Computer Algebra
Independent Integration Tests

Summer 2024

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/341-7.4

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May 18, 2024 Compiled on May 18, 2024 at 9:35am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**179**]. This is test number [341].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (179)	0.00 (0)
Mathematica	97.77 (175)	2.23 (4)
Fricas	88.83 (159)	11.17 (20)
Maple	87.71 (157)	12.29 (22)
Maxima	83.24 (149)	16.76 (30)
Mupad	54.75 (98)	45.25 (81)
Giac	50.28 (90)	49.72 (89)
Sympy	33.52 (60)	66.48 (119)
Reduce	30.17 (54)	69.83 (125)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

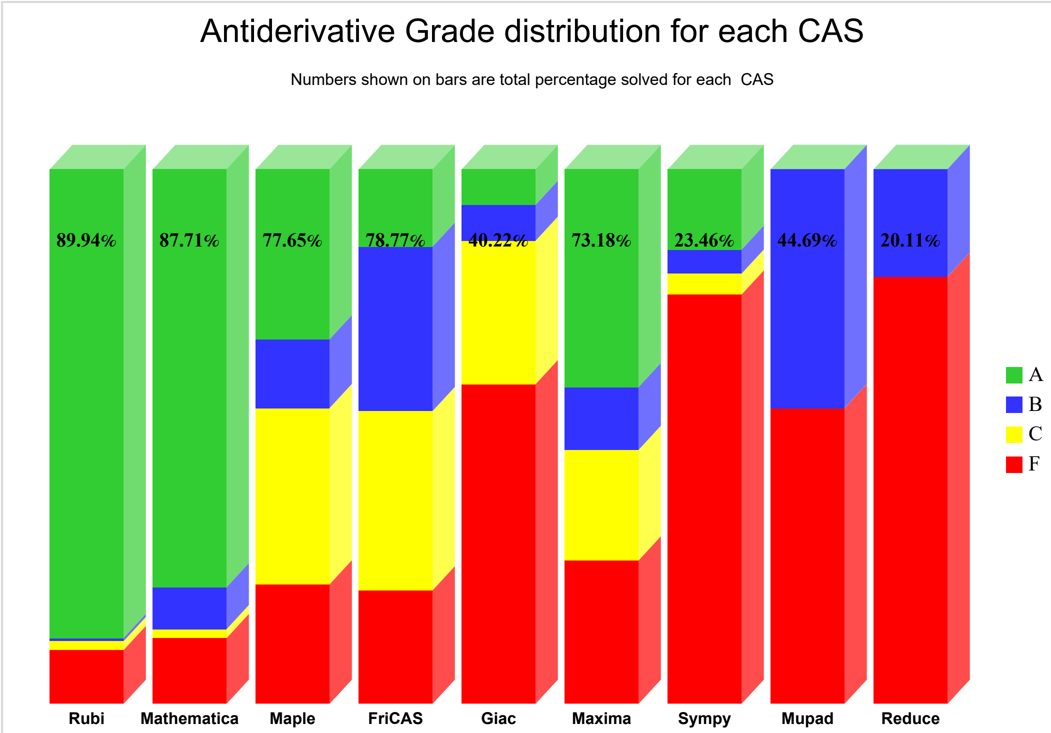
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

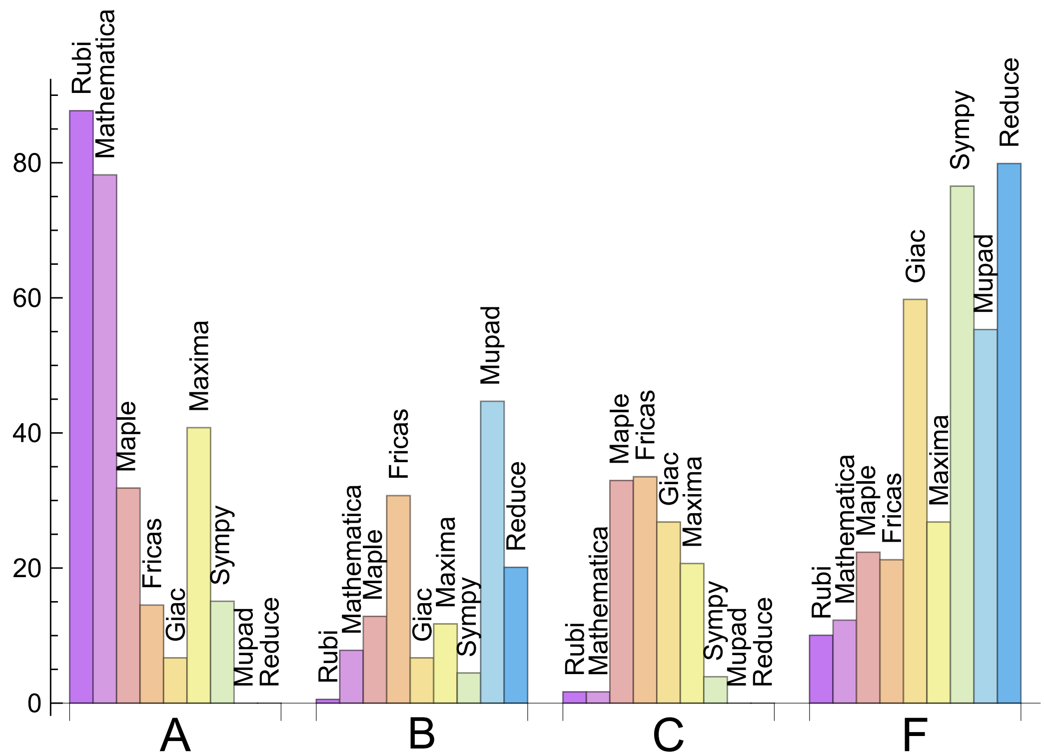
System	% A grade	% B grade	% C grade	% F grade
Rubi	87.709	0.559	1.676	10.056
Mathematica	78.212	7.821	1.676	12.291
Maxima	40.782	11.732	20.670	26.816
Maple	31.844	12.849	32.961	22.346
Sympy	15.084	4.469	3.911	76.536
Fricas	14.525	30.726	33.520	21.229
Giac	6.704	6.704	26.816	59.777
Mupad	0.000	44.693	0.000	55.307
Reduce	0.000	20.112	0.000	79.888

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	20	100.00	0.00	0.00
Maple	22	63.64	36.36	0.00
Maxima	30	100.00	0.00	0.00
Mupad	81	0.00	100.00	0.00
Giac	89	96.63	0.00	3.37
Sympy	119	94.12	5.04	0.84
Reduce	125	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Reduce	0.17
Giac	0.19
Mathematica	0.35
Maxima	0.39
Rubi	0.54
Maple	2.25
Mupad	3.54
Sympy	4.74

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	61.33	1.32	24.50	1.07
Sympy	61.95	1.40	36.50	1.20
Giac	75.07	1.53	55.50	1.19
Maxima	117.65	1.53	78.00	1.13
Rubi	134.06	1.12	84.00	1.01
Mathematica	143.86	1.25	70.00	0.96
Mupad	180.18	2.45	53.00	1.18
Fricas	267.37	2.18	119.00	1.83
Maple	3258.32	35.08	271.00	2.16

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

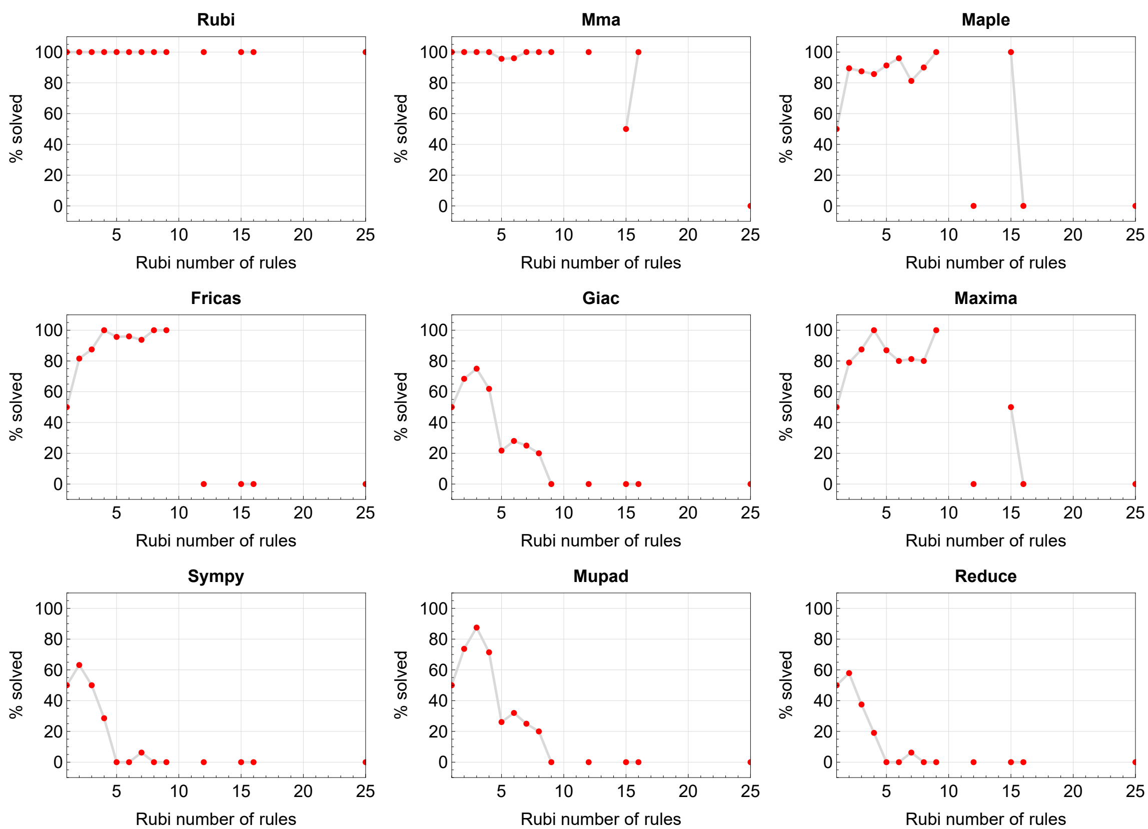


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

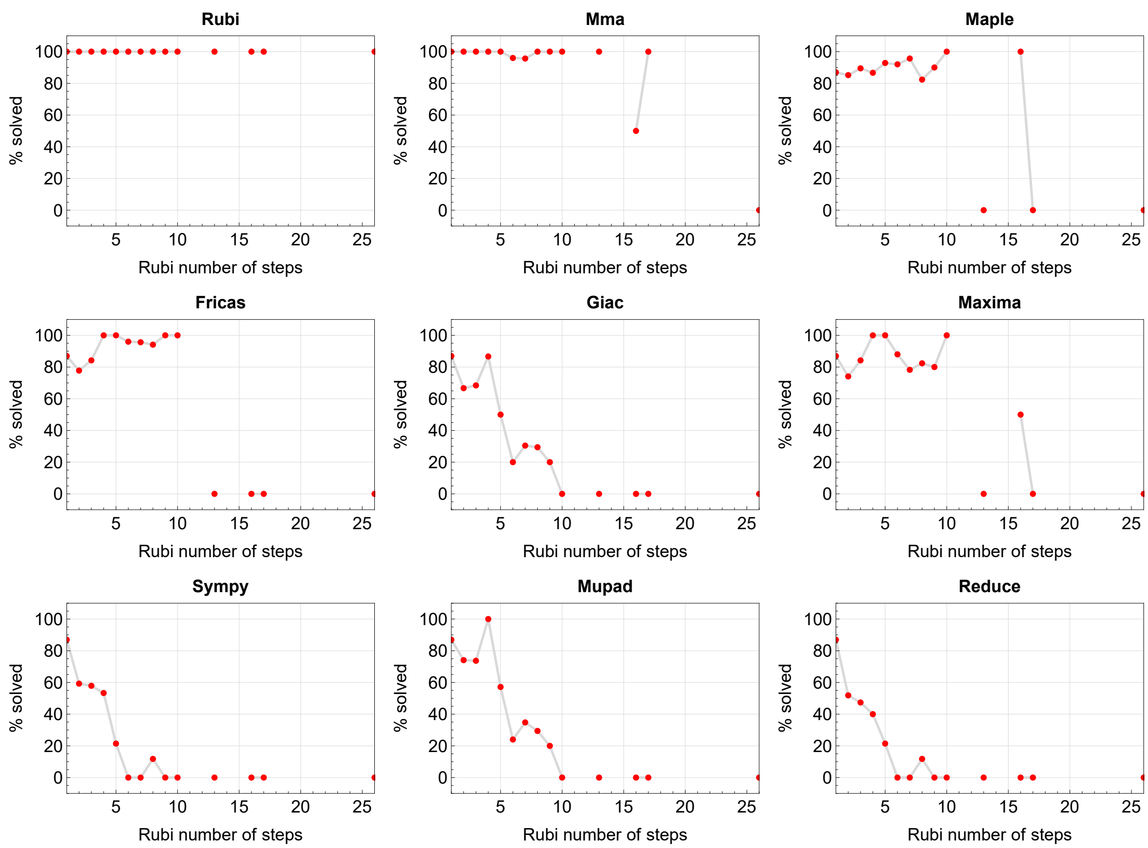


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

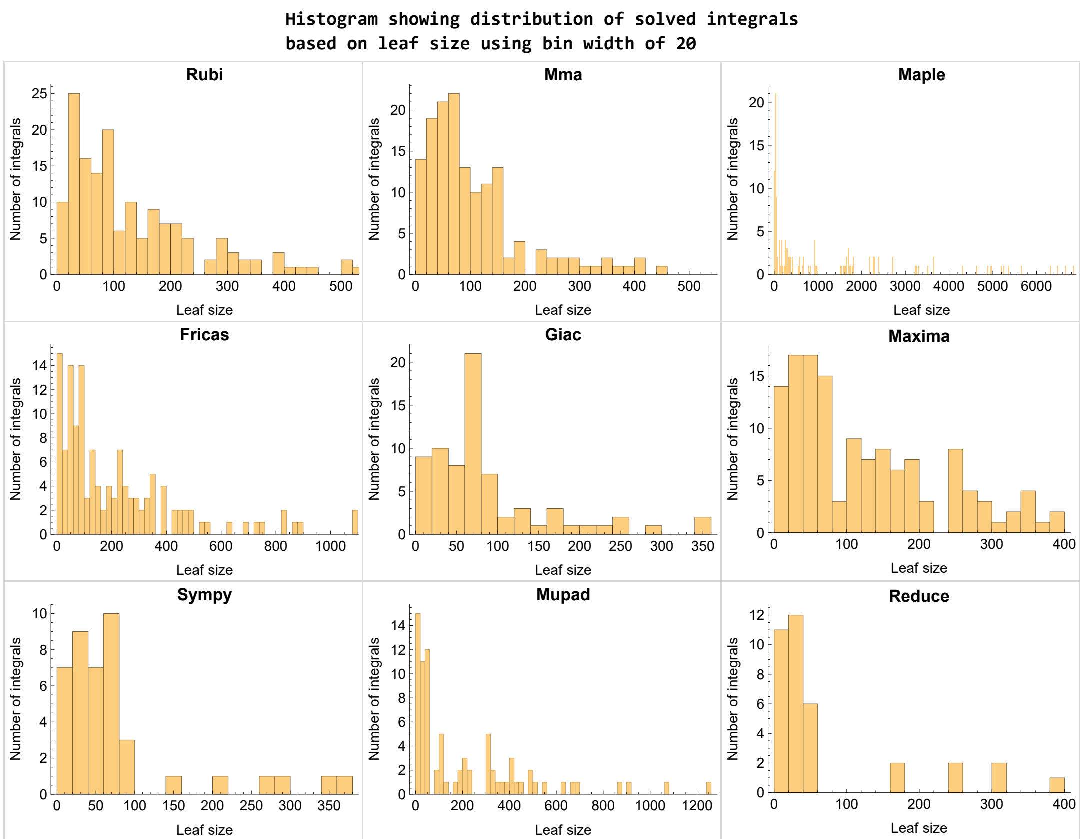


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

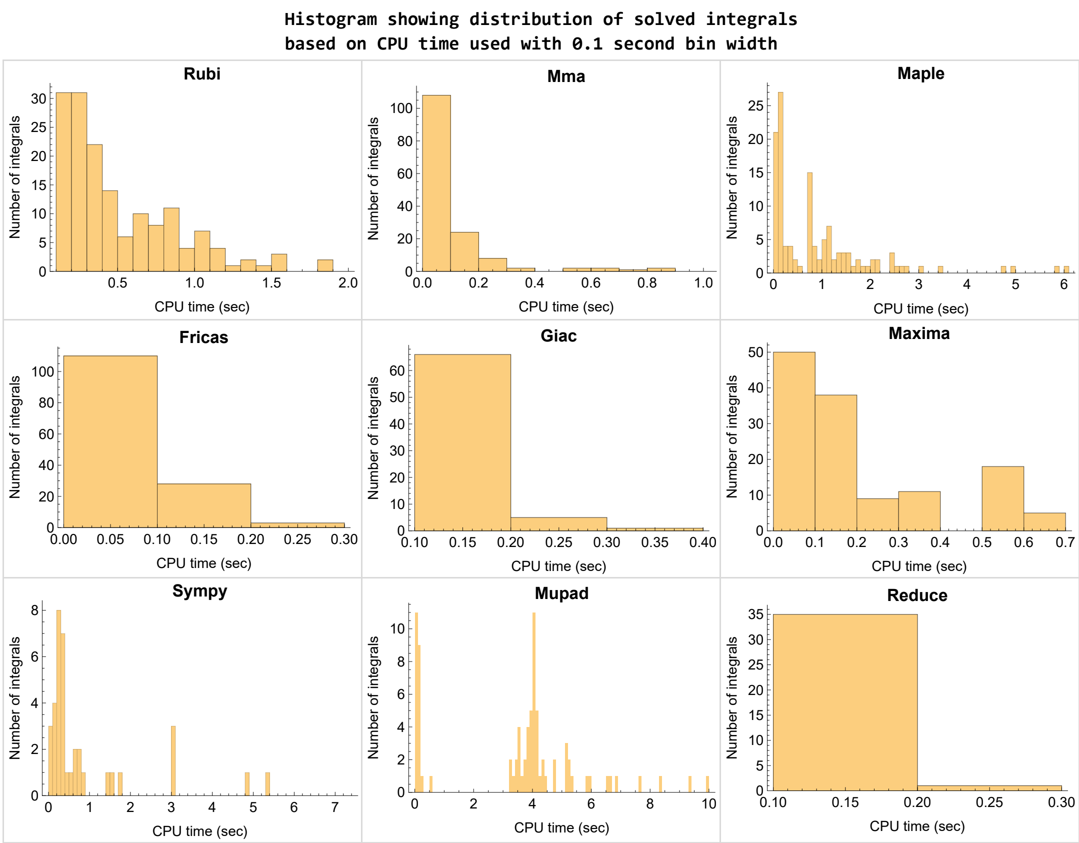


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

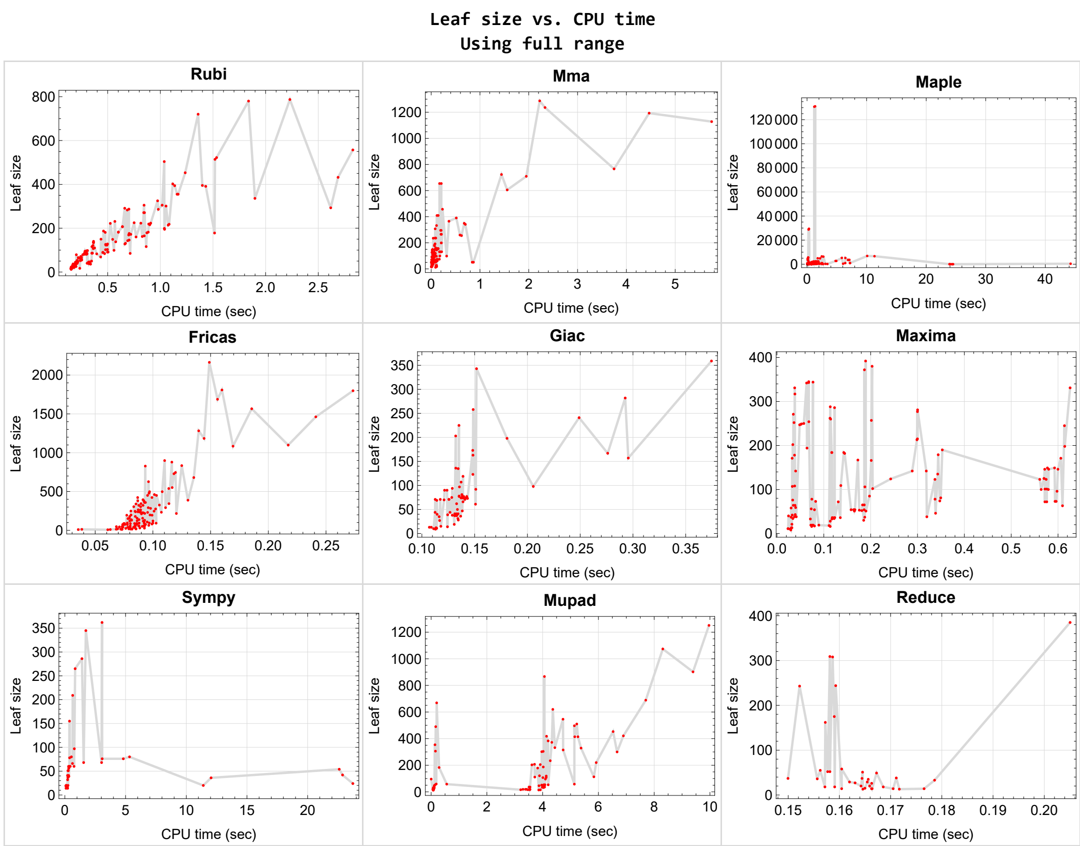


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{1, 5, 6, 85, 90, 95, 99, 104, 109, 114, 118, 122, 126, 131, 135, 139, 143, 159\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{154, 155, 156, 172, 173, 174, 179\}$

Mathematica $\{117, 121, 125, 134, 138, 142, 157, 158, 160, 161\}$

Maple {20, 21, 31, 32, 33, 38, 39, 40, 42, 46, 47, 48, 52, 55, 56, 60, 80, 81, 82, 83, 86, 87, 88, 91, 92, 93, 96, 97, 100, 101, 102, 105, 106, 107, 110, 111, 112, 115, 116, 119, 120, 123, 124, 127, 128, 129, 132, 133, 136, 137, 140, 141, 144, 148, 158, 174, 175, 178, 179}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See **https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/** for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at **https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/**

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

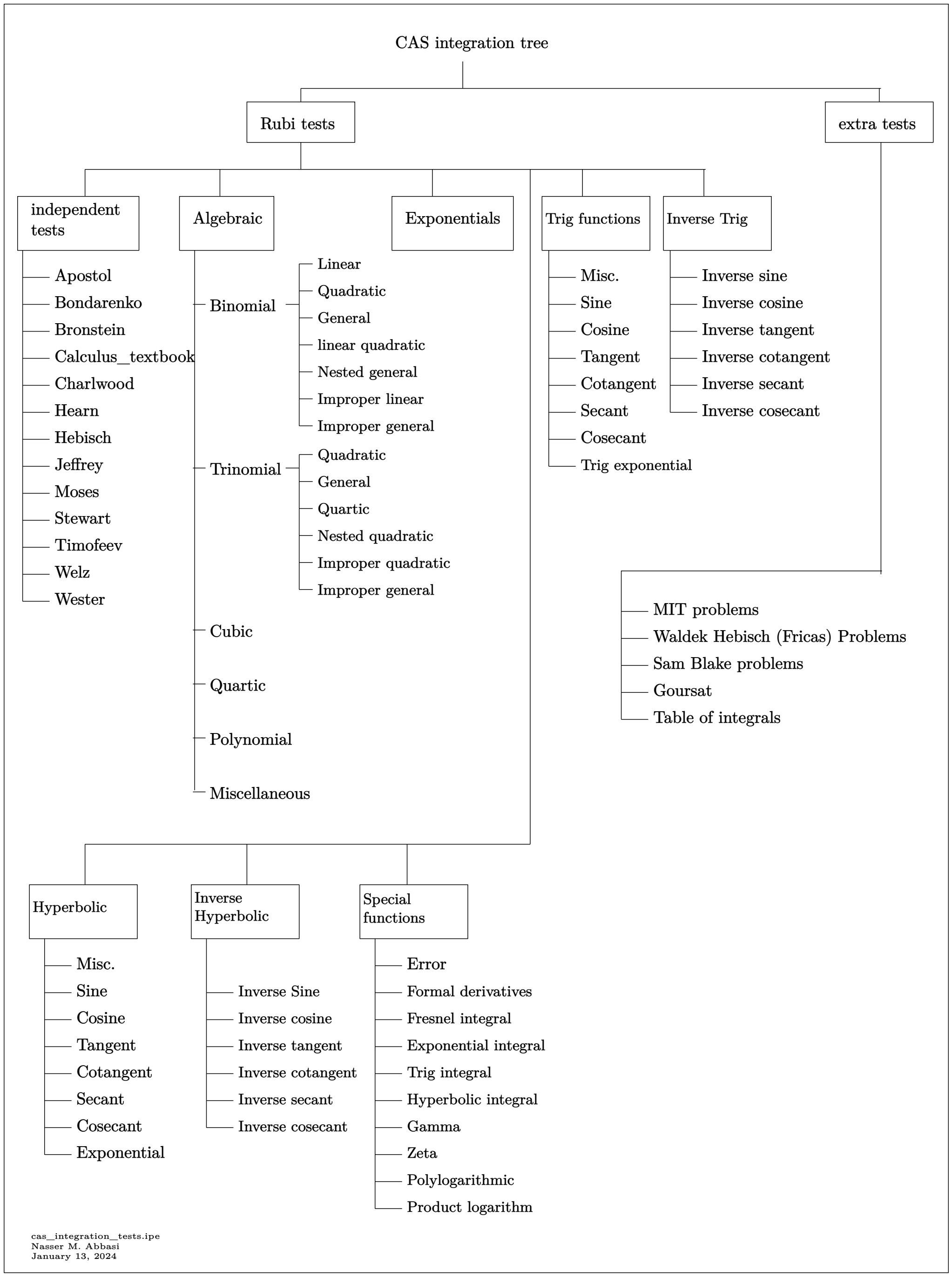
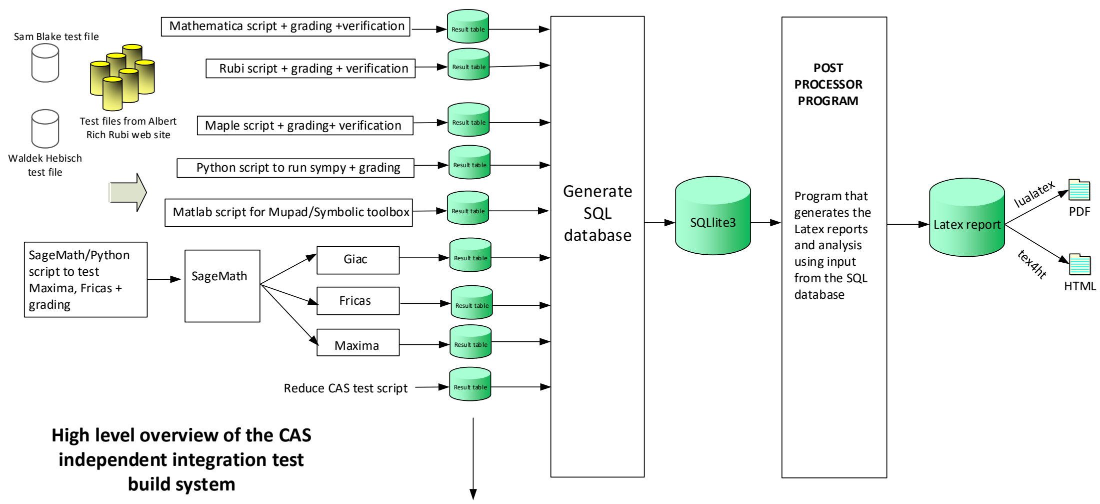


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 136, 137, 138, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179 }

B grade { 170 }

C grade { 79, 80, 81 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 111, 112, 113, 115, 116, 117, 119, 120, 123, 124, 128, 129, 130, 132, 133, 136, 137, 140, 141, 145, 146, 147, 149, 150, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179 }

B grade { 18, 29, 110, 121, 125, 127, 134, 138, 142, 154, 155, 158, 160, 162 }

C grade { 144, 157, 161 }

F normal fail { 2, 3, 148, 156 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 29, 30, 34, 35, 36, 41, 49, 50, 57, 58, 59, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 113, 130, 145, 146, 147, 151, 152, 153, 161, 162, 163, 164, 165, 168, 171, 172, 176, 177 }

B grade { 2, 3, 4, 64, 65, 84, 89, 94, 98, 103, 108, 117, 121, 125, 134, 138, 142, 157, 166, 167, 169, 170, 173 }

C grade { 20, 21, 31, 32, 33, 38, 39, 40, 42, 46, 47, 48, 52, 55, 56, 60, 80, 81, 82, 83, 86, 87, 88, 91, 92, 93, 96, 97, 100, 101, 102, 105, 106, 107, 110, 111, 112, 115, 116, 119, 120, 123, 124, 127, 128, 129, 132, 133, 136, 137, 140, 141, 144, 148, 158, 174, 175, 178, 179 }

F normal fail { 37, 44, 45, 54, 63, 69, 70, 71, 149, 150, 154, 155, 156, 160 }

F(-1) timedout fail { 26, 27, 28, 43, 51, 53, 61, 62 }

F(-2) exception fail { }

Fricas

A grade { 73, 74, 75, 76, 77, 78, 81, 136, 137, 138, 140, 141, 142, 145, 146, 147, 151, 152, 153, 168, 170, 171, 172, 175, 177, 178 }

B grade { 79, 80, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 144, 162, 163, 164, 165, 166, 167, 169, 173, 174, 179 }

C grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 72, 176 }

F normal fail { 2, 3, 4, 37, 45, 54, 63, 69, 70, 71, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 33, 34, 35, 36, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 147, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade { 19, 30, 113, 117, 119, 120, 121, 123, 124, 125, 130, 134, 136, 137, 138, 140, 141, 142, 162, 165, 179 }

C grade { 20, 31, 32, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 145, 146, 148, 151, 152, 153 }

F normal fail { 2, 3, 4, 37, 45, 54, 63, 69, 70, 71, 110, 111, 112, 115, 116, 127, 128, 129, 132, 133, 144, 149, 150, 154, 155, 156, 157, 158, 160, 161 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 68, 73, 74, 75, 76, 77, 78, 174, 176, 177, 178, 179 }

B grade { 7, 8, 9, 10, 12, 13, 14, 72, 171, 172, 173, 175 }

C grade { 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 147, 151, 152, 153 }

F normal fail { 2, 3, 4, 15, 25, 37, 45, 54, 63, 64, 65, 66, 67, 69, 70, 71, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 136, 137, 138, 140, 141, 142, 144, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170 }

F(-1) timedout fail { }

F(-2) exception fail { 145, 146, 168 }

Mupad

A grade { }

B grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 145, 146, 147, 151, 152, 153, 171, 172, 173, 174, 175, 176, 177, 178, 179 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 3, 4, 37, 45, 54, 63, 69, 70, 71, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 136, 137, 138, 140, 141, 142, 144, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170 }

F(-2) exception fail { }

Sympy

A grade { 8, 9, 10, 12, 13, 14, 17, 18, 19, 22, 23, 24, 26, 27, 28, 29, 30, 34, 36, 41, 49, 50, 57, 58, 59, 171, 172 }

B grade { 16, 35, 68, 73, 74, 75, 77, 78 }

C grade { 145, 146, 147, 151, 152, 153, 176 }

F normal fail { 2, 3, 4, 7, 11, 15, 20, 21, 25, 31, 32, 33, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 76, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 136, 137, 138, 140, 141, 142, 144, 148, 149, 150, 154, 155, 156, 162, 163, 164, 165, 166, 167, 168, 169, 170, 175, 177, 179 }

F(-1) timedout fail { 157, 158, 160, 161, 174, 178 }

F(-2) exception fail { 173 }

Reduce

A grade { }

B grade { 7, 10, 12, 13, 14, 19, 22, 23, 24, 30, 34, 35, 36, 41, 49, 50, 57, 58, 59, 67, 68, 72, 75, 77, 78, 145, 146, 147, 151, 152, 153, 171, 172, 173, 176, 177 }

C grade { }

F normal fail { 2, 3, 4, 8, 9, 11, 15, 16, 17, 18, 20, 21, 25, 26, 27, 28, 29, 31, 32, 33, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 73, 74, 76, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 136, 137, 138, 140, 141, 142, 144, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 178, 179 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98	0.98
time (sec)	N/A	0.238	0.131	0.154	0.570	0.105	9.943	0.549	0.225	3.791

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	453	0	1681	0	0	0	0	150	0
N.S.	1	0.98	0.00	3.65	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.236	0.000	0.319	0.000	0.000	0.000	0.000	0.202	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	305	0	926	0	0	0	0	108	0
N.S.	1	1.01	0.00	3.07	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.845	0.000	0.088	0.000	0.000	0.000	0.000	0.180	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	84	98	372	0	0	0	0	64	0
N.S.	1	0.94	1.10	4.18	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.303	0.317	0.068	0.000	0.000	0.000	0.000	0.179	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	63	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	1.58	0.98
time (sec)	N/A	0.239	0.203	0.085	0.188	0.073	4.035	0.252	0.208	3.512

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	246	91	126	38	123	39
N.S.	1	1.00	1.05	0.90	6.15	2.28	3.15	0.95	3.08	0.98
time (sec)	N/A	0.241	0.803	0.092	0.256	0.095	11.897	0.609	0.236	4.283

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	49	38	81	0	90	37	96
N.S.	1	1.00	0.92	1.32	1.03	2.19	0.00	2.43	1.00	2.59
time (sec)	N/A	0.188	0.038	0.177	0.037	0.099	0.000	0.124	0.150	4.100

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	19	19	71	13	19
N.S.	1	1.00	0.87	0.87	0.83	0.83	0.83	3.09	0.57	0.83
time (sec)	N/A	0.161	0.014	0.100	0.077	0.079	0.131	0.117	0.158	3.421

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	19	19	71	11	19
N.S.	1	1.00	0.87	0.87	0.83	0.83	0.83	3.09	0.48	0.83
time (sec)	N/A	0.174	0.012	0.075	0.072	0.069	0.113	0.113	0.172	3.386

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	17	16	14	19	69	14	16
N.S.	1	1.00	1.12	1.06	1.00	0.88	1.19	4.31	0.88	1.00
time (sec)	N/A	0.150	0.006	0.095	0.074	0.068	0.081	0.114	0.166	0.073

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	34	15	0	15	13	59
N.S.	1	1.00	0.90	1.00	1.62	0.71	0.00	0.71	0.62	2.81
time (sec)	N/A	0.181	0.011	0.072	0.034	0.072	0.000	0.118	0.161	0.185

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	17	18	14	70	20	17
N.S.	1	1.00	1.06	1.18	1.00	1.06	0.82	4.12	1.18	1.00
time (sec)	N/A	0.152	0.013	0.095	0.080	0.085	0.096	0.121	0.166	0.093

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	17	19	16	19	71	18	16
N.S.	1	1.00	0.78	0.74	0.83	0.70	0.83	3.09	0.78	0.70
time (sec)	N/A	0.153	0.011	0.092	0.075	0.082	0.199	0.126	0.157	3.213

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	19	19	16	20	71	18	19
N.S.	1	1.00	0.87	0.83	0.83	0.70	0.87	3.09	0.78	0.83
time (sec)	N/A	0.161	0.012	0.092	0.089	0.082	0.258	0.126	0.169	3.295

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	66	62	112	73	264	0	0	15	203
N.S.	1	0.93	0.87	1.58	1.03	3.72	0.00	0.00	0.21	2.86
time (sec)	N/A	0.221	0.126	0.402	0.083	0.089	0.000	0.000	0.158	3.614

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	45	37	37	36	48	78	41	15	36
N.S.	1	1.07	0.88	0.88	0.86	1.14	1.86	0.98	0.36	0.86
time (sec)	N/A	0.193	0.029	24.519	0.122	0.073	0.388	0.134	0.161	3.512

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	45	37	37	36	48	60	41	15	36
N.S.	1	1.07	0.88	0.88	0.86	1.14	1.43	0.98	0.36	0.86
time (sec)	N/A	0.200	0.034	23.956	0.118	0.074	0.303	0.131	0.162	0.115

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	C	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	74	37	36	48	41	41	13	36
N.S.	1	1.00	2.18	1.09	1.06	1.41	1.21	1.21	0.38	1.06
time (sec)	N/A	0.196	0.168	24.158	0.118	0.068	0.218	0.133	0.164	3.548

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	39	20	39	14	33
N.S.	1	1.00	1.00	0.94	2.06	2.44	1.25	2.44	0.88	2.06
time (sec)	N/A	0.155	0.006	0.135	0.117	0.083	0.100	0.124	0.177	0.086

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	53	672	38	36	0	37	15	183
N.S.	1	1.02	1.08	13.71	0.78	0.73	0.00	0.76	0.31	3.73
time (sec)	N/A	0.214	0.077	0.053	0.320	0.084	0.000	0.131	0.164	0.287

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	655	54	41	0	36	15	207
N.S.	1	1.00	0.95	16.79	1.38	1.05	0.00	0.92	0.38	5.31
time (sec)	N/A	0.203	0.034	0.066	0.080	0.080	0.000	0.137	0.152	3.708

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	39	34	38	32	37	38	34
N.S.	1	1.00	1.17	1.08	0.94	1.06	0.89	1.03	1.06	0.94
time (sec)	N/A	0.197	0.026	0.153	0.122	0.079	0.225	0.133	0.171	3.988

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	33	36	40	37	38	35	32
N.S.	1	1.00	1.10	1.06	1.16	1.29	1.19	1.23	1.13	1.03
time (sec)	N/A	0.169	0.030	0.155	0.123	0.078	0.253	0.133	0.166	4.035

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	37	36	36	40	39	38	36	36
N.S.	1	1.00	0.58	0.56	0.56	0.62	0.61	0.59	0.56	0.56
time (sec)	N/A	0.227	0.023	0.177	0.135	0.074	0.330	0.129	0.156	4.070

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	95	97	197	109	627	0	0	15	332
N.S.	1	0.86	0.88	1.79	0.99	5.70	0.00	0.00	0.14	3.02
time (sec)	N/A	0.284	0.108	3.417	0.136	0.096	0.000	0.000	0.158	4.438

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	C	A	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	67	54	0	54	88	97	77	15	53
N.S.	1	1.10	0.89	0.00	0.89	1.44	1.59	1.26	0.25	0.87
time (sec)	N/A	0.248	0.023	0.000	0.165	0.075	0.779	0.140	0.167	4.199

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	C	A	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	67	54	0	54	88	80	77	15	53
N.S.	1	1.10	0.89	0.00	0.89	1.44	1.31	1.26	0.25	0.87
time (sec)	N/A	0.251	0.020	0.000	0.166	0.075	0.538	0.135	0.163	4.082

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	C	A	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	54	0	54	88	60	77	15	53
N.S.	1	1.15	1.02	0.00	1.02	1.66	1.13	1.45	0.28	1.00
time (sec)	N/A	0.247	0.017	0.000	0.158	0.084	0.374	0.138	0.161	0.145

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	C	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	99	54	54	87	41	77	13	53
N.S.	1	1.00	2.91	1.59	1.59	2.56	1.21	2.26	0.38	1.56
time (sec)	N/A	0.193	0.178	23.911	0.164	0.075	0.309	0.143	0.167	4.069

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	76	20	75	14	47
N.S.	1	1.00	1.00	0.94	3.19	4.75	1.25	4.69	0.88	2.94
time (sec)	N/A	0.155	0.007	24.391	0.165	0.076	0.132	0.128	0.170	0.112

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	104	3302	74	75	0	74	15	306
N.S.	1	1.03	1.35	42.88	0.96	0.97	0.00	0.96	0.19	3.97
time (sec)	N/A	0.263	0.083	0.181	0.348	0.074	0.000	0.139	0.166	0.156

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	3256	124	79	0	74	15	372
N.S.	1	1.00	0.91	47.88	1.82	1.16	0.00	1.09	0.22	5.47
time (sec)	N/A	0.248	0.028	0.138	0.243	0.086	0.000	0.139	0.164	4.329

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	66	3235	72	77	0	71	15	383
N.S.	1	1.02	1.10	53.92	1.20	1.28	0.00	1.18	0.25	6.38
time (sec)	N/A	0.250	0.033	0.174	0.131	0.075	0.000	0.141	0.155	4.197

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	56	52	75	51	73	55	51
N.S.	1	1.00	1.09	1.02	0.95	1.36	0.93	1.33	1.00	0.93
time (sec)	N/A	0.254	0.017	0.701	0.181	0.078	0.254	0.143	0.156	3.964

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	49	53	75	56	74	51	48
N.S.	1	1.00	1.61	1.58	1.71	2.42	1.81	2.39	1.65	1.55
time (sec)	N/A	0.166	0.024	0.758	0.176	0.074	0.320	0.137	0.165	4.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	53	54	75	60	74	52	53
N.S.	1	1.00	0.84	0.83	0.84	1.17	0.94	1.16	0.81	0.83
time (sec)	N/A	0.230	0.033	0.730	0.186	0.094	0.442	0.142	0.158	4.040

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	15	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.209	0.087	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	81	87	79	130774	85	90	0	81	15	354
N.S.	1	1.07	0.98	1614.49	1.05	1.11	0.00	1.00	0.19	4.37
time (sec)	N/A	0.300	0.036	1.219	0.197	0.086	0.000	0.138	0.156	0.139

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	55	28786	51	51	0	50	15	234
N.S.	1	1.05	0.98	514.04	0.91	0.91	0.00	0.89	0.27	4.18
time (sec)	N/A	0.249	0.031	0.253	0.190	0.091	0.000	0.134	0.162	4.273

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	4303	30	30	0	28	13	108
N.S.	1	1.00	1.00	138.81	0.97	0.97	0.00	0.90	0.42	3.48
time (sec)	N/A	0.193	0.023	0.090	0.185	0.089	0.000	0.134	0.180	4.128

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	16	17	14	13	12
N.S.	1	1.00	1.00	1.08	1.33	1.33	1.42	1.17	1.08	1.00
time (sec)	N/A	0.153	0.090	0.066	0.114	0.081	0.203	0.124	0.165	0.079

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	972	37	28	0	31	15	113
N.S.	1	1.00	0.66	22.09	0.84	0.64	0.00	0.70	0.34	2.57
time (sec)	N/A	0.215	0.021	7.212	0.187	0.093	0.000	0.134	0.162	5.838

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	81	45	0	65	50	0	62	15	220
N.S.	1	1.25	0.69	0.00	1.00	0.77	0.00	0.95	0.23	3.38
time (sec)	N/A	0.281	0.018	0.000	0.186	0.093	0.000	0.133	0.165	5.921

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	C	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	121	66	0	106	95	0	107	15	300
N.S.	1	1.32	0.72	0.00	1.15	1.03	0.00	1.16	0.16	3.26
time (sec)	N/A	0.360	0.021	0.000	0.188	0.097	0.000	0.137	0.174	6.670

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	45	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.241	0.841	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	98	110	106	131085	179	212	0	135	15	669
N.S.	1	1.12	1.08	1337.60	1.83	2.16	0.00	1.38	0.15	6.83
time (sec)	N/A	0.375	0.073	1.358	0.344	0.090	0.000	0.134	0.163	0.199

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	83	29109	123	151	0	97	15	490
N.S.	1	1.09	1.11	388.12	1.64	2.01	0.00	1.29	0.20	6.53
time (sec)	N/A	0.302	0.038	0.290	0.337	0.078	0.000	0.136	0.167	0.165

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	56	4626	81	95	0	66	15	302
N.S.	1	1.08	1.12	92.52	1.62	1.90	0.00	1.32	0.30	6.04
time (sec)	N/A	0.248	0.053	0.120	0.350	0.078	0.000	0.139	0.174	3.967

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	35	46	53	36	47	33	28
N.S.	1	1.00	0.96	1.25	1.64	1.89	1.29	1.68	1.18	1.00
time (sec)	N/A	0.192	0.111	0.085	0.339	0.084	12.044	0.138	0.179	0.098

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	18	19	20	19	13	14
N.S.	1	1.00	1.00	1.07	1.29	1.36	1.43	1.36	0.93	1.00
time (sec)	N/A	0.155	0.006	0.057	0.111	0.081	11.403	0.131	0.172	3.859

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	88	53	0	78	93	0	78	15	421
N.S.	1	1.26	0.76	0.00	1.11	1.33	0.00	1.11	0.21	6.01
time (sec)	N/A	0.281	0.097	0.000	0.337	0.100	0.000	0.135	0.162	6.896

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	102	135	70	5357	135	157	0	136	15	453
N.S.	1	1.32	0.69	52.52	1.32	1.54	0.00	1.33	0.15	4.44
time (sec)	N/A	0.364	0.058	0.200	0.343	0.086	0.000	0.132	0.164	6.527

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	187	92	0	190	245	0	203	15	689
N.S.	1	1.31	0.64	0.00	1.33	1.71	0.00	1.42	0.10	4.82
time (sec)	N/A	0.460	0.032	0.000	0.353	0.089	0.000	0.132	0.157	7.699

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	51	0	0	0	0	0	48	0
N.S.	1	1.03	0.54	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.299	0.866	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	92	107	114	29460	198	265	0	163	15	867
N.S.	1	1.16	1.24	320.22	2.15	2.88	0.00	1.77	0.16	9.42
time (sec)	N/A	0.363	0.044	0.326	0.614	0.087	0.000	0.148	0.161	4.064

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	81	86	4979	146	195	0	123	15	620
N.S.	1	1.14	1.21	70.13	2.06	2.75	0.00	1.73	0.21	8.73
time (sec)	N/A	0.298	0.050	0.143	0.600	0.077	0.000	0.148	0.162	4.365

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	49	54	96	123	54	92	52	46
N.S.	1	1.11	1.04	1.15	2.04	2.62	1.15	1.96	1.11	0.98
time (sec)	N/A	0.239	0.033	0.095	0.598	0.086	22.601	0.151	0.158	3.868

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	26	63	60	42	61	27	25
N.S.	1	1.00	0.79	0.76	1.85	1.76	1.24	1.79	0.79	0.74
time (sec)	N/A	0.194	0.101	0.082	0.610	0.092	22.874	0.151	0.163	0.097

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	28	44	24	44	14	14
N.S.	1	1.00	1.00	0.94	1.75	2.75	1.50	2.75	0.88	0.88
time (sec)	N/A	0.157	0.007	0.066	0.114	0.071	23.723	0.127	0.166	0.072

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	F	C	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	97	134	74	5659	171	231	0	173	15	902
N.S.	1	1.38	0.76	58.34	1.76	2.38	0.00	1.78	0.15	9.30
time (sec)	N/A	0.366	0.124	0.194	0.606	0.095	0.000	0.148	0.163	9.392

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	181	93	0	245	340	0	258	15	1074
N.S.	1	1.38	0.71	0.00	1.87	2.60	0.00	1.97	0.11	8.20
time (sec)	N/A	0.477	0.033	0.000	0.614	0.085	0.000	0.149	0.157	8.310

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	231	107	0	331	461	0	343	15	1251
N.S.	1	1.36	0.63	0.00	1.95	2.71	0.00	2.02	0.09	7.36
time (sec)	N/A	0.568	0.032	0.000	0.626	0.101	0.000	0.152	0.164	9.965

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	15	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.202	0.107	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	150	146	420	380	828	0	0	15	546
N.S.	1	0.91	0.88	2.55	2.30	5.02	0.00	0.00	0.09	3.31
time (sec)	N/A	0.436	0.070	6.054	0.203	0.093	0.000	0.000	0.165	4.730

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	116	106	277	257	498	0	0	15	418
N.S.	1	0.96	0.88	2.29	2.12	4.12	0.00	0.00	0.12	3.45
time (sec)	N/A	0.350	0.054	2.036	0.202	0.097	0.000	0.000	0.171	4.140

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	163	166	276	0	0	15	304
N.S.	1	1.00	0.87	1.99	2.02	3.37	0.00	0.00	0.18	3.71
time (sec)	N/A	0.274	0.049	1.020	0.201	0.102	0.000	0.000	0.163	4.020

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	77	102	130	0	0	49	205
N.S.	1	1.00	0.85	1.60	2.12	2.71	0.00	0.00	1.02	4.27
time (sec)	N/A	0.215	0.031	0.783	0.204	0.093	0.000	0.000	0.167	3.914

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	65	58	51	27	26	121
N.S.	1	1.00	1.00	1.05	3.25	2.90	2.55	1.35	1.30	6.05
time (sec)	N/A	0.168	0.013	0.779	0.183	0.085	0.286	0.117	0.166	3.886

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	15	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.221	0.093	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	45	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.244	0.043	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	98	67	0	0	0	0	0	48	0
N.S.	1	0.97	0.66	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.308	0.047	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	49	38	81	0	90	37	96
N.S.	1	1.00	0.92	1.32	1.03	2.19	0.00	2.43	1.00	2.59
time (sec)	N/A	0.171	0.018	0.060	0.041	0.086	0.000	0.121	0.164	0.003

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	19	13	13	76	13	13	19
N.S.	1	1.00	0.87	0.83	0.57	0.57	3.30	0.57	0.57	0.83
time (sec)	N/A	0.170	0.019	0.197	0.032	0.073	4.815	0.107	0.157	3.547

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	19	13	13	76	13	11	19
N.S.	1	1.00	0.87	0.83	0.57	0.57	3.30	0.57	0.48	0.83
time (sec)	N/A	0.178	0.012	0.116	0.031	0.038	3.073	0.109	0.162	0.062

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	17	10	10	68	10	14	16
N.S.	1	1.00	1.12	1.06	0.62	0.62	4.25	0.62	0.88	1.00
time (sec)	N/A	0.149	0.005	0.122	0.028	0.035	1.545	0.111	0.160	3.472

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	8	8	0	9	13	58
N.S.	1	1.00	0.90	1.00	0.38	0.38	0.00	0.43	0.62	2.76
time (sec)	N/A	0.174	0.013	0.088	0.030	0.061	0.000	0.113	0.161	0.560

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	11	13	68	12	20	17
N.S.	1	1.00	1.06	1.18	0.65	0.76	4.00	0.71	1.18	1.00
time (sec)	N/A	0.157	0.013	0.124	0.024	0.073	3.024	0.113	0.164	0.081

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	11	11	80	11	18	16
N.S.	1	1.00	0.78	0.87	0.48	0.48	3.48	0.48	0.78	0.70
time (sec)	N/A	0.160	0.012	0.123	0.024	0.063	5.328	0.114	0.159	3.540

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	40	39	36	33	57	0	0	5	0
N.S.	1	1.48	1.44	1.33	1.22	2.11	0.00	0.00	0.19	0.00
time (sec)	N/A	0.305	0.010	0.185	0.071	0.080	0.000	0.000	0.158	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	51	69	70	379	56	87	0	0	7	0
N.S.	1	1.35	1.37	7.43	1.10	1.71	0.00	0.00	0.14	0.00
time (sec)	N/A	0.433	0.009	0.130	0.073	0.081	0.000	0.000	0.160	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	99	91	401	78	117	0	0	9	0
N.S.	1	1.29	1.18	5.21	1.01	1.52	0.00	0.00	0.12	0.00
time (sec)	N/A	0.560	0.014	0.141	0.074	0.092	0.000	0.000	0.163	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	307	395	265	5257	281	899	0	0	17	0
N.S.	1	1.29	0.86	17.12	0.92	2.93	0.00	0.00	0.06	0.00
time (sec)	N/A	1.399	0.211	5.836	0.300	0.110	0.000	0.000	0.168	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	231	305	199	4953	215	745	0	0	15	0
N.S.	1	1.32	0.86	21.44	0.93	3.23	0.00	0.00	0.06	0.00
time (sec)	N/A	1.017	0.209	1.799	0.300	0.120	0.000	0.000	0.162	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	207	131	348	142	551	0	0	13	0
N.S.	1	1.38	0.87	2.32	0.95	3.67	0.00	0.00	0.09	0.00
time (sec)	N/A	0.641	0.170	1.382	0.289	0.117	0.000	0.000	0.171	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.301	3.504	0.145	0.795	0.089	0.846	0.499	0.160	4.276

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	155	199	148	1684	149	450	0	0	18	0
N.S.	1	1.28	0.95	10.86	0.96	2.90	0.00	0.00	0.12	0.00
time (sec)	N/A	1.038	0.113	1.145	0.593	0.103	0.000	0.000	0.164	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	128	165	122	1625	125	381	0	0	18	0
N.S.	1	1.29	0.95	12.70	0.98	2.98	0.00	0.00	0.14	0.00
time (sec)	N/A	0.847	0.061	0.861	0.570	0.092	0.000	0.000	0.173	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	101	131	91	1542	101	322	0	0	16	0
N.S.	1	1.30	0.90	15.27	1.00	3.19	0.00	0.00	0.16	0.00
time (sec)	N/A	0.669	0.060	0.710	0.572	0.088	0.000	0.000	0.163	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	90	63	255	72	238	0	0	14	0
N.S.	1	1.30	0.91	3.70	1.04	3.45	0.00	0.00	0.20	0.00
time (sec)	N/A	0.455	0.080	0.702	0.572	0.087	0.000	0.000	0.160	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.246	2.682	0.123	0.708	0.079	0.523	0.178	0.161	3.843

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	168	218	148	1754	146	423	0	0	20	0
N.S.	1	1.30	0.88	10.44	0.87	2.52	0.00	0.00	0.12	0.00
time (sec)	N/A	1.084	0.118	1.140	0.580	0.099	0.000	0.000	0.167	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	139	182	123	1697	123	359	0	0	20	0
N.S.	1	1.31	0.88	12.21	0.88	2.58	0.00	0.00	0.14	0.00
time (sec)	N/A	0.885	0.063	0.919	0.561	0.093	0.000	0.000	0.164	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	146	93	1616	100	305	0	0	18	0
N.S.	1	1.33	0.85	14.69	0.91	2.77	0.00	0.00	0.16	0.00
time (sec)	N/A	0.704	0.062	0.789	0.564	0.089	0.000	0.000	0.162	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	103	66	271	73	227	0	0	16	0
N.S.	1	1.36	0.87	3.57	0.96	2.99	0.00	0.00	0.21	0.00
time (sec)	N/A	0.498	0.084	0.754	0.594	0.091	0.000	0.000	0.162	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	20	19	17	21	20	19
N.S.	1	1.00	1.11	1.00	1.05	1.00	0.89	1.11	1.05	1.00
time (sec)	N/A	0.247	2.650	0.140	0.730	0.072	0.540	0.186	0.162	4.458

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	303	391	265	5185	277	879	0	0	17	0
N.S.	1	1.29	0.87	17.11	0.91	2.90	0.00	0.00	0.06	0.00
time (sec)	N/A	1.432	0.201	6.431	0.300	0.116	0.000	0.000	0.168	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	229	301	199	4881	213	729	0	0	15	0
N.S.	1	1.31	0.87	21.31	0.93	3.18	0.00	0.00	0.07	0.00
time (sec)	N/A	1.054	0.206	2.191	0.299	0.118	0.000	0.000	0.165	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	207	131	348	142	539	0	0	13	0
N.S.	1	1.38	0.87	2.32	0.95	3.59	0.00	0.00	0.09	0.00
time (sec)	N/A	0.639	0.189	1.422	0.319	0.114	0.000	0.000	0.164	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.290	3.669	0.131	0.801	0.073	2.009	0.491	0.164	3.803

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	152	195	145	1656	146	423	0	0	18	0
N.S.	1	1.28	0.95	10.89	0.96	2.78	0.00	0.00	0.12	0.00
time (sec)	N/A	1.040	0.111	1.184	0.572	0.087	0.000	0.000	0.172	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	126	162	120	1599	123	359	0	0	18	0
N.S.	1	1.29	0.95	12.69	0.98	2.85	0.00	0.00	0.14	0.00
time (sec)	N/A	0.829	0.066	0.943	0.575	0.094	0.000	0.000	0.164	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	100	128	90	1518	100	305	0	0	16	0
N.S.	1	1.28	0.90	15.18	1.00	3.05	0.00	0.00	0.16	0.00
time (sec)	N/A	0.663	0.066	0.727	0.579	0.083	0.000	0.000	0.158	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	89	63	255	72	226	0	0	14	0
N.S.	1	1.29	0.91	3.70	1.04	3.28	0.00	0.00	0.20	0.00
time (sec)	N/A	0.453	0.090	0.716	0.576	0.092	0.000	0.000	0.171	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.239	2.867	0.117	0.720	0.075	1.780	0.171	0.165	3.614

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	165	214	151	1782	149	450	0	0	20	0
N.S.	1	1.30	0.92	10.80	0.90	2.73	0.00	0.00	0.12	0.00
time (sec)	N/A	1.075	0.120	1.199	0.577	0.095	0.000	0.000	0.170	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	179	125	1723	125	381	0	0	20	0
N.S.	1	1.31	0.91	12.58	0.91	2.78	0.00	0.00	0.15	0.00
time (sec)	N/A	0.872	0.064	1.013	0.570	0.087	0.000	0.000	0.164	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	109	143	94	1640	101	322	0	0	18	0
N.S.	1	1.31	0.86	15.05	0.93	2.95	0.00	0.00	0.17	0.00
time (sec)	N/A	0.690	0.067	0.792	0.576	0.091	0.000	0.000	0.166	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	102	66	271	73	239	0	0	16	0
N.S.	1	1.34	0.87	3.57	0.96	3.14	0.00	0.00	0.21	0.00
time (sec)	N/A	0.479	0.066	0.750	0.596	0.085	0.000	0.000	0.161	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	20	19	17	21	20	19
N.S.	1	1.00	1.11	1.00	1.05	1.00	0.89	1.11	1.05	1.00
time (sec)	N/A	0.252	2.835	0.124	0.731	0.081	2.019	0.191	0.162	3.611

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	302	355	654	3640	0	1808	0	0	67	0
N.S.	1	1.18	2.17	12.05	0.00	5.99	0.00	0.00	0.22	0.00
time (sec)	N/A	1.159	0.208	7.110	0.000	0.160	0.000	0.000	0.169	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	234	271	409	2719	0	1282	0	0	46	0
N.S.	1	1.16	1.75	11.62	0.00	5.48	0.00	0.00	0.20	0.00
time (sec)	N/A	0.842	0.144	4.914	0.000	0.140	0.000	0.000	0.167	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	162	182	295	1818	0	834	0	0	25	0
N.S.	1	1.12	1.82	11.22	0.00	5.15	0.00	0.00	0.15	0.00
time (sec)	N/A	0.601	0.205	0.849	0.000	0.125	0.000	0.000	0.159	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	86	127	119	182	498	0	0	9	0
N.S.	1	1.09	1.61	1.51	2.30	6.30	0.00	0.00	0.11	0.00
time (sec)	N/A	0.348	0.018	0.543	0.144	0.108	0.000	0.000	0.153	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.217	0.466	0.094	1.350	0.083	0.524	0.410	0.169	3.640

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	395	522	349	6855	0	2164	0	0	17	0
N.S.	1	1.32	0.88	17.35	0.00	5.48	0.00	0.00	0.04	0.00
time (sec)	N/A	1.534	0.678	10.099	0.000	0.149	0.000	0.000	0.163	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	295	402	259	6481	0	1688	0	0	15	0
N.S.	1	1.36	0.88	21.97	0.00	5.72	0.00	0.00	0.05	0.00
time (sec)	N/A	1.118	0.590	2.473	0.000	0.156	0.000	0.000	0.161	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	287	365	556	372	1184	0	0	13	0
N.S.	1	1.48	1.88	2.87	1.92	6.10	0.00	0.00	0.07	0.00
time (sec)	N/A	0.702	0.364	1.598	0.187	0.144	0.000	0.000	0.161	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.312	0.308	0.115	3.027	0.087	0.791	0.693	0.155	5.344

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	170	218	155	2273	343	344	0	0	21	0
N.S.	1	1.28	0.91	13.37	2.02	2.02	0.00	0.00	0.12	0.00
time (sec)	N/A	0.899	0.146	1.481	0.068	0.090	0.000	0.000	0.171	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	133	171	119	2183	248	292	0	0	19	0
N.S.	1	1.29	0.89	16.41	1.86	2.20	0.00	0.00	0.14	0.00
time (sec)	N/A	0.703	0.074	1.047	0.051	0.111	0.000	0.000	0.168	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	123	766	307	262	217	0	0	17	0
N.S.	1	1.32	8.24	3.30	2.82	2.33	0.00	0.00	0.18	0.00
time (sec)	N/A	0.494	3.748	1.096	0.113	0.120	0.000	0.000	0.164	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	144	36	17	20	21	21
N.S.	1	1.00	1.10	0.90	7.20	1.80	0.85	1.00	1.05	1.05
time (sec)	N/A	0.269	0.588	0.204	3.812	0.088	0.785	0.425	0.160	4.224

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	171	223	156	2383	342	344	0	0	23	0
N.S.	1	1.30	0.91	13.94	2.00	2.01	0.00	0.00	0.13	0.00
time (sec)	N/A	0.909	0.131	1.479	0.063	0.114	0.000	0.000	0.172	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	134	176	120	2285	247	292	0	0	21	0
N.S.	1	1.31	0.90	17.05	1.84	2.18	0.00	0.00	0.16	0.00
time (sec)	N/A	0.704	0.074	1.158	0.048	0.096	0.000	0.000	0.168	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	128	723	320	260	218	0	0	19	0
N.S.	1	1.36	7.69	3.40	2.77	2.32	0.00	0.00	0.20	0.00
time (sec)	N/A	0.498	1.441	1.087	0.115	0.093	0.000	0.000	0.167	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	141	36	17	21	23	22
N.S.	1	1.00	1.10	0.90	6.71	1.71	0.81	1.00	1.10	1.05
time (sec)	N/A	0.270	0.601	0.214	3.811	0.094	0.828	0.499	0.155	3.908

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	302	355	654	3640	0	1566	0	0	67	0
N.S.	1	1.18	2.17	12.05	0.00	5.19	0.00	0.00	0.22	0.00
time (sec)	N/A	1.170	0.172	6.911	0.000	0.186	0.000	0.000	0.166	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	234	271	409	2719	0	1084	0	0	46	0
N.S.	1	1.16	1.75	11.62	0.00	4.63	0.00	0.00	0.20	0.00
time (sec)	N/A	0.850	0.119	4.795	0.000	0.169	0.000	0.000	0.173	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	162	182	295	1818	0	680	0	0	25	0
N.S.	1	1.12	1.82	11.22	0.00	4.20	0.00	0.00	0.15	0.00
time (sec)	N/A	0.602	0.194	0.835	0.000	0.135	0.000	0.000	0.163	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	86	127	119	184	388	0	0	9	0
N.S.	1	1.09	1.61	1.51	2.33	4.91	0.00	0.00	0.11	0.00
time (sec)	N/A	0.355	0.017	0.483	0.143	0.130	0.000	0.000	0.159	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.225	0.082	0.101	1.301	0.078	0.661	0.288	0.164	3.869

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	391	514	341	6661	0	1798	0	0	17	0
N.S.	1	1.31	0.87	17.04	0.00	4.60	0.00	0.00	0.04	0.00
time (sec)	N/A	1.519	0.701	11.325	0.000	0.273	0.000	0.000	0.167	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	293	394	255	6311	0	1462	0	0	15	0
N.S.	1	1.34	0.87	21.54	0.00	4.99	0.00	0.00	0.05	0.00
time (sec)	N/A	1.137	0.622	2.740	0.000	0.241	0.000	0.000	0.166	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	283	390	564	392	1098	0	0	13	0
N.S.	1	1.46	2.01	2.91	2.02	5.66	0.00	0.00	0.07	0.00
time (sec)	N/A	0.686	0.515	2.020	0.189	0.217	0.000	0.000	0.161	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.301	0.322	0.137	3.176	0.080	1.077	0.517	0.183	5.152

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	168	219	155	2383	344	179	0	0	20	0
N.S.	1	1.30	0.92	14.18	2.05	1.07	0.00	0.00	0.12	0.00
time (sec)	N/A	0.890	0.140	1.827	0.077	0.085	0.000	0.000	0.179	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	132	172	119	2285	249	156	0	0	18	0
N.S.	1	1.30	0.90	17.31	1.89	1.18	0.00	0.00	0.14	0.00
time (sec)	N/A	0.720	0.088	1.396	0.052	0.080	0.000	0.000	0.162	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	126	709	307	286	121	0	0	16	0
N.S.	1	1.35	7.62	3.30	3.08	1.30	0.00	0.00	0.17	0.00
time (sec)	N/A	0.480	1.950	1.247	0.123	0.080	0.000	0.000	0.166	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	141	36	17	20	20	21
N.S.	1	1.00	1.10	0.90	7.05	1.80	0.85	1.00	1.00	1.05
time (sec)	N/A	0.255	0.565	0.252	4.359	0.093	1.212	0.304	0.163	5.030

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	169	216	155	2273	345	179	0	0	22	0
N.S.	1	1.28	0.92	13.45	2.04	1.06	0.00	0.00	0.13	0.00
time (sec)	N/A	0.900	0.143	1.924	0.068	0.080	0.000	0.000	0.175	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	133	169	119	2183	250	156	0	0	20	0
N.S.	1	1.27	0.89	16.41	1.88	1.17	0.00	0.00	0.15	0.00
time (sec)	N/A	0.699	0.077	1.635	0.057	0.091	0.000	0.000	0.177	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	123	605	320	288	121	0	0	18	0
N.S.	1	1.31	6.44	3.40	3.06	1.29	0.00	0.00	0.19	0.00
time (sec)	N/A	0.482	1.558	1.582	0.114	0.100	0.000	0.000	0.180	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	144	36	19	21	22	22
N.S.	1	1.00	1.10	0.90	6.86	1.71	0.90	1.00	1.05	1.05
time (sec)	N/A	0.263	0.594	0.252	4.315	0.081	1.096	0.305	0.171	5.002

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	126	160	131	414	0	326	0	0	64	0
N.S.	1	1.27	1.04	3.29	0.00	2.59	0.00	0.00	0.51	0.00
time (sec)	N/A	0.769	0.215	44.246	0.000	0.106	0.000	0.000	0.167	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	291	236	314	331	246	362	0	309	510
N.S.	1	0.98	0.79	1.06	1.11	0.83	1.22	0.00	1.04	1.72
time (sec)	N/A	0.662	0.095	3.073	0.038	0.085	3.061	0.000	0.158	5.224

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	222	192	249	271	195	286	0	244	414
N.S.	1	0.99	0.85	1.11	1.20	0.87	1.27	0.00	1.08	1.84
time (sec)	N/A	0.525	0.080	1.711	0.035	0.099	1.407	0.000	0.159	5.266

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	174	171	138	209	241	175	329
N.S.	1	1.00	0.92	1.24	1.22	0.99	1.49	1.72	1.25	2.35
time (sec)	N/A	0.366	0.058	1.115	0.032	0.088	0.643	0.249	0.159	5.377

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	381	336	0	589	167	0	0	0	174	0
N.S.	1	0.88	0.00	1.55	0.44	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.899	0.000	2.106	0.173	0.000	0.000	0.000	0.161	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	225	161	0	0	0	0	0	111	0
N.S.	1	1.02	0.73	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.751	0.094	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	325	307	0	0	0	0	0	133	0
N.S.	1	1.14	1.08	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.974	0.082	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	286	236	296	317	249	345	359	308	497
N.S.	1	1.09	0.90	1.13	1.21	0.95	1.31	1.37	1.17	1.89
time (sec)	N/A	0.981	0.081	2.401	0.038	0.081	1.730	0.374	0.159	5.139

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	223	183	229	252	198	265	282	243	414
N.S.	1	1.08	0.89	1.11	1.22	0.96	1.29	1.37	1.18	2.01
time (sec)	N/A	0.815	0.068	1.556	0.036	0.081	0.846	0.293	0.152	5.152

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	116	144	141	178	130	155	198	162	315
N.S.	1	1.12	1.38	1.36	1.71	1.25	1.49	1.90	1.56	3.03
time (sec)	N/A	0.866	0.020	0.853	0.037	0.082	0.383	0.181	0.157	4.735

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	85	332	0	0	0	0	0	154	0
N.S.	1	0.81	3.16	0.00	0.00	0.00	0.00	0.00	1.47	0.00
time (sec)	N/A	0.713	0.110	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	178	457	0	0	0	0	0	163	0
N.S.	1	0.90	2.32	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.515	0.232	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	293	0	0	0	0	0	0	324	0
N.S.	1	1.14	0.00	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	2.619	0.000	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	504	1128	953	0	0	0	0	258	0
N.S.	1	0.98	2.20	1.86	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.037	5.747	6.413	0.000	0.000	0.000	0.000	0.166	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	546	787	1287	3508	0	0	0	0	368	0
N.S.	1	1.44	2.36	6.42	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	2.231	2.224	6.881	0.000	0.000	0.000	0.000	0.162	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	93	37	22	26	148	26
N.S.	1	1.00	1.08	1.00	3.88	1.54	0.92	1.08	6.17	1.08
time (sec)	N/A	0.826	0.140	0.337	0.218	0.091	117.218	0.148	0.185	4.835

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	560	780	1236	0	0	0	0	0	115	0
N.S.	1	1.39	2.21	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.838	2.333	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	720	1193	937	0	0	0	0	1231	0
N.S.	1	1.01	1.68	1.32	0.00	0.00	0.00	0.00	1.73	0.00
time (sec)	N/A	1.359	4.467	7.211	0.000	0.000	0.000	0.000	0.194	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	51	22	58	64	0	0	6	0
N.S.	1	1.00	2.04	0.88	2.32	2.56	0.00	0.00	0.24	0.00
time (sec)	N/A	0.199	0.002	0.088	0.030	0.096	0.000	0.000	0.157	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	50	71	54	59	94	0	0	8	0
N.S.	1	0.98	1.39	1.06	1.16	1.84	0.00	0.00	0.16	0.00
time (sec)	N/A	0.347	0.018	0.083	0.040	0.088	0.000	0.000	0.159	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	84	93	71	76	119	0	0	10	0
N.S.	1	1.20	1.33	1.01	1.09	1.70	0.00	0.00	0.14	0.00
time (sec)	N/A	0.470	0.017	0.108	0.034	0.092	0.000	0.000	0.171	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	39	68	49	107	137	0	0	10	0
N.S.	1	0.95	1.66	1.20	2.61	3.34	0.00	0.00	0.24	0.00
time (sec)	N/A	0.210	0.015	0.135	0.033	0.081	0.000	0.000	0.175	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	113	179	108	198	0	0	12	0
N.S.	1	1.00	1.36	2.16	1.30	2.39	0.00	0.00	0.14	0.00
time (sec)	N/A	0.396	0.029	0.141	0.038	0.097	0.000	0.000	0.161	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	138	149	244	142	247	0	0	14	0
N.S.	1	1.16	1.25	2.05	1.19	2.08	0.00	0.00	0.12	0.00
time (sec)	N/A	0.576	0.028	0.180	0.040	0.101	0.000	0.000	0.160	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	149	108	160	202	283	0	0	14	0
N.S.	1	0.89	0.64	0.95	1.20	1.68	0.00	0.00	0.08	0.00
time (sec)	N/A	0.547	0.046	0.737	0.034	0.098	0.000	0.000	0.167	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	432	177	590	194	395	0	0	16	0
N.S.	1	2.00	0.82	2.73	0.90	1.83	0.00	0.00	0.07	0.00
time (sec)	N/A	2.688	0.067	0.744	0.064	0.091	0.000	0.000	0.163	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	557	235	666	254	479	0	0	18	0
N.S.	1	2.07	0.87	2.48	0.94	1.78	0.00	0.00	0.07	0.00
time (sec)	N/A	2.829	0.042	1.188	0.068	0.097	0.000	0.000	0.163	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	21	21	14	44	15	15
N.S.	1	1.00	1.00	0.82	1.24	1.24	0.82	2.59	0.88	0.88
time (sec)	N/A	0.235	0.113	0.125	0.032	0.085	0.230	0.113	0.165	3.919

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	36	39	37	37	58	60	225	385	107
N.S.	1	0.82	0.89	0.84	0.84	1.32	1.36	5.11	8.75	2.43
time (sec)	N/A	0.337	0.014	0.275	0.030	0.088	0.754	0.135	0.205	4.009

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	38	42	118	40	108	0	119	58	58
N.S.	1	0.81	0.89	2.51	0.85	2.30	0.00	2.53	1.23	1.23
time (sec)	N/A	0.322	0.025	2.672	0.025	0.095	0.000	0.139	0.160	5.138

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	112	91	153	794	184	233	0	167	25	187
N.S.	1	0.81	1.37	7.09	1.64	2.08	0.00	1.49	0.22	1.67
time (sec)	N/A	0.504	0.095	2.563	0.117	0.107	0.000	0.276	0.164	4.023

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	60	824	64	92	0	98	25	119
N.S.	1	0.94	1.22	16.82	1.31	1.88	0.00	2.00	0.51	2.43
time (sec)	N/A	0.332	0.044	0.312	0.035	0.103	0.000	0.206	0.167	4.129

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	41	46	37	43	57	66	40	29	28
N.S.	1	0.91	1.02	0.82	0.96	1.27	1.47	0.89	0.64	0.62
time (sec)	N/A	0.311	0.042	0.180	0.039	0.088	0.625	0.114	0.165	0.110

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	41	46	27	42	25	0	35	29	28
N.S.	1	0.91	1.02	0.60	0.93	0.56	0.00	0.78	0.64	0.62
time (sec)	N/A	0.310	0.041	0.186	0.033	0.076	0.000	0.116	0.162	0.071

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	59	939	64	93	0	94	25	111
N.S.	1	0.94	1.20	19.16	1.31	1.90	0.00	1.92	0.51	2.27
time (sec)	N/A	0.319	0.043	0.300	0.039	0.092	0.000	0.129	0.184	3.718

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	A	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	107	91	150	920	184	234	0	157	25	179
N.S.	1	0.85	1.40	8.60	1.72	2.19	0.00	1.47	0.23	1.67
time (sec)	N/A	0.498	0.079	2.470	0.117	0.089	0.000	0.296	0.173	3.821

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [79] had the largest ratio of [2.3333300000000013]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	N/A	1	0	1.00	40	0.000
2	A	7	6	0.98	40	0.150
3	A	6	5	1.01	40	0.125
4	A	3	2	0.94	38	0.053
5	N/A	1	0	1.00	40	0.000
6	N/A	1	0	1.00	40	0.000
7	A	2	2	1.00	11	0.182
8	A	2	2	1.00	11	0.182
9	A	2	2	1.00	9	0.222
10	A	3	2	1.00	7	0.286
11	A	2	2	1.00	11	0.182
12	A	2	2	1.00	11	0.182
13	A	2	2	1.00	11	0.182
14	A	2	2	1.00	11	0.182
15	A	3	3	0.93	13	0.231
16	A	3	3	1.07	13	0.231
17	A	3	3	1.07	13	0.231
18	A	4	3	1.00	11	0.273
19	A	3	2	1.00	9	0.222
20	A	3	3	1.02	13	0.231
21	A	3	3	1.00	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	13	0.231
23	A	1	1	1.00	13	0.077
24	A	2	2	1.00	13	0.154
25	A	4	4	0.86	13	0.308
26	A	4	4	1.10	13	0.308
27	A	4	4	1.10	13	0.308
28	A	5	4	1.15	13	0.308
29	A	4	3	1.00	11	0.273
30	A	3	2	1.00	9	0.222
31	A	4	4	1.03	13	0.308
32	A	4	4	1.00	13	0.308
33	A	4	4	1.02	13	0.308
34	A	4	4	1.00	13	0.308
35	A	1	1	1.00	13	0.077
36	A	2	2	1.00	13	0.154
37	A	1	1	1.00	13	0.077
38	A	6	5	1.07	13	0.385
39	A	5	4	1.05	13	0.308
40	A	4	3	1.00	11	0.273
41	A	3	2	1.00	9	0.222
42	A	5	4	1.00	13	0.308
43	A	6	5	1.25	13	0.385
44	A	7	6	1.32	13	0.462
45	A	2	2	1.00	13	0.154
46	A	7	6	1.12	13	0.462
47	A	6	5	1.09	13	0.385
48	A	5	4	1.08	13	0.308
49	A	4	3	1.00	11	0.273
50	A	3	2	1.00	9	0.222
51	A	6	5	1.26	13	0.385
52	A	7	6	1.32	13	0.462
53	A	8	7	1.31	13	0.538

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	1.03	13	0.231
55	A	7	6	1.16	13	0.462
56	A	6	5	1.14	13	0.385
57	A	5	4	1.11	13	0.308
58	A	4	3	1.00	11	0.273
59	A	3	2	1.00	9	0.222
60	A	7	6	1.38	13	0.462
61	A	8	7	1.38	13	0.538
62	A	9	8	1.36	13	0.615
63	A	1	1	1.00	13	0.077
64	A	7	6	0.91	13	0.462
65	A	6	5	0.96	13	0.385
66	A	5	4	1.00	13	0.308
67	A	4	3	1.00	11	0.273
68	A	3	2	1.00	9	0.222
69	A	1	1	1.00	13	0.077
70	A	2	2	1.00	13	0.154
71	A	3	3	0.97	13	0.231
72	A	2	2	1.00	11	0.182
73	A	2	2	1.00	11	0.182
74	A	2	2	1.00	9	0.222
75	A	3	2	1.00	7	0.286
76	A	2	2	1.00	11	0.182
77	A	2	2	1.00	11	0.182
78	A	2	2	1.00	11	0.182
79	C	8	7	1.48	3	2.333
80	C	9	8	1.35	5	1.600
81	C	10	9	1.29	7	1.286
82	A	7	6	1.29	15	0.400
83	A	6	5	1.32	13	0.385
84	A	5	4	1.38	11	0.364
85	N/A	1	0	1.00	15	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	8	1.28	16	0.500
87	A	8	7	1.29	16	0.438
88	A	7	6	1.30	14	0.429
89	A	6	5	1.30	12	0.417
90	N/A	1	0	1.00	16	0.000
91	A	9	8	1.30	19	0.421
92	A	8	7	1.31	19	0.368
93	A	7	6	1.33	17	0.353
94	A	6	5	1.36	15	0.333
95	N/A	1	0	1.00	19	0.000
96	A	7	6	1.29	15	0.400
97	A	6	5	1.31	13	0.385
98	A	5	4	1.38	11	0.364
99	N/A	1	0	1.00	15	0.000
100	A	9	8	1.28	16	0.500
101	A	8	7	1.29	16	0.438
102	A	7	6	1.28	14	0.429
103	A	6	5	1.29	12	0.417
104	N/A	1	0	1.00	16	0.000
105	A	9	8	1.30	19	0.421
106	A	8	7	1.31	19	0.368
107	A	7	6	1.31	17	0.353
108	A	6	5	1.34	15	0.333
109	N/A	1	0	1.00	19	0.000
110	A	9	8	1.18	15	0.533
111	A	8	7	1.16	15	0.467
112	A	7	6	1.12	13	0.462
113	A	6	5	1.09	7	0.714
114	N/A	1	0	1.00	15	0.000
115	A	7	6	1.32	15	0.400
116	A	6	5	1.36	13	0.385
117	A	5	4	1.48	11	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	N/A	1	0	1.00	15	0.000
119	A	8	7	1.28	20	0.350
120	A	7	6	1.29	18	0.333
121	A	6	5	1.32	16	0.312
122	N/A	1	0	1.00	20	0.000
123	A	8	7	1.30	21	0.333
124	A	7	6	1.31	19	0.316
125	A	6	5	1.36	17	0.294
126	N/A	1	0	1.00	21	0.000
127	A	9	8	1.18	15	0.533
128	A	8	7	1.16	15	0.467
129	A	7	6	1.12	13	0.462
130	A	6	5	1.09	7	0.714
131	N/A	1	0	1.00	15	0.000
132	A	7	6	1.31	15	0.400
133	A	6	5	1.34	13	0.385
134	A	5	4	1.46	11	0.364
135	N/A	1	0	1.00	15	0.000
136	A	8	7	1.30	20	0.350
137	A	7	6	1.30	18	0.333
138	A	6	5	1.35	16	0.312
139	N/A	1	0	1.00	20	0.000
140	A	8	7	1.28	21	0.333
141	A	7	6	1.27	19	0.316
142	A	6	5	1.31	17	0.294
143	N/A	1	0	1.00	21	0.000
144	A	2	2	1.27	24	0.083
145	A	2	2	0.98	27	0.074
146	A	2	2	0.99	27	0.074
147	A	2	2	1.00	25	0.080
148	A	16	15	0.88	27	0.556

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	A	2	2	1.02	27	0.074
150	A	2	2	1.14	27	0.074
151	A	2	2	1.09	27	0.074
152	A	2	2	1.08	27	0.074
153	A	8	7	1.12	24	0.292
154	A	8	7	0.81	27	0.259
155	A	17	16	0.90	27	0.593
156	A	26	25	1.14	27	0.926
157	A	2	2	0.98	22	0.091
158	A	16	15	1.44	21	0.714
159	N/A	8	0	1.00	24	0.000
160	A	13	12	1.39	24	0.500
161	A	2	2	1.01	24	0.083
162	A	3	2	1.00	4	0.500
163	A	5	4	0.98	6	0.667
164	A	6	5	1.20	8	0.625
165	A	3	2	0.95	8	0.250
166	A	5	4	1.00	10	0.400
167	A	6	5	1.16	12	0.417
168	A	9	8	0.89	12	0.667
169	A	6	6	2.00	14	0.429
170	B	6	6	2.07	16	0.375
171	A	1	1	1.00	20	0.050
172	A	5	4	0.82	12	0.333
173	A	5	4	0.81	14	0.286
174	A	8	7	0.81	20	0.350
175	A	7	6	0.94	20	0.300
176	A	4	3	0.91	20	0.150
177	A	4	3	0.91	20	0.150
178	A	7	6	0.94	20	0.300
179	A	9	8	0.85	20	0.400

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Int}\left(\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Defer(Int)((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}\,dx$$

$$\Big\downarrow \; 7234$$

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Sympy [N/A]

Not integrable

Time = 9.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
-Integral((a + b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1),
x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
-integrate((b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

Mupad [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\left(\int \frac{\left(acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)b+a\right)^n}{c^2x^2-1}dx\right)$$

input
```
int((a+b*acoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
- int((acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))*b + a)**n/(c**2*x**2 - 1),x)
```

3.2 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 460

$$\begin{aligned}
&\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad -\frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad +\frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad -\frac{3b^2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad +\frac{3b^2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad -\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}+\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}
\end{aligned}$$

output
```
-2*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arccoth(1-2/(1-(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c-3/2*b*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/2*b*(a+b*arccoth((-c*
x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,1-2*(-c*x+1)^(1/2)/(c*x+1)^(1/2)/((
-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c-3/2*b^2*(a+b*arccoth((-c*x+1)^(1/2)/(c*x
+1)^(1/2)))*polylog(3,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/2*b^2*(a+b
*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2*(-c*x+1)^(1/2)/(c*x+
1)^(1/2)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c-3/4*b^3*polylog(4,1-2/((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/4*b^3*polylog(4,1-2*(-c*x+1)^(1/2)/(c*x+1)^
(1/2)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c
```

Mathematica [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7232, 6449, 6615, 6619, 6623, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$\Big\downarrow$ 6449

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$\Big\downarrow$ 6615

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.}{c}$$

$\Big\downarrow$ 6619

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(b\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.\right.}{}$$

$\Big\downarrow$ 6623

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(b\left(\frac{1}{2}b\int \frac{\text{PolyLog}\left(3,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\text{PolyLo}\right.\right.\right.}{}$$

$\Big\downarrow$ 7164

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(b\left(-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\right.\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
-((2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqr
t[1 - c*x]/Sqrt[1 + c*x])] - 6*b*((-1/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt
[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])]) + b*(-1
/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqr
t[1 - c*x]/Sqrt[1 + c*x])]) - (b*PolyLog[4, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[
1 + c*x])])/4))/2 + (((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyL
og[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x
]))])/2 - b*(-1/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3,
 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])
 - (b*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/S
qrt[1 + c*x]))])/4))/2))/c)
```

Defintions of rubi rules used

rule 6449
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6615
```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a
+ b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyInteg
rand[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1
- c*x))^2, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6623
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(388) = 776$.

Time = 0.32 (sec) , antiderivative size = 1681, normalized size of antiderivative = 3.65

method	result	size
default	Expression too large to display	1681
parts	Expression too large to display	1681

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)-b^3*(-1/c*arccoth((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)
/(c*x+1)^(1/2)))^(1/2))-3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylo
g(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^
(1/2))+6/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,1/(((-c*x+1)^(1
/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1/2))-6/c*polylog(
4,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1
/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1/((-c*x+1)^(1/2)/(c*x
+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2))+1)+3/2/c*arccoth((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*
x+1)^(1/2)/(c*x+1)^(1/2)))-3/2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*pol
ylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)
))+3/4/c*polylog(4,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(
c*x+1)^(1/2)))-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+1/(((-c*x+
1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1/2))-3/c*arc
coth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)
^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1/2))+6/c*arccoth((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x
+1)^(1/2)/(c*x+1)^(1/2)))^(1/2))-6/c*polylog(4,-1/(((-c*x+1)^(1/2)/(c*x+1)
^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1/2)))-3*a*b^2*(-1/c*arcco...
```

Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^3*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccoth(s
qrt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\left(\int \frac{a^3}{c^2x^2-1}\,dx\right) - \int \frac{b^3\operatorname{acoth}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acoth(sqrt(-c*x + 1)/sq
rt(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acoth(sqrt(-c*x +
1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acoth(sqrt(-c
*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*l
og(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^3/c - integrate(1/32*(4
*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x +
1))^3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1)
+ sqrt(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sq
rt(c*x + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) +
(b^3*c*x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*
log(c*x + 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*
x + 1) + sqrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*
b)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqr
t(-c*x + 1)*a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x
+ 1) + sqrt(-c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*
log(sqrt(c*x + 1) + sqrt(-c*x + 1)))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))
/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Giac [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$
$$=\frac{-6\left(\int \frac{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}dx\right)a^2bc-2\left(\int \frac{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{c^2x^2-1}dx\right)b^3c-6\left(\int \frac{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c^2x^2-1}dx\right)a\,b^2c-\log(c^2x-c}{2c}$$

input
```
int((a+b*acoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

output
```
( - 6*int(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1),x)*a**2*b*
c - 2*int(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))**3/(c**2*x**2 - 1),x)*b**3
*c - 6*int(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1),x)*a*b
**2*c - log(c**2*x - c)*a**3 + log(c**2*x + c)*a**3)/(2*c)
```

3.3 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 302

$$\begin{aligned}\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad-\frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad+\frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}\\
&\quad-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}\end{aligned}$$

output
```
-2*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arccoth(1-2/(1-(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c-b*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+b*(a+b*arccoth((-c*x+1)^(1/2)
/(c*x+1)^(1/2)))*polylog(2,1-2*(-c*x+1)^(1/2)/(c*x+1)^(1/2)/((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1))/c-1/2*b^2*polylog(3,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+
1))/c+1/2*b^2*polylog(3,1-2*(-c*x+1)^(1/2)/(c*x+1)^(1/2)/((-c*x+1)^(1/2)/(
c*x+1)^(1/2)+1))/c
```

Mathematica [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7232, 6449, 6615, 6619, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}\,dx$$

$$\Big\downarrow \; 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6449}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\int\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6615}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\int\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.}{c}$$

$$\Big\downarrow \text{6619}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\text{PolyLog}\left(\right.\right.\right.}{}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1}}{\sqrt{c}}\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-((2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 - Sqr
t[1 - c*x]/Sqrt[1 + c*x])] - 4*b*((-1/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt
[1 + c*x]])*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])]) - (b*Poly
Log[3, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4)/2 + (((a + b*ArcCoth[S
qrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*
x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/2 + (b*PolyLog[3, 1 - (2*Sqrt[1 -
c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/4)/2))/c)
```

Defintions of rubi rules used

rule 6449
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6615
```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a
+ b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyInteg
rand[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1
- c*x))^2, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(258) = 516$.

Time = 0.09 (sec) , antiderivative size = 926, normalized size of antiderivative = 3.07

method	result
default	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - b^2 \left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1-\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c} - \frac{2\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) \operatorname{polylog}\left(2, \ldots\right.}{c}\right.$
parts	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - b^2 \left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1-\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c} - \frac{2\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) \operatorname{polylog}\left(2, \ldots\right.}{c}\right.$

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2*(-1/c*arccoth((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^2*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)
/(c*x+1)^(1/2)))^(1/2))-2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(
2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1
/2))+2/c*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(
c*x+1)^(1/2)))^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1/((-
c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2))+1)+1/c*arcc
oth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/
2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))-1/2/c*polylog(3,-1/((-c*x+1)^(1/2)
/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))-1/c*arccoth((-c*x+1)^(
1/2)/(c*x+1)^(1/2))^2*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)
^(1/2)/(c*x+1)^(1/2)))^(1/2))-2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*po
lylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/
2)))^(1/2))+2/c*polylog(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)
^(1/2)/(c*x+1)^(1/2)))^(1/2)))-2*a*b*(-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^
(1/2))*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^
(1/2)))^(1/2))-1/c*polylog(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+
1)^(1/2)/(c*x+1)^(1/2)))^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*
ln(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2))+1)+
1/2/c*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(...
```

Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^2*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccoth(sqr
t(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acoth(sqrt(-c*x + 1)/sq
rt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acoth(sqrt(-c*x + 1)/
sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*lo
g(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*
(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1
))^2 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt
(-c*x + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1
) + sqrt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b
^2)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) +
 (b^2*c*x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(
-c*x + 1)))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)),
x)
```

Giac [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$
$$= \frac{-4\left(\int \frac{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}dx\right)abc-2\left(\int \frac{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c^2x^2-1}dx\right)b^2c-\log(c^2x-c)\,a^2+\log(c^2x+c)\,a^2}{2c}$$

input
```
int((a+b*acoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

output
```
( - 4*int(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1),x)*a*b*c -
 2*int(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1),x)*b**2*c
- log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2)/(2*c)
```

3.4 $\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 89

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{b\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c} + \frac{b\,\mathrm{PolyLog}\left(2,\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

output
```
-a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*b*polylog(2,-(c*x+1)^(1/2)/(-c*x
+1)^(1/2))/c+1/2*b*polylog(2,(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{a\mathrm{arctanh}(cx)}{c} + \frac{b\left(\mathrm{arctanh}(cx)\left(2\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)+\log\left(1-e^{-\mathrm{arctanh}(cx)}\right)-\log\left(1+e^{-\mathrm{arctanh}(cx)}\right)\right)+\mathrm{PolyLog}\left(2,-e\right.\right.}{2c}$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
(a*ArcTanh[c*x])/c + (b*(ArcTanh[c*x]*(2*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*
x]] + Log[1 - E^(-ArcTanh[c*x])] - Log[1 + E^(-ArcTanh[c*x])]) + PolyLog[2
, -E^(-ArcTanh[c*x])] - PolyLog[2, E^(-ArcTanh[c*x])]))/(2*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {7232, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{ 7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{ 6447}$$

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{c}$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (b*PolyLog[2, -(Sqrt[1 + c*x]/Sqrt
[1 - c*x])])/2 - (b*PolyLog[2, Sqrt[1 + c*x]/Sqrt[1 - c*x]])/2)/c)
```

Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(73) = 146$.

Time = 0.07 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.18

method	result
default	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}-b\left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c}-\frac{\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c}+\frac{\operatorname{arccoth}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{}\right.$
parts	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}-b\left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c}-\frac{\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}}}\right)}{c}+\frac{\operatorname{arccoth}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{}\right.$

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

output
```
1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)-b*(-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)
^(1/2))*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)
^(1/2)))^(1/2))-1/c*polylog(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x
+1)^(1/2)/(c*x+1)^(1/2)))^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*ln(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2))+1)
+1/2/c*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*(1+(-c*x+1)^(1/2)/(c*
x+1)^(1/2)))-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+1/(((-c*x+1)^(
1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1/2))-1/c*polylog
(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))^
(1/2)))
```

Fricas [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="fricas")
```

output
```
integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*acoth(sqrt(-c*x + 1)/sqrt(c*x
 + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{a + b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="maxima")
```

output
```
1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
 - (log(c*x + 1) - log(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/c
- 2*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2
- 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*s
qrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) -
 (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)
/c)
```

Giac [F]

$$\int \frac{a + b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{-2\left(\int \frac{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}dx\right)bc-\log(c^2x-c)\,a+\log(c^2x+c)\,a}{2c}$$

input
```
int((a+b*acoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

output
```
( - 2*int(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1),x)*b*c - l
og(c**2*x - c)*a + log(c**2*x + c)*a)/(2*c)
```

3.5 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x
]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - a), x)
```

Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$=-\int \frac{1}{ac^2x^2-a+bc^2x^2\,\text{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\text{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acoth(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

Mupad [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\left(\int \frac{1}{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)b\,c^2x^2-acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)b+a\,c^2x^2-a}dx\right)$$

input
```
int(1/(-c^2*x^2+1)/(a+b*acoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
 - int(1/(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))*b*c**2*x**2 - acoth(sqrt(
- c*x + 1)/sqrt(c*x + 1))*b + a*c**2*x**2 - a),x)
```

3.6 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
,x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\operatorname{arccoth}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}d x$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccoth(sqrt(-c*x + 1)/sqrt
(c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x
 + 1))), x)
```

Sympy [N/A]

Not integrable

Time = 11.90 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx =$$
$$-\int \frac{1}{a^2c^2x^2 - a^2 + 2abc^2x^2\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - 2ab\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + b^2c^2x^2\operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b^2\operatorname{acot}}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,
x)
```

output
```
-Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acoth(sqrt(-c*x + 1)/
sqrt(c*x + 1)) - 2*a*b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**
2*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acoth(sqrt(-c*x + 1)/sqrt(
c*x + 1))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="maxima")
```

output
```
4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x +
1)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(-sqrt(c*x + 1) + sqrt(-c*x +
1)) + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 -
 b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (
b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(-sqrt(c*x + 1) + sqrt(
-c*x + 1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^
2), x)
```

Mupad [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=-\int \frac{1}{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x
)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.08

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx =$$

$$-\left(\int \frac{1}{acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 b^2c^2x^2 - acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 b^2 + 2acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) ab\,c^2x^2 - 2acoth\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) ab$$

input

```
int(1/(-c^2*x^2+1)/(a+b*acoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output

```
- int(1/(acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))**2*b**2*c**2*x**2 - acoth(
sqrt( - c*x + 1)/sqrt(c*x + 1))**2*b**2 + 2*acoth(sqrt( - c*x + 1)/sqrt(c*
x + 1))*a*b*c**2*x**2 - 2*acoth(sqrt( - c*x + 1)/sqrt(c*x + 1))*a*b + a**2
*c**2*x**2 - a**2),x)
```

3.7 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m} + \frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{1+m}\right)$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{2599, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1
 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

method	result
parallelrisch	$-\frac{-x\,x^{m} \operatorname{arccoth}(\tanh(bx+a))m-2 \operatorname{arccoth}(\tanh(bx+a))x\,x^{m}+b\,x^{m}x^{2}}{(1+m)(2+m)}$
risch	$\frac{x\,x^{m}\ln(\mathrm{e}^{bx+a})}{1+m}-\frac{x\left(4bx-4i\pi\,\operatorname{csgn}(i\mathrm{e}^{bx+a})\,\operatorname{csgn}(i\mathrm{e}^{2bx+2a})^{2}+2i\pi\,\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+4i\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+2i\pi\,\operatorname{csgn}\right.}{}$

input

```
int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output

```
-(-x*x^m*arccoth(tanh(b*x+a))*m-2*arccoth(tanh(b*x+a))*x*x^m+b*x^m*x^2)/(1
+m)/(2+m)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$=\frac{(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+2\,a)x)\cosh\left(m\log\left(x\right)\right)+(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am}{2\,(m^2+3\,m+2)}$$

input

```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output

```
1/2*((I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*cosh(m*log(x)) +
 (I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*sinh(m*log(x)))/(m^2
 + 3*m + 2)
```

Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \begin{cases} b\log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m=-2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx & \text{for } m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(
tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**
m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m
**2 + 3*m + 2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\operatorname{arcoth}(\tanh(bx+a))}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m +
2)*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

input
```
int(x^m*acoth(tanh(a + b*x)),x)
```

output
```
(2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)
*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(6*m + 2*m^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^m x(acoth(\tanh(bx+a))\,m+2acoth(\tanh(bx+a))+bx)}{m^2+3m+2}$$

input
```
int(x^m*acoth(tanh(b*x+a)),x)
```

output
```
(x**m*x*(acoth(tanh(a + b*x))*m + 2*acoth(tanh(a + b*x)) + b*x))/(m**2 + 3
*m + 2)
```

3.8 $\int x^2 \coth^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arccoth(tanh(b*x+a))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{12}x^3\left(bx-4\coth^{-1}(\tanh(a+bx))\right)$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^3dx}{3}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{bx^4}{12}$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcCoth[Tanh[a + b*x]])/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{3}$
parallelrisch	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{3}$
parts	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{3}$
risch	$\frac{x^{3} \ln \left({\mathrm e}^{b x +a}\right)}{3}-\frac{b\,x^{4}}{12}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}+1}\right)^{2}}{6}-\frac{i \pi \,x^{3} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}+1}\right)^{3}}{6}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(i {\mathrm e}^{b x +a}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right)^{2}}{6}$ -

input
```
int(x^2*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*b*x^4+1/3*x^3*arccoth(tanh(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{6}i\,\pi x^3 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/6*I*pi*x^3 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}(\tanh(a+bx))}{3}$$

input
```
integrate(x**2*acoth(tanh(b*x+a)),x)
```

output
```
-b*x**4/12 + x**3*acoth(tanh(a + b*x))/3
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{3}\,x^3 \operatorname{arcoth}(\tanh(bx+a))$$

input
```
integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/12*b*x^4 + 1/3*x^3*arccoth(tanh(b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{6}\,x^3 \log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) +
1)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^3\,\mathrm{acoth}(\tanh(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

input
```
int(x^2*acoth(tanh(a + b*x)),x)
```

output
```
(x^3*acoth(tanh(a + b*x)))/3 - (b*x^4)/12
```

Reduce [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = \int acoth(\tanh(bx+a))\,x^2dx$$

input
```
int(x^2*acoth(tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x))*x**2,x)
```

3.9 $\int x \coth^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(\tanh(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arccoth(tanh(b*x+a))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{6}x^2\left(bx - 3\coth^{-1}(\tanh(a+bx))\right)$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6794, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 6794}$$

$$\frac{1}{2}b\int -x^2dx+\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcCoth[Tanh[a + b*x]])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))    Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{2}$
parallelrisch	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{2}$
risch	$\frac{x^{2} \ln \left({\mathrm e}^{b x +a}\right)}{2}-\frac{b\,x^{3}}{6}+\frac{i \pi \,x^{2} \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}+1}\right)^{2}}{8}+\frac{i \pi \,x^{2} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}+1}\right)^{2}}{4}-\frac{i \pi \,x^{2}}{4}-\frac{i \pi \,x^{2} \operatorname{csgn}\left(\ldots\right.}{4}$

input
```
int(x*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*x^3+1/2*x^2*arccoth(tanh(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = \frac{1}{3}\,bx^3 + \frac{1}{4}i\,\pi x^2 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/4*I*pi*x^2 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\coth^{-1}(\tanh(a+bx))\,dx=-\frac{bx^3}{6}+\frac{x^2\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{2}$$

input
```
integrate(x*acoth(tanh(b*x+a)),x)
```

output
```
-b*x**3/6 + x**2*acoth(tanh(a + b*x))/2
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\coth^{-1}(\tanh(a+bx))\,dx=-\frac{1}{6}\,bx^3+\frac{1}{2}\,x^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/6*b*x^3 + 1/2*x^2*arccoth(tanh(b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int x\coth^{-1}(\tanh(a+bx))\,dx=-\frac{1}{6}\,bx^3+\frac{1}{4}\,x^2\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^2\,\mathrm{acoth}(\tanh(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*acoth(tanh(a + b*x)),x)
```

output
```
(x^2*acoth(tanh(a + b*x)))/2 - (b*x^3)/6
```

Reduce [F]

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = \int acoth(\tanh{(bx+a)})\,x dx$$

input
```
int(x*acoth(tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x))*x,x)
```

3.10 $\int \coth^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \coth^{-1}(\tanh(a+bx))\,dx = \frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

output
```
1/2*arccoth(tanh(b*x+a))^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2}{2} + x\coth^{-1}(\tanh(a+bx))$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcCoth[Tanh[a + b*x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]],x]
```

output
```
ArcCoth[Tanh[a + b*x]]^2/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{b\,x^2}{2}+x\,\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)$
derivativedivides	$\frac{\mathrm{arctanh}(\tanh(bx+a))\,\mathrm{arccoth}(\tanh(bx+a))-\frac{\mathrm{arctanh}(\tanh(bx+a))^2}{2}}{b}$
default	$\frac{\mathrm{arctanh}(\tanh(bx+a))\,\mathrm{arccoth}(\tanh(bx+a))-\frac{\mathrm{arctanh}(\tanh(bx+a))^2}{2}}{b}$
parts	$x\,\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)+\frac{-\frac{(bx+a)^2}{2}+(bx+a)a}{b}$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^3 x}{2}-\frac{i\pi\;\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\,\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\,\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)x}{4}+\frac{i\pi\;\mathrm{csgn}}{}$

input
```
int(arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x^2+x*arccoth(tanh(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(\tanh(a+bx))\,dx=\frac{1}{2}\,bx^2+\frac{1}{2}i\,\pi x+ax$$

input
```
integrate(arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + 1/2*I*pi*x + a*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \coth^{-1}(\tanh(a+bx))\,dx = \begin{cases} \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2b} & \text{for } b \neq 0 \\ x \operatorname{acoth}(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((acoth(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*acoth(tanh(a)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{2}bx^2 + x \operatorname{arcoth}(\tanh(bx+a))$$

input
```
integrate(arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 + x*arccoth(tanh(b*x + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{2}bx^2 + \frac{1}{2}x\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)$$

input
```
integrate(arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*x^2 + 1/2*x*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))\,dx = x\,\mathrm{acoth}(\tanh(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(acoth(tanh(a + b*x)),x)
```

output
```
x*acoth(tanh(a + b*x)) - (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{acoth(\tanh(bx+a))^2}{2b}$$

input
```
int(acoth(tanh(b*x+a)),x)
```

output
```
( - acoth(tanh(a + b*x))**2)/(2*b)
```

3.11 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx = bx - \left(bx - \coth^{-1}(\tanh(a+bx))\right) \log(x)$$

output
```
b*x-(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx = bx + \left(-bx + \coth^{-1}(\tanh(a+bx))\right) \log(x)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{2589, 14\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx$$

$$\Big\downarrow \text{ 2589}$$

$$bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx$$

$$\Big\downarrow \text{ 14}$$

$$bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result
default	$\ln(x)\operatorname{arccoth}(\tanh(bx+a))+b(-\ln(x)x+x)$
parts	$\ln(x)\operatorname{arccoth}(\tanh(bx+a))+b(-\ln(x)x+x)$
risch	$\ln(x)\ln\left(e^{bx+a}\right)-\ln(x)xb+bx+\frac{i\pi\left(2\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^{2}-2\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^{3}-\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2}\right.\right.}{}$

input
```
int(arccoth(tanh(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccoth(tanh(b*x+a))+b*(-ln(x)*x+x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = bx + \frac{1}{2}\,(i\,\pi + 2\,a)\log\left(x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + 1/2*(I*pi + 2*a)*log(x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = -b\left(x+\frac{a}{b}\right)\log(x)+b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{arcoth}(\tanh(bx+a))\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arccoth(tanh(b*x + a))*log(x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = bx+\frac{1}{2}\,(i\,\pi+2\,a)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
b*x + 1/2*(I*pi + 2*a)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = b\,x - \ln(x)\,\left(\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2} + b\,x\right)$$

input
```
int(acoth(tanh(a + b*x))/x,x)
```

output
```
b*x - log(x)*(log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = \int \frac{acoth(\tanh(bx+a))}{x}dx$$

input
```
int(acoth(tanh(b*x+a))/x,x)
```

output
```
int(acoth(tanh(a + b*x))/x,x)
```

3.12 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx = -\frac{\coth^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

output
```
-arccoth(tanh(b*x+a))/x+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx = b - \frac{\coth^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

output
```
b - ArcCoth[Tanh[a + b*x]]/x + b*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$b\int \frac{1}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))}{x}$$

$$\Big\downarrow \text{ 14}$$

$$b\log(x)-\frac{\coth^{-1}(\tanh(a+bx))}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{\ln(x)xb-\operatorname{arccoth}(\tanh(bx+a))}{x}$
default	$-\frac{\operatorname{arccoth}(\tanh(bx+a))}{x}+b\ln\left(-bx\right)$
parts	$-\frac{\operatorname{arccoth}(\tanh(bx+a))}{x}+b\ln\left(-bx\right)$
risch	$-\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{x}+\frac{i\pi\ \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)-i\pi\ \operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}-2i\pi\ \operatorname{csgn}}{}$

input
```
int(arccoth(tanh(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*x*b-arccoth(tanh(b*x+a)))/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx=\frac{-i\,\pi+2\,bx\log\left(x\right)-2\,a}{2\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
1/2*(-I*pi + 2*b*x*log(x) - 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\log{(x)} - \frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x}$$

input
```
integrate(acoth(tanh(b*x+a))/x**2,x)
```

output
```
b*log(x) - acoth(tanh(a + b*x))/x
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\log{(x)} - \frac{\operatorname{arcoth}{(\tanh{(bx+a)})}}{x}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - arccoth(tanh(b*x + a))/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\log{(|x|)} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\,\ln(x) - \frac{\operatorname{acoth}(\tanh(a+b\,x))}{x}$$

input
```
int(acoth(tanh(a + b*x))/x^2,x)
```

output
```
b*log(x) - acoth(tanh(a + b*x))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = \frac{-acoth(\tanh(bx+a)) - \log(x)\,bx}{x}$$

input
```
int(acoth(tanh(b*x+a))/x^2,x)
```

output
```
( - (acoth(tanh(a + b*x)) + log(x)*b*x))/x
```

3.13 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx = -\frac{b}{2x} - \frac{\coth^{-1}(\tanh(a+bx))}{2x^2}$$

output
```
-1/2*b/x-1/2*arccoth(tanh(b*x+a))/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx = -\frac{bx+\coth^{-1}(\tanh(a+bx))}{2x^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

output
```
-1/2*(b*x + ArcCoth[Tanh[a + b*x]])/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2} b \int \frac{1}{x^2} dx - \frac{\coth^{-1}(\tanh(a+bx))}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2} - \frac{b}{2x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

output
```
-1/2*b/x - ArcCoth[Tanh[a + b*x]]/(2*x^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{bx+\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
default	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
parts	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{2x^2}-\frac{4bx-2i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^3+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2-i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3+i\pi\,\mathrm{csgn}\left(\frac{}{\mathrm{e}^{2b}}\right.}{}$

input
```
int(arccoth(tanh(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+arccoth(tanh(b*x+a)))/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = \frac{-i\,\pi - 4\,bx - 2\,a}{4\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
1/4*(-I*pi - 4*b*x - 2*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2x} - \frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{2x^2}$$

input
```
integrate(acoth(tanh(b*x+a))/x**3,x)
```

output
```
-b/(2*x) - acoth(tanh(a + b*x))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*b/x - 1/2*arccoth(tanh(b*x + a))/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{4\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="giac")
```

output
```
-1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^
(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

Mupad [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{\mathrm{acoth}(\tanh(a+b\,x))+b\,x}{2\,x^2}$$

input
```
int(acoth(tanh(a + b*x))/x^3,x)
```

output
```
-(acoth(tanh(a + b*x)) + b*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = \frac{-acoth(\tanh(bx+a))+bx}{2x^2}$$

input
```
int(acoth(tanh(b*x+a))/x^3,x)
```

output
```
( - acoth(tanh(a + b*x)) + b*x)/(2*x**2)
```

3.14 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx = -\frac{b}{6x^2} - \frac{\coth^{-1}(\tanh(a+bx))}{3x^3}$$

output
```
-1/6*b/x^2-1/3*arccoth(tanh(b*x+a))/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx = -\frac{bx + 2\coth^{-1}(\tanh(a+bx))}{6x^3}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x^4,x]
```

output
```
-1/6*(b*x + 2*ArcCoth[Tanh[a + b*x]])/x^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{3} b \int \frac{1}{x^3} dx - \frac{\coth^{-1}(\tanh(a+bx))}{3x^3}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x^4,x]
```

output
```
-1/6*b/x^2 - ArcCoth[Tanh[a + b*x]]/(3*x^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{bx+2\,\mathrm{arccoth}(\tanh(bx+a))}{6x^3}$
default	$-\frac{b}{6x^2}-\frac{\mathrm{arccoth}(\tanh(bx+a))}{3x^3}$
parts	$-\frac{b}{6x^2}-\frac{\mathrm{arccoth}(\tanh(bx+a))}{3x^3}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{3x^3}-\frac{2bx-i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3+2i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^2-i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csg}}{}$

input
```
int(arccoth(tanh(b*x+a))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+2*arccoth(tanh(b*x+a)))/x^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx=\frac{-i\,\pi-3\,bx-2\,a}{6\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="fricas")
```

output
```
1/6*(-I*pi - 3*b*x - 2*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6x^2} - \frac{\operatorname{acoth}(\tanh(a+bx))}{3x^3}$$

input
```
integrate(acoth(tanh(b*x+a))/x**4,x)
```

output
```
-b/(6*x**2) - acoth(tanh(a + b*x))/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6\,x^2} - \frac{\operatorname{arcoth}(\tanh(bx+a))}{3\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="maxima")
```

output
```
-1/6*b/x^2 - 1/3*arccoth(tanh(b*x + a))/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6\,x^2} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{6\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="giac")
```

output
```
-1/6*b/x^2 - 1/6*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((
e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^3
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{\operatorname{acoth}(\tanh(a+b\,x))}{3\,x^3} - \frac{b}{6\,x^2}$$

input
```
int(acoth(tanh(a + b*x))/x^4,x)
```

output
```
- acoth(tanh(a + b*x))/(3*x^3) - b/(6*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = \frac{-2acoth(\tanh(bx+a))+bx}{6x^3}$$

input
```
int(acoth(tanh(b*x+a))/x^4,x)
```

output
```
( - 2*acoth(tanh(a + b*x)) + b*x)/(6*x**3)
```

3.15 $\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}$$

output
```
2*b^2*x^(3+m)/(m^3+6*m^2+11*m+6)-2*b*x^(2+m)*arccoth(tanh(b*x+a))/(m^2+3*m
+2)+x^(1+m)*arccoth(tanh(b*x+a))^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{x^{1+m}(2b^2x^2 - 2b(3+m)x\coth^{-1}(\tanh(a+bx)) + (6+5m+m^2)\coth^{-1}(\tanh(a+bx))^2)}{(1+m)(2+m)(3+m)}$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcCoth[Tanh[a + b*x]] + (6 + 5*m +
m^2)*ArcCoth[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1}-\frac{2b\int x^{m+1}\coth^{-1}(\tanh(a+bx))dx}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1}-\frac{2b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))}{m+2}-\frac{b\int x^{m+2}dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1}-\frac{2b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))}{m+2}-\frac{bx^{m+3}}{(m+2)(m+3)}\right)}{m+1}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m) - (2*b*(-((b*x^(3 + m))/((2 +
 m)*(3 + m))) + (x^(2 + m)*ArcCoth[Tanh[a + b*x]])/(2 + m)))/(1 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

method	result
parallelrisch	$-\frac{-6 \operatorname{arccoth}(\tanh(bx+a))^2 x\, x^m + 2x^2 x^m \operatorname{arccoth}(\tanh(bx+a)) bm - 2b^2 x^m x^3 - x\, x^m \operatorname{arccoth}(\tanh(bx+a))^2 m^2 - 5x\, x^m \operatorname{arcc}}{(m^2+3m+2)(3+m)}$
risch	Expression too large to display

input
```
int(x^m*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-6*arccoth(tanh(b*x+a))^2*x*x^m+2*x^2*x^m*arccoth(tanh(b*x+a))*b*m-2*b^2
*x^m*x^3-x*x^m*arccoth(tanh(b*x+a))^2*m^2-5*x*x^m*arccoth(tanh(b*x+a))^2*m
+6*b*arccoth(tanh(b*x+a))*x^m*x^2)/(m^2+3*m+2)/(3+m)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.72

$$\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx =$$
$$-\frac{(\pi^2(m^2+5\,m+6)x - 4\,(b^2m^2+3\,b^2m+2\,b^2)x^3 - 8\,(abm^2+4\,abm+3\,ab)x^2 - 4i\,\pi((bm^2+4\,bm+}{}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-1/4*((pi^2*(m^2 + 5*m + 6)*x - 4*(b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 - 8*(a*b
*m^2 + 4*a*b*m + 3*a*b)*x^2 - 4*I*pi*((b*m^2 + 4*b*m + 3*b)*x^2 + (a*m^2 +
 5*a*m + 6*a)*x) - 4*(a^2*m^2 + 5*a^2*m + 6*a^2)*x)*cosh(m*log(x)) + (pi^2
*(m^2 + 5*m + 6)*x - 4*(b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 - 8*(a*b*m^2 + 4*a*
b*m + 3*a*b)*x^2 - 4*I*pi*((b*m^2 + 4*b*m + 3*b)*x^2 + (a*m^2 + 5*a*m + 6*
a)*x) - 4*(a^2*m^2 + 5*a^2*m + 6*a^2)*x)*sinh(m*log(x)))/(m^3 + 6*m^2 + 11
*m + 6)
```

Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx$$

$$= \begin{cases} b^2\log(x) - \frac{b\operatorname{acoth}(\tanh(a+bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2x^2} \\ \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x}\,dx \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m\operatorname{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m\operatorname{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m\operatorname{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m\operatorname{acoth}}{m^3+6m} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**
2/(2*x**2), Eq(m, -3)), (Integral(acoth(tanh(a + b*x))**2/x**2, x), Eq(m,
-2)), (Integral(acoth(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x*
*m/(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*acoth(tanh(a + b*x))/(m**3
 + 6*m**2 + 11*m + 6) - 6*b*x**2*x**m*acoth(tanh(a + b*x))/(m**3 + 6*m**2
+ 11*m + 6) + m**2*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m +
6) + 5*m*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x*x
**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{2\,b^2x^3x^m}{(m+3)(m+2)(m+1)} - \frac{2\,bx^2x^m\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{(m+2)(m+1)} + \frac{x^{m+1}\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
2*b^2*x^3*x^m/((m + 3)*(m + 2)*(m + 1)) - 2*b*x^2*x^m*arccoth(tanh(b*x + a
))/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))^2/(m + 1)
```

Giac [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx = \int x^m\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.86

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx$$
$$= \frac{4\,b^2\,x^m\,x^3\,(m^2+3\,m+2)}{4\,m^3+24\,m^2+44\,m+24}$$
$$+ \frac{x\,x^m\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2(m^2+5\,m+6)}{4\,m^3+24\,m^2+44\,m+24}$$
$$- \frac{4\,b\,x^m\,x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)(m^2+4\,m+3)}{4\,m^3+24\,m^2+44\,m+24}$$

input
```
int(x^m*acoth(tanh(a + b*x))^2,x)
```

output
```
(4*b^2*x^m*x^3*(3*m + m^2 + 2))/(44*m + 24*m^2 + 4*m^3 + 24) + (x*x^m*(log
(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x)^2*(5*m + m^2 + 6))/(44*m + 24*m^2 + 4*m^3 + 24) - (
4*b*x^m*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(4*m + m^2 + 3))/(44*m + 24*m^2 + 4
*m^3 + 24)
```

Reduce [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx = \int x^m acoth(\tanh(bx+a))^2\,dx$$

input
```
int(x^m*acoth(tanh(b*x+a))^2,x)
```

output
```
int(x**m*acoth(tanh(a + b*x))**2,x)
```

3.16 $\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{b^2x^6}{60} - \frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2$$

output
```
1/60*b^2*x^6-1/10*b*x^5*arccoth(tanh(b*x+a))+1/4*x^4*arccoth(tanh(b*x+a))^
2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{60}x^4\left(b^2x^2 - 6bx\coth^{-1}(\tanh(a+bx)) + 15\coth^{-1}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^4*(b^2*x^2 - 6*b*x*ArcCoth[Tanh[a + b*x]] + 15*ArcCoth[Tanh[a + b*x]]^2
))/60
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b \int x^4 \coth^{-1}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^5dx}{5}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx)) - \frac{bx^6}{30}\right)$$

input
```
Int[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^4*ArcCoth[Tanh[a + b*x]]^2)/4 - (b*(-1/30*(b*x^6) + (x^5*ArcCoth[Tanh[a
 + b*x]])/5))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 24.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{b^{2} x^{6}}{60}-\frac{b\,x^{5} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{10}+\frac{x^{4} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{4}$	37
risch	Expression too large to display	3418

input
```
int(x^3*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*b^2*x^6-1/10*b*x^5*arccoth(tanh(b*x+a))+1/4*x^4*arccoth(tanh(b*x+a))^
2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 - \frac{1}{16}\,\pi^2x^4 + \frac{1}{4}\,a^2x^4 + \frac{1}{20}i\,\pi\left(4\,bx^5 + 5\,ax^4\right)$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 - 1/16*pi^2*x^4 + 1/4*a^2*x^4 + 1/20*I*pi*(4*b*x
^5 + 5*a*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{x^3 \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{x^2 \operatorname{acoth}^4(\tanh(a+bx))}{4b^2} + \frac{x \operatorname{acoth}^5(\tanh(a+bx))}{10b^3} - \frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}^2(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x**3*acoth(tanh(a + b*x))**3/(3*b) - x**2*acoth(tanh(a + b*x))*
*4/(4*b**2) + x*acoth(tanh(a + b*x))**5/(10*b**3) - acoth(tanh(a + b*x))**
6/(60*b**4), Ne(b, 0)), (x**4*acoth(tanh(a))**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{60}\,b^2x^6 - \frac{1}{10}\,bx^5\,\operatorname{arcoth}(\tanh(bx+a)) + \frac{1}{4}\,x^4\,\operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/60*b^2*x^6 - 1/10*b*x^5*arccoth(tanh(b*x + a)) + 1/4*x^4*arccoth(tanh(b*
x + a))^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{6} b^2 x^6 - \frac{1}{5} (-i \pi b - 2 ab) x^5 - \frac{1}{16} (\pi^2 - 4i \pi a - 4 a^2) x^4$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 - 1/5*(-I*pi*b - 2*a*b)*x^5 - 1/16*(pi^2 - 4*I*pi*a - 4*a^2)*x
^4
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2 x^6}{60} - \frac{b x^5 \operatorname{acoth}(\tanh(a+b x))}{10} + \frac{x^4 \operatorname{acoth}(\tanh(a+b x))^2}{4}$$

input
```
int(x^3*acoth(tanh(a + b*x))^2,x)
```

output
```
(x^4*acoth(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*acoth(tanh(a + b*x)
))/10
```

Reduce [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \int acoth(\tanh(bx+a))^2 x^3 dx$$

input
```
int(x^3*acoth(tanh(b*x+a))^2,x)
```

output
```
int(acoth(tanh(a + b*x))**2*x**3,x)
```

3.17 $\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2x^5}{30} - \frac{1}{6}bx^4 \coth^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2$$

output
```
1/30*b^2*x^5-1/6*b*x^4*arccoth(tanh(b*x+a))+1/3*x^3*arccoth(tanh(b*x+a))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{30}x^3 \left(b^2x^2 - 5bx \coth^{-1}(\tanh(a+bx)) + 10 \coth^{-1}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^3*(b^2*x^2 - 5*b*x*ArcCoth[Tanh[a + b*x]] + 10*ArcCoth[Tanh[a + b*x]]^2
))/30
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{3}b \int x^3 \coth^{-1}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{3}b\left(\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^4 dx}{4}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{3}b\left(\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx)) - \frac{bx^5}{20}\right)$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^3*ArcCoth[Tanh[a + b*x]]^2)/3 - (2*b*(-1/20*(b*x^5) + (x^4*ArcCoth[Tanh
[a + b*x]])/4))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 23.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{b^2x^5}{30}-\frac{b\,x^4\operatorname{arccoth}(\tanh(bx+a))}{6}+\frac{x^3\operatorname{arccoth}(\tanh(bx+a))^2}{3}$	37
risch	Expression too large to display	3418

input
```
int(x^2*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*b^2*x^5-1/6*b*x^4*arccoth(tanh(b*x+a))+1/3*x^3*arccoth(tanh(b*x+a))^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int x^2\coth^{-1}(\tanh(a+bx))^2\,dx=\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4-\frac{1}{12}\,\pi^2x^3+\frac{1}{3}\,a^2x^3+\frac{1}{12}i\,\pi\left(3\,bx^4+4\,ax^3\right)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 - 1/12*pi^2*x^3 + 1/3*a^2*x^3 + 1/12*I*pi*(3*b*x
^4 + 4*a*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{x^2 \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{x \operatorname{acoth}^4(\tanh(a+bx))}{6b^2} + \frac{\operatorname{acoth}^5(\tanh(a+bx))}{30b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^2(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x**2*acoth(tanh(a + b*x))**3/(3*b) - x*acoth(tanh(a + b*x))**4/
(6*b**2) + acoth(tanh(a + b*x))**5/(30*b**3), Ne(b, 0)), (x**3*acoth(tanh(
a))**2/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{30}\,b^2x^5 - \frac{1}{6}\,bx^4 \operatorname{arcoth}(\tanh(bx+a)) + \frac{1}{3}\,x^3 \operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/30*b^2*x^5 - 1/6*b*x^4*arccoth(tanh(b*x + a)) + 1/3*x^3*arccoth(tanh(b*x
 + a))^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{5}\,b^2x^5 - \frac{1}{4}\,(-i\,\pi b - 2\,ab)x^4 - \frac{1}{12}\,(\pi^2 - 4i\,\pi a - 4\,a^2)x^3$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 - 1/4*(-I*pi*b - 2*a*b)*x^4 - 1/12*(pi^2 - 4*I*pi*a - 4*a^2)*x
^3
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{b^2\,x^5}{30} - \frac{b\,x^4\,\mathrm{acoth}(\tanh(a+b\,x))}{6} + \frac{x^3\,\mathrm{acoth}(\tanh(a+b\,x))^2}{3}$$

input
```
int(x^2*acoth(tanh(a + b*x))^2,x)
```

output
```
(x^3*acoth(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*acoth(tanh(a + b*x)
))/6
```

Reduce [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx = \int acoth(\tanh(bx+a))^2 x^2 dx$$

input
```
int(x^2*acoth(tanh(b*x+a))^2,x)
```

output
```
int(acoth(tanh(a + b*x))**2*x**2,x)
```

3.18 $\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

output
```
1/3*x*arccoth(tanh(b*x+a))^3/b-1/12*arccoth(tanh(b*x+a))^4/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$$
$$= \frac{(a+bx)\left(-((3a-bx)(a+bx)^2)+4(2a^2+abx-b^2x^2)\coth^{-1}(\tanh(a+bx))-6(a-bx)\coth^{-1}(\tanh(\right.}{12b^2}$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcCo
th[Tanh[a + b*x]] - 6*(a - b*x)*ArcCoth[Tanh[a + b*x]]^2))/(12*b^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\int \coth^{-1}(\tanh(a+bx))^3 dx}{3b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\int \coth^{-1}(\tanh(a+bx))^3 d \coth^{-1}(\tanh(a+bx))}{3b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x*ArcCoth[Tanh[a + b*x]]^3)/(3*b) - ArcCoth[Tanh[a + b*x]]^4/(12*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 24.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{b^{2} x^{4}}{12}+\frac{x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{2}-\frac{b \,x^{3} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{3}$	37
risch	Expression too large to display	3418

input
```
int(x*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*x^4+1/2*x^2*arccoth(tanh(b*x+a))^2-1/3*b*x^3*arccoth(tanh(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{4} \, b^{2} x^{4} + \frac{2}{3} \, a b x^{3} - \frac{1}{8} \, \pi^{2} x^{2} + \frac{1}{2} \, a^{2} x^{2} + \frac{1}{6} i \, \pi \left(2 \, b x^{3} + 3 \, a x^{2}\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 - 1/8*pi^2*x^2 + 1/2*a^2*x^2 + 1/6*I*pi*(2*b*x^3
 + 3*a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x \coth^{-1}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{x \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{\operatorname{acoth}^4(\tanh(a+bx))}{12b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}^2(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x*acoth(tanh(a + b*x))**3/(3*b) - acoth(tanh(a + b*x))**4/(12*b
**2), Ne(b, 0)), (x**2*acoth(tanh(a))**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{12}\,b^2x^4 - \frac{1}{3}\,bx^3 \operatorname{arcoth}(\tanh(bx+a)) + \frac{1}{2}\,x^2 \operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*x^4 - 1/3*b*x^3*arccoth(tanh(b*x + a)) + 1/2*x^2*arccoth(tanh(b*x
 + a))^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{4}\,b^2x^4 - \frac{1}{3}\,(-i\,\pi b - 2\,ab)x^3 - \frac{1}{8}\,(\pi^2 - 4i\,\pi a - 4\,a^2)x^2$$

input
```
integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 - 1/3*(-I*pi*b - 2*a*b)*x^3 - 1/8*(pi^2 - 4*I*pi*a - 4*a^2)*x^
2
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x\coth^{-1}(\tanh(a+bx))^2\,dx=\frac{b^2\,x^4}{12}-\frac{b\,x^3\,\mathrm{acoth}(\tanh(a+b\,x))}{3}+\frac{x^2\,\mathrm{acoth}(\tanh(a+b\,x))^2}{2}$$

input
```
int(x*acoth(tanh(a + b*x))^2,x)
```

output
```
(x^2*acoth(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*acoth(tanh(a + b*x)
))/3
```

Reduce [F]

$$\int x\coth^{-1}(\tanh(a+bx))^2\,dx=\int acoth(\tanh(bx+a))^2\,xdx$$

input
```
int(x*acoth(tanh(b*x+a))^2,x)
```

output
```
int(acoth(tanh(a + b*x))**2*x,x)
```

3.19 $\int \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

output
```
1/3*arccoth(tanh(b*x+a))^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))^2 d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3b}$	15
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3b}$	15
parallelrisch	$\frac{b^2x^3}{3} - b\,x^2 \operatorname{arccoth}(\tanh(bx+a)) + x \operatorname{arccoth}(\tanh(bx+a))^2$	34
risch	Expression too large to display	14844

input
```
int(arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*arccoth(tanh(b*x+a))^3/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}\,b^2x^3 + abx^2 - \frac{1}{4}\,\pi^2x + a^2x + \frac{1}{2}i\,\pi\left(bx^2 + 2\,ax\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 - 1/4*pi^2*x + a^2*x + 1/2*I*pi*(b*x^2 + 2*a*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3b} & \text{for } b \neq 0 \\ x\operatorname{acoth}^2(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((acoth(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*acoth(tanh(a))**2,
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}b^2x^3 - bx^2\operatorname{arcoth}(\tanh(bx+a)) + x\operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 - b*x^2*arccoth(tanh(b*x + a)) + x*arccoth(tanh(b*x + a))^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}b^2x^3 - \frac{1}{2}(-i\pi b - 2ab)x^2 - \frac{1}{4}(\pi^2 - 4i\pi a - 4a^2)x$$

input
```
integrate(arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 - 1/2*(-I*pi*b - 2*a*b)*x^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{b^2\,x^3}{3}-b\,x^2\,\mathrm{acoth}(\tanh(a+b\,x))+x\,\mathrm{acoth}(\tanh(a+b\,x))^2$$

input
```
int(acoth(tanh(a + b*x))^2,x)
```

output
```
x*acoth(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*acoth(tanh(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx=-\frac{acoth(\tanh(bx+a))^3}{3b}$$

input
```
int(acoth(tanh(b*x+a))^2,x)
```

output
```
( - acoth(tanh(a + b*x))**3)/(3*b)
```

3.20 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = -bx\left(bx-\coth^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\coth^{-1}(\tanh(a+bx))^2 + \left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log(x)$$

output
```
-b*x*(b*x-arccoth(tanh(b*x+a)))+1/2*arccoth(tanh(b*x+a))^2+(b*x-arccoth(ta
nh(b*x+a)))^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}(a+bx)^2-(a+bx)\left(a+2bx-2\coth^{-1}(\tanh(a+bx))\right) + \left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log(bx)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

output
```
(a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcCoth[Tanh[a + b*x]]) + (-(b*x)
 + ArcCoth[Tanh[a + b*x]])^2*Log[b*x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx$$

$$\Big\downarrow \text{2589}$$

$$\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^2/2 - (b*x - ArcCoth[Tanh[a + b*x]])*(b*x - (b*x -
ArcCoth[Tanh[a + b*x]])*Log[x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 672, normalized size of antiderivative = 13.71

method	result
risch	$\ln\left(x\right)\ln\left(\mathrm{e}^{bx+a}\right)^{2}+b^{2}x^{2}\ln\left(x\right)-\frac{3b^{2}x^{2}}{2}-2b\ln\left(x\right)\ln\left(\mathrm{e}^{bx+a}\right)x+2b\ln\left(\mathrm{e}^{bx+a}\right)x-\frac{\pi^{2}\left(2\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}}\right.\right.}{}$

input
```
int(arccoth(tanh(b*x+a))^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(exp(b*x+a))^2+b^2*x^2*ln(x)-3/2*b^2*x^2-2*b*ln(x)*ln(exp(b*x+a))*
x+2*b*ln(exp(b*x+a))*x-1/16*Pi^2*(2*csgn(I/(exp(2*b*x+2*a)+1))^2-2*csgn(I/
(exp(2*b*x+2*a)+1))^3-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))+2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-csgn(I*exp(2*b*x+2*a)
)^3+csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-2)^2*ln(x)+1/2*I*Pi*(2*csgn(I/(ex
p(2*b*x+2*a)+1))^2-2*csgn(I/(exp(2*b*x+2*a)+1))^3-csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-csgn(I*ex
p(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^2-csgn(I*exp(2*b*x+2*a))^3+csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2-csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-2)*(l
n(x)*ln(exp(b*x+a))-b*(ln(x)*x-x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + i\,\pi bx + 2\,abx - \frac{1}{4}\left(\pi^2 - 4i\,\pi a - 4\,a^2\right)\log\left(x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

output
```
1/2*b^2*x^2 + I*pi*b*x + 2*a*b*x - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \int \frac{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**2/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 - (i\,\pi b - 2\,ab)x - \frac{1}{4}\left(\pi^2 + 4i\,\pi a - 4\,a^2\right)\log\left(x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 - (I*pi*b - 2*a*b)*x - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + (i\,\pi b + 2\,ab)x - \frac{1}{4}\left(\pi^2 - 4i\,\pi a - 4\,a^2\right)\log\left(x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + (I*pi*b + 2*a*b)*x - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.73

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx$$
$$= \ln(x)\left(\frac{\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2bx\right)^2}{4} - a\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2bx\right)+a^2\right) + \frac{b^2x^2}{2} - bx\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2bx\right)$$

input
```
int(acoth(tanh(a + b*x))^2/x,x)
```

output
```
log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/4 - a*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x) + a^2) + (b^2*x^2)/2 - b*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \int \frac{acoth(\tanh(bx+a))^2}{x}dx$$

input
```
int(acoth(tanh(b*x+a))^2/x,x)
```

output
```
int(acoth(tanh(a + b*x))**2/x,x)
```

3.21 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx = 2b^2x - \frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b\left(bx - \coth^{-1}(\tanh(a+bx))\right)\log(x)$$

output
```
2*b^2*x-arccoth(tanh(b*x+a))^2/x-2*b*(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx = -\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b^2x\log(x) + 2b\coth^{-1}(\tanh(a+bx))(1+\log(x))$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcCoth[Tanh[a + b*x]
]*(1 + Log[x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}dx$$

$$\Big\downarrow \text{ 2599}$$

$$2b\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}$$

$$\Big\downarrow \text{ 2589}$$

$$2b\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}$$

$$\Big\downarrow \text{ 14}$$

$$2b\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^2/x) + 2*b*(b*x - (b*x - ArcCoth[Tanh[a + b*x]])*
Log[x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 655, normalized size of antiderivative = 16.79

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{b x+a}\right)^{2}}{x}-2\ln\left(x\right) x\,b^{2}+2\ln\left(x\right)\ln\left(\mathrm{e}^{b x+a}\right) b+2b^{2}x+\frac{\pi^{2}\left(2\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2 b x+2 a}+1}\right)^{2}-2\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2 b x+2 a}+1}\right)^{3}-\mathrm{cs}\right.}{}$

input
```
int(arccoth(tanh(b*x+a))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln(exp(b*x+a))^2-2*ln(x)*x*b^2+2*ln(x)*ln(exp(b*x+a))*b+2*b^2*x+1/16*
Pi^2*(2*csgn(I/(exp(2*b*x+2*a)+1))^2-2*csgn(I/(exp(2*b*x+2*a)+1))^3-csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2-csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*csgn(I*exp(b*x+a))*
csgn(I*exp(2*b*x+2*a))^2-csgn(I*exp(2*b*x+2*a))^3+csgn(I*exp(2*b*x+2*a))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3-2)^2/x+1/2*I*Pi*(2*csgn(I/(exp(2*b*x+2*a)+1))^2-2*csgn(I/(exp
(2*b*x+2*a)+1))^3-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))+2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-csgn(I*exp(2*b*x+2*a))^3+
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-2)*(-1/x*ln(exp(b*x+a))+b*ln(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = \frac{4\,b^2x^2+\pi^2-4i\,\pi a-4\,a^2-4\,(-i\,\pi bx-2\,abx)\log{(x)}}{4\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*x^2 + pi^2 - 4*I*pi*a - 4*a^2 - 4*(-I*pi*b*x - 2*a*b*x)*log(x))
/x
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = \int \frac{\operatorname{acoth}^2{(\tanh{(a+bx)})}}{x^2}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**2,x)
```

output
```
Integral(acoth(tanh(a + b*x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = 2\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right) - 2\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{x}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

output
```
2*b*arccoth(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x
)/b))*b - arccoth(tanh(b*x + a))^2/x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = b^2x + (i\,\pi b + 2\,ab)\log(x) + \frac{\pi^2 - 4i\,\pi a - 4\,a^2}{4\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

output
```
b^2*x + (I*pi*b + 2*a*b)*log(x) + 1/4*(pi^2 - 4*I*pi*a - 4*a^2)/x
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.31

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = {} & b\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{4\,x} \\ & - b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{4\,x} \\ & + b\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\ln(x) \\ & - b\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\ln(x) \\ & + \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2\,x} - 2\,b^2\,x\,\ln(x)\end{aligned}$$

input
```
int(acoth(tanh(a + b*x))^2/x^2,x)
```

output
```
b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) - 1)) -
log(-2/(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) + b*log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1))*log(x) - b*log(-2/(exp(2*a)*exp(2*b*x) - 1))*lo
g(x) + (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp
(2*a)*exp(2*b*x) - 1)))/(2*x) - 2*b^2*x*log(x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = \int \frac{acoth(\tanh(bx+a))^2}{x^2}dx$$

input
```
int(acoth(tanh(b*x+a))^2/x^2,x)
```

output
```
int(acoth(tanh(a + b*x))**2/x**2,x)
```

3.22 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx = -\frac{b\coth^{-1}(\tanh(a+bx))}{x} - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} + b^2\log(x)$$

output
```
-b*arccoth(tanh(b*x+a))/x-1/2*arccoth(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx = -\frac{2bx\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 - b^2x^2(3+2\log(x))}{2x^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

output
```
-1/2*(2*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 - b^2*x^2*(3
 + 2*Log[x]))/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$b \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$b \left(b \int \frac{1}{x} dx - \frac{\coth^{-1}(\tanh(a+bx))}{x} \right) - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$b \left(b \log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x} \right) - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

output
```
-1/2*ArcCoth[Tanh[a + b*x]]^2/x^2 + b*(-(ArcCoth[Tanh[a + b*x]]/x) + b*Log
[x])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{2b^2x^2\ln(x)-2bx\ \operatorname{arccoth}(\tanh(bx+a))-\operatorname{arccoth}(\tanh(bx+a))^2}{2x^2}$	39
risch	Expression too large to display	3213

input
```
int(arccoth(tanh(b*x+a))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b^2*x^2*ln(x)-2*b*x*arccoth(tanh(b*x+a))-arccoth(tanh(b*x+a))^2)/x^
2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = \frac{8\,b^2x^2\log(x) - 16\,abx + \pi^2 - 4i\,\pi(2\,bx + a) - 4\,a^2}{8\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(8*b^2*x^2*log(x) - 16*a*b*x + pi^2 - 4*I*pi*(2*b*x + a) - 4*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx$$
$$= b^2\log(x) - \frac{b\,\mathrm{acoth}\,(\tanh(a+bx))}{x} - \frac{\mathrm{acoth}^2\,(\tanh(a+bx))}{2x^2}$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**3,x)
```

output
```
b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**2/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\log(x) - \frac{b\,\mathrm{arcoth}\,(\tanh(bx+a))}{x} - \frac{\mathrm{arcoth}\,(\tanh(bx+a))^2}{2\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
```

output
```
b^2*log(x) - b*arccoth(tanh(b*x + a))/x - 1/2*arccoth(tanh(b*x + a))^2/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\log(x) - \frac{8i\,\pi bx + 16\,abx - \pi^2 + 4i\,\pi a + 4\,a^2}{8\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="giac")
```

output
```
b^2*log(x) - 1/8*(8*I*pi*b*x + 16*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\ \ln(x) - \frac{\frac{\mathrm{acoth}(\tanh(a+b\,x))^2}{2} + b\,x\,\mathrm{acoth}(\tanh(a+b\,x))}{x^2}$$

input
```
int(acoth(tanh(a + b*x))^2/x^3,x)
```

output
```
b^2*log(x) - (acoth(tanh(a + b*x))^2/2 + b*x*acoth(tanh(a + b*x)))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = \frac{-acoth(\tanh(bx+a))^2 + 2acoth(\tanh(bx+a))\,bx + 2\log(x)\,b^2x^2}{2x^2}$$

input
```
int(acoth(tanh(b*x+a))^2/x^3,x)
```

output
```
( - acoth(tanh(a + b*x))**2 + 2*acoth(tanh(a + b*x))*b*x + 2*log(x)*b**2*x
**2)/(2*x**2)
```

3.23 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/3*arccoth(tanh(b*x+a))^3/x^3/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2x^2+bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2}{3x^3}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

output
```
-1/3*(b^2*x^2 + b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2)/x^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} \, dx$$

$$\Big\downarrow \text{2598}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3 \left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$-\frac{b^{2} x^{2}+b x \,\operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)+\operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{3 x^{3}}$	33
risch	Expression too large to display	3217

input
```
int(arccoth(tanh(b*x+a))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b^2*x^2+b*x*arccoth(tanh(b*x+a))+arccoth(tanh(b*x+a))^2)/x^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{12\,b^2x^2+12\,abx-\pi^2+2i\,\pi(3\,bx+2\,a)+4\,a^2}{12\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

output
```
-1/12*(12*b^2*x^2 + 12*a*b*x - pi^2 + 2*I*pi*(3*b*x + 2*a) + 4*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2}{3x}-\frac{b\,\mathrm{acoth}\,(\tanh\,(a+bx))}{3x^2}-\frac{\mathrm{acoth}^2\,(\tanh\,(a+bx))}{3x^3}$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**4,x)
```

output
```
-b**2/(3*x) - b*acoth(tanh(a + b*x))/(3*x**2) - acoth(tanh(a + b*x))**2/(3
*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2}{3\,x} - \frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{3\,x^2} - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^2/x - 1/3*b*arccoth(tanh(b*x + a))/x^2 - 1/3*arccoth(tanh(b*x + a))
^2/x^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{12\,b^2x^2 + 6i\,\pi bx + 12\,abx - \pi^2 + 4i\,\pi a + 4\,a^2}{12\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

output
```
-1/12*(12*b^2*x^2 + 6*I*pi*b*x + 12*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2\,x^2 + b\,x\,\mathrm{acoth}(\tanh(a+b\,x)) + \mathrm{acoth}(\tanh(a+b\,x))^2}{3\,x^3}$$

input
```
int(acoth(tanh(a + b*x))^2/x^4,x)
```

output
```
-(acoth(tanh(a + b*x))^2 + b^2*x^2 + b*x*acoth(tanh(a + b*x)))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = \frac{-acoth(\tanh(bx+a))^2 + acoth(\tanh(bx+a))\,bx - b^2x^2}{3x^3}$$

input
```
int(acoth(tanh(b*x+a))^2/x^4,x)
```

output
```
( - acoth(tanh(a + b*x))**2 + acoth(tanh(a + b*x))*b*x - b**2*x**2)/(3*x**
3)
```

3.24 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx = \frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/12*b*arccoth(tanh(b*x+a))^3/x^3/(b*x-arccoth(tanh(b*x+a)))^2+1/4*arccoth
(tanh(b*x+a))^3/x^4/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx = -\frac{b^2x^2+2bx\coth^{-1}(\tanh(a+bx))+3\coth^{-1}(\tanh(a+bx))^2}{12x^4}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

output
```
-1/12*(b^2*x^2 + 2*b*x*ArcCoth[Tanh[a + b*x]] + 3*ArcCoth[Tanh[a + b*x]]^2
)/x^4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}dx}{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{2598}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

output
```
(b*ArcCoth[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcCoth[Tanh[a + b*x]])^2) + A
rcCoth[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{b^{2} x^{2}+2 b x \,\operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)+3 \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{12 x^{4}}$	36
risch	Expression too large to display	3217

input
```
int(arccoth(tanh(b*x+a))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b^2*x^2+2*b*x*arccoth(tanh(b*x+a))+3*arccoth(tanh(b*x+a))^2)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{24\,b^2x^2+32\,abx-3\,\pi^2+4i\,\pi(4\,bx+3\,a)+12\,a^2}{48\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

output
```
-1/48*(24*b^2*x^2 + 32*a*b*x - 3*pi^2 + 4*I*pi*(4*b*x + 3*a) + 12*a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12x^2} - \frac{b\,\mathrm{acoth}\,(\tanh\,(a+bx))}{6x^3} - \frac{\mathrm{acoth}^2\,(\tanh\,(a+bx))}{4x^4}$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**5,x)
```

output
```
-b**2/(12*x**2) - b*acoth(tanh(a + b*x))/(6*x**3) - acoth(tanh(a + b*x))**
2/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12\,x^2} - \frac{b\,\mathrm{arcoth}\,(\tanh\,(bx+a))}{6\,x^3} - \frac{\mathrm{arcoth}\,(\tanh\,(bx+a))^2}{4\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

output
```
-1/12*b^2/x^2 - 1/6*b*arccoth(tanh(b*x + a))/x^3 - 1/4*arccoth(tanh(b*x +
a))^2/x^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{24\,b^2x^2+16i\,\pi bx+32\,abx-3\,\pi^2+12i\,\pi a+12\,a^2}{48\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

output
```
-1/48*(24*b^2*x^2 + 16*I*pi*b*x + 32*a*b*x - 3*pi^2 + 12*I*pi*a + 12*a^2)/
x^4
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{\mathrm{acoth}(\tanh(a+b\,x))^2}{4\,x^4}-\frac{b^2}{12\,x^2}-\frac{b\,\mathrm{acoth}(\tanh(a+b\,x))}{6\,x^3}$$

input
```
int(acoth(tanh(a + b*x))^2/x^5,x)
```

output
```
- acoth(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*acoth(tanh(a + b*x)))
/(6*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = \frac{-3acoth(\tanh(bx+a))^2+2acoth(\tanh(bx+a))\,bx-b^2x^2}{12x^4}$$

input
```
int(acoth(tanh(b*x+a))^2/x^5,x)
```

output
```
( - 3*acoth(tanh(a + b*x))**2 + 2*acoth(tanh(a + b*x))*b*x - b**2*x**2)/(1
2*x**4)
```

3.25 $\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = &-\frac{6b^3x^{4+m}}{(1+m)\,(24+26m+9m^2+m^3)}\\ &+\frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}\\ &-\frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2}\\ &+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^3}{1+m}\end{aligned}$$

output
```
-6*b^3*x^(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^(3+m)*arccoth(tanh(b*x+a)
)/(m^3+6*m^2+11*m+6)-3*b*x^(2+m)*arccoth(tanh(b*x+a))^2/(m^2+3*m+2)+x^(1+m
)*arccoth(tanh(b*x+a))^3/(1+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx$$
$$= \frac{x^{1+m}\left(-6b^3x^3 + 6b^2(4+m)x^2\coth^{-1}(\tanh(a+bx)) - 3b(12+7m+m^2)\,x\coth^{-1}(\tanh(a+bx))^2 + (24}{(1+m)(2+m)(3+m)(4+m)}$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*(1
2 + 7*m + m^2)*x*ArcCoth[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcC
oth[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1} - \frac{3b\int x^{m+1}\coth^{-1}(\tanh(a+bx))^2dx}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1} - \frac{3b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m+2} - \frac{2b\int x^{m+2}\coth^{-1}(\tanh(a+bx))dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1} - \frac{3b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m+2} - \frac{2b\left(\frac{x^{m+3}\coth^{-1}(\tanh(a+bx))}{m+3} - \frac{b\int x^{m+3}dx}{m+3}\right)}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1} - \frac{3b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m+2} - \frac{2b\left(\frac{x^{m+3}\coth^{-1}(\tanh(a+bx))}{m+3} - \frac{bx^{m+4}}{(m+3)(m+4)}\right)}{m+2}\right)}{m+1}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 + m) - (3*b*((x^(2 + m)*ArcCoth[Ta
nh[a + b*x]]^2)/(2 + m) - (2*b*(-((b*x^(4 + m))/((3 + m)*(4 + m))) + (x^(3
 + m)*ArcCoth[Tanh[a + b*x]])/(3 + m)))/(2 + m)))/(1 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{36b\operatorname{arccoth}(\tanh(bx+a))^{2}x^{m}x^{2}-24b^{2}\operatorname{arccoth}(\tanh(bx+a))x^{m}x^{3}-x\,x^{m}\operatorname{arccoth}(\tanh(bx+a))^{3}m^{3}-9x\,x^{m}\operatorname{arccoth}(\tanh\ldots}{}$
risch	Expression too large to display

input
```
int(x^m*arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-(36*b*arccoth(tanh(b*x+a))^2*x^m*x^2-24*b^2*arccoth(tanh(b*x+a))*x^m*x^3-
x*x^m*arccoth(tanh(b*x+a))^3*m^3-9*x*x^m*arccoth(tanh(b*x+a))^3*m^2-26*x*x
^m*arccoth(tanh(b*x+a))^3*m-6*x^3*x^m*arccoth(tanh(b*x+a))*b^2*m+3*x^2*x^m
*arccoth(tanh(b*x+a))^2*b*m^2+21*x^2*x^m*arccoth(tanh(b*x+a))^2*b*m-24*arc
coth(tanh(b*x+a))^3*x^m*x+6*b^3*x^m*x^4)/(1+m)/(m^2+5*m+6)/(4+m)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 627, normalized size of antiderivative = 5.70

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx$$
$$= \frac{(-i\pi^3(m^3+9m^2+26m+24)x+8(b^3m^3+6b^3m^2+11b^3m+6b^3)x^4+24(ab^2m^3+7ab^2m^2+14ab\ldots}{}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/8*((-I*pi^3*(m^3 + 9*m^2 + 26*m + 24)*x + 8*(b^3*m^3 + 6*b^3*m^2 + 11*b^
3*m + 6*b^3)*x^4 + 24*(a*b^2*m^3 + 7*a*b^2*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3
 - 6*pi^2*((b*m^3 + 8*b*m^2 + 19*b*m + 12*b)*x^2 + (a*m^3 + 9*a*m^2 + 26*a
*m + 24*a)*x) + 24*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*a^2*b)*x^2 +
 12*I*pi*((b^2*m^3 + 7*b^2*m^2 + 14*b^2*m + 8*b^2)*x^3 + 2*(a*b*m^3 + 8*a*
b*m^2 + 19*a*b*m + 12*a*b)*x^2 + (a^2*m^3 + 9*a^2*m^2 + 26*a^2*m + 24*a^2)
*x) + 8*(a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*cosh(m*log(x)) + (-I*
pi^3*(m^3 + 9*m^2 + 26*m + 24)*x + 8*(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b
^3)*x^4 + 24*(a*b^2*m^3 + 7*a*b^2*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3 - 6*pi^2
*((b*m^3 + 8*b*m^2 + 19*b*m + 12*b)*x^2 + (a*m^3 + 9*a*m^2 + 26*a*m + 24*a
)*x) + 24*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*a^2*b)*x^2 + 12*I*pi*
((b^2*m^3 + 7*b^2*m^2 + 14*b^2*m + 8*b^2)*x^3 + 2*(a*b*m^3 + 8*a*b*m^2 + 1
9*a*b*m + 12*a*b)*x^2 + (a^2*m^3 + 9*a^2*m^2 + 26*a^2*m + 24*a^2)*x) + 8*(
a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*sinh(m*log(x)))/(m^4 + 10*m^3
+ 35*m^2 + 50*m + 24)
```

Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx$$

$$= \begin{cases} b^3\log(x) - \frac{b^2 \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{b \operatorname{acoth}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3x^3} \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^3}\,dx \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x}\,dx \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m \operatorname{acoth}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m \operatorname{acoth}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m \operatorname{acoth}^2(\tanh(a+bx)}{m^4+10m^3+35m^2+50m+24} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*
x))**2/(2*x**2) - acoth(tanh(a + b*x))**3/(3*x**3), Eq(m, -4)), (Integral(
acoth(tanh(a + b*x))**3/x**3, x), Eq(m, -3)), (Integral(acoth(tanh(a + b*x
))**3/x**2, x), Eq(m, -2)), (Integral(acoth(tanh(a + b*x))**3/x, x), Eq(m,
 -1)), (-6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**2*
m*x**3*x**m*acoth(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*b**2*x**3*x**m*acoth(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m +
24) - 3*b*m**2*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2
 + 50*m + 24) - 21*b*m*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 +
 35*m**2 + 50*m + 24) - 36*b*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*
m**3 + 35*m**2 + 50*m + 24) + m**3*x*x**m*acoth(tanh(a + b*x))**3/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + 9*m**2*x*x**m*acoth(tanh(a + b*x))**3/(m*
*4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*m*x*x**m*acoth(tanh(a + b*x))**3/
(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*x*x**m*acoth(tanh(a + b*x))**3
/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{3\,bx^2x^m\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{(m+2)(m+1)} + \frac{x^{m+1}\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{m+1} - \frac{6\left(\frac{b^2x^4x^m}{(m+4)(m+3)(m+2)} - \frac{bx^3x^m\,\mathrm{arcoth}(\tanh(bx+a))}{(m+3)(m+2)}\right)b}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3*b*x^2*x^m*arccoth(tanh(b*x + a))^2/((m + 2)*(m + 1)) + x^(m + 1)*arccot
h(tanh(b*x + a))^3/(m + 1) - 6*(b^2*x^4*x^m/((m + 4)*(m + 3)*(m + 2)) - b*
x^3*x^m*arccoth(tanh(b*x + a))/((m + 3)*(m + 2)))*b/(m + 1)
```

Giac [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.02

$$\begin{aligned}
&\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx \\
&= \frac{8\,b^3\,x^m\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\
&\quad - \frac{x\,x^m\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3\,\left(m^3+9\,m^2+26\,m+24\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\
&\quad - \frac{12\,b^2\,x^m\,x^3\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(m^3+7\,m^2+14\,m+8\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\
&\quad + \frac{6\,b\,x^m\,x^2\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2\,\left(m^3+8\,m^2+19\,m+12\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}
\end{aligned}$$

input
```
int(x^m*acoth(tanh(a + b*x))^3,x)
```

output
```
(8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4
 + 192) - (x*x^m*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(
400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(log(-2/(exp(2*a
)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + 2*b*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 1
92) + (6*b*x^m*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))
/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192)
```

Reduce [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = \int x^m acoth(\tanh(bx+a))^3\,dx$$

input
```
int(x^m*acoth(tanh(b*x+a))^3,x)
```

output
```
int(x**m*acoth(tanh(a + b*x))**3,x)
```

3.26 $\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{280}b^3x^8 + \frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3$$

output
```
-1/280*b^3*x^8+1/35*b^2*x^7*arccoth(tanh(b*x+a))-1/10*b*x^6*arccoth(tanh(b
*x+a))^2+1/5*x^5*arccoth(tanh(b*x+a))^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{280}x^5\left(b^3x^3 - 8b^2x^2\coth^{-1}(\tanh(a+bx)) + 28bx\coth^{-1}(\tanh(a+bx))^2 - 56\coth^{-1}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/280*(x^5*(b^3*x^3 - 8*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 28*b*x*ArcCoth[T
anh[a + b*x]]^2 - 56*ArcCoth[Tanh[a + b*x]]^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3-\frac{3}{5}b\int x^5 \coth^{-1}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3- \frac{3}{5}b\left(\frac{1}{6}x^6 \coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}b\int x^6 \coth^{-1}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3- \frac{3}{5}b\left(\frac{1}{6}x^6 \coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}b\left(\frac{1}{7}x^7 \coth^{-1}(\tanh(a+bx))-\frac{b\int x^7dx}{7}\right)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3- \frac{3}{5}b\left(\frac{1}{6}x^6 \coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}b\left(\frac{1}{7}x^7 \coth^{-1}(\tanh(a+bx))-\frac{bx^8}{56}\right)\right)$$

input
```
Int[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^5*ArcCoth[Tanh[a + b*x]]^3)/5 - (3*b*((x^6*ArcCoth[Tanh[a + b*x]]^2)/6
- (b*(-1/56*(b*x^8) + (x^7*ArcCoth[Tanh[a + b*x]])/7))/3))/5
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F(-1)]

Timed out.

$$\int x^4 \operatorname{arccoth}(\tanh(bx+a))^3 \, dx$$

input
```
int(x^4*arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^4*arccoth(tanh(b*x+a))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{8}\,b^3x^8 + \frac{3}{7}\,ab^2x^7 + \frac{1}{2}\,a^2bx^6 - \frac{1}{40}i\,\pi^3x^5 + \frac{1}{5}\,a^3x^5 - \frac{1}{40}\,\pi^2\left(5\,bx^6 + 6\,ax^5\right) + \frac{1}{70}i\,\pi\left(15\,b^2x^7 + 35\,abx^6 + 21\,a^2x^5\right)$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 - 1/40*I*pi^3*x^5 + 1/5*a^3*x^
5 - 1/40*pi^2*(5*b*x^6 + 6*a*x^5) + 1/70*I*pi*(15*b^2*x^7 + 35*a*b*x^6 + 2
1*a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = \begin{cases} \frac{x^4 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{x^3 \operatorname{acoth}^5(\tanh(a+bx))}{5b^2} + \frac{x^2 \operatorname{acoth}^6(\tanh(a+bx))}{10b^3} - \frac{x \operatorname{acoth}^7(\tanh(a+bx))}{35b^4} + \frac{\operatorname{acoth}^8(\tanh(a+bx))}{280b^5} \\ \frac{x^5 \operatorname{acoth}^3(\tanh(a))}{5} \end{cases}$$

input
```
integrate(x**4*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x**4*acoth(tanh(a + b*x))**4/(4*b) - x**3*acoth(tanh(a + b*x))*
*5/(5*b**2) + x**2*acoth(tanh(a + b*x))**6/(10*b**3) - x*acoth(tanh(a + b*
x))**7/(35*b**4) + acoth(tanh(a + b*x))**8/(280*b**5), Ne(b, 0)), (x**5*ac
oth(tanh(a))**3/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{10}\,bx^6\,\text{arcoth}\left(\tanh\left(bx+a\right)\right)^2 + \frac{1}{5}\,x^5\,\text{arcoth}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{280}\left(b^2x^8 - 8\,bx^7\,\text{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/10*b*x^6*arccoth(tanh(b*x + a))^2 + 1/5*x^5*arccoth(tanh(b*x + a))^3 -
1/280*(b^2*x^8 - 8*b*x^7*arccoth(tanh(b*x + a)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{8}\,b^3x^8 - \frac{3}{14}\left(-i\,\pi b^2 - 2\,ab^2\right)x^7 - \frac{1}{8}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2b\right)x^6 - \frac{1}{40}\left(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3\right)x^5$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/8*b^3*x^8 - 3/14*(-I*pi*b^2 - 2*a*b^2)*x^7 - 1/8*(pi^2*b - 4*I*pi*a*b -
4*a^2*b)*x^6 - 1/40*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^5
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{b^3 x^8}{280} + \frac{b^2 x^7 \operatorname{acoth}(\tanh(a+b x))}{35} - \frac{b x^6 \operatorname{acoth}(\tanh(a+b x))^2}{10} + \frac{x^5 \operatorname{acoth}(\tanh(a+b x))^3}{5}$$

input
```
int(x^4*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^5*acoth(tanh(a + b*x))^3)/5 - (b^3*x^8)/280 - (b*x^6*acoth(tanh(a + b*x
))^2)/10 + (b^2*x^7*acoth(tanh(a + b*x)))/35
```

Reduce [F]

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx = \int acoth(\tanh(bx+a))^3 x^4 dx$$

input
```
int(x^4*acoth(tanh(b*x+a))^3,x)
```

output
```
int(acoth(tanh(a + b*x))**3*x**4,x)
```

3.27 $\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{140}b^3x^7+\frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3$$

output
```
-1/140*b^3*x^7+1/20*b^2*x^6*arccoth(tanh(b*x+a))-3/20*b*x^5*arccoth(tanh(b
*x+a))^2+1/4*x^4*arccoth(tanh(b*x+a))^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{140}x^4\left(b^3x^3-7b^2x^2\coth^{-1}(\tanh(a+bx)) + 21bx\coth^{-1}(\tanh(a+bx))^2 - 35\coth^{-1}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/140*(x^4*(b^3*x^3 - 7*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 21*b*x*ArcCoth[T
anh[a + b*x]]^2 - 35*ArcCoth[Tanh[a + b*x]]^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3 - \frac{3}{4}b\int x^4 \coth^{-1}(\tanh(a+bx))^2 dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3 - \frac{3}{4}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{5}b\int x^5 \coth^{-1}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3 - \frac{3}{4}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{5}b\left(\frac{1}{6}x^6 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^6dx}{6}\right)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3 - \frac{3}{4}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{5}b\left(\frac{1}{6}x^6 \coth^{-1}(\tanh(a+bx)) - \frac{bx^7}{42}\right)\right)$$

input
```
Int[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^4*ArcCoth[Tanh[a + b*x]]^3)/4 - (3*b*((x^5*ArcCoth[Tanh[a + b*x]]^2)/5
- (2*b*(-1/42*(b*x^7) + (x^6*ArcCoth[Tanh[a + b*x]])/6))/5))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F(-1)]

Timed out.

$$\int x^3 \operatorname{arccoth}(\tanh(bx+a))^3 \, dx$$

input
```
int(x^3*arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^3*arccoth(tanh(b*x+a))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{1}{7} b^3 x^7 + \frac{1}{2} a b^2 x^6 + \frac{3}{5} a^2 b x^5 - \frac{1}{32} i \pi^3 x^4 + \frac{1}{4} a^3 x^4 - \frac{3}{80} \pi^2 (4 b x^5 + 5 a x^4) + \frac{1}{40} i \pi (10 b^2 x^6 + 24 a b x^5 + 15 a^2 x^4)$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 - 1/32*I*pi^3*x^4 + 1/4*a^3*x^
4 - 3/80*pi^2*(4*b*x^5 + 5*a*x^4) + 1/40*I*pi*(10*b^2*x^6 + 24*a*b*x^5 + 1
5*a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = \begin{cases} \frac{x^3 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{3x^2 \operatorname{acoth}^5(\tanh(a+bx))}{20b^2} + \frac{x \operatorname{acoth}^6(\tanh(a+bx))}{20b^3} - \frac{\operatorname{acoth}^7(\tanh(a+bx))}{140b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}^3(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x**3*acoth(tanh(a + b*x))**4/(4*b) - 3*x**2*acoth(tanh(a + b*x)
)**5/(20*b**2) + x*acoth(tanh(a + b*x))**6/(20*b**3) - acoth(tanh(a + b*x)
)**7/(140*b**4), Ne(b, 0)), (x**4*acoth(tanh(a))**3/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{3}{20} b x^5 \operatorname{arcoth}(\tanh(bx+a))^2 + \frac{1}{4} x^4 \operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{140} \left(b^2 x^7 - 7 b x^6 \operatorname{arcoth}(\tanh(bx+a))\right) b$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3/20*b*x^5*arccoth(tanh(b*x + a))^2 + 1/4*x^4*arccoth(tanh(b*x + a))^3 -
1/140*(b^2*x^7 - 7*b*x^6*arccoth(tanh(b*x + a)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{1}{7} b^3 x^7 - \frac{1}{4} \left(-i \pi b^2 - 2 a b^2\right) x^6 - \frac{3}{20} \left(\pi^2 b - 4 i \pi a b - 4 a^2 b\right) x^5 - \frac{1}{32} \left(i \pi^3 + 6 \pi^2 a - 12 i \pi a^2 - 8 a^3\right) x^4$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 - 1/4*(-I*pi*b^2 - 2*a*b^2)*x^6 - 3/20*(pi^2*b - 4*I*pi*a*b -
4*a^2*b)*x^5 - 1/32*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^4
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{b^3\,x^7}{140} + \frac{b^2\,x^6\,\mathrm{acoth}(\tanh(a+b\,x))}{20} - \frac{3\,b\,x^5\,\mathrm{acoth}(\tanh(a+b\,x))^2}{20} + \frac{x^4\,\mathrm{acoth}(\tanh(a+b\,x))^3}{4}$$

input
```
int(x^3*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^4*acoth(tanh(a + b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*acoth(tanh(a + b
*x))^2)/20 + (b^2*x^6*acoth(tanh(a + b*x)))/20
```

Reduce [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = \int acoth(\tanh(bx+a))^3\,x^3dx$$

input
```
int(x^3*acoth(tanh(b*x+a))^3,x)
```

output
```
int(acoth(tanh(a + b*x))**3*x**3,x)
```

3.28 $\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3}$$

output
```
1/4*x^2*arccoth(tanh(b*x+a))^4/b-1/10*x*arccoth(tanh(b*x+a))^5/b^2+1/60*ar
ccoth(tanh(b*x+a))^6/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{60}x^3\left(b^3x^3 - 6b^2x^2 \coth^{-1}(\tanh(a+bx)) + 15bx \coth^{-1}(\tanh(a+bx))^2 - 20 \coth^{-1}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/60*(x^3*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 15*b*x*ArcCoth[Ta
nh[a + b*x]]^2 - 20*ArcCoth[Tanh[a + b*x]]^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int x \coth^{-1}(\tanh(a+bx))^4 dx}{2b}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \coth^{-1}(\tanh(a+bx))^5}{5b} - \frac{\int \coth^{-1}(\tanh(a+bx))^5 dx}{5b}}{2b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \coth^{-1}(\tanh(a+bx))^5}{5b} - \frac{\int \coth^{-1}(\tanh(a+bx))^5 d \coth^{-1}(\tanh(a+bx))}{5b^2}}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \coth^{-1}(\tanh(a+bx))^5}{5b} - \frac{\coth^{-1}(\tanh(a+bx))^6}{30b^2}}{2b}$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^2*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ((x*ArcCoth[Tanh[a + b*x]]^5)/(5*b)
 - ArcCoth[Tanh[a + b*x]]^6/(30*b^2))/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F(-1)]

Timed out.

$$\int x^2 \operatorname{arccoth}(\tanh(bx+a))^3 \, dx$$

input
```
int(x^2*arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^2*arccoth(tanh(b*x+a))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{1}{6} b^3 x^6 + \frac{3}{5} a b^2 x^5 + \frac{3}{4} a^2 b x^4 - \frac{1}{24} i \pi^3 x^3 + \frac{1}{3} a^3 x^3 - \frac{1}{16} \pi^2 (3 b x^4 + 4 a x^3) + \frac{1}{20} i \pi (6 b^2 x^5 + 15 a b x^4 + 10 a^2 x^3)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 - 1/24*I*pi^3*x^3 + 1/3*a^3*x^
3 - 1/16*pi^2*(3*b*x^4 + 4*a*x^3) + 1/20*I*pi*(6*b^2*x^5 + 15*a*b*x^4 + 10
*a^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = \begin{cases} \frac{x^2 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{x \operatorname{acoth}^5(\tanh(a+bx))}{10b^2} + \frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^3(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x**2*acoth(tanh(a + b*x))**4/(4*b) - x*acoth(tanh(a + b*x))**5/
(10*b**2) + acoth(tanh(a + b*x))**6/(60*b**3), Ne(b, 0)), (x**3*acoth(tanh
(a))**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{4}\,bx^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2 + \frac{1}{3}\,x^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{60}\left(b^2x^6 - 6\,bx^5\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/4*b*x^4*arccoth(tanh(b*x + a))^2 + 1/3*x^3*arccoth(tanh(b*x + a))^3 - 1
/60*(b^2*x^6 - 6*b*x^5*arccoth(tanh(b*x + a)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{6}\,b^3x^6 - \frac{3}{10}\left(-i\,\pi b^2 - 2\,ab^2\right)x^5 - \frac{3}{16}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2b\right)x^4 - \frac{1}{24}\left(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3\right)x^3$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 - 3/10*(-I*pi*b^2 - 2*a*b^2)*x^5 - 3/16*(pi^2*b - 4*I*pi*a*b -
 4*a^2*b)*x^4 - 1/24*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^3
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{b^3 x^6}{60} + \frac{b^2 x^5 \operatorname{acoth}(\tanh(a+b x))}{10} - \frac{b x^4 \operatorname{acoth}(\tanh(a+b x))^2}{4} + \frac{x^3 \operatorname{acoth}(\tanh(a+b x))^3}{3}$$

input
```
int(x^2*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^3*acoth(tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*acoth(tanh(a + b*x)
)^2)/4 + (b^2*x^5*acoth(tanh(a + b*x)))/10
```

Reduce [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = \int acoth(\tanh(bx+a))^3 x^2 dx$$

input
```
int(x^2*acoth(tanh(b*x+a))^3,x)
```

output
```
int(acoth(tanh(a + b*x))**3*x**2,x)
```

3.29 $\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

output
```
1/4*x*arccoth(tanh(b*x+a))^4/b-1/20*arccoth(tanh(b*x+a))^5/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(34) = 68.

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$$
$$= \frac{(a+bx)\left((4a-bx)(a+bx)^3 - 5(3a-bx)(a+bx)^2 \coth^{-1}(\tanh(a+bx)) + 10(2a^2+abx-b^2x^2) \coth^{-1}\right.}{20b^2}$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcCoth[Ta
nh[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcCoth[Tanh[a + b*x]]^2 - 10*
(a - b*x)*ArcCoth[Tanh[a + b*x]]^3))/(20*b^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{2599, 2588, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a+bx))^4 dx}{4b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a+bx))^4 d\coth^{-1}(\tanh(a+bx))}{4b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ArcCoth[Tanh[a + b*x]]^5/(20*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 23.91 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

method	result	size
parallelrisch	$\frac{x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{3}}{2}+\frac{b^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right) x^{4}}{4}-\frac{b \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2} x^{3}}{2}-\frac{b^{3} x^{5}}{20}$	54
risch	Expression too large to display	18111

input
```
int(x*arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccoth(tanh(b*x+a))^3+1/4*b^2*arccoth(tanh(b*x+a))*x^4-1/2*b*arcc
oth(tanh(b*x+a))^2*x^3-1/20*b^3*x^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int x \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 - \frac{1}{16}i\,\pi^3x^2 + \frac{1}{2}\,a^3x^2 - \frac{1}{8}\,\pi^2\left(2\,bx^3 + 3\,ax^2\right) + \frac{1}{8}i\,\pi\left(3\,b^2x^4 + 8\,abx^3 + 6\,a^2x^2\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 - 1/16*I*pi^3*x^2 + 1/2*a^3*x^2 -
1/8*pi^2*(2*b*x^3 + 3*a*x^2) + 1/8*I*pi*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2
)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x \coth^{-1}(\tanh(a+bx))^3\,dx = \begin{cases} \frac{x\,\mathrm{acoth}^4\left(\tanh\left(a+bx\right)\right)}{4b} - \frac{\mathrm{acoth}^5\left(\tanh\left(a+bx\right)\right)}{20b^2} & \text{for}\ b \neq 0 \\ \frac{x^2\,\mathrm{acoth}^3\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x*acoth(tanh(a + b*x))**4/(4*b) - acoth(tanh(a + b*x))**5/(20*b
**2), Ne(b, 0)), (x**2*acoth(tanh(a))**3/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{2}\,bx^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2 + \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{20}\left(b^2x^5 - 5\,bx^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*b*x^3*arccoth(tanh(b*x + a))^2 + 1/2*x^2*arccoth(tanh(b*x + a))^3 - 1
/20*(b^2*x^5 - 5*b*x^4*arccoth(tanh(b*x + a)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{5}\,b^3x^5 - \frac{3}{8}\left(-i\,\pi b^2 - 2\,ab^2\right)x^4 - \frac{1}{4}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2b\right)x^3 - \frac{1}{16}\left(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3\right)x^2$$

input
```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 - 3/8*(-I*pi*b^2 - 2*a*b^2)*x^4 - 1/4*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x^3 - 1/16*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^2
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{b^3 x^5}{20} + \frac{b^2 x^4 \operatorname{acoth}(\tanh(a+b x))}{4} - \frac{b x^3 \operatorname{acoth}(\tanh(a+b x))^2}{2} + \frac{x^2 \operatorname{acoth}(\tanh(a+b x))^3}{2}$$

input
```
int(x*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^2*acoth(tanh(a + b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*acoth(tanh(a + b*x)
)^2)/2 + (b^2*x^4*acoth(tanh(a + b*x)))/4
```

Reduce [F]

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx = \int acoth(\tanh(bx+a))^3 x dx$$

input
```
int(x*acoth(tanh(b*x+a))^3,x)
```

output
```
int(acoth(tanh(a + b*x))**3*x,x)
```

3.30 $\int \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

output
```
1/4*arccoth(tanh(b*x+a))^4/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \; 2588$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))^3 d \coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 24.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4b}$
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4b}$
parallelrisch	$-\frac{b^3x^4}{4}-\frac{3b\operatorname{arccoth}(\tanh(bx+a))^2x^2}{2}+x\operatorname{arccoth}(\tanh(bx+a))^3+b^2\operatorname{arccoth}(\tanh(bx+a$
risch	Expression too large to display

input
```
int(arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arccoth(tanh(b*x+a))^4/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.75

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}b^3x^4+ab^2x^3+\frac{3}{2}a^2bx^2-\frac{1}{8}i\pi^3x+a^3x-\frac{3}{8}\pi^2\left(bx^2+2ax\right)+\frac{1}{2}i\pi\left(b^2x^3+3abx^2+3a^2x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 - 1/8*I*pi^3*x + a^3*x - 3/8*pi^2*
(b*x^2 + 2*a*x) + 1/2*I*pi*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \begin{cases} \frac{\operatorname{acoth}^4(\tanh(a+bx))}{4b} & \text{for } b \neq 0 \\ x\operatorname{acoth}^3(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((acoth(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*acoth(tanh(a))**3,
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{3}{2}\,bx^2\operatorname{arcoth}(\tanh(bx+a))^2 + x\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{4}\left(b^2x^4 - 4\,bx^3\operatorname{arcoth}(\tanh(bx+a))\right)b$$

input
```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3/2*b*x^2*arccoth(tanh(b*x + a))^2 + x*arccoth(tanh(b*x + a))^3 - 1/4*(b^
2*x^4 - 4*b*x^3*arccoth(tanh(b*x + a)))*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.69

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}b^3x^4 - \frac{1}{2}\left(-i\pi b^2 - 2ab^2\right)x^3 - \frac{3}{8}\left(\pi^2 b - 4i\pi ab - 4a^2b\right)x^2 - \frac{1}{8}\left(i\pi^3 + 6\pi^2 a - 12i\pi a^2 - 8a^3\right)x$$

input
```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 - 1/2*(-I*pi*b^2 - 2*a*b^2)*x^3 - 3/8*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x^2 - 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{x\,(2\,\mathrm{acoth}(\tanh(a+b\,x))-b\,x)\,\left(b^2\,x^2-2\,b\,x\,\mathrm{acoth}(\tanh(a+b\,x))+2\,\mathrm{acoth}(\tanh(a+b\,x))^2\right)}{4}$$

input
```
int(acoth(tanh(a + b*x))^3,x)
```

output
```
(x*(2*acoth(tanh(a + b*x)) - b*x)*(2*acoth(tanh(a + b*x))^2 + b^2*x^2 - 2*
b*x*acoth(tanh(a + b*x))))/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{acoth(\tanh(bx+a))^4}{4b}$$

input
```
int(acoth(tanh(b*x+a))^3,x)
```

output
```
( - acoth(tanh(a + b*x))**4)/(4*b)
```

3.31 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = {} & bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \\ & -\frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2 \\ & +\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 \\ & -\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\log(x)\end{aligned}$$

output
```
b*x*(b*x-arccoth(tanh(b*x+a)))^2-1/2*(b*x-arccoth(tanh(b*x+a)))*arccoth(ta
nh(b*x+a))^2+1/3*arccoth(tanh(b*x+a))^3-(b*x-arccoth(tanh(b*x+a)))^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}(a+bx)^3 + (a+bx)\left(a^2 - 3a\left(a+bx-\coth^{-1}(\tanh(a+bx))\right) + 3\left(a+bx-\coth^{-1}(\tanh(a+bx))\right)^2\right) - \frac{1}{2}(a+bx)^2\left(2a+3bx-3\coth^{-1}(\tanh(a+bx))\right) + \left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3\log(bx)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

output
```
(a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcCoth[Tanh[a + b*x]]) +
3*(a + b*x - ArcCoth[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*Ar
cCoth[Tanh[a + b*x]]))/2 + (-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[b*x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2590, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 - \left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}dx$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 - (bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-(bx-\coth^{-1}(\tanh(a+bx)))\int\frac{\coth^{-1}(\tanh(a+bx)}{x}\right.$$

$\big\downarrow$ 2589

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 - (bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-(bx-\coth^{-1}(\tanh(a+bx)))\left(bx-(bx-\coth^{-1}(\tan\right.\right.$$

$\big\downarrow$ 14

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 - (bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-(bx-\coth^{-1}(\tanh(a+bx)))\left(bx-\log(x)\left(bx-\coth\right.\right.\right.$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/3 - (b*x - ArcCoth[Tanh[a + b*x]])*(ArcCoth[Tanh[
a + b*x]]^2/2 - (b*x - ArcCoth[Tanh[a + b*x]])*(b*x - (b*x - ArcCoth[Tanh[
a + b*x]])*Log[x]))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 3302, normalized size of antiderivative = 42.88

method	result	size
risch	Expression too large to display	3302

input
```
int(arccoth(tanh(b*x+a))^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(exp(b*x+a))^3-b^3*x^3*ln(x)+3*b^2*ln(x)*ln(exp(b*x+a))*x^2+3*b*ln
(exp(b*x+a))^2*x+11/6*b^3*x^3-3*b*ln(x)*ln(exp(b*x+a))^2*x-9/2*b^2*ln(exp(
b*x+a))*x^2+(-3/4*Pi^2-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+3/4*Pi^2*csgn(I/(exp(2*b
*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))-3/8*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3-3/4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*csgn(I*exp(b*x
+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+
3/2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^5-3/4*Pi^2*csgn(I*exp(2*b*x+2*a))^3-3/
4*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3/16*Pi^2*csgn(I*exp(2*
b*x+2*a))^6-3/16*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-3/4*Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))^6-3/4*csgn(I/(exp(2*b*x+2*a)+1))^4*Pi^2-3/16*P
i^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+3/4*Pi^2*csgn(I*exp(b*x+
a))^3*csgn(I*exp(2*b*x+2*a))^3-9/8*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))^4+3/4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+3/8*Pi^2*
csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3/16*
Pi^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+
3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^3+3/4*Pi^2*cs
gn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 - \frac{3}{4}\pi^2bx + 3a^2bx + \frac{3}{4}i\pi\left(b^2x^2 + 4abx\right) - \frac{1}{8}\left(i\pi^3 + 6\pi^2a - 12i\pi a^2 - 8a^3\right)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 - 3/4*pi^2*b*x + 3*a^2*b*x + 3/4*I*pi*(b^2*x^2
 + 4*a*b*x) - 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log(x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**3/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}b^3x^3 - \frac{3}{4}\left(i\pi b^2 - 2ab^2\right)x^2 - \frac{3}{4}\left(\pi^2b + 4i\pi ab - 4a^2b\right)x + \frac{1}{8}\left(i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3\right)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 - 3/4*(I*pi*b^2 - 2*a*b^2)*x^2 - 3/4*(pi^2*b + 4*I*pi*a*b - 4*
a^2*b)*x + 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\,b^3x^3 - \frac{3}{4}\left(-i\,\pi b^2 - 2\,ab^2\right)x^2 - \frac{3}{4}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2 b\right)x + \frac{1}{8}\left(-i\,\pi^3 - 6\,\pi^2 a + 12i\,\pi a^2 + 8\,a^3\right)\log\left(x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 - 3/4*(-I*pi*b^2 - 2*a*b^2)*x^2 - 3/4*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x + 1/8*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.97

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx$$
$$= \frac{b^3x^3}{3} - \ln(x)\left(\frac{\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^3}{8} - a^3 - \frac{3a\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^2}{4} + \frac{3a^2\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)}{2}\right) - \frac{3b^2x^2\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)}{4} + \frac{3bx\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^2}{4}$$

input
```
int(acoth(tanh(a + b*x))^3/x,x)
```

output
```
(b^3*x^3)/3 - log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/8 - a^3 - (3*a*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 + (3*a^2*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x))/2) - (3*b^2*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/4 + (3*b*x*(log(-2/(exp(2
*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
1)) + 2*b*x)^2)/4
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \int \frac{acoth(\tanh(bx+a))^3}{x}dx$$

input
```
int(acoth(tanh(b*x+a))^3/x,x)
```

output
```
int(acoth(tanh(a + b*x))**3/x,x)
```

3.32 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = -3b^2x\left(bx-\coth^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + 3b\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log(x)$$

output
```
-3*b^2*x*(b*x-arccoth(tanh(b*x+a)))+3/2*b*arccoth(tanh(b*x+a))^2-arccoth(t
anh(b*x+a))^3/x+3*b*(b*x-arccoth(tanh(b*x+a)))^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^3}{x} - 6b^2x\coth^{-1}(\tanh(a+bx))\log(x) + 3b\coth^{-1}(\tanh(a+bx))^2(1+\log(x)) + \frac{3}{2}b^3x^2(-1+2\log(x))$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcCoth[Tanh[a + b*x]]*Log[x] + 3*
b*ArcCoth[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}dx$$

$$\Big\downarrow \text{ 2599}$$

$$3b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{ 2590}$$

$$3b\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{ 2589}$$

$$3b\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{ 14}$$

$$3b\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^3/x) + 3*b*(ArcCoth[Tanh[a + b*x]]^2/2 - (b*x - A
rcCoth[Tanh[a + b*x]])*(b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 3256, normalized size of antiderivative = 47.88

method	result	size
risch	Expression too large to display	3256

input
```
int(arccoth(tanh(b*x+a))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln(exp(b*x+a))^3+3*ln(x)*ln(exp(b*x+a))^2*b+3*b^3*x^2*ln(x)-9/2*b^3*x
^2-6*b^2*ln(x)*ln(exp(b*x+a))*x+6*b^2*ln(exp(b*x+a))*x+(-3/4*Pi^2-3/4*Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3/8*Pi^2*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3/4
*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^2+3/4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+3/2*Pi^2*csgn(I/(exp(2*b*x+2*a)
+1))^5-3/4*Pi^2*csgn(I*exp(2*b*x+2*a))^3-3/4*Pi^2*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^3-3/16*Pi^2*csgn(I*exp(2*b*x+2*a))^6-3/16*Pi^2*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^6-3
/4*csgn(I/(exp(2*b*x+2*a)+1))^4*Pi^2-3/16*Pi^2*csgn(I*exp(b*x+a))^4*csgn(I
*exp(2*b*x+2*a))^2+3/4*Pi^2*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-
9/8*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^4+3/4*Pi^2*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+3/8*Pi^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3/16*Pi^2*csgn(I*exp(2*b*x+2*a))^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)
+1))^2*csgn(I*exp(2*b*x+2*a))^3+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \frac{4\,b^3x^3+24\,ab^2x^2+i\,\pi^3+6\,\pi^2a-8\,a^3+12i\,\pi(b^2x^2-a^2)-6\,(\pi^2bx-4i\,\pi abx-4\,a^2bx)\log{(x)}}{8\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

output
```
1/8*(4*b^3*x^3 + 24*a*b^2*x^2 + I*pi^3 + 6*pi^2*a - 8*a^3 + 12*I*pi*(b^2*x
^2 - a^2) - 6*(pi^2*b*x - 4*I*pi*a*b*x - 4*a^2*b*x)*log(x))/x
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \int \frac{\operatorname{acoth}^3{(\tanh{(a+bx)})}}{x^2}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**2,x)
```

output
```
Integral(acoth(tanh(a + b*x))**3/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = 3\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right) - \frac{3}{2}\left(2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right) - \left(bx^2-2\,(-i\,\pi-2\,a)x+2\left(-\frac{i\,\pi(bx+a)}{b}-\frac{(bx+a)^2}{b}\right)\log\left(\right.\right.\right. - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{x}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

output
```
3*b*arccoth(tanh(b*x + a))^2*log(x) - 3/2*(2*arccoth(tanh(b*x + a))^2*log(
x) - (b*x^2 - 2*(-I*pi - 2*a)*x + 2*(-I*pi*(b*x + a)/b - (b*x + a)^2/b)*lo
g(x) + 2*arccoth(tanh(b*x + a))^2*log(x)/b + 2*(I*pi*a + a^2)*log(x)/b)*b)
*b - arccoth(tanh(b*x + a))^3/x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \frac{1}{2}\,b^3x^2 - \frac{3}{2}\left(-i\,\pi b^2 - 2\,ab^2\right)x - \frac{3}{4}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2 b\right)\log\left(x\right) - \frac{-i\,\pi^3 - 6\,\pi^2 a + 12i\,\pi a^2 + 8\,a^3}{8\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 - 3/2*(-I*pi*b^2 - 2*a*b^2)*x - 3/4*(pi^2*b - 4*I*pi*a*b - 4*a
^2*b)*log(x) - 1/8*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.47

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx$$

$$= \ln(x)\left(3a^2b + \frac{3b\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^2}{4} - 3ab\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)\right)$$

$$+ \frac{\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^3 - 8a^3 - 6a\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-\right.\right.}{8x}$$

$$+ \frac{b^3x^2}{2} - \frac{3b^2x\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)}{2}$$

input
```
int(acoth(tanh(a + b*x))^3/x^2,x)
```

output
```
log(x)*(3*a^2*b + (3*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 - 3*a*b*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a
)*exp(2*b*x) - 1)) + 2*b*x)) + ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3
 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log
(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(8*x) + (b^3*x^2)/2 - (3*b^2*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/2
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \int \frac{acoth(\tanh{(bx+a)})^3}{x^2}dx$$

input
```
int(acoth(tanh(b*x+a))^3/x^2,x)
```

output
```
int(acoth(tanh(a + b*x))**3/x**2,x)
```

3.33 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = 3b^3x - \frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

output
```
3*b^3*x-3/2*b*arccoth(tanh(b*x+a))^2/x-1/2*arccoth(tanh(b*x+a))^3/x^2-3*b^
2*(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = b^3x - \frac{3b\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{x} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{2x^2} + 3b^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

output
```
b^3*x - (3*b*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcCoth[Ta
nh[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3}{2}b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}dx - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{3}{2}b\left(2b\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx - \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3}{2}b\left(2b\left(bx - \left(bx - \coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right) - \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{3}{2}b\left(2b\left(bx - \log(x)\left(bx - \coth^{-1}(\tanh(a+bx))\right)\right) - \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

output
```
-1/2*ArcCoth[Tanh[a + b*x]]^3/x^2 + (3*b*(-(ArcCoth[Tanh[a + b*x]]^2/x) +
2*b*(b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x])))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 3235, normalized size of antiderivative = 53.92

method	result	size
risch	Expression too large to display	3235

input
```
int(arccoth(tanh(b*x+a))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*ln(exp(b*x+a))^3-3/2/x*ln(exp(b*x+a))^2*b-3*ln(x)*x*b^3+3*ln(x)*l
n(exp(b*x+a))*b^2+3*b^3*x+(-3/4*Pi^2-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+3/4*Pi^2*c
sgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))-3/8*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3/4*Pi^2*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*
csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+3/2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^5-3/4*Pi^2*csgn(I*exp(2*
b*x+2*a))^3-3/4*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3/16*Pi^2
*csgn(I*exp(2*b*x+2*a))^6-3/16*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^6-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^6-3/4*csgn(I/(exp(2*b*x+2*a)+1))
^4*Pi^2-3/16*Pi^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+3/4*Pi^2*c
sgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-9/8*Pi^2*csgn(I*exp(b*x+a))^2
*csgn(I*exp(2*b*x+2*a))^4+3/4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a
))^5+3/8*Pi^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2-3/16*Pi^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^4+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))
^3+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(e...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = \frac{16\,b^3x^3 - 48\,a^2bx + i\,\pi^3 + 6\,\pi^2(2\,bx+a) - 8\,a^3 - 12i\,\pi(4\,abx+a^2) - 24\,(-i\,\pi b^2x^2 - 2\,ab^2x^2)\log{(x)}}{16\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

output
```
1/16*(16*b^3*x^3 - 48*a^2*b*x + I*pi^3 + 6*pi^2*(2*b*x + a) - 8*a^3 - 12*I
*pi*(4*a*b*x + a^2) - 24*(-I*pi*b^2*x^2 - 2*a*b^2*x^2)*log(x))/x^2
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^3}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**3,x)
```

output
```
Integral(acoth(tanh(a + b*x))**3/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$$
$$= 3\left(b\operatorname{arcoth}(\tanh(bx+a))\log(x) - \left(b\left(x+\frac{a}{b}\right)\log(x) - b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b$$
$$- \frac{3\,b\operatorname{arcoth}(\tanh(bx+a))^2}{2\,x} - \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{2\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

output
```
3*(b*arccoth(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b)*b - 3/2*b*arccoth(tanh(b*x + a))^2/x - 1/2*arccoth(tanh(b*x + a))^
3/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = b^3x + \frac{3}{2}\left(i\pi b^2 + 2\,ab^2\right)\log(x) + \frac{12\,\pi^2bx - 48i\,\pi abx - 48\,a^2bx + i\,\pi^3 + 6\,\pi^2a - 12i\,\pi a^2 - 8\,a^3}{16\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

output
```
b^3*x + 3/2*(I*pi*b^2 + 2*a*b^2)*log(x) + 1/16*(12*pi^2*b*x - 48*I*pi*a*b*
x - 48*a^2*b*x + I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 383, normalized size of antiderivative = 6.38

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx &= \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^3}{16\,x^2} - \frac{\ln\left(\frac{2\,e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^3}{16\,x^2} \\ &+ \frac{9\,b^2\,\ln\left(\frac{e^{2bx}}{e^{2a}e^{2bx}-1}\right)}{4} - \frac{9\,b^2\,\ln\left(\frac{1}{e^{2a}e^{2bx}-1}\right)}{4} \\ &- \frac{3\,b^3\,x}{2} - \frac{3\,b\ln\left(\frac{2\,e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^2}{8\,x} \\ &+ \frac{3\,b^2\,\ln\left(\frac{2\,e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\ln(x)}{2} - 3\,b^3\,x\,\ln(x) \\ &- \frac{3\,b\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^2}{8\,x} - \frac{3\,b^2\,\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\ln(x)}{2} \\ &- \frac{3\,\ln\left(\frac{2\,e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^2}{16\,x^2} \\ &+ \frac{3\ln\left(\frac{2\,e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^2\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)}{16\,x^2} \\ &+ \frac{3\,b\,\ln\left(\frac{2\,e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)}{4\,x}\end{aligned}$$

input
```
int(acoth(tanh(a + b*x))^3/x^3,x)
```

output
```
log(-2/(exp(2*a)*exp(2*b*x) - 1))^3/(16*x^2) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1))^3/(16*x^2) + (9*b^2*log(exp(2*b*x)/(exp(2*a)*e
xp(2*b*x) - 1)))/4 - (9*b^2*log(1/(exp(2*a)*exp(2*b*x) - 1)))/4 - (3*b^3*x
)/2 - (3*b*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2)/(8*x)
 + (3*b^2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(x))/2
 - 3*b^3*x*log(x) - (3*b*log(-2/(exp(2*a)*exp(2*b*x) - 1))^2)/(8*x) - (3*b
^2*log(-2/(exp(2*a)*exp(2*b*x) - 1))*log(x))/2 - (3*log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1))^2)/(16*
x^2) + (3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2*log(-2/
(exp(2*a)*exp(2*b*x) - 1)))/(16*x^2) + (3*b*log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1)))/(4*x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = \int \frac{acoth(\tanh(bx+a))^3}{x^3}dx$$

input
```
int(acoth(tanh(b*x+a))^3/x^3,x)
```

output
```
int(acoth(tanh(a + b*x))**3/x**3,x)
```

3.34 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx = -\frac{b^2\coth^{-1}(\tanh(a+bx))}{x} - \frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^3\log(x)$$

output
```
-b^2*arccoth(tanh(b*x+a))/x-1/2*b*arccoth(tanh(b*x+a))^2/x^2-1/3*arccoth(t
anh(b*x+a))^3/x^3+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx = \frac{-6b^2x^2\coth^{-1}(\tanh(a+bx)) - 3bx\coth^{-1}(\tanh(a+bx))^2 - 2\coth^{-1}(\tanh(a+bx))^3 + b^3x^3(11+6\log}{6x^3}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

output
```
(-6*b^2*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*x*ArcCoth[Tanh[a + b*x]]^2 - 2*Ar
cCoth[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}dx$$

$$\Big\downarrow \text{ 2599}$$

$$b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}dx-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{ 2599}$$

$$b\left(b\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}dx-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{ 2599}$$

$$b\left(b\left(b\int \frac{1}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{ 14}$$

$$b\left(b\left(b\log(x)-\frac{\coth^{-1}(\tanh(a+bx))}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

output
```
-1/3*ArcCoth[Tanh[a + b*x]]^3/x^3 + b*(-1/2*ArcCoth[Tanh[a + b*x]]^2/x^2 +
 b*(-(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$\frac{6 b^{3} x^{3} \ln \left(x \right)-6 b^{2} x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)-3 b \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2} x -2 \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{3}}{6 x^{3}}$	56
risch	Expression too large to display	17237

input
```
int(arccoth(tanh(b*x+a))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*b^3*x^3*ln(x)-6*b^2*x^2*arccoth(tanh(b*x+a))-3*b*arccoth(tanh(b*x+a
))^2*x-2*arccoth(tanh(b*x+a))^3)/x^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} \, dx = \frac{24 \, b^{3} x^{3} \log \left(x\right) - 72 \, a b^{2} x^{2} - 36 \, a^{2} b x + i \, \pi^{3} + 3 \, \pi^{2} {\left(3 \, b x + 2 \, a\right)} - 8 \, a^{3} - 12 i \, \pi {\left(3 \, b^{2} x^{2} + 3 \, a b x + a^{2}\right)}}{24 \, x^{3}}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

output
```
1/24*(24*b^3*x^3*log(x) - 72*a*b^2*x^2 - 36*a^2*b*x + I*pi^3 + 3*pi^2*(3*b
*x + 2*a) - 8*a^3 - 12*I*pi*(3*b^2*x^2 + 3*a*b*x + a^2))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx = b^3\log(x) - \frac{b^2\operatorname{acoth}(\tanh(a+bx))}{x} - \frac{b\operatorname{acoth}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3x^3}$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**4,x)
```

output
```
b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*x))**2/(2*x
**2) - acoth(tanh(a + b*x))**3/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx = \left(b^2\log(x) - \frac{b\operatorname{arcoth}(\tanh(bx+a))}{x}\right)b - \frac{b\operatorname{arcoth}(\tanh(bx+a))^2}{2x^2} - \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{3x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

output
```
(b^2*log(x) - b*arccoth(tanh(b*x + a))/x)*b - 1/2*b*arccoth(tanh(b*x + a))
^2/x^2 - 1/3*arccoth(tanh(b*x + a))^3/x^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= b^3\log(x) - \frac{36i\,\pi b^2x^2 + 72\,ab^2x^2 - 9\,\pi^2bx + 36i\,\pi abx + 36\,a^2bx - i\,\pi^3 - 6\,\pi^2a + 12i\,\pi a^2 + 8\,a^3}{24\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

output
```
b^3*log(x) - 1/24*(36*I*pi*b^2*x^2 + 72*a*b^2*x^2 - 9*pi^2*b*x + 36*I*pi*a
*b*x + 36*a^2*b*x - I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= b^3\,\ln(x) - \frac{b^2\,x^2\,\mathrm{acoth}(\tanh(a+b\,x)) + \frac{b\,x\,\mathrm{acoth}(\tanh(a+b\,x))^2}{2} + \frac{\mathrm{acoth}(\tanh(a+b\,x))^3}{3}}{x^3}$$

input
```
int(acoth(tanh(a + b*x))^3/x^4,x)
```

output
```
b^3*log(x) - (acoth(tanh(a + b*x))^3/3 + (b*x*acoth(tanh(a + b*x))^2)/2 +
b^2*x^2*acoth(tanh(a + b*x)))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= \frac{-2acoth(\tanh{(bx+a)})^3 + 3acoth(\tanh{(bx+a)})^2 bx - 6acoth(\tanh{(bx+a)})\,b^2x^2 - 6\log(x)\,b^3x^3}{6x^3}$$

input
```
int(acoth(tanh(b*x+a))^3/x^4,x)
```

output
```
( - 2*acoth(tanh(a + b*x))**3 + 3*acoth(tanh(a + b*x))**2*b*x - 6*acoth(ta
nh(a + b*x))*b**2*x**2 - 6*log(x)*b**3*x**3)/(6*x**3)
```

3.35 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/4*arccoth(tanh(b*x+a))^4/x^4/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = -\frac{b^3x^3+b^2x^2\coth^{-1}(\tanh(a+bx))+bx\coth^{-1}(\tanh(a+bx))^2+\coth^{-1}(\tanh(a+bx))^3}{4x^4}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

output
```
-1/4*(b^3*x^3 + b^2*x^2*ArcCoth[Tanh[a + b*x]] + b*x*ArcCoth[Tanh[a + b*x]
]^2 + ArcCoth[Tanh[a + b*x]]^3)/x^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
parallelrisch	$-\frac{b^{3}x^{3}+b^{2}x^{2}\operatorname{arccoth}(\tanh(bx+a))+b\operatorname{arccoth}(\tanh(bx+a))^{2}x+\operatorname{arccoth}(\tanh(bx+a))^{3}}{4x^{4}}$	49
risch	Expression too large to display	17235

input
```
int(arccoth(tanh(b*x+a))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^3*x^3+b^2*x^2*arccoth(tanh(b*x+a))+b*arccoth(tanh(b*x+a))^2*x+arcc
oth(tanh(b*x+a))^3)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{32\,b^3x^3+48\,ab^2x^2+32\,a^2bx-i\,\pi^3-2\,\pi^2(4\,bx+3\,a)+8\,a^3+4i\,\pi(6\,b^2x^2+8\,abx+3\,a^2)}{32\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

output
```
-1/32*(32*b^3*x^3 + 48*a*b^2*x^2 + 32*a^2*b*x - I*pi^3 - 2*pi^2*(4*b*x + 3
*a) + 8*a^3 + 4*I*pi*(6*b^2*x^2 + 8*a*b*x + 3*a^2))/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3}{4x}-\frac{b^2\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{4x^2} - \frac{b\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{4x^3}-\frac{\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**5,x)
```

output
```
-b**3/(4*x) - b**2*acoth(tanh(a + b*x))/(4*x**2) - b*acoth(tanh(a + b*x))*
*2/(4*x**3) - acoth(tanh(a + b*x))**3/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{1}{4}b\left(\frac{b^2}{x}+\frac{b\,\mathrm{arcoth}\,(\tanh\,(bx+a))}{x^2}\right) - \frac{b\,\mathrm{arcoth}\,(\tanh\,(bx+a))^2}{4\,x^3} - \frac{\mathrm{arcoth}\,(\tanh\,(bx+a))^3}{4\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

output
```
-1/4*b*(b^2/x + b*arccoth(tanh(b*x + a))/x^2) - 1/4*b*arccoth(tanh(b*x + a
))^2/x^3 - 1/4*arccoth(tanh(b*x + a))^3/x^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{32\,b^3x^3+24i\,\pi b^2x^2+48\,ab^2x^2-8\,\pi^2bx+32i\,\pi abx+32\,a^2bx-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}{32\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

output
```
-1/32*(32*b^3*x^3 + 24*I*pi*b^2*x^2 + 48*a*b^2*x^2 - 8*pi^2*b*x + 32*I*pi*
a*b*x + 32*a^2*b*x - I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3\,x^3+b^2\,x^2\,\mathrm{acoth}(\tanh(a+b\,x))+b\,x\,\mathrm{acoth}(\tanh(a+b\,x))^2+\mathrm{acoth}(\tanh(a+b\,x))^3}{4\,x^4}$$

input
```
int(acoth(tanh(a + b*x))^3/x^5,x)
```

output
```
-(acoth(tanh(a + b*x))^3 + b^3*x^3 + b*x*acoth(tanh(a + b*x))^2 + b^2*x^2*
acoth(tanh(a + b*x)))/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = \frac{-acoth(\tanh(bx+a))^3+acoth(\tanh(bx+a))^2\,bx-acoth(\tanh(bx+a))\,b^2x^2+b^3x^3}{4x^4}$$

input
```
int(acoth(tanh(b*x+a))^3/x^5,x)
```

output
```
( - acoth(tanh(a + b*x))**3 + acoth(tanh(a + b*x))**2*b*x - acoth(tanh(a +
 b*x))*b**2*x**2 + b**3*x**3)/(4*x**4)
```

3.36 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx = \frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/20*b*arccoth(tanh(b*x+a))^4/x^4/(b*x-arccoth(tanh(b*x+a)))^2+1/5*arccoth
(tanh(b*x+a))^4/x^5/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx = -\frac{b^3x^3 + 2b^2x^2\coth^{-1}(\tanh(a+bx)) + 3bx\coth^{-1}(\tanh(a+bx))^2 + 4\coth^{-1}(\tanh(a+bx))^3}{20x^5}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

output
```
-1/20*(b^3*x^3 + 2*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 3*b*x*ArcCoth[Tanh[a +
 b*x]]^2 + 4*ArcCoth[Tanh[a + b*x]]^3)/x^5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}dx}{5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

output
```
(b*ArcCoth[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcCoth[Tanh[a + b*x]])^2) + A
rcCoth[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{b^{3} x^{3}+2 b^{2} x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)+3 b \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2} x +4 \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{3}}{20 x^{5}}$	53
risch	Expression too large to display	17234

input
```
int(arccoth(tanh(b*x+a))^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b^3*x^3+2*b^2*x^2*arccoth(tanh(b*x+a))+3*b*arccoth(tanh(b*x+a))^2*x
+4*arccoth(tanh(b*x+a))^3)/x^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{40\,b^3x^3+80\,ab^2x^2+60\,a^2bx-2i\,\pi^3-3\,\pi^2(5\,bx+4\,a)+16\,a^3+4i\,\pi(10\,b^2x^2+15\,abx+6\,a^2)}{80\,x^5}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

output
```
-1/80*(40*b^3*x^3 + 80*a*b^2*x^2 + 60*a^2*b*x - 2*I*pi^3 - 3*pi^2*(5*b*x +
 4*a) + 16*a^3 + 4*I*pi*(10*b^2*x^2 + 15*a*b*x + 6*a^2))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{b^3}{20x^2} - \frac{b^2\operatorname{acoth}(\tanh(a+bx))}{10x^3} - \frac{3b\operatorname{acoth}^2(\tanh(a+bx))}{20x^4} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{5x^5}$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**6,x)
```

output
```
-b**3/(20*x**2) - b**2*acoth(tanh(a + b*x))/(10*x**3) - 3*b*acoth(tanh(a +
 b*x))**2/(20*x**4) - acoth(tanh(a + b*x))**3/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{1}{20}\,b\left(\frac{b^2}{x^2} + \frac{2\,b\operatorname{arcoth}(\tanh(bx+a))}{x^3}\right) - \frac{3\,b\operatorname{arcoth}(\tanh(bx+a))^2}{20\,x^4} - \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{5\,x^5}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

output
```
-1/20*b*(b^2/x^2 + 2*b*arccoth(tanh(b*x + a))/x^3) - 3/20*b*arccoth(tanh(b
*x + a))^2/x^4 - 1/5*arccoth(tanh(b*x + a))^3/x^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{40\,b^3x^3+40i\,\pi b^2x^2+80\,ab^2x^2-15\,\pi^2bx+60i\,\pi abx+60\,a^2bx-2i\,\pi^3-12\,\pi^2a+24i\,\pi a^2+16\,a^3}{80\,x^5}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

output
```
-1/80*(40*b^3*x^3 + 40*I*pi*b^2*x^2 + 80*a*b^2*x^2 - 15*pi^2*b*x + 60*I*pi
*a*b*x + 60*a^2*b*x - 2*I*pi^3 - 12*pi^2*a + 24*I*pi*a^2 + 16*a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{\mathrm{acoth}(\tanh(a+b\,x))^3}{5\,x^5}-\frac{b^3}{20\,x^2}-\frac{b^2\,\mathrm{acoth}(\tanh(a+b\,x))}{10\,x^3}-\frac{3\,b\,\mathrm{acoth}(\tanh(a+b\,x))^2}{20\,x^4}$$

input
```
int(acoth(tanh(a + b*x))^3/x^6,x)
```

output
```
- acoth(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*acoth(tanh(a + b*x)
))/(10*x^3) - (3*b*acoth(tanh(a + b*x))^2)/(20*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx$$
$$= \frac{-4acoth(\tanh(bx+a))^3 + 3acoth(\tanh(bx+a))^2 bx - 2acoth(\tanh(bx+a))\, b^2x^2 + b^3x^3}{20x^5}$$

input
```
int(acoth(tanh(b*x+a))^3/x^6,x)
```

output
```
( - 4*acoth(tanh(a + b*x))**3 + 3*acoth(tanh(a + b*x))**2*b*x - 2*acoth(ta
nh(a + b*x))*b**2*x**2 + b**3*x**3)/(20*x**5)
```

3.37 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} \, dx = -\frac{x^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-x^(1+m)*hypergeom([1, 1+m],[2+m],b*x/(b*x-arccoth(tanh(b*x+a))))/(1+m)/(b
*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Integrate[x^m/ArcCoth[Tanh[a + b*x]],x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2595\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \; 2595$$

$$-\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[x^m/ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcCoth[Tanh[
a + b*x]])])/((1 + m)*(b*x - ArcCoth[Tanh[a + b*x]])))
```

Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx+a))}dx$$

input
```
int(x^m/arccoth(tanh(b*x+a)),x)
```

output
```
int(x^m/arccoth(tanh(b*x+a)),x)
```

Fricas [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
integral(x^m/arccoth(tanh(b*x + a)), x)
```

Sympy [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{acoth}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**m/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x**m/acoth(tanh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a)), x)
```

Giac [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

input
```
int(x^m/acoth(tanh(a + b*x)),x)
```

output
```
int(x^m/acoth(tanh(a + b*x)), x)
```

Reduce [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{acoth\,(\tanh\,(bx+a))}dx$$

input
```
int(x^m/acoth(tanh(b*x+a)),x)
```

output
```
int(x**m/acoth(tanh(a + b*x)),x)
```

3.38 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x^3}{3b} + \frac{x^2(bx-\coth^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx-\coth^{-1}(\tanh(a+bx)))^2}{b^3} + \frac{(bx-\coth^{-1}(\tanh(a+bx)))^3 \log(\coth^{-1}(\tanh(a+bx)))}{b^4}$$

output
```
1/3*x^3/b+1/2*x^2*(b*x-arccoth(tanh(b*x+a)))/b^2+x*(b*x-arccoth(tanh(b*x+a
)))^2/b^3+(b*x-arccoth(tanh(b*x+a)))^3*ln(arccoth(tanh(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x^3}{3b} - \frac{x^2(-bx+\coth^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(-bx+\coth^{-1}(\tanh(a+bx)))^2}{b^3} - \frac{(-bx+\coth^{-1}(\tanh(a+bx)))^3 \log(\coth^{-1}(\tanh(a+bx)))}{b^4}$$

input
```
Integrate[x^3/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^3/(3*b) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
 ArcCoth[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log
[ArcCoth[Tanh[a + b*x]]])/b^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2590, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2589}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int\frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\right.}{b}$$

$$\frac{x^3}{3b}$$

$$\Big\downarrow 14$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+$$

$$\frac{x^3}{3b}$$

input
```
Int[x^3/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^3/(3*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x^2/(2*b) + ((b*x - ArcCoth[T
anh[a + b*x]])*(x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a +
 b*x]]])/b^2))/b))/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.22 (sec) , antiderivative size = 130774, normalized size of antiderivative = 1614.49

method	result	size
risch	Expression too large to display	130774

input
```
int(x^3/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{8\,b^3x^3 - 12\,ab^2x^2 - 6\,\pi^2bx + 24\,a^2bx - 6i\,\pi(b^2x^2 - 4\,abx) - 3\,(-i\,\pi^3 - 6\,\pi^2a + 12i\,\pi a^2 + 8\,a^3)\log\left(i\,\pi + \right.}{24\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(8*b^3*x^3 - 12*a*b^2*x^2 - 6*pi^2*b*x + 24*a^2*b*x - 6*I*pi*(b^2*x^2
 - 4*a*b*x) - 3*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)*log(I*pi + 2*b*
x + 2*a))/b^4
```

Sympy [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^3}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x**3/acoth(tanh(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{4\,b^2x^3 - 3\left(-i\,\pi b + 2\,ab\right)x^2 - 3\left(\pi^2 + 4i\,\pi a - 4\,a^2\right)x}{12\,b^3} - \frac{\left(i\,\pi^3 - 6\,\pi^2 a - 12i\,\pi a^2 + 8\,a^3\right)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{8\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*b^2*x^3 - 3*(-I*pi*b + 2*a*b)*x^2 - 3*(pi^2 + 4*I*pi*a - 4*a^2)*x)
/b^3 - 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(-I*pi + 2*b*x + 2
*a)/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3\,b} - \frac{\left(i\,\pi + 2\,a\right)x^2}{4\,b^2} - \frac{\left(\pi^2 - 4i\,\pi a - 4\,a^2\right)x}{4\,b^3} + \frac{\left(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3\right)\log\left(\pi - 2i\,bx - 2i\,a\right)}{8\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/3*x^3/b - 1/4*(I*pi + 2*a)*x^2/b^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*x/b^3
 + 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log(pi - 2*I*b*x - 2*I*a)
/b^4
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.37

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3\,b} + \frac{x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{4\,b^2} + \frac{x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4\,b^3} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3-8\,a^3-}{}$$

input
```
int(x^3/acoth(tanh(a + b*x)),x)
```

output
```
x^3/(3*b) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(4*b^2) + (x*(log(-2/(exp(2*a
)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + 2*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)))/(8*b^4)
```

Reduce [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^3}{acoth\left(\tanh\left(bx+a\right)\right)}dx$$

input
```
int(x^3/acoth(tanh(b*x+a)),x)
```

output
```
int(x**3/acoth(tanh(a + b*x)),x)
```

3.39 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

output
```
1/2*x^2/b+x*(b*x-arccoth(tanh(b*x+a)))/b^2+(b*x-arccoth(tanh(b*x+a)))^2*ln
(arccoth(tanh(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x^2}{2b} - \frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

input
```
Integrate[x^2/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^2/(2*b) - (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{2589}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{14}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log(\coth^{-1}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

input
```
Int[x^2/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^2/(2*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x/b + ((b*x - ArcCoth[Tanh[a
+ b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 28786, normalized size of antiderivative = 514.04

method	result	size
risch	Expression too large to display	28786

input
```
int(x^2/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,b^2x^2 - 2i\,\pi bx - 4\,abx - (\pi^2 - 4i\,\pi a - 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{4\,b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2 - 2*I*pi*b*x - 4*a*b*x - (pi^2 - 4*I*pi*a - 4*a^2)*log(I*pi
 + 2*b*x + 2*a))/b^3
```

Sympy [F]

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^2}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**2/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x**2/acoth(tanh(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{bx^2+(i\pi-2a)x}{2b^2} - \frac{(\pi^2+4i\pi a-4a^2)\log(-i\pi+2bx+2a)}{4b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + (I*pi - 2*a)*x)/b^2 - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi
 + 2*b*x + 2*a)/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2b} - \frac{(i\pi+2a)x}{2b^2} - \frac{(\pi^2-4i\pi a-4a^2)\log(\pi-2ibx-2ia)}{4b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^2/b - 1/2*(I*pi + 2*a)*x/b^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(pi
- 2*I*b*x - 2*I*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.18

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2\,b} + \frac{x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{2\,b^2} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 - 4\,a\left(2\right.\right.}{4\,b^3}$$

input
```
int(x^2/acoth(tanh(a + b*x)),x)
```

output
```
x^2/(2*b) + (x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^2) + (log(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1))
)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/
(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*
x) + 4*a^2))/(4*b^3)
```

Reduce [F]

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^2}{acoth\left(\tanh\left(bx+a\right)\right)}dx$$

input
```
int(x^2/acoth(tanh(b*x+a)),x)
```

output
```
int(x**2/acoth(tanh(a + b*x)),x)
```

3.40 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x}{b} + \frac{(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^2}$$

output
```
x/b+(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(tanh(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^2}$$

input
```
Integrate[x/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x/b - ((-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx$$
$$\Big\downarrow \text{2589}$$
$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}$$
$$\Big\downarrow \text{2588}$$
$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}$$
$$\Big\downarrow \text{14}$$
$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

input
```
Int[x/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 4303, normalized size of antiderivative = 138.81

method	result	size
risch	Expression too large to display	4303

input
```
int(x/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x/b+1/2*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x
+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x
+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*a+4*I*(ln(
exp(b*x+a))-b*x-a)+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+1/4*I/b^2*ln(-2*P
i*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*ex
p(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*a+4*I*(ln(exp(b*x+a))-b*x-a)+2*
Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))-1/4*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2
*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,bx + (-i\,\pi - 2\,a)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x + (-I*pi - 2*a)*log(I*pi + 2*b*x + 2*a))/b^2
```

Sympy [F]

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x/acoth(tanh(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{(-i\,\pi + 2\,a)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
x/b - 1/2*(-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{(i\pi + 2a)\log(\pi - 2ibx - 2ia)}{2b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
x/b - 1/2*(I*pi + 2*a)*log(pi - 2*I*b*x - 2*I*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} + \frac{\ln\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right) - \ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)\right)\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right) + 2\,b\,x\right)}{2\,b^2}$$

input
```
int(x/acoth(tanh(a + b*x)),x)
```

output
```
x/b + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2
/(exp(2*a)*exp(2*b*x) - 1)))*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^2)
```

Reduce [F]

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x}{acoth\,(\tanh\,(bx+a))}dx$$

input
```
int(x/acoth(tanh(b*x+a)),x)
```

output
```
int(x/acoth(tanh(a + b*x)),x)
```

3.41 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx = \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

output
```
ln(arccoth(tanh(b*x+a)))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx = \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

output
```
Log[ArcCoth[Tanh[a + b*x]]]/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log \left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

output
```
Log[ArcCoth[Tanh[a + b*x]]]/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
default	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
parallelrisch	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
risch	$\ln\left(\ln(e^{bx+a}) - \frac{i\pi\left(-2\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2+2\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3+\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\right.}{}\right.$

input
```
int(1/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
ln(arccoth(tanh(b*x+a)))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\log\left(i\,\pi+2\,bx+2\,a\right)}{b}$$

input
```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
log(I*pi + 2*b*x + 2*a)/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \begin{cases} \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((log(acoth(tanh(a + b*x)))/b, Ne(b, 0)), (x/acoth(tanh(a)), True
))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\log\left(-\frac{1}{2}i\pi - bx - a\right)}{b}$$

input
```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
log(-1/2*I*pi - b*x - a)/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\log(\pi - 2i\,bx - 2i\,a)}{b}$$

input
```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
log(pi - 2*I*b*x - 2*I*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\ln(\mathrm{acoth}(\tanh(a+b\,x)))}{b}$$

input
```
int(1/acoth(tanh(a + b*x)),x)
```

output
```
log(acoth(tanh(a + b*x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = -\frac{\log(acoth(\tanh(bx+a)))}{b}$$

input
```
int(1/acoth(tanh(b*x+a)),x)
```

output
```
( - log(acoth(tanh(a + b*x))))/b
```

3.42 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx = -\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

output
```
-ln(x)/(b*x-arccoth(tanh(b*x+a)))+ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(ta
nh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx = \frac{-\log(x)+\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Integrate[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
(-Log[x] + Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{ 2591}$$

$$\frac{b \int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{b \int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
-(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*
x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.21 (sec) , antiderivative size = 972, normalized size of antiderivative = 22.09

method	result	size
risch	Expression too large to display	972

input
```
int(1/x/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-4*I/(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp
(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^3-4*I*b*x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)*l
n(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn
(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*a+4*I*(ln(exp(b*x+a))-b*x
-a)+2*Pi)+4*I/(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)
+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*c
sgn(I*exp(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-P
i*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*cs
gn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{2\left(\log\left(i\pi+2\,bx+2\,a\right)-\log\left(x\right)\right)}{i\pi+2\,a}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
-2*(log(I*pi + 2*b*x + 2*a) - log(x))/(I*pi + 2*a)
```

Sympy [F]

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{x\,\mathrm{acoth}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x/acoth(tanh(b*x+a)),x)
```

output
```
Integral(1/(x*acoth(tanh(a + b*x))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{i\,\pi - 2\,a} - \frac{2\,\log\left(x\right)}{i\,\pi - 2\,a}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*log(-I*pi + 2*b*x + 2*a)/(I*pi - 2*a) - 2*log(x)/(I*pi - 2*a)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))}\,dx = \frac{2i\,\log\left(\pi - 2i\,bx - 2i\,a\right)}{\pi - 2i\,a} - \frac{2i\,\log\left(x\right)}{\pi - 2i\,a}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2*I*log(pi - 2*I*b*x - 2*I*a)/(pi - 2*I*a) - 2*I*log(x)/(pi - 2*I*a)
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.57

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx = -\frac{4\,\mathrm{atanh}\left(\frac{4\,b\,x}{\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x}-1\right)}{\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x}$$

input
```
int(1/(x*acoth(tanh(a + b*x))),x)
```

output
```
-(4*atanh((4*b*x)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) - 1))/(log(-2/(exp(2*a)*exp(2
*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x)
```

Reduce [F]

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)x}dx$$

input
```
int(1/x/acoth(tanh(b*x+a)),x)
```

output
```
int(1/(acoth(tanh(a + b*x))*x),x)
```

3.43 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

output
```
1/x/(b*x-arccoth(tanh(b*x+a)))-b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^2+b*ln(a
rccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx = \frac{-\coth^{-1}(\tanh(a+bx)) + bx\left(1-\log(x)+\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
(-ArcCoth[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcCoth[Tanh[a + b*x]]]))
/(x*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{2591}$$

$$\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{b\left(\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\left(\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])) + (b*(-(Log[x]/(b*x - ArcCoth[Tanh[a
+ b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*x]])))/(
b*x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx+a))}dx$$

input
```
int(1/x^2/arccoth(tanh(b*x+a)),x)
```

output
```
int(1/x^2/arccoth(tanh(b*x+a)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,(i\,\pi - 2\,bx\log\left(i\,\pi + 2\,bx + 2\,a\right) + 2\,bx\log\left(x\right) + 2\,a)}{\pi^2 x - 4i\,\pi ax - 4\,a^2 x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
2*(I*pi - 2*b*x*log(I*pi + 2*b*x + 2*a) + 2*b*x*log(x) + 2*a)/(pi^2*x - 4*
I*pi*a*x - 4*a^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{x^2 \operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/acoth(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**2*acoth(tanh(a + b*x))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{4\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^2+4i\,\pi a-4\,a^2} + \frac{4\,b\log\left(x\right)}{\pi^2+4i\,\pi a-4\,a^2} + \frac{2}{(i\,\pi-2\,a)x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-4*b*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^2
 + 4*I*pi*a - 4*a^2) + 2/((I*pi - 2*a)*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{4\,b\log\left(\pi-2i\,bx-2i\,a\right)}{\pi^2-4i\,\pi a-4\,a^2} + \frac{4\,b\log\left(x\right)}{\pi^2-4i\,\pi a-4\,a^2} + \frac{2}{-i\,\pi x-2\,ax}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-4*b*log(pi - 2*I*b*x - 2*I*a)/(pi^2 - 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^
2 - 4*I*pi*a - 4*a^2) + 2/(-I*pi*x - 2*a*x)
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} \, dx = \frac{2 \ln\left(-\frac{1}{e^{2a} e^{2bx}-1}\right) - 2 \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx}-1}\right) + 4bx + bx \,\mathrm{atan}\left(\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}-1}\right) 1\mathrm{i} - \ln\left(-\frac{2}{e^{2a} e^{2bx}-1}\right) 1\mathrm{i} + bx\, 2\mathrm{i}}{\ln\left(-\frac{2}{e^{2a} e^{2bx}-1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}-1}\right) + 2bx}\right) 8\mathrm{i}}{x \left(\ln\left(-\frac{1}{e^{2a} e^{2bx}-1}\right) - \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx}-1}\right) + 2bx\right)^2}$$

input
```
int(1/(x^2*acoth(tanh(a + b*x))),x)
```

output
```
(2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 4*b*x + b*x*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(
log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x))*8i)/(x*(log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log
((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)
```

Reduce [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} \, dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right) x^2} dx$$

input
```
int(1/x^2/acoth(tanh(b*x+a)),x)
```

output
```
int(1/(acoth(tanh(a + b*x))*x**2),x)
```

3.44 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx = \frac{b}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)} - \frac{b^2 \log(x)}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3} + \frac{b^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3}$$

output
```
b/x/(b*x-arccoth(tanh(b*x+a)))^2+1/2/x^2/(b*x-arccoth(tanh(b*x+a)))-b^2*ln
(x)/(b*x-arccoth(tanh(b*x+a)))^3+b^2*ln(arccoth(tanh(b*x+a)))/(b*x-arccoth
(tanh(b*x+a)))^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{-4bx\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 + b^2x^2\left(3 - 2\log(x) + 2\log\left(\coth^{-1}(\tanh(a+bx)\right.\right.}{2x^2\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3}$$

input
```
Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
(-4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2
*Log[x] + 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b
*x]])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \ 2594$$

$$\frac{b\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \ 2594$$

$$\frac{b\left(\frac{b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\right)}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \ 2591$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\left(\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}+\frac{b\left(\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
1/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])) + (b*(1/(x*(b*x - ArcCoth[Tanh[a
+ b*x]])) + (b*(-(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tan
h[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]
])))/(b*x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

Maple [F]

$$\int \frac{1}{x^3 \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)}dx$$

input
```
int(1/x^3/arccoth(tanh(b*x+a)),x)
```

output
```
int(1/x^3/arccoth(tanh(b*x+a)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{8\,b^2x^2\log\left(i\,\pi+2\,bx+2\,a\right)-8\,b^2x^2\log\left(x\right)-8\,abx-\pi^2-4i\,\pi(bx-a)+4\,a^2}{-i\,\pi^3x^2-6\,\pi^2ax^2+12i\,\pi a^2x^2+8\,a^3x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
-(8*b^2*x^2*log(I*pi + 2*b*x + 2*a) - 8*b^2*x^2*log(x) - 8*a*b*x - pi^2 -
4*I*pi*(b*x - a) + 4*a^2)/(-I*pi^3*x^2 - 6*pi^2*a*x^2 + 12*I*pi*a^2*x^2 +
8*a^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{x^3 \operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**3/acoth(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**3*acoth(tanh(a + b*x))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx &= \frac{8\,b^2 \log\left(-i\,\pi + 2\,bx + 2\,a\right)}{-i\,\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3} \\ &\quad - \frac{8\,b^2 \log\left(x\right)}{-i\,\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3} \\ &\quad - \frac{i\,\pi + 4\,bx - 2\,a}{(\pi^2 + 4i\,\pi a - 4\,a^2)x^2}\end{aligned}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
8*b^2*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3)
- 8*b^2*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - (I*pi + 4*b*x
- 2*a)/((pi^2 + 4*I*pi*a - 4*a^2)*x^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{8i\,b^2\log\left(\pi - 2i\,bx - 2i\,a\right)}{\pi^3 - 6i\,\pi^2 a - 12\,\pi a^2 + 8i\,a^3} + \frac{8i\,b^2\log\left(x\right)}{\pi^3 - 6i\,\pi^2 a - 12\,\pi a^2 + 8i\,a^3} - \frac{-i\,\pi + 4\,bx - 2\,a}{\pi^2 x^2 - 4i\,\pi a x^2 - 4\,a^2 x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-8*I*b^2*log(pi - 2*I*b*x - 2*I*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^
3) + 8*I*b^2*log(x)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) - (-I*pi + 4
*b*x - 2*a)/(pi^2*x^2 - 4*I*pi*a*x^2 - 4*a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2 - \ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,\left(2\,\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 8\,b\,x\right) + \ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2 + 12\,b^2\,x^2 + 8\,b\,x\,\ln}{x^2\,\left(\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \right.}$$

input
```
int(1/(x^3*acoth(tanh(a + b*x))),x)
```

output
```
(log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) - 1))*(2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + 8*b*x) + log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 12*b^2*x^2 + b^2*x^2*atan
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(
2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*16i + 8*b*x
*log(-1/(exp(2*a)*exp(2*b*x) - 1)))/(x^2*(log(-1/(exp(2*a)*exp(2*b*x) - 1)
) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)
```

Reduce [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)x^3}dx$$

input
```
int(1/x^3/acoth(tanh(b*x+a)),x)
```

output
```
int(1/(acoth(tanh(a + b*x))*x**3),x)
```

3.45 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^m}{b\coth^{-1}(\tanh(a+bx))} - \frac{x^m\,\text{Hypergeometric2F1}\left(1,m,1+m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-x^m/b/arccoth(tanh(b*x+a))-x^m*hypergeom([1, m],[1+m],b*x/(b*x-arccoth(ta
nh(b*x+a))))/b/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Integrate[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}dx$$
$$\Big\downarrow \text{ 2599}$$
$$\frac{m\int \frac{x^{m-1}}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$
$$\Big\downarrow \text{ 2595}$$
$$-\frac{x^m \text{Hypergeometric2F1}\left(1,m,m+1,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^m/(b*ArcCoth[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b
*x)/(b*x - ArcCoth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F]

$$\int \frac{x^m}{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(x^m/arccoth(tanh(b*x+a))^2,x)
```

output
```
int(x^m/arccoth(tanh(b*x+a))^2,x)
```

Fricas [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\, dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\, dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
integral(x^m/arccoth(tanh(b*x + a))^2, x)
```

Sympy [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**m/acoth(tanh(a + b*x))**2, x)
```

Maxima [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

Giac [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}\,dx$$

input
```
int(x^m/acoth(tanh(a + b*x))^2,x)
```

output
```
int(x^m/acoth(tanh(a + b*x))^2, x)
```

Reduce [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{-acoth(\tanh\left(bx+a\right))\left(\int \frac{x^m}{acoth(\tanh(bx+a))x}dx\right)m + x^m}{acoth\left(\tanh\left(bx+a\right)\right)b}$$

input
```
int(x^m/acoth(tanh(b*x+a))^2,x)
```

output
```
( - acoth(tanh(a + b*x))*int(x**m/(acoth(tanh(a + b*x))*x),x)*m + x**m)/(a
coth(tanh(a + b*x))*b)
```

3.46 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\begin{aligned}\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx &= \frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} \\ &\quad +\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))} \\ &\quad +\frac{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}\end{aligned}$$

output
```
4/3*x^3/b^2+2*x^2*(b*x-arccoth(tanh(b*x+a)))/b^3+4*x*(b*x-arccoth(tanh(b*x
+a)))^2/b^4-x^4/b/arccoth(tanh(b*x+a))+4*(b*x-arccoth(tanh(b*x+a)))^3*ln(a
rccoth(tanh(b*x+a)))/b^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^3}{3b^2} - \frac{x^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{3x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}{b^5\coth^{-1}(\tanh(a+bx))} - \frac{4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}$$

input
```
Integrate[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
x^3/(3*b^2) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
 ArcCoth[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcCoth[Tanh[a + b*x]])^4/(b^5
*ArcCoth[Tanh[a + b*x]]) - (4*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[ArcC
oth[Tanh[a + b*x]]])/b^5
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2590, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}dx}{b} - \frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$\big\downarrow$ 2590

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$\big\downarrow$ 2589

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$\big\downarrow$ 2588

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$\big\downarrow$ 14

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^4/(b*ArcCoth[Tanh[a + b*x]])) + (4*(x^3/(3*b) + ((b*x - ArcCoth[Tanh[a
 + b*x]])*(x^2/(2*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x/b + ((b*x - ArcC
oth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b))/b))/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.36 (sec) , antiderivative size = 131085, normalized size of antiderivative = 1337.60

method	result	size
risch	Expression too large to display	131085

input
```
int(x^4/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.16

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{16\,b^4x^4 - 32\,ab^3x^3 + 96\,a^2b^2x^2 + 144\,a^3bx - 3\,\pi^4 - 6i\,\pi^3(3\,bx - 4\,a) - 48\,a^4 - 12\,\pi^2(2\,b^2x^2 + 9\,abx - 6\,a}{}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/24*(16*b^4*x^4 - 32*a*b^3*x^3 + 96*a^2*b^2*x^2 + 144*a^3*b*x - 3*pi^4 -
6*I*pi^3*(3*b*x - 4*a) - 48*a^4 - 12*pi^2*(2*b^2*x^2 + 9*a*b*x - 6*a^2) -
8*I*pi*(2*b^3*x^3 - 12*a*b^2*x^2 - 27*a^2*b*x + 12*a^3) - 12*(16*a^3*b*x +
 pi^4 - 2*I*pi^3*(b*x + 4*a) + 16*a^4 - 12*pi^2*(a*b*x + 2*a^2) + 8*I*pi*(
3*a^2*b*x + 4*a^3))*log(I*pi + 2*b*x + 2*a))/(2*b^6*x + I*pi*b^5 + 2*a*b^5
)
```

Sympy [F]

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^4}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**4/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**4/acoth(tanh(a + b*x))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$
$$= \frac{16\,b^4x^4 - 3\,\pi^4 - 24i\,\pi^3a + 72\,\pi^2a^2 + 96i\,\pi a^3 - 48\,a^4 - 16\,(-i\,\pi b^3 + 2\,ab^3)x^3 - 24\,(\pi^2b^2 + 4i\,\pi ab^2 - 4\,a^2}{24\,(2\,b^6x - i\,\pi b^5 + 2\,ab^5)}$$
$$- \frac{(i\,\pi^3 - 6\,\pi^2a - 12i\,\pi a^2 + 8\,a^3)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/24*(16*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 + 96*I*pi*a^3 - 48*a
^4 - 16*(-I*pi*b^3 + 2*a*b^3)*x^3 - 24*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^
2)*x^2 - 18*(-I*pi^3*b + 6*pi^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x)/(2*b^6*x
 - I*pi*b^5 + 2*a*b^5) - 1/2*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log
(-I*pi + 2*b*x + 2*a)/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^3}{3\,b^2} - \frac{\pi^4 - 8i\,\pi^3 a - 24\,\pi^2 a^2 + 32i\,\pi a^3 + 16\,a^4}{8\,(2\,b^6 x + i\,\pi b^5 + 2\,ab^5)} - \frac{(i\,\pi + 2\,a)x^2}{2\,b^3} - \frac{3\,(\pi^2 - 4i\,\pi a - 4\,a^2)x}{4\,b^4} + \frac{(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3)\log(i\,\pi + 2\,bx + 2\,a)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/3*x^3/b^2 - 1/8*(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4)
/(2*b^6*x + I*pi*b^5 + 2*a*b^5) - 1/2*(I*pi + 2*a)*x^2/b^3 - 3/4*(pi^2 - 4
*I*pi*a - 4*a^2)*x/b^4 + 1/2*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log
(I*pi + 2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.83

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \text{Too large to display}$$

input
```
int(x^4/acoth(tanh(a + b*x))^2,x)
```

output
```
x^3/(3*b^2) - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*
x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32
*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(8*a*b^4 + 8*b^5*x - 4*b^4*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x))) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^3)
 + (3*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/(4*b^4) + (log(log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)))/(2*b^5)
```

Reduce [F]

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^4}{acoth\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(x^4/acoth(tanh(b*x+a))^2,x)
```

output
```
int(x**4/acoth(tanh(a + b*x))**2,x)
```

3.47 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{3x^2}{2b^2}+\frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

output
```
3/2*x^2/b^2+3*x*(b*x-arccoth(tanh(b*x+a)))/b^3-x^3/b/arccoth(tanh(b*x+a))+
3*(b*x-arccoth(tanh(b*x+a)))^2*ln(arccoth(tanh(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^2}{2b^2}-\frac{2x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{b^4\coth^{-1}(\tanh(a+bx))} + \frac{3\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

input
```
Integrate[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
x^2/(2*b^2) - (2*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcC
oth[Tanh[a + b*x]])^3/(b^4*ArcCoth[Tanh[a + b*x]]) + (3*(-(b*x) + ArcCoth[
Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int\frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \ 14$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log(\coth^{-1}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^3/(b*ArcCoth[Tanh[a + b*x]])) + (3*(x^2/(2*b) + ((b*x - ArcCoth[Tanh[a
 + b*x]])*(x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]
]])/b^2))/b))/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 29109, normalized size of antiderivative = 388.12

method	result	size
risch	Expression too large to display	29109

input
```
int(x^3/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$
$$= \frac{4\,b^3x^3 - 12\,ab^2x^2 - 16\,a^2bx - i\,\pi^3 + 2\,\pi^2(2\,bx - 3\,a) + 8\,a^3 - 2i\,\pi(3\,b^2x^2 + 8\,abx - 6\,a^2) + 3\,(8\,a^2bx - i}{4\,(2\,b^5x + i\,\pi b^4 + 2\,ab^4)}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^3*x^3 - 12*a*b^2*x^2 - 16*a^2*b*x - I*pi^3 + 2*pi^2*(2*b*x - 3*a)
 + 8*a^3 - 2*I*pi*(3*b^2*x^2 + 8*a*b*x - 6*a^2) + 3*(8*a^2*b*x - I*pi^3 -
2*pi^2*(b*x + 3*a) + 8*a^3 + 4*I*pi*(2*a*b*x + 3*a^2))*log(I*pi + 2*b*x +
2*a))/(2*b^5*x + I*pi*b^4 + 2*a*b^4)
```

Sympy [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^3}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**3/acoth(tanh(a + b*x))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\,b^3x^3 + i\,\pi^3 - 6\,\pi^2 a - 12i\,\pi a^2 + 8\,a^3 - 6\,(-i\,\pi b^2 + 2\,ab^2)x^2 + 4\,(\pi^2 b + 4i\,\pi ab - 4\,a^2 b)x}{4\,(2\,b^5 x - i\,\pi b^4 + 2\,ab^4)} - \frac{3\,(\pi^2 + 4i\,\pi a - 4\,a^2)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{4\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/4*(4*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 - 6*(-I*pi*b^2 +
2*a*b^2)*x^2 + 4*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x)/(2*b^5*x - I*pi*b^4 +
2*a*b^4) - 3/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi + 2*b*x + 2*a)/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{i\pi^3+6\pi^2a-12i\pi a^2-8a^3}{4(2b^5x+i\pi b^4+2ab^4)}+\frac{x^2}{2b^2}+\frac{(-i\pi-2a)x}{b^3} - \frac{3(\pi^2-4i\pi a-4a^2)\log(i\pi+2bx+2a)}{4b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-1/4*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)/(2*b^5*x + I*pi*b^4 + 2*a*b
^4) + 1/2*x^2/b^2 + (-I*pi - 2*a)*x/b^3 - 3/4*(pi^2 - 4*I*pi*a - 4*a^2)*lo
g(I*pi + 2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 490, normalized size of antiderivative = 6.53

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^2}{2b^2} + \frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)-\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\right)\left(3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2-12a\right.}{4b^4} - \frac{\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-\right.\right.}{4b\left(2ab^3+2b^4x-b^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(\right.\right.\right.} + \frac{x\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+2bx\right)}{b^3}$$

input
```
int(x^3/acoth(tanh(a + b*x))^2,x)
```

output
```
x^2/(2*b^2) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
- log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2
 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 12*a^2))/(4*b^4) - ((2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 +
 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(4*b*(2*a*b^3 + 2*b^4*x - b^3*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x))) + (x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/b^3
```

Reduce [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^3}{acoth\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(x^3/acoth(tanh(b*x+a))^2,x)
```

output
```
int(x**3/acoth(tanh(a + b*x))**2,x)
```

3.48 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

output
```
2*x/b^2-x^2/b/arccoth(tanh(b*x+a))+2*(b*x-arccoth(tanh(b*x+a)))*ln(arccoth
(tanh(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{bx - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{\coth^{-1}(\tanh(a+bx))} + 2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

input
```
Integrate[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(b*x - (-(b*x) + ArcCoth[Tanh[a + b*x]])^2/ArcCoth[Tanh[a + b*x]] + 2*(b*x
 - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{2\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{2\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log(\coth^{-1}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^2/(b*ArcCoth[Tanh[a + b*x]])) + (2*(x/b + ((b*x - ArcCoth[Tanh[a + b*x
]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 4626, normalized size of antiderivative = 92.52

method	result	size
risch	Expression too large to display	4626

input
```
int(x^2/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-4*I*x^2/b/(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-2*Pi*csgn(
I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(
2*b*x+2*a))-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)+2*x
/b^2+I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*
a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*
a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*a+4*I*(ln(exp
(b*x+a))-b*x-a)+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+1/2*I/b^3*ln(-2*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*
x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*ex...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\,b^2x^2 + 4\,abx + \pi^2 + 2i\,\pi(bx - 2\,a) - 4\,a^2 - 2\,(4\,abx - \pi^2 + 2i\,\pi(bx + 2\,a) + 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,(2\,b^4x + i\,\pi b^3 + 2\,ab^3)}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/2*(4*b^2*x^2 + 4*a*b*x + pi^2 + 2*I*pi*(b*x - 2*a) - 4*a^2 - 2*(4*a*b*x
- pi^2 + 2*I*pi*(b*x + 2*a) + 4*a^2)*log(I*pi + 2*b*x + 2*a))/(2*b^4*x + I
*pi*b^3 + 2*a*b^3)
```

Sympy [F]

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^2}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(x**2/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**2/acoth(tanh(a + b*x))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\,b^2x^2 + \pi^2 + 4i\,\pi a - 4\,a^2 - 2\,(i\,\pi b - 2\,ab)x}{2\,(2\,b^4x - i\,\pi b^3 + 2\,ab^3)} - \frac{(-i\,\pi + 2\,a)\log(-i\,\pi + 2\,bx + 2\,a)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/2*(4*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b - 2*a*b)*x)/(2*b^4*x
- I*pi*b^3 + 2*a*b^3) - (-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{\pi^2 - 4i\,\pi a - 4\,a^2}{2\,(2\,b^4x + i\,\pi b^3 + 2\,ab^3)} + \frac{x}{b^2} - \frac{(i\,\pi + 2\,a)\log(i\,\pi + 2\,bx + 2\,a)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/2*(pi^2 - 4*I*pi*a - 4*a^2)/(2*b^4*x + I*pi*b^3 + 2*a*b^3) + x/b^2 - (I*
pi + 2*a)*log(I*pi + 2*b*x + 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 302, normalized size of antiderivative = 6.04

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x}{b^2} - \frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2-4\,a\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2}{2\,b\left(2\,a\,b^2+2\,b^3\,x-b^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\right)} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{b^3}$$

input
```
int(x^2/acoth(tanh(a + b*x))^2,x)
```

output
```
x/b^2 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x) + 4*a^2)/(2*b*(2*a*b^2 + 2*b^3*x - b^2*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x))) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - l
og(-2/(exp(2*a)*exp(2*b*x) - 1)))*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/b^3
```

Reduce [F]

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^2}{acoth\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(x^2/acoth(tanh(b*x+a))^2,x)
```

output
```
int(x**2/acoth(tanh(a + b*x))**2,x)
```

3.49 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx = -\frac{x}{b\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

output
```
-x/b/arccoth(tanh(b*x+a))+ln(arccoth(tanh(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx = \frac{1 - \frac{bx}{\coth^{-1}(\tanh(a+bx))} + \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

input
```
Integrate[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(1 - (b*x)/ArcCoth[Tanh[a + b*x]] + Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2599, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b} - \frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2} - \frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2} - \frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x/(b*ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/b^2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))\operatorname{arccoth}(\tanh(bx+a))-bx}{b^2\operatorname{arccoth}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(2\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3-\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+\dots}\right.\right.}$

input
```
int(x/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(arccoth(tanh(b*x+a)))*arccoth(tanh(b*x+a))-b*x)/b^2/arccoth(tanh(b*x+a
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{i\,\pi + (i\,\pi + 2\,bx + 2\,a)\log\left(i\,\pi + 2\,bx + 2\,a\right) + 2\,a}{2\,b^3x + i\,\pi b^2 + 2\,ab^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
(I*pi + (I*pi + 2*b*x + 2*a)*log(I*pi + 2*b*x + 2*a) + 2*a)/(2*b^3*x + I*p
i*b^2 + 2*a*b^2)
```

Sympy [A] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \begin{cases} -\frac{x}{b\,\mathrm{acoth}\,(\tanh\,(a+bx))} + \frac{\log\,(\mathrm{acoth}\,(\tanh\,(a+bx)))}{b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2\,\mathrm{acoth}^2\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((-x/(b*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**2, N
e(b, 0)), (x**2/(2*acoth(tanh(a))**2), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{-i\,\pi + 2\,a}{2\,b^3x - i\,\pi b^2 + 2\,ab^2} + \frac{\log\,(-i\,\pi + 2\,bx + 2\,a)}{b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
(-I*pi + 2*a)/(2*b^3*x - I*pi*b^2 + 2*a*b^2) + log(-I*pi + 2*b*x + 2*a)/b^
2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{-i\pi - 2a}{2b^3x + i\pi b^2 + 2ab^2} + \frac{\log(i\pi + 2bx + 2a)}{b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-(-I*pi - 2*a)/(2*b^3*x + I*pi*b^2 + 2*a*b^2) + log(I*pi + 2*b*x + 2*a)/b^
2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{\ln(\mathrm{acoth}(\tanh(a+b\,x)))}{b^2} - \frac{x}{b\,\mathrm{acoth}(\tanh(a+b\,x))}$$

input
```
int(x/acoth(tanh(a + b*x))^2,x)
```

output
```
log(acoth(tanh(a + b*x)))/b^2 - x/(b*acoth(tanh(a + b*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{acoth(\tanh(bx+a))\log(acoth(\tanh(bx+a))) + bx}{acoth(\tanh(bx+a))\,b^2}$$

input
```
int(x/acoth(tanh(b*x+a))^2,x)
```

output
```
(acoth(tanh(a + b*x))*log(acoth(tanh(a + b*x))) + b*x)/(acoth(tanh(a + b*x
))*b**2)
```

3.50 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

output
```
-1/b/arccoth(tanh(b*x+a))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^(-2),x]
```

output
```
-(1/(b*ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^(-2),x]
```

output
```
-(1/(b*ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{1}{b\,\operatorname{arccoth}(\tanh(bx+a))}$
default	$-\frac{1}{b\,\operatorname{arccoth}(\tanh(bx+a))}$
parallelrisch	$-\frac{1}{b\,\operatorname{arccoth}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(2\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^{3}-\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}+\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie}{e^{2}}\right.\right.}$

input
```
int(1/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/arccoth(tanh(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{2}{2\,b^2x+i\,\pi b+2\,ab}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-2/(2*b^2*x + I*pi*b + 2*a*b)
```

Sympy [A] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \begin{cases} -\frac{1}{b\,\mathrm{acoth}\,(\tanh\,(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\mathrm{acoth}^2\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((-1/(b*acoth(tanh(a + b*x))), Ne(b, 0)), (x/acoth(tanh(a))**2, T
rue))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4}{-2\,(i\,\pi+2\,bx+2\,a)b}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
4/((-2*I*pi - 4*b*x - 4*a)*b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{2}{2\,b^2x+i\,\pi b+2\,ab}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-2/(2*b^2*x + I*pi*b + 2*a*b)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)}$$

input
```
int(1/acoth(tanh(a + b*x))^2,x)
```

output
```
-1/(b*acoth(tanh(a + b*x)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)b}$$

input
```
int(1/acoth(tanh(b*x+a))^2,x)
```

output
```
1/(acoth(tanh(a + b*x))*b)
```

3.51 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} \, dx = -\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{(bx-\coth^{-1}(\tanh(a+bx)))^2} - \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{(bx-\coth^{-1}(\tanh(a+bx)))^2}$$

output
```
-1/(b*x-arccoth(tanh(b*x+a)))/arccoth(tanh(b*x+a))+ln(x)/(b*x-arccoth(tanh
(b*x+a)))^2-ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{-bx+\coth^{-1}(\tanh(a+bx))\left(1+\log(bx)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{\coth^{-1}(\tanh(a+bx))\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
(-(b*x) + ArcCoth[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcCoth[Tanh[a + b*x]
]]))/(ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \ 2594$$

$$-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \ 2591$$

$$-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \ 14$$

$$-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \ 2588$$

$$-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \; 14$$

$$-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log(\coth^{-1}(\tanh(a+bx)))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
-(1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(
b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCot
h[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx+a))^2} dx$$

input
```
int(1/x/arccoth(tanh(b*x+a))^2,x)
```

output
```
int(1/x/arccoth(tanh(b*x+a))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx = -\frac{4 (-i\pi + (i\pi + 2bx + 2a)\log(i\pi + 2bx + 2a) + (-i\pi - 2bx - 2a)\log(x) - 2a)}{8a^2bx - i\pi^3 - 2\pi^2(bx + 3a) + 8a^3 + 4i\pi(2abx + 3a^2)}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-4*(-I*pi + (I*pi + 2*b*x + 2*a)*log(I*pi + 2*b*x + 2*a) + (-I*pi - 2*b*x
- 2*a)*log(x) - 2*a)/(8*a^2*b*x - I*pi^3 - 2*pi^2*(b*x + 3*a) + 8*a^3 + 4*
I*pi*(2*a*b*x + 3*a^2))
```

Sympy [F]

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx = \int \frac{1}{x \operatorname{acoth}^2(\tanh(a+bx))} dx$$

input
```
integrate(1/x/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x*acoth(tanh(a + b*x))**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4 \log\left(-i\,\pi + 2\,bx + 2\,a\right)}{\pi^2 + 4i\,\pi a - 4\,a^2} - \frac{4 \log\left(x\right)}{\pi^2 + 4i\,\pi a - 4\,a^2} - \frac{4}{\pi^2 + 4i\,\pi a - 4\,a^2 - 2\left(-i\,\pi b + 2\,ab\right)x}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
4*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 + 4*
I*pi*a - 4*a^2) - 4/(pi^2 + 4*I*pi*a - 4*a^2 - 2*(-I*pi*b + 2*a*b)*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4 \log\left(i\,\pi + 2\,bx + 2\,a\right)}{\pi^2 - 4i\,\pi a - 4\,a^2} - \frac{4 \log\left(x\right)}{\pi^2 - 4i\,\pi a - 4\,a^2} + \frac{4}{2i\,\pi bx + 4\,abx - \pi^2 + 4i\,\pi a + 4\,a^2}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
4*log(I*pi + 2*b*x + 2*a)/(pi^2 - 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 - 4*I
*pi*a - 4*a^2) + 4/(2*I*pi*b*x + 4*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 421, normalized size of antiderivative = 6.01

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{4\,\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-4\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+8\,b\,x+\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,1\mathrm{i}-\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,1\mathrm{i}+}{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2}\right.}{\left(\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\,\left(\ln\left(-\frac{}{e^{2\,a}\,e}\right.\right.}$$

input
```
int(1/(x*acoth(tanh(a + b*x))^2),x)
```

output
```
-(4*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 4*log((exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 8*b*x + log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1
i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2
*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x))*8i - atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i
- log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b
*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
))*(log(2) + log(-1/(exp(2*a)*exp(2*b*x) - 1)))*8i)/((log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log
(-1/(exp(2*a)*exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^2)
```

Reduce [F]

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)^2 x}dx$$

input
```
int(1/x/acoth(tanh(b*x+a))^2,x)
```

output
```
int(1/(acoth(tanh(a + b*x))**2*x),x)
```

3.52 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 102

$$\begin{aligned}
&\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx \\
&= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))} \\
&\quad + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))} \\
&\quad + \frac{2b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3} - \frac{2b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}
\end{aligned}$$

output
```
-2*b/(b*x-arccoth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))+1/x/(b*x-arccoth(ta
nh(b*x+a)))/arccoth(tanh(b*x+a))+2*b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^3-2*
b*ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{-b^2x^2 + \coth^{-1}(\tanh(a+bx))^2 + 2bx \coth^{-1}(\tanh(a+bx)) \left(\log(x) - \log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))}$$

input
```
Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
(-(b^2*x^2) + ArcCoth[Tanh[a + b*x]]^2 + 2*b*x*ArcCoth[Tanh[a + b*x]]*(Log
[x] - Log[ArcCoth[Tanh[a + b*x]]]))/(x*(b*x - ArcCoth[Tanh[a + b*x]])^3*Ar
cCoth[Tanh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2602, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx$$

$$\Big\downarrow \; 2602$$

$$\frac{2b \int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \; 2594$$

$$\frac{2b \left(-\frac{\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{1}{\left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))}\right)}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2591}$$

$$\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2b\left(-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{2b\left(-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*(-(1/((
b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x -
ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh
[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]
])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 5357, normalized size of antiderivative = 52.52

output too large to display

input
```
int(1/x^2/arccoth(tanh(b*x+a))^2,x)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{4\left(8\,abx - \pi^2 + 4i\,\pi(bx+a) + 4\,a^2 - 4\left(2\,b^2x^2 + i\,\pi bx + 2\,abx\right)\log\left(i\,\pi + 2\,bx + 2\,a\right) + 4\left(2\,b^2x^2 + i\,\pi b\right.\right.}{16\,a^3bx^2 + \pi^4x + 16\,a^4x - 2i\,\pi^3(bx^2 + 4\,ax) - 12\,\pi^2(abx^2 + 2\,a^2x) + 8i\,\pi(3\,a^2bx^2 + 4\,a}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-4*(8*a*b*x - pi^2 + 4*I*pi*(b*x + a) + 4*a^2 - 4*(2*b^2*x^2 + I*pi*b*x +
2*a*b*x)*log(I*pi + 2*b*x + 2*a) + 4*(2*b^2*x^2 + I*pi*b*x + 2*a*b*x)*log(
x))/(16*a^3*b*x^2 + pi^4*x + 16*a^4*x - 2*I*pi^3*(b*x^2 + 4*a*x) - 12*pi^2
*(a*b*x^2 + 2*a^2*x) + 8*I*pi*(3*a^2*b*x^2 + 4*a^3*x))
```

Sympy [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x^2 \operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**2*acoth(tanh(a + b*x))**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{16\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3}+\frac{16\,b\log\left(x\right)}{-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3}\\&\quad-\frac{4\left(i\,\pi-4\,bx-2\,a\right)}{2\left(\pi^2b+4i\,\pi ab-4\,a^2b\right)x^2-\left(i\,\pi^3-6\,\pi^2a-12i\,\pi a^2+8\,a^3\right)x}\end{aligned}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-16*b*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3)
+ 16*b*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 4*(I*pi - 4*b*x
 - 2*a)/(2*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x^2 - (I*pi^3 - 6*pi^2*a - 12*I
*pi*a^2 + 8*a^3)*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{16i\,b\log\left(i\,\pi+2\,bx+2\,a\right)}{\pi^3-6i\,\pi^2a-12\,\pi a^2+8i\,a^3}-\frac{16i\,b\log\left(x\right)}{\pi^3-6i\,\pi^2a-12\,\pi a^2+8i\,a^3}+\frac{8\,b}{2\,\pi^2bx-8i\,\pi abx-8\,a^2bx+i\,\pi^3+6\,\pi^2a-12i\,\pi a^2-8\,a^3}+\frac{4}{\pi^2x-4i\,\pi ax-4\,a^2x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
16*I*b*log(I*pi + 2*b*x + 2*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) -
 16*I*b*log(x)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) + 8*b/(2*pi^2*b*x
 - 8*I*pi*a*b*x - 8*a^2*b*x + I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3) + 4
/(pi^2*x - 4*I*pi*a*x - 4*a^2*x)
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 453, normalized size of antiderivative = 4.44

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)^2-\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,\left(8\,\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+b\,x\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,1{}\mathrm{i}-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,1{}\mathrm{i}+b\,x}{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x}\right.\right.}{x\,\left(\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\right)}$$

input
```
int(1/(x^2*acoth(tanh(a + b*x))^2),x)
```

output
```
(4*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1))*(8*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + b*x*atan((log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*ex
p(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*32i) + 4*log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 - 16*b^2*x^2 + b*x*log(-1/(e
xp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + 2*b*x))*32i)/(x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log(-1/(exp(2*a)*exp(2*b*x) -
1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)
```

Reduce [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)^2 x^2} dx$$

input
```
int(1/x^2/acoth(tanh(b*x+a))^2,x)
```

output
```
int(1/(acoth(tanh(a + b*x))**2*x**2),x)
```

3.53 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\begin{aligned}
&\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx \\
&= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))} \\
&\quad + \frac{3b}{2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))} \\
&\quad + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))} \\
&\quad + \frac{3b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4} - \frac{3b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}
\end{aligned}$$

output

```
-3*b^2/(b*x-arccoth(tanh(b*x+a)))^3/arccoth(tanh(b*x+a))+3/2*b/x/(b*x-arcc
oth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))+1/2/x^2/(b*x-arccoth(tanh(b*x+a))
)/arccoth(tanh(b*x+a))+3*b^2*ln(x)/(b*x-arccoth(tanh(b*x+a)))^4-3*b^2*ln(a
rccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} dx = -\frac{2b^3x^3 - 6bx \coth^{-1}(\tanh(a+bx))^2 + \coth^{-1}(\tanh(a+bx))^3 - 3b^2x^2 \coth^{-1}(\tanh(a+bx)) \left(-1+2\log\right.}{2x^2 \coth^{-1}(\tanh(a+bx)) \left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}$$

input
```
Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
-1/2*(2*b^3*x^3 - 6*b*x*ArcCoth[Tanh[a + b*x]]^2 + ArcCoth[Tanh[a + b*x]]^
3 - 3*b^2*x^2*ArcCoth[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a
 + b*x]]]))/(x^2*ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^
4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2602, 2602, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} dx}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b\left(\frac{2b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\Big\downarrow$ 2594

$$\frac{3b\left(\frac{2b\left(-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\Big\downarrow$ 2591

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\Big\downarrow$ 14

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\Big\downarrow$ 2588

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}\right.}\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow 14$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+$$

$$\frac{3b\left(\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{2b\left(-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{}{bx-\text{co}}}{bx-\coth^{-1}(\tanh(a+}\right.}{bx-\coth^{-1}(\tanh(a+bx))}\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
1/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (3*b*(1/
(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*(-(1/((b*
x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x - Ar
cCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a
 + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]])
))/(2*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx+a))^2} dx$$

input
```
int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

output
```
int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} dx = \frac{2\left(48\,ab^2x^2 + 24\,a^2bx + i\,\pi^3 - 6\,\pi^2(bx-a) - 8\,a^3 + 12i\,\pi(2\,b^2x^2 + 2\,abx - a^2) - 24\left(2\,b^3x^3 + i\,\pi b^2x^2 + \right.\right.}{32\,a^4bx^3 + i\,\pi^5x^2 + 32\,a^5x^2 + 2\,\pi^4(bx^3 + 5\,ax^2) - 8i\,\pi^3(2\,abx^3 + 5\,a^2x^2) - 16\,\pi}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
2*(48*a*b^2*x^2 + 24*a^2*b*x + I*pi^3 - 6*pi^2*(b*x - a) - 8*a^3 + 12*I*pi
*(2*b^2*x^2 + 2*a*b*x - a^2) - 24*(2*b^3*x^3 + I*pi*b^2*x^2 + 2*a*b^2*x^2)
*log(I*pi + 2*b*x + 2*a) + 24*(2*b^3*x^3 + I*pi*b^2*x^2 + 2*a*b^2*x^2)*log
(x))/(32*a^4*b*x^3 + I*pi^5*x^2 + 32*a^5*x^2 + 2*pi^4*(b*x^3 + 5*a*x^2) -
8*I*pi^3*(2*a*b*x^3 + 5*a^2*x^2) - 16*pi^2*(3*a^2*b*x^3 + 5*a^3*x^2) + 16*
I*pi*(4*a^3*b*x^3 + 5*a^4*x^2))
```

Sympy [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x^3 \operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(1/x**3/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**3*acoth(tanh(a + b*x))**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.33

$$\begin{aligned}&\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx \\ &= -\frac{48\,b^2\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^4+8i\,\pi^3 a-24\,\pi^2 a^2-32i\,\pi a^3+16\,a^4}+\frac{48\,b^2\log\left(x\right)}{\pi^4+8i\,\pi^3 a-24\,\pi^2 a^2-32i\,\pi a^3+16\,a^4} \\ &\quad+\frac{2\,(24\,b^2x^2+\pi^2+4i\,\pi a-4\,a^2-6\,(i\,\pi b-2\,ab)x)}{2\,(i\,\pi^3 b-6\,\pi^2 ab-12i\,\pi a^2 b+8\,a^3 b)x^3+(\pi^4+8i\,\pi^3 a-24\,\pi^2 a^2-32i\,\pi a^3+16\,a^4)x^2}\end{aligned}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-48*b^2*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*p
i*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi
*a^3 + 16*a^4) + 2*(24*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 6*(I*pi*b - 2*a
*b)*x)/(2*(I*pi^3*b - 6*pi^2*a*b - 12*I*pi*a^2*b + 8*a^3*b)*x^3 + (pi^4 +
8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 + 16*a^4)*x^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))^2}\,dx$$
$$= -\frac{48\,b^2\log\left(i\,\pi+2\,bx+2\,a\right)}{\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4}+\frac{48\,b^2\log\left(x\right)}{\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4}$$
$$+\frac{16\,b^2}{-2i\,\pi^3bx-12\,\pi^2abx+24i\,\pi a^2bx+16\,a^3bx+\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4}$$
$$-\frac{4\left(i\,\pi-8\,bx+2\,a\right)}{-2i\,\pi^3x^2-12\,\pi^2ax^2+24i\,\pi a^2x^2+16\,a^3x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-48*b^2*log(I*pi + 2*b*x + 2*a)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi
*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*
a^3 + 16*a^4) + 16*b^2/(-2*I*pi^3*b*x - 12*pi^2*a*b*x + 24*I*pi*a^2*b*x +
16*a^3*b*x + pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4) - 4*(
I*pi - 8*b*x + 2*a)/(-2*I*pi^3*x^2 - 12*pi^2*a*x^2 + 24*I*pi*a^2*x^2 + 16*
a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 689, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*acoth(tanh(a + b*x))^2),x)
```

output
```
(2*log(-1/(exp(2*a)*exp(2*b*x) - 1))^3 - 2*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) - 1))^3 - 6*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 - 1))*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 + 6*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1))^2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 32*b^3*x^3
 + 24*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 24*b*x*
log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + 24*b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1))
 - b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*atan((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*e
xp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*96i - 48*b*x*log(
(exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-1/(exp(2*a)*exp(2*b*
x) - 1)) + b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) -
 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*96i)/(x^2*(log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(
log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x)^4)
```

Reduce [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)^2 x^3} dx$$

input
```
int(1/x^3/acoth(tanh(b*x+a))^2,x)
```

output
```
int(1/(acoth(tanh(a + b*x))**2*x**3),x)
```

3.54 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m}\,\text{Hypergeometric2F1}\left(1,-1+m,m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-1/2*x^m/b/arccoth(tanh(b*x+a))^2-1/2*m*x^(-1+m)/b^2/arccoth(tanh(b*x+a))-
1/2*m*x^(-1+m)*hypergeom([1, -1+m],[m],b*x/(b*x-arccoth(tanh(b*x+a))))/b^2
/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(3,1+m,2+m,-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}$$

input
```
Integrate[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{m\int \frac{x^{m-1}}{\coth^{-1}(\tanh(a+bx))^2}dx}{2b}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2599$$

$$\frac{m\left(-\frac{(1-m)\int \frac{x^{m-2}}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^{m-1}}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2595$$

$$\frac{m\left(-\frac{x^{m-1}\text{Hypergeometric2F1}\left(1,m-1,m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b(bx-\coth^{-1}(\tanh(a+bx)))}-\frac{x^{m-1}}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^m/(b*ArcCoth[Tanh[a + b*x]]^2) + (m*(-(x^(-1 + m)/(b*ArcCoth[Tanh[a
 + b*x]])) - (x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcC
oth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))))/(2*b)
```

Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F]

$$\int \frac{x^m}{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^3}dx$$

input
```
int(x^m/arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^m/arccoth(tanh(b*x+a))^3,x)
```

Fricas [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
integral(x^m/arccoth(tanh(b*x + a))^3, x)
```

Sympy [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(x**m/acoth(tanh(a + b*x))**3, x)
```

Maxima [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

Giac [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^3}\,dx$$

input
```
int(x^m/acoth(tanh(a + b*x))^3,x)
```

output
```
int(x^m/acoth(tanh(a + b*x))^3, x)
```

Reduce [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{-acoth(\tanh\left(bx+a\right))^2\left(\int \frac{x^m}{acoth(\tanh(bx+a))^2 x}dx\right)m+x^m}{2acoth\left(\tanh\left(bx+a\right)\right)^2 b}$$

input
```
int(x^m/acoth(tanh(b*x+a))^3,x)
```

output
```
( - acoth(tanh(a + b*x))**2*int(x**m/(acoth(tanh(a + b*x))**2*x),x)*m + x*
*m)/(2*acoth(tanh(a + b*x))**2*b)
```

3.55 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{3x^2}{b^3} + \frac{6x\left(bx - \coth^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{6\left(bx - \coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}$$

output
```
3*x^2/b^3+6*x*(b*x-arccoth(tanh(b*x+a)))/b^4-1/2*x^4/b/arccoth(tanh(b*x+a)
)^2-2*x^3/b^2/arccoth(tanh(b*x+a))+6*(b*x-arccoth(tanh(b*x+a)))^2*ln(arcco
th(tanh(b*x+a)))/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$
\begin{aligned}
&\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx \\
&= \frac{x^2}{2b^3} - \frac{3x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^4} \\
&\quad + \frac{4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{b^5\coth^{-1}(\tanh(a+bx))} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}{2b^5\coth^{-1}(\tanh(a+bx))^2} \\
&\quad + \frac{6\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}
\end{aligned}
$$

input
```
Integrate[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
x^2/(2*b^3) - (3*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + A
rcCoth[Tanh[a + b*x]])^3)/(b^5*ArcCoth[Tanh[a + b*x]]) - (-(b*x) + ArcCoth
[Tanh[a + b*x]])^4/(2*b^5*ArcCoth[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2599, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$
\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx
$$

$$
\Big\downarrow \text{2599}
$$

$$
\frac{2\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}dx}{b} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}
$$

$$
\Big\downarrow \text{2599}
$$

$$\frac{2\left(\frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2590

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2589

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2588

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 14

$$\frac{2\left(\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log(\coth^{-1}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^4/(b*ArcCoth[Tanh[a + b*x]]^2) + (2*(-(x^3/(b*ArcCoth[Tanh[a + b*x]
])) + (3*(x^2/(2*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x/b + ((b*x - ArcCo
th[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b))/b))/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 29460, normalized size of antiderivative = 320.22

method	result	size
risch	Expression too large to display	29460

input
```
int(x^4/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.88

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{16\,b^4x^4 - 64\,ab^3x^3 - 176\,a^2b^2x^2 + 32\,a^3bx + 7\,\pi^4 - 4i\,\pi^3(bx + 14\,a) + 112\,a^4 + 4\,\pi^2(11\,b^2x^2 - 6\,abx - 4}{}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/8*(16*b^4*x^4 - 64*a*b^3*x^3 - 176*a^2*b^2*x^2 + 32*a^3*b*x + 7*pi^4 - 4
*I*pi^3*(b*x + 14*a) + 112*a^4 + 4*pi^2*(11*b^2*x^2 - 6*a*b*x - 42*a^2) -
16*I*pi*(2*b^3*x^3 + 11*a*b^2*x^2 - 3*a^2*b*x - 14*a^3) + 12*(16*a^2*b^2*x
^2 + 32*a^3*b*x + pi^4 - 4*I*pi^3*(b*x + 2*a) + 16*a^4 - 4*pi^2*(b^2*x^2 +
 6*a*b*x + 6*a^2) + 16*I*pi*(a*b^2*x^2 + 3*a^2*b*x + 2*a^3))*log(I*pi + 2*
b*x + 2*a))/(4*b^7*x^2 + 8*a*b^6*x - pi^2*b^5 + 4*a^2*b^5 + 4*I*pi*(b^6*x
+ a*b^5))
```

Sympy [F]

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^4}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**4/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(x**4/acoth(tanh(a + b*x))**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.15

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{16\,b^4x^4 + 7\,\pi^4 + 56i\,\pi^3a - 168\,\pi^2a^2 - 224i\,\pi a^3 + 112\,a^4 - 32\,(-i\,\pi b^3 + 2\,ab^3)x^3 + 44\,(\pi^2b^2 + 4i\,\pi ab^2 - \cdots}{8\,(4\,b^7x^2 - \pi^2b^5 - 4i\,\pi ab^5 + 4\,a^2b^5 - 4\,(i\,\pi b^6 - 2\,ab^6)x \cdots} - \frac{3\,(\pi^2 + 4i\,\pi a - 4\,a^2)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/8*(16*b^4*x^4 + 7*pi^4 + 56*I*pi^3*a - 168*pi^2*a^2 - 224*I*pi*a^3 + 112
*a^4 - 32*(-I*pi*b^3 + 2*a*b^3)*x^3 + 44*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*
b^2)*x^2 - 4*(-I*pi^3*b + 6*pi^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x)/(4*b^7*
x^2 - pi^2*b^5 - 4*I*pi*a*b^5 + 4*a^2*b^5 - 4*(I*pi*b^6 - 2*a*b^6)*x) - 3/
2*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi + 2*b*x + 2*a)/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.77

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{16\,\pi^3 bx - 96i\,\pi^2 abx - 192\,\pi a^2 bx + 128i\,a^3 bx + 7i\,\pi^4 + 56\,\pi^3 a - 168i\,\pi^2 a^2 - 224\,\pi a^3 + 112i\,a^4}{-32i\,b^7x^2 + 32\,\pi b^6 x - 64i\,ab^6 x + 8i\,\pi^2 b^5 + 32\,\pi ab^5 - 32i\,a^2 b^5} + \frac{x^2}{2\,b^3} - \frac{3\,(i\,\pi + 2\,a)x}{2\,b^4} - \frac{3\,(\pi^2 - 4i\,\pi a - 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-(16*pi^3*b*x - 96*I*pi^2*a*b*x - 192*pi*a^2*b*x + 128*I*a^3*b*x + 7*I*pi^
4 + 56*pi^3*a - 168*I*pi^2*a^2 - 224*pi*a^3 + 112*I*a^4)/(-32*I*b^7*x^2 +
32*pi*b^6*x - 64*I*a*b^6*x + 8*I*pi^2*b^5 + 32*pi*a*b^5 - 32*I*a^2*b^5) +
1/2*x^2/b^3 - 3/2*(I*pi + 2*a)*x/b^4 - 3/2*(pi^2 - 4*I*pi*a - 4*a^2)*log(I
*pi + 2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 867, normalized size of antiderivative = 9.42

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/acoth(tanh(a + b*x))^3,x)
```

output
```
((7*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a
)*exp(2*b*x) - 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
)^3 - 32*a^3 - 24*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 48*a^2*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x)))/(2*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + x*(16
*a*b^5 - 8*b^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 -
 8*a*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + x^2/(2*b^3) + (log(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x)
 - 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(...
```

Reduce [F]

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^4}{acoth\left(\tanh\left(bx+a\right)\right)^3}dx$$

input
```
int(x^4/acoth(tanh(b*x+a))^3,x)
```

output
```
int(x**4/acoth(tanh(a + b*x))**3,x)
```

3.56 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

output
```
3*x/b^3-1/2*x^3/b/arccoth(tanh(b*x+a))^2-3/2*x^2/b^2/arccoth(tanh(b*x+a))+
3*(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(tanh(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{b^3x^3 + 3b^2x^2\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^3\left(5 + 6\log\left(\coth^{-1}(\tanh(a+bx))\right)\right) - bx}{2b^4\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Integrate[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*(b^3*x^3 + 3*b^2*x^2*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^
3*(5 + 6*Log[ArcCoth[Tanh[a + b*x]]]) - b*x*ArcCoth[Tanh[a + b*x]]^2*(11 +
 6*Log[ArcCoth[Tanh[a + b*x]]]))/(b^4*ArcCoth[Tanh[a + b*x]]^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}dx}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\left(\frac{2\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3\left(\frac{2\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{3\left(\frac{2\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 14$$

$$\frac{3\left(\frac{2\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^3/(b*ArcCoth[Tanh[a + b*x]]^2) + (3*(-(x^2/(b*ArcCoth[Tanh[a + b*x]
])) + (2*(x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]
])/b^2))/b))/(2*b)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 4979, normalized size of antiderivative = 70.13

method	result	size
risch	Expression too large to display	4979

input
```
int(x^3/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2*I*(6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2-6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)
+1))^3-3*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+3*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3*Pi*x^2*csgn(I*exp(b*x+a))^2*csgn
(I*exp(2*b*x+2*a))+6*Pi*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-3*
Pi*x^2*csgn(I*exp(2*b*x+2*a))^3+3*Pi*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3*Pi*x^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3-12*I*x^2*ln(exp(b*x+a))-6*Pi*x^2-4*I*x^3*b)/b^2/(2*Pi*csgn(I/
(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2-Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*Pi*csgn(I*exp(b*x+a))*
csgn(I*exp(2*b*x+2*a))^2-Pi*csgn(I*exp(2*b*x+2*a))^3+Pi*csgn(I*exp(2*b*x+2
*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^3-2*Pi-4*I*ln(exp(b*x+a)))^2+3*x/b^3+3/2*I/b^4*ln(-2*P
i*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.75

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{16\,b^3x^3 + 32\,ab^2x^2 - 32\,a^2bx + 5i\,\pi^3 + 2\,\pi^2(4\,bx + 15\,a) - 40\,a^3 + 4i\,\pi(4\,b^2x^2 - 8\,abx - 15\,a^2) - 6\,(8\,ab^2 \cdots}{4\,(4\,b^6x^2 + 8\,ab^5x - \pi^2b^4 + 4\,a^2 \cdots}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/4*(16*b^3*x^3 + 32*a*b^2*x^2 - 32*a^2*b*x + 5*I*pi^3 + 2*pi^2*(4*b*x + 1
5*a) - 40*a^3 + 4*I*pi*(4*b^2*x^2 - 8*a*b*x - 15*a^2) - 6*(8*a*b^2*x^2 + 1
6*a^2*b*x - I*pi^3 - 2*pi^2*(2*b*x + 3*a) + 8*a^3 + 4*I*pi*(b^2*x^2 + 4*a*
b*x + 3*a^2))*log(I*pi + 2*b*x + 2*a))/(4*b^6*x^2 + 8*a*b^5*x - pi^2*b^4 +
 4*a^2*b^4 + 4*I*pi*(b^5*x + a*b^4))
```

Sympy [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^3}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(x**3/acoth(tanh(a + b*x))**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{16\,b^3x^3 - 5i\,\pi^3 + 30\,\pi^2a + 60i\,\pi a^2 - 40\,a^3 - 16\,(i\,\pi b^2 - 2\,ab^2)x^2 + 8\,(\pi^2b + 4i\,\pi ab - 4\,a^2b)x}{4\,(4\,b^6x^2 - \pi^2b^4 - 4i\,\pi ab^4 + 4\,a^2b^4 - 4\,(i\,\pi b^5 - 2\,ab^5)x)} - \frac{3\,(-i\,\pi + 2\,a)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/4*(16*b^3*x^3 - 5*I*pi^3 + 30*pi^2*a + 60*I*pi*a^2 - 40*a^3 - 16*(I*pi*b
^2 - 2*a*b^2)*x^2 + 8*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x)/(4*b^6*x^2 - pi^2
*b^4 - 4*I*pi*a*b^4 + 4*a^2*b^4 - 4*(I*pi*b^5 - 2*a*b^5)*x) - 3/2*(-I*pi +
 2*a)*log(-I*pi + 2*b*x + 2*a)/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.73

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{12\,\pi^2 bx - 48i\,\pi abx - 48\,a^2bx + 5i\,\pi^3 + 30\,\pi^2 a - 60i\,\pi a^2 - 40\,a^3}{4\,(4\,b^6x^2 + 4i\,\pi b^5x + 8\,ab^5x - \pi^2b^4 + 4i\,\pi ab^4 + 4\,a^2b^4)} + \frac{x}{b^3} + \frac{3\,(-i\,\pi - 2\,a)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/4*(12*pi^2*b*x - 48*I*pi*a*b*x - 48*a^2*b*x + 5*I*pi^3 + 30*pi^2*a - 60*
I*pi*a^2 - 40*a^3)/(4*b^6*x^2 + 4*I*pi*b^5*x + 8*a*b^5*x - pi^2*b^4 + 4*I*
pi*a*b^4 + 4*a^2*b^4) + x/b^3 + 3/2*(-I*pi - 2*a)*log(I*pi + 2*b*x + 2*a)/
b^4
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 620, normalized size of antiderivative = 8.73

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(x^3/acoth(tanh(a + b*x))^3,x)
```

output
```
x/b^3 - (x*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x)
- 1)) + 2*b*x) + 12*a^2) - (5*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3
- 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(
-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
+ 2*b*x)))/(4*b))/(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + x*(8*a*b^4 - 4
*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + 4*a^2*b^3 + 4*b^5*x^2 - 4*a*b^3*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp
(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*log(-2/(exp(2
*a)*exp(2*b*x) - 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) + 6*b*x))/(2*b^4)
```

Reduce [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^3}{acoth\left(\tanh\left(bx+a\right)\right)^3}dx$$

input
```
int(x^3/acoth(tanh(b*x+a))^3,x)
```

output
```
int(x**3/acoth(tanh(a + b*x))**3,x)
```

3.57 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

output
```
-1/2*x^2/b/arccoth(tanh(b*x+a))^2-x/b^2/arccoth(tanh(b*x+a))+ln(arccoth(ta
nh(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{3 - \frac{b^2x^2}{\coth^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\coth^{-1}(\tanh(a+bx))} + 2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{2b^3}$$

input
```
Integrate[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(3 - (b^2*x^2)/ArcCoth[Tanh[a + b*x]]^2 - (2*b*x)/ArcCoth[Tanh[a + b*x]] +
 2*Log[ArcCoth[Tanh[a + b*x]]])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}dx}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^2/(b*ArcCoth[Tanh[a + b*x]]^2) + (-(x/(b*ArcCoth[Tanh[a + b*x]])) +
 Log[ArcCoth[Tanh[a + b*x]]]/b^2)/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$\frac{-b^{2} x^{2}+2 \ln \left(\operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)\right) \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)^{2}-2 b x \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)}{2 b^{3} \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)^{2}}$	54
risch	Expression too large to display	956

input
```
int(x^2/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^2+2*ln(arccoth(tanh(b*x+a)))*arccoth(tanh(b*x+a))^2-2*b*x*arcc
oth(tanh(b*x+a)))/b^3/arccoth(tanh(b*x+a))^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.62

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{16\,abx - 3\,\pi^2 + 4i\,\pi(2\,bx + 3\,a) + 12\,a^2 + 2\,(4\,b^2x^2 + 8\,abx - \pi^2 + 4i\,\pi(bx + a) + 4\,a^2)\log\left(i\,\pi + 2\,bx + \right.}{2\,(4\,b^5x^2 + 8\,ab^4x - \pi^2b^3 + 4\,a^2b^3 + 4i\,\pi(b^4x + ab^3))}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(16*a*b*x - 3*pi^2 + 4*I*pi*(2*b*x + 3*a) + 12*a^2 + 2*(4*b^2*x^2 + 8*
a*b*x - pi^2 + 4*I*pi*(b*x + a) + 4*a^2)*log(I*pi + 2*b*x + 2*a))/(4*b^5*x
^2 + 8*a*b^4*x - pi^2*b^3 + 4*a^2*b^3 + 4*I*pi*(b^4*x + a*b^3))
```

Sympy [A] (verification not implemented)

Time = 22.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \begin{cases} -\frac{x^2}{2b\,\operatorname{acoth}^2(\tanh(a+bx))} - \frac{x}{b^2\,\operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3\,\operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-x**2/(2*b*acoth(tanh(a + b*x))**2) - x/(b**2*acoth(tanh(a + b*
x))) + log(acoth(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*acoth(tanh(a))*
*3), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{3\pi^2 + 12i\pi a - 12a^2 - 8(-i\pi b + 2ab)x}{2(4b^5x^2 - \pi^2b^3 - 4i\pi ab^3 + 4a^2b^3 - 4(i\pi b^4 - 2ab^4)x)} + \frac{\log(-i\pi + 2bx + 2a)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*(3*pi^2 + 12*I*pi*a - 12*a^2 - 8*(-I*pi*b + 2*a*b)*x)/(4*b^5*x^2 - pi
^2*b^3 - 4*I*pi*a*b^3 + 4*a^2*b^3 - 4*(I*pi*b^4 - 2*a*b^4)*x) + log(-I*pi
+ 2*b*x + 2*a)/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{8\pi bx - 16i\,abx + 3i\pi^2 + 12\pi a - 12i\,a^2}{8i\,b^5x^2 - 8\pi b^4x + 16i\,ab^4x - 2i\pi^2b^3 - 8\pi ab^3 + 8i\,a^2b^3} + \frac{\log(i\pi + 2bx + 2a)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-(8*pi*b*x - 16*I*a*b*x + 3*I*pi^2 + 12*pi*a - 12*I*a^2)/(8*I*b^5*x^2 - 8*
pi*b^4*x + 16*I*a*b^4*x - 2*I*pi^2*b^3 - 8*pi*a*b^3 + 8*I*a^2*b^3) + log(I
*pi + 2*b*x + 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{\ln\left(\mathrm{acoth}(\tanh(a+b\,x))\right)}{b^3} - \frac{\frac{b^2\,x^2}{2}+b\,x\,\mathrm{acoth}(\tanh(a+b\,x))}{b^3\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(x^2/acoth(tanh(a + b*x))^3,x)
```

output
```
log(acoth(tanh(a + b*x)))/b^3 - ((b^2*x^2)/2 + b*x*acoth(tanh(a + b*x)))/(
b^3*acoth(tanh(a + b*x))^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{-2acoth(\tanh(bx+a))^2\log(acoth(\tanh(bx+a))) - 2acoth(\tanh(bx+a))\,bx + b^2x^2}{2acoth(\tanh(bx+a))^2\,b^3}$$

input
```
int(x^2/acoth(tanh(b*x+a))^3,x)
```

output
```
( - 2*acoth(tanh(a + b*x))**2*log(acoth(tanh(a + b*x))) - 2*acoth(tanh(a +
 b*x))*b*x + b**2*x**2)/(2*acoth(tanh(a + b*x))**2*b**3)
```

3.58 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx = -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}$$

output
```
-1/2*x/b/arccoth(tanh(b*x+a))^2-1/2/b^2/arccoth(tanh(b*x+a))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx = -\frac{bx+\coth^{-1}(\tanh(a+bx))}{2b^2\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Integrate[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*(b*x + ArcCoth[Tanh[a + b*x]])/(b^2*ArcCoth[Tanh[a + b*x]]^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}dx}{2b} - \frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}d\coth^{-1}(\tanh(a+bx))}{2b^2} - \frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x/(b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{bx+\operatorname{arccoth}(\tanh(bx+a))}{2b^2\operatorname{arccoth}(\tanh(bx+a))^2}$
risch	$\frac{2i\left(2\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2-2\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3-\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2}}\right)\right)}{b^2\left(2\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2-2\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3-\pi\,\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi\,\operatorname{csgn}\left(\frac{i}{e^{2b}}\right)\right)}$

input
```
int(x/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+arccoth(tanh(b*x+a)))/b^2/arccoth(tanh(b*x+a))^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx=\frac{-i\,\pi-4\,bx-2\,a}{4\,b^4x^2+8\,ab^3x-\pi^2b^2+4\,a^2b^2+4i\,\pi(b^3x+ab^2)}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
(-I*pi - 4*b*x - 2*a)/(4*b^4*x^2 + 8*a*b^3*x - pi^2*b^2 + 4*a^2*b^2 + 4*I*
pi*(b^3*x + a*b^2))
```

Sympy [A] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \begin{cases} -\frac{x}{2b\operatorname{acoth}^2(\tanh(a+bx))} - \frac{1}{2b^2\operatorname{acoth}(\tanh(a+bx))} & \text{for } b\neq 0 \\ \frac{x^2}{2\operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-x/(2*b*acoth(tanh(a + b*x))**2) - 1/(2*b**2*acoth(tanh(a + b*x
))), Ne(b, 0)), (x**2/(2*acoth(tanh(a))**3), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{-i\pi + 4bx + 2a}{4b^4x^2 - \pi^2b^2 - 4i\pi ab^2 + 4a^2b^2 - 4(i\pi b^3 - 2ab^3)x}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-(-I*pi + 4*b*x + 2*a)/(4*b^4*x^2 - pi^2*b^2 - 4*I*pi*a*b^2 + 4*a^2*b^2 -
4*(I*pi*b^3 - 2*a*b^3)*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{i\pi + 4bx + 2a}{4b^4x^2 + 4i\pi b^3x + 8ab^3x - \pi^2b^2 + 4i\pi ab^2 + 4a^2b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-(I*pi + 4*b*x + 2*a)/(4*b^4*x^2 + 4*I*pi*b^3*x + 8*a*b^3*x - pi^2*b^2 + 4
*I*pi*a*b^2 + 4*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{\mathrm{acoth}(\tanh(a+b\,x))+b\,x}{2\,b^2\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(x/acoth(tanh(a + b*x))^3,x)
```

output
```
-(acoth(tanh(a + b*x)) + b*x)/(2*b^2*acoth(tanh(a + b*x))^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{-acoth(\tanh(bx+a))+bx}{2acoth\left(\tanh\left(bx+a\right)\right)^2 b^2}$$

input
```
int(x/acoth(tanh(b*x+a))^3,x)
```

output
```
( - acoth(tanh(a + b*x)) + b*x)/(2*acoth(tanh(a + b*x))**2*b**2)
```

3.59 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

output
```
-1/2/b/arccoth(tanh(b*x+a))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

output
```
-1/2*1/(b*ArcCoth[Tanh[a + b*x]]^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx$$

$$\Big\downarrow \; 2588$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

output
```
-1/2*1/(b*ArcCoth[Tanh[a + b*x]]^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{1}{2b \operatorname{arccoth}(\tanh(bx+a))^2}$
default	$-\frac{1}{2b \operatorname{arccoth}(\tanh(bx+a))^2}$
parallelrisch	$-\frac{1}{2b \operatorname{arccoth}(\tanh(bx+a))^2}$
risch	$\frac{}{b\left(2\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2b}}{e^{2bx}}\right.}$

input
```
int(1/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/arccoth(tanh(b*x+a))^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{2}{4\,b^3x^2+8\,ab^2x-\pi^2b+4\,a^2b+4i\,\pi(b^2x+ab)}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-2/(4*b^3*x^2 + 8*a*b^2*x - pi^2*b + 4*a^2*b + 4*I*pi*(b^2*x + a*b))
```

Sympy [A] (verification not implemented)

Time = 23.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \begin{cases} -\frac{1}{2b\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)} & \text{for } b \neq 0 \\ \frac{x}{\mathrm{acoth}^3\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-1/(2*b*acoth(tanh(a + b*x))**2), Ne(b, 0)), (x/acoth(tanh(a))*
*3, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{2}{\left(\pi^2 - 4i\,\pi(bx+a) - 4\,(bx+a)^2\right)b}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
2/((pi^2 - 4*I*pi*(b*x + a) - 4*(b*x + a)^2)*b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{2i}{4i\,b^3x^2 - 4\,\pi b^2x + 8i\,ab^2x - i\,\pi^2 b - 4\,\pi ab + 4i\,a^2b}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-2*I/(4*I*b^3*x^2 - 4*pi*b^2*x + 8*I*a*b^2*x - I*pi^2*b - 4*pi*a*b + 4*I*a
^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\,b\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(1/acoth(tanh(a + b*x))^3,x)
```

output
```
-1/(2*b*acoth(tanh(a + b*x))^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{2acoth\left(\tanh\left(bx+a\right)\right)^2 b}$$

input
```
int(1/acoth(tanh(b*x+a))^3,x)
```

output
```
1/(2*acoth(tanh(a + b*x))**2*b)
```

3.60 $\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} - \frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}$$

output
```
-1/2/(b*x-arccoth(tanh(b*x+a)))/arccoth(tanh(b*x+a))^2+1/(b*x-arccoth(tanh
(b*x+a)))^2/arccoth(tanh(b*x+a))-ln(x)/(b*x-arccoth(tanh(b*x+a)))^3+ln(arc
coth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{b^2x^2 - 4bx \coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 \left(3 + 2\log(bx) - 2\log\left(\coth^{-1}(\tanh(a+bx))\right.\right.}{2\coth^{-1}(\tanh(a+bx))^2 \left(-bx + \coth^{-1}(\tanh(a+bx))\right)^3}$$

input
```
Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
(b^2*x^2 - 4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2*(3 + 2*
Log[b*x] - 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*ArcCoth[Tanh[a + b*x]]^2*(-(
b*x) + ArcCoth[Tanh[a + b*x]])^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx$$

$$\Big\downarrow \; 2594$$

$$-\frac{\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2}dx}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx - \coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2594$$

$$-\frac{-\frac{\int \frac{1}{x \coth^{-1}(\tanh(a+bx))}dx}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{1}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx - \coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2591}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$-\frac{-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
-1/2*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) - (-(1/((
b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x -
ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh
[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]
)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 5659, normalized size of antiderivative = 58.34

output too large to display

input
```
int(1/x/arccoth(tanh(b*x+a))^3,x)
```

output
```
result too large to display
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.38

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{4\,(8\,abx - 3\,\pi^2 + 4i\,\pi(bx+3\,a) + 12\,a^2 - 2\,(4\,b^2x^2 + 8\,abx - \pi^2 + 4i\,\pi(bx+a) + 4\,a^2)\log\left(i\,\pi + 2\,bx + \right.}{32\,a^3b^2x^2 + 64\,a^4bx + i\,\pi^5 + 2\,\pi^4(2\,bx+5\,a) + 32\,a^5 - 4i\,\pi^3(b^2x^2 + 8\,abx + 10\,a^2) - 8\,\pi^2(3\,ab^2x^2}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
4*(8*a*b*x - 3*pi^2 + 4*I*pi*(b*x + 3*a) + 12*a^2 - 2*(4*b^2*x^2 + 8*a*b*x
 - pi^2 + 4*I*pi*(b*x + a) + 4*a^2)*log(I*pi + 2*b*x + 2*a) + 2*(4*b^2*x^2
 + 8*a*b*x - pi^2 + 4*I*pi*(b*x + a) + 4*a^2)*log(x))/(32*a^3*b^2*x^2 + 64
*a^4*b*x + I*pi^5 + 2*pi^4*(2*b*x + 5*a) + 32*a^5 - 4*I*pi^3*(b^2*x^2 + 8*
a*b*x + 10*a^2) - 8*pi^2*(3*a*b^2*x^2 + 12*a^2*b*x + 10*a^3) + 16*I*pi*(3*
a^2*b^2*x^2 + 8*a^3*b*x + 5*a^4))
```

Sympy [F]

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x \operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

input
```
integrate(1/x/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x*acoth(tanh(a + b*x))**3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx$$
$$= \frac{4(-3i\pi + 4bx + 6a)}{\pi^4 + 8i\pi^3 a - 24\pi^2 a^2 - 32i\pi a^3 + 16a^4 - 4(\pi^2 b^2 + 4i\pi a b^2 - 4a^2 b^2)x^2 - 4(-i\pi^3 b + 6\pi^2 ab + 12i\pi a^2 b}$$
$$+ \frac{8\log(-i\pi + 2bx + 2a)}{-i\pi^3 + 6\pi^2 a + 12i\pi a^2 - 8a^3} - \frac{8\log(x)}{-i\pi^3 + 6\pi^2 a + 12i\pi a^2 - 8a^3}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
4*(-3*I*pi + 4*b*x + 6*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 +
 16*a^4 - 4*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^2)*x^2 - 4*(-I*pi^3*b + 6*p
i^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x) + 8*log(-I*pi + 2*b*x + 2*a)/(-I*pi^
3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 8*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*
pi*a^2 - 8*a^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.78

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx$$
$$= \frac{4(-3i\pi - 4bx - 6a)}{4\pi^2 b^2 x^2 - 16i\pi a b^2 x^2 - 16a^2 b^2 x^2 + 4i\pi^3 bx + 24\pi^2 abx - 48i\pi a^2 bx - 32a^3 bx - \pi^4 + 8i\pi^3 a + 24\pi^2 a^2 +}$$
$$- \frac{8i\log(i\pi + 2bx + 2a)}{\pi^3 - 6i\pi^2 a - 12\pi a^2 + 8i a^3} + \frac{8i\log(x)}{\pi^3 - 6i\pi^2 a - 12\pi a^2 + 8i a^3}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output

```
4*(-3*I*pi - 4*b*x - 6*a)/(4*pi^2*b^2*x^2 - 16*I*pi*a*b^2*x^2 - 16*a^2*b^2
*x^2 + 4*I*pi^3*b*x + 24*pi^2*a*b*x - 48*I*pi*a^2*b*x - 32*a^3*b*x - pi^4
+ 8*I*pi^3*a + 24*pi^2*a^2 - 32*I*pi*a^3 - 16*a^4) - 8*I*log(I*pi + 2*b*x
+ 2*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) + 8*I*log(x)/(pi^3 - 6*I*
pi^2*a - 12*pi*a^2 + 8*I*a^3)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 902, normalized size of antiderivative = 9.30

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input

```
int(1/(x*acoth(tanh(a + b*x))^3),x)
```

output

```
- (16*atanh((16*(4*b*x - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x))/((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(
-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x) + 4*a^2))*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/16 - (a*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b
*x) - 1)) + 2*b*x))/4 + a^2/4))/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3))/(log(-2/(ex
p(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 - 1)) + 2*b*x)^3 - (12/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) - (16*b*x)/((2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2))/(
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/...
```

Reduce [F]

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)^3 x} dx$$

input
```
int(1/x/acoth(tanh(b*x+a))^3,x)
```

output
```
int(1/(acoth(tanh(a + b*x))**3*x),x)
```

3.61 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 131

$$\begin{aligned}\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2} \\ &+ \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2} \\ &+ \frac{3b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))} \\ &- \frac{3b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4} + \frac{3b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}\end{aligned}$$

output

```
-3/2*b/(b*x-arccoth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))^2+1/x/(b*x-arccot
h(tanh(b*x+a)))/arccoth(tanh(b*x+a))^2+3*b/(b*x-arccoth(tanh(b*x+a)))^3/ar
ccoth(tanh(b*x+a))-3*b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^4+3*b*ln(arccoth(t
anh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{b^3x^3 - 6b^2x^2\coth^{-1}(\tanh(a+bx)) + 2\coth^{-1}(\tanh(a+bx))^3 + 3bx\coth^{-1}(\tanh(a+bx))^2\left(1+2\log\right.}{2x\coth^{-1}(\tanh(a+bx))^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}$$

input
```
Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
-1/2*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]
]^3 + 3*b*x*ArcCoth[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a
+ b*x]]]))/(x*ArcCoth[Tanh[a + b*x]]^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])^4
)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2602, 2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \; 2602$$

$$\frac{3b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2594$$

$$\frac{3b\left(-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2594}$$

$$\frac{3b\left(-\frac{-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2591}$$

$$\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{3b\left(-\frac{\frac{-\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{}{2(bx-\coth^{-1}(\tan}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 14$$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+$$

$$\frac{3b\left(-\frac{1}{2(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))^2}-\frac{-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{}{bx-\cot}}{bx-\coth^{-1}(\tanh(a+b}}{bx-\coth^{-1}(\tanh(a+bx))}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b*(-1/2
*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) - (-(1/((b*x
- ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x - ArcC
oth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a +
 b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/
(b*x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx+a))^3}\,dx$$

input
```
int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

output
```
int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{8\,(24\,ab^2x^2+36\,a^2bx-i\,\pi^3-3\,\pi^2(3\,bx+2\,a)+8\,a^3+12i\,\pi(b^2x^2+3\,abx+a^2)-6\,(4\,b^3x^3+8\,ab^2x^2}{64\,a^4b^2x^3+128\,a^5bx^2-\pi^6x+64\,a^6x+4i\,\pi^5(bx^2+3\,ax)+4\,\pi^4(b^2x^3+10\,abx^2+15\,a^2}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-8*(24*a*b^2*x^2 + 36*a^2*b*x - I*pi^3 - 3*pi^2*(3*b*x + 2*a) + 8*a^3 + 12
*I*pi*(b^2*x^2 + 3*a*b*x + a^2) - 6*(4*b^3*x^3 + 8*a*b^2*x^2 - pi^2*b*x +
4*a^2*b*x + 4*I*pi*(b^2*x^2 + a*b*x))*log(I*pi + 2*b*x + 2*a) + 6*(4*b^3*x
^3 + 8*a*b^2*x^2 - pi^2*b*x + 4*a^2*b*x + 4*I*pi*(b^2*x^2 + a*b*x))*log(x)
)/(64*a^4*b^2*x^3 + 128*a^5*b*x^2 - pi^6*x + 64*a^6*x + 4*I*pi^5*(b*x^2 +
3*a*x) + 4*pi^4*(b^2*x^3 + 10*a*b*x^2 + 15*a^2*x) - 32*I*pi^3*(a*b^2*x^3 +
 5*a^2*b*x^2 + 5*a^3*x) - 16*pi^2*(6*a^2*b^2*x^3 + 20*a^3*b*x^2 + 15*a^4*x
) + 64*I*pi*(2*a^3*b^2*x^3 + 5*a^4*b*x^2 + 3*a^5*x))
```

Sympy [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^2 \operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

input
```
integrate(1/x**2/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**2*acoth(tanh(a + b*x))**3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{48\,b\log(-i\pi + 2\,bx + 2\,a)}{\pi^4 + 8i\,\pi^3 a - 24\,\pi^2 a^2 - 32i\,\pi a^3 + 16\,a^4} - \frac{48\,b\log(x)}{\pi^4 + 8i\,\pi^3 a - 24\,\pi^2 a^2 - 32i\,\pi a^3 + 16\,a^4} - \frac{8\,(12\,b^2x^2 - \pi^2 - 4i\,\pi a + 4\,a^2 - 9\,(i\,\pi b - 2\,ab)x)}{4\,(i\,\pi^3 b^2 - 6\,\pi^2 ab^2 - 12i\,\pi a^2 b^2 + 8\,a^3 b^2)x^3 + 4\,(\pi^4 b + 8i\,\pi^3 ab - 24\,\pi^2 a^2 b - 32i\,\pi a^3 b + 16\,a^4 b)x^2 - (i\,\pi}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
48*b*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a
^3 + 16*a^4) - 48*b*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3
+ 16*a^4) - 8*(12*b^2*x^2 - pi^2 - 4*I*pi*a + 4*a^2 - 9*(I*pi*b - 2*a*b)*x
)/(4*(I*pi^3*b^2 - 6*pi^2*a*b^2 - 12*I*pi*a^2*b^2 + 8*a^3*b^2)*x^3 + 4*(pi
^4*b + 8*I*pi^3*a*b - 24*pi^2*a^2*b - 32*I*pi*a^3*b + 16*a^4*b)*x^2 - (I*p
i^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5)*x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{48\,b\log\left(i\,\pi+2\,bx+2\,a\right)}{\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4}-\frac{48\,b\log\left(x\right)}{\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4}$$
$$+\frac{16\left(8\,b^2x+5i\,\pi b+10\,ab\right)}{8i\,\pi^3b^2x^2+48\,\pi^2ab^2x^2-96i\,\pi a^2b^2x^2-64\,a^3b^2x^2-8\,\pi^4bx+64i\,\pi^3abx+192\,\pi^2a^2bx-256i\,\pi a^3bx-}$$
$$+\frac{8}{i\,\pi^3x+6\,\pi^2ax-12i\,\pi a^2x-8\,a^3x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
48*b*log(I*pi + 2*b*x + 2*a)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^
3 + 16*a^4) - 48*b*log(x)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 +
 16*a^4) + 16*(8*b^2*x + 5*I*pi*b + 10*a*b)/(8*I*pi^3*b^2*x^2 + 48*pi^2*a*
b^2*x^2 - 96*I*pi*a^2*b^2*x^2 - 64*a^3*b^2*x^2 - 8*pi^4*b*x + 64*I*pi^3*a*
b*x + 192*pi^2*a^2*b*x - 256*I*pi*a^3*b*x - 128*a^4*b*x - 2*I*pi^5 - 20*pi
^4*a + 80*I*pi^3*a^2 + 160*pi^2*a^3 - 160*I*pi*a^4 - 64*a^5) + 8/(I*pi^3*x
 + 6*pi^2*a*x - 12*I*pi*a^2*x - 8*a^3*x)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 1074, normalized size of antiderivative = 8.20

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*acoth(tanh(a + b*x))^3),x)
```

output
```
(8/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x) - (72*b*x)/((2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2) + (96*b^2*x^2)/((log(-2
/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + 2*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x) + 4*a^2)))/(x*((2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4
*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/
(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2) + x^2*(8*a*b - 4*b*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x)) + 4*b^2*x^3) + (96*b*atanh(((2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)
) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*exp(2*...
```

Reduce [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} \, dx = \int \frac{1}{acoth\,(\tanh\,(bx+a))^3\,x^2} dx$$

input
```
int(1/x^2/acoth(tanh(b*x+a))^3,x)
```

output
```
int(1/(acoth(tanh(a + b*x))**3*x**2),x)
```

3.62 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 170

$$\begin{aligned}
&\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx \\
&= -\frac{3b^2}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))^2} \\
&\quad + \frac{2b}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2} \\
&\quad + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2} \\
&\quad + \frac{6b^2}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^4 \coth^{-1}(\tanh(a+bx))} \\
&\quad - \frac{6b^2 \log(x)}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^5} + \frac{6b^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^5}
\end{aligned}$$

output

```
-3*b^2/(b*x-arccoth(tanh(b*x+a)))^3/arccoth(tanh(b*x+a))^2+2*b/x/(b*x-arcc
oth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))^2+1/2/x^2/(b*x-arccoth(tanh(b*x+a
)))/arccoth(tanh(b*x+a))^2+6*b^2/(b*x-arccoth(tanh(b*x+a)))^4/arccoth(tanh
(b*x+a))-6*b^2*ln(x)/(b*x-arccoth(tanh(b*x+a)))^5+6*b^2*ln(arccoth(tanh(b*
x+a)))/(b*x-arccoth(tanh(b*x+a)))^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$$
$$= \frac{-b^4x^4 + 8b^3x^3 \coth^{-1}(\tanh(a+bx)) - 8bx \coth^{-1}(\tanh(a+bx))^3 + \coth^{-1}(\tanh(a+bx))^4 - 12b^2x^2 \text{co}}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)^5 \coth^{-1}(\tanh(a+}$$

input
```
Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
(-(b^4*x^4) + 8*b^3*x^3*ArcCoth[Tanh[a + b*x]] - 8*b*x*ArcCoth[Tanh[a + b*
x]]^3 + ArcCoth[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcCoth[Tanh[a + b*x]]^2*(Lo
g[x] - Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])
^5*ArcCoth[Tanh[a + b*x]]^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2602, 2602, 2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b\left(\frac{3b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}dx}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 2594

$$\frac{2b\left(\frac{3b\left(-\frac{\int\frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\cdots\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 2594

$$\frac{2b\left(\frac{3b\left(-\frac{-\frac{\int\frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{\cdots}{x(bx\cdots}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 2591

$$\frac{2b\left(\frac{3b\left(-\frac{-\frac{\frac{b\int\frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int\frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-\cdots}}\cdots\right.}{bx-\coth^{-1}(\tanh(a+bx))}\cdots\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$2b\left(\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-\cdots}}\cdots\right.}{bx-\coth^{-1}(\tanh(a+bx))}\cdots\right.$$

$$\overline{\qquad\qquad bx-\coth^{-1}(\tanh(a+bx))\qquad\cdots}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$2b\left(\frac{3b\left(-\frac{-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\cdots}{2\left(bx-\coth^{-1}(\tan\cdots\right.}\cdots\right.}{bx-\coth^{-1}(\tanh(a+bx))}\cdots\right.$$

$$\overline{\qquad\qquad bx-\coth^{-1}(\tanh(a+bx))\qquad\cdots}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+$$

$$2b\left(\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{3b\left(-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\text{co}\cdots}\cdots}{\cdots}\right.}{bx-\coth^{-1}(\tanh(a\cdots}\cdots\right.$$

$$\overline{\qquad\qquad bx-\coth^{-1}(\tanh(a+bx))\qquad\cdots}$$

input
```
Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
1/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (2*b*(
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b*(-1/2
*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) - (-(1/((b*x
- ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x - ArcC
oth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a +
 b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/
(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

output
```
int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.71

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{4\left(192\,ab^3x^3 + 288\,a^2b^2x^2 + 64\,a^3bx - \pi^4 - 8i\,\pi^3(bx-a) - 16\,a^4 - 24\,\pi^2(3\,b^2x^2 + 2\,abx - a^2) + 32i\,\pi(3\right.}{128\,a^5b^2x^4 + 256\,a^6bx^3 - i\,\pi^7x^2 + 128\,a^7x^2 - 2\,\pi^6(2\,bx^3 + 7\,ax^2) + 4i\,\pi^5(b^2x^4 + 12}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
4*(192*a*b^3*x^3 + 288*a^2*b^2*x^2 + 64*a^3*b*x - pi^4 - 8*I*pi^3*(b*x - a
) - 16*a^4 - 24*pi^2*(3*b^2*x^2 + 2*a*b*x - a^2) + 32*I*pi*(3*b^3*x^3 + 9*
a*b^2*x^2 + 3*a^2*b*x - a^3) - 48*(4*b^4*x^4 + 8*a*b^3*x^3 - pi^2*b^2*x^2
+ 4*a^2*b^2*x^2 + 4*I*pi*(b^3*x^3 + a*b^2*x^2))*log(I*pi + 2*b*x + 2*a) +
48*(4*b^4*x^4 + 8*a*b^3*x^3 - pi^2*b^2*x^2 + 4*a^2*b^2*x^2 + 4*I*pi*(b^3*x
^3 + a*b^2*x^2))*log(x))/(128*a^5*b^2*x^4 + 256*a^6*b*x^3 - I*pi^7*x^2 + 1
28*a^7*x^2 - 2*pi^6*(2*b*x^3 + 7*a*x^2) + 4*I*pi^5*(b^2*x^4 + 12*a*b*x^3 +
 21*a^2*x^2) + 40*pi^4*(a*b^2*x^4 + 6*a^2*b*x^3 + 7*a^3*x^2) - 80*I*pi^3*(
2*a^2*b^2*x^4 + 8*a^3*b*x^3 + 7*a^4*x^2) - 32*pi^2*(10*a^3*b^2*x^4 + 30*a^
4*b*x^3 + 21*a^5*x^2) + 64*I*pi*(5*a^4*b^2*x^4 + 12*a^5*b*x^3 + 7*a^6*x^2)
)
```

Sympy [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\, dx = \int \frac{1}{x^3 \operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\, dx$$

input
```
integrate(1/x**3/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**3*acoth(tanh(a + b*x))**3), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\, dx = \frac{192\, b^2 \log\left(-i\,\pi + 2\,bx + 2\,a\right)}{i\,\pi^5 - 10\,\pi^4 a - 40i\,\pi^3 a^2 + 80\,\pi^2 a^3 + 80i\,\pi a^4 - 32\,a^5} - \frac{192\, b^2 \log\left(x\right)}{i\,\pi^5 - 10\,\pi^4 a - 40i\,\pi^3 a^2 + 80\,\pi^2 a^3 + 80i\,\pi a^4 - 32\,a^5} + \frac{4\,(96\,b^3 x^3 - i\,\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3 - 72\,(i\,\pi}{4\,(\pi^4 b^2 + 8i\,\pi^3 ab^2 - 24\,\pi^2 a^2 b^2 - 32i\,\pi a^3 b^2 + 16\,a^4 b^2)x^4 - 4\,(i\,\pi^5 b - 10\,\pi^4 ab - 40i\,\pi^3 a^2 b + 80\,\pi^2 a^3 b +}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
192*b^2*log(-I*pi + 2*b*x + 2*a)/(I*pi^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*
pi^2*a^3 + 80*I*pi*a^4 - 32*a^5) - 192*b^2*log(x)/(I*pi^5 - 10*pi^4*a - 40
*I*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5) + 4*(96*b^3*x^3 - I*pi^3
 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3 - 72*(I*pi*b^2 - 2*a*b^2)*x^2 - 8*(pi^2*
b + 4*I*pi*a*b - 4*a^2*b)*x)/(4*(pi^4*b^2 + 8*I*pi^3*a*b^2 - 24*pi^2*a^2*b
^2 - 32*I*pi*a^3*b^2 + 16*a^4*b^2)*x^4 - 4*(I*pi^5*b - 10*pi^4*a*b - 40*I*
pi^3*a^2*b + 80*pi^2*a^3*b + 80*I*pi*a^4*b - 32*a^5*b)*x^3 - (pi^6 + 12*I*
pi^5*a - 60*pi^4*a^2 - 160*I*pi^3*a^3 + 240*pi^2*a^4 + 192*I*pi*a^5 - 64*a
^6)*x^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx = \frac{192i b^2 \log(i\pi + 2bx + 2a)}{\pi^5 - 10i\pi^4 a - 40\pi^3 a^2 + 80i\pi^2 a^3 + 80\pi a^4 - 32i a^5} - \frac{192i b^2 \log(x)}{\pi^5 - 10i\pi^4 a - 40\pi^3 a^2 + 80i\pi^2 a^3 + 80\pi a^4 - 32i a^5} - \frac{4(i\pi - 12bx + 2a)}{\pi^4 x^2 - 8i\pi^3 a x^2 - 24\pi^2 a^2 x^2 + 32i\pi a^3 x^2 + 16a^4 x^2} + \frac{16(12b^3 x +}{4\pi^4 b^2 x^2 - 32i\pi^3 a b^2 x^2 - 96\pi^2 a^2 b^2 x^2 + 128i\pi a^3 b^2 x^2 + 64a^4 b^2 x^2 + 4i\pi^5 bx + 40\pi^4 abx - 160i\pi^3 a^2 bx}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
192*I*b^2*log(I*pi + 2*b*x + 2*a)/(pi^5 - 10*I*pi^4*a - 40*pi^3*a^2 + 80*I
*pi^2*a^3 + 80*pi*a^4 - 32*I*a^5) - 192*I*b^2*log(x)/(pi^5 - 10*I*pi^4*a -
 40*pi^3*a^2 + 80*I*pi^2*a^3 + 80*pi*a^4 - 32*I*a^5) - 4*(I*pi - 12*b*x +
2*a)/(pi^4*x^2 - 8*I*pi^3*a*x^2 - 24*pi^2*a^2*x^2 + 32*I*pi*a^3*x^2 + 16*a
^4*x^2) + 16*(12*b^3*x + 7*I*pi*b^2 + 14*a*b^2)/(4*pi^4*b^2*x^2 - 32*I*pi^
3*a*b^2*x^2 - 96*pi^2*a^2*b^2*x^2 + 128*I*pi*a^3*b^2*x^2 + 64*a^4*b^2*x^2
+ 4*I*pi^5*b*x + 40*pi^4*a*b*x - 160*I*pi^3*a^2*b*x - 320*pi^2*a^3*b*x + 3
20*I*pi*a^4*b*x + 128*a^5*b*x - pi^6 + 12*I*pi^5*a + 60*pi^4*a^2 - 160*I*p
i^3*a^3 - 240*pi^2*a^4 + 192*I*pi*a^5 + 64*a^6)
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 1251, normalized size of antiderivative = 7.36

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx = \text{Too large to display}$$

input
```
int(1/(x^3*acoth(tanh(a + b*x))^3),x)
```

output
```
(4/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x) + (32*b*x)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - (2
88*b^2*x^2)/((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^
2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2)) + (384*b^3*x^3)/((log(-
2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x) + 4*a^2)))/(x^3*(8*a*b - 4*b*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x)) + x^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*
b*x) - 1)) + 2*b*x) + 4*a^2) + 4*b^2*x^4) - (384*b^2*atanh((4*b*x*((2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e...
```

Reduce [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{1}{acoth\left(\tanh\left(bx+a\right)\right)^3 x^3}dx$$

input
```
int(1/x^3/acoth(tanh(b*x+a))^3,x)
```

output
```
int(1/(acoth(tanh(a + b*x))**3*x**3),x)
```

3.63 $\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{1+n} \text{Hypergeometric2F1}\left(-m, 1+n, 2+n, -\frac{\coth^{-1}(\text{t}}{bx-\coth^{-1}}\right.}{b(1+n)}$$

output
```
x^m*arccoth(tanh(b*x+a))^(1+n)*hypergeom([-m, 1+n],[2+n],-arccoth(tanh(b*x
+a))/(b*x-arccoth(tanh(b*x+a))))/b/(1+n)/((b*x/(b*x-arccoth(tanh(b*x+a))))
^m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{x^{1+m} \coth^{-1}(\tanh(a+bx))^n \left(1+\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)^{-n} \text{Hypergeometric2F1}\left(1+m, -n, 2+m, -\frac{}{-b}\right.}{1+m}$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^(1 + m)*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -(
(b*x)/(-(b*x) + ArcCoth[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + A
rcCoth[Tanh[a + b*x]]))^n)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2604}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{2604}$$

$$\frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(-m,n+1,n+2,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^m*ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(
ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)
/(b*x - ArcCoth[Tanh[a + b*x]]))^m)
```

Defintions of rubi rules used

rule 2604
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[u^m*(v^(n + 1)/(b*(n + 1)*(b*(u/(b*u - a*v)))^m))*Hype
rgeometric2F1[-m, n + 1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^n \, dx$$

input
```
int(x^m*arccoth(tanh(b*x+a))^n,x)
```

output
```
int(x^m*arccoth(tanh(b*x+a))^n,x)
```

Fricas [F]

$$\int x^m \coth^{-1}(\tanh(a + bx))^n \, dx = \int x^m \operatorname{arcoth}(\tanh(bx + a))^n \, dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral(x^m*arccoth(tanh(b*x + a))^n, x)
```

Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a + bx))^n \, dx = \int x^m \operatorname{acoth}^n(\tanh(a + bx)) \, dx$$

input
```
integrate(x**m*acoth(tanh(b*x+a))**n,x)
```

output
```
Integral(x**m*acoth(tanh(a + b*x))**n, x)
```

Maxima [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

Giac [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{acoth}(\tanh(a+b\,x))^n\,dx$$

input
```
int(x^m*acoth(tanh(a + b*x))^n,x)
```

output
```
int(x^m*acoth(tanh(a + b*x))^n, x)
```

Reduce [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m acoth(\tanh(bx+a))^n\,dx$$

input
```
int(x^m*acoth(tanh(b*x+a))^n,x)
```

output
```
int(x**m*acoth(tanh(a + b*x))**n,x)
```

3.64 $\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx$

Optimal result

Integrand size = 13, antiderivative size = 165

$$\begin{aligned}\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx =& \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} \\ &- \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} \\ &+ \frac{12x^2 \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)} \\ &- \frac{24x \coth^{-1}(\tanh(a+bx))^{4+n}}{b^4(1+n)(2+n)(3+n)(4+n)} \\ &+ \frac{24 \coth^{-1}(\tanh(a+bx))^{5+n}}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)}\end{aligned}$$

output
```
x^4*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-4*x^3*arccoth(tanh(b*x+a))^(2+n)/b^
2/(1+n)/(2+n)+12*x^2*arccoth(tanh(b*x+a))^(3+n)/b^3/(1+n)/(2+n)/(3+n)-24*x
*arccoth(tanh(b*x+a))^(4+n)/b^4/(1+n)/(2+n)/(3+n)/(4+n)+24*arccoth(tanh(b*
x+a))^(5+n)/b^5/(1+n)/(2+n)/(3+n)/(4+n)/(5+n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx$$
$$= \frac{\coth^{-1}(\tanh(a+bx))^{1+n}\left(b^4(120+154n+71n^2+14n^3+n^4)\,x^4 - 4b^3(60+47n+12n^2+n^3)\,x^3\coth^{-}\cdots\right.}{b^5(1\cdots}$$

input
```
Integrate[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)
*x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcCoth[Tanh[a + b*x]] + 12*b^
2*(20 + 9*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcCoth[T
anh[a + b*x]]^3 + 24*ArcCoth[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 +
n)*(4 + n)*(5 + n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^4\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\int x^3\coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^4\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{3\int x^2\coth^{-1}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\int x\coth^{-1}(\tanh(a+bx))^{n+3}dx}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+4}}{b(n+4)}-\frac{\int\coth^{-1}(\tanh(a+bx))^{n+4}dx}{b(n+4)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+4}}{b(n+4)}-\frac{\int\coth^{-1}(\tanh(a+bx))^{n+4}d\coth^{-1}(\tanh(a+bx))}{b^2(n+4)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+4}}{b(n+4)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+5}}{b^2(n+4)(n+5)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

input
```
Int[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*((x^3*ArcCoth[Tanh[a
 + b*x]]^(2 + n))/(b*(2 + n)) - (3*((x^2*ArcCoth[Tanh[a + b*x]]^(3 + n))/(
b*(3 + n)) - (2*((x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b*(4 + n)) - ArcCoth[
Tanh[a + b*x]]^(5 + n)/(b^2*(4 + n)*(5 + n))))/(b*(3 + n))))/(b*(2 + n)))
/(b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(165) = 330$.

Time = 6.05 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.55

method	result
parallelrisch	$-\frac{-240 \operatorname{arccoth}(\tanh(bx+a))^n \operatorname{arccoth}(\tanh(bx+a))^3 x^2 b^2+120 \operatorname{arccoth}(\tanh(bx+a))^n \operatorname{arccoth}(\tanh(bx+a))^4 x b-120 \operatorname{arcco}}{}$
risch	Expression too large to display

input
```
int(x^4*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-240*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^3*x^2*b^2+120*arccoth(t
anh(b*x+a))^n*arccoth(tanh(b*x+a))^4*x*b-120*arccoth(tanh(b*x+a))^n*arccot
h(tanh(b*x+a))*x^4*b^4+240*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^2*x
^3*b^3-71*x^4*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^4*n^2+4*x^3*ar
ccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a))^n*b^3*n^3-154*x^4*arccoth(tanh(b
*x+a))*arccoth(tanh(b*x+a))^n*b^4*n+48*x^3*arccoth(tanh(b*x+a))^2*arccoth(
tanh(b*x+a))^n*b^3*n^2+188*x^3*arccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a))
^n*b^3*n-12*x^2*arccoth(tanh(b*x+a))^3*arccoth(tanh(b*x+a))^n*b^2*n^2-108*
x^2*arccoth(tanh(b*x+a))^3*arccoth(tanh(b*x+a))^n*b^2*n+24*x*arccoth(tanh(
b*x+a))^4*arccoth(tanh(b*x+a))^n*b*n-x^4*arccoth(tanh(b*x+a))*arccoth(tanh
(b*x+a))^n*b^4*n^4-14*x^4*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^4*
n^3-24*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^5)/(n^4+10*n^3+35*n^2+5
0*n+24)/(5+n)/b^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 828, normalized size of antiderivative = 5.02

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx = \text{Too large to display}$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
-1/4*((96*a^4*b*n*x - 4*(b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24
*b^5)*x^5 - 3*I*pi^5 + 6*pi^4*(b*n*x - 5*a) - 96*a^5 - 4*(a*b^4*n^4 + 6*a*
b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 - 6*I*pi^3*(8*a*b*n*x - (b^2*n^2 +
 b^2*n)*x^2 - 20*a^2) + 16*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3
 - 4*pi^2*(36*a^2*b*n*x + (b^3*n^3 + 3*b^3*n^2 + 2*b^3*n)*x^3 - 60*a^3 - 9
*(a*b^2*n^2 + a*b^2*n)*x^2) - 48*(a^3*b^2*n^2 + a^3*b^2*n)*x^2 + 2*I*pi*(9
6*a^3*b*n*x - (b^4*n^4 + 6*b^4*n^3 + 11*b^4*n^2 + 6*b^4*n)*x^4 - 120*a^4 +
 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 36*(a^2*b^2*n^2 + a^2*b^2*n
)*x^2))*cosh(n*log(1/2*I*pi + b*x + a)) + (96*a^4*b*n*x - 4*(b^5*n^4 + 10*
b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 - 3*I*pi^5 + 6*pi^4*(b*n*x -
 5*a) - 96*a^5 - 4*(a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^
4 - 6*I*pi^3*(8*a*b*n*x - (b^2*n^2 + b^2*n)*x^2 - 20*a^2) + 16*(a^2*b^3*n^
3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3 - 4*pi^2*(36*a^2*b*n*x + (b^3*n^3 + 3
*b^3*n^2 + 2*b^3*n)*x^3 - 60*a^3 - 9*(a*b^2*n^2 + a*b^2*n)*x^2) - 48*(a^3*
b^2*n^2 + a^3*b^2*n)*x^2 + 2*I*pi*(96*a^3*b*n*x - (b^4*n^4 + 6*b^4*n^3 + 1
1*b^4*n^2 + 6*b^4*n)*x^4 - 120*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*
n)*x^3 - 36*(a^2*b^2*n^2 + a^2*b^2*n)*x^2))*sinh(n*log(1/2*I*pi + b*x + a)
))/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [F]

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx = \text{Too large to display}$$

input
```
integrate(x**4*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**5*acoth(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*acoth(tanh(a +
 b*x))**4) - x**3/(3*b**2*acoth(tanh(a + b*x))**3) - x**2/(2*b**3*acoth(ta
nh(a + b*x))**2) - x/(b**4*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)
))/b**5, Eq(n, -5)), (Integral(x**4/acoth(tanh(a + b*x))**4, x), Eq(n, -4)
), (Integral(x**4/acoth(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/
acoth(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/acoth(tanh(a + b*x
)), x), Eq(n, -1)), (b**4*n**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*
x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 14*b**4*n**3*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x
))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 71*b**4*n**2*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x)
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 154*b**4*n*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
120*b**5) + 120*b**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b*
*5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) - 4*b**3*n**3*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*
b**5) - 48*b**3*n**2*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +...
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.30

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{(4\,(n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5-3i\,\pi^5+30\,\pi^4a+120i\,\pi^3a^2-240\,\pi^2a^3-240i\,\pi a^4+96\,a^5-2}{}$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(4*(n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 - 3*I*pi^5 + 30*pi^4*a + 12
0*I*pi^3*a^2 - 240*pi^2*a^3 - 240*I*pi*a^4 + 96*a^5 - 2*(I*pi*(n^4 + 6*n^3
 + 11*n^2 + 6*n)*b^4 - 2*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4)*x^4 + 4*(pi^2
*(n^3 + 3*n^2 + 2*n)*b^3 + 4*I*pi*(n^3 + 3*n^2 + 2*n)*a*b^3 - 4*(n^3 + 3*n
^2 + 2*n)*a^2*b^3)*x^3 - 6*(-I*pi^3*(n^2 + n)*b^2 + 6*pi^2*(n^2 + n)*a*b^2
 + 12*I*pi*(n^2 + n)*a^2*b^2 - 8*(n^2 + n)*a^3*b^2)*x^2 - 6*(pi^4*b*n + 8*
I*pi^3*a*b*n - 24*pi^2*a^2*b*n - 32*I*pi*a^3*b*n + 16*a^4*b*n)*x)*(cosh(-n
*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(n + 2
)*n^5 + 15*2^(n + 2)*n^4 + 85*2^(n + 2)*n^3 + 225*2^(n + 2)*n^2 + 137*2^(n
 + 3)*n + 15*2^(n + 5))*b^5)
```

Giac [F]

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx = \int x^4 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^4*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.31

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx = -\left(\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)^5}{4\,b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} - \frac{x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120} + \frac{3\,n\,x\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)^4}{2\,b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{n\,x^4\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(n^3+6\,n^2+11\,n+6\right)}{2\,b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{3\,n\,x^2\,\left(n+1\right)\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{2\,b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{n\,x^3\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)^2\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

input
```
int(x^4*acoth(tanh(a + b*x))^n,x)
```

output

```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5)/(4*b^5*(274*n
+ 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(
-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
 120)) + (n*x^4*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(-
2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
120)) + (n*x^3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n +
 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

Reduce [F]

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx = \int acoth(\tanh(bx+a))^n x^4 dx$$

input

```
int(x^4*acoth(tanh(b*x+a))^n,x)
```

output

```
int(acoth(tanh(a + b*x))**n*x**4,x)
```

3.65 $\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal result

Integrand size = 13, antiderivative size = 121

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^3 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)} - \frac{6 \coth^{-1}(\tanh(a+bx))^{4+n}}{b^4(1+n)(2+n)(3+n)(4+n)}$$

output
```
x^3*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-3*x^2*arccoth(tanh(b*x+a))^(2+n)/b^
2/(1+n)/(2+n)+6*x*arccoth(tanh(b*x+a))^(3+n)/b^3/(1+n)/(2+n)/(3+n)-6*arcco
th(tanh(b*x+a))^(4+n)/b^4/(1+n)/(2+n)/(3+n)/(4+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \left(b^3(24+26n+9n^2+n^3)\,x^3 - 3b^2(12+7n+n^2)\,x^2 \coth^{-1}(\tanh(a+bx)) + 6}{b^4(1+n)(2+n)(3+n)(4+n)}$$

input
```
Integrate[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2
*(12 + 7*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcCoth[Tanh[
a + b*x]]^2 - 6*ArcCoth[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
 + n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{3 \int x^2 \coth^{-1}(\tanh(a+bx))^{n+1} dx}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{3\left(\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{2\int x \coth^{-1}(\tanh(a+bx))^{n+2} dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\int\coth^{-1}(\tanh(a+bx))^{n+3}dx}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \textbf{2588}$$

$$\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\int\coth^{-1}(\tanh(a+bx))^{n+3}d\coth^{-1}(\tanh(a+bx))}{b^2(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \textbf{15}$$

$$\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+4}}{b^2(n+3)(n+4)}\right)}{b(n+2)}\right)}{b(n+1)}$$

input
```
Int[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*((x^2*ArcCoth[Tanh[a
 + b*x]]^(2 + n))/(b*(2 + n)) - (2*((x*ArcCoth[Tanh[a + b*x]]^(3 + n))/(b*
(3 + n)) - ArcCoth[Tanh[a + b*x]]^(4 + n)/(b^2*(3 + n)*(4 + n)))))/(b*(2 +
n))))/(b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(121) = 242$.

Time = 2.04 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.29

method	result
parallelrisch	$-\frac{-x^{3} \operatorname{arccoth}(\tanh(bx+a)) \operatorname{arccoth}(\tanh(bx+a))^{n} b^{3} n^{3}-9x^{3} \operatorname{arccoth}(\tanh(bx+a)) \operatorname{arccoth}(\tanh(bx+a))^{n} b^{3} n^{2}-26x^{3} \operatorname{arcc}}{}$
risch	Expression too large to display

input
```
int(x^3*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-x^3*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^3*n^3-9*x^3*arccoth(t
anh(b*x+a))*arccoth(tanh(b*x+a))^n*b^3*n^2-26*x^3*arccoth(tanh(b*x+a))*arc
coth(tanh(b*x+a))^n*b^3*n+3*x^2*arccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a)
)^n*b^2*n^2+21*x^2*arccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a))^n*b^2*n-6*x
*arccoth(tanh(b*x+a))^3*arccoth(tanh(b*x+a))^n*b*n-24*arccoth(tanh(b*x+a))
^n*arccoth(tanh(b*x+a))*x^3*b^3+36*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x
+a))^2*x^2*b^2-24*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^3*x*b+6*arcc
oth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^4)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.12

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{(48\,a^3bnx + 8\,(b^4n^3 + 6\,b^4n^2 + 11\,b^4n + 6\,b^4)x^4 - 3\,\pi^4 - 6i\,\pi^3(bnx - 4\,a) - 48\,a^4 + 8\,(ab^3n^3 + 3\,ab^3n^2 +}{}$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
1/8*((48*a^3*b*n*x + 8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 3*pi
^4 - 6*I*pi^3*(b*n*x - 4*a) - 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^
3*n)*x^3 - 6*pi^2*(6*a*b*n*x - (b^2*n^2 + b^2*n)*x^2 - 12*a^2) - 24*(a^2*b
^2*n^2 + a^2*b^2*n)*x^2 + 4*I*pi*(18*a^2*b*n*x + (b^3*n^3 + 3*b^3*n^2 + 2*
b^3*n)*x^3 - 24*a^3 - 6*(a*b^2*n^2 + a*b^2*n)*x^2))*cosh(n*log(1/2*I*pi +
b*x + a)) + (48*a^3*b*n*x + 8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4
 - 3*pi^4 - 6*I*pi^3*(b*n*x - 4*a) - 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 +
 2*a*b^3*n)*x^3 - 6*pi^2*(6*a*b*n*x - (b^2*n^2 + b^2*n)*x^2 - 12*a^2) - 24
*(a^2*b^2*n^2 + a^2*b^2*n)*x^2 + 4*I*pi*(18*a^2*b*n*x + (b^3*n^3 + 3*b^3*n
^2 + 2*b^3*n)*x^3 - 24*a^3 - 6*(a*b^2*n^2 + a*b^2*n)*x^2))*sinh(n*log(1/2*
I*pi + b*x + a)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \text{Too large to display}$$

input
```
integrate(x**3*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**4*acoth(tanh(a))**n/4, Eq(b, 0)), (-x**3/(3*b*acoth(tanh(a +
 b*x))**3) - x**2/(2*b**2*acoth(tanh(a + b*x))**2) - x/(b**3*acoth(tanh(a
+ b*x))) + log(acoth(tanh(a + b*x)))/b**4, Eq(n, -4)), (Integral(x**3/acot
h(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**3/acoth(tanh(a + b*x))**
2, x), Eq(n, -2)), (Integral(x**3/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b
**3*n**3*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**3*n**2*x**3*acoth(
tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + 26*b**3*n*x**3*acoth(tanh(a + b*x))*acoth(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24
*b**4) + 24*b**3*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n
**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*b**2*n**2*x**
2*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*acoth(tanh(a + b*
x))**2*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 36*b**2*x**2*acoth(tanh(a + b*x))**2*acoth(tanh(a +
 b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
+ 6*b*n*x*acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*acoth(tanh(a + b*
x))**3*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2...
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.12

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{(8\,(n^3+6\,n^2+11\,n+6)b^4x^4-3\,\pi^4-24i\,\pi^3a+72\,\pi^2a^2+96i\,\pi a^3-48\,a^4-4\,(i\,\pi(n^3+3\,n^2+2\,n)b^3-}{}$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(8*(n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 +
 96*I*pi*a^3 - 48*a^4 - 4*(I*pi*(n^3 + 3*n^2 + 2*n)*b^3 - 2*(n^3 + 3*n^2 +
 2*n)*a*b^3)*x^3 + 6*(pi^2*(n^2 + n)*b^2 + 4*I*pi*(n^2 + n)*a*b^2 - 4*(n^2
 + n)*a^2*b^2)*x^2 - 6*(-I*pi^3*b*n + 6*pi^2*a*b*n + 12*I*pi*a^2*b*n - 8*a
^3*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x
+ 2*a)))/((2^(n + 3)*n^4 + 5*2^(n + 4)*n^3 + 35*2^(n + 3)*n^2 + 25*2^(n +
4)*n + 3*2^(n + 6))*b^4)
```

Giac [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n\,dx=\int x^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.45

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n\,dx = -\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{3\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^4}{8\,b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{3\,n\,x\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^3}{4\,b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{n\,x^3\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(n^2+3\,n+2\right)}{2\,b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,n\,x^2\,\left(n+1\right)\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2}{4\,b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

input
```
int(x^3*acoth(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4)/(8*b^4*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) - (x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*
n^2 + 10*n^3 + n^4 + 24) + (3*n*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)/(4*b^3*(50
*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (n*x^3*(log(-2/(exp(2*a)*exp(2*b*x) -
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(3*n
 + n^2 + 2))/(2*b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*n*x^2*(n + 1)*
(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) - 1)) + 2*b*x)^2)/(4*b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

Reduce [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \int acoth(\tanh(bx+a))^n x^3 dx$$

input
```
int(x^3*acoth(tanh(b*x+a))^n,x)
```

output
```
int(acoth(tanh(a + b*x))**n*x**3,x)
```

3.66 $\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^2 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}$$

output
```
x^2*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-2*x*arccoth(tanh(b*x+a))^(2+n)/b^2/
(1+n)/(2+n)+2*arccoth(tanh(b*x+a))^(3+n)/b^3/(1+n)/(2+n)/(3+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \left(b^2(6+5n+n^2)\, x^2 - 2b(3+n)x \coth^{-1}(\tanh(a+bx)) + 2 \coth^{-1}(\tanh(a+b\right.}{b^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*A
rcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*
(3 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\int x \coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+2}d\coth^{-1}(\tanh(a+bx))}{b^2(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+3}}{b^2(n+2)(n+3)}\right)}{b(n+1)}$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^2*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*((x*ArcCoth[Tanh[a +
 b*x]]^(2 + n))/(b*(2 + n)) - ArcCoth[Tanh[a + b*x]]^(3 + n)/(b^2*(2 + n)*
(3 + n))))/(b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.99

method	result
parallelrisch	$-\frac{-2\operatorname{arccoth}(\tanh(bx+a))^{n}\operatorname{arccoth}(\tanh(bx+a))^{3}-6\operatorname{arccoth}(\tanh(bx+a))^{n}\operatorname{arccoth}(\tanh(bx+a))x^{2}b^{2}+6\operatorname{arccoth}(\tanh(bx}{}$
risch	Expression too large to display

input
```
int(x^2*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-2*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^3-6*arccoth(tanh(b*x+a))^
n*arccoth(tanh(b*x+a))*x^2*b^2+6*arccoth(tanh(b*x+a))^n*x*arccoth(tanh(b*x
+a))^2*b-x^2*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^2*n^2-5*x^2*arc
coth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^2*n+2*x*arccoth(tanh(b*x+a))^2*
arccoth(tanh(b*x+a))^n*b*n)/(n^2+3*n+2)/(3+n)/b^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.37

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = -\frac{(8\,a^2bnx - 4\,(b^3n^2 + 3\,b^3n + 2\,b^3)x^3 + i\,\pi^3 - 2\,\pi^2(bnx - 3\,a) - 8\,a^3 - 4\,(ab^2n^2 + ab^2n)x^2 + 2i\,\pi(4\,abn$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
-1/4*((8*a^2*b*n*x - 4*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + I*pi^3 - 2*pi^2*(
b*n*x - 3*a) - 8*a^3 - 4*(a*b^2*n^2 + a*b^2*n)*x^2 + 2*I*pi*(4*a*b*n*x - (
b^2*n^2 + b^2*n)*x^2 - 6*a^2))*cosh(n*log(1/2*I*pi + b*x + a)) + (8*a^2*b*
n*x - 4*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + I*pi^3 - 2*pi^2*(b*n*x - 3*a) -
8*a^3 - 4*(a*b^2*n^2 + a*b^2*n)*x^2 + 2*I*pi*(4*a*b*n*x - (b^2*n^2 + b^2*n
)*x^2 - 6*a^2))*sinh(n*log(1/2*I*pi + b*x + a)))/(b^3*n^3 + 6*b^3*n^2 + 11
*b^3*n + 6*b^3)
```

Sympy [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx$$
$$= \begin{cases} \frac{x^3 \operatorname{acoth}^n(\tanh(a))}{3} \\ -\frac{x^2}{2b \operatorname{acoth}^2(\tanh(a+bx))} - \frac{x}{b^2 \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} \\ \int \frac{x^2}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx \\ \int \frac{x^2}{\operatorname{acoth}(\tanh(a+bx))}\,dx \\ \frac{b^2n^2x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{5b^2nx^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{6b^2x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acot}}{b^3n^3+6b^3n^2+11b^3n} \end{cases}$$

input
```
integrate(x**2*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**3*acoth(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*acoth(tanh(a +
 b*x))**2) - x/(b**2*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**
3, Eq(n, -3)), (Integral(x**2/acoth(tanh(a + b*x))**2, x), Eq(n, -2)), (In
tegral(x**2/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*acoth(ta
nh(a + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n
+ 6*b**3) + 5*b**2*n*x**2*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*acoth(tanh(a + b
*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3
) - 2*b*n*x*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) - 6*b*x*acoth(tanh(a + b*x))**2*acoth(tan
h(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*acoth(ta
nh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3
*n + 6*b**3), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.02

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx$$
$$= \frac{(4(n^2+3n+2)b^3x^3 + i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3 - 2(i\pi(n^2+n)b^2 - 2(n^2+n)ab^2)x^2 + 2(\pi^2bn +}{(2^{n+2}n^3 + 3\cdot 2^{n+3}n^2 + 11\cdot 2^{n-}}$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(4*(n^2 + 3*n + 2)*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 - 2*(
I*pi*(n^2 + n)*b^2 - 2*(n^2 + n)*a*b^2)*x^2 + 2*(pi^2*b*n + 4*I*pi*a*b*n -
 4*a^2*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*
b*x + 2*a)))/((2^(n + 2)*n^3 + 3*2^(n + 3)*n^2 + 11*2^(n + 2)*n + 3*2^(n +
 3))*b^3)
```

Giac [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^2 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.71

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{4\,b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{n\,x\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{2\,b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{n\,x^2\,\left(n+1\right)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

input
```
int(x^2*acoth(tanh(a + b*x))^n,x)
```

output

```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n
^2 + n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n +
1)*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

Reduce [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = \int acoth(\tanh(bx+a))^n x^2 dx$$

input

```
int(x^2*acoth(tanh(b*x+a))^n,x)
```

output

```
int(acoth(tanh(a + b*x))**n*x**2,x)
```

3.67 $\int x \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

output
```
x*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-arccoth(tanh(b*x+a))^(2+n)/b^2/(1+n)/
(2+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{(b(2+n)x - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^{1+n}}{b^2(1+n)(2+n)}$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
((b*(2 + n)*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b
^2*(1 + n)*(2 + n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+1}d\coth^{-1}(\tanh(a+bx))}{b^2(n+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcCoth[Tanh[a + b*x]]^(2
 + n)/(b^2*(1 + n)*(2 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

method	result
parallelrisch	$-\frac{\operatorname{arccoth}(\tanh(bx+a))^{n}\operatorname{arccoth}(\tanh(bx+a))^{2}-x\operatorname{arccoth}(\tanh(bx+a))\operatorname{arccoth}(\tanh(bx+a))^{n}bn-2\operatorname{arccoth}(\tanh(bx+a))}{b^{2}(1+n)(2+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln(\mathrm{e}^{bx+a})-\frac{i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\operatorname{csgn}\left(\right.\right.}{}\right.}{2b(1+n)}$

input
```
int(x*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^2-x*arccoth(tanh(b*x+a))*arc
coth(tanh(b*x+a))^n*b*n-2*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))*x*b)
/b^2/(1+n)/(2+n)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.71

$$\int x\coth^{-1}(\tanh(a+bx))^{n}\,dx$$
$$=\frac{(4\,abnx+4\,(b^{2}n+b^{2})x^{2}+\pi^{2}+2i\,\pi(bnx-2\,a)-4\,a^{2})\cosh\left(n\log\left(\frac{1}{2}i\,\pi+bx+a\right)\right)+(4\,abnx+4\,(b^{2}n}{4\,(b^{2}n^{2}+3\,b^{2}n+2\,b^{2})}$$

input
```
integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
1/4*((4*a*b*n*x + 4*(b^2*n + b^2)*x^2 + pi^2 + 2*I*pi*(b*n*x - 2*a) - 4*a^
2)*cosh(n*log(1/2*I*pi + b*x + a)) + (4*a*b*n*x + 4*(b^2*n + b^2)*x^2 + pi
^2 + 2*I*pi*(b*n*x - 2*a) - 4*a^2)*sinh(n*log(1/2*I*pi + b*x + a)))/(b^2*n
^2 + 3*b^2*n + 2*b^2)
```

Sympy [F]

$$\int x\coth^{-1}(\tanh(a+bx))^n\,dx$$
$$= \begin{cases} \frac{x^2\operatorname{acoth}^n(\tanh(a))}{2} \\ -\frac{x}{b\operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^2} \\ \int \frac{x}{\operatorname{acoth}(\tanh(a+bx))}\,dx \\ \frac{bnx\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} + \frac{2bx\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} - \frac{\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a-}{b^2n^2+3b^2n+2b^2} \end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**2*acoth(tanh(a))**n/2, Eq(b, 0)), (-x/(b*acoth(tanh(a + b*x)
)) + log(acoth(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/acoth(tanh(a
+ b*x)), x), Eq(n, -1)), (b*n*x*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))*
*n/(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*acoth(tanh(a + b*x))*acoth(tanh
(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - acoth(tanh(a + b*x))**2*ac
oth(tanh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.12

$$\int x\coth^{-1}(\tanh(a+bx))^n\,dx$$
$$= \frac{(4\,b^2(n+1)x^2+\pi^2+4i\,\pi a-4\,a^2-2\,(i\,\pi bn-2\,abn)x)(\cosh(-n\log(-i\,\pi+2\,bx+2\,a))-\sinh(-n\log}{(2^{n+2}n^2+3\cdot 2^{n+2}n+2^{n+3})b^2}$$

input
```
integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(4*b^2*(n + 1)*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b*n - 2*a*b*n)*x)*(
cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2
^(n + 2)*n^2 + 3*2^(n + 2)*n + 2^(n + 3))*b^2)
```

Giac [F]

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \int x \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

input
```
integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.27

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2} - \frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n \left(\frac{\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2}{4\,b^2\,\left(n^2+3\,n+2\right)} - \frac{x^2\,\left(n+1\right)}{n^2+3\,n+2} + \frac{n\,x\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{2\,b\,\left(n^2+3\,n+2\right)}\right)$$

input
```
int(x*acoth(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/(4*b^2*(3*n + n^2
+ 2)) - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(log(-2/(exp(2*a)*exp(2*b*x)
- 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(
2*b*(3*n + n^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{acoth(\tanh(bx+a))^n \, acoth(\tanh(bx+a)) \left(-acoth(\tanh(bx+a)) - bnx - 2bx\right)}{b^2 (n^2+3n+2)}$$

input
```
int(x*acoth(tanh(b*x+a))^n,x)
```

output
```
(acoth(tanh(a + b*x))**n*acoth(tanh(a + b*x))*( - acoth(tanh(a + b*x)) - b
*n*x - 2*b*x))/(b**2*(n**2 + 3*n + 2))
```

3.68 $\int \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

output
```
arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))^n d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b(1+n)}$
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b(1+n)}$
parallelrisch	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{n} \operatorname{arccoth}(\tanh(bx+a))}{b(1+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\ \mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\right.}{}\right.}{2b(1}$

input
```
int(arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{(i\pi+2bx+2a)\cosh\left(n\log\left(\frac{1}{2}i\pi+bx+a\right)\right)+(i\pi+2bx+2a)\sinh\left(n\log\left(\frac{1}{2}i\pi+bx+a\right)\right)}{2(bn+b)}$$

input
```
integrate(arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
1/2*((I*pi + 2*b*x + 2*a)*cosh(n*log(1/2*I*pi + b*x + a)) + (I*pi + 2*b*x
+ 2*a)*sinh(n*log(1/2*I*pi + b*x + a)))/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \begin{cases} \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{for } b = 0 \wedge n = -1 \\ x \operatorname{acoth}^n(\tanh(a)) & \text{for } b = 0 \\ \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } n = -1 \\ \frac{\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x/acoth(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*acoth(tanh(a))**n,
Eq(b, 0)), (log(acoth(tanh(a + b*x)))/b, Eq(n, -1)), (acoth(tanh(a + b*x))
*acoth(tanh(a + b*x))**n/(b*n + b), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{(-i\pi+2bx+2a)(\cosh(-n\log(-i\pi+2bx+2a)) - \sinh(-n\log(-i\pi+2bx+2a)))}{(2^{n+1}n+2^{n+1})b}$$

input
```
integrate(arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(-I*pi + 2*b*x + 2*a)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*
pi + 2*b*x + 2*a)))/((2^(n + 1)*n + 2^(n + 1))*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{\left(\frac{1}{2}\log\left(-e^{(2bx+2a)}\right)\right)^{n+1}}{b(n+1)}$$

input
```
integrate(arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
(1/2*log(-e^(2*b*x + 2*a)))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.05

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \left(\frac{1}{2}\right)^n \left(\frac{x}{n+1} - \frac{\frac{\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)}{2} + b\,x}{b\,(n+1)}\right) \left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right) - \ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)\right)^n$$

input
```
int(acoth(tanh(a + b*x))^n,x)
```

output
```
(1/2)^n*(x/(n + 1) - (log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)/(b*(n + 1)))*(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x)
 - 1)))^n
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx=-\frac{acoth(\tanh(bx+a))^n\,acoth(\tanh(bx+a))}{b(n+1)}$$

input
```
int(acoth(tanh(b*x+a))^n,x)
```

output
```
( - acoth(tanh(a + b*x))**n*acoth(tanh(a + b*x)))/(b*(n + 1))
```

3.69 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
arccoth(tanh(b*x+a))^(1+n)*hypergeom([1, 1+n],[2+n],-arccoth(tanh(b*x+a))/
(b*x-arccoth(tanh(b*x+a))))/(1+n)/(b*x-arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^n \left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(-n,-n,1-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcCoth[Tan
h[a + b*x]]/(b*x)])/(n*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx$$

$$\Big\downarrow \; 2595$$

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCo
th[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcCot
h[Tanh[a + b*x]]))
```

Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x}dx$$

input
```
int(arccoth(tanh(b*x+a))^n/x,x)
```

output
```
int(arccoth(tanh(b*x+a))^n/x,x)
```

Fricas [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

output
```
integral(arccoth(tanh(b*x + a))^n/x, x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{acoth}^{n}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**n/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))**n/x, x)
```

Maxima [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x, x)
```

Giac [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="giac")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{acoth}(\tanh(a+b\,x))^n}{x}\,dx$$

input
```
int(acoth(tanh(a + b*x))^n/x,x)
```

output
```
int(acoth(tanh(a + b*x))^n/x, x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{acoth(\tanh(bx+a))^n}{x}dx$$

input
```
int(acoth(tanh(b*x+a))^n/x,x)
```

output
```
int(acoth(tanh(a + b*x))**n/x,x)
```

3.70 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} \, dx = -\frac{\coth^{-1}(\tanh(a+bx))^n}{x} + \frac{b \coth^{-1}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1,n,1+n,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

output
```
-arccoth(tanh(b*x+a))^n/x+b*arccoth(tanh(b*x+a))^n*hypergeom([1, n],[1+n],
-arccoth(tanh(b*x+a))/(b*x-arccoth(tanh(b*x+a))))/(b*x-arccoth(tanh(b*x+a)
))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^n \left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(1-n,-n,2-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right.}{(-1+n)x}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcCoth[
Tanh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} \, dx$$

$$\Big\downarrow \text{2599}$$

$$bn \int \frac{\coth^{-1}(\tanh(a+bx))^{n-1}}{x} dx - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

$$\Big\downarrow \text{2595}$$

$$\frac{b \coth^{-1}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1,n,n+1,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^n/x) + (b*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric
2F1[1, n, 1 + n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))]
)/(b*x - ArcCoth[Tanh[a + b*x]])
```

Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F]

$$\int \frac{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}dx$$

input
```
int(arccoth(tanh(b*x+a))^n/x^2,x)
```

output
```
int(arccoth(tanh(b*x+a))^n/x^2,x)
```

Fricas [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

output
```
integral(arccoth(tanh(b*x + a))^n/x^2, x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**n/x**2,x)
```

output
```
Integral(acoth(tanh(a + b*x))**n/x**2, x)
```

Maxima [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^2}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

Giac [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^2}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\mathrm{acoth}(\tanh(a+b\,x))^n}{x^2}\,dx$$

input
```
int(acoth(tanh(a + b*x))^n/x^2,x)
```

output
```
int(acoth(tanh(a + b*x))^n/x^2, x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \frac{-acoth(\tanh{(bx+a)})^n - \left(\int \frac{acoth(\tanh(bx+a))^n}{acoth(\tanh(bx+a))x}dx\right)bnx}{x}$$

input
```
int(acoth(tanh(b*x+a))^n/x^2,x)
```

output
```
( - (acoth(tanh(a + b*x))**n + int(acoth(tanh(a + b*x))**n/(acoth(tanh(a +
 b*x))*x),x)*b*n*x))/x
```

3.71 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} \, dx = -\frac{bn \coth^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{b^2 n \coth^{-1}(\tanh(a+bx))^{-1+n} \text{Hypergeometric2F1}\left(1, -1+n, n, -\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-1/2*b*n*arccoth(tanh(b*x+a))^(-1+n)/x-1/2*arccoth(tanh(b*x+a))^n/x^2+b^2*
n*arccoth(tanh(b*x+a))^(-1+n)*hypergeom([1, -1+n],[n],-arccoth(tanh(b*x+a)
)/(b*x-arccoth(tanh(b*x+a))))/(2*b*x-2*arccoth(tanh(b*x+a)))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^n \left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(2-n, -n, 3-n, 1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(-2+n)x^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcCoth[
Tanh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{2} bn \int \frac{\coth^{-1}(\tanh(a+bx))^{n-1}}{x^2} dx - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{2} bn \left(-b(1-n) \int \frac{\coth^{-1}(\tanh(a+bx))^{n-2}}{x} dx - \frac{\coth^{-1}(\tanh(a+bx))^{n-1}}{x}\right) - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}$$

$$\Big\downarrow \text{ 2595}$$

$$\frac{1}{2} bn \left(\frac{b \coth^{-1}(\tanh(a+bx))^{n-1} \text{Hypergeometric2F1}\left(1, n-1, n, -\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+b}{x}\right. \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

output
```
-1/2*ArcCoth[Tanh[a + b*x]]^n/x^2 + (b*n*(-(ArcCoth[Tanh[a + b*x]]^(-1 + n
)/x) + (b*ArcCoth[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n, n,
-(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*x - ArcCoth[
Tanh[a + b*x]])))/2
```

Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [F]

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^n}{x^3}\,dx$$

input
```
int(arccoth(tanh(b*x+a))^n/x^3,x)
```

output
```
int(arccoth(tanh(b*x+a))^n/x^3,x)
```

Fricas [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
```

output
```
integral(arccoth(tanh(b*x + a))^n/x^3, x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**n/x**3,x)
```

output
```
Integral(acoth(tanh(a + b*x))**n/x**3, x)
```

Maxima [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

Giac [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="giac")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{acoth}(\tanh(a+b\,x))^n}{x^3}\,dx$$

input
```
int(acoth(tanh(a + b*x))^n/x^3,x)
```

output
```
int(acoth(tanh(a + b*x))^n/x^3, x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \frac{-acoth(\tanh\left(bx+a\right))^n - \left(\int \frac{acoth(\tanh(bx+a))^n}{acoth(\tanh(bx+a))x^2}dx\right) bn\,x^2}{2x^2}$$

input
```
int(acoth(tanh(b*x+a))^n/x^3,x)
```

output
```
( - (acoth(tanh(a + b*x))**n + int(acoth(tanh(a + b*x))**n/(acoth(tanh(a +
b*x))*x**2),x)*b*n*x**2))/(2*x**2)
```

3.72 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m}+\frac{x(-bx+\coth^{-1}(\tanh(a+bx)))}{1+m}\right)$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1
 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

method	result
parallelrisch	$-\frac{-x\,x^{m} \operatorname{arccoth}(\tanh(bx+a))m-2 \operatorname{arccoth}(\tanh(bx+a))x\,x^{m}+b\,x^{m}x^{2}}{(1+m)(2+m)}$
risch	$\frac{x\,x^{m} \ln(\mathrm{e}^{bx+a})}{1+m}-\frac{x\left(4bx-4i\pi \operatorname{csgn}(i\mathrm{e}^{bx+a}) \operatorname{csgn}(i\mathrm{e}^{2bx+2a})^{2}+2i\pi \operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+4i\pi \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+2i\pi \operatorname{csgn}\right.}{}$

input
```
int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-(-x*x^m*arccoth(tanh(b*x+a))*m-2*arccoth(tanh(b*x+a))*x*x^m+b*x^m*x^2)/(1
+m)/(2+m)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+2\,a)x)\cosh\left(m\log\left(x\right)\right)+(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am}{2\,(m^2+3\,m+2)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*((I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*cosh(m*log(x)) +
 (I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*sinh(m*log(x)))/(m^2
 + 3*m + 2)
```

Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \begin{cases} b\log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m=-2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx & \text{for } m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(
tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**
m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m
**2 + 3*m + 2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1} \operatorname{arcoth}(\tanh(bx+a))}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m +
2)*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

input
```
int(x^m*acoth(tanh(a + b*x)),x)
```

output
```
(2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)
*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(6*m + 2*m^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$=\frac{x^m x(acoth(\tanh{(bx+a)})\,m+2acoth(\tanh{(bx+a)})+bx)}{m^2+3m+2}$$

input
```
int(x^m*acoth(tanh(b*x+a)),x)
```

output
```
(x**m*x*(acoth(tanh(a + b*x))*m + 2*acoth(tanh(a + b*x)) + b*x))/(m**2 + 3
*m + 2)
```

3.73 $\int x^2 \coth^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\coth(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arccoth(coth(b*x+a))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = -\frac{1}{12}x^3\left(bx - 4\coth^{-1}(\coth(a+bx))\right)$$

input
```
Integrate[x^2*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcCoth[Coth[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{1}{3}x^3 \coth^{-1}(\coth(a+bx)) - \frac{b\int x^3dx}{3}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{3}x^3 \coth^{-1}(\coth(a+bx)) - \frac{bx^4}{12}$$

input
```
Int[x^2*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcCoth[Coth[a + b*x]])/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{x^3(bx-4\,\operatorname{arccoth}(\coth(bx+a)))}{12}$
default	$-\frac{b\,x^4}{12}+\frac{x^3\,\operatorname{arccoth}(\coth(bx+a))}{3}$
parts	$-\frac{b\,x^4}{12}+\frac{x^3\,\operatorname{arccoth}(\coth(bx+a))}{3}$
risch	$\frac{x^3\ln(\mathrm{e}^{bx+a})}{3}-\frac{b\,x^4}{12}+\frac{i\pi\,x^3\,\mathrm{csgn}(i\mathrm{e}^{bx+a})\,\mathrm{csgn}(i\mathrm{e}^{2bx+2a})^2}{6}+\frac{i\pi\,x^3\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^2}{12}-\frac{i\pi\,x^3\,\mathrm{csg}}{}$

input
```
int(x^2*arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*x^3*(b*x-4*arccoth(coth(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2\coth^{-1}(\coth(a+bx))\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 4.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \begin{cases} \frac{x^3 \operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{3} & \text{for } a = \log\left(-e^{-bx}\right) \\ \frac{x^3 \operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{3} & \text{for } a = \log\left(e^{-bx}\right) \\ -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acoth(coth(b*x+a)),x)
```

output
```
Piecewise((x**3*acoth(coth(b*x + log(-exp(-b*x))))/3, Eq(a, log(-exp(-b*x)
))), (x**3*acoth(coth(b*x + log(exp(-b*x))))/3, Eq(a, log(exp(-b*x)))), (-
b*x**4/12 + x**3*acoth(1/tanh(a + b*x))/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{x^3\,\mathrm{acoth}(\coth(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

input
```
int(x^2*acoth(coth(a + b*x)),x)
```

output
```
(x^3*acoth(coth(a + b*x)))/3 - (b*x^4)/12
```

Reduce [F]

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \int acoth(\coth(bx+a))\,x^2dx$$

input
```
int(x^2*acoth(coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x))*x**2,x)
```

3.74 $\int x \coth^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \coth^{-1}(\coth(a+bx))\,dx = -\frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(\coth(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arccoth(coth(b*x+a))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \coth^{-1}(\coth(a+bx))\,dx = -\frac{1}{6}x^2\left(bx - 3\coth^{-1}(\coth(a+bx))\right)$$

input
```
Integrate[x*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcCoth[Coth[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6796, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{ 6796}$$

$$\frac{1}{2}b\int -x^2dx+\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

input
```
Int[x*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcCoth[Coth[a + b*x]])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{x^{2}\left(bx-3\,\mathrm{arccoth}\left(\coth\left(bx+a\right)\right)\right)}{6}$
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\,\mathrm{arccoth}\left(\coth\left(bx+a\right)\right)}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\,\mathrm{arccoth}\left(\coth\left(bx+a\right)\right)}{2}$
risch	$\frac{x^{2}\ln\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{b\,x^{3}}{6}+\frac{i\pi\,x^{2}\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\,\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}}{8}+\frac{i\pi\,x^{2}\,\mathrm{csgn}\left(i\,\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\,\mathrm{e}^{2bx+2a}\right)^{2}}{4}-\frac{i\pi\,x^{2}\,\mathrm{csg}}{}$

input
```
int(x*arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^2*(b*x-3*arccoth(coth(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x\coth^{-1}(\coth(a+bx))\,dx=\frac{1}{3}x^{3}b+\frac{1}{2}x^{2}a$$

input
```
integrate(x*arccoth(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*x^3*b + 1/2*x^2*a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 3.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int x\coth^{-1}(\coth(a+bx))\,dx = \begin{cases} \frac{x^2\operatorname{acoth}(\coth(bx+\log(-e^{-bx})))}{2} & \text{for } a=\log(-e^{-bx}) \\ \frac{x^2\operatorname{acoth}(\coth(bx+\log(e^{-bx})))}{2} & \text{for } a=\log(e^{-bx}) \\ -\frac{bx^3}{6}+\frac{x^2\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(coth(b*x+a)),x)
```

output
```
Piecewise(((x**2*acoth(coth(b*x + log(-exp(-b*x))))/2, Eq(a, log(-exp(-b*x)
))), (x**2*acoth(coth(b*x + log(exp(-b*x))))/2, Eq(a, log(exp(-b*x)))), (-
b*x**3/6 + x**2*acoth(1/tanh(a + b*x))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x\coth^{-1}(\coth(a+bx))\,dx = \frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate(x*arccoth(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x\coth^{-1}(\coth(a+bx))\,dx = \frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate(x*arccoth(coth(b*x+a)),x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \coth^{-1}(\coth(a+bx))\,dx = \frac{x^2\,\mathrm{acoth}(\coth(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*acoth(coth(a + b*x)),x)
```

output
```
(x^2*acoth(coth(a + b*x)))/2 - (b*x^3)/6
```

Reduce [F]

$$\int x \coth^{-1}(\coth(a+bx))\,dx = \int acoth(\coth(bx+a))\,x dx$$

input
```
int(x*acoth(coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x))*x,x)
```

3.75 $\int \coth^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \coth^{-1}(\coth(a+bx))\,dx=\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

output
```
1/2*arccoth(coth(b*x+a))^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \coth^{-1}(\coth(a+bx))\,dx=-\frac{bx^2}{2}+x\coth^{-1}(\coth(a+bx))$$

input
```
Integrate[ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcCoth[Coth[a + b*x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \coth^{-1}(\coth(a+bx))d\coth^{-1}(\coth(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

input
```
Int[ArcCoth[Coth[a + b*x]],x]
```

output
```
ArcCoth[Coth[a + b*x]]^2/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{x(bx-2\,\mathrm{arccoth}(\coth(bx+a)))}{2}$
derivativedivides	$\frac{\mathrm{arctanh}(\coth(bx+a))\,\mathrm{arccoth}(\coth(bx+a))-\frac{\mathrm{arctanh}(\coth(bx+a))^2}{2}}{b}$
default	$\frac{\mathrm{arctanh}(\coth(bx+a))\,\mathrm{arccoth}(\coth(bx+a))-\frac{\mathrm{arctanh}(\coth(bx+a))^2}{2}}{b}$
parts	$x\,\mathrm{arccoth}\left(\coth\left(bx+a\right)\right)+\frac{-\frac{(bx+a)^2}{2}+(bx+a)a}{b}$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)+\frac{i\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^2\pi x}{4}-\frac{i\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+}\right.}{4}$

input
```
int(arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(b*x-2*arccoth(coth(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \coth^{-1}(\coth(a+bx))\,dx=\frac{1}{2}x^2b+xa$$

input
```
integrate(arccoth(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*x^2*b + x*a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.

Time = 1.55 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \coth^{-1}(\coth(a+bx))\,dx = \begin{cases} x\,\mathrm{acoth}\left(\coth\left(a\right)\right) & \text{for } b=0 \\ x\,\mathrm{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right) & \text{for } a=\log\left(-e^{-bx}\right) \\ x\,\mathrm{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right) & \text{for } a=\log\left(e^{-bx}\right) \\ \frac{\mathrm{acoth}^2\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2b} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(coth(b*x+a)),x)
```

output
```
Piecewise((x*acoth(coth(a)), Eq(b, 0)), (x*acoth(coth(b*x + log(-exp(-b*x)
))), Eq(a, log(-exp(-b*x)))), (x*acoth(coth(b*x + log(exp(-b*x)))), Eq(a,
log(exp(-b*x)))), (acoth(1/tanh(a + b*x))**2/(2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{2}\,bx^2 + ax$$

input
```
integrate(arccoth(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{2}bx^2 + ax$$

input
```
integrate(arccoth(coth(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\coth(a+bx))\,dx = x\,\mathrm{acoth}(\coth(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(acoth(coth(a + b*x)),x)
```

output
```
x*acoth(coth(a + b*x)) - (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(\coth(a+bx))\,dx = -\frac{acoth(\coth(bx+a))^2}{2b}$$

input
```
int(acoth(coth(b*x+a)),x)
```

output
```
( - acoth(coth(a + b*x))**2)/(2*b)
```

3.76 $\int \frac{\coth^{-1}(\coth(a+bx))}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x} dx = bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \log(x)$$

output
```
b*x-(b*x-arccoth(coth(b*x+a)))*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x} dx = bx + \left(-bx + \coth^{-1}(\coth(a+bx))\right) \log(x)$$

input
```
Integrate[ArcCoth[Coth[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcCoth[Coth[a + b*x]])*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x} \, dx$$

$$\Big\downarrow \text{ 2589}$$

$$bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \int \frac{1}{x} dx$$

$$\Big\downarrow \text{ 14}$$

$$bx - \log(x) \left(bx - \coth^{-1}(\coth(a+bx))\right)$$

input
```
Int[ArcCoth[Coth[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcCoth[Coth[a + b*x]])*Log[x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result
default	$\ln(x)\,\mathrm{arccoth}(\coth(bx+a))+b(-\ln(x)\,x+x)$
parts	$\ln(x)\,\mathrm{arccoth}(\coth(bx+a))+b(-\ln(x)\,x+x)$
risch	$\ln(x)\ln\left(\mathrm{e}^{bx+a}\right)-\ln(x)\,xb+bx+\frac{i\pi\left(\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}-\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)\right.}{}$

input
```
int(arccoth(coth(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccoth(coth(b*x+a))+b*(-ln(x)*x+x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = bx + a\log(x)$$

input
```
integrate(arccoth(coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + a*log(x)
```

Sympy [F]

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}(\coth(a+bx))}{x}\,dx$$

input
```
integrate(acoth(coth(b*x+a))/x,x)
```

output
```
Integral(acoth(coth(a + b*x))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = bx + a\log\left(x\right)$$

input
```
integrate(arccoth(coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
b*x + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = bx + a\log\left(|x|\right)$$

input
```
integrate(arccoth(coth(b*x+a))/x,x, algorithm="giac")
```

output
```
b*x + a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = b\,x - \frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2} + \frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2} - b\,x\,\ln\left(x\right)$$

input
```
int(acoth(coth(a + b*x))/x,x)
```

output
```
b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)
```

Reduce [F]

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = \int \frac{acoth(\coth(bx+a))}{x}dx$$

input
```
int(acoth(coth(b*x+a))/x,x)
```

output
```
int(acoth(coth(a + b*x))/x,x)
```

3.77 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx = -\frac{\coth^{-1}(\coth(a+bx))}{x} + b\log(x)$$

output
```
-arccoth(coth(b*x+a))/x+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx = b - \frac{\coth^{-1}(\coth(a+bx))}{x} + b\log(x)$$

input
```
Integrate[ArcCoth[Coth[a + b*x]]/x^2,x]
```

output
```
b - ArcCoth[Coth[a + b*x]]/x + b*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} \, dx$$

$$\Big\downarrow \text{2599}$$

$$b \int \frac{1}{x} dx - \frac{\coth^{-1}(\coth(a+bx))}{x}$$

$$\Big\downarrow \text{14}$$

$$b \log(x) - \frac{\coth^{-1}(\coth(a+bx))}{x}$$

input
```
Int[ArcCoth[Coth[a + b*x]]/x^2,x]
```

output
```
-(ArcCoth[Coth[a + b*x]]/x) + b*Log[x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{\ln(x)xb-\operatorname{arccoth}(\coth(bx+a))}{x}$
default	$-\frac{\operatorname{arccoth}(\coth(bx+a))}{x}+b\ln\left(-bx\right)$
parts	$-\frac{\operatorname{arccoth}(\coth(bx+a))}{x}+b\ln\left(-bx\right)$
risch	$-\frac{\ln(e^{bx+a})}{x}+\frac{i\pi\ \mathrm{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(ie^{2bx+2a}\right)-i\pi\ \mathrm{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right)^2+i\pi\ \mathrm{csg}}{}$

input
```
int(arccoth(coth(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*x*b-arccoth(coth(b*x+a)))/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx=\frac{bx\log\left(x\right)-a}{x}$$

input
```
integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - a)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(14) = 28.

Time = 3.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.00

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = \begin{cases} -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{x} & \text{for } a=\log\left(-e^{-bx}\right) \\ -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{x} & \text{for } a=\log\left(e^{-bx}\right) \\ b\log(x)-\frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(coth(b*x+a))/x**2,x)
```

output
```
Piecewise((-acoth(coth(b*x + log(-exp(-b*x))))/x, Eq(a, log(-exp(-b*x)))),
 (-acoth(coth(b*x + log(exp(-b*x))))/x, Eq(a, log(exp(-b*x)))), (b*log(x)
- acoth(1/tanh(a + b*x))/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = b\log(x)-\frac{a}{x}$$

input
```
integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - a/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = b\log(|x|)-\frac{a}{x}$$

input
```
integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - a/x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = b\,\ln(x) - \frac{\mathrm{acoth}(\coth(a+b\,x))}{x}$$

input
```
int(acoth(coth(a + b*x))/x^2,x)
```

output
```
b*log(x) - acoth(coth(a + b*x))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = \frac{-acoth(\coth(bx+a)) - \log(x)\,bx}{x}$$

input
```
int(acoth(coth(b*x+a))/x^2,x)
```

output
```
( - (acoth(coth(a + b*x)) + log(x)*b*x))/x
```

3.78 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx = -\frac{b}{2x} - \frac{\coth^{-1}(\coth(a+bx))}{2x^2}$$

output
```
-1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx = -\frac{bx+\coth^{-1}(\coth(a+bx))}{2x^2}$$

input
```
Integrate[ArcCoth[Coth[a + b*x]]/x^3,x]
```

output
```
-1/2*(b*x + ArcCoth[Coth[a + b*x]])/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\int \frac{1}{x^2}dx - \frac{\coth^{-1}(\coth(a+bx))}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2} - \frac{b}{2x}$$

input

```
Int[ArcCoth[Coth[a + b*x]]/x^3,x]
```

output

```
-1/2*b/x - ArcCoth[Coth[a + b*x]]/(2*x^2)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\coth(bx+a))}{2x^2}$
parallelrisch	$\frac{-bx-\operatorname{arccoth}(\coth(bx+a))}{2x^2}$
parts	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\coth(bx+a))}{2x^2}$
risch	$-\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{2x^2}-\frac{4bx+i\pi\ \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^2-i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3-i\pi\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+i\pi}{}$

input
```
int(arccoth(coth(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx=-\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(19) = 38$.

Time = 5.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = \begin{cases} -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a=\log\left(-e^{-bx}\right) \\ -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a=\log\left(e^{-bx}\right) \\ -\frac{b}{2x}-\frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(coth(b*x+a))/x**3,x)
```

output
```
Piecewise((-acoth(coth(b*x + log(-exp(-b*x))))/(2*x**2), Eq(a, log(-exp(-b
*x)))), (-acoth(coth(b*x + log(exp(-b*x))))/(2*x**2), Eq(a, log(exp(-b*x))
)), (-b/(2*x) - acoth(1/tanh(a + b*x))/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx=-\frac{\mathrm{acoth}(\coth(a+b\,x))+b\,x}{2\,x^2}$$

input
```
int(acoth(coth(a + b*x))/x^3,x)
```

output
```
-(acoth(coth(a + b*x)) + b*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx=\frac{-acoth(\coth{(bx+a)})+bx}{2x^2}$$

input
```
int(acoth(coth(b*x+a))/x^3,x)
```

output
```
( - acoth(coth(a + b*x)) + b*x)/(2*x**2)
```

3.79 $\int \coth^{-1}(\cosh(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 27

$$\int \coth^{-1}(\cosh(x))\,dx = x\coth^{-1}(\cosh(x)) - 2x\text{arctanh}(e^x) - \text{PolyLog}(2,-e^x) + \text{PolyLog}(2,e^x)$$

output
```
x*arccoth(cosh(x))-2*x*arctanh(exp(x))-polylog(2,-exp(x))+polylog(2,exp(x)
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \coth^{-1}(\cosh(x))\,dx = x\coth^{-1}(\cosh(x)) + x(\log(1-e^x) - \log(1+e^x)) - \text{PolyLog}(2,-e^x) + \text{PolyLog}(2,e^x)$$

input
```
Integrate[ArcCoth[Cosh[x]],x]
```

output
```
x*ArcCoth[Cosh[x]] + x*(Log[1 - E^x] - Log[1 + E^x]) - PolyLog[2, -E^x] +
PolyLog[2, E^x]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 2.333$, Rules used = {6826, 25, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{ 6826}$$

$$x\coth^{-1}(\cosh(x))-\int -x\text{csch}(x)dx$$

$$\Big\downarrow \text{ 25}$$

$$\int x\text{csch}(x)dx+x\coth^{-1}(\cosh(x))$$

$$\Big\downarrow \text{ 3042}$$

$$x\coth^{-1}(\cosh(x))+\int ix\csc(ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$x\coth^{-1}(\cosh(x))+i\int x\csc(ix)dx$$

$$\Big\downarrow \text{ 4670}$$

$$x\coth^{-1}(\cosh(x))+i\left(i\int \log\left(1-e^x\right)dx-i\int \log\left(1+e^x\right)dx+2ix\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x\coth^{-1}(\cosh(x))+i\left(i\int e^{-x}\log\left(1-e^x\right)de^x-i\int e^{-x}\log\left(1+e^x\right)de^x+2ix\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{ 2838}$$

$$x\coth^{-1}(\cosh(x))+i(2ix\text{arctanh}(e^x)+i\,\text{PolyLog}\left(2,-e^x\right)-i\,\text{PolyLog}\left(2,e^x\right))$$

input
```
Int[ArcCoth[Cosh[x]],x]
```

output
```
x*ArcCoth[Cosh[x]] + I*((2*I)*x*ArcTanh[E^x] + I*PolyLog[2, -E^x] - I*Poly
Log[2, E^x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6826
```
Int[ArcCoth[u_], x_Symbol] :> Simp[x*ArcCoth[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

method	result
default	$x\ \operatorname{arccoth}\left(\cosh\left(x\right)\right)+x\ln\left(1-{\mathrm e}^{x}\right)+\operatorname{polylog}\left(2,{\mathrm e}^{x}\right)-x\ln\left(1+{\mathrm e}^{x}\right)-\operatorname{polylog}\left(2,-{\mathrm e}^{x}\right)$
parts	$x\ \operatorname{arccoth}\left(\cosh\left(x\right)\right)+x\ln\left(1-{\mathrm e}^{x}\right)+\operatorname{polylog}\left(2,{\mathrm e}^{x}\right)-x\ln\left(1+{\mathrm e}^{x}\right)-\operatorname{polylog}\left(2,-{\mathrm e}^{x}\right)$
risch	$\frac{i\pi\ \operatorname{csgn}\left(i{\mathrm e}^{-x}\right)\operatorname{csgn}\left(i{\mathrm e}^{-x}\left(1+{\mathrm e}^{x}\right)^{2}\right)^{2}x}{4}+\frac{i\pi\operatorname{csgn}\left(i\left({\mathrm e}^{x}-1\right)^{2}\right)^{3}x}{4}+\frac{i\pi\operatorname{csgn}\left(i\left({\mathrm e}^{x}-1\right)\right)^{2}\operatorname{csgn}\left(i\left({\mathrm e}^{x}-1\right)^{2}\right)x}{4}+\frac{i\pi\ \operatorname{csgn}\left(i\left(1+{\mathrm e}^{x}\right)^{2}\right)\ \mathrm{cs}}{4}$

input
```
int(arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(cosh(x))+x*ln(1-exp(x))+polylog(2,exp(x))-x*ln(1+exp(x))-polylog
(2,-exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\begin{aligned}\int \coth^{-1}(\cosh(x))\,dx =& \frac{1}{2}\,x\log\left(\frac{\cosh\left(x\right)+1}{\cosh\left(x\right)-1}\right)-x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\\&+x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)\\&+\operatorname{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\operatorname{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))\end{aligned}$$

input
```
integrate(arccoth(cosh(x)),x, algorithm="fricas")
```

output
```
1/2*x*log((cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*
log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) -
sinh(x))
```

Sympy [F]

$$\int \coth^{-1}(\cosh(x))\,dx = \int \operatorname{acoth}(\cosh(x))\,dx$$

input
```
integrate(acoth(cosh(x)),x)
```

output
```
Integral(acoth(cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \coth^{-1}(\cosh(x))\,dx = x \operatorname{arcoth}(\cosh(x)) - x\log(e^x+1) + x\log(-e^x+1) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

input
```
integrate(arccoth(cosh(x)),x, algorithm="maxima")
```

output
```
x*arccoth(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilo
g(e^x)
```

Giac [F]

$$\int \coth^{-1}(\cosh(x))\,dx = \int \operatorname{arcoth}(\cosh(x))\,dx$$

input
```
integrate(arccoth(cosh(x)),x, algorithm="giac")
```

output
```
integrate(arccoth(cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(\cosh(x))\,dx = \int \mathrm{acoth}(\cosh(x))\,dx$$

input
```
int(acoth(cosh(x)),x)
```

output
```
int(acoth(cosh(x)), x)
```

Reduce [F]

$$\int \coth^{-1}(\cosh(x))\,dx = \int acoth(\cosh(x))\,dx$$

input
```
int(acoth(cosh(x)),x)
```

output
```
int(acoth(cosh(x)),x)
```

3.80 $\int x \coth^{-1}(\cosh(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 51

$$\int x \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2\coth^{-1}(\cosh(x)) - x^2\text{arctanh}(e^x) - x\,\text{PolyLog}(2,-e^x) + x\,\text{PolyLog}(2,e^x) + \text{PolyLog}(3,-e^x) - \text{PolyLog}(3,e^x)$$

output
```
1/2*x^2*arccoth(cosh(x))-x^2*arctanh(exp(x))-x*polylog(2,-exp(x))+x*polylo
g(2,exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int x \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2\coth^{-1}(\cosh(x)) + \frac{1}{2}x^2\log(1-e^x) - \frac{1}{2}x^2\log(1+e^x) - x\,\text{PolyLog}(2,-e^x) + x\,\text{PolyLog}(2,e^x) + \text{PolyLog}(3,-e^x) - \text{PolyLog}(3,e^x)$$

input
```
Integrate[x*ArcCoth[Cosh[x]],x]
```

output
```
(x^2*ArcCoth[Cosh[x]])/2 + (x^2*Log[1 - E^x])/2 - (x^2*Log[1 + E^x])/2 - x
*PolyLog[2, -E^x] + x*PolyLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {6828, 25, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6828}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))-\frac{1}{2}\int -x^2\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int x^2\text{csch}(x)dx+\frac{1}{2}x^2\coth^{-1}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+\frac{1}{2}\int ix^2\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+\frac{1}{2}i\int x^2\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+\frac{1}{2}i\left(2i\int x\log\left(1-e^x\right)dx-2i\int x\log\left(1+e^x\right)dx+2ix^2\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+$$
$$\frac{1}{2}i\left(-2i\left(\int \text{PolyLog}\left(2,-e^x\right)dx-x\,\text{PolyLog}\left(2,-e^x\right)\right)+2i\left(\int \text{PolyLog}\left(2,e^x\right)dx-x\,\text{PolyLog}\left(2,e^x\right)\right)+2ix^2\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+$$
$$\frac{1}{2}i\left(-2i\left(\int e^{-x}\operatorname{PolyLog}(2,-e^x)\,de^x-x\operatorname{PolyLog}(2,-e^x)\right)+2i\left(\int e^{-x}\operatorname{PolyLog}(2,e^x)\,de^x-x\operatorname{PolyLog}(2,e^x\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+$$
$$\frac{1}{2}i(2ix^2\text{arctanh}(e^x)-2i(\operatorname{PolyLog}(3,-e^x)-x\operatorname{PolyLog}(2,-e^x))+2i(\operatorname{PolyLog}(3,e^x)-x\operatorname{PolyLog}(2,e^x)))$$

input
```
Int[x*ArcCoth[Cosh[x]],x]
```

output
```
(x^2*ArcCoth[Cosh[x]])/2 + (I/2)*((2*I)*x^2*ArcTanh[E^x] - (2*I)*(-(x*Poly
Log[2, -E^x]) + PolyLog[3, -E^x]) + (2*I)*(-(x*PolyLog[2, E^x]) + PolyLog[
3, E^x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6828
```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCoth[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
 &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 379, normalized size of antiderivative = 7.43

method	result
risch	$-\frac{x^2 \ln(\mathrm{e}^x-1)}{2} - x\,\mathrm{polylog}(2,-\mathrm{e}^x) + \mathrm{polylog}(3,-\mathrm{e}^x) - \frac{i\pi\left(\mathrm{csgn}(i(1+\mathrm{e}^x))^2\,\mathrm{csgn}\left(i(1+\mathrm{e}^x)^2\right) - 2\,\mathrm{csgn}(i(1+\mathrm{e}^x))\mathrm{cs}\right.}{}$

input
```
int(x*arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*ln(exp(x)-1)-x*polylog(2,-exp(x))+polylog(3,-exp(x))-1/8*I*Pi*(cs
gn(I*(1+exp(x)))^2*csgn(I*(1+exp(x))^2)-2*csgn(I*(1+exp(x)))*csgn(I*(1+exp
(x))^2)^2+csgn(I*(1+exp(x))^2)^3+csgn(I*(1+exp(x))^2)*csgn(I*exp(-x))*csgn
(I*exp(-x)*(1+exp(x))^2)-csgn(I*(1+exp(x))^2)*csgn(I*exp(-x)*(1+exp(x))^2)
^2-csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)+2*csgn(I*(exp(x)-1))*csgn(I*(
exp(x)-1)^2)^2-csgn(I*(exp(x)-1)^2)^3-csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))
*csgn(I*exp(-x)*(exp(x)-1)^2)+csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-
1)^2)^2-csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(x))^2)^2+csgn(I*exp(-x))*csg
n(I*exp(-x)*(exp(x)-1)^2)^2+csgn(I*exp(-x)*(1+exp(x))^2)^3-csgn(I*exp(-x)*
(exp(x)-1)^2)^3)*x^2+1/2*x^2*ln(1-exp(x))+x*polylog(2,exp(x))-polylog(3,ex
p(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\begin{aligned}\int x\coth^{-1}(\cosh(x))\,dx &= \frac{1}{4}\,x^2\log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2}\,x^2\log(\cosh(x)+\sinh(x)+1)\\ &\quad+\frac{1}{2}\,x^2\log(-\cosh(x)-\sinh(x)+1)\\ &\quad+x\mathrm{Li}_2(\cosh(x)+\sinh(x)) - x\mathrm{Li}_2(-\cosh(x)-\sinh(x))\\ &\quad-\mathrm{polylog}(3,\cosh(x)+\sinh(x))\\ &\quad+\mathrm{polylog}(3,-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(x*arccoth(cosh(x)),x, algorithm="fricas")
```

output
```
1/4*x^2*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
 1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x
*dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -c
osh(x) - sinh(x))
```

Sympy [F]

$$\int x \coth^{-1}(\cosh(x))\,dx = \int x\,\mathrm{acoth}\,(\cosh(x))\,dx$$

input
```
integrate(x*acoth(cosh(x)),x)
```

output
```
Integral(x*acoth(cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2\,\mathrm{arcoth}\,(\cosh(x)) - \frac{1}{2}x^2\log(e^x+1) + \frac{1}{2}x^2\log(-e^x+1) - x\mathrm{Li}_2(-e^x) + x\mathrm{Li}_2(e^x) + \mathrm{Li}_3(-e^x) - \mathrm{Li}_3(e^x)$$

input
```
integrate(x*arccoth(cosh(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) -
x*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

Giac [F]

$$\int x \coth^{-1}(\cosh(x))\,dx = \int x\,\mathrm{arcoth}\,(\cosh(x))\,dx$$

input
```
integrate(x*arccoth(cosh(x)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(\cosh(x))\,dx=\int x\,\mathrm{acoth}(\cosh(x))\,dx$$

input
```
int(x*acoth(cosh(x)),x)
```

output
```
int(x*acoth(cosh(x)), x)
```

Reduce [F]

$$\int x\coth^{-1}(\cosh(x))\,dx=\int \mathit{acoth}(\cosh(x))\,x dx$$

input
```
int(x*acoth(cosh(x)),x)
```

output
```
int(acoth(cosh(x))*x,x)
```

3.81 $\int x^2 \coth^{-1}(\cosh(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \frac{1}{3}x^3\coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\text{arctanh}(e^x) - x^2\,\text{PolyLog}(2,-e^x) + x^2\,\text{PolyLog}(2,e^x) + 2x\,\text{PolyLog}(3,-e^x) - 2x\,\text{PolyLog}(3,e^x) - 2\,\text{PolyLog}(4,-e^x) + 2\,\text{PolyLog}(4,e^x)$$

output
```
1/3*x^3*arccoth(cosh(x))-2/3*x^3*arctanh(exp(x))-x^2*polylog(2,-exp(x))+x^
2*polylog(2,exp(x))+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog
(4,-exp(x))+2*polylog(4,exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \frac{1}{3}\left(x^3\coth^{-1}(\cosh(x)) + x^3\log(1-e^x) - x^3\log(1+e^x) - 3x^2\,\text{PolyLog}(2,-e^x) + 3x^2\,\text{PolyLog}(2,e^x) + 6x\,\text{PolyLog}(3,-e^x) - 6x\,\text{PolyLog}(3,e^x) - 6\,\text{PolyLog}(4,-e^x) + 6\,\text{PolyLog}(4,e^x)\right)$$

input
```
Integrate[x^2*ArcCoth[Cosh[x]],x]
```

output
```
(x^3*ArcCoth[Cosh[x]] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLo
g[2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3,
 E^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {6828, 25, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6828}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))-\frac{1}{3}\int -x^3\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int x^3\text{csch}(x)dx+\frac{1}{3}x^3\coth^{-1}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+\frac{1}{3}\int ix^3\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+\frac{1}{3}i\int x^3\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+$$
$$\frac{1}{3}i\left(3i\int x^2\log\left(1-e^x\right)dx-3i\int x^2\log\left(1+e^x\right)dx+2ix^3\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+$$
$$\frac{1}{3}i\left(-3i\left(2\int x\,\text{PolyLog}\left(2,-e^x\right)dx-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\int x\,\text{PolyLog}\left(2,e^x\right)dx-x^2\,\text{PolyLog}\left(2,e^x\right)\right)\right.$$

$\Big\downarrow$ 7163

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+$$
$$\frac{1}{3}i\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int\text{PolyLog}\left(3,-e^x\right)dx\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e^x\right)-\right.\right.\right.$$

$\Big\downarrow$ 2720

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+$$
$$\frac{1}{3}i\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int e^{-x}\,\text{PolyLog}\left(3,-e^x\right)de^x\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e\right.\right.\right.\right.$$

$\Big\downarrow$ 7143

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+$$
$$\frac{1}{3}i(2ix^3\text{arctanh}(e^x)-3i(2(x\,\text{PolyLog}\left(3,-e^x\right)-\text{PolyLog}\left(4,-e^x\right))-x^2\,\text{PolyLog}\left(2,-e^x\right))+3i(2(x\,\text{PolyLog}\left(3,\right.$$

input
```
Int[x^2*ArcCoth[Cosh[x]],x]
```

output
```
(x^3*ArcCoth[Cosh[x]])/3 + (I/3)*((2*I)*x^3*ArcTanh[E^x] - (3*I)*(-(x^2*Po
lyLog[2, -E^x]) + 2*(x*PolyLog[3, -E^x] - PolyLog[4, -E^x])) + (3*I)*(-(x^
2*PolyLog[2, E^x]) + 2*(x*PolyLog[3, E^x] - PolyLog[4, E^x])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6828
```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCoth[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
 &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 5.21

method	result
risch	$-\frac{x^{3} \ln \left(\mathrm{e}^{x}-1\right)}{3}-x^{2} \operatorname{polylog}\left(2,-\mathrm{e}^{x}\right)+2 x \operatorname{polylog}\left(3,-\mathrm{e}^{x}\right)-2 \operatorname{polylog}\left(4,-\mathrm{e}^{x}\right)-\frac{i \pi \left(\operatorname{csgn}\left(i\left(1+\mathrm{e}^{x}\right)\right)^{2} \operatorname{csg}\right.}{}$

input
```
int(x^2*arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*ln(exp(x)-1)-x^2*polylog(2,-exp(x))+2*x*polylog(3,-exp(x))-2*poly
log(4,-exp(x))-1/12*I*Pi*(csgn(I*(1+exp(x)))^2*csgn(I*(1+exp(x))^2)-2*csgn
(I*(1+exp(x)))*csgn(I*(1+exp(x))^2)^2+csgn(I*(1+exp(x))^2)^3+csgn(I*(1+exp
(x))^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(x))^2)-csgn(I*(1+exp(x))^2)*
csgn(I*exp(-x)*(1+exp(x))^2)^2-csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)+2
*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2-csgn(I*(exp(x)-1)^2)^3-csgn(I*(
exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)+csgn(I*(exp(x)-1
)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2-csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(
x))^2)^2+csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2+csgn(I*exp(-x)*(1+
exp(x))^2)^3-csgn(I*exp(-x)*(exp(x)-1)^2)^3)*x^3+1/3*x^3*ln(1-exp(x))+x^2*
polylog(2,exp(x))-2*x*polylog(3,exp(x))+2*polylog(4,exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.52

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \frac{1}{6}\,x^3\log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3}\,x^3\log(\cosh(x)+\sinh(x)+1) + \frac{1}{3}\,x^3\log(-\cosh(x)-\sinh(x)+1) + x^2\mathrm{Li}_2(\cosh(x)+\sinh(x)) - x^2\mathrm{Li}_2(-\cosh(x)-\sinh(x)) - 2\,x\mathrm{polylog}(3,\cosh(x)+\sinh(x)) + 2\,x\mathrm{polylog}(3,-\cosh(x)-\sinh(x)) + 2\,\mathrm{polylog}(4,\cosh(x)+\sinh(x)) - 2\,\mathrm{polylog}(4,-\cosh(x)-\sinh(x))$$

input
```
integrate(x^2*arccoth(cosh(x)),x, algorithm="fricas")
```

output
```
1/6*x^3*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) -
 x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*p
olylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylo
g(4, -cosh(x) - sinh(x))
```

Sympy [F]

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \int x^2\,\mathrm{acoth}\,(\cosh(x))\,dx$$

input
```
integrate(x**2*acoth(cosh(x)),x)
```

output
```
Integral(x**2*acoth(cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int x^2 \coth^{-1}(\cosh(x))\, dx = \frac{1}{3}\, x^3 \operatorname{arcoth}\left(\cosh\left(x\right)\right) - \frac{1}{3}\, x^3 \log\left(e^x + 1\right) + \frac{1}{3}\, x^3 \log\left(-e^x + 1\right) - x^2 \mathrm{Li}_2(-e^x) + x^2 \mathrm{Li}_2(e^x) + 2\, x \mathrm{Li}_3(-e^x) - 2\, x \mathrm{Li}_3(e^x) - 2\, \mathrm{Li}_4(-e^x) + 2\, \mathrm{Li}_4(e^x)$$

input
```
integrate(x^2*arccoth(cosh(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) -
x^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e
^x) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

Giac [F]

$$\int x^2 \coth^{-1}(\cosh(x))\, dx = \int x^2 \operatorname{arcoth}\left(\cosh\left(x\right)\right)\, dx$$

input
```
integrate(x^2*arccoth(cosh(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(\cosh(x))\, dx = \int x^2\, \mathrm{acoth}(\cosh(x))\, dx$$

input
```
int(x^2*acoth(cosh(x)),x)
```

output
```
int(x^2*acoth(cosh(x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \int acoth(\cosh(x))\,x^2dx$$

input
```
int(x^2*acoth(cosh(x)),x)
```

output
```
int(acoth(cosh(x))*x**2,x)
```

3.82 $\int x^2 \coth^{-1}(c + d\tanh(a + bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 307

$$\begin{aligned}\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = {} & \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) \\ & + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ & - \frac{1}{6}x^3\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ & + \frac{x^2\,\text{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ & - \frac{x^2\,\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ & - \frac{x\,\text{PolyLog}\left(3,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b^2} \\ & + \frac{x\,\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b^2} \\ & + \frac{\text{PolyLog}\left(4,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^3} \\ & - \frac{\text{PolyLog}\left(4,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*tanh(b*x+a))+1/6*x^3*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/6*x^3*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,-(1-c-d
)*exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c
-d))/b-1/4*x*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(
3,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,-(1-c-d)*exp(2*b*x+2*
a)/(1-c+d))/b^3-1/8*polylog(4,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.86

$$\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c-d)e^{-2(a+bx)}}{-1+c+d}\right) - 4b^3x^3\log\left(1+\frac{(1+c-d)e^{-2(a+bx)}}{1+c+d}\right) - 6b^2x^2\,\text{PolyLog}\left(2,\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right.}{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + (-1 + c - d)/((-
1 + c + d)*E^(2*(a + b*x)))] - 4*b^3*x^3*Log[1 + (1 + c - d)/((1 + c + d)*
E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (1 - c + d)/((-1 + c + d)*E^(2*(a
 + b*x)))] + 6*b^2*x^2*PolyLog[2, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)
))] - 6*b*x*PolyLog[3, (1 - c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + 6*b*x
*PolyLog[3, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)))] - 3*PolyLog[4, (1
- c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, (-1 - c + d)/((1 +
 c + d)*E^(2*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6798, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6798}$$

$$\frac{1}{3}b(-c-d+1)\int \frac{e^{2a+2bx}x^3}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx-\frac{1}{3}b(c+d+1)\int \frac{e^{2a+2bx}x^3}{c+(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{2b(-c-d+1)}\right)-\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}\right)-\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(}{}\right.}{2}\right.}{2b(-c-d+1)}\right.\quad \frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+}{c}\right.}{2b}\right.}{2b(c+d+1)}\right.\quad \frac{1}{3}b(c+d+$$

$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x}{}\right.}{2b(-c-d+1)}\right.\quad \frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{Pol}}{}\right.}{2b(c+d+1)}\right.\quad \frac{1}{3}b(c+d+$$

$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-}{}\right.}{2b}}{2b(-c-d+1)}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^2}{c-d+}\right.}{2b}}{2b(c+d+1)}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (b*(1 - c - d)*((x^3*Log[1 + ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) - (3*(-1/2*(x^2*Po
lyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/b + ((x*PolyLog[3,
 -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(2*b) - PolyLog[4, -(((1 -
 c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2))/b))/(2*b*(1 - c - d))))/3
- (b*(1 + c + d)*((x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])
/(2*b*(1 + c + d)) - (3*(-1/2*(x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*
x))/(1 + c - d))])/b + ((x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 +
 c - d))])/(2*b) - PolyLog[4, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))
]/(4*b^2))/b))/(2*b*(1 + c + d))))/3
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6798
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*
((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
 d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.84 (sec) , antiderivative size = 5257, normalized size of antiderivative = 17.12

method	result	size
risch	Expression too large to display	5257

input
```
int(x^2*arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(263) = 526.

Time = 0.11 (sec) , antiderivative size = 899, normalized size of antiderivative = 2.93

$$\int x^2 \coth^{-1}(c + d\tanh(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1
))*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sq
rt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*
(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sq
rt(-(c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c
 + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) -
a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c -
 d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x +
 a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d
 - 1))) + 6*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) +
 sinh(b*x + a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(
b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1
))*(cosh(b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 +
 a^3)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) + (b^3*x^3 + a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a...
```

Sympy [F]

$$\int x^2 \coth^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(c + d\tanh\left(a + bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*tanh(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)$$
$$-\frac{1}{18}bd\left(\frac{4b^3x^3\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_3(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1})+3}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x
 + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c -
d + 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d)
- (4*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*
dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d
 - 1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x +
2*a)/(c - d - 1)))/(b^4*d))
```

Giac [F]

$$\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx = \int x^2\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = \int x^2\,\mathrm{acoth}(c+d\tanh(a+b\,x))\,\,dx$$

input
```
int(x^2*acoth(c + d*tanh(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*tanh(a + b*x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = \int acoth(\tanh(bx+a)\,d+c)\,x^2dx$$

input
```
int(x^2*acoth(c+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + c)*x**2,x)
```

3.83 $\int x \coth^{-1}(c+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 231

$$\begin{aligned}\int x\coth^{-1}(c+d\tanh(a+bx))\,dx=&\ \frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))\\&+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)\\&-\frac{1}{4}x^2\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)\\&+\frac{x\operatorname{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b}\\&-\frac{x\operatorname{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}\\&-\frac{\operatorname{PolyLog}\left(3,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^2}\\&+\frac{\operatorname{PolyLog}\left(3,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*tanh(b*x+a))+1/4*x^2*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/4*x^2*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,-(1-c-d)*
exp(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))
/b-1/8*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,-(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int x \coth^{-1}(c+d\tanh(a+bx))\,dx$$
$$= \frac{4b^2x^2\coth^{-1}(c+d\tanh(a+bx)) + 2b^2x^2\log\left(1+\frac{(-1+c-d)e^{-2(a+bx)}}{-1+c+d}\right) - 2b^2x^2\log\left(1+\frac{(1+c-d)e^{-2(a+bx)}}{1+c+d}\right) -$$

input
```
Integrate[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Tanh[a + b*x]] + 2*b^2*x^2*Log[1 + (-1 + c - d)/(
(-1 + c + d)*E^(2*(a + b*x)))] - 2*b^2*x^2*Log[1 + (1 + c - d)/((1 + c + d
)*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (1 - c + d)/((-1 + c + d)*E^(2*(a +
 b*x)))] + 2*b*x*PolyLog[2, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)))] -
PolyLog[3, (1 - c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + PolyLog[3, (-1 -
c + d)/((1 + c + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6798, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6798}$$

$$\frac{1}{2}b(-c-d+1)\int \frac{e^{2a+2bx}x^2}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx-\frac{1}{2}b(c+d+1)\int \frac{e^{2a+2bx}x^2}{c+(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$\big\downarrow$ 7143

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (b*(1 - c - d)*((x^2*Log[1 + ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) - (-1/2*(x*PolyLog
[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/b + PolyLog[3, -(((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2))/(b*(1 - c - d))))/2 - (b*(1
 + c + d)*((x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(2*b*(
1 + c + d)) - (-1/2*(x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c -
 d))])/b + PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b^2
))/(b*(1 + c + d))))/2
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6798
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*
((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
 d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 4953, normalized size of antiderivative = 21.44

method	result	size
risch	Expression too large to display	4953

input
```
int(x*arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b
*x+2*a)-1)*d-exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*
a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1))*csgn(I
*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2
*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x
+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(e
xp(2*b*x+2*a)+1))^2-csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(
2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x
+2*a)-1)/(exp(2*b*x+2*a)+1))^2+csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a
)-1)*d+exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*
d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^2+csgn(I*((exp(2*b*x+2*a)+1)*c+(ex
p(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))^3-csgn(I*((exp(2*b
*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^3)
*x^2-1/2/b^2*c*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1
))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((
-exp(b*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*
(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-
d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/4/b^2*d/(c+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(197) = 394$.

Time = 0.12 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.23

$$\int x \coth^{-1}(c + d \tanh(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh
(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/
(c - d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b
*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d
 - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c
 + d - 1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x
^2 - a^2)*log(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
)) + 1) - (b^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, s
qrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog
(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*p
olylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)...
```

Sympy [F]

$$\int x \coth^{-1}(c + d \tanh(a + bx))\, dx = \int x \operatorname{acoth}\left(c + d \tanh\left(a + bx\right)\right) dx$$

input
```
integrate(x*acoth(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*tanh(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.93

$$\int x \coth^{-1}(c + d \tanh(a + bx))\, dx = -\frac{1}{8} bd \left(\frac{2 b^2 x^2 \log\left(\frac{(c+d+1)e^{(2 bx+2 a)}}{c-d+1} + 1\right) + 2 bx \mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2 bx+2 a)}}{c-d+1}\right) - \mathrm{Li}_3\left(-\frac{(c+d+1)e^{(2 bx+2 a)}}{c-d+1}\right)}{b^3 d} - \frac{2 b^2 x^2 \mathrm{l}}{} \right. + \frac{1}{2} x^2 \operatorname{arcoth}\left(d \tanh\left(bx + a\right) + c\right)$$

input
```
integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d +
 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(
c - d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d
)) + 1/2*x^2*arccoth(d*tanh(b*x + a) + c)
```

Giac [F]

$$\int x \coth^{-1}(c+d\tanh(a+bx))\,dx = \int x \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(c+d\tanh(a+bx))\,dx = \int x\,\mathrm{acoth}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(x*acoth(c + d*tanh(a + b*x)),x)
```

output
```
int(x*acoth(c + d*tanh(a + b*x)), x)
```

Reduce [F]

$$\int x \coth^{-1}(c+d\tanh(a+bx))\,dx = \int acoth(\tanh\left(bx+a\right)d+c)\,x dx$$

input
```
int(x*acoth(c+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + c)*x,x)
```

3.84 $\int \coth^{-1}(c+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 150

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) + \frac{\operatorname{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} - \frac{\operatorname{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}$$

output
```
x*arccoth(c+d*tanh(b*x+a))+1/2*x*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*
x*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,-(1-c-d)*exp(2*b*x+2*
a)/(1-c+d))/b-1/4*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = x\coth^{-1}(c+d\tanh(a+bx)) + \frac{2bx\left(\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\operatorname{PolyLo}}{4b}$$

input
```
Integrate[ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])
+ PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2,
-(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(4*b)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6790, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6790}$$

$$b(-c-d+1)\int \frac{e^{2a+2bx}x}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx - b(c+d+1)\int \frac{e^{2a+2bx}x}{c+(c+d+1)e^{2a+2bx}-d+1}dx + x\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$b(-c-d+1)\left(\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{2b(-c-d+1)}\right)-b(c+d+1)\left(\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{2b(c+d+1)}\right)+x\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$b(-c-d+1)\left(\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)de^{2a+2bx}}{4b^2(-c-d+1)}\right)-b(c+d+1)\left(\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)de^{2a+2bx}}{4b^2(c+d+1)}\right)+x\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$b(-c-d+1)\left(\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2(-c-d+1)}+\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}\right)-b(c+d+1)\left(\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2(c+d+1)}+\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Tanh[a + b*x]] + b*(1 - c - d)*((x*Log[1 + ((1 - c - d)*E^
(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) + PolyLog[2, -(((1 - c - d)
*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2*(1 - c - d))) - b*(1 + c + d)*((x*L
og[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(2*b*(1 + c + d)) + Pol
yLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b^2*(1 + c + d)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6790
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + (Simp[b*(1 - c - d)   Int[x*(E^(2*a + 2*b*
x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*(1 + c + d)
  Int[x*(E^(2*a + 2*b*x)/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 1.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

method	result
derivativedivides	$-\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+\cdots}{-1-c\cdots}\right)}{2}\cdots\right.}{\cdots}$
default	$-\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+\cdots}{-1-c\cdots}\right)}{2}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int(arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arccoth(c+
d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*d^2*(1/d*(-1/2*dilog((-d*tanh(b*
x+a)-c-1)/(-1-c-d))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c-1)/(-1-c
-d))+1/2*dilog((-d*tanh(b*x+a)-c+1)/(1-c-d))+1/2*ln(-d*tanh(b*x+a)+d)*ln((
-d*tanh(b*x+a)-c+1)/(1-c-d)))-1/d*(1/2*dilog((-d*tanh(b*x+a)-c+1)/(1-c+d))
+1/2*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c+1)/(1-c+d))-1/2*dilog((-d*t
anh(b*x+a)-c-1)/(-1-c+d))-1/2*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c-1)
/(-1-c+d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(128) = 256$.

Time = 0.12 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.67

$$\int \coth^{-1}(c + d\tanh(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
 a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1
)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*
(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sq
rt(-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c +
 d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*
log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d -
 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c +
d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(s
qrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x
+ a)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) +
 1) - dilog(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))
) - dilog(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 + dilog(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) +
 dilog(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{acoth}\left(c+d\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(c+d*tanh(b*x+a)),x)
```

output
```
Integral(acoth(c + d*tanh(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = -\frac{1}{4}bd\left(\frac{2\,bx\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(\cdots\right.}{b^2d}\right. + x\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1
)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + c)
```

Giac [F]

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{acoth}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(acoth(c + d*tanh(a + b*x)),x)
```

output
```
int(acoth(c + d*tanh(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}(c + d\tanh(a + bx))\, dx = \int acoth(\tanh(bx + a)\, d + c)\, dx$$

input
```
int(acoth(c+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + c),x)
```

3.85 $\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(c+d*tanh(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+c)}{x}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*tanh(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(c+d\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*tanh(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*tanh(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}(c+d\tanh(a+b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*tanh(a + b*x))/x,x)
```

output
```
int(acoth(c + d*tanh(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\tanh(a + bx))}{x} \, dx = \int \frac{acoth(\tanh(bx + a)\, d + c)}{x} dx$$

input
```
int(acoth(c+d*tanh(b*x+a))/x,x)
```

output
```
int(acoth(tanh(a + b*x)*d + c)/x,x)
```

3.86 $\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 155

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4 \log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^3 \operatorname{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} + \frac{3x^2 \operatorname{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^2} - \frac{3x \operatorname{PolyLog}\left(4,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^3} + \frac{3 \operatorname{PolyLog}\left(5,-\left((1+d)e^{2a+2bx}\right)\right)}{16b^4}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1+d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1+d)
*exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3+3/16*polyl
og(5,-(1+d)*exp(2*b*x+2*a))/b^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{4b^4x^4\coth^{-1}(1+d+d\tanh(a+bx)) - 2b^4x^4\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + 4b^3x^3\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 + d + d*Tanh[a + b*x]] - 2*b^4*x^4*Log[1 + 1/((1 + d)
*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))]
+ 6*b^2*x^2*PolyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))] + 6*b*x*PolyLog[4,
-(1/((1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x)
)))])/(16*b^4)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6794, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{ 6794}$$

$$\frac{1}{4}b\int \frac{x^4}{e^{2a+2bx}(d+1)+1}dx + \frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\int \frac{e^{2a+2bx}x^4}{e^{2a+2bx}(d+1)+1}dx\right) + \frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\int x^3\log\left(e^{2a+2bx}(d+1)+1\right)dx}{b(d+1)}\right)\right)+\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\int x^2\operatorname{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x^3\operatorname{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int x\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)dx}{b}\right)}{2b}-\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \operatorname{Poly}}{b}}{}\right.}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a}\cdots}{\cdots}}{\cdots}\right)}{2b}\cdots\right)}{b\cdots}\right)\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7143

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLo}\cdots}{\cdots}}{b}\right)}{2b}\cdots\right)}{b(d+1)}\right)\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 + (b*(x^5/5 - (1 + d)*((x^4*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (2*(-1/2*(x^3*PolyLog[2, -((1
 + d)*E^(2*a + 2*b*x))])/b + (3*((x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x)
)])/(2*b) - ((x*PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))])/(2*b) - PolyLog[5,
 -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b)))/(b*(1 + d)))))/4
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 1684, normalized size of antiderivative = 10.86

method	result	size
risch	Expression too large to display	1684

input
```
int(x^3*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/20*b*x^5+1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*d*a
^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^3*d/(1+d)*ln(1+(1+d)*exp(2*
b*x+2*a))*x*a^3-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3-3/8/b^4/(
1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^4+3/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b
*x+2*a))*x^2-1/4/b^4/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^3-3/8/b^3/(1
+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*e
xp(2*b*x+2*a))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^4
/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*
(-d-1)^(1/2))-1/4*x^4*ln(exp(b*x+a))-1/8/b^4*a^4/(1+d)*ln(d*exp(2*b*x+2*a)
+exp(2*b*x+2*a)+1)-1/16*(I*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I*e
xp(2*b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)
*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp
(2*b*x+2*a))^2-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-I*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^3+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)+ex
p(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a
)+1))+I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^
3+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(135) = 270$.

Time = 0.10 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.90

$$\int x^3 \coth^{-1}(1 + d + d\tanh(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 + 5*b^4*x^4*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d
 + 1)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a)
+ 2*(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sq
rt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2
*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*
sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*
sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/
2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*
log(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog
(5, 1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -
1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int x^3 \coth^{-1}(1 + d + d\tanh(a + bx))\, dx = \int x^3 \operatorname{acoth}\left(d \tanh\left(a + bx\right) + d + 1\right) dx$$

input
```
integrate(x**3*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x**3*acoth(d*tanh(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right) + \frac{1}{40}\left(\frac{2x^5}{d} - \frac{5\left(2b^4x^4\log\left((d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-6b^2x^2\mathrm{Li}_3(-(d+1)e^{(2bx+2a)})\right.}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d +
 1)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*
d
```

Giac [F]

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^3 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^3\,\mathrm{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x^3*acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x^3*acoth(d + d*tanh(a + b*x) + 1), x)
```

Reduce [F]

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int acoth(\tanh(bx+a)\,d+d+1)\,x^3dx$$

input
```
int(x^3*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + d + 1)*x**3,x)
```

3.87 $\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^2\,\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} + \frac{x\,\text{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{4b^2} - \frac{\text{PolyLog}\left(4,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1+d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1+d)*e
xp(2*b*x+2*a))/b^2-1/8*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(1+d+d\tanh(a+bx)) - 4b^3x^3\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right) + 6bx}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 + d + d*Tanh[a + b*x]] - 4*b^3*x^3*Log[1 + 1/((1 + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))]
+ 6*b*x*PolyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[4, -(1/((1
+ d)*E^(2*(a + b*x))))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6794, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6794}$$

$$\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}(d+1)+1}dx + \frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}b\left(\frac{x^4}{4} - (d+1)\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}(d+1)+1}dx\right) + \frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\int x^2\log\left(e^{2a+2bx}(d+1)+1\right)dx}{2b(d+1)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}}{b}-\frac{x^2\,\text{Po}}{}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)de^{2}}{4b^2}}{b}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLo}}{}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + d + d*Tanh[a + b*x]])/3 + (b*(x^4/4 - (1 + d)*((x^3*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (3*(-1/2*(x^2*PolyLog[2, -((1
 + d)*E^(2*a + 2*b*x))])/b + ((x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(
2*b) - PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b*(1 + d))))
)/3
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 1625, normalized size of antiderivative = 12.70

method	result	size
risch	Expression too large to display	1625

input
```
int(x^2*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a
))*a^2*x-1/2/b^2*d*a^2/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^2*d*a^2
/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/8/b^3/(1+d)*polylog(4,-(1+d)*exp(
2*b*x+2*a))-1/6/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3+1/4/b^2*d/(1+d)*polyl
og(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(
1/2))-1/2/b^2*a^2/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^3/(1+d
)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)
^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2
/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/6/b^3*d*a^3/(1+d)*ln(d*exp(2*b*x
+2*a)+exp(2*b*x+2*a)+1)+1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2*x-1/4
/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/3/b^3*d/(1+d)*ln(1+(1+d)
*exp(2*b*x+2*a))*a^3+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2-
1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1-exp
(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1
/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/6/b^3*a^3/(1+d)*ln(d*e
xp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a
))*x^2+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3+1/4/b^3/(1+d)*polylog(
2,-(1+d)*exp(2*b*x+2*a))*a^2-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3+1/
4/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/8/b^3*d/(1+d)*polylog(4,-
(1+d)*exp(2*b*x+2*a))-1/12*(I*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(111) = 222$.

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.98

$$\int x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) \, dx$$
$$= \frac{b^4x^4 + 2\,b^3x^3 \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - }{}$$

input
```
integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*b^3*x^3*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1
)*sinh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*
(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d
- 4)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sq
rt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt
(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^2 \operatorname{acoth}(d\tanh(a+bx)+d+1)\,dx$$

input
```
integrate(x**2*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*tanh(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arcoth}(d\tanh(bx+a)+d+1) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left((d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left(-(d+1)e^{(2\cdots}\right.\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2
*b*x + 2*a)))/(b^4*d))*b*d
```

Giac [F]

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^2\,\mathrm{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x^2*acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d + d*tanh(a + b*x) + 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int acoth\left(\tanh\left(bx+a\right)d+d+1\right)x^2dx$$

input
```
int(x^2*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + d + 1)*x**2,x)
```

3.88 $\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\begin{aligned}\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx &= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) \\ &\quad -\frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) \\ &\quad -\frac{x\,\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} \\ &\quad +\frac{\text{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^2}\end{aligned}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1+d)*exp(2*
b*x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{2b^2x^2\left(2\coth^{-1}(1+d+d\tanh(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLo}}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6794, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{ 6794}$$
$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}(d+1)+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{ 2615}$$
$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}(d+1)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int x\log\left(e^{2a+2bx}(d+1)+1\right)dx}{b(d+1)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\int \text{PolyLog}(2,-((d+1)e^{2a+2bx}))dx}{2b}-\frac{x\,\text{PolyLog}(2,-((d+1)e^{2a+2bx}))}{2b}}{b(d+1)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 2720

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,-((d+1)e^{2a+2bx}))de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,-((d+1)e}{2b}}{b(d+1)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 7143

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\text{PolyLog}(3,-((d+1)e^{2a+2bx}))}{4b^2}-\frac{x\,\text{PolyLog}(2,-((d+1)e^{2a+2bx}))}{2b}}{b(d+1)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + d + d*Tanh[a + b*x]])/2 + (b*(x^3/3 - (1 + d)*((x^2*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (-1/2*(x*PolyLog[2, -((1 + d)
*E^(2*a + 2*b*x))])/b + PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/(b
*(1 + d)))))/2
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 1542, normalized size of antiderivative = 15.27

method	result	size
risch	Expression too large to display	1542

input
```
int(x*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*x^3-1/2*x^2*ln(exp(b*x+a))+1/2/b*d*a/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^
(1/2))+1/2/b*d*a/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b*d/(1+d)*ln(1+
(1+d)*exp(2*b*x+2*a))*a*x-1/4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8/b
^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))-1/4/b^2*d/(1+d)*polylog(2,-(1+
d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b*a
/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)
*(-d-1)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d
*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/4/b^2*d/(1+d)*ln(1+(1+d)*exp(2
*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x+1/2/b^2*d*
a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b/(1+d)*ln(1+(1+d)*exp(2*b*x+
2*a))*a*x-1/4/b^2*d*a^2/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/2/b^
2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a
)*(-d-1)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a
/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/4/b^2*a^2/(1+d)*ln(d*exp(2*b*x+2
*a)+exp(2*b*x+2*a)+1)-1/4/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(
1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,-(1+d)*exp
(2*b*x+2*a))*a-1/4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8/b^2/(1+d)*poly
log(3,-(1+d)*exp(2*b*x+2*a))-1/8*(I*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi
*csgn(I*exp(2*b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(87) = 174$.

Time = 0.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.19

$$\int x\coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{2\,b^3x^3 + 3\,b^2x^2\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right) -}{}$$

input
```
integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*b^2*x^2*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x +
 a) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1
)*sinh(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d -
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(
-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int x\coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x\,\mathrm{acoth}\,(d\tanh(a+bx)+d+1)\,dx$$

input
```
integrate(x*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x*acoth(d*tanh(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$=\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-(d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right.$$
$$+\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)$$

input
```
integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d
))*b*d + 1/2*x^2*arccoth(d*tanh(b*x + a) + d + 1)
```

Giac [F]

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int x\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int x\,\mathrm{acoth}(d+d\tanh(a+b\,x)+1)\;dx$$

input
```
int(x*acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x*acoth(d + d*tanh(a + b*x) + 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1 + d + d \tanh(a + bx)) \, dx = \int acoth(\tanh(bx + a) \, d + d + 1) \, x dx$$

input
```
int(x*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + d + 1)*x,x)
```

3.89 $\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1+d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1+d)*exp(2*b*x+2*a))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx = x\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{-2bx\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + \text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Tanh[a + b*x]] + (-2*b*x*Log[1 + 1/((1 + d)*E^(2*(a +
b*x)))] + PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/(4*b)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6786, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6786}$$

$$b\int \frac{x}{e^{2a+2bx}(d+1)+1}dx+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left(\frac{x^2}{2}-(d+1)\int \frac{e^{2a+2bx}x}{e^{2a+2bx}(d+1)+1}dx\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int \log\left(e^{2a+2bx}(d+1)+1\right)dx}{2b(d+1)}\right)\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int e^{-2a-2bx}\log\left(e^{2a+2bx}(d+1)+1\right)de^{2a+2bx}}{4b^2(d+1)}\right)\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2838}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2(d+1)}+\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}\right)\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Tanh[a + b*x]] + b*(x^2/2 - (1 + d)*((x*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) + PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]
/(4*b^2*(1 + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6786
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(61) = 122$.

Time = 0.70 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}+\frac{d^2\left(\frac{\ln(d+d\tanh(bx+}{4}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}+\frac{d^2\left(\frac{\ln(d+d\tanh(bx+}{4}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(1+d+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arccoth(
1+d+d*tanh(b*x+a))*d*ln(d+d*tanh(b*x+a))+1/2*d^2*(1/d*(1/4*ln(d+d*tanh(b*x
+a))^2-1/2*dilog(1/2*d*tanh(b*x+a)+1/2*d+1)-1/2*ln(d+d*tanh(b*x+a))*ln(1/2
*d*tanh(b*x+a)+1/2*d+1))-1/d*(1/2*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)+1/2*ln(
-d*tanh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a)-d)/d)-1/2*dilog((-d*tanh(b*x+a)-
d-2)/(-2*d-2))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d-2)/(-2*d-2)))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(60) = 120$.

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.45

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{b^2x^2 + bx\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+\sqrt{-}\right.}{}$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + b*x*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*
sinh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + s
inh(b*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))
))/b
```

Sympy [F]

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(acoth(d*tanh(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx &= \frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left((d+1)e^{(2bx+2a)}+1\right)+\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)}{b^2d}\right)\\ &\quad + x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\end{aligned}$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1
)*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + d + 1)
```

Giac [F]

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int \operatorname{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(acoth(d + d*tanh(a + b*x) + 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int acoth(\tanh\left(bx+a\right)d+d+1)\,dx$$

input
```
int(acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
int(acoth(tanh(a + b*x)*d + d + 1),x)
```

3.90 $\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+d+1)}{x} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\coth^{-1}(d\tanh(a+bx)+d+1)}{x}dx$$

input
```
Int[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))}{x}dx$$

input
```
int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

output
```
int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+d+d*tanh(b*x+a))/x,x)
```

output
```
Integral(acoth(d*tanh(a + b*x) + d + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}(d+d\tanh(a+b\,x)+1)}{x}\,dx$$

input
```
int(acoth(d + d*tanh(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d + d*tanh(a + b*x) + 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{acoth(\tanh(bx+a)\,d+d+1)}{x}dx$$

input
```
int(acoth(1+d+d*tanh(b*x+a))/x,x)
```

output
```
int(acoth(tanh(a + b*x)*d + d + 1)/x,x)
```

3.91 $\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 168

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x^3\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b} + \frac{3x^2\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2} - \frac{3x\,\text{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3} + \frac{3\,\text{PolyLog}\left(5,-\left((1-d)e^{2a+2bx}\right)\right)}{16b^4}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1-d-d*tanh(b*x+a))-1/8*x^4*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1-d)
*exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3+3/16*polyl
og(5,-(1-d)*exp(2*b*x+2*a))/b^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{4b^4x^4\coth^{-1}(1-d-d\tanh(a+bx)) - 2b^4x^4\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + 4b^3x^3\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x^2}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 - d - d*Tanh[a + b*x]] - 2*b^4*x^4*Log[1 - 1/((-1 + d
)*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] +
 6*b^2*x^2*PolyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))] + 6*b*x*PolyLog[4, 1/(
(-1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/(
16*b^4)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6794, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6794}$$

$$\frac{1}{4}b\int \frac{x^4}{e^{2a+2bx}(1-d)+1}dx + \frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\int \frac{e^{2a+2bx}x^4}{e^{2a+2bx}(1-d)+1}dx\right) + \frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\int x^3\log\left(e^{2a+2bx}(1-d)+1\right)dx}{b(1-d)}\right)\right)+\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\int x^2\,\mathrm{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x^3\,\mathrm{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}\right.}{b(1-d)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\,\mathrm{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int x\,\mathrm{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)dx}{b}\right)}{2b}-\right.}{b(1-d)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\,\mathrm{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\,\mathrm{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \mathrm{Poly}}{}}{b}\right.}{2b}\right.}{b(1-d}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 2720

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a}\ldots}{\ldots}}{\ldots}\right)}{2b}\ldots\right)}{b\ldots}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\big\downarrow$ 7143

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\operatorname{PolyLo}\ldots}{\ldots}}{b}\right)}{2b}\ldots\right)}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 - d - d*Tanh[a + b*x]])/4 + (b*(x^5/5 - (1 - d)*((x^4*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (2*(-1/2*(x^3*PolyLog[2, -((1
 - d)*E^(2*a + 2*b*x))])/b + (3*((x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x)
)])/(2*b) - ((x*PolyLog[4, -((1 - d)*E^(2*a + 2*b*x))])/(2*b) - PolyLog[5,
 -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b)))/(b*(1 - d)))))/4
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 1754, normalized size of antiderivative = 10.44

method	result	size
risch	Expression too large to display	1754

input
```
int(x^3*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/20*b*x^5+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d/(d
-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(
d-1)^(1/2))*x-1/16*(-I*Pi*csgn(I*exp(2*b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2-
2*I*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2
*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))-I*Pi*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-I*P
i*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^3-I*Pi*cs
gn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp
(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I*d*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x
+2*a))^2-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I*d)*csgn(I*d
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(
d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+2*ln(d))*x^4-1/4*x^4*ln(exp(b*x+a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(135) = 270$.

Time = 0.10 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.52

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 - 5\,b^4x^4\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 2}{}$$

input
```
integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(
b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a)
+ sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x
 + a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x
+ a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a)
 + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(
b*x + a))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 5*(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + 120*polylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*pol
ylog(5, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\int x^3\,\mathrm{acoth}\,(d\tanh(a+bx)+d-1)\,dx$$

input
```
integrate(x**3*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(x**3*acoth(d*tanh(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\frac{1}{4}\,x^4\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$
$$+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3((d-1)e^{(2}}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^4*arccoth(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*l
og(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a)
) - 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d -
1)*e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Giac [F]

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int x^3\,\mathrm{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -x^3\,\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x^3*acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x^3*acoth(d + d*tanh(a + b*x) - 1), x)
```

Reduce [F]

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\left(\int acoth(\tanh(bx+a)\,d+d-1)\,x^3dx\right)$$

input
```
int(x^3*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
- int(acoth(tanh(a + b*x)*d + d - 1)*x**3,x)
```

3.92 $\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 139

$$\begin{aligned}\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) \\ &\quad -\frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) \\ &\quad -\frac{x^2\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b} \\ &\quad +\frac{x\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2} \\ &\quad -\frac{\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}\end{aligned}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1-d-d*tanh(b*x+a))-1/6*x^3*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1-d)*e
xp(2*b*x+2*a))/b^2-1/8*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{8b^3x^3\coth^{-1}(1-d-d\tanh(a+bx))-4b^3x^3\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)+6b^2x^2\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+6bx\,\text{P}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 - d - d*Tanh[a + b*x]] - 4*b^3*x^3*Log[1 - 1/((-1 + d
)*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] +
 6*b*x*PolyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, 1/((-1 + d)
*E^(2*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {6794, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \; 6794$$

$$\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}(1-d)+1}dx+\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}(1-d)+1}dx\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\int x^2\log\left(e^{2a+2bx}(1-d)+1\right)dx}{2b(1-d)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 3011

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-((1-d)e^{2a+2bx}))dx}{b}-\frac{x^2\,\text{PolyLog}(2,-((1-d)e^{2a+2bx}))}{2b}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}}{b}-\frac{x^2\,\text{Po}}{}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 2720

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)de^{2}}{4b^2}}{b}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7143

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLo}}{}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - d - d*Tanh[a + b*x]])/3 + (b*(x^4/4 - (1 - d)*((x^3*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (3*(-1/2*(x^2*PolyLog[2, -((1
 - d)*E^(2*a + 2*b*x))])/b + ((x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(
2*b) - PolyLog[4, -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b*(1 - d))))
)/3
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 1697, normalized size of antiderivative = 12.21

method	result	size
risch	Expression too large to display	1697

input
```
int(x^2*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+
2*a))+1/6/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3+1/6*x^3*ln(d*exp(2*b*x+2*a)
-exp(2*b*x+2*a)-1)-1/8/b^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/4/b/(
d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2-1/3/b^3/(d-1)*ln(1-(d-1)*exp(2*b*
x+2*a))*a^3-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1
)*polylog(3,(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1
)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*
dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)
^(1/2))-1/6/b^3*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-1/6*d/(d-1
)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(d
-1)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/6/b^3*d*a
^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-1/4/b*d/(d-1)*polylog(2,(d-
1)*exp(2*b*x+2*a))*x^2+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/
b^3*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,
(d-1)*exp(2*b*x+2*a))*x-1/2/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2*x+1/2
/b^2*a^2/(d-1)*x*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*a^2/(d-1)*x*ln(1+exp
(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/
b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/12*(-I*Pi*csgn(I*exp(2*b*x+
2*a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)-1))^2-2*I*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.58

$$\int x^2 \coth^{-1}(1 - d - d\tanh(a + bx))\, dx$$
$$= \frac{b^4x^4 - 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x \cdots}{\cdots}$$

input
```
integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x
 + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) + 2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + s
inh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 12*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*po
lylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\int x^2 \operatorname{acoth}(d\tanh(a+bx)+d-1)\,dx$$

input
```
integrate(x**2*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(x**2*acoth(d*tanh(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\frac{1}{3}x^3 \operatorname{arcoth}(d\tanh(bx+a)+d-1) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left(-(d-1)e^{(2bx+2a)}+1\right) + 6b^2x^2\operatorname{Li}_2\left((d-1)e^{(2bx+2a)}\right) - 6bx\operatorname{Li}_3\left((d-1)e^{(2bx\right.}\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*x^3*arccoth(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*l
og(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*
b*x + 2*a)))/(b^4*d))*b*d
```

Giac [F]

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int x^2\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -x^2\operatorname{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x^2*acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x^2*acoth(d + d*tanh(a + b*x) - 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\left(\int acoth(\tanh\left(bx+a\right)d+d-1)\,x^2dx\right)$$

input
```
int(x^2*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
- int(acoth(tanh(a + b*x)*d + d - 1)*x**2,x)
```

3.93 $\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b} + \frac{\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1-d-d*tanh(b*x+a))-1/4*x^2*ln(1+(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1-d)*exp(2*
b*x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\coth^{-1}(1-d-d\tanh(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*
(a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3
, 1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6794, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6794}$$

$$\frac{1}{2}b\int\frac{x^2}{e^{2a+2bx}(1-d)+1}dx+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\int\frac{e^{2a+2bx}x^2}{e^{2a+2bx}(1-d)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int x\log\left(e^{2a+2bx}(1-d)+1\right)dx}{b(1-d)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\int \text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}}{b(1-d)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\left((1-d)e\right.\right.}{2b}}{b(1-d)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}}{b(1-d)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - d - d*Tanh[a + b*x]])/2 + (b*(x^3/3 - (1 - d)*((x^2*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (-1/2*(x*PolyLog[2, -((1 - d)
*E^(2*a + 2*b*x))])/b + PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/(b
*(1 - d)))))/2
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.79 (sec) , antiderivative size = 1616, normalized size of antiderivative = 14.69

method	result	size
risch	Expression too large to display	1616

input
```
int(x*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1+exp
(b*x+a)*(d-1)^(1/2))+1/6*b*x^3-1/2/b*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a*
x+1/2/b*d*a/(d-1)*x*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b*d*a/(d-1)*x*ln(1+ex
p(b*x+a)*(d-1)^(1/2))-1/2*x^2*ln(exp(b*x+a))+1/4*x^2*ln(d*exp(2*b*x+2*a)-e
xp(2*b*x+2*a)-1)-1/8*(-I*Pi*csgn(I*exp(2*b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^
2-2*I*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x
+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))-I*Pi*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-I
*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^3-I*Pi*
csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*e
xp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I*d*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b
*x+2*a))^2-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I*d)*csgn(I
*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(89) = 178$.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.77

$$\int x \coth^{-1}(1 - d - d \tanh(a + bx)) \, dx = \frac{2 b^3 x^3 - 3 b^2 x^2 \log\left(\frac{d \cosh(bx+a)+d \sinh(bx+a)}{(d-2)\cosh(bx+a)+d \sinh(bx+a)}\right) - 6 bx \mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6 bx\mathrm{I}}{}$$

input
```
integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
 + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*
sqrt(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a)
 - 2*sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)
)) + 6*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int x \coth^{-1}(1 - d - d \tanh(a + bx)) \, dx = -\int x \operatorname{acoth}(d \tanh(a + bx) + d - 1) \, dx$$

input
```
integrate(x*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(x*acoth(d*tanh(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right) - \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$

input
```
integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d)
)*b*d - 1/2*x^2*arccoth(d*tanh(b*x + a) + d - 1)
```

Giac [F]

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int x\,\mathrm{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -x\,\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x*acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x*acoth(d + d*tanh(a + b*x) - 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\left(\int acoth(\tanh(bx+a)\,d+d-1)\,xdx\right)$$

input
```
int(x*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
 - int(acoth(tanh(a + b*x)*d + d - 1)*x,x)
```

3.94 $\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx=\frac{bx^2}{2}+x\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1-d-d*tanh(b*x+a))-1/2*x*ln(1+(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1-d)*exp(2*b*x+2*a))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx=x\coth^{-1}(1-d-d\tanh(a+bx))+\frac{-2bx\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Tanh[a + b*x]] + (-2*b*x*Log[1 - 1/((-1 + d)*E^(2*(a +
 b*x)))] + PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/(4*b)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6786, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \ 6786$$

$$b\int \frac{x}{e^{2a+2bx}(1-d)+1}dx+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2615$$

$$b\left(\frac{x^2}{2}-(1-d)\int \frac{e^{2a+2bx}x}{e^{2a+2bx}(1-d)+1}dx\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2620$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int \log\left(e^{2a+2bx}(1-d)+1\right)dx}{2b(1-d)}\right)\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2715$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int e^{-2a-2bx}\log\left(e^{2a+2bx}(1-d)+1\right)de^{2a+2bx}}{4b^2(1-d)}\right)\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2838$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2(1-d)}+\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}\right)\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Tanh[a + b*x]] + b*(x^2/2 - (1 - d)*((x*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) + PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]
/(4*b^2*(1 - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6786
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(68) = 136.

Time = 0.75 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.57

method	result
derivativedivides	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\ln(-d\tanh(}{4}\right.}{}}{}$
default	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\ln(-d\tanh(}{4}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(-1/2*arccoth(1-d-d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)+1/2*arccoth
(1-d-d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(1/d*(1/4*ln(-d*tanh(b*
x+a)-d)^2-1/2*dilog(-1/2*d*tanh(b*x+a)-1/2*d+1)-1/2*ln(-d*tanh(b*x+a)-d)*l
n(-1/2*d*tanh(b*x+a)-1/2*d+1))-1/d*(1/2*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)+1
/2*ln(-d*tanh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a)-d)/d)-1/2*dilog((-d*tanh(b
*x+a)-d+2)/(-2*d+2))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d+2)/(-2*
d+2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(63) = 126.

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.99

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{b^2x^2-bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\sqrt{}\right.}{}$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*
x + a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(
-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(acoth(d*tanh(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}\,bd\left(\frac{2\,x^2}{d} - \frac{2\,bx\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right) - x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1
)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*tanh(b*x + a) + d - 1)
```

Giac [F]

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx=\int \operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx=\int -\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-acoth(d + d*tanh(a + b*x) - 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx=-\left(\int acoth(\tanh\left(bx+a\right)d+d-1)\,dx\right)$$

input
```
int(acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
- int(acoth(tanh(a + b*x)*d + d - 1),x)
```

3.95 $\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x}dx$$

input
```
Int[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(1-d-d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

output
```
int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = -\int \frac{\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

input
```
integrate(acoth(1-d-d*tanh(b*x+a))/x,x)
```

output
```
-Integral(acoth(d*tanh(a + b*x) + d - 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-integrate(arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tanh(b*x + a) - d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx=\int -\frac{\operatorname{acoth}(d+d\tanh(a+b\,x)-1)}{x}\,dx$$

input
```
int(-acoth(d + d*tanh(a + b*x) - 1)/x,x)
```

output
```
int(-acoth(d + d*tanh(a + b*x) - 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = -\left(\int \frac{acoth(\tanh(bx+a)\,d+d-1)}{x}dx\right)$$

input
```
int(acoth(1-d-d*tanh(b*x+a))/x,x)
```

output
```
- int(acoth(tanh(a + b*x)*d + d - 1)/x,x)
```

3.96 $\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx$

Optimal result

Integrand size = 15, antiderivative size = 303

$$\begin{aligned}\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx &= \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) \\ &+ \frac{1}{6}x^3 \log\left(1 - \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &- \frac{1}{6}x^3 \log\left(1 - \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+ \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &- \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &- \frac{x \operatorname{PolyLog}\left(3, \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b^2} \\ &+ \frac{x \operatorname{PolyLog}\left(3, \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b^2} \\ &+ \frac{\operatorname{PolyLog}\left(4, \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^3} \\ &- \frac{\operatorname{PolyLog}\left(4, \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*coth(b*x+a))+1/6*x^3*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/6*x^3*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,(1-c-d)
*exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d
))/b-1/4*x*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,(
1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,(1-c-d)*exp(2*b*x+2*a)/(1
-c+d))/b^3-1/8*polylog(4,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) - \frac{-4b^3x^3\log\left(1+\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)+4b^3x^3\log\left(1+\frac{(-1-c+d)e^{-2(a+bx)}}{1+c+d}\right)+6b^2x^2\,\mathrm{PolyLog}\left(2,\frac{(-1+c-d)e^{-2(a-}{-1+c+d}\right.}{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Coth[a + b*x]])/3 - (-4*b^3*x^3*Log[1 + (1 - c + d)/((-
1 + c + d)*E^(2*(a + b*x)))] + 4*b^3*x^3*Log[1 + (-1 - c + d)/((1 + c + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (-1 + c - d)/((-1 + c + d)*E^(2*
(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x
)))] + 6*b*x*PolyLog[3, (-1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] - 6*b
*x*PolyLog[3, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, (-
1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] - 3*PolyLog[4, (1 + c - d)/((1
+ c + d)*E^(2*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6800, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \; 6800$$

$$-\frac{1}{3}b(-c-d+1)\int \frac{e^{2a+2bx}x^3}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+\frac{1}{3}b(c+d+1)\int \frac{e^{2a+2bx}x^3}{c-(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\int x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{3}b(c+d+1)\left(\frac{3\int x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \; 3011$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \; 7163$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c\cdots}{\cdots}\right)}{2b(-c-\cdots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)\cdots}{c-d\cdots}\right)}{2b(c+d+1\cdots}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\cdots}{\cdots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\cdots}{\cdots}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Coth[a + b*x]])/3 - (b*(1 - c - d)*(-1/2*(x^3*Log[1 - (
(1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) + (3*(-1/2*(x^2
*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/b + ((x*PolyLog[3,
 ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b) - PolyLog[4, ((1 - c -
d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2))/b))/(2*b*(1 - c - d))))/3 + (b*(
1 + c + d)*(-1/2*(x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/
(b*(1 + c + d)) + (3*(-1/2*(x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(
1 + c - d)])/b + ((x*PolyLog[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]
)/(2*b) - PolyLog[4, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2))/b
))/(2*b*(1 + c + d))))/3
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6800
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b
*((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c
- d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.43 (sec) , antiderivative size = 5185, normalized size of antiderivative = 17.11

method	result	size
risch	Expression too large to display	5185

input
```
int(x^2*arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(259) = 518$.

Time = 0.12 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.90

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d -
 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c +
 d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((
c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d
+ 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*lo
g(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1
)*sqrt((c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2
*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1)))
+ 6*b*x*polylog(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 6*b*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) +
 sinh(b*x + a))) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*
x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))
*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sq
rt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^
3 + a^3)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + ...
```

Sympy [F]

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(c + d\coth\left(a + bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*coth(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*coth(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)$$
$$-\frac{1}{18}bd\left(\frac{4\,b^3x^3\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-6\,bx\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)+3\,\mathrm{I}}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d +
1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x
 + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d
 + 1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) -
(4*b^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*d
ilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d -
1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)
/(c - d - 1)))/(b^4*d))
```

Giac [F]

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\,dx = \int x^2\,\mathrm{acoth}(c + d\coth(a + b\,x))\,dx$$

input
```
int(x^2*acoth(c + d*coth(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*coth(a + b*x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\,dx = \int acoth(\coth(bx + a)\,d + c)\,x^2 dx$$

input
```
int(x^2*acoth(c+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + c)*x**2,x)
```

3.97 $\int x \coth^{-1}(c+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 229

$$\begin{aligned}\int x\coth^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx)) \\ &\quad+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &\quad-\frac{1}{4}x^2\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &\quad+\frac{x\,\mathrm{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &\quad-\frac{x\,\mathrm{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &\quad-\frac{\mathrm{PolyLog}\left(3,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^2} \\ &\quad+\frac{\mathrm{PolyLog}\left(3,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*coth(b*x+a))+1/4*x^2*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/4*x^2*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,(1-c-d)*e
xp(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
-1/8*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx$$
$$= \frac{4b^2x^2 \coth^{-1}(c + d \coth(a + bx)) + 2b^2x^2 \log\left(1 + \frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right) - 2b^2x^2 \log\left(1 + \frac{(-1-c+d)e^{-2(a+bx)}}{1+c+d}\right) - }{}$$

input
```
Integrate[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Coth[a + b*x]] + 2*b^2*x^2*Log[1 + (1 - c + d)/((
-1 + c + d)*E^(2*(a + b*x)))] - 2*b^2*x^2*Log[1 + (-1 - c + d)/((1 + c + d
)*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-1 + c - d)/((-1 + c + d)*E^(2*(a
+ b*x)))] + 2*b*x*PolyLog[2, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x)))] -
PolyLog[3, (-1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] + PolyLog[3, (1 +
c - d)/((1 + c + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6800, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d \coth(a + bx) + c) \, dx$$

$$\Big\downarrow \text{6800}$$

$$-\frac{1}{2}b(-c-d+1)\int \frac{e^{2a+2bx}x^2}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+\frac{1}{2}b(c+d+1)\int \frac{e^{2a+2bx}x^2}{c-(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\int x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\int x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\operatorname{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\operatorname{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

input
```
Int[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCoth[c + d*Coth[a + b*x]])/2 - (b*(1 - c - d)*(-1/2*(x^2*Log[1 - (
(1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) + (-1/2*(x*Poly
Log[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/b + PolyLog[3, ((1 - c
- d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2))/(b*(1 - c - d))))/2 + (b*(1 +
c + d)*(-1/2*(x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(b*(
1 + c + d)) + (-1/2*(x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d
)])/b + PolyLog[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2))/(b*
(1 + c + d))))/2
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6800
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b
*((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c
- d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 4881, normalized size of antiderivative = 21.31

method	result	size
risch	Expression too large to display	4881

input
```
int(x*arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*a^2/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)
+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a^2/(c+d-1)*ln((exp(b*x+a)*c
+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))
+1/4/b^2*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*c/(c+d-1
)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*c/(c+d-1)*polylog(2,
(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a-1/2/b^2*d*a/(c+d-1)*dilog((-exp(b*x+a)*c
-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))
+1/4*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/4*c/(c+d-1)*ln(1
-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*e
xp(2*b*x+2*a)/(c-d-1))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/
(c-d-1))-1/4/b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b/(c
+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x-1/4/b^2/(c+d-1)*polylog(
2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a+1/2/b^2*a/(c+d-1)*dilog((-exp(b*x+a)*c
-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))
+1/2/b^2*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2
)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/4/b^2*a^2/(c+d-1)*ln(c*exp(2*b*x+
2*a)+d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-c+d+1)-1/4/b^2/(1+c+d)*polylog(2,(1+c
+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/8/b^2*c/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*
x+2*a)/(1+c-d))+1/8/b^2*d/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d)
)-1/4/b^2*a^2/(1+c+d)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b*x+2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(195) = 390$.

Time = 0.12 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.18

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x
 + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c -
d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x +
a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*
cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d -
1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sin
h(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2
)*log(sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 (b^2*x^2 - a^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d +
 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c
+ d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt
((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(...
```

Sympy [F]

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx=\int x\,\mathrm{acoth}\,(c+d\coth\,(a+bx))\,dx$$

input
```
integrate(x*acoth(c+d*coth(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*coth(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx=$$
$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^3d}-\frac{2\,b^2x^2\log\ldots}{\ldots}\right.$$
$$+\frac{1}{2}\,x^2\,\mathrm{arcoth}\,(d\coth\,(bx+a)+c)$$

input
```
integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2
*b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d +
1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d))
 + 1/2*x^2*arccoth(d*coth(b*x + a) + c)
```

Giac [F]

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx = \int x \operatorname{arcoth}(d \coth(bx + a) + c) \, dx$$

input
```
integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx = \int x \operatorname{acoth}(c + d \coth(a + b x)) \, dx$$

input
```
int(x*acoth(c + d*coth(a + b*x)),x)
```

output
```
int(x*acoth(c + d*coth(a + b*x)), x)
```

Reduce [F]

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx = \int acoth(\coth(bx + a) d + c) x dx$$

input
```
int(x*acoth(c+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + c)*x,x)
```

3.98 $\int \coth^{-1}(c+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 150

$$\begin{aligned}\int \coth^{-1}(c+d\coth(a+bx))\,dx &= x\coth^{-1}(c+d\coth(a+bx)) \\ &+ \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &- \frac{1}{2}x\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+ \frac{\text{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &- \frac{\text{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}\end{aligned}$$

output

```
x*arccoth(c+d*coth(b*x+a))+1/2*x*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*
x*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,(1-c-d)*exp(2*b*x+2*a
)/(1-c+d))/b-1/4*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(c + d\coth(a + bx))\,dx = x\coth^{-1}(c + d\coth(a + bx)) - \frac{-2bx\left(\log\left(1 - \frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - \log\left(1 - \frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)\right) - \text{PolyLog}\left(2, \frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) + \text{PolyLo}}{4b}$$

input
```
Integrate[ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a +
b*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])
 - PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((
1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])/(4*b)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6792, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\coth(a + bx) + c)\,dx$$

$$\Big\downarrow \text{6792}$$

$$-b(-c - d + 1)\int \frac{e^{2a+2bx}x}{-c - (-c - d + 1)e^{2a+2bx} + d + 1}dx + b(c + d + 1)\int \frac{e^{2a+2bx}x}{c - (c + d + 1)e^{2a+2bx} - d + 1}dx + x\coth^{-1}(d\coth(a + bx) + c)$$

$$\Big\downarrow \text{2620}$$

$$-b(-c-d+1)\left(\frac{\int \log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(\frac{\int \log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$-b(-c-d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$-b(-c-d+1)\left(-\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Coth[a + b*x]] - b*(1 - c - d)*(-1/2*(x*Log[1 - ((1 - c -
d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) - PolyLog[2, ((1 - c - d
)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2*(1 - c - d))) + b*(1 + c + d)*(-1/2
*(x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(b*(1 + c + d)) -
PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2*(1 + c + d)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6792
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + (-Simp[b*(1 - c - d)   Int[x*(E^(2*a + 2*b
*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b*(1 + c + d)
   Int[x*(E^(2*a + 2*b*x)/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x))), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 1.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)}{1-c-d}\right.}{2}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)}{1-c-d}\right.}{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccoth(c+
d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*d^2*(1/d*(1/2*dilog((-d*coth(b*x
+a)-c+1)/(1-c-d))+1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c+1)/(1-c-d)
)-1/2*dilog((-d*coth(b*x+a)-c-1)/(-1-c-d))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d
*coth(b*x+a)-c-1)/(-1-c-d)))-1/d*(1/2*dilog((-d*coth(b*x+a)-c+1)/(1-c+d))+
1/2*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c+1)/(1-c+d))-1/2*dilog((-d*co
th(b*x+a)-c-1)/(-1-c+d))-1/2*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c-1)/
(-1-c+d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(128) = 256$.

Time = 0.11 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.59

$$\int \coth^{-1}(c + d\coth(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) +
(c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1
)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(
c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqr
t((c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d
 - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log
(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)
*sqrt((c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)
/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*lo
g(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - di
log(sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog
(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(s
qrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqr
t((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{acoth}\left(c+d\coth\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(c+d*coth(b*x+a)),x)
```

output
```
Integral(acoth(c + d*coth(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = -\frac{1}{4}bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\right.$$
$$+\,x\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)$$

input
```
integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog
((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d -
1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + c)
```

Giac [F]

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{acoth}(c+d\coth(a+b\,x))\,dx$$

input
```
int(acoth(c + d*coth(a + b*x)),x)
```

output
```
int(acoth(c + d*coth(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = \int acoth(\coth(bx+a)\,d+c)\,dx$$

input
```
int(acoth(c+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + c),x)
```

3.99 $\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\text{Int}\left(\frac{\coth^{-1}(c+d\coth(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(c+d*coth(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\coth(a+bx)+c)}{x} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\coth^{-1}(d\coth(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*coth(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*coth(b*x + a) + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}\left(c+d\coth\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*coth(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*coth(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\coth\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{acoth}(c + d\coth(a + b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*coth(a + b*x))/x,x)
```

output
```
int(acoth(c + d*coth(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\coth(a + bx))}{x}\,dx = \int \frac{acoth(\coth(bx + a)\,d + c)}{x}dx$$

input
```
int(acoth(c+d*coth(b*x+a))/x,x)
```

output
```
int(acoth(coth(a + b*x)*d + c)/x,x)
```

3.100 $\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 152

$$\begin{aligned}\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx &= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) \\ &\quad - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) \\ &\quad - \frac{x^3\operatorname{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b} \\ &\quad + \frac{3x^2\operatorname{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{8b^2} \\ &\quad - \frac{3x\operatorname{PolyLog}\left(4,(1+d)e^{2a+2bx}\right)}{8b^3} \\ &\quad + \frac{3\operatorname{PolyLog}\left(5,(1+d)e^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1+d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1+d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,(1+d)*exp(2*b*x+2*a))/b^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{4b^4x^4\coth^{-1}(1+d+d\coth(a+bx)) - 2b^4x^4\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + 4b^3x^3\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 + d + d*Coth[a + b*x]] - 2*b^4*x^4*Log[1 - 1/((1 + d)
*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + 6
*b^2*x^2*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))] + 6*b*x*PolyLog[4, 1/((1
+ d)*E^(2*(a + b*x)))] + 3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/(16*b^
4)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6796, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6796}$$

$$\frac{1}{4}b\int \frac{x^4}{1-(d+1)e^{2a+2bx}}dx + \frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{4}b\left((d+1)\int \frac{e^{2a+2bx}x^4}{1-(d+1)e^{2a+2bx}}dx + \frac{x^5}{5}\right) + \frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{4}b\left((d+1)\left(\frac{2\int x^3\log\left(1-(d+1)e^{2a+2bx}\right)dx}{b(d+1)}-\frac{x^4\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\int x^2\operatorname{PolyLog}(2,(d+1)e^{2a+2bx})dx}{2b}-\frac{x^3\operatorname{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{b(d+1)}-\frac{x^4\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^5}{5}\right)$$
$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int x\operatorname{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{b(d+1)}-\frac{x^4\log\left(1-(\right.}{2b(}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int\operatorname{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)dx}{2b}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(d+1)e^{2a+}}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 2720

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}\right)}{2b}-\frac{x^3\,\text{Poly}}{}\right.}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 7143

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{4b^2}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}(2,(d+1)e^{2a+2bx}}{2b}\right.}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 + d + d*Coth[a + b*x]])/4 + (b*(x^5/5 + (1 + d)*(-1/2*(x^4*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (2*(-1/2*(x^3*PolyLog[2, (
1 + d)*E^(2*a + 2*b*x)])/b + (3*((x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])
/(2*b) - ((x*PolyLog[4, (1 + d)*E^(2*a + 2*b*x)])/(2*b) - PolyLog[5, (1 +
d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b)))/(b*(1 + d)))))/4
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 1656, normalized size of antiderivative = 10.89

method	result	size
risch	Expression too large to display	1656

input
```
int(x^3*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)*d)*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)-I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(
2*b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2
*a)-1))^2*csgn(I/(exp(2*b*x+2*a)-1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)-1)*d)^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+I*Pi*csgn(I/(exp(2
*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+2*I*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)
)^3+I*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a
)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1))+I*Pi*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)*d)^2*csgn(I*d)+I*Pi*csgn(I/(exp(2
*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-I*Pi*csgn(I*(d*e
xp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*
a)+exp(2*b*x+2*a)-1))^2+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2*c
sgn(I*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)*d)^3-I
*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*ln(d))*x^4+1/20*b*x^5-3/
8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(1+d)*ln(1-exp
(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/
2/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^3+1/2/b^4*d*a^4/(1+d)*ln(1-exp(
b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(132) = 264.

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.78

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 + 5\,b^4x^4\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 2\cdots}{\cdots}$$

input
```
integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 + 5*b^4*x^4*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(
b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a)
+ sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x
 + a) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x
+ a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a)
 + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(
b*x + a))) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 5*(b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + 120*polylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*pol
ylog(5, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^3\,\mathrm{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x**3*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x**3*acoth(d*coth(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right) + \frac{1}{40}\left(\frac{2x^5}{d} - \frac{5\left(2b^4x^4\log\left(-(d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\text{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6b^2x^2\text{Li}_3((d+1)e^{(2}\right.}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Giac [F]

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^3 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^3\,\mathrm{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x^3*acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x^3*acoth(d + d*coth(a + b*x) + 1), x)
```

Reduce [F]

$$\int x^3 \coth^{-1}(1 + d + d\coth(a + bx))\, dx = \int acoth(\coth (bx + a)\, d + d + 1)\, x^3 dx$$

input
```
int(x^3*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + d + 1)*x**3,x)
```

3.101 $\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^2\,\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b} + \frac{x\,\text{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4,(1+d)e^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1+d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1+d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(1+d+d\coth(a+bx)) - 4b^3x^3\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right) + 6bx\operatorname{Po}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 + d + d*Coth[a + b*x]] - 4*b^3*x^3*Log[1 - 1/((1 + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + 6
*b*x*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, 1/((1 + d)*E^(
2*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6796, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\coth(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{ 6796}$$
$$\frac{1}{3}b\int \frac{x^3}{1-(d+1)e^{2a+2bx}}dx + \frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{ 2615}$$
$$\frac{1}{3}b\left((d+1)\int \frac{e^{2a+2bx}x^3}{1-(d+1)e^{2a+2bx}}dx + \frac{x^4}{4}\right) + \frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\int x^2\log\left(1-(d+1)e^{2a+2bx}\right)dx}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log}{}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + d + d*Coth[a + b*x]])/3 + (b*(x^4/4 + (1 + d)*(-1/2*(x^3*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (3*(-1/2*(x^2*PolyLog[2, (
1 + d)*E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(2*b
) - PolyLog[4, (1 + d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*(1 + d)))))/3
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 1599, normalized size of antiderivative = 12.69

method	result	size
risch	Expression too large to display	1599

input
```
int(x^2*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+
d)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b/(1+d)*po
lylog(2,(1+d)*exp(2*b*x+2*a))*x^2+1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))
*a^3+1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polyl
og(3,(1+d)*exp(2*b*x+2*a))*x+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b
*x+2*a)-1)-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(1+d)*poly
log(4,(1+d)*exp(2*b*x+2*a))-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))
-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/12*(-I*Pi*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1)*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*cs
gn(I*d)-I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))*csgn(I*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a))^3-I*Pi*csgn(I/(
exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2*csgn(I/(exp(2*b*x
+2*a)-1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)*d)^2*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)-1))+I*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2
*a)+exp(2*b*x+2*a)-1))^3+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^
2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+I*Pi*csgn(I*(d*exp(2*b*
x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(
2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)-1)*d)^2*csgn(I*d)+I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)-1))^2-I*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(109) = 218$.

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.85

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x}{}$$

input
```
integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*b^3*x^3*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x
 + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) + 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + s
inh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
+ a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*po
lylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^2 \operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x**2*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*coth(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left(-(d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left((d+1)e^{(2bx\right.\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

Giac [F]

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^2\,\mathrm{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x^2*acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d + d*coth(a + b*x) + 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int acoth(\coth\left(bx+a\right)d+d+1)\,x^2dx$$

input
```
int(x^2*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + d + 1)*x**2,x)
```

3.102 $\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 100

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x\,\mathrm{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b} + \frac{\mathrm{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{8b^2}$$

output

```
1/6*b*x^3+1/2*x^2*arccoth(1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1+d)*exp(2*b*
x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx$$

$$= \frac{2b^2x^2\left(2\coth^{-1}(1+d+d\coth(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(\right.}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6796, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \; 6796$$

$$\frac{1}{2}b\int \frac{x^2}{1-(d+1)e^{2a+2bx}}dx+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{2}b\left((d+1)\int \frac{e^{2a+2bx}x^2}{1-(d+1)e^{2a+2bx}}dx+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\int x\log\left(1-(d+1)e^{2a+2bx}\right)dx}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\int \text{PolyLog}(2,(d+1)e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,(d+1)e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\text{PolyLog}(3,(d+1)e^{2a+2bx})}{4b^2}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + d + d*Coth[a + b*x]])/2 + (b*(x^3/3 + (1 + d)*(-1/2*(x^2*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (-1/2*(x*PolyLog[2, (1 + d
)*E^(2*a + 2*b*x)])/b + PolyLog[3, (1 + d)*E^(2*a + 2*b*x)]/(4*b^2))/(b*(1
 + d)))))/2
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.73 (sec) , antiderivative size = 1518, normalized size of antiderivative = 15.18

method	result	size
risch	Expression too large to display	1518

input
```
int(x*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*d*a/(1+d)*x*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/6*b*x^3-1/2*x^2*ln(exp(b*
x+a))-1/2/b*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a*x+1/2/b*d*a/(1+d)*x*ln(1-
exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*a^2/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*
a)-1)-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/8/b^2*d/(1+d)*poly
log(3,(1+d)*exp(2*b*x+2*a))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))
+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1-ex
p(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/
b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,(1+d)*exp(2
*b*x+2*a))*x-1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^2-1/8*(-I*Pi*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
-1))*csgn(I*d)-I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a))^3-I*Pi*c
sgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2*csgn(I/(ex
p(2*b*x+2*a)-1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)*d)^2*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+I*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(
2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x
+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+I*Pi*csgn(I*(d*e
xp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*
a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1))+I*Pi*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)-1)*d)^2*csgn(I*d)+I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csg...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(86) = 172$.

Time = 0.08 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.05

$$\int x \coth^{-1}(1 + d + d\coth(a + bx))\,dx = \frac{2\,b^3x^3 + 3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,bx\mathrm{L}}{}$$

input
```
integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*b^2*x^2*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
 + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*
sqrt(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a)
 - 2*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)
)) + 6*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int x \coth^{-1}(1 + d + d\coth(a + bx))\,dx = \int x\,\mathrm{acoth}\,(d\coth(a + bx) + d + 1)\,dx$$

input
```
integrate(x*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x*acoth(d*coth(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$=\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3((d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)$$
$$+\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)$$

input
```
integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d)
)*b*d + 1/2*x^2*arccoth(d*coth(b*x + a) + d + 1)
```

Giac [F]

$$\int x\coth^{-1}(1+d+d\coth(a+bx))\,dx=\int x\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(1+d+d\coth(a+bx))\,dx=\int x\,\mathrm{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x*acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x*acoth(d + d*coth(a + b*x) + 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1 + d + d \coth(a + bx))\, dx = \int acoth(\coth(bx + a)\, d + d + 1)\, x dx$$

input
```
int(x*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + d + 1)*x,x)
```

3.103 $\int \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{\operatorname{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1+d+d*coth(b*x+a))-1/2*x*ln(1-(1+d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1+d)*exp(2*b*x+2*a))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = x\coth^{-1}(1+d+d\coth(a+bx)) + \frac{-2bx\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)+\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Coth[a + b*x]] + (-2*b*x*Log[1 - 1/((1 + d)*E^(2*(a +
b*x)))] + PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/(4*b)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6788, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6788}$$

$$b\int \frac{x}{1-(d+1)e^{2a+2bx}}dx+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left((d+1)\int \frac{e^{2a+2bx}x}{1-(d+1)e^{2a+2bx}}dx+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left((d+1)\left(\frac{\int \log\left(1-(d+1)e^{2a+2bx}\right)dx}{2b(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left((d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2838}$$

$$b\left((d+1)\left(-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b^2(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Coth[a + b*x]] + b*(x^2/2 + (1 + d)*(-1/2*(x*Log[1 - (
1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]
/(4*b^2*(1 + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6788
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(61) = 122.

Time = 0.72 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}+\frac{d^2\left(\frac{\ln(d+d\coth(bx+a}{4}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}+\frac{d^2\left(\frac{\ln(d+d\coth(bx+a}{4}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(1+d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccoth(
1+d+d*coth(b*x+a))*d*ln(d+d*coth(b*x+a))+1/2*d^2*(1/d*(1/4*ln(d+d*coth(b*x
+a))^2-1/2*dilog(1/2*d*coth(b*x+a)+1/2*d+1)-1/2*ln(d+d*coth(b*x+a))*ln(1/2
*d*coth(b*x+a)+1/2*d+1))-1/d*(1/2*dilog(-1/2*(-d*coth(b*x+a)-d)/d)+1/2*ln(
-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)-1/2*dilog((-d*coth(b*x+a)-
d-2)/(-2*d-2))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d-2)/(-2*d-2)))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(60) = 120.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.28

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{b^2x^2+bx\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d+1)\cosh\,(bx+a)+2\,(d+1)\sinh\,(bx+a)+2\,\sqrt{}\right.}{}$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + b*x*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*
x + a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(
-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx=\int \operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(acoth(d*coth(a + b*x) + d + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\begin{aligned}&\int \coth^{-1}(1+d+d\coth(a+bx))\,dx\\&=\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)\\&\quad+x\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\end{aligned}$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1
)*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + d + 1)
```

Giac [F]

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int \operatorname{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(acoth(d + d*coth(a + b*x) + 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int acoth(\coth\left(bx+a\right)d+d+1)\,dx$$

input
```
int(acoth(1+d+d*coth(b*x+a)),x)
```

output
```
int(acoth(coth(a + b*x)*d + d + 1),x)
```

3.104 $\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} \, dx = \text{Int}\left(\frac{\coth^{-1}(1+d+d\coth(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1+d+d*coth(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} \, dx = \int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} \, dx$$

input
```
Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\coth(a+bx)+d+1)}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\coth^{-1}(d\coth(a+bx)+d+1)}{x}dx$$

input
```
Int[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(1+d+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

output
```
int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+d+d*coth(b*x+a))/x,x)
```

output
```
Integral(acoth(d*coth(a + b*x) + d + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx=\int \frac{\mathrm{acoth}(d+d\coth(a+b\,x)+1)}{x}\,dx$$

input
```
int(acoth(d + d*coth(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d + d*coth(a + b*x) + 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{acoth(\coth(bx+a)\,d+d+1)}{x}dx$$

input
```
int(acoth(1+d+d*coth(b*x+a))/x,x)
```

output
```
int(acoth(coth(a + b*x)*d + d + 1)/x,x)
```

3.105 $\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3 \operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{3x^2 \operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2} - \frac{3x \operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3} + \frac{3 \operatorname{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1-d-d*coth(b*x+a))-1/8*x^4*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1-d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,(1-d)*exp(2*b*x+2*a))/b^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{4b^4x^4\coth^{-1}(1-d-d\coth(a+bx)) - 2b^4x^4\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + 4b^3x^3\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 - d - d*Coth[a + b*x]] - 2*b^4*x^4*Log[1 + 1/((-1 + d
)*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x)))
] + 6*b^2*x^2*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))] + 6*b*x*PolyLog[
4, -(1/((-1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[5, -(1/((-1 + d)*E^(2*(a +
 b*x))))])/(16*b^4)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6796, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{ 6796}$$

$$\frac{1}{4}b\int \frac{x^4}{1-(1-d)e^{2a+2bx}}dx + \frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{4}b\left((1-d)\int \frac{e^{2a+2bx}x^4}{1-(1-d)e^{2a+2bx}}dx + \frac{x^5}{5}\right) + \frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\int x^3\log\left(1-(1-d)e^{2a+2bx}\right)dx}{b(1-d)}-\frac{x^4\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^5}{5}\right)+$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\int x^2\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})dx}{2b}-\frac{x^3\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{b(1-d)}-\frac{x^4\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^5}{5}\right)$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}(3,(1-d)e^{2a+2bx})}{2b}-\frac{\int x\operatorname{PolyLog}(3,(1-d)e^{2a+2bx})dx}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{b(1-d)}-\frac{x^4\log(1-(}{2b(}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}(3,(1-d)e^{2a+2bx})}{2b}-\frac{\frac{x\operatorname{PolyLog}(4,(1-d)e^{2a+2bx})}{2b}-\frac{\int\operatorname{PolyLog}(4,(1-d)e^{2a+2bx})dx}{2b}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(1-d)e^{2a-}}{2b}\right.}{b(1-d)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 2720

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}\right)}{2b}-\frac{x^3\operatorname{Poly}}{}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\operatorname{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{4b^2}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(1-d)e^{2a+2bx}}{2b}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 - d - d*Coth[a + b*x]])/4 + (b*(x^5/5 + (1 - d)*(-1/2*(x^4*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (2*(-1/2*(x^3*PolyLog[2, (
1 - d)*E^(2*a + 2*b*x)])/b + (3*((x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])
/(2*b) - ((x*PolyLog[4, (1 - d)*E^(2*a + 2*b*x)])/(2*b) - PolyLog[5, (1 -
d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b)))/(b*(1 - d)))))/4
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 1782, normalized size of antiderivative = 10.80

method	result	size
risch	Expression too large to display	1782

input
```
int(x^3*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/20*b*x^5-1/4*x^4*ln(exp(b*x+a))+1/8/b^4*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-ex
p(2*b*x+2*a)+1)-3/16/b^4/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+1/8/(d-1)*
ln(1+(d-1)*exp(2*b*x+2*a))*x^4-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/
2))*x-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^4*d*a^4/(d-1)
*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1
/2))+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^3/(d-
1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*
x+2*a))*x^3-3/8/b^4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(d-1)
*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp
(2*b*x+2*a))*a^3-3/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^
3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1-exp(
b*x+a)*(1-d)^(1/2))*x-1/2/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3+1/2
/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3-1/8/b^4*d*a^4/(d-1)*ln(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)+1)+1/8*x^4*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)+
3/16/b^4*d/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+3/8/b^4/(d-1)*ln(1+(d-1)
*exp(2*b*x+2*a))*a^4-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2+1/
4/b^4/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3+3/8/b^3/(d-1)*polylog(4,-
(d-1)*exp(2*b*x+2*a))*x-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2
/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1+exp(
b*x+a)*(1-d)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(138) = 276$.

Time = 0.10 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.73

$$\int x^3 \coth^{-1}(1 - d - d\coth(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d
 - 1)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a)
+ 2*(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sq
rt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2
*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*
sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*
sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/
2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*
log(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog
(5, 1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -
1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int x^3 \coth^{-1}(1 - d - d\coth(a + bx))\,dx = -\int x^3 \operatorname{acoth}\left(d\coth\left(a + bx\right) + d - 1\right) dx$$

input
```
integrate(x**3*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(x**3*acoth(d*coth(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\frac{1}{4}\,x^4 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right) + \frac{1}{40}\left(\frac{2\,x^5}{d} - \frac{5\left(2\,b^4x^4\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3(-(d-1)e\right.}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^4*arccoth(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*l
og((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a)
) - 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d
- 1)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b
*d
```

Giac [F]

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int x^3 \operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int -x^3\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x^3*acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x^3*acoth(d + d*coth(a + b*x) - 1), x)
```

Reduce [F]

$$\int x^3 \coth^{-1}(1 - d - d\coth(a + bx))\, dx = -\left(\int acoth(\coth(bx + a)\, d + d - 1)\, x^3 dx\right)$$

input
```
int(x^3*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
- int(acoth(coth(a + b*x)*d + d - 1)*x**3,x)
```

3.106 $\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^2\operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{x\operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1-d-d*coth(b*x+a))-1/6*x^3*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1-d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$=\frac{8b^3x^3\coth^{-1}(1-d-d\coth(a+bx))-4b^3x^3\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)+6b^2x^2\,\mathrm{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+6bx}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 - d - d*Coth[a + b*x]] - 4*b^3*x^3*Log[1 + 1/((-1 + d
)*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))
] + 6*b*x*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[4, -(1/(
(-1 + d)*E^(2*(a + b*x))))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6796, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6796}$$

$$\frac{1}{3}b\int \frac{x^3}{1-(1-d)e^{2a+2bx}}dx+\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}b\left((1-d)\int \frac{e^{2a+2bx}x^3}{1-(1-d)e^{2a+2bx}}dx+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\int x^2\log\left(1-(1-d)e^{2a+2bx}\right)dx}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^4}{4}\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^4}{4}\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{2b}\right)}{2b(1-d)}-\frac{x^3\log}{}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - d - d*Coth[a + b*x]])/3 + (b*(x^4/4 + (1 - d)*(-1/2*(x^3*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (3*(-1/2*(x^2*PolyLog[2, (
1 - d)*E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(2*b
) - PolyLog[4, (1 - d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*(1 - d)))))/3
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.01 (sec) , antiderivative size = 1723, normalized size of antiderivative = 12.58

method	result	size
risch	Expression too large to display	1723

input
```
int(x^2*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))+1/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x
+2*a))+1/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/4/b/(d-1)*polylog(2,-(d-
1)*exp(2*b*x+2*a))*x^2-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3-1/4/b^
3/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(3,-(d-1
)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3
*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+
a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/6/b^3*
a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/8/b^3*d/(d-1)*polylog(4,
-(d-1)*exp(2*b*x+2*a))-1/6*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/6*x^3*
ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)+1/2/b^2*a^2/(d-1)*x*ln(1+exp(b*x+a)*
(1-d)^(1/2))+1/2/b^2*a^2/(d-1)*x*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^
3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(
1-d)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/6/b^3*d*
a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/4/b*d/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*x^2+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/
4/b^3*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog
(3,-(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(
1/2))-1/2/b^2*d*a^2/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*d*a^2/(d-
1)*x*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a
))*a^2*x-1/12*(I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(114) = 228$.

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.78

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{b^4x^4 - 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a))\right) -}{}$$

input
```
integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(
b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1
)*sinh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*
(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d
+ 4)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sq
rt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt
(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\int x^2\,\mathrm{acoth}\,(d\coth(a+bx)+d-1)\,dx$$

input
```
integrate(x**2*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(x**2*acoth(d*coth(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\frac{1}{3}x^3\,\mathrm{arcoth}\,(d\coth(bx+a)+d-1) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left((d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left(-(d-1)e^{(2\right.\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*x^3*arccoth(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*l
og((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(
2*b*x + 2*a)))/(b^4*d))*b*d
```

Giac [F]

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int -x^2\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x^2*acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x^2*acoth(d + d*coth(a + b*x) - 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\left(\int acoth(\coth\left(bx+a\right)d+d-1)\,x^2dx\right)$$

input
```
int(x^2*acoth(1-d-d*coth(b*x+a)),x)
```

output

```
- int(acoth(coth(a + b*x)*d + d - 1)*x**2,x)
```

3.107 $\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1-d-d*coth(b*x+a))-1/4*x^2*ln(1-(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1-d)*exp(2*b*
x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\coth^{-1}(1-d-d\coth(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*
(a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLo
g[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {6796, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{ 6796}$$

$$\frac{1}{2}b\int\frac{x^2}{1-(1-d)e^{2a+2bx}}dx+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}b\left((1-d)\int\frac{e^{2a+2bx}x^2}{1-(1-d)e^{2a+2bx}}dx+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\int x\log\left(1-(1-d)e^{2a+2bx}\right)dx}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\int \text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,(1-d)e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\text{PolyLog}(3,(1-d)e^{2a+2bx})}{4b^2}-\frac{x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - d - d*Coth[a + b*x]])/2 + (b*(x^3/3 + (1 - d)*(-1/2*(x^2*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (-1/2*(x*PolyLog[2, (1 - d
)*E^(2*a + 2*b*x)])/b + PolyLog[3, (1 - d)*E^(2*a + 2*b*x)]/(4*b^2))/(b*(1
 - d)))))/2
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.79 (sec) , antiderivative size = 1640, normalized size of antiderivative = 15.05

method	result	size
risch	Expression too large to display	1640

input
```
int(x*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*x^3-1/2*x^2*ln(exp(b*x+a))-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+
2*a))*x-1/4/b^2*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(
1+(d-1)*exp(2*b*x+2*a))*a*x-1/2/b*a/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))-1
/2/b*a/(d-1)*x*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/4/b^2*d*a^2/(d-1)*ln(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)+1)-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))
-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+ex
p(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/8/
b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(
2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^2*d*a
^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*
(1-d)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a
/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b*d*a/(d-1)*x*ln(1-exp(b*x+a)*(
1-d)^(1/2))-1/2/b*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a*x+1/2/b*d*a/(d-1)*x
*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/4/b^2/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a
))*a+1/4/b^2*a^2/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/4*d/(d-1)*l
n(1+(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a
^2+1/4*x^2*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/8*(I*Pi*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-I*Pi*csgn(I*exp(2*
b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a
)+1))^2*csgn(I/(exp(2*b*x+2*a)-1))-I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(90) = 180$.

Time = 0.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.95

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{2\,b^3x^3 - 3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) -}{}$$

input
```
integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x +
 a) + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
(b*x + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1
)*sinh(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d +
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(
-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx = -\int x\,\mathrm{acoth}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(x*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(x*acoth(d*coth(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{24}\left(\frac{4\,x^3}{d} - \frac{3\left(2\,b^2x^2\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-(d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right. - \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)$$

input
```
integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d
))*b*d - 1/2*x^2*arccoth(d*coth(b*x + a) + d - 1)
```

Giac [F]

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx = \int x\,\mathrm{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx = \int -x\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x*acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x*acoth(d + d*coth(a + b*x) - 1), x)
```

Reduce [F]

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx=-\left(\int acoth(\coth\left(bx+a\right)d+d-1)\,xdx\right)$$

input
```
int(x*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
- int(acoth(coth(a + b*x)*d + d - 1)*x,x)
```

3.108 $\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^2}{2} + x\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1-d-d*coth(b*x+a))-1/2*x*ln(1-(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1-d)*exp(2*b*x+2*a))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = x\coth^{-1}(1-d-d\coth(a+bx)) + \frac{-2bx\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Coth[a + b*x]] + (-2*b*x*Log[1 + 1/((-1 + d)*E^(2*(a +
 b*x)))] + PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/(4*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6788, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{ 6788}$$

$$b\int \frac{x}{1-(1-d)e^{2a+2bx}}dx+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$b\left((1-d)\int \frac{e^{2a+2bx}x}{1-(1-d)e^{2a+2bx}}dx+\frac{x^2}{2}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$b\left((1-d)\left(\frac{\int \log\left(1-(1-d)e^{2a+2bx}\right)dx}{2b(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+ x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2715}$$

$$b\left((1-d)\left(\frac{\int e^{-2a-2bx}\log\left(1-(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+ x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2838}$$

$$b\left((1-d)\left(-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b^2(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+ x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Coth[a + b*x]] + b*(x^2/2 + (1 - d)*(-1/2*(x*Log[1 - (
1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]
/(4*b^2*(1 - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6788
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(68) = 136$.

Time = 0.75 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.57

method	result
derivativedivides	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{d\,c}{}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{d\,c}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(-1/2*arccoth(1-d-d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)+1/2*arccoth
(1-d-d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)-1/2*d^2*(1/d*(-1/2*dilog(-1/2*d
*coth(b*x+a)-1/2*d+1)-1/2*ln(-d*coth(b*x+a)-d)*ln(-1/2*d*coth(b*x+a)-1/2*d
+1)+1/4*ln(-d*coth(b*x+a)-d)^2)-1/d*(1/2*dilog(-1/2*(-d*coth(b*x+a)-d)/d)+
1/2*ln(-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)-1/2*dilog((-d*coth(
b*x+a)-d+2)/(-2*d+2))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d+2)/(-2
*d+2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.14

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{b^2x^2 - bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) + a\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)+\sqrt{-\cdots}\right.}{\cdots}$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a
) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*
sinh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + s
inh(b*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))
))/b
```

Sympy [F]

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\int \operatorname{acoth}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(acoth(d*coth(a + b*x) + d - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right) - x\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1
)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*coth(b*x + a) + d - 1)
```

Giac [F]

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int \operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int -\operatorname{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-acoth(d + d*coth(a + b*x) - 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\left(\int acoth(\coth\left(bx+a\right)d+d-1)\,dx\right)$$

input
```
int(acoth(1-d-d*coth(b*x+a)),x)
```

output
```
- int(acoth(coth(a + b*x)*d + d - 1),x)
```

3.109 $\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(1-d-d\coth(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1-d-d*coth(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\coth(a+bx))-d+1)}{x}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\coth^{-1}(d(-\coth(a+bx))-d+1)}{x}dx$$

input
```
Int[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(1-d-d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

output
```
int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = -\int \frac{\operatorname{acoth}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

input
```
integrate(acoth(1-d-d*coth(b*x+a))/x,x)
```

output
```
-Integral(acoth(d*coth(a + b*x) + d - 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
-integrate(arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*coth(b*x + a) - d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int -\frac{\operatorname{acoth}(d+d\coth(a+b\,x)-1)}{x}\,dx$$

input
```
int(-acoth(d + d*coth(a + b*x) - 1)/x,x)
```

output
```
int(-acoth(d + d*coth(a + b*x) - 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = -\left(\int \frac{acoth(\coth(bx+a)\,d+d-1)}{x}dx\right)$$

input
```
int(acoth(1-d-d*coth(b*x+a))/x,x)
```

output
```
- int(acoth(coth(a + b*x)*d + d - 1)/x,x)
```

3.110 $\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 302

$$\begin{aligned}\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = {} & \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} \\ & + \frac{i(e+fx)^4 \arctan\left(e^{2i(a+bx)}\right)}{4f} \\ & - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ & + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ & + \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ & - \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2} \\ & + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ & - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3} \\ & - \frac{3f^3 \operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{16b^4} \\ & + \frac{3f^3 \operatorname{PolyLog}\left(5,ie^{2i(a+bx)}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arccoth(tan(b*x+a))/f+1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*poly
log(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))
/b^2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*p
olylog(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b
*x+a)))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5
,I*exp(2*I*(b*x+a)))/b^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.

Time = 0.21 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\coth^{-1}(\tan(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right) - 12b^4e^2fx^2\log\left(1-ie^{2i(a+bx)}\right) - 8b^4ef^2x^3\log\left(1-ie^{2i(a+bx)}\right) - 2b^4f^3x^4}{}$$

input
```
Integrate[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Tan[a + b*x]])/4 +
(-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*
E^((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*
b^4*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I
)*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*
f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*
(a + b*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] +
 (4*I)*b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*Pol
yLog[3, (-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I
)*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2
*e^2*f*PolyLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^(
(2*I)*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I
)*b*e*f^2*PolyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4,
(-I)*E^((2*I)*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))
] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I
)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6806, 3042, 4669, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6806}$$

$$\frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f}-\frac{b\int (e+fx)^4 \sec(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f}-\frac{b\int (e+fx)^4 \csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f}-\frac{b\left(-\frac{2f\int (e+fx)^3\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{2f\int (e+fx)^3\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}}{2b}\right.}{b}\right.}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-$$

$$\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\int(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyL}\ldots}{}\right.}{}\right.}{4f}$$

$$\downarrow \text{7163}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-$$

$$\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{if\int\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2i(a\ldots}{2b}\right.}{2b}\right.}{b}\right.}{}$$

$$\downarrow \text{2720}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-$$

$$b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2}{}\right.}{2b}\right.}{b}\right.$$

$\downarrow$ 7143

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-$$

$$b\left(-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right.}{b}\right.}{2b}\right.}{b}\right.$$

input

```
Int[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) - (b*(((-I)*(e + f*x)^4*ArcTan[E
^((2*I)*(a + b*x))])/b + (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (((3*I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, (-I)*E^((
2*I)*(a + b*x))])/b + (I*f*(((-1/2*I)*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(
a + b*x))])/b + (f*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))
/b - (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (((3*
I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (I*f
*(((-1/2*I)*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[5,
 I*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b))/(4*f)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6806
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.11 (sec) , antiderivative size = 3640, normalized size of antiderivative = 12.05

method	result	size
risch	Expression too large to display	3640

input
```
int((f*x+e)^3*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
3/8*I*f^2/b^3*e*polylog(4,-I*exp(2*I*(b*x+a)))+1/2*I/b*e^3*dilog(((-I)^(1/
2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e^3*dilog(((-I)^(1/2)+exp(I*(b*x+a)
))/(-I)^(1/2))-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylo
g(5,I*exp(2*I*(b*x+a)))/b^4-3/8*I*f^3/b^3*polylog(4,I*exp(2*I*(b*x+a)))*x+
1/4*I*f^3/b*polylog(2,I*exp(2*I*(b*x+a)))*x^3-1/2*I*f^3/b^4*a^3*dilog(((-I
)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)+ex
p(I*(b*x+a)))/(-I)^(1/2))-3/8*I*f^2/b^3*e*polylog(4,I*exp(2*I*(b*x+a)))-3/
2*f^2/b^2*e*a^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-3/4*I*f^2/b^3
*e*a^2*polylog(2,I*exp(2*I*(b*x+a)))+3/2*I*f^2/b^3*e*a^2*dilog(((-I)^(1/2)
-exp(I*(b*x+a)))/(-I)^(1/2))+3/2*f^2/b^3*a^3*e*ln(1-exp(I*(b*x+a))*(-1)^(3
/4))-3/2*f/b^2*a^2*e^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-3/2*f/b^2*a^2*e^2*l
n(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*f^3/b^3*a^3*ln(1+exp(I*(b*x+a))*(-1)^(3
/4))*x-1/2*f^3/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x-1/2*f^2/b^3*a^3*e
*ln(-exp(2*I*(b*x+a))+I)+3/4*f/b^2*a^2*e^2*ln(-exp(2*I*(b*x+a))+I)+1/2*f^3
/b^3*ln(1+I*exp(2*I*(b*x+a)))*x*a^3-f^2/b^3*e*ln(1+I*exp(2*I*(b*x+a)))*a^3
+1/2*f^3/b^3*a^3*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x+1/2*f^3/b^3*
a^3*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x+3/2*f/b^2*e^2*a^2*ln(((-I
)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+3/2*f/b^2*e^2*a^2*ln(((-I)^(1/2)+exp(I
*(b*x+a)))/(-I)^(1/2))-1/2*f^3/b^3*a^3*ln(1-I*exp(2*I*(b*x+a)))*x+1/2*f^2/
b^3*e*a^3*ln(exp(2*I*(b*x+a))+I)-1/16*I*Pi*(csgn(I*(exp(2*I*(b*x+a))+I)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1808 vs. $2(236) = 472$.

Time = 0.16 (sec) , antiderivative size = 1808, normalized size of antiderivative = 5.99

$$\int (e + fx)^3 \coth^{-1}(\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(ta
n(b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a)
+ I)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b
*x + a) + I)/(tan(b*x + a)^2 + 1)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2
 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x
 + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f
^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*t
an(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^
3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x + a)^2
 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 +
3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3*x^
4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*
b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b
*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f
+ 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a)
+ I - 1)/(tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*
b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)
/(tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f...
```

Sympy [F]

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \int (e+fx)^3 \operatorname{acoth}(\tan(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*acoth(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*acoth(tan(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)^3 \operatorname{arcoth}(\tan(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a
)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e
*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x +
 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*
x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^
3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2
*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1), x)
```

Giac [F]

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)^3 \operatorname{arcoth}(\tan(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*arccoth(tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \int \mathrm{acoth}(\tan(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

input
```
int(acoth(tan(a + b*x))*(e + f*x)^3,x)
```

output
```
int(acoth(tan(a + b*x))*(e + f*x)^3, x)
```

Reduce [F]

$$\begin{aligned}\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = &\left(\int acoth(\tan(bx+a))\,dx\right)e^3 \\ &+ \left(\int acoth(\tan(bx+a))\,x^3dx\right)f^3 \\ &+ 3\left(\int acoth(\tan(bx+a))\,x^2dx\right)e\,f^2 \\ &+ 3\left(\int acoth(\tan(bx+a))\,xdx\right)e^2f\end{aligned}$$

input
```
int((f*x+e)^3*acoth(tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)),x)*e**3 + int(acoth(tan(a + b*x))*x**3,x)*f**3 + 3
*int(acoth(tan(a + b*x))*x**2,x)*e*f**2 + 3*int(acoth(tan(a + b*x))*x,x)*e
**2*f
```

3.111 $\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 234

$$\begin{aligned}\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx &= \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} \\ &+ \frac{i(e+fx)^3 \arctan\left(e^{2i(a+bx)}\right)}{3f} \\ &- \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{f(e+fx) \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2} \\ &- \frac{f(e+fx) \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2} \\ &+ \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3*arccoth(tan(b*x+a))/f+1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*poly
log(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b
^2-1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.75

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\tan(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\,\mathrm{lo}}{}$$

input
```
Integrate[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*L
og[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x
))] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 +
I*E^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4
*b^3*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLo
g[2, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*
I)*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*
PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4
, (-I)*E^((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])
/(24*b^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6806, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6806}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\int (e+fx)^3\sec(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\int (e+fx)^3\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{3f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\left(-\frac{3f\int (e+fx)^2\log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{3f\int (e+fx)^2\log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{3f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int (e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int (e+fx)\text{PolyLog}\left(2,ie\right.}{b}\right.}{2b}\right.}{3f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} - \frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{if\int \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2i\cdots}\right.}{2b}\right.}{\cdots}\right.}{3f}$$

$\Big\downarrow$ 2720

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} - \frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{3f\left(\frac{i(e+\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\Big\downarrow$ 7143

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} - \frac{b\left(-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}\cdots\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) - (b*(((-I)*(e + f*x)^3*ArcTan[E
^((2*I)*(a + b*x))])/b + (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (I*f*(((-1/2*I)*(e + f*x)*PolyLog[3, (-I)*E^((2*I)*(a +
b*x))])/b + (f*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) -
 (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (I*f*(((-
1/2*I)*(e + f*x)*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[4, I*E^
((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b)))/(3*f)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6806
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.91 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.62

method	result	size
risch	Expression too large to display	2719

input
```
int((f*x+e)^2*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*f^2*polylog(4,-I*exp(2*I*(b*x+a)))/b^3+1/2*I*f*e/b*polylog(2,I*exp(2
*I*(b*x+a)))*x+1/2*I*f*e/b^2*polylog(2,I*exp(2*I*(b*x+a)))*a-I*f/b^2*a*e*d
ilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-I*f/b^2*a*e*dilog(((-I)^(1/2)
+exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3+1
/6*f^2*ln(1+I*exp(2*I*(b*x+a)))*x^3+1/6/f*e^3*ln(-exp(2*I*(b*x+a))+I)+1/2*
e^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x+1/2*e^2*ln(1-exp(I*(b*x+a))*(-1)^(3/
4))*x+f*e/b*ln(1+I*exp(2*I*(b*x+a)))*a*x-f/b*a*e*ln(1+exp(I*(b*x+a))*(-1)^
(3/4))*x-f/b*a*e*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x+I*f/b^2*a*e*dilog(1+exp
(I*(b*x+a))*(-1)^(3/4))+I*f/b^2*a*e*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2
*I*f*e/b*polylog(2,-I*exp(2*I*(b*x+a)))*x-1/2*I*f*e/b^2*polylog(2,-I*exp(2
*I*(b*x+a)))*a+1/6*(f*x+e)^3/f*ln(exp(2*I*(b*x+a))+I)+1/4*f*e/b^2*polylog(
3,-I*exp(2*I*(b*x+a)))-1/6*f^2/b^3*a^3*ln(-exp(2*I*(b*x+a))+I)+1/2*f*e*ln(
1+I*exp(2*I*(b*x+a)))*x^2+1/2*f^2/b^3*a^3*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+
1/2*f^2/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/4*f^2/b^2*polylog(3,-I*e
xp(2*I*(b*x+a)))*x-1/3*f^2/b^3*ln(1+I*exp(2*I*(b*x+a)))*a^3-1/2/b*a*e^2*ln
(-exp(2*I*(b*x+a))+I)+1/2/b*e^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*a+1/2/b*e^
2*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*a-1/2*I/b*e^2*dilog(1+exp(I*(b*x+a))*(-1
)^(3/4))-1/2*I/b*e^2*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*f*e*ln(1-I*exp
(2*I*(b*x+a)))*x^2-1/2*f^2*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))/b^3*
a^3-1/2*f^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))/b^3*a^3+1/3*f^2/...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(180) = 360$.

Time = 0.14 (sec) , antiderivative size = 1282, normalized size of antiderivative = 5.48

$$\int (e + fx)^2 \coth^{-1}(\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x
+ a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/
(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x
+ a) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 -
2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*
f*x - I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/
(tan(b*x + a)^2 + 1) + 1) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)
*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2
+ 1) + 1) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-(-(I - 1)
*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I
*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-(-(I - 1)*tan(b*x + a)^2
- 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 4*(b^3*f^2*x^3 + 3*b
^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I +
1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a
*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*
x + a) + I - 1)/(tan(b*x + a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3
*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a
)^2 + 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3
*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1
)/(tan(b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x ...
```

Sympy [F]

$$\int (e+fx)^2\coth^{-1}(\tan(a+bx))\,dx=\int (e+fx)^2\operatorname{acoth}\left(\tan\left(a+bx\right)\right)dx$$

input
```
integrate((f*x+e)**2*acoth(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*acoth(tan(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*
x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x
)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2
) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*
cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))
/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*arccoth(tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{acoth}(\tan(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

input
```
int(acoth(tan(a + b*x))*(e + f*x)^2,x)
```

output
```
int(acoth(tan(a + b*x))*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \left(\int acoth(\tan(bx+a))\,dx\right)e^2 + \left(\int acoth(\tan(bx+a))\,x^2dx\right)f^2 + 2\left(\int acoth(\tan(bx+a))\,xdx\right)ef$$

input
```
int((f*x+e)^2*acoth(tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)),x)*e**2 + int(acoth(tan(a + b*x))*x**2,x)*f**2 + 2
*int(acoth(tan(a + b*x))*x,x)*e*f
```

3.112 $\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 162

$$\begin{aligned}\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx =& \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} \\ &+ \frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{2f} \\ &- \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{f\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ &- \frac{f\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}\end{aligned}$$

output
```
1/2*(f*x+e)^2*arccoth(tan(b*x+a))/f+1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(
2,I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*pol
ylog(3,I*exp(2*I*(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = ex\coth^{-1}(\tan(a+bx))+\frac{1}{2}fx^2\coth^{-1}(\tan(a+bx))$$
$$-\frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)\right.}{8b}$$
$$+\frac{f(4ib^2x^2\arctan(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\operatorname{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx)}{}$$

input
```
Integrate[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

output
```
e*x*ArcCoth[Tan[a + b*x]] + (f*x^2*ArcCoth[Tan[a + b*x]])/2 - (e*((-4*a +
Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x)
)]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*
I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^
2*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2,
 I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*
(a + b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a
+ b*x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6806, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 6806}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\int (e+fx)^2\sec(2a+2bx)dx}{2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\int(e+fx)^2\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \; 4669$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(-\frac{f\int(e+fx)\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{f\int(e+fx)\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{2f}$$

$$\Big\downarrow \; 3011$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\cdots}{2f}$$

$$\Big\downarrow \; 2720$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\cdots}{\cdots}\right)}{b}\cdots}{2f}$$

$$\Big\downarrow \; 7143$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\,\mathrm{Po}\cdots}{\cdots}\right)}{b}\cdots}{2f}$$

input

```
Int[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) - (b*(((-I)*(e + f*x)^2*ArcTan[E
^((2*I)*(a + b*x))])/b + (f*(((I/2)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a
+ b*x))])/b - (f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b - (f*((
(I/2)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (f*PolyLog[3, I*E^(
(2*I)*(a + b*x))])/(4*b^2)))/b))/(2*f)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6806
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.85 (sec) , antiderivative size = 1818, normalized size of antiderivative = 11.22

method	result	size
risch	Expression too large to display	1818

input
```
int((f*x+e)*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)
))/b^2-1/4*ln(exp(2*I*(b*x+a))-I)*f*x^2+1/4*f*ln(1+I*exp(2*I*(b*x+a)))*x^2
-1/4/b^2*f*ln(1-I*exp(2*I*(b*x+a)))*a^2-1/2*e/b*ln(((-I)^(1/2)-exp(I*(b*x+
a)))/(-I)^(1/2))*a-1/2*e/b*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*a+1/
2*I*e/b*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*dilog(((-I
)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*f/b^2*a^2*ln(((-I)^(1/2)-exp(I*(b*
x+a)))/(-I)^(1/2))+1/2*f/b^2*a^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2)
)+1/4/b^2*f*ln(1+I*exp(2*I*(b*x+a)))*a^2+1/2*e/b*ln(1+exp(I*(b*x+a))*(-1)^
(3/4))*a+1/2*e/b*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*a-1/2*I*e/b*dilog(1+exp(I
*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*f/b
^2*a^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*f/b^2*a^2*ln(1-exp(I*(b*x+a))*(
-1)^(3/4))-1/4*I*Pi*(csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csg
n((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))-csgn((1+I)*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))+1))^2-csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(
2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))+csgn(I/
(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))+1))+csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp
(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+csgn(I*(exp(2*I*(b*x+a))-I))*csgn
(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-csgn(I*(exp(2*I*(b*x+a)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(130) = 260$.

Time = 0.13 (sec) , antiderivative size = 834, normalized size of antiderivative = 5.15

$$\int (e + fx) \coth^{-1}(\tan(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/16*(2*(-I*b*f*x - I*b*e)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(-I*b*f*x - I*b*e)*dilog(-((I + 1
)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(
I*b*f*x + I*b*e)*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1) + 1) + 2*(I*b*f*x + I*b*e)*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*f*x^2
+ 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I + 1)*tan(b*
x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e -
a^2*f)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x +
a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*f*x^2
+ 2*b^2*e*x + 2*a*b*e - a^2*f)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a
^2*f)*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x +
 a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(
b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(b^2*f*x^
2 + 2*b^2*e*x)*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - f*polylog(3, (
I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylo...
```

Sympy [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = \int (e+fx)\,\text{acoth}\,(\tan\,(a+bx))\,dx$$

input
```
integrate((f*x+e)*acoth(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)*acoth(tan(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)\,\text{arcoth}\,(\tan\,(bx+a))\;dx$$

input
```
integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*si
n(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin
(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)
*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)\,\text{arcoth}\,(\tan\,(bx+a))\;dx$$

input
```
integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*arccoth(tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx=\int \mathrm{acoth}(\tan(a+b\,x))\ (e+f\,x)\ dx$$

input
```
int(acoth(tan(a + b*x))*(e + f*x),x)
```

output
```
int(acoth(tan(a + b*x))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx=\left(\int acoth(\tan(bx+a))\,dx\right)e+\left(\int acoth(\tan(bx+a))\,xdx\right)f$$

input
```
int((f*x+e)*acoth(tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)),x)*e + int(acoth(tan(a + b*x))*x,x)*f
```

3.113 $\int \coth^{-1}(\tan(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \coth^{-1}(\tan(a+bx))\,dx = x\coth^{-1}(\tan(a+bx)) + ix\arctan\left(e^{2i(a+bx)}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

output
```
x*arccoth(tan(b*x+a))+I*x*arctan(exp(2*I*(b*x+a)))-1/4*I*polylog(2,-I*exp(
2*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \coth^{-1}(\tan(a+bx))\,dx = x\coth^{-1}(\tan(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+}{8b}$$

input
```
Integrate[ArcCoth[Tan[a + b*x]],x]
```

output
```
x*ArcCoth[Tan[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*
x))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*
x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*
(a + b*x))]))/(8*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6802, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6802}$$

$$x\coth^{-1}(\tan(a+bx)) - b\int x\sec(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\coth^{-1}(\tan(a+bx)) - b\int x\csc\left(2a+2bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$x\coth^{-1}(\tan(a+bx)) - b\left(-\frac{\int \log\left(1-ie^{2i(a+bx)}\right)dx}{2b} + \frac{\int \log\left(1+ie^{2i(a+bx)}\right)dx}{2b} - \frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\coth^{-1}(\tan(a+bx)) - b\left(\frac{i\int e^{-2i(a+bx)}\log\left(1-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i\int e^{-2i(a+bx)}\log\left(1+ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{ix\arctan\left(e^{2i(a+bx}\right.}{b}\right.$$

$$\Big\downarrow \text{2838}$$

$$x\coth^{-1}(\tan(a+bx)) - b\left(-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b} + \frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[ArcCoth[Tan[a + b*x]],x]
```

output
```
x*ArcCoth[Tan[a + b*x]] - b*(((-I)*x*ArcTan[E^((2*I)*(a + b*x))])/b + ((I/
4)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b^2 - ((I/4)*PolyLog[2, I*E^((2*I
)*(a + b*x))])/b^2)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6802
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Tan[a + b
*x]], x] - Simp[b   Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

method	result
parts	$x \operatorname{arccoth}(\tan(bx+a)) + \frac{\frac{(bx+a)\ln\left(1+ie^{2i(bx+a)}\right)}{2} - \frac{(bx+a)\ln\left(1-ie^{2i(bx+a)}\right)}{2} - \frac{i\,\mathrm{dilog}\left(1+ie^{2i(bx+a)}\right)}{4} + \frac{i\,\mathrm{dilog}}{}}{b}$
derivativedivides	$\frac{\arctan(\tan(bx+a))\operatorname{arccoth}(\tan(bx+a)) + \frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan(bx+a)^2}\right)}{2} - \frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan}{1+\tan(}\right.}{2}}{b}$
default	$\frac{\arctan(\tan(bx+a))\operatorname{arccoth}(\tan(bx+a)) + \frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan(bx+a)^2}\right)}{2} - \frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan}{1+\tan(}\right.}{2}}{b}$
risch	Expression too large to display

input
```
int(arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(tan(b*x+a))+1/b*(1/2*(b*x+a)*ln(1+I*exp(2*I*(b*x+a)))-1/2*(b*x+a
)*ln(1-I*exp(2*I*(b*x+a)))-1/4*I*dilog(1+I*exp(2*I*(b*x+a)))+1/4*I*dilog(1
-I*exp(2*I*(b*x+a)))+1/2*a*ln(sec(2*b*x+2*a)+tan(2*b*x+2*a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(57) = 114.

Time = 0.11 (sec) , antiderivative size = 498, normalized size of antiderivative = 6.30

$$\int \coth^{-1}(\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((
I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*
a*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2
+ 1)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b
*x + a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a)
 - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2
 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I -
1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*lo
g(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)
) - 2*a*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x +
 a)^2 + 1)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(
tan(b*x + a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x +
a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
+ 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*t
an(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

Sympy [F]

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{acoth}\left(\tan\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(tan(b*x+a)),x)
```

output
```
Integral(acoth(tan(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(57) = 114.

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.30

$$\int \coth^{-1}(\tan(a+bx))\,dx$$
$$= \frac{4\left(bx+a\right)\operatorname{arcoth}\left(\tan\left(bx+a\right)\right)+\left(\arctan\left(\frac{1}{2}\tan\left(bx+a\right)+\frac{1}{2},\frac{1}{2}\tan\left(bx+a\right)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\tan\left(bx\right.\right.}{}$$

input
```
integrate(arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan
(b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2)
- I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/
2)*tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*
I + 1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Giac [F]

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{arcoth}(\tan(bx+a))\,dx$$

input
```
integrate(arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{acoth}(\tan(a+b\,x))\,dx$$

input
```
int(acoth(tan(a + b*x)),x)
```

output
```
int(acoth(tan(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int acoth(\tan(bx+a))\,dx$$

input
```
int(acoth(tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)),x)
```

3.114 $\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \text{Int}\left(\frac{\coth^{-1}(\tan(a+bx))}{e+fx},x\right)$$

output
```
Defer(Int)(arccoth(tan(b*x+a))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

input
```
Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}dx$$

input
```
Int[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(\tan\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arccoth(tan(b*x+a))/(f*x+e),x)
```

output
```
int(arccoth(tan(b*x+a))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arcoth}\left(\tan\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arccoth(tan(b*x + a))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{acoth}\left(\tan\left(a+bx\right)\right)}{e+fx}\,dx$$

input
```
integrate(acoth(tan(b*x+a))/(f*x+e),x)
```

output
```
Integral(acoth(tan(a + b*x))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arcoth}\left(\tan\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arcoth}\left(\tan\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acoth}(\tan(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(acoth(tan(a + b*x))/(e + f*x),x)
```

output
```
int(acoth(tan(a + b*x))/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{acoth(\tan{(bx+a)})}{fx+e}dx$$

input
```
int(acoth(tan(b*x+a))/(f*x+e),x)
```

output
```
int(acoth(tan(a + b*x))/(e + f*x),x)
```

3.115 $\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 395

$$\begin{aligned}\int x^2 \coth^{-1}(c+d\tan(a+bx))\,dx &= \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) \\ &\quad+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ &\quad-\frac{1}{6}x^3\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ &\quad-\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ &\quad+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b} \\ &\quad+\frac{x\operatorname{PolyLog}\left(3,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b^2} \\ &\quad-\frac{x\operatorname{PolyLog}\left(3,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b^2} \\ &\quad+\frac{i\operatorname{PolyLog}\left(4,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{8b^3} \\ &\quad-\frac{i\operatorname{PolyLog}\left(4,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*tan(b*x+a))+1/6*x^3*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/
(1-c-I*d))-1/6*x^3*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x^2*
polylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x^2*polylog(2,-
(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/4*x*polylog(3,-(1-c+I*d)*exp(2
*I*a+2*I*b*x)/(1-c-I*d))/b^2-1/4*x*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)
/(1+c+I*d))/b^2+1/8*I*polylog(4,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b
^3-1/8*I*polylog(4,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^3
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(c+d\tan(a+bx))\,dx = \frac{8b^3x^3\coth^{-1}(c+d\tan(a+bx))+4b^3x^3\log\left(1+\frac{(-1+c+id)e^{-2i(a+bx)}}{-1+c-id}\right)-4b^3x^3\log\left(1+\frac{(1+c+id)e^{-2i(a+bx)}}{1+c-id}\right)-}{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[c + d*Tan[a + b*x]] + 4*b^3*x^3*Log[1 + (-1 + c + I*d)/
((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 4*b^3*x^3*Log[1 + (1 + c + I*d)/((
1 + c - I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (1 - c - I*d
)/((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-1 - c
 - I*d)/((1 + c - I*d)*E^((2*I)*(a + b*x)))] + 6*b*x*PolyLog[3, (1 - c - I
*d)/((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 6*b*x*PolyLog[3, (-1 - c - I*d
)/((1 + c - I*d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLog[4, (1 - c - I*d)/((
-1 + c - I*d)*E^((2*I)*(a + b*x)))] + (3*I)*PolyLog[4, (-1 - c - I*d)/((1
+ c - I*d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6822, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6822}$$

$$-\frac{1}{3}b(ic+d+i)\int \frac{e^{2ia+2ibx}x^3}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{3}b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x^3}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{2b(-d+i(1-c))}\right)-\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{2(bd+i(bc+b))}\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{b}\right)}{2b(-d+i(1-c))}\right)$$

$$\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{b}\right)}{2(bd+i(bc+b))}\right)+$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow\ 7163$$

$$\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{i\int\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{2b}-\frac{ix}{}\right.}{b}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{i\int\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{2b}-\frac{ix\,\text{PolyLo}}{}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ib}}{-c-id+1}\right.}{4b^2}\right.}{}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)de^{2}}{4b^2}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyL}}{}\right.}{b}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-\right.}{}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (b*(I*(1 - c) - d)*((x^3*Log[1 + ((1
 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d))
 - (3*(((I/2)*x^2*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1
- c - I*d))])/b - (I*(((-1/2*I)*x*PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a +
 (2*I)*b*x))/(1 - c - I*d))])/b + PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a +
 (2*I)*b*x))/(1 - c - I*d))]/(4*b^2)))/b))/(2*b*(I*(1 - c) - d))))/3 - (b*
(I + I*c + d)*((x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
 + I*d)])/(2*(I*(b + b*c) + b*d)) - (3*(((I/2)*x^2*PolyLog[2, -(((1 + c -
I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b - (I*(((-1/2*I)*x*PolyLog
[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog
[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(4*b^2)))/b)
)/(2*(I*(b + b*c) + b*d))))/3
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6822
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 + c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 - c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.10 (sec) , antiderivative size = 6855, normalized size of antiderivative = 17.35

method	result	size
risch	Expression too large to display	6855

input
```
int(x^2*arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2164 vs. $2(279) = 558$.

Time = 0.15 (sec) , antiderivative size = 2164, normalized size of antiderivative = 5.48

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(8*b^3*x^3*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 6
*I*b^2*x^2*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d
 + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2
 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2
*dilog(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^
2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(2
*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c -
 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*
tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(2*((-I*(c -
 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d +
I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x
+ a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b*
x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) -
2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a^3*log(((I*(c + 1)*d - d^2)*tan(b*x +
a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c
+ 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2
 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)
/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c
^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/...
```

Sympy [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(c + d\tan\left(a + bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*tan(a + b*x)), x)
```

Maxima [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*
b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2
 - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 -
2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 -
2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*
x + 2*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*
x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2
*a) - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))
/(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*
cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x
 + 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 +
2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*
a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)
*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*si
n(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a
) + 1), x)
```

Giac [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\, dx = \int x^2 \operatorname{arcoth}\left(d\tan\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\, dx = \int x^2 \operatorname{acoth}(c + d\tan(a + b\,x))\, dx$$

input
```
int(x^2*acoth(c + d*tan(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*tan(a + b*x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\, dx = \int acoth(\tan\left(bx + a\right) d + c)\, x^2 dx$$

input
```
int(x^2*acoth(c+d*tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)*d + c)*x**2,x)
```

3.116 $\int x \coth^{-1}(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 295

$$\begin{aligned}\int x \coth^{-1}(c+d\tan(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) \\ &+ \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ &- \frac{1}{4}x^2\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ &- \frac{ix\,\text{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ &+ \frac{ix\,\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b} \\ &+ \frac{\text{PolyLog}\left(3,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{8b^2} \\ &- \frac{\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*tan(b*x+a))+1/4*x^2*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/
(1-c-I*d))-1/4*x^2*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*po
lylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*polylog(2,-(1+c
-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*polylog(3,-(1-c+I*d)*exp(2*I*a+2
*I*b*x)/(1-c-I*d))/b^2-1/8*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*
d))/b^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.88

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx$$
$$=\frac{4b^2x^2\coth^{-1}(c+d\tan(a+bx))+2b^2x^2\log\left(1+\frac{(-1+c+id)e^{-2i(a+bx)}}{-1+c-id}\right)-2b^2x^2\log\left(1+\frac{(1+c+id)e^{-2i(a+bx)}}{1+c-id}\right)-}{}$$

input
```
Integrate[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Tan[a + b*x]] + 2*b^2*x^2*Log[1 + (-1 + c + I*d)/
((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 2*b^2*x^2*Log[1 + (1 + c + I*d)/((
1 + c - I*d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (1 - c - I*d)/((
-1 + c - I*d)*E^((2*I)*(a + b*x)))] - (2*I)*b*x*PolyLog[2, (-1 - c - I*d)/
((1 + c - I*d)*E^((2*I)*(a + b*x)))] + PolyLog[3, (1 - c - I*d)/((-1 + c -
 I*d)*E^((2*I)*(a + b*x)))] - PolyLog[3, (-1 - c - I*d)/((1 + c - I*d)*E^(
(2*I)*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6822, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6822}$$

$$-\frac{1}{2}b(ic+d+i)\int \frac{e^{2ia+2ibx}x^2}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{2}b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x^2}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{2b}}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{2b}}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\mathrm{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)de}{4b^2}}{b(-d+i(1-c))}\right.$$

$$\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\mathrm{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)de^{2ia+2i}}{4b^2}}{bd+i(bc+b)}\right.$$

$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{\mathrm{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}}{b(-d+i(1-c))}\right)-$$

$$\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{\mathrm{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}}{bd+i(bc+b)}\right)+$$

$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

input

```
Int[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

output

```
(x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (b*(I*(1 - c) - d)*((x^2*Log[1 + ((1
 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d))
 - (((I/2)*x*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c -
 I*d))])/b - PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c -
 I*d))]/(4*b^2))/(b*(I*(1 - c) - d))))/2 - (b*(I + I*c + d)*((x^2*Log[1 +
((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/(2*(I*(b + b*c) +
b*d)) - (((I/2)*x*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1
+ c + I*d))])/b - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1
+ c + I*d))]/(4*b^2))/(I*(b + b*c) + b*d)))/2
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6822
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 + c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 - c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 6481, normalized size of antiderivative = 21.97

method	result	size
risch	Expression too large to display	6481

input
```
int(x*arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1688 vs. 2(209) = 418.

Time = 0.16 (sec) , antiderivative size = 1688, normalized size of antiderivative = 5.72

$$\int x \coth^{-1}(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(4*b^2*x^2*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2
*I*b*x*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (
I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d
^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2
*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c
 + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1
)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((I*(c - 1)
*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^
2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)
^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*I*b*x*dilog(2*((-I*(c - 1)*d - d^2)*tan
(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c -
I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*c + 1) + 1) - 2*a^2*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 +
 I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*
x + a)^2 + 1)) - 2*a^2*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(
c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x +
a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c -
1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2
 + 1)) + 2*a^2*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d
 + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 ...
```

Sympy [F]

$$\int x \coth^{-1}(c + d\tan(a + bx))\,dx = \int x\,\mathrm{acoth}\left(c + d\tan\left(a + bx\right)\right)dx$$

input
```
integrate(x*acoth(c+d*tan(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*tan(a + b*x)), x)
```

Maxima [F]

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{arcoth}\,(d\tan\,(bx+a)+c)\,\,dx$$

input
```
integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*si
n(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1
)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2
*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 -
 d^2 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)
*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c
^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 +
2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 -
1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 -
 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 +
(c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2
 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)
*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x
 + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log(
(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) +
(c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c
+ 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2
*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2
*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c
 + 1)
```

Giac [F]

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{arcoth}\,(d\tan\,(bx+a)+c)\,\,dx$$

input
```
integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(c+d\tan(a+bx))\,dx = \int x\,\mathrm{acoth}(c+d\tan(a+b\,x))\;dx$$

input
```
int(x*acoth(c + d*tan(a + b*x)),x)
```

output
```
int(x*acoth(c + d*tan(a + b*x)), x)
```

Reduce [F]

$$\int x \coth^{-1}(c+d\tan(a+bx))\,dx = \int acoth(\tan(bx+a)\,d+c)\,x dx$$

input
```
int(x*acoth(c+d*tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)*d + c)*x,x)
```

3.117 $\int \coth^{-1}(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 194

$$\begin{aligned}\int \coth^{-1}(c+d\tan(a+bx))\,dx = {} & x\coth^{-1}(c+d\tan(a+bx)) \\ & +\frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ & -\frac{1}{2}x\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ & -\frac{i\,\text{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ & +\frac{i\,\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b}\end{aligned}$$

output
```
x*arccoth(c+d*tan(b*x+a))+1/2*x*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d
))-1/2*x*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*polylog(2,-(1-
c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*polylog(2,-(1+c-I*d)*exp(2*I*
a+2*I*b*x)/(1+c+I*d))/b
```

Mathematica [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.88

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx = x\Bigg(\coth^{-1}(c+d\tan(a+bx))$$
$$+\frac{2a\log(1-c-d\tan(a+bx))-i\log(1-i\tan(a+bx))\log\left(\frac{-1+c+d\tan(a+bx)}{-1+c-id}\right)+i\log(1+i\tan(a+bx$$

input
```
Integrate[ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
x*(ArcCoth[c + d*Tan[a + b*x]] + (2*a*Log[1 - c - d*Tan[a + b*x]] - I*Log[
1 - I*Tan[a + b*x]]*Log[(-1 + c + d*Tan[a + b*x])/(-1 + c - I*d)] + I*Log[
1 + I*Tan[a + b*x]]*Log[(-1 + c + d*Tan[a + b*x])/(-1 + c + I*d)] - 2*a*Lo
g[1 + c + d*Tan[a + b*x]] + I*Log[1 - I*Tan[a + b*x]]*Log[(1 + c + d*Tan[a
 + b*x])/(1 + c - I*d)] - I*Log[1 + I*Tan[a + b*x]]*Log[(1 + c + d*Tan[a +
 b*x])/(1 + c + I*d)] + I*PolyLog[2, -((d*(-I + Tan[a + b*x]))/(-1 + c + I
*d))] - I*PolyLog[2, -((d*(-I + Tan[a + b*x]))/(1 + c + I*d))] - I*PolyLog
[2, -((d*(I + Tan[a + b*x]))/(-1 + c - I*d))] + I*PolyLog[2, -((d*(I + Tan
[a + b*x]))/(1 + c - I*d))])/(4*a - (2*I)*Log[1 - I*Tan[a + b*x]] + (2*I)*
Log[1 + I*Tan[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6814, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6814}$$

$$-b(ic+d+i)\int \frac{e^{2ia+2ibx}x}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+x\coth^{-1}(d\tan(a+bx)+c)$$

$\Big\downarrow$ 2620

$$b(-d+i(1-c))\left(\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\int \log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{2b(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\int \log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{2(bd+i(bc+b))}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

$\Big\downarrow$ 2715

$$b(-d+i(1-c))\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)de^{2ia+2ibx}}{4b^2(-d+i(1-c))}+\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)de^{2ia+2ibx}}{4b(bd+i(bc+b))}+\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

$\Big\downarrow$ 2838

$$b(-d+i(1-c))\left(\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{i\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{i\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b(bd+i(bc+b))}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

input

```
Int[ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Tan[a + b*x]] + b*(I*(1 - c) - d)*((x*Log[1 + ((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d)) - ((I/4)
*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(b^
2*(I*(1 - c) - d))) - b*(I + I*c + d)*((x*Log[1 + ((1 + c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 + c + I*d)])/(2*(I*(b + b*c) + b*d)) - ((I/4)*PolyLog[2
, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(b*(I*(b + b*
c) + b*d)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6814
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + (-Simp[I*b*(1 + c - I*d)   Int[x*(E^(2*I*a +
 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp
[I*b*(1 - c + I*d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d
)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)
^2, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(164) = 328.

Time = 1.60 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.87

method	result
derivativedivides	$\frac{d\arctan(\tan(bx+a))\,\mathrm{arccoth}(c+d\tan(bx+a))+d^2\left(\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\frac{\mathrm{arcta}}{}\right.}{}$
default	$\frac{d\arctan(\tan(bx+a))\,\mathrm{arccoth}(c+d\tan(bx+a))+d^2\left(\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\frac{\mathrm{arcta}}{}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(d*arctan(tan(b*x+a))*arccoth(c+d*tan(b*x+a))+d^2*(1/2*arctan(-(c+d*
tan(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)-1/2*arctan(-(c+d*t
an(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)+1/4*I*ln(d*((c+d*ta
n(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-ln((
I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d)))/d+1/4*I*(dilog((I*d-d*((c+d*ta
n(b*x+a))/d-c/d))/(I*d+c-1))-dilog((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I
*d)))/d-1/4*I*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*tan(b*x+
a))/d-c/d))/(1+c+I*d))-ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d-1
/4*I*(dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))-dilog((I*d+d*((c+d
*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1184 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 1184, normalized size of antiderivative = 6.10

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
 + a)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*
c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2
 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*log(-2*((
-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c +
1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(-2*((I*(c - 1)*d -
 d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 -
2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 +
 c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x
+ a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*ta
n(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 -
2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*
d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 +
1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I
*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) -
2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 +
 I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a*lo
g(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2
 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - I*dilog(2...
```

Sympy [F]

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{acoth}\left(c + d\tan\left(a + bx\right)\right)dx$$

input
```
integrate(acoth(c+d*tan(b*x+a)),x)
```

output
```
Integral(acoth(c + d*tan(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(136) = 272.

Time = 0.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.92

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \frac{4\left(bx + a\right)\operatorname{arcoth}\left(d\tan\left(bx + a\right) + c\right) + \left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2c+1}, \frac{(c+1)d\tan(bx+a)+c^2+2c+1}{c^2+d^2+2c+1}\right) - \arctan\left(\frac{d^2 \cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a)
+ (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c +
1)/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 +
d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c
+ 1)*d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((d^2*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d
^2 - 2*c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(
-(I*d*tan(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-
I*c + d + I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

Giac [F]

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{arcoth}\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{acoth}(c + d\tan(a + b\,x))\,dx$$

input
```
int(acoth(c + d*tan(a + b*x)),x)
```

output
```
int(acoth(c + d*tan(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \int acoth(\tan\left(bx + a\right)d + c)\,dx$$

input
```
int(acoth(c+d*tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)*d + c),x)
```

3.118 $\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(c+d\tan(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(c+d*tan(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tan(a+bx)+c)}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\coth^{-1}(d\tan(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*tan(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*tan(b*x + a) + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}\left(c+d\tan\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*tan(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*tan(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\mathrm{acoth}(c+d\tan(a+b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*tan(a + b*x))/x,x)
```

output
```
int(acoth(c + d*tan(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{acoth(\tan(bx+a)\,d+c)}{x}dx$$

input
```
int(acoth(c+d*tan(b*x+a))/x,x)
```

output
```
int(acoth(tan(a + b*x)*d + c)/x,x)
```

3.119 $\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 170

$$\begin{aligned}\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx &= \frac{1}{12} ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id + d\tan(a + bx)) \\ &\quad - \frac{1}{6}x^3 \log\left(1 + (1 - id)e^{2ia + 2ibx}\right) \\ &\quad + \frac{ix^2 \operatorname{PolyLog}\left(2, -\left((1 - id)e^{2ia + 2ibx}\right)\right)}{4b} \\ &\quad - \frac{x \operatorname{PolyLog}\left(3, -\left((1 - id)e^{2ia + 2ibx}\right)\right)}{4b^2} \\ &\quad - \frac{i \operatorname{PolyLog}\left(4, -\left((1 - id)e^{2ia + 2ibx}\right)\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*p
olylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1-I*d)*exp(2*I
*a+2*I*b*x))/b^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \frac{1}{3}x^3 \coth^{-1}(1 - id + d\tan(a + bx))$$
$$- \frac{4b^3x^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{i+d}\right) + 6ib^2x^2 \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{i+d}\right) + 6bx \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{i+d}\right) - 3i \operatorname{Poly}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6818, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\tan(a + bx) - id + 1)\,dx$$

$$\Big\downarrow \text{6818}$$

$$\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}(1 - id) + 1}dx + \frac{1}{3}x^3 \coth^{-1}(d\tan(a + bx) - id + 1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4} - (1 - id)\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}(1 - id) + 1}dx\right) + \frac{1}{3}x^3 \coth^{-1}(d\tan(a + bx) - id + 1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\int x^2\log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{2b(d+i)}\right)\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia\cdots}\right.\right.}{b}\right.}{2b(d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\left((1-id)e^{2ia+\cdots}\right.\right.}{2b}\right.}{}\right.}{2b(d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\left(\cdots\right.\right.}{4\cdots}\right.}{}\right.}{2b(d+\cdots}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\left((1-id)e^{2ia+2ib\right.\right.}}{4b^2}\right.}{}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

input
```
Int[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 + (I/3)*b*(x^4/4 - (1 - I*d)*((x
^3*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - (3*(((I/2)*
x^2*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - (I*(((-1/2*I)*x*
PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b + PolyLog[4, -((1 - I*
d)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2)))/b))/(2*b*(I + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 2273, normalized size of antiderivative = 13.37

method	result	size
risch	Expression too large to display	2273

input
```
int(x^2*arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*b*x^4-1/2/b^2*d*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x
+1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2*x-1/2/b^2*d*a^2/(I+d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2*I/b^3*d*a^2/(I+d)*dilog(1-I*
exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b^3*d*a^2/(I+d)*dilog(1+I*exp(I*(b*
x+a))*(-I*(I+d))^(1/2))-1/4*I/b^3*a^2*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b
*x+a)))+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2*I/b^2/
(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2*x-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(
I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(
-I*(I+d))^(1/2))*x+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d+I)-1/4/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x+1/3/b^3*d/(
I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2/b^3*d*a^3/(I+d)*ln(1-I*exp(I*(
b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*d*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I
+d))^(1/2))-1/4*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/8*I/b^
3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))-1/2*I/b^3*a^3/(I+d)*ln(1-I*e
xp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*
(-I*(I+d))^(1/2))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d+I)+1/3*I/b^3/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3+1/6*x^3*ln(I*ex
p(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/6*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x
+a)))*x^3-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2
/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*p...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(118) = 236.

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.02

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx$$
$$= \frac{i\,b^4x^4 + 2\,b^3x^3\log\left(\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i}\right.}{}$$

input
```
integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*b^3*x^3*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*
b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) +
 6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*l
og(1/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + 2*a^3*lo
g(1/2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 12*b*x*po
lylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqr
t(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*
e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x
 + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I
*polylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(d\tan\left(a + bx\right) - id + 1\right)dx$$

input
```
integrate(x**2*acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*tan(a + b*x) - I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(118) = 236.

Time = 0.07 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.02

$$\int x^2\coth^{-1}(1-id+d\tan(a+bx))\,dx$$
$$= \frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d+1)}{b^2} - \frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(-4i(bx+a)^3+9i(bx-\right.}{}}{}$$

input
```
integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b
*x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2
)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog
((I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*
x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^
2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog
(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x +
 2*I*a)))/b^2)/b
```

Giac [F]

$$\int x^2\coth^{-1}(1-id+d\tan(a+bx))\,dx = \int x^2\operatorname{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x^2\,\mathrm{acoth}(d\tan(a + b\,x) + 1 - d\,1\mathrm{i})\,dx$$

input
```
int(x^2*acoth(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(x^2*acoth(d*tan(a + b*x) - d*1i + 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int acoth(\tan(bx + a)\,d - di + 1)\,x^2 dx$$

input
```
int(x^2*acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)*d - d*i + 1)*x**2,x)
```

3.120 $\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right) + \frac{ix\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b} - \frac{\text{PolyLog}\left(3,-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylo
g(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6818, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \text{6818}$$

$$\frac{1}{2}ib\int\frac{x^2}{e^{2ia+2ibx}(1-id)+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\int\frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}(1-id)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\int x\log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{b(d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{i\int\text{PolyLog}(2,-((1-id)e^{2ia+2ibx})}{2b}}{b(d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,-((1-id)}{4b^2}}{b(d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{\text{PolyLog}(3,-((1-id)e^{2ia+2ibx}))}{4b^2}}{b(d+i)}\right)\right)$$
$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

input
```
Int[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 + (I/2)*b*(x^3/3 - (1 - I*d)*((x
^2*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - (((I/2)*x*P
olyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d
)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2))/(b*(I + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 2183, normalized size of antiderivative = 16.41

method	result	size
risch	Expression too large to display	2183

input
```
int(x*arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b*x^3+1/4*I/b^2*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a-1/4/b^
2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/2/b^2*d*a^2/(I
+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b^2*d*a^2/(I+d)*ln(1+I*exp
(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)
))*a^2-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/4
*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1-I
*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a)
)*(-I*(I+d))^(1/2))+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2)
)*x-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylo
g(2,I*(I+d)*exp(2*I*(b*x+a)))*a-1/8/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*
(b*x+a)))+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b
^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4*d/(I+d)*ln(1-I*(
I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2-
1/8*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))+1/2/b*d*a/(I+d)*ln(1-I
*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b*d*a/(I+d)*ln(1+I*exp(I*(b*x+a))*
(-I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a*x-1/2*I
/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a*x+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(
b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*d*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(
-I*(I+d))^(1/2))-1/2*I/b^2*d*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(
1/2))+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/8*(I*Pi*c...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(93) = 186$.

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.20

$$\int x \coth^{-1}(1 - id + d\tan(a + bx))\,dx$$
$$= \frac{2i\,b^3x^3 + 3\,b^2x^2\log\left(\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\right.}{}$$

input
```
integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x +
 I*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(1
/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) - 3*a^2*log(1/
2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 3*(b^2*x^2 -
a^2)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(
-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d -
4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/
b^2
```

Sympy [F]

$$\int x \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x\,\mathrm{acoth}\,(d\tan(a + bx) - id + 1)\,dx$$

input
```
integrate(x*acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x*acoth(d*tan(a + b*x) - I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(93) = 186.

Time = 0.05 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.86

$$\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\,\mathrm{arcoth}(d\tan(bx+a)-i\,d+1)}{b} - \frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d + 1)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I
*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*
polylog(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Giac [F]

$$\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx = \int x\,\mathrm{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

input
```
integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int x\,\mathrm{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\,dx$$

input
```
int(x*acoth(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(x*acoth(d*tan(a + b*x) - d*1i + 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int acoth(\tan(bx+a)\,d-di+1)\,xdx$$

input
```
int(x*acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)*d - d*i + 1)*x,x)
```

3.121 $\int \coth^{-1}(1-id+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{2}ibx^2 + x\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) + \frac{i\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccoth(1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 766 vs. 2(93) = 186.

Time = 3.75 (sec) , antiderivative size = 766, normalized size of antiderivative = 8.24

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = x\coth^{-1}(1-id+d\tan(a+bx)) + \frac{x\left(2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx))) - \log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))((2i+d)\cos(a+bx)+id\sin(a+bx))}{2(i+d)}\right)\log(1-\right.}{((2i+d)\cos(a+bx)+id\sin(a+bx))\left(\frac{i\log(1+i\tan(bx))\sec(bx)(d\cos(a)+i(2i+d)\sin(a))}{(2i+d)\cos(a+bx)+id\sin(a+bx)}+\text{lo}\right.}$$

input
```
Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*
x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b
*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*
((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[
1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec
[b*x]*(d*Cos[a] + I*(2*I + d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*
(I + d))] + PolyLog[2, -1/2*((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*S
in[a])*(-I + Tan[b*x]))])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*
x] + Sin[b*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[
a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]]*Sec[b*x]*(d*Cos[a] +
I*(2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1
- I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[
a + b*x] + I*d*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x]) + (Log[(Sec[b*x]*((2
 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x
]^2)/(-I + Tan[b*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I +
d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]
*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I +
d))]*(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a
 + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I
+ Tan[a + b*x])*(2 - I*d + d*Tan[a + b*x]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6810, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \text{ 6810}$$

$$ib\int \frac{x}{e^{2ia+2ibx}(1-id)+1}dx+x\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}-(1-id)\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}(1-id)+1}dx\right)+x\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\int \log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{2b(d+i)}\right)\right)+ x\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{i\int e^{-2ia-2ibx}\log\left(e^{2ia+2ibx}(1-id)+1\right)de^{2ia+2ibx}}{4b^2(d+i)}+\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2838}$$

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{i\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2(d+i)}\right)\right)+ x\coth^{-1}(d\tan(a+bx)-id+1)$$

input
```
Int[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d + d*Tan[a + b*x]] + I*b*(x^2/2 - (1 - I*d)*((x*Log[1 + (
1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - ((I/4)*PolyLog[2, -((1
- I*d)*E^((2*I)*a + (2*I)*b*x))])/(b^2*(I + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6810
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(76) = 152$.

Time = 1.10 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.30

method	result
derivativedivides	$\frac{\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}-\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(1-\frac{id}{2}\right.}{}\right.}{}\right.}{}$
default	$\frac{\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}-\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(1-\frac{id}{2}\right.}{}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*I*arccoth(1-I*d+d*tan(b*x+a))*d*ln(I*d+d*tan(b*x+a))-1/2*I*arcc
oth(1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))+1/2*d^2*(-I/d*(-1/2*dilog(
1-1/2*I*d+1/2*d*tan(b*x+a))-1/2*ln(-I*d+d*tan(b*x+a))*ln(1-1/2*I*d+1/2*d*t
an(b*x+a))+1/4*ln(-I*d+d*tan(b*x+a))^2)+I/d*(1/2*dilog(1/2*I*(-I*d+d*tan(b
*x+a))/d)+1/2*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/2*dil
og(I*(I*d+d*tan(b*x+a)-I*(2*I+2*d))/(2*I+2*d))-1/2*ln(I*d+d*tan(b*x+a))*ln
(I*(I*d+d*tan(b*x+a)-I*(2*I+2*d))/(2*I+2*d)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.33

$$\int \coth^{-1}(1 - id + d\tan(a + bx))\,dx$$
$$= \frac{i\,b^2x^2 + bx\log\left(\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}+1\right) - (bx+a)\,\mathrm{l}}{}$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + b*x*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1)
- (b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*(
d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + a*log(1/2*(2*(d + I
)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) + I*dilog(1/2*sqrt(4*I*d -
 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [F]

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx=\int \operatorname{acoth}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

input
```
integrate(acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(acoth(d*tan(a + b*x) - I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.82

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx=$$

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d+2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)-d\left(\frac{2i\left(\log(d\tan(bx+a)-i\,d+2)\log\left(-\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}+1\right)+\mathrm{Li}_2\left(\cdots\right)\right)}{d}\cdots\right)}{\cdots}$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) -
I)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-1/2*(I*d*tan(b*x + a) +
 d + 2*I)/(d + I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d + 2*I)/(d + I)))/
d - (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b
*x + a) - I)^2)/d + 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x
 + a) - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d - 2*I*(log(tan(b*x +
a) - I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/
d) - 8*(b*x + a)*arccoth(d*tan(b*x + a) - I*d + 1))/b
```

Giac [F]

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int \operatorname{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int \operatorname{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\,dx$$

input
```
int(acoth(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(acoth(d*tan(a + b*x) - d*1i + 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int acoth(\tan\left(bx+a\right)d-di+1)\,dx$$

input
```
int(acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
int(acoth(tan(a + b*x)*d - d*i + 1),x)
```

3.122 $\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\coth^{-1}(1-id+d\tan(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tan(a+bx)-id+1)}{x}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\coth^{-1}(d\tan(a+bx)-id+1)}{x}dx$$

input
```
Int[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1-id+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\coth^{-1}(1 - id + d\tan(a + bx))}{x} \, dx = \int \frac{\operatorname{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)}{x} \, dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{-1}(1 - id + d\tan(a + bx))}{x} \, dx = \int \frac{\operatorname{acoth}\left(d\tan\left(a + bx\right) - id + 1\right)}{x} \, dx$$

input
```
integrate(acoth(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
Integral(acoth(d*tan(a + b*x) - I*d + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 144, normalized size of antiderivative = 7.20

$$\int \frac{\coth^{-1}(1 - id + d\tan(a + bx))}{x} \, dx = \int \frac{\operatorname{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)}{x} \, dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-
d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x
)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) - I*d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\mathrm{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})}{x}\,dx$$

input
```
int(acoth(d*tan(a + b*x) - d*1i + 1)/x,x)
```

output
```
int(acoth(d*tan(a + b*x) - d*1i + 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1 - id + d\tan(a + bx))}{x} \, dx = \int \frac{acoth(\tan(bx + a) \, d - di + 1)}{x} dx$$

input
```
int(acoth(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
int(acoth(tan(a + b*x)*d - d*i + 1)/x,x)
```

3.123 $\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\begin{aligned}\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx)) \\ &\quad -\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) \\ &\quad +\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b} \\ &\quad -\frac{x\operatorname{PolyLog}\left(3,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b^2} \\ &\quad -\frac{i\operatorname{PolyLog}\left(4,-\left((1+id)e^{2ia+2ibx}\right)\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1+I*d-d*tan(b*x+a))-1/6*x^3*ln(1+(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*p
olylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1+I*d)*exp(2*I
*a+2*I*b*x))/b^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx)) - \frac{4b^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\operatorname{PolyLog}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)
*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a
+ b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6818, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{6818}$$

$$\frac{1}{3}ib\int\frac{x^3}{e^{2ia+2ibx}(id+1)+1}dx+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\int\frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}(id+1)+1}dx\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\int x^2\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{2b(-d+i)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia}\right.\right.}{b}\right.}{2b(-d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\left((id+1)e^{2ia+}\right.\right.}{2b}\right.}{}\right.}{2b(-d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\left(\right.\right.}{4}\right.}{}\right.}{2b(-d+}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\left((id+1)e^{2ia+2ib}\right.\right.}{4b^2}\right.}{}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

input
```
Int[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 + (I/3)*b*(x^4/4 - (1 + I*d)*((x
^3*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - (3*(((I/2)*
x^2*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b - (I*(((-1/2*I)*x*
PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b + PolyLog[4, -((1 + I*
d)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2)))/b))/(2*b*(I - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 2383, normalized size of antiderivative = 13.94

method	result	size
risch	Expression too large to display	2383

input
```
int(x^2*arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*b*x^4-1/2/b^2*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^2*x+1/2/b^
2*d*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/2/b^2*d*a^2/(-
d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/2*I/b^2*a^2/(-d+I)*ln(1-
I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/4*I/b^3*a^2*d/(-d+I)*polylog(2,I*(
-d+I)*exp(2*I*(b*x+a)))+1/2*I/b^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a
^2*x-1/2*I/b^3*d*a^2/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/
2*I/b^3*d*a^2/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b*d
/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/2*I/b^2*a^2/(-d+I)*ln(1
+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(
b*x+a))-exp(2*I*(b*x+a))*d+I)+1/2/b^3*d*a^3/(-d+I)*ln(1+I*exp(I*(b*x+a))*(
-I*(-d+I))^(1/2))+1/2/b^3*d*a^3/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(
1/2))-1/3/b^3*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^3+1/4/b^2*d/(-d+I
)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I
*(b*x+a))*(-I*(-d+I))^(1/2))-1/2*I/b^3*a^3/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-
I*(-d+I))^(1/2))+1/8*I/b^3*d/(-d+I)*polylog(4,I*(-d+I)*exp(2*I*(b*x+a)))-1
/4*I/b^2/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))*x+1/6*I/b^3*a^3/(-d+I
)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/3*I/b^3/(-d+I)*ln(1-I*(-d+
I)*exp(2*I*(b*x+a)))*a^3+1/8/b^3/(-d+I)*polylog(4,I*(-d+I)*exp(2*I*(b*x+a)
))+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)+1/6*d/(-d+I)*ln(1-I
*(-d+I)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(-d+I)*dilog(1+I*exp(I*(b*x+a...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(119) = 238.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.01

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{i\,b^4x^4-2\,b^3x^3\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)+6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)+6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)}{}$$

input
```
integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 - 2*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
+ 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) +
 6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*
log(1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + 2*a^3*
log(1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 12*b*x
*polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2
*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d
 - 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e
^(I*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)
) - 12*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x^2\,\mathrm{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(x**2*acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
Integral(x**2*acoth(-d*tan(a + b*x) + I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(119) = 238$.

Time = 0.06 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.00

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx =$$
$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(
b*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^
2)*arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilo
g((-I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(
b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a
)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polyl
og(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b
*x + 2*I*a)))/b^2)/b
```

Giac [F]

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x^2\,\mathrm{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

input
```
int(x^2*acoth(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d*1i - d*tan(a + b*x) + 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = -\left(\int acoth(\tan(bx+a)\,d-di-1)\,x^2dx\right)$$

input
```
int(x^2*acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
- int(acoth(tan(a + b*x)*d - d*i - 1)*x**2,x)
```

3.124 $\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\begin{aligned}\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx)) \\ &\quad -\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right) \\ &\quad +\frac{ix\operatorname{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b} \\ &\quad -\frac{\operatorname{PolyLog}\left(3,-\left((1+id)e^{2ia+2ibx}\right)\right)}{8b^2}\end{aligned}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1+I*d-d*tan(b*x+a))-1/4*x^2*ln(1+(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylo
g(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx$$
$$=\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))$$
$$-\frac{2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*
x)))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6818, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{6818}$$

$$\frac{1}{2}ib\int\frac{x^2}{e^{2ia+2ibx}(id+1)+1}dx+\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\int\frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}(id+1)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\int x\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{b(-d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}}{b(-d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\left((id+1)\right.\right.}{4b^2}}{b(-d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(3,-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2}}{b(-d+i)}\right)\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

input
```
Int[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 + (I/2)*b*(x^3/3 - (1 + I*d)*((x
^2*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - (((I/2)*x*P
olyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d
)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2))/(b*(I - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 2285, normalized size of antiderivative = 17.05

method	result	size
risch	Expression too large to display	2285

input
```
int(x*arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b*x^3-1/2/b^2*d*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+
1/4/b^2*a^2*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/4/b^2*d
/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/2/b^2*d*a^2/(-d+I)*ln(1+I*ex
p(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b^2*a^2/(-d+I)*ln(I*exp(2*I*(b*x+a))
-exp(2*I*(b*x+a))*d+I)-1/4*I/b^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^
2+1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2*I/b^2*
a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/4*x^2*ln(exp(2*I*(b*
x+a))*d-I*exp(2*I*(b*x+a))-I)-1/8*(I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp
(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp
(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)*d)*csgn(I*d
)+I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a
))/(exp(2*I*(b*x+a))+1)*d)^2+2*I*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*
(b*x+a)))^2+I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)*d)*csgn(exp(
2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)*d)^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(
I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1
))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^
2-I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)*d)*csgn(exp(2*I*(b*x+a
))/(exp(2*I*(b*x+a))+1)*d)-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b
*x+a)))-I*Pi*csgn(I*exp(2*I*(b*x+a)))^3+I*Pi*csgn(I*exp(2*I*(b*x+a))/(e...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(94) = 188$.

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.18

$$\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{2i\,b^3x^3 - 3\,b^2x^2\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-\right.}{}$$

input
```
integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*
x + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
 I*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(
1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) - 3*a^2*log(
1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 3*(b^2*x^2
 - a^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*
log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*
I*d - 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
I*a)))/b^2
```

Sympy [F]

$$\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x\,\mathrm{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(x*acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
Integral(x*acoth(-d*tan(a + b*x) + I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(94) = 188$.

Time = 0.05 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.84

$$\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx = -\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\right.}{}}{}$$

input
```
integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d - 1)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2
*I*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) +
 3*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 +
 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Giac [F]

$$\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 + id - d\tan(a + bx))\,dx = \int x\,\mathrm{acoth}(1 - d\tan(a + b\,x) + d\,1\mathrm{i})\,dx$$

input
```
int(x*acoth(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(x*acoth(d*1i - d*tan(a + b*x) + 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1 + id - d\tan(a + bx))\,dx = -\left(\int acoth(\tan(bx + a)\,d - di - 1)\,xdx\right)$$

input
```
int(x*acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
- int(acoth(tan(a + b*x)*d - d*i - 1)*x,x)
```

3.125 $\int \coth^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=\frac{1}{2}ibx^2+x\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{i\,\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

output

```
1/2*I*b*x^2+x*arccoth(1+I*d-d*tan(b*x+a))-1/2*x*ln(1+(1+I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 723 vs. 2(94) = 188.

Time = 1.44 (sec) , antiderivative size = 723, normalized size of antiderivative = 7.69

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=x\coth^{-1}(1+id-d\tan(a+bx))-\frac{x\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))((-2i+d)\cos(a+bx)+id\sin(a+bx))}{2(-i+d)}\right)\log(\cdots\right.}{((-2i+d)\cos(a+bx)+id\sin(a+bx))\left(\frac{i\log(1-i\tan(bx))\sec(bx)(d\cos(a)+(2+id)\sin(a))}{(-2i+d)\cos(a+bx)+id\sin(a+bx)}+\frac{\log(1+i\tan(b\cdots}{(-2\cdots}\right.}$$

input
```
Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b
*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a +
 b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*
x]*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*L
og[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (
Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2
*(-I + d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] +
Sin[b*x]))/(((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan
[b*x]]*Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] +
I*d*Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I +
 d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]
*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec
[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 +
I*d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Se
c[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(
-I + d))]*(-I + Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)
*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]
) + (2*I)*b*x*(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6810, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{ 6810}$$

$$ib\int \frac{x}{e^{2ia+2ibx}(id+1)+1}dx+x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$ib\left(\frac{x^2}{2}-(1+id)\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}(id+1)+1}dx\right)+x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \ 2620$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\int \log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \ 2715$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{i\int e^{-2ia-2ibx}\log\left(e^{2ia+2ibx}(id+1)+1\right)de^{2ia+2ibx}}{4b^2(-d+i)}+\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \ 2838$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{i\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2(-d+i)}\right)\right)+ x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

input
```
Int[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d - d*Tan[a + b*x]] + I*b*(x^2/2 - (1 + I*d)*((x*Log[1 + (
1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - ((I/4)*PolyLog[2, -((1
+ I*d)*E^((2*I)*a + (2*I)*b*x))])/(b^2*(I - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6810
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(77) = 154$.

Time = 1.09 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.40

method	result
derivativedivides	$-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(id-d\tan(bx+a))}{2}-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\frac{\ln(id-d\,\mathrm{ta}}{4}\right.}{}\right.}{}$
default	$-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(id-d\tan(bx+a))}{2}-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\frac{\ln(id-d\,\mathrm{ta}}{4}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(1/2*I*arccoth(1+I*d-d*tan(b*x+a))*d*ln(I*d-d*tan(b*x+a))-1/2*I*arc
coth(1+I*d-d*tan(b*x+a))*d*ln(-I*d-d*tan(b*x+a))-1/2*d^2*(-I/d*(1/4*ln(I*d
-d*tan(b*x+a))^2-1/2*dilog(1+1/2*I*d-1/2*d*tan(b*x+a))-1/2*ln(I*d-d*tan(b*
x+a))*ln(1+1/2*I*d-1/2*d*tan(b*x+a)))+I/d*(1/2*dilog(-1/2*I*(I*d-d*tan(b*x
+a))/d)+1/2*ln(-I*d-d*tan(b*x+a))*ln(-1/2*I*(I*d-d*tan(b*x+a))/d)-1/2*dilo
g(I*(-I*d-d*tan(b*x+a)-I*(2*I-2*d))/(2*I-2*d))-1/2*ln(-I*d-d*tan(b*x+a))*l
n(I*(-I*d-d*tan(b*x+a)-I*(2*I-2*d))/(2*I-2*d)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(66) = 132.

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.32

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{i\,b^2x^2 - bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+1\right) - (bx+a)\log\left(-\frac{1}{2}\sqrt{\right.}$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 - b*x*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a
) - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1)
- (b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*
(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + a*log(1/2*(2*(d -
 I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) + I*dilog(1/2*sqrt(-4*I
*d - 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))
/b
```

Sympy [F]

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=\int \operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
Integral(acoth(-d*tan(a + b*x) + I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(66)=132$.

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.77

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)+d\left(-\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}+1\right)+\mathrm{L}\right.}{d}\right.}{}$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) -
I)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-1/2*(I*d*tan(b*x + a)
+ d - 2*I)/(d - I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d - 2*I)/(d - I)))
/d + (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(
b*x + a) - I)^2)/d - 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b
*x + a) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d + 2*I*(log(tan(b*x +
 a) - I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))
/d) + 8*(b*x + a)*arccoth(d*tan(b*x + a) - I*d - 1))/b
```

Giac [F]

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=\int \operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=\int \operatorname{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\,dx$$

input
```
int(acoth(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(acoth(d*1i - d*tan(a + b*x) + 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=-\left(\int acoth(\tan\left(bx+a\right)d-di-1)\,dx\right)$$

input
```
int(acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
- int(acoth(tan(a + b*x)*d - d*i - 1),x)
```

3.126 $\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\text{Int}\left(\frac{\coth^{-1}(1+id-d\tan(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x}dx$$

input
```
Int[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}(1+id-d\tan(bx+a))}{x}dx$$

input
```
int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

output
```
int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+I*d-d*tan(b*x+a))/x,x)
```

output
```
Integral(acoth(-d*tan(a + b*x) + I*d + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 141, normalized size of antiderivative = 6.71

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*
a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tan(b*x + a) + I*d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\mathrm{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(acoth(d*1i - d*tan(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d*1i - d*tan(a + b*x) + 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = -\left(\int \frac{acoth(\tan(bx+a)\,d-di-1)}{x}dx\right)$$

input
```
int(acoth(1+I*d-d*tan(b*x+a))/x,x)
```

output
```
- int(acoth(tan(a + b*x)*d - d*i - 1)/x,x)
```

3.127 $\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 302

$$\begin{aligned}\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx &= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} \\ &+ \frac{i(e+fx)^4 \arctan\left(e^{2i(a+bx)}\right)}{4f} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ &- \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3f^3 \operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{16b^4} \\ &+ \frac{3f^3 \operatorname{PolyLog}\left(5,ie^{2i(a+bx)}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arccoth(cot(b*x+a))/f+1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*poly
log(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))
/b^2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*p
olylog(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b
*x+a)))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5
,I*exp(2*I*(b*x+a)))/b^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.

Time = 0.17 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\coth^{-1}(\cot(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)-12b^4e^2fx^2\log\left(1-ie^{2i(a+bx)}\right)-8b^4ef^2x^3\log\left(1-ie^{2i(a+bx)}\right)-2b^4f^3x^4}{}$$

input
```
Integrate[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Cot[a + b*x]])/4 +
(-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*
E^((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*
b^4*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I
)*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*
f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*
(a + b*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] +
 (4*I)*b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*Pol
yLog[3, (-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I
)*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2
*e^2*f*PolyLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^(
(2*I)*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I
)*b*e*f^2*PolyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4,
(-I)*E^((2*I)*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))
] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I
)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6808, 3042, 4669, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 6808}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\int (e+fx)^4 \sec(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\int (e+fx)^4 \csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(-\frac{2f\int (e+fx)^3\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{2f\int (e+fx)^3\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\right.}{2b}}{b}\right.}{4f}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\int(e+fx)\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\,\text{PolyL}\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{4f}$$

$$\Big\downarrow \; 7163$$

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{if\int\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\,\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\,\text{PolyLog}\left(3,-ie^{2i(a\cdots}\right.}{2b}\cdots\right.}{2b}\cdots\right.}{b}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 2720$$

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}-$$

$$b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2}{}\right.}{2b}\right.}{b}\right.$$

$\big\downarrow$ 7143

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}-$$

$$b\left(-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right.}{b}\right.}{2b}\right.}{b}\right.$$

input
```
Int[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) - (b*(((-I)*(e + f*x)^4*ArcTan[E
^((2*I)*(a + b*x))])/b + (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (((3*I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, (-I)*E^((
2*I)*(a + b*x))])/b + (I*f*(((-1/2*I)*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(
a + b*x))])/b + (f*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))
/b - (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (((3*
I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (I*f
*(((-1/2*I)*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[5,
 I*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b))/(4*f)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6808
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.91 (sec) , antiderivative size = 3640, normalized size of antiderivative = 12.05

method	result	size
risch	Expression too large to display	3640

input
```
int((f*x+e)^3*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
3/8*I*f^2/b^3*e*polylog(4,-I*exp(2*I*(b*x+a)))+3/2*I*f^2/b^3*e*a^2*dilog((
(-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-3/2*I*f/b^2*e^2*a*dilog(((-I)^(1/2)
-exp(I*(b*x+a)))/(-I)^(1/2))-3/2*I*f/b^2*e^2*a*dilog(((-I)^(1/2)+exp(I*(b*
x+a)))/(-I)^(1/2))-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*po
lylog(5,I*exp(2*I*(b*x+a)))/b^4-3/8*I*f^3/b^3*polylog(4,I*exp(2*I*(b*x+a))
)*x+1/4*I*f^3/b*polylog(2,I*exp(2*I*(b*x+a)))*x^3-3/8*I*f^2/b^3*e*polylog(
4,I*exp(2*I*(b*x+a)))-1/2/b*e^3*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))
*a-1/2/b*e^3*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*a-1/2*f^3/b^4*a^4*
ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*f^3/b^4*a^4*ln(1-exp(I*(b*x+a))*(-1)^(
3/4))-1/8/f*e^4*ln(exp(2*I*(b*x+a))+I)-3/4*I*f^2/b^3*e*a^2*polylog(2,I*exp
(2*I*(b*x+a)))-1/2*f^2/b^3*a^3*e*ln(-exp(2*I*(b*x+a))+I)+3/4*f/b^2*a^2*e^2
*ln(-exp(2*I*(b*x+a))+I)+1/2*f^3/b^3*ln(1+I*exp(2*I*(b*x+a)))*x*a^3-f^2/b^
3*e*ln(1+I*exp(2*I*(b*x+a)))*a^3-3/2*I*f^2/b^3*a^2*e*dilog(1-exp(I*(b*x+a)
)*(-1)^(3/4))-1/2*f^3/b^3*a^3*ln(1-I*exp(2*I*(b*x+a)))*x+1/2*f^2/b^3*e*a^3
*ln(exp(2*I*(b*x+a))+I)-1/2/b*a*e^3*ln(-exp(2*I*(b*x+a))+I)-1/2*f^2*ln(exp
(2*I*(b*x+a))-I)*x^3*e-3/4*f*ln(exp(2*I*(b*x+a))-I)*x^2*e^2-3/8*f^3/b^2*po
lylog(3,I*exp(2*I*(b*x+a)))*x^2-3/8*f/b^2*e^2*polylog(3,I*exp(2*I*(b*x+a))
)+3/8*f/b^2*e^2*polylog(3,-I*exp(2*I*(b*x+a)))-3/2*f^2/b^3*e*a^3*ln(((-I)^
(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-3/2*f^2/b^3*e*a^3*ln(((-I)^(1/2)+exp(I*(
b*x+a)))/(-I)^(1/2))+1/2*f^3/b^4*a^4*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1566 vs. 2(236) = 472.

Time = 0.19 (sec) , antiderivative size = 1566, normalized size of antiderivative = 5.19

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output

```
-1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*pol
ylog(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2
*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - s
in(2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x
 - I*b^3*e^3)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(-I*b^3*f^3
*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(I*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a)) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^
3*e^2*f*x + I*b^3*e^3)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(
I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-I*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 +
6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) +
1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2
*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(c
os(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x
^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3
*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^
4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3
- 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 ...
```

Sympy [F]

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int (e+fx)^3 \operatorname{acoth}(\cot(a+bx))\,dx$$

input

```
integrate((f*x+e)**3*acoth(cot(b*x+a)),x)
```

output

```
Integral((e + f*x)**3*acoth(cot(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^3 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a
)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e
*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x +
 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*
x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^
3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2
*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1), x)
```

Giac [F]

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^3 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*arccoth(cot(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

input
```
int(acoth(cot(a + b*x))*(e + f*x)^3,x)
```

output
```
int(acoth(cot(a + b*x))*(e + f*x)^3, x)
```

Reduce [F]

$$\begin{aligned}\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx &= \left(\int acoth(\cot(bx+a))\,dx\right)e^3 \\ &+ \left(\int acoth(\cot(bx+a))\,x^3dx\right)f^3 \\ &+ 3\left(\int acoth(\cot(bx+a))\,x^2dx\right)e\,f^2 \\ &+ 3\left(\int acoth(\cot(bx+a))\,xdx\right)e^2f\end{aligned}$$

input
```
int((f*x+e)^3*acoth(cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)),x)*e**3 + int(acoth(cot(a + b*x))*x**3,x)*f**3 + 3
*int(acoth(cot(a + b*x))*x**2,x)*e*f**2 + 3*int(acoth(cot(a + b*x))*x,x)*e
**2*f
```

3.128 $\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 234

$$\begin{aligned}\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx &= \frac{(e+fx)^3 \coth^{-1}(\cot(a+bx))}{3f} \\ &+ \frac{i(e+fx)^3 \arctan\left(e^{2i(a+bx)}\right)}{3f} \\ &- \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{f(e+fx) \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2} \\ &- \frac{f(e+fx) \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2} \\ &+ \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3*arccoth(cot(b*x+a))/f+1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*poly
log(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b
^2-1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.75

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\cot(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\,\mathrm{lo}}{}$$

input
```
Integrate[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*L
og[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x
))] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 +
I*E^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4
*b^3*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLo
g[2, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*
I)*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*
PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4
, (-I)*E^((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])
/(24*b^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6808, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{6808}$$

$$\frac{(e+fx)^3 \coth^{-1}(\cot(a+bx))}{3f}-\frac{b\int (e+fx)^3 \sec(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3 \coth^{-1}(\cot(a+bx))}{3f}-\frac{b\int (e+fx)^3 \csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{3f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^3 \coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(-\frac{3f\int (e+fx)^2 \log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{3f\int (e+fx)^2 \log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)^3 \arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{3f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^3 \coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int (e+fx)\operatorname{PolyLog}\left(2,ie\right.}{b}\right.}{2b}\right.}{3f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{if\int\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2i\ldots}\right.}{2b}\right.}{}\right.}{3f}$$

$\downarrow$ 2720

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+\ldots}{}\right.}{}\right.}{}$$

$\downarrow$ 7143

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) - (b*(((-I)*(e + f*x)^3*ArcTan[E
^((2*I)*(a + b*x))])/b + (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (I*f*(((-1/2*I)*(e + f*x)*PolyLog[3, (-I)*E^((2*I)*(a +
b*x))])/b + (f*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) -
 (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (I*f*(((-
1/2*I)*(e + f*x)*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[4, I*E^
((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b)))/(3*f)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6808
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.80 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.62

method	result	size
risch	Expression too large to display	2719

input
```
int((f*x+e)^2*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*f^2*polylog(4,-I*exp(2*I*(b*x+a)))/b^3+1/2*I*f*e/b*polylog(2,I*exp(2
*I*(b*x+a)))*x+1/2*I*f*e/b^2*polylog(2,I*exp(2*I*(b*x+a)))*a-1/8*I*f^2*pol
ylog(4,I*exp(2*I*(b*x+a)))/b^3-1/2*f^2/b^2*a^2*ln(((-I)^(1/2)-exp(I*(b*x+a
)))/(-I)^(1/2))*x-1/2*f^2/b^2*a^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2
))*x+f*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))/b^2*a^2*e+f*ln(((-I)^(1/
2)+exp(I*(b*x+a)))/(-I)^(1/2))/b^2*a^2*e+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/
2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)+exp(I*(
b*x+a)))/(-I)^(1/2))-1/2*I*f^2/b^3*a^2*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+
1/6*f^2*ln(1+I*exp(2*I*(b*x+a)))*x^3+1/6/f*e^3*ln(-exp(2*I*(b*x+a))+I)+f*e
/b*ln(1+I*exp(2*I*(b*x+a)))*a*x-1/2*I*f*e/b*polylog(2,-I*exp(2*I*(b*x+a)))
*x-1/2*I*f*e/b^2*polylog(2,-I*exp(2*I*(b*x+a)))*a+I*f/b^2*a*e*dilog(1+exp(
I*(b*x+a))*(-1)^(3/4))+I*f/b^2*a*e*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-f/b*
a*e*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x-f/b*a*e*ln(1-exp(I*(b*x+a))*(-1)^(3/
4))*x+1/6*(f*x+e)^3/f*ln(exp(2*I*(b*x+a))+I)+1/2*f^2/b^3*a^3*ln(1-exp(I*(b
*x+a))*(-1)^(3/4))+1/2*f^2/b^3*a^3*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2/b*e
^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*a+1/2/b*e^2*ln(1-exp(I*(b*x+a))*(-1)^(3
/4))*a-1/2*I/b*e^2*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e^2*dilog(1-
exp(I*(b*x+a))*(-1)^(3/4))+1/4*f*e/b^2*polylog(3,-I*exp(2*I*(b*x+a)))-1/6*
f^2/b^3*a^3*ln(-exp(2*I*(b*x+a))+I)+1/2*f*e*ln(1+I*exp(2*I*(b*x+a)))*x^2+1
/4*f^2/b^2*polylog(3,-I*exp(2*I*(b*x+a)))*x-1/3*f^2/b^3*ln(1+I*exp(2*I*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(180) = 360$.

Time = 0.17 (sec) , antiderivative size = 1084, normalized size of antiderivative = 4.63

$$\int (e + fx)^2 \coth^{-1}(\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2
*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*
dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2
*e*f*x - I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(I*b^
2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a)) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-I*cos(2*
b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^
2*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - sin
(2*b*x + 2*a) + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(cos(2*b*
x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^
2)*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 4*(b^3*f^2*x^3 + 3*b^3
*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*
b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^
3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*x + 2*a) - si
n(2*b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b
^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
 + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2
*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 4*(...
```

Sympy [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int (e+fx)^2 \operatorname{acoth}(\cot(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*acoth(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*acoth(cot(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*
x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x
)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2
) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*
cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))
/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*arccoth(cot(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

input
```
int(acoth(cot(a + b*x))*(e + f*x)^2,x)
```

output
```
int(acoth(cot(a + b*x))*(e + f*x)^2, x)
```

Reduce [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \left(\int acoth(\cot(bx+a))\,dx\right)e^2 + \left(\int acoth(\cot(bx+a))\,x^2dx\right)f^2 + 2\left(\int acoth(\cot(bx+a))\,xdx\right)ef$$

input
```
int((f*x+e)^2*acoth(cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)),x)*e**2 + int(acoth(cot(a + b*x))*x**2,x)*f**2 + 2
*int(acoth(cot(a + b*x))*x,x)*e*f
```

3.129 $\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 162

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} + \frac{f\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} - \frac{f\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}$$

output
```
1/2*(f*x+e)^2*arccoth(cot(b*x+a))/f+1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(
2,I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*pol
ylog(3,I*exp(2*I*(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = ex\coth^{-1}(\cot(a+bx)) + \frac{1}{2}fx^2\coth^{-1}(\cot(a+bx))$$
$$-\frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)\right.}{8b}$$
$$+\frac{f(4ib^2x^2\arctan(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\,\mathrm{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx)}{}$$

input
```
Integrate[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

output
```
e*x*ArcCoth[Cot[a + b*x]] + (f*x^2*ArcCoth[Cot[a + b*x]])/2 - (e*((-4*a +
Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x)
)]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*
I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^
2*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2,
 I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*
(a + b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a
+ b*x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6808, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \; 6808$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\int (e+fx)^2\sec(2a+2bx)dx}{2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\int(e+fx)^2\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(-\frac{f\int(e+fx)\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{f\int(e+fx)\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{2f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\right.}{2f}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\ldots}{}\right.}{b}\right.}{2f}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\operatorname{Po}\ldots}{}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcCoth[Cot[a + b*x]])/(2*f) - (b*(((-I)*(e + f*x)^2*ArcTan[E
^((2*I)*(a + b*x))])/b + (f*(((I/2)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a
+ b*x))])/b - (f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b - (f*((
(I/2)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (f*PolyLog[3, I*E^(
(2*I)*(a + b*x))])/(4*b^2)))/b))/(2*f)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6808
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 1818, normalized size of antiderivative = 11.22

method	result	size
risch	Expression too large to display	1818

input
```
int((f*x+e)*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)
))/b^2-1/4*ln(exp(2*I*(b*x+a))-I)*f*x^2+1/4*f*ln(1+I*exp(2*I*(b*x+a)))*x^2
-1/4/b^2*f*ln(1-I*exp(2*I*(b*x+a)))*a^2-1/2*e/b*ln(((-I)^(1/2)-exp(I*(b*x+
a)))/(-I)^(1/2))*a-1/2*e/b*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*a+1/
2*I*e/b*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*dilog(((-I
)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*f/b^2*a^2*ln(((-I)^(1/2)-exp(I*(b*
x+a)))/(-I)^(1/2))+1/2*f/b^2*a^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2)
)+1/2*f/b*a*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x+1/2*f/b*a*ln(((-I
)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2*I*f/b^2*a*dilog(((-I)^(1/2)-exp(
I*(b*x+a)))/(-I)^(1/2))-1/2*I*f/b^2*a*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-
I)^(1/2))-1/2*e*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2*e*ln(((-I
)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x+1/4/b^2*f*ln(1+I*exp(2*I*(b*x+a)))*a
^2+1/2*e/b*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*a+1/2*e/b*ln(1-exp(I*(b*x+a))*(
-1)^(3/4))*a-1/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(
1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*f/b^2*a^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))
-1/2*f/b^2*a^2*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/4/b^2*f*a^2*ln(exp(2*I*(b
*x+a))+I)+1/2/b*e*a*ln(exp(2*I*(b*x+a))+I)-1/2*ln(exp(2*I*(b*x+a))-I)*e*x-
1/2*f/b*a*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x-1/2*f/b*a*ln(1-exp(I*(b*x+a))*
(-1)^(3/4))*x+1/2*I*f/b^2*a*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I*f/b^2
*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/4*f*ln(1-I*exp(2*I*(b*x+a)))*x^...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 680, normalized size of antiderivative = 4.20

$$\int (e + fx) \coth^{-1}(\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/16*(2*(-I*b*f*x - I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
 2*(-I*b*f*x - I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 2*(I*
b*f*x + I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 2*(I*b*f*x
+ I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^2*f*x^2 + 2*
b^2*e*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) -
 sin(2*b*x + 2*a) + 1)) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*sin
(2*b*x + 2*a) + I) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a) + I) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^
2*f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^2
*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) -
2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a) + 1) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a) + I) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a) + I) - f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + f*polylo
g(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x +
2*a) + sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x +
2*a)))/b^2
```

Sympy [F]

$$\int (e + fx) \coth^{-1}(\cot(a + bx))\, dx = \int (e + fx) \operatorname{acoth}(\cot(a + bx))\, dx$$

input
```
integrate((f*x+e)*acoth(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)*acoth(cot(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx=\int (fx+e)\,\mathrm{arcoth}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*si
n(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin
(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)
*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx=\int (fx+e)\,\mathrm{arcoth}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*arccoth(cot(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx=\int \mathrm{acoth}(\cot(a+b\,x))\,\left(e+f\,x\right)\,dx$$

input
```
int(acoth(cot(a + b*x))*(e + f*x),x)
```

output
```
int(acoth(cot(a + b*x))*(e + f*x), x)
```

Reduce [F]

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx=\left(\int acoth(\cot(bx+a))\,dx\right)e+\left(\int acoth(\cot(bx+a))\,xdx\right)f$$

input
```
int((f*x+e)*acoth(cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)),x)*e + int(acoth(cot(a + b*x))*x,x)*f
```

3.130 $\int \coth^{-1}(\cot(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \coth^{-1}(\cot(a+bx))\,dx = x\coth^{-1}(\cot(a+bx)) + ix\arctan\left(e^{2i(a+bx)}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

output
```
x*arccoth(cot(b*x+a))+I*x*arctan(exp(2*I*(b*x+a)))-1/4*I*polylog(2,-I*exp(
2*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \coth^{-1}(\cot(a+bx))\,dx = x\coth^{-1}(\cot(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+}{8b}$$

input
```
Integrate[ArcCoth[Cot[a + b*x]],x]
```

output
```
x*ArcCoth[Cot[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*
x))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*
x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*
(a + b*x))]))/(8*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6804, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{6804}$$

$$x\coth^{-1}(\cot(a+bx))-b\int x\sec(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\coth^{-1}(\cot(a+bx))-b\int x\csc\left(2a+2bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$x\coth^{-1}(\cot(a+bx))-b\left(-\frac{\int \log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{\int \log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\coth^{-1}(\cot(a+bx))-b\left(\frac{i\int e^{-2i(a+bx)}\log\left(1-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i\int e^{-2i(a+bx)}\log\left(1+ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\coth^{-1}(\cot(a+bx))-b\left(-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[ArcCoth[Cot[a + b*x]],x]
```

output
```
x*ArcCoth[Cot[a + b*x]] - b*(((-I)*x*ArcTan[E^((2*I)*(a + b*x))])/b + ((I/
4)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b^2 - ((I/4)*PolyLog[2, I*E^((2*I
)*(a + b*x))])/b^2)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6804
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Cot[a + b
*x]], x] - Simp[b   Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

method	result
parts	$x \operatorname{arccoth}(\cot(bx+a)) + \frac{\frac{(bx+a)\ln\left(1+i e^{2i(bx+a)}\right)}{2} - \frac{(bx+a)\ln\left(1-i e^{2i(bx+a)}\right)}{2} - \frac{i \operatorname{dilog}\left(1+i e^{2i(bx+a)}\right)}{4} + \frac{i \operatorname{dilog}}{}}{b}$
derivativedivides	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(\cot(bx+a)) - \frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\ln\left(1+\frac{i(1+i\cot(bx+a))^2}{\cot(bx+a)^2+1}\right)}{2} + \frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a\right.}{}}{b}$
default	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(\cot(bx+a)) - \frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\ln\left(1+\frac{i(1+i\cot(bx+a))^2}{\cot(bx+a)^2+1}\right)}{2} + \frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a\right.}{}}{b}$
risch	Expression too large to display

input
```
int(arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(cot(b*x+a))+1/b*(1/2*(b*x+a)*ln(1+I*exp(2*I*(b*x+a)))-1/2*(b*x+a
)*ln(1-I*exp(2*I*(b*x+a)))-1/4*I*dilog(1+I*exp(2*I*(b*x+a)))+1/4*I*dilog(1
-I*exp(2*I*(b*x+a)))+1/2*a*ln(sec(2*b*x+2*a)+tan(2*b*x+2*a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(57) = 114.

Time = 0.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 4.91

$$\int \coth^{-1}(\cot(a+bx))\,dx$$
$$= \frac{4\,bx\log\left(\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right) + 2\,a\log\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+i\right) - 2\,a\log\left(\cos\left(2\,bx\right.\right.}{}$$

input
```
integrate(arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
 - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)
+ I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*lo
g(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x
 + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*
a) + 1) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-c
os(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilo
g(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) -
sin(2*b*x + 2*a)))/b
```

Sympy [F]

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+bx))\,dx$$

input
```
integrate(acoth(cot(b*x+a)),x)
```

output
```
Integral(acoth(cot(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(57) = 114.

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.33

$$\int \coth^{-1}(\cot(a+bx))\,dx$$
$$= \frac{4(bx+a)\operatorname{arcoth}\left(\frac{1}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{1}{2}\tan(bx+a)+\frac{1}{2},\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\tan(bx+a\right.\right.}{}$$

input
```
integrate(arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2
, 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x +
 a) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + t
an(b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2
) - I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I -
1/2)*tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/
2*I + 1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Giac [F]

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate(arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(cot(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+b\,x))\,dx$$

input
```
int(acoth(cot(a + b*x)),x)
```

output
```
int(acoth(cot(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int acoth(\cot(bx+a))\,dx$$

input
```
int(acoth(cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)),x)
```

3.131 $\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx = \text{Int}\left(\frac{\coth^{-1}(\cot(a+bx))}{e+fx}, x\right)$$

output
```
Defer(Int)(arccoth(cot(b*x+a))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx = \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

input
```
Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}dx$$

input
```
Int[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(\cot\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arccoth(cot(b*x+a))/(f*x+e),x)
```

output
```
int(arccoth(cot(b*x+a))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e}\,dx$$

input
```
integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arccoth(cot(b*x + a))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acoth}(\cot(a+bx))}{e+fx}\,dx$$

input
```
integrate(acoth(cot(b*x+a))/(f*x+e),x)
```

output
```
Integral(acoth(cot(a + b*x))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e}\,dx$$

input
```
integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arcoth}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{acoth}(\cot(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(acoth(cot(a + b*x))/(e + f*x),x)
```

output
```
int(acoth(cot(a + b*x))/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{acoth(\cot(bx+a))}{fx+e}dx$$

input
```
int(acoth(cot(b*x+a))/(f*x+e),x)
```

output
```
int(acoth(cot(a + b*x))/(e + f*x),x)
```

3.132 $\int x^2 \coth^{-1}(c + d \cot(a + bx)) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 391

$$\begin{aligned}\int x^2 \coth^{-1}(c + d\cot(a + bx)) \, dx = {} & \frac{1}{3}x^3 \coth^{-1}(c + d\cot(a + bx)) \\ & + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\ & - \frac{1}{6}x^3 \log\left(1 - \frac{(1 + c + id)e^{2ia+2ibx}}{1 + c - id}\right) \\ & - \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ & + \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b} \\ & + \frac{x \operatorname{PolyLog}\left(3, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b^2} \\ & - \frac{x \operatorname{PolyLog}\left(3, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b^2} \\ & + \frac{i \operatorname{PolyLog}\left(4, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{8b^3} \\ & - \frac{i \operatorname{PolyLog}\left(4, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*cot(b*x+a))+1/6*x^3*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/
(1-c+I*d))-1/6*x^3*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x^2*
polylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x^2*polylog(2,(1
+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/4*x*polylog(3,(1-c-I*d)*exp(2*I*
a+2*I*b*x)/(1-c+I*d))/b^2-1/4*x*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+
c-I*d))/b^2+1/8*I*polylog(4,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^3-1/
8*I*polylog(4,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^3
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(c+d\cot(a+bx)) + 4b^3x^3\log\left(1+\frac{(1-c+id)e^{-2i(a+bx)}}{-1+c+id}\right) - 4b^3x^3\log\left(1+\frac{(-1-c+id)e^{-2i(a+bx)}}{1+c+id}\right) +}{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[c + d*Cot[a + b*x]] + 4*b^3*x^3*Log[1 + (1 - c + I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 4*b^3*x^3*Log[1 + (-1 - c + I*d)/((
1 + c + I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-1 + c - I*
d)/((-1 + c + I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (1 + c
 - I*d)/((1 + c + I*d)*E^((2*I)*(a + b*x)))] + 6*b*x*PolyLog[3, (-1 + c -
I*d)/((-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 6*b*x*PolyLog[3, (1 + c - I*d
)/((1 + c + I*d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLog[4, (-1 + c - I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] + (3*I)*PolyLog[4, (1 + c - I*d)/((1
+ c + I*d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6824, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \ 6824$$

$$-\frac{1}{3}b(-ic+d+i)\int \frac{e^{2ia+2ibx}x^3}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{3}b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x^3}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \ 2620$$

$$-\frac{1}{3}b(-ic+d+i)\left(\frac{3\int x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{3}b(-d+i(c+1))\left(\frac{3\int x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \ 3011$$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right)}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\text{lo}}{}\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(\right.}{2(}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ib}}{-c+id+1}\right.}{2b}\right.}{b}\right.}{2b(d+i(1-c))}\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right.}{b}\right.}{2(-bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow\ 7143$$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right)}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\left(1\right.}{2}\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\right.}{2(-bd}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Cot[a + b*x]])/3 - (b*(I - I*c + d)*(-1/2*(x^3*Log[1 -
((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)
) + (3*(((I/2)*x^2*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 -
 c + I*d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1 - c - I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c + I*d)])/b + PolyLog[4, ((1 - c - I*d)*E^((2*I)*a + (2*I)*
b*x))/(1 - c + I*d)]/(4*b^2)))/b))/(2*b*(I*(1 - c) + d))))/3 + (b*(I*(1 +
c) - d)*(-1/2*(x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
- I*d)])/(I*(b + b*c) - b*d) + (3*(((I/2)*x^2*PolyLog[2, ((1 + c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1
+ c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[4, ((1 + c
 + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(4*b^2)))/b))/(2*(I*(b + b
*c) - b*d))))/3
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6824
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 - c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 + c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.32 (sec) , antiderivative size = 6661, normalized size of antiderivative = 17.04

method	result	size
risch	Expression too large to display	6661

input
```
int(x^2*arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(275) = 550$.

Time = 0.27 (sec) , antiderivative size = 1798, normalized size of antiderivative = 4.60

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(8*b^3*x^3*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c
^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^
2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^
2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c +
I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*di
log(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c
^2 + d^2 - 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c -
 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2
*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*l
og(1/2*c^2 + I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^
3*log(1/2*c^2 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*
x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4
*a^3*log(-1/2*c^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(
2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2)
 - 4*a^3*log(-1/2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c...
```

Sympy [F]

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx = \int x^2 \operatorname{acoth}(c+d\cot(a+bx))\,dx$$

input
```
integrate(x**2*acoth(c+d*cot(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*cot(a + b*x)), x)
```

Maxima [F]

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx = \int x^2 \operatorname{arcoth}(d\cot(bx+a)+c)\,dx$$

input
```
integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*
b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2
 - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 -
2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 -
2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*
x + 2*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x +
 2*a)^2 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*c
os(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*c
os(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x
+ 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2
*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a
) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*
d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin
(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a)
 + 1), x)
```

Giac [F]

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx=\int x^2 \operatorname{arcoth}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx=\int x^2 \operatorname{acoth}(c+d\cot(a+b\,x))\,dx$$

input
```
int(x^2*acoth(c + d*cot(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*cot(a + b*x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx=\int acoth(\cot\left(bx+a\right)d+c)\,x^2dx$$

input
```
int(x^2*acoth(c+d*cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)*d + c)*x**2,x)
```

3.133 $\int x \coth^{-1}(c + d\cot(a + bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 293

$$\begin{aligned}\int x \coth^{-1}(c + d\cot(a + bx))\,dx = {} & \frac{1}{2}x^2 \coth^{-1}(c + d\cot(a + bx)) \\ & + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\ & - \frac{1}{4}x^2 \log\left(1 - \frac{(1 + c + id)e^{2ia+2ibx}}{1 + c - id}\right) \\ & - \frac{ix \operatorname{PolyLog}\left(2, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ & + \frac{ix \operatorname{PolyLog}\left(2, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b} \\ & + \frac{\operatorname{PolyLog}\left(3, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{8b^2} \\ & - \frac{\operatorname{PolyLog}\left(3, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*cot(b*x+a))+1/4*x^2*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/
(1-c+I*d))-1/4*x^2*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*po
lylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*polylog(2,(1+c+I
*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*polylog(3,(1-c-I*d)*exp(2*I*a+2*I*
b*x)/(1-c+I*d))/b^2-1/8*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/
b^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.87

$$\int x \coth^{-1}(c + d\cot(a + bx))\,dx$$
$$= \frac{4b^2x^2\coth^{-1}(c + d\cot(a + bx)) + 2b^2x^2\log\left(1 + \frac{(1-c+id)e^{-2i(a+bx)}}{-1+c+id}\right) - 2b^2x^2\log\left(1 + \frac{(-1-c+id)e^{-2i(a+bx)}}{1+c+id}\right) +}{}$$

input
```
Integrate[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Cot[a + b*x]] + 2*b^2*x^2*Log[1 + (1 - c + I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 2*b^2*x^2*Log[1 + (-1 - c + I*d)/((
1 + c + I*d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-1 + c - I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - (2*I)*b*x*PolyLog[2, (1 + c - I*d)/
((1 + c + I*d)*E^((2*I)*(a + b*x)))] + PolyLog[3, (-1 + c - I*d)/((-1 + c
+ I*d)*E^((2*I)*(a + b*x)))] - PolyLog[3, (1 + c - I*d)/((1 + c + I*d)*E^(
(2*I)*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6824, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \; 6824$$

$$-\frac{1}{2}b(-ic+d+i)\int \frac{e^{2ia+2ibx}x^2}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{2}b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x^2}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\int x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\int x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 3011$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2b}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2720$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)$$

$$\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b^2}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)$$

$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 7143$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+$$

$$\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+$$

$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

input

```
Int[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

output

```
(x^2*ArcCoth[c + d*Cot[a + b*x]])/2 - (b*(I - I*c + d)*(-1/2*(x^2*Log[1 -
((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)
) + (((I/2)*x*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c +
I*d)])/b - PolyLog[3, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d
)]/(4*b^2))/(b*(I*(1 - c) + d))))/2 + (b*(I*(1 + c) - d)*(-1/2*(x^2*Log[1
- ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(I*(b + b*c) - b
*d) + (((I/2)*x*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
- I*d)])/b - PolyLog[3, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I
*d)]/(4*b^2))/(I*(b + b*c) - b*d)))/2
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6824
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 - c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 + c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.74 (sec) , antiderivative size = 6311, normalized size of antiderivative = 21.54

method	result	size
risch	Expression too large to display	6311

input
```
int(x*arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. 2(207) = 414.

Time = 0.24 (sec) , antiderivative size = 1462, normalized size of antiderivative = 4.99

$$\int x \coth^{-1}(c + d\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(4*b^2*x^2*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 +
 d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 +
2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2
*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 +
d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2
*c + 1) + 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*
b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(
c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c
^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 +
I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(
I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^
2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/
2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)
 + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2...
```

Sympy [F]

$$\int x \coth^{-1}(c + d\cot(a + bx))\,dx = \int x \operatorname{acoth}\left(c + d\cot\left(a + bx\right)\right)dx$$

input
```
integrate(x*acoth(c+d*cot(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*cot(a + b*x)), x)
```

Maxima [F]

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{arcoth}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)
*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*
cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 -
d^2 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*
d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2
*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1
)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 -
1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (
c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2
+ 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*
d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x
+ 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((
c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (
c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c +
 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*
b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*
b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c
+ 1)
```

Giac [F]

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{arcoth}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{acoth}(c+d\cot(a+b\,x))\;dx$$

input
```
int(x*acoth(c + d*cot(a + b*x)),x)
```

output
```
int(x*acoth(c + d*cot(a + b*x)), x)
```

Reduce [F]

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\int acoth(\cot(bx+a)\,d+c)\,xdx$$

input
```
int(x*acoth(c+d*cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)*d + c)*x,x)
```

3.134 $\int \coth^{-1}(c+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 194

$$\begin{aligned}\int \coth^{-1}(c+d\cot(a+bx))\,dx &= x\coth^{-1}(c+d\cot(a+bx)) \\ &\quad+\frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \\ &\quad-\frac{1}{2}x\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right) \\ &\quad-\frac{i\,\text{PolyLog}\left(2,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ &\quad+\frac{i\,\text{PolyLog}\left(2,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b}\end{aligned}$$

output

```
x*arccoth(c+d*cot(b*x+a))+1/2*x*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d
))-1/2*x*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*polylog(2,(1-c
-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*polylog(2,(1+c+I*d)*exp(2*I*a+
2*I*b*x)/(1+c-I*d))/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 390 vs. 2(194) = 388.

Time = 0.52 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.01

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx = x\left(\coth^{-1}(c+d\cot(a+bx)) + \frac{2a\log(d+(-1+c)\tan(a+bx)) + i\log(1+i\tan(a+bx))\log\left(-\frac{i(d+(-1+c)\tan(a+bx))}{-1+c-id}\right) - i\log(1-i\tan(a+bx))\ldots}{\ldots}\right.$$

input
```
Integrate[ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
x*(ArcCoth[c + d*Cot[a + b*x]] + (2*a*Log[d + (-1 + c)*Tan[a + b*x]] + I*L
og[1 + I*Tan[a + b*x]]*Log[((-I)*(d + (-1 + c)*Tan[a + b*x]))/(-1 + c - I*
d)] - I*Log[1 - I*Tan[a + b*x]]*Log[(I*(d + (-1 + c)*Tan[a + b*x]))/(-1 +
c + I*d)] - 2*a*Log[d + (1 + c)*Tan[a + b*x]] + I*Log[1 - I*Tan[a + b*x]]*
Log[(I*(d + (1 + c)*Tan[a + b*x]))/(1 + c + I*d)] - I*Log[1 + I*Tan[a + b*
x]]*Log[(d + (1 + c)*Tan[a + b*x])/(I*(1 + c) + d)] - I*PolyLog[2, ((-1 +
c)*(1 - I*Tan[a + b*x]))/(-1 + c + I*d)] + I*PolyLog[2, ((1 + c)*(1 - I*Ta
n[a + b*x]))/(1 + c + I*d)] + I*PolyLog[2, ((-1 + c)*(1 + I*Tan[a + b*x]))
/(-1 + c - I*d)] - I*PolyLog[2, ((1 + c)*(1 + I*Tan[a + b*x]))/(1 + c - I*
d)])/(4*a - (2*I)*Log[1 - I*Tan[a + b*x]] + (2*I)*Log[1 + I*Tan[a + b*x]])
)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6816, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\cot(a+bx)+c)\,dx$$

$\big\downarrow$ 6816

$$-b(-ic+d+i)\int \frac{e^{2ia+2ibx}x}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+x\coth^{-1}(d\cot(a+bx)+c)$$

$\big\downarrow$ 2620

$$-b(-ic+d+i)\left(\frac{\int \log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(\frac{\int \log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

$\big\downarrow$ 2715

$$-b(-ic+d+i)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

$\big\downarrow$ 2838

$$-b(-ic+d+i)\left(\frac{i\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(\frac{i\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Cot[a + b*x]] - b*(I - I*c + d)*(-1/2*(x*Log[1 - ((1 - c -
 I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)) + ((I/4
)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b^2*
(I*(1 - c) + d))) + b*(I*(1 + c) - d)*(-1/2*(x*Log[1 - ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(I*(b + b*c) - b*d) + ((I/4)*PolyLog[
2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(b*(I*(b + b*c)
 - b*d)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6816
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + (-Simp[I*b*(1 - c - I*d)   Int[x*(E^(2*I*a +
 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp
[I*b*(1 + c + I*d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d
)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)
^2, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(164) = 328$.

Time = 2.02 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.91

method	result
derivativedivides	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(c+d\cot(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}\right.}{}$
default	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(c+d\cot(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}\right.}{}$
risch	Expression too large to display

input

```
int(arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output

```
1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arccoth(c+d*cot(b*x+a))-d^2*(1/2*arc
tan(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)-1/2*arct
an(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+1/4*I*ln(
d*((c+d*cot(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I*d+
c-1))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d)))/d+1/4*I*(dilog((I*d-
d*((c+d*cot(b*x+a))/d-c/d))/(I*d+c-1))-dilog((I*d+d*((c+d*cot(b*x+a))/d-c/
d))/(1-c+I*d)))/d-1/4*I*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+
d*cot(b*x+a))/d-c/d))/(1+c+I*d))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-
c-1)))/d-1/4*I*(dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(1+c+I*d))-dilog((I
*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1)))/d))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(136) = 272$.

Time = 0.22 (sec) , antiderivative size = 1098, normalized size of antiderivative = 5.66

$$\int \coth^{-1}(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(
2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1
)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c -
1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
 I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c
+ 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2
 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(
c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c
^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2
 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2
+ 2*c + 1)) - 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2
- (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c
- 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c +
 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2
*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*...
```

Sympy [F]

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx=\int \operatorname{acoth}\left(c+d\cot\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(c+d*cot(b*x+a)),x)
```

output
```
Integral(acoth(c + d*cot(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(136) = 272.

Time = 0.19 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.02

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx$$
$$=\frac{4\left(bx+a\right)\operatorname{arcoth}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+\left(c^2+2\,c+1\right)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)}{}\right.\right.}{}$$

input
```
integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2
 + 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) +
d^2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x
 + a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 -
2*c + 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x +
a) + (c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x +
 a)*log((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/
(c^2 + d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c +
d + I)) - I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dil
og(-((c - 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*t
an(b*x + a) + I*c + I)/(-I*c + d - I)))/b
```

Giac [F]

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx=\int \operatorname{arcoth}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx=\int \operatorname{acoth}(c+d\cot(a+b\,x))\,dx$$

input
```
int(acoth(c + d*cot(a + b*x)),x)
```

output
```
int(acoth(c + d*cot(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx=\int acoth(\cot\left(bx+a\right)d+c)\,dx$$

input
```
int(acoth(c+d*cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)*d + c),x)
```

3.135 $\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(c+d\cot(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(c+d*cot(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Cot[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\cot(a+bx)+c)}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\coth^{-1}(d\cot(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*cot(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*cot(b*x + a) + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(c+d\cot\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*cot(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*cot(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}(c+d\cot(a+b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*cot(a + b*x))/x,x)
```

output
```
int(acoth(c + d*cot(a + b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\cot(a + bx))}{x} \, dx = \int \frac{acoth(\cot(bx + a)\, d + c)}{x} dx$$

input
```
int(acoth(c+d*cot(b*x+a))/x,x)
```

output
```
int(acoth(cot(a + b*x)*d + c)/x,x)
```

3.136 $\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{ix^2\,\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b} - \frac{x\,\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i\,\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1+I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\,\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\,\text{Poly}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)
*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*
(a + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*
I)*PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6820, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{ 6820}$$

$$\frac{1}{3}ib\int \frac{x^3}{1-(id+1)e^{2ia+2ibx}}dx+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\int \frac{e^{2ia+2ibx}x^3}{1-(id+1)e^{2ia+2ibx}}dx\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\int x^2\log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{2b(-d+i)}-\frac{x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\int x\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})dx}{b}\right)}{2b(-d+i)}-\frac{x^3\log\left(1-(1+id)e^{2ia+2i\right.}}{2b(-d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(-d+i)}-\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e\right.}{2b}\right.}{b}\right.}{2b(-d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,(id+1)e^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(-d+i)}-\frac{x^3 l}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

input
```
Int[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 + (I/3)*b*(x^4/4 + (1 + I*d)*(-1
/2*(x^3*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + (3*(((I/
2)*x^2*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*
PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)]/(4*b^2)))/b))/(2*b*(I - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 2383, normalized size of antiderivative = 14.18

method	result	size
risch	Expression too large to display	2383

input
```
int(x^2*arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(-d+I)*pol
ylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(-d+I)*dilog(1-I*exp(I*
(b*x+a))*(I*(-d+I))^(1/2))-1/2/b^3*a^2/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(I*
(-d+I))^(1/2))+1/6*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x^3-1/6*I/(-d+
I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x^3+1/12*I*b*x^4-1/2/b^2*d/(-d+I)*ln(1+
I*(-d+I)*exp(2*I*(b*x+a)))*a^2*x+1/2/b^2*d*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a)
)*(I*(-d+I))^(1/2))*x+1/2/b^2*d*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I)
)^(1/2))*x+1/2*I/b^2/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*a^2*x-1/4*I/b*
d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/2*I/b^2*a^2/(-d+I)*ln
(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*
(b*x+a))*(I*(-d+I))^(1/2))*x-1/2*I/b^3*d*a^2/(-d+I)*dilog(1-I*exp(I*(b*x+a
))*(I*(-d+I))^(1/2))-1/2*I/b^3*d*a^2/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(I*(-
d+I))^(1/2))+1/4*I/b^3*a^2*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))+
1/8/b^3/(-d+I)*polylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))-1/3*x^3*ln(exp(I*(b*x
+a)))-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/4
/b^2*d/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))*x-1/3/b^3*d/(-d+I)*ln(
1+I*(-d+I)*exp(2*I*(b*x+a)))*a^3+1/2/b^3*d*a^3/(-d+I)*ln(1-I*exp(I*(b*x+a)
)*(I*(-d+I))^(1/2))+1/2/b^3*d*a^3/(-d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I))^
(1/2))+1/8*I/b^3*d/(-d+I)*polylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))-1/4*I/b^2/
(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))*x+1/6*I/b^3*a^3/(-d+I)*ln(...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.07

$$\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\, dx$$
$$= \frac{2i\, b^4 x^4 + 4\, b^3 x^3 \log\left(\frac{((d-i)e^{(2i\, bx+2i\, a)}+i)e^{(-2i\, bx-2i\, a)}}{d}\right) + 6i\, b^2 x^2 \mathrm{Li}_2\left(-(-i\, d - 1)e^{(2i\, bx+2i\, a)}\right) - 2i\, a^4 + 4\, a^3 \log}{}$$

input
```
integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2
*I*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*poly
log(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e
^(2*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b
^3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\, dx = \int x^2\, \mathrm{acoth}\,(d\cot{(a + bx)} + id + 1)\, dx$$

input
```
integrate(x**2*acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*cot(a + b*x) + I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(119) = 238$.

Time = 0.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.05

$$\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\, dx$$
$$= \frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d+1)}{b^2} - \frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a\right.}{}}{}$$

input
```
integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(b
*x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)
*arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2
*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((
I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2
+ 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3
, (I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2
*I*a)))/b^2)/b
```

Giac [F]

$$\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\, dx = \int x^2 \operatorname{arcoth}\left(d\cot\left(bx + a\right) + i\, d + 1\right)\, dx$$

input
```
integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int x^2\,\mathrm{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})\,dx$$

input
```
int(x^2*acoth(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int acoth(\cot(bx+a)\,d+di+1)\,x^2dx$$

input
```
int(x^2*acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)*d + d*i + 1)*x**2,x)
```

3.137 $\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\begin{aligned}\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))\\ &\quad-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)\\ &\quad+\frac{ix\,\mathrm{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}\\ &\quad-\frac{\mathrm{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}\end{aligned}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog
(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx)) - \frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a +
 b*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6820, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{6820}$$

$$\frac{1}{2}ib\int \frac{x^2}{1-(id+1)e^{2ia+2ibx}}dx+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\int \frac{e^{2ia+2ibx}x^2}{1-(id+1)e^{2ia+2ibx}}dx\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\int x\log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,(id+1)e^{2ia+2ibx})dx}{2b}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,(id+1)e^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id\right.}{2b(-d+}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{\text{PolyLog}(3,(id+1)e^{2ia+2ibx})}{4b^2}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

input
```
Int[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 + (I/2)*b*(x^3/3 + (1 + I*d)*(-1
/2*(x^2*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + (((I/2)*
x*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*
E^((2*I)*a + (2*I)*b*x)]/(4*b^2))/(b*(I - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.40 (sec) , antiderivative size = 2285, normalized size of antiderivative = 17.31

method	result	size
risch	Expression too large to display	2285

input
```
int(x*arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d/(exp(2*I
*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*
I*(b*x+a)))^2*csgn(I*d)+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*
(b*x+a)))^3+I*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(
b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d/
(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)-I*Pi*csgn(I*exp(I*(b*x+a)
))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(ex
p(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+2*I*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I
*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)
/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a
)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(d/(exp(2*I*(
b*x+a))-1)*exp(2*I*(b*x+a)))^3-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*
(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-I*Pi*csgn(I*exp(2*
I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2
*I*(b*x+a)))+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*exp
(2*I*(b*x+a)))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-I*Pi
*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(I*(exp(2*I*(b*x
+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{4i\,b^3x^3+6\,b^2x^2\log\left(\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)+4i\,a^3+6i\,bx\mathrm{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\right.}{24\,b^2}$$

input
```
integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a
)) - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a
^2)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*
I*b*x + 2*I*a)))/b^2
```

Sympy [F]

$$\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int x\,\mathrm{acoth}\,(d\cot\,(a+bx)+id+1)\,dx$$

input
```
integrate(x*acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(x*acoth(d*cot(a + b*x) + I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(94) = 188$.

Time = 0.05 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\mathrm{arcoth}(d\cot(bx+a)+i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a\right.}{}}{}$$

input
```
integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d + 1)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I
*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*
sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*po
lylog(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Giac [F]

$$\int x \coth^{-1}(1 + id + d\cot(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(d\cot\left(bx + a\right) + i\,d + 1\right)\,dx$$

input
```
integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 + id + d\cot(a + bx))\,dx = \int x\,\mathrm{acoth}(d\cot(a + b\,x) + 1 + d\,1\mathrm{i})\,dx$$

input
```
int(x*acoth(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(x*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1 + id + d\cot(a + bx))\,dx = \int acoth(\cot(bx + a)\,d + di + 1)\,xdx$$

input
```
int(x*acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)*d + d*i + 1)*x,x)
```

3.138 $\int \coth^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}ibx^2 + x\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{i\,\mathrm{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccoth(1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1+I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 709 vs. 2(93) = 186.

Time = 1.95 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.62

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = x\coth^{-1}(1+id+d\cot(a+bx)) + \frac{x\csc^2(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\right.}{(i+\cot(a+bx))(2+id+d\cot(a+bx))\left(2ibx+\log\left(1+\frac{1}{2}\sec(bx)((-2-id)\cos(a)+d\sin(a))(\cos(\right.\right.}$$

input
```
Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b
*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[
a + b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*
Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b
*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
I*PolyLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*
Sin[a])*(I + Tan[b*x]))/(2*(-I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] +
 I*Sin[b*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x +
Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a +
 b*x]))/2] - Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I
*d)*Sin[a + b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Lo
g[1 + I*Tan[b*x]]))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1
- I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] +
(-2*I + d)*Sin[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d
)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[
(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
)/2]*Tan[b*x] - I*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan
[b*x]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6812, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{ 6812}$$

$$ib\int \frac{x}{1-(id+1)e^{2ia+2ibx}}dx+x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$ib\left(\frac{x^2}{2}+(1+id)\int \frac{e^{2ia+2ibx}x}{1-(id+1)e^{2ia+2ibx}}dx\right)+x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(\frac{\int \log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{2b(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2838}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(\frac{i\operatorname{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{4b^2(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d\cot(a+bx)+id+1)$$

input
```
Int[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + I*b*(x^2/2 + (1 + I*d)*(-1/2*(x*Log[
1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + ((I/4)*PolyLog[2, (1
 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b^2*(I - d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6812
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(76) = 152$.

Time = 1.25 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.30

method	result
derivativedivides	$-\frac{i \operatorname{arccoth}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i \operatorname{arccoth}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}+\frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(1+\frac{i}{2}\right.}{}\right.}{}\right.}{}$
default	$-\frac{i \operatorname{arccoth}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i \operatorname{arccoth}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}+\frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(1+\frac{i}{2}\right.}{}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*I*arccoth(1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))+1/2*I*arc
coth(1+I*d+d*cot(b*x+a))*d*ln(-I*d+d*cot(b*x+a))+1/2*d^2*(-I/d*(-1/2*dilog
(1+1/2*I*d+1/2*d*cot(b*x+a))-1/2*ln(I*d+d*cot(b*x+a))*ln(1+1/2*I*d+1/2*d*c
ot(b*x+a))+1/4*ln(I*d+d*cot(b*x+a))^2)+I/d*(1/2*dilog(-1/2*I*(I*d+d*cot(b*
x+a))/d)+1/2*ln(-I*d+d*cot(b*x+a))*ln(-1/2*I*(I*d+d*cot(b*x+a))/d)-1/2*dil
og(I*(-I*d+d*cot(b*x+a)-I*(2*I-2*d))/(2*I-2*d))-1/2*ln(-I*d+d*cot(b*x+a))*
ln(I*(-I*d+d*cot(b*x+a)-I*(2*I-2*d))/(2*I-2*d)))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{2i\,b^2x^2+2\,bx\log\left(\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)-2i\,a^2-2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a}{4\,b}$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*I*b^2*x^2 + 2*b*x*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) +
1) + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d -
 1)*e^(2*I*b*x + 2*I*a)))/b
```

Sympy [F]

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int \operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(acoth(d*cot(a + b*x) + I*d + 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(66) = 132$.

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.08

$$\int \coth^{-1}(1 + id + d\cot(a + bx))\, dx =$$

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d+2)\tan(bx+a)+d)}{d} - \frac{\log(i\,\tan(bx+a)+1)}{d}\right) + d\left(-\frac{2i\left(\log((i\,d+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}+1\right.\right.}{d}\right.}{}$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a
) + 1)/d) + d*(-2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*
x + a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*
I*d + 2)))/d - 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*
x + a) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d + (2*I*l
og((I*d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x +
 a) + 1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2
) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arccoth(I*d + d/tan(
b*x + a) + 1))/b
```

Giac [F]

$$\int \coth^{-1}(1 + id + d\cot(a + bx))\, dx = \int \operatorname{arcoth}\left(d\cot\left(bx + a\right) + i\,d + 1\right)\, dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx=\int \mathrm{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})\;dx$$

input
```
int(acoth(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(acoth(d*1i + d*cot(a + b*x) + 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx=\int acoth(\cot(bx+a)\,d+di+1)\,dx$$

input
```
int(acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
int(acoth(cot(a + b*x)*d + d*i + 1),x)
```

3.139 $\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(1+id+d\cot(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\cot(a+bx)+id+1)}{x}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\coth^{-1}(d\cot(a+bx)+id+1)}{x}dx$$

input
```
Int[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1+id+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
Integral(acoth(d*cot(a + b*x) + I*d + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*
a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + I*d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(acoth(d*1i + d*cot(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d*1i + d*cot(a + b*x) + 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{acoth(\cot(bx+a)\,d+di+1)}{x}dx$$

input
```
int(acoth(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
int(acoth(cot(a + b*x)*d + d*i + 1)/x,x)
```

3.140 $\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\, dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\, dx = \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d\cot(a + bx)) - \frac{1}{6} x^3 \log\left(1 - (1 - id)e^{2ia + 2ibx}\right) + \frac{ix^2 \operatorname{PolyLog}\left(2, (1 - id)e^{2ia + 2ibx}\right)}{4b} - \frac{x \operatorname{PolyLog}\left(3, (1 - id)e^{2ia + 2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, (1 - id)e^{2ia + 2ibx}\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1-I*d-d*cot(b*x+a))-1/6*x^3*ln(1-(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1-I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int x^2 \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+6bx\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)-3i\operatorname{PolyLog}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*
d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6820, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{6820}$$

$$\frac{1}{3}ib\int \frac{x^3}{1-(1-id)e^{2ia+2ibx}}dx+\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\int \frac{e^{2ia+2ibx}x^3}{1-(1-id)e^{2ia+2ibx}}dx\right)+\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\int x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{2b(d+i)}-\frac{x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{i\int x\,\text{PolyLog}(2,(1-id)e^{2ia+2ibx})dx}{b}\right)}{2b(d+i)}-\frac{x^3\log\left(1-(1-id)e^{2ia+2i\right.}}{2b(d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{i\int\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)dx}{2b}-\frac{ix\,\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(d+i)}-\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,(1-id)e\right.}{2b}\right.}{b}\right.}{2b(d+i)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(d+i)}-\frac{x^3\mathrm{l}}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

input
```
Int[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 + (I/3)*b*(x^4/4 + (1 - I*d)*(-1
/2*(x^3*Log[1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + (3*(((I/
2)*x^2*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*
PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)]/(4*b^2)))/b))/(2*b*(I + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.92 (sec) , antiderivative size = 2273, normalized size of antiderivative = 13.45

method	result	size
risch	Expression too large to display	2273

input
```
int(x^2*arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2
*I*(b*x+a)))^2+I*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(
2*I*(b*x+a))-1))^3+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*cs
gn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I*exp(2*I*(b*x+a))
)*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn((I*exp(2*I*(b*
x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(d/(exp(2*I*(
b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b
*x+a)))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(I/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(
b*x+a)))*csgn(I*d)-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+I*
Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*
csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))-I*Pi*
csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
-I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d-I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x
+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+
a))-1))^2-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^2*csgn(I*d)-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b
*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-I*Pi*csgn(I*ex...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx$$
$$= \frac{2i\,b^4x^4 - 4\,b^3x^3\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 2i\,a^4 + 4\,a^3\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a}}{d+i}\right.}{}$$

input
```
integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I
*a^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylo
g(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(
2*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^
3
```

Sympy [F]

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx = -\int x^2\,\mathrm{acoth}\,(d\cot(a + bx) + id - 1)\,dx$$

input
```
integrate(x**2*acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
-Integral(x**2*acoth(d*cot(a + b*x) + I*d - 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(120) = 240$.

Time = 0.07 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.04

$$\int x^2 \coth^{-1}(1-id-d\cot(a+bx))\,dx =$$
$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx\right.}{}}{}$$

input
```
integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(
b*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2
)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilo
g((-I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(
b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a
)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polyl
og(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b
*x + 2*I*a)))/b^2)/b
```

Giac [F]

$$\int x^2 \coth^{-1}(1-id-d\cot(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int -x^2\,\mathrm{acoth}(d\cot(a + b\,x) - 1 + d\,1\mathrm{i})\;dx$$

input
```
int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx = -\left(\int acoth(\cot(bx + a)\,d + di - 1)\,x^2dx\right)$$

input
```
int(x^2*acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
- int(acoth(cot(a + b*x)*d + d*i - 1)*x**2,x)
```

3.141 $\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \frac{1}{6}ibx^3 + \frac{1}{2}x^2\coth^{-1}(1 - id - d\cot(a + bx)) - \frac{1}{4}x^2\log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{ix\,\text{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{\text{PolyLog}\left(3, (1 - id)e^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1-I*d-d*cot(b*x+a))-1/4*x^2*ln(1-(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog
(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int x\coth^{-1}(1-id-d\cot(a+bx))\,dx$$
$$=\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))$$
$$-\frac{2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+2ibx\,\mathrm{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+\mathrm{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*
d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6820, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{ 6820}$$

$$\frac{1}{2}ib\int\frac{x^2}{1-(1-id)e^{2ia+2ibx}}dx+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\int\frac{e^{2ia+2ibx}x^2}{1-(1-id)e^{2ia+2ibx}}dx\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\int x\log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)dx}{2b}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}}{b(d+i)}-\frac{x^2\log\left(1-(1-i\right.}{2b(d+}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

input
```
Int[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 + (I/2)*b*(x^3/3 + (1 - I*d)*(-1
/2*(x^2*Log[1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + (((I/2)*
x*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*
E^((2*I)*a + (2*I)*b*x)]/(4*b^2))/(b*(I + d))))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 2183, normalized size of antiderivative = 16.41

method	result	size
risch	Expression too large to display	2183

input
```
int(x*arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b*x^3-1/8*(I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a)
)-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*
d-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b
*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I*exp(2*
I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn((I*e
xp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(d/
(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*
exp(2*I*(b*x+a)))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-I*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d/(exp(2*I*(b*x+a))-1)
*exp(2*I*(b*x+a)))*csgn(I*d)-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x
+a)))^3+I*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*
x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))
-1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*
I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(I*exp(2*I*(b*x+a))+ex
p(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp
(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*c
sgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+I*Pi*csgn(I*d/(exp(2*I*(b*x+a
))-1)*exp(2*I*(b*x+a)))^2*csgn(I*d)-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(
exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-I*Pi...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17

$$\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx$$
$$= \frac{4i\,b^3x^3 - 6\,b^2x^2\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) + 4i\,a^3 + 6i\,bx\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 6\,a^2\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-}{d+i}\right.}{24\,b^2}$$

input
```
integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a))
 - 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2
)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*
b*x + 2*I*a)))/b^2
```

Sympy [F]

$$\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx = -\int x\,\mathrm{acoth}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

input
```
integrate(x*acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
-Integral(x*acoth(d*cot(a + b*x) + I*d - 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(95) = 190.

Time = 0.06 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx =$$
$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\mathrm{arcoth}(d\cot(bx+a)+i\,d-1)}{b} + \frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(b\right.}{}}{}$$

input
```
integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d - 1)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2
*I*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) +
 3*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 +
 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Giac [F]

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(-d\cot\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int -x\,\mathrm{acoth}(d\cot(a + b\,x) - 1 + d\,1\mathrm{i})\,dx$$

input
```
int(-x*acoth(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-x*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

Reduce [F]

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\, dx = -\left(\int acoth(\cot(bx + a)\, d + di - 1)\, x dx\right)$$

input
```
int(x*acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
- int(acoth(cot(a + b*x)*d + d*i - 1)*x,x)
```

3.142 $\int \coth^{-1}(1-id-d\cot(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}ibx^2 + x\coth^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{i\,\mathrm{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccoth(1-I*d-d*cot(b*x+a))-1/2*x*ln(1-(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 605 vs. 2(94) = 188.

Time = 1.56 (sec) , antiderivative size = 605, normalized size of antiderivative = 6.44

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = x\coth^{-1}(1-id-d\cot(a+bx)) + \frac{x\csc^2(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))(d\cos(a+bx)+i(2i+d)\sin(a}{2(i+d)}\right.\right.}{(i+\cot(a+bx))(-2+id+d\cot(a}$$

input
```
Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b
*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[
a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I
*Log[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*
x]] - I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] +
 I*Sin[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a]
 + d*Sin[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*
x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[
1 - I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x]
+ I*(2*I + d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Co
s[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*
Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Si
n[a])]*Sec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Se
c[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(
I + Tan[b*x])))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6812, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{6812}$$

$$ib\int \frac{x}{1-(1-id)e^{2ia+2ibx}}dx+x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}+(1-id)\int \frac{e^{2ia+2ibx}x}{1-(1-id)e^{2ia+2ibx}}dx\right)+x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(\frac{\int \log \left(1-(1-id)e^{2ia+2ibx}\right)dx}{2b(d+i)}-\frac{x\log \left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(-\frac{i\int e^{-2ia-2ibx}\log \left(1-(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2(d+i)}-\frac{x\log \left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2838}$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(\frac{i\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b^2(d+i)}-\frac{x\log \left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

input

```
Int[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output

```
x*ArcCoth[1 - I*d - d*Cot[a + b*x]] + I*b*(x^2/2 + (1 - I*d)*(-1/2*(x*Log[
1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + ((I/4)*PolyLog[2, (1
 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b^2*(I + d))))
```

Defintions of rubi rules used

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6812
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(77) = 154$.

Time = 1.58 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.40

method	result
derivativedivides	$-\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(-id-d\cot(bx+a))}{2}-\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\frac{\ln(-id-d\cot \cdots}{4}\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(-id-d\cot(bx+a))}{2}-\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\frac{\ln(-id-d\cot \cdots}{4}\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(1/2*I*arccoth(1-I*d-d*cot(b*x+a))*d*ln(-I*d-d*cot(b*x+a))-1/2*I*ar
ccoth(1-I*d-d*cot(b*x+a))*d*ln(I*d-d*cot(b*x+a))-1/2*d^2*(-I/d*(1/4*ln(-I*
d-d*cot(b*x+a))^2-1/2*dilog(1-1/2*I*d-1/2*d*cot(b*x+a))-1/2*ln(-I*d-d*cot(
b*x+a))*ln(1-1/2*I*d-1/2*d*cot(b*x+a)))+I/d*(1/2*dilog(1/2*I*(-I*d-d*cot(b
*x+a))/d)+1/2*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/2*dil
og(I*(I*d-d*cot(b*x+a)-I*(2*I+2*d))/(2*I+2*d))-1/2*ln(I*d-d*cot(b*x+a))*ln
(I*(I*d-d*cot(b*x+a)-I*(2*I+2*d))/(2*I+2*d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{2i\,b^2x^2 - 2\,bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) - 2i\,a^2 - 2\,(bx+a)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right) + 2\,a\log\left(\frac{(d+i)e^{(2i\,bx}}{d+}\right.}{4\,b}$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*I*b^2*x^2 - 2*b*x*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2
*I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
 + 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)
*e^(2*I*b*x + 2*I*a)))/b
```

Sympy [F]

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = -\int \operatorname{acoth}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

input
```
integrate(acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
-Integral(acoth(d*cot(a + b*x) + I*d - 1), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.06

$$\int \coth^{-1}(1 - id - d\cot(a + bx))\,dx = -\frac{4\,(bx+a)d\left(\frac{\log((i\,d-2)\tan(bx+a)+d)}{d} - \frac{\log(i\,\tan(bx+a)+1)}{d}\right) - d\left(\frac{2i\left(\log((i\,d-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}+1\right)\cdots}{d}\right.}{\cdots}$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a
) + 1)/d) - d*(2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x
 + a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I
*d - 2)))/d + 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*
x + a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d - (2*I*lo
g((I*d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x +
a) + 1)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2)
 + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arccoth(I*d + d/tan(b
*x + a) - 1))/b
```

Giac [F]

$$\int \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int \operatorname{arcoth}\left(-d\cot\left(bx+a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx=\int -\mathrm{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\,dx$$

input
```
int(-acoth(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-acoth(d*1i + d*cot(a + b*x) - 1), x)
```

Reduce [F]

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx=-\left(\int acoth(\cot(bx+a)\,d+di-1)\,dx\right)$$

input
```
int(acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
- int(acoth(cot(a + b*x)*d + d*i - 1),x)
```

3.143 $\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\coth^{-1}(1-id-d\cot(a+bx))}{x},x\right)$$

output
```
Defer(Int)(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x}\,dx$$
$$\big\downarrow \text{ 7299}$$
$$\int \frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x}dx$$

input
```
Int[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1-id-d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

output
```
int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=-\int \frac{\operatorname{acoth}\left(d\cot\left(a+bx\right)+id-1\right)}{x}\,dx$$

input
```
integrate(acoth(1-I*d-d*cot(b*x+a))/x,x)
```

output
```
-Integral(acoth(d*cot(a + b*x) + I*d - 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 4.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.86

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-
d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x
)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*cot(b*x + a) - I*d + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int -\frac{\operatorname{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(-acoth(d*1i + d*cot(a + b*x) - 1)/x,x)
```

output
```
int(-acoth(d*1i + d*cot(a + b*x) - 1)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx = -\left(\int \frac{acoth(\cot\left(bx+a\right)d+di-1)}{x}dx\right)$$

input
```
int(acoth(1-I*d-d*cot(b*x+a))/x,x)
```

output
```
- int(acoth(cot(a + b*x)*d + d*i - 1)/x,x)
```

3.144 $\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = \frac{a(d+e\log(fx^m))^2}{2em}+\frac{b(d+e\log(fx^m))\operatorname{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n} - \frac{b(d+e\log(fx^m))\operatorname{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n} + \frac{bem\operatorname{PolyLog}\left(3,-\frac{x^{-n}}{c}\right)}{2n^2}-\frac{bem\operatorname{PolyLog}\left(3,\frac{x^{-n}}{c}\right)}{2n^2}$$

output
```
1/2*a*(d+e*ln(f*x^m))^2/e/m+1/2*b*(d+e*ln(f*x^m))*polylog(2,-1/c/(x^n))/n-
1/2*b*(d+e*ln(f*x^m))*polylog(2,1/c/(x^n))/n+1/2*b*e*m*polylog(3,-1/c/(x^n
))/n^2-1/2*b*e*m*polylog(3,1/c/(x^n))/n^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$= -\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n^2}$$
$$+\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)(d+e\log(fx^m))}{n}$$
$$-\frac{1}{2}\left(a+b\coth^{-1}(cx^n)-b\text{arctanh}(cx^n)\right)\log(x)\left(em\log(x)-2(d+e\log(fx^m))\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
-((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*
x^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2
*x^(2*n)]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCoth[c*x^n] - b*ArcTanh[c*x^n
])*Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{d(a+b\coth^{-1}(cx^n))}{x}+\frac{e\log(fx^m)(a+b\coth^{-1}(cx^n))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{bd\,\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\,\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{be\,\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)\log(fx^m)}{2n}-\frac{be\,\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)\log(fx^m)}{2n}+\frac{bem\,\text{PolyLog}\left(3,-\frac{x^{-n}}{c}\right)}{2n^2}-\frac{bem\,\text{PolyLog}\left(3,\frac{x^{-n}}{c}\right)}{2n^2}$$

input
```
Int[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + (b*d*PolyLog[2, -(1/(c*x^n))])/(2*
n) + (b*e*Log[f*x^m]*PolyLog[2, -(1/(c*x^n))])/(2*n) - (b*d*PolyLog[2, 1/(
c*x^n)])/(2*n) - (b*e*Log[f*x^m]*PolyLog[2, 1/(c*x^n)])/(2*n) + (b*e*m*Pol
yLog[3, -(1/(c*x^n))])/(2*n^2) - (b*e*m*PolyLog[3, 1/(c*x^n)])/(2*n^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.25 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.29

method	result
risch	$\frac{\left(-\frac{ie\pi\,\text{csgn}(if)\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)}{4}+\frac{ie\pi\,\text{csgn}(if)\,\text{csgn}(if\,x^m)^2}{4}+\frac{ie\pi\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)^2}{4}-\frac{ie\pi\,\text{csgn}(if\,x^m)^3}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)(-b\,\text{dil}}{n}$

input
```
int((a+b*arccoth(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m
)^3+1/2*e*ln(f)+1/2*d)/n*(-b*dilog(c*x^n+1)+2*a*ln(x^n)-ln(c*x^n)*ln(c*x^n
-1)*b-dilog(c*x^n)*b)-1/2*e*b*m/n*ln(x)*polylog(2,-c*x^n)+1/2*e*b*m/n^2*po
lylog(3,-c*x^n)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)-1/2*e*b/n*dilog(c*x^n+1)*
ln(x^m)+1/2*e*a/m*ln(x^m)^2+1/4*e*b*ln(c*x^n-1)*m*ln(x)^2-1/2*e*b*ln(x^m)*
ln(c*x^n-1)*ln(x)-1/4*e*b*m*ln(x)^2*ln(1-c*x^n)+1/2*e*b*m/n*ln(x)*polylog(
2,c*x^n)-1/2*e*b*m/n^2*polylog(3,c*x^n)+1/2*e*b*ln(1-c*x^n)*ln(x)*ln(x^m)+
1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln
(x^m)+1/2*e*b/n*dilog(c*x^n)*m*ln(x)-1/2*e*b/n*dilog(c*x^n)*ln(x^m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(114) = 228$.

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.59

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx$$

$$= \frac{2\,aemn^2\log(x)^2 - 2\,bem\mathrm{polylog}(3, c\cosh(n\log(x)) + c\sinh(n\log(x))) + 2\,bem\mathrm{polylog}(3, -c\cosh(n\,\mathrm{l}}{}$$

input
```
integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

output
```
1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n
*log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(
b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*l
og(x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)
) - c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)
*log(x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^
2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*
log(x)) + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 +
 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log((c*cosh(n*log(x)) + c*sinh(n*log
(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

Sympy [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(a + b\operatorname{acoth}(cx^n))(d + e\log(fx^m))}{x}\,dx$$

input
```
integrate((a+b*acoth(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

output
```
Integral((a + b*acoth(c*x**n))*(d + e*log(f*x**m))/x, x)
```

Maxima [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(b\operatorname{arcoth}(cx^n) + a)(e\log(fx^m) + d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*l
og(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2
- 2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n - 1) + inte
grate(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*lo
g(f) + d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

Giac [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(b\operatorname{arcoth}(cx^n) + a)(e\log(fx^m) + d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(a + b\,\mathrm{acoth}(c\,x^n))\,(d + e\,\ln(f\,x^m))}{x}\,dx$$

input
```
int(((a + b*acoth(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

output
```
int(((a + b*acoth(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

Reduce [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx$$
$$= \frac{2\left(\int \frac{acoth(x^n c)}{x}dx\right)bdm + 2\left(\int \frac{acoth(x^n c)\log(x^m f)}{x}dx\right)bem + \log(x^m f)^2 ae + 2\log(x)\,adm}{2m}$$

input
```
int((a+b*acoth(c*x^n))*(d+e*log(f*x^m))/x,x)
```

output
```
(2*int(acoth(x**n*c)/x,x)*b*d*m + 2*int((acoth(x**n*c)*log(x**m*f))/x,x)*b
*e*m + log(x**m*f)**2*a*e + 2*log(x)*a*d*m)/(2*m)
```

3.145 $\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 297

$$\begin{aligned}
&\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx \\
&=\frac{b(6d-11e)x}{36c^5}-\frac{23bex}{45c^5}+\frac{b(6d-5e)x^3}{108c^3}-\frac{8bex^3}{135c^3}+\frac{b(3d-e)x^5}{90c} \\
&\quad-\frac{bex^5}{75c}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2} \\
&\quad-\frac{1}{18}ex^6\left(a+b\coth^{-1}(cx)\right)-\frac{b(6d-11e)\text{arctanh}(cx)}{36c^6}+\frac{23be\text{arctanh}(cx)}{45c^6} \\
&\quad+\frac{bex\log\left(1-c^2x^2\right)}{6c^5}+\frac{bex^3\log\left(1-c^2x^2\right)}{18c^3}+\frac{bex^5\log\left(1-c^2x^2\right)}{30c} \\
&\quad-\frac{e\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{6c^6}+\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)
\end{aligned}$$

output
```
1/36*b*(6*d-11*e)*x/c^5-23/45*b*e*x/c^5+1/108*b*(6*d-5*e)*x^3/c^3-8/135*b*
e*x^3/c^3+1/90*b*(3*d-e)*x^5/c-1/75*b*e*x^5/c-1/6*e*x^2*(a+b*arccoth(c*x))
/c^4-1/12*e*x^4*(a+b*arccoth(c*x))/c^2-1/18*e*x^6*(a+b*arccoth(c*x))-1/36*
b*(6*d-11*e)*arctanh(c*x)/c^6+23/45*b*e*arctanh(c*x)/c^6+1/6*b*e*x*ln(-c^2
*x^2+1)/c^5+1/18*b*e*x^3*ln(-c^2*x^2+1)/c^3+1/30*b*e*x^5*ln(-c^2*x^2+1)/c-
1/6*e*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^6+1/6*x^6*(a+b*arccoth(c*x))*(d+
e*ln(-c^2*x^2+1))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.79

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{30bc(10d-49e)x-300ac^2ex^2+10bc^3(10d-19e)x^3-150ac^4ex^4+4bc^5(15d-11e)x^5+100ac^6(3d-e}{}$$

input
```
Integrate[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(30*b*c*(10*d - 49*e)*x - 300*a*c^2*e*x^2 + 10*b*c^3*(10*d - 19*e)*x^3 - 1
50*a*c^4*e*x^4 + 4*b*c^5*(15*d - 11*e)*x^5 + 100*a*c^6*(3*d - e)*x^6 - 50*
b*c^2*x^2*(-6*c^4*d*x^4 + e*(6 + 3*c^2*x^2 + 2*c^4*x^4))*ArcCoth[c*x] + 15
*(10*b*d - 20*a*e - 49*b*e)*Log[1 - c*x] - 15*(10*b*d + 20*a*e - 49*b*e)*L
og[1 + c*x] + 20*e*(15*a*c^6*x^6 + b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + 15
*b*(-1 + c^6*x^6)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(1800*c^6)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$
$$\Big\downarrow \text{6646}$$
$$-bc\int\left(\frac{(3d-e)x^6}{18\left(1-c^2x^2\right)}-\frac{ex^4}{12c^2\left(1-c^2x^2\right)}-\frac{ex^2}{6c^4\left(1-c^2x^2\right)}-\frac{e\left(c^4x^4+c^2x^2+1\right)\log\left(1-c^2x^2\right)}{6c^6}\right)dx-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}+\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{6c^6}-\frac{1}{18}ex^6\left(a+b\coth^{-1}(cx)\right)$$

$$\big\downarrow \text{2009}$$

$$-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}+\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{6c^6}-\frac{1}{18}ex^6\left(a+b\coth^{-1}(cx)\right)-bc\left(\frac{(3d-e)\text{arctanh}(cx)}{18c^7}-\frac{137e\text{arctanh}(cx)}{180c^7}-\frac{x(3d-e)}{18c^6}+\frac{137ex}{180c^6}-\frac{x^3(3d-e)}{54c^4}+\frac{47ex^3}{540c^4}-\frac{x^5(3d-e)}{90c^2}+\frac{ex^5}{75c^2}\right.$$

input
```
Int[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
-1/6*(e*x^2*(a + b*ArcCoth[c*x]))/c^4 - (e*x^4*(a + b*ArcCoth[c*x]))/(12*c
^2) - (e*x^6*(a + b*ArcCoth[c*x]))/18 - (e*(a + b*ArcCoth[c*x])*Log[1 - c^
2*x^2])/(6*c^6) + (x^6*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/6 -
b*c*(-1/18*((3*d - e)*x)/c^6 + (137*e*x)/(180*c^6) - ((3*d - e)*x^3)/(54*c
^4) + (47*e*x^3)/(540*c^4) - ((3*d - e)*x^5)/(90*c^2) + (e*x^5)/(75*c^2) +
 ((3*d - e)*ArcTanh[c*x])/(18*c^7) - (137*e*ArcTanh[c*x])/(180*c^7) - (e*x
*Log[1 - c^2*x^2])/(6*c^6) - (e*x^3*Log[1 - c^2*x^2])/(18*c^4) - (e*x^5*Lo
g[1 - c^2*x^2])/(30*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.06

method	result
parallelrisch	$\frac{-150\ \mathrm{arccoth}(cx)bd-150\ln\left(-c^2x^2+1\right)ae-150\ \mathrm{arccoth}(cx)b\,c^2e\,x^2+150\ln\left(-c^2x^2+1\right)bcex-150\ \mathrm{arccoth}(cx)\ln\left(-c^2x^2+1\right)b}{}$
orering	$\frac{\left(50c^{10}x^{10}+224c^8x^8+1587x^6c^6-15146x^4c^4+37905c^2x^2-26460\right)\left(a+b\ \mathrm{arccoth}(cx)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{90c^8x^2\left(c^2x^2+3\right)}-\frac{\left(61c^{10}x^{10}+368\right.}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/900*(-150*arccoth(c*x)*b*d-150*ln(-c^2*x^2+1)*a*e-150*arccoth(c*x)*b*c^2
*e*x^2+150*ln(-c^2*x^2+1)*b*c*e*x-150*arccoth(c*x)*ln(-c^2*x^2+1)*b*e+150*
b*c*d*x-150*a*c^2*e*x^2+150*a*e*ln(-c^2*x^2+1)*x^6*c^6+150*b*arccoth(c*x)*
x^6*c^6*d-50*b*arccoth(c*x)*x^6*c^6*e+50*e*b*ln(-c^2*x^2+1)*x^3*c^3+30*b*e
*ln(-c^2*x^2+1)*x^5*c^5-75*e*b*arccoth(c*x)*x^4*c^4-735*b*x*e*c-95*b*e*x^3
*c^3+30*b*c^5*d*x^5-22*b*c^5*e*x^5-75*a*e*x^4*c^4+50*b*c^3*d*x^3+735*arcco
th(c*x)*b*e+150*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^6*c^6-150*a*e+150*a*c^6*
d*x^6-50*a*c^6*e*x^6)/c^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.83

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$
$$-\frac{150\,ac^4ex^4-100\,(3\,ac^6d-ac^6e)x^6+300\,ac^2ex^2-4\,(15\,bc^5d-11\,bc^5e)x^5-10\,(10\,bc^3d-19\,bc^3e)x^3}{}$$

input
```
integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/1800*(150*a*c^4*e*x^4 - 100*(3*a*c^6*d - a*c^6*e)*x^6 + 300*a*c^2*e*x^2
 - 4*(15*b*c^5*d - 11*b*c^5*e)*x^5 - 10*(10*b*c^3*d - 19*b*c^3*e)*x^3 - 30
*(10*b*c*d - 49*b*c*e)*x - 20*(15*a*c^6*e*x^6 + 3*b*c^5*e*x^5 + 5*b*c^3*e*
x^3 + 15*b*c*e*x - 15*a*e)*log(-c^2*x^2 + 1) + 5*(15*b*c^4*e*x^4 - 10*(3*b
*c^6*d - b*c^6*e)*x^6 + 30*b*c^2*e*x^2 + 30*b*d - 147*b*e - 30*(b*c^6*e*x^
6 - b*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1)))/c^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.22

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^6}{6}+\frac{aex^6\log\left(-c^2x^2+1\right)}{6}-\frac{aex^6}{18}-\frac{aex^4}{12c^2}-\frac{aex^2}{6c^4}-\frac{ae\log\left(-c^2x^2+1\right)}{6c^6}+\frac{bdx^6\operatorname{acoth}(cx)}{6}+\frac{bex^6\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{6}-\\ \frac{dx^6\left(a+\frac{i\pi b}{2}\right)}{6}\end{cases}$$

input
```
integrate(x**5*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**6/6 + a*e*x**6*log(-c**2*x**2 + 1)/6 - a*e*x**6/18 - a*e
*x**4/(12*c**2) - a*e*x**2/(6*c**4) - a*e*log(-c**2*x**2 + 1)/(6*c**6) + b
*d*x**6*acoth(c*x)/6 + b*e*x**6*log(-c**2*x**2 + 1)*acoth(c*x)/6 - b*e*x**
6*acoth(c*x)/18 + b*d*x**5/(30*c) + b*e*x**5*log(-c**2*x**2 + 1)/(30*c) -
11*b*e*x**5/(450*c) - b*e*x**4*acoth(c*x)/(12*c**2) + b*d*x**3/(18*c**3) +
 b*e*x**3*log(-c**2*x**2 + 1)/(18*c**3) - 19*b*e*x**3/(180*c**3) - b*e*x**
2*acoth(c*x)/(6*c**4) + b*d*x/(6*c**5) + b*e*x*log(-c**2*x**2 + 1)/(6*c**5
) - 49*b*e*x/(60*c**5) - b*d*acoth(c*x)/(6*c**6) - b*e*log(-c**2*x**2 + 1)
*acoth(c*x)/(6*c**6) + 49*b*e*acoth(c*x)/(60*c**6), Ne(c, 0)), (d*x**6*(a
+ I*pi*b/2)/6, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.11

$$\int x^5(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \frac{1}{6}\,adx^6 + \frac{1}{36}\left(6\,x^6\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6}+\frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)be\,\mathrm{arcoth}\,(cx) + \frac{1}{180}\left(30\,x^6\,\mathrm{arcoth}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)bd + \frac{1}{36}\left(6\,x^6\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6}+\frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)ae - \frac{(4\,(-15i\,\pi c^5+11\,c^5)x^5+10\,(-10i\,\pi c^3+19\,c^3)x^3+30\,(-10i\,\pi c+49\,c)x+5\,(30i\,\pi-12\,c^5x^5-20\,c^3}{1800\,c^6}$$

input
```
integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/6*a*d*x^6 + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^6 + 3*c^2*x^4
+ 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*b*e*arccoth(c*x) + 1/180*(30*x^6*a
rccoth(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^
7 + 15*log(c*x - 1)/c^7))*b*d + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^
4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*e - 1/1800*(4*
(-15*I*pi*c^5 + 11*c^5)*x^5 + 10*(-10*I*pi*c^3 + 19*c^3)*x^3 + 30*(-10*I*p
i*c + 49*c)*x + 5*(30*I*pi - 12*c^5*x^5 - 20*c^3*x^3 - 60*c*x - 147)*log(c
*x + 1) + 5*(-30*I*pi - 12*c^5*x^5 - 20*c^3*x^3 - 60*c*x + 147)*log(c*x -
1))*b*e/c^6
```

Giac [F(-2)]

Exception generated.

$$\int x^5(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.72

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$\begin{aligned}
&= \ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^6}{6}+\frac{b\,e\,x}{6\,c^5}+\frac{b\,e\,x^5}{30\,c}+\frac{b\,e\,x^3}{18\,c^3}\right)\\
&\quad -\ln\left(\frac{1}{c\,x}+1\right)\,\left(\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e}{12\,c^6}-\frac{b\,e\,x^6}{12}\right)-\frac{b\,d\,x^6}{12}+\frac{b\,e\,x^6}{36}+\frac{b\,e\,x^4}{24\,c^2}+\frac{b\,e\,x^2}{12\,c^4}\right)\\
&\quad +\ln\left(1-\frac{1}{c\,x}\right)\,\left(\frac{\frac{b\,d\,x^7}{6}-\frac{b\,c^2\,d\,x^9}{6}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,e\,x^7}{36}+\frac{b\,e\,x^5}{12\,c^2}-\frac{b\,e\,x^3}{6\,c^4}+\frac{b\,c^2\,e\,x^9}{18}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right.\\
&\quad \left.+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^7}{6}-\frac{b\,c^2\,e\,x^9}{6}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\left(x-c^2\,x^3\right)}{12\,c^6\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right)\\
&\quad +x^4\,\left(\frac{a\,\left(3\,d-e\right)}{12\,c^2}-\frac{a\,d}{4\,c^2}\right)+x^3\,\left(\frac{b\,\left(15\,d-11\,e\right)}{270\,c^3}-\frac{7\,b\,e}{108\,c^3}\right)\\
&\quad +x\,\left(\frac{\frac{b\,\left(15\,d-11\,e\right)}{90\,c^3}-\frac{7\,b\,e}{36\,c^3}}{c^2}-\frac{b\,e}{2\,c^5}\right)+\frac{a\,x^6\,\left(3\,d-e\right)}{18}\\
&\quad +\frac{x^2\,\left(\frac{a\,\left(3\,d-e\right)}{3\,c^2}-\frac{a\,d}{c^2}\right)}{2\,c^2}-\frac{\ln\left(c\,x-1\right)\,\left(20\,a\,e-10\,b\,d+49\,b\,e\right)}{120\,c^6}\\
&\quad -\frac{\ln\left(c\,x+1\right)\,\left(20\,a\,e+10\,b\,d-49\,b\,e\right)}{120\,c^6}+\frac{b\,x^5\,\left(15\,d-11\,e\right)}{450\,c}
\end{aligned}$$

input
```
int(x^5*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - c^2*x^2)*((a*e*x^6)/6 + (b*e*x)/(6*c^5) + (b*e*x^5)/(30*c) + (b*e*
x^3)/(18*c^3)) - log(1/(c*x) + 1)*(log(1 - c^2*x^2)*((b*e)/(12*c^6) - (b*e
*x^6)/12) - (b*d*x^6)/12 + (b*e*x^6)/36 + (b*e*x^4)/(24*c^2) + (b*e*x^2)/(
12*c^4)) + log(1 - 1/(c*x))*(((b*d*x^7)/6 - (b*c^2*d*x^9)/6)/(2*(x + c*x^2
)*(c*x - 1)) + ((b*e*x^7)/36 + (b*e*x^5)/(12*c^2) - (b*e*x^3)/(6*c^4) + (b
*c^2*e*x^9)/18)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^7)/6
 - (b*c^2*e*x^9)/6))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1 - c^2*x^2)*(x
- c^2*x^3))/(12*c^6*(x + c*x^2)*(c*x - 1))) + x^4*((a*(3*d - e))/(12*c^2)
- (a*d)/(4*c^2)) + x^3*((b*(15*d - 11*e))/(270*c^3) - (7*b*e)/(108*c^3)) +
 x*(((b*(15*d - 11*e))/(90*c^3) - (7*b*e)/(36*c^3))/c^2 - (b*e)/(2*c^5)) +
 (a*x^6*(3*d - e))/18 + (x^2*((a*(3*d - e))/(3*c^2) - (a*d)/c^2))/(2*c^2)
- (log(c*x - 1)*(20*a*e - 10*b*d + 49*b*e))/(120*c^6) - (log(c*x + 1)*(20*
a*e + 10*b*d - 49*b*e))/(120*c^6) + (b*x^5*(15*d - 11*e))/(450*c)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.04

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= \frac{150acoth(cx)\log(-c^2x^2+1)\,bc^6ex^6-150acoth(cx)\log(-c^2x^2+1)\,be+150acoth(cx)\,bc^6dx^6-50acot}{}$$

input
```
int(x^5*(a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(150*acoth(c*x)*log( - c**2*x**2 + 1)*b*c**6*e*x**6 - 150*acoth(c*x)*log(
- c**2*x**2 + 1)*b*e + 150*acoth(c*x)*b*c**6*d*x**6 - 50*acoth(c*x)*b*c**6
*e*x**6 - 75*acoth(c*x)*b*c**4*e*x**4 - 150*acoth(c*x)*b*c**2*e*x**2 - 150
*acoth(c*x)*b*d + 735*acoth(c*x)*b*e + 150*log( - c**2*x**2 + 1)*a*c**6*e*
x**6 - 150*log( - c**2*x**2 + 1)*a*e - 30*log( - c**2*x**2 + 1)*b*c**5*e*x
**5 - 50*log( - c**2*x**2 + 1)*b*c**3*e*x**3 - 150*log( - c**2*x**2 + 1)*b
*c*e*x + 150*a*c**6*d*x**6 - 50*a*c**6*e*x**6 - 75*a*c**4*e*x**4 - 150*a*c
**2*e*x**2 - 30*b*c**5*d*x**5 + 22*b*c**5*e*x**5 - 50*b*c**3*d*x**3 + 95*b
*c**3*e*x**3 - 150*b*c*d*x + 735*b*c*e*x)/(900*c**6)
```

3.146 $\int x^3 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\begin{aligned}
&\int x^3 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx \\
&= \frac{b(2d-3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d-e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2 \left(a+b \coth^{-1}(cx)\right)}{4c^2} \\
&\quad - \frac{1}{8} ex^4 \left(a+b \coth^{-1}(cx)\right) - \frac{b(2d-3e)\text{arctanh}(cx)}{8c^4} \\
&\quad + \frac{2be\text{arctanh}(cx)}{3c^4} + \frac{bex \log \left(1-c^2x^2\right)}{4c^3} + \frac{bex^3 \log \left(1-c^2x^2\right)}{12c} \\
&\quad - \frac{e\left(a+b \coth^{-1}(cx)\right) \log \left(1-c^2x^2\right)}{4c^4} + \frac{1}{4} x^4 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right)
\end{aligned}$$

output
```
1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3+1/24*b*(2*d-e)*x^3/c-1/18*b*e*x^3/c-1/
4*e*x^2*(a+b*arccoth(c*x))/c^2-1/8*e*x^4*(a+b*arccoth(c*x))-1/8*b*(2*d-3*e
)*arctanh(c*x)/c^4+2/3*b*e*arctanh(c*x)/c^4+1/4*b*e*x*ln(-c^2*x^2+1)/c^3+1
/12*b*e*x^3*ln(-c^2*x^2+1)/c-1/4*e*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^4+1
/4*x^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{6bc(6d-25e)x-36ac^2ex^2+2bc^3(6d-7e)x^3+18ac^4(2d-e)x^4-18bc^2x^2(-2c^2dx^2+e(2+c^2x^2))\coth}{}$$

input
```
Integrate[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^
4*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcCoth[c*
x] + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e
)*Log[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x
^4)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6646}$$

$$-bc\int\left(-\frac{\left(2e-c^2(2d-e)x^2\right)x^2}{8c^2\left(1-c^2x^2\right)}-\frac{e\left(c^2x^2+1\right)\log\left(1-c^2x^2\right)}{4c^4}\right)dx+\frac{1}{4}x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2(a+b\coth^{-1}(cx))}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4(a+b\coth^{-1}(cx))-bc\left(\frac{(2d-3e)\text{arctanh}(cx)}{8c^5}-\frac{2e\text{arctanh}(cx)}{3c^5}-\frac{x(2d-3e)}{8c^4}+\frac{2ex}{3c^4}-\frac{x^3(2d-e)}{24c^2}+\frac{ex^3}{18c^2}-\frac{ex^3\log\left(1-c^2x^2\right)}{12c^2}-\frac{e\right.$$

input
```
Int[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
-1/4*(e*x^2*(a + b*ArcCoth[c*x]))/c^2 - (e*x^4*(a + b*ArcCoth[c*x]))/8 - (
e*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcCoth[c*x
])*(d + e*Log[1 - c^2*x^2]))/4 - b*c*(-1/8*((2*d - 3*e)*x)/c^4 + (2*e*x)/(
3*c^4) - ((2*d - e)*x^3)/(24*c^2) + (e*x^3)/(18*c^2) + ((2*d - 3*e)*ArcTan
h[c*x])/(8*c^5) - (2*e*ArcTanh[c*x])/(3*c^5) - (e*x*Log[1 - c^2*x^2])/(4*c
^4) - (e*x^3*Log[1 - c^2*x^2])/(12*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{-18\ \mathrm{arccoth}(cx)bd-18\ln(-c^2x^2+1)ae-18\ \mathrm{arccoth}(cx)b\,c^2e\,x^2+18\ln(-c^2x^2+1)bcex-18\ \mathrm{arccoth}(cx)\ln(-c^2x^2+1)be+18}{}$
orering	$\frac{(9c^8x^8+76x^6c^6-497x^4c^4+1570c^2x^2-1350)(a+b\ \mathrm{arccoth}(cx))(d+e\ln(-c^2x^2+1))}{12c^6x^2(c^2x^2+3)}-\frac{(19c^8x^8+274x^6c^6-2162x^4c^4+6210}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/72*(-18*arccoth(c*x)*b*d-18*ln(-c^2*x^2+1)*a*e-18*arccoth(c*x)*b*c^2*e*x
^2+18*ln(-c^2*x^2+1)*b*c*e*x-18*arccoth(c*x)*ln(-c^2*x^2+1)*b*e+18*b*c*d*x
-18*a*c^2*e*x^2+6*e*b*ln(-c^2*x^2+1)*x^3*c^3-9*e*b*arccoth(c*x)*x^4*c^4-75
*b*x*e*c-7*b*e*x^3*c^3+18*a*c^4*d*x^4-9*a*e*x^4*c^4+6*b*c^3*d*x^3+75*arcco
th(c*x)*b*e+18*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^4*c^4+18*a*e*ln(-c^2*x^2+
1)*x^4*c^4+18*b*arccoth(c*x)*x^4*c^4*d-18*a*e)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.87

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$
$$-\frac{36\,ac^2ex^2-18\,(2\,ac^4d-ac^4e)x^4-2\,(6\,bc^3d-7\,bc^3e)x^3-6\,(6\,bcd-25\,bce)x-12\,(3\,ac^4ex^4+bc^3ex^3}{}$$

input
```
integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b
*c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 +
 3*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d -
b*c^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*l
og((c*x + 1)/(c*x - 1)))/c^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.27

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$=\begin{cases} \frac{adx^4}{4}+\frac{aex^4\log\left(-c^2x^2+1\right)}{4}-\frac{aex^4}{8}-\frac{aex^2}{4c^2}-\frac{ae\log\left(-c^2x^2+1\right)}{4c^4}+\frac{bdx^4\operatorname{acoth}(cx)}{4}+\frac{bex^4\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{4}-\frac{bex^4\operatorname{aco}}{8} \\ \frac{dx^4\left(a+\frac{i\pi b}{2}\right)}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*
x**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*acoth(c*x)/4 +
 b*e*x**4*log(-c**2*x**2 + 1)*acoth(c*x)/4 - b*e*x**4*acoth(c*x)/8 + b*d*x
**3/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e
*x**2*acoth(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*
c**3) - 25*b*e*x/(24*c**3) - b*d*acoth(c*x)/(4*c**4) - b*e*log(-c**2*x**2
+ 1)*acoth(c*x)/(4*c**4) + 25*b*e*acoth(c*x)/(24*c**4), Ne(c, 0)), (d*x**4
*(a + I*pi*b/2)/4, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.20

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{4}\,adx^4$$
$$+\frac{1}{8}\left(2\,x^4\log\left(-c^2x^2+1\right)-c^2\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)be\,\mathrm{arcoth}\left(cx\right)$$
$$+\frac{1}{24}\left(6\,x^4\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bd$$
$$+\frac{1}{8}\left(2\,x^4\log\left(-c^2x^2+1\right)-c^2\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)ae$$
$$-\frac{(2\left(-6i\,\pi c^3+7\,c^3\right)x^3+6\left(-6i\,\pi c+25\,c\right)x+3\left(6i\,\pi-4\,c^3x^3-12\,cx-25\right)\log\left(cx+1\right)+3\left(-6i\,\pi-4\right.}{144\,c^4}$$

input
```
integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 +
2*log(c^2*x^2 - 1)/c^6))*b*e*arccoth(c*x) + 1/24*(6*x^4*arccoth(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/
8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
1)/c^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x
 + 3*(6*I*pi - 4*c^3*x^3 - 12*c*x - 25)*log(c*x + 1) + 3*(-6*I*pi - 4*c^3*
x^3 - 12*c*x + 25)*log(c*x - 1))*b*e/c^4
```

Giac [F(-2)]

Exception generated.

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.84

$$\int x^3(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,e\,x^5}{8}-\frac{b\,e\,x^3}{4\,c^2}+\frac{b\,c^2\,e\,x^7}{8}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,d\,x^5}{4}-\frac{b\,c^2\,d\,x^7}{4}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^5}{4}-\frac{b\,c^2\,e\,x^7}{4}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\left(x-c^2\,x^3\right)}{8\,c^4\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right)+x\,\left(\frac{b\,\left(6\,d-7\,e\right)}{24\,c^3}-\frac{3\,b\,e}{4\,c^3}\right)+\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^4}{4}+\frac{b\,e\,x}{4\,c^3}+\frac{b\,e\,x^3}{12\,c}\right)-\ln\left(\frac{1}{c\,x}+1\right)\,\left(\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e}{8\,c^4}-\frac{b\,e\,x^4}{8}\right)-\frac{b\,d\,x^4}{8}+\frac{b\,e\,x^4}{16}+\frac{b\,e\,x^2}{8\,c^2}\right)+x^2\,\left(\frac{a\,\left(2\,d-e\right)}{4\,c^2}-\frac{a\,d}{2\,c^2}\right)+\frac{a\,x^4\,\left(2\,d-e\right)}{8}-\frac{\ln\left(c\,x-1\right)\,\left(12\,a\,e-6\,b\,d+25\,b\,e\right)}{48\,c^4}-\frac{\ln\left(c\,x+1\right)\,\left(12\,a\,e+6\,b\,d-25\,b\,e\right)}{48\,c^4}+\frac{b\,x^3\,\left(6\,d-7\,e\right)}{72\,c}$$

input
```
int(x^3*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - 1/(c*x))*(((b*e*x^5)/8 - (b*e*x^3)/(4*c^2) + (b*c^2*e*x^7)/8)/(2*(
x + c*x^2)*(c*x - 1)) + ((b*d*x^5)/4 - (b*c^2*d*x^7)/4)/(2*(x + c*x^2)*(c*
x - 1)) + (log(1 - c^2*x^2)*((b*e*x^5)/4 - (b*c^2*e*x^7)/4))/(2*(x + c*x^2
)*(c*x - 1)) - (b*e*log(1 - c^2*x^2)*(x - c^2*x^3))/(8*c^4*(x + c*x^2)*(c*
x - 1))) + x*((b*(6*d - 7*e))/(24*c^3) - (3*b*e)/(4*c^3)) + log(1 - c^2*x^
2)*((a*e*x^4)/4 + (b*e*x)/(4*c^3) + (b*e*x^3)/(12*c)) - log(1/(c*x) + 1)*(
log(1 - c^2*x^2)*((b*e)/(8*c^4) - (b*e*x^4)/8) - (b*d*x^4)/8 + (b*e*x^4)/1
6 + (b*e*x^2)/(8*c^2)) + x^2*((a*(2*d - e))/(4*c^2) - (a*d)/(2*c^2)) + (a*
x^4*(2*d - e))/8 - (log(c*x - 1)*(12*a*e - 6*b*d + 25*b*e))/(48*c^4) - (lo
g(c*x + 1)*(12*a*e + 6*b*d - 25*b*e))/(48*c^4) + (b*x^3*(6*d - 7*e))/(72*c
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{18acoth(cx)\log(-c^2x^2+1)\,bc^4e\,x^4-18acoth(cx)\log(-c^2x^2+1)\,be+18acoth(cx)\,bc^4d\,x^4-9acoth(cx}{}$$

input
```
int(x^3*(a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(18*acoth(c*x)*log( - c**2*x**2 + 1)*b*c**4*e*x**4 - 18*acoth(c*x)*log( -
c**2*x**2 + 1)*b*e + 18*acoth(c*x)*b*c**4*d*x**4 - 9*acoth(c*x)*b*c**4*e*x
**4 - 18*acoth(c*x)*b*c**2*e*x**2 - 18*acoth(c*x)*b*d + 75*acoth(c*x)*b*e
+ 18*log( - c**2*x**2 + 1)*a*c**4*e*x**4 - 18*log( - c**2*x**2 + 1)*a*e -
6*log( - c**2*x**2 + 1)*b*c**3*e*x**3 - 18*log( - c**2*x**2 + 1)*b*c*e*x +
 18*a*c**4*d*x**4 - 9*a*c**4*e*x**4 - 18*a*c**2*e*x**2 - 6*b*c**3*d*x**3 +
 7*b*c**3*e*x**3 - 18*b*c*d*x + 75*b*c*e*x)/(72*c**4)
```

3.147 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\begin{aligned}\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= \frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)\\&\quad-\frac{b(d-e)\text{arctanh}(cx)}{2c^2}+\frac{be\text{arctanh}(cx)}{c^2}+\frac{bex\log\left(1-c^2x^2\right)}{2c}\\&\quad-\frac{e\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{2c^2}\end{aligned}$$

output
```
1/2*b*(d-e)*x/c-b*e*x/c+1/2*d*x^2*(a+b*arccoth(c*x))-1/2*e*x^2*(a+b*arccot
h(c*x))-1/2*b*(d-e)*arctanh(c*x)/c^2+b*e*arctanh(c*x)/c^2+1/2*b*e*x*ln(-c^
2*x^2+1)/c-1/2*e*(-c^2*x^2+1)*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \frac{2bc(d-3e)x+2ac^2(d-e)x^2+2bc^2(d-e)x^2\coth^{-1}(cx)+(b(d-3e)-2ae)\log(1-cx)-(b(d-3e)}{4c^2}$$

input
```
Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcCoth[c*x
] + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x
] + 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcCoth[c*x])*Log[1 - c^2*x^2]
)/(4*c^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{ 6646}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(1-c^2x^2\right)}-\frac{e\log\left(1-c^2x^2\right)}{2c^2}\right)dx-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e(1-c^2x^2)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-bc\left(\frac{(d-e)\text{arctanh}(cx)}{2c^3}-\frac{e\text{arctanh}(cx)}{c^3}-\frac{x(d-e)}{2c^2}-\frac{ex\log\left(1-c^2x^2\right)}{2c^2}+\frac{ex}{c^2}\right)$$

input
```
Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(a + b*ArcCoth[c*x]))/2 - (e*(1 -
c^2*x^2)*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(2*c^2) - b*c*(-1/2*((d -
e)*x)/c^2 + (e*x)/c^2 + ((d - e)*ArcTanh[c*x])/(2*c^3) - (e*ArcTanh[c*x])/
c^3 - (e*x*Log[1 - c^2*x^2])/(2*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{\operatorname{arccoth}(cx)\ln(-c^2x^2+1)b\,c^2e\,x^2+\operatorname{arccoth}(cx)b\,c^2d\,x^2-\operatorname{arccoth}(cx)b\,c^2e\,x^2+\ln(-c^2x^2+1)a\,c^2e\,x^2+a\,c^2d\,x^2-a\,c^2e\,x^2+\ln(\cdots}{2c^2}$
orering	$\frac{(x^6c^6+9c^2x^2-18)(a+b\operatorname{arccoth}(cx))(d+e\ln(-c^2x^2+1))}{c^4x^2(c^2x^2+3)}-\frac{(x^6c^6-2x^4c^4+21c^2x^2-36)\left((a+b\operatorname{arccoth}(cx))(d+e\ln(-c^2x\cdots\right.}{2c^4x^2(c^2\cdots}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arccoth(c*x)*ln(-c^2*x^2+1)*b*c^2*e*x^2+arccoth(c*x)*b*c^2*d*x^2-arcc
oth(c*x)*b*c^2*e*x^2+ln(-c^2*x^2+1)*a*c^2*e*x^2+a*c^2*d*x^2-a*c^2*e*x^2+ln
(-c^2*x^2+1)*b*c*e*x+b*c*d*x-3*b*x*e*c-arccoth(c*x)*ln(-c^2*x^2+1)*b*e-arc
coth(c*x)*b*d+3*arccoth(c*x)*b*e-ln(-c^2*x^2+1)*a*e)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{2\,(ac^2d-ac^2e)x^2+2\,(bcd-3\,bce)x+2\,(ac^2ex^2+bcex-ae)\log\left(-c^2x^2+1\right)+\left((bc^2d-bc^2e)x^2-bd+\cdots\right.}{4\,c^2}$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/4*(2*(a*c^2*d - a*c^2*e)*x^2 + 2*(b*c*d - 3*b*c*e)*x + 2*(a*c^2*e*x^2 +
b*c*e*x - a*e)*log(-c^2*x^2 + 1) + ((b*c^2*d - b*c^2*e)*x^2 - b*d + 3*b*e
+ (b*c^2*e*x^2 - b*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1)))/c^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log\left(-c^2x^2+1\right)}{2}-\frac{aex^2}{2}-\frac{ae\log\left(-c^2x^2+1\right)}{2c^2}+\frac{bdx^2\operatorname{acoth}(cx)}{2}+\frac{bex^2\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{2}-\frac{bex^2\operatorname{acoth}(cx)}{2}+\\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2}\end{cases}$$

input
```
integrate(x*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*
log(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*acoth(c*x)/2 + b*e*x**2*log(-c**2*
x**2 + 1)*acoth(c*x)/2 - b*e*x**2*acoth(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-
c**2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*acoth(c*x)/(2*c**2) - b*e*log(-
c**2*x**2 + 1)*acoth(c*x)/(2*c**2) + 3*b*e*acoth(c*x)/(2*c**2), Ne(c, 0)),
 (d*x**2*(a + I*pi*b/2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\operatorname{arcoth}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd$$
$$-\frac{(c^2x^2-(c^2x^2-1)\log(-c^2x^2+1)-1)be\operatorname{arcoth}(cx)}{2\,c^2}$$
$$-\frac{(c^2x^2-(c^2x^2-1)\log(-c^2x^2+1)-1)ae}{2\,c^2}$$
$$-\frac{(3\,cx-(cx+1)\log(cx+1)-(cx-1)\log(-cx+1))be}{2\,c^2}$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)
*b*e*arccoth(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1
)*a*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))
*b*e/c^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= -\frac{1}{4}bex^2\log\left(-cx+1\right)^2 - \frac{1}{4}(-i\pi bd+i\pi be-2\,ad+2\,ae)x^2 + \frac{1}{4}\left(bex^2-\frac{be}{c^2}\right)\log\left(cx+1\right)^2 - \frac{1}{4}\left((-i\pi be-bd-2\,ae+be)x^2-\frac{2\,bex}{c}\right)\log\left(cx+1\right) - \frac{be\log\left(cx-1\right)^2}{4\,c^2} - \frac{1}{4}\left((-i\pi be+bd-2\,ae-be)x^2-\frac{2\,bex}{c}-\frac{2\,be\log\left(cx-1\right)}{c^2}\right)\log\left(-cx+1\right) + \frac{(bd-3\,be)x}{2\,c} + \frac{(-i\pi be-bd-2\,ae+3\,be)\log\left(cx+1\right)}{4\,c^2} + \frac{(-i\pi be+bd-2\,ae-3\,be)\log\left(cx-1\right)}{4\,c^2}$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

output
```
-1/4*b*e*x^2*log(-c*x + 1)^2 - 1/4*(-I*pi*b*d + I*pi*b*e - 2*a*d + 2*a*e)*
x^2 + 1/4*(b*e*x^2 - b*e/c^2)*log(c*x + 1)^2 - 1/4*((-I*pi*b*e - b*d - 2*a
*e + b*e)*x^2 - 2*b*e*x/c)*log(c*x + 1) - 1/4*b*e*log(c*x - 1)^2/c^2 - 1/4
*((-I*pi*b*e + b*d - 2*a*e - b*e)*x^2 - 2*b*e*x/c - 2*b*e*log(c*x - 1)/c^2
)*log(-c*x + 1) + 1/2*(b*d - 3*b*e)*x/c + 1/4*(-I*pi*b*e - b*d - 2*a*e + 3
*b*e)*log(c*x + 1)/c^2 + 1/4*(-I*pi*b*e + b*d - 2*a*e - 3*b*e)*log(c*x - 1
)/c^2
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.35

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$=\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,d\,x^3}{2}-\frac{b\,c^2\,d\,x^5}{2}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{\frac{b\,e\,x^3}{2}-\frac{b\,c^2\,e\,x^5}{2}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^3}{2}-\frac{b\,c^2\,e\,x^5}{2}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\left(x-c^2\,x^3\right)}{4\,c^2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right)+\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^2}{2}+\frac{b\,e\,x}{2\,c}\right)-\ln\left(\frac{1}{c\,x}+1\right)\,\left(\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e}{4\,c^2}-\frac{b\,e\,x^2}{4}\right)-\frac{b\,d\,x^2}{4}+\frac{b\,e\,x^2}{4}\right)+\frac{a\,x^2\,\left(d-e\right)}{2}-\frac{\ln\left(c\,x+1\right)\,\left(2\,a\,e+b\,d-3\,b\,e\right)}{4\,c^2}-\frac{\ln\left(c\,x-1\right)\,\left(2\,a\,e-b\,d+3\,b\,e\right)}{4\,c^2}+\frac{b\,x\,\left(d-3\,e\right)}{2\,c}$$

input
```
int(x*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - 1/(c*x))*(((b*d*x^3)/2 - (b*c^2*d*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)
) - ((b*e*x^3)/2 - (b*c^2*e*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c
^2*x^2)*((b*e*x^3)/2 - (b*c^2*e*x^5)/2))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*
log(1 - c^2*x^2)*(x - c^2*x^3))/(4*c^2*(x + c*x^2)*(c*x - 1))) + log(1 - c
^2*x^2)*((a*e*x^2)/2 + (b*e*x)/(2*c)) - log(1/(c*x) + 1)*(log(1 - c^2*x^2)
*((b*e)/(4*c^2) - (b*e*x^2)/4) - (b*d*x^2)/4 + (b*e*x^2)/4) + (a*x^2*(d -
e))/2 - (log(c*x + 1)*(2*a*e + b*d - 3*b*e))/(4*c^2) - (log(c*x - 1)*(2*a*
e - b*d + 3*b*e))/(4*c^2) + (b*x*(d - 3*e))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \frac{acoth(cx)\log(-c^2x^2+1)\,b\,c^2e\,x^2-acoth(cx)\log(-c^2x^2+1)\,be+acoth(cx)\,b\,c^2d\,x^2-acoth(cx)\,b\,c^2e\,x^2}{}$$

input
```
int(x*(a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
(acoth(c*x)*log( - c**2*x**2 + 1)*b*c**2*e*x**2 - acoth(c*x)*log( - c**2*x
**2 + 1)*b*e + acoth(c*x)*b*c**2*d*x**2 - acoth(c*x)*b*c**2*e*x**2 - acoth
(c*x)*b*d + 3*acoth(c*x)*b*e + log( - c**2*x**2 + 1)*a*c**2*e*x**2 - log(
- c**2*x**2 + 1)*a*e - log( - c**2*x**2 + 1)*b*c*e*x + a*c**2*d*x**2 - a*c
**2*e*x**2 - b*c*d*x + 3*b*c*e*x)/(2*c**2)
```

3.148 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 381

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx \\
&= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x) \\
&\quad -be\log\left(\frac{c+\frac{1}{x}}{c}\right)\operatorname{PolyLog}\left(2,\frac{c+\frac{1}{x}}{c}\right)+be\log\left(1-\frac{1}{cx}\right)\operatorname{PolyLog}\left(2,1-\frac{1}{cx}\right) \\
&\quad +\frac{1}{2}bd\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right)+\frac{1}{2}be\log\left(-c^2x^2\right)\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}be\left(\log\left(1-\frac{1}{cx}\right)\right. \\
&\qquad \left.+\log\left(1+\frac{1}{cx}\right)+\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)\right)\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right) \\
&\quad -\frac{1}{2}bd\operatorname{PolyLog}\left(2,\frac{1}{cx}\right)-\frac{1}{2}be\log\left(-c^2x^2\right)\operatorname{PolyLog}\left(2,\frac{1}{cx}\right)+\frac{1}{2}be\left(\log\left(1-\frac{1}{cx}\right)\right. \\
&\qquad \left.+\log\left(1+\frac{1}{cx}\right)+\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)\right)\operatorname{PolyLog}\left(2,\frac{1}{cx}\right) \\
&\quad -\frac{1}{2}ae\operatorname{PolyLog}\left(2,c^2x^2\right)+be\operatorname{PolyLog}\left(3,\frac{c+\frac{1}{x}}{c}\right) \\
&\quad -be\operatorname{PolyLog}\left(3,1-\frac{1}{cx}\right)+be\operatorname{PolyLog}\left(3,-\frac{1}{cx}\right)-be\operatorname{PolyLog}\left(3,\frac{1}{cx}\right)
\end{aligned}$$

output
```
-1/2*b*e*ln(1+1/c/x)^2*ln(-1/c/x)+1/2*b*e*ln(1-1/c/x)^2*ln(1/c/x)+a*d*ln(x
)-b*e*ln((c+1/x)/c)*polylog(2,(c+1/x)/c)+b*e*ln(1-1/c/x)*polylog(2,1-1/c/x
)+1/2*b*d*polylog(2,-1/c/x)+1/2*b*e*ln(-c^2*x^2)*polylog(2,-1/c/x)-1/2*b*e
*(ln(1-1/c/x)+ln(1+1/c/x)+ln(-c^2*x^2)-ln(-c^2*x^2+1))*polylog(2,-1/c/x)-1
/2*b*d*polylog(2,1/c/x)-1/2*b*e*ln(-c^2*x^2)*polylog(2,1/c/x)+1/2*b*e*(ln(
1-1/c/x)+ln(1+1/c/x)+ln(-c^2*x^2)-ln(-c^2*x^2+1))*polylog(2,1/c/x)-1/2*a*e
*polylog(2,c^2*x^2)+b*e*polylog(3,(c+1/x)/c)-b*e*polylog(3,1-1/c/x)+b*e*po
lylog(3,-1/c/x)-b*e*polylog(3,1/c/x)
```

Mathematica [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x}\,dx = \int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x}\,dx$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.88, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6642, 6447, 6640, 2838, 6638, 2904, 2843, 27, 2881, 27, 2821, 6447, 6632, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\coth^{-1}(cx))\,(e\log(1 - c^2x^2) + d)}{x}\,dx$$

$$\Big\downarrow \text{6642}$$

$$e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{x}dx+d\int \frac{a+b\coth^{-1}(cx)}{x}dx$$

$\downarrow$ 6447

$$e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{x}dx+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 6640

$$e\left(a\int \frac{\log\left(1-c^2x^2\right)}{x}dx+b\int \frac{\coth^{-1}(cx)\log\left(1-c^2x^2\right)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 2838

$$e\left(b\int \frac{\coth^{-1}(cx)\log\left(1-c^2x^2\right)}{x}dx-\frac{1}{2}a\,\text{PolyLog}\left(2,c^2x^2\right)\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 6638

$$e\Bigg(b\Bigg(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int \frac{\coth^{-1}(cx)}{x}dx\right)+\int \frac{\coth^{-1}(cx)\,\mathrm{l}}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 2904

$$e\Bigg(b\Bigg(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int \frac{\coth^{-1}(cx)}{x}dx\right)+\int \frac{\coth^{-1}(cx)\,\mathrm{l}}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 2843

$$e\Bigg(b\Bigg(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int \frac{\coth^{-1}(cx)}{x}dx\right)+\int \frac{\coth^{-1}(cx)\,\mathrm{l}}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 27$$

$$e\left(b\left(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int\frac{\coth^{-1}(cx)}{x}dx\right)+\int\frac{\coth^{-1}(cx)\,\mathrm{l}}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\Big)$$

$$\Big\downarrow \ 2881$$

$$e\left(b\left(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int\frac{\coth^{-1}(cx)}{x}dx\right)+\int\frac{\coth^{-1}(cx)\,\mathrm{l}}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\Big)$$

$$\Big\downarrow \ 27$$

$$e\left(b\left(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int\frac{\coth^{-1}(cx)}{x}dx\right)+\int\frac{\coth^{-1}(cx)\,\mathrm{l}}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\Big)$$

$$\Big\downarrow \ 2821$$

$$e\left(b\left(-\left(\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int\frac{\coth^{-1}(cx)}{x}dx\right)+\int\frac{\coth^{-1}(cx)\,\mathrm{l}}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\Big)$$

$$\Big\downarrow \ 6447$$

$$e\left(b\left(\int\frac{\coth^{-1}(cx)\log\left(-c^2x^2\right)}{x}dx+\frac{1}{2}\left(\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)-2\left(\int x\,\mathrm{PolyLog}\left(2,1-\frac{1}{cx}\right)d\left(1-\frac{1}{cx}\right)-\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\Big)$$

$$\Big\downarrow \ 6632$$

$$e\left(b\left(-\frac{1}{2}\int\frac{\log\left(1-\frac{1}{cx}\right)\log\left(-c^2x^2\right)}{x}dx+\frac{1}{2}\int\frac{\log\left(1+\frac{1}{cx}\right)\log\left(-c^2x^2\right)}{x}dx+\frac{1}{2}\left(\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)-2\left(\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\Big)$$

$$\Big\downarrow \text{2821}$$

$$e\left(b\left(\frac{1}{2}\left(\text{PolyLog}\left(2,-\frac{1}{cx}\right)\log\left(-c^2x^2\right)-2\int\frac{\text{PolyLog}\left(2,-\frac{1}{cx}\right)}{x}dx\right)+\frac{1}{2}\left(2\int\frac{\text{PolyLog}\left(2,\frac{1}{cx}\right)}{x}dx-\text{PolyLog}\left(\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$e\left(b\left(-\left(\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\right.\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

output
```
d*(a*Log[x] + (b*PolyLog[2, -(1/(c*x))])/2 - (b*PolyLog[2, 1/(c*x)])/2) +
e*(-1/2*(a*PolyLog[2, c^2*x^2]) + b*(-((Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)
] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*(PolyLog[2, -(1/(c*x))]/2 - PolyLo
g[2, 1/(c*x)]/2)) + (Log[1 - 1/(c*x)]^2*Log[1/(c*x)] - 2*(-(Log[1 - 1/(c*x
)]*PolyLog[2, 1 - 1/(c*x)]) + PolyLog[3, 1 - 1/(c*x)]))/2 + (-(Log[1 + 1/(
c*x)]^2*Log[-(1/(c*x))]) + 2*(-(Log[1 + 1/(c*x)]*PolyLog[2, 1 + 1/(c*x)])
+ PolyLog[3, 1 + 1/(c*x)]))/2 + (Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))] +
2*PolyLog[3, -(1/(c*x))])/2 + (-(Log[-(c^2*x^2)]*PolyLog[2, 1/(c*x)]) - 2*
PolyLog[3, 1/(c*x)])/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6632
```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Si
mp[1/2   Int[Log[d*x^m]*(Log[1 + 1/(c*x^n)]/x), x], x] - Simp[1/2   Int[Log
[d*x^m]*(Log[1 - 1/(c*x^n)]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

rule 6638
```
Int[(ArcCoth[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp
[(Log[f + g*x^2] - Log[(-c^2)*x^2] - Log[1 - 1/(c*x)] - Log[1 + 1/(c*x)])
 Int[ArcCoth[c*x]/x, x], x] + (Int[Log[(-c^2)*x^2]*(ArcCoth[c*x]/x), x] + S
imp[1/2   Int[Log[1 + 1/(c*x)]^2/x, x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)
]^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

rule 6640
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcCoth[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x
^2]*(ArcCoth[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 6642
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcCoth[c*x])/x, x], x] + Si
mp[e   Int[Log[f + g*x^2]*((a + b*ArcCoth[c*x])/x), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.11 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{\left(-2 i \pi b e \,\mathrm{csgn}\left(i \left(c x -1\right) \left(c x +1\right)\right)^{2}-i \pi b e \,\mathrm{csgn}\left(i \left(c x -1\right)\right) \mathrm{csgn}\left(i \left(c x +1\right)\right) \mathrm{csgn}\left(i \left(c x -1\right) \left(c x +1\right)\right)+i \pi b e \,\mathrm{csgn}\left(i \left(c x -1\right)\right) \mathrm{csgn}\left(i \left(c x -1\right) \left(c x\right.\right.\right.}{}$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2-I*Pi*b*e*csgn(I*(c*x-1))*csgn(
I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)
*(c*x+1))^2+I*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+I*Pi*b*e*cs
gn(I*(c*x-1)*(c*x+1))^3+2*I*e*Pi*b-4*a*e+2*b*d)*(dilog(c*x)+ln(c*x-1)*ln(c
*x))+1/2*a*(-2*I*e*Pi*csgn(I*(c*x-1)*(c*x+1))^2-I*e*Pi*csgn(I*(c*x-1))*csg
n(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+I*e*Pi*csgn(I*(c*x-1))*csgn(I*(c*x-1)
*(c*x+1))^2+I*e*Pi*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+I*e*Pi*csgn(I
*(c*x-1)*(c*x+1))^3+2*I*e*Pi+2*d)*ln(c*x)-1/2*ln(c*x)*ln(c*x-1)^2*b*e-poly
log(2,-c*x+1)*ln(c*x-1)*b*e+polylog(3,-c*x+1)*b*e-(-1/2*I*Pi*b*e*csgn(I*(c
*x-1)*(c*x+1))^2-1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-
1)*(c*x+1))+1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*P
i*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x-1
)*(c*x+1))^3+1/2*I*e*Pi*b+a*e+1/2*b*d)*dilog(c*x+1)+1/2*ln(-c*x)*ln(c*x+1)
^2*b*e+polylog(2,c*x+1)*ln(c*x+1)*b*e-polylog(3,c*x+1)*b*e
```

Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas"
)
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x, x)
```

Sympy [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x}\,dx = \int \frac{(a + b\operatorname{acoth}(cx))\,(d + e\log(-c^2x^2 + 1))}{x}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.44

$$\begin{aligned}\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x}\,dx &= i\,\pi ae\log(x) \\ &\quad - \frac{1}{2}\left(\log(cx-1)^2\log(cx) + 2\operatorname{Li}_2(-cx+1)\log(cx-1) - 2\operatorname{Li}_3(-cx+1)\right)be \\ &\quad + \frac{1}{2}\left(\log(cx+1)^2\log(-cx) + 2\operatorname{Li}_2(cx+1)\log(cx+1) - 2\operatorname{Li}_3(cx+1)\right)be \\ &\quad + ad\log(x) - \frac{1}{2}(i\,\pi be + bd - 2\,ae)(\log(cx-1)\log(cx) + \operatorname{Li}_2(-cx+1)) \\ &\quad - \frac{1}{2}(-i\,\pi be - bd - 2\,ae)(\log(cx+1)\log(-cx) + \operatorname{Li}_2(cx+1))\end{aligned}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima"
)
```

output
```
I*pi*a*e*log(x) - 1/2*(log(c*x - 1)^2*log(c*x) + 2*dilog(-c*x + 1)*log(c*x
 - 1) - 2*polylog(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*di
log(c*x + 1)*log(c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*
(I*pi*b*e + b*d - 2*a*e)*(log(c*x - 1)*log(c*x) + dilog(-c*x + 1)) - 1/2*(
-I*pi*b*e - b*d - 2*a*e)*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))
```

Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(b\,\mathrm{arcoth}(cx)+a)(e\log(-c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x, x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx &= -\left(\int \frac{acoth(cx)}{c^2x^3-x}dx\right)bd-\left(\int \frac{\log(-c^2x^2+1)}{c^2x^3-x}dx\right)ae \\ &+\left(\int \frac{acoth(cx)\log(-c^2x^2+1)\,x}{c^2x^2-1}dx\right)b\,c^2e-\left(\int \frac{acoth(cx)\log(-c^2x^2+1)}{c^2x^3-x}dx\right)be \\ &+\left(\int \frac{acoth(cx)\,x}{c^2x^2-1}dx\right)b\,c^2d+\frac{\log(-c^2x^2+1)^2\,ae}{4}+\log(x)\,ad\end{aligned}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x)
```

output
```
( - 4*int(acoth(c*x)/(c**2*x**3 - x),x)*b*d - 4*int(log( - c**2*x**2 + 1)/
(c**2*x**3 - x),x)*a*e + 4*int((acoth(c*x)*log( - c**2*x**2 + 1)*x)/(c**2*
x**2 - 1),x)*b*c**2*e - 4*int((acoth(c*x)*log( - c**2*x**2 + 1))/(c**2*x**
3 - x),x)*b*e + 4*int((acoth(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2*d + log( -
c**2*x**2 + 1)**2*a*e + 4*log(x)*a*d)/4
```

3.149 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = -\frac{c^2e\left(a+b\coth^{-1}(cx)\right)^2}{2b}-bc^2e\,\mathrm{arctanh}(cx) -\frac{1}{2}bc^2e\,\mathrm{arctanh}(cx)^2+bc^2e\,\mathrm{arctanh}(cx)\log\left(\frac{2}{1-cx}\right) -\frac{bc(d+e\log(1-c^2x^2))}{2x}-\frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{2x^2} +\frac{1}{2}bc^2\,\mathrm{arctanh}(cx)\,(d+e\log(1-c^2x^2))-c^2e\left(a+b\coth^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right) +\frac{1}{2}bc^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}bc^2e\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)$$

output
```
-1/2*c^2*e*(a+b*arccoth(c*x))^2/b-b*c^2*e*arctanh(c*x)-1/2*b*c^2*e*arctanh
(c*x)^2+b*c^2*e*arctanh(c*x)*ln(2/(-c*x+1))-1/2*b*c*(d+e*ln(-c^2*x^2+1))/x
-1/2*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2+1/2*b*c^2*arctanh(c*x)*(d
+e*ln(-c^2*x^2+1))-c^2*e*(a+b*arccoth(c*x))*ln(2-2/(c*x+1))+1/2*b*c^2*e*po
lylog(2,1-2/(-c*x+1))+1/2*b*c^2*e*polylog(2,-1+2/(c*x+1))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx$$
$$=\frac{1}{2}\left(-\frac{ad}{x^2}-2ac^2e\log(x)+(a+b)c^2e\log(1-cx)+(a-b)c^2e\log(1+cx)\right.$$
$$-\frac{bd(2\coth^{-1}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))}{2x^2}$$
$$-\frac{e(a+bcx+(b-bc^2x^2)\coth^{-1}(cx))\log(1-c^2x^2)}{x^2}$$
$$\left.-bc^2e\left(\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

output
```
(-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^
2*e*Log[1 + c*x] - (b*d*(2*ArcCoth[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*
Log[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcCoth[c*x])*Lo
g[1 - c^2*x^2])/x^2 - b*c^2*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2, 1/(c*x)
]))/2
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{x^3}\,dx$$
$$\Big\downarrow \; 6648$$

$$2c^2e\int \left(\frac{bc^2x\text{arctanh}(cx)}{2\left(1-c^2x^2\right)}-\frac{a+bcx+b\coth^{-1}(cx)}{2x\left(1-c^2x^2\right)}\right)dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{2x}$$

$$\Big\downarrow \text{ 2009}$$

$$2c^2e\left(\frac{1}{4}(a+b)\log(1-cx)+\frac{1}{4}(a-b)\log(cx+1)-\frac{1}{2}a\log(x)-\frac{1}{4}b\text{arctanh}(cx)^2+\frac{1}{2}b\text{arctanh}(cx)\log\left(\frac{2}{1-cx}\right)\right.$$
$$-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{2x}$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[1 - c^2*x^2]))/x - ((a + b*ArcCoth[c*x])*(d + e*Log[1
 - c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/2 +
2*c^2*e*(-1/4*(b*ArcCoth[c*x]^2) - (b*ArcTanh[c*x]^2)/4 - (a*Log[x])/2 + (
b*ArcTanh[c*x]*Log[2/(1 - c*x)])/2 + ((a + b)*Log[1 - c*x])/4 + ((a - b)*L
og[1 + c*x])/4 - (b*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)])/2 + (b*PolyLog[2, 1
 - 2/(1 - c*x)])/4 + (b*PolyLog[2, -1 + 2/(1 + c*x)])/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(a+b\operatorname{arccoth}(cx))\,(d+e\ln(-c^2x^2+1))}{x^3}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(a+b\operatorname{acoth}(cx))\,(d+e\log(-c^2x^2+1))}{x^3}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxim
a")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d +
 1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e - 1/4
*b*e*(log(c*x + 1)^2/x^2 - 2*integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi
+ (I*pi*c + c)*x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^4 +
 x^3), x)) - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log{(1-c^2x^2)})}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log{(1-c^2x^2)})}{x^3}\,dx = \frac{acoth(cx)\,b\,c^2d\,x^2-acoth(cx)\,bd+2\left(\int \frac{acoth(cx)\log(-c^2x^2+1)}{x^3}dx\right)be\,x^2+\log(-c^2x^2+1)\,a\,c^2e\,x^2-\log(-c^2}{2x^2}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x)
```

output
```
(acoth(c*x)*b*c**2*d*x**2 - acoth(c*x)*b*d + 2*int((acoth(c*x)*log( - c**2
*x**2 + 1))/x**3,x)*b*e*x**2 + log( - c**2*x**2 + 1)*a*c**2*e*x**2 - log(
- c**2*x**2 + 1)*a*e - 2*log(x)*a*c**2*e*x**2 - a*d + b*c*d*x)/(2*x**2)
```

3.150 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 285

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx \\
&= \frac{5bc^3e}{12x}+\frac{c^2e(a+b\coth^{-1}(cx))}{4x^2}-\frac{c^4e(a+b\coth^{-1}(cx))^2}{4b}-\frac{11}{12}bc^4e\,\mathrm{arctanh}(cx) \\
&\quad -\frac{1}{4}bc^4e\,\mathrm{arctanh}(cx)^2+\frac{1}{2}bc^4e\,\mathrm{arctanh}(cx)\log\left(\frac{2}{1-cx}\right)-\frac{bc(d+e\log(1-c^2x^2))}{12x^3} \\
&\quad -\frac{bc^3(d+e\log(1-c^2x^2))}{4x}-\frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{4x^4} \\
&\quad +\frac{1}{4}bc^4\,\mathrm{arctanh}(cx)\,(d+e\log(1-c^2x^2))-\frac{1}{2}c^4e(a+b\coth^{-1}(cx))\log\left(2-\frac{2}{1+cx}\right) \\
&\quad +\frac{1}{4}bc^4e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{4}bc^4e\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)
\end{aligned}$$

output
```
5/12*b*c^3*e/x+1/4*c^2*e*(a+b*arccoth(c*x))/x^2-1/4*c^4*e*(a+b*arccoth(c*x
))^2/b-11/12*b*c^4*e*arctanh(c*x)-1/4*b*c^4*e*arctanh(c*x)^2+1/2*b*c^4*e*a
rctanh(c*x)*ln(2/(-c*x+1))-1/12*b*c*(d+e*ln(-c^2*x^2+1))/x^3-1/4*b*c^3*(d+
e*ln(-c^2*x^2+1))/x-1/4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4+1/4*b*
c^4*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/2*c^4*e*(a+b*arccoth(c*x))*ln(2-2/
(c*x+1))+1/4*b*c^4*e*polylog(2,1-2/(-c*x+1))+1/4*b*c^4*e*polylog(2,-1+2/(c
*x+1))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx$$
$$= -\frac{ad}{4x^4}+\frac{ac^2e}{4x^2}+\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}\left(3ac^4e+4bc^4e\right)\log(1-cx)$$
$$-\frac{1}{2}bc^4e\left(-\frac{\coth^{-1}(cx)}{2c^2x^2}+\frac{1}{2}\left(-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)$$
$$+bc^4d\left(-\frac{\coth^{-1}(cx)}{4c^4x^4}+\frac{1}{4}\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)$$
$$+\frac{1}{12}\left(3ac^4e-4bc^4e\right)\log(1+cx)$$
$$+\frac{e\left(-3a-bcx-3bc^3x^3-3b\coth^{-1}(cx)+3bc^4x^4\coth^{-1}(cx)\right)\log(1-c^2x^2)}{12x^4}$$
$$-\frac{1}{4}bc^4e\left(\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right)-\operatorname{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

output
```
-1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2
+ ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcCoth[c*x]/
(c^2*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d
*(-1/4*ArcCoth[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]
/2 + Log[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(
-3*a - b*c*x - 3*b*c^3*x^3 - 3*b*ArcCoth[c*x] + 3*b*c^4*x^4*ArcCoth[c*x])*
Log[1 - c^2*x^2])/(12*x^4) - (b*c^4*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2,
 1/(c*x)]))/4
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x^5}\,dx$$

$$\Big\downarrow \text{6648}$$

$$2c^2e\int\left(\frac{bc^4x\text{arctanh}(cx)}{4\,(1-c^2x^2)}-\frac{3bc^3x^3+bcx+3a+3b\coth^{-1}(cx)}{12x^3\,(1-c^2x^2)}\right)dx-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{4}bc^4\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc(e\log\left(1-c^2x^2\right)+d)}{12x^3}-\frac{bc^3\left(e\log\left(1-c^2x^2\right)+d\right)}{4x}$$

$$\Big\downarrow \text{2009}$$

$$2c^2e\left(\frac{1}{24}c^2(3a+4b)\log(1-cx)+\frac{1}{24}c^2(3a-4b)\log(cx+1)-\frac{1}{4}ac^2\log(x)+\frac{a}{8x^2}-\frac{1}{8}bc^2\text{arctanh}(cx)^2-\frac{1}{8}bc^2\text{ar}\right.$$
$$-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{4}bc^4\text{arctanh}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc(e\log\left(1-c^2x^2\right)+d)}{12x^3}-\frac{bc^3\left(e\log\left(1-c^2x^2\right)+d\right)}{4x}$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

output
```
-1/12*(b*c*(d + e*Log[1 - c^2*x^2]))/x^3 - (b*c^3*(d + e*Log[1 - c^2*x^2])
)/(4*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(4*x^4) + (b*c^4
*ArcTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/4 + 2*c^2*e*(a/(8*x^2) + (5*b*c)/(
24*x) + (b*ArcCoth[c*x])/(8*x^2) - (b*c^2*ArcCoth[c*x]^2)/8 - (b*c^2*ArcTa
nh[c*x])/8 - (b*c^2*ArcTanh[c*x]^2)/8 - (a*c^2*Log[x])/4 + (b*c^2*ArcTanh[
c*x]*Log[2/(1 - c*x)])/4 + ((3*a + 4*b)*c^2*Log[1 - c*x])/24 + ((3*a - 4*b
)*c^2*Log[1 + c*x])/24 - (b*c^2*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)])/4 + (b*
c^2*PolyLog[2, 1 - 2/(1 - c*x)])/8 + (b*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/
8)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arccoth}\,(cx))\,(d+e\ln{(-c^2x^2+1)})}{x^5}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log{(1-c^2x^2)})}{x^5}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log{(-c^2x^2+1)}+d)}{x^5}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^5, x)
```

Sympy [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log(-c^2x^2+1))}{x^5}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxim
a")
```

output
```
1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c
- 6*arccoth(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/
x^2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e - 1/8*b*e*(log(c*x + 1)^2/x^4 - 4*in
tegrate(-1/2*(2*(c*x + 1)*log(c*x - 1)^2 - (2*I*pi + (2*I*pi*c + c)*x)*log
(c*x + 1) + 2*(I*pi + I*pi*c*x)*log(c*x - 1))/(c*x^6 + x^5), x)) - 1/4*a*d
/x^4
```

Giac [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{acoth}(c\,x)\right)\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^5}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^5,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^5, x)
```

Reduce [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = \frac{3acoth(cx)\,b\,c^4 d\,x^4-3acoth(cx)\,bd+12\left(\int \frac{acoth(cx)\log(-c^2x^2+1)}{x^5}dx\right)be\,x^4+3\log(-c^2x^2+1)\,a\,c^4 e\,x^4-3\,\mathrm{l}}{12x^4}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x)
```

output
```
(3*acoth(c*x)*b*c**4*d*x**4 - 3*acoth(c*x)*b*d + 12*int((acoth(c*x)*log( -
 c**2*x**2 + 1))/x**5,x)*b*e*x**4 + 3*log( - c**2*x**2 + 1)*a*c**4*e*x**4
- 3*log( - c**2*x**2 + 1)*a*e - 6*log(x)*a*c**4*e*x**4 + 3*a*c**2*e*x**2 -
 3*a*d + 3*b*c**3*d*x**3 + b*c*d*x)/(12*x**4)
```

3.151 $\int x^4(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 263

$$\begin{aligned}\int x^4(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= -\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{9bex^4}{200c}-\frac{2bex\coth^{-1}(cx)}{5c^4}-\frac{2ex^3(a+b\coth^{-1}(cx))}{15c^2}\\&\quad-\frac{2}{25}ex^5(a+b\coth^{-1}(cx))+\frac{e(a+b\coth^{-1}(cx))^2}{5bc^5}-\frac{137be\log(1-c^2x^2)}{300c^5}\\&\quad-\frac{be\log^2(1-c^2x^2)}{20c^5}+\frac{bx^2(d+e\log(1-c^2x^2))}{10c^3}+\frac{bx^4(d+e\log(1-c^2x^2))}{20c}\\&\quad+\frac{1}{5}x^5(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\log(1-c^2x^2)(d+e\log(1-c^2x^2))}{10c^5}\end{aligned}$$

output

```
-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-9/200*b*e*x^4/c-2/5*b*e*x*arccoth(c*x)/c
^4-2/15*e*x^3*(a+b*arccoth(c*x))/c^2-2/25*e*x^5*(a+b*arccoth(c*x))+1/5*e*(
a+b*arccoth(c*x))^2/b/c^5-137/300*b*e*ln(-c^2*x^2+1)/c^5-1/20*b*e*ln(-c^2*
x^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(-c^2*x^2+1))/c^3+1/20*b*x^4*(d+e*ln(-c^2*x
^2+1))/c+1/5*x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/10*b*ln(-c^2*x^
2+1)*(d+e*ln(-c^2*x^2+1))/c^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.90

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-240acex+2bc^2(30d-77e)x^2-80ac^3ex^3+3bc^4(10d-9e)x^4+24ac^5(5d-2e)x^5-8bcx(-15c^4dx^4+\cdots}{}$$

input
```
Integrate[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
 - 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15
+ 5*c^2*x^2 + 3*c^4*x^4))*ArcCoth[c*x] + 120*b*e*ArcCoth[c*x]^2 + 2*(30*b*
d - 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 +
 c*x] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcCoth[
c*x])*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \; 6648$$

$$2c^2e\int\left(\frac{4ac^3x^6+4bc^3\coth^{-1}(cx)x^6+bc^2x^5+2bx^3}{20c^3\left(1-c^2x^2\right)}+\frac{bx\log\left(1-c^2x^2\right)}{10c^5\left(1-c^2x^2\right)}\right)dx+\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}$$

$$\Big\downarrow \; 2009$$

$$2c^2e\left(\frac{a\text{arctanh}(cx)}{5c^7}-\frac{ax}{5c^6}-\frac{ax^3}{15c^4}-\frac{ax^5}{25c^2}+\frac{b\coth^{-1}(cx)^2}{10c^7}-\frac{bx\coth^{-1}(cx)}{5c^6}-\frac{77bx^2}{600c^5}-\frac{bx^3\coth^{-1}(cx)}{15c^4}-\frac{9bx^4}{400c^3}\right.$$
$$\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}+$$
$$\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}$$

input
```
Int[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*Log[1 - c^2*x^2]
))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/5 + (b*Log
[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5) + 2*c^2*e*(-1/5*(a*x)/c^6
 - (77*b*x^2)/(600*c^5) - (a*x^3)/(15*c^4) - (9*b*x^4)/(400*c^3) - (a*x^5)
/(25*c^2) - (b*x*ArcCoth[c*x])/(5*c^6) - (b*x^3*ArcCoth[c*x])/(15*c^4) - (
b*x^5*ArcCoth[c*x])/(25*c^2) + (b*ArcCoth[c*x]^2)/(10*c^7) + (a*ArcTanh[c*
x])/(5*c^7) - (137*b*Log[1 - c^2*x^2])/(600*c^7) - (b*Log[1 - c^2*x^2]^2)/
(40*c^7))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{-274\ln(-c^2x^2+1)be-154be+60bd-240aecx-27be\,x^4c^4-154be\,x^2c^2-48a\,c^5e\,x^5-80ae\,x^3c^3+30be\ln(-c^2x^2+1)x^4c^4+120a}{}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/600*(-274*ln(-c^2*x^2+1)*b*e-154*b*e+60*b*d-240*a*e*c*x-27*b*e*x^4*c^4-1
54*b*e*x^2*c^2-48*a*c^5*e*x^5-80*a*e*x^3*c^3+30*b*e*ln(-c^2*x^2+1)*x^4*c^4
+120*a*e*ln(-c^2*x^2+1)*x^5*c^5+120*b*arccoth(c*x)*x^5*c^5*d-48*b*arccoth(
c*x)*x^5*c^5*e+60*x^2*ln(-c^2*x^2+1)*b*c^2*e+30*b*c^4*d*x^4+60*b*c^2*d*x^2
+120*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^5*c^5-240*arccoth(c*x)*b*c*e*x+30*e
*b*ln(-c^2*x^2+1)^2+120*e*b*arccoth(c*x)^2+60*ln(-c^2*x^2+1)*b*d+240*arcco
th(c*x)*a*e+120*a*c^5*d*x^5-80*e*b*arccoth(c*x)*x^3*c^3)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$
$$-\frac{80\,ac^3ex^3-24\,(5\,ac^5d-2\,ac^5e)x^5-3\,(10\,bc^4d-9\,bc^4e)x^4+240\,acex-30\,be\log\left(-c^2x^2+1\right)^2-30\,b}{}$$

input
```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d -
9*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*log(-c^2*x^2 + 1)^2 - 30*b*e*log((c*
x + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5
+ 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*log(-c^2*x^2 + 1) -
4*(15*b*c^5*e*x^5*log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*
c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*log((c*x + 1)/(c*x - 1)))/c^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.31

$$\int x^4(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$=\begin{cases}\frac{adx^5}{5}+\frac{aex^5\log\left(-c^2x^2+1\right)}{5}-\frac{2aex^5}{25}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\operatorname{acoth}(cx)}{5c^5}+\frac{bdx^5\operatorname{acoth}(cx)}{5}+\frac{bex^5\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{5}- \\ \frac{dx^5\left(a+\frac{i\pi b}{2}\right)}{5}\end{cases}$$

input
```
integrate(x**4*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2
*a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*acoth(c*x)/(5*c**5) + b*d*x
**5*acoth(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*acoth(c*x)/5 - 2*b*e*x**5*
acoth(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*
b*e*x**4/(200*c) - 2*b*e*x**3*acoth(c*x)/(15*c**2) + b*d*x**2/(10*c**3) +
b*e*x**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*
acoth(c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x*
*2 + 1)**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*acoth(
c*x)**2/(5*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.21

$$\int x^4(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{5}\,adx^5$$
$$+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)be$$
$$+\frac{1}{20}\left(4\,x^5\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)bd$$
$$+\frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)ae$$
$$-\frac{(3\,(-10i\,\pi c^4+9\,c^4)x^4+2\,(-30i\,\pi c^2+77\,c^2)x^2+2\,(-30i\,\pi-15\,c^4x^4-30\,c^2x^2-60\,\log\left(cx-1\right)+1}{600\,c^5}$$

input
```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x
^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*e*arccoth(c
*x) + 1/20*(4*x^5*arccoth(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2
- 1)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^
2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/60
0*(3*(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 + 2*(-30*I
*pi - 15*c^4*x^4 - 30*c^2*x^2 - 60*log(c*x - 1) + 137)*log(c*x + 1) + 2*(-
30*I*pi - 15*c^4*x^4 - 30*c^2*x^2 + 137)*log(c*x - 1))*b*e/c^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.37

$$
\begin{aligned}
&\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx \\
&= -\frac{1}{10}bex^5\log(-cx+1)^2-\frac{1}{50}(-5i\pi bd+2i\pi be-10\,ad+4\,ae)x^5 \\
&\quad+\frac{(10\,bd-9\,be)x^4}{200\,c}+\frac{1}{10}\left(bex^5+\frac{be}{c^5}\right)\log(cx+1)^2-\frac{(i\pi be+2\,ae)x^3}{15\,c^2} \\
&\quad-\frac{1}{300}\left(6\,(-5i\pi be-5\,bd-10\,ae+2\,be)x^5-\frac{15\,bex^4}{c}+\frac{20\,bex^3}{c^2}-\frac{30\,bex^2}{c^3}+\frac{60\,bex}{c^4}\right)\log(cx \\
&\quad+1) \\
&\quad-\frac{1}{300}\left(6\,(-5i\pi be+5\,bd-10\,ae-2\,be)x^5-\frac{15\,bex^4}{c}-\frac{20\,bex^3}{c^2}-\frac{30\,bex^2}{c^3}-\frac{60\,bex}{c^4}-\frac{60\,be\log(cx-1}{c^5}\right. \\
&\quad+1)+\frac{(30\,bd-77\,be)x^2}{300\,c^3} \\
&\quad-\frac{be\log(cx-1)^2}{10\,c^5}-\frac{(i\pi be+2\,ae)x}{5\,c^4}+\frac{(30i\pi be+30\,bd+60\,ae-137\,be)\log(cx+1)}{300\,c^5} \\
&\quad+\frac{(-30i\pi be+30\,bd-60\,ae-137\,be)\log(cx-1)}{300\,c^5}
\end{aligned}
$$

input

```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output

```
-1/10*b*e*x^5*log(-c*x + 1)^2 - 1/50*(-5*I*pi*b*d + 2*I*pi*b*e - 10*a*d +
4*a*e)*x^5 + 1/200*(10*b*d - 9*b*e)*x^4/c + 1/10*(b*e*x^5 + b*e/c^5)*log(c
*x + 1)^2 - 1/15*(I*pi*b*e + 2*a*e)*x^3/c^2 - 1/300*(6*(-5*I*pi*b*e - 5*b*
d - 10*a*e + 2*b*e)*x^5 - 15*b*e*x^4/c + 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 +
 60*b*e*x/c^4)*log(c*x + 1) - 1/300*(6*(-5*I*pi*b*e + 5*b*d - 10*a*e - 2*b
*e)*x^5 - 15*b*e*x^4/c - 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 - 60*b*e*x/c^4 -
60*b*e*log(c*x - 1)/c^5)*log(-c*x + 1) + 1/300*(30*b*d - 77*b*e)*x^2/c^3 -
 1/10*b*e*log(c*x - 1)^2/c^5 - 1/5*(I*pi*b*e + 2*a*e)*x/c^4 + 1/300*(30*I*
pi*b*e + 30*b*d + 60*a*e - 137*b*e)*log(c*x + 1)/c^5 + 1/300*(-30*I*pi*b*e
 + 30*b*d - 60*a*e - 137*b*e)*log(c*x - 1)/c^5
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.89

$$\int x^4 \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= \ln\left(\frac{1}{c\,x}+1\right)\left(\frac{b\,d\,x^5}{10}-\frac{\frac{2\,b\,e\,c^5\,x^5}{5}+\frac{2\,b\,e\,c^3\,x^3}{3}+2\,b\,e\,c\,x}{10\,c^5}+\frac{b\,e\,x^5\,\ln\left(1-c^2\,x^2\right)}{10}\right)$$
$$+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,d\,x^6}{5}-\frac{b\,c^2\,d\,x^8}{5}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{4\,b\,e\,x^6}{75}+\frac{4\,b\,e\,x^4}{15\,c^2}-\frac{2\,b\,e\,x^2}{5\,c^4}+\frac{2\,b\,c^2\,e\,x^8}{25}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right.$$
$$\left.+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^6}{5}-\frac{b\,c^2\,e\,x^8}{5}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{10\,c^5}\right)$$
$$+x^3\left(\frac{a\,\left(5\,d-2\,e\right)}{15\,c^2}-\frac{a\,d}{3\,c^2}\right)+x^2\left(\frac{b\,\left(10\,d-9\,e\right)}{100\,c^3}-\frac{b\,e}{6\,c^3}\right)+\frac{x\,\left(\frac{a\,\left(5\,d-2\,e\right)}{5\,c^2}-\frac{a\,d}{c^2}\right)}{c^2}$$
$$+\frac{a\,x^5\,\left(5\,d-2\,e\right)}{25}+c^2\,\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^5}{5\,c^2}+\frac{b\,e\,x^4}{20\,c^3}+\frac{b\,e\,x^2}{10\,c^5}\right)$$
$$-\frac{\ln\left(c\,x-1\right)\,\left(60\,a\,e-30\,b\,d+137\,b\,e\right)}{300\,c^5}+\frac{\ln\left(c\,x+1\right)\,\left(60\,a\,e+30\,b\,d-137\,b\,e\right)}{300\,c^5}$$
$$+\frac{b\,e\ln\left(\frac{1}{c\,x}+1\right)^2}{20\,c^5}+\frac{b\,e\ln\left(1-\frac{1}{c\,x}\right)^2}{20\,c^5}+\frac{b\,e\ln\left(1-c^2\,x^2\right)^2}{20\,c^5}+\frac{b\,x^4\,\left(10\,d-9\,e\right)}{200\,c}$$

input
```
int(x^4*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1/(c*x) + 1)*((b*d*x^5)/10 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3 + (2*b*c^5
*e*x^5)/5)/(10*c^5) + (b*e*x^5*log(1 - c^2*x^2))/10) + log(1 - 1/(c*x))*((
(b*d*x^6)/5 - (b*c^2*d*x^8)/5)/(2*(x + c*x^2)*(c*x - 1)) + ((4*b*e*x^6)/75
 + (4*b*e*x^4)/(15*c^2) - (2*b*e*x^2)/(5*c^4) + (2*b*c^2*e*x^8)/25)/(2*(x
+ c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^6)/5 - (b*c^2*e*x^8)/5))/(
2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(10*c^5)) + x^3*((a*(5*d
 - 2*e))/(15*c^2) - (a*d)/(3*c^2)) + x^2*((b*(10*d - 9*e))/(100*c^3) - (b*
e)/(6*c^3)) + (x*((a*(5*d - 2*e))/(5*c^2) - (a*d)/c^2))/c^2 + (a*x^5*(5*d
- 2*e))/25 + c^2*log(1 - c^2*x^2)*((a*e*x^5)/(5*c^2) + (b*e*x^4)/(20*c^3)
+ (b*e*x^2)/(10*c^5)) - (log(c*x - 1)*(60*a*e - 30*b*d + 137*b*e))/(300*c^
5) + (log(c*x + 1)*(60*a*e + 30*b*d - 137*b*e))/(300*c^5) + (b*e*log(1/(c*
x) + 1)^2)/(20*c^5) + (b*e*log(1 - 1/(c*x))^2)/(20*c^5) + (b*e*log(1 - c^2
*x^2)^2)/(20*c^5) + (b*x^4*(10*d - 9*e))/(200*c)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.17

$$\int x^4 \left(a + b \coth^{-1}(cx)\right) \left(d + e \log\left(1 - c^2 x^2\right)\right) \, dx$$
$$= \frac{-120acoth(cx)^2 be + 120acoth(cx)\log(-c^2x^2+1)\, b\, c^5 e\, x^5 + 120acoth(cx)\, b\, c^5 d\, x^5 - 48acoth(cx)\, b\, c^5 e\, x^5}{}$$

input
```
int(x^4*(a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
( - 120*acoth(c*x)**2*b*e + 120*acoth(c*x)*log( - c**2*x**2 + 1)*b*c**5*e*
x**5 + 120*acoth(c*x)*b*c**5*d*x**5 - 48*acoth(c*x)*b*c**5*e*x**5 - 80*aco
th(c*x)*b*c**3*e*x**3 - 240*acoth(c*x)*b*c*e*x - 30*log( - c**2*x**2 + 1)*
*2*b*e + 120*log( - c**2*x**2 + 1)*a*c**5*e*x**5 + 120*log( - c**2*x**2 +
1)*a*e - 30*log( - c**2*x**2 + 1)*b*c**4*e*x**4 - 60*log( - c**2*x**2 + 1)
*b*c**2*e*x**2 - 60*log( - c**2*x**2 + 1)*b*d + 274*log( - c**2*x**2 + 1)*
b*e - 240*log(c**2*x - c)*a*e + 120*a*c**5*d*x**5 - 48*a*c**5*e*x**5 - 80*
a*c**3*e*x**3 - 240*a*c*e*x - 30*b*c**4*d*x**4 + 27*b*c**4*e*x**4 - 60*b*c
**2*d*x**2 + 154*b*c**2*e*x**2)/(600*c**5)
```

3.152 $\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\begin{aligned}
&\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx\\
&=-\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2bex\coth^{-1}(cx)}{3c^2}-\frac{2}{9}ex^3\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{3bc^3}\\
&\quad-\frac{11be\log\left(1-c^2x^2\right)}{18c^3}-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}\\
&\quad+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\log\left(1-c^2x^2\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{6c^3}
\end{aligned}$$

output
```
-2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/3*b*e*x*arccoth(c*x)/c^2-2/9*e*x^3*(a+b*ar
ccoth(c*x))+1/3*e*(a+b*arccoth(c*x))^2/b/c^3-11/18*b*e*ln(-c^2*x^2+1)/c^3-
1/12*b*e*ln(-c^2*x^2+1)^2/c^3+1/6*b*x^2*(d+e*ln(-c^2*x^2+1))/c+1/3*x^3*(a+
b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/6*b*ln(-c^2*x^2+1)*(d+e*ln(-c^2*x^2
+1))/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.89

$$\int x^2 \left(a + b\coth^{-1}(cx)\right)\left(d + e\log\left(1 - c^2x^2\right)\right)\,dx$$
$$= \frac{-24acex + 2bc^2(3d - 5e)x^2 + 4ac^3(3d - 2e)x^3 + 4bcx(3c^2dx^2 - 2e(3 + c^2x^2))\coth^{-1}(cx) + 12be\coth^{-1}}{}$$

input
```
Integrate[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x
*(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcCoth[c*x] + 12*b*e*ArcCoth[c*x]^2 +
2*(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1
 + c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcCoth[c*x])*Log[1 - c^2*x^2
] + 3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a + b\coth^{-1}(cx)\right)\left(e\log\left(1 - c^2x^2\right) + d\right)\,dx$$

$$\Big\downarrow \text{ 6648}$$

$$2c^2e\int \left(\frac{\left(2cx\coth^{-1}(cx)b + b + 2acx\right)x^3}{6c\left(1 - c^2x^2\right)} + \frac{b\log\left(1 - c^2x^2\right)x}{6c^3\left(1 - c^2x^2\right)}\right)dx + \frac{1}{3}x^3\left(a + b\coth^{-1}(cx)\right)\left(e\log\left(1 - c^2x^2\right) + d\right) + \frac{bx^2\left(e\log\left(1 - c^2x^2\right) + d\right)}{6c} + \frac{b\log\left(1 - c^2x^2\right)\left(e\log\left(1 - c^2x^2\right) + d\right)}{6c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$2c^2e\left(\frac{a\text{arctanh}(cx)}{3c^5}-\frac{ax}{3c^4}-\frac{ax^3}{9c^2}+\frac{b\coth^{-1}(cx)^2}{6c^5}-\frac{bx\coth^{-1}(cx)}{3c^4}-\frac{5bx^2}{36c^3}-\frac{bx^3\coth^{-1}(cx)}{9c^2}-\frac{b\log^2\left(1-c^2x^2\right.}{24c^5}\right.$$
$$\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}+$$
$$\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}$$

input
```
Int[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*
Log[1 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^
3) + 2*c^2*e*(-1/3*(a*x)/c^4 - (5*b*x^2)/(36*c^3) - (a*x^3)/(9*c^2) - (b*x
*ArcCoth[c*x])/(3*c^4) - (b*x^3*ArcCoth[c*x])/(9*c^2) + (b*ArcCoth[c*x]^2)
/(6*c^5) + (a*ArcTanh[c*x])/(3*c^5) - (11*b*Log[1 - c^2*x^2])/(36*c^5) - (
b*Log[1 - c^2*x^2]^2)/(24*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{12\,\mathrm{arccoth}(cx)bd+12\ln(cx-1)bd-24aecx+12ae\ln(-c^2x^2+1)x^3c^3+12b\,\mathrm{arccoth}(cx)x^3c^3d-10be\,x^2c^2-8ae\,x^3c^3+6x^2\ln(-}{}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^2*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(12*arccoth(c*x)*b*d+12*ln(c*x-1)*b*d-24*a*e*c*x+12*a*e*ln(-c^2*x^2+1
)*x^3*c^3+12*b*arccoth(c*x)*x^3*c^3*d-10*b*e*x^2*c^2-8*a*e*x^3*c^3+6*x^2*l
n(-c^2*x^2+1)*b*c^2*e+6*b*c^2*d*x^2+12*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^3
*c^3-44*ln(c*x-1)*b*e-24*arccoth(c*x)*b*c*e*x+3*e*b*ln(-c^2*x^2+1)^2+12*e*
b*arccoth(c*x)^2+24*arccoth(c*x)*a*e+12*x^3*a*d*c^3-44*arccoth(c*x)*b*e-8*
e*b*arccoth(c*x)*x^3*c^3)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.96

$$\int x^2(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = -\frac{24\,acex-4\,(3\,ac^3d-2\,ac^3e)x^3-3\,be\log\left(-c^2x^2+1\right)^2-3\,be\log\left(\frac{cx+1}{cx-1}\right)^2-2\,(3\,bc^2d-5\,bc^2e)x^2-2\,(}{}$$

input
```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*log(-c^2*x^2 + 1
)^2 - 3*b*e*log((c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 - 2
*(6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(-c^2*x^2 + 1) - 2*(3
*b*c^3*e*x^3*log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3 +
 6*a*e)*log((c*x + 1)/(c*x - 1)))/c^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.29

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^3\log\left(-c^2x^2+1\right)}{3}-\frac{2aex^3}{9}-\frac{2aex}{3c^2}+\frac{2ae\,\mathrm{acoth}\,(cx)}{3c^3}+\frac{bdx^3\,\mathrm{acoth}\,(cx)}{3}+\frac{bex^3\log\left(-c^2x^2+1\right)\mathrm{acoth}\,(cx)}{3}-\frac{2bex^3\,\mathrm{acot}}{9}\\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3}\end{cases}$$

input
```
integrate(x**2*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*
a*e*x/(3*c**2) + 2*a*e*acoth(c*x)/(3*c**3) + b*d*x**3*acoth(c*x)/3 + b*e*x
**3*log(-c**2*x**2 + 1)*acoth(c*x)/3 - 2*b*e*x**3*acoth(c*x)/9 + b*d*x**2/
(6*c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*a
coth(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2
 + 1)**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*acoth(c*x)
**2/(3*c**3), Ne(c, 0)), (d*x**3*(a + I*pi*b/2)/3, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.22

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{3}\,adx^3$$
$$+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)be\,\mathrm{arcoth}\,(cx)$$
$$+\frac{1}{6}\left(2\,x^3\,\mathrm{arcoth}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bd$$
$$+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)ae$$
$$+\frac{((3i\,\pi c^2-5\,c^2)x^2+(3i\,\pi+3\,c^2x^2+6\,\log\left(cx-1\right)-11)\log\left(cx+1\right)+(3i\,\pi+3\,c^2x^2-11)\log\left(cx-\right.}{18\,c^3}$$

input
```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 -
3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arccoth(c*x) + 1/6*(2*x^3*ar
ccoth(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2
*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x -
1)/c^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*lo
g(c*x - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*
e/c^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.37

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= -\frac{1}{6}\,bex^3\log\left(-cx+1\right)^2 - \frac{1}{18}\left(-3i\,\pi bd + 2i\,\pi be - 6\,ad + 4\,ae\right)x^3 + \frac{1}{6}\left(bex^3 + \frac{be}{c^3}\right)\log\left(cx+1\right)^2 + \frac{\left(3\,bd - 5\,be\right)x^2}{18\,c} - \frac{1}{18}\left(\left(-3i\,\pi be - 3\,bd - 6\,ae + 2\,be\right)x^3 - \frac{3\,bex^2}{c} + \frac{6\,bex}{c^2}\right)\log\left(cx+1\right) - \frac{1}{18}\left(\left(-3i\,\pi be + 3\,bd - 6\,ae - 2\,be\right)x^3 - \frac{3\,bex^2}{c} - \frac{6\,bex}{c^2} - \frac{6\,be\log\left(cx-1\right)}{c^3}\right)\log\left(-cx+1\right) - \frac{be\log\left(cx-1\right)^2}{6\,c^3} - \frac{\left(i\,\pi be + 2\,ae\right)x}{3\,c^2} + \frac{\left(3i\,\pi be + 3\,bd + 6\,ae - 11\,be\right)\log\left(cx+1\right)}{18\,c^3} + \frac{\left(-3i\,\pi be + 3\,bd - 6\,ae - 11\,be\right)\log\left(cx-1\right)}{18\,c^3}$$

input
```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
-1/6*b*e*x^3*log(-c*x + 1)^2 - 1/18*(-3*I*pi*b*d + 2*I*pi*b*e - 6*a*d + 4*
a*e)*x^3 + 1/6*(b*e*x^3 + b*e/c^3)*log(c*x + 1)^2 + 1/18*(3*b*d - 5*b*e)*x
^2/c - 1/18*((-3*I*pi*b*e - 3*b*d - 6*a*e + 2*b*e)*x^3 - 3*b*e*x^2/c + 6*b
*e*x/c^2)*log(c*x + 1) - 1/18*((-3*I*pi*b*e + 3*b*d - 6*a*e - 2*b*e)*x^3 -
 3*b*e*x^2/c - 6*b*e*x/c^2 - 6*b*e*log(c*x - 1)/c^3)*log(-c*x + 1) - 1/6*b
*e*log(c*x - 1)^2/c^3 - 1/3*(I*pi*b*e + 2*a*e)*x/c^2 + 1/18*(3*I*pi*b*e +
3*b*d + 6*a*e - 11*b*e)*log(c*x + 1)/c^3 + 1/18*(-3*I*pi*b*e + 3*b*d - 6*a
*e - 11*b*e)*log(c*x - 1)/c^3
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.01

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\ln\left(\frac{1}{c\,x}+1\right)\left(\frac{b\,d\,x^3}{6}-\frac{\frac{2\,b\,e\,c^3\,x^3}{3}+2\,b\,e\,c\,x}{6\,c^3}+\frac{b\,e\,x^3\,\ln\left(1-c^2\,x^2\right)}{6}\right)+x\left(\frac{a\,(3\,d-2\,e)}{3\,c^2}-\frac{a\,d}{c^2}\right)+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{4\,b\,e\,x^4}{9}-\frac{2\,b\,e\,x^2}{3\,c^2}+\frac{2\,b\,c^2\,e\,x^6}{9}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,d\,x^4}{3}-\frac{b\,c^2\,d\,x^6}{3}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^4}{3}-\frac{b\,c^2\,e\,x^6}{3}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{6\,c^3}\right)+\frac{a\,x^3\,(3\,d-2\,e)}{9}+c^2\,\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^3}{3\,c^2}+\frac{b\,e\,x^2}{6\,c^3}\right)-\frac{\ln\left(c\,x-1\right)\,\left(6\,a\,e-3\,b\,d+11\,b\,e\right)}{18\,c^3}+\frac{\ln\left(c\,x+1\right)\,\left(6\,a\,e+3\,b\,d-11\,b\,e\right)}{18\,c^3}+\frac{b\,e\ln\left(\frac{1}{c\,x}+1\right)^2}{12\,c^3}+\frac{b\,e\ln\left(1-\frac{1}{c\,x}\right)^2}{12\,c^3}+\frac{b\,e\ln\left(1-c^2\,x^2\right)^2}{12\,c^3}+\frac{b\,x^2\,(3\,d-5\,e)}{18\,c}$$

input
```
int(x^2*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1/(c*x) + 1)*((b*d*x^3)/6 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3)/(6*c^3) +
(b*e*x^3*log(1 - c^2*x^2))/6) + x*((a*(3*d - 2*e))/(3*c^2) - (a*d)/c^2) +
log(1 - 1/(c*x))*(((4*b*e*x^4)/9 - (2*b*e*x^2)/(3*c^2) + (2*b*c^2*e*x^6)/9
)/(2*(x + c*x^2)*(c*x - 1)) + ((b*d*x^4)/3 - (b*c^2*d*x^6)/3)/(2*(x + c*x^
2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^4)/3 - (b*c^2*e*x^6)/3))/(2*(x +
 c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(6*c^3)) + (a*x^3*(3*d - 2*e))
/9 + c^2*log(1 - c^2*x^2)*((a*e*x^3)/(3*c^2) + (b*e*x^2)/(6*c^3)) - (log(c
*x - 1)*(6*a*e - 3*b*d + 11*b*e))/(18*c^3) + (log(c*x + 1)*(6*a*e + 3*b*d
- 11*b*e))/(18*c^3) + (b*e*log(1/(c*x) + 1)^2)/(12*c^3) + (b*e*log(1 - 1/(
c*x))^2)/(12*c^3) + (b*e*log(1 - c^2*x^2)^2)/(12*c^3) + (b*x^2*(3*d - 5*e)
)/(18*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.18

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-12acoth(cx)^2\,be+12acoth(cx)\log(-c^2x^2+1)\,bc^3e\,x^3+12acoth(cx)\,bc^3d\,x^3-8acoth(cx)\,bc^3e\,x^3-2}{}$$

input
```
int(x^2*(a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
( - 12*acoth(c*x)**2*b*e + 12*acoth(c*x)*log( - c**2*x**2 + 1)*b*c**3*e*x*
*3 + 12*acoth(c*x)*b*c**3*d*x**3 - 8*acoth(c*x)*b*c**3*e*x**3 - 24*acoth(c
*x)*b*c*e*x - 3*log( - c**2*x**2 + 1)**2*b*e + 12*log( - c**2*x**2 + 1)*a*
c**3*e*x**3 + 12*log( - c**2*x**2 + 1)*a*e - 6*log( - c**2*x**2 + 1)*b*c**
2*e*x**2 - 6*log( - c**2*x**2 + 1)*b*d + 22*log( - c**2*x**2 + 1)*b*e - 24
*log(c**2*x - c)*a*e + 12*a*c**3*d*x**3 - 8*a*c**3*e*x**3 - 24*a*c*e*x - 6
*b*c**2*d*x**2 + 10*b*c**2*e*x**2)/(36*c**3)
```

3.153 $\int \left(a+b\coth^{-1}(cx)\right) \left(d+e\log\left(1-c^2x^2\right)\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \left(a+b\coth^{-1}(cx)\right) \left(d+e\log\left(1-c^2x^2\right)\right) \, dx = -2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c}+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\left(d+e\log\left(1-c^2x^2\right)\right)^2}{4ce}$$

output
```
-2*a*e*x-2*b*e*x*arccoth(c*x)+e*(a+b*arccoth(c*x))^2/b/c-b*e*ln(-c^2*x^2+1
)/c+x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/4*b*(d+e*ln(-c^2*x^2+1))^2
/c/e
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=adx-2aex+bdx\coth^{-1}(cx)-2bex\coth^{-1}(cx)+\frac{be\coth^{-1}(cx)^2}{c}$$
$$+\frac{2ae\text{arctanh}(cx)}{c}+\frac{bd\log\left(1-c^2x^2\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c}$$
$$+aex\log\left(1-c^2x^2\right)+bex\coth^{-1}(cx)\log\left(1-c^2x^2\right)+\frac{be\log^2\left(1-c^2x^2\right)}{4c}$$

input
```
Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] - 2*b*e*x*ArcCoth[c*x] + (b*e*ArcCoth
[c*x]^2)/c + (2*a*e*ArcTanh[c*x])/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*
Log[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcCoth[c*x]*Log[1 -
c^2*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6636, 2925, 2837, 2738, 6543, 2009, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{ 6636}$$

$$2c^2e\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{1-c^2x^2}dx-bc\int \frac{x\left(d+e\log\left(1-c^2x^2\right)\right)}{1-c^2x^2}dx+x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{ 2925}$$

$$2c^2e\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{1-c^2x^2}dx^2+x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)$$

$\Big\downarrow$ 2837

$$2c^2e\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{1-c^2x^2}dx+\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^2}d(1-c^2x^2)}{2c}+x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)$$

$\Big\downarrow$ 2738

$$2c^2e\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{1-c^2x^2}dx+x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$\Big\downarrow$ 6543

$$2c^2e\left(\frac{\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int \left(a+b\coth^{-1}(cx)\right)dx}{c^2}\right)+x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$\Big\downarrow$ 2009

$$2c^2e\left(\frac{\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{c^2}\right)+x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$\Big\downarrow$ 6511

$$x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+2c^2e\left(\frac{\left(a+b\coth^{-1}(cx)\right)^2}{2bc^3}-\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{c^2}\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

input
```
Int[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(d + e*Log[1 - c^2*x^
2])^2)/(4*c*e) + 2*c^2*e*((a + b*ArcCoth[c*x])^2/(2*b*c^3) - (a*x + b*x*Ar
cCoth[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6636
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp
[2*e*g   Int[x^2*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b
, c, d, e, f, g}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{4be\ln(-c^2x^2+1)\,\mathrm{arccoth}(cx)xc+4b\,\mathrm{arccoth}(cx)xcd-8\,\mathrm{arccoth}(cx)bcex+4aex\ln(-c^2x^2+1)c+4xadc-8aecx+4eb\,\mathrm{arccoth}(}{4c}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x*c+4*b*arccoth(c*x)*x*c*d-8*arccot
h(c*x)*b*c*e*x+4*a*e*x*ln(-c^2*x^2+1)*c+4*x*a*d*c-8*a*e*c*x+4*e*b*arccoth(
c*x)^2+e*b*ln(-c^2*x^2+1)^2+8*arccoth(c*x)*a*e+2*ln(-c^2*x^2+1)*b*d-4*ln(-
c^2*x^2+1)*b*e)/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.25

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{be\log\left(-c^2x^2+1\right)^2+be\log\left(\frac{cx+1}{cx-1}\right)^2+4\,(acd-2\,ace)x+2\,(2\,acex+bd-2\,be)\log\left(-c^2x^2+1\right)+2\,(bce}{4\,c}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/4*(b*e*log(-c^2*x^2 + 1)^2 + b*e*log((c*x + 1)/(c*x - 1))^2 + 4*(a*c*d -
 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*log(-c^2*x^2 + 1) + 2*(b*c*e*x*l
og(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*log((c*x + 1)/(c*x - 1)))/
c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases} adx+aex\log\left(-c^2x^2+1\right)-2aex+\frac{2ae\,\mathrm{acoth}\,(cx)}{c}+bdx\,\mathrm{acoth}\,(cx)+bex\log\left(-c^2x^2+1\right)\mathrm{acoth}\,(cx)-2 \\ dx\left(a+\frac{i\pi b}{2}\right) \end{cases}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*acoth(c*x)/
c + b*d*x*acoth(c*x) + b*e*x*log(-c**2*x**2 + 1)*acoth(c*x) - 2*b*e*x*acot
h(c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c)
- b*e*log(-c**2*x**2 + 1)/c + b*e*acoth(c*x)**2/c, Ne(c, 0)), (d*x*(a + I*
pi*b/2), True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.71

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)be\,\mathrm{arcoth}\,(cx)$$
$$-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)ae$$
$$+adx+\frac{(2\,cx\,\mathrm{arcoth}\,(cx)+\log\left(-c^2x^2+1\right))bd}{2\,c}$$
$$+\frac{((i\pi+2\log(cx-1)-2)\log(cx+1)+(i\pi-2)\log(cx-1))be}{2\,c}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

output
```
-(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1
))*b*e*arccoth(c*x) - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)
 - x*log(-c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x
^2 + 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2
)*log(c*x - 1))*b*e/c
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.90

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=-\frac{1}{2}bex\log\left(-cx+1\right)^2-\frac{1}{2}(-i\pi be-bd-2\,ae+2\,be)x\log\left(cx+1\right)$$
$$+\frac{1}{2}\left(bex+\frac{be}{c}\right)\log\left(cx+1\right)^2-\frac{be\log\left(cx-1\right)^2}{2\,c}-\frac{1}{2}(-i\pi bd+2i\pi be-2\,ad+4\,ae)x$$
$$-\frac{1}{2}\left((-i\pi be+bd-2\,ae-2\,be)x-\frac{2\,be\log\left(cx-1\right)}{c}\right)\log\left(-cx+1\right)$$
$$+\frac{(i\pi be+bd+2\,ae-2\,be)\log\left(cx+1\right)}{2\,c}+\frac{(-i\pi be+bd-2\,ae-2\,be)\log\left(cx-1\right)}{2\,c}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

output
```
-1/2*b*e*x*log(-c*x + 1)^2 - 1/2*(-I*pi*b*e - b*d - 2*a*e + 2*b*e)*x*log(c
*x + 1) + 1/2*(b*e*x + b*e/c)*log(c*x + 1)^2 - 1/2*b*e*log(c*x - 1)^2/c -
1/2*(-I*pi*b*d + 2*I*pi*b*e - 2*a*d + 4*a*e)*x - 1/2*((-I*pi*b*e + b*d - 2
*a*e - 2*b*e)*x - 2*b*e*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*(I*pi*b*e + b*
d + 2*a*e - 2*b*e)*log(c*x + 1)/c + 1/2*(-I*pi*b*e + b*d - 2*a*e - 2*b*e)*
log(c*x - 1)/c
```

Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.03

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\ln\left(\frac{1}{c\,x}+1\right)\left(\frac{b\,d\,x}{2}-b\,e\,x+\frac{b\,e\,x\,\ln\left(1-c^2\,x^2\right)}{2}\right)$$
$$+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{b\,d\,x^2-b\,c^2\,d\,x^4}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{2\,b\,e\,x^2-2\,b\,c^2\,e\,x^4}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right.$$
$$\left.+\frac{\ln\left(1-c^2\,x^2\right)\,\left(b\,e\,x^2-b\,c^2\,e\,x^4\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{2\,c}\right)$$
$$+a\,x\,\left(d-2\,e\right)+\frac{\ln\left(c\,x+1\right)\,\left(2\,a\,e+b\,d-2\,b\,e\right)}{2\,c}-\frac{\ln\left(c\,x-1\right)\,\left(2\,a\,e-b\,d+2\,b\,e\right)}{2\,c}$$
$$+\frac{b\,e\ln\left(\frac{1}{c\,x}+1\right)^2}{4\,c}+\frac{b\,e\ln\left(1-\frac{1}{c\,x}\right)^2}{4\,c}+\frac{b\,e\ln\left(1-c^2\,x^2\right)^2}{4\,c}+a\,e\,x\,\ln\left(1-c^2\,x^2\right)$$

input
```
int((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1/(c*x) + 1)*((b*d*x)/2 - b*e*x + (b*e*x*log(1 - c^2*x^2))/2) + log(1
- 1/(c*x))*((b*d*x^2 - b*c^2*d*x^4)/(2*(x + c*x^2)*(c*x - 1)) - (2*b*e*x^2
 - 2*b*c^2*e*x^4)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*(b*e*x^2 -
 b*c^2*e*x^4))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(2*c)) +
 a*x*(d - 2*e) + (log(c*x + 1)*(2*a*e + b*d - 2*b*e))/(2*c) - (log(c*x - 1
)*(2*a*e - b*d + 2*b*e))/(2*c) + (b*e*log(1/(c*x) + 1)^2)/(4*c) + (b*e*log
(1 - 1/(c*x))^2)/(4*c) + (b*e*log(1 - c^2*x^2)^2)/(4*c) + a*e*x*log(1 - c^
2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.56

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-4acoth(cx)^2\,be+4acoth(cx)\log(-c^2x^2+1)\,bcex+4acoth(cx)\,bcdx-8acoth(cx)\,bcex-\log(-c^2x^2+}{}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1)),x)
```

output
```
( - 4*acoth(c*x)**2*b*e + 4*acoth(c*x)*log( - c**2*x**2 + 1)*b*c*e*x + 4*a
coth(c*x)*b*c*d*x - 8*acoth(c*x)*b*c*e*x - log( - c**2*x**2 + 1)**2*b*e +
4*log( - c**2*x**2 + 1)*a*c*e*x + 4*log( - c**2*x**2 + 1)*a*e - 2*log( - c
**2*x**2 + 1)*b*d + 4*log( - c**2*x**2 + 1)*b*e - 8*log(c**2*x - c)*a*e +
4*a*c*d*x - 8*a*c*e*x)/(4*c)
```

3.154 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b} - \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x} + \frac{1}{2}bc\left(d+e\log(1-c^2x^2)\right)\log\left(1-\frac{1}{1-c^2x^2}\right) - \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)$$

output
```
-c*e*(a+b*arccoth(c*x))^2/b-(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x+1/2*
b*c*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(-c^2*
x^2+1))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(105) = 210.

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{4ad+4bd\coth^{-1}(cx)+4bcex\coth^{-1}(cx)^2+8acex\,\mathrm{arctanh}(cx)-4bcdx\log(x)-bcex\log^2\left(-\frac{1}{c}+x\right)}{}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

output
```
-1/4*(4*a*d + 4*b*d*ArcCoth[c*x] + 4*b*c*e*x*ArcCoth[c*x]^2 + 8*a*c*e*x*Ar
cTanh[c*x] - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c
^(-1) + x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[
x]*Log[1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*
Log[x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2]
+ 4*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2]
+ 2*b*c*e*x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*
Log[1 - c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x
] - 2*b*c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2
])/x
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6644, 2925, 2858, 27, 2779, 2838, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x^2}dx$$

$$\Big\downarrow \text{6644}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x\left(1-c^2x^2\right)}dx-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^2\left(1-c^2x^2\right)}dx^2-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2858}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^4}d(1-c^2x^2)}{2c}-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2779}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right)-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2838}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{1}{2}bc\left(e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right)$$

$$\Big\downarrow \text{6511}$$

$$-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce(a+b\coth^{-1}(cx))^2}{b}-\frac{1}{2}bc\left(e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

output
```
-((c*e*(a + b*ArcCoth[c*x])^2)/b) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c
^2*x^2]))/x - (b*c*(-(Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2])) + e*PolyLo
g[2, x^(-2)]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^2}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log(-c^2x^2+1))}{x^2}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxim
a")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d - (c^2*(log(
c*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e - 1/2*b*e*(log(c*x
 + 1)^2/x - integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (I*pi*c + 2*c)*
x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^3 + x^2), x)) - a*
d/x
```

Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \frac{2acoth(cx)^2\,bcex - 2acoth(cx)\log(-c^2x^2+1)\,be - 2acoth(cx)\,bd + 2\left(\int \frac{\log(-c^2x^2+1)}{c^2x^3-x}dx\right)bcex - 2\log(-c}{2}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x)
```

output

```
(2*acoth(c*x)**2*b*c*e*x - 2*acoth(c*x)*log( - c**2*x**2 + 1)*b*e - 2*acot
h(c*x)*b*d + 2*int(log( - c**2*x**2 + 1)/(c**2*x**3 - x),x)*b*c*e*x - 2*lo
g( - c**2*x**2 + 1)*a*c*e*x - 2*log( - c**2*x**2 + 1)*a*e + log( - c**2*x*
*2 + 1)*b*c*d*x + 4*log(c**2*x - c)*a*c*e*x - 2*log(x)*b*c*d*x - 2*a*d)/(2
*x)
```

3.155 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 197

$$\begin{aligned}\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx &= \frac{2c^2e(a+b\coth^{-1}(cx))}{3x} - \frac{c^3e(a+b\coth^{-1}(cx))^2}{3b} - bc^3e\log(x) + \frac{1}{3}bc^3e\log(1-c^2x^2) \\ &\quad - \frac{bc(1-c^2x^2)\,(d+e\log(1-c^2x^2))}{6x^2} - \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{3x^3} \\ &\quad + \frac{1}{6}bc^3(d+e\log(1-c^2x^2))\log\left(1-\frac{1}{1-c^2x^2}\right) - \frac{1}{6}bc^3e\,\mathrm{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)\end{aligned}$$

output
```
2/3*c^2*e*(a+b*arccoth(c*x))/x-1/3*c^3*e*(a+b*arccoth(c*x))^2/b-b*c^3*e*ln
(x)+1/3*b*c^3*e*ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/x
^2-1/3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*ln(-c^2*
x^2+1))*ln(1-1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 457 vs. $2(197) = 394$.

Time = 0.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.32

$$\int \frac{(a + b\coth^{-1}(cx))(d + e\log(1 - c^2x^2))}{x^4}\,dx$$

$$= \frac{1}{6}\left(-\frac{2ad}{x^3} - \frac{bcd}{x^2} + \frac{4ac^2e}{x} - \frac{2bd\coth^{-1}(cx)}{x^3} + \frac{4bc^2e\coth^{-1}(cx)}{x} - 2bc^3e\coth^{-1}(cx)^2 - 4ac^3e\text{arctanh}(cx) - 4bc^3e\log\left(\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\right) + 2bc^3d\log(x) - 2bc^3e\log(x) + \frac{1}{2}bc^3e\log^2\left(-\frac{1}{c}+x\right) + \frac{1}{2}bc^3e\log^2\left(\frac{1}{c}+x\right) + bc^3e\log\left(\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1-cx)\right) - 2bc^3e\log(x)\log(1-cx) + bc^3e\log\left(-\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1+cx)\right) - 2bc^3e\log(x)\log(1+cx) - bc^3d\log(1-c^2x^2) + bc^3e\log(1-c^2x^2) - \frac{2ae\log(1-c^2x^2)}{x^3} - \frac{bce\log(1-c^2x^2)}{x^2} - \frac{2be\coth^{-1}(cx)\log(1-c^2x^2)}{x^3} + 2bc^3e\log(x)\log(1-c^2x^2) - bc^3e\log\left(-\frac{1}{c}+x\right)\log(1-c^2x^2) - bc^3e\log\left(\frac{1}{c}+x\right)\log(1-c^2x^2) - 2bc^3e\,\text{PolyLog}(2,-cx) - 2bc^3e\,\text{PolyLog}(2,cx) + bc^3e\,\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right) + bc^3e\,\text{PolyLog}\left(2,\frac{1}{2}(1+cx)\right)\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

output
```
((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - (2*b*d*ArcCoth[c*x])/x^3 + (
4*b*c^2*e*ArcCoth[c*x])/x - 2*b*c^3*e*ArcCoth[c*x]^2 - 4*a*c^3*e*ArcTanh[c
*x] - 4*b*c^3*e*Log[1/Sqrt[1 - 1/(c^2*x^2)]] + 2*b*c^3*d*Log[x] - 2*b*c^3*
e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 + (b*c^3*e*Log[c^(-1) + x]^2)/2
+ b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 - c*x]
 + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 + c*
x] - b*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 -
c^2*x^2])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcCoth[c*x]*Log[1 -
 c^2*x^2])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) +
 x]*Log[1 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*
e*PolyLog[2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2
- (c*x)/2] + b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

Rubi [A] (warning: unable to verify)

Time = 1.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6644, 2925, 2858, 27, 2789, 2751, 16, 2779, 2838, 6545, 6453, 243, 47, 14, 16, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x^4}dx$$

$$\Big\downarrow \text{6644}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{3}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^3\left(1-c^2x^2\right)}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2925}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{6}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^4\left(1-c^2x^2\right)}dx^2-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{b\int \frac{d+e\log\left(1-c^2x^2\right)}{x^6}d\left(1-c^2x^2\right)}{6c}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{1}{6}bc^3\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d\left(1-c^2x^2\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2789$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d\left(1-c^2x^2\right)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^4}d\left(1-c^2x^2\right)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2751$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d\left(1-c^2x^2\right)-e\int \frac{1}{c^2x^2}d\left(1-c^2x^2\right)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \; 16$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d\left(1-c^2x^2\right)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}+e\log\left(c^2x^2\right)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2779$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{1}{6}bc^3\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d\left(1-c^2x^2\right)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^{\cdots}}\right.\right.$$

$$\Big\downarrow \text{6545}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\int \frac{a+b\coth^{-1}(cx)}{x^2}dx\right)-$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^{\cdots}}\right.\right.$$

$$\Big\downarrow \text{6453}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x\left(1-c^2x^2\right)}dx-\frac{a+b\coth^{-1}(cx)}{x}\right)-$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^{\cdots}}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2\left(1-c^2x^2\right)}dx^2-\frac{a+b\coth^{-1}(cx)}{x}\right)-$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^{\cdots}}\right.\right.$$

$$\Big\downarrow \text{47}$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\coth^{-1}(cx)}{x}\right)-$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^{\cdots}}\right.\right.$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{3}c^2e\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\coth^{-1}(cx)}{x}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\operatorname{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \; 16$$

$$-\frac{2}{3}c^2e\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\operatorname{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \; 6511$$

$$-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{2}{3}c^2e\left(\frac{c\left(a+b\coth^{-1}(cx)\right)^2}{2b}-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{1}{6}bc^3\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\operatorname{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

output
```
(-2*c^2*e*(-((a + b*ArcCoth[c*x])/x) + (c*(a + b*ArcCoth[c*x])^2)/(2*b) +
(b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2))/3 - ((a + b*ArcCoth[c*x])*(d + e*L
og[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(e*Log[c^2*x^2] + ((1 - c^2*x^2)*(d + e
*Log[1 - c^2*x^2]))/(c^2*x^2) - Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2]) +
 e*PolyLog[2, x^(-2)]))/6
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a+b\operatorname{arccoth}(cx))\,(d+e\ln(-c^2x^2+1))}{x^4}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log(-c^2x^2+1)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^4, x)
```

Sympy [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x^4}\,dx = \int \frac{(a + b\,\mathrm{acoth}\,(cx))\,(d + e\log(-c^2x^2 + 1))}{x^4}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

Maxima [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x^4}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx) + a)(e\log(-c^2x^2 + 1) + d)}{x^4}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxim
a")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arccoth(c*x)/x^3
)*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 +
1)/x^3)*a*e - 1/6*b*e*(log(c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x + 1)*
log(c*x - 1)^2 - (3*I*pi + (3*I*pi*c + 2*c)*x)*log(c*x + 1) + 3*(I*pi + I*
pi*c*x)*log(c*x - 1))/(c*x^5 + x^4), x)) - 1/3*a*d/x^3
```

Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^4}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^4,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \frac{-2acoth(cx)\,b\,c^3d\,x^3-2acoth(cx)\,bd+6\left(\int \frac{acoth(cx)\log(-c^2x^2+1)}{x^4}dx\right)be\,x^3-2\log(-c^2x^2+1)\,a\,c^3e\,x^3-2}{}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x)
```

output
```
( - 2*acoth(c*x)*b*c**3*d*x**3 - 2*acoth(c*x)*b*d + 6*int((acoth(c*x)*log(
 - c**2*x**2 + 1))/x**4,x)*b*e*x**3 - 2*log( - c**2*x**2 + 1)*a*c**3*e*x**
3 - 2*log( - c**2*x**2 + 1)*a*e + 4*log(c**2*x - c)*a*c**3*e*x**3 + 2*log(
c**2*x - c)*b*c**3*d*x**3 - 2*log(x)*b*c**3*d*x**3 + 4*a*c**2*e*x**2 - 2*a
*d + b*c*d*x)/(6*x**3)
```

3.156 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx = \frac{7bc^3e}{60x^2}+\frac{2c^2e(a+b\coth^{-1}(cx))}{15x^3}+\frac{2c^4e(a+b\coth^{-1}(cx))}{5x}-\frac{c^5e(a+b\coth^{-1}(cx))^2}{5b} - \frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log(1-c^2x^2)-\frac{bc(d+e\log(1-c^2x^2))}{20x^4} - \frac{bc^3(1-c^2x^2)(d+e\log(1-c^2x^2))}{10x^2}-\frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{5x^5} + \frac{1}{10}bc^5(d+e\log(1-c^2x^2))\log\left(1-\frac{1}{1-c^2x^2}\right)-\frac{1}{10}bc^5e\,\mathrm{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)$$

output
```
7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*arccoth(c*x))/x^3+2/5*c^4*e*(a+b*arccoth(
c*x))/x-1/5*c^5*e*(a+b*arccoth(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*l
n(-c^2*x^2+1)-1/20*b*c*(d+e*ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d
+e*ln(-c^2*x^2+1))/x^2-1/5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5+1/1
0*b*c^5*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1
/(-c^2*x^2+1))
```

Mathematica [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx = \int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

output
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.62 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.14, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.926$, Rules used = {6644, 2925, 2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838, 6545, 6453, 243, 54, 2009, 6545, 6453, 243, 47, 14, 16, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{x^6}\,dx$$

$$\Big\downarrow \ 6644$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx+\frac{1}{5}bc\int \frac{d+e\log(1-c^2x^2)}{x^5(1-c^2x^2)}dx-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \ 2925$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx+\frac{1}{10}bc\int \frac{d+e\log(1-c^2x^2)}{x^6(1-c^2x^2)}dx^2-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \ 2858$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{b\int \frac{d+e\log\left(1-c^2x^2\right)}{x^8}d(1-c^2x^2)}{10c}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 27

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\int \frac{d+e\log\left(1-c^2x^2\right)}{c^6x^8}d(1-c^2x^2)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2789

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^6x^6}d(1-c^2x^2)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2756

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\int \frac{1}{c^4x^6}d(1-c^2x^2)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 54

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\int \left(\frac{1}{c^2x^2}+\frac{1}{x^2}+\frac{1}{c^4x^4}\right)d(1-c^2x^2)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2009

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(c^2x^2\right)+\log\left(1-c^2x^2\right)\right)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2789}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)+\int \frac{d+e\log(1-c^2x^2)}{c^4x^4}d(1-c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(c^2x^2)+\log\right.\right.$$
$$\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \text{2751}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)-e\int \frac{1}{c^2x^2}d(1-c^2x^2)+\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\frac{1}{2}e\left(\right.\right.$$
$$\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)+\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$
$$\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \text{2779}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx-$$
$$\frac{1}{10}bc^5\left(e\int \frac{\log(1-\frac{1}{x^2})}{x^2}d(1-c^2x^2)+\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)\right.$$
$$\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{6545}$$

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\int \frac{a+b\coth^{-1}(cx)}{x^4}dx\right)-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{6453}$$

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{3}bc\int \frac{1}{x^3(1-c^2x^2)}dx-\frac{a+b\coth^{-1}(cx)}{3x^3}\right)-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int \frac{1}{x^4(1-c^2x^2)}dx^2-\frac{a+b\coth^{-1}(cx)}{3x^3}\right)-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{54}$$

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int \left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\coth^{-1}(cx)}{3x^3}\right)-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log(x^2)-c^2\log(1-c^2x^2)-\frac{1}{x^2}\right)\right)-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{6545}$$

$$-\frac{2}{5}c^2e\Bigg(c^2\Bigg(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\int\frac{a+b\coth^{-1}(cx)}{x^2}dx\Bigg)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\Big(c^2\log\left(x^2\right)-c^2\log\left(\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\Bigg(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\Bigg(\frac{1}{c^2x^2}-\log$$

$$\Big\downarrow \text{6453}$$

$$-\frac{2}{5}c^2e\Bigg(c^2\Bigg(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+bc\int\frac{1}{x\left(1-c^2x^2\right)}dx-\frac{a+b\coth^{-1}(cx)}{x}\Bigg)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\Big(c^2$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\Bigg(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\Bigg(\frac{1}{c^2x^2}-\log$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{5}c^2e\Bigg(c^2\Bigg(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{1}{x^2\left(1-c^2x^2\right)}dx^2-\frac{a+b\coth^{-1}(cx)}{x}\Bigg)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\Bigg(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\Bigg(\frac{1}{c^2x^2}-\log$$

$$\Big\downarrow \text{47}$$

$$-\frac{2}{5}c^2e\Bigg(c^2\Bigg(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\Bigg(c^2\int\frac{1}{1-c^2x^2}dx^2+\int\frac{1}{x^2}dx^2\Bigg)-\frac{a+b\coth^{-1}(cx)}{x}\Bigg)-\frac{a+b\coth}{3x^3}$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\Bigg(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\Bigg(\frac{1}{c^2x^2}-\log$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{5}c^2e\Bigg(c^2\Bigg(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\Bigg(c^2\int\frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\Bigg)-\frac{a+b\coth^{-1}(cx)}{x}\Bigg)-\frac{a+b\coth^{-}}{3x^3}$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\Bigg(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\Bigg(\frac{1}{c^2x^2}-\log$$

$$\Big\downarrow \; 16$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}$$

$$\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$

$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \; 6511$$

$$-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$

$$\frac{2}{5}c^2e\left(c^2\left(\frac{c(a+b\coth^{-1}(cx))^2}{2b}-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}\right.$$

$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

output
```
-1/5*((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5 - (2*c^2*e*(-1/3*
(a + b*ArcCoth[c*x])/x^3 + c^2*(-((a + b*ArcCoth[c*x])/x) + (c*(a + b*ArcC
oth[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2) + (b*c*(-x^(-2
) + c^2*Log[x^2] - c^2*Log[1 - c^2*x^2]))/6))/5 - (b*c^5*(e*Log[c^2*x^2] -
 (e*(1/(c^2*x^2) - Log[c^2*x^2] + Log[1 - c^2*x^2]))/2 + (d + e*Log[1 - c^
2*x^2])/(2*c^4*x^4) + ((1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(c^2*x^2) -
 Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2]) + e*PolyLog[2, x^(-2)]))/10
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccoth}\,(cx))\,(d+e\ln(-c^2x^2+1))}{x^6}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx$$
$$=\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^6, x)
```

Sympy [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x^6}\,dx = \int \frac{(a + b\,\mathrm{acoth}\,(cx))\,(d + e\log(-c^2x^2 + 1))}{x^6}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

Maxima [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x^6}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx) + a)(e\log(-c^2x^2 + 1) + d)}{x^6}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxim
a")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
 4*arccoth(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1)
- 2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e - 1/10*b*e*(lo
g(c*x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x + 1)*log(c*x - 1)^2 - (5*I*pi
+ (5*I*pi*c + 2*c)*x)*log(c*x + 1) + 5*(I*pi + I*pi*c*x)*log(c*x - 1))/(c*
x^7 + x^6), x)) - 1/5*a*d/x^5
```

Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^6}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^6,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^6, x)
```

Reduce [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(1 - c^2x^2))}{x^6}\,dx$$
$$= \frac{12acoth(cx)^2\,b\,c^5e\,x^5 - 12acoth(cx)\log(-c^2x^2 + 1)\,be + 24acoth(cx)\,b\,c^4e\,x^4 + 8acoth(cx)\,b\,c^2e\,x^2 - 12a}{}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x)
```

output
```
(12*acoth(c*x)**2*b*c**5*e*x**5 - 12*acoth(c*x)*log( - c**2*x**2 + 1)*b*e
+ 24*acoth(c*x)*b*c**4*e*x**4 + 8*acoth(c*x)*b*c**2*e*x**2 - 12*acoth(c*x)
*b*d + 12*int(log( - c**2*x**2 + 1)/(c**2*x**3 - x),x)*b*c**5*e*x**5 - 12*
log( - c**2*x**2 + 1)*a*c**5*e*x**5 - 12*log( - c**2*x**2 + 1)*a*e + 6*log
( - c**2*x**2 + 1)*b*c**5*d*x**5 - 25*log( - c**2*x**2 + 1)*b*c**5*e*x**5
+ 6*log( - c**2*x**2 + 1)*b*c**3*e*x**3 + 3*log( - c**2*x**2 + 1)*b*c*e*x
+ 24*log(c**2*x - c)*a*c**5*e*x**5 - 12*log(x)*b*c**5*d*x**5 + 50*log(x)*b
*c**5*e*x**5 + 24*a*c**4*e*x**4 + 8*a*c**2*e*x**2 - 12*a*d + 6*b*c**3*d*x*
*3 - 7*b*c**3*e*x**3 + 3*b*c*d*x)/(60*x**5)
```

3.157 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 512

$$
\begin{aligned}
&\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx\\
&=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)\\
&\quad+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\text{arctanh}(cx)}{2c^2}-\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(\frac{2}{1+cx}\right)}{c^2g}\\
&\quad+\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-\sqrt{g}\right)(1+cx)}\right)}{2c^2g}\\
&\quad+\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{\left(c\sqrt{-f}+\sqrt{g}\right)(1+cx)}\right)}{2c^2g}\\
&\quad+\frac{bex\log\left(f+gx^2\right)}{2c}+\frac{e\left(f+gx^2\right)\left(a+b\coth^{-1}(cx)\right)\log\left(f+gx^2\right)}{2g}\\
&\quad-\frac{be(c^2f+g)\,\text{arctanh}(cx)\log\left(f+gx^2\right)}{2c^2g}+\frac{be(c^2f+g)\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2g}\\
&\quad-\frac{be(c^2f+g)\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-\sqrt{g}\right)(1+cx)}\right)}{4c^2g}\\
&\quad-\frac{be(c^2f+g)\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{\left(c\sqrt{-f}+\sqrt{g}\right)(1+cx)}\right)}{4c^2g}
\end{aligned}
$$

output
```
1/2*b*(d-e)*x/c-b*e*x/c+1/2*d*x^2*(a+b*arccoth(c*x))-1/2*e*x^2*(a+b*arccot
h(c*x))+b*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))/c/g^(1/2)-1/2*b*(d-e)*arctan
h(c*x)/c^2-b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1))/c^2/g+1/2*b*e*(c^2*f+g
)*arctanh(c*x)*ln(2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))/(c*x+1
))/c^2/g+1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(
-f)^(1/2)+g^(1/2))/(c*x+1))/c^2/g+1/2*b*e*x*ln(g*x^2+f)/c+1/2*e*(g*x^2+f)*
(a+b*arccoth(c*x))*ln(g*x^2+f)/g-1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(g*x^2+f
)/c^2/g+1/2*b*e*(c^2*f+g)*polylog(2,1-2/(c*x+1))/c^2/g-1/4*b*e*(c^2*f+g)*p
olylog(2,1-2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))/(c*x+1))/c^2/
g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+g
^(1/2))/(c*x+1))/c^2/g
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.75 (sec) , antiderivative size = 1128, normalized size of antiderivative = 2.20

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Too large to display}$$

input
```
Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(2*b*c*d*g*x - 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 - 2*b*d*g*A
rcCoth[c*x] + 2*b*e*g*ArcCoth[c*x] + 2*b*c^2*d*g*x^2*ArcCoth[c*x] - 2*b*c^
2*e*g*x^2*ArcCoth[c*x] + 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f
]] - (4*I)*b*c^2*e*f*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2
*f*g)]*x)] - (4*I)*b*e*g*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-
(c^2*f*g)]*x)] - 4*b*c^2*e*f*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] - 4
*b*e*g*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] + 2*b*c^2*e*f*ArcCoth[c*x
]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt
[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + 2*b*e*g*ArcCoth[c*x]*Log
[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^
2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - (2*I)*b*c^2*e*f*ArcSin[Sqrt[g
/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x
])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - (2*I)*b*e*g
*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^
(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))]
 + 2*b*c^2*e*f*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(
2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))]
+ 2*b*e*g*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*Arc
Coth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + (2*
I)*b*c^2*e*f*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*...
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$$

$$\Big\downarrow \text{6646}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(1-c^2x^2\right)}+\frac{e\left(gx^2+f\right)\log\left(gx^2+f\right)}{2g(1-cx)(cx+1)}\right)dx+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)\left(a+b\coth^{-1}(cx)\right)}{2g}-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)$$

$\downarrow$ 2009

$$\frac{1}{2}dx^2(a+b\coth^{-1}(cx))+\frac{e(f+gx^2)\log(f+gx^2)(a+b\coth^{-1}(cx))}{2g}-\frac{1}{2}ex^2(a+b\coth^{-1}(cx))-bc\left(-\frac{e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c^2\sqrt{g}}+\frac{(d-e)\text{arctanh}(cx)}{2c^3}+\frac{e\text{arctanh}(cx)(c^2f+g)\log(f+gx^2)}{2c^3g}+\frac{e\text{arctanh}(cx)(c^2f+}{c^3g}\right.$$

input
```
Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(a + b*ArcCoth[c*x]))/2 + (e*(f +
g*x^2)*(a + b*ArcCoth[c*x])*Log[f + g*x^2])/(2*g) - b*c*(-1/2*((d - e)*x)/
c^2 + (e*x)/c^2 - (e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(c^2*Sqrt[g]) +
((d - e)*ArcTanh[c*x])/(2*c^3) + (e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*
x)])/(c^3*g) - (e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x)
)/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(2*c^3*g) - (e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])
/(2*c^3*g) - (e*x*Log[f + g*x^2])/(2*c^2) + (e*(c^2*f + g)*ArcTanh[c*x]*Lo
g[f + g*x^2])/(2*c^3*g) - (e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c
^3*g) + (e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqr
t[-f] - Sqrt[g])*(1 + c*x))])/(4*c^3*g) + (e*(c^2*f + g)*PolyLog[2, 1 - (2
*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*c^3*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
 + 1)/2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(448) = 896$.

Time = 6.41 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{3bex}{2c}+\frac{bxd}{2c}-\frac{b\,x^{2}e\ln(cx+1)}{4}+\frac{efb\arctan\left(\frac{xg}{\sqrt{fg}}\right)}{c\sqrt{fg}}-\frac{eb\ln(cx-1)\ln\left(\frac{c\sqrt{-fg}-g(cx-1)-g}{c\sqrt{-fg}-g}\right)f}{4g}-\frac{eb\ln(cx-1)\ln\left(\frac{c\sqrt{-fg}+g(c}{c\sqrt{-fg}}\right)}{4g}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
-3/2*b*e*x/c+1/2/c*b*x*d-1/4*b*x^2*e*ln(c*x+1)+e/c*f*b/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))-1/4*e*b/g*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-
f*g)^(1/2)-g))*f-1/4*e*b/g*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-
f*g)^(1/2)+g))*f+1/4*e*b/g*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-
f*g)^(1/2)+g))*f+1/4*e*b/g*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-
f*g)^(1/2)-g))*f+1/4*d*b*ln(c*x+1)*x^2-1/4*d/c^2*b*ln(c*x+1)-1/4*d*b*ln(c*
x-1)*x^2+1/4*d/c^2*b*ln(c*x-1)+(1/4*b*x^2*e*ln(c*x+1)+1/4*e*(-b*x^2*ln(c*x
-1)*c^2+2*a*c^2*x^2+2*b*c*x+b*ln(c*x-1)-b*ln(c*x+1))/c^2)*ln(g*x^2+f)+1/2*
a*d*x^2+1/4*e*b*ln(c*x-1)*x^2-1/4*e/c^2*b*ln(c*x-1)-1/4*e/c^2*b*dilog((c*(
-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))-1/4*e/c^2*b*dilog((c*(-f*g)^(
1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))-1/2*e*a*x^2+1/4*e/c^2*b*dilog((c*(-f
*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4*e/c^2*b*dilog((c*(-f*g)^(1/
2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))-1/4*e*b/g*dilog((c*(-f*g)^(1/2)-g*(c*x
-1)-g)/(c*(-f*g)^(1/2)-g))*f-1/4*e*b/g*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/
(c*(-f*g)^(1/2)+g))*f+1/2*e*f/g*a*ln(g*x^2+f)+1/4/c^2*b*e*ln(c*x+1)+1/4*e*
b/g*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f+1/4*e*b/g*dil
og((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f-1/4*e/c^2*b*ln(c*x-1
)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))-1/4*e/c^2*b*ln(c*x-1
)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))+1/4*e/c^2*b*ln(c*x+1
)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4*e/c^2*b*ln(c*...
```

Fricas [F]

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d)x\,dx$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*x*arccoth(c*x) + a*d*x + (b*e*x*arccoth(c*x) + a*e*x)*log(g*x
^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d)x\,dx$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*d - 1/4*(2*c^2*g*integrate(x^3*log(c*x + 1)/(c^2*g*x^2
+ c^2*f), x) - 2*c^2*g*integrate(x^3*log(c*x - 1)/(c^2*g*x^2 + c^2*f), x)
- 2*c*g*(-I*f*(log(I*g*x/sqrt(f*g) + 1) - log(-I*g*x/sqrt(f*g) + 1))/(sqrt
(f*g)*c^2*g) - 2*x/(c^2*g)) - 2*g*integrate(x*log(c*x + 1)/(c^2*g*x^2 + c^
2*f), x) + 2*g*integrate(x*log(c*x - 1)/(c^2*g*x^2 + c^2*f), x) - (2*c*x +
 (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(c*x - 1))*log(g*x^2 + f)/c
^2)*b*e - 1/2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g
```

Giac [F]

$$\int x(a + b\coth^{-1}(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx = \int (b\,\mathrm{arcoth}\,(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)x\,dx$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a + b\coth^{-1}(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx = \int x\,(a + b\,\mathrm{acoth}(c\,x))\, \left(d + e\,\ln\left(g\,x^2 + f\right)\right) dx$$

input
```
int(x*(a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int(x*(a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

Reduce [F]

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\frac{acoth(cx)\log(g\,x^2+f)\,b\,c^2ef+acoth(cx)\log(g\,x^2+f)\,b\,c^2eg\,x^2+acoth(cx)\,b\,c^2dg\,x^2-acoth(cx)\,b\,c^2e\text{…}}{}$$

input
```
int(x*(a+b*acoth(c*x))*(d+e*log(g*x^2+f)),x)
```

output
```
(acoth(c*x)*log(f + g*x**2)*b*c**2*e*f + acoth(c*x)*log(f + g*x**2)*b*c**2
*e*g*x**2 + acoth(c*x)*b*c**2*d*g*x**2 - acoth(c*x)*b*c**2*e*g*x**2 - acot
h(c*x)*b*d*g + acoth(c*x)*b*e*g - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sq
rt(f)))*b*c*e - int(log(f + g*x**2)/(c**2*x**2 - 1),x)*b*c**3*e*f - int(lo
g(f + g*x**2)/(c**2*x**2 - 1),x)*b*c*e*g + log(f + g*x**2)*a*c**2*e*f + lo
g(f + g*x**2)*a*c**2*e*g*x**2 - log(f + g*x**2)*b*c*e*g*x + a*c**2*d*g*x**
2 - a*c**2*e*g*x**2 - b*c*d*g*x + 3*b*c*e*g*x)/(2*c**2*g)
```

3.158 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$

Optimal result

Integrand size = 21, antiderivative size = 546

$$\begin{aligned}
&\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx \\
&= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{g}} \\
&\quad+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{1}{cx}\right)}{\sqrt{g}}+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{\left(ic\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{g}} \\
&\quad-\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(1+cx)}{\left(ic\sqrt{f}+\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{g}}-\frac{be\log\left(1-c^2x^2\right)}{c} \\
&\quad+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{b\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c} \\
&\quad+\frac{be\,\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right)}{2c}-\frac{ibe\sqrt{f}\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{\left(ic\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{g}} \\
&\quad+\frac{ibe\sqrt{f}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(1+cx)}{\left(ic\sqrt{f}+\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{g}}
\end{aligned}$$

output
```
-2*a*e*x-2*b*e*x*arccoth(c*x)+2*a*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))/g^(1
/2)-b*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(1-1/c/x)/g^(1/2)+b*e*f^(1/2)*
arctan(g^(1/2)*x/f^(1/2))*ln(1+1/c/x)/g^(1/2)+b*e*f^(1/2)*arctan(g^(1/2)*x
/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*(-c*x+1)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*
g^(1/2)*x))/g^(1/2)-b*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(
1/2)*(c*x+1)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(1/2)-b*e*ln(-
c^2*x^2+1)/c+x*(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))+1/2*b*ln(g*(-c^2*x^2+1
)/(c^2*f+g))*(d+e*ln(g*x^2+f))/c+1/2*b*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g)
)/c-1/2*I*b*e*f^(1/2)*polylog(2,1+2*f^(1/2)*g^(1/2)*(-c*x+1)/(I*c*f^(1/2)-
g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(1/2)+1/2*I*b*e*f^(1/2)*polylog(2,1-2*f^
(1/2)*g^(1/2)*(c*x+1)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1287 vs. $2(546) = 1092$.

Time = 2.22 (sec) , antiderivative size = 1287, normalized size of antiderivative = 2.36

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output

```
a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/S
qrt[f]])/Sqrt[g] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b
*e*(x*ArcCoth[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*(-4*c*x
*ArcCoth[c*x] + 4*Log[1/(c*Sqrt[1 - 1/(c^2*x^2)]*x)] + (Sqrt[c^2*f*g]*((-2
*I)*ArcCos[(c^2*f - g)/(c^2*f + g)]*ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + 4*ArcC
oth[c*x]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - (ArcCos[(c^2*f - g)/(c^2*f + g)]
+ 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[((2*I)*g*(I*c^2*f + Sqrt[c^2*f*g])*
(-1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] - (ArcCos[(c^
2*f - g)/(c^2*f + g)] - 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[(2*g*(c^2*f +
 I*Sqrt[c^2*f*g])*(1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)
))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)]
+ ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcCoth[c
*x]*Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])
] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] +
ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcCoth[c*x]*Sqrt[c^2*f*g])
/(Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])]
+ I*(-PolyLog[2, ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(g - (I*Sqrt[c^2*f*
g])/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] + PolyLog[2, ((c^
2*f - g + (2*I)*Sqrt[c^2*f*g])*(I*g + Sqrt[c^2*f*g]/(c*x)))/((c^2*f + g)*(
(-I)*g + Sqrt[c^2*f*g]/(c*x)))])))/g))/(2*c) - (b*e*g*(((-Log[-c^(-1) +...
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6636, 2925, 2841, 2840, 2838, 6543, 2009, 6537, 218, 6535, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$

$$\Big\downarrow \text{ 6636}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-bc\int \frac{x\left(d+e\log\left(gx^2+f\right)\right)}{1-c^2x^2}dx+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{ 2925}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{1-c^2x^2}dx^2+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2841}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{eg\int \frac{\log\left(\frac{g\left(1-c^2x^2\right)}{fc^2+g}\right)}{gx^2+f}dx^2}{c^2}-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2840}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{e\int \frac{\log\left(1-\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{x^2}d\left(gx^2+f\right)}{c^2}-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$-2eg\left(\frac{\int \left(a+b\coth^{-1}(cx)\right)dx}{g}-\frac{f\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx}{g}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\int\frac{a+b\coth^{-1}(cx)}{gx^2+f}dx}{g}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)- \frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 6537

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(a\int\frac{1}{gx^2+f}dx+b\int\frac{\coth^{-1}(cx)}{gx^2+f}dx\right)}{g}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)- \frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 218

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\int\frac{\coth^{-1}(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)- \frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 6535

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\int\frac{\log\left(1+\frac{1}{cx}\right)}{gx^2+f}dx-\frac{1}{2}\int\frac{\log\left(1-\frac{1}{cx}\right)}{gx^2+f}dx\right)+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+ x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)- \frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 2920

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(c-\frac{1}{x}\right)x^2}dx}{c}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\frac{c\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{f}\sqrt{g}\left(c+\frac{1}{x}\right.}}{c}\right.\right.\right.}{g}\right.$$

$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$

$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow 27$$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(c-\frac{1}{x}\right)x^2}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(c+\frac{1}{x}\right)x^2}}{\sqrt{f}\sqrt{g}}\right.\right.\right.}{g}\right.$$

$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$

$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow 2005$$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx-1)}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx+1)}}{\sqrt{f}\sqrt{g}}\right.\right.\right.}{g}\right.$$

$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$

$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow 5411$$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\left(\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{cx-1}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\right)dx}{\sqrt{f}\sqrt{g}}-\frac{\log(1-\frac{1}{cx})\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\right.\right.}{}\right.$$

$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-$$

$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 2009

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}\left(-\frac{\log(1-\frac{1}{cx})\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\ldots\right.}{}\right.\right.\right.}{}\right.$$

$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-$$

$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

input
```
Int[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) - (b*c*(-((Log[(g*(1 - c^2*x
^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/c^2) - (e*PolyLog[2, (c^2*(f + g
*x^2))/(c^2*f + g)])/c^2))/2 - 2*e*g*((a*x + b*x*ArcCoth[c*x] + (b*Log[1 -
 c^2*x^2])/(2*c))/g - (f*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]
) + b*((-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/(Sqrt[f]*Sqrt[g])
) + (-(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)
]) + ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*
Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] - (I/2)*PolyLog[2, ((-I)*Sqrt
[g]*x)/Sqrt[f]] + (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] + (I/2)*PolyLog[
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] - (I/2)*PolyLog[2, 1 + (2*Sqrt
[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])
/(Sqrt[f]*Sqrt[g]))/2 + ((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/(S
qrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] -
I*Sqrt[g]*x)] - ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*
x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] + (I/2)*PolyLog[2,
((-I)*Sqrt[g]*x)/Sqrt[f]] - (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] - (I/2
)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] + (I/2)*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g]))/2)))/g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 6535
```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)]/(d
+ e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 6537
```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcCoth[c*x]/(d + e*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6636
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp
[2*e*g   Int[x^2*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b
, c, d, e, f, g}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.88 (sec) , antiderivative size = 3508, normalized size of antiderivative = 6.42

method	result	size
risch	Expression too large to display	3508

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
x*a*d-2*a*e*x+a*e*x*ln(g*x^2+f)-1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f
+((c*x+1)^2-2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*csgn(I*c^2)^3*ln(c*x+1)*x-1/4*I*
e*b*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-
2*c*x-1)*g))^2*x-1/4*I*e*b*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x
-1)*g))^3*x-1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-
1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(
c^2*f+((c*x+1)^2-2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*csgn(I*c^2)^3*ln(c*x+1)-1/4
*I*e*b/c*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^3+1/4*I*e*
b*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*x-1/4*I*e*b*P
i*csgn(I*c^2)^3*ln(c*x-1)*x+1/4*I*e*b*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)
*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*ln(c*x-1)*
csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3*x-1/4*I*e*b/c*Pi*csgn(I/c^2)*c
sgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((
c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2+1/4*I*e*
b/c*Pi*csgn(I*c^2)^3*ln(c*x-1)+4*b*e/c+1/2*d/c*b*ln(c*x+1)+1/2*d/c*b*ln(c*
x-1)-1/4*I*e*b/c*Pi*ln(c*x-1)*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3+
I*e*b/c*Pi*csgn(I*c)*csgn(I*c^2)^2-1/2*I*e*b/c*Pi*csgn(I*c)^2*csgn(I*c^2)-
1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*
(c^2*f+((c*x-1)^2+2*c*x-1)*g))*x-1/4*I*e*b*Pi*csgn(I/c^2)*ln(c*x-1)*csgn(I
/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*x-1/4*I*e*b*Pi*csgn(I*c)^2*csgn(I...
```

Fricas [F]

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = \int (b\,\mathrm{arcoth}\,(cx)+a)\big(e\log\left(gx^2+f\right)+d\big)\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\coth^{-1}(cx)) (d+e\log(f+gx^2)) dx = \text{Timed out}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\coth^{-1}(cx)) (d+e\log(f+gx^2)) dx$$
$$= \int (b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d) dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
(2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 + f))*a*e
 + a*d*x + 1/2*b*e*(((c*x + 1)*log(c*x + 1) - (c*x - 1)*log(c*x - 1))*log(
g*x^2 + f)/c - integrate(2*((c*g*x^2 + g*x)*log(c*x + 1) - (c*g*x^2 - g*x)
*log(c*x - 1))/(c*g*x^2 + c*f), x)) + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x
^2 + 1))*b*d/c
```

Giac [F]

$$\int (a+b\coth^{-1}(cx)) (d+e\log(f+gx^2)) dx$$
$$= \int (b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d) dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int \left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

input
```
int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

Reduce [F]

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\frac{4acoth(cx)^2\,b\,c^2ef+4acoth(cx)\log(g\,x^2+f)\,bcegx+4acoth(cx)\,bcdgx-8acoth(cx)\,bcegx+8\sqrt{g}\,\sqrt{f}\,a}{}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(g*x^2+f)),x)
```

output
```
(4*acoth(c*x)**2*b*c**2*e*f + 4*acoth(c*x)*log(f + g*x**2)*b*c*e*g*x + 4*a
coth(c*x)*b*c*d*g*x - 8*acoth(c*x)*b*c*e*g*x + 8*sqrt(g)*sqrt(f)*atan((g*x
)/(sqrt(g)*sqrt(f)))*a*c*e - 8*int(acoth(c*x)/(c**2*f*x**2 + c**2*g*x**4 -
 f - g*x**2),x)*b*c**3*e*f**2 - 8*int(acoth(c*x)/(c**2*f*x**2 + c**2*g*x**
4 - f - g*x**2),x)*b*c*e*f*g - 4*int((log(f + g*x**2)*x)/(c**2*f*x**2 + c*
*2*g*x**4 - f - g*x**2),x)*b*c**2*e*f*g - 4*int((log(f + g*x**2)*x)/(c**2*
f*x**2 + c**2*g*x**4 - f - g*x**2),x)*b*e*g**2 - 2*log(c**2*x - c)*b*d*g +
 4*log(c**2*x - c)*b*e*g - 2*log(c**2*x + c)*b*d*g + 4*log(c**2*x + c)*b*e
*g - log(f + g*x**2)**2*b*e*g + 4*log(f + g*x**2)*a*c*e*g*x + 4*a*c*d*g*x
- 8*a*c*e*g*x)/(4*c*g)
```

3.159 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx &= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right) \\ &\quad +\frac{1}{2}bd\,\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}bd\,\mathrm{PolyLog}\left(2,\frac{1}{cx}\right) \\ &\quad +\frac{1}{2}ae\,\mathrm{PolyLog}\left(2,1+\frac{gx^2}{f}\right)+be\mathrm{Int}\left(\frac{\coth^{-1}(cx)\log\left(f+gx^2\right)}{x},x\right)\end{aligned}$$

output

```
a*d*ln(x)+1/2*a*e*ln(-g*x^2/f)*ln(g*x^2+f)+1/2*b*d*polylog(2,-1/c/x)-1/2*b
*d*polylog(2,1/c/x)+1/2*a*e*polylog(2,1+g*x^2/f)+b*e*Defer(Int)(arccoth(c*
x)*ln(g*x^2+f)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{6642}$$

$$d\int \frac{a+b\coth^{-1}(cx)}{x}dx+e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(gx^2+f\right)}{x}dx$$

$$\Big\downarrow \text{6447}$$

$$e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(gx^2+f\right)}{x}dx+d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{6640}$$

$$e\left(a\int \frac{\log\left(gx^2+f\right)}{x}dx+b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2904}$$

$$e\left(\frac{1}{2}a\int \frac{\log\left(gx^2+f\right)}{x^2}dx^2+b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2841}$$

$$e\left(\frac{1}{2}a\left(\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-g\int \frac{\log\left(-\frac{gx^2}{f}\right)}{gx^2+f}dx^2\right)+b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$e\left(b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\right)\right)+d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7299}$$

$$e\left(b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\right)\right)+d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(g\,x^{2}+f\right)\right)}{x}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log\left(gx^2+f\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f)
)/x, x)
```

Sympy [N/A]

Not integrable

Time = 117.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log(f+gx^2))}{x}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(f + g*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.88

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + integrate(1/2*b*e*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))*log(
g*x^2 + f)/x + 1/2*b*d*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))/x + a*e*log(
g*x^2 + f)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln(g\,x^2+f)\right)}{x}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.17

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \left(\int \frac{acoth(cx)}{g\,x^3+fx}dx\right)bdf + \left(\int \frac{\log(g\,x^2+f)}{g\,x^3+fx}dx\right)aef + \left(\int \frac{acoth(cx)\log(g\,x^2+f)\,x}{g\,x^2+f}dx\right)beg + \left(\int \frac{acoth(cx)\log(g\,x^2+f)}{g\,x^3+fx}dx\right)bef + \left(\int \frac{acoth(cx)\,x}{g\,x^2+f}dx\right)bdg + \frac{\log(g\,x^2+f)^2\,ae}{4} + \log(x)\,ad$$

input
```
int((a+b*acoth(c*x))*(d+e*log(g*x^2+f))/x,x)
```

output
```
(4*int(acoth(c*x)/(f*x + g*x**3),x)*b*d*f + 4*int(log(f + g*x**2)/(f*x + g
*x**3),x)*a*e*f + 4*int((acoth(c*x)*log(f + g*x**2)*x)/(f + g*x**2),x)*b*e
*g + 4*int((acoth(c*x)*log(f + g*x**2))/(f*x + g*x**3),x)*b*e*f + 4*int((a
coth(c*x)*x)/(f + g*x**2),x)*b*d*g + log(f + g*x**2)**2*a*e + 4*log(x)*a*d
)/4
```

3.160 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 560

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx \\
&= \frac{2ae\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} \\
&\quad + \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(ic\sqrt{f}-\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}} \\
&\quad - \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(1+cx)}{(ic\sqrt{f}+\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}} \\
&\quad - \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right) \\
&\quad - \frac{1}{2}bc\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right) - \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right) \\
&\quad + \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,1+\frac{gx^2}{f}\right) - \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(ic\sqrt{f}-\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}} \\
&\quad + \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(1+cx)}{(ic\sqrt{f}+\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}}
\end{aligned}$$

output
```
2*a*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))/f^(1/2)-b*e*g^(1/2)*arctan(g^(1/2)
*x/f^(1/2))*ln(1-1/c/x)/f^(1/2)+b*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(1
+1/c/x)/f^(1/2)+b*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2
)*(-c*x+1)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)-b*e*g^(1/2
)*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*(c*x+1)/(I*c*f^(1/2)+g^(1
/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)-(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x
+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+f))-1/2*b*c*ln(g*(-c^2*x^2+1)/(c^2*f+g
))*(d+e*ln(g*x^2+f))-1/2*b*c*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))+1/2*b*c*
e*polylog(2,1+g*x^2/f)-1/2*I*b*e*g^(1/2)*polylog(2,1+2*f^(1/2)*g^(1/2)*(-c
*x+1)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)+1/2*I*b*e*g^(1/
2)*polylog(2,1-2*f^(1/2)*g^(1/2)*(c*x+1)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*
g^(1/2)*x))/f^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1236 vs. $2(560) = 1120$.

Time = 2.33 (sec) , antiderivative size = 1236, normalized size of antiderivative = 2.21

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-((a*d)/x) - (b*d*ArcCoth[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2]
)/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2
]/x) + (b*e*(-(((2*ArcCoth[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[
f + g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*
Sqrt[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (
I*Sqrt[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x
))/(c*Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*
x))/(c*Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g
]*x))/(c*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log
[1 - c^2*x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x)
)/(I*c*Sqrt[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt
[f] + Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g]
)] + PolyLog[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyL
og[2, (I*Sqrt[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) - (c*g*((2*I)*ArcCos
[(c^2*f - g)/(c^2*f + g)]*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] - 4*ArcCoth[c*x]
*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcT
an[(c*f)/(Sqrt[c^2*f*g]*x)])*Log[(2*g*(c^2*f - I*Sqrt[c^2*f*g])*(-1 + c*x)
)/((c^2*f + g)*(I*Sqrt[c^2*f*g] + c*g*x))] + (ArcCos[(c^2*f - g)/(c^2*f +
g)] - 2*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)])*Log[(2*g*(c^2*f + I*Sqrt[c^2*f*g]
)*(1 + c*x))/((c^2*f + g)*(I*Sqrt[c^2*f*g] + c*g*x))] - (ArcCos[(c^2*f ...
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6644, 2925, 2863, 2009, 6537, 218, 6535, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{6644}$$

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx+bc\int \frac{d+e\log\left(gx^2+f\right)}{x\left(1-c^2x^2\right)}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{x^2\left(1-c^2x^2\right)}dx^2-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$\Big\downarrow$ 2863

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \left(\frac{d+e\log\left(gx^2+f\right)}{x^2}-\frac{c^2\left(d+e\log\left(gx^2+f\right)\right)}{c^2x^2-1}\right)dx^2-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$\Big\downarrow$ 2009

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 6537

$$2eg\left(a\int \frac{1}{gx^2+f}dx+b\int \frac{\coth^{-1}(cx)}{gx^2+f}dx\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 218

$$2eg\left(b\int \frac{\coth^{-1}(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 6535

$$2eg\left(b\left(\frac{1}{2}\int \frac{\log\left(1+\frac{1}{cx}\right)}{gx^2+f}dx-\frac{1}{2}\int \frac{\log\left(1-\frac{1}{cx}\right)}{gx^2+f}dx\right)+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\Big\downarrow$ 2920

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(c-\frac{1}{x})x^2}dx}{c}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(c+\frac{1}{x})x^2}dx}{c}+\frac{\log\left(\frac{1}{cx}+1\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$$\Big\downarrow 27$$

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{(c-\frac{1}{x})x^2}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{(c+\frac{1}{x})x^2}dx}{\sqrt{f}\sqrt{g}}+\frac{\log\left(\frac{1}{cx}+1\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$$\Big\downarrow 2005$$

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx-1)}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx+1)}dx}{\sqrt{f}\sqrt{g}}+\frac{\log\left(\frac{1}{cx}+1\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$$\Big\downarrow 5411$$

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int\left(\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{cx-1}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\right)dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}-\frac{c\arctan\left(\right.}{cx+}\right.}{\sqrt{f}\sqrt{g}}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\downarrow$ 2009

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}\left(-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)-\text{arc}}{}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-(((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x) + (b*c*(Log[-((g*x^2)/f
)]*(d + e*Log[f + g*x^2]) - Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[
f + g*x^2]) - e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f + g)] + e*PolyLog[2, 1
 + (g*x^2)/f]))/2 + 2*e*g*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g
]) + b*((-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/(Sqrt[f]*Sqrt[g]
)) + (-(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)]) + ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c
*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] - (I/2)*PolyLog[2, ((-I)*Sqr
t[g]*x)/Sqrt[f]] + (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] + (I/2)*PolyLog
[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] - (I/2)*PolyLog[2, 1 + (2*Sqr
t[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(Sqrt[f]*Sqrt[g]))/2 + ((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/(
Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] -
 I*Sqrt[g]*x)] - ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c
*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] + (I/2)*PolyLog[2,
 ((-I)*Sqrt[g]*x)/Sqrt[f]] - (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] - (I/
2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] + (I/2)*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x))])/(Sqrt[f]*Sqrt[g]))/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 6535
```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)]/(d
+ e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 6537
```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcCoth[c*x]/(d + e*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

Maple [F]

$$\int \frac{(a + b\,\mathrm{arccoth}\,(cx))\,(d + e\ln(g\,x^2 + f))}{x^2}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f)
)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d + (2*g*arcta
n(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + 1/2*b*e*integrate((lo
g(1/(c*x) + 1) - log(-1/(c*x) + 1))*log(g*x^2 + f)/x^2, x) - a*d/x
```

Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x^2}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,(d+e\,\ln(g\,x^2+f))}{x^2}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x^2}\,dx$$
$$= \frac{-acoth(cx)\,bcdfx - acoth(cx)\,bdf + 2\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right)aex + \left(\int \frac{acoth(cx)\log(g\,x^2+f)}{x^2}dx\right)befx + \log(c^2}{fx}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(g*x^2+f))/x^2,x)
```

output
```
( - acoth(c*x)*b*c*d*f*x - acoth(c*x)*b*d*f + 2*sqrt(g)*sqrt(f)*atan((g*x)
/(sqrt(g)*sqrt(f)))*a*e*x + int((acoth(c*x)*log(f + g*x**2))/x**2,x)*b*e*f
*x + log(c**2*x - c)*b*c*d*f*x - log(f + g*x**2)*a*e*f - log(x)*b*c*d*f*x
- a*d*f)/(f*x)
```

3.161 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 712

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx$$

$$= \frac{bce\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f}$$
$$+ bc^2e\text{arctanh}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{beg\coth^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{2f}$$
$$- \frac{1}{2}bc^2e\text{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})\,(1+cx)}\right)$$
$$- \frac{beg\coth^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{2f}$$
$$- \frac{1}{2}bc^2e\text{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})\,(1+cx)}\right) - \frac{aeg\log(f+gx^2)}{2f}$$
$$- \frac{bc(d+e\log(f+gx^2))}{2x} - \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{2x^2}$$
$$+ \frac{1}{2}bc^2\text{arctanh}(cx)\,(d+e\log(f+gx^2)) + \frac{beg\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)}{2f}$$
$$- \frac{beg\,\text{PolyLog}\left(2,\frac{1}{cx}\right)}{2f} - \frac{1}{2}bc^2e\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)$$
$$- \frac{beg\,\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2f} + \frac{1}{4}bc^2e\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})\,(1+cx)}\right)$$
$$+ \frac{beg\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{4f}$$
$$+ \frac{1}{4}bc^2e\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})\,(1+cx)}\right)$$
$$+ \frac{beg\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{4f}$$

output
```
b*c*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))/f^(1/2)+a*e*g*ln(x)/f+b*e*g*arccot
h(c*x)*ln(2/(c*x+1))/f+b*c^2*e*arctanh(c*x)*ln(2/(c*x+1))-1/2*b*e*g*arccot
h(c*x)*ln(2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))/(c*x+1))/f-1/2
*b*c^2*e*arctanh(c*x)*ln(2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^(1/2))
/(c*x+1))-1/2*b*e*g*arccoth(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/
2)+g^(1/2))/(c*x+1))/f-1/2*b*c^2*e*arctanh(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)
*x)/(c*(-f)^(1/2)+g^(1/2))/(c*x+1))-1/2*a*e*g*ln(g*x^2+f)/f-1/2*b*c*(d+e*l
n(g*x^2+f))/x-1/2*(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*b*c^2*arcta
nh(c*x)*(d+e*ln(g*x^2+f))+1/2*b*e*g*polylog(2,-1/c/x)/f-1/2*b*e*g*polylog(
2,1/c/x)/f-1/2*b*c^2*e*polylog(2,1-2/(c*x+1))-1/2*b*e*g*polylog(2,1-2/(c*x
+1))/f+1/4*b*c^2*e*polylog(2,1-2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-g^
(1/2))/(c*x+1))+1/4*b*e*g*polylog(2,1-2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(
1/2)-g^(1/2))/(c*x+1))/f+1/4*b*c^2*e*polylog(2,1-2*c*((-f)^(1/2)+g^(1/2)*x
)/(c*(-f)^(1/2)+g^(1/2))/(c*x+1))+1/4*b*e*g*polylog(2,1-2*c*((-f)^(1/2)+g^
(1/2)*x)/(c*(-f)^(1/2)+g^(1/2))/(c*x+1))/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.47 (sec) , antiderivative size = 1193, normalized size of antiderivative = 1.68

$$\int \frac{(a + b \coth^{-1}(cx)) (d + e \log (f + gx^2))}{x^3} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
(-2*a*d*f - 2*b*c*d*f*x - 2*b*d*f*ArcCoth[c*x] + 2*b*c^2*d*f*x^2*ArcCoth[c
*x] + 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + (4*I)*b*c^
2*e*f*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f*g)]*x)]
+ (4*I)*b*e*g*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f*
g)]*x)] + 4*b*c^2*e*f*x^2*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] + 4*b*
e*g*x^2*ArcCoth[c*x]*Log[1 + E^(-2*ArcCoth[c*x])] - 2*b*c^2*e*f*x^2*ArcCot
h[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2
*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - 2*b*e*g*x^2*ArcCoth
[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*
Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + (2*I)*b*c^2*e*f*x^2*
ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(
2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))]
+ (2*I)*b*e*g*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[
c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*
x])*(c^2*f + g))] - 2*b*c^2*e*f*x^2*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCo
th[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth
[c*x])*(c^2*f + g))] - 2*b*e*g*x^2*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCot
h[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[
c*x])*(c^2*f + g))] - (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[
(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(...
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 6648}$$

$$-2eg\int\left(\frac{bc^2x\operatorname{arctanh}(cx)}{2\left(gx^2+f\right)}-\frac{a+bcx+b\coth^{-1}(cx)}{2x\left(gx^2+f\right)}\right)dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2\operatorname{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

$$\downarrow \text{2009}$$

$$-2eg\left(\frac{a\log\left(f+gx^2\right)}{4f}-\frac{a\log(x)}{2f}-\frac{bc\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}+\frac{bc^2\text{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)}{4g}+\frac{bc^2\text{arctanh}(cx\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[f + g*x^2]))/x - ((a + b*ArcCoth[c*x])*(d + e*Log[f +
 g*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]))/2 - 2*e*g*
(-1/2*(b*c*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (a*Log[x])/(2*
f) - (b*ArcCoth[c*x]*Log[2/(1 + c*x)])/(2*f) - (b*c^2*ArcTanh[c*x]*Log[2/(
1 + c*x)])/(2*g) + (b*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sq
rt[-f] - Sqrt[g])*(1 + c*x))])/(4*f) + (b*c^2*ArcTanh[c*x]*Log[(2*c*(Sqrt[
-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(4*g) + (b*ArcCoth[
c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])
/(4*f) + (b*c^2*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f]
 + Sqrt[g])*(1 + c*x))])/(4*g) + (a*Log[f + g*x^2])/(4*f) - (b*PolyLog[2,
-(1/(c*x))])/(4*f) + (b*PolyLog[2, 1/(c*x)])/(4*f) + (b*PolyLog[2, 1 - 2/(
1 + c*x)])/(4*f) + (b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(4*g) - (b*PolyLog[
2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(
8*f) - (b*c^2*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - S
qrt[g])*(1 + c*x))])/(8*g) - (b*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x)
)/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(8*f) - (b*c^2*PolyLog[2, 1 - (2*c*
(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(8*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 7.21 (sec) , antiderivative size = 937, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{ad}{2x^2}+\frac{aeg\ln(x)}{f}-\frac{aeg\ln(g\,x^2+f)}{2f}+\frac{db\,c^2\ln(cx+1)}{4}-\frac{db\ln(cx+1)}{4x^2}-\frac{db\,c^2\ln(cx-1)}{4}+\frac{db\ln(cx-1)}{4x^2}+\Big(-\frac{be\ln(cx+}{4x^2}$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*d/x^2+a*e*g*ln(x)/f-1/2*a*e*g*ln(g*x^2+f)/f+1/4*d*b*c^2*ln(c*x+1)-1
/4*d*b*ln(c*x+1)/x^2-1/4*d*b*c^2*ln(c*x-1)+1/4*d*b*ln(c*x-1)/x^2+(-1/4*b*e
/x^2*ln(c*x+1)-1/4*e*(b*x^2*ln(c*x-1)*c^2-b*c^2*ln(c*x+1)*x^2+2*b*c*x-b*ln
(c*x-1)+2*a)/x^2)*ln(g*x^2+f)+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-
1)-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-
1)+g)/(c*(-f*g)^(1/2)+g))*c^2+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-g*(c*x-1)-
g)/(c*(-f*g)^(1/2)-g))+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(
-f*g)^(1/2)+g))-1/2*g*b*e/f*dilog(c*x)-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(
c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))-1/2*g*b*e/f*dilog(c*x+1)-1/2*d*b*c/x-1/4*b
*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b
*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2-1/4*g
*b*e/f*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*b*e*dilo
g((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*dilog((c*(-
f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1
/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)+g*(
c*x-1)+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)
-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)
+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+g*e*b*c/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/
2))+1/4*g*b*e/f*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-
g))+1/4*g*b*e/f*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/...
```

Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = \int \frac{(b\operatorname{arcoth}(cx)+a)(e\log(gx^2+f)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f)
)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx$$
$$= \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(gx^2+f)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d -
 1/2*(g*(log(g*x^2 + f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e - 1/4*(2
*c^2*g*integrate(x^2*log(c*x + 1)/(g*x^3 + f*x), x) - 2*c^2*g*integrate(x^
2*log(c*x - 1)/(g*x^3 + f*x), x) + 2*I*c*g*(log(I*g*x/sqrt(f*g) + 1) - log
(-I*g*x/sqrt(f*g) + 1))/sqrt(f*g) - 2*g*integrate(log(c*x + 1)/(g*x^3 + f*
x), x) + 2*g*integrate(log(c*x - 1)/(g*x^3 + f*x), x) + (2*c*x - (c^2*x^2
- 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1))*log(g*x^2 + f)/x^2)*b*e -
1/2*a*d/x^2
```

Giac [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(gx^2+f\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = \int \frac{\left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*acoth(c*x))*(d+e*log(g*x^2+f))/x^3,x)
```

output

```
( - acoth(c*x)*log(f + g*x**2)*b*c**2*e*f**2 + acoth(c*x)*log(f + g*x**2)*
b*e*f*g + acoth(c*x)*b*c**4*d*f**2*x**2 + acoth(c*x)*b*c**4*e*f**2*x**2 -
acoth(c*x)*b*c**2*d*f**2 - acoth(c*x)*b*c**2*d*f*g*x**2 - acoth(c*x)*b*c**
2*e*f**2 - acoth(c*x)*b*c**2*e*f*g*x**2 + acoth(c*x)*b*d*f*g + acoth(c*x)*
b*e*f*g + 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*b*c*e*g*x**2 + 2
*int(acoth(c*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 - c**2*f**2*x**3 - 2*c**2*
f*g*x**5 - c**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*c**4*e*f**4*x**2 -
4*int(acoth(c*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 - c**2*f**2*x**3 - 2*c**2
*f*g*x**5 - c**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*c**2*e*f**3*g*x**2
 + 2*int(acoth(c*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 - c**2*f**2*x**3 - 2*c
**2*f*g*x**5 - c**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*e*f**2*g**2*x**
2 - 2*int(acoth(c*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 - c**2*f**2*x - 2*c**
2*f*g*x**3 - c**2*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**6*e*f**4*x**2 + 4
*int(acoth(c*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 - c**2*f**2*x - 2*c**2*f*g
*x**3 - c**2*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**4*e*f**3*g*x**2 - 2*in
t(acoth(c*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 - c**2*f**2*x - 2*c**2*f*g*x*
*3 - c**2*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**2*e*f**2*g**2*x**2 + int(
log(f + g*x**2)/(c**4*f**2*x**4 + c**4*f*g*x**6 - c**2*f**2*x**2 - 2*c**2*
f*g*x**4 - c**2*g**2*x**6 + f*g*x**2 + g**2*x**4),x)*b*c**5*e*f**4*x**2 -
int(log(f + g*x**2)/(c**4*f**2*x**4 + c**4*f*g*x**6 - c**2*f**2*x**2 - ...
```

3.162 $\int \coth^{-1}(e^x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \coth^{-1}(e^x)\, dx = \frac{\text{PolyLog}(2,-e^{-x})}{2} - \frac{\text{PolyLog}(2,e^{-x})}{2}$$

output
```
1/2*polylog(2,-exp(-x))-1/2*polylog(2,exp(-x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \coth^{-1}(e^x)\, dx = x\coth^{-1}(e^x) + \frac{1}{2}x\log(1-e^x) - \frac{1}{2}x\log(1+e^x) - \frac{\text{PolyLog}(2,-e^x)}{2} + \frac{\text{PolyLog}(2,e^x)}{2}$$

input
```
Integrate[ArcCoth[E^x],x]
```

output
```
x*ArcCoth[E^x] + (x*Log[1 - E^x])/2 - (x*Log[1 + E^x])/2 - PolyLog[2, -E^x
]/2 + PolyLog[2, E^x]/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2720, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(e^x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}\coth^{-1}(e^x)\,de^x$$

$$\Big\downarrow \text{6447}$$

$$\frac{\text{PolyLog}(2,-e^{-x})}{2}-\frac{\text{PolyLog}(2,e^{-x})}{2}$$

input
```
Int[ArcCoth[E^x],x]
```

output
```
PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{x\ln(e^{x}-1)}{2}-\frac{\text{dilog}(e^{x})}{2}-\frac{\text{dilog}(1+e^{x})}{2}$	22
derivativedivides	$\ln(e^{x})\operatorname{arccoth}(e^{x})-\frac{\text{dilog}(e^{x})}{2}-\frac{\text{dilog}(1+e^{x})}{2}-\frac{\ln(e^{x})\ln(1+e^{x})}{2}$	31
default	$\ln(e^{x})\operatorname{arccoth}(e^{x})-\frac{\text{dilog}(e^{x})}{2}-\frac{\text{dilog}(1+e^{x})}{2}-\frac{\ln(e^{x})\ln(1+e^{x})}{2}$	31
parts	$x\operatorname{arccoth}(e^{x})+\frac{x\ln(1-e^{x})}{2}+\frac{\text{polylog}(2,e^{x})}{2}-\frac{x\ln(1+e^{x})}{2}-\frac{\text{polylog}(2,-e^{x})}{2}$	39

input
```
int(arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*ln(exp(x)-1)-1/2*dilog(exp(x))-1/2*dilog(1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int \coth^{-1}(e^x)\,dx = \frac{1}{2}x\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1)+\frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}\operatorname{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}\operatorname{Li}_2(-\cosh(x)-\sinh(x))$$

input
```
integrate(arccoth(exp(x)),x, algorithm="fricas")
```

output
```
1/2*x*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cos
h(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x
) + sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \coth^{-1}(e^x)\ dx = \int \operatorname{acoth}(e^x)\,dx$$

input
```
integrate(acoth(exp(x)),x)
```

output
```
Integral(acoth(exp(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \coth^{-1}(e^x)\ dx = &-\frac{1}{2}x(\log(e^x+1)-\log(e^x-1))+x\operatorname{arcoth}(e^x)\\&+\frac{1}{2}\log(-e^x)\log(e^x+1)-\frac{1}{2}x\log(e^x-1)\\&+\frac{1}{2}\operatorname{Li}_2(e^x+1)-\frac{1}{2}\operatorname{Li}_2(-e^x+1)\end{aligned}$$

input
```
integrate(arccoth(exp(x)),x, algorithm="maxima")
```

output
```
-1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arccoth(e^x) + 1/2*log(-e^x)*log(
e^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

Giac [F]

$$\int \coth^{-1}(e^x)\ dx = \int \operatorname{arcoth}(e^x)\ dx$$

input
```
integrate(arccoth(exp(x)),x, algorithm="giac")
```

output
```
integrate(arccoth(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(e^x)\, dx = \int \mathrm{acoth}(\mathrm{e}^x)\, dx$$

input
```
int(acoth(exp(x)),x)
```

output
```
int(acoth(exp(x)), x)
```

Reduce [F]

$$\int \coth^{-1}(e^x)\, dx = \int acoth(e^x)\, dx$$

input
```
int(acoth(exp(x)),x)
```

output
```
int(acoth(e**x),x)
```

3.163 $\int x \coth^{-1}(e^x)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 51

$$\int x \coth^{-1}(e^x)\, dx = \frac{1}{2}x \operatorname{PolyLog}\left(2,-e^{-x}\right) - \frac{1}{2}x \operatorname{PolyLog}\left(2,e^{-x}\right) + \frac{\operatorname{PolyLog}\left(3,-e^{-x}\right)}{2} - \frac{\operatorname{PolyLog}\left(3,e^{-x}\right)}{2}$$

output
```
1/2*x*polylog(2,-exp(-x))-1/2*x*polylog(2,exp(-x))+1/2*polylog(3,-exp(-x))
-1/2*polylog(3,exp(-x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int x \coth^{-1}(e^x)\, dx = \frac{1}{4}\left(2x^2 \coth^{-1}(e^x) + x^2 \log(1-e^x) - x^2 \log(1+e^x) - 2x \operatorname{PolyLog}(2,-e^x) + 2x \operatorname{PolyLog}(2,e^x) + 2 \operatorname{PolyLog}(3,-e^x) - 2 \operatorname{PolyLog}(3,e^x)\right)$$

input
```
Integrate[x*ArcCoth[E^x],x]
```

output
```
(2*x^2*ArcCoth[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
 -E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6768, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}\left(e^x\right) dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2}\int x \log\left(1+e^{-x}\right) dx - \frac{1}{2}\int x \log\left(1-e^{-x}\right) dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(x \operatorname{PolyLog}\left(2,-e^{-x}\right) - \int \operatorname{PolyLog}\left(2,-e^{-x}\right) dx\right) + \frac{1}{2}\left(\int \operatorname{PolyLog}\left(2,e^{-x}\right) dx - x \operatorname{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\int e^x \operatorname{PolyLog}\left(2,-e^{-x}\right) de^{-x} + x \operatorname{PolyLog}\left(2,-e^{-x}\right)\right) + \frac{1}{2}\left(-\int e^x \operatorname{PolyLog}\left(2,e^{-x}\right) de^{-x} - x \operatorname{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(x \operatorname{PolyLog}\left(2,-e^{-x}\right) + \operatorname{PolyLog}\left(3,-e^{-x}\right)\right) + \frac{1}{2}\left(-x \operatorname{PolyLog}\left(2,e^{-x}\right) - \operatorname{PolyLog}\left(3,e^{-x}\right)\right)$$

input
```
Int[x*ArcCoth[E^x],x]
```

output
```
(x*PolyLog[2, -E^(-x)] + PolyLog[3, -E^(-x)])/2 + (-(x*PolyLog[2, E^(-x)])
 - PolyLog[3, E^(-x)])/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{x^2 \ln(e^x-1)}{4}-\frac{x \,\mathrm{polylog}(2,-e^x)}{2}+\frac{\mathrm{polylog}(3,-e^x)}{2}+\frac{x^2 \ln(1-e^x)}{4}+\frac{x \,\mathrm{polylog}(2,e^x)}{2}-\frac{\mathrm{polylog}(3,e^x)}{2}$
default	$\frac{x^2 \,\mathrm{arccoth}(e^x)}{2}+\frac{x^2 \ln(1-e^x)}{4}+\frac{x \,\mathrm{polylog}(2,e^x)}{2}-\frac{\mathrm{polylog}(3,e^x)}{2}-\frac{x^2 \ln(1+e^x)}{4}-\frac{x \,\mathrm{polylog}(2,-e^x)}{2}+\frac{\mathrm{polylog}(3,-e^x)}{2}$
parts	$\frac{x^2 \,\mathrm{arccoth}(e^x)}{2}+\frac{x^2 \ln(1-e^x)}{4}+\frac{x \,\mathrm{polylog}(2,e^x)}{2}-\frac{\mathrm{polylog}(3,e^x)}{2}-\frac{x^2 \ln(1+e^x)}{4}-\frac{x \,\mathrm{polylog}(2,-e^x)}{2}+\frac{\mathrm{polylog}(3,-e^x)}{2}$

input
```
int(x*arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*ln(exp(x)-1)-1/2*x*polylog(2,-exp(x))+1/2*polylog(3,-exp(x))+1/4*
x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*polylog(3,exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(37) = 74.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.84

$$\begin{aligned}\int x \coth^{-1}(e^x)\,dx = {} & \frac{1}{4}x^2\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right) \\ & -\frac{1}{4}x^2\log(\cosh(x)+\sinh(x)+1) \\ & +\frac{1}{4}x^2\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x\mathrm{Li}_2(\cosh(x)+\sinh(x)) \\ & -\frac{1}{2}x\mathrm{Li}_2(-\cosh(x)-\sinh(x))-\frac{1}{2}\mathrm{polylog}(3,\cosh(x)+\sinh(x)) \\ & +\frac{1}{2}\mathrm{polylog}(3,-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(x*arccoth(exp(x)),x, algorithm="fricas")
```

output
```
1/4*x^2*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log
(cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilo
g(cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, co
sh(x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

Sympy [F]

$$\int x \coth^{-1}(e^x)\ dx = \int x \operatorname{acoth}(e^x)\, dx$$

input
```
integrate(x*acoth(exp(x)),x)
```

output
```
Integral(x*acoth(exp(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x \coth^{-1}(e^x)\ dx = \frac{1}{2}\,x^2 \operatorname{arcoth}(e^x) - \frac{1}{4}\,x^2 \log(e^x+1) + \frac{1}{4}\,x^2 \log(-e^x+1) - \frac{1}{2}\,x\mathrm{Li}_2(-e^x) + \frac{1}{2}\,x\mathrm{Li}_2(e^x) + \frac{1}{2}\,\mathrm{Li}_3(-e^x) - \frac{1}{2}\,\mathrm{Li}_3(e^x)$$

input
```
integrate(x*arccoth(exp(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*
x*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e
^x)
```

Giac [F]

$$\int x \coth^{-1}(e^x)\ dx = \int x \operatorname{arcoth}(e^x)\ dx$$

input
```
integrate(x*arccoth(exp(x)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}\left(e^x\right)\,dx=\int x\,\mathrm{acoth}(\mathrm{e}^x)\,dx$$

input
```
int(x*acoth(exp(x)),x)
```

output
```
int(x*acoth(exp(x)), x)
```

Reduce [F]

$$\int x\coth^{-1}\left(e^x\right)\,dx=\int acoth(e^x)\,x dx$$

input
```
int(x*acoth(exp(x)),x)
```

output
```
int(acoth(e**x)*x,x)
```

3.164 $\int x^2 \coth^{-1}(e^x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 70

$$\int x^2 \coth^{-1}(e^x)\,dx = \frac{1}{2}x^2 \operatorname{PolyLog}\left(2,-e^{-x}\right) - \frac{1}{2}x^2 \operatorname{PolyLog}\left(2,e^{-x}\right) + x \operatorname{PolyLog}\left(3,-e^{-x}\right) - x \operatorname{PolyLog}\left(3,e^{-x}\right) + \operatorname{PolyLog}\left(4,-e^{-x}\right) - \operatorname{PolyLog}\left(4,e^{-x}\right)$$

output
```
1/2*x^2*polylog(2,-exp(-x))-1/2*x^2*polylog(2,exp(-x))+x*polylog(3,-exp(-x
))-x*polylog(3,exp(-x))+polylog(4,-exp(-x))-polylog(4,exp(-x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int x^2 \coth^{-1}(e^x)\,dx = \frac{1}{6}\left(2x^3 \coth^{-1}(e^x) + x^3 \log(1-e^x) - x^3 \log(1+e^x) - 3x^2 \operatorname{PolyLog}(2,-e^x) + 3x^2 \operatorname{PolyLog}(2,e^x) + 6x \operatorname{PolyLog}(3,-e^x) - 6x \operatorname{PolyLog}(3,e^x) - 6 \operatorname{PolyLog}(4,-e^x) + 6 \operatorname{PolyLog}(4,e^x)\right)$$

input
```
Integrate[x^2*ArcCoth[E^x],x]
```

output
```
(2*x^3*ArcCoth[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[
2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E
^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6768, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}\left(e^x\right) dx$$

$$\Big\downarrow \text{ 6768}$$

$$\frac{1}{2}\int x^2 \log\left(1+e^{-x}\right) dx - \frac{1}{2}\int x^2 \log\left(1-e^{-x}\right) dx$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}\left(x^2 \text{PolyLog}\left(2,-e^{-x}\right) - 2\int x \text{PolyLog}\left(2,-e^{-x}\right) dx\right) + \frac{1}{2}\left(2\int x \text{PolyLog}\left(2,e^{-x}\right) dx - x^2 \text{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{1}{2}\left(x^2 \text{PolyLog}\left(2,-e^{-x}\right) - 2\left(\int \text{PolyLog}\left(3,-e^{-x}\right) dx - x \text{PolyLog}\left(3,-e^{-x}\right)\right)\right) + \frac{1}{2}\left(2\left(\int \text{PolyLog}\left(3,e^{-x}\right) dx - x \text{PolyLog}\left(3,e^{-x}\right)\right) - x^2 \text{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}\left(x^2 \text{PolyLog}\left(2,-e^{-x}\right) - 2\left(-\int e^x \text{PolyLog}\left(3,-e^{-x}\right) de^{-x} - x \text{PolyLog}\left(3,-e^{-x}\right)\right)\right) + \frac{1}{2}\left(2\left(-\int e^x \text{PolyLog}\left(3,e^{-x}\right) de^{-x} - x \text{PolyLog}\left(3,e^{-x}\right)\right) - x^2 \text{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}\left(x^2 \operatorname{PolyLog}\left(2,-e^{-x}\right)-2\left(-x \operatorname{PolyLog}\left(3,-e^{-x}\right)-\operatorname{PolyLog}\left(4,-e^{-x}\right)\right)\right)+\frac{1}{2}\left(2\left(-x \operatorname{PolyLog}\left(3,e^{-x}\right)-\operatorname{PolyLog}\left(4,e^{-x}\right)\right)-x^2 \operatorname{PolyLog}\left(2,e^{-x}\right)\right)$$

input
```
Int[x^2*ArcCoth[E^x],x]
```

output
```
(x^2*PolyLog[2, -E^(-x)] - 2*(-(x*PolyLog[3, -E^(-x)]) - PolyLog[4, -E^(-x
)]))/2 + (-(x^2*PolyLog[2, E^(-x)]) + 2*(-(x*PolyLog[3, E^(-x)]) - PolyLog
[4, E^(-x)]))/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x^3 \ln(e^x-1)}{6} - \frac{x^2 \operatorname{polylog}(2,-e^x)}{2} + x \operatorname{polylog}(3,-e^x) - \operatorname{polylog}(4,-e^x) + \frac{x^3 \ln(1-e^x)}{6} + \frac{x^2 \operatorname{polylog}(2,e^x)}{2}$
default	$\frac{x^3 \operatorname{arccoth}(e^x)}{3} + \frac{x^3 \ln(1-e^x)}{6} + \frac{x^2 \operatorname{polylog}(2,e^x)}{2} - x \operatorname{polylog}(3,e^x) + \operatorname{polylog}(4,e^x) - \frac{x^3 \ln(1+e^x)}{6} - \frac{x^2 \operatorname{poly}}{}$
parts	$\frac{x^3 \operatorname{arccoth}(e^x)}{3} + \frac{x^3 \ln(1-e^x)}{6} + \frac{x^2 \operatorname{polylog}(2,e^x)}{2} - x \operatorname{polylog}(3,e^x) + \operatorname{polylog}(4,e^x) - \frac{x^3 \ln(1+e^x)}{6} - \frac{x^2 \operatorname{poly}}{}$

input
```
int(x^2*arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*ln(exp(x)-1)-1/2*x^2*polylog(2,-exp(x))+x*polylog(3,-exp(x))-poly
log(4,-exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2,exp(x))-x*polylog(3,
exp(x))+polylog(4,exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int x^2 \coth^{-1}(e^x)\, dx = \frac{1}{6}x^3 \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{6}x^3 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{6}x^3 \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2}x^2 \mathrm{Li}_2(\cosh(x) + \sinh(x)) - \frac{1}{2}x^2 \mathrm{Li}_2(-\cosh(x) - \sinh(x)) - x\mathrm{polylog}(3, \cosh(x) + \sinh(x)) + x\mathrm{polylog}(3, -\cosh(x) - \sinh(x)) + \mathrm{polylog}(4, \cosh(x) + \sinh(x)) - \mathrm{polylog}(4, -\cosh(x) - \sinh(x))$$

input
```
integrate(x^2*arccoth(exp(x)),x, algorithm="fricas")
```

output
```
1/6*x^3*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log
(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*di
log(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3,
cosh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x)
 + sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

Sympy [F]

$$\int x^2 \coth^{-1}(e^x)\, dx = \int x^2 \operatorname{acoth}(e^x)\, dx$$

input
```
integrate(x**2*acoth(exp(x)),x)
```

output
```
Integral(x**2*acoth(exp(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int x^2 \coth^{-1}(e^x)\,dx = \frac{1}{3}x^3 \operatorname{arcoth}(e^x) - \frac{1}{6}x^3 \log(e^x+1) + \frac{1}{6}x^3 \log(-e^x+1) - \frac{1}{2}x^2 \mathrm{Li}_2(-e^x) + \frac{1}{2}x^2 \mathrm{Li}_2(e^x) + x\mathrm{Li}_3(-e^x) - x\mathrm{Li}_3(e^x) - \mathrm{Li}_4(-e^x) + \mathrm{Li}_4(e^x)$$

input
```
integrate(x^2*arccoth(exp(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*
x^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e
^x) - polylog(4, -e^x) + polylog(4, e^x)
```

Giac [F]

$$\int x^2 \coth^{-1}(e^x)\,dx = \int x^2 \operatorname{arcoth}(e^x)\,dx$$

input
```
integrate(x^2*arccoth(exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(e^x)\,dx = \int x^2 \operatorname{acoth}(e^x)\,dx$$

input
```
int(x^2*acoth(exp(x)),x)
```

output
```
int(x^2*acoth(exp(x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}(e^x)\ dx = \int acoth(e^x)\, x^2 dx$$

input
```
int(x^2*acoth(exp(x)),x)
```

output
```
int(acoth(e**x)*x**2,x)
```

3.165 $\int \coth^{-1}\left(e^{a+bx}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \coth^{-1}\left(e^{a+bx}\right) dx = \frac{\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b} - \frac{\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

output
```
1/2*polylog(2,-exp(-b*x-a))/b-1/2*polylog(2,exp(-b*x-a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \coth^{-1}\left(e^{a+bx}\right) dx = \frac{bx\left(2\coth^{-1}\left(e^{a+bx}\right)+\log\left(1-e^{a+bx}\right)-\log\left(1+e^{a+bx}\right)\right)-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

input
```
Integrate[ArcCoth[E^(a + b*x)],x]
```

output
```
(b*x*(2*ArcCoth[E^(a + b*x)] + Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)]
) - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\coth^{-1}\left(e^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{6447}$$

$$\frac{\frac{1}{2}\text{PolyLog}\left(2,-e^{-a-bx}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{-a-bx}\right)}{b}$$

input
```
Int[ArcCoth[E^(a + b*x)],x]
```

output
```
(PolyLog[2, -E^(-a - b*x)]/2 - PolyLog[2, E^(-a - b*x)]/2)/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\operatorname{dilog}(\mathrm{e}^{bx+a}+1)}{2b}-\frac{\ln(\mathrm{e}^{bx+a}-1)\ln(\mathrm{e}^{bx+a})}{2b}-\frac{\operatorname{dilog}(\mathrm{e}^{bx+a})}{2b}$
derivativedivides	$\frac{\ln(\mathrm{e}^{bx+a})\operatorname{arccoth}(\mathrm{e}^{bx+a})-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}}{b}$
default	$\frac{\ln(\mathrm{e}^{bx+a})\operatorname{arccoth}(\mathrm{e}^{bx+a})-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}}{b}$
parts	$x\operatorname{arccoth}\left(\mathrm{e}^{bx+a}\right)+\frac{\frac{(bx+a)\ln\left(1-\mathrm{e}^{bx+a}\right)}{2}+\frac{\operatorname{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{2}-\frac{(bx+a)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{2}+a\ \mathrm{arcta}}{b}$

input
```
int(arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*dilog(exp(b*x+a)+1)-1/2/b*ln(exp(b*x+a)-1)*ln(exp(b*x+a))-1/2/b*dil
og(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.34

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx$$
$$=\frac{bx\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-a\log\left(\cosh\left(bx+a\right)+\sinh\right.}{}$$

input
```
integrate(arccoth(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
 + a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x
+ a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) +
 1) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b
*x + a)))/b
```

Sympy [F]

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{acoth}\left(e^{a+bx}\right)dx$$

input
```
integrate(acoth(exp(b*x+a)),x)
```

output
```
Integral(acoth(exp(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(33) = 66.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \frac{(bx+a)\operatorname{arcoth}\left(e^{(bx+a)}\right)}{b} - \frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e^{(bx+a}\right.}{2b}$$

input
```
integrate(arccoth(exp(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)*arccoth(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) -
log(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)
*log(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/
b
```

Giac [F]

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{arcoth}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(arccoth(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(acoth(exp(a + b*x)),x)
```

output
```
int(acoth(exp(a + b*x)), x)
```

Reduce [F]

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int acoth\left(e^{bx+a}\right)dx$$

input
```
int(acoth(exp(b*x+a)),x)
```

output
```
int(acoth(e**(a + b*x)),x)
```

3.166 $\int x \coth^{-1}\left(e^{a+bx}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx = \frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{2b} - \frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{2b} + \frac{\operatorname{PolyLog}\left(3,-e^{-a-bx}\right)}{2b^2} - \frac{\operatorname{PolyLog}\left(3,e^{-a-bx}\right)}{2b^2}$$

output
```
1/2*x*polylog(2,-exp(-b*x-a))/b-1/2*x*polylog(2,exp(-b*x-a))/b+1/2*polylog
(3,-exp(-b*x-a))/b^2-1/2*polylog(3,exp(-b*x-a))/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx = \frac{2b^2x^2 \coth^{-1}\left(e^{a+bx}\right) + b^2x^2 \log\left(1-e^{a+bx}\right) - b^2x^2 \log\left(1+e^{a+bx}\right) - 2bx \operatorname{PolyLog}\left(2,-e^{a+bx}\right) + 2bx \operatorname{Poly}}{4b^2}$$

input
```
Integrate[x*ArcCoth[E^(a + b*x)],x]
```

output
```
(2*b^2*x^2*ArcCoth[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*L
og[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^
(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^
2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6768, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2}\int x \log\left(1+e^{-a-bx}\right) dx-\frac{1}{2}\int x \log\left(1-e^{-a-bx}\right) dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{\int \operatorname{PolyLog}\left(2,-e^{-a-bx}\right) dx}{b}\right)+\frac{1}{2}\left(\frac{\int \operatorname{PolyLog}\left(2,e^{-a-bx}\right) dx}{b}-\frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{\int e^{a+bx} \operatorname{PolyLog}\left(2,-e^{-a-bx}\right) de^{-a-bx}}{b^2}+\frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}\right)+\frac{1}{2}\left(-\frac{\int e^{a+bx} \operatorname{PolyLog}\left(2,e^{-a-bx}\right) de^{-a-bx}}{b^2}-\frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\frac{\operatorname{PolyLog}\left(3,-e^{-a-bx}\right)}{b^2}+\frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}\right)+\frac{1}{2}\left(-\frac{\operatorname{PolyLog}\left(3,e^{-a-bx}\right)}{b^2}-\frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

input
```
Int[x*ArcCoth[E^(a + b*x)],x]
```

output
```
((x*PolyLog[2, -E^(-a - b*x)])/b + PolyLog[3, -E^(-a - b*x)]/b^2)/2 + (-((
x*PolyLog[2, E^(-a - b*x)])/b) - PolyLog[3, E^(-a - b*x)]/b^2)/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(71) = 142.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.16

method	result
default	$\frac{x^2 \operatorname{arccoth}(e^{bx+a})}{2} + \frac{-a^2 \operatorname{arctanh}(e^{bx+a}) + \frac{(bx+a)^2 \ln(1-e^{bx+a})}{2} + (bx+a) \operatorname{polylog}(2, e^{bx+a}) - \operatorname{polylog}(3, e^{bx+a}) - \frac{(bx+a)^2 \ln(e^{bx+}}{2}$
parts	$\frac{x^2 \operatorname{arccoth}(e^{bx+a})}{2} + \frac{-a^2 \operatorname{arctanh}(e^{bx+a}) + \frac{(bx+a)^2 \ln(1-e^{bx+a})}{2} + (bx+a) \operatorname{polylog}(2, e^{bx+a}) - \operatorname{polylog}(3, e^{bx+a}) - \frac{(bx+a)^2 \ln(e^{bx+}}{2}$
risch	$-\frac{x^2 \ln(e^{bx+a}-1)}{4} + \frac{\ln(1-e^{bx+a})x^2}{4} + \frac{\ln(1-e^{bx+a})ax}{2b} + \frac{\ln(1-e^{bx+a})a^2}{4b^2} + \frac{\operatorname{polylog}(2, e^{bx+a})x}{2b} + \frac{a^2 \ln(e^{bx+a}-1)}{4b^2} + \text{pc}$

input
```
int(x*arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccoth(exp(b*x+a))+1/2/b^2*(-a^2*arctanh(exp(b*x+a))+1/2*(b*x+a)^
2*ln(1-exp(b*x+a))+(b*x+a)*polylog(2,exp(b*x+a))-polylog(3,exp(b*x+a))-1/2
*(b*x+a)^2*ln(exp(b*x+a)+1)-(b*x+a)*polylog(2,-exp(b*x+a))+polylog(3,-exp(
b*x+a))-a*(b*x+a)*ln(1-exp(b*x+a))+a*(b*x+a)*ln(exp(b*x+a)+1)+a*polylog(2,
-exp(b*x+a))-a*polylog(2,exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(69) = 138.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.39

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx$$
$$= \frac{b^2x^2 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^2x^2 \log(\cosh(bx+a) + \sinh(bx+a) + 1) + 2\,bx\mathrm{Li}_2(\cosh(bx+a) +}{}$$

input
```
integrate(x*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*
x + a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log
(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

Sympy [F]

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{acoth}\left(e^{a}e^{bx}\right)dx$$

input
```
integrate(x*acoth(exp(b*x+a)),x)
```

output
```
Integral(x*acoth(exp(a)*exp(b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx=\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(e^{(bx+a)}\right)-\frac{1}{4}\,b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3}-\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2(}{b^3}\right.$$

input
```
integrate(x*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*
x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^
(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
)
```

Giac [F]

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{arcoth}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x*arccoth(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(x*acoth(exp(a + b*x)),x)
```

output
```
int(x*acoth(exp(a + b*x)), x)
```

Reduce [F]

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx=\int acoth\left(e^{bx+a}\right)xdx$$

input
```
int(x*acoth(exp(b*x+a)),x)
```

output
```
int(acoth(e**(a + b*x))*x,x)
```

3.167 $\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 119

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = \frac{x^2 \operatorname{PolyLog}\left(2, -e^{-a-bx}\right)}{2b} - \frac{x^2 \operatorname{PolyLog}\left(2, e^{-a-bx}\right)}{2b} + \frac{x \operatorname{PolyLog}\left(3, -e^{-a-bx}\right)}{b^2} - \frac{x \operatorname{PolyLog}\left(3, e^{-a-bx}\right)}{b^2} + \frac{\operatorname{PolyLog}\left(4, -e^{-a-bx}\right)}{b^3} - \frac{\operatorname{PolyLog}\left(4, e^{-a-bx}\right)}{b^3}$$

output
```
1/2*x^2*polylog(2,-exp(-b*x-a))/b-1/2*x^2*polylog(2,exp(-b*x-a))/b+x*polyl
og(3,-exp(-b*x-a))/b^2-x*polylog(3,exp(-b*x-a))/b^2+polylog(4,-exp(-b*x-a)
)/b^3-polylog(4,exp(-b*x-a))/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = \frac{2b^3x^3 \coth^{-1}\left(e^{a+bx}\right) + b^3x^3 \log\left(1 - e^{a+bx}\right) - b^3x^3 \log\left(1 + e^{a+bx}\right) - 3b^2x^2 \operatorname{PolyLog}\left(2, -e^{a+bx}\right) + 3b^2x^2 \text{I}}{}$$

input
```
Integrate[x^2*ArcCoth[E^(a + b*x)],x]
```

output
```
(2*b^3*x^3*ArcCoth[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*L
og[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyL
og[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(
a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3
)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6768, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2}\int x^2 \log\left(1+e^{-a-bx}\right) dx - \frac{1}{2}\int x^2 \log\left(1-e^{-a-bx}\right) dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{x^2 \text{PolyLog}\left(2,-e^{-a-bx}\right)}{b} - \frac{2\int x \text{PolyLog}\left(2,-e^{-a-bx}\right) dx}{b}\right) + \frac{1}{2}\left(\frac{2\int x \text{PolyLog}\left(2,e^{-a-bx}\right) dx}{b} - \frac{x^2 \text{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2}\left(\frac{x^2 \text{PolyLog}\left(2,-e^{-a-bx}\right)}{b} - \frac{2\left(\frac{\int \text{PolyLog}\left(3,-e^{-a-bx}\right)dx}{b} - \frac{x \text{PolyLog}\left(3,-e^{-a-bx}\right)}{b}\right)}{b}\right) + \frac{1}{2}\left(\frac{2\left(\frac{\int \text{PolyLog}\left(3,e^{-a-bx}\right)dx}{b} - \frac{x \text{PolyLog}\left(3,e^{-a-bx}\right)}{b}\right)}{b} - \frac{x^2 \text{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{x^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\int e^{a+bx} \operatorname{PolyLog}(3,-e^{-a-bx})de^{-a-bx}}{b^2}-\frac{x \operatorname{PolyLog}(3,-e^{-a-bx})}{b}\right)}{b}\right)+\frac{1}{2}\left(\frac{2\left(-\frac{\int e^{a+bx} \operatorname{PolyLog}(3,e^{-a-bx})de^{-a-bx}}{b^2}-\frac{x \operatorname{PolyLog}(3,e^{-a-bx})}{b}\right)}{b}-\frac{x^2 \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}\left(\frac{x^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\operatorname{PolyLog}(4,-e^{-a-bx})}{b^2}-\frac{x \operatorname{PolyLog}(3,-e^{-a-bx})}{b}\right)}{b}\right)+\frac{1}{2}\left(\frac{2\left(-\frac{\operatorname{PolyLog}(4,e^{-a-bx})}{b^2}-\frac{x \operatorname{PolyLog}(3,e^{-a-bx})}{b}\right)}{b}-\frac{x^2 \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

input
```
Int[x^2*ArcCoth[E^(a + b*x)],x]
```

output
```
((x^2*PolyLog[2, -E^(-a - b*x)])/b - (2*(-((x*PolyLog[3, -E^(-a - b*x)])/b
) - PolyLog[4, -E^(-a - b*x)]/b^2))/b)/2 + (-((x^2*PolyLog[2, E^(-a - b*x)
])/b) + (2*(-((x*PolyLog[3, E^(-a - b*x)])/b) - PolyLog[4, E^(-a - b*x)]/b
^2))/b)/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(109) = 218$.

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{x^3\ln(e^{bx+a}-1)}{6}+\frac{\ln(1-e^{bx+a})x^3}{6}-\frac{\ln(1-e^{bx+a})x\,a^2}{2b^2}+\frac{\mathrm{polylog}(2,e^{bx+a})x^2}{2b}-\frac{\ln(1-e^{bx+a})a^3}{3b^3}-\frac{a^3\ln(e^{bx+a}-1)}{6b^3}-$
default	$\frac{x^3\,\mathrm{arccoth}(e^{bx+a})}{3}+$ [numerator: $\frac{(bx+a)^3\ln(1-e^{bx+a})}{2}+\frac{3(bx+a)^2\,\mathrm{polylog}(2,e^{bx+a})}{2}-3(bx+a)\,\mathrm{polylog}(3,e^{bx+a})+3\,\mathrm{polylog}(4,e^{bx+a})-(bx+a)^3$ …]
parts	$\frac{x^3\,\mathrm{arccoth}(e^{bx+a})}{3}+$ [numerator: $\frac{(bx+a)^3\ln(1-e^{bx+a})}{2}+\frac{3(bx+a)^2\,\mathrm{polylog}(2,e^{bx+a})}{2}-3(bx+a)\,\mathrm{polylog}(3,e^{bx+a})+3\,\mathrm{polylog}(4,e^{bx+a})-(bx+a)^3$ …]

input
```
int(x^2*arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*ln(exp(b*x+a)-1)+1/6*ln(1-exp(b*x+a))*x^3-1/2/b^2*ln(1-exp(b*x+a)
)*x*a^2+1/2/b*polylog(2,exp(b*x+a))*x^2-1/3/b^3*ln(1-exp(b*x+a))*a^3-1/6/b
^3*a^3*ln(exp(b*x+a)-1)-1/2/b^3*a^2*dilog(exp(b*x+a))-1/b^2*polylog(3,exp(
b*x+a))*x-1/2/b^3*polylog(2,exp(b*x+a))*a^2+1/b^3*polylog(4,exp(b*x+a))-1/
2/b*polylog(2,-exp(b*x+a))*x^2+1/b^2*polylog(3,-exp(b*x+a))*x+1/2/b^3*poly
log(2,-exp(b*x+a))*a^2-1/2/b^3*dilog(exp(b*x+a)+1)*a^2-1/b^3*polylog(4,-ex
p(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(107) = 214$.

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.08

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx$$
$$= \frac{b^3x^3 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3x^3 \log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 3\,b^2x^2 \mathrm{Li}_2(\cosh\left(bx+a\right)}{}$$

input
```
integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x
^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) -
 sinh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polyl
og(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - s
inh(b*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) +
6*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a)
- sinh(b*x + a)))/b^3
```

Sympy [F]

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = \int x^2 \operatorname{acoth}\left(e^a e^{bx}\right) dx$$

input
```
integrate(x**2*acoth(exp(b*x+a)),x)
```

output
```
Integral(x**2*acoth(exp(a)*exp(b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = \frac{1}{3} x^3 \operatorname{arcoth}\left(e^{(bx+a)}\right) - \frac{1}{6} b \left(\frac{b^3 x^3 \log\left(e^{(bx+a)}+1\right) + 3 b^2 x^2 \mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6 bx \mathrm{Li}_3(-e^{(bx+a)}) + 6 \mathrm{Li}_4(-e^{(bx+a)})}{b^4} - \frac{b^3 x^3 \log\left(-e\right.}{}\right.$$

input
```
integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^
2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4,
-e^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b
*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

Giac [F]

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = \int x^2 \operatorname{arcoth}\left(e^{(bx+a)}\right) dx$$

input
```
integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) \, dx = \int x^2 \operatorname{acoth}\left(e^{a+b\,x}\right) \, dx$$

input
```
int(x^2*acoth(exp(a + b*x)),x)
```

output
```
int(x^2*acoth(exp(a + b*x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) \, dx = \int acoth\left(e^{bx+a}\right) x^2 dx$$

input
```
int(x^2*acoth(exp(b*x+a)),x)
```

output
```
int(acoth(e**(a + b*x))*x**2,x)
```

3.168 $\int \coth^{-1}\left(a+bf^{c+dx}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 168

$$\int \coth^{-1}\left(a+bf^{c+dx}\right) dx = -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\text{PolyLog}\left(2,1-\frac{2}{1+a+bf^{c+dx}}\right)}{2d\log(f)} - \frac{\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)(1+a+bf^{c+dx})}\right)}{2d\log(f)}$$

output
```
-arccoth(a+b*f^(d*x+c))*ln(2/(1+a+b*f^(d*x+c)))/d/ln(f)+arccoth(a+b*f^(d*x
+c))*ln(2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c)))/d/ln(f)+1/2*polylog(2,1-2/(
1+a+b*f^(d*x+c)))/d/ln(f)-1/2*polylog(2,1-2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*
x+c)))/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{dx\log(f)\left(2\coth^{-1}\left(a+bf^{c+dx}\right)+\log\left(\frac{-1+a+bf^{c+dx}}{-1+a}\right)-\log\left(\frac{1+a+bf^{c+dx}}{1+a}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-\operatorname{Po}}{2d\log(f)}$$

input
```
Integrate[ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(d*x*Log[f]*(2*ArcCoth[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(
-1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c +
d*x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2720, 6662, 25, 27, 6473, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$\Big\downarrow \ 2720$$
$$\frac{\int f^{-c-dx}\coth^{-1}\left(bf^{c+dx}+a\right)df^{c+dx}}{d\log(f)}$$
$$\Big\downarrow \ 6662$$
$$\frac{\int f^{-c-dx}\coth^{-1}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$
$$\Big\downarrow \ 25$$
$$-\frac{\int -f^{-c-dx}\coth^{-1}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{f^{-c-dx}\coth^{-1}(bf^{c+dx}+a)}{b}d(bf^{c+dx}+a)}{d\log(f)}$$

$$\Big\downarrow \text{6473}$$

$$-\frac{-\int \frac{\log\left(\frac{2}{bf^{c+dx}+a+1}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c}\right.}{d\log(f)}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{-\int \frac{\log\left(\frac{2}{bf^{c+dx}+a+1}\right)}{1-\frac{2}{bf^{c+dx}+a+1}}d\frac{1}{bf^{c+dx}+a+1}+\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+c}\right.}{d\log(f)}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{bf^{c+dx}+a+1}\right)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right.}{d\log(f)}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{bf^{c+dx}+a+1}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)(bf^{c+dx}+a+1)}\right)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+}\right.}{d\log(f)}$$

input
```
Int[ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
-((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))] - ArcCoth[a +
 b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))] -
 PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/2 + PolyLog[2, 1 - (2*b*f^(c +
d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/2)/(d*Log[f]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6473
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcCoth
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\ln(-b f^{dx+c}) \operatorname{arccoth}(a+b f^{dx+c})+\frac{\operatorname{dilog}\left(\frac{-b f^{dx+c}-a+1}{1-a}\right)}{2}+\frac{\ln(-b f^{dx+c}) \ln\left(\frac{-b f^{dx+c}-a+1}{1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{-b f^{dx+c}-a-1}{-a-1}\right.}{2}}{d \ln(f)}$
default	$\frac{\ln(-b f^{dx+c}) \operatorname{arccoth}(a+b f^{dx+c})+\frac{\operatorname{dilog}\left(\frac{-b f^{dx+c}-a+1}{1-a}\right)}{2}+\frac{\ln(-b f^{dx+c}) \ln\left(\frac{-b f^{dx+c}-a+1}{1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{-b f^{dx+c}-a-1}{-a-1}\right.}{2}}{d \ln(f)}$
risch	$-\frac{x \ln(a+b f^{dx+c}-1)}{2}+\frac{\operatorname{dilog}\left(\frac{a+b f^{dx} f^{c}-1}{a-1}\right)}{2 \ln(f) d}+\frac{\ln\left(\frac{a+b f^{dx} f^{c}-1}{a-1}\right) x}{2}+\frac{\ln\left(\frac{a+b f^{dx} f^{c}-1}{a-1}\right) c}{2d}-\frac{c \ln(a+b f^{dx} f^{c}}{2d}$

input
```
int(arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)*(ln(-b*f^(d*x+c))*arccoth(a+b*f^(d*x+c))+1/2*dilog((-b*f^(d*x+c)
-a+1)/(1-a))+1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a+1)/(1-a))-1/2*dilog((
-b*f^(d*x+c)-a-1)/(-a-1))-1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a-1)/(-a-1
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.68

$$\int \coth^{-1}\left(a+bf^{c+dx}\right) dx$$
$$= \frac{dx \log(f) \log\left(\frac{b \cosh((dx+c)\log(f))+b \sinh((dx+c)\log(f))+a+1}{b \cosh((dx+c)\log(f))+b \sinh((dx+c)\log(f))+a-1}\right)+c \log\left(b \cosh\left((dx+c)\log(f)\right)+b \sinh\left((dx+c\right.\right.}{}$$

input
```
integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d*x*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c
*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) -
 c*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)
 - (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f
)) + a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*
sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x +
 c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

Sympy [F]

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \operatorname{acoth}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(acoth(a+b*f**(d*x+c)),x)
```

output
```
Integral(acoth(a + b*f**(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{(dx+c)\operatorname{arcoth}\left(bf^{dx+c}+a\right)}{d}-\frac{(dx+c)b\left(\frac{\log\left(bf^{dx+c}+a+1\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)}{b}\right)\log\left(f\right)-b\left(\frac{\log\left(bf^{dx+c}+a+1\right)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{bf^{dx+c}+a+1}{a+1}\right.}{b}\right.}{2\,d\log\left(f\right)}$$

input
```
integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)*arccoth(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x +
c) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x +
c) + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x +
c) + a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c)
+ a - 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log
(f))
```

Giac [F(-2)]

Exception generated.

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,2,0,0,0]%%%}+%%%{2,[0,1,1,1,1,0]%%%}+%%%{-2,[0,1,1,0,0,0]%%%}+%%%{1,[0,
1,0,2,0,1]%%
```

Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \int \mathrm{acoth}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(acoth(a + b*f^(c + d*x)),x)
```

output
```
int(acoth(a + b*f^(c + d*x)), x)
```

Reduce [F]

$$\int \coth^{-1}\left(a + bf^{c+dx}\right) \, dx = \int acoth\left(f^{dx+c}b + a\right) dx$$

input
```
int(acoth(a+b*f^(d*x+c)),x)
```

output
```
int(acoth(f**(c + d*x)*b + a),x)
```

3.169 $\int x \coth^{-1}\left(a+bf^{c+dx}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\begin{aligned}\int x \coth^{-1}\left(a+bf^{c+dx}\right) \, dx &= \frac{1}{4}x^2 \log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2 \log\left(1+\frac{bf^{c+dx}}{1+a}\right)\\ &\quad -\frac{1}{4}x^2 \log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2 \log\left(1+\frac{1}{a+bf^{c+dx}}\right)\\ &\quad +\frac{x \operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x \operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)}\\ &\quad -\frac{\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{2d^2\log^2(f)}\end{aligned}$$

output

```
1/4*x^2*ln(1-b*f^(d*x+c)/(1-a))-1/4*x^2*ln(1+b*f^(d*x+c)/(1+a))-1/4*x^2*ln
(1-1/(a+b*f^(d*x+c)))+1/4*x^2*ln(1+1/(a+b*f^(d*x+c)))+1/2*x*polylog(2,b*f^
(d*x+c)/(1-a))/d/ln(f)-1/2*x*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-1/2*pol
ylog(3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+1/2*polylog(3,-b*f^(d*x+c)/(1+a))/d^
2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2d^2x^2\coth^{-1}\left(a+bf^{c+dx}\right)\log^2(f)+d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+2dx}{}$$

input
```
Integrate[x*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(2*d^2*x^2*ArcCoth[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 +
(b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 +
a)] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*
PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-
1 + a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6768, 3031, 25, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$\Big\downarrow \; 6768$$
$$\frac{1}{2}\int x\log\left(1+\frac{1}{bf^{c+dx}+a}\right)dx-\frac{1}{2}\int x\log\left(1-\frac{1}{bf^{c+dx}+a}\right)dx$$
$$\Big\downarrow \; 3031$$
$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{bdf^{c+dx}x^2\log(f)}{\left(-bf^{c+dx}-a+1\right)\left(bf^{c+dx}+a\right)}dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+ \frac{1}{2}\left(\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)-\frac{1}{2}\int -\frac{bdf^{c+dx}x^2\log(f)}{\left(bf^{c+dx}+a\right)\left(bf^{c+dx}+a+1\right)}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{bdf^{c+dx}x^2\log(f)}{(-bf^{c+dx}-a+1)\,(bf^{c+dx}+a)}dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{bdf^{c+dx}x^2\log(f)}{(bf^{c+dx}+a)\,(bf^{c+dx}+a+1)}dx+\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{2}bd\log(f)\int \frac{f^{c+dx}x^2}{(-bf^{c+dx}-a+1)\,(bf^{c+dx}+a)}dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}bd\log(f)\int \frac{f^{c+dx}x^2}{(bf^{c+dx}+a)\,(bf^{c+dx}+a+1)}dx+\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}\left(\frac{1}{2}bd\log(f)\int \left(\frac{x^2f^{c+dx}}{-bf^{c+dx}-a-1}+\frac{x^2f^{c+dx}}{bf^{c+dx}+a}\right)dx+\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)+\frac{1}{2}\left(-\frac{1}{2}bd\log(f)\int \left(\frac{x^2f^{c+dx}}{-bf^{c+dx}-a+1}+\frac{x^2f^{c+dx}}{bf^{c+dx}+a}\right)dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{2}bd\log(f)\left(\frac{2\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{bd^3\log^3(f)}-\frac{2\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}-\frac{2x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{bd^2\log^2(f)}+\frac{2x\,\text{PolyLog}\left(2,\right.}{bd^2\log^2(}\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}bd\log(f)\left(-\frac{2\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}+\frac{2\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{bd^3\log^3(f)}+\frac{2x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a}\right)}{bd^2\log^2(f)}-\frac{2x\,\text{PolyLog}}{bd^2\,\text{l}}\right.\right.$$

input
```
Int[x*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(-1/2*(x^2*Log[1 - (a + b*f^(c + d*x))^(-1)]) - (b*d*Log[f]*(-((x^2*Log[1
- (b*f^(c + d*x))/(1 - a)])/(b*d*Log[f])) + (x^2*Log[1 + (b*f^(c + d*x))/a
])/(b*d*Log[f]) - (2*x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(b*d^2*Log[f]^
2) + (2*x*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2) + (2*PolyLog[
3, (b*f^(c + d*x))/(1 - a)])/(b*d^3*Log[f]^3) - (2*PolyLog[3, -((b*f^(c +
d*x))/a)])/(b*d^3*Log[f]^3)))/2)/2 + ((x^2*Log[1 + (a + b*f^(c + d*x))^(-1
)])/2 + (b*d*Log[f]*((x^2*Log[1 + (b*f^(c + d*x))/a])/(b*d*Log[f]) - (x^2*
Log[1 + (b*f^(c + d*x))/(1 + a)])/(b*d*Log[f]) + (2*x*PolyLog[2, -((b*f^(c
 + d*x))/a)])/(b*d^2*Log[f]^2) - (2*x*PolyLog[2, -((b*f^(c + d*x))/(1 + a)
)])/(b*d^2*Log[f]^2) - (2*PolyLog[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^
3) + (2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(b*d^3*Log[f]^3)))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(200) = 400$.

Time = 0.74 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.73

method	result
risch	$-\frac{x^2 \ln(a+b f^{dx+c}-1)}{4}+\frac{x^2 \ln(1+a+b f^{dx+c})}{4}-\frac{\ln\left(1-\frac{b f^{dx} f^{c}}{-a-1}\right) x^2}{4}-\frac{\ln\left(1-\frac{b f^{dx} f^{c}}{-a-1}\right) c x}{2d}-\frac{\ln\left(1-\frac{b f^{dx} f^{c}}{-a-1}\right) c^2}{4d^2}-$ polylog

input
```
int(x*arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*ln(a+b*f^(d*x+c)-1)+1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*ln(1-b*f^(d*x
)*f^c/(-a-1))*x^2-1/2/d*ln(1-b*f^(d*x)*f^c/(-a-1))*c*x-1/4/d^2*ln(1-b*f^(d
*x)*f^c/(-a-1))*c^2-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-a-1))*x-1/2/ln(f
)/d^2*polylog(2,b*f^(d*x)*f^c/(-a-1))*c+1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x
)*f^c/(-a-1))-1/4/d^2*c^2*ln(b*f^(d*x)*f^c+a+1)+1/2/ln(f)/d^2*c*dilog((b*f
^(d*x)*f^c+a+1)/(1+a))+1/2/d*c*ln((b*f^(d*x)*f^c+a+1)/(1+a))*x+1/2/d^2*c^2
*ln((b*f^(d*x)*f^c+a+1)/(1+a))+1/4*ln(1-b*f^(d*x)*f^c/(1-a))*x^2+1/2/d*ln(
1-b*f^(d*x)*f^c/(1-a))*c*x+1/4/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*c^2+1/2/ln(f)
/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/
(1-a))*c-1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))+1/4/d^2*c^2*ln(a+b
*f^(d*x)*f^c-1)-1/2/ln(f)/d^2*c*dilog((a+b*f^(d*x)*f^c-1)/(a-1))-1/2/d*c*l
n((a+b*f^(d*x)*f^c-1)/(a-1))*x-1/2/d^2*c^2*ln((a+b*f^(d*x)*f^c-1)/(a-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(193) = 386$.

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.83

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right) dx$$
$$= \frac{d^2x^2 \log(f)^2 \log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right) - c^2 \log\left(b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx\right.\right.}{}$$

input
```
integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(d^2*x^2*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1
)) - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*
log(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a
 - 1)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)
*log(f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 -
c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a
 + 1)/(a + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) +
b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x +
c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d
*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

Sympy [F]

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\, dx = \int x \operatorname{acoth}\left(a+bf^{c+dx}\right) dx$$

input
```
integrate(x*acoth(a+b*f**(d*x+c)),x)
```

output
```
Integral(x*acoth(a + b*f**(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\, dx =$$
$$-\frac{1}{4}\,bd\left(\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)-2\,\mathrm{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)}{bd^3\log(f)^3}-\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a-1}+\right.}{}\right.$$
$$+\frac{1}{2}\,x^2\operatorname{arcoth}\left(bf^{dx+c}+a\right)$$

input
```
integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((d^2*x^2*log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f)^2 + 2*d*x*dilog(-
b*f^(d*x)*f^c/(a + 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d
^3*log(f)^3) - (d^2*x^2*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f)^2 + 2*d*x*di
log(-b*f^(d*x)*f^c/(a - 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))
/(b*d^3*log(f)^3))*log(f) + 1/2*x^2*arccoth(b*f^(d*x + c) + a)
```

Giac [F]

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{arcoth}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{acoth}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x*acoth(a + b*f^(c + d*x)),x)
```

output
```
int(x*acoth(a + b*f^(c + d*x)), x)
```

Reduce [F]

$$\int x \coth^{-1}\left(a + bf^{c+dx}\right) dx = \int acoth\left(f^{dx+c}b + a\right) x dx$$

input
```
int(x*acoth(a+b*f^(d*x+c)),x)
```

output
```
int(acoth(f**(c + d*x)*b + a)*x,x)
```

3.170 $\int x^2 \coth^{-1}\left(a + b f^{c+dx}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 269

$$\begin{aligned}\int x^2 \coth^{-1}\left(a + b f^{c+dx}\right) dx &= \frac{1}{6}x^3 \log\left(1 - \frac{b f^{c+dx}}{1-a}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{b f^{c+dx}}{1+a}\right) \\ &\quad - \frac{1}{6}x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6}x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\ &\quad + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)} - \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{1+a}\right)}{2d \log(f)} \\ &\quad - \frac{x \operatorname{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{d^2 \log^2(f)} + \frac{x \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{1+a}\right)}{d^2 \log^2(f)} \\ &\quad + \frac{\operatorname{PolyLog}\left(4, \frac{b f^{c+dx}}{1-a}\right)}{d^3 \log^3(f)} - \frac{\operatorname{PolyLog}\left(4, -\frac{b f^{c+dx}}{1+a}\right)}{d^3 \log^3(f)}\end{aligned}$$

output
```
1/6*x^3*ln(1-b*f^(d*x+c)/(1-a))-1/6*x^3*ln(1+b*f^(d*x+c)/(1+a))-1/6*x^3*ln
(1-1/(a+b*f^(d*x+c)))+1/6*x^3*ln(1+1/(a+b*f^(d*x+c)))+1/2*x^2*polylog(2,b*
f^(d*x+c)/(1-a))/d/ln(f)-1/2*x^2*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-x*p
olylog(3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+x*polylog(3,-b*f^(d*x+c)/(1+a))/d^
2/ln(f)^2+polylog(4,b*f^(d*x+c)/(1-a))/d^3/ln(f)^3-polylog(4,-b*f^(d*x+c)/
(1+a))/d^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{2d^3x^3\coth^{-1}\left(a+bf^{c+dx}\right)\log^3(f)+d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+3d^2}{}$$

input
```
Integrate[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(2*d^3*x^3*ArcCoth[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 +
(b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 +
a)] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x
^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[
3, -((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x)
)/(1 + a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((
b*f^(c + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 557 vs. 2(269) = 538.

Time = 2.83 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6768, 3031, 25, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$\Big\downarrow \text{6768}$$
$$\frac{1}{2}\int x^2\log\left(1+\frac{1}{bf^{c+dx}+a}\right)dx-\frac{1}{2}\int x^2\log\left(1-\frac{1}{bf^{c+dx}+a}\right)dx$$
$$\Big\downarrow \text{3031}$$

$$\frac{1}{2}\left(\frac{1}{3}\int -\frac{bdf^{c+dx}x^3\log(f)}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)-\frac{1}{3}\int -\frac{bdf^{c+dx}x^3\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{bdf^{c+dx}x^3\log(f)}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{3}\int \frac{bdf^{c+dx}x^3\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx+\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{1}{3}bd\log(f)\int \frac{f^{c+dx}x^3}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{3}bd\log(f)\int \frac{f^{c+dx}x^3}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx+\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$$\Big\downarrow \ 7293$$

$$\frac{1}{2}\left(\frac{1}{3}bd\log(f)\int \left(\frac{x^3f^{c+dx}}{-bf^{c+dx}-a-1}+\frac{x^3f^{c+dx}}{bf^{c+dx}+a}\right)dx+\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)+\frac{1}{2}\left(-\frac{1}{3}bd\log(f)\int \left(\frac{x^3f^{c+dx}}{-bf^{c+dx}-a+1}+\frac{x^3f^{c+dx}}{bf^{c+dx}+a}\right)dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{1}{3}bd\log(f)\left(-\frac{6\,\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{bd^4\log^4(f)}+\frac{6\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a}\right)}{bd^4\log^4(f)}+\frac{6x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{bd^3\log^3(f)}-\frac{6x\,\text{PolyLog}\left(\right.}{bd^3\log}\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{3}bd\log(f)\left(\frac{6\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a}\right)}{bd^4\log^4(f)}-\frac{6\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{bd^4\log^4(f)}-\frac{6x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}+\frac{6x\,\text{PolyLog}\left(\right.}{bd^3\log}\right.\right.$$

input
```
Int[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(-1/3*(x^3*Log[1 - (a + b*f^(c + d*x))^(-1)]) - (b*d*Log[f]*(-((x^3*Log[1
- (b*f^(c + d*x))/(1 - a)])/(b*d*Log[f])) + (x^3*Log[1 + (b*f^(c + d*x))/a
])/(b*d*Log[f]) - (3*x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(b*d^2*Log[f
]^2) + (3*x^2*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2) + (6*x*Po
lyLog[3, (b*f^(c + d*x))/(1 - a)])/(b*d^3*Log[f]^3) - (6*x*PolyLog[3, -((b
*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) - (6*PolyLog[4, (b*f^(c + d*x))/(1 - a
)])/(b*d^4*Log[f]^4) + (6*PolyLog[4, -((b*f^(c + d*x))/a)])/(b*d^4*Log[f]^
4)))/3)/2 + ((x^3*Log[1 + (a + b*f^(c + d*x))^(-1)])/3 + (b*d*Log[f]*((x^3
*Log[1 + (b*f^(c + d*x))/a])/(b*d*Log[f]) - (x^3*Log[1 + (b*f^(c + d*x))/(
1 + a)])/(b*d*Log[f]) + (3*x^2*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Lo
g[f]^2) - (3*x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(b*d^2*Log[f]^2)
- (6*x*PolyLog[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) + (6*x*PolyLog[3
, -((b*f^(c + d*x))/(1 + a))])/(b*d^3*Log[f]^3) + (6*PolyLog[4, -((b*f^(c
+ d*x))/a)])/(b*d^4*Log[f]^4) - (6*PolyLog[4, -((b*f^(c + d*x))/(1 + a))])
/(b*d^4*Log[f]^4)))/3)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(257) = 514$.

Time = 1.19 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.48

method	result
risch	$-\frac{x^3\ln(a+b\,f^{dx+c}-1)}{6}+\frac{x^3\ln(1+a+b\,f^{dx+c})}{6}-\frac{\ln\left(1-\frac{b\,f^{dx}f^c}{-a-1}\right)x^3}{6}+\frac{\ln\left(1-\frac{b\,f^{dx}f^c}{-a-1}\right)x\,c^2}{2d^2}+\frac{\ln\left(1-\frac{b\,f^{dx}f^c}{-a-1}\right)c^3}{3d^3}-$ polyl

input
```
int(x^2*arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*ln(a+b*f^(d*x+c)-1)+1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*ln(1-b*f^(d*x
)*f^c/(-a-1))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-a-1))*x*c^2+1/3/d^3*ln(1-b*
f^(d*x)*f^c/(-a-1))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-a-1))*x^2+1/
2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-a-1))*c^2+1/ln(f)^2/d^2*polylog(3,b*
f^(d*x)*f^c/(-a-1))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(-a-1))+1/6/d^
3*c^3*ln(b*f^(d*x)*f^c+a+1)-1/2/ln(f)/d^3*c^2*dilog((b*f^(d*x)*f^c+a+1)/(1
+a))-1/2/d^2*c^2*ln((b*f^(d*x)*f^c+a+1)/(1+a))*x-1/2/d^3*c^3*ln((b*f^(d*x)
*f^c+a+1)/(1+a))+1/6*ln(1-b*f^(d*x)*f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*
f^c/(1-a))*x*c^2-1/3/d^3*ln(1-b*f^(d*x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog
(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c
^2-1/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,
b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*ln(a+b*f^(d*x)*f^c-1)+1/2/ln(f)/d^3*c^2*d
ilog((a+b*f^(d*x)*f^c-1)/(a-1))+1/2/d^2*c^2*ln((a+b*f^(d*x)*f^c-1)/(a-1))*
x+1/2/d^3*c^3*ln((a+b*f^(d*x)*f^c-1)/(a-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.78

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right) dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(d^3*x^3*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1
)) - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f))
 + a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f
)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*c
osh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*l
og(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 -
 (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)
*log(f)) + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c
)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*poly
log(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6
*d*x*log(f)*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f
)))/(a - 1)) - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*
log(f)))/(a + 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x
+ c)*log(f)))/(a - 1)))/(d^3*log(f)^3)
```

Sympy [F]

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right) dx = \int x^2 \operatorname{acoth}\left(a+bf^{c+dx}\right) dx$$

input
```
integrate(x**2*acoth(a+b*f**(d*x+c)),x)
```

output
```
Integral(x**2*acoth(a + b*f**(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.94

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right) dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(bf^{dx+c}+a\right) - \frac{1}{6}bd\left(\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^3+3d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)^2-6dx\log(f)\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a+1})+6\mathrm{Li}_4(\cdots}{bd^4\log(f)^4}\right.$$

input
```
integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*log(b*f^(d*x)*f^c/(
a + 1) + 1)*log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f)^2 -
6*d*x*log(f)*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*
f^c/(a + 1)))/(b*d^4*log(f)^4) - (d^3*x^3*log(b*f^(d*x)*f^c/(a - 1) + 1)*l
og(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f)^2 - 6*d*x*log(f)*
polylog(3, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))
/(b*d^4*log(f)^4))*log(f)
```

Giac [F]

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right) dx = \int x^2 \operatorname{arcoth}\left(bf^{dx+c}+a\right) dx$$

input
```
integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x^2\,\mathrm{acoth}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x^2*acoth(a + b*f^(c + d*x)),x)
```

output
```
int(x^2*acoth(a + b*f^(c + d*x)), x)
```

Reduce [F]

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int acoth\left(f^{dx+c}b+a\right)x^2dx$$

input
```
int(x^2*acoth(a+b*f^(d*x+c)),x)
```

output
```
int(acoth(f**(c + d*x)*b + a)*x**2,x)
```

3.171 $\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

output
```
-1/2*ln(1-2*arccoth(x))/a/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(-1+2\coth^{-1}(x)\right)}{2ab}$$

input
```
Integrate[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

output
```
-1/2*Log[-1 + 2*ArcCoth[x]]/(a*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}dx$$

$$\Big\downarrow \text{6509}$$

$$-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

input
```
Int[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

output
```
-1/2*Log[1 - 2*ArcCoth[x]]/(a*b)
```

Defintions of rubi rules used

rule 6509
```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$-\frac{\ln\left(\operatorname{arccoth}(x)-\frac{1}{2}\right)}{2ab}$	14
default	$-\frac{\ln(2b\ \operatorname{arccoth}(x)-b)}{2ab}$	19
risch	$-\frac{\ln(-1+\ln(x+1)-\ln(x-1))}{2ab}$	22

input
```
int(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(arccoth(x)-1/2)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx=-\frac{\log\left(\log\left(\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="fricas")
```

output
```
-1/2*log(log((x + 1)/(x - 1)) - 1)/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx=-\frac{\log\left(\operatorname{acoth}(x)-\frac{1}{2}\right)}{2ab}$$

input
```
integrate(1/(-a*x**2+a)/(b-2*b*acoth(x)),x)
```

output
```
-log(acoth(x) - 1/2)/(2*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(\log\left(x+1\right)-\log\left(x-1\right)-1\right)}{2\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="maxima")
```

output
```
-1/2*log(log(x + 1) - log(x - 1) - 1)/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(\frac{1}{4}\,\pi^2(\mathrm{sgn}(x+1)\,\mathrm{sgn}(x-1)-1)^2+\left(\log\left(\frac{|x+1|}{|x-1|}\right)-1\right)^2\right)}{4\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="giac")
```

output
```
-1/4*log(1/4*pi^2*(sgn(x + 1)*sgn(x - 1) - 1)^2 + (log(abs(x + 1)/abs(x -
1)) - 1)^2)/(a*b)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\ln\left(2\,\mathrm{acoth}(x)-1\right)}{2\,a\,b}$$

input
```
int(1/((a - a*x^2)*(b - 2*b*acoth(x))),x)
```

output
```
-log(2*acoth(x) - 1)/(2*a*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = \frac{\log(2acoth(x)-1)}{2ab}$$

input
```
int(1/(-a*x^2+a)/(b-2*b*acoth(x)),x)
```

output
```
log(2*acoth(x) - 1)/(2*a*b)
```

3.172 $\int x^3 \coth^{-1}(a+bx^4)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int x^3 \coth^{-1}(a+bx^4)\,dx = \frac{(a+bx^4)\coth^{-1}(a+bx^4)}{4b} + \frac{\log\left(1-(a+bx^4)^2\right)}{8b}$$

output
```
1/4*(b*x^4+a)*arccoth(b*x^4+a)/b+1/8*ln(1-(b*x^4+a)^2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^3 \coth^{-1}(a+bx^4)\,dx = \frac{2(a+bx^4)\coth^{-1}(a+bx^4)+\log\left(1-(a+bx^4)^2\right)}{8b}$$

input
```
Integrate[x^3*ArcCoth[a + b*x^4],x]
```

output
```
(2*(a + b*x^4)*ArcCoth[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7266, 6654, 6437, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}\left(a+bx^4\right) dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{4}\int \coth^{-1}\left(bx^4+a\right) dx^4$$

$$\Big\downarrow \text{6654}$$

$$\frac{\int \coth^{-1}\left(bx^4+a\right) d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{6437}$$

$$\frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)-\int \frac{bx^4+a}{1-x^8}d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{240}$$

$$\frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)+\frac{1}{2}\log\left(1-x^8\right)}{4b}$$

input
```
Int[x^3*ArcCoth[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcCoth[a + b*x^4] + Log[1 - x^8]/2)/(4*b)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{(b\,x^4+a)\,\operatorname{arccoth}(b\,x^4+a)+\frac{\ln\left((b\,x^4+a)^2-1\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\,\operatorname{arccoth}(b\,x^4+a)+\frac{\ln\left((b\,x^4+a)^2-1\right)}{2}}{4b}$
parts	$\frac{x^4\operatorname{arccoth}(b\,x^4+a)}{4}+b\left(\frac{(1-a)\ln(b\,x^4+a-1)}{8b^2}+\frac{(1+a)\ln(b\,x^4+a+1)}{8b^2}\right)$
parallelrisch	$-\frac{-\operatorname{arccoth}(b\,x^4+a)x^4b^2-\operatorname{arccoth}(b\,x^4+a)ab-\ln(b\,x^4+a-1)b-b\operatorname{arccoth}(b\,x^4+a)}{4b^2}$
risch	$\frac{x^4\ln(b\,x^4+a+1)}{8}-\frac{x^4\ln(b\,x^4+a-1)}{8}-\frac{\ln(-b\,x^4-a+1)a}{8b}+\frac{\ln(b\,x^4+a+1)a}{8b}+\frac{\ln(-b\,x^4-a+1)}{8b}+\frac{\ln(b\,x^4+a+}{8b}$

input
```
int(x^3*arccoth(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arccoth(b*x^4+a)+1/2*ln((b*x^4+a)^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int x^3\coth^{-1}\left(a+bx^4\right)\,dx = \frac{bx^4\log\left(\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8\,b}$$

input
```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(b*x^4*log((b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a +
1) - (a - 1)*log(b*x^4 + a - 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int x^3\coth^{-1}\left(a+bx^4\right)\,dx = \begin{cases}\frac{a\operatorname{acoth}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acoth}\left(a+bx^4\right)}{4}+\frac{\log\left(a+bx^4+1\right)}{4b}-\frac{\operatorname{acoth}\left(a+bx^4\right)}{4b} & \text{for } b\neq 0\\ \frac{x^4\operatorname{acoth}(a)}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*acoth(b*x**4+a),x)
```

output
```
Piecewise((a*acoth(a + b*x**4)/(4*b) + x**4*acoth(a + b*x**4)/4 + log(a +
b*x**4 + 1)/(4*b) - acoth(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*acoth(a)/4,
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx = \frac{2\,(bx^4+a)\,\mathrm{arcoth}\,(bx^4+a)+\log\left(-(bx^4+a)^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*(b*x^4 + a)*arccoth(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(40) = 80$.

Time = 0.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 5.11

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx$$

$$= \frac{1}{8}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx^4+a+1|}{|bx^4+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx^4+a+1}{bx^4+a-1}-1\right|\right)}{b^2}+\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{(bx^4+a+1)b}{bx^4+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{(bx^4+a+1)b}{bx^4+a-1}-b}}-1}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1}-1\right)}\right)$$

input
```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*((a + 1)*b - (a - 1)*b)*(log(abs(b*x^4 + a + 1)/abs(b*x^4 + a - 1))/b^
2 - log(abs((b*x^4 + a + 1)/(b*x^4 + a - 1) - 1))/b^2 + log(-(1/(a - ((b*x
^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 +
 a - 1) - b)) + 1)/(1/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a -
1)*b/((b*x^4 + a + 1)*b/(b*x^4 + a - 1) - b)) - 1))/(b^2*((b*x^4 + a + 1)/
(b*x^4 + a - 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx = \frac{x^4\,\ln\left(\frac{b\,x^4+a+1}{b\,x^4+a}\right)}{8} - \frac{x^4\,\ln\left(\frac{b\,x^4+a-1}{b\,x^4+a}\right)}{8} + \frac{\ln\left(b\,x^4+a-1\right)}{8\,b} + \frac{\ln\left(b\,x^4+a+1\right)}{8\,b} - \frac{a\,\ln\left(b\,x^4+a-1\right)}{8\,b} + \frac{a\,\ln\left(b\,x^4+a+1\right)}{8\,b}$$

input
```
int(x^3*acoth(a + b*x^4),x)
```

output
```
(x^4*log((a + b*x^4 + 1)/(a + b*x^4)))/8 - (x^4*log((a + b*x^4 - 1)/(a + b
*x^4)))/8 + log(a + b*x^4 - 1)/(8*b) + log(a + b*x^4 + 1)/(8*b) - (a*log(a
 + b*x^4 - 1))/(8*b) + (a*log(a + b*x^4 + 1))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 385, normalized size of antiderivative = 8.75

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx$$

$$= \frac{2acoth(b\,x^4+a)\,a + 2acoth(b\,x^4+a)\,b\,x^4 - \log\left(\frac{\sqrt{a-1}\,(a^2-1)^{\frac{1}{4}} - b^{\frac{1}{4}}\sqrt{2(a^2-1)^{\frac{1}{4}}a - 2(a^2-1)^{\frac{1}{4}} - \sqrt{-a+1}\,\sqrt{a^2-1} + \sqrt{-a+1}\,\cdots}}{\sqrt{a-1}}\right.}{}$$

input
```
int(x^3*acoth(b*x^4+a),x)
```

output

```
(2*acoth(a + b*x**4)*a + 2*acoth(a + b*x**4)*b*x**4 - log((sqrt(a - 1)*(a*
*2 - 1)**(1/4) - b**(1/4)*sqrt(2*(a**2 - 1)**(1/4)*a - 2*(a**2 - 1)**(1/4)
 - sqrt( - a + 1)*sqrt(a**2 - 1) + sqrt( - a + 1)*a - sqrt( - a + 1))*x +
sqrt(b)*sqrt(a - 1)*x**2)/sqrt(a - 1)) - log((sqrt(a - 1)*(a**2 - 1)**(1/4
) - b**(1/4)*sqrt(2*(a**2 - 1)**(1/4)*a - 2*(a**2 - 1)**(1/4) + sqrt( - a
+ 1)*sqrt(a**2 - 1) - sqrt( - a + 1)*a + sqrt( - a + 1))*x + sqrt(b)*sqrt(
a - 1)*x**2)/sqrt(a - 1)) - log((sqrt(a - 1)*(a**2 - 1)**(1/4) + b**(1/4)*
sqrt(2*(a**2 - 1)**(1/4)*a - 2*(a**2 - 1)**(1/4) - sqrt( - a + 1)*sqrt(a**
2 - 1) + sqrt( - a + 1)*a - sqrt( - a + 1))*x + sqrt(b)*sqrt(a - 1)*x**2)/
sqrt(a - 1)) - log((sqrt(a - 1)*(a**2 - 1)**(1/4) + b**(1/4)*sqrt(2*(a**2
- 1)**(1/4)*a - 2*(a**2 - 1)**(1/4) + sqrt( - a + 1)*sqrt(a**2 - 1) - sqrt
( - a + 1)*a + sqrt( - a + 1))*x + sqrt(b)*sqrt(a - 1)*x**2)/sqrt(a - 1)))
/(8*b)
```

3.173 $\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx=\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}+\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

output
```
(a+b*x^n)*arccoth(a+b*x^n)/b/n+1/2*ln(1-(a+b*x^n)^2)/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx=\frac{2(a+bx^n)\coth^{-1}(a+bx^n)+\log\left(1-(a+bx^n)^2\right)}{2bn}$$

input
```
Integrate[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 6654, 6437, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\coth^{-1}\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \; 7266$$

$$\frac{\int \coth^{-1}\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \; 6654$$

$$\frac{\int \coth^{-1}\left(bx^n+a\right)d(bx^n+a)}{bn}$$

$$\Big\downarrow \; 6437$$

$$\frac{(a+bx^n)\coth^{-1}\left(a+bx^n\right)-\int \frac{bx^n+a}{1-x^{2n}}d(bx^n+a)}{bn}$$

$$\Big\downarrow \; 240$$

$$\frac{(a+bx^n)\coth^{-1}\left(a+bx^n\right)+\frac{1}{2}\log\left(1-x^{2n}\right)}{bn}$$

input
```
Int[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - x^(2*n)]/2)/(b*n)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(45) = 90$.

Time = 2.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.51

method	result	size
risch	$\frac{x^{n} \ln(a+b\,x^{n}+1)}{2n}-\frac{x^{n} \ln(a+b\,x^{n}-1)}{2n}-\frac{\ln(x^{n}+\frac{a-1}{b})a}{2nb}+\frac{\ln(x^{n}+\frac{1+a}{b})a}{2nb}+\frac{\ln(x^{n}+\frac{a-1}{b})}{2nb}+\frac{\ln(x^{n}+\frac{1+a}{b})}{2nb}$	118

input
```
int(x^(-1+n)*arccoth(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*x^n*ln(a+b*x^n+1)-1/2/n*x^n*ln(a+b*x^n-1)-1/2/n/b*ln(x^n+(a-1)/b)*a+
1/2/n/b*ln(x^n+(1+a)/b)*a+1/2/n/b*ln(x^n+(a-1)/b)+1/2/n/b*ln(x^n+(1+a)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(45) = 90$.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.30

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx$$
$$= \frac{(a+1)\log(b\cosh(n\log(x))+b\sinh(n\log(x))+a+1)-(a-1)\log(b\cosh(n\log(x))+b\sinh(n\log}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*lo
g(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sin
h(n*log(x)))*log((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*l
og(x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+n)*acoth(a+b*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx=\frac{2\,(bx^n+a)\,\mathrm{arcoth}\,(bx^n+a)+\log\left(-(bx^n+a)^2+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arccoth(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx=\frac{((a+1)b-(a-1)b)\left(\frac{\log\left(\frac{|bx^n+a+1|}{|bx^n+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx^n+a+1}{bx^n+a-1}-1\right|\right)}{b^2}+\frac{\log\left(\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1}-1\right)}\right)}{2\,n}$$

input
```
integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x^n + a + 1)/abs(b*x^n + a - 1))/b^
2 - log(abs((b*x^n + a + 1)/(b*x^n + a - 1) - 1))/b^2 + log((b*x^n + a + 1
)/(b*x^n + a - 1))/(b^2*((b*x^n + a + 1)/(b*x^n + a - 1) - 1)))/n
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int x^{-1+n}\coth^{-1}\left(a+bx^n\right)\,dx=\frac{\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n-1\right)}{2}+a\,\mathrm{acoth}\left(a+b\,x^n\right)}{b\,n}+\frac{x^n\,\mathrm{acoth}\left(a+b\,x^n\right)}{n}$$

input
```
int(x^(n - 1)*acoth(a + b*x^n),x)
```

output
```
(log(a^2 + b^2*x^(2*n) + 2*a*b*x^n - 1)/2 + a*acoth(a + b*x^n))/(b*n) + (x
^n*acoth(a + b*x^n))/n
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int x^{-1+n}\coth^{-1}\left(a+bx^n\right)\,dx=\frac{2x^n\,acoth(x^nb+a)\,b+2acoth(x^nb+a)\,a-\log(x^{2n}b^2+2x^nab+a^2-1)}{2bn}$$

input
```
int(x^(-1+n)*acoth(a+b*x^n),x)
```

output
```
(2*x**n*acoth(x**n*b + a)*b + 2*acoth(x**n*b + a)*a - log(x**(2*n)*b**2 +
2*x**n*a*b + a**2 - 1))/(2*b*n)
```

3.174 $\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(4+3\sqrt{2}-\sqrt{2}e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arccoth(sinh(c*(b*x+a)))/b/c+1/2*(1-2^(1/2))*ln(3-2*2^(1/2)
-exp(2*c*(b*x+a)))/b/c+1/2*(1+2^(1/2))*ln(4+3*2^(1/2)-2^(1/2)*exp(2*c*(b*x
+a)))/b/c
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.37

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx = \frac{-2e^{c(a+bx)}\coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right) - 2\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right) + 2\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right) + \log(}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

output
```
(-2*E^(c*(a + b*x))*ArcCoth[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2
*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 +
E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
 + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {7281, 6830, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\sinh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\sinh(ac+bcx)) - \int \frac{e^{ac+bxc}\cosh(ac+bxc)}{1-\sinh^2(ac+bxc)}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\sinh(ac+bcx)) - \int -\frac{2e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc} + e^{ac+bcx} \coth^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{1+e^{2ac+2bxc}}{1-5e^{2ac+2bxc}}de^{2ac+2bxc} + e^{ac+bcx} \coth^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{\int \left(-\frac{1+\sqrt{2}}{2\left(-ac-bxc+2\sqrt{2}+3\right)}-\frac{1-\sqrt{2}}{2\left(-ac-bxc-2\sqrt{2}+3\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(1-\sqrt{2})\log\left(-ac-bcx-2\sqrt{2}+3\right)+\frac{1}{2}(1+\sqrt{2})\log\left(-ac-bcx+2\sqrt{2}+3\right)+e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Sinh[a*c + b*c*x]] + ((1 - Sqrt[2])*Log[3 - 2*Sqr
t[2] - a*c - b*c*x])/2 + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2] - a*c - b*c*x])/
2)/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.56 (sec) , antiderivative size = 794, normalized size of antiderivative = 7.09

method	result	size
risch	Expression too large to display	794

input
```
int(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/c/b*P
i*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(b*x+a)))*c
sgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*exp(c*(b*x+a
))+1/4*I/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp(
-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I
/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+
2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a)))
*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*
x+a))-1/4*I/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*ex
p(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1
))*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2
*exp(c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I
*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b
*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/2/c/b*exp(c*(b*x+a))*ln(exp
(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/2/c/b*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^
2)*2^(1/2)-1/2/c/b*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2/b*a+1/2/c/
b*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)+1/2/c/b*ln(exp(2*c*(b*x+a))-(2^(1/2)-
1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(93) = 186$.

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx$$
$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+\cdots}{\cosh(bcx+ac)^2\cdots}\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)
^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2)
+ 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*
x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(
cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x +
 a*c)^2)))/(b*c)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx=\text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*acoth(sinh(b*c*x+a*c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.64

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx=\frac{\operatorname{arcoth}(\sinh(bcx+ac))\,e^{((bx+a)c)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(
2) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqr
t(2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))
/(b*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*
log(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.49

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc} + \frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(
e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*
sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c)
 - 6)) + log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.67

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx$$
$$=\frac{\ln\left(6\sqrt{2}\,e^{2\,c\,(a+b\,x)}-2\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}+1\right)}{2\,b\,c}$$
$$-\frac{e^{a\,c+b\,c\,x}\ln\left(1-\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{2\,b\,c}$$
$$-\frac{\ln\left(2\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}-6\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}-1\right)}{2\,b\,c}$$
$$+\frac{\ln\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}+1\right)e^{a\,c+b\,c\,x}}{2\,b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(sinh(a*c + b*c*x)),x)
```

output
```
(log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2) - 8*exp(2*c*(a + b*x)))*(2^(
1/2) + 1))/(2*b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(b*c*x)*exp(a*c))/2
- (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b
*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (log(1/((exp
(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2
*b*c)
```

Reduce [F]

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx=e^{ac}\left(\int e^{bcx}acoth(\sinh(bcx+ac))\,dx\right)$$

input
```
int(exp(c*(b*x+a))*acoth(sinh(b*c*x+a*c)),x)
```

output
```
e**(a*c)*int(e**(b*c*x)*acoth(sinh(a*c + b*c*x)),x)
```

3.175 $\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arccoth(cosh(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{c(a+bx)} \coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcCoth[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Lo
g[1 - E^(2*c*(a + b*x))])/(b*c)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 6830, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\coth^{-1}(\cosh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\cosh(ac+bcx))-\int -e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\coth^{-1}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\coth^{-1}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\cosh(ac+bcx))-2\int \frac{e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2ac+2bcx}\right)+e^{ac+bcx}\coth^{-1}(\cosh(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Cosh[a*c + b*c*x]] + Log[1 - E^(2*a*c + 2*b*c*x)]
)/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 824, normalized size of antiderivative = 16.82

method	result	size
risch	Expression too large to display	824

input
```
int(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-1/2*I/c/b*Pi*csgn(I*(exp(c*(b*x+
a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*
exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))+1/2*I/c/b*Pi*csgn(I
*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))+1/4*I/c
/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(
b*x+a))*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(c*(b
*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))
-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*c
sgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/
4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1
/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))
-1)^2)^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(
exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)
^2)^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b
*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(
b*x+a)))*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a
))+1)^2)*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*e
xp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/c/b*exp(c*(b*x+a))
*ln(exp(c*(b*x+a))-1)-2/b*a+1/c/b*ln(-1+exp(2*c*(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - s
inh(b*c*x + a*c))))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acoth}(\cosh(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(cosh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(cosh(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{\operatorname{arcoth}(\cosh(bcx+ac))\,e^{((bx+a)c)}}{bc} + \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1
)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(47) = 94.

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.00

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2\,bc} + \frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(
e^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a
*c) - 1))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.43

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}-1\right)}{b\,c} - \frac{e^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{2\,b\,c} + \frac{\ln\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}+1\right)\,e^{a\,c+b\,c\,x}}{2\,b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(cosh(a*c + b*c*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp
(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) + (log(1/((exp(
b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*
b*c)
```

Reduce [F]

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = e^{ac}\left(\int e^{bcx}\,acoth(\cosh(bcx+ac))\,dx\right)$$

input
```
int(exp(c*(b*x+a))*acoth(cosh(b*c*x+a*c)),x)
```

output
```
e**(a*c)*int(e**(b*c*x)*acoth(cosh(a*c + b*c*x)),x)
```

3.176 $\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx = -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}$$

output
```
-exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arccoth(tanh(c*(b*x+a)))/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx = \frac{e^{c(a+bx)}\left(-1+\coth^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*(-1 + ArcCoth[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b
*x)))]))/(b*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7281, 6830, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\tanh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 6830$$

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(ac+bcx)) - \int e^{ac+bxc}d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 2624$$

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(ac+bcx)) - e^{ac+bcx}}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

output
```
(-E^(a*c + b*c*x) + E^(a*c + b*c*x)*ArcCoth[Tanh[a*c + b*c*x]])/(b*c)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result
parallelrisch	$-\frac{-{\mathrm e}^{c (b x +a)} \operatorname{arccoth}\left(\tanh \left(c (b x +a)\right)\right)+{\mathrm e}^{c (b x +a)}}{c b}$
risch	$\frac{{\mathrm e}^{c (b x +a)} \ln \left({\mathrm e}^{c (b x +a)}\right)}{c b}+\frac{i \left(2 \pi \,\operatorname{csgn}\left(\frac{i}{1+{\mathrm e}^{2 c (b x +a)}}\right)^{2}-2 \pi \,\operatorname{csgn}\left(\frac{i}{1+{\mathrm e}^{2 c (b x +a)}}\right)^{3}-\pi \,\operatorname{csgn}\left(\frac{i}{1+{\mathrm e}^{2 c (b x +a)}}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 c (b x +a)}\right) \mathrm{cs}$

input
```
int(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-(-exp(c*(b*x+a))*arccoth(tanh(c*(b*x+a)))+exp(c*(b*x+a)))/c/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx = \frac{(i\,\pi + 2\,bcx + 2\,ac - 2)\cosh\left(bcx + ac\right) + (i\,\pi + 2\,bcx + 2\,ac - 2)\sinh\left(bcx + ac\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((I*pi + 2*b*c*x + 2*a*c - 2)*cosh(b*c*x + a*c) + (I*pi + 2*b*c*x + 2*
a*c - 2)*sinh(b*c*x + a*c))/(b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx = \begin{cases} \frac{i\pi x}{2} & \text{for } b = 0 \wedge c = 0 \\ xe^{ac} \operatorname{acoth}\left(\tanh\left(ac\right)\right) & \text{for } b = 0 \\ \frac{i\pi x}{2} & \text{for } c = 0 \\ \frac{e^{ac}e^{bcx} \operatorname{acoth}\left(\tanh\left(ac+bcx\right)\right)}{bc} - \frac{e^{ac}e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(c*(b*x+a))*acoth(tanh(b*c*x+a*c)),x)
```

output
```
Piecewise((I*pi*x/2, Eq(b, 0) & Eq(c, 0)), (x*exp(a*c)*acoth(tanh(a*c)), E
q(b, 0)), (I*pi*x/2, Eq(c, 0)), (exp(a*c)*exp(b*c*x)*acoth(tanh(a*c + b*c*
x))/(b*c) - exp(a*c)*exp(b*c*x)/(b*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx = \frac{\operatorname{arcoth}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{e^{\left(bcx+ac\right)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \frac{\left(e^{(bcx)}\log\left(-e^{(2\,bcx+2\,ac)}\right)-2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,(\mathrm{acoth}(\tanh(a\,c+b\,c\,x))-1)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(tanh(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*(acoth(tanh(a*c + b*c*x)) - 1))/(b*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \frac{e^{bcx+ac}(acoth(\tanh(bcx+ac))+1)}{bc}$$

input
```
int(exp(c*(b*x+a))*acoth(tanh(b*c*x+a*c)),x)
```

output
```
(e**(a*c + b*c*x)*(acoth(tanh(a*c + b*c*x)) + 1))/(b*c)
```

3.177 $\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx = -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}$$

output
```
-exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arccoth(coth(c*(b*x+a)))/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx = \frac{e^{c(a+bx)}\left(-1+\coth^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*(-1 + ArcCoth[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b
*x)))]))/(b*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7281, 6830, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\coth(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(ac+bcx)) - \int e^{ac+bxc}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(ac+bcx)) - e^{ac+bcx}}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

output
```
(-E^(a*c + b*c*x) + E^(a*c + b*c*x)*ArcCoth[Coth[a*c + b*c*x]])/(b*c)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{\mathrm{e}^{c(bx+a)}(-1+\operatorname{arccoth}(\coth(c(bx+a))))}{cb}$
default	$\frac{(bcx+ac)\mathrm{e}^{bcx+ac}-\mathrm{e}^{bcx+ac}+\mathrm{e}^{bcx+ac}(\operatorname{arccoth}(\coth(bcx+ac))-bcx-ac)}{cb}$
risch	$\frac{\mathrm{e}^{c(bx+a)}\ln\left(\mathrm{e}^{c(bx+a)}\right)}{cb}-\frac{i\left(\pi\,\mathrm{csgn}\left(i\mathrm{e}^{c(bx+a)}\right)^{2}\mathrm{csgn}\left(i\mathrm{e}^{2c(bx+a)}\right)-2\pi\,\mathrm{csgn}\left(i\mathrm{e}^{c(bx+a)}\right)\mathrm{csgn}\left(i\mathrm{e}^{2c(bx+a)}\right)^{2}+\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2c(bx+a)}\right.\right.}{}$

input
```
int(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/c/b*exp(c*(b*x+a))*(-1+arccoth(coth(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx=\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
(b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\mathrm{acoth}\,(\coth\,(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(coth(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(coth(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx = \frac{ae^{(bcx+ac)}}{b} + \frac{\left(bcxe^{(ac)} - e^{(ac)}\right)e^{(bcx)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
a*e^(b*c*x + a*c)/b + (b*c*x*e^(a*c) - e^(a*c))*e^(b*c*x)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx = \frac{(b^2c^2x + abc^2 - bc)e^{(bcx+ac)}}{b^2c^2}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx=\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\left(\mathrm{acoth}(\coth(a\,c+b\,c\,x))-1\right)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(coth(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*(acoth(coth(a*c + b*c*x)) - 1))/(b*c)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx=\frac{e^{bcx+ac}(acoth(\coth(bcx+ac))+1)}{bc}$$

input
```
int(exp(c*(b*x+a))*acoth(coth(b*c*x+a*c)),x)
```

output
```
(e**(a*c + b*c*x)*(acoth(coth(a*c + b*c*x)) + 1))/(b*c)
```

3.178 $\int e^{c(a+bx)} \coth^{-1}(\text{sech}(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{c(a+bx)} \coth^{-1}(\text{sech}(ac+bcx))\,dx = \frac{e^{ac+bcx}\coth^{-1}(\text{sech}(c(a+bx)))}{bc} + \frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arccoth(sech(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int e^{c(a+bx)} \coth^{-1}(\text{sech}(ac+bcx))\,dx = \frac{e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Lo
g[1 - E^(2*c*(a + b*x))])/(b*c)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 6830, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\coth^{-1}(\operatorname{sech}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))-\int -e^{ac+bxc}\operatorname{csch}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int e^{ac+bxc}\operatorname{csch}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))-2\int \frac{e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2ac+2bcx}\right)+e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Sech[a*c + b*c*x]] + Log[1 - E^(2*a*c + 2*b*c*x)]
)/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 939, normalized size of antiderivative = 19.16

method	result	size
risch	Expression too large to display	939

input
```
int(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(c*(b*x+a))-1)^2/(1+
exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)
^2)*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(c*(b*x+a))+1)^2/(1+exp(2*c*(b
*x+a))))*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*(
exp(c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*cs
gn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4
*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))-1/2*I/c/b*Pi*csgn(
I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/4*I/
c/b*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*
x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I/(1+exp(2*c*(b*x+a))
))*csgn(I*(exp(c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-1/4*I/
c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(
exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b*x+a))+1)^2/(1+exp(2*c*(b*x+a))))^2*e
xp(c*(b*x+a))+1/2*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a
))+1)^2)^2*exp(c*(b*x+a))+1/2*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2/(1+exp(
2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2/(
1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I/(1+exp(2*c*(b*x+
a))))*csgn(I*(exp(c*(b*x+a))+1)^2/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1
/2*I/c/b*exp(c*(b*x+a))*Pi-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2/(1+exp
(2*c*(b*x+a))))^3*exp(c*(b*x+a))-1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/
(cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*acoth(sech(b*c*x+a*c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\operatorname{arcoth}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{bc} + \frac{\log\left(e^{\left(bcx+ac\right)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1
)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}+1}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}\right)+2\,e^{(-ac)}\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)\right)e^{(ac)}}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(e^(b*c*x)*log(-(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) + 1)/(e^(2*b*
c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(abs(e^(2*b*c*x + 2
*a*c) - 1)))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}-1\right)}{b\,c} - \frac{e^{b\,c\,x}\,e^{a\,c}\,\ln\left(1-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}-\frac{e^{b\,c\,x}\,e^{a\,c}}{2}\right)}{2\,b\,c} + \frac{e^{b\,c\,x}\,e^{a\,c}\,\ln\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}+1\right)}{2\,b\,c}$$

input
```
int(acoth(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(b*c*x)*exp(a*c)*log(1 - (exp
(-b*c*x)*exp(-a*c))/2 - (exp(b*c*x)*exp(a*c))/2))/(2*b*c) + (exp(b*c*x)*ex
p(a*c)*log((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2 + 1))/(2*b*
c)
```

Reduce [F]

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = e^{ac}\left(\int e^{bcx} acoth(\operatorname{sech}(bcx+ac))\,dx\right)$$

input
```
int(exp(c*(b*x+a))*acoth(sech(b*c*x+a*c)),x)
```

output
```
e**(a*c)*int(e**(b*c*x)*acoth(sech(a*c + b*c*x)),x)
```

3.179 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{csch}(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arccoth(csch(c*(b*x+a)))/b/c+1/2*(1-2^(1/2))*ln(3-2*2^(1/2)
-exp(2*c*(b*x+a)))/b/c+1/2*(1+2^(1/2))*ln(3+2*2^(1/2)-exp(2*c*(b*x+a)))/b/
c
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{2e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right) - 2\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right) + 2\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right) + \log\left(1-2e^{c(a+bx)}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

output
```
(2*E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] -
 2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1
+ E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x)
)] + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7281, 6830, 25, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\text{csch}(ac+bcx)) \, dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\text{csch}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\text{csch}(ac+bcx)) - \int -\frac{e^{ac+bxc} \coth(ac+bxc)\text{csch}(ac+bxc)}{1-\text{csch}^2(ac+bxc)}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e^{ac+bxc} \coth(ac+bxc)\text{csch}(ac+bxc)}{1-\text{csch}^2(ac+bxc)}d(ac+bxc) + e^{ac+bcx} \coth^{-1}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{ac+bxc}(1+e^{2ac+2bxc})}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc} + e^{ac+bcx} \coth^{-1}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}(1+e^{2ac+2bxc})}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc} + e^{ac+bcx} \coth^{-1}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{1+e^{2ac+2bxc}}{1-5e^{2ac+2bxc}}de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{\int \left(-\frac{1+\sqrt{2}}{2\left(-ac-bxc+2\sqrt{2}+3\right)}-\frac{1-\sqrt{2}}{2\left(-ac-bxc-2\sqrt{2}+3\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(1-\sqrt{2})\log\left(-ac-bcx-2\sqrt{2}+3\right)+\frac{1}{2}(1+\sqrt{2})\log\left(-ac-bcx+2\sqrt{2}+3\right)+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Csch[a*c + b*c*x]] + ((1 - Sqrt[2])*Log[3 - 2*Sqr
t[2] - a*c - b*c*x])/2 + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2] - a*c - b*c*x])/
2)/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 920, normalized size of antiderivative = 8.60

method	result	size
risch	Expression too large to display	920

input
```
int(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)-1/2*I/c/b*e
xp(c*(b*x+a))*Pi+1/4*I/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1
))*csgn(I/(-1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a)
)+1)/(-1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(-exp(2*c*(
b*x+a))+2*exp(c*(b*x+a))+1)/(-1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+1/4*I/
c/b*Pi*csgn(I/(-1+exp(2*c*(b*x+a))))*csgn(I/(-1+exp(2*c*(b*x+a)))*(exp(2*c
*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(-exp(
2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a
))+1)/(-1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(2*c
*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(-1+exp(2*c*(b*x+a))))*csgn(I/(-1+ex
p(2*c*(b*x+a)))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))+1/2*
I/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(-1+exp(2*c*(b*x+a)
)))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I/(-1+exp(2*c*(b*x+a))))*csgn(I*(-e
xp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(-1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a)
)+1/4*I/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(-1+ex
p(2*c*(b*x+a)))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/
4*I/c/b*Pi*csgn(I/(-1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))
-1))^3*exp(c*(b*x+a))-1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(
b*x+a))-1)+1/2/c/b*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/c/b*ln(e
xp(2*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2/b*a+1/2/c/b*ln(exp(2*c*(b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(90) = 180$.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.19

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx$$

$$= \frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bc\ldots}{\cosh(bcx+ac\ldots}\right.}{2\,bc\ldots}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/
(sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c
)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2)
 + 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c
*x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/
(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x
+ a*c)^2)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acoth}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(csch(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(csch(a*c + b*c*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\begin{aligned}\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx &= \frac{\operatorname{arcoth}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} \\ &\quad+ \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc} \\ &\quad- \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc} \\ &\quad+ \frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc} \\ &\quad+ \frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(
2) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqr
t(2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))
/(b*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*
log(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{e^{(bcx+ac)}-e^{(-bcx-ac)}+2}{e^{(bcx+ac)}-e^{(-bcx-ac)}-2}\right)}{2\,bc} + \frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*
x + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sqrt(2)
+ 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)) +
 log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\mathrm{e}^{a\,c+b\,c\,x}\,\ln\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}+1\right)}{2\,b\,c} - \frac{\mathrm{e}^{a\,c+b\,c\,x}\,\ln\left(\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+1\right)}{2\,b\,c} + \frac{\ln\left(6\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}-2\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(2\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}-6\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

input
```
int(acoth(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
(exp(a*c + b*c*x)*log((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2
+ 1))/(2*b*c) - (exp(a*c + b*c*x)*log((exp(-b*c*x)*exp(-a*c))/2 - (exp(b*c
*x)*exp(a*c))/2 + 1))/(2*b*c) + (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1
/2) - 8*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(2*2^(1/2) - 8*ex
p(2*c*(a + b*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

Reduce [F]

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = e^{ac}\left(\int e^{bcx}\,acoth(\operatorname{csch}(bcx+ac))\,dx\right)$$

input
```
int(exp(c*(b*x+a))*acoth(csch(b*c*x+a*c)),x)
```

output
```
e**(a*c)*int(e**(b*c*x)*acoth(csch(a*c + b*c*x)),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#    "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file