x^2*((a*(e - 2*d + 2*e*(
log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(4*c^2) + (a*(d - e*(log
(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(2*c^2)) - x*((b*(7*e - 6*d
 + 6*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(24*c^3) + (3*b*
e)/(4*c^3)) - (a*x^4*(e - 2*d + 2*e*(log(c*x + 1) + log(1 - c*x) - log(1 -
 c^2*x^2))))/8 - (log((x*(12*a*e - 6*b*d + 25*b*e + 6*b*e*(log(c*x + 1) +
log(1 - c*x) - log(1 - c^2*x^2))))/(24*c^2) - (25*b*e - 6*b*d + 6*b*e*(log
(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/(24*c^3) - (a*e*x)/(2*c^2))*
(12*a*e - 6*b*d + 25*b*e + 6*b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^
2*x^2))))/(48*c^4) - (log((x*(12*a*e + 6*b*d - 25*b*e - 6*b*e*(log(c*x + 1
) + log(1 - c*x) - log(1 - c^2*x^2))))/(24*c^2) - (25*b*e - 6*b*d + 6*b*e*
(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/(24*c^3) - (a*e*x)/(2*c^
2))*(12*a*e + 6*b*d - 25*b*e - 6*b*e*(log(c*x + 1) + log(1 - c*x) - log(1
- c^2*x^2))))/(48*c^4) + c*log(c*x + 1)*((x^4*(4*a*e + 2*b*d - b*e - 2*b*e
*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(16*c) + (b*e*x)/(4...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

$$\int x^3(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{18atanh(cx)\log(-c^2x^2+1)\,b\,c^4e\,x^4-18atanh(cx)\log(-c^2x^2+1)\,be+18atanh(cx)\,b\,c^4d\,x^4-9atanh(c}{}$$

input
```
int(x^3*(a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(18*atanh(c*x)*log( - c**2*x**2 + 1)*b*c**4*e*x**4 - 18*atanh(c*x)*log( -
c**2*x**2 + 1)*b*e + 18*atanh(c*x)*b*c**4*d*x**4 - 9*atanh(c*x)*b*c**4*e*x
**4 - 18*atanh(c*x)*b*c**2*e*x**2 - 18*atanh(c*x)*b*d + 75*atanh(c*x)*b*e
+ 18*log( - c**2*x**2 + 1)*a*c**4*e*x**4 - 18*log( - c**2*x**2 + 1)*a*e +
6*log( - c**2*x**2 + 1)*b*c**3*e*x**3 + 18*log( - c**2*x**2 + 1)*b*c*e*x +
 18*a*c**4*d*x**4 - 9*a*c**4*e*x**4 - 18*a*c**2*e*x**2 + 6*b*c**3*d*x**3 -
 7*b*c**3*e*x**3 + 18*b*c*d*x - 75*b*c*e*x)/(72*c**4)
```

3.523 $\int x^2(a+b\,\mathbf{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\begin{aligned}&\int x^2(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx\\&=-\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2bex\text{arctanh}(cx)}{3c^2}-\frac{2}{9}ex^3(a+b\text{arctanh}(cx))+\frac{e(a+b\text{arctanh}(cx))^2}{3bc^3}\\&\quad-\frac{11be\log(1-c^2x^2)}{18c^3}-\frac{be\log^2(1-c^2x^2)}{12c^3}+\frac{bx^2(d+e\log(1-c^2x^2))}{6c}\\&\quad+\frac{1}{3}x^3(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\log(1-c^2x^2)(d+e\log(1-c^2x^2))}{6c^3}\end{aligned}$$

output

```
-2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/3*b*e*x*arctanh(c*x)/c^2-2/9*e*x^3*(a+b*ar
ctanh(c*x))+1/3*e*(a+b*arctanh(c*x))^2/b/c^3-11/18*b*e*ln(-c^2*x^2+1)/c^3-
1/12*b*e*ln(-c^2*x^2+1)^2/c^3+1/6*b*x^2*(d+e*ln(-c^2*x^2+1))/c+1/3*x^3*(a+
b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/6*b*ln(-c^2*x^2+1)*(d+e*ln(-c^2*x^2
+1))/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.89

$$\int x^2(a+b\text{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-24acex+2bc^2(3d-5e)x^2+4ac^3(3d-2e)x^3+4bcx(3c^2dx^2-2e(3+c^2x^2))\text{arctanh}(cx)+12be\text{arctan}}{}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x
*(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcTanh[c*x] + 12*b*e*ArcTanh[c*x]^2 +
2*(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1
 + c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcTanh[c*x])*Log[1 - c^2*x^2
] + 3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6647, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arctanh}(cx)) \left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6647}$$

$$2c^2e\int\left(\frac{(2cx\text{arctanh}(cx)b+b+2acx)x^3}{6c\left(1-c^2x^2\right)}+\frac{b\log\left(1-c^2x^2\right)x}{6c^3\left(1-c^2x^2\right)}\right)dx+\frac{1}{3}x^3(a+ b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\operatorname{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+$$
$$2c^2e\left(\frac{a\operatorname{arctanh}(cx)}{3c^5}-\frac{ax}{3c^4}-\frac{ax^3}{9c^2}+\frac{b\operatorname{arctanh}(cx)^2}{6c^5}-\frac{bx\operatorname{arctanh}(cx)}{3c^4}-\frac{bx^3\operatorname{arctanh}(cx)}{9c^2}-\frac{5bx^2}{36c^3}-\frac{b\log^2\left(1-c^2\right.}{24c^5}\right.$$
$$\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}$$

input
```
Int[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcTanh[c*x])*(d + e*
Log[1 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^
3) + 2*c^2*e*(-1/3*(a*x)/c^4 - (5*b*x^2)/(36*c^3) - (a*x^3)/(9*c^2) + (a*A
rcTanh[c*x])/(3*c^5) - (b*x*ArcTanh[c*x])/(3*c^4) - (b*x^3*ArcTanh[c*x])/(
9*c^2) + (b*ArcTanh[c*x]^2)/(6*c^5) - (11*b*Log[1 - c^2*x^2])/(36*c^5) - (
b*Log[1 - c^2*x^2]^2)/(24*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6647
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{-44\operatorname{arctanh}(cx)be+12a\,c^3d\,x^3+6b\,c^2d\,x^2+12\ln(cx-1)bd+12\operatorname{arctanh}(cx)bd+12db\operatorname{arctanh}(cx)x^3c^3-24acex-8a\,c^3e\,x^3-1}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^2*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(-44*arctanh(c*x)*b*e+12*a*c^3*d*x^3+6*b*c^2*d*x^2+12*ln(c*x-1)*b*d+1
2*arctanh(c*x)*b*d+12*d*b*arctanh(c*x)*x^3*c^3-24*a*c*e*x-8*a*c^3*e*x^3-10
*b*c^2*e*x^2+24*arctanh(c*x)*a*e+12*e*b*arctanh(c*x)^2+3*e*b*ln(-c^2*x^2+1
)^2-8*e*b*arctanh(c*x)*x^3*c^3-24*e*b*arctanh(c*x)*x*c+6*x^2*ln(-c^2*x^2+1
)*b*c^2*e+12*x^3*arctanh(c*x)*ln(-c^2*x^2+1)*b*e*c^3-44*ln(c*x-1)*b*e+12*x
^3*ln(-c^2*x^2+1)*a*e*c^3)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97

$$\int x^2(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$

$$-\frac{24\,acex-4\,(3\,ac^3d-2\,ac^3e)x^3-3\,be\log\left(-c^2x^2+1\right)^2-3\,be\log\left(-\frac{cx+1}{cx-1}\right)^2-2\,(3\,bc^2d-5\,bc^2e)x^2-2$$

input
```
integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*log(-c^2*x^2 + 1
)^2 - 3*b*e*log(-(c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 -
2*(6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(-c^2*x^2 + 1) - 2*(
3*b*c^3*e*x^3*log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3
+ 6*a*e)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.25

$$\int x^2(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^3\log\left(-c^2x^2+1\right)}{3}-\frac{2aex^3}{9}-\frac{2aex}{3c^2}+\frac{2ae\,\text{atanh}\,(cx)}{3c^3}+\frac{bdx^3\,\text{atanh}\,(cx)}{3}+\frac{bex^3\log\left(-c^2x^2+1\right)\text{atanh}\,(cx)}{3}-\frac{2bex^3\,\text{atan}}{9}\\ \frac{adx^3}{3}\end{cases}$$

input
```
integrate(x**2*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*
a*e*x/(3*c**2) + 2*a*e*atanh(c*x)/(3*c**3) + b*d*x**3*atanh(c*x)/3 + b*e*x
**3*log(-c**2*x**2 + 1)*atanh(c*x)/3 - 2*b*e*x**3*atanh(c*x)/9 + b*d*x**2/
(6*c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*a
tanh(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2
 + 1)**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*atanh(c*x)
**2/(3*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.22

$$\int x^2(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{3}\,adx^3$$
$$+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)be\,\text{artanh}\left(cx\right)$$
$$+\frac{1}{6}\left(2\,x^3\,\text{artanh}\left(cx\right)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bd$$
$$+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)ae$$
$$+\frac{((3i\,\pi c^2-5\,c^2)x^2+(3i\,\pi+3\,c^2x^2+6\,\log\left(cx-1\right)-11)\log\left(cx+1\right)+(3i\,\pi+3\,c^2x^2-11)\log\left(cx-\right.}{18\,c^3}$$

input
```
integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 -
3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arctanh(c*x) + 1/6*(2*x^3*ar
ctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2
*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x -
1)/c^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*lo
g(c*x - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*
e/c^3
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.18

$$\int x^2(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=-\frac{1}{6}\,bex^3\log\left(-cx+1\right)^2+\frac{1}{9}\left(3\,ad-2\,ae\right)x^3+\frac{1}{6}\left(bex^3+\frac{be}{c^3}\right)\log\left(cx+1\right)^2$$
$$+\frac{(3\,bd-5\,be)x^2}{18\,c}+\frac{1}{18}\left((3\,bd+6\,ae-2\,be)x^3+\frac{3\,bex^2}{c}-\frac{6\,bex}{c^2}\right)\log\left(cx+1\right)$$
$$-\frac{1}{18}\left((3\,bd-6\,ae-2\,be)x^3-\frac{3\,bex^2}{c}-\frac{6\,bex}{c^2}-\frac{6\,be\log\left(cx-1\right)}{c^3}\right)\log\left(-cx+1\right)$$
$$-\frac{2\,aex}{3\,c^2}-\frac{be\log\left(cx-1\right)^2}{6\,c^3}+\frac{(3\,bd+6\,ae-11\,be)\log\left(cx+1\right)}{18\,c^3}$$
$$+\frac{(3\,bd-6\,ae-11\,be)\log\left(cx-1\right)}{18\,c^3}$$

input
```
integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
-1/6*b*e*x^3*log(-c*x + 1)^2 + 1/9*(3*a*d - 2*a*e)*x^3 + 1/6*(b*e*x^3 + b*
e/c^3)*log(c*x + 1)^2 + 1/18*(3*b*d - 5*b*e)*x^2/c + 1/18*((3*b*d + 6*a*e
- 2*b*e)*x^3 + 3*b*e*x^2/c - 6*b*e*x/c^2)*log(c*x + 1) - 1/18*((3*b*d - 6*
a*e - 2*b*e)*x^3 - 3*b*e*x^2/c - 6*b*e*x/c^2 - 6*b*e*log(c*x - 1)/c^3)*log
(-c*x + 1) - 2/3*a*e*x/c^2 - 1/6*b*e*log(c*x - 1)^2/c^3 + 1/18*(3*b*d + 6*
a*e - 11*b*e)*log(c*x + 1)/c^3 + 1/18*(3*b*d - 6*a*e - 11*b*e)*log(c*x - 1
)/c^3
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.50

$$\int x^2(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{a\,d\,x^3}{3}-\frac{2\,a\,e\,x^3}{9}+\frac{b\,d\,x^3\,\ln\left(c\,x+1\right)}{6}-\frac{b\,d\,x^3\,\ln\left(1-c\,x\right)}{6}-\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)}{9}+\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)}{9}+\frac{b\,e\ln\left(c\,x+1\right)^2}{6\,c^3}+\frac{b\,e\ln\left(1-c\,x\right)^2}{6\,c^3}-\frac{2\,a\,e\,x}{3\,c^2}+\frac{b\,d\,x^2}{6\,c}-\frac{5\,b\,e\,x^2}{18\,c}+\frac{a\,e\,x^3\,\ln\left(1-c^2\,x^2\right)}{3}-\frac{a\,e\,\ln\left(c\,x-1\right)}{3\,c^3}+\frac{a\,e\,\ln\left(c\,x+1\right)}{3\,c^3}+\frac{b\,d\,\ln\left(c\,x-1\right)}{6\,c^3}+\frac{b\,d\,\ln\left(c\,x+1\right)}{6\,c^3}-\frac{11\,b\,e\,\ln\left(c\,x-1\right)}{18\,c^3}-\frac{11\,b\,e\,\ln\left(c\,x+1\right)}{18\,c^3}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{3\,c^2}\right)}{6\,c^3}-\frac{b\,e\,x\,\ln\left(c\,x+1\right)}{3\,c^2}+\frac{b\,e\,x\,\ln\left(1-c\,x\right)}{3\,c^2}+\frac{b\,e\,x^2\,\ln\left(1-c^2\,x^2\right)}{6\,c}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e-2\,a\,c\,e\,x}{3\,c^2}\right)\,\ln\left(1-c^2\,x^2\right)}{6\,c^3}+\frac{b\,e\,\ln\left(-\frac{2\,a\,e+2\,a\,c\,e\,x}{3\,c^2}\right)\,\ln\left(1-c^2\,x^2\right)}{6\,c^3}+\frac{b\,e\,x^3\,\ln\left(c\,x+1\right)\,\ln\left(1-c^2\,x^2\right)}{6}-\frac{b\,e\,x^3\,\ln\left(1-c\,x\right)\,\ln\left(1-c^2\,x^2\right)}{6}$$

input
```
int(x^2*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
(a*d*x^3)/3 - (2*a*e*x^3)/9 + (b*d*x^3*log(c*x + 1))/6 - (b*d*x^3*log(1 -
c*x))/6 - (b*e*x^3*log(c*x + 1))/9 + (b*e*x^3*log(1 - c*x))/9 + (b*e*log(c
*x + 1)^2)/(6*c^3) + (b*e*log(1 - c*x)^2)/(6*c^3) - (2*a*e*x)/(3*c^2) + (b
*d*x^2)/(6*c) - (5*b*e*x^2)/(18*c) + (a*e*x^3*log(1 - c^2*x^2))/3 - (a*e*l
og(c*x - 1))/(3*c^3) + (a*e*log(c*x + 1))/(3*c^3) + (b*d*log(c*x - 1))/(6*
c^3) + (b*d*log(c*x + 1))/(6*c^3) - (11*b*e*log(c*x - 1))/(18*c^3) - (11*b
*e*log(c*x + 1))/(18*c^3) - (b*e*log(c*x + 1)*log(-(2*a*e - 2*a*c*e*x)/(3*
c^2)))/(6*c^3) - (b*e*log(c*x + 1)*log(-(2*a*e + 2*a*c*e*x)/(3*c^2)))/(6*c
^3) - (b*e*log(1 - c*x)*log(-(2*a*e - 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b*e*
log(1 - c*x)*log(-(2*a*e + 2*a*c*e*x)/(3*c^2)))/(6*c^3) - (b*e*x*log(c*x +
 1))/(3*c^2) + (b*e*x*log(1 - c*x))/(3*c^2) + (b*e*x^2*log(1 - c^2*x^2))/(
6*c) + (b*e*log(-(2*a*e - 2*a*c*e*x)/(3*c^2))*log(1 - c^2*x^2))/(6*c^3) +
(b*e*log(-(2*a*e + 2*a*c*e*x)/(3*c^2))*log(1 - c^2*x^2))/(6*c^3) + (b*e*x^
3*log(c*x + 1)*log(1 - c^2*x^2))/6 - (b*e*x^3*log(1 - c*x)*log(1 - c^2*x^2
))/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.18

$$\int x^2(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= \frac{12atanh(cx)^2\,be+12atanh(cx)\log(-c^2x^2+1)\,b\,c^3e\,x^3+12atanh(cx)\,b\,c^3d\,x^3-8atanh(cx)\,b\,c^3e\,x^3-24}{}$$

input
```
int(x^2*(a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(12*atanh(c*x)**2*b*e + 12*atanh(c*x)*log( - c**2*x**2 + 1)*b*c**3*e*x**3
+ 12*atanh(c*x)*b*c**3*d*x**3 - 8*atanh(c*x)*b*c**3*e*x**3 - 24*atanh(c*x)
*b*c*e*x + 3*log( - c**2*x**2 + 1)**2*b*e + 12*log( - c**2*x**2 + 1)*a*c**
3*e*x**3 + 12*log( - c**2*x**2 + 1)*a*e + 6*log( - c**2*x**2 + 1)*b*c**2*e
*x**2 + 6*log( - c**2*x**2 + 1)*b*d - 22*log( - c**2*x**2 + 1)*b*e - 24*lo
g(c**2*x - c)*a*e + 12*a*c**3*d*x**3 - 8*a*c**3*e*x**3 - 24*a*c*e*x + 6*b*
c**2*d*x**2 - 10*b*c**2*e*x**2)/(36*c**3)
```

3.524 $\int x(a+b\mathbf{arctanh}(cx)) \left(d+e\log \left(1-c^2x^2\right)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\begin{aligned}
&\int x(a+b\text{arctanh}(cx)) \left(d+e\log \left(1-c^2x^2\right)\right) \, dx \\
&= \frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\text{arctanh}(cx)}{2c^2}+\frac{be\text{arctanh}(cx)}{c^2} \\
&\quad +\frac{1}{2}dx^2(a+b\text{arctanh}(cx))-\frac{1}{2}ex^2(a+b\text{arctanh}(cx)) \\
&\quad +\frac{bex\log \left(1-c^2x^2\right)}{2c}-\frac{e\left(1-c^2x^2\right)(a+b\text{arctanh}(cx))\log \left(1-c^2x^2\right)}{2c^2}
\end{aligned}$$

output

```
1/2*b*(d-e)*x/c-b*e*x/c-1/2*b*(d-e)*arctanh(c*x)/c^2+b*e*arctanh(c*x)/c^2+
1/2*d*x^2*(a+b*arctanh(c*x))-1/2*e*x^2*(a+b*arctanh(c*x))+1/2*b*e*x*ln(-c^
2*x^2+1)/c-1/2*e*(-c^2*x^2+1)*(a+b*arctanh(c*x))*ln(-c^2*x^2+1)/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{2bc(d-3e)x+2ac^2(d-e)x^2+2bc^2(d-e)x^2\text{arctanh}(cx)+(b(d-3e)-2ae)\log(1-cx)-(b(d-3e)}{4c^2}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcTanh[c*x
] + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x
] + 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcTanh[c*x])*Log[1 - c^2*x^2]
)/(4*c^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6645, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \; 6645$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(1-c^2x^2\right)}-\frac{e\log\left(1-c^2x^2\right)}{2c^2}\right)dx-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)(a+b\text{arctanh}(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))-\frac{1}{2}ex^2(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \; 2009$$

$$-\frac{e(1-c^2x^2)\log(1-c^2x^2)(a+b\text{arctanh}(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))-\frac{1}{2}ex^2(a+b\text{arctanh}(cx))-bc\left(\frac{(d-e)\text{arctanh}(cx)}{2c^3}-\frac{e\text{arctanh}(cx)}{c^3}-\frac{x(d-e)}{2c^2}-\frac{ex\log(1-c^2x^2)}{2c^2}+\frac{ex}{c^2}\right)$$

input
```
Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcTanh[c*x]))/2 - (e*x^2*(a + b*ArcTanh[c*x]))/2 - (e*(1 -
c^2*x^2)*(a + b*ArcTanh[c*x])*Log[1 - c^2*x^2])/(2*c^2) - b*c*(-1/2*((d -
e)*x)/c^2 + (e*x)/c^2 + ((d - e)*ArcTanh[c*x])/(2*c^3) - (e*ArcTanh[c*x])/
c^3 - (e*x*Log[1 - c^2*x^2])/(2*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6645
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{\text{arctanh}(cx)\ln(-c^2x^2+1)bc^2ex^2+x^2\text{arctanh}(cx)bc^2d-\text{arctanh}(cx)bc^2ex^2+\ln(-c^2x^2+1)ac^2ex^2+ac^2dx^2-ac^2ex^2+\ln(}{2c^2}$
orering	$\frac{(c^6x^6+9c^2x^2-18)(a+b\text{ arctanh}(cx))(d+e\ln(-c^2x^2+1))}{x^2c^4(c^2x^2+3)}-\frac{(c^6x^6-2c^4x^4+21c^2x^2-36)\left((a+b\text{ arctanh}(cx))(d+e\ln(-c^2\right.}{2c^4x^2(c^2}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctanh(c*x)*ln(-c^2*x^2+1)*b*c^2*e*x^2+x^2*arctanh(c*x)*b*c^2*d-arct
anh(c*x)*b*c^2*e*x^2+ln(-c^2*x^2+1)*a*c^2*e*x^2+a*c^2*d*x^2-a*c^2*e*x^2+ln
(-c^2*x^2+1)*b*c*e*x+b*c*d*x-3*b*c*e*x-arctanh(c*x)*ln(-c^2*x^2+1)*b*e-arc
tanh(c*x)*b*d+3*arctanh(c*x)*b*e-ln(-c^2*x^2+1)*a*e)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{2\left(ac^2d-ac^2e\right)x^2+2\left(bcd-3\,bce\right)x+2\left(ac^2ex^2+bcex-ae\right)\log\left(-c^2x^2+1\right)+\left(\left(bc^2d-bc^2e\right)x^2-bd+\right.}{4\,c^2}$$

input
```
integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/4*(2*(a*c^2*d - a*c^2*e)*x^2 + 2*(b*c*d - 3*b*c*e)*x + 2*(a*c^2*e*x^2 +
b*c*e*x - a*e)*log(-c^2*x^2 + 1) + ((b*c^2*d - b*c^2*e)*x^2 - b*d + 3*b*e
+ (b*c^2*e*x^2 - b*e)*log(-c^2*x^2 + 1))*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log\left(-c^2x^2+1\right)}{2}-\frac{aex^2}{2}-\frac{ae\log\left(-c^2x^2+1\right)}{2c^2}+\frac{bdx^2\,\text{atanh}\,(cx)}{2}+\frac{bex^2\log\left(-c^2x^2+1\right)\text{atanh}\,(cx)}{2}-\frac{bex^2\,\text{atanh}\,(cx)}{2}\\\frac{adx^2}{2}\end{cases}$$

input
```
integrate(x*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*
log(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atanh(c*x)/2 + b*e*x**2*log(-c**2*
x**2 + 1)*atanh(c*x)/2 - b*e*x**2*atanh(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-
c**2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*atanh(c*x)/(2*c**2) - b*e*log(-
c**2*x**2 + 1)*atanh(c*x)/(2*c**2) + 3*b*e*atanh(c*x)/(2*c**2), Ne(c, 0)),
 (a*d*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\text{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}+\frac{\log\,(cx-1)}{c^3}\right)\right)bd$$
$$-\frac{(c^2x^2-(c^2x^2-1)\log\,(-c^2x^2+1)-1)be\,\text{artanh}\,(cx)}{2\,c^2}$$
$$-\frac{(c^2x^2-(c^2x^2-1)\log\,(-c^2x^2+1)-1)ae}{2\,c^2}$$
$$-\frac{(3\,cx-(cx+1)\log\,(cx+1)-(cx-1)\log\,(-cx+1))be}{2\,c^2}$$

input
```
integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)
*b*e*arctanh(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1
)*a*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))
*b*e/c^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= -\frac{1}{4}\,bex^2\log\left(-cx+1\right)^2+\frac{1}{2}\,(ad-ae)x^2+\frac{1}{4}\left(bex^2-\frac{be}{c^2}\right)\log\left(cx+1\right)^2$$
$$+\frac{1}{4}\left((bd+2\,ae-be)x^2+\frac{2\,bex}{c}\right)\log\left(cx+1\right)-\frac{be\log\left(cx-1\right)^2}{4\,c^2}$$
$$-\frac{1}{4}\left((bd-2\,ae-be)x^2-\frac{2\,bex}{c}-\frac{2\,be\log\left(cx-1\right)}{c^2}\right)\log\left(-cx+1\right)$$
$$+\frac{(bd-3\,be)x}{2\,c}-\frac{(bd+2\,ae-3\,be)\log\left(cx+1\right)}{4\,c^2}+\frac{(bd-2\,ae-3\,be)\log\left(cx-1\right)}{4\,c^2}$$

input
```
integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

output
```
-1/4*b*e*x^2*log(-c*x + 1)^2 + 1/2*(a*d - a*e)*x^2 + 1/4*(b*e*x^2 - b*e/c^
2)*log(c*x + 1)^2 + 1/4*((b*d + 2*a*e - b*e)*x^2 + 2*b*e*x/c)*log(c*x + 1)
 - 1/4*b*e*log(c*x - 1)^2/c^2 - 1/4*((b*d - 2*a*e - b*e)*x^2 - 2*b*e*x/c -
 2*b*e*log(c*x - 1)/c^2)*log(-c*x + 1) + 1/2*(b*d - 3*b*e)*x/c - 1/4*(b*d
+ 2*a*e - 3*b*e)*log(c*x + 1)/c^2 + 1/4*(b*d - 2*a*e - 3*b*e)*log(c*x - 1)
/c^2
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.98

$$
\begin{aligned}
&\int x(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx \\
&=\ln(1-cx)^2\left(\frac{be}{4c^2}-\frac{bex^2}{4}\right)-\ln(cx+1)^2\left(\frac{be}{4c^2}-\frac{bex^2}{4}\right) \\
&\quad+\ln(1-cx)\left(\frac{x^2\left(ae-\frac{bd}{2}+\frac{be}{2}+\frac{be\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)}{2}\right)}{2}+\frac{bex}{2c}\right) \\
&\quad+c\ln(cx+1)\left(\frac{x^2\left(2ae+bd-be-be\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)\right)}{4c}\right. \\
&\qquad\left.+\frac{bex}{2c^2}\right)-\frac{ax^2\left(e-d+e\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)\right)}{2} \\
&\quad-\frac{\ln\left(\frac{x\left(2ae+bd-3be-be\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)\right)}{2}-\frac{3be-bd+be\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)}{2c}-aex\right)(2a}{4c^2} \\
&\quad-\frac{\ln\left(\frac{x\left(2ae-bd+3be+be\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)\right)}{2}-\frac{3be-bd+be\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)}{2c}-aex\right)(2a}{4c^2} \\
&\quad-\frac{bx\left(3e-d+e\left(\ln(cx+1)+\ln(1-cx)-\ln(1-c^2x^2)\right)\right)}{2c}
\end{aligned}
$$

input

```
int(x*(a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output

```
log(1 - c*x)^2*((b*e)/(4*c^2) - (b*e*x^2)/4) - log(c*x + 1)^2*((b*e)/(4*c^
2) - (b*e*x^2)/4) + log(1 - c*x)*((x^2*(a*e - (b*d)/2 + (b*e)/2 + (b*e*(lo
g(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/2))/2 + (b*e*x)/(2*c)) + c*
log(c*x + 1)*((x^2*(2*a*e + b*d - b*e - b*e*(log(c*x + 1) + log(1 - c*x) -
 log(1 - c^2*x^2))))/(4*c) + (b*e*x)/(2*c^2)) - (a*x^2*(e - d + e*(log(c*x
 + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/2 - (log((x*(2*a*e + b*d - 3*b*
e - b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/2 - (3*b*e - b*
d + b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2)))/(2*c) - a*e*x)*(
2*a*e + b*d - 3*b*e - b*e*(log(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))
))/(4*c^2) - (log((x*(2*a*e - b*d + 3*b*e + b*e*(log(c*x + 1) + log(1 - c*
x) - log(1 - c^2*x^2))))/2 - (3*b*e - b*d + b*e*(log(c*x + 1) + log(1 - c*
x) - log(1 - c^2*x^2)))/(2*c) - a*e*x)*(2*a*e - b*d + 3*b*e + b*e*(log(c*x
 + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(4*c^2) - (b*x*(3*e - d + e*(lo
g(c*x + 1) + log(1 - c*x) - log(1 - c^2*x^2))))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$= \frac{atanh(cx)\log(-c^2x^2+1)\,b\,c^2e\,x^2-atanh(cx)\log(-c^2x^2+1)\,be+atanh(cx)\,b\,c^2d\,x^2-atanh(cx)\,b\,c^2e\,x}{}$$

input
```
int(x*(a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(atanh(c*x)*log( - c**2*x**2 + 1)*b*c**2*e*x**2 - atanh(c*x)*log( - c**2*x
**2 + 1)*b*e + atanh(c*x)*b*c**2*d*x**2 - atanh(c*x)*b*c**2*e*x**2 - atanh
(c*x)*b*d + 3*atanh(c*x)*b*e + log( - c**2*x**2 + 1)*a*c**2*e*x**2 - log(
- c**2*x**2 + 1)*a*e + log( - c**2*x**2 + 1)*b*c*e*x + a*c**2*d*x**2 - a*c
**2*e*x**2 + b*c*d*x - 3*b*c*e*x)/(2*c**2)
```

3.525 $\int (a+b\,\mathrm{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int (a+b\,\mathrm{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right)\,dx = -2aex-2bex\,\mathrm{arctanh}(cx)+\frac{e(a+b\,\mathrm{arctanh}(cx))^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c} + x(a+b\,\mathrm{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\left(d+e\log\left(1-c^2x^2\right)\right)^2}{4ce}$$

output
```
-2*a*e*x-2*b*e*x*arctanh(c*x)+e*(a+b*arctanh(c*x))^2/b/c-b*e*ln(-c^2*x^2+1
)/c+x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))+1/4*b*(d+e*ln(-c^2*x^2+1))^2
/c/e
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

$$\int (a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= adx-2aex+\frac{2ae\text{arctanh}(cx)}{c}+bdx\text{arctanh}(cx)-2bex\text{arctanh}(cx)$$
$$+\frac{be\text{arctanh}(cx)^2}{c}+\frac{bd\log\left(1-c^2x^2\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c}$$
$$+aex\log\left(1-c^2x^2\right)+bex\text{arctanh}(cx)\log\left(1-c^2x^2\right)+\frac{be\log^2\left(1-c^2x^2\right)}{4c}$$

input
```
Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + (2*a*e*ArcTanh[c*x])/c + b*d*x*ArcTanh[c*x] - 2*b*e*x*Ar
cTanh[c*x] + (b*e*ArcTanh[c*x]^2)/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*
Log[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcTanh[c*x]*Log[1 -
c^2*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6635, 2925, 2837, 2738, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6635}$$

$$2c^2e\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx-bc\int \frac{x\left(d+e\log\left(1-c^2x^2\right)\right)}{1-c^2x^2}dx+x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{2925}$$

$$2c^2e\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{1-c^2x^2}dx^2+x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{2837}$$

$$2c^2e\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx+\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^2}d(1-c^2x^2)}{2c}+x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{2738}$$

$$2c^2e\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx+x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{6542}$$

$$2c^2e\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int (a+b\text{arctanh}(cx))dx}{c^2}\right)+x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{2009}$$

$$2c^2e\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)+x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{6510}$$

$$x(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)+2c^2e\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

input
```
Int[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(d + e*Log[1 - c^2*x^
2])^2)/(4*c*e) + 2*c^2*e*((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (a*x + b*x*Ar
cTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6635
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp
[2*e*g   Int[x^2*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b
, c, d, e, f, g}, x]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{4x\ \mathrm{arctanh}(cx)\ln(-c^2x^2+1)bec+4bcdx\ \mathrm{arctanh}(cx)-8eb\ \mathrm{arctanh}(cx)xc+4x\ln(-c^2x^2+1)aec+4adxc-8acex+4eb\ \mathrm{arctanh}}{4c}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*x*arctanh(c*x)*ln(-c^2*x^2+1)*b*e*c+4*b*c*d*x*arctanh(c*x)-8*e*b*ar
ctanh(c*x)*x*c+4*x*ln(-c^2*x^2+1)*a*e*c+4*a*d*x*c-8*a*c*e*x+4*e*b*arctanh(
c*x)^2+e*b*ln(-c^2*x^2+1)^2+8*arctanh(c*x)*a*e+2*b*d*ln(-c^2*x^2+1)-4*ln(-
c^2*x^2+1)*b*e)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

$$\int (a+b\mathrm{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{be\log\left(-c^2x^2+1\right)^2+be\log\left(-\frac{cx+1}{cx-1}\right)^2+4\,(acd-2\,ace)x+2\,(2\,acex+bd-2\,be)\log\left(-c^2x^2+1\right)+2\,(bc}{4\,c}$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/4*(b*e*log(-c^2*x^2 + 1)^2 + b*e*log(-(c*x + 1)/(c*x - 1))^2 + 4*(a*c*d
- 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*log(-c^2*x^2 + 1) + 2*(b*c*e*x*
log(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*log(-(c*x + 1)/(c*x - 1))
)/c
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.42

$$\int (a+b\text{arctanh}(cx)) \left(d+e\log \left(1-c^2x^2\right)\right) \, dx = \begin{cases} adx + aex\log\left(-c^2x^2+1\right) - 2aex + \frac{2ae\,\text{atanh}\,(cx)}{c} + bdx\,\text{atanh}\,(cx) + bex\log\left(-c^2x^2+1\right)\text{atanh}\,(cx) - \\ adx \end{cases}$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atanh(c*x)/
c + b*d*x*atanh(c*x) + b*e*x*log(-c**2*x**2 + 1)*atanh(c*x) - 2*b*e*x*atan
h(c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c)
- b*e*log(-c**2*x**2 + 1)/c + b*e*atanh(c*x)**2/c, Ne(c, 0)), (a*d*x, True
))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.71

$$\begin{aligned} \int (a+b\text{arctanh}(cx)) \left(d+e\log \left(1-c^2x^2\right)\right) \, dx &= -\left(c^2\left(\frac{2\,x}{c^2} - \frac{\log\left(cx+1\right)}{c^3} + \frac{\log\left(cx-1\right)}{c^3}\right) - x\log\left(-c^2x^2+1\right)\right)be\,\text{artanh}\,(cx) \\ &\quad - \left(c^2\left(\frac{2\,x}{c^2} - \frac{\log\left(cx+1\right)}{c^3} + \frac{\log\left(cx-1\right)}{c^3}\right) - x\log\left(-c^2x^2+1\right)\right)ae \\ &\quad + adx + \frac{\left(2\,cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)bd}{2\,c} \\ &\quad + \frac{\left(\left(i\,\pi + 2\,\log\left(cx-1\right) - 2\right)\log\left(cx+1\right) + \left(i\,\pi - 2\right)\log\left(cx-1\right)\right)be}{2\,c} \end{aligned}$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

output
```
-(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1
))*b*e*arctanh(c*x) - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)
 - x*log(-c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x
^2 + 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2
)*log(c*x - 1))*b*e/c
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int (a+b\operatorname{arctanh}(cx)) \left(d+e\log \left(1-c^2x^2\right)\right) \, dx \\
&= -\frac{1}{2}\,bex\log\left(-cx+1\right)^2+\frac{1}{2}\,(bd+2\,ae-2\,be)x\log\left(cx+1\right) \\
&\quad +\frac{1}{2}\left(bex+\frac{be}{c}\right)\log\left(cx+1\right)^2-\frac{be\log\left(cx-1\right)^2}{2\,c}+(ad-2\,ae)x \\
&\quad -\frac{1}{2}\left((bd-2\,ae-2\,be)x-\frac{2\,be\log\left(cx-1\right)}{c}\right)\log\left(-cx+1\right) \\
&\quad +\frac{(bd+2\,ae-2\,be)\log\left(cx+1\right)}{2\,c}+\frac{(bd-2\,ae-2\,be)\log\left(cx-1\right)}{2\,c}
\end{aligned}$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

output
```
-1/2*b*e*x*log(-c*x + 1)^2 + 1/2*(b*d + 2*a*e - 2*b*e)*x*log(c*x + 1) + 1/
2*(b*e*x + b*e/c)*log(c*x + 1)^2 - 1/2*b*e*log(c*x - 1)^2/c + (a*d - 2*a*e
)*x - 1/2*((b*d - 2*a*e - 2*b*e)*x - 2*b*e*log(c*x - 1)/c)*log(-c*x + 1) +
 1/2*(b*d + 2*a*e - 2*b*e)*log(c*x + 1)/c + 1/2*(b*d - 2*a*e - 2*b*e)*log(
c*x - 1)/c
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.70

$$\int (a+b\operatorname{arctanh}(cx)) \left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= a\,d\,x-2\,a\,e\,x+\frac{b\,e\ln\left(c\,x+1\right)^2}{2\,c}+\frac{b\,e\ln\left(1-c\,x\right)^2}{2\,c}+a\,e\,x\,\ln\left(1-c^2\,x^2\right)$$
$$+\frac{b\,d\,x\,\ln\left(c\,x+1\right)}{2}-\frac{b\,d\,x\,\ln\left(1-c\,x\right)}{2}-b\,e\,x\,\ln\left(c\,x+1\right)$$
$$+b\,e\,x\,\ln\left(1-c\,x\right)-\frac{a\,e\,\ln\left(c\,x-1\right)}{c}+\frac{a\,e\,\ln\left(c\,x+1\right)}{c}$$
$$+\frac{b\,d\,\ln\left(c\,x-1\right)}{2\,c}+\frac{b\,d\,\ln\left(c\,x+1\right)}{2\,c}-\frac{b\,e\,\ln\left(c\,x-1\right)}{c}-\frac{b\,e\,\ln\left(c\,x+1\right)}{c}$$
$$+\frac{b\,e\,x\,\ln\left(c\,x+1\right)\,\ln\left(1-c^2\,x^2\right)}{2}-\frac{b\,e\,x\,\ln\left(1-c\,x\right)\,\ln\left(1-c^2\,x^2\right)}{2}$$
$$+\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\ln\left(-2\,a\,e-2\,a\,c\,e\,x\right)}{2\,c}+\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\ln\left(2\,a\,c\,e\,x-2\,a\,e\right)}{2\,c}$$
$$-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(-2\,a\,e-2\,a\,c\,e\,x\right)}{2\,c}-\frac{b\,e\,\ln\left(c\,x+1\right)\,\ln\left(2\,a\,c\,e\,x-2\,a\,e\right)}{2\,c}$$
$$-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(-2\,a\,e-2\,a\,c\,e\,x\right)}{2\,c}-\frac{b\,e\,\ln\left(1-c\,x\right)\,\ln\left(2\,a\,c\,e\,x-2\,a\,e\right)}{2\,c}$$

input
```
int((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
a*d*x - 2*a*e*x + (b*e*log(c*x + 1)^2)/(2*c) + (b*e*log(1 - c*x)^2)/(2*c)
+ a*e*x*log(1 - c^2*x^2) + (b*d*x*log(c*x + 1))/2 - (b*d*x*log(1 - c*x))/2
 - b*e*x*log(c*x + 1) + b*e*x*log(1 - c*x) - (a*e*log(c*x - 1))/c + (a*e*l
og(c*x + 1))/c + (b*d*log(c*x - 1))/(2*c) + (b*d*log(c*x + 1))/(2*c) - (b*
e*log(c*x - 1))/c - (b*e*log(c*x + 1))/c + (b*e*x*log(c*x + 1)*log(1 - c^2
*x^2))/2 - (b*e*x*log(1 - c*x)*log(1 - c^2*x^2))/2 + (b*e*log(1 - c^2*x^2)
*log(- 2*a*e - 2*a*c*e*x))/(2*c) + (b*e*log(1 - c^2*x^2)*log(2*a*c*e*x - 2
*a*e))/(2*c) - (b*e*log(c*x + 1)*log(- 2*a*e - 2*a*c*e*x))/(2*c) - (b*e*lo
g(c*x + 1)*log(2*a*c*e*x - 2*a*e))/(2*c) - (b*e*log(1 - c*x)*log(- 2*a*e -
 2*a*c*e*x))/(2*c) - (b*e*log(1 - c*x)*log(2*a*c*e*x - 2*a*e))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.55

$$\int (a+b\text{arctanh}(cx)) \left(d+e\log \left(1-c^2x^2\right)\right) \, dx$$
$$= \frac{4atanh(cx)^2 be + 4atanh(cx)\log(-c^2x^2+1)\, bcex + 4atanh(cx)\, bcdx - 8atanh(cx)\, bcex + \log(-c^2x^2+1)}{}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(4*atanh(c*x)**2*b*e + 4*atanh(c*x)*log( - c**2*x**2 + 1)*b*c*e*x + 4*atan
h(c*x)*b*c*d*x - 8*atanh(c*x)*b*c*e*x + log( - c**2*x**2 + 1)**2*b*e + 4*l
og( - c**2*x**2 + 1)*a*c*e*x + 4*log( - c**2*x**2 + 1)*a*e + 2*log( - c**2
*x**2 + 1)*b*d - 4*log( - c**2*x**2 + 1)*b*e - 8*log(c**2*x - c)*a*e + 4*a
*c*d*x - 8*a*c*e*x)/(4*c)
```

3.526 $\int \frac{(a+b\,\mathbf{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 216

$$\begin{aligned}
&\int \frac{(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx \\
&= ad\log(x)-\frac{1}{2}be\log(cx)\log^2(1-cx) \\
&\quad+\frac{1}{2}be\log(-cx)\log^2(1+cx)-\frac{1}{2}bd\operatorname{PolyLog}(2,-cx) \\
&\quad+\frac{1}{2}be\left(\log(1-cx)+\log(1+cx)-\log\left(1-c^2x^2\right)\right)\operatorname{PolyLog}(2,-cx) \\
&\quad+\frac{1}{2}bd\operatorname{PolyLog}(2,cx)-\frac{1}{2}be\left(\log(1-cx)+\log(1+cx)-\log\left(1-c^2x^2\right)\right)\operatorname{PolyLog}(2,cx) \\
&\quad-\frac{1}{2}ae\operatorname{PolyLog}\left(2,c^2x^2\right)-be\log(1-cx)\operatorname{PolyLog}(2,1-cx) \\
&\quad+be\log(1+cx)\operatorname{PolyLog}(2,1+cx)+be\operatorname{PolyLog}(3,1-cx)-be\operatorname{PolyLog}(3,1+cx)
\end{aligned}$$

output
```
a*d*ln(x)-1/2*b*e*ln(c*x)*ln(-c*x+1)^2+1/2*b*e*ln(-c*x)*ln(c*x+1)^2-1/2*b*
d*polylog(2,-c*x)+1/2*b*e*(ln(-c*x+1)+ln(c*x+1)-ln(-c^2*x^2+1))*polylog(2,
-c*x)+1/2*b*d*polylog(2,c*x)-1/2*b*e*(ln(-c*x+1)+ln(c*x+1)-ln(-c^2*x^2+1))
*polylog(2,c*x)-1/2*a*e*polylog(2,c^2*x^2)-b*e*ln(-c*x+1)*polylog(2,-c*x+1
)+b*e*ln(c*x+1)*polylog(2,c*x+1)+b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1
)
```

Mathematica [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(a+b\operatorname{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6641, 6446, 6639, 2838, 6637, 2843, 2881, 2821, 6446, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arctanh}(cx))\,(e\log(1-c^2x^2)+d)}{x}\,dx$$

$$\Big\downarrow \text{6641}$$

$$e\int \frac{(a+b\operatorname{arctanh}(cx))\log(1-c^2x^2)}{x}dx+d\int \frac{a+b\operatorname{arctanh}(cx)}{x}dx$$

$$\Big\downarrow \text{6446}$$

$$e\int \frac{(a+b\operatorname{arctanh}(cx))\log(1-c^2x^2)}{x}dx+ d\left(a\log(x)-\frac{1}{2}b\operatorname{PolyLog}(2,-cx)+\frac{1}{2}b\operatorname{PolyLog}(2,cx)\right)$$

$$\Big\downarrow \text{6639}$$

$$e\left(a\int \frac{\log \left(1-c^2x^2\right)}{x}dx+b\int \frac{\text{arctanh}(cx)\log \left(1-c^2x^2\right)}{x}dx\right)+$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\big\downarrow$ 2838

$$e\left(b\int \frac{\text{arctanh}(cx)\log \left(1-c^2x^2\right)}{x}dx-\frac{1}{2}a\,\text{PolyLog}\left(2,c^2x^2\right)\right)+$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\big\downarrow$ 6637

$$e\left(b\left(-\left(\left(-\log \left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\int \frac{\text{arctanh}(cx)}{x}dx\right)-\frac{1}{2}\int \frac{\log^2(1-cx)}{x}dx+\frac{1}{2}\int \frac{\log^2(\right.\right.$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\big\downarrow$ 2843

$$e\left(b\left(-\left(\left(-\log \left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\int \frac{\text{arctanh}(cx)}{x}dx\right)+\frac{1}{2}\left(-2c\int \frac{\log(cx)\log(1-cx)}{1-cx}dx\right.\right.\right.$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\big\downarrow$ 2881

$$e\left(b\left(-\left(\left(-\log \left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\int \frac{\text{arctanh}(cx)}{x}dx\right)+\frac{1}{2}\left(2\int \frac{\log(cx)\log(1-cx)}{1-cx}d(1-\right.\right.\right.$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\big\downarrow$ 2821

$$e\left(b\left(-\left(\left(-\log \left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\int \frac{\text{arctanh}(cx)}{x}dx\right)+\frac{1}{2}\left(2\left(\int \frac{\text{PolyLog}(2,1-cx)}{1-cx}d(1-\right.\right.\right.\right.$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\big\downarrow$ 6446

$$e\left(b\left(\frac{1}{2}\left(2\left(\int \frac{\text{PolyLog}(2,1-cx)}{1-cx}d(1-cx)-\text{PolyLog}(2,1-cx)\log(1-cx)\right)-\log(cx)\log^2(1-cx)\right)+\frac{1}{2}\left(\log\right.\right.\right.$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$\downarrow$ 7143

$$e\left(b\left(-\left(\left(\frac{\text{PolyLog}(2,cx)}{2}-\frac{\text{PolyLog}(2,-cx)}{2}\right)\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\right)+\frac{1}{2}(2(\text{PolyLo}\right.\right.$$
$$d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

output
```
d*(a*Log[x] - (b*PolyLog[2, -(c*x)])/2 + (b*PolyLog[2, c*x])/2) + e*(-1/2*
(a*PolyLog[2, c^2*x^2]) + b*(-((Log[1 - c*x] + Log[1 + c*x] - Log[1 - c^2*
x^2])*(-1/2*PolyLog[2, -(c*x)] + PolyLog[2, c*x]/2)) + (-(Log[c*x]*Log[1 -
 c*x]^2) + 2*(-(Log[1 - c*x]*PolyLog[2, 1 - c*x]) + PolyLog[3, 1 - c*x]))/
2 + (Log[-(c*x)]*Log[1 + c*x]^2 - 2*(-(Log[1 + c*x]*PolyLog[2, 1 + c*x]) +
 PolyLog[3, 1 + c*x]))/2))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6637
```
Int[(ArcTanh[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp
[(Log[f + g*x^2] - Log[1 - c*x] - Log[1 + c*x])   Int[ArcTanh[c*x]/x, x], x
] + (-Simp[1/2   Int[Log[1 - c*x]^2/x, x], x] + Simp[1/2   Int[Log[1 + c*x]
^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

rule 6639
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTanh[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x
^2]*(ArcTanh[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 6641
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcTanh[c*x])/x, x], x] + Si
mp[e   Int[Log[f + g*x^2]*((a + b*ArcTanh[c*x])/x), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.18 (sec) , antiderivative size = 1227, normalized size of antiderivative = 5.68

method	result	size
risch	Expression too large to display	1227

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-8*a*d+4*I*Pi*a*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*
x+1))-4*I*Pi*a*e*csgn(I*(c*x-1)*(c*x+1))^3-4*Pi^2*b*e*csgn(I*(c*x-1))^2*cs
gn(I*(c*x-1)*(c*x+1))^2-2*Pi^2*b*e*csgn(I*(c*x-1))^4*csgn(I*(c*x-1)*(c*x+1
))^2+4*Pi^2*b*e*csgn(I*(c*x-1))^2+4*Pi^2*b*e*csgn(I*(c*x-1)*(c*x+1))^2+2*P
i^2*b*e*csgn(I*(c*x-1))^4*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+2*Pi^2*b
*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-4*Pi^2*b*e*csgn
(I*(c*x-1))^3-2*Pi^2*b*e*csgn(I*(c*x-1)*(c*x+1))^3-2*Pi^2*b*e*csgn(I*(c*x-
1))^3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-4*I*Pi*a*e*csgn(I*(c*x-1))*c
sgn(I*(c*x-1)*(c*x+1))^2-4*I*Pi*a*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1)
)^2-2*Pi^2*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2
-2*Pi^2*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-2*Pi^2*b*e*csgn(I*(c
*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+4*I*Pi*b*d*csgn(I*(c*x-1))^3+8*I*Pi*a*e*c
sgn(I*(c*x-1)*(c*x+1))^2-4*I*Pi*b*d*csgn(I*(c*x-1))^2-8*I*Pi*a*e+4*I*Pi*b*
d-4*Pi^2*b*e+2*Pi^2*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(
c*x+1))^2+2*Pi^2*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(c*x+1))^3+6*Pi^2*b*
e*csgn(I*(c*x-1))^3*csgn(I*(c*x-1)*(c*x+1))^2-2*Pi^2*b*e*csgn(I*(c*x-1))^3
*csgn(I*(c*x-1)*(c*x+1))^3)*ln(c*x)-1/8*(-4*I*Pi*b*e*csgn(I*(c*x-1))^2-4*I
*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2+4*I*Pi*b*e*csgn(I*(c*x-1))^3-2*I*Pi*b*e*
csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+2*I*Pi*b*e*csgn(I*
(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+2*I*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c...
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
 1))/x, x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(a+b\,\text{atanh}\,(cx))\,(d+e\log(-c^2x^2+1))}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx$$
$$= -\frac{1}{2}\left(\log(cx)\log(-cx+1)^2 + 2\,\text{Li}_2(-cx+1)\log(-cx+1) - 2\,\text{Li}_3(-cx+1)\right)be$$
$$+ \frac{1}{2}\left(\log(cx+1)^2\log(-cx) + 2\,\text{Li}_2(cx+1)\log(cx+1) - 2\,\text{Li}_3(cx+1)\right)be$$
$$+ ad\log(x) - \frac{1}{2}(bd-2\,ae)(\log(cx)\log(-cx+1) + \text{Li}_2(-cx+1))$$
$$+ \frac{1}{2}(bd+2\,ae)(\log(cx+1)\log(-cx) + \text{Li}_2(cx+1))$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima"
)
```

output
```
-1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polyl
og(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*dilog(c*x + 1)*lo
g(c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*(b*d - 2*a*e)*(
log(c*x)*log(-c*x + 1) + dilog(-c*x + 1)) + 1/2*(b*d + 2*a*e)*(log(c*x + 1
)*log(-c*x) + dilog(c*x + 1))
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(b\,\text{artanh}(cx)+a)(e\log(-c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log(1 - c^2x^2))}{x}\,dx$$
$$= \int \frac{(a + b\,\text{atanh}(c\,x))\,\left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x, x)
```

Reduce [F]

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log(1 - c^2x^2))}{x}\,dx$$
$$= -\left(\int \frac{atanh(cx)}{c^2x^3 - x}dx\right)bd - \left(\int \frac{\log(-c^2x^2 + 1)}{c^2x^3 - x}dx\right)ae + \left(\int \frac{atanh(cx)\log(-c^2x^2 + 1)\,x}{c^2x^2 - 1}dx\right)b\,c^2e - \left(\int \frac{atanh(cx)\log(-c^2x^2 + 1)}{c^2x^3 - x}dx\right)be + \left(\int \frac{atanh(cx)\,x}{c^2x^2 - 1}dx\right)b\,c^2d + \frac{\log(-c^2x^2 + 1)^2\,ae}{4} + \log(x)\,ad$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x)
```

output
```
( - 4*int(atanh(c*x)/(c**2*x**3 - x),x)*b*d - 4*int(log( - c**2*x**2 + 1)/
(c**2*x**3 - x),x)*a*e + 4*int((atanh(c*x)*log( - c**2*x**2 + 1)*x)/(c**2*
x**2 - 1),x)*b*c**2*e - 4*int((atanh(c*x)*log( - c**2*x**2 + 1))/(c**2*x**
3 - x),x)*b*e + 4*int((atanh(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2*d + log( -
c**2*x**2 + 1)**2*a*e + 4*log(x)*a*d)/4
```

3.527 $\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{ce(a+b\text{arctanh}(cx))^2}{b} - \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x} + \frac{1}{2}bc(d+e\log(1-c^2x^2))\log\left(1-\frac{1}{1-c^2x^2}\right) - \frac{1}{2}bce\,\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)$$

output
```
-c*e*(a+b*arctanh(c*x))^2/b-(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x+1/2*
b*c*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(-c^2*
x^2+1))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(105) = 210.

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{4ad+4bd\text{arctanh}(cx)+8acex\text{arctanh}(cx)+4bcex\text{arctanh}(cx)^2-4bcdx\log(x)-bcex\log^2\left(-\frac{1}{c}+x\right)}{}$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

output
```
-1/4*(4*a*d + 4*b*d*ArcTanh[c*x] + 8*a*c*e*x*ArcTanh[c*x] + 4*b*c*e*x*ArcT
anh[c*x]^2 - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c
^(-1) + x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[
x]*Log[1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*
Log[x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2]
+ 4*b*e*ArcTanh[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2]
+ 2*b*c*e*x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*
Log[1 - c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x
] - 2*b*c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2
])/x
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6643, 2925, 2858, 27, 2779, 2838, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx)) \left(e\log \left(1-c^2x^2\right)+d\right)}{x^2}dx$$

$$\Big\downarrow \text{6643}$$

$$-2c^2e\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int \frac{d+e\log \left(1-c^2x^2\right)}{x \left(1-c^2x^2\right)}dx-\frac{(a+b\text{arctanh}(cx)) \left(e\log \left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$-2c^2e\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{d+e\log \left(1-c^2x^2\right)}{x^2 \left(1-c^2x^2\right)}dx^2-\frac{(a+b\text{arctanh}(cx)) \left(e\log \left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2858}$$

$$-2c^2e\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^4}d(1-c^2x^2)}{2c}-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$-2c^2e\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2779}$$

$$-2c^2e\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2838}$$

$$-2c^2e\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{1}{2}bc\left(e\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right)$$

$$\Big\downarrow \text{6510}$$

$$-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce(a+b\text{arctanh}(cx))^2}{b}-\frac{1}{2}bc\left(e\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right)$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

output
```
-((c*e*(a + b*ArcTanh[c*x])^2)/b) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c
^2*x^2]))/x - (b*c*(-(Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2])) + e*PolyLo
g[2, x^(-2)]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6643
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln\left(-c^2x^2+1\right))}{x^2}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx))\,(d+e\log\left(1-c^2x^2\right))}{x^2}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="frica
s")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx)) (d + e \log (1 - c^2x^2))}{x^2} dx = \int \frac{(a + b \operatorname{atanh}(cx)) (d + e \log (-c^2x^2 + 1))}{x^2} dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx)) (d + e \log (1 - c^2x^2))}{x^2} dx = \int \frac{(b \operatorname{artanh}(cx) + a)(e \log (-c^2x^2 + 1) + d)}{x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxim
a")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - (c^2*(log(
c*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e + 1/2*b*e*(log(-c*
x + 1)^2/x - integrate(-((c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(
c*x^3 - x^2), x)) - a*d/x
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac"
)
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^2,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = \frac{-2atanh(cx)^2\,bcex-2atanh(cx)\log(-c^2x^2+1)\,be-2atanh(cx)\,bd-2\left(\int\frac{\log(-c^2x^2+1)}{c^2x^3-x}dx\right)bcex-2\log(}{}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x)
```

output
```
( - 2*atanh(c*x)**2*b*c*e*x - 2*atanh(c*x)*log( - c**2*x**2 + 1)*b*e - 2*a
tanh(c*x)*b*d - 2*int(log( - c**2*x**2 + 1)/(c**2*x**3 - x),x)*b*c*e*x - 2
*log( - c**2*x**2 + 1)*a*c*e*x - 2*log( - c**2*x**2 + 1)*a*e - log( - c**2
*x**2 + 1)*b*c*d*x + 4*log(c**2*x - c)*a*c*e*x + 2*log(x)*b*c*d*x - 2*a*d)
/(2*x)
```

3.528 $\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 157

$$\begin{aligned}
&\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^3}\,dx \\
&= -ac^2e\log(x)+\frac{1}{2}(a+b)c^2e\log(1-cx)+\frac{1}{2}(a-b)c^2e\log(1+cx) \\
&\quad -\frac{bc(d+e\log(1-c^2x^2))}{2x}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(d+e\log\left(1-c^2x^2\right)\right) \\
&\quad -\frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{2x^2} \\
&\quad +\frac{1}{2}bc^2e\,\text{PolyLog}(2,-cx)-\frac{1}{2}bc^2e\,\text{PolyLog}(2,cx)
\end{aligned}$$

output
```
-a*c^2*e*ln(x)+1/2*(a+b)*c^2*e*ln(-c*x+1)+1/2*(a-b)*c^2*e*ln(c*x+1)-1/2*b*
c*(d+e*ln(-c^2*x^2+1))/x+1/2*b*c^2*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/2*(
a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2+1/2*b*c^2*e*polylog(2,-c*x)-1/2
*b*c^2*e*polylog(2,c*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = \frac{1}{2}\left(-\frac{ad}{x^2}-2ac^2e\log(x)+(a+b)c^2e\log(1-cx)+(a-b)c^2e\log(1+cx) - \frac{bd(2\text{arctanh}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))}{2x^2} - \frac{e(a+bcx+(b-bc^2x^2)\,\text{arctanh}(cx))\log\left(1-c^2x^2\right)}{x^2} + bc^2e(\text{PolyLog}(2,-cx)-\text{PolyLog}(2,cx))\right)$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

output
```
(-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^
2*e*Log[1 + c*x] - (b*d*(2*ArcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*
Log[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcTanh[c*x])*Lo
g[1 - c^2*x^2])/x^2 + b*c^2*e*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]))/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6647, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 6647}$$

$$2c^2e\int \left(-\frac{a+bcx}{2x(1-c^2x^2)}-\frac{b\text{arctanh}(cx)}{2x}\right)dx-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{2x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+2c^2e\left(\frac{1}{4}(a+b)\log(1-cx)+\frac{1}{4}(a-b)\log(cx+1)-\frac{1}{2}a\log(x)+\frac{1}{4}b\,\text{PolyLog}(2,-cx)-\frac{1}{4}b\,\text{PolyLog}(2,cx)\right)+\frac{1}{2}bc^2\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{2x}$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[1 - c^2*x^2]))/x + (b*c^2*ArcTanh[c*x]*(d + e*Log[1 -
 c^2*x^2]))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(2*x^2) +
2*c^2*e*(-1/2*(a*Log[x]) + ((a + b)*Log[1 - c*x])/4 + ((a - b)*Log[1 + c*x
])/4 + (b*PolyLog[2, -(c*x)])/4 - (b*PolyLog[2, c*x])/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6647
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(a+b \operatorname{arctanh}(cx))\,(d+e\ln(-c^2x^2+1))}{x^3}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(b\,\text{artanh}(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="frica
s")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(a+b\,\text{atanh}(cx))\,(d+e\log(-c^2x^2+1))}{x^3}\,dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx$$
$$=\int \frac{(b\,\text{artanh}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxim
a")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d +
 1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e + 1/4
*b*e*(log(-c*x + 1)^2/x^2 - 2*integrate(-((c*x - 1)*log(c*x + 1)^2 - c*x*l
og(-c*x + 1))/(c*x^4 - x^3), x)) - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx$$
$$=\int \frac{(b\,\text{artanh}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac"
)
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = \int \frac{\left(a+b\,\text{atanh}(c\,x)\right)\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^3,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = \frac{atanh(cx)\,b\,c^2d\,x^2-atanh(cx)\,bd+2\left(\int\frac{atanh(cx)\log(-c^2x^2+1)}{x^3}dx\right)be\,x^2+\log(-c^2x^2+1)\,a\,c^2e\,x^2-\log(-c}{2x^2}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x)
```

output
```
(atanh(c*x)*b*c**2*d*x**2 - atanh(c*x)*b*d + 2*int((atanh(c*x)*log( - c**2
*x**2 + 1))/x**3,x)*b*e*x**2 + log( - c**2*x**2 + 1)*a*c**2*e*x**2 - log(
- c**2*x**2 + 1)*a*e - 2*log(x)*a*c**2*e*x**2 - a*d - b*c*d*x)/(2*x**2)
```

3.529 $\int \frac{(a+b\,\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 197

$$\begin{aligned}
&\int \frac{(a+b\,\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx\\
&= \frac{2c^2e(a+b\,\text{arctanh}(cx))}{3x} - \frac{c^3e(a+b\,\text{arctanh}(cx))^2}{3b} - bc^3e\log(x) + \frac{1}{3}bc^3e\log\left(1-c^2x^2\right)\\
&\quad - \frac{bc(1-c^2x^2)\left(d+e\log\left(1-c^2x^2\right)\right)}{6x^2} - \frac{(a+b\,\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}\\
&\quad + \frac{1}{6}bc^3\left(d+e\log\left(1-c^2x^2\right)\right)\log\left(1-\frac{1}{1-c^2x^2}\right) - \frac{1}{6}bc^3e\,\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)
\end{aligned}$$

output
```
2/3*c^2*e*(a+b*arctanh(c*x))/x-1/3*c^3*e*(a+b*arctanh(c*x))^2/b-b*c^3*e*ln
(x)+1/3*b*c^3*e*ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/x
^2-1/3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*ln(-c^2*
x^2+1))*ln(1-1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 460 vs. $2(197) = 394$.

Time = 0.21 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.34

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx$$
$$\begin{aligned}&=\frac{1}{6}\left(-\frac{2ad}{x^3}-\frac{bcd}{x^2}+\frac{4ac^2e}{x}-4ac^3e\text{arctanh}(cx)-\frac{2bd\text{arctanh}(cx)}{x^3}+\frac{4bc^2e\text{arctanh}(cx)}{x}\right.\\&\quad-2bc^3e\text{arctanh}(cx)^2+2bc^3d\log(x)-2bc^3e\log(x)+\frac{1}{2}bc^3e\log^2\left(-\frac{1}{c}+x\right)\\&\quad+\frac{1}{2}bc^3e\log^2\left(\frac{1}{c}+x\right)+bc^3e\log\left(\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1-cx)\right)-2bc^3e\log(x)\log(1-cx)\\&\quad+bc^3e\log\left(-\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1+cx)\right)-2bc^3e\log(x)\log(1+cx)\\&\quad-4bc^3e\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-bc^3d\log\left(1-c^2x^2\right)+bc^3e\log\left(1-c^2x^2\right)\\&\quad-\frac{2ae\log\left(1-c^2x^2\right)}{x^3}-\frac{bce\log\left(1-c^2x^2\right)}{x^2}-\frac{2be\text{arctanh}(cx)\log\left(1-c^2x^2\right)}{x^3}\\&\quad+2bc^3e\log(x)\log\left(1-c^2x^2\right)-bc^3e\log\left(-\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)\\&\quad-bc^3e\log\left(\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)-2bc^3e\,\text{PolyLog}(2,-cx)-2bc^3e\,\text{PolyLog}(2,cx)\\&\quad\left.+bc^3e\,\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)+bc^3e\,\text{PolyLog}\left(2,\frac{1}{2}(1+cx)\right)\right)\end{aligned}$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

output
```
((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - 4*a*c^3*e*ArcTanh[c*x] - (2*
b*d*ArcTanh[c*x])/x^3 + (4*b*c^2*e*ArcTanh[c*x])/x - 2*b*c^3*e*ArcTanh[c*x
]^2 + 2*b*c^3*d*Log[x] - 2*b*c^3*e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2
 + (b*c^3*e*Log[c^(-1) + x]^2)/2 + b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2
] - 2*b*c^3*e*Log[x]*Log[1 - c*x] + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)
/2] - 2*b*c^3*e*Log[x]*Log[1 + c*x] - 4*b*c^3*e*Log[(c*x)/Sqrt[1 - c^2*x^2
]] - b*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 -
c^2*x^2])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcTanh[c*x]*Log[1 -
 c^2*x^2])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) +
 x]*Log[1 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*
e*PolyLog[2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2
- (c*x)/2] + b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

Rubi [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {6643, 2925, 2858, 27, 2789, 2751, 16, 2779, 2838, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx)) \left(e\log \left(1-c^2x^2\right)+d\right)}{x^4}dx$$

$$\Big\downarrow \text{6643}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{3}bc\int \frac{d+e\log \left(1-c^2x^2\right)}{x^3\left(1-c^2x^2\right)}dx-\frac{(a+b\text{arctanh}(cx)) \left(e\log \left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2925}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{6}bc\int \frac{d+e\log \left(1-c^2x^2\right)}{x^4\left(1-c^2x^2\right)}dx^2-\frac{(a+b\text{arctanh}(cx)) \left(e\log \left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^6}d(1-c^2x^2)}{6c}-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2789}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^4}d(1-c^2x^2)\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2751}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)-e\int \frac{1}{c^2x^2}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}+e\log\left(c^2x^2\right)\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2779}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow \text{6544}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\int \frac{a+b\text{arctanh}(cx)}{x^2}dx\right)-$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)-$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow \text{47}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \ 16$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \ 6510$$

$$-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{2}{3}c^2e\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

output
```
(-2*c^2*e*(-((a + b*ArcTanh[c*x])/x) + (c*(a + b*ArcTanh[c*x])^2)/(2*b) +
(b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2))/3 - ((a + b*ArcTanh[c*x])*(d + e*L
og[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(e*Log[c^2*x^2] + ((1 - c^2*x^2)*(d + e
*Log[1 - c^2*x^2]))/(c^2*x^2) - Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2]) +
 e*PolyLog[2, x^(-2)]))/6
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6643
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a + b\,\mathrm{arctanh}\,(cx))\,(d + e\ln(-c^2x^2 + 1))}{x^4}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

Fricas [F]

$$\int \frac{(a + b\mathrm{arctanh}(cx))\,(d + e\log(1 - c^2x^2))}{x^4}\,dx$$
$$= \int \frac{(b\,\mathrm{artanh}\,(cx) + a)(e\log(-c^2x^2 + 1) + d)}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="frica
s")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^4, x)
```

Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log(1 - c^2x^2))}{x^4}\,dx = \int \frac{(a + b\,\text{atanh}\,(cx))\,(d + e\log(-c^2x^2 + 1))}{x^4}\,dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log(1 - c^2x^2))}{x^4}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)(e\log(-c^2x^2 + 1) + d)}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxim
a")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 +
1)/x^3)*a*e + 1/6*b*e*(log(-c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x - 1)
*log(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x^5 - x^4), x)) - 1/3*a*d/x^3
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log{(1-c^2x^2)})}{x^4}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log{(-c^2x^2+1)}+d)}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac"
)
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log{(1-c^2x^2)})}{x^4}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+e\,\ln{(1-c^2\,x^2)}\right)}{x^4}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^4,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log{(1-c^2x^2)})}{x^4}\,dx = \frac{-2atanh(cx)\,b\,c^3d\,x^3-2atanh(cx)\,bd+6\left(\int \frac{atanh(cx)\log(-c^2x^2+1)}{x^4}dx\right)be\,x^3-2\log(-c^2x^2+1)\,a\,c^3e\,x^3-}{}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x)
```

output
```
( - 2*atanh(c*x)*b*c**3*d*x**3 - 2*atanh(c*x)*b*d + 6*int((atanh(c*x)*log(
 - c**2*x**2 + 1))/x**4,x)*b*e*x**3 - 2*log( - c**2*x**2 + 1)*a*c**3*e*x**
3 - 2*log( - c**2*x**2 + 1)*a*e + 4*log(c**2*x - c)*a*c**3*e*x**3 - 2*log(
c**2*x - c)*b*c**3*d*x**3 + 2*log(x)*b*c**3*d*x**3 + 4*a*c**2*e*x**2 - 2*a
*d - b*c*d*x)/(6*x**3)
```

3.530 $\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 244

$$\begin{aligned}
&\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx \\
&= \frac{ac^2e}{4x^2}+\frac{5bc^3e}{12x}-\frac{1}{4}bc^4e\text{arctanh}(cx)+\frac{bc^2e\text{arctanh}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x) \\
&\quad+\frac{1}{12}(3a+4b)c^4e\log(1-cx)+\frac{1}{12}(3a-4b)c^4e\log(1+cx) \\
&\quad-\frac{bc(d+e\log(1-c^2x^2))}{12x^3}-\frac{bc^3(d+e\log(1-c^2x^2))}{4x} \\
&\quad+\frac{1}{4}bc^4\text{arctanh}(cx)\left(d+e\log\left(1-c^2x^2\right)\right)-\frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{4x^4} \\
&\quad+\frac{1}{4}bc^4e\,\text{PolyLog}(2,-cx)-\frac{1}{4}bc^4e\,\text{PolyLog}(2,cx)
\end{aligned}$$

output
```
1/4*a*c^2*e/x^2+5/12*b*c^3*e/x-1/4*b*c^4*e*arctanh(c*x)+1/4*b*c^2*e*arctan
h(c*x)/x^2-1/2*a*c^4*e*ln(x)+1/12*(3*a+4*b)*c^4*e*ln(-c*x+1)+1/12*(3*a-4*b
)*c^4*e*ln(c*x+1)-1/12*b*c*(d+e*ln(-c^2*x^2+1))/x^3-1/4*b*c^3*(d+e*ln(-c^2
*x^2+1))/x+1/4*b*c^4*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/4*(a+b*arctanh(c*
x))*(d+e*ln(-c^2*x^2+1))/x^4+1/4*b*c^4*e*polylog(2,-c*x)-1/4*b*c^4*e*polyl
og(2,c*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx$$
$$=-\frac{ad}{4x^4}+\frac{ac^2e}{4x^2}+\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}\left(3ac^4e+4bc^4e\right)\log(1-cx)$$
$$-\frac{1}{2}bc^4e\left(-\frac{\text{arctanh}(cx)}{2c^2x^2}+\frac{1}{2}\left(-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)$$
$$+bc^4d\left(-\frac{\text{arctanh}(cx)}{4c^4x^4}+\frac{1}{4}\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)$$
$$+\frac{1}{12}\left(3ac^4e-4bc^4e\right)\log(1+cx)$$
$$+\frac{e(-3a-bcx-3bc^3x^3-3b\text{arctanh}(cx)+3bc^4x^4\text{arctanh}(cx))\log\left(1-c^2x^2\right)}{12x^4}$$
$$-\frac{1}{4}bc^4e(-\operatorname{PolyLog}(2,-cx)+\operatorname{PolyLog}(2,cx))$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

output
```
-1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2
+ ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcTanh[c*x]/
(c^2*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d
*(-1/4*ArcTanh[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]
/2 + Log[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(
-3*a - b*c*x - 3*b*c^3*x^3 - 3*b*ArcTanh[c*x] + 3*b*c^4*x^4*ArcTanh[c*x])*
Log[1 - c^2*x^2])/(12*x^4) - (b*c^4*e*(-PolyLog[2, -(c*x)] + PolyLog[2, c*
x]))/4
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6647, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x^5}\,dx$$

$$\Big\downarrow \text{ 6647}$$

$$2c^2e\int\left(-\frac{3bc^3x^3+bcx+3a}{12x^3\left(1-c^2x^2\right)}-\frac{b\left(c^2x^2+1\right)\text{arctanh}(cx)}{4x^3}\right)dx-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{4}bc^4\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{12x^3}-\frac{bc^3\left(e\log\left(1-c^2x^2\right)+d\right)}{4x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+2c^2e\left(\frac{1}{24}c^2(3a+4b)\log(1-cx)+\frac{1}{24}c^2(3a-4b)\log(cx+1)-\frac{1}{4}ac^2\log(x)+\frac{a}{8x^2}-\frac{1}{8}bc^2\text{arctanh}(cx)+\frac{b\text{arctan}}{8x}\right.$$
$$\frac{1}{4}bc^4\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{12x^3}-\frac{bc^3\left(e\log\left(1-c^2x^2\right)+d\right)}{4x}$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

output
```
-1/12*(b*c*(d + e*Log[1 - c^2*x^2]))/x^3 - (b*c^3*(d + e*Log[1 - c^2*x^2])
)/(4*x) + (b*c^4*ArcTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/4 - ((a + b*ArcTan
h[c*x])*(d + e*Log[1 - c^2*x^2]))/(4*x^4) + 2*c^2*e*(a/(8*x^2) + (5*b*c)/(
24*x) - (b*c^2*ArcTanh[c*x])/8 + (b*ArcTanh[c*x])/(8*x^2) - (a*c^2*Log[x])
/4 + ((3*a + 4*b)*c^2*Log[1 - c*x])/24 + ((3*a - 4*b)*c^2*Log[1 + c*x])/24
 + (b*c^2*PolyLog[2, -(c*x)])/8 - (b*c^2*PolyLog[2, c*x])/8)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6647
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln\,(-c^2x^2+1))}{x^5}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx))\,(d+e\log\,(1-c^2x^2))}{x^5}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx)+a)(e\log\,(-c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="frica
s")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^5, x)
```

Sympy [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx))\,(d+e\log\,(1-c^2x^2))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atanh}\,(cx))\,(d+e\log\,(-c^2x^2+1))}{x^5}\,dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxim
a")
```

output
```
1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c
- 6*arctanh(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/
x^2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e + 1/8*b*e*(log(-c*x + 1)^2/x^4 - 4*i
ntegrate(-1/2*(2*(c*x - 1)*log(c*x + 1)^2 - c*x*log(-c*x + 1))/(c*x^6 - x^
5), x)) - 1/4*a*d/x^4
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac"
)
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{atanh}(c\,x)\right)\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^5} \,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^5,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = \frac{3atanh(cx)\,b\,c^4d\,x^4-3atanh(cx)\,bd+12\left(\int \frac{atanh(cx)\log(-c^2x^2+1)}{x^5}dx\right)be\,x^4+3\log(-c^2x^2+1)\,a\,c^4e\,x^4-3}{12x^4}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x)
```

output
```
(3*atanh(c*x)*b*c**4*d*x**4 - 3*atanh(c*x)*b*d + 12*int((atanh(c*x)*log( -
 c**2*x**2 + 1))/x**5,x)*b*e*x**4 + 3*log( - c**2*x**2 + 1)*a*c**4*e*x**4
- 3*log( - c**2*x**2 + 1)*a*e - 6*log(x)*a*c**4*e*x**4 + 3*a*c**2*e*x**2 -
 3*a*d - 3*b*c**3*d*x**3 - b*c*d*x)/(12*x**4)
```

3.531 $\int \frac{(a+b\text{arctanh}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\begin{aligned}
&\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx \\
&= \frac{7bc^3e}{60x^2}+\frac{2c^2e(a+b\text{arctanh}(cx))}{15x^3}+\frac{2c^4e(a+b\text{arctanh}(cx))}{5x}-\frac{c^5e(a+b\text{arctanh}(cx))^2}{5b} \\
&\quad -\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log\left(1-c^2x^2\right)-\frac{bc(d+e\log(1-c^2x^2))}{20x^4} \\
&\quad -\frac{bc^3(1-c^2x^2)\left(d+e\log\left(1-c^2x^2\right)\right)}{10x^2}-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5} \\
&\quad +\frac{1}{10}bc^5\left(d+e\log\left(1-c^2x^2\right)\right)\log\left(1-\frac{1}{1-c^2x^2}\right)-\frac{1}{10}bc^5e\,\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)
\end{aligned}$$

output

```
7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*arctanh(c*x))/x^3+2/5*c^4*e*(a+b*arctanh(
c*x))/x-1/5*c^5*e*(a+b*arctanh(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*l
n(-c^2*x^2+1)-1/20*b*c*(d+e*ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d
+e*ln(-c^2*x^2+1))/x^2-1/5*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5+1/1
0*b*c^5*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1
/(-c^2*x^2+1))
```

Mathematica [F]

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

output
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.14, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.926$, Rules used = {6643, 2925, 2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838, 6544, 6452, 243, 54, 2009, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x^6}dx$$

$$\Big\downarrow \text{6643}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx+\frac{1}{5}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^5\left(1-c^2x^2\right)}dx-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2925}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx+\frac{1}{10}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^6\left(1-c^2x^2\right)}dx^2-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^8}d(1-c^2x^2)}{10c}-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 27

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{1}{10}bc^5\int \frac{d+e\log\left(1-c^2x^2\right)}{c^6x^8}d(1-c^2x^2)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2789

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^6x^6}d(1-c^2x^2)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2756

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\int \frac{1}{c^4x^6}d(1-c^2x^2)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 54

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\int \left(\frac{1}{c^2x^2}+\frac{1}{x^2}+\frac{1}{c^4x^4}\right)d(1-c^2x^2)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2009

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(c^2x^2\right)+\log\left(1-c^2x^2\right)\right)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2789

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^4}d(1-c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(c^2x^2\right)+\log\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2751

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)-e\int \frac{1}{c^2x^2}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\frac{1}{2}e\left(\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 16

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2779

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2838

$$-\frac{2}{5}c^2e\int \frac{a+b\text{arctanh}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$\Big\downarrow$ 6544

$$-\frac{2}{5}c^2e\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\int\frac{a+b\text{arctanh}(cx)}{x^4}dx\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$\downarrow$ 6452

$$-\frac{2}{5}c^2e\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{3}bc\int\frac{1}{x^3(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$\downarrow$ 243

$$-\frac{2}{5}c^2e\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int\frac{1}{x^4(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$\downarrow$ 54

$$-\frac{2}{5}c^2e\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int\left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$\downarrow$ 2009

$$-\frac{2}{5}c^2e\left(c^2\int\frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)\right)-\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$\downarrow$ 6544

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\int\frac{a+b\text{arctanh}(cx)}{x^2}dx\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int\frac{1}{x\left(1-c^2x^2\right)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{1}{x^2\left(1-c^2x^2\right)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{47}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\int\frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arct}}{3x}\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arcta}}{3x^3}\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{16}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}\right.$$
$$\frac{(a+b\text{arctanh}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{6510}$$

$$-\frac{(a+b\text{arctanh}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$
$$\frac{2}{5}c^2e\left(c^2\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}\right.$$
$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

output
```
-1/5*((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5 - (2*c^2*e*(-1/3*
(a + b*ArcTanh[c*x])/x^3 + c^2*(-((a + b*ArcTanh[c*x])/x) + (c*(a + b*ArcT
anh[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2) + (b*c*(-x^(-2
) + c^2*Log[x^2] - c^2*Log[1 - c^2*x^2]))/6))/5 - (b*c^5*(e*Log[c^2*x^2] -
 (e*(1/(c^2*x^2) - Log[c^2*x^2] + Log[1 - c^2*x^2]))/2 + (d + e*Log[1 - c^
2*x^2])/(2*c^4*x^4) + ((1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(c^2*x^2) -
 Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2]) + e*PolyLog[2, x^(-2)]))/10
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6643
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln\left(-c^2x^2+1\right))}{x^6}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="frica
s")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^6, x)
```

Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log\left(1-c^2x^2\right))}{x^6}\,dx$$
$$=\int \frac{(a+b\,\text{atanh}\,(cx))\,(d+e\log\left(-c^2x^2+1\right))}{x^6}\,dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log\left(1-c^2x^2\right))}{x^6}\,dx$$
$$=\int \frac{(b\,\text{artanh}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxim
a")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
 4*arctanh(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1)
- 2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e + 1/10*b*e*(lo
g(-c*x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x - 1)*log(c*x + 1)^2 - 2*c*x*l
og(-c*x + 1))/(c*x^7 - x^6), x)) - 1/5*a*d/x^5
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac"
)
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^6}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^6,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(1 - c^2*x^2)))/x^6, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = \frac{-12atanh(cx)^2\,b\,c^5e\,x^5-12atanh(cx)\log(-c^2x^2+1)\,be+24atanh(cx)\,b\,c^4e\,x^4+8atanh(cx)\,b\,c^2e\,x^2-}{}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x)
```

output
```
( - 12*atanh(c*x)**2*b*c**5*e*x**5 - 12*atanh(c*x)*log( - c**2*x**2 + 1)*b
*e + 24*atanh(c*x)*b*c**4*e*x**4 + 8*atanh(c*x)*b*c**2*e*x**2 - 12*atanh(c
*x)*b*d - 12*int(log( - c**2*x**2 + 1)/(c**2*x**3 - x),x)*b*c**5*e*x**5 -
12*log( - c**2*x**2 + 1)*a*c**5*e*x**5 - 12*log( - c**2*x**2 + 1)*a*e - 6*
log( - c**2*x**2 + 1)*b*c**5*d*x**5 + 25*log( - c**2*x**2 + 1)*b*c**5*e*x*
*5 - 6*log( - c**2*x**2 + 1)*b*c**3*e*x**3 - 3*log( - c**2*x**2 + 1)*b*c*e
*x + 24*log(c**2*x - c)*a*c**5*e*x**5 + 12*log(x)*b*c**5*d*x**5 - 50*log(x
)*b*c**5*e*x**5 + 24*a*c**4*e*x**4 + 8*a*c**2*e*x**2 - 12*a*d - 6*b*c**3*d
*x**3 + 7*b*c**3*e*x**3 - 3*b*c*d*x)/(60*x**5)
```

3.532 $\int x(a+b\,\mathrm{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 512

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\text{arctanh}(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))-\frac{1}{2}ex^2(a+b\text{arctanh}(cx))-\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(\frac{2}{1+cx}\right)}{c^2g}+\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{2c^2g}+\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{2c^2g}+\frac{bex\log\left(f+gx^2\right)}{2c}-\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(f+gx^2\right)}{2c^2g}+\frac{e(f+gx^2)\,(a+b\text{arctanh}(cx))\log\left(f+gx^2\right)}{2g}+\frac{be(c^2f+g)\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2g}-\frac{be(c^2f+g)\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{4c^2g}-\frac{be(c^2f+g)\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{4c^2g}$$

output
```
1/2*b*(d-e)*x/c-b*e*x/c+b*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))/c/g^(1/2)-1/
2*b*(d-e)*arctanh(c*x)/c^2+1/2*d*x^2*(a+b*arctanh(c*x))-1/2*e*x^2*(a+b*arc
tanh(c*x))-b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1))/c^2/g+1/2*b*e*(c^2*f+g
)*arctanh(c*x)*ln(2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))/(c*x+1
))/c^2/g+1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(
-f)^(1/2)+g^(1/2))/(c*x+1))/c^2/g+1/2*b*e*x*ln(g*x^2+f)/c-1/2*b*e*(c^2*f+g
)*arctanh(c*x)*ln(g*x^2+f)/c^2/g+1/2*e*(g*x^2+f)*(a+b*arctanh(c*x))*ln(g*x
^2+f)/g+1/2*b*e*(c^2*f+g)*polylog(2,1-2/(c*x+1))/c^2/g-1/4*b*e*(c^2*f+g)*p
olylog(2,1-2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))/(c*x+1))/c^2/
g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+g
^(1/2))/(c*x+1))/c^2/g
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 1145, normalized size of antiderivative = 2.24

$$\int x(a + b\text{arctanh}(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx = \text{Too large to display}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(2*b*c*d*g*x - 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 + 4*b*c*e*S
qrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - 2*b*d*g*ArcTanh[c*x] + 2*b*e*
g*ArcTanh[c*x] + 2*b*c^2*d*g*x^2*ArcTanh[c*x] - 2*b*c^2*e*g*x^2*ArcTanh[c*
x] - (4*I)*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqr
t[-(c^2*f*g)]] - (4*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*
g*x)/Sqrt[-(c^2*f*g)]] - 4*b*c^2*e*f*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*
x])] - 4*b*e*g*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - (2*I)*b*c^2*e*f
*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (
-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*
f + g))] - (2*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(
2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^
(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*ArcTanh[c*x]*Log[(c^2*(1 + E^
(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E
^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*e*g*ArcTanh[c*x]*Log[(c^2*(1 + E^(2*
ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2
*ArcTanh[c*x])*(c^2*f + g))] + (2*I)*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f
+ g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g +
 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + (2*I)*b*e*g*ArcSi
n[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E
^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f +...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6645, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)dx$$

$$\Big\downarrow \text{6645}$$

$$-bc\int\left(\frac{(d-e)x^2}{2(1-c^2x^2)}+\frac{e(gx^2+f)\log(gx^2+f)}{2g(1-cx)(cx+1)}\right)dx+\frac{1}{2}dx^2(a+b\text{arctanh}(cx))+\frac{e(f+gx^2)\log(f+gx^2)(a+b\text{arctanh}(cx))}{2g}-\frac{1}{2}ex^2(a+b\text{arctanh}(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}dx^2(a+b\text{arctanh}(cx))+\frac{e(f+gx^2)\log(f+gx^2)(a+b\text{arctanh}(cx))}{2g}-\frac{1}{2}ex^2(a+b\text{arctanh}(cx))-bc\left(-\frac{e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c^2\sqrt{g}}+\frac{(d-e)\text{arctanh}(cx)}{2c^3}+\frac{e\text{arctanh}(cx)(c^2f+g)\log(f+gx^2)}{2c^3g}+\frac{e\text{arctanh}(cx)(c^2f+}{c^3g}\right.$$

input
```
Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcTanh[c*x]))/2 - (e*x^2*(a + b*ArcTanh[c*x]))/2 + (e*(f +
g*x^2)*(a + b*ArcTanh[c*x])*Log[f + g*x^2])/(2*g) - b*c*(-1/2*((d - e)*x)/
c^2 + (e*x)/c^2 - (e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(c^2*Sqrt[g]) +
((d - e)*ArcTanh[c*x])/(2*c^3) + (e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*
x)])/(c^3*g) - (e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x)
)/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(2*c^3*g) - (e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])
/(2*c^3*g) - (e*x*Log[f + g*x^2])/(2*c^2) + (e*(c^2*f + g)*ArcTanh[c*x]*Lo
g[f + g*x^2])/(2*c^3*g) - (e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c
^3*g) + (e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqr
t[-f] - Sqrt[g])*(1 + c*x))])/(4*c^3*g) + (e*(c^2*f + g)*PolyLog[2, 1 - (2
*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*c^3*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6645
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcTanh[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(448) = 896$.

Time = 14.13 (sec) , antiderivative size = 970, normalized size of antiderivative = 1.89

method	result
risch	$\frac{bdx}{2c} + \frac{bd\ln(-cx+1)}{4c^2} - \frac{bd\ln(cx+1)}{4c^2} - \frac{ae\,x^2}{2} - \frac{3bex}{2c} + \frac{ad\,x^2}{2} + \frac{be\ln(cx+1)}{4c^2} + \frac{eb\ln(cx+1)\ln\left(\frac{c\sqrt{-fg}-(cx+1)g+g}{c\sqrt{-fg}+g}\right)f}{4g} +$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*d*x/c+1/4*b*d*ln(-c*x+1)/c^2-1/4*b*d*ln(c*x+1)/c^2-1/2*a*e*x^2-3/2*b
*e*x/c+1/2*a*d*x^2+1/4/c^2*b*e*ln(c*x+1)+1/4*e*b/g*ln(c*x+1)*ln((c*(-f*g)^
(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f+1/4*e*b/g*ln(c*x+1)*ln((c*(-f*g)^
(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f-1/4*e*b/g*ln(-c*x+1)*ln((c*(-f*g)
^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f-1/4*e*b/g*ln(-c*x+1)*ln((c*(-f*
g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f+1/c*e*f*b/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))-1/4*b*e*x^2*ln(c*x+1)-1/4/c^2*b*e*ln(-c*x+1)+1/2*e/g*f*a*
ln(g*x^2+f)+1/4/c^2*e*b*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g
)^(1/2)+g))+1/4/c^2*e*b*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g
)^(1/2)-g))-1/4/c^2*e*b*ln(-c*x+1)*ln((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f
*g)^(1/2)+g))-1/4/c^2*e*b*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(
-f*g)^(1/2)-g))-1/4*e*b/g*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1
/2)+g))*f-1/4*e*b/g*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g)
)*f+1/4*e*b/g*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f+1/4
*e*b/g*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f+(1/4*b*e*x
^2*ln(c*x+1)-1/4*e*(b*c^2*x^2*ln(-c*x+1)-2*a*c^2*x^2-2*b*c*x+b*ln(c*x+1)-b
*ln(-c*x+1))/c^2)*ln(g*x^2+f)+1/4*e*b*ln(-c*x+1)*x^2-1/4/c^2*e*b*dilog((c*
(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4/c^2*e*b*dilog((c*(-f*g)
^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+1/4/c^2*e*b*dilog((c*(-f*g)^(1/2)
-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4/c^2*e*b*dilog((c*(-f*g)^(1/2)+(c*...
```

Fricas [F]

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx = \int (b\,\text{artanh}\,(cx)+a)\big(e\log\left(gx^2+f\right)+d\big)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*x*arctanh(c*x) + a*d*x + (b*e*x*arctanh(c*x) + a*e*x)*log(g*x
^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\,\text{artanh}\,(cx)+a)\big(e\log\left(gx^2+f\right)+d\big)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*d - 1/4*(2*c^2*g*integrate(x^3*log(c*x + 1)/(c^2*g*x^2
+ c^2*f), x) - 2*c^2*g*integrate(x^3*log(-c*x + 1)/(c^2*g*x^2 + c^2*f), x)
 - 2*c*g*(-I*f*(log(I*g*x/sqrt(f*g) + 1) - log(-I*g*x/sqrt(f*g) + 1))/(sqr
t(f*g)*c^2*g) - 2*x/(c^2*g)) - 2*g*integrate(x*log(c*x + 1)/(c^2*g*x^2 + c
^2*f), x) + 2*g*integrate(x*log(-c*x + 1)/(c^2*g*x^2 + c^2*f), x) - (2*c*x
 + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(-c*x + 1))*log(g*x^2 + f
)/c^2)*b*e - 1/2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g
```

Giac [F]

$$\int x(a+b\text{arctanh}(cx)) \left(d+e\log \left(f+gx^2\right)\right) \, dx = \int (b\,\text{artanh}\,(cx)+a)\left(e\log \left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arctanh}(cx)) \left(d+e\log \left(f+gx^2\right)\right) \, dx = \int x\,\left(a+b\,\text{atanh}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right) \,dx$$

input
```
int(x*(a + b*atanh(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int(x*(a + b*atanh(c*x))*(d + e*log(f + g*x^2)), x)
```

Reduce [F]

$$\int x(a+b\text{arctanh}(cx)) \left(d+e\log \left(f+gx^2\right)\right) \, dx = \frac{2\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right)bce+atanh(cx)\log(g\,x^2+f)\,b\,c^2ef+atanh(cx)\log(g\,x^2+f)\,b\,c^2eg\,x^2+atanh(c$$

input
```
int(x*(a+b*atanh(c*x))*(d+e*log(g*x^2+f)),x)
```

output

```
(2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*b*c*e + atanh(c*x)*log(f
+ g*x**2)*b*c**2*e*f + atanh(c*x)*log(f + g*x**2)*b*c**2*e*g*x**2 + atanh(
c*x)*b*c**2*d*g*x**2 - atanh(c*x)*b*c**2*e*g*x**2 - atanh(c*x)*b*d*g + ata
nh(c*x)*b*e*g + int(log(f + g*x**2)/(c**2*x**2 - 1),x)*b*c**3*e*f + int(lo
g(f + g*x**2)/(c**2*x**2 - 1),x)*b*c*e*g + log(f + g*x**2)*a*c**2*e*f + lo
g(f + g*x**2)*a*c**2*e*g*x**2 + log(f + g*x**2)*b*c*e*g*x + a*c**2*d*g*x**
2 - a*c**2*e*g*x**2 + b*c*d*g*x - 3*b*c*e*g*x)/(2*c**2*g)
```

3.533 $\int (a+b\text{arctanh}(cx)) \left(d+e\log\left(f+gx^2\right)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 599

$$\begin{aligned}
&\int (a+b\text{arctanh}(cx)) \left(d+e\log\left(f+gx^2\right)\right) dx \\
&= -2aex+\frac{2ae\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\text{arctanh}(cx) \\
&+\frac{be\sqrt{-f}\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\log(1+cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}} \\
&+\frac{be\sqrt{-f}\log(1+cx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}x\right)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}} \\
&-\frac{be\log\left(1-c^2x^2\right)}{c}+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right) \\
&+\frac{b\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}+\frac{be\sqrt{-f}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}} \\
&-\frac{be\sqrt{-f}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(1+cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}} \\
&-\frac{be\sqrt{-f}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(1+cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\,\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)}{2c}
\end{aligned}$$

output
```
-2*a*e*x+2*a*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))/g^(1/2)-2*b*e*x*arctanh(c
*x)+1/2*b*e*(-f)^(1/2)*ln(-c*x+1)*ln(c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2
)-g^(1/2)))/g^(1/2)-1/2*b*e*(-f)^(1/2)*ln(c*x+1)*ln(c*((-f)^(1/2)-g^(1/2)*
x)/(c*(-f)^(1/2)+g^(1/2)))/g^(1/2)+1/2*b*e*(-f)^(1/2)*ln(c*x+1)*ln(c*((-f)
^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2)))/g^(1/2)-1/2*b*e*(-f)^(1/2)*ln(-c
*x+1)*ln(c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+g^(1/2)))/g^(1/2)-b*e*ln(-
c^2*x^2+1)/c+x*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))+1/2*b*ln(g*(-c^2*x^2+1
)/(c^2*f+g))*(d+e*ln(g*x^2+f))/c+1/2*b*e*(-f)^(1/2)*polylog(2,-g^(1/2)*(-c
*x+1)/(c*(-f)^(1/2)-g^(1/2)))/g^(1/2)-1/2*b*e*(-f)^(1/2)*polylog(2,g^(1/2)
*(-c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/g^(1/2)+1/2*b*e*(-f)^(1/2)*polylog(2,-g^
(1/2)*(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/g^(1/2)-1/2*b*e*(-f)^(1/2)*polylog(2
,g^(1/2)*(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/g^(1/2)+1/2*b*e*polylog(2,c^2*(g*
x^2+f)/(c^2*f+g))/c
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 1251, normalized size of antiderivative = 2.09

$$\int (a + b\text{arctanh}(cx)) \left(d + e\log\left(f + gx^2\right)\right) \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + b*
d*x*ArcTanh[c*x] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b
*e*(x*ArcTanh[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] - (b*e*g*(((-L
og[-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2*x^2])*Log[f + g*x^2])/(2*g
) + (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt
[g]/c)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)])/
(2*g) + (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqr
t[g]/c)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)])/(2
*g) + (Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt
[g]/c)] + PolyLog[2, (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)])/(
2*g) + (Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g
]/c)] + PolyLog[2, (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)])/(2*g))
)/c - (b*e*(4*c*x*ArcTanh[c*x] - 4*Log[1/Sqrt[1 - c^2*x^2]] + (Sqrt[c^2*f*
g]*((-2*I)*ArcCos[(-(c^2*f) + g)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]
] + 4*ArcTan[Sqrt[c^2*f*g]/(c*g*x)]*ArcTanh[c*x] - (ArcCos[(-(c^2*f) + g)/
(c^2*f + g)] - 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(g + I*Sqrt[c
^2*f*g])*(1 + c*x))/((c^2*f + g)*(c^2*f + I*c*Sqrt[c^2*f*g]*x))] - (ArcCos
[(-(c^2*f) + g)/(c^2*f + g)] + 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2
*f*(I*g + Sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f + g)*((-I)*c^2*f + c*Sqrt[c^2
*f*g]*x))] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f...
```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6635, 2925, 2841, 2840, 2838, 6542, 2009, 6536, 218, 6534, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(cx)) \left(d+e\log \left(f+gx^2\right)\right) \, dx$$

$$\Big\downarrow \text{ 6635}$$

$$-2eg\int \frac{x^2(a+b\text{arctanh}(cx))}{gx^2+f}dx-bc\int \frac{x\left(d+e\log \left(gx^2+f\right)\right)}{1-c^2x^2}dx+x(a+b\text{arctanh}(cx)) \left(d+e\log \left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{ 2925}$$

$$-2eg\int \frac{x^2(a+b\text{arctanh}(cx))}{gx^2+f}dx-\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{1-c^2x^2}dx^2+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2841}$$

$$-2eg\int \frac{x^2(a+b\text{arctanh}(cx))}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{eg\int \frac{\log\left(\frac{g\left(1-c^2x^2\right)}{fc^2+g}\right)}{gx^2+f}dx^2}{c^2}-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2840}$$

$$-2eg\int \frac{x^2(a+b\text{arctanh}(cx))}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{e\int \frac{\log\left(1-\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{x^2}d(gx^2+f)}{c^2}-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2eg\int \frac{x^2(a+b\text{arctanh}(cx))}{gx^2+f}dx+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{6542}$$

$$-2eg\left(\frac{\int (a+b\text{arctanh}(cx))dx}{g}-\frac{f\int \frac{a+b\text{arctanh}(cx)}{gx^2+f}dx}{g}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2eg\left(\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{g}-\frac{f\int\frac{a+b\text{arctanh}(cx)}{gx^2+f}dx}{g}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 6536$$

$$-2eg\left(\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{g}-\frac{f\left(a\int\frac{1}{gx^2+f}dx+b\int\frac{\text{arctanh}(cx)}{gx^2+f}dx\right)}{g}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 218$$

$$-2eg\left(\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{g}-\frac{f\left(b\int\frac{\text{arctanh}(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 6534$$

$$-2eg\left(\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{g}-\frac{f\left(b\left(\frac{1}{2}\int\frac{\log(cx+1)}{gx^2+f}dx-\frac{1}{2}\int\frac{\log(1-cx)}{gx^2+f}dx\right)+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 2856$$

$$-2eg\left(\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{g}-\frac{f\left(b\left(\frac{1}{2}\int\left(\frac{\sqrt{-f}\log(cx+1)}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}\log(cx+1)}{2f(\sqrt{g}x+\sqrt{-f})}\right)dx-\frac{1}{2}\int\left(\frac{\sqrt{-f}\log(1-cx)}{2f(\sqrt{-f}-\sqrt{g}x)}\right.\right.\right.}{g}\right.$$

$$x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 2009$$

$$-2eg\left(\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(1-cx)}{\sqrt{-f}c+\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}\right.\right.\right.}{}\right.$$

$$x(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

input
```
Int[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]) - 2*e*g*((a*x + b*x*ArcTanh[
c*x] + (b*Log[1 - c^2*x^2])/(2*c))/g - (f*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])
/(Sqrt[f]*Sqrt[g]) + b*((-1/2*(Log[1 - c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))
/(c*Sqrt[-f] - Sqrt[g])])/(Sqrt[-f]*Sqrt[g]) + (Log[1 - c*x]*Log[(c*(Sqrt[
-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - PolyLog[
2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt[g]))]/(2*Sqrt[-f]*Sqrt[g]) + P
olyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[g])
)/2 + ((Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])
])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*
Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - PolyLog[2, -((Sqrt[g]*(1 + c*
x))/(c*Sqrt[-f] - Sqrt[g]))]/(2*Sqrt[-f]*Sqrt[g]) + PolyLog[2, (Sqrt[g]*(1
 + c*x))/(c*Sqrt[-f] + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[g]))/2)))/g) - (b*c*(-((
Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/c^2) - (e*PolyL
og[2, (c^2*(f + g*x^2))/(c^2*f + g)])/c^2))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6534
```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + c*x]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - c*x]/(d + e*x^2)
, x], x] /; FreeQ[{c, d, e}, x]
```

rule 6536
```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTanh[c*x]/(d + e*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6635
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp
[2*e*g   Int[x^2*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b
, c, d, e, f, g}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.32 (sec) , antiderivative size = 3550, normalized size of antiderivative = 5.93

method	result	size
risch	Expression too large to display	3550

input
```
int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/2/c*ln(-c*x+1)*b*d-2*a*e*x+4*b*e/c-b*e/c*ln(c*x+1)+a*e*x*ln(g*x^2+
f)+1/2*d*b*ln(c*x+1)*x-1/2*d*b*ln(-c*x+1)*x+e*b*ln(-c*x+1)*x-b*e*x*ln(c*x+
1)-b*d/c-1/2*I*e*b/c*Pi*csgn(I*c^2)^3-1/4*I*e*b/c*Pi*ln(-c*x+1)*csgn(I/c^2
)*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x
-1)*g))+1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I/c^2)*csgn(I*(c^2*f+((c*x-1)^2+2*c*x
-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*x-1/4*I*e*b/c*Pi*ln(c*x+
1)*csgn(I/c^2)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c
*x+1)^2-2*c*x-1)*g))-1/4*I*e*b*Pi*ln(c*x+1)*csgn(I/c^2)*csgn(I*(c^2*f+((c*
x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*x-1/4*I*e*b/
c*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2-1/4*I*e*b/c*P
i*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x
-1)*g))^2+1/4*I*e*b/c*Pi*ln(-c*x+1)*csgn(I*c^2)^3-1/4*I*e*b/c*Pi*ln(-c*x+1
)*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3+1/4*I*e*b*Pi*csgn(I/c^2)*csg
n(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*x-1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I*
c^2)^3*x+1/4*I*e*b*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^
2*f+((c*x-1)^2+2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*ln(-c*x+1)*csgn(I/c^2*(c^2*f+
((c*x-1)^2+2*c*x-1)*g))^3*x-1/4*I*e*b*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c*x
+1)^2-2*c*x-1)*g))^3*x-1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x+
1)^2-2*c*x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*c
sgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*ln(c*x+1)*csg...
```

Fricas [F]

$$\int (a+b\text{arctanh}(cx)) \left(d+e\log \left(f+gx^2\right)\right) \, dx = \int (b\,\text{artanh}\,(cx)+a)\big(e\log \left(gx^2+f\right)+d\big)\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\operatorname{arctanh}(cx)) \left(d + e \log \left(f + gx^2\right)\right) \, dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\operatorname{arctanh}(cx)) \left(d + e \log \left(f + gx^2\right)\right) \, dx = \int (b \operatorname{artanh}(cx) + a)\left(e \log \left(gx^2 + f\right) + d\right) dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
(2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 + f))*a*e
 + a*d*x + 1/2*b*e*(((c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*log
(g*x^2 + f)/c + integrate(-2*((c*g*x^2 + g*x)*log(c*x + 1) - (c*g*x^2 - g*
x)*log(-c*x + 1))/(c*g*x^2 + c*f), x)) + 1/2*(2*c*x*arctanh(c*x) + log(-c^
2*x^2 + 1))*b*d/c
```

Giac [F]

$$\int (a + b\operatorname{arctanh}(cx)) \left(d + e \log \left(f + gx^2\right)\right) \, dx = \int (b \operatorname{artanh}(cx) + a)\left(e \log \left(gx^2 + f\right) + d\right) dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arctanh}(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx$$
$$= \int (a + b\,\text{atanh}(c\,x)) \left(d + e\,\ln\left(g\,x^2 + f\right)\right) dx$$

input
```
int((a + b*atanh(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int((a + b*atanh(c*x))*(d + e*log(f + g*x^2)), x)
```

Reduce [F]

$$\int (a + b\text{arctanh}(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx$$
$$= \frac{8\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right)ace - 4atanh(cx)^2\,b\,c^2ef + 4atanh(cx)\log(g\,x^2 + f)\,bcegx + 4atanh(cx)\,bcdgx -}{}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(g*x^2+f)),x)
```

output
```
(8*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*a*c*e - 4*atanh(c*x)**2*b
*c**2*e*f + 4*atanh(c*x)*log(f + g*x**2)*b*c*e*g*x + 4*atanh(c*x)*b*c*d*g*
x - 8*atanh(c*x)*b*c*e*g*x - 8*int(atanh(c*x)/(c**2*f*x**2 + c**2*g*x**4 -
 f - g*x**2),x)*b*c**3*e*f**2 - 8*int(atanh(c*x)/(c**2*f*x**2 + c**2*g*x**
4 - f - g*x**2),x)*b*c*e*f*g + 4*int((log(f + g*x**2)*x)/(c**2*f*x**2 + c*
*2*g*x**4 - f - g*x**2),x)*b*c**2*e*f*g + 4*int((log(f + g*x**2)*x)/(c**2*
f*x**2 + c**2*g*x**4 - f - g*x**2),x)*b*e*g**2 + 2*log(c**2*x - c)*b*d*g -
 4*log(c**2*x - c)*b*e*g + 2*log(c**2*x + c)*b*d*g - 4*log(c**2*x + c)*b*e
*g + log(f + g*x**2)**2*b*e*g + 4*log(f + g*x**2)*a*c*e*g*x + 4*a*c*d*g*x
- 8*a*c*e*g*x)/(4*c*g)
```

3.534 $\int \frac{(a+b\text{arctanh}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}
&\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx \\
&= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right) \\
&\quad -\frac{1}{2}bd\,\text{PolyLog}(2,-cx)+\frac{1}{2}bd\,\text{PolyLog}(2,cx) \\
&\quad +\frac{1}{2}ae\,\text{PolyLog}\left(2,1+\frac{gx^2}{f}\right)+be\text{Int}\left(\frac{\text{arctanh}(cx)\log\left(f+gx^2\right)}{x},x\right)
\end{aligned}$$

output
```
a*d*ln(x)+1/2*a*e*ln(-g*x^2/f)*ln(g*x^2+f)-1/2*b*d*polylog(2,-c*x)+1/2*b*d
*polylog(2,c*x)+1/2*a*e*polylog(2,1+g*x^2/f)+b*e*Defer(Int)(arctanh(c*x)*l
n(g*x^2+f)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{ 6641}$$

$$d\int \frac{a+b\text{arctanh}(cx)}{x}dx+e\int \frac{(a+b\text{arctanh}(cx))\log\left(gx^2+f\right)}{x}dx$$

$$\Big\downarrow \text{ 6446}$$

$$e\int \frac{(a+b\text{arctanh}(cx))\log\left(gx^2+f\right)}{x}dx+ d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$$\Big\downarrow \text{6639}$$

$$e\left(a\int \frac{\log \left(gx^2+f\right)}{x}dx+b\int \frac{\text{arctanh}(cx)\log \left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$$\Big\downarrow \text{2904}$$

$$e\left(\frac{1}{2}a\int \frac{\log \left(gx^2+f\right)}{x^2}dx^2+b\int \frac{\text{arctanh}(cx)\log \left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$$\Big\downarrow \text{2841}$$

$$e\left(\frac{1}{2}a\left(\log \left(-\frac{gx^2}{f}\right)\log \left(f+gx^2\right)-g\int \frac{\log \left(-\frac{gx^2}{f}\right)}{gx^2+f}dx^2\right)+b\int \frac{\text{arctanh}(cx)\log \left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$$\Big\downarrow \text{2752}$$

$$e\left(b\int \frac{\text{arctanh}(cx)\log \left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log \left(-\frac{gx^2}{f}\right)\log \left(f+gx^2\right)\right)\right)+d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

$$\Big\downarrow \text{7299}$$

$$e\left(b\int \frac{\text{arctanh}(cx)\log \left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log \left(-\frac{gx^2}{f}\right)\log \left(f+gx^2\right)\right)\right)+d\left(a\log(x)-\frac{1}{2}b\,\text{PolyLog}(2,-cx)+\frac{1}{2}b\,\text{PolyLog}(2,cx)\right)$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,\operatorname{arctanh}(cx))\,(d+e\ln(g\,x^2+f))}{x}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\mathrm{arctanh}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f)
)/x, x)
```

Sympy [N/A]

Not integrable

Time = 116.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\mathrm{arctanh}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(a+b\,\mathrm{atanh}\,(cx))\,(d+e\log(f+gx^2))}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

output
```
Integral((a + b*atanh(c*x))*(d + e*log(f + g*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.21

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log{(f + gx^2)})}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)(e\log{(gx^2 + f)} + d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + integrate(1/2*b*e*(log(c*x + 1) - log(-c*x + 1))*log(g*x^2 +
f)/x + 1/2*b*d*(log(c*x + 1) - log(-c*x + 1))/x + a*e*log(g*x^2 + f)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log{(f + gx^2)})}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)(e\log{(gx^2 + f)} + d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(a+b\,\mathrm{atanh}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.17

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \left(\int \frac{atanh(cx)}{g\,x^3+fx}dx\right)bdf + \left(\int \frac{\log(g\,x^2+f)}{g\,x^3+fx}dx\right)aef + \left(\int \frac{atanh(cx)\log(g\,x^2+f)\,x}{g\,x^2+f}dx\right)beg + \left(\int \frac{atanh(cx)\log(g\,x^2+f)}{g\,x^3+fx}dx\right)bef + \left(\int \frac{atanh(cx)\,x}{g\,x^2+f}dx\right)bdg + \frac{\log(g\,x^2+f)^2\,ae}{4} + \log(x)\,ad$$

input
```
int((a+b*atanh(c*x))*(d+e*log(g*x^2+f))/x,x)
```

output
```
(4*int(atanh(c*x)/(f*x + g*x**3),x)*b*d*f + 4*int(log(f + g*x**2)/(f*x + g
*x**3),x)*a*e*f + 4*int((atanh(c*x)*log(f + g*x**2)*x)/(f + g*x**2),x)*b*e
*g + 4*int((atanh(c*x)*log(f + g*x**2))/(f*x + g*x**3),x)*b*e*f + 4*int((a
tanh(c*x)*x)/(f + g*x**2),x)*b*d*g + log(f + g*x**2)**2*a*e + 4*log(x)*a*d
)/4
```

3.535 $\int \frac{(a+b\text{arctanh}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 613

$$\begin{aligned}
&\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx \\
&= \frac{2ae\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\log(1-cx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad + \frac{be\sqrt{g}\log(1+cx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{be\sqrt{g}\log(1+cx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad + \frac{be\sqrt{g}\log(1-cx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x} \\
&\quad + \frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right) - \frac{1}{2}bc\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right) \\
&\quad - \frac{be\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{be\sqrt{g}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad - \frac{be\sqrt{g}\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(1+cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{be\sqrt{g}\,\text{PolyLog}\left(2,\frac{\sqrt{g}(1+cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad - \frac{1}{2}bce\,\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right) + \frac{1}{2}bce\,\text{PolyLog}\left(2,1+\frac{gx^2}{f}\right)
\end{aligned}$$

output
```
2*a*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))/f^(1/2)-1/2*b*e*g^(1/2)*ln(-c*x+1)
*ln(c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2)))/(-f)^(1/2)+1/2*b*e*g^
(1/2)*ln(c*x+1)*ln(c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)+g^(1/2)))/(-f)^(
1/2)-1/2*b*e*g^(1/2)*ln(c*x+1)*ln(c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)-g
^(1/2)))/(-f)^(1/2)+1/2*b*e*g^(1/2)*ln(-c*x+1)*ln(c*((-f)^(1/2)+g^(1/2)*x)
/(c*(-f)^(1/2)+g^(1/2)))/(-f)^(1/2)-(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x
+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+f))-1/2*b*c*ln(g*(-c^2*x^2+1)/(c^2*f+g
))*(d+e*ln(g*x^2+f))-1/2*b*e*g^(1/2)*polylog(2,-g^(1/2)*(-c*x+1)/(c*(-f)^(
1/2)-g^(1/2)))/(-f)^(1/2)+1/2*b*e*g^(1/2)*polylog(2,g^(1/2)*(-c*x+1)/(c*(-
f)^(1/2)+g^(1/2)))/(-f)^(1/2)-1/2*b*e*g^(1/2)*polylog(2,-g^(1/2)*(c*x+1)/(
c*(-f)^(1/2)-g^(1/2)))/(-f)^(1/2)+1/2*b*e*g^(1/2)*polylog(2,g^(1/2)*(c*x+1
)/(c*(-f)^(1/2)+g^(1/2)))/(-f)^(1/2)-1/2*b*c*e*polylog(2,c^2*(g*x^2+f)/(c^
2*f+g))+1/2*b*c*e*polylog(2,1+g*x^2/f)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 1226, normalized size of antiderivative = 2.00

$$\int \frac{(a + b\text{arctanh}(cx)) (d + e \log (f + gx^2))}{x^2} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-((a*d)/x) - (b*d*ArcTanh[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2]
)/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2
]/x) + (b*e*(-(((2*ArcTanh[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[
f + g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*
Sqrt[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (
I*Sqrt[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x
))/(c*Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*
x))/(c*Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g
]*x))/(c*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log
[1 - c^2*x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x)
)/(I*c*Sqrt[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt
[f] + Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g]
)] + PolyLog[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyL
og[2, (I*Sqrt[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]]) + (c*g*((2*I)*ArcCos
[(-(c^2*f) + g)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - 4*ArcTan[(c*f
)/(Sqrt[c^2*f*g]*x)]*ArcTanh[c*x] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] +
2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[((2*I)*c*f*(I*g + Sqrt[c^2*f*g])*(-1
+ c*x))/((c^2*f + g)*(c*f + I*Sqrt[c^2*f*g]*x))] + (ArcCos[(-(c^2*f) + g)/
(c^2*f + g)] - 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c*f*(g + I*Sqrt[c^2
*f*g])*(1 + c*x))/((c^2*f + g)*(c*f + I*Sqrt[c^2*f*g]*x))] - (ArcCos[(-...
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6643, 2925, 2863, 2009, 6536, 218, 6534, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx)) \left(d+e\log\left(f+gx^2\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{6643}$$

$$2eg\int \frac{a+b\text{arctanh}(cx)}{gx^2+f}dx+bc\int \frac{d+e\log\left(gx^2+f\right)}{x\left(1-c^2x^2\right)}dx-\frac{(a+b\text{arctanh}(cx)) \left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2eg\int \frac{a+b\text{arctanh}(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{x^2\left(1-c^2x^2\right)}dx^2-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$\Big\downarrow$ 2863

$$2eg\int \frac{a+b\text{arctanh}(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \left(\frac{d+e\log\left(gx^2+f\right)}{x^2}-\frac{c^2\left(d+e\log\left(gx^2+f\right)\right)}{c^2x^2-1}\right)dx^2-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$\Big\downarrow$ 2009

$$2eg\int \frac{a+b\text{arctanh}(cx)}{gx^2+f}dx-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 6536

$$2eg\left(a\int \frac{1}{gx^2+f}dx+b\int \frac{\text{arctanh}(cx)}{gx^2+f}dx\right)-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 218

$$2eg\left(b\int \frac{\text{arctanh}(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 6534

$$2eg\left(b\left(\frac{1}{2}\int \frac{\log(cx+1)}{gx^2+f}dx-\frac{1}{2}\int \frac{\log(1-cx)}{gx^2+f}dx\right)+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 2856

$$2eg\left(b\left(\frac{1}{2}\int\left(\frac{\sqrt{-f}\log(cx+1)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log(cx+1)}{2f\left(\sqrt{g}x+\sqrt{-f}\right)}\right)dx-\frac{1}{2}\int\left(\frac{\sqrt{-f}\log(1-cx)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log(1-cx)}{2f\left(\sqrt{g}x+\sqrt{-f}\right)}\right)dx\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(1-cx)}{\sqrt{-f}c+\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log(1-cx)\log\left(\frac{c(\sqrt{-f}-\ldots}{c\sqrt{-f}-\ldots}\right)}{2\sqrt{-f}\sqrt{g}}\right.\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-(((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x) + 2*e*g*((a*ArcTan[(Sqr
t[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) + b*((-1/2*(Log[1 - c*x]*Log[(c*(Sqrt[
-f] - Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])]))/(Sqrt[-f]*Sqrt[g]) + (Log[1 - c
*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]*Sq
rt[g]) - PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt[g]))]/(2*Sqrt
[-f]*Sqrt[g]) + PolyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] + Sqrt[g])]/(2*
Sqrt[-f]*Sqrt[g]))/2 + ((Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sq
rt[-f] + Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1 + c*x]*Log[(c*(Sqrt[-f]
+ Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - PolyLog[2, -
((Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] - Sqrt[g]))]/(2*Sqrt[-f]*Sqrt[g]) + PolyL
og[2, (Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[g]))/2)
) + (b*c*(Log[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]) - Log[(g*(1 - c^2*x^2))
/(c^2*f + g)]*(d + e*Log[f + g*x^2]) - e*PolyLog[2, (c^2*(f + g*x^2))/(c^2
*f + g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6534
```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + c*x]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - c*x]/(d + e*x^2)
, x], x] /; FreeQ[{c, d, e}, x]
```

rule 6536
```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTanh[c*x]/(d + e*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6643
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcTanh[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcTanh[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arctanh}\,(cx))\,(d+e\ln\,(g\,x^2+f))}{x^2}dx$$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

output
```
int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx))\,(d+e\log\,(f+gx^2))}{x^2}\,dx$$
$$=\int \frac{(b\,\mathrm{artanh}\,(cx)+a)(e\log\,(gx^2+f)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f)
)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx)) \left(d + e \log \left(f + gx^2\right)\right)}{x^2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx)) \left(d + e \log \left(f + gx^2\right)\right)}{x^2} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)(e \log(gx^2 + f) + d)}{x^2} \, dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d + (2*g*arcta
n(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + 1/2*b*e*integrate((lo
g(c*x + 1) - log(-c*x + 1))*log(g*x^2 + f)/x^2, x) - a*d/x
```

Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx)) \left(d + e \log \left(f + gx^2\right)\right)}{x^2} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)(e \log(gx^2 + f) + d)}{x^2} \, dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx)) (d + e \log (f + gx^2))}{x^2} \, dx = \int \frac{(a + b\,\text{atanh}(c\,x)) \, (d + e\,\ln (g\,x^2 + f))}{x^2} \, dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a + b\text{arctanh}(cx)) (d + e \log (f + gx^2))}{x^2} \, dx = \frac{2\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right) aex - atanh(cx)\,bcdfx - atanh(cx)\,bdf + \left(\int \frac{atanh(cx)\log(g\,x^2+f)}{x^2} dx\right) befx - \log(c^2x}{fx}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(g*x^2+f))/x^2,x)
```

output
```
(2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*a*e*x - atanh(c*x)*b*c*d*
f*x - atanh(c*x)*b*d*f + int((atanh(c*x)*log(f + g*x**2))/x**2,x)*b*e*f*x
- log(c**2*x - c)*b*c*d*f*x - log(f + g*x**2)*a*e*f + log(x)*b*c*d*f*x - a
*d*f)/(f*x)
```

3.536 $\int \frac{(a+b\,\mathrm{arctanh}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 511

$$
\begin{aligned}
&\int \frac{(a+b\,\mathrm{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx \\
&= \frac{bce\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(\frac{2}{1+cx}\right)}{f} \\
&-\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{2f} \\
&-\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{2f}-\frac{aeg\log\left(f+gx^2\right)}{2f} \\
&-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\mathrm{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right) \\
&-\frac{(a+b\,\mathrm{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2e\,\mathrm{PolyLog}(2,-cx) \\
&-\frac{be(c^2f+g)\,\mathrm{PolyLog}(2,-cx)}{2f}-\frac{1}{2}bc^2e\,\mathrm{PolyLog}(2,cx)+\frac{be(c^2f+g)\,\mathrm{PolyLog}(2,cx)}{2f} \\
&-\frac{be(c^2f+g)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2f}+\frac{be(c^2f+g)\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{4f} \\
&+\frac{be(c^2f+g)\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{4f}
\end{aligned}
$$

output
```
b*c*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))/f^(1/2)+a*e*g*ln(x)/f+b*e*(c^2*f+g
)*arctanh(c*x)*ln(2/(c*x+1))/f-1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)
^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))/(c*x+1))/f-1/2*b*e*(c^2*f+g)*arct
anh(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+g^(1/2))/(c*x+1))/f-1
/2*a*e*g*ln(g*x^2+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x+1/2*b*c^2*arctanh(c*x)*
(d+e*ln(g*x^2+f))-1/2*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*b*c^2*e
*polylog(2,-c*x)-1/2*b*e*(c^2*f+g)*polylog(2,-c*x)/f-1/2*b*c^2*e*polylog(2
,c*x)+1/2*b*e*(c^2*f+g)*polylog(2,c*x)/f-1/2*b*e*(c^2*f+g)*polylog(2,1-2/(
c*x+1))/f+1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)
^(1/2)-g^(1/2))/(c*x+1))/f+1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+g
^(1/2)*x)/(c*(-f)^(1/2)+g^(1/2))/(c*x+1))/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.37

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
(-2*a*d*f - 2*b*c*d*f*x + 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/S
qrt[f]] - 2*b*d*f*ArcTanh[c*x] + 2*b*c^2*d*f*x^2*ArcTanh[c*x] + (4*I)*b*c^
2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g
)]] + (4*I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sq
rt[-(c^2*f*g)]] + 4*b*e*g*x^2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] +
4*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + (2*I)*b*c^2*e*
f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*
f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*
(c^2*f + g))] + (2*I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2
*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f
*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log
[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(
c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - 2*b*e*g*x^2*ArcTanh[c*x]*Lo
g[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-
(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - (2*I)*b*c^2*e*f*x^2*ArcSin
[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^
(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))
] - (2*I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*A
rcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*
ArcTanh[c*x])*(c^2*f + g))] - 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[(c^2*(1 ...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6647, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$$

$$\Big\downarrow \text{6647}$$

$$-2eg\int\left(-\frac{a+bcx}{2x\left(gx^2+f\right)}-\frac{b(1-c^2x^2)\operatorname{arctanh}(cx)}{2x\left(gx^2+f\right)}\right)dx-\frac{(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2\operatorname{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

$\Big\downarrow$ 2009

$$-2eg\left(\frac{a\log\left(f+gx^2\right)}{4f}-\frac{a\log(x)}{2f}-\frac{bc\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{b\operatorname{arctanh}(cx)\left(c^2f+g\right)\log\left(\frac{2}{cx+1}\right)}{2fg}+\frac{b\operatorname{arctanh}(cx)\left(c^2f\right.}{}\right.$$
$$-\frac{(a+b\operatorname{arctanh}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2\operatorname{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

input
```
Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[f + g*x^2]))/x + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g
*x^2]))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - 2*e*g*
(-1/2*(b*c*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (a*Log[x])/(2*
f) - (b*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*x)])/(2*f*g) + (b*(c^2*f + g
)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1
 + c*x))])/(4*f*g) + (b*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt
[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*f*g) + (a*Log[f + g*x^2])/
(4*f) + (b*PolyLog[2, -(c*x)])/(4*f) - (b*PolyLog[2, c*x])/(4*f) + (b*(c^2
*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(4*f*g) - (b*(c^2*f + g)*PolyLog[2, 1
 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(8*f*
g) - (b*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-
f] + Sqrt[g])*(1 + c*x))])/(8*f*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6647
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(445) = 890$.

Time = 12.71 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.88

method	result
risch	$-\frac{bcd}{2x}-\frac{aeg\ln\left(g\,x^{2}+f\right)}{2f}+\frac{aeg\ln\left(x\right)}{f}-\frac{b\,c^{2}d\ln\left(-cx+1\right)}{4}-\frac{ad}{2x^{2}}-\frac{\ln\left(cx+1\right)bd}{4x^{2}}+\frac{bd\ln\left(-cx+1\right)}{4x^{2}}-\frac{gbe\ln\left(cx+1\right)\ln\left(\frac{c\sqrt{-f}}{}\right.}{4f}$

input
```
int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*c*d/x-1/2*a*e*g*ln(g*x^2+f)/f+a*e*g*ln(x)/f-1/4*b*c^2*d*ln(-c*x+1)-
1/2*a*d/x^2-1/4/x^2*ln(c*x+1)*b*d+1/4/x^2*b*d*ln(-c*x+1)-1/4*g*b*e/f*ln(c*
x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+g*e*b*c/(f*g)^(1/
2)*arctan(x*g/(f*g)^(1/2))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-f*g)^(1/2)-(-c*x
+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c
*x+1)*g-g)/(c*(-f*g)^(1/2)-g))-1/2*g*b*e/f*dilog(c*x+1)-1/4*b*e*ln(c*x+1)*
ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2-1/4*b*e*ln(c*x+1)*
ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*g*b*e/f*dilog(
(c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*dilog((c*(-f*
g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+1/2*g*b*e/f*dilog(-c*x+1)+1/4*b*
e*ln(-c*x+1)*ln((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2+1/4*
b*e*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2+1/
4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4*g*b*
e/f*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+(-1/4*b*e/x^2*
ln(c*x+1)-1/4*e*(b*c^2*x^2*ln(-c*x+1)-b*c^2*ln(c*x+1)*x^2+2*b*c*x-b*ln(-c*
x+1)+2*a)/x^2)*ln(g*x^2+f)-1/4*b*e*c^2*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/
(c*(-f*g)^(1/2)+g))-1/4*b*e*c^2*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*
g)^(1/2)-g))+1/4*b*e*c^2*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/
2)+g))+1/4*b*e*c^2*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))
-1/4*d*b*c^2*ln(c*x)+1/4*d*b*c^2*ln(c*x+1)+1/4*d*b*c^2*ln(-c*x)-1/4*g*b...
```

Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx)) (d + e\log(f + gx^2))}{x^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)(e\log(gx^2 + f) + d)}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

output
```
integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f)
)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log(f + gx^2))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))\,(d + e\log(f + gx^2))}{x^3}\,dx = \int \frac{(b\,\text{artanh}(cx) + a)(e\log(gx^2 + f) + d)}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d -
 1/2*(g*(log(g*x^2 + f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e - 1/4*(2
*c^2*g*integrate(x^2*log(c*x + 1)/(g*x^3 + f*x), x) - 2*c^2*g*integrate(x^
2*log(-c*x + 1)/(g*x^3 + f*x), x) + 2*I*c*g*(log(I*g*x/sqrt(f*g) + 1) - lo
g(-I*g*x/sqrt(f*g) + 1))/sqrt(f*g) - 2*g*integrate(log(c*x + 1)/(g*x^3 + f
*x), x) + 2*g*integrate(log(-c*x + 1)/(g*x^3 + f*x), x) + (2*c*x - (c^2*x^
2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x + 1))*log(g*x^2 + f)/x^2)*b*e
 - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)(e\log(gx^2+f)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))\,\left(d+e\,\ln(g\,x^2+f)\right)}{x^3}\,dx$$

input
```
int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

output
```
int(((a + b*atanh(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\text{arctanh}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*atanh(c*x))*(d+e*log(g*x^2+f))/x^3,x)
```

output

```
( - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*b*c*e*g*x**2 - atanh(c
*x)*log(f + g*x**2)*b*c**2*e*f**2 + atanh(c*x)*log(f + g*x**2)*b*e*f*g + a
tanh(c*x)*b*c**4*d*f**2*x**2 + atanh(c*x)*b*c**4*e*f**2*x**2 - atanh(c*x)*
b*c**2*d*f**2 - atanh(c*x)*b*c**2*d*f*g*x**2 - atanh(c*x)*b*c**2*e*f**2 -
atanh(c*x)*b*c**2*e*f*g*x**2 + atanh(c*x)*b*d*f*g + atanh(c*x)*b*e*f*g + 2
*int(atanh(c*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 - c**2*f**2*x**3 - 2*c**2*
f*g*x**5 - c**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*c**4*e*f**4*x**2 -
4*int(atanh(c*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 - c**2*f**2*x**3 - 2*c**2
*f*g*x**5 - c**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*c**2*e*f**3*g*x**2
 + 2*int(atanh(c*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 - c**2*f**2*x**3 - 2*c
**2*f*g*x**5 - c**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*e*f**2*g**2*x**
2 - 2*int(atanh(c*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 - c**2*f**2*x - 2*c**
2*f*g*x**3 - c**2*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**6*e*f**4*x**2 + 4
*int(atanh(c*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 - c**2*f**2*x - 2*c**2*f*g
*x**3 - c**2*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**4*e*f**3*g*x**2 - 2*in
t(atanh(c*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 - c**2*f**2*x - 2*c**2*f*g*x*
*3 - c**2*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**2*e*f**2*g**2*x**2 - int(
log(f + g*x**2)/(c**4*f**2*x**4 + c**4*f*g*x**6 - c**2*f**2*x**2 - 2*c**2*
f*g*x**4 - c**2*g**2*x**6 + f*g*x**2 + g**2*x**4),x)*b*c**5*e*f**4*x**2 +
int(log(f + g*x**2)/(c**4*f**2*x**4 + c**4*f*g*x**6 - c**2*f**2*x**2 - ...
```

3.537 $\int \frac{\text{arctanh}(cx)(a+b\text{arctanh}(cx))}{(1+cx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{\text{arctanh}(cx)(a+b\text{arctanh}(cx))}{(1+cx)^2}\,dx = -\frac{a+b}{2c(1+cx)}+\frac{(a+b)\text{arctanh}(cx)}{2c} - \frac{(a+b)\text{arctanh}(cx)}{c(1+cx)}-\frac{b(1-cx)\text{arctanh}(cx)^2}{2c(1+cx)}$$

output
```
-1/2*(a+b)/c/(c*x+1)+1/2*(a+b)*arctanh(c*x)/c-(a+b)*arctanh(c*x)/c/(c*x+1)
-1/2*b*(-c*x+1)*arctanh(c*x)^2/c/(c*x+1)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\text{arctanh}(cx)(a+b\text{arctanh}(cx))}{(1+cx)^2}\,dx = -\frac{4(a+b)\text{arctanh}(cx)-2b(-1+cx)\text{arctanh}(cx)^2+(a+b)(2+(1+cx)\log(1-cx)-(1+cx)\log(1+}{4c(1+cx)}$$

input
```
Integrate[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

output
```
-1/4*(4*(a + b)*ArcTanh[c*x] - 2*b*(-1 + c*x)*ArcTanh[c*x]^2 + (a + b)*(2
+ (1 + c*x)*Log[1 - c*x] - (1 + c*x)*Log[1 + c*x]))/(c*(1 + c*x))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(cx)(a+b\text{arctanh}(cx))}{(cx+1)^2} \, dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \frac{\text{arctanh}(cx)(a+b\text{arctanh}(cx))}{(cx+1)^2}d(cx)}{c}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(\frac{b\text{arctanh}(cx)^2}{(cx+1)^2}+\frac{a\text{arctanh}(cx)}{(cx+1)^2}\right)d(cx)}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}a\text{arctanh}(cx)-\frac{a\text{arctanh}(cx)}{cx+1}-\frac{a}{2(cx+1)}+\frac{1}{2}b\text{arctanh}(cx)^2-\frac{b\text{arctanh}(cx)^2}{cx+1}+\frac{1}{2}b\text{arctanh}(cx)-\frac{b\text{arctanh}(cx)}{cx+1}-\frac{}{2(}}{c}$$

input
```
Int[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

output
```
(-1/2*a/(1 + c*x) - b/(2*(1 + c*x)) + (a*ArcTanh[c*x])/2 + (b*ArcTanh[c*x]
)/2 - (a*ArcTanh[c*x])/(1 + c*x) - (b*ArcTanh[c*x])/(1 + c*x) + (b*ArcTanh
[c*x]^2)/2 - (b*ArcTanh[c*x]^2)/(1 + c*x))/c
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
parallelrisch	$-\frac{-b \operatorname{arctanh}(cx)^2 xc - x \operatorname{arctanh}(cx) ac - \operatorname{arctanh}(cx) bcx - acx - bcx + b \operatorname{arctanh}(cx)^2 + a \operatorname{arctanh}(cx) + b \operatorname{arctanh}(cx)}{2(cx+1)c}$
derivativedivides	$a\left(-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{\ln(cx-1)}{4}-\frac{1}{2(cx+1)}+\frac{\ln(cx+1)}{4}\right)+b\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)}{cx+1}+\frac{\operatorname{arctanh}(cx)\ln(c}{2}\right.$
default	$a\left(-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{\ln(cx-1)}{4}-\frac{1}{2(cx+1)}+\frac{\ln(cx+1)}{4}\right)+b\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)}{cx+1}+\frac{\operatorname{arctanh}(cx)\ln(c}{2}\right.$
parts	$\frac{a\left(-\frac{\operatorname{arctanh}(cx)}{cx+1}-\frac{\ln(cx-1)}{4}-\frac{1}{2(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{c}+\frac{b\left(-\frac{\operatorname{arctanh}(cx)^2}{cx+1}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)}{cx+1}+\frac{\operatorname{arctanh}(cx)}{2}\right.}{}$
risch	$\frac{b(cx-1)\ln(cx+1)^2}{8(cx+1)c}-\frac{(bcx\ln(-cx+1)-b\ln(-cx+1)+2a+2b)\ln(cx+1)}{4(cx+1)c}-\frac{-bcx\ln(-cx+1)^2+2\ln(cx-1)acx+2\ln(}{}$
orering	$-\frac{(4x^3c^3-3c^2x^2-4cx+3)\operatorname{arctanh}(cx)(a+b\operatorname{arctanh}(cx))}{2c(cx+1)^2}-\frac{(cx+1)^2(cx-1)(7cx-5)\left(\frac{c(a+b\operatorname{arctanh}(cx))}{(-c^2x^2+1)(cx+1)^2}+\frac{\operatorname{arcta}}{(-c^2x^2}\right.}{4c^2}$

input
```
int(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*arctanh(c*x)^2*x*c-x*arctanh(c*x)*a*c-arctanh(c*x)*b*c*x-a*c*x-b*
c*x+b*arctanh(c*x)^2+a*arctanh(c*x)+b*arctanh(c*x))/(c*x+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(cx)(a + b\operatorname{arctanh}(cx))}{(1+cx)^2}\,dx = \frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)^2 + 2\left((a+b)cx-a-b\right)\log\left(-\frac{cx+1}{cx-1}\right) - 4\,a - 4\,b}{8\,(c^2x+c)}$$

input
```
integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="fricas")
```

output
```
1/8*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1))^2 + 2*((a + b)*c*x - a - b)*log
(-(c*x + 1)/(c*x - 1)) - 4*a - 4*b)/(c^2*x + c)
```

Sympy [F]

$$\int \frac{\operatorname{arctanh}(cx)(a + b\operatorname{arctanh}(cx))}{(1+cx)^2}\,dx = \int \frac{(a + b\operatorname{atanh}(cx))\operatorname{atanh}(cx)}{(cx+1)^2}\,dx$$

input
```
integrate(atanh(c*x)*(a+b*atanh(c*x))/(c*x+1)**2,x)
```

output
```
Integral((a + b*atanh(c*x))*atanh(c*x)/(c*x + 1)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.90

$$\int \frac{\operatorname{arctanh}(cx)(a + b\operatorname{arctanh}(cx))}{(1+cx)^2}\,dx = -\frac{1}{8}\left(bc\left(\frac{2}{c^4x+c^3} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) + 2\,a\left(\frac{2}{c^3x+c^2} - \frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2}\right) + -\right.$$
$$-\frac{1}{4}\left(\left(c\left(\frac{2}{c^3x+c^2} - \frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2}\right) + \frac{4\operatorname{artanh}(cx)}{c^2x+c}\right)b + \frac{4\,a}{c^2x+c}\right)\operatorname{artanh}(cx)$$

input
```
integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="maxima")
```

output
```
-1/8*(b*c*(2/(c^4*x + c^3) - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) + 2*a*(2
/(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + (-2*I*pi*b + (I*pi
*b + (I*pi*b*c - b*c)*x + b)*log(c*x + 1) + (-I*pi*b + (-I*pi*b*c + b*c)*x
 - b)*log(c*x - 1) + 2*b)/(c^3*x + c^2))*c - 1/4*((c*(2/(c^3*x + c^2) - lo
g(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*b + 4*a/(
c^2*x + c))*arctanh(c*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{arctanh}(cx)(a+b\operatorname{arctanh}(cx))}{(1+cx)^2}\,dx = \frac{1}{8}c\left(\frac{(cx-1)b\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx+1)c^2}+\frac{2\,(cx-1)(a+b)\log\left(-\frac{cx+1}{cx-1}\right)}{(cx+1)c^2}+\frac{2\,(cx-1)(a+b)}{(cx+1)c^2}\right)$$

input
```
integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="giac")
```

output
```
1/8*c*((c*x - 1)*b*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)*c^2) + 2*(c*x -
1)*(a + b)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)*c^2) + 2*(c*x - 1)*(a + b)
/((c*x + 1)*c^2))
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(cx)(a+b\operatorname{arctanh}(cx))}{(1+cx)^2}\,dx = \frac{a\,\mathrm{atanh}(c\,x)+b\,\mathrm{atanh}(c\,x)+b\,{\mathrm{atanh}(c\,x)}^2}{2\,c} - \frac{a+b+2\,a\,\mathrm{atanh}(c\,x)+2\,b\,\mathrm{atanh}(c\,x)+2\,b\,{\mathrm{atanh}(c\,x)}^2}{2\,x\,c^2+2\,c}$$

input
```
int((atanh(c*x)*(a + b*atanh(c*x)))/(c*x + 1)^2,x)
```

output
```
(a*atanh(c*x) + b*atanh(c*x) + b*atanh(c*x)^2)/(2*c) - (a + b + 2*a*atanh(
c*x) + 2*b*atanh(c*x) + 2*b*atanh(c*x)^2)/(2*c + 2*c^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \frac{\text{arctanh}(cx)(a+b\text{arctanh}(cx))}{(1+cx)^2}\,dx$$
$$= \frac{2atanh(cx)^2\,bcx - 2atanh(cx)^2\,b + 4atanh(cx)\,acx + 4atanh(cx)\,bcx + \log(cx-1)\,acx + \log(cx-1)\,a +$$

input
```
int(atanh(c*x)*(a+b*atanh(c*x))/(c*x+1)^2,x)
```

output
```
(2*atanh(c*x)**2*b*c*x - 2*atanh(c*x)**2*b + 4*atanh(c*x)*a*c*x + 4*atanh(
c*x)*b*c*x + log(c*x - 1)*a*c*x + log(c*x - 1)*a + log(c*x - 1)*b*c*x + lo
g(c*x - 1)*b - log(c*x + 1)*a*c*x - log(c*x + 1)*a - log(c*x + 1)*b*c*x -
log(c*x + 1)*b + 2*a*c*x + 2*b*c*x)/(4*c*(c*x + 1))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file