Computer Algebra Independent Integration Tests

Summer 2024

8-Special-functions/350-8.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**311**]. This is test number [350].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (311)	0.00 (0)
Mathematica	96.46 (300)	3.54 (11)
Fricas	82.96 (258)	17.04 (53)
Mupad	65.27 (203)	34.73 (108)
Sympy	63.02 (196)	36.98 (115)
Maple	60.45 (188)	39.55 (123)
Reduce	47.91 (149)	52.09 (162)
Maxima	45.02 (140)	54.98 (171)
Giac	43.73 (136)	56.27 (175)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

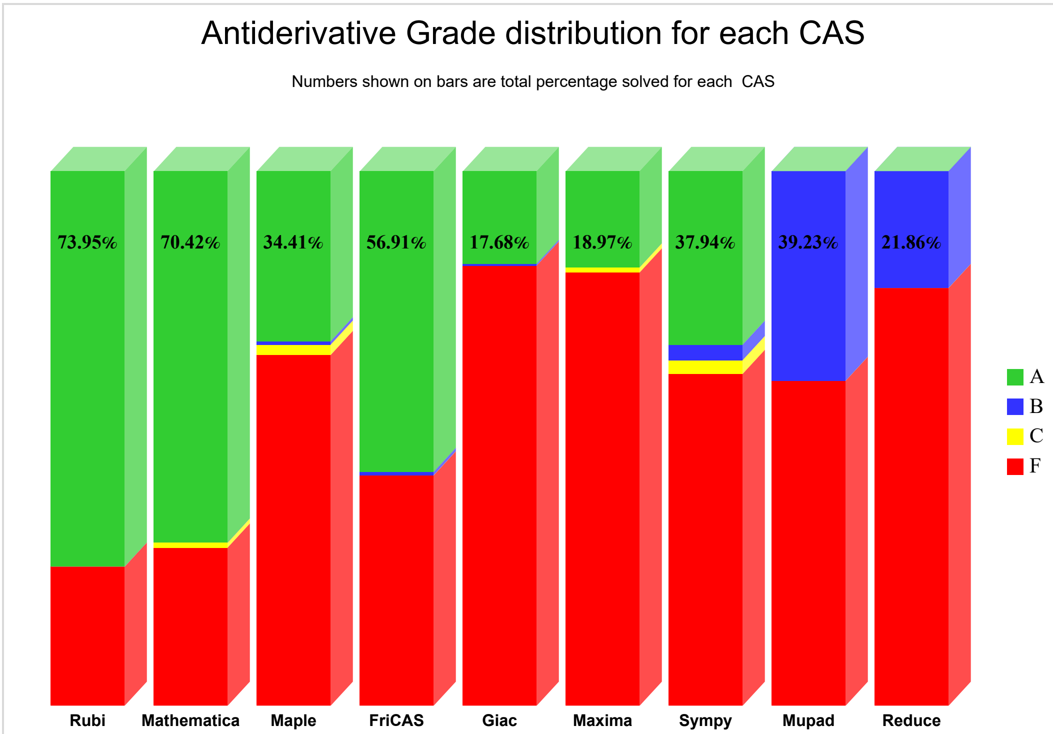
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

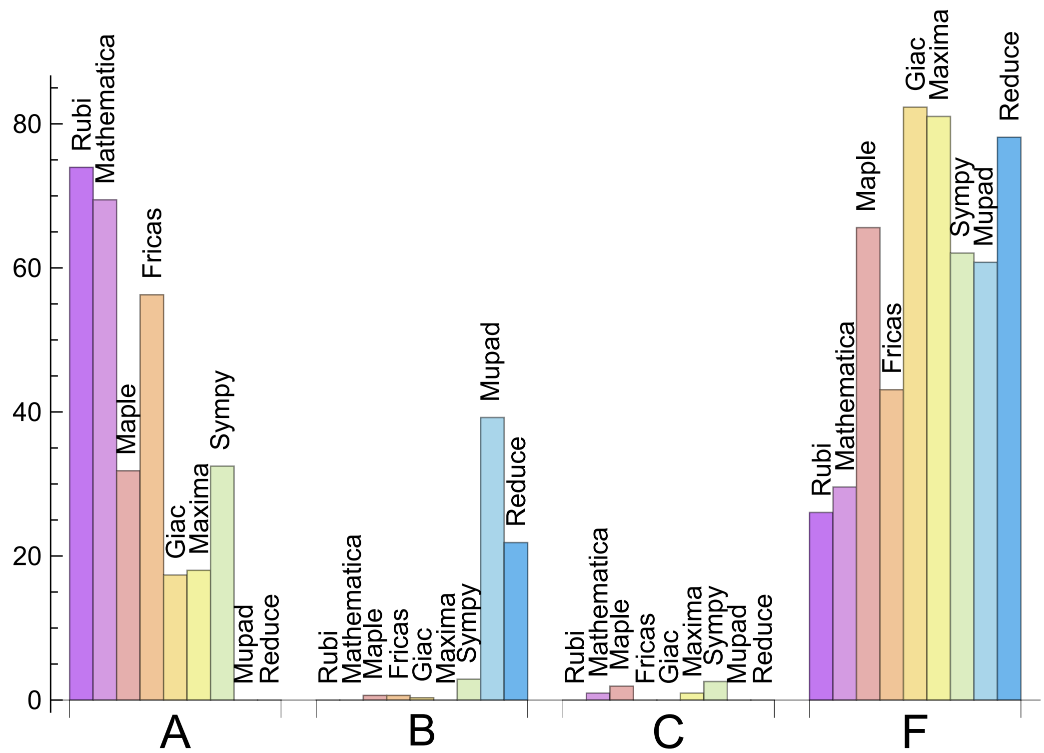
System	% A grade	% B grade	% C grade	% F grade
Rubi	73.955	0.000	0.000	26.045
Mathematica	69.453	0.000	0.965	29.582
Fricas	56.270	0.643	0.000	43.087
Sympy	32.476	2.894	2.572	62.058
Maple	31.833	0.643	1.929	65.595
Maxima	18.006	0.000	0.965	81.029
Giac	17.363	0.322	0.000	82.315
Mupad	0.000	39.228	0.000	60.772
Reduce	0.000	21.865	0.000	78.135

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	11	100.00	0.00	0.00
Fricas	53	100.00	0.00	0.00
Mupad	108	0.00	100.00	0.00
Sympy	115	86.96	13.04	0.00
Maple	123	99.19	0.81	0.00
Reduce	162	100.00	0.00	0.00
Maxima	171	100.00	0.00	0.00
Giac	175	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.09
Giac	0.12
Reduce	0.16
Mathematica	0.18
Maple	0.44
Rubi	0.50
Mupad	3.24
Sympy	9.72

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	28.27	1.08	20.00	1.05
Giac	44.20	1.04	19.00	1.06
Reduce	51.60	2.23	34.00	1.30
Sympy	57.50	1.02	24.00	0.97
Maple	59.65	0.99	26.50	0.95
Mupad	59.75	1.06	24.00	1.05
Mathematica	60.60	0.94	51.00	1.00
Fricas	71.12	1.09	51.00	1.06
Rubi	80.27	1.06	60.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

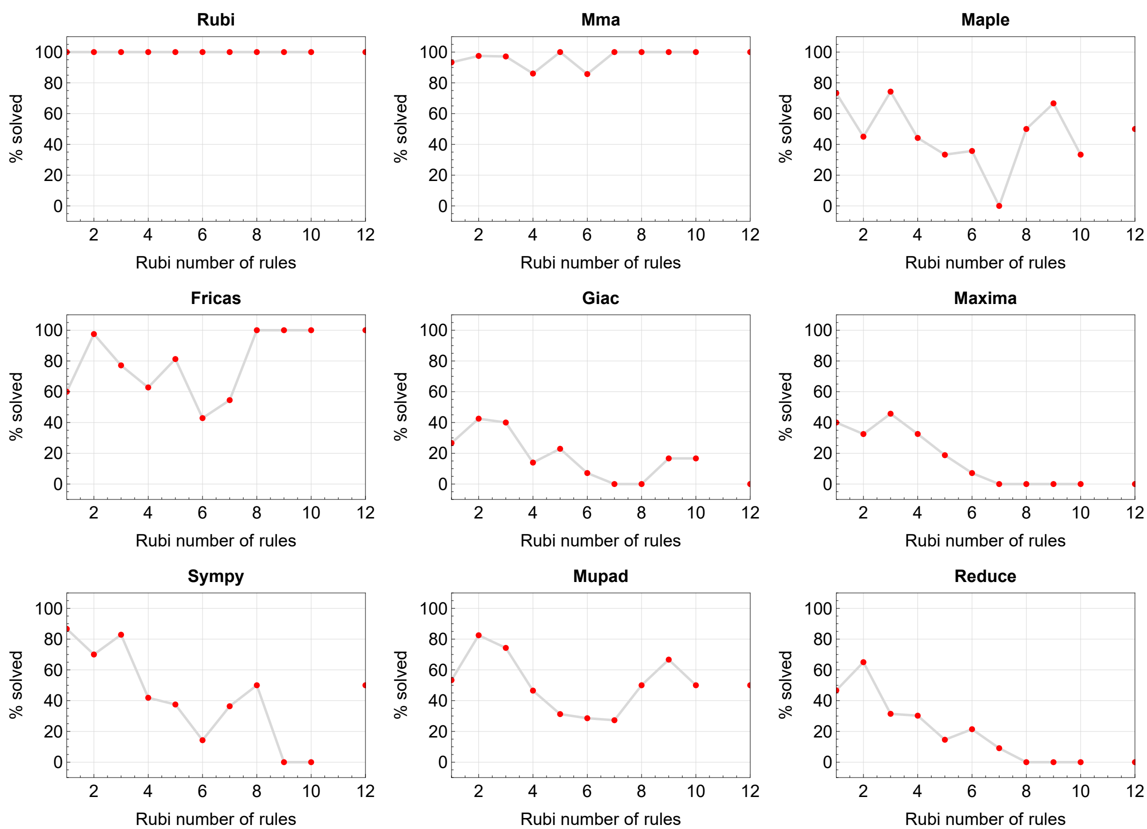


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

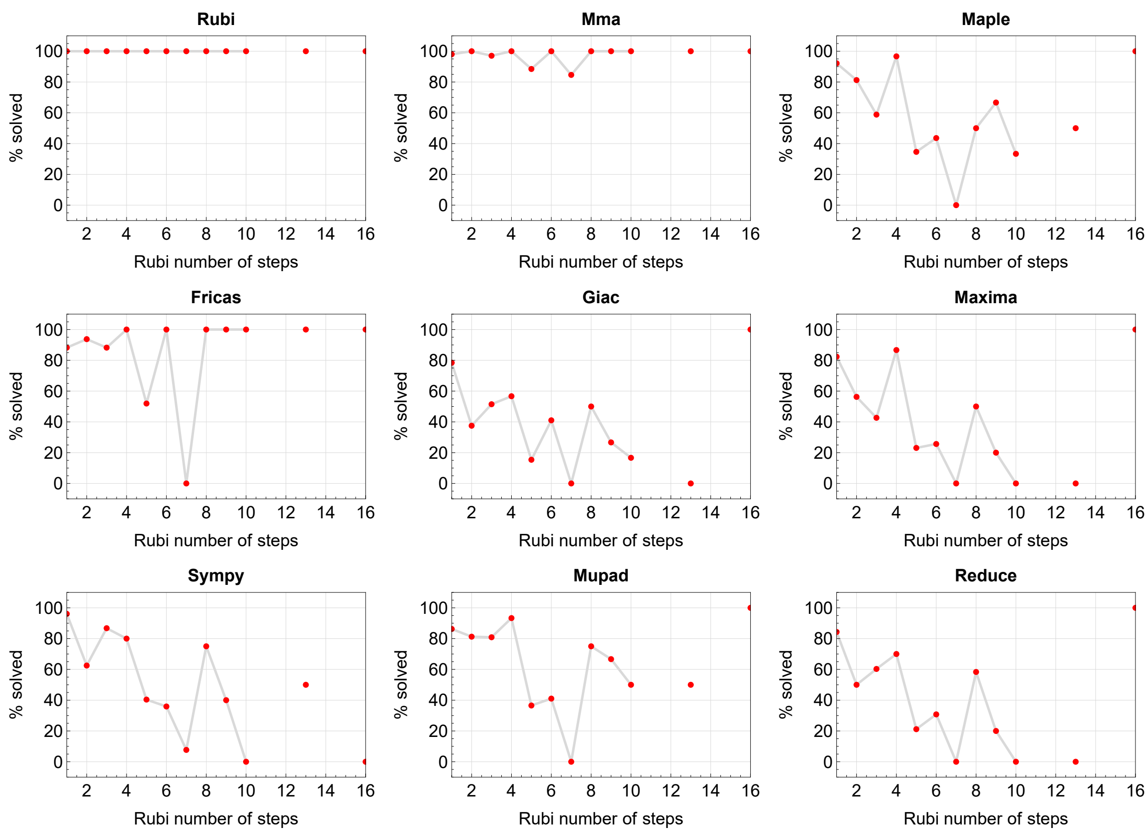


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

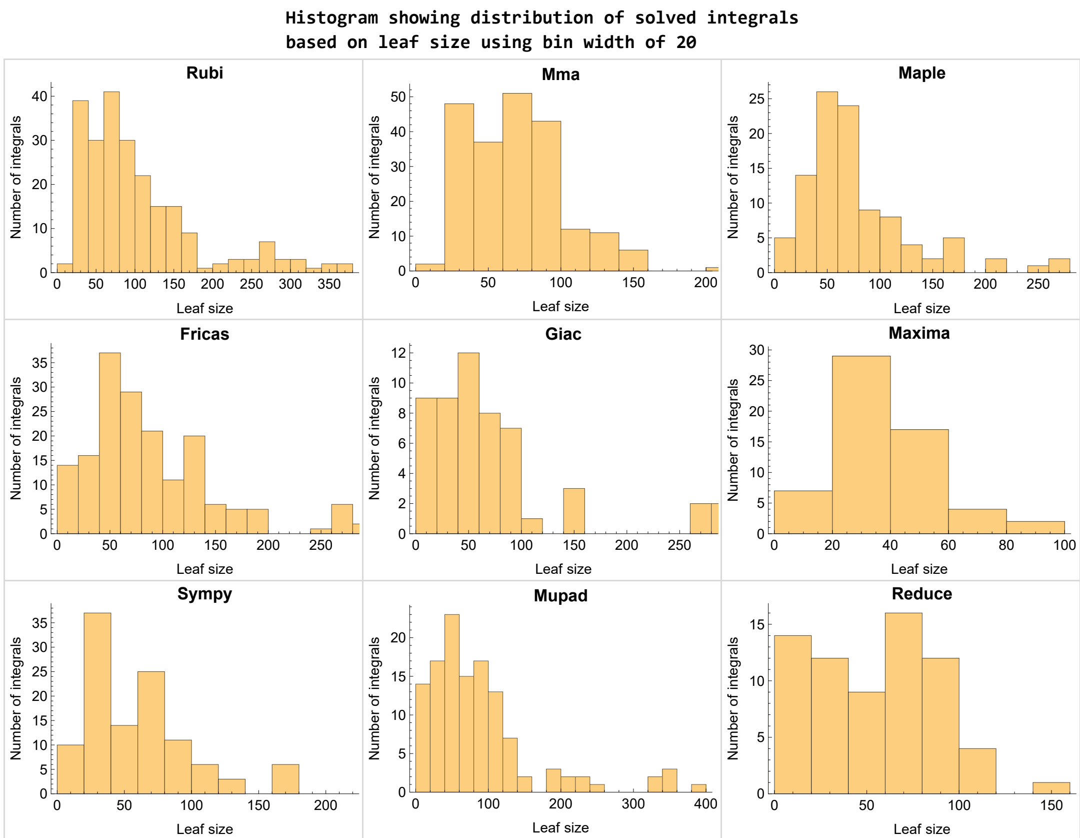


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

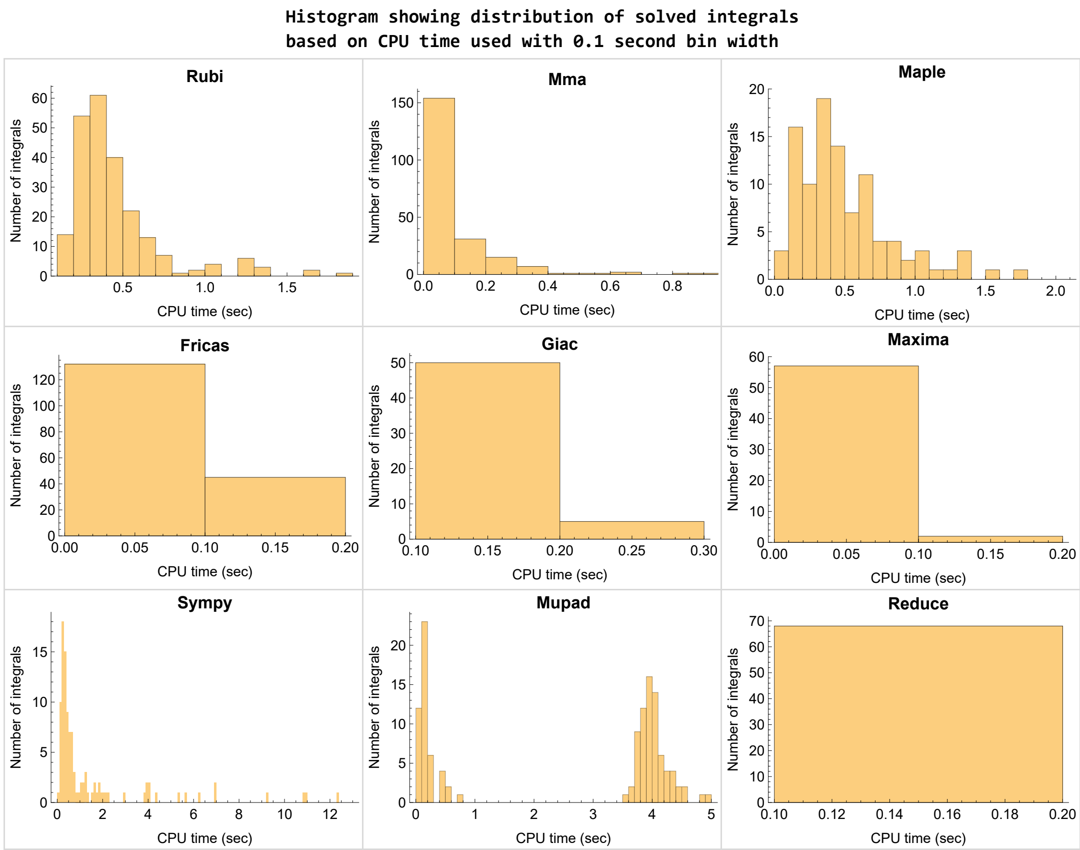


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

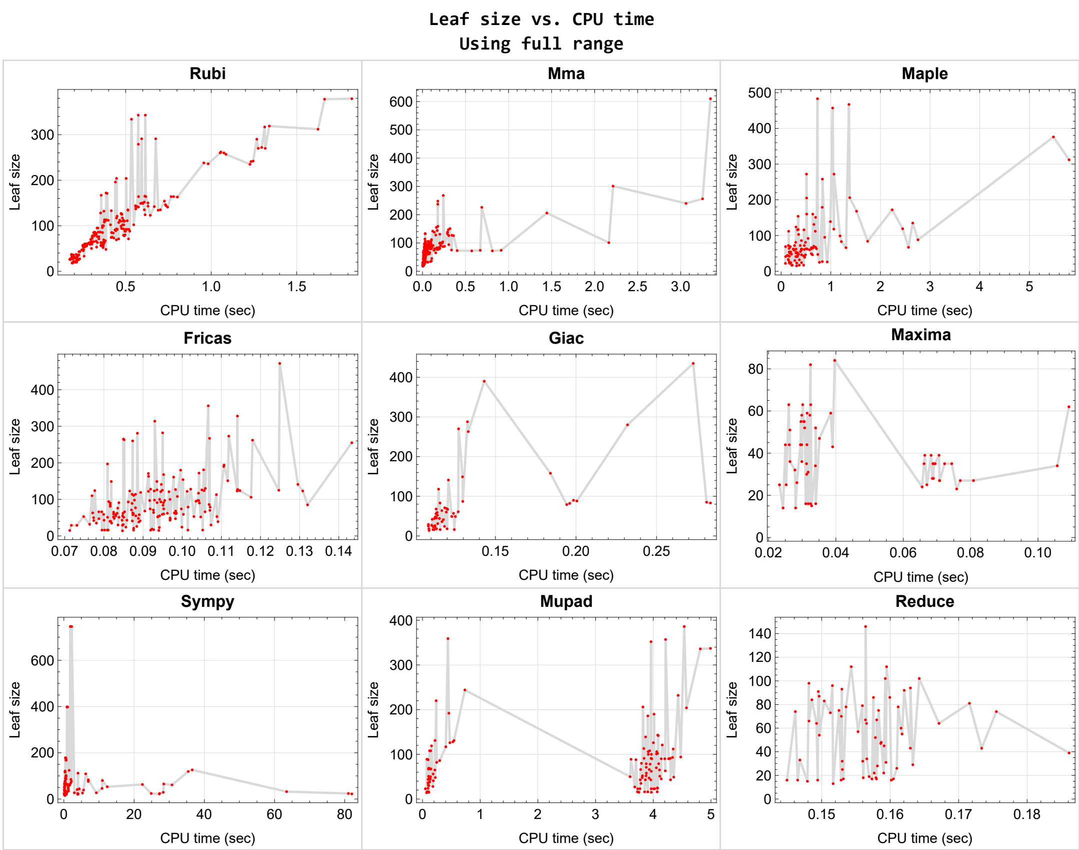


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{19, 20, 21, 25, 32, 33, 34, 38, 39, 56, 57, 58, 59, 60, 61, 62, 63, 78, 79, 80, 88, 89, 90, 91, 92, 93, 94, 122, 123, 124, 128, 135, 136, 137, 141, 142, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 225, 226, 227, 231, 238, 239, 240, 244, 245, 262, 263, 264, 265, 266, 267, 268, 269, 286, 287, 288, 296, 297, 298, 299, 300, 301, 302}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{\}$

Maple $\{110, 118, 119, 270, 271, 272, 303\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See **https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/** for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at **https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/**

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

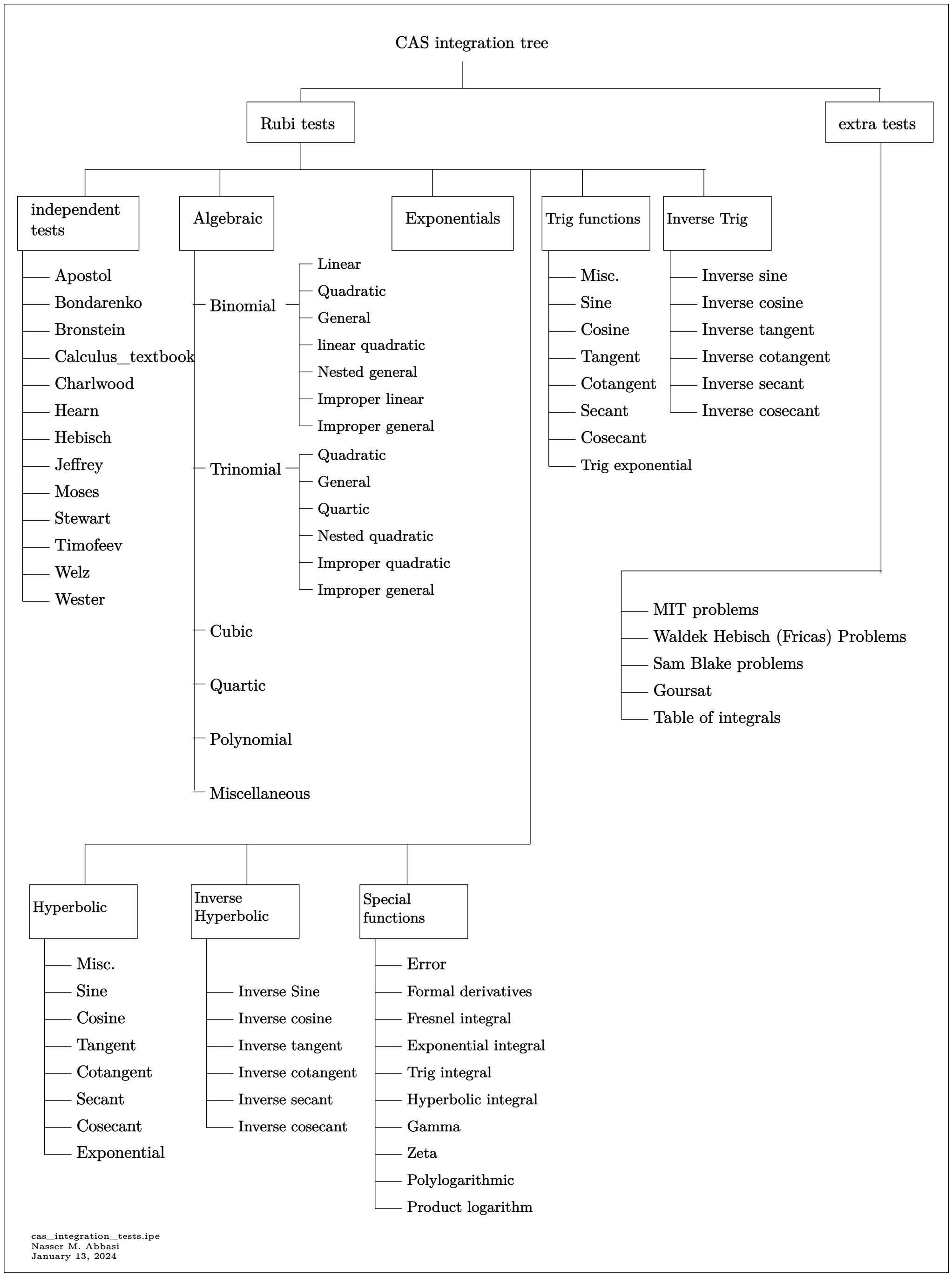
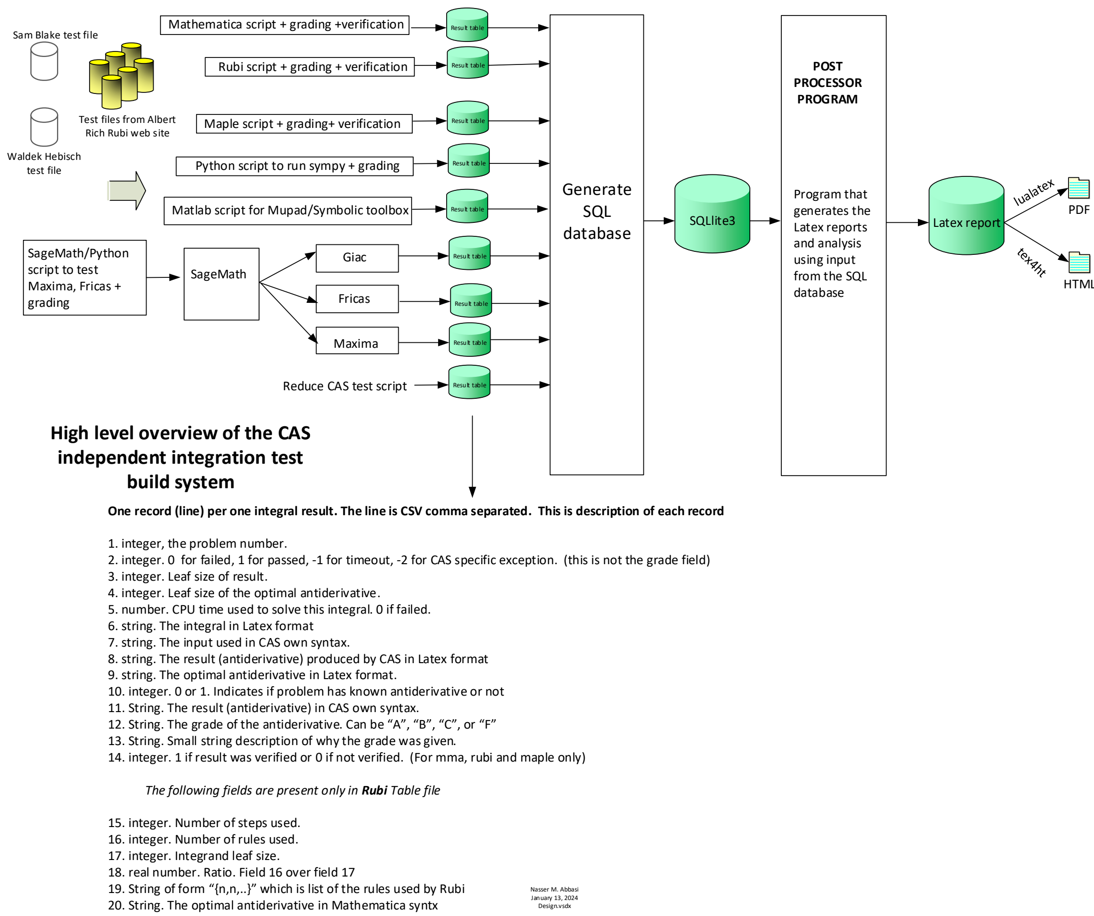


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303, 304, 305, 306, 307, 308, 309, 310, 311 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 232, 233, 234, 235, 236, 237, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303, 308, 309, 310, 311 }

B grade { }

C grade { 280, 281, 282 }

F normal fail { 72, 98, 99, 175, 201, 202, 241, 304, 305, 306, 307 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 22, 23, 24, 29, 30, 31, 37, 43, 47, 48, 50, 51, 53, 54, 55, 64, 65, 66, 75, 76, 77, 83, 87, 95, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 132, 133, 134, 140, 146, 151, 156, 157, 158, 167, 168, 169, 178, 179, 180, 186, 198, 210, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 249, 270, 271, 272, 303 }

B grade { 150, 190 }

C grade { 14, 207, 208, 209, 211, 213 }

F normal fail { 26, 27, 28, 35, 36, 40, 41, 42, 44, 45, 46, 52, 67, 68, 69, 70, 71, 72, 73, 74, 81, 82, 84, 85, 86, 96, 97, 98, 99, 100, 101, 102, 103, 107, 129, 130, 131, 138, 139, 143, 144, 145, 147, 148, 149, 152, 153, 154, 155, 170, 171, 172, 173, 174, 175, 176, 177, 184, 185, 187, 188, 189, 199, 200, 201, 202, 203, 204, 205, 206, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-1) timedout fail { 49 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 95, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 167, 168, 169, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 198, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303 }

B grade { 140, 146 }

C grade { }

F normal fail { 4, 67, 68, 69, 70, 71, 72, 73, 74, 96, 97, 98, 99, 100, 101, 102, 103, 107, 170, 171, 172, 173, 174, 175, 176, 177, 199, 200, 201, 202, 203, 204, 205, 206, 210, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 31, 43, 47, 48, 49, 50, 51, 55, 64, 65, 66, 77, 83, 87, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 146, 186, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 249 }

B grade { }

C grade { 207, 208, 209 }

F normal fail { 4, 15, 16, 17, 22, 23, 24, 26, 27, 28, 29, 30, 35, 36, 37, 40, 41, 42, 44, 45, 46, 52, 53, 54, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 82, 84, 85, 86, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 118, 119, 120, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 210, 221, 222, 223, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 83, 86, 87, 104, 105, 106, 111, 112, 113, 114, 118, 119, 120, 121, 143, 144, 145, 146, 151, 152, 153, 154, 155, 186 }

B grade { 150 }

C grade { }

F normal fail { 4, 5, 6, 7, 22, 23, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 44, 45, 46, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 81, 82, 84, 85, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 115, 116, 117, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 140, 147, 148, 149, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 303, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 29, 30, 31, 35, 36, 37, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 75, 76, 77, 81, 82, 83, 86, 87, 95, 104, 105, 106, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 146, 150, 151, 152, 153, 154, 155, 167, 168, 169, 184, 185, 186, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 249, 253, 254, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 283, 284, 285, 289, 290, 291, 294, 295, 303 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 26, 27, 28, 40, 41, 42, 44, 45, 46, 67, 68, 69, 70, 71, 72, 73, 74, 84, 85, 96, 97, 98, 99, 100, 101, 102, 103, 107, 110, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 147, 148, 149, 156, 157, 158, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 210, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 250, 251, 252, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 292, 293, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 22, 23, 24, 47, 48, 49, 50, 51, 64, 65, 66, 67, 68, 71, 72, 73, 81, 82, 83, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 125, 126, 127, 150, 151, 152, 153, 154, 167, 168, 169, 170, 171, 177, 184, 185, 186, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 223, 224, 228, 229, 230, 253, 254, 255, 256, 257, 271, 272, 273, 274, 275, 278, 279, 280, 289, 290, 291, 303 }

B grade { 15, 16, 52, 118, 119, 155, 221, 222, 258 }

C grade { 74, 174, 175, 176, 218, 219, 220, 281 }

F normal fail { 26, 27, 28, 29, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 75, 76, 77, 84, 85, 87, 95, 96, 97, 98, 99, 100, 101, 102, 103, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 178, 179, 180, 187, 188, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 260, 261, 283, 284, 285, 292, 293, 295, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-1) timedout fail { 69, 70, 86, 90, 172, 173, 189, 193, 259, 270, 276, 277, 282, 294, 298 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 5, 8, 9, 10, 11, 12, 13, 14, 18, 47, 48, 49, 50, 51, 52, 64, 65, 66, 83, 104, 105, 106, 108, 111, 112, 113, 114, 115, 116, 117, 121, 150, 151, 152, 153, 154, 155, 167, 168, 169, 186, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 224, 253, 254, 255, 256, 257, 258, 270, 271, 272, 289, 290, 291, 303 }

C grade { }

F normal fail { 4, 6, 7, 15, 16, 17, 22, 23, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 81, 82, 84, 85, 86, 87, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 109, 110, 118, 119, 120, 125, 126, 127, 129, 130, 131, 132, 133, 134, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 187, 188, 189, 190, 198, 199, 200, 201, 202, 203, 204, 205, 206, 210, 218, 219, 220, 221, 222, 223, 228, 229, 230, 232, 233, 234, 235, 236, 237, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 292, 293, 294, 295, 304, 305, 306, 307, 308, 309, 310, 311 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	113	72	60	63	63	88	64	84	108
N.S.	1	1.18	0.75	0.62	0.66	0.66	0.92	0.67	0.88	1.12
time (sec)	N/A	0.384	0.019	0.573	0.026	0.087	0.545	0.117	0.149	0.142

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	63	52	55	55	65	56	74	88
N.S.	1	1.20	0.89	0.73	0.77	0.77	0.92	0.79	1.04	1.24
time (sec)	N/A	0.313	0.017	0.328	0.030	0.083	0.308	0.113	0.146	0.098

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	55	42	41	44	42	39	44	64	48
N.S.	1	1.20	0.91	0.89	0.96	0.91	0.85	0.96	1.39	1.04
time (sec)	N/A	0.251	0.025	0.304	0.029	0.077	0.203	0.115	0.149	3.809

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	23	0	0	26	0	10	0
N.S.	1	1.00	1.00	0.72	0.00	0.00	0.81	0.00	0.31	0.00
time (sec)	N/A	0.188	0.004	0.315	0.000	0.000	0.373	0.000	0.159	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	42	41	35	41	36	0	64	67
N.S.	1	1.12	1.00	0.98	0.83	0.98	0.86	0.00	1.52	1.60
time (sec)	N/A	0.252	0.027	0.328	0.066	0.087	0.251	0.000	0.157	0.105

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	63	62	35	55	60	0	78	88
N.S.	1	1.06	0.89	0.87	0.49	0.77	0.85	0.00	1.10	1.24
time (sec)	N/A	0.304	0.015	0.415	0.069	0.088	0.372	0.000	0.164	3.700

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	100	73	70	35	62	87	0	88	113
N.S.	1	1.04	0.76	0.73	0.36	0.65	0.91	0.00	0.92	1.18
time (sec)	N/A	0.364	0.017	0.658	0.069	0.083	0.609	0.000	0.184	3.836

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	112	72	63	52	59	99	52	74	52
N.S.	1	1.03	0.66	0.58	0.48	0.54	0.91	0.48	0.68	0.48
time (sec)	N/A	0.398	0.014	0.611	0.034	0.080	0.648	0.114	0.176	3.813

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	66	55	44	51	75	44	64	44
N.S.	1	1.02	0.79	0.65	0.52	0.61	0.89	0.52	0.76	0.52
time (sec)	N/A	0.352	0.013	0.389	0.025	0.081	0.395	0.112	0.167	0.097

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	60	41	47	36	43	51	36	52	36
N.S.	1	1.02	0.69	0.80	0.61	0.73	0.86	0.61	0.88	0.61
time (sec)	N/A	0.267	0.018	0.739	0.030	0.080	0.243	0.117	0.158	0.096

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	25	29	24	23	39	23
N.S.	1	1.00	1.00	0.92	0.96	1.12	0.92	0.88	1.50	0.88
time (sec)	N/A	0.173	0.005	0.411	0.023	0.073	0.172	0.112	0.186	3.731

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	24	30	24	24	29	24
N.S.	1	1.00	1.00	1.00	0.92	1.15	0.92	0.92	1.12	0.92
time (sec)	N/A	0.217	0.010	0.928	0.066	0.084	0.681	0.118	0.163	4.075

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	46	27	48	54	45	70	45
N.S.	1	1.00	0.84	0.82	0.48	0.86	0.96	0.80	1.25	0.80
time (sec)	N/A	0.264	0.034	0.491	0.077	0.083	1.272	0.113	0.153	4.139

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	63	27	60	76	68	78	65
N.S.	1	1.00	0.77	0.78	0.33	0.74	0.94	0.84	0.96	0.80
time (sec)	N/A	0.327	0.024	0.713	0.081	0.095	2.146	0.113	0.154	4.108

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	291	248	467	0	265	746	390	208	337
N.S.	1	1.01	0.86	1.62	0.00	0.92	2.58	1.35	0.72	1.17
time (sec)	N/A	0.676	0.178	1.366	0.000	0.085	2.211	0.143	0.185	4.992

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	204	138	272	0	163	398	263	167	204
N.S.	1	1.06	0.72	1.42	0.00	0.85	2.07	1.37	0.87	1.06
time (sec)	N/A	0.503	0.137	1.065	0.000	0.091	1.076	0.133	0.153	4.576

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	133	88	112	0	91	178	149	122	126
N.S.	1	1.13	0.75	0.95	0.00	0.77	1.51	1.26	1.03	1.07
time (sec)	N/A	0.416	0.077	0.688	0.000	0.095	0.566	0.130	0.154	0.475

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	32	31	47	53	59	87	48
N.S.	1	1.00	0.97	0.89	0.86	1.31	1.47	1.64	2.42	1.33
time (sec)	N/A	0.194	0.015	0.367	0.032	0.079	0.254	0.114	0.150	0.222

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.14
time (sec)	N/A	0.195	0.398	0.147	0.135	0.082	1.236	0.121	0.155	4.227

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	66	27	14	16	27	16
N.S.	1	1.00	1.14	1.00	4.71	1.93	1.00	1.14	1.93	1.14
time (sec)	N/A	0.261	0.583	0.211	0.122	0.084	12.943	0.155	0.153	4.363

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	94	38	14	16	38	16
N.S.	1	1.00	1.14	1.00	6.71	2.71	1.00	1.14	2.71	1.14
time (sec)	N/A	0.441	1.486	0.184	0.125	0.091	88.112	0.139	0.153	4.946

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	270	106	160	0	98	168	0	12	142
N.S.	1	1.52	0.60	0.90	0.00	0.55	0.94	0.00	0.07	0.80
time (sec)	N/A	1.316	0.032	0.638	0.000	0.093	0.619	0.000	0.155	4.087

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	164	90	116	0	81	117	0	12	101
N.S.	1	1.30	0.71	0.92	0.00	0.64	0.93	0.00	0.10	0.80
time (sec)	N/A	0.780	0.026	0.460	0.000	0.082	0.333	0.000	0.159	4.056

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	64	72	0	59	65	0	10	67
N.S.	1	1.20	0.90	1.01	0.00	0.83	0.92	0.00	0.14	0.94
time (sec)	N/A	0.418	0.031	0.385	0.000	0.083	0.211	0.000	0.150	0.146

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.179	0.023	0.020	0.058	0.076	1.142	0.118	0.150	3.950

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	74	63	0	0	65	0	0	47	0
N.S.	1	1.10	0.94	0.00	0.00	0.97	0.00	0.00	0.70	0.00
time (sec)	N/A	0.415	0.022	0.000	0.000	0.082	0.000	0.000	0.150	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	144	97	0	0	94	0	0	47	0
N.S.	1	1.15	0.78	0.00	0.00	0.75	0.00	0.00	0.38	0.00
time (sec)	N/A	0.736	0.056	0.000	0.000	0.081	0.000	0.000	0.159	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	241	133	0	0	114	0	0	47	0
N.S.	1	1.36	0.75	0.00	0.00	0.64	0.00	0.00	0.27	0.00
time (sec)	N/A	1.233	0.031	0.000	0.000	0.087	0.000	0.000	0.166	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	260	106	131	0	111	0	0	12	131
N.S.	1	1.58	0.64	0.79	0.00	0.67	0.00	0.00	0.07	0.79
time (sec)	N/A	1.053	0.066	0.661	0.000	0.088	0.000	0.000	0.155	0.218

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	149	88	95	0	90	0	0	12	90
N.S.	1	1.32	0.78	0.84	0.00	0.80	0.00	0.00	0.11	0.80
time (sec)	N/A	0.588	0.052	0.878	0.000	0.082	0.000	0.000	0.154	3.974

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	56	48	62	63	0	0	8	44
N.S.	1	1.07	1.00	0.86	1.11	1.12	0.00	0.00	0.14	0.79
time (sec)	N/A	0.290	0.025	0.351	0.109	0.098	0.000	0.000	0.155	0.133

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	37	12	10	12	44	12
N.S.	1	1.00	1.20	1.00	3.70	1.20	1.00	1.20	4.40	1.20
time (sec)	N/A	0.184	0.027	0.023	0.062	0.072	0.951	0.116	0.150	3.852

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	37	12	10	12	47	12
N.S.	1	1.00	1.20	1.00	3.70	1.20	1.00	1.20	4.70	1.20
time (sec)	N/A	0.186	0.027	0.023	0.057	0.078	1.097	0.122	0.153	3.846

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	37	12	10	12	47	12
N.S.	1	1.00	1.20	1.00	3.70	1.20	1.00	1.20	4.70	1.20
time (sec)	N/A	0.184	0.028	0.024	0.057	0.074	1.359	0.117	0.150	4.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	343	226	0	0	281	0	0	49	359
N.S.	1	0.91	0.60	0.00	0.00	0.75	0.00	0.00	0.13	0.96
time (sec)	N/A	0.615	0.689	0.000	0.000	0.089	0.000	0.000	0.160	0.441

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	171	132	0	0	171	0	0	27	186
N.S.	1	0.91	0.70	0.00	0.00	0.91	0.00	0.00	0.14	0.99
time (sec)	N/A	0.391	0.290	0.000	0.000	0.091	0.000	0.000	0.154	3.906

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	59	0	91	0	0	10	79
N.S.	1	1.00	0.93	0.83	0.00	1.28	0.00	0.00	0.14	1.11
time (sec)	N/A	0.516	0.008	0.405	0.000	0.094	0.000	0.000	0.152	3.879

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.204	0.041	0.135	0.071	0.080	3.102	0.157	0.153	3.861

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	74	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	4.62	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.216	0.077	0.186	0.068	0.079	12.251	0.220	0.152	3.931

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	88	0	0	125	0	85	20	0
N.S.	1	1.00	0.86	0.00	0.00	1.23	0.00	0.83	0.20	0.00
time (sec)	N/A	0.547	0.256	0.000	0.000	0.095	0.000	0.281	0.153	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	111	84	0	0	121	0	83	18	0
N.S.	1	1.18	0.89	0.00	0.00	1.29	0.00	0.88	0.19	0.00
time (sec)	N/A	0.509	0.189	0.000	0.000	0.095	0.000	0.283	0.157	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	122	0	79	16	0
N.S.	1	1.00	0.86	0.00	0.00	1.31	0.00	0.85	0.17	0.00
time (sec)	N/A	0.446	0.172	0.000	0.000	0.091	0.000	0.194	0.154	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	79	62	58	119	0	67	20	121
N.S.	1	0.98	1.22	0.95	0.89	1.83	0.00	1.03	0.31	1.86
time (sec)	N/A	0.294	0.098	0.527	0.032	0.089	0.000	0.120	0.154	4.156

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	80	0	0	126	0	0	20	0
N.S.	1	1.00	0.87	0.00	0.00	1.37	0.00	0.00	0.22	0.00
time (sec)	N/A	0.508	0.174	0.000	0.000	0.105	0.000	0.000	0.152	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	110	77	0	0	124	0	0	20	0
N.S.	1	1.16	0.81	0.00	0.00	1.31	0.00	0.00	0.21	0.00
time (sec)	N/A	0.493	0.169	0.000	0.000	0.106	0.000	0.000	0.157	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	141	127	0	0	180	0	0	24	0
N.S.	1	1.13	1.02	0.00	0.00	1.44	0.00	0.00	0.19	0.00
time (sec)	N/A	0.574	0.343	0.000	0.000	0.100	0.000	0.000	0.159	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	19	16	16	16
N.S.	1	1.00	1.00	0.81	0.76	0.76	0.90	0.76	0.76	0.76
time (sec)	N/A	0.214	0.009	0.453	0.034	0.105	0.425	0.111	0.153	4.069

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	19	16	16	93
N.S.	1	1.00	1.00	0.81	0.76	0.76	0.90	0.76	0.76	4.43
time (sec)	N/A	0.202	0.006	0.366	0.032	0.080	0.209	0.112	0.149	4.334

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	15	15	17	16	15	15
N.S.	1	1.00	1.00	0.00	0.75	0.75	0.85	0.80	0.75	0.75
time (sec)	N/A	0.206	0.010	0.000	0.033	0.071	0.197	0.124	0.148	3.968

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	17	16	16	16
N.S.	1	1.00	1.00	0.81	0.76	0.76	0.81	0.76	0.76	0.76
time (sec)	N/A	0.210	0.007	0.213	0.032	0.081	0.316	0.116	0.146	0.099

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	19	16	16	16
N.S.	1	1.00	1.00	0.81	0.76	0.76	0.90	0.76	0.76	0.76
time (sec)	N/A	0.202	0.007	0.225	0.031	0.081	0.477	0.112	0.145	3.906

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	24	63	23	25	23
N.S.	1	1.00	1.00	0.00	0.00	0.86	2.25	0.82	0.89	0.82
time (sec)	N/A	0.210	0.010	0.000	0.000	0.086	1.625	0.111	0.153	4.009

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	319	138	312	0	260	0	270	475	244
N.S.	1	1.12	0.48	1.09	0.00	0.91	0.00	0.95	1.67	0.86
time (sec)	N/A	1.340	0.291	5.799	0.000	0.087	0.000	0.127	0.154	0.735

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	165	99	168	0	149	0	141	221	131
N.S.	1	1.06	0.64	1.08	0.00	0.96	0.00	0.91	1.43	0.85
time (sec)	N/A	0.612	0.201	1.519	0.000	0.094	0.000	0.121	0.152	0.546

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	67	47	62	0	48	19	47
N.S.	1	1.00	0.89	1.18	0.82	1.09	0.00	0.84	0.33	0.82
time (sec)	N/A	0.249	0.026	0.695	0.035	0.085	0.000	0.113	0.157	3.927

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	17	18	21	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.00	1.06	1.24	1.06
time (sec)	N/A	0.212	0.105	0.071	0.081	0.080	2.990	0.112	0.145	4.277

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	21	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.24	1.06
time (sec)	N/A	0.548	0.145	0.194	0.081	0.085	6.223	0.113	0.149	4.979

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	204	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	12.00	1.06
time (sec)	N/A	1.135	0.176	0.175	0.085	0.076	33.685	0.115	0.153	4.989

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	21	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.24	1.06
time (sec)	N/A	0.848	0.196	0.085	0.084	0.080	46.328	0.112	0.150	5.148

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	21	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.24	1.06
time (sec)	N/A	0.397	0.150	0.135	0.157	0.082	8.667	0.108	0.150	4.941

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	15	15	15	15	18	15
N.S.	1	1.00	1.14	0.93	1.07	1.07	1.07	1.07	1.29	1.07
time (sec)	N/A	0.189	0.022	0.070	0.088	0.083	2.164	0.118	0.161	4.592

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	21	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.24	1.06
time (sec)	N/A	0.405	0.148	0.081	0.083	0.080	3.260	0.112	0.149	4.735

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	21	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.24	1.06
time (sec)	N/A	0.825	0.210	0.178	0.086	0.081	14.471	0.111	0.149	4.705

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	135	73	88	82	74	119	118	98	91
N.S.	1	1.14	0.62	0.75	0.69	0.63	1.01	1.00	0.83	0.77
time (sec)	N/A	0.513	0.035	2.756	0.032	0.095	35.422	0.115	0.148	3.963

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	86	57	66	59	55	76	71	66	65
N.S.	1	1.09	0.72	0.84	0.75	0.70	0.96	0.90	0.84	0.82
time (sec)	N/A	0.340	0.025	1.306	0.031	0.085	6.962	0.118	0.148	0.215

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	51	34	35	34	31	33	31
N.S.	1	1.00	0.92	1.38	0.92	0.95	0.92	0.84	0.89	0.84
time (sec)	N/A	0.210	0.016	0.300	0.034	0.082	1.310	0.114	0.147	0.100

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	26	0	23	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.81	0.00	0.72	0.00
time (sec)	N/A	0.217	0.063	0.000	0.000	0.000	5.307	0.000	0.152	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	34	0	0	0	29	0	23	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.41	0.00	0.32	0.00
time (sec)	N/A	0.343	0.094	0.000	0.000	0.000	28.307	0.000	0.150	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	36	0	0	0	0	0	94	0
N.S.	1	1.02	0.31	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.481	0.103	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	127	100	0	0	0	0	0	23	0
N.S.	1	1.07	0.84	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.473	0.205	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	80	0	0	0	24	0	23	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.32	0.00	0.30	0.00
time (sec)	N/A	0.334	0.144	0.000	0.000	0.000	27.265	0.000	0.147	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	0	0	0	0	22	0	20	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.76	0.00	0.69	0.00
time (sec)	N/A	0.197	0.000	0.000	0.000	0.000	4.077	0.000	0.148	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	74	0	0	0	46	0	23	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.70	0.00	0.35	0.00
time (sec)	N/A	0.327	0.133	0.000	0.000	0.000	10.877	0.000	0.150	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	112	100	0	0	0	24	0	23	0
N.S.	1	0.97	0.87	0.00	0.00	0.00	0.21	0.00	0.20	0.00
time (sec)	N/A	0.465	0.231	0.000	0.000	0.000	81.041	0.000	0.156	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	236	86	119	0	97	0	0	21	192
N.S.	1	1.75	0.64	0.88	0.00	0.72	0.00	0.00	0.16	1.42
time (sec)	N/A	0.982	0.055	2.448	0.000	0.096	0.000	0.000	0.162	0.457

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	125	68	83	0	76	0	0	21	106
N.S.	1	1.39	0.76	0.92	0.00	0.84	0.00	0.00	0.23	1.18
time (sec)	N/A	0.478	0.036	1.213	0.000	0.084	0.000	0.000	0.157	4.061

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	39	34	43	0	0	19	43
N.S.	1	1.00	0.91	0.91	0.79	1.00	0.00	0.00	0.44	1.00
time (sec)	N/A	0.227	0.014	0.474	0.106	0.086	0.000	0.000	0.147	3.863

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	19	19	15	19	21	19
N.S.	1	1.00	1.11	1.00	1.06	1.06	0.83	1.06	1.17	1.06
time (sec)	N/A	0.204	0.062	0.061	0.060	0.073	1.634	0.107	0.148	3.910

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	19	19	17	19	21	19
N.S.	1	1.00	1.11	1.00	1.06	1.06	0.94	1.06	1.17	1.06
time (sec)	N/A	0.492	0.099	0.152	0.065	0.078	3.411	0.110	0.155	4.265

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	19	19	17	19	21	19
N.S.	1	1.00	1.11	1.00	1.06	1.06	0.94	1.06	1.17	1.06
time (sec)	N/A	0.872	0.113	0.168	0.081	0.096	16.120	0.114	0.150	4.120

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	142	85	0	0	74	109	0	21	90
N.S.	1	1.27	0.76	0.00	0.00	0.66	0.97	0.00	0.19	0.80
time (sec)	N/A	0.667	0.021	0.000	0.000	0.100	6.227	0.000	0.150	4.200

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	52	60	0	21	80
N.S.	1	1.00	0.89	0.00	0.00	0.83	0.95	0.00	0.33	1.27
time (sec)	N/A	0.355	0.029	0.000	0.000	0.105	1.143	0.000	0.164	3.796

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	15	14	13	41
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	0.78	0.72	2.28
time (sec)	N/A	0.191	0.005	0.312	0.024	0.085	0.299	0.109	0.152	0.187

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	53	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	1.02	0.00	0.00	0.40	0.00
time (sec)	N/A	0.368	0.014	0.000	0.000	0.094	0.000	0.000	0.150	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	123	85	0	0	84	0	0	21	0
N.S.	1	1.14	0.79	0.00	0.00	0.78	0.00	0.00	0.19	0.00
time (sec)	N/A	0.644	0.048	0.000	0.000	0.095	0.000	0.000	0.153	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	379	240	0	0	267	0	288	23	386
N.S.	1	1.11	0.70	0.00	0.00	0.78	0.00	0.84	0.07	1.13
time (sec)	N/A	1.822	3.062	0.000	0.000	0.107	0.000	0.133	0.159	4.537

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	95	82	139	84	100	0	87	21	89
N.S.	1	1.10	0.95	1.62	0.98	1.16	0.00	1.01	0.24	1.03
time (sec)	N/A	0.337	0.079	0.993	0.040	0.094	0.000	0.130	0.159	3.615

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	19	20	23	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.00	1.05	1.21	1.05
time (sec)	N/A	0.226	0.147	0.066	0.087	0.075	6.537	0.116	0.152	4.013

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	23	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.21	1.05
time (sec)	N/A	1.346	0.231	0.200	0.087	0.083	25.501	0.123	0.151	4.201

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	0	20	23	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	0.00	1.05	1.21	1.05
time (sec)	N/A	3.819	0.292	0.106	0.085	0.084	0.000	0.123	0.148	5.519

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	23	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.21	1.05
time (sec)	N/A	0.678	0.241	0.161	0.086	0.092	57.754	0.122	0.149	5.238

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	17	17	17	17	20	17
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.06	1.06	1.25	1.06
time (sec)	N/A	0.195	0.028	0.079	0.086	0.071	5.257	0.122	0.149	4.975

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	23	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.21	1.05
time (sec)	N/A	0.508	0.251	0.108	0.086	0.098	8.590	0.117	0.147	4.502

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	23	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.21	1.05
time (sec)	N/A	2.579	0.321	0.214	0.088	0.082	76.808	0.132	0.160	4.659

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	67	0	66	0	0	47	52
N.S.	1	1.00	1.00	1.08	0.00	1.06	0.00	0.00	0.76	0.84
time (sec)	N/A	0.284	0.081	2.565	0.000	0.079	0.000	0.000	0.161	3.876

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	0	0	0	0	0	18	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.377	0.051	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	0	0	0	0	0	22	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.377	0.047	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	0	0	0	0	0	0	18	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.374	0.000	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	0	0	0	0	0	0	20	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.366	0.000	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	17	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.369	0.028	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	0	0	0	0	0	21	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.356	0.030	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	73	0	0	0	0	0	17	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.364	0.403	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	0	0	0	0	0	19	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.363	0.337	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	113	62	81	63	71	92	69	102	78
N.S.	1	1.18	0.65	0.84	0.66	0.74	0.96	0.72	1.06	0.81
time (sec)	N/A	0.376	0.041	0.293	0.030	0.087	0.447	0.120	0.164	3.946

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	54	64	55	63	68	61	92	58
N.S.	1	1.20	0.76	0.90	0.77	0.89	0.96	0.86	1.30	0.82
time (sec)	N/A	0.314	0.038	0.162	0.031	0.077	0.261	0.127	0.162	3.880

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	54	43	44	44	50	42	49	81	38
N.S.	1	1.17	0.93	0.96	0.96	1.09	0.91	1.07	1.76	0.83
time (sec)	N/A	0.263	0.032	0.124	0.030	0.077	0.172	0.123	0.172	0.116

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	0	0	0	36	0	15	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	1.03	0.00	0.43	0.00
time (sec)	N/A	0.223	0.008	0.000	0.000	0.000	0.425	0.000	0.155	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	48	40	42	35	43	34	0	75	38
N.S.	1	1.20	1.00	1.05	0.88	1.08	0.85	0.00	1.88	0.95
time (sec)	N/A	0.254	0.028	0.167	0.074	0.085	0.220	0.000	0.153	0.125

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	53	62	35	58	60	0	90	71
N.S.	1	1.06	0.75	0.87	0.49	0.82	0.85	0.00	1.27	1.00
time (sec)	N/A	0.305	0.025	0.247	0.072	0.083	0.300	0.000	0.152	3.879

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	96	100	62	81	35	66	87	0	100	0
N.S.	1	1.04	0.65	0.84	0.36	0.69	0.91	0.00	1.04	0.00
time (sec)	N/A	0.370	0.034	0.491	0.069	0.079	0.523	0.000	0.153	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	112	73	86	52	68	102	57	93	90
N.S.	1	1.03	0.67	0.79	0.48	0.62	0.94	0.52	0.85	0.83
time (sec)	N/A	0.387	0.014	0.408	0.031	0.090	0.590	0.124	0.153	4.139

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	66	69	44	60	78	49	83	70
N.S.	1	1.02	0.79	0.82	0.52	0.71	0.93	0.58	0.99	0.83
time (sec)	N/A	0.319	0.013	0.185	0.026	0.082	0.329	0.125	0.150	4.053

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	60	42	49	36	52	54	41	73	50
N.S.	1	1.02	0.71	0.83	0.61	0.88	0.92	0.69	1.24	0.85
time (sec)	N/A	0.259	0.018	0.608	0.026	0.082	0.196	0.118	0.151	0.140

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	26	35	24	26	54	24
N.S.	1	1.00	1.00	0.93	0.96	1.30	0.89	0.96	2.00	0.89
time (sec)	N/A	0.175	0.002	0.164	0.028	0.078	0.135	0.112	0.150	0.095

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	25	32	20	0	32	25
N.S.	1	1.00	1.00	0.93	0.93	1.19	0.74	0.00	1.19	0.93
time (sec)	N/A	0.231	0.012	0.771	0.067	0.076	0.711	0.000	0.153	4.072

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	49	46	27	51	48	0	79	46
N.S.	1	1.00	0.88	0.82	0.48	0.91	0.86	0.00	1.41	0.82
time (sec)	N/A	0.272	0.028	0.339	0.071	0.084	1.226	0.000	0.156	0.120

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	66	27	62	70	0	91	66
N.S.	1	1.00	0.90	0.81	0.33	0.77	0.86	0.00	1.12	0.81
time (sec)	N/A	0.320	0.018	0.612	0.071	0.099	1.985	0.000	0.150	3.930

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	292	291	268	483	0	314	746	435	329	352
N.S.	1	1.00	0.92	1.65	0.00	1.08	2.55	1.49	1.13	1.21
time (sec)	N/A	0.593	0.242	0.733	0.000	0.093	1.868	0.273	0.159	3.961

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	194	204	159	272	0	197	398	280	256	220
N.S.	1	1.05	0.82	1.40	0.00	1.02	2.05	1.44	1.32	1.13
time (sec)	N/A	0.446	0.178	0.511	0.000	0.081	0.891	0.232	0.160	0.237

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	132	104	124	0	110	178	158	179	119
N.S.	1	1.11	0.87	1.04	0.00	0.92	1.50	1.33	1.50	1.00
time (sec)	N/A	0.373	0.093	0.300	0.000	0.077	0.461	0.184	0.151	0.157

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	33	32	53	53	60	112	49
N.S.	1	1.00	1.14	0.89	0.86	1.43	1.43	1.62	3.03	1.32
time (sec)	N/A	0.183	0.016	0.144	0.028	0.075	0.237	0.119	0.154	0.116

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	19	12	16	30	16
N.S.	1	1.00	1.14	1.00	1.14	1.36	0.86	1.14	2.14	1.14
time (sec)	N/A	0.187	0.093	0.160	0.121	0.077	0.975	0.121	0.156	3.773

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	30	14	16	77	16
N.S.	1	1.00	1.14	1.00	1.14	2.14	1.00	1.14	5.50	1.14
time (sec)	N/A	0.249	0.292	0.229	0.117	0.095	10.163	0.155	0.156	3.802

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	41	14	16	160	16
N.S.	1	1.00	1.14	1.00	1.14	2.93	1.00	1.14	11.43	1.14
time (sec)	N/A	0.383	0.579	0.211	0.115	0.089	72.855	0.125	0.154	3.953

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	272	155	160	0	149	172	0	129	143
N.S.	1	1.53	0.87	0.90	0.00	0.84	0.97	0.00	0.72	0.80
time (sec)	N/A	1.295	0.161	0.515	0.000	0.082	0.621	0.000	0.159	4.077

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	164	144	116	0	124	121	0	119	102
N.S.	1	1.30	1.14	0.92	0.00	0.98	0.96	0.00	0.94	0.81
time (sec)	N/A	0.766	0.171	0.329	0.000	0.078	0.337	0.000	0.158	4.075

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	85	99	72	0	91	68	0	105	68
N.S.	1	1.18	1.38	1.00	0.00	1.26	0.94	0.00	1.46	0.94
time (sec)	N/A	0.418	0.098	0.270	0.000	0.084	0.209	0.000	0.156	0.137

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	20	8	12	27	12
N.S.	1	1.00	1.20	1.00	1.20	2.00	0.80	1.20	2.70	1.20
time (sec)	N/A	0.177	0.112	0.026	0.062	0.078	1.062	0.113	0.155	3.900

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	75	63	0	0	98	0	0	132	0
N.S.	1	1.12	0.94	0.00	0.00	1.46	0.00	0.00	1.97	0.00
time (sec)	N/A	0.421	0.022	0.000	0.000	0.088	0.000	0.000	0.155	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	146	97	0	0	141	0	0	146	0
N.S.	1	1.17	0.78	0.00	0.00	1.13	0.00	0.00	1.17	0.00
time (sec)	N/A	0.733	0.057	0.000	0.000	0.095	0.000	0.000	0.162	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	242	133	0	0	168	0	0	156	0
N.S.	1	1.37	0.75	0.00	0.00	0.95	0.00	0.00	0.88	0.00
time (sec)	N/A	1.245	0.032	0.000	0.000	0.095	0.000	0.000	0.154	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	262	108	205	0	154	0	0	110	0
N.S.	1	1.59	0.65	1.24	0.00	0.93	0.00	0.00	0.67	0.00
time (sec)	N/A	1.056	0.101	0.514	0.000	0.100	0.000	0.000	0.152	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	150	88	151	0	123	0	0	100	0
N.S.	1	1.33	0.78	1.34	0.00	1.09	0.00	0.00	0.88	0.00
time (sec)	N/A	0.597	0.060	0.694	0.000	0.087	0.000	0.000	0.155	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	56	48	0	85	0	0	75	0
N.S.	1	1.07	1.00	0.86	0.00	1.52	0.00	0.00	1.34	0.00
time (sec)	N/A	0.297	0.031	0.209	0.000	0.082	0.000	0.000	0.154	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	20	10	12	69	12
N.S.	1	1.00	1.20	1.00	1.20	2.00	1.00	1.20	6.90	1.20
time (sec)	N/A	0.191	0.095	0.026	0.074	0.081	1.001	0.109	0.153	3.786

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	20	10	12	138	12
N.S.	1	1.00	1.20	1.00	1.20	2.00	1.00	1.20	13.80	1.20
time (sec)	N/A	0.185	0.102	0.032	0.061	0.080	1.122	0.119	0.153	3.761

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	20	10	12	147	12
N.S.	1	1.00	1.20	1.00	1.20	2.00	1.00	1.20	14.70	1.20
time (sec)	N/A	0.185	0.107	0.025	0.054	0.099	1.377	0.110	0.160	3.619

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	343	610	0	0	472	0	0	369	0
N.S.	1	0.91	1.63	0.00	0.00	1.26	0.00	0.00	0.98	0.00
time (sec)	N/A	0.574	3.346	0.000	0.000	0.125	0.000	0.000	0.163	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	172	301	0	0	273	0	0	249	0
N.S.	1	0.91	1.59	0.00	0.00	1.44	0.00	0.00	1.32	0.00
time (sec)	N/A	0.385	2.214	0.000	0.000	0.112	0.000	0.000	0.153	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	59	0	141	0	0	144	0
N.S.	1	1.00	0.93	0.83	0.00	1.99	0.00	0.00	2.03	0.00
time (sec)	N/A	0.459	0.084	0.263	0.000	0.130	0.000	0.000	0.163	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	28	14	18	50	18
N.S.	1	1.00	1.12	1.00	1.12	1.75	0.88	1.12	3.12	1.12
time (sec)	N/A	0.202	0.457	0.151	0.050	0.099	2.498	0.123	0.163	3.581

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	39	15	18	143	18
N.S.	1	1.00	1.12	1.00	1.12	2.44	0.94	1.12	8.94	1.12
time (sec)	N/A	0.204	0.294	0.184	0.052	0.085	11.590	0.171	0.163	3.762

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	87	0	0	130	0	90	28	0
N.S.	1	1.00	0.85	0.00	0.00	1.27	0.00	0.88	0.27	0.00
time (sec)	N/A	0.482	0.238	0.000	0.000	0.106	0.000	0.198	0.166	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	111	80	0	0	126	0	88	26	0
N.S.	1	1.18	0.85	0.00	0.00	1.34	0.00	0.94	0.28	0.00
time (sec)	N/A	0.487	0.198	0.000	0.000	0.094	0.000	0.201	0.162	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	77	0	0	123	0	82	20	0
N.S.	1	1.00	0.84	0.00	0.00	1.34	0.00	0.89	0.22	0.00
time (sec)	N/A	0.451	0.180	0.000	0.000	0.131	0.000	0.196	0.162	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	93	63	59	128	0	83	25	100
N.S.	1	0.98	1.41	0.95	0.89	1.94	0.00	1.26	0.38	1.52
time (sec)	N/A	0.303	0.104	0.506	0.038	0.092	0.000	0.116	0.165	3.914

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	128	0	0	29	0
N.S.	1	1.00	0.87	0.00	0.00	1.38	0.00	0.00	0.31	0.00
time (sec)	N/A	0.486	0.201	0.000	0.000	0.102	0.000	0.000	0.163	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	110	79	0	0	125	0	0	32	0
N.S.	1	1.16	0.83	0.00	0.00	1.32	0.00	0.00	0.34	0.00
time (sec)	N/A	0.485	0.193	0.000	0.000	0.125	0.000	0.000	0.167	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	142	126	0	0	194	0	0	60	0
N.S.	1	1.13	1.00	0.00	0.00	1.54	0.00	0.00	0.48	0.00
time (sec)	N/A	0.567	0.360	0.000	0.000	0.111	0.000	0.000	0.176	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	43	0	31	24	42	28	16
N.S.	1	1.00	1.00	2.05	0.00	1.48	1.14	2.00	1.33	0.76
time (sec)	N/A	0.209	0.010	0.452	0.000	0.085	0.389	0.111	0.158	3.726

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	30	0	23	24	29	22	16
N.S.	1	1.00	1.00	1.43	0.00	1.10	1.14	1.38	1.05	0.76
time (sec)	N/A	0.202	0.007	0.276	0.000	0.094	0.171	0.108	0.159	0.079

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	17	24	18	17	15
N.S.	1	1.00	1.00	0.00	0.00	0.85	1.20	0.90	0.85	0.75
time (sec)	N/A	0.213	0.012	0.000	0.000	0.093	0.249	0.120	0.160	3.752

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	19	20	18	22	16
N.S.	1	1.00	1.00	0.00	0.00	0.90	0.95	0.86	1.05	0.76
time (sec)	N/A	0.201	0.007	0.000	0.000	0.092	0.391	0.113	0.158	0.104

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	26	22	18	26	16
N.S.	1	1.00	1.00	0.00	0.00	1.24	1.05	0.86	1.24	0.76
time (sec)	N/A	0.204	0.007	0.000	0.000	0.088	0.721	0.109	0.161	0.090

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	30	60	27	31	23
N.S.	1	1.00	1.00	0.00	0.00	1.07	2.14	0.96	1.11	0.82
time (sec)	N/A	0.213	0.010	0.000	0.000	0.107	1.671	0.110	0.159	3.770

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	317	138	376	0	356	0	0	693	0
N.S.	1	1.12	0.49	1.33	0.00	1.26	0.00	0.00	2.45	0.00
time (sec)	N/A	1.313	0.291	5.483	0.000	0.107	0.000	0.000	0.166	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	164	99	206	0	190	0	0	309	0
N.S.	1	1.06	0.64	1.33	0.00	1.23	0.00	0.00	1.99	0.00
time (sec)	N/A	0.618	0.218	1.382	0.000	0.111	0.000	0.000	0.163	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	92	0	70	0	0	34	0
N.S.	1	1.00	0.88	1.61	0.00	1.23	0.00	0.00	0.60	0.00
time (sec)	N/A	0.250	0.030	0.543	0.000	0.107	0.000	0.000	0.165	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	17	18	31	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.00	1.06	1.82	1.06
time (sec)	N/A	0.217	0.359	0.078	0.079	0.099	3.065	0.113	0.163	3.763

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	19	18	51	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.12	1.06	3.00	1.06
time (sec)	N/A	0.552	0.460	0.213	0.081	0.093	6.127	0.113	0.161	4.019

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	19	18	268	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.12	1.06	15.76	1.06
time (sec)	N/A	1.170	0.536	0.226	0.077	0.098	31.259	0.116	0.164	4.019

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	26	19	18	75	18
N.S.	1	1.00	1.12	0.94	1.06	1.53	1.12	1.06	4.41	1.06
time (sec)	N/A	0.873	0.583	0.073	0.079	0.102	53.034	0.108	0.165	4.015

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	26	19	18	57	18
N.S.	1	1.00	1.12	0.94	1.06	1.53	1.12	1.06	3.35	1.06
time (sec)	N/A	0.400	0.447	0.145	0.081	0.105	10.605	0.112	0.174	4.024

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	15	18	15	15	39	15
N.S.	1	1.00	1.14	0.93	1.07	1.29	1.07	1.07	2.79	1.07
time (sec)	N/A	0.196	0.024	0.069	0.075	0.106	2.659	0.111	0.161	3.902

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	19	18	37	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.12	1.06	2.18	1.06
time (sec)	N/A	0.410	0.481	0.079	0.090	0.100	3.095	0.110	0.171	3.908

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	19	18	37	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.12	1.06	2.18	1.06
time (sec)	N/A	0.846	0.576	0.177	0.086	0.093	14.016	0.111	0.164	4.141

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	134	73	135	0	97	126	0	146	94
N.S.	1	1.14	0.62	1.14	0.00	0.82	1.07	0.00	1.24	0.80
time (sec)	N/A	0.513	0.033	2.650	0.000	0.092	36.559	0.000	0.156	4.476

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	85	58	99	0	68	83	0	96	63
N.S.	1	1.06	0.72	1.24	0.00	0.85	1.04	0.00	1.20	0.79
time (sec)	N/A	0.365	0.025	1.187	0.000	0.098	6.969	0.000	0.152	4.209

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	51	0	38	41	0	45	30
N.S.	1	1.00	1.00	1.42	0.00	1.06	1.14	0.00	1.25	0.83
time (sec)	N/A	0.225	0.018	0.171	0.000	0.089	1.276	0.000	0.159	0.110

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	0	39	0	35	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.81	0.00	0.73	0.00
time (sec)	N/A	0.385	0.108	0.000	0.000	0.000	5.613	0.000	0.162	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	65	0	0	0	61	0	59	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.69	0.00	0.67	0.00
time (sec)	N/A	0.524	0.154	0.000	0.000	0.000	30.795	0.000	0.162	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	83	0	0	0	0	0	141	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.690	0.168	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	150	147	0	0	0	0	0	73	0
N.S.	1	1.09	1.07	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.630	0.301	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	104	0	0	0	65	0	55	0
N.S.	1	1.04	1.09	0.00	0.00	0.00	0.68	0.00	0.58	0.00
time (sec)	N/A	0.473	0.235	0.000	0.000	0.000	28.461	0.000	0.162	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	44	0	38	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.88	0.00	0.76	0.00
time (sec)	N/A	0.300	0.000	0.000	0.000	0.000	4.374	0.000	0.159	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	89	99	0	0	0	80	0	41	0
N.S.	1	1.11	1.24	0.00	0.00	0.00	1.00	0.00	0.51	0.00
time (sec)	N/A	0.442	0.200	0.000	0.000	0.000	10.982	0.000	0.160	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	135	151	0	0	0	22	0	41	0
N.S.	1	1.01	1.13	0.00	0.00	0.00	0.16	0.00	0.31	0.00
time (sec)	N/A	0.609	0.320	0.000	0.000	0.000	82.024	0.000	0.160	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	238	87	172	0	121	0	0	69	0
N.S.	1	1.76	0.64	1.27	0.00	0.90	0.00	0.00	0.51	0.00
time (sec)	N/A	0.957	0.097	2.236	0.000	0.097	0.000	0.000	0.161	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	126	69	118	0	90	0	0	61	0
N.S.	1	1.40	0.77	1.31	0.00	1.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.475	0.067	1.061	0.000	0.104	0.000	0.000	0.157	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	53	0	47	0	0	51	0
N.S.	1	1.00	0.91	1.23	0.00	1.09	0.00	0.00	1.19	0.00
time (sec)	N/A	0.232	0.019	0.303	0.000	0.098	0.000	0.000	0.158	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	19	22	15	19	35	19
N.S.	1	1.00	1.11	1.00	1.06	1.22	0.83	1.06	1.94	1.06
time (sec)	N/A	0.203	0.163	0.069	0.060	0.085	1.687	0.112	0.158	4.178

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	19	22	17	19	79	19
N.S.	1	1.00	1.11	1.00	1.06	1.22	0.94	1.06	4.39	1.06
time (sec)	N/A	0.450	0.222	0.194	0.055	0.090	3.294	0.115	0.158	4.174

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	19	22	17	19	85	19
N.S.	1	1.00	1.11	1.00	1.06	1.22	0.94	1.06	4.72	1.06
time (sec)	N/A	0.840	0.218	0.207	0.060	0.085	12.093	0.108	0.159	4.215

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	143	112	0	0	97	112	0	81	90
N.S.	1	1.28	1.00	0.00	0.00	0.87	1.00	0.00	0.72	0.80
time (sec)	N/A	0.631	0.104	0.000	0.000	0.104	4.031	0.000	0.157	4.266

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	0	0	66	63	0	72	49
N.S.	1	1.00	1.25	0.00	0.00	1.05	1.00	0.00	1.14	0.78
time (sec)	N/A	0.335	0.089	0.000	0.000	0.106	1.003	0.000	0.160	4.355

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	22	14	21	17	25	19	14
N.S.	1	1.00	1.00	1.22	0.78	1.17	0.94	1.39	1.06	0.78
time (sec)	N/A	0.186	0.005	0.171	0.028	0.088	0.297	0.108	0.157	0.072

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	77	0	0	69	0
N.S.	1	1.00	1.00	0.00	0.00	1.45	0.00	0.00	1.30	0.00
time (sec)	N/A	0.330	0.013	0.000	0.000	0.088	0.000	0.000	0.158	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	124	85	0	0	122	0	0	87	0
N.S.	1	1.15	0.79	0.00	0.00	1.13	0.00	0.00	0.81	0.00
time (sec)	N/A	0.613	0.047	0.000	0.000	0.101	0.000	0.000	0.165	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	378	256	0	0	328	0	0	54	0
N.S.	1	1.11	0.75	0.00	0.00	0.96	0.00	0.00	0.16	0.00
time (sec)	N/A	1.663	3.254	0.000	0.000	0.114	0.000	0.000	0.157	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	95	81	179	0	108	0	0	36	0
N.S.	1	1.10	0.94	2.08	0.00	1.26	0.00	0.00	0.42	0.00
time (sec)	N/A	0.333	0.087	0.828	0.000	0.099	0.000	0.000	0.160	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	23	19	20	33	20
N.S.	1	1.00	1.11	0.95	1.05	1.21	1.00	1.05	1.74	1.05
time (sec)	N/A	0.220	0.493	0.112	0.082	0.113	6.486	0.119	0.159	4.275

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	23	20	20	53	20
N.S.	1	1.00	1.11	0.95	1.05	1.21	1.05	1.05	2.79	1.05
time (sec)	N/A	1.349	0.642	0.251	0.083	0.092	23.380	0.123	0.158	4.386

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	28	0	20	77	20
N.S.	1	1.00	1.11	0.95	1.05	1.47	0.00	1.05	4.05	1.05
time (sec)	N/A	3.817	0.836	0.062	0.079	0.080	0.000	0.118	0.165	4.266

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	28	20	20	59	20
N.S.	1	1.00	1.11	0.95	1.05	1.47	1.05	1.05	3.11	1.05
time (sec)	N/A	0.693	0.589	0.150	0.081	0.102	44.255	0.121	0.160	4.313

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	17	20	17	17	41	17
N.S.	1	1.00	1.12	0.94	1.06	1.25	1.06	1.06	2.56	1.06
time (sec)	N/A	0.196	0.032	0.075	0.074	0.096	4.321	0.122	0.156	4.170

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	23	20	20	39	20
N.S.	1	1.00	1.11	0.95	1.05	1.21	1.05	1.05	2.05	1.05
time (sec)	N/A	0.509	0.682	0.089	0.079	0.096	7.447	0.118	0.159	4.347

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	23	20	20	39	20
N.S.	1	1.00	1.11	0.95	1.05	1.21	1.05	1.05	2.05	1.05
time (sec)	N/A	2.611	0.825	0.198	0.078	0.093	66.321	0.119	0.166	4.417

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	84	0	71	0	0	91	0
N.S.	1	1.00	1.00	1.40	0.00	1.18	0.00	0.00	1.52	0.00
time (sec)	N/A	0.275	0.061	1.743	0.000	0.093	0.000	0.000	0.158	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	94	0	0	0	0	0	34	0
N.S.	1	1.01	1.03	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.519	0.305	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	101	0	0	0	0	0	38	0
N.S.	1	1.01	1.11	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.501	2.165	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	0	0	0	0	0	0	34	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.499	0.000	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	0	0	0	0	0	0	38	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.495	0.000	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	78	83	0	0	0	0	0	32	0
N.S.	1	1.04	1.11	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.493	0.109	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	84	0	0	0	0	0	36	0
N.S.	1	1.03	1.09	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.487	0.109	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	78	94	0	0	0	0	0	32	0
N.S.	1	1.04	1.25	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.491	0.081	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	101	0	0	0	0	0	36	0
N.S.	1	1.04	1.31	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.497	0.081	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	110	64	62	63	62	88	0	86	108
N.S.	1	1.18	0.69	0.67	0.68	0.67	0.95	0.00	0.92	1.16
time (sec)	N/A	0.377	0.021	0.300	0.033	0.096	0.482	0.000	0.160	0.123

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	83	51	54	55	53	65	0	67	89
N.S.	1	1.20	0.74	0.78	0.80	0.77	0.94	0.00	0.97	1.29
time (sec)	N/A	0.318	0.023	0.164	0.030	0.109	0.269	0.000	0.156	0.085

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	54	39	44	44	41	39	0	48	43
N.S.	1	1.20	0.87	0.98	0.98	0.91	0.87	0.00	1.07	0.96
time (sec)	N/A	0.256	0.023	0.126	0.031	0.089	0.165	0.000	0.159	4.300

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	0	0	24	0	14	0
N.S.	1	1.00	1.00	0.71	0.00	0.00	0.77	0.00	0.45	0.00
time (sec)	N/A	0.185	0.003	0.087	0.000	0.000	0.338	0.000	0.162	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	37	43	39	40	34	0	47	69
N.S.	1	1.12	0.92	1.08	0.98	1.00	0.85	0.00	1.18	1.72
time (sec)	N/A	0.252	0.019	0.154	0.071	0.082	0.216	0.000	0.159	0.106

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	72	51	62	39	52	60	0	67	89
N.S.	1	1.04	0.74	0.90	0.57	0.75	0.87	0.00	0.97	1.29
time (sec)	N/A	0.304	0.019	0.243	0.066	0.094	0.329	0.000	0.158	0.077

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	96	64	72	39	61	87	0	86	108
N.S.	1	1.03	0.69	0.77	0.42	0.66	0.94	0.00	0.92	1.16
time (sec)	N/A	0.363	0.019	0.468	0.068	0.088	0.540	0.000	0.158	3.938

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	57	62	51	59	99	0	94	51
N.S.	1	1.03	0.54	0.59	0.49	0.56	0.94	0.00	0.90	0.49
time (sec)	N/A	0.375	0.026	0.407	0.026	0.100	0.590	0.000	0.163	0.105

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	49	54	43	51	75	0	75	43
N.S.	1	1.02	0.60	0.67	0.53	0.63	0.93	0.00	0.93	0.53
time (sec)	N/A	0.327	0.024	0.193	0.039	0.085	0.333	0.000	0.158	3.774

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	41	46	35	43	49	0	55	35
N.S.	1	1.02	0.72	0.81	0.61	0.75	0.86	0.00	0.96	0.61
time (sec)	N/A	0.264	0.022	0.630	0.031	0.097	0.216	0.000	0.162	3.787

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	25	29	22	0	32	23
N.S.	1	1.00	1.00	0.92	0.96	1.12	0.85	0.00	1.23	0.88
time (sec)	N/A	0.177	0.006	0.195	0.025	0.072	0.072	0.000	0.156	0.050

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	23	29	32	0	14	23
N.S.	1	1.00	1.00	1.08	0.92	1.16	1.28	0.00	0.56	0.92
time (sec)	N/A	0.224	0.010	0.830	0.076	0.085	0.641	0.000	0.163	4.025

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	46	28	48	63	0	68	43
N.S.	1	1.00	0.93	0.85	0.52	0.89	1.17	0.00	1.26	0.80
time (sec)	N/A	0.261	0.017	0.411	0.069	0.089	1.101	0.000	0.158	3.957

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	65	28	58	85	0	88	62
N.S.	1	1.00	0.78	0.83	0.36	0.74	1.09	0.00	1.13	0.79
time (sec)	N/A	0.306	0.021	0.642	0.069	0.087	2.009	0.000	0.162	3.966

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	237	457	0	263	746	0	141	357
N.S.	1	1.00	0.85	1.64	0.00	0.94	2.67	0.00	0.51	1.28
time (sec)	N/A	0.574	0.178	1.038	0.000	0.085	1.875	0.000	0.162	4.214

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	196	142	258	0	161	398	0	114	190
N.S.	1	1.05	0.76	1.39	0.00	0.87	2.14	0.00	0.61	1.02
time (sec)	N/A	0.440	0.118	0.832	0.000	0.098	0.939	0.000	0.159	4.013

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	128	78	117	0	89	178	0	83	106
N.S.	1	1.11	0.68	1.02	0.00	0.77	1.55	0.00	0.72	0.92
time (sec)	N/A	0.353	0.057	0.345	0.000	0.087	0.466	0.000	0.160	3.912

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	31	30	45	51	0	60	46
N.S.	1	1.00	0.94	0.89	0.86	1.29	1.46	0.00	1.71	1.31
time (sec)	N/A	0.184	0.011	0.149	0.031	0.097	0.176	0.000	0.162	0.133

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	22	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.57	1.14
time (sec)	N/A	0.187	0.350	0.143	0.117	0.084	0.878	0.105	0.161	3.936

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	33	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	2.36	1.14
time (sec)	N/A	0.246	0.541	0.172	0.106	0.087	10.279	0.107	0.163	4.139

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	38	14	16	44	16
N.S.	1	1.00	1.14	1.00	1.14	2.71	1.00	1.14	3.14	1.14
time (sec)	N/A	0.381	0.589	0.167	0.106	0.089	75.355	0.105	0.165	5.937

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	270	99	154	0	97	168	0	15	139
N.S.	1	1.54	0.57	0.88	0.00	0.55	0.96	0.00	0.09	0.79
time (sec)	N/A	1.275	0.029	0.342	0.000	0.104	0.621	0.000	0.161	3.867

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	163	82	112	0	79	116	0	15	100
N.S.	1	1.31	0.66	0.90	0.00	0.64	0.94	0.00	0.12	0.81
time (sec)	N/A	0.802	0.022	0.152	0.000	0.107	0.358	0.000	0.159	3.968

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	63	69	0	58	63	0	13	66
N.S.	1	1.18	0.89	0.97	0.00	0.82	0.89	0.00	0.18	0.93
time (sec)	N/A	0.427	0.013	0.089	0.000	0.089	0.217	0.000	0.159	3.782

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	15	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.50	1.20
time (sec)	N/A	0.187	0.023	0.015	0.057	0.096	1.205	0.107	0.159	3.704

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	60	0	0	64	0	0	15	0
N.S.	1	1.12	0.92	0.00	0.00	0.98	0.00	0.00	0.23	0.00
time (sec)	N/A	0.430	0.018	0.000	0.000	0.093	0.000	0.000	0.160	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	141	97	0	0	93	0	0	15	0
N.S.	1	1.15	0.79	0.00	0.00	0.76	0.00	0.00	0.12	0.00
time (sec)	N/A	0.745	0.022	0.000	0.000	0.086	0.000	0.000	0.156	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	235	114	0	0	113	0	0	15	0
N.S.	1	1.35	0.66	0.00	0.00	0.65	0.00	0.00	0.09	0.00
time (sec)	N/A	1.227	0.028	0.000	0.000	0.109	0.000	0.000	0.164	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	260	105	0	0	114	0	0	15	0
N.S.	1	1.60	0.65	0.00	0.00	0.70	0.00	0.00	0.09	0.00
time (sec)	N/A	1.074	0.036	0.000	0.000	0.088	0.000	0.000	0.172	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	149	87	0	0	95	0	0	15	0
N.S.	1	1.34	0.78	0.00	0.00	0.86	0.00	0.00	0.14	0.00
time (sec)	N/A	0.600	0.023	0.000	0.000	0.104	0.000	0.000	0.157	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	54	0	0	66	0	0	11	0
N.S.	1	1.09	1.00	0.00	0.00	1.22	0.00	0.00	0.20	0.00
time (sec)	N/A	0.291	0.011	0.000	0.000	0.085	0.000	0.000	0.159	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	15	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.50	1.20
time (sec)	N/A	0.182	0.026	0.017	0.054	0.100	1.101	0.108	0.160	3.822

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	15	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.50	1.20
time (sec)	N/A	0.185	0.026	0.017	0.053	0.090	1.283	0.100	0.160	3.796

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	15	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.50	1.20
time (sec)	N/A	0.185	0.028	0.016	0.054	0.111	1.560	0.106	0.159	3.815

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	366	334	0	0	0	282	0	0	60	0
N.S.	1	0.91	0.00	0.00	0.00	0.77	0.00	0.00	0.16	0.00
time (sec)	N/A	0.534	0.000	0.000	0.000	0.095	0.000	0.000	0.172	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	167	128	0	0	172	0	0	35	0
N.S.	1	0.91	0.70	0.00	0.00	0.93	0.00	0.00	0.19	0.00
time (sec)	N/A	0.357	0.155	0.000	0.000	0.104	0.000	0.000	0.165	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	64	0	0	93	0	0	15	0
N.S.	1	1.01	0.94	0.00	0.00	1.37	0.00	0.00	0.22	0.00
time (sec)	N/A	0.446	0.036	0.000	0.000	0.095	0.000	0.000	0.161	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	23	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.44	1.12
time (sec)	N/A	0.202	0.040	0.145	0.053	0.087	2.639	0.104	0.163	3.626

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	29	15	18	34	18
N.S.	1	1.00	1.12	1.00	1.12	1.81	0.94	1.12	2.12	1.12
time (sec)	N/A	0.204	0.069	0.139	0.055	0.081	15.562	0.111	0.163	3.937

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	90	0	0	127	0	0	25	0
N.S.	1	1.00	0.88	0.00	0.00	1.25	0.00	0.00	0.25	0.00
time (sec)	N/A	0.489	0.211	0.000	0.000	0.098	0.000	0.000	0.162	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	110	81	0	0	123	0	0	23	0
N.S.	1	1.18	0.87	0.00	0.00	1.32	0.00	0.00	0.25	0.00
time (sec)	N/A	0.471	0.193	0.000	0.000	0.114	0.000	0.000	0.166	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	78	0	0	123	0	0	21	0
N.S.	1	1.00	0.86	0.00	0.00	1.35	0.00	0.00	0.23	0.00
time (sec)	N/A	0.437	0.164	0.000	0.000	0.093	0.000	0.000	0.164	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	63	83	61	58	117	0	0	25	112
N.S.	1	0.98	1.30	0.95	0.91	1.83	0.00	0.00	0.39	1.75
time (sec)	N/A	0.290	0.056	0.467	0.030	0.104	0.000	0.000	0.162	4.345

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	0	0	127	0	0	25	0
N.S.	1	1.00	0.87	0.00	0.00	1.35	0.00	0.00	0.27	0.00
time (sec)	N/A	0.476	0.176	0.000	0.000	0.114	0.000	0.000	0.155	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	112	80	0	0	124	0	0	25	0
N.S.	1	1.18	0.84	0.00	0.00	1.31	0.00	0.00	0.26	0.00
time (sec)	N/A	0.487	0.186	0.000	0.000	0.115	0.000	0.000	0.158	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	142	126	0	0	181	0	0	28	0
N.S.	1	1.13	1.00	0.00	0.00	1.44	0.00	0.00	0.22	0.00
time (sec)	N/A	0.559	0.307	0.000	0.000	0.105	0.000	0.000	0.158	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	16	19	0	18	16
N.S.	1	1.00	1.00	0.00	0.00	0.76	0.90	0.00	0.86	0.76
time (sec)	N/A	0.206	0.009	0.000	0.000	0.081	0.417	0.000	0.156	0.103

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	16	19	0	17	91
N.S.	1	1.00	1.00	0.00	0.00	0.76	0.90	0.00	0.81	4.33
time (sec)	N/A	0.200	0.005	0.000	0.000	0.092	0.175	0.000	0.158	4.293

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	15	24	0	16	15
N.S.	1	1.00	1.00	0.00	0.00	0.75	1.20	0.00	0.80	0.75
time (sec)	N/A	0.208	0.011	0.000	0.000	0.094	0.209	0.000	0.160	0.111

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	16	24	0	18	16
N.S.	1	1.00	1.00	0.00	0.00	0.76	1.14	0.00	0.86	0.76
time (sec)	N/A	0.207	0.006	0.000	0.000	0.088	0.446	0.000	0.158	0.099

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	16	26	0	17	16
N.S.	1	1.00	1.00	0.00	0.00	0.76	1.24	0.00	0.81	0.76
time (sec)	N/A	0.208	0.006	0.000	0.000	0.080	0.794	0.000	0.153	3.851

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	24	63	0	34	23
N.S.	1	1.00	1.00	0.00	0.00	0.86	2.25	0.00	1.21	0.82
time (sec)	N/A	0.216	0.010	0.000	0.000	0.100	1.719	0.000	0.157	3.964

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	290	131	0	0	255	0	0	24	232
N.S.	1	1.13	0.51	0.00	0.00	0.99	0.00	0.00	0.09	0.90
time (sec)	N/A	1.267	0.202	0.000	0.000	0.143	0.000	0.000	0.162	4.431

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	152	91	0	0	151	0	0	24	128
N.S.	1	1.07	0.64	0.00	0.00	1.06	0.00	0.00	0.17	0.90
time (sec)	N/A	0.609	0.123	0.000	0.000	0.112	0.000	0.000	0.162	0.528

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	0	0	61	0	0	22	51
N.S.	1	1.00	0.89	0.00	0.00	1.15	0.00	0.00	0.42	0.96
time (sec)	N/A	0.252	0.015	0.000	0.000	0.098	0.000	0.000	0.161	4.114

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	17	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.00	1.06	1.41	1.06
time (sec)	N/A	0.219	0.098	0.045	0.079	0.093	3.678	0.105	0.159	4.329

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.41	1.06
time (sec)	N/A	0.543	0.128	0.146	0.081	0.091	6.248	0.110	0.158	4.945

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.41	1.06
time (sec)	N/A	1.071	0.163	0.168	0.082	0.092	34.353	0.106	0.159	5.062

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	320	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	18.82	1.06
time (sec)	N/A	0.818	0.193	0.066	0.078	0.094	56.828	0.114	0.163	5.455

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	117	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	6.88	1.06
time (sec)	N/A	0.391	0.145	0.121	0.084	0.086	10.807	0.106	0.164	5.136

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	15	15	15	15	21	15
N.S.	1	1.00	1.14	0.93	1.07	1.07	1.07	1.07	1.50	1.07
time (sec)	N/A	0.193	0.018	0.060	0.076	0.101	2.331	0.107	0.158	4.572

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.41	1.06
time (sec)	N/A	0.395	0.142	0.062	0.078	0.095	3.073	0.106	0.155	4.380

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.41	1.06
time (sec)	N/A	0.827	0.209	0.157	0.084	0.107	14.346	0.104	0.157	4.865

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	107	121	68	103	0	79	0	0	112	82
N.S.	1	1.13	0.64	0.96	0.00	0.74	0.00	0.00	1.05	0.77
time (sec)	N/A	0.450	0.036	0.352	0.000	0.102	0.000	0.000	0.159	0.246

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	76	51	72	0	59	63	0	78	56
N.S.	1	1.07	0.72	1.01	0.00	0.83	0.89	0.00	1.10	0.79
time (sec)	N/A	0.343	0.020	0.148	0.000	0.095	22.368	0.000	0.161	0.184

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	0	40	27	0	43	27
N.S.	1	1.00	1.00	1.28	0.00	1.25	0.84	0.00	1.34	0.84
time (sec)	N/A	0.199	0.011	0.091	0.000	0.082	2.996	0.000	0.163	3.680

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	24	0	25	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	0.83	0.00
time (sec)	N/A	0.204	0.014	0.000	0.000	0.000	3.926	0.000	0.156	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	32	0	0	0	27	0	84	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.42	0.00	1.29	0.00
time (sec)	N/A	0.313	0.013	0.000	0.000	0.000	9.247	0.000	0.160	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	34	0	0	0	32	0	119	0
N.S.	1	1.02	0.32	0.00	0.00	0.00	0.30	0.00	1.13	0.00
time (sec)	N/A	0.459	0.015	0.000	0.000	0.000	63.445	0.000	0.157	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	164	52	0	0	0	0	0	147	0
N.S.	1	1.11	0.35	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.575	0.021	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	117	43	0	0	0	0	0	112	0
N.S.	1	1.07	0.39	0.00	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.446	0.018	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	36	0	0	0	22	0	77	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.31	0.00	1.10	0.00
time (sec)	N/A	0.295	0.014	0.000	0.000	0.000	27.188	0.000	0.172	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	0	20	0	22	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.74	0.00	0.81	0.00
time (sec)	N/A	0.191	0.009	0.000	0.000	0.000	3.923	0.000	0.157	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	26	0	0	0	41	0	25	0
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.68	0.00	0.42	0.00
time (sec)	N/A	0.303	0.025	0.000	0.000	0.000	3.807	0.000	0.163	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	102	29	0	0	0	24	0	25	0
N.S.	1	0.97	0.28	0.00	0.00	0.00	0.23	0.00	0.24	0.00
time (sec)	N/A	0.411	0.016	0.000	0.000	0.000	24.886	0.000	0.158	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	29	0	0	0	0	0	25	0
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.558	0.017	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	257	95	0	0	106	0	0	26	206
N.S.	1	1.78	0.66	0.00	0.00	0.74	0.00	0.00	0.18	1.43
time (sec)	N/A	1.086	0.048	0.000	0.000	0.118	0.000	0.000	0.165	3.821

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	137	77	0	0	86	0	0	26	117
N.S.	1	1.41	0.79	0.00	0.00	0.89	0.00	0.00	0.27	1.21
time (sec)	N/A	0.559	0.032	0.000	0.000	0.107	0.000	0.000	0.162	0.404

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	0	0	49	0	0	24	50
N.S.	1	1.00	0.89	0.00	0.00	1.04	0.00	0.00	0.51	1.06
time (sec)	N/A	0.244	0.010	0.000	0.000	0.097	0.000	0.000	0.159	3.599

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	19	20	26	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.00	1.05	1.37	1.05
time (sec)	N/A	0.219	0.072	0.045	0.086	0.086	2.614	0.103	0.155	3.672

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	26	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.37	1.05
time (sec)	N/A	0.515	0.109	0.145	0.086	0.113	4.579	0.104	0.158	4.294

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	26	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.37	1.05
time (sec)	N/A	0.985	0.148	0.152	0.080	0.082	20.106	0.111	0.158	4.432

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	154	78	0	0	74	124	0	102	126
N.S.	1	1.27	0.64	0.00	0.00	0.61	1.02	0.00	0.84	1.04
time (sec)	N/A	0.726	0.027	0.000	0.000	0.092	1.515	0.000	0.159	4.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	53	68	0	57	86
N.S.	1	1.00	0.84	0.00	0.00	0.77	0.99	0.00	0.83	1.25
time (sec)	N/A	0.371	0.014	0.000	0.000	0.103	0.517	0.000	0.155	0.299

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	16	19	0	17	91
N.S.	1	1.00	1.00	0.00	0.00	0.76	0.90	0.00	0.81	4.33
time (sec)	N/A	0.194	0.001	0.000	0.000	0.101	0.165	0.000	0.157	3.840

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	0	0	55	0	0	26	0
N.S.	1	1.00	0.95	0.00	0.00	0.93	0.00	0.00	0.44	0.00
time (sec)	N/A	0.369	0.017	0.000	0.000	0.094	0.000	0.000	0.158	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	135	91	0	0	85	0	0	26	0
N.S.	1	1.14	0.77	0.00	0.00	0.72	0.00	0.00	0.22	0.00
time (sec)	N/A	0.703	0.027	0.000	0.000	0.132	0.000	0.000	0.158	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	312	206	0	0	262	0	0	28	336
N.S.	1	1.03	0.68	0.00	0.00	0.86	0.00	0.00	0.09	1.11
time (sec)	N/A	1.624	1.445	0.000	0.000	0.118	0.000	0.000	0.161	4.816

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	0	100	0	0	26	79
N.S.	1	1.00	0.94	0.00	0.00	1.28	0.00	0.00	0.33	1.01
time (sec)	N/A	0.317	0.055	0.000	0.000	0.097	0.000	0.000	0.165	4.094

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	19	20	28	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.00	1.05	1.47	1.05
time (sec)	N/A	0.225	0.126	0.052	0.079	0.100	6.769	0.107	0.156	4.475

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	28	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.47	1.05
time (sec)	N/A	1.280	0.191	0.176	0.078	0.106	27.398	0.105	0.161	4.482

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	0	20	28	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	0.00	1.05	1.47	1.05
time (sec)	N/A	3.558	0.298	0.071	0.078	0.091	0.000	0.107	0.163	5.466

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	28	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.47	1.05
time (sec)	N/A	0.660	0.235	0.148	0.081	0.084	46.701	0.110	0.169	5.454

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	17	17	17	17	25	17
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.06	1.06	1.56	1.06
time (sec)	N/A	0.208	0.025	0.065	0.074	0.088	4.033	0.105	0.163	5.148

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	28	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.47	1.05
time (sec)	N/A	0.509	0.218	0.079	0.081	0.092	7.336	0.108	0.158	4.500

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	20	20	20	28	20
N.S.	1	1.00	1.11	0.95	1.05	1.05	1.05	1.05	1.47	1.05
time (sec)	N/A	2.518	0.292	0.226	0.080	0.113	62.329	0.108	0.158	4.507

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	41	0	39	53	0	43	28
N.S.	1	1.00	1.00	1.24	0.00	1.18	1.61	0.00	1.30	0.85
time (sec)	N/A	0.257	0.045	0.296	0.000	0.109	12.348	0.000	0.173	0.180

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	0	0	0	0	0	0	22	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.380	0.000	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	0	0	0	0	0	0	23	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.379	0.000	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	22	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.367	0.000	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.368	0.000	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	0	0	0	0	0	21	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.360	0.671	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	0	0	22	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.361	0.573	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	0	0	0	0	0	21	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.364	0.914	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	0	0	23	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.355	0.812	0.000	0.000	0.000	0.000	0.000	0.162	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [22] had the largest ratio of [1.19999999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.18	8	0.625
2	A	4	4	1.20	8	0.500
3	A	3	3	1.20	6	0.500
4	A	1	1	1.00	8	0.125
5	A	3	3	1.12	8	0.375
6	A	4	4	1.06	8	0.500
7	A	5	5	1.04	8	0.625
8	A	5	5	1.03	8	0.625
9	A	4	4	1.02	8	0.500
10	A	3	3	1.02	8	0.375
11	A	1	1	1.00	4	0.250
12	A	2	2	1.00	8	0.250
13	A	3	3	1.00	8	0.375
14	A	4	4	1.00	8	0.500
15	A	3	3	1.01	14	0.214
16	A	3	3	1.06	14	0.214
17	A	3	3	1.13	12	0.250
18	A	1	1	1.00	6	0.167
19	N/A	1	0	1.00	14	0.000
20	N/A	2	0	1.00	14	0.000
21	N/A	4	0	1.00	14	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	13	12	1.52	10	1.200
23	A	9	8	1.30	10	0.800
24	A	6	5	1.20	8	0.625
25	N/A	1	0	1.00	10	0.000
26	A	6	5	1.10	10	0.500
27	A	9	8	1.15	10	0.800
28	A	13	12	1.36	10	1.200
29	A	10	10	1.58	10	1.000
30	A	6	6	1.32	10	0.600
31	A	4	4	1.07	6	0.667
32	N/A	1	0	1.00	10	0.000
33	N/A	1	0	1.00	10	0.000
34	N/A	1	0	1.00	10	0.000
35	A	3	2	0.91	16	0.125
36	A	3	2	0.91	14	0.143
37	A	5	4	1.00	8	0.500
38	N/A	1	0	1.00	16	0.000
39	N/A	1	0	1.00	16	0.000
40	A	6	5	1.00	17	0.294
41	A	6	5	1.18	15	0.333
42	A	6	5	1.00	13	0.385
43	A	4	3	0.98	17	0.176
44	A	6	5	1.00	17	0.294
45	A	6	5	1.16	17	0.294
46	A	6	5	1.13	19	0.263
47	A	3	2	1.00	19	0.105
48	A	3	2	1.00	17	0.118
49	A	3	2	1.00	19	0.105
50	A	3	2	1.00	19	0.105
51	A	3	2	1.00	19	0.105
52	A	3	2	1.00	19	0.105
53	A	9	9	1.12	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	5	1.06	17	0.294
55	A	2	2	1.00	15	0.133
56	N/A	1	0	1.00	17	0.000
57	N/A	4	0	1.00	17	0.000
58	N/A	8	0	1.00	17	0.000
59	N/A	6	0	1.00	17	0.000
60	N/A	3	0	1.00	17	0.000
61	N/A	1	0	1.00	14	0.000
62	N/A	3	0	1.00	17	0.000
63	N/A	6	0	1.00	17	0.000
64	A	6	6	1.14	19	0.316
65	A	4	4	1.09	19	0.211
66	A	2	2	1.00	17	0.118
67	A	1	1	1.00	19	0.053
68	A	3	3	1.00	19	0.158
69	A	5	5	1.02	19	0.263
70	A	5	5	1.07	19	0.263
71	A	3	3	1.00	19	0.158
72	A	1	1	1.00	16	0.062
73	A	3	3	1.00	19	0.158
74	A	5	5	0.97	19	0.263
75	A	9	9	1.75	18	0.500
76	A	5	5	1.39	18	0.278
77	A	2	2	1.00	16	0.125
78	N/A	1	0	1.00	18	0.000
79	N/A	4	0	1.00	18	0.000
80	N/A	8	0	1.00	18	0.000
81	A	8	7	1.27	18	0.389
82	A	5	4	1.00	18	0.222
83	A	3	2	1.00	15	0.133
84	A	5	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	8	7	1.14	18	0.389
86	A	10	10	1.11	19	0.526
87	A	3	3	1.10	17	0.176
88	N/A	1	0	1.00	19	0.000
89	N/A	6	0	1.00	19	0.000
90	N/A	16	0	1.00	19	0.000
91	N/A	5	0	1.00	19	0.000
92	N/A	1	0	1.00	16	0.000
93	N/A	3	0	1.00	19	0.000
94	N/A	9	0	1.00	19	0.000
95	A	1	1	1.00	40	0.025
96	A	5	4	1.00	18	0.222
97	A	5	4	1.00	18	0.222
98	A	5	4	1.00	18	0.222
99	A	5	4	1.00	18	0.222
100	A	5	4	1.00	15	0.267
101	A	5	4	1.00	16	0.250
102	A	5	4	1.00	15	0.267
103	A	5	4	1.00	16	0.250
104	A	5	5	1.18	8	0.625
105	A	4	4	1.20	8	0.500
106	A	3	3	1.17	6	0.500
107	A	2	2	1.00	8	0.250
108	A	3	3	1.20	8	0.375
109	A	4	4	1.06	8	0.500
110	A	5	5	1.04	8	0.625
111	A	5	5	1.03	8	0.625
112	A	4	4	1.02	8	0.500
113	A	3	3	1.02	8	0.375
114	A	1	1	1.00	4	0.250
115	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	3	3	1.00	8	0.375
117	A	4	4	1.00	8	0.500
118	A	3	3	1.00	14	0.214
119	A	3	3	1.05	14	0.214
120	A	3	3	1.11	12	0.250
121	A	1	1	1.00	6	0.167
122	N/A	1	0	1.00	14	0.000
123	N/A	2	0	1.00	14	0.000
124	N/A	4	0	1.00	14	0.000
125	A	13	12	1.53	10	1.200
126	A	9	8	1.30	10	0.800
127	A	6	5	1.18	8	0.625
128	N/A	1	0	1.00	10	0.000
129	A	6	5	1.12	10	0.500
130	A	9	8	1.17	10	0.800
131	A	13	12	1.37	10	1.200
132	A	10	10	1.59	10	1.000
133	A	6	6	1.33	10	0.600
134	A	4	4	1.07	6	0.667
135	N/A	1	0	1.00	10	0.000
136	N/A	1	0	1.00	10	0.000
137	N/A	1	0	1.00	10	0.000
138	A	3	2	0.91	16	0.125
139	A	3	2	0.91	14	0.143
140	A	5	4	1.00	8	0.500
141	N/A	1	0	1.00	16	0.000
142	N/A	1	0	1.00	16	0.000
143	A	6	5	1.00	17	0.294
144	A	6	5	1.18	15	0.333
145	A	6	5	1.00	13	0.385
146	A	4	3	0.98	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	6	5	1.00	17	0.294
148	A	6	5	1.16	17	0.294
149	A	6	5	1.13	19	0.263
150	A	3	2	1.00	19	0.105
151	A	3	2	1.00	17	0.118
152	A	3	2	1.00	19	0.105
153	A	3	2	1.00	19	0.105
154	A	3	2	1.00	19	0.105
155	A	3	2	1.00	19	0.105
156	A	9	9	1.12	17	0.529
157	A	5	5	1.06	17	0.294
158	A	2	2	1.00	15	0.133
159	N/A	1	0	1.00	17	0.000
160	N/A	4	0	1.00	17	0.000
161	N/A	8	0	1.00	17	0.000
162	N/A	6	0	1.00	17	0.000
163	N/A	3	0	1.00	17	0.000
164	N/A	1	0	1.00	14	0.000
165	N/A	3	0	1.00	17	0.000
166	N/A	6	0	1.00	17	0.000
167	A	6	6	1.14	19	0.316
168	A	4	4	1.06	19	0.211
169	A	2	2	1.00	17	0.118
170	A	3	3	1.00	19	0.158
171	A	5	5	1.00	19	0.263
172	A	7	7	1.00	19	0.368
173	A	7	7	1.09	19	0.368
174	A	5	5	1.04	19	0.263
175	A	3	3	1.00	16	0.188
176	A	5	5	1.11	19	0.263
177	A	7	7	1.01	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
178	A	9	9	1.76	18	0.500
179	A	5	5	1.40	18	0.278
180	A	2	2	1.00	16	0.125
181	N/A	1	0	1.00	18	0.000
182	N/A	4	0	1.00	18	0.000
183	N/A	8	0	1.00	18	0.000
184	A	8	7	1.28	18	0.389
185	A	5	4	1.00	18	0.222
186	A	3	2	1.00	15	0.133
187	A	5	4	1.00	18	0.222
188	A	8	7	1.15	18	0.389
189	A	10	10	1.11	19	0.526
190	A	3	3	1.10	17	0.176
191	N/A	1	0	1.00	19	0.000
192	N/A	6	0	1.00	19	0.000
193	N/A	16	0	1.00	19	0.000
194	N/A	5	0	1.00	19	0.000
195	N/A	1	0	1.00	16	0.000
196	N/A	3	0	1.00	19	0.000
197	N/A	9	0	1.00	19	0.000
198	A	1	1	1.00	40	0.025
199	A	7	6	1.01	18	0.333
200	A	7	6	1.01	18	0.333
201	A	7	6	1.04	18	0.333
202	A	7	6	1.04	18	0.333
203	A	7	6	1.04	15	0.400
204	A	7	6	1.03	16	0.375
205	A	7	6	1.04	15	0.400
206	A	7	6	1.04	16	0.375
207	A	5	5	1.18	8	0.625
208	A	4	4	1.20	8	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
209	A	3	3	1.20	6	0.500
210	A	1	1	1.00	8	0.125
211	A	3	3	1.12	8	0.375
212	A	4	4	1.04	8	0.500
213	A	5	5	1.03	8	0.625
214	A	5	5	1.03	8	0.625
215	A	4	4	1.02	8	0.500
216	A	3	3	1.02	8	0.375
217	A	1	1	1.00	4	0.250
218	A	2	2	1.00	8	0.250
219	A	3	3	1.00	8	0.375
220	A	4	4	1.00	8	0.500
221	A	3	3	1.00	14	0.214
222	A	3	3	1.05	14	0.214
223	A	3	3	1.11	12	0.250
224	A	1	1	1.00	6	0.167
225	N/A	1	0	1.00	14	0.000
226	N/A	2	0	1.00	14	0.000
227	N/A	4	0	1.00	14	0.000
228	A	13	12	1.54	10	1.200
229	A	9	8	1.31	10	0.800
230	A	6	5	1.18	8	0.625
231	N/A	1	0	1.00	10	0.000
232	A	6	5	1.12	10	0.500
233	A	9	8	1.15	10	0.800
234	A	13	12	1.35	10	1.200
235	A	10	10	1.60	10	1.000
236	A	6	6	1.34	10	0.600
237	A	4	4	1.09	6	0.667
238	N/A	1	0	1.00	10	0.000
239	N/A	1	0	1.00	10	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	N/A	1	0	1.00	10	0.000
241	A	3	2	0.91	16	0.125
242	A	3	2	0.91	14	0.143
243	A	5	4	1.01	8	0.500
244	N/A	1	0	1.00	16	0.000
245	N/A	1	0	1.00	16	0.000
246	A	6	5	1.00	17	0.294
247	A	6	5	1.18	15	0.333
248	A	6	5	1.00	13	0.385
249	A	4	3	0.98	17	0.176
250	A	6	5	1.00	17	0.294
251	A	6	5	1.18	17	0.294
252	A	6	5	1.13	19	0.263
253	A	3	2	1.00	18	0.111
254	A	3	2	1.00	16	0.125
255	A	3	2	1.00	18	0.111
256	A	3	2	1.00	18	0.111
257	A	3	2	1.00	18	0.111
258	A	3	2	1.00	18	0.111
259	A	9	9	1.13	17	0.529
260	A	5	5	1.07	17	0.294
261	A	2	2	1.00	15	0.133
262	N/A	1	0	1.00	17	0.000
263	N/A	4	0	1.00	17	0.000
264	N/A	8	0	1.00	17	0.000
265	N/A	6	0	1.00	17	0.000
266	N/A	3	0	1.00	17	0.000
267	N/A	1	0	1.00	14	0.000
268	N/A	3	0	1.00	17	0.000
269	N/A	6	0	1.00	17	0.000
270	A	6	6	1.13	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	4	1.07	18	0.222
272	A	2	2	1.00	16	0.125
273	A	1	1	1.00	18	0.056
274	A	3	3	1.00	18	0.167
275	A	5	5	1.02	18	0.278
276	A	7	7	1.11	18	0.389
277	A	5	5	1.07	18	0.278
278	A	3	3	1.00	18	0.167
279	A	1	1	1.00	15	0.067
280	A	3	3	1.00	18	0.167
281	A	5	5	0.97	18	0.278
282	A	7	7	1.00	18	0.389
283	A	9	9	1.78	19	0.474
284	A	5	5	1.41	19	0.263
285	A	2	2	1.00	17	0.118
286	N/A	1	0	1.00	19	0.000
287	N/A	4	0	1.00	19	0.000
288	N/A	8	0	1.00	19	0.000
289	A	8	7	1.27	19	0.368
290	A	5	4	1.00	19	0.211
291	A	3	2	1.00	16	0.125
292	A	5	4	1.00	19	0.211
293	A	8	7	1.14	19	0.368
294	A	10	10	1.03	19	0.526
295	A	3	3	1.00	17	0.176
296	N/A	1	0	1.00	19	0.000
297	N/A	6	0	1.00	19	0.000
298	N/A	16	0	1.00	19	0.000
299	N/A	5	0	1.00	19	0.000
300	N/A	1	0	1.00	16	0.000
301	N/A	3	0	1.00	19	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	N/A	9	0	1.00	19	0.000
303	A	1	1	1.00	40	0.025
304	A	5	4	1.00	18	0.222
305	A	5	4	1.00	18	0.222
306	A	5	4	1.00	18	0.222
307	A	5	4	1.00	18	0.222
308	A	5	4	1.00	15	0.267
309	A	5	4	1.00	16	0.250
310	A	5	4	1.00	15	0.267
311	A	5	4	1.00	16	0.250

CHAPTER 3

Listing of integrals

3.1 $\int x^5 \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int x^5\text{erf}(bx)\,dx=\frac{5e^{-b^2x^2}x}{8b^5\sqrt{\pi}}+\frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}}+\frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}}-\frac{5\text{erf}(bx)}{16b^6}+\frac{1}{6}x^6\text{erf}(bx)$$

output
```
5/8*x/b^5/exp(b^2*x^2)/Pi^(1/2)+5/12*x^3/b^3/exp(b^2*x^2)/Pi^(1/2)+1/6*x^5
/b/exp(b^2*x^2)/Pi^(1/2)-5/16*erf(b*x)/b^6+1/6*x^6*erf(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

$$\int x^5\text{erf}(bx)\,dx=\frac{e^{-b^2x^2}\left(30bx+20b^3x^3+8b^5x^5+e^{b^2x^2}\sqrt{\pi}(-15+8b^6x^6)\,\text{erf}(bx)\right)}{48b^6\sqrt{\pi}}$$

input
```
Integrate[x^5*Erf[b*x],x]
```

output
```
(30*b*x + 20*b^3*x^3 + 8*b^5*x^5 + E^(b^2*x^2)*Sqrt[Pi]*(-15 + 8*b^6*x^6)*
Erf[b*x])/(48*b^6*E^(b^2*x^2)*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6915, 2641, 2641, 2641, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{1}{6}x^6\text{erf}(bx)-\frac{b\int e^{-b^2x^2}x^6dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erf}(bx)-\frac{b\left(\frac{5\int e^{-b^2x^2}x^4dx}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erf}(bx)-\frac{b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2dx}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erf}(bx)-\frac{b\left(\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}dx}{2b^2}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{6}x^6\text{erf}(bx) - \frac{b\left(\frac{5\left(\frac{3\left(\frac{\sqrt{\pi}\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}$$

input
```
Int[x^5*Erf[b*x],x]
```

output
```
(x^6*Erf[b*x])/6 - (b*(-1/2*x^5/(b^2*E^(b^2*x^2)) + (5*(-1/2*x^3/(b^2*E^(b
^2*x^2)) + (3*(-1/2*x/(b^2*E^(b^2*x^2)) + (Sqrt[Pi]*Erf[b*x])/(4*b^3)))/(2
*b^2)))/(2*b^2)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result	size
meijerg	$\frac{\frac{xb\left(28b^4x^4+70b^2x^2+105\right)e^{-b^2x^2}}{84}-\frac{\left(-56b^6x^6+105\right)\operatorname{erf}(bx)\sqrt{\pi}}{168}}{2b^6\sqrt{\pi}}$	60
parallelrisch	$\frac{8\operatorname{erf}(bx)x^6b^6\sqrt{\pi}+8\,e^{-b^2x^2}x^5b^5+20\,e^{-b^2x^2}x^3b^3+30\,e^{-b^2x^2}bx-15\operatorname{erf}(bx)\sqrt{\pi}}{48b^6\sqrt{\pi}}$	81
derivativedivides	$\frac{\frac{\operatorname{erf}(bx)b^6x^6}{6}-\frac{-\frac{e^{-b^2x^2}x^5b^5}{2}-\frac{5\,e^{-b^2x^2}x^3b^3}{4}-\frac{15\,e^{-b^2x^2}bx}{8}+\frac{15\operatorname{erf}(bx)\sqrt{\pi}}{16}}{3\sqrt{\pi}}}{b^6}$	83
default	$\frac{\frac{\operatorname{erf}(bx)b^6x^6}{6}-\frac{-\frac{e^{-b^2x^2}x^5b^5}{2}-\frac{5\,e^{-b^2x^2}x^3b^3}{4}-\frac{15\,e^{-b^2x^2}bx}{8}+\frac{15\operatorname{erf}(bx)\sqrt{\pi}}{16}}{3\sqrt{\pi}}}{b^6}$	83
parts	$\frac{x^6\operatorname{erf}(bx)}{6}-\frac{b\left(-\frac{x^5e^{-b^2x^2}}{2b^2}+\frac{-\frac{5x^3e^{-b^2x^2}}{4b^2}+\frac{5\left(-\frac{3x\,e^{-b^2x^2}}{4b^2}+\frac{3\sqrt{\pi}\operatorname{erf}(bx)}{8b^3}\right)}{2b^2}}{b^2}\right)}{3\sqrt{\pi}}$	91
orering	$\frac{\left(4b^6x^6-10b^4x^4-25b^2x^2-45\right)\operatorname{erf}(bx)}{24b^6}+\frac{\left(4b^4x^4+10b^2x^2+15\right)\left(5x^4\operatorname{erf}(bx)+\frac{2x^5e^{-b^2x^2}b}{\sqrt{\pi}}\right)}{48x^4b^6}$	91

input
```
int(x^5*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^6/Pi^(1/2)*(1/84*x*b*(28*b^4*x^4+70*b^2*x^2+105)*exp(-b^2*x^2)-1/168
*(-56*b^6*x^6+105)*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int x^5\operatorname{erf}(bx)\,dx=\frac{2\sqrt{\pi}(4\,b^5x^5+10\,b^3x^3+15\,bx)e^{(-b^2x^2)}-(15\,\pi-8\,\pi b^6x^6)\operatorname{erf}(bx)}{48\,\pi b^6}$$

input
```
integrate(x^5*erf(b*x),x, algorithm="fricas")
```

output
```
1/48*(2*sqrt(pi)*(4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*e^(-b^2*x^2) - (15*pi -
 8*pi*b^6*x^6)*erf(b*x))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int x^5\mathrm{erf}(bx)\,dx = \begin{cases} \frac{x^6\,\mathrm{erf}\,(bx)}{6} + \frac{x^5e^{-b^2x^2}}{6\sqrt{\pi}b} + \frac{5x^3e^{-b^2x^2}}{12\sqrt{\pi}b^3} + \frac{5xe^{-b^2x^2}}{8\sqrt{\pi}b^5} - \frac{5\,\mathrm{erf}\,(bx)}{16b^6} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*erf(b*x),x)
```

output
```
Piecewise((x**6*erf(b*x)/6 + x**5*exp(-b**2*x**2)/(6*sqrt(pi)*b) + 5*x**3*
exp(-b**2*x**2)/(12*sqrt(pi)*b**3) + 5*x*exp(-b**2*x**2)/(8*sqrt(pi)*b**5)
 - 5*erf(b*x)/(16*b**6), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int x^5\mathrm{erf}(bx)\,dx = \frac{1}{6}\,x^6\,\mathrm{erf}\,(bx) + \frac{b\left(\frac{2\,(4\,b^4x^5+10\,b^2x^3+15\,x)e^{\left(-b^2x^2\right)}}{b^6} - \frac{15\,\sqrt{\pi}\,\mathrm{erf}(bx)}{b^7}\right)}{48\,\sqrt{\pi}}$$

input
```
integrate(x^5*erf(b*x),x, algorithm="maxima")
```

output
```
1/6*x^6*erf(b*x) + 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-b^2*x^2)/
b^6 - 15*sqrt(pi)*erf(b*x)/b^7)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int x^5\mathrm{erf}(bx)\,dx = \frac{1}{6}\,x^6\,\mathrm{erf}\,(bx) + \frac{b\left(\frac{2\,(4\,b^4x^5+10\,b^2x^3+15\,x)e^{\left(-b^2x^2\right)}}{b^6} + \frac{15\,\sqrt{\pi}\,\mathrm{erf}(-bx)}{b^7}\right)}{48\,\sqrt{\pi}}$$

input
```
integrate(x^5*erf(b*x),x, algorithm="giac")
```

output
```
1/6*x^6*erf(b*x) + 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-b^2*x^2)/
b^6 + 15*sqrt(pi)*erf(-b*x)/b^7)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int x^5 \mathrm{erf}(bx)\,dx = \frac{x^6\,\mathrm{erf}(b\,x)}{6} - \frac{5\,b\,x^7}{16\,\left(b^2\,x^2\right)^{7/2}} + \frac{x^5\,\mathrm{e}^{-b^2\,x^2}}{6\,b\,\sqrt{\pi}} + \frac{5\,x^3\,\mathrm{e}^{-b^2\,x^2}}{12\,b^3\,\sqrt{\pi}} + \frac{5\,x\,\mathrm{e}^{-b^2\,x^2}}{8\,b^5\,\sqrt{\pi}} + \frac{5\,b\,x^7\,\mathrm{erfc}\left(\sqrt{b^2\,x^2}\right)}{16\,\left(b^2\,x^2\right)^{7/2}}$$

input
```
int(x^5*erf(b*x),x)
```

output
```
(x^6*erf(b*x))/6 - (5*b*x^7)/(16*(b^2*x^2)^(7/2)) + (x^5*exp(-b^2*x^2))/(6
*b*pi^(1/2)) + (5*x^3*exp(-b^2*x^2))/(12*b^3*pi^(1/2)) + (5*x*exp(-b^2*x^2
))/(8*b^5*pi^(1/2)) + (5*b*x^7*erfc((b^2*x^2)^(1/2)))/(16*(b^2*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int x^5 \mathrm{erf}(bx)\,dx = \frac{8e^{b^2x^2}\mathrm{erf}(bx)\,b^6\pi\,x^6 - 15e^{b^2x^2}\mathrm{erf}(bx)\,\pi + 8\sqrt{\pi}\,b^5x^5 + 20\sqrt{\pi}\,b^3x^3 + 30\sqrt{\pi}\,bx}{48e^{b^2x^2}b^6\pi}$$

input
```
int(x^5*erf(b*x),x)
```

output
```
(8*e**(b**2*x**2)*erf(b*x)*b**6*pi*x**6 - 15*e**(b**2*x**2)*erf(b*x)*pi +
8*sqrt(pi)*b**5*x**5 + 20*sqrt(pi)*b**3*x**3 + 30*sqrt(pi)*b*x)/(48*e**(b*
*2*x**2)*b**6*pi)
```

3.2 $\int x^3 \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int x^3\text{erf}(bx)\,dx=\frac{3e^{-b^2x^2}x}{8b^3\sqrt{\pi}}+\frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}}-\frac{3\text{erf}(bx)}{16b^4}+\frac{1}{4}x^4\text{erf}(bx)$$

output
```
3/8*x/b^3/exp(b^2*x^2)/Pi^(1/2)+1/4*x^3/b/exp(b^2*x^2)/Pi^(1/2)-3/16*erf(b
*x)/b^4+1/4*x^4*erf(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x^3\text{erf}(bx)\,dx=e^{-b^2x^2}\left(\frac{3x}{8b^3\sqrt{\pi}}+\frac{x^3}{4b\sqrt{\pi}}\right)-\frac{3\text{erf}(bx)}{16b^4}+\frac{1}{4}x^4\text{erf}(bx)$$

input
```
Integrate[x^3*Erf[b*x],x]
```

output
```
((3*x)/(8*b^3*Sqrt[Pi]) + x^3/(4*b*Sqrt[Pi]))/E^(b^2*x^2) - (3*Erf[b*x])/(
16*b^4) + (x^4*Erf[b*x])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6915, 2641, 2641, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{erf}(bx)\,dx$$

$$\Big\downarrow \ 6915$$

$$\frac{1}{4}x^4\text{erf}(bx)-\frac{b\int e^{-b^2x^2}x^4dx}{2\sqrt{\pi}}$$

$$\Big\downarrow \ 2641$$

$$\frac{1}{4}x^4\text{erf}(bx)-\frac{b\left(\frac{3\int e^{-b^2x^2}x^2dx}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \ 2641$$

$$\frac{1}{4}x^4\text{erf}(bx)-\frac{b\left(\frac{3\left(\frac{\int e^{-b^2x^2}dx}{2b^2}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \ 2634$$

$$\frac{1}{4}x^4\text{erf}(bx)-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2\sqrt{\pi}}$$

input
```
Int[x^3*Erf[b*x],x]
```

output
```
(x^4*Erf[b*x])/4 - (b*(-1/2*x^3/(b^2*E^(b^2*x^2)) + (3*(-1/2*x/(b^2*E^(b^2
*x^2)) + (Sqrt[Pi]*Erf[b*x])/(4*b^3)))/(2*b^2)))/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
meijerg	$\frac{\frac{x b \left(10 b^{2} x^{2}+15\right) {\mathrm e}^{-b^{2} x^{2}}}{20}-\frac{\left(-20 b^{4} x^{4}+15\right) \operatorname{erf}\left(b x \right) \sqrt{\pi}}{40}}{2 b^{4} \sqrt{\pi}}$	52
parallelrisch	$\frac{4 \operatorname{erf}\left(b x \right) x^{4} \sqrt{\pi}\, b^{4}+4 \,{\mathrm e}^{-b^{2} x^{2}} x^{3} b^{3}+6 \,{\mathrm e}^{-b^{2} x^{2}} b x -3 \operatorname{erf}\left(b x \right) \sqrt{\pi}}{16 \sqrt{\pi}\, b^{4}}$	64
derivativedivides	$\frac{\frac{\operatorname{erf}\left(b x \right) b^{4} x^{4}}{4}-\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{3} b^{3}}{2}-\frac{3 \,{\mathrm e}^{-b^{2} x^{2}} b x}{4}+\frac{3 \operatorname{erf}\left(b x \right) \sqrt{\pi}}{8}}{2 \sqrt{\pi}}}{b^{4}}$	65
default	$\frac{\frac{\operatorname{erf}\left(b x \right) b^{4} x^{4}}{4}-\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{3} b^{3}}{2}-\frac{3 \,{\mathrm e}^{-b^{2} x^{2}} b x}{4}+\frac{3 \operatorname{erf}\left(b x \right) \sqrt{\pi}}{8}}{2 \sqrt{\pi}}}{b^{4}}$	65
parts	$\frac{x^{4} \operatorname{erf}\left(b x \right)}{4}-\frac{b \left(-\frac{x^{3} {\mathrm e}^{-b^{2} x^{2}}}{2 b^{2}}+\frac{-\frac{3 x \,{\mathrm e}^{-b^{2} x^{2}}}{4 b^{2}}+\frac{3 \sqrt{\pi}\, \operatorname{erf}\left(b x \right)}{8 b^{3}}}{b^{2}}\right)}{2 \sqrt{\pi}}$	68
orering	$\frac{\left(2 b^{4} x^{4}-3 b^{2} x^{2}-6\right) \operatorname{erf}\left(b x \right)}{8 b^{4}}+\frac{\left(2 b^{2} x^{2}+3\right) \left(3 x^{2} \operatorname{erf}\left(b x \right)+\frac{2 x^{3} {\mathrm e}^{-b^{2} x^{2}} b}{\sqrt{\pi}}\right)}{16 x^{2} b^{4}}$	75

input
```
int(x^3*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4/Pi^(1/2)*(1/20*x*b*(10*b^2*x^2+15)*exp(-b^2*x^2)-1/40*(-20*b^4*x^4
+15)*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int x^3\mathrm{erf}(bx)\,dx = \frac{2\sqrt{\pi}(2\,b^3x^3+3\,bx)e^{(-b^2x^2)}-(3\,\pi-4\,\pi b^4x^4)\,\mathrm{erf}\,(bx)}{16\,\pi b^4}$$

input
```
integrate(x^3*erf(b*x),x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(pi)*(2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b^4*x^4)*
erf(b*x))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x^3\mathrm{erf}(bx)\,dx = \begin{cases} \frac{x^4\,\mathrm{erf}\,(bx)}{4}+\frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b}+\frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3}-\frac{3\,\mathrm{erf}\,(bx)}{16b^4} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erf(b*x),x)
```

output
```
Piecewise((x**4*erf(b*x)/4 + x**3*exp(-b**2*x**2)/(4*sqrt(pi)*b) + 3*x*exp
(-b**2*x**2)/(8*sqrt(pi)*b**3) - 3*erf(b*x)/(16*b**4), Ne(b, 0)), (0, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int x^3\text{erf}(bx)\,dx = \frac{1}{4}\,x^4\,\text{erf}\,(bx) + \frac{b\left(\frac{2\,(2\,b^2x^3+3\,x)e^{\left(-b^2x^2\right)}}{b^4} - \frac{3\,\sqrt{\pi}\,\text{erf}(bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

input
```
integrate(x^3*erf(b*x),x, algorithm="maxima")
```

output
```
1/4*x^4*erf(b*x) + 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4 - 3*sqrt(p
i)*erf(b*x)/b^5)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int x^3\text{erf}(bx)\,dx = \frac{1}{4}\,x^4\,\text{erf}\,(bx) + \frac{b\left(\frac{2\,(2\,b^2x^3+3\,x)e^{\left(-b^2x^2\right)}}{b^4} + \frac{3\,\sqrt{\pi}\,\text{erf}(-bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

input
```
integrate(x^3*erf(b*x),x, algorithm="giac")
```

output
```
1/4*x^4*erf(b*x) + 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4 + 3*sqrt(p
i)*erf(-b*x)/b^5)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int x^3\text{erf}(bx)\,dx = \frac{x^4\,\text{erf}(b\,x)}{4} - \frac{3\,b\,x^5}{16\,(b^2\,x^2)^{5/2}} + \frac{x^3\,\mathrm{e}^{-b^2\,x^2}}{4\,b\,\sqrt{\pi}} + \frac{3\,x\,\mathrm{e}^{-b^2\,x^2}}{8\,b^3\,\sqrt{\pi}} + \frac{3\,b\,x^5\,\text{erfc}\left(\sqrt{b^2\,x^2}\right)}{16\,(b^2\,x^2)^{5/2}}$$

input
```
int(x^3*erf(b*x),x)
```

output
```
(x^4*erf(b*x))/4 - (3*b*x^5)/(16*(b^2*x^2)^(5/2)) + (x^3*exp(-b^2*x^2))/(4
*b*pi^(1/2)) + (3*x*exp(-b^2*x^2))/(8*b^3*pi^(1/2)) + (3*b*x^5*erfc((b^2*x
^2)^(1/2)))/(16*(b^2*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int x^3\text{erf}(bx)\,dx = \frac{4e^{b^2x^2}\text{erf}(bx)\,b^4\pi\,x^4 - 3e^{b^2x^2}\text{erf}(bx)\,\pi + 4\sqrt{\pi}\,b^3x^3 + 6\sqrt{\pi}\,bx}{16e^{b^2x^2}b^4\pi}$$

input
```
int(x^3*erf(b*x),x)
```

output
```
(4*e**(b**2*x**2)*erf(b*x)*b**4*pi*x**4 - 3*e**(b**2*x**2)*erf(b*x)*pi + 4
*sqrt(pi)*b**3*x**3 + 6*sqrt(pi)*b*x)/(16*e**(b**2*x**2)*b**4*pi)
```

3.3 $\int x\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 46

$$\int x\operatorname{erf}(bx)\,dx = \frac{e^{-b^2x^2}x}{2b\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{4b^2} + \frac{1}{2}x^2\operatorname{erf}(bx)$$

output
```
1/2*x/b/exp(b^2*x^2)/Pi^(1/2)-1/4*erf(b*x)/b^2+1/2*x^2*erf(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x\operatorname{erf}(bx)\,dx = \frac{1}{4}\left(\frac{2e^{-b^2x^2}x}{b\sqrt{\pi}} + \left(-\frac{1}{b^2} + 2x^2\right)\operatorname{erf}(bx)\right)$$

input
```
Integrate[x*Erf[b*x],x]
```

output
```
((2*x)/(b*E^(b^2*x^2)*Sqrt[Pi]) + (-b^(-2) + 2*x^2)*Erf[b*x])/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6915, 2641, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{1}{2}x^2\text{erf}(bx)-\frac{b\int e^{-b^2x^2}x^2dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{2}x^2\text{erf}(bx)-\frac{b\left(\frac{\int e^{-b^2x^2}dx}{2b^2}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}x^2\text{erf}(bx)-\frac{b\left(\frac{\sqrt{\pi}\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{\sqrt{\pi}}$$

input
```
Int[x*Erf[b*x],x]
```

output
```
(x^2*Erf[b*x])/2 - (b*(-1/2*x/(b^2*E^(b^2*x^2)) + (Sqrt[Pi]*Erf[b*x])/(4*b
^3)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{\mathrm{e}^{-b^{2} x^{2}} b x-\frac{\left(-6 b^{2} x^{2}+3\right) \operatorname{erf}\left(b x\right) \sqrt{\pi}}{6}}{2 b^{2} \sqrt{\pi}}$	41
parts	$\frac{x^{2} \operatorname{erf}\left(b x\right)}{2}-\frac{b \left(-\frac{x\,\mathrm{e}^{-b^{2} x^{2}}}{2 b^{2}}+\frac{\sqrt{\pi}\, \operatorname{erf}\left(b x\right)}{4 b^{3}}\right)}{\sqrt{\pi}}$	45
orering	$\frac{\left(b^{2} x^{2}-1\right) \operatorname{erf}\left(b x\right)}{2 b^{2}}+\frac{\operatorname{erf}\left(b x\right)+\frac{2\,\mathrm{e}^{-b^{2} x^{2}} b x}{\sqrt{\pi}}}{4 b^{2}}$	46
derivativedivides	$\frac{\frac{\operatorname{erf}\left(b x\right) b^{2} x^{2}}{2}-\frac{-\frac{\mathrm{e}^{-b^{2} x^{2}} b x}{2}+\frac{\operatorname{erf}\left(b x\right) \sqrt{\pi}}{4}}{\sqrt{\pi}}}{b^{2}}$	47
default	$\frac{\frac{\operatorname{erf}\left(b x\right) b^{2} x^{2}}{2}-\frac{-\frac{\mathrm{e}^{-b^{2} x^{2}} b x}{2}+\frac{\operatorname{erf}\left(b x\right) \sqrt{\pi}}{4}}{\sqrt{\pi}}}{b^{2}}$	47
parallelrisch	$\frac{2 x^{2} \operatorname{erf}\left(b x\right) \sqrt{\pi}\, b^{2}+2\,\mathrm{e}^{-b^{2} x^{2}} b x-\operatorname{erf}\left(b x\right) \sqrt{\pi}}{4 \sqrt{\pi}\, b^{2}}$	47

input
```
int(x*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2/Pi^(1/2)*(exp(-b^2*x^2)*b*x-1/6*(-6*b^2*x^2+3)*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x\mathrm{erf}(bx)\,dx = \frac{2\sqrt{\pi}bxe^{(-b^2x^2)} - (\pi - 2\,\pi b^2x^2)\,\mathrm{erf}\,(bx)}{4\,\pi b^2}$$

input
```
integrate(x*erf(b*x),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(pi)*b*x*e^(-b^2*x^2) - (pi - 2*pi*b^2*x^2)*erf(b*x))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int x\mathrm{erf}(bx)\,dx = \begin{cases} \frac{x^2\,\mathrm{erf}\,(bx)}{2} + \frac{xe^{-b^2x^2}}{2\sqrt{\pi}b} - \frac{\mathrm{erf}\,(bx)}{4b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*erf(b*x),x)
```

output
```
Piecewise((x**2*erf(b*x)/2 + x*exp(-b**2*x**2)/(2*sqrt(pi)*b) - erf(b*x)/(
4*b**2), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x\text{erf}(bx)\,dx = \frac{1}{2}\,x^2\,\text{erf}\,(bx) + \frac{b\left(\frac{2\,xe^{\left(-b^2x^2\right)}}{b^2} - \frac{\sqrt{\pi}\,\text{erf}(bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

input
```
integrate(x*erf(b*x),x, algorithm="maxima")
```

output
```
1/2*x^2*erf(b*x) + 1/4*b*(2*x*e^(-b^2*x^2)/b^2 - sqrt(pi)*erf(b*x)/b^3)/sq
rt(pi)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x\text{erf}(bx)\,dx = \frac{1}{2}\,x^2\,\text{erf}\,(bx) + \frac{b\left(\frac{2\,xe^{\left(-b^2x^2\right)}}{b^2} + \frac{\sqrt{\pi}\,\text{erf}(-bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

input
```
integrate(x*erf(b*x),x, algorithm="giac")
```

output
```
1/2*x^2*erf(b*x) + 1/4*b*(2*x*e^(-b^2*x^2)/b^2 + sqrt(pi)*erf(-b*x)/b^3)/s
qrt(pi)
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int x\text{erf}(bx)\,dx = \frac{x^2\,\text{erf}(b\,x)}{2} + \frac{b\,\text{erfi}\left(x\,\sqrt{-b^2}\right)}{4\,{\left(-b^2\right)}^{3/2}} + \frac{x\,\mathrm{e}^{-b^2\,x^2}}{2\,b\,\sqrt{\pi}}$$

input
```
int(x*erf(b*x),x)
```

output
```
(x^2*erf(b*x))/2 + (b*erfi(x*(-b^2)^(1/2)))/(4*(-b^2)^(3/2)) + (x*exp(-b^2
*x^2))/(2*b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int x\mathrm{erf}(bx)\,dx = \frac{2e^{b^2x^2}\mathrm{erf}(bx)\,b^2\pi\,x^2 - e^{b^2x^2}\mathrm{erf}(bx)\,\pi + 2\sqrt{\pi}\,bx}{4e^{b^2x^2}b^2\pi}$$

input
```
int(x*erf(b*x),x)
```

output
```
(2*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 - e**(b**2*x**2)*erf(b*x)*pi + 2*s
qrt(pi)*b*x)/(4*e**(b**2*x**2)*b**2*pi)
```

3.4 $\int \frac{\text{erf}(bx)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \frac{\text{erf}(bx)}{x} \, dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
2*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}(bx)}{x} \, dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]/x,x]
```

output
```
(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6912}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)}{x} \, dx$$

$$\Big\downarrow \text{6912}$$

$$\frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[Erf[b*x]/x,x]
```

output
```
(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 6912
```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result	size
meijerg	$\frac{2bx\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}\right],\left[\frac{3}{2},\frac{3}{2}\right],-b^2x^2\right)}{\sqrt{\pi}}$	23

input
```
int(erf(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
2*b*x*hypergeom([1/2,1/2],[3/2,3/2],-b^2*x^2)/Pi^(1/2)
```

Fricas [F]

$$\int \frac{\operatorname{erf}(bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx)}{x}\,dx$$

input
```
integrate(erf(b*x)/x,x, algorithm="fricas")
```

output
```
integral(erf(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{erf}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\begin{matrix} \frac{1}{2}, \frac{1}{2} \\ \frac{3}{2}, \frac{3}{2} \end{matrix}\middle| -b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(erf(b*x)/x,x)
```

output
```
2*b*x*hyper((1/2, 1/2), (3/2, 3/2), -b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int \frac{\operatorname{erf}(bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx)}{x}\,dx$$

input
```
integrate(erf(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)/x, x)
```

Giac [F]

$$\int \frac{\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}\,(bx)}{x}\,dx$$

input
```
integrate(erf(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erf(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}(b\,x)}{x}\,dx$$

input
```
int(erf(b*x)/x,x)
```

output
```
int(erf(b*x)/x, x)
```

Reduce [F]

$$\int \frac{\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}(bx)}{x}dx$$

input
```
int(erf(b*x)/x,x)
```

output
```
int(erf(b*x)/x,x)
```

3.5 $\int \frac{\text{erf}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \frac{\text{erf}(bx)}{x^3}\,dx = -\frac{be^{-b^2x^2}}{\sqrt{\pi}x} - b^2\text{erf}(bx) - \frac{\text{erf}(bx)}{2x^2}$$

output
```
-b/exp(b^2*x^2)/Pi^(1/2)/x-b^2*erf(b*x)-1/2*erf(b*x)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}(bx)}{x^3}\,dx = -\frac{be^{-b^2x^2}}{\sqrt{\pi}x} - b^2\text{erf}(bx) - \frac{\text{erf}(bx)}{2x^2}$$

input
```
Integrate[Erf[b*x]/x^3,x]
```

output
```
-(b/(E^(b^2*x^2)*Sqrt[Pi]*x)) - b^2*Erf[b*x] - Erf[b*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6915, 2643, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)}{x^3} \, dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{b \int \frac{e^{-b^2x^2}}{x^2}dx}{\sqrt{\pi}} - \frac{\text{erf}(bx)}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(-2b^2 \int e^{-b^2x^2}dx - \frac{e^{-b^2x^2}}{x}\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(\sqrt{\pi}(-b)\text{erf}(bx) - \frac{e^{-b^2x^2}}{x}\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)}{2x^2}$$

input
```
Int[Erf[b*x]/x^3,x]
```

output
```
-1/2*Erf[b*x]/x^2 + (b*(-(1/(E^(b^2*x^2)*x)) - b*Sqrt[Pi]*Erf[b*x]))/Sqrt[
Pi]
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{\operatorname{erf}(bx)}{2x^2}+\frac{b\left(-\frac{e^{-b^2x^2}}{x}-b\sqrt{\pi}\,\operatorname{erf}(bx)\right)}{\sqrt{\pi}}$	41
parallelrisch	$-\frac{2x^2\operatorname{erf}(bx)\sqrt{\pi}\,b^2+2\,e^{-b^2x^2}bx+\operatorname{erf}(bx)\sqrt{\pi}}{2\sqrt{\pi}\,x^2}$	46
derivativedivides	$b^2\left(-\frac{\operatorname{erf}(bx)}{2b^2x^2}+\frac{-\frac{e^{-b^2x^2}}{xb}-\operatorname{erf}(bx)\sqrt{\pi}}{\sqrt{\pi}}\right)$	50
default	$b^2\left(-\frac{\operatorname{erf}(bx)}{2b^2x^2}+\frac{-\frac{e^{-b^2x^2}}{xb}-\operatorname{erf}(bx)\sqrt{\pi}}{\sqrt{\pi}}\right)$	50
meijerg	$\frac{b^2\left(-\frac{2\,e^{-b^2x^2}}{xb}-\frac{\left(2b^2x^2+1\right)\operatorname{erf}(bx)\sqrt{\pi}}{x^2b^2}\right)}{2\sqrt{\pi}}$	52
orering	$\frac{\left(-b^2x^3-2x\right)\operatorname{erf}(bx)}{x^3}-\frac{x^2\left(\frac{2\,e^{-b^2x^2}b}{\sqrt{\pi}\,x^3}-\frac{3\operatorname{erf}(bx)}{x^4}\right)}{2}$	55

input
```
int(erf(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*erf(b*x)/x^2+1/Pi^(1/2)*b*(-1/x*exp(-b^2*x^2)-b*Pi^(1/2)*erf(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = -\frac{2\,\sqrt{\pi}bxe^{\left(-b^2x^2\right)} + \left(\pi + 2\,\pi b^2x^2\right)\operatorname{erf}\left(bx\right)}{2\,\pi x^2}$$

input
```
integrate(erf(b*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(pi)*b*x*e^(-b^2*x^2) + (pi + 2*pi*b^2*x^2)*erf(b*x))/(pi*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = -b^2\operatorname{erf}\left(bx\right) - \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{erf}\left(bx\right)}{2x^2}$$

input
```
integrate(erf(b*x)/x**3,x)
```

output
```
-b**2*erf(b*x) - b*exp(-b**2*x**2)/(sqrt(pi)*x) - erf(b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = -\frac{b^2\sqrt{x^2}\Gamma\left(-\frac{1}{2}, b^2x^2\right)}{2\sqrt{\pi}x} - \frac{\operatorname{erf}(bx)}{2x^2}$$

input
```
integrate(erf(b*x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^2*sqrt(x^2)*gamma(-1/2, b^2*x^2)/(sqrt(pi)*x) - 1/2*erf(b*x)/x^2
```

Giac [F]

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(bx)}{x^3}\,dx$$

input
```
integrate(erf(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = \frac{b\operatorname{erfc}\left(\sqrt{b^2x^2}\right)\sqrt{b^2x^2}}{x} - \frac{b\sqrt{b^2x^2}}{x} - \frac{b\,e^{-b^2x^2}}{x\sqrt{\pi}} - \frac{\operatorname{erf}(b\,x)}{2\,x^2}$$

input
```
int(erf(b*x)/x^3,x)
```

output
```
(b*erfc((b^2*x^2)^(1/2))*(b^2*x^2)^(1/2))/x - (b*(b^2*x^2)^(1/2))/x - (b*e
xp(-b^2*x^2))/(x*pi^(1/2)) - erf(b*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{erf}(bx)}{x^3}\,dx = \frac{-2e^{b^2x^2}\operatorname{erf}(bx)\,b^2\pi\,x^2 - e^{b^2x^2}\operatorname{erf}(bx)\,\pi - 2\sqrt{\pi}\,bx}{2e^{b^2x^2}\pi\,x^2}$$

input
```
int(erf(b*x)/x^3,x)
```

output
```
( - 2*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 - e**(b**2*x**2)*erf(b*x)*pi -
2*sqrt(pi)*b*x)/(2*e**(b**2*x**2)*pi*x**2)
```

3.6 $\int \frac{\mathbf{erf}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int \frac{\operatorname{erf}(bx)}{x^5}\,dx = -\frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} + \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} + \frac{1}{3}b^4\operatorname{erf}(bx) - \frac{\operatorname{erf}(bx)}{4x^4}$$

output
```
-1/6*b/exp(b^2*x^2)/Pi^(1/2)/x^3+1/3*b^3/exp(b^2*x^2)/Pi^(1/2)/x+1/3*b^4*e
rf(b*x)-1/4*erf(b*x)/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{erf}(bx)}{x^5}\,dx = e^{-b^2x^2}\left(-\frac{b}{6\sqrt{\pi}x^3} + \frac{b^3}{3\sqrt{\pi}x}\right) + \frac{1}{3}b^4\operatorname{erf}(bx) - \frac{\operatorname{erf}(bx)}{4x^4}$$

input
```
Integrate[Erf[b*x]/x^5,x]
```

output
```
(-1/6*b/(Sqrt[Pi]*x^3) + b^3/(3*Sqrt[Pi]*x))/E^(b^2*x^2) + (b^4*Erf[b*x])/
3 - Erf[b*x]/(4*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6915, 2643, 2643, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)}{x^5} \, dx$$

$$\Big\downarrow \text{ 6915}$$

$$\frac{b \int \frac{e^{-b^2x^2}}{x^4}dx}{2\sqrt{\pi}} - \frac{\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}}{x^2}dx - \frac{e^{-b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}} - \frac{\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{3}b^2\left(-2b^2 \int e^{-b^2x^2}dx - \frac{e^{-b^2x^2}}{x}\right) - \frac{e^{-b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}} - \frac{\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{b\left(-\frac{2}{3}b^2\left(\sqrt{\pi}(-b)\text{erf}(bx) - \frac{e^{-b^2x^2}}{x}\right) - \frac{e^{-b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}} - \frac{\text{erf}(bx)}{4x^4}$$

input
```
Int[Erf[b*x]/x^5,x]
```

output
```
-1/4*Erf[b*x]/x^4 + (b*(-1/3*1/(E^(b^2*x^2)*x^3) - (2*b^2*(-(1/(E^(b^2*x^2
)*x)) - b*Sqrt[Pi]*Erf[b*x]))/3))/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result	size
meijerg	$\frac{b^4\left(-\frac{4\left(-\frac{b^2x^2}{2}+\frac{1}{4}\right)e^{-b^2x^2}}{3x^3b^3}-\frac{\left(-4b^4x^4+3\right)\operatorname{erf}(bx)\sqrt{\pi}}{6x^4b^4}\right)}{2\sqrt{\pi}}$	62
parts	$-\frac{\operatorname{erf}(bx)}{4x^4}+\frac{b\left(-\frac{e^{-b^2x^2}}{3x^3}-\frac{2b^2\left(-\frac{e^{-b^2x^2}}{x}-b\sqrt{\pi}\operatorname{erf}(bx)\right)}{3}\right)}{2\sqrt{\pi}}$	62
parallelrisch	$\frac{4\operatorname{erf}(bx)x^4\sqrt{\pi}\,b^4+4\,e^{-b^2x^2}x^3b^3-2\,e^{-b^2x^2}bx-3\operatorname{erf}(bx)\sqrt{\pi}}{12\sqrt{\pi}\,x^4}$	64
derivativedivides	$b^4\left(-\frac{\operatorname{erf}(bx)}{4b^4x^4}+\frac{-\frac{e^{-b^2x^2}}{3b^3x^3}+\frac{2\,e^{-b^2x^2}}{3xb}+\frac{2\operatorname{erf}(bx)\sqrt{\pi}}{3}}{2\sqrt{\pi}}\right)$	69
default	$b^4\left(-\frac{\operatorname{erf}(bx)}{4b^4x^4}+\frac{-\frac{e^{-b^2x^2}}{3b^3x^3}+\frac{2\,e^{-b^2x^2}}{3xb}+\frac{2\operatorname{erf}(bx)\sqrt{\pi}}{3}}{2\sqrt{\pi}}\right)$	69
orering	$\frac{\left(\frac{1}{3}b^4x^5+\frac{5}{6}b^2x^3-\frac{2}{3}x\right)\operatorname{erf}(bx)}{x^5}+\frac{(2b^2x^2-1)x^2\left(\frac{2\,e^{-b^2x^2}b}{\sqrt{\pi}\,x^5}-\frac{5\operatorname{erf}(bx)}{x^6}\right)}{12}$	73

input
```
int(erf(b*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2/Pi^(1/2)*b^4*(-4/3/x^3/b^3*(-1/2*b^2*x^2+1/4)*exp(-b^2*x^2)-1/6/x^4/b^
4*(-4*b^4*x^4+3)*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{erf}(bx)}{x^5}\,dx = \frac{2\sqrt{\pi}(2\,b^3x^3-bx)e^{(-b^2x^2)}-(3\,\pi-4\,\pi b^4x^4)\operatorname{erf}(bx)}{12\,\pi x^4}$$

input
```
integrate(erf(b*x)/x^5,x, algorithm="fricas")
```

output
```
1/12*(2*sqrt(pi)*(2*b^3*x^3 - b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b^4*x^4)*er
f(b*x))/(pi*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{erf}(bx)}{x^5} \, dx = \frac{b^4 \operatorname{erf}(bx)}{3} + \frac{b^3 e^{-b^2x^2}}{3\sqrt{\pi}x} - \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\operatorname{erf}(bx)}{4x^4}$$

input
```
integrate(erf(b*x)/x**5,x)
```

output
```
b**4*erf(b*x)/3 + b**3*exp(-b**2*x**2)/(3*sqrt(pi)*x) - b*exp(-b**2*x**2)/
(6*sqrt(pi)*x**3) - erf(b*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{erf}(bx)}{x^5} \, dx = -\frac{b^4 (x^2)^{\frac{3}{2}} \Gamma\left(-\frac{3}{2}, b^2x^2\right)}{4\sqrt{\pi}x^3} - \frac{\operatorname{erf}(bx)}{4\,x^4}$$

input
```
integrate(erf(b*x)/x^5,x, algorithm="maxima")
```

output
```
-1/4*b^4*(x^2)^(3/2)*gamma(-3/2, b^2*x^2)/(sqrt(pi)*x^3) - 1/4*erf(b*x)/x^
4
```

Giac [F]

$$\int \frac{\operatorname{erf}(bx)}{x^5} \, dx = \int \frac{\operatorname{erf}(bx)}{x^5} \, dx$$

input
```
integrate(erf(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erf(b*x)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{erf}(bx)}{x^5}\,dx = \frac{b\,(b^2\,x^2)^{3/2}}{3\,x^3} - \frac{\operatorname{erf}(b\,x)}{4\,x^4} + \frac{b^3\,\mathrm{e}^{-b^2\,x^2}}{3\,x\,\sqrt{\pi}} - \frac{b\,\mathrm{e}^{-b^2\,x^2}}{6\,x^3\,\sqrt{\pi}} - \frac{b\,\operatorname{erfc}\left(\sqrt{b^2\,x^2}\right)\,(b^2\,x^2)^{3/2}}{3\,x^3}$$

input
```
int(erf(b*x)/x^5,x)
```

output
```
(b*(b^2*x^2)^(3/2))/(3*x^3) - erf(b*x)/(4*x^4) + (b^3*exp(-b^2*x^2))/(3*x*
pi^(1/2)) - (b*exp(-b^2*x^2))/(6*x^3*pi^(1/2)) - (b*erfc((b^2*x^2)^(1/2))*
(b^2*x^2)^(3/2))/(3*x^3)
```

Reduce [F]

$$\int \frac{\operatorname{erf}(bx)}{x^5}\,dx = \frac{-3e^{b^2x^2}\operatorname{erf}(bx)\,\pi - 4\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^3x^4 - 2\sqrt{\pi}\,bx}{12e^{b^2x^2}\pi\,x^4}$$

input
```
int(erf(b*x)/x^5,x)
```

output
```
( - 3*e**(b**2*x**2)*erf(b*x)*pi - 4*sqrt(pi)*e**(b**2*x**2)*int(1/(e**(b*
*2*x**2)*x**2),x)*b**3*x**4 - 2*sqrt(pi)*b*x)/(12*e**(b**2*x**2)*pi*x**4)
```

3.7 $\int \frac{\text{erf}(bx)}{x^7} dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int \frac{\text{erf}(bx)}{x^7} dx = -\frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5} + \frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3} - \frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x} - \frac{4}{45}b^6\text{erf}(bx) - \frac{\text{erf}(bx)}{6x^6}$$

output
```
-1/15*b/exp(b^2*x^2)/Pi^(1/2)/x^5+2/45*b^3/exp(b^2*x^2)/Pi^(1/2)/x^3-4/45*
b^5/exp(b^2*x^2)/Pi^(1/2)/x-4/45*b^6*erf(b*x)-1/6*erf(b*x)/x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{\text{erf}(bx)}{x^7} dx = \frac{e^{-b^2x^2}\left(-6bx + 4b^3x^3 - 8b^5x^5 - e^{b^2x^2}\sqrt{\pi}(15 + 8b^6x^6)\,\text{erf}(bx)\right)}{90\sqrt{\pi}x^6}$$

input
```
Integrate[Erf[b*x]/x^7,x]
```

output
```
(-6*b*x + 4*b^3*x^3 - 8*b^5*x^5 - E^(b^2*x^2)*Sqrt[Pi]*(15 + 8*b^6*x^6)*Er
f[b*x])/(90*E^(b^2*x^2)*Sqrt[Pi]*x^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6915, 2643, 2643, 2643, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)}{x^7} \, dx$$

$$\Big\downarrow \text{ 6915}$$

$$\frac{b \int \frac{e^{-b^2x^2}}{x^6}dx}{3\sqrt{\pi}} - \frac{\text{erf}(bx)}{6x^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{5}b^2 \int \frac{e^{-b^2x^2}}{x^4}dx - \frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}} - \frac{\text{erf}(bx)}{6x^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}}{x^2}dx - \frac{e^{-b^2x^2}}{3x^3}\right) - \frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}} - \frac{\text{erf}(bx)}{6x^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2 \int e^{-b^2x^2}dx - \frac{e^{-b^2x^2}}{x}\right) - \frac{e^{-b^2x^2}}{3x^3}\right) - \frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}} - \frac{\text{erf}(bx)}{6x^6}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(\sqrt{\pi}(-b)\text{erf}(bx) - \frac{e^{-b^2x^2}}{x}\right) - \frac{e^{-b^2x^2}}{3x^3}\right) - \frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}} - \frac{\text{erf}(bx)}{6x^6}$$

input
```
Int[Erf[b*x]/x^7,x]
```

output
```
-1/6*Erf[b*x]/x^6 + (b*(-1/5*1/(E^(b^2*x^2)*x^5) - (2*b^2*(-1/3*1/(E^(b^2*
x^2)*x^3) - (2*b^2*(-(1/(E^(b^2*x^2)*x)) - b*Sqrt[Pi]*Erf[b*x]))/3))/5))/(
3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73

method	result	size
meijerg	$\frac{b^{6}\left(-\frac{4\left(\frac{2}{9}b^{4}x^{4}-\frac{1}{9}b^{2}x^{2}+\frac{1}{6}\right)\mathrm{e}^{-b^{2}x^{2}}}{5x^{5}b^{5}}-\frac{\left(8b^{6}x^{6}+15\right)\mathrm{erf}(bx)\sqrt{\pi}}{45x^{6}b^{6}}\right)}{2\sqrt{\pi}}$	70
parallelrisch	$-\frac{8\,\mathrm{erf}(bx)x^{6}b^{6}\sqrt{\pi}+8\,\mathrm{e}^{-b^{2}x^{2}}x^{5}b^{5}-4\,\mathrm{e}^{-b^{2}x^{2}}x^{3}b^{3}+6\,\mathrm{e}^{-b^{2}x^{2}}bx+15\,\mathrm{erf}(bx)\sqrt{\pi}}{90\sqrt{\pi}\,x^{6}}$	81
parts	$-\frac{\mathrm{erf}(bx)}{6x^{6}}+\frac{b\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}}{5x^{5}}-\frac{2b^{2}\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}}{3x^{3}}-\frac{2b^{2}\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}}{x}-b\sqrt{\pi}\,\mathrm{erf}(bx)\right)}{3}\right)}{5}\right)}{3\sqrt{\pi}}$	82
derivativedivides	$b^{6}\left(-\frac{\mathrm{erf}(bx)}{6b^{6}x^{6}}+\frac{-\frac{\mathrm{e}^{-b^{2}x^{2}}}{5b^{5}x^{5}}+\frac{2\,\mathrm{e}^{-b^{2}x^{2}}}{15b^{3}x^{3}}-\frac{4\,\mathrm{e}^{-b^{2}x^{2}}}{15xb}-\frac{4\,\mathrm{erf}(bx)\sqrt{\pi}}{15}}{3\sqrt{\pi}}\right)$	87
default	$b^{6}\left(-\frac{\mathrm{erf}(bx)}{6b^{6}x^{6}}+\frac{-\frac{\mathrm{e}^{-b^{2}x^{2}}}{5b^{5}x^{5}}+\frac{2\,\mathrm{e}^{-b^{2}x^{2}}}{15b^{3}x^{3}}-\frac{4\,\mathrm{e}^{-b^{2}x^{2}}}{15xb}-\frac{4\,\mathrm{erf}(bx)\sqrt{\pi}}{15}}{3\sqrt{\pi}}\right)$	87
orering	$\frac{\left(-\frac{4}{45}b^{6}x^{7}-\frac{14}{45}b^{4}x^{5}+\frac{7}{45}b^{2}x^{3}-\frac{2}{5}x\right)\mathrm{erf}(bx)}{x^{7}}-\frac{\left(4b^{4}x^{4}-2b^{2}x^{2}+3\right)x^{2}\left(\frac{2\,\mathrm{e}^{-b^{2}x^{2}}b}{\sqrt{\pi}\,x^{7}}-\frac{7\,\mathrm{erf}(bx)}{x^{8}}\right)}{90}$	89

input
```
int(erf(b*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/2/Pi^(1/2)*b^6*(-4/5/x^5/b^5*(2/9*b^4*x^4-1/9*b^2*x^2+1/6)*exp(-b^2*x^2)
-1/45/x^6/b^6*(8*b^6*x^6+15)*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{\mathrm{erf}(bx)}{x^{7}}\,dx=-\frac{2\sqrt{\pi}(4\,b^{5}x^{5}-2\,b^{3}x^{3}+3\,bx)e^{(-b^{2}x^{2})}+(15\,\pi+8\,\pi b^{6}x^{6})\,\mathrm{erf}\,(bx)}{90\,\pi x^{6}}$$

input
```
integrate(erf(b*x)/x^7,x, algorithm="fricas")
```

output
```
-1/90*(2*sqrt(pi)*(4*b^5*x^5 - 2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) + (15*pi +
8*pi*b^6*x^6)*erf(b*x))/(pi*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{erf}(bx)}{x^7}\,dx = -\frac{4b^6\operatorname{erf}(bx)}{45} - \frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x} + \frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3} - \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\operatorname{erf}(bx)}{6x^6}$$

input
```
integrate(erf(b*x)/x**7,x)
```

output
```
-4*b**6*erf(b*x)/45 - 4*b**5*exp(-b**2*x**2)/(45*sqrt(pi)*x) + 2*b**3*exp(
-b**2*x**2)/(45*sqrt(pi)*x**3) - b*exp(-b**2*x**2)/(15*sqrt(pi)*x**5) - er
f(b*x)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int \frac{\operatorname{erf}(bx)}{x^7}\,dx = -\frac{b^6(x^2)^{\frac{5}{2}}\Gamma\left(-\frac{5}{2},b^2x^2\right)}{6\sqrt{\pi}x^5} - \frac{\operatorname{erf}(bx)}{6\,x^6}$$

input
```
integrate(erf(b*x)/x^7,x, algorithm="maxima")
```

output
```
-1/6*b^6*(x^2)^(5/2)*gamma(-5/2, b^2*x^2)/(sqrt(pi)*x^5) - 1/6*erf(b*x)/x^
6
```

Giac [F]

$$\int \frac{\operatorname{erf}(bx)}{x^7}\,dx = \int \frac{\operatorname{erf}(bx)}{x^7}\,dx$$

input
```
integrate(erf(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(erf(b*x)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{erf}(bx)}{x^7}\,dx = -\frac{\operatorname{erf}(b\,x)}{6\,x^6} - \frac{3\,b\,\mathrm{e}^{-b^2\,x^2} - 2\,b^3\,x^2\,\mathrm{e}^{-b^2\,x^2} + 4\,b^5\,x^4\,\mathrm{e}^{-b^2\,x^2} + 4\,b^5\,\sqrt{\pi}\,\sqrt{b^2}\,{\left(x^2\right)}^{5/2} - 4\,b^5\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{b^2}\,\sqrt{x^2}\right)\,\sqrt{b^2}\,{\left(x^2\right)}^{5/2}}{45\,x^5\,\sqrt{\pi}}$$

input
```
int(erf(b*x)/x^7,x)
```

output
```
- erf(b*x)/(6*x^6) - (3*b*exp(-b^2*x^2) - 2*b^3*x^2*exp(-b^2*x^2) + 4*b^5*
x^4*exp(-b^2*x^2) + 4*b^5*pi^(1/2)*(b^2)^(1/2)*(x^2)^(5/2) - 4*b^5*pi^(1/2
)*erfc((b^2)^(1/2)*(x^2)^(1/2))*(b^2)^(1/2)*(x^2)^(5/2))/(45*x^5*pi^(1/2))
```

Reduce [F]

$$\int \frac{\operatorname{erf}(bx)}{x^7}\,dx = \frac{-15e^{b^2x^2}\operatorname{erf}(bx)\,\pi + 8\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^5x^6 + 4\sqrt{\pi}\,b^3x^3 - 6\sqrt{\pi}\,bx}{90e^{b^2x^2}\pi\,x^6}$$

input
```
int(erf(b*x)/x^7,x)
```

output
```
( - 15*e**(b**2*x**2)*erf(b*x)*pi + 8*sqrt(pi)*e**(b**2*x**2)*int(1/(e**(b
**2*x**2)*x**2),x)*b**5*x**6 + 4*sqrt(pi)*b**3*x**3 - 6*sqrt(pi)*b*x)/(90*
e**(b**2*x**2)*pi*x**6)
```

3.8 $\int x^6 \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 109

$$\int x^6 \text{erf}(bx)\,dx = \frac{6e^{-b^2x^2}}{7b^7\sqrt{\pi}} + \frac{6e^{-b^2x^2}x^2}{7b^5\sqrt{\pi}} + \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erf}(bx)$$

output
```
6/7/b^7/exp(b^2*x^2)/Pi^(1/2)+6/7*x^2/b^5/exp(b^2*x^2)/Pi^(1/2)+3/7*x^4/b^
3/exp(b^2*x^2)/Pi^(1/2)+1/7*x^6/b/exp(b^2*x^2)/Pi^(1/2)+1/7*x^7*erf(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.66

$$\int x^6 \text{erf}(bx)\,dx = \frac{e^{-b^2x^2}\left(6 + 6b^2x^2 + 3b^4x^4 + b^6x^6 + b^7e^{b^2x^2}\sqrt{\pi}x^7\text{erf}(bx)\right)}{7b^7\sqrt{\pi}}$$

input
```
Integrate[x^6*Erf[b*x],x]
```

output
```
(6 + 6*b^2*x^2 + 3*b^4*x^4 + b^6*x^6 + b^7*E^(b^2*x^2)*Sqrt[Pi]*x^7*Erf[b*
x])/(7*b^7*E^(b^2*x^2)*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6915, 2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{erf}(bx) \, dx$$

$$\Big\downarrow \; 6915$$

$$\frac{1}{7}x^7\text{erf}(bx)-\frac{2b\int e^{-b^2x^2}x^7dx}{7\sqrt{\pi}}$$

$$\Big\downarrow \; 2641$$

$$\frac{1}{7}x^7\text{erf}(bx)-\frac{2b\left(\frac{3\int e^{-b^2x^2}x^5dx}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}$$

$$\Big\downarrow \; 2641$$

$$\frac{1}{7}x^7\text{erf}(bx)-\frac{2b\left(\frac{3\left(\frac{2\int e^{-b^2x^2}x^3dx}{b^2}-\frac{x^4e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}$$

$$\Big\downarrow \; 2641$$

$$\frac{1}{7}x^7\text{erf}(bx)-\frac{2b\left(\frac{3\left(\frac{2\left(\frac{\int e^{-b^2x^2}xdx}{b^2}-\frac{x^2e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}$$

$$\Big\downarrow \; 2638$$

$$\frac{1}{7}x^7\operatorname{erf}(bx) - \frac{2b\left(\frac{3\left(\frac{2\left(-\frac{x^2e^{-b^2x^2}}{2b^2}-\frac{e^{-b^2x^2}}{2b^4}\right)}{b^2}-\frac{x^4e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}$$

input
```
Int[x^6*Erf[b*x],x]
```

output
```
(-2*b*(-1/2*x^6/(b^2*E^(b^2*x^2)) + (3*(-1/2*x^4/(b^2*E^(b^2*x^2)) + (2*(-
1/2*1/(b^4*E^(b^2*x^2)) - x^2/(2*b^2*E^(b^2*x^2))))/b^2))/b^2))/(7*Sqrt[Pi
]) + (x^7*Erf[b*x])/7
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

method	result	size
meijerg	$\frac{-\frac{12}{7}+\frac{\left(4b^{6}x^{6}+12b^{4}x^{4}+24b^{2}x^{2}+24\right)e^{-b^{2}x^{2}}}{14}+\frac{2x^{7}b^{7}\operatorname{erf}(bx)\sqrt{\pi}}{7}}{2b^{7}\sqrt{\pi}}$	63
parallelrisch	$\frac{x^{7}b^{7}\operatorname{erf}(bx)\sqrt{\pi}+e^{-b^{2}x^{2}}x^{6}b^{6}+3\,e^{-b^{2}x^{2}}x^{4}b^{4}+6\,e^{-b^{2}x^{2}}x^{2}b^{2}+6\,e^{-b^{2}x^{2}}}{7b^{7}\sqrt{\pi}}$	85
derivativedivides	$\frac{\frac{\operatorname{erf}(bx)b^{7}x^{7}}{7}-\frac{2\left(-\frac{e^{-b^{2}x^{2}}x^{6}b^{6}}{2}-\frac{3\,e^{-b^{2}x^{2}}x^{4}b^{4}}{2}-3\,e^{-b^{2}x^{2}}x^{2}b^{2}-3\,e^{-b^{2}x^{2}}\right)}{7\sqrt{\pi}}}{b^{7}}$	90
default	$\frac{\frac{\operatorname{erf}(bx)b^{7}x^{7}}{7}-\frac{2\left(-\frac{e^{-b^{2}x^{2}}x^{6}b^{6}}{2}-\frac{3\,e^{-b^{2}x^{2}}x^{4}b^{4}}{2}-3\,e^{-b^{2}x^{2}}x^{2}b^{2}-3\,e^{-b^{2}x^{2}}\right)}{7\sqrt{\pi}}}{b^{7}}$	90
parts	$\frac{x^{7}\operatorname{erf}(bx)}{7}-\frac{2b\left(-\frac{x^{6}e^{-b^{2}x^{2}}}{2b^{2}}+\frac{-\frac{3x^{4}e^{-b^{2}x^{2}}}{2b^{2}}+\frac{3\left(-\frac{x^{2}e^{-b^{2}x^{2}}}{b^{2}}-\frac{e^{-b^{2}x^{2}}}{b^{4}}\right)}{b^{2}}}{b^{2}}\right)}{7\sqrt{\pi}}$	95
orering	$\frac{\left(x^{8}b^{8}-3b^{6}x^{6}-9b^{4}x^{4}-18b^{2}x^{2}-18\right)\operatorname{erf}(bx)}{7b^{8}x}+\frac{\left(b^{6}x^{6}+3b^{4}x^{4}+6b^{2}x^{2}+6\right)\left(6x^{5}\operatorname{erf}(bx)+\frac{2x^{6}e^{-b^{2}x^{2}}b}{\sqrt{\pi}}\right)}{14b^{8}x^{6}}$	108

input
```
int(x^6*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^7/Pi^(1/2)*(-12/7+1/14*(4*b^6*x^6+12*b^4*x^4+24*b^2*x^2+24)*exp(-b^2
*x^2)+2/7*x^7*b^7*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int x^{6}\operatorname{erf}(bx)\,dx=\frac{\pi b^{7}x^{7}\operatorname{erf}(bx)+\sqrt{\pi}(b^{6}x^{6}+3\,b^{4}x^{4}+6\,b^{2}x^{2}+6)e^{(-b^{2}x^{2})}}{7\,\pi b^{7}}$$

input
```
integrate(x^6*erf(b*x),x, algorithm="fricas")
```

output
```
1/7*(pi*b^7*x^7*erf(b*x) + sqrt(pi)*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*
e^(-b^2*x^2))/(pi*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int x^6 \text{erf}(bx)\,dx = \begin{cases} \frac{x^7 \operatorname{erf}(bx)}{7} + \frac{x^6 e^{-b^2x^2}}{7\sqrt{\pi}b} + \frac{3x^4 e^{-b^2x^2}}{7\sqrt{\pi}b^3} + \frac{6x^2 e^{-b^2x^2}}{7\sqrt{\pi}b^5} + \frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**6*erf(b*x),x)
```

output
```
Piecewise((x**7*erf(b*x)/7 + x**6*exp(-b**2*x**2)/(7*sqrt(pi)*b) + 3*x**4*
exp(-b**2*x**2)/(7*sqrt(pi)*b**3) + 6*x**2*exp(-b**2*x**2)/(7*sqrt(pi)*b**
5) + 6*exp(-b**2*x**2)/(7*sqrt(pi)*b**7), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int x^6 \text{erf}(bx)\,dx = \frac{1}{7}\,x^7 \operatorname{erf}(bx) + \frac{(b^6x^6 + 3\,b^4x^4 + 6\,b^2x^2 + 6)e^{(-b^2x^2)}}{7\,\sqrt{\pi}b^7}$$

input
```
integrate(x^6*erf(b*x),x, algorithm="maxima")
```

output
```
1/7*x^7*erf(b*x) + 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(-b^2*x^2)/
(sqrt(pi)*b^7)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int x^6 \text{erf}(bx)\,dx = \frac{1}{7}\,x^7\,\text{erf}\,(bx) + \frac{(b^6x^6 + 3\,b^4x^4 + 6\,b^2x^2 + 6)e^{(-b^2x^2)}}{7\,\sqrt{\pi}b^7}$$

input
```
integrate(x^6*erf(b*x),x, algorithm="giac")
```

output
```
1/7*x^7*erf(b*x) + 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(-b^2*x^2)/
(sqrt(pi)*b^7)
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int x^6 \text{erf}(bx)\,dx = \frac{x^7\,\text{erf}(b\,x)}{7} + \frac{e^{-b^2\,x^2}\,(b^6\,x^6 + 3\,b^4\,x^4 + 6\,b^2\,x^2 + 6)}{7\,b^7\,\sqrt{\pi}}$$

input
```
int(x^6*erf(b*x),x)
```

output
```
(x^7*erf(b*x))/7 + (exp(-b^2*x^2)*(6*b^2*x^2 + 3*b^4*x^4 + b^6*x^6 + 6))/(
7*b^7*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^6 \text{erf}(bx)\,dx = \frac{e^{b^2x^2}\text{erf}(bx)\,b^7\pi\,x^7 + \sqrt{\pi}\,b^6x^6 + 3\sqrt{\pi}\,b^4x^4 + 6\sqrt{\pi}\,b^2x^2 + 6\sqrt{\pi}}{7e^{b^2x^2}b^7\pi}$$

input
```
int(x^6*erf(b*x),x)
```

output
```
(e**(b**2*x**2)*erf(b*x)*b**7*pi*x**7 + sqrt(pi)*b**6*x**6 + 3*sqrt(pi)*b*
*4*x**4 + 6*sqrt(pi)*b**2*x**2 + 6*sqrt(pi))/(7*e**(b**2*x**2)*b**7*pi)
```

3.9 $\int x^4 \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 84

$$\int x^4 \operatorname{erf}(bx)\,dx = \frac{2e^{-b^2x^2}}{5b^5\sqrt{\pi}} + \frac{2e^{-b^2x^2}x^2}{5b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}} + \frac{1}{5}x^5\operatorname{erf}(bx)$$

output
```
2/5/b^5/exp(b^2*x^2)/Pi^(1/2)+2/5*x^2/b^3/exp(b^2*x^2)/Pi^(1/2)+1/5*x^4/b/
exp(b^2*x^2)/Pi^(1/2)+1/5*x^5*erf(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int x^4 \operatorname{erf}(bx)\,dx = e^{-b^2x^2}\left(\frac{2}{5b^5\sqrt{\pi}} + \frac{2x^2}{5b^3\sqrt{\pi}} + \frac{x^4}{5b\sqrt{\pi}}\right) + \frac{1}{5}x^5\operatorname{erf}(bx)$$

input
```
Integrate[x^4*Erf[b*x],x]
```

output
```
(2/(5*b^5*Sqrt[Pi]) + (2*x^2)/(5*b^3*Sqrt[Pi]) + x^4/(5*b*Sqrt[Pi]))/E^(b^
2*x^2) + (x^5*Erf[b*x])/5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6915, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erf}(bx) \, dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{1}{5}x^5 \text{erf}(bx) - \frac{2b \int e^{-b^2x^2}x^5 dx}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{5}x^5 \text{erf}(bx) - \frac{2b\left(\frac{2\int e^{-b^2x^2}x^3 dx}{b^2} - \frac{x^4 e^{-b^2x^2}}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{5}x^5 \text{erf}(bx) - \frac{2b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x dx}{b^2} - \frac{x^2 e^{-b^2x^2}}{2b^2}\right)}{b^2} - \frac{x^4 e^{-b^2x^2}}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{5}x^5 \text{erf}(bx) - \frac{2b\left(\frac{2\left(-\frac{x^2 e^{-b^2x^2}}{2b^2} - \frac{e^{-b^2x^2}}{2b^4}\right)}{b^2} - \frac{x^4 e^{-b^2x^2}}{2b^2}\right)}{5\sqrt{\pi}}$$

input
```
Int[x^4*Erf[b*x],x]
```

output
```
(-2*b*(-1/2*x^4/(b^2*E^(b^2*x^2)) + (2*(-1/2*1/(b^4*E^(b^2*x^2)) - x^2/(2*
b^2*E^(b^2*x^2))))/b^2))/(5*Sqrt[Pi]) + (x^5*Erf[b*x])/5
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
meijerg	$\frac{-\frac{4}{5}+\frac{2\left(3b^4x^4+6b^2x^2+6\right)\mathrm{e}^{-b^2x^2}}{15}+\frac{2b^5x^5\,\mathrm{erf}(bx)\sqrt{\pi}}{5}}{2b^5\sqrt{\pi}}$	55
parallelrisch	$\frac{b^5x^5\,\mathrm{erf}(bx)\sqrt{\pi}+\mathrm{e}^{-b^2x^2}x^4b^4+2\,\mathrm{e}^{-b^2x^2}x^2b^2+2\,\mathrm{e}^{-b^2x^2}}{5b^5\sqrt{\pi}}$	68
derivativedivides	$\frac{\frac{\mathrm{erf}(bx)b^5x^5}{5}-\frac{2\left(-\frac{\mathrm{e}^{-b^2x^2}x^4b^4}{2}-\mathrm{e}^{-b^2x^2}x^2b^2-\mathrm{e}^{-b^2x^2}\right)}{5\sqrt{\pi}}}{b^5}$	72
default	$\frac{\frac{\mathrm{erf}(bx)b^5x^5}{5}-\frac{2\left(-\frac{\mathrm{e}^{-b^2x^2}x^4b^4}{2}-\mathrm{e}^{-b^2x^2}x^2b^2-\mathrm{e}^{-b^2x^2}\right)}{5\sqrt{\pi}}}{b^5}$	72
parts	$\frac{x^5\,\mathrm{erf}(bx)}{5}-\frac{2b\left(-\frac{x^4\mathrm{e}^{-b^2x^2}}{2b^2}+\frac{-\frac{x^2\mathrm{e}^{-b^2x^2}}{b^2}-\frac{\mathrm{e}^{-b^2x^2}}{b^4}}{b^2}\right)}{5\sqrt{\pi}}$	72
orering	$\frac{\left(b^6x^6-2b^4x^4-4b^2x^2-4\right)\mathrm{erf}(bx)}{5b^6x}+\frac{\left(b^4x^4+2b^2x^2+2\right)\left(4x^3\,\mathrm{erf}(bx)+\frac{2x^4\mathrm{e}^{-b^2x^2}b}{\sqrt{\pi}}\right)}{10b^6x^4}$	92

input
```
int(x^4*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^5/Pi^(1/2)*(-4/5+2/15*(3*b^4*x^4+6*b^2*x^2+6)*exp(-b^2*x^2)+2/5*b^5*
x^5*erf(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int x^4\mathrm{erf}(bx)\,dx=\frac{\pi b^5x^5\,\mathrm{erf}\,(bx)+\sqrt{\pi}(b^4x^4+2\,b^2x^2+2)e^{(-b^2x^2)}}{5\,\pi b^5}$$

input
```
integrate(x^4*erf(b*x),x, algorithm="fricas")
```

output
```
1/5*(pi*b^5*x^5*erf(b*x) + sqrt(pi)*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)
)/(pi*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int x^4 \operatorname{erf}(bx)\, dx = \begin{cases} \frac{x^5 \operatorname{erf}(bx)}{5} + \frac{x^4 e^{-b^2x^2}}{5\sqrt{\pi}b} + \frac{2x^2 e^{-b^2x^2}}{5\sqrt{\pi}b^3} + \frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*erf(b*x),x)
```

output
```
Piecewise((x**5*erf(b*x)/5 + x**4*exp(-b**2*x**2)/(5*sqrt(pi)*b) + 2*x**2*
exp(-b**2*x**2)/(5*sqrt(pi)*b**3) + 2*exp(-b**2*x**2)/(5*sqrt(pi)*b**5), N
e(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int x^4 \operatorname{erf}(bx)\, dx = \frac{1}{5}\, x^5 \operatorname{erf}(bx) + \frac{(b^4x^4 + 2\, b^2x^2 + 2)e^{(-b^2x^2)}}{5\, \sqrt{\pi}b^5}$$

input
```
integrate(x^4*erf(b*x),x, algorithm="maxima")
```

output
```
1/5*x^5*erf(b*x) + 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b^
5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int x^4 \operatorname{erf}(bx)\, dx = \frac{1}{5}\, x^5 \operatorname{erf}(bx) + \frac{(b^4x^4 + 2\, b^2x^2 + 2)e^{(-b^2x^2)}}{5\, \sqrt{\pi}b^5}$$

input
```
integrate(x^4*erf(b*x),x, algorithm="giac")
```

output
```
1/5*x^5*erf(b*x) + 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b^
5)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int x^4\mathrm{erf}(bx)\,dx=\frac{x^5\,\mathrm{erf}(b\,x)}{5}+\frac{\mathrm{e}^{-b^2\,x^2}\,\left(b^4\,x^4+2\,b^2\,x^2+2\right)}{5\,b^5\,\sqrt{\pi}}$$

input
```
int(x^4*erf(b*x),x)
```

output
```
(x^5*erf(b*x))/5 + (exp(-b^2*x^2)*(2*b^2*x^2 + b^4*x^4 + 2))/(5*b^5*pi^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int x^4\mathrm{erf}(bx)\,dx=\frac{e^{b^2x^2}\mathrm{erf}(bx)\,b^5\pi\,x^5+\sqrt{\pi}\,b^4x^4+2\sqrt{\pi}\,b^2x^2+2\sqrt{\pi}}{5e^{b^2x^2}b^5\pi}$$

input
```
int(x^4*erf(b*x),x)
```

output
```
(e**(b**2*x**2)*erf(b*x)*b**5*pi*x**5 + sqrt(pi)*b**4*x**4 + 2*sqrt(pi)*b*
*2*x**2 + 2*sqrt(pi))/(5*e**(b**2*x**2)*b**5*pi)
```

3.10 $\int x^2 \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x^2 \operatorname{erf}(bx)\,dx = \frac{e^{-b^2x^2}}{3b^3\sqrt{\pi}} + \frac{e^{-b^2x^2}x^2}{3b\sqrt{\pi}} + \frac{1}{3}x^3\operatorname{erf}(bx)$$

output
```
1/3/b^3/exp(b^2*x^2)/Pi^(1/2)+1/3*x^2/b/exp(b^2*x^2)/Pi^(1/2)+1/3*x^3*erf(
b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int x^2 \operatorname{erf}(bx)\,dx = \frac{1}{3}\left(\frac{e^{-b^2x^2}(1+b^2x^2)}{b^3\sqrt{\pi}} + x^3\operatorname{erf}(bx)\right)$$

input
```
Integrate[x^2*Erf[b*x],x]
```

output
```
((1 + b^2*x^2)/(b^3*E^(b^2*x^2)*Sqrt[Pi]) + x^3*Erf[b*x])/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6915, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erf}(bx) \, dx$$

$$\Big\downarrow \text{ 6915}$$

$$\frac{1}{3} x^3 \text{erf}(bx) - \frac{2b \int e^{-b^2 x^2} x^3 dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{3} x^3 \text{erf}(bx) - \frac{2b \left(\frac{\int e^{-b^2 x^2} x dx}{b^2} - \frac{x^2 e^{-b^2 x^2}}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{1}{3} x^3 \text{erf}(bx) - \frac{2b \left(-\frac{x^2 e^{-b^2 x^2}}{2b^2} - \frac{e^{-b^2 x^2}}{2b^4}\right)}{3\sqrt{\pi}}$$

input
```
Int[x^2*Erf[b*x],x]
```

output
```
(-2*b*(-1/2*1/(b^4*E^(b^2*x^2)) - x^2/(2*b^2*E^(b^2*x^2))))/(3*Sqrt[Pi]) +
 (x^3*Erf[b*x])/3
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

method	result	size
meijerg	$\frac{-\frac{2}{3}+\frac{\left(2 b^{2} x^{2}+2\right) {\mathrm e}^{-b^{2} x^{2}}}{3}+\frac{2 b^{3} x^{3} \operatorname{erf}\left(b x \right) \sqrt{\pi}}{3}}{2 b^{3} \sqrt{\pi}}$	47
parallelrisch	$\frac{b^{3} x^{3} \operatorname{erf}\left(b x \right) \sqrt{\pi}+{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}+{\mathrm e}^{-b^{2} x^{2}}}{3 b^{3} \sqrt{\pi}}$	49
parts	$\frac{x^{3} \operatorname{erf}\left(b x \right)}{3}-\frac{2 b \left(-\frac{x^{2} {\mathrm e}^{-b^{2} x^{2}}}{2 b^{2}}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{2 b^{4}}\right)}{3 \sqrt{\pi}}$	49
derivativedivides	$\frac{\frac{\operatorname{erf}\left(b x \right) b^{3} x^{3}}{3}-\frac{2 \left(-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}}{2}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{2}\right)}{3 \sqrt{\pi}}}{b^{3}}$	54
default	$\frac{\frac{\operatorname{erf}\left(b x \right) b^{3} x^{3}}{3}-\frac{2 \left(-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}}{2}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{2}\right)}{3 \sqrt{\pi}}}{b^{3}}$	54
orering	$\frac{\left(b^{4} x^{4}-b^{2} x^{2}-1\right) \operatorname{erf}\left(b x \right)}{3 b^{4} x}+\frac{\left(b^{2} x^{2}+1\right) \left(2 x \operatorname{erf}\left(b x \right)+\frac{2 x^{2} {\mathrm e}^{-b^{2} x^{2}} b}{\sqrt{\pi}}\right)}{6 b^{4} x^{2}}$	74

input
```
int(x^2*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3/Pi^(1/2)*(-2/3+1/3*(2*b^2*x^2+2)*exp(-b^2*x^2)+2/3*b^3*x^3*erf(b*x
)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^2\text{erf}(bx)\,dx = \frac{\pi b^3x^3\,\text{erf}\,(bx) + \sqrt{\pi}(b^2x^2+1)e^{(-b^2x^2)}}{3\,\pi b^3}$$

input
```
integrate(x^2*erf(b*x),x, algorithm="fricas")
```

output
```
1/3*(pi*b^3*x^3*erf(b*x) + sqrt(pi)*(b^2*x^2 + 1)*e^(-b^2*x^2))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x^2\text{erf}(bx)\,dx = \begin{cases} \frac{x^3\,\text{erf}\,(bx)}{3} + \frac{x^2e^{-b^2x^2}}{3\sqrt{\pi}b} + \frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*erf(b*x),x)
```

output
```
Piecewise((x**3*erf(b*x)/3 + x**2*exp(-b**2*x**2)/(3*sqrt(pi)*b) + exp(-b*
*2*x**2)/(3*sqrt(pi)*b**3), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int x^2\text{erf}(bx)\,dx = \frac{1}{3}\,x^3\,\text{erf}\,(bx) + \frac{(b^2x^2+1)e^{(-b^2x^2)}}{3\,\sqrt{\pi}b^3}$$

input
```
integrate(x^2*erf(b*x),x, algorithm="maxima")
```

output
```
1/3*x^3*erf(b*x) + 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int x^2\text{erf}(bx)\,dx = \frac{1}{3}\,x^3\,\text{erf}\,(bx) + \frac{(b^2x^2+1)e^{(-b^2x^2)}}{3\,\sqrt{\pi}b^3}$$

input
```
integrate(x^2*erf(b*x),x, algorithm="giac")
```

output
```
1/3*x^3*erf(b*x) + 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int x^2\text{erf}(bx)\,dx = \frac{x^3\,\text{erf}(b\,x)}{3} + \frac{\mathrm{e}^{-b^2\,x^2}\,\left(b^2\,x^2+1\right)}{3\,b^3\,\sqrt{\pi}}$$

input
```
int(x^2*erf(b*x),x)
```

output
```
(x^3*erf(b*x))/3 + (exp(-b^2*x^2)*(b^2*x^2 + 1))/(3*b^3*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int x^2\text{erf}(bx)\,dx = \frac{e^{b^2x^2}\text{erf}(bx)\,b^3\pi\,x^3 + \sqrt{\pi}\,b^2x^2 + \sqrt{\pi}}{3e^{b^2x^2}b^3\pi}$$

input
```
int(x^2*erf(b*x),x)
```

output
```
(e**(b**2*x**2)*erf(b*x)*b**3*pi*x**3 + sqrt(pi)*b**2*x**2 + sqrt(pi))/(3*
e**(b**2*x**2)*b**3*pi)
```

3.11 $\int \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \operatorname{erf}(bx)\,dx = \frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\operatorname{erf}(bx)$$

output
```
1/b/exp(b^2*x^2)/Pi^(1/2)+x*erf(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{erf}(bx)\,dx = \frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\operatorname{erf}(bx)$$

input
```
Integrate[Erf[b*x],x]
```

output
```
1/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx) \, dx$$

$$\Big\downarrow \text{6903}$$

$$\frac{e^{-b^2x^2}}{\sqrt{\pi}b} + x\text{erf}(bx)$$

input
```
Int[Erf[b*x],x]
```

output
```
1/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]
```

Defintions of rubi rules used

rule 6903
```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b),
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
parts	$x\ \mathrm{erf}\left(bx\right)+\frac{\mathrm{e}^{-b^{2} x^{2}}}{\sqrt{\pi}\, b}$	24
orering	$x\ \mathrm{erf}\left(bx\right)+\frac{\mathrm{e}^{-b^{2} x^{2}}}{\sqrt{\pi}\, b}$	24
derivativedivides	$\frac{\mathrm{erf}(bx) bx+\frac{\mathrm{e}^{-b^{2} x^{2}}}{\sqrt{\pi}}}{b}$	26
default	$\frac{\mathrm{erf}(bx) bx+\frac{\mathrm{e}^{-b^{2} x^{2}}}{\sqrt{\pi}}}{b}$	26
parallelrisch	$\frac{bx\ \mathrm{erf}(bx)\sqrt{\pi}+\mathrm{e}^{-b^{2} x^{2}}}{\sqrt{\pi}\, b}$	28
meijerg	$\frac{-2+2\,\mathrm{e}^{-b^{2} x^{2}}+2bx\ \mathrm{erf}(bx)\sqrt{\pi}}{2\sqrt{\pi}\, b}$	33

input
```
int(erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
x*erf(b*x)+1/Pi^(1/2)/b*exp(-b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \mathrm{erf}(bx)\, dx = \frac{\pi bx\, \mathrm{erf}\left(bx\right)+\sqrt{\pi}e^{\left(-b^{2}x^{2}\right)}}{\pi b}$$

input
```
integrate(erf(b*x),x, algorithm="fricas")
```

output
```
(pi*b*x*erf(b*x) + sqrt(pi)*e^(-b^2*x^2))/(pi*b)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \operatorname{erf}(bx)\,dx = \begin{cases} x\operatorname{erf}(bx) + \frac{e^{-b^2x^2}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(erf(b*x),x)
```

output
```
Piecewise((x*erf(b*x) + exp(-b**2*x**2)/(sqrt(pi)*b), Ne(b, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \operatorname{erf}(bx)\,dx = \frac{bx\operatorname{erf}(bx) + \frac{e^{(-b^2x^2)}}{\sqrt{\pi}}}{b}$$

input
```
integrate(erf(b*x),x, algorithm="maxima")
```

output
```
(b*x*erf(b*x) + e^(-b^2*x^2)/sqrt(pi))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \operatorname{erf}(bx)\,dx = x\operatorname{erf}(bx) + \frac{e^{(-b^2x^2)}}{\sqrt{\pi}b}$$

input
```
integrate(erf(b*x),x, algorithm="giac")
```

output
```
x*erf(b*x) + e^(-b^2*x^2)/(sqrt(pi)*b)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \operatorname{erf}(bx)\,dx = x\operatorname{erf}(b\,x) + \frac{\mathrm{e}^{-b^2\,x^2}}{b\,\sqrt{\pi}}$$

input
```
int(erf(b*x),x)
```

output
```
x*erf(b*x) + exp(-b^2*x^2)/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \operatorname{erf}(bx)\,dx = \frac{\sqrt{\pi}\,e^{b^2x^2}\operatorname{erf}(bx)\,bx + 1}{\sqrt{\pi}\,e^{b^2x^2}b}$$

input
```
int(erf(b*x),x)
```

output
```
(sqrt(pi)*e**(b**2*x**2)*erf(b*x)*b*x + 1)/(sqrt(pi)*e**(b**2*x**2)*b)
```

3.12 $\int \frac{\operatorname{erf}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = -\frac{\operatorname{erf}(bx)}{x} + \frac{b\,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-erf(b*x)/x+b*Ei(-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = -\frac{\operatorname{erf}(bx)}{x} + \frac{b\,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]/x^2,x]
```

output
```
-(Erf[b*x]/x) + (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6915, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6915}$$

$$\frac{2b \int \frac{e^{-b^2x^2}}{x}dx}{\sqrt{\pi}} - \frac{\text{erf}(bx)}{x}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{b \,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)}{x}$$

input
```
Int[Erf[b*x]/x^2,x]
```

output
```
-(Erf[b*x]/x) + (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
parts	$-\frac{\operatorname{erf}(bx)}{x}-\frac{b\,\mathrm{expIntegral}_1(b^2x^2)}{\sqrt{\pi}}$	26
derivativedivides	$b\left(-\frac{\operatorname{erf}(bx)}{bx}-\frac{\mathrm{expIntegral}_1(b^2x^2)}{\sqrt{\pi}}\right)$	30
default	$b\left(-\frac{\operatorname{erf}(bx)}{bx}-\frac{\mathrm{expIntegral}_1(b^2x^2)}{\sqrt{\pi}}\right)$	30
meijerg	$\frac{b\left(2\gamma-4+4\ln(x)+4\ln(b)-\frac{2b^2x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{3}{2}\right],\left[2,2,\frac{5}{2}\right],-b^2x^2\right)}{3}\right)}{2\sqrt{\pi}}$	45

input
```
int(erf(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-erf(b*x)/x-1/Pi^(1/2)*b*Ei(1,b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx=\frac{\sqrt{\pi}bx\mathrm{Ei}(-b^2x^2)-\pi\operatorname{erf}(bx)}{\pi x}$$

input
```
integrate(erf(b*x)/x^2,x, algorithm="fricas")
```

output
```
(sqrt(pi)*b*x*Ei(-b^2*x^2) - pi*erf(b*x))/(pi*x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = -\frac{b\,\mathrm{E}_1\left(b^2x^2\right)}{\sqrt{\pi}} + \frac{\operatorname{erfc}(bx)}{x} - \frac{1}{x}$$

input
```
integrate(erf(b*x)/x**2,x)
```

output
```
-b*expint(1, b**2*x**2)/sqrt(pi) + erfc(b*x)/x - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = \frac{b\mathrm{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{x}$$

input
```
integrate(erf(b*x)/x^2,x, algorithm="maxima")
```

output
```
b*Ei(-b^2*x^2)/sqrt(pi) - erf(b*x)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = \frac{b\mathrm{Ei}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{x}$$

input
```
integrate(erf(b*x)/x^2,x, algorithm="giac")
```

output
```
b*Ei(-b^2*x^2)/sqrt(pi) - erf(b*x)/x
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = \frac{b\,\operatorname{ei}(-b^2\,x^2)}{\sqrt{\pi}} - \frac{\operatorname{erf}(b\,x)}{x}$$

input
```
int(erf(b*x)/x^2,x)
```

output
```
(b*ei(-b^2*x^2))/pi^(1/2) - erf(b*x)/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erf}(bx)}{x^2}\,dx = \frac{\sqrt{\pi}\,ei(-b^2x^2)\,bx - \operatorname{erf}(bx)\,\pi}{\pi x}$$

input
```
int(erf(b*x)/x^2,x)
```

output
```
(sqrt(pi)*ei( - b**2*x**2)*b*x - erf(b*x)*pi)/(pi*x)
```

3.13 $\int \frac{\operatorname{erf}(bx)}{x^4} dx$

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{\operatorname{erf}(bx)}{x^4} dx = -\frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\operatorname{erf}(bx)}{3x^3} - \frac{b^3 \operatorname{ExpIntegralEi}(-b^2x^2)}{3\sqrt{\pi}}$$

output
```
-1/3*b/exp(b^2*x^2)/Pi^(1/2)/x^2-1/3*erf(b*x)/x^3-1/3*b^3*Ei(-b^2*x^2)/Pi^
(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{erf}(bx)}{x^4} dx = -\frac{\operatorname{erf}(bx) + \frac{bx\left(e^{-b^2x^2}+b^2x^2 \operatorname{ExpIntegralEi}(-b^2x^2)\right)}{\sqrt{\pi}}}{3x^3}$$

input
```
Integrate[Erf[b*x]/x^4,x]
```

output
```
-1/3*(Erf[b*x] + (b*x*(E^(-(b^2*x^2)) + b^2*x^2*ExpIntegralEi[-(b^2*x^2)])
)/Sqrt[Pi])/x^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6915, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(bx)}{x^4} \, dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{2b \int \frac{e^{-b^2x^2}}{x^3} dx}{3\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2b \left(b^2 \left(-\int \frac{e^{-b^2x^2}}{x} dx\right) - \frac{e^{-b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2b \left(-\frac{1}{2} b^2 \operatorname{ExpIntegralEi}\left(-b^2x^2\right) - \frac{e^{-b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{3x^3}$$

input
```
Int[Erf[b*x]/x^4,x]
```

output
```
-1/3*Erf[b*x]/x^3 + (2*b*(-1/2*1/(E^(b^2*x^2)*x^2) - (b^2*ExpIntegralEi[-(
b^2*x^2)])/2))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
parts	$-\frac{\operatorname{erf}(bx)}{3x^3}+\frac{2b\left(-\frac{e^{-b^2x^2}}{2x^2}+\frac{b^2\,\mathrm{expIntegral}_1\left(b^2x^2\right)}{2}\right)}{3\sqrt{\pi}}$	46
derivativedivides	$b^3\left(-\frac{\operatorname{erf}(bx)}{3b^3x^3}+\frac{-\frac{e^{-b^2x^2}}{3x^2b^2}+\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{3}}{\sqrt{\pi}}\right)$	53
default	$b^3\left(-\frac{\operatorname{erf}(bx)}{3b^3x^3}+\frac{-\frac{e^{-b^2x^2}}{3x^2b^2}+\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{3}}{\sqrt{\pi}}\right)$	53
meijerg	$\frac{b^3\left(-\frac{2}{b^2x^2}+\frac{10}{9}-\frac{2\gamma}{3}-\frac{4\ln(x)}{3}-\frac{4\ln(b)}{3}+\frac{b^2x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{2}\right],\left[2,3,\frac{7}{2}\right],-b^2x^2\right)}{5}\right)}{2\sqrt{\pi}}$	55

input
```
int(erf(b*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*erf(b*x)/x^3+2/3/Pi^(1/2)*b*(-1/2/x^2*exp(-b^2*x^2)+1/2*b^2*Ei(1,b^2*
x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\text{erf}(bx)}{x^4}\,dx = -\frac{\pi\,\text{erf}\,(bx) + \sqrt{\pi}\left(b^3x^3\text{Ei}(-b^2x^2) + bxe^{(-b^2x^2)}\right)}{3\,\pi x^3}$$

input
```
integrate(erf(b*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(pi*erf(b*x) + sqrt(pi)*(b^3*x^3*Ei(-b^2*x^2) + b*x*e^(-b^2*x^2)))/(p
i*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{\text{erf}(bx)}{x^4}\,dx = \frac{b^3\,\text{E}_1\,(b^2x^2)}{3\sqrt{\pi}} - \frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} + \frac{\text{erfc}\,(bx)}{3x^3} - \frac{1}{3x^3}$$

input
```
integrate(erf(b*x)/x**4,x)
```

output
```
b**3*expint(1, b**2*x**2)/(3*sqrt(pi)) - b*exp(-b**2*x**2)/(3*sqrt(pi)*x**
2) + erfc(b*x)/(3*x**3) - 1/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{erf}(bx)}{x^4}\,dx = -\frac{b^3\Gamma\left(-1, b^2x^2\right)}{3\sqrt{\pi}} - \frac{\operatorname{erf}\left(bx\right)}{3\,x^3}$$

input
```
integrate(erf(b*x)/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^3*gamma(-1, b^2*x^2)/sqrt(pi) - 1/3*erf(b*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erf}(bx)}{x^4}\,dx = -\frac{\left(b^2x^2\operatorname{Ei}\left(-b^2x^2\right) + e^{\left(-b^2x^2\right)}\right)b}{3\sqrt{\pi}x^2} - \frac{\operatorname{erf}\left(bx\right)}{3\,x^3}$$

input
```
integrate(erf(b*x)/x^4,x, algorithm="giac")
```

output
```
-1/3*(b^2*x^2*Ei(-b^2*x^2) + e^(-b^2*x^2))*b/(sqrt(pi)*x^2) - 1/3*erf(b*x)
/x^3
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erf}(bx)}{x^4}\,dx = -\frac{\operatorname{erf}(b\,x)}{3\,x^3} - \frac{b^3\operatorname{ei}\left(-b^2\,x^2\right)}{3\sqrt{\pi}} - \frac{b\,\mathrm{e}^{-b^2\,x^2}}{3\,x^2\sqrt{\pi}}$$

input
```
int(erf(b*x)/x^4,x)
```

output
```
- erf(b*x)/(3*x^3) - (b^3*ei(-b^2*x^2))/(3*pi^(1/2)) - (b*exp(-b^2*x^2))/(
3*x^2*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{erf}(bx)}{x^4}\,dx = \frac{-\sqrt{\pi}\,e^{b^2x^2}\,ei(-b^2x^2)\,b^3x^3 - e^{b^2x^2}\operatorname{erf}(bx)\,\pi - \sqrt{\pi}\,bx}{3e^{b^2x^2}\pi\,x^3}$$

input
```
int(erf(b*x)/x^4,x)
```

output
```
( - (sqrt(pi)*e**(b**2*x**2)*ei( - b**2*x**2)*b**3*x**3 + e**(b**2*x**2)*e
rf(b*x)*pi + sqrt(pi)*b*x))/(3*e**(b**2*x**2)*pi*x**3)
```

3.14 $\int \frac{\text{erf}(bx)}{x^6} dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int \frac{\text{erf}(bx)}{x^6} dx = -\frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} + \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} - \frac{\text{erf}(bx)}{5x^5} + \frac{b^5 \text{ExpIntegralEi}(-b^2x^2)}{10\sqrt{\pi}}$$

output
```
-1/10*b/exp(b^2*x^2)/Pi^(1/2)/x^4+1/10*b^3/exp(b^2*x^2)/Pi^(1/2)/x^2-1/5*e
rf(b*x)/x^5+1/10*b^5*Ei(-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{\text{erf}(bx)}{x^6} dx = \frac{be^{-b^2x^2}x(-1+b^2x^2) - 2\sqrt{\pi}\text{erf}(bx) + b^5x^5 \text{ExpIntegralEi}(-b^2x^2)}{10\sqrt{\pi}x^5}$$

input
```
Integrate[Erf[b*x]/x^6,x]
```

output
```
((b*x*(-1 + b^2*x^2))/E^(b^2*x^2) - 2*Sqrt[Pi]*Erf[b*x] + b^5*x^5*ExpInteg
ralEi[-(b^2*x^2)])/(10*Sqrt[Pi]*x^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6915, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)}{x^6} \, dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{2b \int \frac{e^{-b^2x^2}}{x^5}dx}{5\sqrt{\pi}} - \frac{\text{erf}(bx)}{5x^5}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2b\left(-\frac{1}{2}b^2 \int \frac{e^{-b^2x^2}}{x^3}dx - \frac{e^{-b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erf}(bx)}{5x^5}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2b\left(-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}}{x}dx\right) - \frac{e^{-b^2x^2}}{2x^2}\right) - \frac{e^{-b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erf}(bx)}{5x^5}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2b\left(-\frac{1}{2}b^2\left(-\frac{1}{2}b^2 \text{ExpIntegralEi}\left(-b^2x^2\right) - \frac{e^{-b^2x^2}}{2x^2}\right) - \frac{e^{-b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erf}(bx)}{5x^5}$$

input
```
Int[Erf[b*x]/x^6,x]
```

output
```
-1/5*Erf[b*x]/x^5 + (2*b*(-1/4*1/(E^(b^2*x^2)*x^4) - (b^2*(-1/2*1/(E^(b^2*
x^2)*x^2) - (b^2*ExpIntegralEi[-(b^2*x^2)])/2))/2))/(5*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

method	result	size
meijerg	$\frac{b^{5}\left(-\frac{1}{b^{4}x^{4}}+\frac{2}{3b^{2}x^{2}}-\frac{19}{50}+\frac{\gamma}{5}+\frac{2\ln(x)}{5}+\frac{2\ln(b)}{5}-\frac{b^{2}x^{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{2}\right],\left[2,4,\frac{9}{2}\right],-b^{2}x^{2}\right)}{21}\right)}{2\sqrt{\pi}}$	63
parts	$-\frac{\mathrm{erf}(bx)}{5x^{5}}+\frac{2b\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}}{4x^{4}}-\frac{b^{2}\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}}{2x^{2}}+\frac{b^{2}\,\mathrm{expIntegral}_{1}\left(b^{2}x^{2}\right)}{2}\right)}{2}\right)}{5\sqrt{\pi}}$	66
derivativedivides	$b^{5}\left(-\frac{\mathrm{erf}(bx)}{5b^{5}x^{5}}+\frac{-\frac{\mathrm{e}^{-b^{2}x^{2}}}{10b^{4}x^{4}}+\frac{\mathrm{e}^{-b^{2}x^{2}}}{10x^{2}b^{2}}-\frac{\mathrm{expIntegral}_{1}\left(b^{2}x^{2}\right)}{10}}{\sqrt{\pi}}\right)$	71
default	$b^{5}\left(-\frac{\mathrm{erf}(bx)}{5b^{5}x^{5}}+\frac{-\frac{\mathrm{e}^{-b^{2}x^{2}}}{10b^{4}x^{4}}+\frac{\mathrm{e}^{-b^{2}x^{2}}}{10x^{2}b^{2}}-\frac{\mathrm{expIntegral}_{1}\left(b^{2}x^{2}\right)}{10}}{\sqrt{\pi}}\right)$	71

input
```
int(erf(b*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2/Pi^(1/2)*b^5*(-1/b^4/x^4+2/3/b^2/x^2-19/50+1/5*gamma+2/5*ln(x)+2/5*ln(
b)-1/21*b^2*x^2*hypergeom([1,1,7/2],[2,4,9/2],-b^2*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{erf}(bx)}{x^6}\,dx = -\frac{2\,\pi\operatorname{erf}(bx) - \sqrt{\pi}\left(b^5x^5\operatorname{Ei}(-b^2x^2) + (b^3x^3 - bx)e^{\left(-b^2x^2\right)}\right)}{10\,\pi x^5}$$

input
```
integrate(erf(b*x)/x^6,x, algorithm="fricas")
```

output
```
-1/10*(2*pi*erf(b*x) - sqrt(pi)*(b^5*x^5*Ei(-b^2*x^2) + (b^3*x^3 - b*x)*e^
(-b^2*x^2)))/(pi*x^5)
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{erf}(bx)}{x^6}\,dx = -\frac{b^5\operatorname{E}_1\left(b^2x^2\right)}{10\sqrt{\pi}} + \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} - \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} + \frac{\operatorname{erfc}(bx)}{5x^5} - \frac{1}{5x^5}$$

input
```
integrate(erf(b*x)/x**6,x)
```

output
```
-b**5*expint(1, b**2*x**2)/(10*sqrt(pi)) + b**3*exp(-b**2*x**2)/(10*sqrt(p
i)*x**2) - b*exp(-b**2*x**2)/(10*sqrt(pi)*x**4) + erfc(b*x)/(5*x**5) - 1/(
5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.33

$$\int \frac{\operatorname{erf}(bx)}{x^6} \, dx = -\frac{b^5\Gamma(-2, b^2x^2)}{5\sqrt{\pi}} - \frac{\operatorname{erf}(bx)}{5x^5}$$

input
```
integrate(erf(b*x)/x^6,x, algorithm="maxima")
```

output
```
-1/5*b^5*gamma(-2, b^2*x^2)/sqrt(pi) - 1/5*erf(b*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{erf}(bx)}{x^6} \, dx = -\frac{\operatorname{erf}(bx)}{5x^5} + \frac{b^{10}x^4\operatorname{Ei}(-b^2x^2) + b^8x^2e^{(-b^2x^2)} - b^6e^{(-b^2x^2)}}{10\sqrt{\pi}b^5x^4}$$

input
```
integrate(erf(b*x)/x^6,x, algorithm="giac")
```

output
```
-1/5*erf(b*x)/x^5 + 1/10*(b^10*x^4*Ei(-b^2*x^2) + b^8*x^2*e^(-b^2*x^2) - b
^6*e^(-b^2*x^2))/(sqrt(pi)*b^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erf}(bx)}{x^6} \, dx = \frac{b^5 \operatorname{ei}(-b^2 x^2)}{10\sqrt{\pi}} - \frac{\operatorname{erf}(b x)}{5x^5} - \frac{\frac{b\,e^{-b^2 x^2}}{2} - \frac{b^3 x^2 e^{-b^2 x^2}}{2}}{5x^4\sqrt{\pi}}$$

input
```
int(erf(b*x)/x^6,x)
```

output
```
(b^5*ei(-b^2*x^2))/(10*pi^(1/2)) - erf(b*x)/(5*x^5) - ((b*exp(-b^2*x^2))/2
 - (b^3*x^2*exp(-b^2*x^2))/2)/(5*x^4*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{erf}(bx)}{x^6}\,dx = \frac{\sqrt{\pi}\,e^{b^2x^2}\,ei(-b^2x^2)\,b^5x^5 - 2e^{b^2x^2}\operatorname{erf}(bx)\,\pi + \sqrt{\pi}\,b^3x^3 - \sqrt{\pi}\,bx}{10e^{b^2x^2}\pi\,x^5}$$

input
```
int(erf(b*x)/x^6,x)
```

output
```
(sqrt(pi)*e**(b**2*x**2)*ei( - b**2*x**2)*b**5*x**5 - 2*e**(b**2*x**2)*erf
(b*x)*pi + sqrt(pi)*b**3*x**3 - sqrt(pi)*b*x)/(10*e**(b**2*x**2)*pi*x**5)
```

3.15 $\int (c+dx)^3 \mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 289

$$\begin{aligned}\int (c+dx)^3\operatorname{erf}(a+bx)\,dx &= \frac{d^2(bc-ad)e^{-(a+bx)^2}}{b^4\sqrt{\pi}}+\frac{(bc-ad)^3e^{-(a+bx)^2}}{b^4\sqrt{\pi}}\\ &+\frac{3d^3e^{-(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}}+\frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}}\\ &+\frac{d^2(bc-ad)e^{-(a+bx)^2}(a+bx)^2}{b^4\sqrt{\pi}}+\frac{d^3e^{-(a+bx)^2}(a+bx)^3}{4b^4\sqrt{\pi}}\\ &-\frac{3d^3\operatorname{erf}(a+bx)}{16b^4}-\frac{3d(bc-ad)^2\operatorname{erf}(a+bx)}{4b^4}\\ &-\frac{(bc-ad)^4\operatorname{erf}(a+bx)}{4b^4d}+\frac{(c+dx)^4\operatorname{erf}(a+bx)}{4d}\end{aligned}$$

output
```
d^2*(-a*d+b*c)/b^4/exp((b*x+a)^2)/Pi^(1/2)+(-a*d+b*c)^3/b^4/exp((b*x+a)^2)
/Pi^(1/2)+3/8*d^3*(b*x+a)/b^4/exp((b*x+a)^2)/Pi^(1/2)+3/2*d*(-a*d+b*c)^2*(
b*x+a)/b^4/exp((b*x+a)^2)/Pi^(1/2)+d^2*(-a*d+b*c)*(b*x+a)^2/b^4/exp((b*x+a
)^2)/Pi^(1/2)+1/4*d^3*(b*x+a)^3/b^4/exp((b*x+a)^2)/Pi^(1/2)-3/16*d^3*erf(b
*x+a)/b^4-3/4*d*(-a*d+b*c)^2*erf(b*x+a)/b^4-1/4*(-a*d+b*c)^4*erf(b*x+a)/b^
4/d+1/4*(d*x+c)^4*erf(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$\int (c+dx)^3\text{erf}(a+bx)\,dx$$
$$=\frac{e^{-(a+bx)^2}\left(-2a(5+2a^2)\,d^3+2bd^2(8(1+a^2)\,c+(3+2a^2)\,dx)-4ab^2d(6c^2+4cdx+d^2x^2)+4b^3(4c^3+6\right.}{}$$

input
```
Integrate[(c + d*x)^3*Erf[a + b*x],x]
```

output
```
(-2*a*(5 + 2*a^2)*d^3 + 2*b*d^2*(8*(1 + a^2)*c + (3 + 2*a^2)*d*x) - 4*a*b^
2*d*(6*c^2 + 4*c*d*x + d^2*x^2) + 4*b^3*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 +
 d^3*x^3) - E^(a + b*x)^2*Sqrt[Pi]*(12*b^2*c^2*d - 16*a^3*b*c*d^2 + 3*d^3
+ 4*a^4*d^3 - 8*a*(2*b^3*c^3 + 3*b*c*d^2) + 12*a^2*(2*b^2*c^2*d + d^3) - 4
*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Erf[a + b*x])/(16*b^4*
E^(a + b*x)^2*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6915, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{erf}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6915}$$

$$\frac{(c+dx)^4\text{erf}(a+bx)}{4d}-\frac{b\int e^{-(a+bx)^2}(c+dx)^4dx}{2\sqrt{\pi}d}$$

$$\Big\downarrow \text{ 2656}$$

$$\frac{(c+dx)^4\text{erf}(a+bx)}{4d}-$$
$$\frac{b\int\left(\frac{e^{-(a+bx)^2}(bc-ad)^4}{b^4}+\frac{4de^{-(a+bx)^2}(a+bx)(bc-ad)^3}{b^4}+\frac{6d^2e^{-(a+bx)^2}(a+bx)^2(bc-ad)^2}{b^4}+\frac{4d^3e^{-(a+bx)^2}(a+bx)^3(bc-ad)}{b^4}+\frac{d^4e^{-(a+}\dots\right.}{2\sqrt{\pi}d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^4\text{erf}(a+bx)}{4d}-$$
$$\frac{b\left(-\frac{2d^3e^{-(a+bx)^2}(bc-ad)}{b^5}-\frac{2d^3e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{b^5}+\frac{3\sqrt{\pi}d^2(bc-ad)^2\text{erf}(a+bx)}{2b^5}-\frac{3d^2e^{-(a+bx)^2}(a+bx)(bc-ad)^2}{b^5}+\frac{\sqrt{\pi}(bc-ad}{2}\dots\right.}{2\sqrt{\pi}d}$$

input
```
Int[(c + d*x)^3*Erf[a + b*x],x]
```

output
```
((c + d*x)^4*Erf[a + b*x])/(4*d) - (b*((-2*d^3*(b*c - a*d))/(b^5*E^(a + b*
x)^2) - (2*d*(b*c - a*d)^3)/(b^5*E^(a + b*x)^2) - (3*d^4*(a + b*x))/(4*b^5
*E^(a + b*x)^2) - (3*d^2*(b*c - a*d)^2*(a + b*x))/(b^5*E^(a + b*x)^2) - (2
*d^3*(b*c - a*d)*(a + b*x)^2)/(b^5*E^(a + b*x)^2) - (d^4*(a + b*x)^3)/(2*b
^5*E^(a + b*x)^2) + (3*d^4*Sqrt[Pi]*Erf[a + b*x])/(8*b^5) + (3*d^2*(b*c -
a*d)^2*Sqrt[Pi]*Erf[a + b*x])/(2*b^5) + ((b*c - a*d)^4*Sqrt[Pi]*Erf[a + b*
x])/(2*b^5)))/(2*d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{\frac{\operatorname{erf}(bx+a)(ad-cb-d(bx+a))^4}{4b^3d} - \frac{\frac{a^4d^4\sqrt{\pi}\operatorname{erf}(bx+a)}{2} + \frac{b^4c^4\sqrt{\pi}\operatorname{erf}(bx+a)}{2} + d^4\left(-\frac{e^{-(bx+a)^2}(bx+a)^3}{2} - \frac{3(bx+a)e^{-(bx+a)^2}}{4} + \frac{3\sqrt{\pi}}{}\right.}{}}{}$
default	$\frac{\frac{\operatorname{erf}(bx+a)(ad-cb-d(bx+a))^4}{4b^3d} - \frac{\frac{a^4d^4\sqrt{\pi}\operatorname{erf}(bx+a)}{2} + \frac{b^4c^4\sqrt{\pi}\operatorname{erf}(bx+a)}{2} + d^4\left(-\frac{e^{-(bx+a)^2}(bx+a)^3}{2} - \frac{3(bx+a)e^{-(bx+a)^2}}{4} + \frac{3\sqrt{\pi}}{}\right.}{}}{}$
orering	$-\frac{(-4b^5d^4x^5-20b^5cd^3x^4-40b^5c^2d^2x^3-40b^5c^3dx^2+4a^4bd^4x-16a^3b^2cd^3x+24a^2b^3c^2d^2x-16ab^4c^3dx-16b^5c^4x+6}{}$
parallelrisch	$\frac{-16x e^{-(bx+a)^2} a b^2 c d^2+16d^2c\operatorname{erf}(bx+a)x^3\sqrt{\pi}b^4+24c^2d\operatorname{erf}(bx+a)x^2\sqrt{\pi}b^4+16\sqrt{\pi}\operatorname{erf}(bx+a)a^3bcd^2-24\sqrt{\pi}\operatorname{erf}(b}{}$
parts	$\frac{\operatorname{erf}(bx+a)d^3x^4}{4} + \operatorname{erf}(bx+a)d^2cx^3 + \frac{3\operatorname{erf}(bx+a)dc^2x^2}{2} + \operatorname{erf}(bx+a)c^3x + \frac{\operatorname{erf}(bx+a)c^4}{4d} - \frac{b\left(\right.}{}$

input
```
int((d*x+c)^3*erf(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*erf(b*x+a)*(a*d-c*b-d*(b*x+a))^4/b^3/d-1/2/Pi^(1/2)/b^3/d*(1/2*a^
4*d^4*Pi^(1/2)*erf(b*x+a)+1/2*b^4*c^4*Pi^(1/2)*erf(b*x+a)+d^4*(-1/2/exp((b
*x+a)^2)*(b*x+a)^3-3/4*(b*x+a)/exp((b*x+a)^2)+3/8*Pi^(1/2)*erf(b*x+a))-4*a
*d^4*(-1/2/exp((b*x+a)^2)*(b*x+a)^2-1/2/exp((b*x+a)^2))+6*a^2*d^4*(-1/2*(b
*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))+2*a^3*d^4/exp((b*x+a)^2)-2*a
*b^3*c^3*d*Pi^(1/2)*erf(b*x+a)+3*a^2*b^2*c^2*d^2*Pi^(1/2)*erf(b*x+a)-2*a^3
*b*c*d^3*Pi^(1/2)*erf(b*x+a)+4*b*c*d^3*(-1/2/exp((b*x+a)^2)*(b*x+a)^2-1/2/
exp((b*x+a)^2))+6*b^2*c^2*d^2*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*er
f(b*x+a))-2*b^3*c^3*d/exp((b*x+a)^2)-12*a*b*c*d^3*(-1/2*(b*x+a)/exp((b*x+a
)^2)+1/4*Pi^(1/2)*erf(b*x+a))+6*a*b^2*c^2*d^2/exp((b*x+a)^2)-6*a^2*b*c*d^3
/exp((b*x+a)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int (c+dx)^3 \operatorname{erf}(a+bx)\,dx$$
$$= \frac{2\sqrt{\pi}(2\,b^3d^3x^3 + 8\,b^3c^3 - 12\,ab^2c^2d + 8\,(a^2+1)bcd^2 - (2\,a^3+5\,a)d^3 + 2\,(4\,b^3cd^2 - ab^2d^3)x^2 + (12\,b^3c^2d}{}$$

input
```
integrate((d*x+c)^3*erf(b*x+a),x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(pi)*(2*b^3*d^3*x^3 + 8*b^3*c^3 - 12*a*b^2*c^2*d + 8*(a^2 + 1)
*b*c*d^2 - (2*a^3 + 5*a)*d^3 + 2*(4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (12*b^3*c
^2*d - 8*a*b^2*c*d^2 + (2*a^2 + 3)*b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x - a^2)
+ (4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi*b^
4*c^3*x + pi*(16*a*b^3*c^3 - 12*(2*a^2 + 1)*b^2*c^2*d + 8*(2*a^3 + 3*a)*b*
c*d^2 - (4*a^4 + 12*a^2 + 3)*d^3))*erf(b*x + a))/(pi*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(258) = 516$.

Time = 2.21 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.58

$$\int (c+dx)^3\text{erf}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*erf(b*x+a),x)
```

output
```
Piecewise((-a**4*d**3*erf(a + b*x)/(4*b**4) + a**3*c*d**2*erf(a + b*x)/b**
3 - a**3*d**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b**4) -
 3*a**2*c**2*d*erf(a + b*x)/(2*b**2) + a**2*c*d**2*exp(-a**2)*exp(-b**2*x*
*2)*exp(-2*a*b*x)/(sqrt(pi)*b**3) + a**2*d**3*x*exp(-a**2)*exp(-b**2*x**2)
*exp(-2*a*b*x)/(4*sqrt(pi)*b**3) - 3*a**2*d**3*erf(a + b*x)/(4*b**4) + a*c
**3*erf(a + b*x)/b - 3*a*c**2*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(
2*sqrt(pi)*b**2) - a*c*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sq
rt(pi)*b**2) - a*d**3*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqr
t(pi)*b**2) + 3*a*c*d**2*erf(a + b*x)/(2*b**3) - 5*a*d**3*exp(-a**2)*exp(-
b**2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b**4) + c**3*x*erf(a + b*x) + 3*c**2*
d*x**2*erf(a + b*x)/2 + c*d**2*x**3*erf(a + b*x) + d**3*x**4*erf(a + b*x)/
4 + c**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) + 3*c**2*d*
x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) + c*d**2*x**2*ex
p(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) + d**3*x**3*exp(-a**2)
*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b) - 3*c**2*d*erf(a + b*x)/(4*b
**2) + c*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b**3) + 3
*d**3*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b**3) - 3*d**
3*erf(a + b*x)/(16*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x
**3 + d**3*x**4/4)*erf(a), True))
```

Maxima [F]

$$\int (c+dx)^3\text{erf}(a+bx)\,dx = \int (dx+c)^3\,\text{erf}\,(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*erf(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(d^3*x^4 + 4*c*d^2*x^3 + 6*c^2*d*x^2 + 4*c^3*x)*erf(b*x + a) - 1/2*int
egrate((b*d^3*x^4 + 4*b*c*d^2*x^3 + 6*b*c^2*d*x^2 + 4*b*c^3*x)*e^(-b^2*x^2
 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.35

$$\int (c+dx)^3\text{erf}(a+bx)\,dx = \frac{(dx+c)^4\,\text{erf}\,(bx+a)}{4\,d} + \frac{4\sqrt{\pi}c^4\,\text{erf}\,\left(-b\left(x+\frac{a}{b}\right)\right) - 16\left(\frac{\sqrt{\pi}a\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)c^3d + \frac{12\left(\frac{\sqrt{\pi}\left(2\,a^2+1\right)\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\right.\right.}{}\right.}{b}}{}$$

input
```
integrate((d*x+c)^3*erf(b*x+a),x, algorithm="giac")
```

output
```
1/4*(d*x + c)^4*erf(b*x + a)/d + 1/16*(4*sqrt(pi)*c^4*erf(-b*(x + a/b)) -
16*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c^3*d
 + 12*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^
(-b^2*x^2 - 2*a*b*x - a^2)/b)*c^2*d^2/b - 8*(sqrt(pi)*(2*a^3 + 3*a)*erf(-b
*(x + a/b))/b - 2*(b^2*(x + a/b)^2 - 3*a*b*(x + a/b) + 3*a^2 + 1)*e^(-b^2*
x^2 - 2*a*b*x - a^2)/b)*c*d^3/b^2 + (sqrt(pi)*(4*a^4 + 12*a^2 + 3)*erf(-b*
(x + a/b))/b + 2*(2*b^3*(x + a/b)^3 - 8*a*b^2*(x + a/b)^2 + 12*a^2*b*(x +
a/b) - 8*a^3 + 3*b*(x + a/b) - 8*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*d^4/b^
3)/(sqrt(pi)*d)
```

Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.17

$$\int (c+dx)^3\text{erf}(a+bx)\,dx = \text{erf}(a+b\,x)\,\left(c^3\,x+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{d^4\,x^4}{4}\right) - \frac{\frac{e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,\left(\frac{5\,a\,d^3}{4}+\frac{a^3\,d^3}{2}-2\,b^3\,c^3-b\,\left(2\,c\,a^2\,d^2+2\,c\,d^2\right)+3\,a\,b^2\,c^2\,d\right)}{b^4} - \frac{x\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,\left(2\,a^2\,d^3-8\,a\,b\,c\,d^2+12\,b^2\,c^2\,d+3\,d^3\right)}{4\,b^3} - }{2\,\sqrt{\pi}} + \frac{\text{erfi}(a\,1\text{i}+b\,x\,1\text{i})\,\left(4\,a^4\,d^3-16\,a^3\,b\,c\,d^2+24\,a^2\,b^2\,c^2\,d+12\,a^2\,d^3-16\,a\,b^3\,c^3-24\,a\,b\,c\,d^2+12\,b^2\,c^2\,d+\right.}{16\,b^4}$$

input
```
int(erf(a + b*x)*(c + d*x)^3,x)
```

output
```
erf(a + b*x)*(c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3) - ((exp(-
 a^2 - b^2*x^2 - 2*a*b*x)*((5*a*d^3)/4 + (a^3*d^3)/2 - 2*b^3*c^3 - b*(2*c*
d^2 + 2*a^2*c*d^2) + 3*a*b^2*c^2*d))/b^4 - (x*exp(- a^2 - b^2*x^2 - 2*a*b*
x)*(3*d^3 + 2*a^2*d^3 + 12*b^2*c^2*d - 8*a*b*c*d^2))/(4*b^3) - (d^3*x^3*ex
p(- a^2 - b^2*x^2 - 2*a*b*x))/(2*b) + (x^2*exp(- a^2 - b^2*x^2 - 2*a*b*x)*
(a*d^3 - 4*b*c*d^2))/(2*b^2))/(2*pi^(1/2)) + (erfi(a*1i + b*x*1i)*(3*d^3 +
 12*a^2*d^3 + 4*a^4*d^3 - 16*a*b^3*c^3 + 12*b^2*c^2*d + 24*a^2*b^2*c^2*d -
 24*a*b*c*d^2 - 16*a^3*b*c*d^2)*1i)/(16*b^4)
```

Reduce [F]

$$\int (c+dx)^3 \operatorname{erf}(a+bx)\,dx = \frac{\sqrt{\pi}\, e^{b^2x^2+2abx+a^2} \operatorname{erf}(bx+a)\, a\, c^3 + \sqrt{\pi}\, e^{b^2x^2+2abx+a^2} \operatorname{erf}(bx+a)\, b\, c^3 x + \sqrt{\pi}\, e^{b^2x^2+2abx+a^2} \left(\int \operatorname{erf}(bx+a)\, x^3 d\right.}{\sqrt{\pi}\, e^{b^2x^2-}}$$

input
```
int((d*x+c)^3*erf(b*x+a),x)
```

output
```
(sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c**3 + sqrt(pi)*e
**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c**3*x + sqrt(pi)*e**(a**2 +
 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**3,x)*b*d**3 + 3*sqrt(pi)*e**(a**
2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**2,x)*b*c*d**2 + 3*sqrt(pi)*e*
*(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*c**2*d + c**3)/(sqrt
(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.16 $\int (c+dx)^2 \mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 192

$$\begin{aligned}\int (c+dx)^2\operatorname{erf}(a+bx)\,dx = {} & \frac{d^2e^{-(a+bx)^2}}{3b^3\sqrt{\pi}}+\frac{(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}} \\ & +\frac{d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} \\ & +\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}}-\frac{d(bc-ad)\operatorname{erf}(a+bx)}{2b^3} \\ & -\frac{(bc-ad)^3\operatorname{erf}(a+bx)}{3b^3d}+\frac{(c+dx)^3\operatorname{erf}(a+bx)}{3d}\end{aligned}$$

output
```
1/3*d^2/b^3/exp((b*x+a)^2)/Pi^(1/2)+(-a*d+b*c)^2/b^3/exp((b*x+a)^2)/Pi^(1/
2)+d*(-a*d+b*c)*(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)+1/3*d^2*(b*x+a)^2/b^3/
exp((b*x+a)^2)/Pi^(1/2)-1/2*d*(-a*d+b*c)*erf(b*x+a)/b^3-1/3*(-a*d+b*c)^3*e
rf(b*x+a)/b^3/d+1/3*(d*x+c)^3*erf(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int (c+dx)^2\text{erf}(a+bx)\,dx$$
$$=\frac{\frac{2e^{-(a+bx)^2}\left((1+a^2)d^2-abd(3c+dx)+b^2(3c^2+3cdx+d^2x^2)\right)}{\sqrt{\pi}}+\left(-3bcd-6a^2bcd+2a^3d^2+3a(2b^2c^2+d^2)+2b^3x(3c^2+\right.}{6b^3}$$

input
```
Integrate[(c + d*x)^2*Erf[a + b*x],x]
```

output
```
((2*((1 + a^2)*d^2 - a*b*d*(3*c + d*x) + b^2*(3*c^2 + 3*c*d*x + d^2*x^2)))
/(E^(a + b*x)^2*Sqrt[Pi]) + (-3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + 3*a*(2*b
^2*c^2 + d^2) + 2*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Erf[a + b*x])/(6*b^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6915, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{erf}(a+bx)\,dx$$
$$\Big\downarrow \text{6915}$$
$$\frac{(c+dx)^3\text{erf}(a+bx)}{3d}-\frac{2b\int e^{-(a+bx)^2}(c+dx)^3dx}{3\sqrt{\pi}d}$$
$$\Big\downarrow \text{2656}$$
$$\frac{(c+dx)^3\text{erf}(a+bx)}{3d}-\frac{2b\int \left(\frac{e^{-(a+bx)^2}(bc-ad)^3}{b^3}+\frac{3de^{-(a+bx)^2}(a+bx)(bc-ad)^2}{b^3}+\frac{3d^2e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{b^3}+\frac{d^3e^{-(a+bx)^2}(a+bx)^3}{b^3}\right)dx}{3\sqrt{\pi}d}$$

$\downarrow$ 2009

$$\frac{(c+dx)^3\text{erf}(a+bx)}{3d}-\frac{2b\left(\frac{3\sqrt{\pi}d^2(bc-ad)\text{erf}(a+bx)}{4b^4}-\frac{3d^2e^{-(a+bx)^2}(a+bx)(bc-ad)}{2b^4}+\frac{\sqrt{\pi}(bc-ad)^3\text{erf}(a+bx)}{2b^4}-\frac{3de^{-(a+bx)^2}(bc-ad)^2}{2b^4}-\frac{d^3e^{-(a+bx)^2}}{2b^4}-d^3e\right.}{3\sqrt{\pi}d}$$

input
```
Int[(c + d*x)^2*Erf[a + b*x],x]
```

output
```
((c + d*x)^3*Erf[a + b*x])/(3*d) - (2*b*(-1/2*d^3/(b^4*E^(a + b*x)^2) - (3
*d*(b*c - a*d)^2)/(2*b^4*E^(a + b*x)^2) - (3*d^2*(b*c - a*d)*(a + b*x))/(2
*b^4*E^(a + b*x)^2) - (d^3*(a + b*x)^2)/(2*b^4*E^(a + b*x)^2) + (3*d^2*(b*
c - a*d)*Sqrt[Pi]*Erf[a + b*x])/(4*b^4) + ((b*c - a*d)^3*Sqrt[Pi]*Erf[a +
b*x])/(2*b^4)))/(3*d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.42

method	result
parallelrisch	$\frac{2d^2 \operatorname{erf}(bx+a)x^3\sqrt{\pi}\,b^3+6cd\,x^2\operatorname{erf}(bx+a)\sqrt{\pi}\,b^3+6c^2x\,\operatorname{erf}(bx+a)\sqrt{\pi}\,b^3+2\sqrt{\pi}\,\operatorname{erf}(bx+a)a^3d^2-6\sqrt{\pi}\,\operatorname{erf}(bx+a)a^2bcd+\cdots}{\cdots}$
derivativedivides	$\frac{-\frac{\operatorname{erf}(bx+a)(ad-cb-d(bx+a))^3}{3b^2d}+\frac{\frac{a^3d^3\sqrt{\pi}\,\operatorname{erf}(bx+a)}{3}-\frac{b^3c^3\sqrt{\pi}\,\operatorname{erf}(bx+a)}{3}-\frac{2d^3\left(-\frac{e^{-(bx+a)^2}(bx+a)^2}{2}-\frac{e^{-(bx+a)^2}}{2}\right)}{3}+2a\,d^3\left(\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{\operatorname{erf}(bx+a)(ad-cb-d(bx+a))^3}{3b^2d}+\frac{\frac{a^3d^3\sqrt{\pi}\,\operatorname{erf}(bx+a)}{3}-\frac{b^3c^3\sqrt{\pi}\,\operatorname{erf}(bx+a)}{3}-\frac{2d^3\left(-\frac{e^{-(bx+a)^2}(bx+a)^2}{2}-\frac{e^{-(bx+a)^2}}{2}\right)}{3}+2a\,d^3\left(\cdots\right.}{\cdots}}{\cdots}$
orering	$\frac{(2b^4d^3x^4+8b^4c\,d^2x^3+12b^4c^2d\,x^2+2a^3b\,d^3x-6a^2b^2c\,d^2x+6a\,b^3c^2dx+6b^4c^3x+2a^3bc\,d^2-6a^2b^2c^2d+6a\,b^3c^3-2b^2d^3x^2\cdots}{6b^4(dx+c)}$
parts	$\frac{\operatorname{erf}(bx+a)d^2x^3}{3}+\operatorname{erf}(bx+a)\,dc\,x^2+\operatorname{erf}(bx+a)\,c^2x+\frac{\operatorname{erf}(bx+a)c^3}{3d}-\frac{2b\left(\frac{c^3\sqrt{\pi}\,\operatorname{erf}(bx+a)}{2b}+e^{-a^2}d\cdots\right.}{\cdots}$

input
```
int((d*x+c)^2*erf(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*d^2*erf(b*x+a)*x^3*Pi^(1/2)*b^3+6*c*d*x^2*erf(b*x+a)*Pi^(1/2)*b^3+6
*c^2*x*erf(b*x+a)*Pi^(1/2)*b^3+2*Pi^(1/2)*erf(b*x+a)*a^3*d^2-6*Pi^(1/2)*er
f(b*x+a)*a^2*b*c*d+6*Pi^(1/2)*erf(b*x+a)*a*b^2*c^2+2*d^2*exp(-(b*x+a)^2)*x
^2*b^2-2*x*exp(-(b*x+a)^2)*a*b*d^2+6*x*exp(-(b*x+a)^2)*b^2*c*d+3*Pi^(1/2)*
erf(b*x+a)*a*d^2-3*Pi^(1/2)*erf(b*x+a)*b*c*d+2*exp(-(b*x+a)^2)*a^2*d^2-6*e
xp(-(b*x+a)^2)*a*b*c*d+6*exp(-(b*x+a)^2)*b^2*c^2+2*exp(-(b*x+a)^2)*d^2)/Pi
^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int (c+dx)^2 \operatorname{erf}(a+bx)\,dx$$
$$= \frac{2\sqrt{\pi}(b^2d^2x^2 + 3\,b^2c^2 - 3\,abcd + (a^2+1)d^2 + (3\,b^2cd - abd^2)x)e^{(-b^2x^2-2\,abx-a^2)} + (2\,\pi b^3d^2x^3 + 6\,\pi b^3cdx}{6\,\pi b^3}$$

input
```
integrate((d*x+c)^2*erf(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(pi)*(b^2*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 + 1)*d^2 + (3*
b^2*c*d - a*b*d^2)*x)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (2*pi*b^3*d^2*x^3 + 6
*pi*b^3*c*d*x^2 + 6*pi*b^3*c^2*x + pi*(6*a*b^2*c^2 - 3*(2*a^2 + 1)*b*c*d +
 (2*a^3 + 3*a)*d^2))*erf(b*x + a))/(pi*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(165) = 330$.

Time = 1.08 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.07

$$\int (c+dx)^2 \operatorname{erf}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3d^2\operatorname{erf}(a+bx)}{3b^3} - \frac{a^2cd\operatorname{erf}(a+bx)}{b^2} + \frac{a^2d^2e^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^3} + \frac{ac^2\operatorname{erf}(a+bx)}{b} - \frac{acde^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b^2} - \frac{ad^2xe^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\operatorname{erf}(a) \end{cases}$$

input
```
integrate((d*x+c)**2*erf(b*x+a),x)
```

output
```
Piecewise((a**3*d**2*erf(a + b*x)/(3*b**3) - a**2*c*d*erf(a + b*x)/b**2 +
a**2*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b**3) + a*c
**2*erf(a + b*x)/b - a*c*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(
pi)*b**2) - a*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*
b**2) + a*d**2*erf(a + b*x)/(2*b**3) + c**2*x*erf(a + b*x) + c*d*x**2*erf(
a + b*x) + d**2*x**3*erf(a + b*x)/3 + c**2*exp(-a**2)*exp(-b**2*x**2)*exp(
-2*a*b*x)/(sqrt(pi)*b) + c*d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(s
qrt(pi)*b) + d**2*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi
)*b) - c*d*erf(a + b*x)/(2*b**2) + d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*
a*b*x)/(3*sqrt(pi)*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*er
f(a), True))
```

Maxima [F]

$$\int (c+dx)^2\text{erf}(a+bx)\,dx=\int (dx+c)^2\,\text{erf}\,(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*erf(b*x+a),x, algorithm="maxima")
```

output
```
1/3*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)*erf(b*x + a) - 1/3*integrate(2*(b*d^2*
x^3 + 3*b*c*d*x^2 + 3*b*c^2*x)*e^(-b^2*x^2 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.37

$$\int (c+dx)^2\text{erf}(a+bx)\,dx=\frac{(dx+c)^3\,\text{erf}\,(bx+a)}{3\,d}+\frac{2\,\sqrt{\pi}c^3\,\text{erf}\,\left(-b\left(x+\frac{a}{b}\right)\right)-6\left(\frac{\sqrt{\pi}a\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)c^2d+\frac{3\left(\frac{\sqrt{\pi}\left(2\,a^2+1\right)\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(b\left(x+\frac{a}{b}\right)\right.}{}\right.}{b}}{6\,\sqrt{\pi}d}$$

input
```
integrate((d*x+c)^2*erf(b*x+a),x, algorithm="giac")
```

output
```
1/3*(d*x + c)^3*erf(b*x + a)/d + 1/6*(2*sqrt(pi)*c^3*erf(-b*(x + a/b)) - 6
*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c^2*d +
 3*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b
^2*x^2 - 2*a*b*x - a^2)/b)*c*d^2/b - (sqrt(pi)*(2*a^3 + 3*a)*erf(-b*(x + a
/b))/b - 2*(b^2*(x + a/b)^2 - 3*a*b*(x + a/b) + 3*a^2 + 1)*e^(-b^2*x^2 - 2
*a*b*x - a^2)/b)*d^3/b^2)/(sqrt(pi)*d)
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.06

$$\int (c+dx)^2\text{erf}(a+bx)\,dx$$
$$= \frac{\frac{e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(a^2\,d^2-3\,a\,b\,c\,d+3\,b^2\,c^2+d^2)}{b^3}+\frac{d^2\,x^2\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}}{b}-\frac{x\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(a\,d^2-3\,b\,c\,d)}{b^2}}{3\,\sqrt{\pi}}$$
$$+\text{erf}(a+b\,x)\left(c^2\,x+c\,d\,x^2+\frac{d^2\,x^3}{3}\right)$$
$$-\frac{\text{erfi}(a\,1\text{i}+b\,x\,1\text{i})\,\left(2\,a^3\,d^2-6\,a^2\,b\,c\,d+6\,a\,b^2\,c^2+3\,a\,d^2-3\,b\,c\,d\right)\,1\text{i}}{6\,b^3}$$

input
```
int(erf(a + b*x)*(c + d*x)^2,x)
```

output
```
((exp(- a^2 - b^2*x^2 - 2*a*b*x)*(d^2 + a^2*d^2 + 3*b^2*c^2 - 3*a*b*c*d))/
b^3 + (d^2*x^2*exp(- a^2 - b^2*x^2 - 2*a*b*x))/b - (x*exp(- a^2 - b^2*x^2
- 2*a*b*x)*(a*d^2 - 3*b*c*d))/b^2)/(3*pi^(1/2)) + erf(a + b*x)*(c^2*x + (d
^2*x^3)/3 + c*d*x^2) - (erfi(a*1i + b*x*1i)*(3*a*d^2 + 2*a^3*d^2 + 6*a*b^2
*c^2 - 3*b*c*d - 6*a^2*b*c*d)*1i)/(6*b^3)
```

Reduce [F]

$$\int (c+dx)^2 \operatorname{erf}(a+bx)\,dx$$
$$= \frac{\sqrt{\pi}\, e^{b^2x^2+2abx+a^2} \operatorname{erf}(bx+a)\, a\, c^2 + \sqrt{\pi}\, e^{b^2x^2+2abx+a^2} \operatorname{erf}(bx+a)\, b\, c^2 x + \sqrt{\pi}\, e^{b^2x^2+2abx+a^2} \left(\int \operatorname{erf}(bx+a)\, x^2 \right.}{\sqrt{\pi}\, e^{b^2x^2+2abx+a^2}\, b}$$

input
```
int((d*x+c)^2*erf(b*x+a),x)
```

output
```
(sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c**2 + sqrt(pi)*e
**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c**2*x + sqrt(pi)*e**(a**2 +
 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**2,x)*b*d**2 + 2*sqrt(pi)*e**(a**
2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*c*d + c**2)/(sqrt(pi)*e**
(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.17 $\int (c+dx)\mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int (c+dx)\text{erf}(a+bx)\,dx=\frac{(bc-ad)e^{-(a+bx)^2}}{b^2\sqrt{\pi}}+\frac{de^{-(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}}-\frac{d\text{erf}(a+bx)}{4b^2}-\frac{(bc-ad)^2\text{erf}(a+bx)}{2b^2d}+\frac{(c+dx)^2\text{erf}(a+bx)}{2d}$$

output
```
(-a*d+b*c)/b^2/exp((b*x+a)^2)/Pi^(1/2)+1/2*d*(b*x+a)/b^2/exp((b*x+a)^2)/Pi
^(1/2)-1/4*d*erf(b*x+a)/b^2-1/2*(-a*d+b*c)^2*erf(b*x+a)/b^2/d+1/2*(d*x+c)^
2*erf(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int (c+dx)\text{erf}(a+bx)\,dx=\frac{e^{-(a+bx)^2}\left(4bc-2ad+2bdx-e^{(a+bx)^2}\sqrt{\pi}(-4abc+d+2a^2d-4b^2cx-2b^2dx^2)\,\text{erf}(a+bx)\right)}{4b^2\sqrt{\pi}}$$

input
```
Integrate[(c + d*x)*Erf[a + b*x],x]
```

output
```
(4*b*c - 2*a*d + 2*b*d*x - E^(a + b*x)^2*Sqrt[Pi]*(-4*a*b*c + d + 2*a^2*d
- 4*b^2*c*x - 2*b^2*d*x^2)*Erf[a + b*x])/(4*b^2*E^(a + b*x)^2*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6915, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{erf}(a+bx)\,dx$$

$$\Big\downarrow \text{6915}$$

$$\frac{(c+dx)^2\text{erf}(a+bx)}{2d}-\frac{b\int e^{-(a+bx)^2}(c+dx)^2dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2656}$$

$$\frac{(c+dx)^2\text{erf}(a+bx)}{2d}-\frac{b\int \left(\frac{e^{-(a+bx)^2}(bc-ad)^2}{b^2}+\frac{2de^{-(a+bx)^2}(a+bx)(bc-ad)}{b^2}+\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{b^2}\right)dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^2\text{erf}(a+bx)}{2d}-\frac{b\left(\frac{\sqrt{\pi}(bc-ad)^2\text{erf}(a+bx)}{2b^3}-\frac{de^{-(a+bx)^2}(bc-ad)}{b^3}+\frac{\sqrt{\pi}d^2\text{erf}(a+bx)}{4b^3}-\frac{d^2e^{-(a+bx)^2}(a+bx)}{2b^3}\right)}{\sqrt{\pi}d}$$

input
```
Int[(c + d*x)*Erf[a + b*x],x]
```

output
```
((c + d*x)^2*Erf[a + b*x])/(2*d) - (b*(-((d*(b*c - a*d))/(b^3*E^(a + b*x)^
2)) - (d^2*(a + b*x))/(2*b^3*E^(a + b*x)^2) + (d^2*Sqrt[Pi]*Erf[a + b*x])/
(4*b^3) + ((b*c - a*d)^2*Sqrt[Pi]*Erf[a + b*x])/(2*b^3)))/(d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6915
```
Int[Erf[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
c + d*x)^(m + 1)*(Erf[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(m
+ 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\operatorname{erf}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-d\left(-\frac{(bx+a)\mathrm{e}^{-(bx+a)^2}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}(bx+a)}{4}\right)+\mathrm{e}^{-(bx+a)^2}bc-\mathrm{e}^{-(bx+a)^2}ad}{\sqrt{\pi}\,b}}{b}$
default	$\frac{-\frac{\operatorname{erf}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-d\left(-\frac{(bx+a)\mathrm{e}^{-(bx+a)^2}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}(bx+a)}{4}\right)+\mathrm{e}^{-(bx+a)^2}bc-\mathrm{e}^{-(bx+a)^2}ad}{\sqrt{\pi}\,b}}{b}$
parallelrisch	$\frac{2d\,x^2\operatorname{erf}(bx+a)\sqrt{\pi}\,b^2+4x\,\operatorname{erf}(bx+a)c\sqrt{\pi}\,b^2-2\sqrt{\pi}\,\operatorname{erf}(bx+a)a^2d+4\sqrt{\pi}\,\operatorname{erf}(bx+a)abc+2\,\mathrm{e}^{-(bx+a)^2}bdx-\operatorname{erf}(bx+a)d\sqrt{}}{4\sqrt{\pi}\,b^2}$
orering	$-\frac{\left(-2b^3d^2x^3-6b^3cd\,x^2+2a^2b\,d^2x-4a\,b^2cdx-4b^3c^2x+2a^2bcd-4a\,b^2c^2+2b\,d^2x-a\,d^2+3bcd\right)\operatorname{erf}(bx+a)}{4b^3(dx+c)}-\frac{(-xbd+a}{}$
parts	$\frac{\operatorname{erf}(bx+a)d\,x^2}{2}+\operatorname{erf}\left(bx+a\right)cx-\frac{b\left(\mathrm{e}^{-a^2}d\left(-\frac{x\,\mathrm{e}^{-b^2x^2-2bxa}}{2b^2}-\frac{a\left(-\frac{\mathrm{e}^{-b^2x^2-2bxa}}{2b^2}-\frac{a\sqrt{\pi}\,\mathrm{e}^{a^2}\operatorname{erf}(bx+a)}{2b^2}\right)}{b}+\frac{\sqrt{\pi}}{}\right.\right.}{\sqrt{\pi}}$

input
```
int((d*x+c)*erf(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-erf(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/Pi^(1/2)/b*
(-d*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf(b*x+a))+c*b/exp((b*x+a)^
2)-d*a/exp((b*x+a)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int (c+dx)\text{erf}(a+bx)\,dx$$
$$=\frac{2\sqrt{\pi}(bdx+2\,bc-ad)e^{(-b^2x^2-2\,abx-a^2)}+(2\,\pi b^2dx^2+4\,\pi b^2cx+\pi(4\,abc-(2\,a^2+1)d))\,\text{erf}\,(bx+a)}{4\,\pi b^2}$$

input
```
integrate((d*x+c)*erf(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(pi)*(b*d*x + 2*b*c - a*d)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (2*pi
*b^2*d*x^2 + 4*pi*b^2*c*x + pi*(4*a*b*c - (2*a^2 + 1)*d))*erf(b*x + a))/(p
i*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.51

$$\int (c+dx)\text{erf}(a+bx)\,dx$$
$$=\begin{cases}-\frac{a^2d\,\text{erf}\,(a+bx)}{2b^2}+\frac{ac\,\text{erf}\,(a+bx)}{b}-\frac{ade^{-a^2}e^{-b^2x^2}e^{-2abx}}{2\sqrt{\pi}b^2}+cx\,\text{erf}\,(a+bx)+\frac{dx^2\,\text{erf}\,(a+bx)}{2}+\frac{ce^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b}+\frac{dxe^{-}}{} \\ \left(cx+\frac{dx^2}{2}\right)\text{erf}\,(a)\end{cases}$$

input
```
integrate((d*x+c)*erf(b*x+a),x)
```

output
```
Piecewise((-a**2*d*erf(a + b*x)/(2*b**2) + a*c*erf(a + b*x)/b - a*d*exp(-a
**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b**2) + c*x*erf(a + b*x) +
d*x**2*erf(a + b*x)/2 + c*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(p
i)*b) + d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) - d*er
f(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*erf(a), True))
```

Maxima [F]

$$\int (c+dx)\operatorname{erf}(a+bx)\,dx=\int (dx+c)\operatorname{erf}(bx+a)\,dx$$

input
```
integrate((d*x+c)*erf(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(d*x^2 + 2*c*x)*erf(b*x + a) - integrate((b*d*x^2 + 2*b*c*x)*e^(-b^2*x
^2 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int (c+dx)\operatorname{erf}(a+bx)\,dx=\frac{1}{2}\left(dx^2+2\,cx\right)\operatorname{erf}(bx+a)-\frac{4\sqrt{\pi}\left(\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)c-\frac{\sqrt{\pi}\left(\frac{\sqrt{\pi}\left(2\,a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(b\left(x+\frac{a}{b}\right)-2\,a\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}\right)d}{b}}{4\,\pi}$$

input
```
integrate((d*x+c)*erf(b*x+a),x, algorithm="giac")
```

output
```
1/2*(d*x^2 + 2*c*x)*erf(b*x + a) - 1/4*(4*sqrt(pi)*(sqrt(pi)*a*erf(-b*(x +
 a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)*c - sqrt(pi)*(sqrt(pi)*(2*a^2 +
 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^
2)/b)*d/b)/pi
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int (c+dx)\text{erf}(a+bx)\,dx = \text{erf}(a+b\,x)\,\left(\frac{d\,x^2}{2}+c\,x\right) - \frac{\frac{\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}\,\left(\frac{a\,d}{2}-b\,c\right)}{b^2}-\frac{d\,x\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}}{2\,b}}{\sqrt{\pi}} + \frac{\sqrt{\pi}\,\text{erfi}(a\,1\mathrm{i}+b\,x\,1\mathrm{i})\,\left(\frac{2\,d\,a^2+d}{2\,\sqrt{\pi}}-\frac{2\,a\,b\,c}{\sqrt{\pi}}\right)\,1\mathrm{i}}{2\,b^2}$$

input
```
int(erf(a + b*x)*(c + d*x),x)
```

output
```
erf(a + b*x)*(c*x + (d*x^2)/2) - ((exp(- a^2 - b^2*x^2 - 2*a*b*x)*((a*d)/2
 - b*c))/b^2 - (d*x*exp(- a^2 - b^2*x^2 - 2*a*b*x))/(2*b))/pi^(1/2) + (pi^
(1/2)*erfi(a*1i + b*x*1i)*((d + 2*a^2*d)/(2*pi^(1/2)) - (2*a*b*c)/pi^(1/2)
)*1i)/(2*b^2)
```

Reduce [F]

$$\int (c+dx)\text{erf}(a+bx)\,dx = \frac{\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,ac+\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,bcx+\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int \text{erf}(bx+a)\,xdx\right)}{\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}b}$$

input
```
int((d*x+c)*erf(b*x+a),x)
```

output
```
(sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c + sqrt(pi)*e**(
a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c*x + sqrt(pi)*e**(a**2 + 2*a*b
*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*d + c)/(sqrt(pi)*e**(a**2 + 2*a*b*
x + b**2*x**2)*b)
```

3.18 $\int \mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \operatorname{erf}(a+bx)\,dx=\frac{e^{-(a+bx)^2}}{b\sqrt{\pi}}+\frac{(a+bx)\operatorname{erf}(a+bx)}{b}$$

output
```
1/b/exp((b*x+a)^2)/Pi^(1/2)+(b*x+a)*erf(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \operatorname{erf}(a+bx)\,dx=\frac{e^{-(a+bx)^2}}{b\sqrt{\pi}}+\left(\frac{a}{b}+x\right)\operatorname{erf}(a+bx)$$

input
```
Integrate[Erf[a + b*x],x]
```

output
```
1/(b*E^(a + b*x)^2*Sqrt[Pi]) + (a/b + x)*Erf[a + b*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(a+bx)\,dx$$

$$\Big\downarrow \text{6903}$$

$$\frac{(a+bx)\text{erf}(a+bx)}{b}+\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

input
```
Int[Erf[a + b*x],x]
```

output
```
1/(b*E^(a + b*x)^2*Sqrt[Pi]) + ((a + b*x)*Erf[a + b*x])/b
```

Defintions of rubi rules used

rule 6903
```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b),
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\operatorname{erf}(bx+a)(bx+a)+\frac{e^{-(bx+a)^2}}{\sqrt{\pi}}}{b}$	32
default	$\frac{\operatorname{erf}(bx+a)(bx+a)+\frac{e^{-(bx+a)^2}}{\sqrt{\pi}}}{b}$	32
orering	$\frac{(bx+a)\operatorname{erf}(bx+a)}{b}+\frac{e^{-(bx+a)^2}}{b\sqrt{\pi}}$	34
parallelrisch	$\frac{x\operatorname{erf}(bx+a)\sqrt{\pi}\,b+a\operatorname{erf}(bx+a)\sqrt{\pi}+e^{-(bx+a)^2}}{\sqrt{\pi}\,b}$	42
parts	$x\operatorname{erf}(bx+a)-\frac{2b\left(-\frac{e^{-b^2x^2-2bxa-a^2}}{2b^2}-\frac{a\sqrt{\pi}\operatorname{erf}(bx+a)}{2b^2}\right)}{\sqrt{\pi}}$	57

input
```
int(erf(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(erf(b*x+a)*(b*x+a)+1/Pi^(1/2)*exp(-(b*x+a)^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \operatorname{erf}(a+bx)\,dx=\frac{(\pi bx+\pi a)\operatorname{erf}(bx+a)+\sqrt{\pi}e^{\left(-b^2x^2-2\,abx-a^2\right)}}{\pi b}$$

input
```
integrate(erf(b*x+a),x, algorithm="fricas")
```

output
```
((pi*b*x + pi*a)*erf(b*x + a) + sqrt(pi)*e^(-b^2*x^2 - 2*a*b*x - a^2))/(pi
*b)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \operatorname{erf}(a+bx)\,dx = \begin{cases} \frac{a\operatorname{erf}(a+bx)}{b} + x\operatorname{erf}(a+bx) + \frac{e^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b} & \text{for } b\neq 0 \\ x\operatorname{erf}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(erf(b*x+a),x)
```

output
```
Piecewise((a*erf(a + b*x)/b + x*erf(a + b*x) + exp(-a**2)*exp(-b**2*x**2)*
exp(-2*a*b*x)/(sqrt(pi)*b), Ne(b, 0)), (x*erf(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \operatorname{erf}(a+bx)\,dx = \frac{(bx+a)\operatorname{erf}(bx+a) + \frac{e^{\left(-(bx+a)^2\right)}}{\sqrt{\pi}}}{b}$$

input
```
integrate(erf(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*erf(b*x + a) + e^(-(b*x + a)^2)/sqrt(pi))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \operatorname{erf}(a+bx)\,dx = x\operatorname{erf}(bx+a) - \frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}$$

input
```
integrate(erf(b*x+a),x, algorithm="giac")
```

output
```
x*erf(b*x + a) - (sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x -
 a^2)/b)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \operatorname{erf}(a+bx)\,dx = x\operatorname{erf}(a+b\,x)+\frac{a\operatorname{erf}(a+b\,x)}{b}+\frac{\mathrm{e}^{-b^2\,x^2}\,\mathrm{e}^{-a^2}\,\mathrm{e}^{-2\,a\,b\,x}}{b\,\sqrt{\pi}}$$

input
```
int(erf(a + b*x),x)
```

output
```
x*erf(a + b*x) + (a*erf(a + b*x))/b + (exp(-b^2*x^2)*exp(-a^2)*exp(-2*a*b*
x))/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.42

$$\int \operatorname{erf}(a+bx)\,dx = \frac{\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,a+\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,bx+1}{\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}b}$$

input
```
int(erf(b*x+a),x)
```

output
```
(sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a + sqrt(pi)*e**(a*
*2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*x + 1)/(sqrt(pi)*e**(a**2 + 2*a*b
*x + b**2*x**2)*b)
```

3.19 $\int \frac{\mathbf{erf}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx=\operatorname{Int}\left(\frac{\operatorname{erf}(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(erf(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Erf[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Erf[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{6924}$$
$$\int \frac{\text{erf}(a+bx)}{c+dx}dx$$

input
```
Int[Erf[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx+a\right)}{dx+c}dx$$

input
```
int(erf(b*x+a)/(d*x+c),x)
```

output
```
int(erf(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{erf}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{erf}\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c),x)
```

output
```
Integral(erf(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{erf}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erf}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erf}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(erf(a + b*x)/(c + d*x),x)
```

output
```
int(erf(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erf}(bx+a)}{dx+c}dx$$

input
```
int(erf(b*x+a)/(d*x+c),x)
```

output
```
int(erf(a + b*x)/(c + d*x),x)
```

3.20 $\int \frac{\text{erf}(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{erf}(a+bx)}{(c+dx)^2}\,dx = -\frac{\text{erf}(a+bx)}{d(c+dx)} + \frac{2b\text{Int}\left(\frac{e^{-(a+bx)^2}}{c+dx},x\right)}{d\sqrt{\pi}}$$

output
```
-erf(b*x+a)/d/(d*x+c)+2*b*Defer(Int)(1/exp((b*x+a)^2)/(d*x+c),x)/d/Pi^(1/2
)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erf}(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Erf[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Erf[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 6915$$

$$\frac{2b\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{\sqrt{\pi}d}-\frac{\text{erf}(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \; 2654$$

$$\frac{2b\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{\sqrt{\pi}d}-\frac{\text{erf}(a+bx)}{d(c+dx)}$$

input
```
Int[Erf[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx+a\right)}{\left(dx+c\right)^2}dx$$

input
```
int(erf(b*x+a)/(d*x+c)^2,x)
```

output
```
int(erf(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\text{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erf}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 12.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erf}\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(erf(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 4.71

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*b*integrate(e^(-b^2*x^2 - 2*a*b*x)/(sqrt(pi)*d^2*x*e^(a^2) + sqrt(pi)*c*
d*e^(a^2)), x) - erf(b*x + a)/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(erf(a + b*x)/(c + d*x)^2,x)
```

output
```
int(erf(a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(erf(b*x+a)/(d*x+c)^2,x)
```

output
```
int(erf(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.21 $\int \frac{\mathbf{erf}(a+bx)}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3} \, dx = -\frac{\operatorname{erf}(a+bx)}{2d(c+dx)^2} + \frac{b\operatorname{Int}\left(\frac{e^{-(a+bx)^2}}{(c+dx)^2}, x\right)}{d\sqrt{\pi}}$$

output
```
-1/2*erf(b*x+a)/d/(d*x+c)^2+b*Defer(Int)(1/exp((b*x+a)^2)/(d*x+c)^2,x)/d/P
i^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3} \, dx = \int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3} \, dx$$

input
```
Integrate[Erf[a + b*x]/(c + d*x)^3,x]
```

output
```
Integrate[Erf[a + b*x]/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \ 6915$$

$$\frac{b\int \frac{e^{-(a+bx)^2}}{(c+dx)^2}dx}{\sqrt{\pi}d}-\frac{\operatorname{erf}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \ 2650$$

$$\frac{b\left(-\frac{2b^2\int e^{-(a+bx)^2}dx}{d^2}+\frac{2b(bc-ad)\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{-(a+bx)^2}}{d(c+dx)}\right)}{\sqrt{\pi}d}-\frac{\operatorname{erf}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \ 2634$$

$$\frac{b\left(\frac{2b(bc-ad)\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{-(a+bx)^2}}{d(c+dx)}-\frac{\sqrt{\pi}b\operatorname{erf}(a+bx)}{d^2}\right)}{\sqrt{\pi}d}-\frac{\operatorname{erf}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \ 2654$$

$$\frac{b\left(\frac{2b(bc-ad)\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{-(a+bx)^2}}{d(c+dx)}-\frac{\sqrt{\pi}b\operatorname{erf}(a+bx)}{d^2}\right)}{\sqrt{\pi}d}-\frac{\operatorname{erf}(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Erf[a + b*x]/(c + d*x)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx+a)}{(dx+c)^3}dx$$

input
```
int(erf(b*x+a)/(d*x+c)^3,x)
```

output
```
int(erf(b*x+a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erf}(bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [N/A]

Not integrable

Time = 88.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(erf(a + b*x)/(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 6.71

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erf}(bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
b*integrate(e^(-b^2*x^2 - 2*a*b*x)/(sqrt(pi)*d^3*x^2*e^(a^2) + 2*sqrt(pi)*
c*d^2*x*e^(a^2) + sqrt(pi)*c^2*d*e^(a^2)), x) - 1/2*erf(b*x + a)/(d^3*x^2
+ 2*c*d^2*x + c^2*d)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erf}(bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(erf(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erf}(a+b\,x)}{(c+d\,x)^3}\,dx$$

input
```
int(erf(a + b*x)/(c + d*x)^3,x)
```

output
```
int(erf(a + b*x)/(c + d*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{erf}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erf}(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(erf(b*x+a)/(d*x+c)^3,x)
```

output
```
int(erf(a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.22 $\int x^5 \mathbf{erf}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 178

$$\int x^5\text{erf}(bx)^2\,dx = \frac{11e^{-2b^2x^2}}{12b^6\pi}+\frac{7e^{-2b^2x^2}x^2}{12b^4\pi}+\frac{e^{-2b^2x^2}x^4}{6b^2\pi}+\frac{5e^{-b^2x^2}x\text{erf}(bx)}{4b^5\sqrt{\pi}}+\frac{5e^{-b^2x^2}x^3\text{erf}(bx)}{6b^3\sqrt{\pi}}+\frac{e^{-b^2x^2}x^5\text{erf}(bx)}{3b\sqrt{\pi}}-\frac{5\text{erf}(bx)^2}{16b^6}+\frac{1}{6}x^6\text{erf}(bx)^2$$

output
```
11/12/b^6/exp(2*b^2*x^2)/Pi+7/12*x^2/b^4/exp(2*b^2*x^2)/Pi+1/6*x^4/b^2/exp
(2*b^2*x^2)/Pi+5/4*x*erf(b*x)/b^5/exp(b^2*x^2)/Pi^(1/2)+5/6*x^3*erf(b*x)/b
^3/exp(b^2*x^2)/Pi^(1/2)+1/3*x^5*erf(b*x)/b/exp(b^2*x^2)/Pi^(1/2)-5/16*erf
(b*x)^2/b^6+1/6*x^6*erf(b*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int x^5\text{erf}(bx)^2\,dx = \frac{e^{-2b^2x^2}\left(44+28b^2x^2+8b^4x^4+4be^{b^2x^2}\sqrt{\pi}x(15+10b^2x^2+4b^4x^4)\,\text{erf}(bx)+e^{2b^2x^2}\pi(-15+8b^6x^6)\,\text{erf}(bx)\right.}{48b^6\pi}$$

input
```
Integrate[x^5*Erf[b*x]^2,x]
```

output
```
(44 + 28*b^2*x^2 + 8*b^4*x^4 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(15 + 10*b^2*x^2
 + 4*b^4*x^4)*Erf[b*x] + E^(2*b^2*x^2)*Pi*(-15 + 8*b^6*x^6)*Erf[b*x]^2)/(4
8*b^6*E^(2*b^2*x^2)*Pi)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6918, 6939, 2641, 2641, 2638, 6939, 2641, 2638, 6939, 2638, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{erf}(bx)^2 \, dx$$

$$\Big\downarrow \text{6918}$$

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\int e^{-b^2x^2}x^6\text{erf}(bx)dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6939}$$

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}x^5dx}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erf}(bx)dx}{2b^2}+\frac{\frac{\int e^{-2b^2x^2}x^3dx}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erf}(bx)dx}{2b^2}+\frac{\frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2}-\frac{x^2e^{-2b^2x^2}}{4b^2}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erf}(bx)dx}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 6939

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}x^3dx}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 2641

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2}+\frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2}-\frac{x^2e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b} \cdots}{3\sqrt{\pi}}$$

$\Big\downarrow$ 2638

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 6939

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-x^2e \cdots}{\cdots}}{3\sqrt{\pi}}$$

$\downarrow$ 2638

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}\cdots}{\cdots}\right.}{3\sqrt{\pi}}$$

$\downarrow$ 6927

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(\frac{5\left(\frac{3\left(\frac{\sqrt{\pi}\int\text{erf}(bx)d\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}\cdots}{\cdots}\right.}{3\sqrt{\pi}}$$

$\downarrow$ 15

$$\frac{1}{6}x^6\text{erf}(bx)^2-\frac{2b\left(-\frac{x^5e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}+\frac{5\left(-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}+\frac{3\left(\frac{\sqrt{\pi}\text{erf}(bx)^2}{8b^3}-\frac{xe^{-b^2x}\cdots}{2\cdots}\right.}{2b^2\cdots}\right.}{2b^2}\right.}{3\sqrt{\pi}}$$

input
```
Int[x^5*Erf[b*x]^2,x]
```

output
```
(x^6*Erf[b*x]^2)/6 - (2*b*((-1/4*x^4/(b^2*E^(2*b^2*x^2)) + (-1/8*1/(b^4*E^
(2*b^2*x^2)) - x^2/(4*b^2*E^(2*b^2*x^2)))/b^2)/(b*Sqrt[Pi]) - (x^5*Erf[b*x
])/(2*b^2*E^(b^2*x^2)) + (5*((-1/8*1/(b^4*E^(2*b^2*x^2)) - x^2/(4*b^2*E^(2
*b^2*x^2)))/(b*Sqrt[Pi]) - (x^3*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(-1/4*1
/(b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (Sqrt[P
i]*Erf[b*x]^2)/(8*b^3)))/(2*b^2)))/(2*b^2)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{8\operatorname{erf}(bx)^2x^6b^6\pi^{\frac{3}{2}}+16\,\mathrm{e}^{-b^2x^2}\operatorname{erf}(bx)x^5b^5\pi+8\,\mathrm{e}^{-2b^2x^2}x^4b^4\sqrt{\pi}+40\,\mathrm{e}^{-b^2x^2}x^3\operatorname{erf}(bx)b^3\pi+28\,\mathrm{e}^{-2b^2x^2}x^2b^2\sqrt{\pi}+60\,\mathrm{e}^{-b^2x^2}x\ \mathrm{e}}{48b^6\pi^{\frac{3}{2}}}$
orering	$\frac{(8b^8x^8-30b^6x^6-55b^4x^4-40b^2x^2+165)\operatorname{erf}(bx)^2}{48b^8x^2}+\frac{(6b^6x^6+7b^4x^4-9b^2x^2-55)\left(5x^4\operatorname{erf}(bx)^2+\frac{4x^5\operatorname{erf}(bx)\mathrm{e}^{-b^2x^2}b}{\sqrt{\pi}}\right)}{48b^8x^6}+-\ (2$

input
```
int(x^5*erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*erf(b*x)^2*x^6*b^6*Pi^(3/2)+16*exp(-b^2*x^2)*erf(b*x)*x^5*b^5*Pi+8
*exp(-b^2*x^2)^2*x^4*b^4*Pi^(1/2)+40*exp(-b^2*x^2)*x^3*erf(b*x)*b^3*Pi+28*
exp(-b^2*x^2)^2*x^2*b^2*Pi^(1/2)+60*exp(-b^2*x^2)*x*erf(b*x)*b*Pi-15*erf(b
*x)^2*Pi^(3/2)+44*exp(-b^2*x^2)^2*Pi^(1/2))/b^6/Pi^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.55

$$\int x^5\operatorname{erf}(bx)^2\,dx$$
$$=\frac{4\sqrt{\pi}(4\,b^5x^5+10\,b^3x^3+15\,bx)\operatorname{erf}(bx)\,e^{(-b^2x^2)}-(15\,\pi-8\,\pi b^6x^6)\operatorname{erf}(bx)^2+4\,(2\,b^4x^4+7\,b^2x^2+11)e^{(-}}{48\,\pi b^6}$$

input
```
integrate(x^5*erf(b*x)^2,x, algorithm="fricas")
```

output
```
1/48*(4*sqrt(pi)*(4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*erf(b*x)*e^(-b^2*x^2) -
 (15*pi - 8*pi*b^6*x^6)*erf(b*x)^2 + 4*(2*b^4*x^4 + 7*b^2*x^2 + 11)*e^(-2*
b^2*x^2))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.94

$$\int x^5 \text{erf}(bx)^2\,dx$$
$$= \begin{cases} \frac{x^6 \operatorname{erf}^2(bx)}{6} + \frac{x^5 e^{-b^2x^2} \operatorname{erf}(bx)}{3\sqrt{\pi}b} + \frac{x^4 e^{-2b^2x^2}}{6\pi b^2} + \frac{5x^3 e^{-b^2x^2} \operatorname{erf}(bx)}{6\sqrt{\pi}b^3} + \frac{7x^2 e^{-2b^2x^2}}{12\pi b^4} + \frac{5xe^{-b^2x^2} \operatorname{erf}(bx)}{4\sqrt{\pi}b^5} - \frac{5\operatorname{erf}^2(bx)}{16b^6} + \frac{11e^{-2b^2x}}{12\pi b^6} \\ 0 \end{cases}$$

input
```
integrate(x**5*erf(b*x)**2,x)
```

output
```
Piecewise((x**6*erf(b*x)**2/6 + x**5*exp(-b**2*x**2)*erf(b*x)/(3*sqrt(pi)*
b) + x**4*exp(-2*b**2*x**2)/(6*pi*b**2) + 5*x**3*exp(-b**2*x**2)*erf(b*x)/
(6*sqrt(pi)*b**3) + 7*x**2*exp(-2*b**2*x**2)/(12*pi*b**4) + 5*x*exp(-b**2*
x**2)*erf(b*x)/(4*sqrt(pi)*b**5) - 5*erf(b*x)**2/(16*b**6) + 11*exp(-2*b**
2*x**2)/(12*pi*b**6), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^5 \text{erf}(bx)^2\,dx = \int x^5 \operatorname{erf}(bx)^2\,dx$$

input
```
integrate(x^5*erf(b*x)^2,x, algorithm="maxima")
```

output
```
-1/6*integrate((4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-2*b^2*x^2), x)/(pi*b^4)
 + 1/48*((8*sqrt(pi)*b^6*x^6 - 15*sqrt(pi))*erf(b*x)^2 + 4*(4*b^5*x^5 + 10
*b^3*x^3 + 15*b*x)*erf(b*x)*e^(-b^2*x^2))/(sqrt(pi)*b^6)
```

Giac [F]

$$\int x^5\text{erf}(bx)^2\,dx = \int x^5\,\text{erf}\,(bx)^2\,\,dx$$

input
```
integrate(x^5*erf(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*erf(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80

$$\int x^5\text{erf}(bx)^2\,dx = \frac{x^6\,\text{erf}(b\,x)^2}{6} + \frac{\frac{11\,e^{-2\,b^2\,x^2}}{12} - \frac{5\,\pi\,\text{erf}(b\,x)^2}{16} + \frac{7\,b^2\,x^2\,e^{-2\,b^2\,x^2}}{12} + \frac{b^4\,x^4\,e^{-2\,b^2\,x^2}}{6} + \frac{5\,b^3\,x^3\,\sqrt{\pi}\,e^{-b^2\,x^2}\,\text{erf}(b\,x)}{6} + \frac{b^5\,x^5\,\sqrt{\pi}\,e^{-b^2\,x^2}\,\text{erf}(b\,x)}{3} + \frac{5\,b\,x\,\sqrt{}}{}}{b^6\,\pi}$$

input
```
int(x^5*erf(b*x)^2,x)
```

output
```
(x^6*erf(b*x)^2)/6 + ((11*exp(-2*b^2*x^2))/12 - (5*pi*erf(b*x)^2)/16 + (7*
b^2*x^2*exp(-2*b^2*x^2))/12 + (b^4*x^4*exp(-2*b^2*x^2))/6 + (5*b^3*x^3*pi^
(1/2)*exp(-b^2*x^2)*erf(b*x))/6 + (b^5*x^5*pi^(1/2)*exp(-b^2*x^2)*erf(b*x)
)/3 + (5*b*x*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/4)/(b^6*pi)
```

Reduce [F]

$$\int x^5\text{erf}(bx)^2\,dx = \int \text{erf}(bx)^2\,x^5dx$$

input
```
int(x^5*erf(b*x)^2,x)
```

output
```
int(erf(b*x)**2*x**5,x)
```

3.23 $\int x^3\mathbf{erf}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 126

$$\int x^3\text{erf}(bx)^2\,dx=\frac{e^{-2b^2x^2}}{2b^4\pi}+\frac{e^{-2b^2x^2}x^2}{4b^2\pi}+\frac{3e^{-b^2x^2}x\text{erf}(bx)}{4b^3\sqrt{\pi}}+\frac{e^{-b^2x^2}x^3\text{erf}(bx)}{2b\sqrt{\pi}}-\frac{3\text{erf}(bx)^2}{16b^4}+\frac{1}{4}x^4\text{erf}(bx)^2$$

output
```
1/2/b^4/exp(2*b^2*x^2)/Pi+1/4*x^2/b^2/exp(2*b^2*x^2)/Pi+3/4*x*erf(b*x)/b^3
/exp(b^2*x^2)/Pi^(1/2)+1/2*x^3*erf(b*x)/b/exp(b^2*x^2)/Pi^(1/2)-3/16*erf(b
*x)^2/b^4+1/4*x^4*erf(b*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int x^3\text{erf}(bx)^2\,dx=\frac{e^{-2b^2x^2}\left(8+4b^2x^2+4be^{b^2x^2}\sqrt{\pi}x(3+2b^2x^2)\,\text{erf}(bx)+e^{2b^2x^2}\pi(-3+4b^4x^4)\,\text{erf}(bx)^2\right)}{16b^4\pi}$$

input
```
Integrate[x^3*Erf[b*x]^2,x]
```

output
```
(8 + 4*b^2*x^2 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x] + E^(
2*b^2*x^2)*Pi*(-3 + 4*b^4*x^4)*Erf[b*x]^2)/(16*b^4*E^(2*b^2*x^2)*Pi)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6918, 6939, 2641, 2638, 6939, 2638, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{erf}(bx)^2 \, dx$$

$$\Big\downarrow \text{6918}$$

$$\frac{1}{4}x^4\text{erf}(bx)^2 - \frac{b\int e^{-b^2x^2}x^4\text{erf}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6939}$$

$$\frac{1}{4}x^4\text{erf}(bx)^2 - \frac{b\left(\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2} + \frac{\int e^{-2b^2x^2}x^3dx}{\sqrt{\pi}b} - \frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{4}x^4\text{erf}(bx)^2 - \frac{b\left(\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2} + \frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2} - \frac{x^2e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b} - \frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{4}x^4\text{erf}(bx)^2 - \frac{b\left(\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2} - \frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2} + \frac{-\frac{x^2e^{-2b^2x^2}}{4b^2} - \frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6939}$$

$$\frac{1}{4}x^4\text{erf}(bx)^2-\frac{b\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \; 2638$$

$$\frac{1}{4}x^4\text{erf}(bx)^2-\frac{b\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \; 6927$$

$$\frac{1}{4}x^4\text{erf}(bx)^2-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}\int \text{erf}(bx)d\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{4}x^4\text{erf}(bx)^2-\frac{b\left(-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}+\frac{3\left(\frac{\sqrt{\pi}\text{erf}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}\right)}{\sqrt{\pi}}$$

input
```
Int[x^3*Erf[b*x]^2,x]
```

output
```
(x^4*Erf[b*x]^2)/4 - (b*((-1/8*1/(b^4*E^(2*b^2*x^2)) - x^2/(4*b^2*E^(2*b^2
*x^2)))/(b*Sqrt[Pi]) - (x^3*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(-1/4*1/(b^
3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (Sqrt[Pi]*E
rf[b*x]^2)/(8*b^3)))/(2*b^2)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{4\,\mathrm{erf}(bx)^2x^4\pi^{\frac{3}{2}}b^4+8\,\mathrm{e}^{-b^2x^2}x^3\,\mathrm{erf}(bx)b^3\pi+4\,\mathrm{e}^{-2b^2x^2}x^2b^2\sqrt{\pi}+12\,\mathrm{e}^{-b^2x^2}x\,\mathrm{erf}(bx)b\pi-3\,\mathrm{erf}(bx)^2\pi^{\frac{3}{2}}+8\,\mathrm{e}^{-2b^2x^2}\sqrt{\pi}}{16\pi^{\frac{3}{2}}b^4}$
orering	$\frac{(4b^6x^6-9b^4x^4-12b^2x^2+12)\,\mathrm{erf}(bx)^2}{16b^6x^2}+\frac{(3b^4x^4+2b^2x^2-6)\left(3x^2\,\mathrm{erf}(bx)^2+\frac{4x^3\,\mathrm{erf}(bx)\mathrm{e}^{-b^2x^2}b}{\sqrt{\pi}}\right)}{16b^6x^4}+\frac{(b^2x^2+2)\left(6x\,\mathrm{erf}(bx)^2\right.}{}$

input
```
int(x^3*erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*erf(b*x)^2*x^4*Pi^(3/2)*b^4+8*exp(-b^2*x^2)*x^3*erf(b*x)*b^3*Pi+4*
exp(-b^2*x^2)^2*x^2*b^2*Pi^(1/2)+12*exp(-b^2*x^2)*x*erf(b*x)*b*Pi-3*erf(b*
x)^2*Pi^(3/2)+8*exp(-b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int x^3\mathrm{erf}(bx)^2\,dx = \frac{4\,\sqrt{\pi}(2\,b^3x^3+3\,bx)\,\mathrm{erf}\,(bx)\,e^{(-b^2x^2)}-(3\,\pi-4\,\pi b^4x^4)\,\mathrm{erf}\,(bx)^2+4\,(b^2x^2+2)e^{(-2\,b^2x^2)}}{16\,\pi b^4}$$

input
```
integrate(x^3*erf(b*x)^2,x, algorithm="fricas")
```

output
```
1/16*(4*sqrt(pi)*(2*b^3*x^3 + 3*b*x)*erf(b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*
b^4*x^4)*erf(b*x)^2 + 4*(b^2*x^2 + 2)*e^(-2*b^2*x^2))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int x^3\text{erf}(bx)^2\,dx$$
$$= \begin{cases} \frac{x^4\operatorname{erf}^2(bx)}{4} + \frac{x^3e^{-b^2x^2}\operatorname{erf}(bx)}{2\sqrt{\pi}b} + \frac{x^2e^{-2b^2x^2}}{4\pi b^2} + \frac{3xe^{-b^2x^2}\operatorname{erf}(bx)}{4\sqrt{\pi}b^3} - \frac{3\operatorname{erf}^2(bx)}{16b^4} + \frac{e^{-2b^2x^2}}{2\pi b^4} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erf(b*x)**2,x)
```

output
```
Piecewise((x**4*erf(b*x)**2/4 + x**3*exp(-b**2*x**2)*erf(b*x)/(2*sqrt(pi)*
b) + x**2*exp(-2*b**2*x**2)/(4*pi*b**2) + 3*x*exp(-b**2*x**2)*erf(b*x)/(4*
sqrt(pi)*b**3) - 3*erf(b*x)**2/(16*b**4) + exp(-2*b**2*x**2)/(2*pi*b**4),
Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^3\text{erf}(bx)^2\,dx = \int x^3\operatorname{erf}(bx)^2\,dx$$

input
```
integrate(x^3*erf(b*x)^2,x, algorithm="maxima")
```

output
```
-1/2*integrate((2*b^2*x^3 + 3*x)*e^(-2*b^2*x^2), x)/(pi*b^2) - 1/16*((3*pi
 - 4*pi*b^4*x^4)*erf(b*x)^2 - 4*(2*sqrt(pi)*b^3*x^3 + 3*sqrt(pi)*b*x)*erf(
b*x)*e^(-b^2*x^2))/(pi*b^4)
```

Giac [F]

$$\int x^3 \text{erf}(bx)^2 \, dx = \int x^3 \,\text{erf}\,(bx)^2 \, dx$$

input
```
integrate(x^3*erf(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*erf(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^3 \text{erf}(bx)^2 \, dx = \frac{x^4 \,\text{erf}(b\,x)^2}{4} + \frac{\frac{\mathrm{e}^{-2\,b^2\,x^2}}{2} - \frac{3\,\pi\,\text{erf}(b\,x)^2}{16} + \frac{b^2\,x^2\,\mathrm{e}^{-2\,b^2\,x^2}}{4} + \frac{b^3\,x^3\,\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{2} + \frac{3\,b\,x\,\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{4}}{b^4\,\pi}$$

input
```
int(x^3*erf(b*x)^2,x)
```

output
```
(x^4*erf(b*x)^2)/4 + (exp(-2*b^2*x^2)/2 - (3*pi*erf(b*x)^2)/16 + (b^2*x^2*
exp(-2*b^2*x^2))/4 + (b^3*x^3*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/2 + (3*b*x*
pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/4)/(b^4*pi)
```

Reduce [F]

$$\int x^3 \text{erf}(bx)^2 \, dx = \int \text{erf}(bx)^2 \, x^3 dx$$

input
```
int(x^3*erf(b*x)^2,x)
```

output

```
int(erf(b*x)**2*x**3,x)
```

3.24 $\int x\mathbf{erf}(bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int x\mathrm{erf}(bx)^2\,dx = \frac{e^{-2b^2x^2}}{2b^2\pi}+\frac{e^{-b^2x^2}x\mathrm{erf}(bx)}{b\sqrt{\pi}}-\frac{\mathrm{erf}(bx)^2}{4b^2}+\frac{1}{2}x^2\mathrm{erf}(bx)^2$$

output
```
1/2/b^2/exp(2*b^2*x^2)/Pi+x*erf(b*x)/b/exp(b^2*x^2)/Pi^(1/2)-1/4*erf(b*x)^
2/b^2+1/2*x^2*erf(b*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int x\mathrm{erf}(bx)^2\,dx = \frac{2e^{-2b^2x^2}+4be^{-b^2x^2}\sqrt{\pi}x\mathrm{erf}(bx)+\pi(-1+2b^2x^2)\,\mathrm{erf}(bx)^2}{4b^2\pi}$$

input
```
Integrate[x*Erf[b*x]^2,x]
```

output
```
(2/E^(2*b^2*x^2) + (4*b*Sqrt[Pi]*x*Erf[b*x])/E^(b^2*x^2) + Pi*(-1 + 2*b^2*
x^2)*Erf[b*x]^2)/(4*b^2*Pi)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6918, 6939, 2638, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erf}(bx)^2\,dx$$

$$\Big\downarrow \text{6918}$$

$$\frac{1}{2}x^2\text{erf}(bx)^2-\frac{2b\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6939}$$

$$\frac{1}{2}x^2\text{erf}(bx)^2-\frac{2b\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{2}x^2\text{erf}(bx)^2-\frac{2b\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6927}$$

$$\frac{1}{2}x^2\text{erf}(bx)^2-\frac{2b\left(\frac{\sqrt{\pi}\int \text{erf}(bx)d\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}x^2\text{erf}(bx)^2-\frac{2b\left(\frac{\sqrt{\pi}\text{erf}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}$$

input
```
Int[x*Erf[b*x]^2,x]
```

output
```
(x^2*Erf[b*x]^2)/2 - (2*b*(-1/4*1/(b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erf[b*
x])/(2*b^2*E^(b^2*x^2)) + (Sqrt[Pi]*Erf[b*x]^2)/(8*b^3)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{2\,\mathrm{erf}(bx)^2 x^2 \pi^{\frac{3}{2}} b^2+4\,\mathrm{e}^{-b^2x^2} x\ \mathrm{erf}(bx) b\pi-\mathrm{erf}(bx)^2 \pi^{\frac{3}{2}}+2\,\mathrm{e}^{-2b^2x^2}\sqrt{\pi}}{4\pi^{\frac{3}{2}} b^2}$
orering	$\frac{(4b^4x^4-5b^2x^2+1)\,\mathrm{erf}(bx)^2}{8b^4x^2}+\frac{(3b^2x^2-1)\left(\mathrm{erf}(bx)^2+\frac{4x\ \mathrm{erf}(bx)\mathrm{e}^{-b^2x^2}b}{\sqrt{\pi}}\right)}{8b^4x^2}+\frac{\frac{8\ \mathrm{erf}(bx)\mathrm{e}^{-b^2x^2}b}{\sqrt{\pi}}+\frac{8x\,\mathrm{e}^{-2b^2x^2}b^2}{\pi}-\frac{8x^2\ \mathrm{erf}(bx)b}{\sqrt{\pi}}}{16b^4x}$

input
```
int(x*erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*erf(b*x)^2*x^2*Pi^(3/2)*b^2+4*exp(-b^2*x^2)*x*erf(b*x)*b*Pi-erf(b*x
)^2*Pi^(3/2)+2*exp(-b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int x\mathrm{erf}(bx)^2\,dx=\frac{4\sqrt{\pi}bx\,\mathrm{erf}\,(bx)\,e^{(-b^2x^2)}-(\pi-2\,\pi b^2x^2)\,\mathrm{erf}\,(bx)^2+2\,e^{(-2\,b^2x^2)}}{4\,\pi b^2}$$

input
```
integrate(x*erf(b*x)^2,x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(pi)*b*x*erf(b*x)*e^(-b^2*x^2) - (pi - 2*pi*b^2*x^2)*erf(b*x)^2
 + 2*e^(-2*b^2*x^2))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x\mathrm{erf}(bx)^2\,dx=\begin{cases}\frac{x^2\,\mathrm{erf}^2\,(bx)}{2}+\frac{xe^{-b^2x^2}\,\mathrm{erf}\,(bx)}{\sqrt{\pi}b}-\frac{\mathrm{erf}^2\,(bx)}{4b^2}+\frac{e^{-2b^2x^2}}{2\pi b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x*erf(b*x)**2,x)
```

output
```
Piecewise((x**2*erf(b*x)**2/2 + x*exp(-b**2*x**2)*erf(b*x)/(sqrt(pi)*b) -
erf(b*x)**2/(4*b**2) + exp(-2*b**2*x**2)/(2*pi*b**2), Ne(b, 0)), (0, True)
)
```

Maxima [F]

$$\int x\text{erf}(bx)^2\,dx=\int x\,\text{erf}\,(bx)^2\,\,dx$$

input
```
integrate(x*erf(b*x)^2,x, algorithm="maxima")
```

output
```
-2*integrate(x*e^(-2*b^2*x^2), x)/pi + 1/4*(4*b*x*erf(b*x)*e^(-b^2*x^2) +
(2*sqrt(pi)*b^2*x^2 - sqrt(pi))*erf(b*x)^2)/(sqrt(pi)*b^2)
```

Giac [F]

$$\int x\text{erf}(bx)^2\,dx=\int x\,\text{erf}\,(bx)^2\,\,dx$$

input
```
integrate(x*erf(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*erf(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int x\mathrm{erf}(bx)^2\,dx=\frac{\frac{\mathrm{e}^{-2\,b^2\,x^2}}{2}+b\,x\,\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}\,\mathrm{erf}(b\,x)}{b^2\,\pi}-\frac{\frac{\mathrm{erf}(b\,x)^2}{4}-\frac{b^2\,x^2\,\mathrm{erf}(b\,x)^2}{2}}{b^2}$$

input
```
int(x*erf(b*x)^2,x)
```

output
```
(exp(-2*b^2*x^2)/2 + b*x*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/(b^2*pi) - (erf(
b*x)^2/4 - (b^2*x^2*erf(b*x)^2)/2)/b^2
```

Reduce [F]

$$\int x\mathrm{erf}(bx)^2\,dx=\int \mathrm{erf}(bx)^2\,xdx$$

input
```
int(x*erf(b*x)^2,x)
```

output
```
int(erf(b*x)**2*x,x)
```

3.25 $\int \frac{\text{erf}(bx)^2}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erf}(bx)^2}{x} dx = \text{Int}\left(\frac{\text{erf}(bx)^2}{x}, x\right)$$

output
```
Defer(Int)(erf(b*x)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x} dx = \int \frac{\text{erf}(bx)^2}{x} dx$$

input
```
Integrate[Erf[b*x]^2/x,x]
```

output
```
Integrate[Erf[b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)^2}{x} \, dx$$
$$\Big\downarrow \text{6924}$$
$$\int \frac{\text{erf}(bx)^2}{x}dx$$

input
```
Int[Erf[b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx\right)^{2}}{x}dx$$

input
```
int(erf(b*x)^2/x,x)
```

output
```
int(erf(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erf(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral(erf(b*x)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erf}(bx)^2}{x}\,dx = \int \frac{\operatorname{erf}^2\left(bx\right)}{x}\,dx$$

input
```
integrate(erf(b*x)**2/x,x)
```

output
```
Integral(erf(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erf(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x}\,dx = \int \frac{\text{erf}\,(bx)^2}{x}\,dx$$

input
```
integrate(erf(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x}\,dx = \int \frac{\text{erf}(b\,x)^2}{x}\,dx$$

input
```
int(erf(b*x)^2/x,x)
```

output
```
int(erf(b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x}\,dx = \int \frac{\operatorname{erf}(bx)^2}{x}dx$$

input
```
int(erf(b*x)^2/x,x)
```

output
```
int(erf(b*x)**2/x,x)
```

3.26 $\int \frac{\mathbf{erf}(bx)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{\text{erf}(bx)^2}{x^3} dx = -\frac{2be^{-b^2x^2}\text{erf}(bx)}{\sqrt{\pi}x} - b^2\text{erf}(bx)^2 - \frac{\text{erf}(bx)^2}{2x^2} + \frac{2b^2 \text{ ExpIntegralEi}\left(-2b^2x^2\right)}{\pi}$$

output
```
-2*b*erf(b*x)/exp(b^2*x^2)/Pi^(1/2)/x-b^2*erf(b*x)^2-1/2*erf(b*x)^2/x^2+2*
b^2*Ei(-2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\text{erf}(bx)^2}{x^3} dx = -\frac{2be^{-b^2x^2}\text{erf}(bx)}{\sqrt{\pi}x} + \left(-b^2 - \frac{1}{2x^2}\right) \text{erf}(bx)^2 + \frac{2b^2 \text{ ExpIntegralEi}\left(-2b^2x^2\right)}{\pi}$$

input
```
Integrate[Erf[b*x]^2/x^3,x]
```

output
```
(-2*b*Erf[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) + (-b^2 - 1/(2*x^2))*Erf[b*x]^2 +
 (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6918, 6945, 2639, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(bx)^2}{x^3}dx$$

$$\Big\downarrow \text{6918}$$

$$\frac{2b\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}dx}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{6945}$$

$$\frac{2b\left(-2b^2\int e^{-b^2x^2}\operatorname{erf}(bx)dx+\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2b\left(-2b^2\int e^{-b^2x^2}\operatorname{erf}(bx)dx-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{6927}$$

$$\frac{2b\left(-\sqrt{\pi}b\int \operatorname{erf}(bx)d\operatorname{erf}(bx)-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2b\left(-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\operatorname{erf}(bx)^2\right)}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)^2}{2x^2}$$

input
```
Int[Erf[b*x]^2/x^3,x]
```

output
```
-1/2*Erf[b*x]^2/x^2 + (2*b*(-(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[
b*x]^2)/2 + (b*ExpIntegralEi[-2*b^2*x^2])/Sqrt[Pi]))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
 + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^3} dx$$

input
```
int(erf(b*x)^2/x^3,x)
```

output
```
int(erf(b*x)^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{erf}(bx)^2}{x^3} dx = \frac{4 b^2 x^2 \operatorname{Ei}(-2 b^2 x^2) - 4 \sqrt{\pi} bx \operatorname{erf}(bx) e^{(-b^2 x^2)} - (\pi + 2 \pi b^2 x^2) \operatorname{erf}(bx)^2}{2 \pi x^2}$$

input
```
integrate(erf(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*b^2*x^2*Ei(-2*b^2*x^2) - 4*sqrt(pi)*b*x*erf(b*x)*e^(-b^2*x^2) - (pi
 + 2*pi*b^2*x^2)*erf(b*x)^2)/(pi*x^2)
```

Sympy [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^3} dx = \int \frac{\operatorname{erf}^2(bx)}{x^3} dx$$

input
```
integrate(erf(b*x)**2/x**3,x)
```

output
```
Integral(erf(b*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^3} \, dx = \int \frac{\operatorname{erf}(bx)^2}{x^3} \, dx$$

input
```
integrate(erf(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
2*b*integrate(erf(b*x)*e^(-b^2*x^2)/x^2, x)/sqrt(pi) - 1/2*erf(b*x)^2/x^2
```

Giac [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^3} \, dx = \int \frac{\operatorname{erf}(bx)^2}{x^3} \, dx$$

input
```
integrate(erf(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erf}(bx)^2}{x^3} \, dx = \int \frac{\operatorname{erf}(b\,x)^2}{x^3} \, dx$$

input
```
int(erf(b*x)^2/x^3,x)
```

output
```
int(erf(b*x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^3}\,dx = \frac{-\operatorname{erf}(bx)^2\pi + 4\sqrt{\pi}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^2}dx\right)b\,x^2}{2\pi\,x^2}$$

input
```
int(erf(b*x)^2/x^3,x)
```

output
```
( - erf(b*x)**2*pi + 4*sqrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x**2),x)*b*x*
*2)/(2*pi*x**2)
```

3.27 $\int \frac{\text{erf}(bx)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 125

$$\int \frac{\text{erf}(bx)^2}{x^5}\,dx = -\frac{b^2e^{-2b^2x^2}}{3\pi x^2} - \frac{be^{-b^2x^2}\text{erf}(bx)}{3\sqrt{\pi}x^3} + \frac{2b^3e^{-b^2x^2}\text{erf}(bx)}{3\sqrt{\pi}x} + \frac{1}{3}b^4\text{erf}(bx)^2 - \frac{\text{erf}(bx)^2}{4x^4} - \frac{4b^4\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{3\pi}$$

output
```
-1/3*b^2/exp(2*b^2*x^2)/Pi/x^2-1/3*b*erf(b*x)/exp(b^2*x^2)/Pi^(1/2)/x^3+2/
3*b^3*erf(b*x)/exp(b^2*x^2)/Pi^(1/2)/x+1/3*b^4*erf(b*x)^2-1/4*erf(b*x)^2/x
^4-4/3*b^4*Ei(-2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{\text{erf}(bx)^2}{x^5}\,dx = \frac{\frac{4be^{-b^2x^2}x(-1+2b^2x^2)\text{erf}(bx)}{\sqrt{\pi}} + (-3+4b^4x^4)\,\text{erf}(bx)^2 - \frac{4b^2x^2\left(e^{-2b^2x^2}+4b^2x^2\,\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)}{\pi}}{12x^4}$$

input
```
Integrate[Erf[b*x]^2/x^5,x]
```

output
```
((4*b*x*(-1 + 2*b^2*x^2)*Erf[b*x])/(E^(b^2*x^2)*Sqrt[Pi]) + (-3 + 4*b^4*x^
4)*Erf[b*x]^2 - (4*b^2*x^2*(E^(-2*b^2*x^2) + 4*b^2*x^2*ExpIntegralEi[-2*b^
2*x^2]))/Pi)/(12*x^4)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6918, 6945, 2643, 2639, 6945, 2639, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)^2}{x^5} \, dx$$

$$\Big\downarrow \text{6918}$$

$$\frac{b \int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}dx}{\sqrt{\pi}} - \frac{\text{erf}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{6945}$$

$$\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx + \frac{2b\int \frac{e^{-2b^2x^2}}{x^3}dx}{3\sqrt{\pi}} - \frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx + \frac{2b\left(-2b^2\int \frac{e^{-2b^2x^2}}{x}dx - \frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{2639}$$

$$\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx - \frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3} + \frac{2b\left(b^2(-\text{ExpIntegralEi}(-2b^2x^2)) - \frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{6945}$$

$$\frac{b\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx+\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\right.}{3\sqrt{\pi}}}{\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{4x^4}$$

$\Big\downarrow$ 2639

$$\frac{b\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-\right.\right.\right.}{3\sqrt{\pi}}}{\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{4x^4}$$

$\Big\downarrow$ 6927

$$\frac{b\left(-\frac{2}{3}b^2\left(-\sqrt{\pi}b\int \text{erf}(bx)d\text{erf}(bx)-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-\right.\right.\right.}{3\sqrt{\pi}}}{\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{4x^4}$$

$\Big\downarrow$ 15

$$\frac{b\left(-\frac{2}{3}b^2\left(-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\text{erf}(bx)^2\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\right.}{3\sqrt{\pi}}}{\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{4x^4}$$

input
```
Int[Erf[b*x]^2/x^5,x]
```

output
```
-1/4*Erf[b*x]^2/x^4 + (b*(-1/3*Erf[b*x]/(E^(b^2*x^2)*x^3) + (2*b*(-1/2*1/(
E^(2*b^2*x^2)*x^2) - b^2*ExpIntegralEi[-2*b^2*x^2]))/(3*Sqrt[Pi]) - (2*b^2
*(-(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegral
Ei[-2*b^2*x^2])/Sqrt[Pi]))/3))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}dx$$

input
```
int(erf(b*x)^2/x^5,x)
```

output
```
int(erf(b*x)^2/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx = -\frac{16\,b^4x^4\operatorname{Ei}(-2\,b^2x^2) + 4\,b^2x^2e^{(-2\,b^2x^2)} - 4\,\sqrt{\pi}(2\,b^3x^3 - bx)\operatorname{erf}(bx)\,e^{(-b^2x^2)} + (3\,\pi - 4\,\pi b^4x^4)\operatorname{erf}(bx)^2}{12\,\pi x^4}$$

input
```
integrate(erf(b*x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(16*b^4*x^4*Ei(-2*b^2*x^2) + 4*b^2*x^2*e^(-2*b^2*x^2) - 4*sqrt(pi)*(
2*b^3*x^3 - b*x)*erf(b*x)*e^(-b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*erf(b*x)^2)
/(pi*x^4)
```

Sympy [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erf}^2(bx)}{x^5}\,dx$$

input
```
integrate(erf(b*x)**2/x**5,x)
```

output
```
Integral(erf(b*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x^5}\,dx$$

input
```
integrate(erf(b*x)^2/x^5,x, algorithm="maxima")
```

output
```
b*integrate(erf(b*x)*e^(-b^2*x^2)/x^4, x)/sqrt(pi) - 1/4*erf(b*x)^2/x^4
```

Giac [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x^5}\,dx$$

input
```
integrate(erf(b*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erf}(b\,x)^2}{x^5}\,dx$$

input
```
int(erf(b*x)^2/x^5,x)
```

output
```
int(erf(b*x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^5}\,dx = \frac{-\operatorname{erf}(bx)^2\,\pi + 4\sqrt{\pi}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^4}dx\right)b\,x^4}{4\pi\,x^4}$$

input
```
int(erf(b*x)^2/x^5,x)
```

output
```
( - erf(b*x)**2*pi + 4*sqrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x**4),x)*b*x*
*4)/(4*pi*x**4)
```

3.28 $\int \frac{\text{erf}(bx)^2}{x^7} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 177

$$\int \frac{\text{erf}(bx)^2}{x^7} \, dx = -\frac{b^2 e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4 e^{-2b^2x^2}}{9\pi x^2} - \frac{2be^{-b^2x^2}\text{erf}(bx)}{15\sqrt{\pi}x^5} + \frac{4b^3 e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x^3} - \frac{8b^5 e^{-b^2x^2}\text{erf}(bx)}{45\sqrt{\pi}x} - \frac{4}{45}b^6\text{erf}(bx)^2 - \frac{\text{erf}(bx)^2}{6x^6} + \frac{28b^6 \text{ ExpIntegralEi}\left(-2b^2x^2\right)}{45\pi}$$

output

```
-1/15*b^2/exp(2*b^2*x^2)/Pi/x^4+2/9*b^4/exp(2*b^2*x^2)/Pi/x^2-2/15*b*erf(b
*x)/exp(b^2*x^2)/Pi^(1/2)/x^5+4/45*b^3*erf(b*x)/exp(b^2*x^2)/Pi^(1/2)/x^3-
8/45*b^5*erf(b*x)/exp(b^2*x^2)/Pi^(1/2)/x-4/45*b^6*erf(b*x)^2-1/6*erf(b*x)
^2/x^6+28/45*b^6*Ei(-2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{\text{erf}(bx)^2}{x^7} \, dx$$
$$= \frac{e^{-2b^2x^2} \left(-6b^2x^2 + 20b^4x^4 - 4be^{b^2x^2}\sqrt{\pi}x(3 - 2b^2x^2 + 4b^4x^4)\,\text{erf}(bx) - e^{2b^2x^2}\pi(15 + 8b^6x^6)\,\text{erf}(bx)^2 + 56b^6\right.}{90\pi x^6}$$

input
```
Integrate[Erf[b*x]^2/x^7,x]
```

output
```
(-6*b^2*x^2 + 20*b^4*x^4 - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 - 2*b^2*x^2 + 4*b
^4*x^4)*Erf[b*x] - E^(2*b^2*x^2)*Pi*(15 + 8*b^6*x^6)*Erf[b*x]^2 + 56*b^6*E
^(2*b^2*x^2)*x^6*ExpIntegralEi[-2*b^2*x^2])/(90*E^(2*b^2*x^2)*Pi*x^6)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6918, 6945, 2643, 2643, 2639, 6945, 2643, 2639, 6945, 2639, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)^2}{x^7} \, dx$$

$$\Big\downarrow \text{ 6918}$$

$$\frac{2b\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^6}dx}{3\sqrt{\pi}} - \frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 6945}$$

$$\frac{2b\left(-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}dx + \frac{2b\int \frac{e^{-2b^2x^2}}{x^5}dx}{5\sqrt{\pi}} - \frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}\right)}{3\sqrt{\pi}} - \frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2b\left(-\frac{2}{5}b^2\int\frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}dx+\frac{2b\left(b^2\left(-\int\frac{e^{-2b^2x^2}}{x^3}dx\right)-\frac{e^{-2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2b\left(-\frac{2}{5}b^2\int\frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}dx+\frac{2b\left(-\left(b^2\left(-2b^2\int\frac{e^{-2b^2x^2}}{x}dx-\frac{e^{-2b^2x^2}}{2x^2}\right)\right)-\frac{e^{-2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{2b\left(-\frac{2}{5}b^2\int\frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}+\frac{2b\left(-\left(b^2\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)\right)-\frac{e^{-2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 6945}$$

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int\frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx+\frac{2b\int\frac{e^{-2b^2x^2}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}+\frac{2b\left(-\left(b^2\left(b^2\left(-\text{ExpIntegralEi}\left(-2b\right.\right.\right.\right.\right.}{5\sqrt{\pi}}}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int\frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx+\frac{2b\left(-2b^2\int\frac{e^{-2b^2x^2}}{x}dx-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}+\frac{2b\left(-\left(b^2\left(b^2\left(-\right.\right.\right.\right.}{}}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int\frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{5x^5}+\frac{2b\left(\right.}{}\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

$\Big\downarrow$ 6945

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx+\frac{2b\int\frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(\right.\right.\right.}{3\sqrt{\pi}}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2639

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIn}\right.\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

$\Big\downarrow$ 6927

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-\sqrt{\pi}b\int\text{erf}(bx)d\text{erf}(bx)-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIn}\right.\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

$\Big\downarrow$ 15

$$\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\text{erf}(bx)^2\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(\right.\right.\right.}{3\sqrt{\pi}}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erf}(bx)^2}{6x^6}$$

input
```
Int[Erf[b*x]^2/x^7,x]
```

output
```
-1/6*Erf[b*x]^2/x^6 + (2*b*(-1/5*Erf[b*x]/(E^(b^2*x^2)*x^5) + (2*b*(-1/4*1
/(E^(2*b^2*x^2)*x^4) - b^2*(-1/2*1/(E^(2*b^2*x^2)*x^2) - b^2*ExpIntegralEi
[-2*b^2*x^2])))/(5*Sqrt[Pi]) - (2*b^2*(-1/3*Erf[b*x]/(E^(b^2*x^2)*x^3) + (
2*b*(-1/2*1/(E^(2*b^2*x^2)*x^2) - b^2*ExpIntegralEi[-2*b^2*x^2]))/(3*Sqrt[
Pi]) - (2*b^2*(-(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (
b*ExpIntegralEi[-2*b^2*x^2])/Sqrt[Pi]))/3))/5))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^7} dx$$

input
```
int(erf(b*x)^2/x^7,x)
```

output
```
int(erf(b*x)^2/x^7,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{erf}(bx)^2}{x^7} dx = \frac{56 b^6 x^6 \operatorname{Ei}(-2 b^2 x^2) - 4 \sqrt{\pi}(4 b^5 x^5 - 2 b^3 x^3 + 3 bx) \operatorname{erf}(bx) e^{(-b^2 x^2)} - (15 \pi + 8 \pi b^6 x^6) \operatorname{erf}(bx)^2 + 2 (10 b^4}{90 \pi x^6}$$

input
```
integrate(erf(b*x)^2/x^7,x, algorithm="fricas")
```

output
```
1/90*(56*b^6*x^6*Ei(-2*b^2*x^2) - 4*sqrt(pi)*(4*b^5*x^5 - 2*b^3*x^3 + 3*b*
x)*erf(b*x)*e^(-b^2*x^2) - (15*pi + 8*pi*b^6*x^6)*erf(b*x)^2 + 2*(10*b^4*x
^4 - 3*b^2*x^2)*e^(-2*b^2*x^2))/(pi*x^6)
```

Sympy [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx = \int \frac{\operatorname{erf}^2(bx)}{x^7}\,dx$$

input
```
integrate(erf(b*x)**2/x**7,x)
```

output
```
Integral(erf(b*x)**2/x**7, x)
```

Maxima [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx = \int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx$$

input
```
integrate(erf(b*x)^2/x^7,x, algorithm="maxima")
```

output
```
2/3*b*integrate(erf(b*x)*e^(-b^2*x^2)/x^6, x)/sqrt(pi) - 1/6*erf(b*x)^2/x^
6
```

Giac [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx = \int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx$$

input
```
integrate(erf(b*x)^2/x^7,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx = \int \frac{\operatorname{erf}(b\,x)^2}{x^7}\,dx$$

input
```
int(erf(b*x)^2/x^7,x)
```

output
```
int(erf(b*x)^2/x^7, x)
```

Reduce [F]

$$\int \frac{\operatorname{erf}(bx)^2}{x^7}\,dx = \frac{-\operatorname{erf}(bx)^2\,\pi + 4\sqrt{\pi}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^6}dx\right)b\,x^6}{6\pi\,x^6}$$

input
```
int(erf(b*x)^2/x^7,x)
```

output
```
( - erf(b*x)**2*pi + 4*sqrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x**6),x)*b*x*
*6)/(6*pi*x**6)
```

3.29 $\int x^4 \text{erf}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 165

$$\int x^4 \text{erf}(bx)^2 \, dx = \frac{11 e^{-2b^2x^2} x}{20 b^4 \pi} + \frac{e^{-2b^2x^2} x^3}{5 b^2 \pi} + \frac{4 e^{-b^2x^2} \text{erf}(bx)}{5 b^5 \sqrt{\pi}} + \frac{4 e^{-b^2x^2} x^2 \text{erf}(bx)}{5 b^3 \sqrt{\pi}} + \frac{2 e^{-b^2x^2} x^4 \text{erf}(bx)}{5 b \sqrt{\pi}} + \frac{1}{5} x^5 \text{erf}(bx)^2 - \frac{43 \text{erf}\left(\sqrt{2} bx\right)}{40 b^5 \sqrt{2\pi}}$$

output
```
11/20*x/b^4/exp(2*b^2*x^2)/Pi+1/5*x^3/b^2/exp(2*b^2*x^2)/Pi+4/5*erf(b*x)/b
^5/exp(b^2*x^2)/Pi^(1/2)+4/5*x^2*erf(b*x)/b^3/exp(b^2*x^2)/Pi^(1/2)+2/5*x^
4*erf(b*x)/b/exp(b^2*x^2)/Pi^(1/2)+1/5*x^5*erf(b*x)^2-43/80*erf(2^(1/2)*b*
x)/b^5*2^(1/2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$\int x^4 \text{erf}(bx)^2 \, dx = \frac{4 b e^{-2b^2x^2} x (11 + 4 b^2 x^2) + 32 e^{-b^2x^2} \sqrt{\pi} (2 + 2 b^2 x^2 + b^4 x^4) \text{erf}(bx) + 16 b^5 \pi x^5 \text{erf}(bx)^2 - 43 \sqrt{2\pi} \text{erf}\left(\sqrt{2} bx\right)}{80 b^5 \pi}$$

input
```
Integrate[x^4*Erf[b*x]^2,x]
```

output
```
((4*b*x*(11 + 4*b^2*x^2))/E^(2*b^2*x^2) + (32*Sqrt[Pi]*(2 + 2*b^2*x^2 + b^
4*x^4)*Erf[b*x])/E^(b^2*x^2) + 16*b^5*Pi*x^5*Erf[b*x]^2 - 43*Sqrt[2*Pi]*Er
f[Sqrt[2]*b*x])/(80*b^5*Pi)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.58, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6918, 6939, 2641, 2641, 2634, 6939, 2641, 2634, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erf}(bx)^2 \, dx$$

$$\Big\downarrow \text{6918}$$

$$\frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4b\int e^{-b^2x^2}x^5\text{erf}(bx)dx}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{6939}$$

$$\frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2} + \frac{\int e^{-2b^2x^2}x^4dx}{\sqrt{\pi}b} - \frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2} + \frac{\frac{3\int e^{-2b^2x^2}x^2dx}{4b^2} - \frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b} - \frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2} + \frac{\frac{3\left(\frac{\int e^{-2b^2x^2}dx}{4b^2} - \frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2} - \frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b} - \frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{5}x^5\text{erf}(bx)^2-\frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 6939

$$\frac{1}{5}x^5\text{erf}(bx)^2-\frac{4b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\int e^{-2b^2x^2}x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 2641

$$\frac{1}{5}x^5\text{erf}(bx)^2-\frac{4b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 2634

$$\frac{1}{5}x^5\text{erf}(bx)^2-\frac{4b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 6936

$$\frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4b\left(\frac{2\left(\frac{\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}}{\sqrt{\pi}b}\right.}{5\sqrt{\pi}}$$

$$\downarrow 2634$$

$$\frac{1}{5}x^5\text{erf}(bx)^2 - \frac{4b\left(-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{2\left(-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\text{erf}(\sqrt{2}bx)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2}}{4b^2}\right.}{4b^2}}{\sqrt{\pi}b}\right.}{5\sqrt{\pi}}$$

input
```
Int[x^4*Erf[b*x]^2,x]
```

output
```
(x^5*Erf[b*x]^2)/5 - (4*b*(-1/2*(x^4*Erf[b*x])/(b^2*E^(b^2*x^2)) + (-1/4*x
^3/(b^2*E^(2*b^2*x^2)) + (3*(-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[
Sqrt[2]*b*x])/(8*b^3)))/(4*b^2))/(b*Sqrt[Pi]) + (2*(-1/2*(x^2*Erf[b*x])/(b
^2*E^(b^2*x^2)) + (-1/2*Erf[b*x]/(b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2*S
qrt[2]*b^2))/b^2 + (-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b
*x])/(8*b^3))/(b*Sqrt[Pi])))/b^2))/(5*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\operatorname{erf}(bx)^2 b^5 x^5}{5}-\frac{4\operatorname{erf}(bx)\left(-\frac{e^{-b^2x^2}x^4b^4}{2}-e^{-b^2x^2}x^2b^2-e^{-b^2x^2}\right)}{5\sqrt{\pi}}+\frac{-\frac{43\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{80}+\frac{11\,e^{-2b^2x^2}bx}{20}+\frac{e^{-2b^2x^2}b^3x^3}{5}}{\pi}}{b^5}$
default	$\frac{\frac{\operatorname{erf}(bx)^2 b^5 x^5}{5}-\frac{4\operatorname{erf}(bx)\left(-\frac{e^{-b^2x^2}x^4b^4}{2}-e^{-b^2x^2}x^2b^2-e^{-b^2x^2}\right)}{5\sqrt{\pi}}+\frac{-\frac{43\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{80}+\frac{11\,e^{-2b^2x^2}bx}{20}+\frac{e^{-2b^2x^2}b^3x^3}{5}}{\pi}}{b^5}$

input
```
int(x^4*erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*erf(b*x)^2*b^5*x^5-4/5*erf(b*x)/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^4
*x^4-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))+4/5/Pi*(-43/64*2^(1/2)*Pi^(1/2)*
erf(2^(1/2)*b*x)+11/16/exp(b^2*x^2)^2*b*x+1/4/exp(b^2*x^2)^2*b^3*x^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.67

$$\int x^4 \operatorname{erf}(bx)^2\,dx = \frac{16\,\pi b^6 x^5 \operatorname{erf}(bx)^2 + 32\,\sqrt{\pi}(b^5x^4+2\,b^3x^2+2\,b)\operatorname{erf}(bx)\,e^{(-b^2x^2)} - 43\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right) + 4\,(4\,b^4x}{80\,\pi b^6}$$

input
```
integrate(x^4*erf(b*x)^2,x, algorithm="fricas")
```

output
```
1/80*(16*pi*b^6*x^5*erf(b*x)^2 + 32*sqrt(pi)*(b^5*x^4 + 2*b^3*x^2 + 2*b)*e
rf(b*x)*e^(-b^2*x^2) - 43*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)
*x) + 4*(4*b^4*x^3 + 11*b^2*x)*e^(-2*b^2*x^2))/(pi*b^6)
```

Sympy [F]

$$\int x^4 \mathrm{erf}(bx)^2 \, dx = \int x^4 \, \mathrm{erf}^2 \left(bx\right) dx$$

input
```
integrate(x**4*erf(b*x)**2,x)
```

output
```
Integral(x**4*erf(b*x)**2, x)
```

Maxima [F]

$$\int x^4 \mathrm{erf}(bx)^2 \, dx = \int x^4 \, \mathrm{erf}\left(bx\right)^2 \, dx$$

input
```
integrate(x^4*erf(b*x)^2,x, algorithm="maxima")
```

output
```
-1/5*integrate(4*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-2*b^2*x^2), x)/(pi*b^4) + 1
/5*(sqrt(pi)*b^5*x^5*erf(b*x)^2 + 2*(b^4*x^4 + 2*b^2*x^2 + 2)*erf(b*x)*e^(
-b^2*x^2))/(sqrt(pi)*b^5)
```

Giac [F]

$$\int x^4 \mathrm{erf}(bx)^2 \, dx = \int x^4 \, \mathrm{erf}\left(bx\right)^2 \, dx$$

input
```
integrate(x^4*erf(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*erf(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

$$\int x^4 \text{erf}(bx)^2 \, dx = \frac{x^5 \operatorname{erf}(b\,x)^2}{5} + \frac{\frac{4\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}\operatorname{erf}(b\,x)}{5} + \frac{b^3\,x^3\,\mathrm{e}^{-2\,b^2\,x^2}}{5} - \frac{43\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\,b\,x\right)}{80} + \frac{11\,b\,x\,\mathrm{e}^{-2\,b^2\,x^2}}{20} + \frac{4\,b^2\,x^2\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}\operatorname{erf}(b\,x)}{5} + \frac{2\,b^4\,x^4\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}}{5}}{b^5\,\pi}$$

input
```
int(x^4*erf(b*x)^2,x)
```

output
```
(x^5*erf(b*x)^2)/5 + ((4*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/5 + (b^3*x^3*exp
(-2*b^2*x^2))/5 - (43*2^(1/2)*pi^(1/2)*erf(2^(1/2)*b*x))/80 + (11*b*x*exp(
-2*b^2*x^2))/20 + (4*b^2*x^2*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/5 + (2*b^4*x
^4*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/5)/(b^5*pi)
```

Reduce [F]

$$\int x^4 \text{erf}(bx)^2 \, dx = \int \operatorname{erf}(bx)^2 \, x^4 dx$$

input
```
int(x^4*erf(b*x)^2,x)
```

output
```
int(erf(b*x)**2*x**4,x)
```

3.30 $\int x^2\mathbf{erf}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int x^2\text{erf}(bx)^2\,dx=\frac{e^{-2b^2x^2}x}{3b^2\pi}+\frac{2e^{-b^2x^2}\text{erf}(bx)}{3b^3\sqrt{\pi}}+\frac{2e^{-b^2x^2}x^2\text{erf}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erf}(bx)^2-\frac{5\text{erf}\left(\sqrt{2}bx\right)}{6b^3\sqrt{2\pi}}$$

output
```
1/3*x/b^2/exp(2*b^2*x^2)/Pi+2/3*erf(b*x)/b^3/exp(b^2*x^2)/Pi^(1/2)+2/3*x^2
*erf(b*x)/b/exp(b^2*x^2)/Pi^(1/2)+1/3*x^3*erf(b*x)^2-5/12*erf(2^(1/2)*b*x)
/b^3*2^(1/2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int x^2\text{erf}(bx)^2\,dx=\frac{4be^{-2b^2x^2}x+8e^{-b^2x^2}\sqrt{\pi}(1+b^2x^2)\,\text{erf}(bx)+4b^3\pi x^3\text{erf}(bx)^2-5\sqrt{2\pi}\text{erf}\left(\sqrt{2}bx\right)}{12b^3\pi}$$

input
```
Integrate[x^2*Erf[b*x]^2,x]
```

output
```
((4*b*x)/E^(2*b^2*x^2) + (8*Sqrt[Pi]*(1 + b^2*x^2)*Erf[b*x])/E^(b^2*x^2) +
 4*b^3*Pi*x^3*Erf[b*x]^2 - 5*Sqrt[2*Pi]*Erf[Sqrt[2]*b*x])/(12*b^3*Pi)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6918, 6939, 2641, 2634, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erf}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6918}$$

$$\frac{1}{3}x^3\text{erf}(bx)^2 - \frac{4b\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 6939}$$

$$\frac{1}{3}x^3\text{erf}(bx)^2 - \frac{4b\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\int e^{-2b^2x^2}x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{3}x^3\text{erf}(bx)^2 - \frac{4b\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{1}{3}x^3\text{erf}(bx)^2 - \frac{4b\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 6936}$$

$$\frac{1}{3}x^3\text{erf}(bx)^2 - \frac{4b\left(\frac{\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{3}x^3\text{erf}(bx)^2-\frac{4b\left(-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\text{erf}(\sqrt{2}bx)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

input
```
Int[x^2*Erf[b*x]^2,x]
```

output
```
(x^3*Erf[b*x]^2)/3 - (4*b*(-1/2*(x^2*Erf[b*x])/(b^2*E^(b^2*x^2)) + (-1/2*E
rf[b*x]/(b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2))/b^2 + (-1/4*
x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi]
)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6918
```
Int[Erf[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erf[b*x]^2/(
m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erf[b*x])/E^(b
^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\operatorname{erf}(bx)^2 b^3 x^3}{3}-\frac{4\operatorname{erf}(bx)\left(-\frac{\mathrm{e}^{-b^2x^2}x^2b^2}{2}-\frac{\mathrm{e}^{-b^2x^2}}{2}\right)}{3\sqrt{\pi}}+\frac{-\frac{5\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{12}+\frac{\mathrm{e}^{-2b^2x^2}bx}{3}}{\pi}}{b^3}$	95
default	$\frac{\frac{\operatorname{erf}(bx)^2 b^3 x^3}{3}-\frac{4\operatorname{erf}(bx)\left(-\frac{\mathrm{e}^{-b^2x^2}x^2b^2}{2}-\frac{\mathrm{e}^{-b^2x^2}}{2}\right)}{3\sqrt{\pi}}+\frac{-\frac{5\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{12}+\frac{\mathrm{e}^{-2b^2x^2}bx}{3}}{\pi}}{b^3}$	95

input
```
int(x^2*erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*erf(b*x)^2*b^3*x^3-4/3*erf(b*x)/Pi^(1/2)*(-1/2*b^2*x^2/exp(b^2*
x^2)-1/2/exp(b^2*x^2))+4/3/Pi*(-5/16*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)+1/4
/exp(b^2*x^2)^2*b*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int x^2\operatorname{erf}(bx)^2\,dx = \frac{4\,\pi b^4x^3\operatorname{erf}\left(bx\right)^2+4\,b^2xe^{\left(-2\,b^2x^2\right)}+8\,\sqrt{\pi}(b^3x^2+b)\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}-5\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)}{12\,\pi b^4}$$

input
```
integrate(x^2*erf(b*x)^2,x, algorithm="fricas")
```

output
```
1/12*(4*pi*b^4*x^3*erf(b*x)^2 + 4*b^2*x*e^(-2*b^2*x^2) + 8*sqrt(pi)*(b^3*x
^2 + b)*erf(b*x)*e^(-b^2*x^2) - 5*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*s
qrt(b^2)*x))/(pi*b^4)
```

Sympy [F]

$$\int x^2 \mathrm{erf}(bx)^2 \, dx = \int x^2 \operatorname{erf}^2(bx) \, dx$$

input
```
integrate(x**2*erf(b*x)**2,x)
```

output
```
Integral(x**2*erf(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \mathrm{erf}(bx)^2 \, dx = \int x^2 \operatorname{erf}(bx)^2 \, dx$$

input
```
integrate(x^2*erf(b*x)^2,x, algorithm="maxima")
```

output
```
-1/3*integrate(4*(b^2*x^2 + 1)*e^(-2*b^2*x^2), x)/(pi*b^2) + 1/3*(pi*b^3*x
^3*erf(b*x)^2 + 2*(sqrt(pi)*b^2*x^2 + sqrt(pi))*erf(b*x)*e^(-b^2*x^2))/(pi
*b^3)
```

Giac [F]

$$\int x^2\text{erf}(bx)^2\,dx = \int x^2\,\text{erf}\,(bx)^2\;dx$$

input
```
integrate(x^2*erf(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*erf(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int x^2\text{erf}(bx)^2\,dx$$
$$= \frac{x^3\,\text{erf}(b\,x)^2}{3} + \frac{\frac{2\,\sqrt{\pi}\,\text{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{3} - \frac{5\,\sqrt{2}\,\sqrt{\pi}\,\text{erf}\left(\sqrt{2}\,b\,x\right)}{12} + \frac{b\,x\,\text{e}^{-2\,b^2\,x^2}}{3} + \frac{2\,b^2\,x^2\,\sqrt{\pi}\,\text{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{3}}{b^3\,\pi}$$

input
```
int(x^2*erf(b*x)^2,x)
```

output
```
(x^3*erf(b*x)^2)/3 + ((2*pi^(1/2)*exp(-b^2*x^2)*erf(b*x))/3 - (5*2^(1/2)*p
i^(1/2)*erf(2^(1/2)*b*x))/12 + (b*x*exp(-2*b^2*x^2))/3 + (2*b^2*x^2*pi^(1/
2)*exp(-b^2*x^2)*erf(b*x))/3)/(b^3*pi)
```

Reduce [F]

$$\int x^2\text{erf}(bx)^2\,dx = \int \text{erf}(bx)^2\,x^2dx$$

input
```
int(x^2*erf(b*x)^2,x)
```

output
```
int(erf(b*x)**2*x**2,x)
```

3.31 $\int \mathbf{erf}(bx)^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int \operatorname{erf}(bx)^2 \, dx = \frac{2e^{-b^2x^2}\operatorname{erf}(bx)}{b\sqrt{\pi}} + x\operatorname{erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\operatorname{erf}\left(\sqrt{2}bx\right)}{b}$$

output
```
2*erf(b*x)/b/exp(b^2*x^2)/Pi^(1/2)+x*erf(b*x)^2-2^(1/2)/Pi^(1/2)*erf(2^(1/
2)*b*x)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \operatorname{erf}(bx)^2 \, dx = \frac{2e^{-b^2x^2}\operatorname{erf}(bx)}{b\sqrt{\pi}} + x\operatorname{erf}(bx)^2 - \frac{\sqrt{\frac{2}{\pi}}\operatorname{erf}\left(\sqrt{2}bx\right)}{b}$$

input
```
Integrate[Erf[b*x]^2,x]
```

output
```
(2*Erf[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]) + x*Erf[b*x]^2 - (Sqrt[2/Pi]*Erf[Sqr
t[2]*b*x])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6906, 27, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6906}$$

$$x\text{erf}(bx)^2 - \frac{4\int be^{-b^2x^2}x\text{erf}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 27}$$

$$x\text{erf}(bx)^2 - \frac{4b\int e^{-b^2x^2}x\text{erf}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 6936}$$

$$x\text{erf}(bx)^2 - \frac{4b\left(\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b} - \frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2634}$$

$$x\text{erf}(bx)^2 - \frac{4b\left(\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2} - \frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

input
```
Int[Erf[b*x]^2,x]
```

output
```
x*Erf[b*x]^2 - (4*b*(-1/2*Erf[b*x]/(b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2
*Sqrt[2]*b^2)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6906
```
Int[Erf[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]^2/
b), x] - Simp[4/Sqrt[Pi]   Int[(a + b*x)*(Erf[a + b*x]/E^(a + b*x)^2), x],
x] /; FreeQ[{a, b}, x]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\operatorname{erf}(bx)^{2} bx+\frac{2 \operatorname{erf}(bx) e^{-b^{2} x^{2}}}{\sqrt{\pi}}-\frac{\sqrt{2}\, \operatorname{erf}\left(\sqrt{2}\, bx\right)}{\sqrt{\pi}}}{b}$	48
default	$\frac{\operatorname{erf}(bx)^{2} bx+\frac{2 \operatorname{erf}(bx) e^{-b^{2} x^{2}}}{\sqrt{\pi}}-\frac{\sqrt{2}\, \operatorname{erf}\left(\sqrt{2}\, bx\right)}{\sqrt{\pi}}}{b}$	48

input
```
int(erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(erf(b*x)^2*b*x+2*erf(b*x)/Pi^(1/2)*exp(-b^2*x^2)-1/Pi^(1/2)*2^(1/2)*e
rf(2^(1/2)*b*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \operatorname{erf}(bx)^2\,dx = \frac{\pi b^2 x \operatorname{erf}(bx)^2 + 2\sqrt{\pi}b\operatorname{erf}(bx)\,e^{(-b^2x^2)} - \sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)}{\pi b^2}$$

input
```
integrate(erf(b*x)^2,x, algorithm="fricas")
```

output
```
(pi*b^2*x*erf(b*x)^2 + 2*sqrt(pi)*b*erf(b*x)*e^(-b^2*x^2) - sqrt(2)*sqrt(p
i)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F]

$$\int \operatorname{erf}(bx)^2\,dx = \int \operatorname{erf}^2(bx)\,dx$$

input
```
integrate(erf(b*x)**2,x)
```

output
```
Integral(erf(b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \operatorname{erf}(bx)^2\,dx = \frac{\left(\sqrt{\pi}bx\operatorname{erf}(bx)^2 e^{(b^2x^2)} + 2\operatorname{erf}(bx)\right)e^{(-b^2x^2)}}{\sqrt{\pi}b} - \frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}bx\right)}{\sqrt{\pi}b}$$

input
```
integrate(erf(b*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(pi)*b*x*erf(b*x)^2*e^(b^2*x^2) + 2*erf(b*x))*e^(-b^2*x^2)/(sqrt(pi)*
b) - sqrt(2)*erf(sqrt(2)*b*x)/(sqrt(pi)*b)
```

Giac [F]

$$\int \operatorname{erf}(bx)^2\,dx = \int \operatorname{erf}(bx)^2\,dx$$

input
```
integrate(erf(b*x)^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \operatorname{erf}(bx)^2\,dx = x \operatorname{erf}(b\,x)^2 + \frac{2\,e^{-b^2\,x^2}\operatorname{erf}(b\,x) - \sqrt{2}\operatorname{erf}\left(\sqrt{2}\,b\,x\right)}{b\,\sqrt{\pi}}$$

input
```
int(erf(b*x)^2,x)
```

output
```
x*erf(b*x)^2 + (2*exp(-b^2*x^2)*erf(b*x) - 2^(1/2)*erf(2^(1/2)*b*x))/(b*pi
^(1/2))
```

Reduce [F]

$$\int \operatorname{erf}(bx)^2\,dx = \int \operatorname{erf}(bx)^2\,dx$$

input
```
int(erf(b*x)^2,x)
```

output
```
int(erf(b*x)**2,x)
```

3.32 $\int \frac{\text{erf}(bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erf}(bx)^2}{x^2}\,dx = \text{Int}\left(\frac{\text{erf}(bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(erf(b*x)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x^2}\,dx = \int \frac{\text{erf}(bx)^2}{x^2}\,dx$$

input
```
Integrate[Erf[b*x]^2/x^2,x]
```

output
```
Integrate[Erf[b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)^2}{x^2} dx$$
$$\Big\downarrow \text{ 6924}$$
$$\int \frac{\text{erf}(bx)^2}{x^2}dx$$

input
```
Int[Erf[b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx\right)^2}{x^2}dx$$

input
```
int(erf(b*x)^2/x^2,x)
```

output
```
int(erf(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx$$

input
```
integrate(erf(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(erf(b*x)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erf}^2(bx)}{x^2}\,dx$$

input
```
integrate(erf(b*x)**2/x**2,x)
```

output
```
Integral(erf(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx$$

input
```
integrate(erf(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
4*b*integrate(erf(b*x)*e^(-b^2*x^2)/x, x)/sqrt(pi) - erf(b*x)^2/x
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x^2}\,dx$$

input
```
integrate(erf(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erf}(b\,x)^2}{x^2}\,dx$$

input
```
int(erf(b*x)^2/x^2,x)
```

output
```
int(erf(b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.40

$$\int \frac{\operatorname{erf}(bx)^2}{x^2}\,dx = \frac{-\operatorname{erf}(bx)^2\,\pi + 4\sqrt{\pi}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x}dx\right)bx}{\pi x}$$

input
```
int(erf(b*x)^2/x^2,x)
```

output
```
( - erf(b*x)**2*pi + 4*sqrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x),x)*b*x)/(p
i*x)
```

3.33 $\int \frac{\mathbf{erf}(bx)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erf}(bx)^2}{x^4}\,dx = \text{Int}\left(\frac{\text{erf}(bx)^2}{x^4},x\right)$$

output
```
Defer(Int)(erf(b*x)^2/x^4,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x^4}\,dx = \int \frac{\text{erf}(bx)^2}{x^4}\,dx$$

input
```
Integrate[Erf[b*x]^2/x^4,x]
```

output
```
Integrate[Erf[b*x]^2/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 6924}$$

$$\int \frac{\text{erf}(bx)^2}{x^4}dx$$

input
```
Int[Erf[b*x]^2/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx\right)^2}{x^4}dx$$

input
```
int(erf(b*x)^2/x^4,x)
```

output
```
int(erf(b*x)^2/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^4} \, dx = \int \frac{\operatorname{erf}(bx)^2}{x^4} \, dx$$

input
```
integrate(erf(b*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(erf(b*x)^2/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx)^2}{x^4} \, dx = \int \frac{\operatorname{erf}^2(bx)}{x^4} \, dx$$

input
```
integrate(erf(b*x)**2/x**4,x)
```

output
```
Integral(erf(b*x)**2/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \frac{\operatorname{erf}(bx)^2}{x^4} \, dx = \int \frac{\operatorname{erf}(bx)^2}{x^4} \, dx$$

input
```
integrate(erf(b*x)^2/x^4,x, algorithm="maxima")
```

output
```
4/3*b*integrate(erf(b*x)*e^(-b^2*x^2)/x^3, x)/sqrt(pi) - 1/3*erf(b*x)^2/x^
3
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(erf(b*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erf}(b\,x)^2}{x^4}\,dx$$

input
```
int(erf(b*x)^2/x^4,x)
```

output
```
int(erf(b*x)^2/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

$$\int \frac{\operatorname{erf}(bx)^2}{x^4}\,dx = \frac{-\operatorname{erf}(bx)^2\,\pi + 4\sqrt{\pi}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^3}dx\right)b\,x^3}{3\pi\,x^3}$$

input
```
int(erf(b*x)^2/x^4,x)
```

output
```
( - erf(b*x)**2*pi + 4*sqrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x**3),x)*b*x*
*3)/(3*pi*x**3)
```

3.34 $\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx=\operatorname{Int}\left(\frac{\operatorname{erf}(bx)^2}{x^6},x\right)$$

output
```
Defer(Int)(erf(b*x)^2/x^6,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx=\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx$$

input
```
Integrate[Erf[b*x]^2/x^6,x]
```

output
```
Integrate[Erf[b*x]^2/x^6, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)^2}{x^6} \, dx$$
$$\Big\downarrow \ 6924$$
$$\int \frac{\text{erf}(bx)^2}{x^6}dx$$

input
```
Int[Erf[b*x]^2/x^6,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx\right)^{2}}{x^{6}}dx$$

input
```
int(erf(b*x)^2/x^6,x)
```

output
```
int(erf(b*x)^2/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x^6}\,dx$$

input
```
integrate(erf(b*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral(erf(b*x)^2/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erf}^2\left(bx\right)}{x^6}\,dx$$

input
```
integrate(erf(b*x)**2/x**6,x)
```

output
```
Integral(erf(b*x)**2/x**6, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erf}\left(bx\right)^2}{x^6}\,dx$$

input
```
integrate(erf(b*x)^2/x^6,x, algorithm="maxima")
```

output
```
4/5*b*integrate(erf(b*x)*e^(-b^2*x^2)/x^5, x)/sqrt(pi) - 1/5*erf(b*x)^2/x^
5
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x^6}\,dx = \int \frac{\text{erf}\left(bx\right)^2}{x^6}\,dx$$

input
```
integrate(erf(b*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(erf(b*x)^2/x^6, x)
```

Mupad [N/A]

Not integrable

Time = 4.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erf}(bx)^2}{x^6}\,dx = \int \frac{\text{erf}(b\,x)^2}{x^6}\,dx$$

input
```
int(erf(b*x)^2/x^6,x)
```

output
```
int(erf(b*x)^2/x^6, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

$$\int \frac{\operatorname{erf}(bx)^2}{x^6}\,dx = \frac{-\operatorname{erf}(bx)^2\,\pi + 4\sqrt{\pi}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^5}dx\right) b\,x^5}{5\pi\,x^5}$$

input
```
int(erf(b*x)^2/x^6,x)
```

output
```
( - erf(b*x)**2*pi + 4*sqrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x**5),x)*b*x*
*5)/(5*pi*x**5)
```

3.35 $\int (c+dx)^2\mathbf{erf}(a+bx)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 375

$$\begin{aligned}\int (c+dx)^2\text{erf}(a+bx)^2\,dx &= \frac{d(bc-ad)e^{-2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{-2(a+bx)^2}(a+bx)}{3b^3\pi} \\ &+\frac{2d^2e^{-(a+bx)^2}\text{erf}(a+bx)}{3b^3\sqrt{\pi}} \\ &+\frac{2(bc-ad)^2e^{-(a+bx)^2}\text{erf}(a+bx)}{b^3\sqrt{\pi}} \\ &+\frac{2d(bc-ad)e^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)}{b^3\sqrt{\pi}} \\ &+\frac{2d^2e^{-(a+bx)^2}(a+bx)^2\text{erf}(a+bx)}{3b^3\sqrt{\pi}} \\ &-\frac{d(bc-ad)\text{erf}(a+bx)^2}{2b^3}+\frac{(bc-ad)^2(a+bx)\text{erf}(a+bx)^2}{b^3} \\ &+\frac{d(bc-ad)(a+bx)^2\text{erf}(a+bx)^2}{b^3} \\ &+\frac{d^2(a+bx)^3\text{erf}(a+bx)^2}{3b^3} \\ &-\frac{(bc-ad)^2\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^3}-\frac{5d^2\text{erf}\left(\sqrt{2}(a+bx)\right)}{6b^3\sqrt{2\pi}}\end{aligned}$$

output
```
d*(-a*d+b*c)/b^3/exp(2*(b*x+a)^2)/Pi+1/3*d^2*(b*x+a)/b^3/exp(2*(b*x+a)^2)/
Pi+2/3*d^2*erf(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)+2*(-a*d+b*c)^2*erf(b*x+a
)/b^3/exp((b*x+a)^2)/Pi^(1/2)+2*d*(-a*d+b*c)*(b*x+a)*erf(b*x+a)/b^3/exp((b
*x+a)^2)/Pi^(1/2)+2/3*d^2*(b*x+a)^2*erf(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)
-1/2*d*(-a*d+b*c)*erf(b*x+a)^2/b^3+(-a*d+b*c)^2*(b*x+a)*erf(b*x+a)^2/b^3+d
*(-a*d+b*c)*(b*x+a)^2*erf(b*x+a)^2/b^3+1/3*d^2*(b*x+a)^3*erf(b*x+a)^2/b^3-
(-a*d+b*c)^2*2^(1/2)/Pi^(1/2)*erf(2^(1/2)*(b*x+a))/b^3-5/12*d^2*erf(2^(1/2
)*(b*x+a))/b^3*2^(1/2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.60

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx = \frac{\frac{4de^{-2(a+bx)^2}(3bc-2ad+bdx)}{\pi}+\frac{8e^{-(a+bx)^2}\left((1+a^2)d^2-abd(3c+dx)+b^2(3c^2+3cdx+d^2x^2)\right)\text{erf}(a+bx)}{\sqrt{\pi}}+2(-3bcd-6a^2bcd+2a^3$$

input
```
Integrate[(c + d*x)^2*Erf[a + b*x]^2,x]
```

output
```
((4*d*(3*b*c - 2*a*d + b*d*x))/(E^(2*(a + b*x)^2)*Pi) + (8*((1 + a^2)*d^2
- a*b*d*(3*c + d*x) + b^2*(3*c^2 + 3*c*d*x + d^2*x^2))*Erf[a + b*x])/(E^(a
 + b*x)^2*Sqrt[Pi]) + 2*(-3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + 3*a*(2*b^2*c
^2 + d^2) + 2*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Erf[a + b*x]^2 - (12*b^2*
c^2 - 24*a*b*c*d + (5 + 12*a^2)*d^2)*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/(1
2*b^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \text{erf}(a+bx)^2 \, dx$$

$$\Big\downarrow \; 6921$$

$$\frac{\int \left((bc-ad)^2 \text{erf}(a+bx)^2 + d^2 (a+bx)^2 \text{erf}(a+bx)^2 + 2d(bc-ad)(a+bx)\text{erf}(a+bx)^2\right) d(a+bx)}{b^3}$$

$$\Big\downarrow \; 2009$$

$$d(a+bx)^2(bc-ad)\text{erf}(a+bx)^2 + (a+bx)(bc-ad)^2\text{erf}(a+bx)^2 + \frac{2de^{-(a+bx)^2}(a+bx)(bc-ad)\text{erf}(a+bx)}{\sqrt{\pi}} - \frac{1}{2}d(bc-ad$$

input
```
Int[(c + d*x)^2*Erf[a + b*x]^2,x]
```

output
```
((d*(b*c - a*d))/(E^(2*(a + b*x)^2)*Pi) + (d^2*(a + b*x))/(3*E^(2*(a + b*x
)^2)*Pi) + (2*d^2*Erf[a + b*x])/(3*E^(a + b*x)^2*Sqrt[Pi]) + (2*(b*c - a*d
)^2*Erf[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) + (2*d*(b*c - a*d)*(a + b*x)*Er
f[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) + (2*d^2*(a + b*x)^2*Erf[a + b*x])/(3
*E^(a + b*x)^2*Sqrt[Pi]) - (d*(b*c - a*d)*Erf[a + b*x]^2)/2 + (b*c - a*d)^
2*(a + b*x)*Erf[a + b*x]^2 + d*(b*c - a*d)*(a + b*x)^2*Erf[a + b*x]^2 + (d
^2*(a + b*x)^3*Erf[a + b*x]^2)/3 - (b*c - a*d)^2*Sqrt[2/Pi]*Erf[Sqrt[2]*(a
 + b*x)] - (5*d^2*Erf[Sqrt[2]*(a + b*x)])/(6*Sqrt[2*Pi]))/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6921
```
Int[Erf[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
1/b^(m + 1)    Subst[Int[ExpandIntegrand[Erf[x]^2, (b*c - a*d + d*x)^m, x],
x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)^2 \operatorname{erf}(bx+a)^2 \, dx$$

input
```
int((d*x+c)^2*erf(b*x+a)^2,x)
```

output
```
int((d*x+c)^2*erf(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.75

$$\int (c+dx)^2 \operatorname{erf}(a+bx)^2 \, dx = -\frac{\sqrt{2}\sqrt{\pi}(12\,b^2c^2 - 24\,abcd + (12\,a^2+5)d^2)\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) - 8\,\sqrt{\pi}(b^3d^2x^2 + 3\,b^3c^2 - 3\,ab^2cd + (a$$

input
```
integrate((d*x+c)^2*erf(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2)*sqrt(pi)*(12*b^2*c^2 - 24*a*b*c*d + (12*a^2 + 5)*d^2)*sqrt(
b^2)*erf(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 8*sqrt(pi)*(b^3*d^2*x^2 + 3*b^3*
c^2 - 3*a*b^2*c*d + (a^2 + 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x)*erf(b*x +
 a)*e^(-b^2*x^2 - 2*a*b*x - a^2) - 2*(2*pi*b^4*d^2*x^3 + 6*pi*b^4*c*d*x^2
+ 6*pi*b^4*c^2*x + pi*(6*a*b^3*c^2 - 3*(2*a^2 + 1)*b^2*c*d + (2*a^3 + 3*a)
*b*d^2))*erf(b*x + a)^2 - 4*(b^2*d^2*x + 3*b^2*c*d - 2*a*b*d^2)*e^(-2*b^2*
x^2 - 4*a*b*x - 2*a^2))/(pi*b^4)
```

Sympy [F]

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx = \int (c+dx)^2\,\text{erf}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*erf(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*erf(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx = \int (dx+c)^2\,\text{erf}\,(bx+a)^2\,\,dx$$

input
```
integrate((d*x+c)^2*erf(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)*erf(b*x + a)^2 - 1/3*integrate(4*(b*d^
2*x^3 + 3*b*c*d*x^2 + 3*b*c^2*x)*erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x - a^2)
, x)/sqrt(pi)
```

Giac [F]

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx = \int (dx+c)^2\,\text{erf}\,(bx+a)^2\,\,dx$$

input
```
integrate((d*x+c)^2*erf(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*erf(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.96

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx$$
$$= \frac{\text{erf}(a+b\,x)^2\,\left(\frac{a\,d^2}{2}-b\,\left(c\,d\,a^2+\frac{c\,d}{2}\right)+\frac{a^3\,d^2}{3}+a\,b^2\,c^2\right)}{b^3}+c^2\,x\,\text{erf}(a+b\,x)^2$$
$$+\frac{d^2\,x^3\,\text{erf}(a+b\,x)^2}{3}-\frac{\mathrm{e}^{-2\,a^2-4\,a\,b\,x-2\,b^2\,x^2}\,(2\,a\,d^2-3\,b\,c\,d)}{3\,b^3\,\pi}+c\,d\,x^2\,\text{erf}(a+b\,x)^2$$
$$+\frac{2\,\text{erf}(a+b\,x)\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(a^2\,d^2-3\,a\,b\,c\,d+3\,b^2\,c^2+d^2)}{3\,b^3\,\sqrt{\pi}}$$
$$-\frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}\,(a+b\,x)\right)\,(12\,a^2\,d^2-24\,a\,b\,c\,d+12\,b^2\,c^2+5\,d^2)}{12\,b^3\,\sqrt{\pi}}$$
$$+\frac{d^2\,x\,\mathrm{e}^{-2\,a^2-4\,a\,b\,x-2\,b^2\,x^2}}{3\,b^2\,\pi}+\frac{2\,d^2\,x^2\,\text{erf}(a+b\,x)\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}}{3\,b\,\sqrt{\pi}}$$
$$-\frac{2\,x\,\text{erf}(a+b\,x)\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(a\,d^2-3\,b\,c\,d)}{3\,b^2\,\sqrt{\pi}}$$

input
```
int(erf(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(erf(a + b*x)^2*((a*d^2)/2 - b*((c*d)/2 + a^2*c*d) + (a^3*d^2)/3 + a*b^2*c
^2))/b^3 + c^2*x*erf(a + b*x)^2 + (d^2*x^3*erf(a + b*x)^2)/3 - (exp(- 2*a^
2 - 2*b^2*x^2 - 4*a*b*x)*(2*a*d^2 - 3*b*c*d))/(3*b^3*pi) + c*d*x^2*erf(a +
 b*x)^2 + (2*erf(a + b*x)*exp(- a^2 - b^2*x^2 - 2*a*b*x)*(d^2 + a^2*d^2 +
3*b^2*c^2 - 3*a*b*c*d))/(3*b^3*pi^(1/2)) - (2^(1/2)*erf(2^(1/2)*(a + b*x))
*(5*d^2 + 12*a^2*d^2 + 12*b^2*c^2 - 24*a*b*c*d))/(12*b^3*pi^(1/2)) + (d^2*
x*exp(- 2*a^2 - 2*b^2*x^2 - 4*a*b*x))/(3*b^2*pi) + (2*d^2*x^2*erf(a + b*x)
*exp(- a^2 - b^2*x^2 - 2*a*b*x))/(3*b*pi^(1/2)) - (2*x*erf(a + b*x)*exp(-
a^2 - b^2*x^2 - 2*a*b*x)*(a*d^2 - 3*b*c*d))/(3*b^2*pi^(1/2))
```

Reduce [F]

$$\int (c+dx)^2\text{erf}(a+bx)^2\,dx = \left(\int \text{erf}(bx+a)^2\,dx\right)c^2 + \left(\int \text{erf}(bx+a)^2\,x^2dx\right)d^2 + 2\left(\int \text{erf}(bx+a)^2\,xdx\right)cd$$

input
```
int((d*x+c)^2*erf(b*x+a)^2,x)
```

output
```
int(erf(a + b*x)**2,x)*c**2 + int(erf(a + b*x)**2*x**2,x)*d**2 + 2*int(erf
(a + b*x)**2*x,x)*c*d
```

3.36 $\int (c+dx)\text{erf}(a+bx)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 188

$$\begin{aligned}\int (c+dx)\text{erf}(a+bx)^2\,dx = {} & \frac{de^{-2(a+bx)^2}}{2b^2\pi}+\frac{2(bc-ad)e^{-(a+bx)^2}\text{erf}(a+bx)}{b^2\sqrt{\pi}} \\ & +\frac{de^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)}{b^2\sqrt{\pi}}-\frac{d\text{erf}(a+bx)^2}{4b^2} \\ & +\frac{(bc-ad)(a+bx)\text{erf}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{erf}(a+bx)^2}{2b^2} \\ & -\frac{(bc-ad)\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^2}\end{aligned}$$

output
```
1/2*d/b^2/exp(2*(b*x+a)^2)/Pi+2*(-a*d+b*c)*erf(b*x+a)/b^2/exp((b*x+a)^2)/P
i^(1/2)+d*(b*x+a)*erf(b*x+a)/b^2/exp((b*x+a)^2)/Pi^(1/2)-1/4*d*erf(b*x+a)^
2/b^2+(-a*d+b*c)*(b*x+a)*erf(b*x+a)^2/b^2+1/2*d*(b*x+a)^2*erf(b*x+a)^2/b^2
-(-a*d+b*c)*2^(1/2)/Pi^(1/2)*erf(2^(1/2)*(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.70

$$\int (c+dx)\text{erf}(a+bx)^2\,dx$$
$$= \frac{2de^{-2(a+bx)^2}+4e^{-(a+bx)^2}\sqrt{\pi}(2bc-ad+bdx)\text{erf}(a+bx)+\pi\left(4abc-d-2a^2d+4b^2cx+2b^2dx^2\right)\text{erf}(a+}{4b^2\pi}$$

input
```
Integrate[(c + d*x)*Erf[a + b*x]^2,x]
```

output
```
((2*d)/E^(2*(a + b*x)^2) + (4*Sqrt[Pi]*(2*b*c - a*d + b*d*x)*Erf[a + b*x])
/E^(a + b*x)^2 + Pi*(4*a*b*c - d - 2*a^2*d + 4*b^2*c*x + 2*b^2*d*x^2)*Erf[
a + b*x]^2 + 4*(-(b*c) + a*d)*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)])/(4*b^2*Pi
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{erf}(a+bx)^2\,dx$$
$$\Big\downarrow \text{ 6921}$$
$$\frac{\int \left((bc-ad)\text{erf}(a+bx)^2+d(a+bx)\text{erf}(a+bx)^2\right)d(a+bx)}{b^2}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{(a+bx)(bc-ad)\text{erf}(a+bx)^2+\frac{2e^{-(a+bx)^2}(bc-ad)\text{erf}(a+bx)}{\sqrt{\pi}}-\sqrt{\frac{2}{\pi}}(bc-ad)\text{erf}\left(\sqrt{2}(a+bx)\right)+\frac{1}{2}d(a+bx)^2\text{erf}(a+b}{b^2}$$

input
```
Int[(c + d*x)*Erf[a + b*x]^2,x]
```

output
```
(d/(2*E^(2*(a + b*x)^2)*Pi) + (2*(b*c - a*d)*Erf[a + b*x])/(E^(a + b*x)^2*
Sqrt[Pi]) + (d*(a + b*x)*Erf[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) - (d*Erf[a
 + b*x]^2)/4 + (b*c - a*d)*(a + b*x)*Erf[a + b*x]^2 + (d*(a + b*x)^2*Erf[a
 + b*x]^2)/2 - (b*c - a*d)*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6921
```
Int[Erf[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
1/b^(m + 1)   Subst[Int[ExpandIntegrand[Erf[x]^2, (b*c - a*d + d*x)^m, x],
x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)\,\mathrm{erf}\,(bx+a)^2\,dx$$

input
```
int((d*x+c)*erf(b*x+a)^2,x)
```

output
```
int((d*x+c)*erf(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.91

$$\int (c+dx)\mathrm{erf}(a+bx)^2\,dx = -\frac{4\sqrt{2}\sqrt{\pi}\sqrt{b^2}(bc-ad)\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)-4\sqrt{\pi}(b^2dx+2b^2c-abd)\,\mathrm{erf}\,(bx+a)\,e^{(-b^2x^2-2abx-a^2)}-(2\cdots}{4\pi b^3}$$

input
```
integrate((d*x+c)*erf(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(2)*sqrt(pi)*sqrt(b^2)*(b*c - a*d)*erf(sqrt(2)*sqrt(b^2)*(b*x
+ a)/b) - 4*sqrt(pi)*(b^2*d*x + 2*b^2*c - a*b*d)*erf(b*x + a)*e^(-b^2*x^2
- 2*a*b*x - a^2) - (2*pi*b^3*d*x^2 + 4*pi*b^3*c*x + pi*(4*a*b^2*c - (2*a^2
 + 1)*b*d))*erf(b*x + a)^2 - 2*b*d*e^(-2*b^2*x^2 - 4*a*b*x - 2*a^2))/(pi*b
^3)
```

Sympy [F]

$$\int (c+dx)\text{erf}(a+bx)^2\,dx = \int (c+dx)\,\text{erf}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*erf(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*erf(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)\text{erf}(a+bx)^2\,dx = \int (dx+c)\,\text{erf}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*erf(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(d*x^2 + 2*c*x)*erf(b*x + a)^2 - integrate(2*(b*d*x^2 + 2*b*c*x)*erf(b
*x + a)*e^(-b^2*x^2 - 2*a*b*x - a^2), x)/sqrt(pi)
```

Giac [F]

$$\int (c+dx)\text{erf}(a+bx)^2\,dx=\int (dx+c)\,\text{erf}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*erf(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*erf(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.99

$$\int (c+dx)\text{erf}(a+bx)^2\,dx=\frac{d\,x^2\,\text{erf}(a+b\,x)^2}{2}-\frac{\text{erf}(a+b\,x)^2\,(2\,d\,a^2-4\,b\,c\,a+d)}{4\,b^2}+c\,x\,\text{erf}(a+b\,x)^2+\frac{d\,e^{-2\,a^2-4\,a\,b\,x-2\,b^2\,x^2}}{2\,b^2\,\pi}-\frac{\text{erf}(a+b\,x)\,\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(a\,d-2\,b\,c)}{b^2\,\sqrt{\pi}}+\frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}\,(a+b\,x)\right)\,(a\,d-b\,c)}{b^2\,\sqrt{\pi}}+\frac{d\,x\,\text{erf}(a+b\,x)\,\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}}{b\,\sqrt{\pi}}$$

input
```
int(erf(a + b*x)^2*(c + d*x),x)
```

output
```
(d*x^2*erf(a + b*x)^2)/2 - (erf(a + b*x)^2*(d + 2*a^2*d - 4*a*b*c))/(4*b^2
) + c*x*erf(a + b*x)^2 + (d*exp(- 2*a^2 - 2*b^2*x^2 - 4*a*b*x))/(2*b^2*pi)
 - (erf(a + b*x)*exp(- a^2 - b^2*x^2 - 2*a*b*x)*(a*d - 2*b*c))/(b^2*pi^(1/
2)) + (2^(1/2)*erf(2^(1/2)*(a + b*x))*(a*d - b*c))/(b^2*pi^(1/2)) + (d*x*e
rf(a + b*x)*exp(- a^2 - b^2*x^2 - 2*a*b*x))/(b*pi^(1/2))
```

Reduce [F]

$$\int (c+dx)\text{erf}(a+bx)^2\,dx = \left(\int \text{erf}(bx+a)^2\,dx\right)c + \left(\int \text{erf}(bx+a)^2\,xdx\right)d$$

input
```
int((d*x+c)*erf(b*x+a)^2,x)
```

output
```
int(erf(a + b*x)**2,x)*c + int(erf(a + b*x)**2*x,x)*d
```

3.37 $\int \text{erf}(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int \text{erf}(a+bx)^2\,dx=\frac{2e^{-(a+bx)^2}\text{erf}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\text{erf}(a+bx)^2}{b}-\frac{\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b}$$

output
```
2*erf(b*x+a)/b/exp((b*x+a)^2)/Pi^(1/2)+(b*x+a)*erf(b*x+a)^2/b-2^(1/2)/Pi^(
1/2)*erf(2^(1/2)*(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \text{erf}(a+bx)^2\,dx=\frac{\frac{2e^{-(a+bx)^2}\text{erf}(a+bx)}{\sqrt{\pi}}+(a+bx)\text{erf}(a+bx)^2-\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b}$$

input
```
Integrate[Erf[a + b*x]^2,x]
```

output
```
((2*Erf[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) + (a + b*x)*Erf[a + b*x]^2 - Sq
rt[2/Pi]*Erf[Sqrt[2]*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6906, 7281, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 6906}$$

$$\frac{(a+bx)\text{erf}(a+bx)^2}{b} - \frac{4\int e^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{(a+bx)\text{erf}(a+bx)^2}{b} - \frac{4\int e^{-(a+bx)^2}(a+bx)\text{erf}(a+bx)d(a+bx)}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 6936}$$

$$\frac{(a+bx)\text{erf}(a+bx)^2}{b} - \frac{4\left(\frac{\int e^{-2(a+bx)^2}d(a+bx)}{\sqrt{\pi}} - \frac{1}{2}e^{-(a+bx)^2}\text{erf}(a+bx)\right)}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{(a+bx)\text{erf}(a+bx)^2}{b} - \frac{4\left(\frac{\text{erf}\left(\sqrt{2}(a+bx)\right)}{2\sqrt{2}} - \frac{1}{2}e^{-(a+bx)^2}\text{erf}(a+bx)\right)}{\sqrt{\pi}b}$$

input
```
Int[Erf[a + b*x]^2,x]
```

output
```
((a + b*x)*Erf[a + b*x]^2)/b - (4*(-1/2*Erf[a + b*x]/E^(a + b*x)^2 + Erf[S
qrt[2]*(a + b*x)]/(2*Sqrt[2])))/(b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6906
```
Int[Erf[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]^2/
b), x] - Simp[4/Sqrt[Pi]   Int[(a + b*x)*(Erf[a + b*x]/E^(a + b*x)^2), x],
x] /; FreeQ[{a, b}, x]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{erf}(bx+a)^2(bx+a)+\frac{2\operatorname{erf}(bx+a)e^{-(bx+a)^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,(bx+a)\right)}{\sqrt{\pi}}}{b}$	59
default	$\frac{\operatorname{erf}(bx+a)^2(bx+a)+\frac{2\operatorname{erf}(bx+a)e^{-(bx+a)^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,(bx+a)\right)}{\sqrt{\pi}}}{b}$	59

input
```
int(erf(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(erf(b*x+a)^2*(b*x+a)+2*erf(b*x+a)/Pi^(1/2)*exp(-(b*x+a)^2)-1/Pi^(1/2)
*2^(1/2)*erf(2^(1/2)*(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int \operatorname{erf}(a+bx)^2\,dx$$
$$=\frac{2\sqrt{\pi}b\operatorname{erf}(bx+a)\,e^{\left(-b^2x^2-2\,abx-a^2\right)}+\left(\pi b^2x+\pi ab\right)\operatorname{erf}(bx+a)^2-\sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{\pi b^2}$$

input
```
integrate(erf(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*sqrt(pi)*b*erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x - a^2) + (pi*b^2*x + pi*a
*b)*erf(b*x + a)^2 - sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*(b*x
 + a)/b))/(pi*b^2)
```

Sympy [F]

$$\int \operatorname{erf}(a+bx)^2\,dx=\int \operatorname{erf}^2(a+bx)\,dx$$

input
```
integrate(erf(b*x+a)**2,x)
```

output
```
Integral(erf(a + b*x)**2, x)
```

Maxima [F]

$$\int \operatorname{erf}(a+bx)^2\,dx=\int \operatorname{erf}(bx+a)^2\,dx$$

input
```
integrate(erf(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*erf(b*x + a)^2 - 4*b*integrate(x*erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x - a^
2), x)/sqrt(pi)
```

Giac [F]

$$\int \operatorname{erf}(a+bx)^2\,dx = \int \operatorname{erf}\left(bx+a\right)^2\,dx$$

input
```
integrate(erf(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \operatorname{erf}(a+bx)^2\,dx = x\operatorname{erf}(a+b\,x)^2 + \frac{a\operatorname{erf}(a+b\,x)^2}{b} - \frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,(a+b\,x)\right)}{b\,\sqrt{\pi}} + \frac{2\operatorname{erf}(a+b\,x)\ \mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}}{b\,\sqrt{\pi}}$$

input
```
int(erf(a + b*x)^2,x)
```

output
```
x*erf(a + b*x)^2 + (a*erf(a + b*x)^2)/b - (2^(1/2)*erf(2^(1/2)*(a + b*x)))
/(b*pi^(1/2)) + (2*erf(a + b*x)*exp(- a^2 - b^2*x^2 - 2*a*b*x))/(b*pi^(1/2
))
```

Reduce [F]

$$\int \operatorname{erf}(a+bx)^2\,dx = \int \operatorname{erf}(bx+a)^2\,dx$$

input
```
int(erf(b*x+a)^2,x)
```

output
```
int(erf(a + b*x)**2,x)
```

3.38 $\int \frac{\text{erf}(a+bx)^2}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{erf}(a+bx)^2}{c+dx} \, dx = \text{Int}\left(\frac{\text{erf}(a+bx)^2}{c+dx}, x\right)$$

output
```
Defer(Int)(erf(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erf}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{erf}(a+bx)^2}{c+dx} \, dx$$

input
```
Integrate[Erf[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Erf[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(a+bx)^2}{c+dx}\,dx$$

$$\Big\downarrow \text{ 6924}$$

$$\int \frac{\operatorname{erf}(a+bx)^2}{c+dx}dx$$

input
```
Int[Erf[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(erf(b*x+a)^2/(d*x+c),x)
```

output
```
int(erf(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erf}(a+bx)^2}{c+dx}\,dx=\int \frac{\operatorname{erf}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erf(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{erf}(a+bx)^2}{c+dx}\,dx=\int \frac{\operatorname{erf}^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(erf(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(erf(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erf}(a+bx)^2}{c+dx}\,dx=\int \frac{\operatorname{erf}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erf(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)^2/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{erf}(a+bx)^2}{c+dx}\,dx=\int \frac{\mathrm{erf}\,(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(erf(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{erf}(a+bx)^2}{c+dx}\,dx=\int \frac{\mathrm{erf}(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(erf(a + b*x)^2/(c + d*x),x)
```

output
```
int(erf(a + b*x)^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erf}(a+bx)^2}{c+dx}\,dx = \int \frac{\operatorname{erf}(bx+a)^2}{dx+c}dx$$

input
```
int(erf(b*x+a)^2/(d*x+c),x)
```

output
```
int(erf(a + b*x)**2/(c + d*x),x)
```

3.39 $\int \frac{\mathbf{erf}(a+bx)^2}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{erf}(a+bx)^2}{(c+dx)^2} \, dx = \text{Int}\left(\frac{\text{erf}(a+bx)^2}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(erf(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erf}(a+bx)^2}{(c+dx)^2} \, dx = \int \frac{\text{erf}(a+bx)^2}{(c+dx)^2} \, dx$$

input
```
Integrate[Erf[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Erf[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(a+bx)^2}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 6924}$$

$$\int \frac{\text{erf}(a+bx)^2}{(c+dx)^2}dx$$

input
```
Int[Erf[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(erf(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(erf(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{erf}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erf(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 12.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{erf}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(erf(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(erf(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 4.62

$$\int \frac{\operatorname{erf}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erf(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
4*b*integrate(erf(b*x + a)*e^(-b^2*x^2 - 2*a*b*x)/(sqrt(pi)*d^2*x*e^(a^2)
+ sqrt(pi)*c*d*e^(a^2)), x) - erf(b*x + a)^2/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erf}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erf(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erf}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}(a+b\,x)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(erf(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(erf(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{erf}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erf}(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(erf(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(erf(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.40 $\int x^2 \mathbf{erf}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int x^2\text{erf}(d(a+b\log(cx^n)))\,dx = \frac{1}{3}x^3\text{erf}(d(a+b\log(cx^n))) - \frac{1}{3}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}x^3(cx^n)^{-3/n}\text{erf}\left(\frac{2abd^2-\frac{3}{n}+2b^2d^2\log(cx^n)}{2bd}\right)$$

output
```
1/3*x^3*erf(d*(a+b*ln(c*x^n)))-1/3*exp(1/4*(-12*a*b*d^2*n+9)/b^2/d^2/n^2)*
x^3*erf(1/2*(2*a*b*d^2-3/n+2*b^2*d^2*ln(c*x^n))/b/d)/((c*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int x^2\operatorname{erf}(d(a+b\log(cx^n)))\,dx=\frac{1}{3}\left(x^3\operatorname{erf}(d(a+b\log(cx^n)))-e^{-\frac{3\left(\frac{-\frac{3}{d^2}+4abn}{b^2}+4n\log(cx^n)\right)}{4n^2}}x^3\operatorname{erf}\left(ad-\frac{3}{2bdn}+bd\log(cx^n)\right)\right)$$

input
```
Integrate[x^2*Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*Erf[d*(a + b*Log[c*x^n])] - (x^3*Erf[a*d - 3/(2*b*d*n) + b*d*Log[c*x^
n]])/E^((3*((-3/d^2 + 4*a*b*n)/b^2 + 4*n*Log[c*x^n]))/(4*n^2)))/3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6955, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\operatorname{erf}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6955}$$

$$\frac{1}{3}x^3\operatorname{erf}(d(a+b\log(cx^n)))-\frac{2bdn\int e^{-d^2(a+b\log(cx^n))^2}x^2dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2712}$$

$$\frac{1}{3}x^3\text{erf}(d(a+b\log(cx^n)))-\frac{2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{2-2abd^2n}dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2706}$$

$$\frac{1}{3}x^3\text{erf}(d(a+b\log(cx^n)))-\frac{2bdx^3(cx^n)^{-3/n}\int\exp\left(-a^2d^2-b^2\log^2(cx^n)d^2+\frac{(3-2abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{3}x^3\text{erf}(d(a+b\log(cx^n)))-\frac{2bdx^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\int\exp\left(-\frac{\left(2abd^2+2b^2\log(cx^n)d^2-\frac{3}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{3}x^3\text{erf}(d(a+b\log(cx^n)))-\frac{1}{3}x^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\text{erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{3}{n}}{2bd}\right)$$

input
```
Int[x^2*Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*Erf[d*(a + b*Log[c*x^n])])/3 - (E^((9 - 12*a*b*d^2*n)/(4*b^2*d^2*n^2)
)*x^3*Erf[(2*a*b*d^2 - 3/n + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/(3*(c*x^n)^(3
/n))
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6955
```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(e*x)^(m + 1)*(Erf[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x] -
 Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2
, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^2 \operatorname{erf}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int(x^2*erf(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*erf(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.23

$$\int x^2\operatorname{erf}(d(a+b\log\left(cx^n\right)))\,dx=\frac{1}{3}\,x^3\operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)$$
$$-\frac{1}{3}\,\sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n-3\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{3\left(4\,b^2d^2n\log(c)+4\,abd^2n-3\right)}{4\,b^2d^2n^2}\right)}$$

input
```
integrate(x^2*erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/3*x^3*erf(b*d*log(c*x^n) + a*d) - 1/3*sqrt(b^2*d^2*n^2)*erf(1/2*(2*b^2*d
^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 3)*sqrt(b^2*d^2*n^2)/(b
^2*d^2*n^2))*e^(-3/4*(4*b^2*d^2*n*log(c) + 4*a*b*d^2*n - 3)/(b^2*d^2*n^2))
```

Sympy [F]

$$\int x^2 \text{erf}(d(a+b\log(cx^n)))\,dx=\int x^2\,\text{erf}\,(ad+bd\log(cx^n))\,dx$$

input
```
integrate(x**2*erf(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*erf(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \text{erf}(d(a+b\log(cx^n)))\,dx=\int x^2\,\text{erf}\,((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
1/3*x^3*erf(b*d*log(x^n) + (b*log(c) + a)*d) - 2/3*b*d*n*integrate(x^2*e^(
-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d^2*log(x^n)^2 - 2*a*b
*d^2*log(x^n) - a^2*d^2), x)/(sqrt(pi)*c^(2*a*b*d^2))
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int x^2\text{erf}(d(a+b\log(cx^n)))\,dx$$
$$=\frac{1}{3}x^3\,\text{erf}\left(bdn\log(x)+bd\log(c)+ad\right)$$
$$+\frac{\text{erf}\left(-bdn\log(x)-bd\log(c)-ad+\frac{3}{2\,bdn}\right)e^{\left(-\frac{3\,a}{bn}+\frac{9}{4\,b^2d^2n^2}\right)}}{3\,c^{\frac{3}{n}}}$$

input
```
integrate(x^2*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
1/3*x^3*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/3*erf(-b*d*n*log(x) - b*d
*log(c) - a*d + 3/2/(b*d*n))*e^(-3*a/(b*n) + 9/4/(b^2*d^2*n^2))/c^(3/n)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{erf}(d(a+b\log(cx^n)))\,dx=\int x^2\,\text{erf}\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x^2*erf(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*erf(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2\text{erf}(d(a+b\log(cx^n)))\,dx=\int \text{erf}(\log(x^nc)\,bd+ad)\,x^2dx$$

input
```
int(x^2*erf(d*(a+b*log(c*x^n))),x)
```

output
```
int(erf(log(x**n*c)*b*d + a*d)*x**2,x)
```

3.41 $\int x\text{erf}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x\text{erf}(d(a+b\log(cx^n)))\,dx = \frac{1}{2}x^2\text{erf}(d(a+b\log(cx^n))) - \frac{1}{2}e^{\frac{1-2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{-2/n}\text{erf}\left(\frac{abd^2-\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right)$$

output
```
1/2*x^2*erf(d*(a+b*ln(c*x^n)))-1/2*exp((-2*a*b*d^2*n+1)/b^2/d^2/n^2)*x^2*e
rf((a*b*d^2-1/n+b^2*d^2*ln(c*x^n))/b/d)/((c*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int x\text{erf}(d(a+b\log(cx^n)))\,dx = \frac{1}{2}\left(x^2\text{erf}(d(a+b\log(cx^n))) - e^{-\frac{\frac{-\frac{1}{d^2}+2abn}{b^2}+2n\log(cx^n)}{n^2}}x^2\text{erf}\left(ad-\frac{1}{bdn}+bd\log(cx^n)\right)\right)$$

input
```
Integrate[x*Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*Erf[d*(a + b*Log[c*x^n])] - (x^2*Erf[a*d - 1/(b*d*n) + b*d*Log[c*x^n]
])/E^(((-d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n])/n^2))/2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6955, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erf}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6955}$$

$$\frac{1}{2}x^2\text{erf}(d(a+b\log(cx^n)))-\frac{bdn\int e^{-d^2(a+b\log(cx^n))^2}xdx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2712}$$

$$\frac{1}{2}x^2\text{erf}(d(a+b\log(cx^n)))-\frac{bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{1-2abd^2n}dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2706}$$

$$\frac{1}{2}x^2\text{erf}(d(a+b\log(cx^n)))-\frac{bdx^2(cx^n)^{2\left(abd^2-\frac{1}{n}\right)-2abd^2}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)\,d^2+\frac{2(1-abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{2}x^2\text{erf}(d(a+b\log(cx^n)))-\frac{bdx^2e^{\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2\left(abd^2-\frac{1}{n}\right)-2abd^2}\int \exp\left(-\frac{\left(abd^2+b^2\log(cx^n)d^2-\frac{1}{n}\right)^2}{b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}x^2\text{erf}(d(a+b\log(cx^n)))-\frac{1}{2}x^2e^{\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2\left(abd^2-\frac{1}{n}\right)-2abd^2}\text{erf}\left(\frac{abd^2+b^2d^2\log(cx^n)-\frac{1}{n}}{bd}\right)$$

input
```
Int[x*Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*Erf[d*(a + b*Log[c*x^n])])/2 - (E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x
^2*(c*x^n)^(-2*a*b*d^2 + 2*(a*b*d^2 - n^(-1)))*Erf[(a*b*d^2 - n^(-1) + b^2
*d^2*Log[c*x^n])/(b*d)])/2
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6955
```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(e*x)^(m + 1)*(Erf[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x] -
 Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2
, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x\,\mathrm{erf}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(x*erf(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*erf(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int x\mathrm{erf}(d(a+b\log\left(cx^{n}\right)))\,dx=\frac{1}{2}\,x^{2}\,\mathrm{erf}\left(bd\log\left(cx^{n}\right)+ad\right)\\-\frac{1}{2}\,\sqrt{b^{2}d^{2}n^{2}}\,\mathrm{erf}\left(\frac{\left(b^{2}d^{2}n^{2}\log\left(x\right)+b^{2}d^{2}n\log\left(c\right)+abd^{2}n-1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{2\,b^{2}d^{2}n\log\left(c\right)+2\,abd^{2}n-1}{b^{2}d^{2}n^{2}}\right)}$$

input
```
integrate(x*erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/2*x^2*erf(b*d*log(c*x^n) + a*d) - 1/2*sqrt(b^2*d^2*n^2)*erf((b^2*d^2*n^2
*log(x) + b^2*d^2*n*log(c) + a*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2
))*e^(-(2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 1)/(b^2*d^2*n^2))
```

Sympy [F]

$$\int x\mathrm{erf}(d(a+b\log\left(cx^{n}\right)))\,dx=\int x\,\mathrm{erf}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

input
```
integrate(x*erf(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*erf(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\text{erf}(d(a+b\log(cx^n)))\,dx=\int x\,\text{erf}\left((b\log(cx^n)+a)d\right)\,dx$$

input
```
integrate(x*erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
1/2*x^2*erf(b*d*log(x^n) + (b*log(c) + a)*d) - b*d*n*integrate(x*e^(-b^2*d
^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d^2*log(x^n)^2 - 2*a*b*d^2*l
og(x^n) - a^2*d^2), x)/(sqrt(pi)*c^(2*a*b*d^2))
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int x\text{erf}(d(a+b\log(cx^n)))\,dx\\&=\frac{1}{2}x^2\,\text{erf}\left(bdn\log(x)+bd\log(c)+ad\right)\\&\quad+\frac{\text{erf}\left(-bdn\log(x)-bd\log(c)-ad+\frac{1}{bdn}\right)e^{\left(-\frac{2a}{bn}+\frac{1}{b^2d^2n^2}\right)}}{2\,c^{\frac{2}{n}}}\end{aligned}$$

input
```
integrate(x*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
1/2*x^2*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/2*erf(-b*d*n*log(x) - b*d
*log(c) - a*d + 1/(b*d*n))*e^(-2*a/(b*n) + 1/(b^2*d^2*n^2))/c^(2/n)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{erf}(d(a+b\log(cx^n)))\,dx=\int x\,\text{erf}\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x*erf(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*erf(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\text{erf}(d(a+b\log(cx^n)))\,dx=\int \text{erf}(\log(x^nc)\,bd+ad)\,xdx$$

input
```
int(x*erf(d*(a+b*log(c*x^n))),x)
```

output
```
int(erf(log(x**n*c)*b*d + a*d)*x,x)
```

3.42 $\int \text{erf}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \text{erf}(d(a+b\log(cx^n)))\,dx = x\text{erf}(d(a+b\log(cx^n))) - e^{\frac{1-4abd^2n}{4b^2d^2n^2}}x(cx^n)^{-1/n}\text{erf}\left(\frac{2abd^2-\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)$$

output
```
x*erf(d*(a+b*ln(c*x^n)))-exp(1/4*(-4*a*b*d^2*n+1)/b^2/d^2/n^2)*x*erf(1/2*(
2*a*b*d^2-1/n+2*b^2*d^2*ln(c*x^n))/b/d)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \text{erf}(d(a+b\log(cx^n)))\,dx = x\text{erf}(d(a+b\log(cx^n))) - e^{-\frac{\frac{-\frac{1}{d^2}+4abn}{b^2}+4n\log(cx^n)}{4n^2}}x\text{erf}\left(ad-\frac{1}{2bdn}+bd\log(cx^n)\right)$$

input
```
Integrate[Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
x*Erf[d*(a + b*Log[c*x^n])] - (x*Erf[a*d - 1/(2*b*d*n) + b*d*Log[c*x^n]])/
E^(((-d^(-2) + 4*a*b*n)/b^2 + 4*n*Log[c*x^n])/(4*n^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6951, 2710, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erf}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6951}$$

$$x\operatorname{erf}(d(a+b\log(cx^n)))-\frac{2bdn\int e^{-d^2(a+b\log(cx^n))^2}dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2710}$$

$$x\operatorname{erf}(d(a+b\log(cx^n)))-\frac{2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{-2abd^2n}dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2706}$$

$$x\operatorname{erf}(d(a+b\log(cx^n)))-\frac{2bdx(cx^n)^{-1/n}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2+\frac{(1-2abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2664}$$

$$x\operatorname{erf}(d(a+b\log(cx^n)))-\frac{2bdx(cx^n)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\int \exp\left(-\frac{\left(2abd^2+2b^2\log(cx^n)d^2-\frac{1}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$x\operatorname{erf}(d(a+b\log(cx^n)))-x(cx^n)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\operatorname{erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)$$

input
```
Int[Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
x*Erf[d*(a + b*Log[c*x^n])] - (E^((1 - 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*x*Erf
[(2*a*b*d^2 - n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 6951
```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*E
rf[d*(a + b*Log[c*x^n])], x] - Simp[2*b*d*(n/Sqrt[Pi])   Int[1/E^(d*(a + b*
Log[c*x^n]))^2, x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Maple [F]

$$\int \operatorname{erf}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(erf(d*(a+b*ln(c*x^n))),x)
```

output
```
int(erf(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \operatorname{erf}(d(a+b\log\left(cx^{n}\right)))\,dx = -\sqrt{b^{2}d^{2}n^{2}}\operatorname{erf}\left(\frac{\left(2\,b^{2}d^{2}n^{2}\log\left(x\right)+2\,b^{2}d^{2}n\log\left(c\right)+2\,abd^{2}n-1\right)\sqrt{b^{2}d^{2}n^{2}}}{2\,b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{4\,b^{2}d^{2}n\log(c)+4\,abd^{2}n-1}{4\,b^{2}d^{2}n^{2}}\right)} + x\operatorname{erf}\left(bd\log\left(cx^{n}\right)+ad\right)$$

input
```
integrate(erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-sqrt(b^2*d^2*n^2)*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*
a*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(-1/4*(4*b^2*d^2*n*log(c
) + 4*a*b*d^2*n - 1)/(b^2*d^2*n^2)) + x*erf(b*d*log(c*x^n) + a*d)
```

Sympy [F]

$$\int \operatorname{erf}(d(a+b\log\left(cx^{n}\right)))\,dx = \int \operatorname{erf}\left(d(a+b\log\left(cx^{n}\right))\right)dx$$

input
```
integrate(erf(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(erf(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \operatorname{erf}(d(a+b\log(cx^n)))\,dx=\int \operatorname{erf}\left((b\log(cx^n)+a)d\right)\,dx$$

input
```
integrate(erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
-2*b*d*n*integrate(e^(-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*
d^2*log(x^n)^2 - 2*a*b*d^2*log(x^n) - a^2*d^2), x)/(sqrt(pi)*c^(2*a*b*d^2)
) + x*erf(b*d*log(x^n) + (b*log(c) + a)*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \operatorname{erf}(d(a+b\log(cx^n)))\,dx &= x\operatorname{erf}\left(bdn\log(x)+bd\log(c)+ad\right)\\&\quad+\frac{\operatorname{erf}\left(-bdn\log(x)-bd\log(c)-ad+\frac{1}{2\,bdn}\right)e^{\left(-\frac{a}{bn}+\frac{1}{4\,b^2d^2n^2}\right)}}{c^{\left(\frac{1}{n}\right)}}\end{aligned}$$

input
```
integrate(erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
x*erf(b*d*n*log(x) + b*d*log(c) + a*d) + erf(-b*d*n*log(x) - b*d*log(c) -
a*d + 1/2/(b*d*n))*e^(-a/(b*n) + 1/4/(b^2*d^2*n^2))/c^(1/n)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erf}(d(a+b\log{(cx^n)}))\,dx = \int \operatorname{erf}\left(d\,(a+b\,\ln{(c\,x^n)})\right)\,dx$$

input
```
int(erf(d*(a + b*log(c*x^n))),x)
```

output
```
int(erf(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \operatorname{erf}(d(a+b\log{(cx^n)}))\,dx = \int \operatorname{erf}(\log(x^n c)\,bd+ad)\,dx$$

input
```
int(erf(d*(a+b*log(c*x^n))),x)
```

output
```
int(erf(log(x**n*c)*b*d + a*d),x)
```

3.43 $\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x}\,dx = \frac{e^{-d^2(a+b\log(cx^n))^2}}{bdn\sqrt{\pi}} + \frac{\text{erf}(d(a+b\log(cx^n)))\,(a+b\log(cx^n))}{bn}$$

output
```
1/b/d/exp(d^2*(a+b*ln(c*x^n))^2)/n/Pi^(1/2)+erf(d*(a+b*ln(c*x^n)))*(a+b*ln
(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\frac{e^{-d^2\left(a^2+b^2\log^2(cx^n)\right)}(cx^n)^{-2abd^2}}{bd\sqrt{\pi}} + \text{erf}(d(a+b\log(cx^n)))\left(\frac{a}{b}+\log(cx^n)\right)}{n}$$

input
```
Integrate[Erf[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(1/(b*d*E^(d^2*(a^2 + b^2*Log[c*x^n]^2))*Sqrt[Pi]*(c*x^n)^(2*a*b*d^2)) + E
rf[d*(a + b*Log[c*x^n])]*(a/b + Log[c*x^n]))/n
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3039, 7281, 6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{erf}(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \operatorname{erf}(ad+b\log(cx^n)d)d(ad+b\log(cx^n)d)}{bdn}$$

$$\Big\downarrow \text{6903}$$

$$\frac{(ad+bd\log(cx^n))\operatorname{erf}(ad+bd\log(cx^n))+\frac{e^{-(ad+bd\log(cx^n))^2}}{\sqrt{\pi}}}{bdn}$$

input
```
Int[Erf[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(1/(E^(a*d + b*d*Log[c*x^n])^2*Sqrt[Pi]) + Erf[a*d + b*d*Log[c*x^n]]*(a*d
+ b*d*Log[c*x^n]))/(b*d*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 6903
```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b),
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\operatorname{erf}(ad+bd\ln(cx^n))(ad+bd\ln(cx^n))+\frac{e^{-(ad+bd\ln(cx^n))^2}}{\sqrt{\pi}}}{nbd}$
default	$\frac{\operatorname{erf}(ad+bd\ln(cx^n))(ad+bd\ln(cx^n))+\frac{e^{-(ad+bd\ln(cx^n))^2}}{\sqrt{\pi}}}{nbd}$
parts	$\ln(x)\operatorname{erf}(d(a+b\ln(cx^n))) - \frac{2dbn\left(-\frac{e^{-\ln(x)^2b^2d^2n^2-2d^2(b(\ln(cx^n)-n\ln(x))+a)nb\ln(x)-d^2(b(\ln(cx^n)-n}{2b^2d^2n^2}\right.}{}$

input
```
int(erf(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b/d*(erf(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))+1/Pi^(1/2)*exp(-(a*d+b
*d*ln(c*x^n))^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x}\,dx = \frac{(\pi bdn\log(x) + \pi bd\log(c) + \pi ad)\operatorname{erf}(bd\log(cx^n) + ad) + \sqrt{\pi}e^{\left(-b^2d^2n^2\log(x)^2 - b^2d^2\log(c)^2 - 2\,abd^2\log(c) - a^2d^2 - \cdots\right.}}{\pi bdn}$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*erf(b*d*log(c*x^n) + a*d) + sq
rt(pi)*e^(-b^2*d^2*n^2*log(x)^2 - b^2*d^2*log(c)^2 - 2*a*b*d^2*log(c) - a^
2*d^2 - 2*(b^2*d^2*n*log(c) + a*b*d^2*n)*log(x)))/(pi*b*d*n)
```

Sympy [F]

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\operatorname{erf}(ad + bd\log(cx^n))}{x}\,dx$$

input
```
integrate(erf(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(erf(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x}\,dx = \frac{(b\log(cx^n) + a)d\operatorname{erf}((b\log(cx^n) + a)d) + \frac{e^{\left(-(b\log(cx^n)+a)^2d^2\right)}}{\sqrt{\pi}}}{bdn}$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
((b*log(c*x^n) + a)*d*erf((b*log(c*x^n) + a)*d) + e^(-(b*log(c*x^n) + a)^2
*d^2)/sqrt(pi))/(b*d*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x}\,dx = \frac{(bdn\log(x)+bd\log(c)+ad)\operatorname{erf}(bdn\log(x)+bd\log(c)+ad)+\frac{e^{\left(-(bdn\log(x)+bd\log(c)+ad)^2\right)}}{\sqrt{\pi}}}{bdn}$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
((b*d*n*log(x) + b*d*log(c) + a*d)*erf(b*d*n*log(x) + b*d*log(c) + a*d) +
e^(-(b*d*n*log(x) + b*d*log(c) + a*d)^2)/sqrt(pi))/(b*d*n)
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\ln(c\,x^n)\,\operatorname{erf}(a\,d+b\,d\,\ln(c\,x^n))}{n} + \frac{a\,d\,\operatorname{erfi}\left(a\,\sqrt{-d^2}+b\,\ln(c\,x^n)\,\sqrt{-d^2}\right)}{b\,n\,\sqrt{-d^2}} + \frac{\mathrm{e}^{-b^2\,d^2\,\ln(c\,x^n)^2}\,\mathrm{e}^{-a^2\,d^2}}{b\,d\,n\,\sqrt{\pi}\,(c\,x^n)^{2\,a\,b\,d^2}}$$

input
```
int(erf(d*(a + b*log(c*x^n)))/x,x)
```

output
```
(log(c*x^n)*erf(a*d + b*d*log(c*x^n)))/n + (a*d*erfi(a*(-d^2)^(1/2) + b*lo
g(c*x^n)*(-d^2)^(1/2)))/(b*n*(-d^2)^(1/2)) + (exp(-b^2*d^2*log(c*x^n)^2)*e
xp(-a^2*d^2))/(b*d*n*pi^(1/2)*(c*x^n)^(2*a*b*d^2))
```

Reduce [F]

$$\int \frac{\operatorname{erf}(d(a + b\log(cx^n)))}{x}\,dx = \int \frac{\operatorname{erf}(\log(x^n c)\,bd + ad)}{x}dx$$

input
```
int(erf(d*(a+b*log(c*x^n)))/x,x)
```

output
```
int(erf(log(x**n*c)*b*d + a*d)/x,x)
```

3.44 $\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\text{erf}(d(a+b\log(cx^n)))}{x} + \frac{e^{\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\text{erf}\left(\frac{2abd^2+\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)}{x}$$

output
```
-erf(d*(a+b*ln(c*x^n)))/x+exp(1/4/b^2/d^2/n^2+a/b/n)*(c*x^n)^(1/n)*erf(1/2
*(2*a*b*d^2+1/n+2*b^2*d^2*ln(c*x^n))/b/d)/x
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{-\text{erf}(d(a+b\log(cx^n)))+e^{\frac{\frac{\frac{1}{d^2}+4abn}{b^2}+4n\log(cx^n)}{4n^2}}\,\text{erf}\left(ad+\frac{1}{2bdn}+bd\log(cx^n)\right)}{x}$$

input
```
Integrate[Erf[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
(-Erf[d*(a + b*Log[c*x^n])] + E^(((d^(-2) + 4*a*b*n)/b^2 + 4*n*Log[c*x^n])
/(4*n^2))*Erf[a*d + 1/(2*b*d*n) + b*d*Log[c*x^n]])/x
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6955, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx$$

$$\Big\downarrow \text{6955}$$

$$\frac{2bdn\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^2}dx}{\sqrt{\pi}}-\frac{\text{erf}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2712}$$

$$\frac{2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{-2abnd^2-2}dx}{\sqrt{\pi}}-\frac{\text{erf}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2706}$$

$$\frac{2bd(cx^n)^{\frac{1}{n}}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2-\frac{(2abnd^2+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}x}-\frac{\text{erf}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2664}$$

$$\frac{2bd(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}}\int \exp\left(-\frac{\left(2abd^2+2b^2\log(cx^n)d^2+\frac{1}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}x}-\frac{\text{erf}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2634}$$

$$\frac{(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}}\text{erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)}{x}-\frac{\text{erf}(d(a+b\log(cx^n)))}{x}$$

input
```
Int[Erf[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-(Erf[d*(a + b*Log[c*x^n])]/x) + (E^(1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)^
n^(-1)*Erf[(2*a*b*d^2 + n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6955
```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(e*x)^(m + 1)*(Erf[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x] -
 Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2
, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erf}(d(a+b\ln(c\,x^n)))}{x^2}dx$$

input
```
int(erf(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(erf(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{\sqrt{b^2d^2n^2}x\operatorname{erf}\left(\frac{(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n+1)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(\frac{4\,b^2d^2n\log(c)+4\,abd^2n+1}{4\,b^2d^2n^2}\right)} - \operatorname{erf}(bd\log(cx^n)+ad)}{x}$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
(sqrt(b^2*d^2*n^2)*x*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) +
2*a*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*n*log(
c) + 4*a*b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d))/x
```

Sympy [F]

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erf}(ad+bd\log(cx^n))}{x^2}\,dx$$

input
```
integrate(erf(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(erf(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{erf}\left((b\log(cx^n)+a)d\right)}{x^2}\,dx$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
2*b*d*n*integrate(e^(-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d
^2*log(x^n)^2 - 2*a*b*d^2*log(x^n) - a^2*d^2)/x^2, x)/(sqrt(pi)*c^(2*a*b*d
^2)) - erf(b*d*log(x^n) + (b*log(c) + a)*d)/x
```

Giac [F]

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{erf}\left((b\log(cx^n)+a)d\right)}{x^2}\,dx$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(erf((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\text{erf}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}{x^2}\,dx$$

input
```
int(erf(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(erf(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erf}(\log(x^n c)\,bd+ad)}{x^2}dx$$

input
```
int(erf(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
int(erf(log(x**n*c)*b*d + a*d)/x**2,x)
```

3.45 $\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\operatorname{erf}(d(a+b\log(cx^n)))}{2x^2} + \frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\operatorname{erf}\left(\frac{1+abd^2n+b^2d^2n\log(cx^n)}{bdn}\right)}{2x^2}$$

output
```
-1/2*erf(d*(a+b*ln(c*x^n)))/x^2+1/2*exp((2*a*b*d^2*n+1)/b^2/d^2/n^2)*(c*x^
n)^(2/n)*erf((1+a*b*d^2*n+b^2*d^2*n*ln(c*x^n))/b/d/n)/x^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{-\operatorname{erf}(d(a+b\log(cx^n))) + e^{\frac{\frac{\frac{1}{d^2}+2abn}{b^2}+2n\log(cx^n)}{n^2}}\operatorname{erf}\left(ad+\frac{1}{bdn}+bd\log(cx^n)\right)}{2x^2}$$

input
```
Integrate[Erf[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(-Erf[d*(a + b*Log[c*x^n])] + E^(((d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n])
/n^2)*Erf[a*d + 1/(b*d*n) + b*d*Log[c*x^n]])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6955, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(d(a+b\log(cx^n)))}{x^3}dx$$

$$\Big\downarrow \text{6955}$$

$$\frac{bdn\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^3}dx}{\sqrt{\pi}}-\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2712}$$

$$\frac{bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{-2abnd^2-3}dx}{\sqrt{\pi}}-\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2706}$$

$$\frac{bd(cx^n)^{2\left(abd^2+\frac{1}{n}\right)-2abd^2}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)\,d^2-\frac{2(abnd^2+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}x^2}-\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2664}$$

$$\frac{bde^{\frac{2abd^2n+1}{b^2d^2n^2}}(cx^n)^{2\left(abd^2+\frac{1}{n}\right)-2abd^2}\int \exp\left(-\frac{(abnd^2+b^2n\log(cx^n)d^2+1)^2}{b^2d^2n^2}\right)d\log(cx^n)}{\sqrt{\pi}x^2}-\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{e^{\frac{2abd^2n+1}{b^2d^2n^2}}(cx^n)^{2\left(abd^2+\frac{1}{n}\right)-2abd^2}\text{erf}\left(\frac{abd^2n+b^2d^2n\log(cx^n)+1}{bdn}\right)}{2x^2}-\frac{\text{erf}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Int[Erf[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*Erf[d*(a + b*Log[c*x^n])]/x^2 + (E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*
(c*x^n)^(-2*a*b*d^2 + 2*(a*b*d^2 + n^(-1)))*Erf[(1 + a*b*d^2*n + b^2*d^2*n
*Log[c*x^n])/(b*d*n)])/(2*x^2)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6955
```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(e*x)^(m + 1)*(Erf[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x] -
 Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2
, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erf}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(erf(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(erf(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{erf}(d(a+b\log\left(cx^{n}\right)))}{x^{3}}\,dx = \frac{\sqrt{b^{2}d^{2}n^{2}}x^{2}\operatorname{erf}\left(\frac{\left(b^{2}d^{2}n^{2}\log(x)+b^{2}d^{2}n\log(c)+abd^{2}n+1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(\frac{2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n+1}{b^{2}d^{2}n^{2}}\right)}-\operatorname{erf}\left(bd\log\left(cx^{n}\right)+ad\right)}{2\,x^{2}}$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
1/2*(sqrt(b^2*d^2*n^2)*x^2*erf((b^2*d^2*n^2*log(x) + b^2*d^2*n*log(c) + a*
b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^((2*b^2*d^2*n*log(c) + 2*a
*b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F]

$$\int \frac{\operatorname{erf}(d(a+b\log\left(cx^{n}\right)))}{x^{3}}\,dx = \int \frac{\operatorname{erf}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

input
```
integrate(erf(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(erf(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erf}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
b*d*n*integrate(e^(-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d^2
*log(x^n)^2 - 2*a*b*d^2*log(x^n) - a^2*d^2)/x^3, x)/(sqrt(pi)*c^(2*a*b*d^2
)) - 1/2*erf(b*d*log(x^n) + (b*log(c) + a)*d)/x^2
```

Giac [F]

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erf}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(erf(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(erf((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erf}(d\,(a+b\,\ln(c\,x^n)))}{x^3}\,dx$$

input
```
int(erf(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(erf(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{erf}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erf}(\log(x^n c)\,bd+ad)}{x^3}dx$$

input

```
int(erf(d*(a+b*log(c*x^n)))/x^3,x)
```

output

```
int(erf(log(x**n*c)*b*d + a*d)/x**3,x)
```

3.46 $\int (ex)^m \mathbf{erf}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int (ex)^m \text{erf}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\text{erf}(d(a+b\log(cx^n)))}{e(1+m)} + \frac{e^{\frac{(1+m)\left(1+m-4abd^2n\right)}{4b^2d^2n^2}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\text{erf}\left(\frac{1+m-2abd^2n-2b^2d^2n\log(cx^n)}{2bdn}\right)}{1+m}$$

output

```
(e*x)^(1+m)*erf(d*(a+b*ln(c*x^n)))/e/(1+m)+exp(1/4*(1+m)*(-4*a*b*d^2*n+m+1
)/b^2/d^2/n^2)*x*(e*x)^m*erf(1/2*(1+m-2*a*b*d^2*n-2*b^2*d^2*n*ln(c*x^n))/b
/d/n)/(1+m)/((c*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int (ex)^m \text{erf}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^m\left(x\text{erf}(d(a+b\log(cx^n))) - e^{\frac{(1+m)\left(1+m-4abd^2n+4b^2d^2n^2\log(x)-4b^2d^2n\log(cx^n)\right)}{4b^2d^2n^2}}x^{-m}\text{erf}\left(ad-\frac{1+m-2b^2d^2n\log(cx^n)}{2bdn}\right.\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*(x*Erf[d*(a + b*Log[c*x^n])] - (E^(((1 + m)*(1 + m - 4*a*b*d^2*n
+ 4*b^2*d^2*n^2*Log[x] - 4*b^2*d^2*n*Log[c*x^n]))/(4*b^2*d^2*n^2))*Erf[a*d
 - (1 + m - 2*b^2*d^2*n*Log[c*x^n])/(2*b*d*n)])/x^m))/(1 + m)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6955, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \text{erf}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{ 6955}$$

$$\frac{(ex)^{m+1}\text{erf}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdn\int e^{-d^2(a+b\log(cx^n))^2}(ex)^m dx}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \text{ 2712}$$

$$\frac{(ex)^{m+1}\text{erf}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdn(ex)^m\left(cx^n\right)^{-2abd^2}x^{2abd^2n-m}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{m-2abd^2n}dx}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \text{ 2706}$$

$$\frac{(ex)^{m+1}\text{erf}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdx(ex)^m\left(cx^n\right)^{-\frac{-2abd^2n+m+1}{n}-2abd^2}\int \exp\left(-a^2d^2-b^2\log^2\left(cx^n\right)d^2+\frac{\left(-2abnd^2+m+1\right)\log(cx^n)}{n}\right)d\log\left(cx^n\right)}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{(ex)^{m+1}\text{erf}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdx(ex)^m\exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right)(cx^n)^{-\frac{-2abd^2n+m+1}{n}-2abd^2}\int\exp\left(-\frac{(-2abnd^2-2b^2n\log(cx^n)d^2+m+1)^2}{4b^2d^2n^2}\right)d\log(cx}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{x(ex)^m\exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right)(cx^n)^{-\frac{-2abd^2n+m+1}{n}-2abd^2}\text{erf}\left(\frac{-2abd^2n-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}+\frac{(ex)^{m+1}\text{erf}(d(a+b\log(cx^n)))}{e(m+1)}$$

input
```
Int[(e*x)^m*Erf[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*Erf[d*(a + b*Log[c*x^n])])/(e*(1 + m)) + (E^(((1 + m)*(1 +
m - 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*x*(e*x)^m*(c*x^n)^(-2*a*b*d^2 - (1 + m
- 2*a*b*d^2*n)/n)*Erf[(1 + m - 2*a*b*d^2*n - 2*b^2*d^2*n*Log[c*x^n])/(2*b*
d*n)])/(1 + m)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6955
```
Int[Erf[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(e*x)^(m + 1)*(Erf[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x] -
 Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))^2
, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m \operatorname{erf}\left(d(a + b\ln\left(c\,x^n\right))\right) dx$$

input
```
int((e*x)^m*erf(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*erf(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.44

$$\int (ex)^m \operatorname{erf}(d(a + b\log\left(cx^n\right)))\ dx$$
$$= \frac{x \operatorname{erf}\left(bd\log\left(cx^n\right) + ad\right) e^{(m\log(e) + m\log(x))} - \sqrt{b^2d^2n^2} \operatorname{erf}\left(\frac{(2\,b^2d^2n^2\log(x) + 2\,b^2d^2n\log(c) + 2\,abd^2n - m - 1)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right) e^{\left(\right.}}{m + 1}$$

input
```
integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
(x*erf(b*d*log(c*x^n) + a*d)*e^(m*log(e) + m*log(x)) - sqrt(b^2*d^2*n^2)*e
rf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - m - 1)*s
qrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*m*n^2*log(e) - 4*(b^2*d^
2*m + b^2*d^2)*n*log(c) + m^2 - 4*(a*b*d^2*m + a*b*d^2)*n + 2*m + 1)/(b^2*
d^2*n^2)))/(m + 1)
```

Sympy [F]

$$\int (ex)^m \operatorname{erf}(d(a+b\log(cx^n)))\,dx=\int (ex)^m \operatorname{erf}(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*erf(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*erf(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \operatorname{erf}(d(a+b\log(cx^n)))\,dx=\int (ex)^m \operatorname{erf}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
e^m*x*x^m*erf(b*d*log(x^n) + (b*log(c) + a)*d)/(m + 1) - 2*b*d*e^m*n*integ
rate(e^(-b^2*d^2*log(c)^2 - 2*b^2*d^2*log(c)*log(x^n) - b^2*d^2*log(x^n)^2
 - 2*a*b*d^2*log(x^n) - a^2*d^2 + m*log(x)), x)/(sqrt(pi)*c^(2*a*b*d^2)*(m
 + 1))
```

Giac [F]

$$\int (ex)^m \text{erf}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \,\text{erf}\left((b\log(cx^n)+a)d\right)\,dx$$

input
```
integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*erf((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \text{erf}(d(a+b\log(cx^n)))\,dx = \int \text{erf}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(erf(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(erf(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \text{erf}(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \text{erf}(\log(x^n c)\,bd+ad)\,dx\right)$$

input
```
int((e*x)^m*erf(d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*erf(log(x**n*c)*b*d + a*d),x)
```

3.47 $\int e^{c-b^2x^2}\mathbf{erf}(bx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int e^{c-b^2x^2}\text{erf}(bx)^2\,dx=\frac{e^c\sqrt{\pi}\text{erf}(bx)^3}{6b}$$

output
```
1/6*exp(c)*Pi^(1/2)*erf(b*x)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c-b^2x^2}\text{erf}(bx)^2\,dx=\frac{e^c\sqrt{\pi}\text{erf}(bx)^3}{6b}$$

input
```
Integrate[E^(c - b^2*x^2)*Erf[b*x]^2,x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^3)/(6*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-b^2x^2}\text{erf}(bx)^2\,dx$$

$$\Big\downarrow \text{ 6927}$$

$$\frac{\sqrt{\pi}e^c\int \text{erf}(bx)^2d\text{erf}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}e^c\text{erf}(bx)^3}{6b}$$

input
```
Int[E^(c - b^2*x^2)*Erf[b*x]^2,x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^3)/(6*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{e^c\sqrt{\pi}\ \mathrm{erf}(bx)^3}{6b}$	17

input
```
int(exp(-b^2*x^2+c)*erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(c)*Pi^(1/2)*erf(b*x)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\mathrm{erf}(bx)^2\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(bx\right)^3e^c}{6\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x)^2,x, algorithm="fricas")
```

output
```
1/6*sqrt(pi)*erf(b*x)^3*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{c-b^2x^2}\mathrm{erf}(bx)^2\,dx = \begin{cases} \frac{\sqrt{\pi}e^c\,\mathrm{erf}^3\left(bx\right)}{6b} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)*erf(b*x)**2,x)
```

output
```
Piecewise((sqrt(pi)*exp(c)*erf(b*x)**3/(6*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)^2\,dx=\frac{\sqrt{\pi}\operatorname{erf}(bx)^3\,e^c}{6\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x)^2,x, algorithm="maxima")
```

output
```
1/6*sqrt(pi)*erf(b*x)^3*e^c/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)^2\,dx=\frac{\sqrt{\pi}\operatorname{erf}(bx)^3\,e^c}{6\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(pi)*erf(b*x)^3*e^c/b
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)^2\,dx=\frac{\sqrt{\pi}\,\mathrm{e}^c\operatorname{erf}(b\,x)^3}{6\,b}$$

input
```
int(exp(c - b^2*x^2)*erf(b*x)^2,x)
```

output
```
(pi^(1/2)*exp(c)*erf(b*x)^3)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\text{erf}(bx)^2\,dx = \frac{\sqrt{\pi}\,e^c\text{erf}(bx)^3}{6b}$$

input
```
int(exp(-b^2*x^2+c)*erf(b*x)^2,x)
```

output
```
(sqrt(pi)*e**c*erf(b*x)**3)/(6*b)
```

3.48 $\int e^{c-b^2x^2} \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{e^c\sqrt{\pi}\operatorname{erf}(bx)^2}{4b}$$

output
```
1/4*exp(c)*Pi^(1/2)*erf(b*x)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{e^c\sqrt{\pi}\operatorname{erf}(bx)^2}{4b}$$

input
```
Integrate[E^(c - b^2*x^2)*Erf[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-b^2x^2}\text{erf}(bx)\,dx$$

$$\Big\downarrow \ 6927$$

$$\frac{\sqrt{\pi}e^c\int \text{erf}(bx)d\text{erf}(bx)}{2b}$$

$$\Big\downarrow \ 15$$

$$\frac{\sqrt{\pi}e^c\text{erf}(bx)^2}{4b}$$

input
```
Int[E^(c - b^2*x^2)*Erf[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{e^{c}\sqrt{\pi}\, \mathrm{erf}(bx)^{2}}{4b}$	17

input
```
int(exp(-b^2*x^2+c)*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(c)*Pi^(1/2)*erf(b*x)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\mathrm{erf}(bx)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(bx\right)^{2}e^{c}}{4\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*erf(b*x)^2*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{c-b^2x^2}\mathrm{erf}(bx)\,dx=\begin{cases}\frac{\sqrt{\pi}e^{c}\,\mathrm{erf}^{2}\left(bx\right)}{4b} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)*erf(b*x),x)
```

output
```
Piecewise((sqrt(pi)*exp(c)*erf(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{\sqrt{\pi}\operatorname{erf}(bx)^2 e^c}{4b}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(b*x)^2*e^c/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{\sqrt{\pi}\operatorname{erf}(bx)^2 e^c}{4b}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x),x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(b*x)^2*e^c/b
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 4.43

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(x\sqrt{b^2}\right)^2 e^c}{4b} - \frac{\sqrt{\pi}\,e^c\operatorname{erfi}\left(\frac{b^2x}{\sqrt{-b^2}}\right)\operatorname{erf}(b\,x)}{2\sqrt{-b^2}} + \frac{b\sqrt{\pi}\operatorname{erf}\left(x\sqrt{b^2}\right)e^c\operatorname{erfi}\left(\frac{b^2x}{\sqrt{-b^2}}\right)}{2\sqrt{b^2}\sqrt{-b^2}}$$

input
```
int(exp(c - b^2*x^2)*erf(b*x),x)
```

output
```
(pi^(1/2)*erf(x*(b^2)^(1/2))^2*exp(c))/(4*b) - (pi^(1/2)*exp(c)*erfi((b^2*
x)/(-b^2)^(1/2))*erf(b*x))/(2*(-b^2)^(1/2)) + (b*pi^(1/2)*erf(x*(b^2)^(1/2
))*exp(c)*erfi((b^2*x)/(-b^2)^(1/2)))/(2*(b^2)^(1/2)*(-b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)\,dx=\frac{\sqrt{\pi}\,e^c\operatorname{erf}(bx)^2}{4b}$$

input
```
int(exp(-b^2*x^2+c)*erf(b*x),x)
```

output
```
(sqrt(pi)*e**c*erf(b*x)**2)/(4*b)
```

3.49 $\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)}\,dx = \frac{e^c\sqrt{\pi}\log(\operatorname{erf}(bx))}{2b}$$

output
```
1/2*exp(c)*Pi^(1/2)*ln(erf(b*x))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)}\,dx = \frac{e^c\sqrt{\pi}\log(\operatorname{erf}(bx))}{2b}$$

input
```
Integrate[E^(c - b^2*x^2)/Erf[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Log[Erf[b*x]])/(2*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6927, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c-b^2x^2}}{\text{erf}(bx)}dx$$

$$\Big\downarrow \text{ 6927}$$

$$\frac{\sqrt{\pi}e^c\int \frac{1}{\text{erf}(bx)}d\text{erf}(bx)}{2b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\sqrt{\pi}e^c\log(\text{erf}(bx))}{2b}$$

input
```
Int[E^(c - b^2*x^2)/Erf[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Log[Erf[b*x]])/(2*b)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [F(-1)]

Timed out.

$$\int \frac{\mathrm{e}^{-b^2x^2+c}}{\mathrm{erf}\,(bx)}dx$$

input
```
int(exp(-b^2*x^2+c)/erf(b*x),x)
```

output
```
int(exp(-b^2*x^2+c)/erf(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c-b^2x^2}}{\mathrm{erf}(bx)}\,dx = \frac{\sqrt{\pi}e^c \log\left(\mathrm{erf}\,(bx)\right)}{2\,b}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x),x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*e^c*log(erf(b*x))/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{c-b^2x^2}}{\mathrm{erf}(bx)}\,dx = \frac{\sqrt{\pi}e^c \log\left(\mathrm{erf}\,(bx)\right)}{2b}$$

input
```
integrate(exp(-b**2*x**2+c)/erf(b*x),x)
```

output
```
sqrt(pi)*exp(c)*log(erf(b*x))/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)}\,dx = \frac{\sqrt{\pi}e^c \log\left(\operatorname{erf}\left(bx\right)\right)}{2\,b}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*e^c*log(erf(b*x))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)}\,dx = \frac{\sqrt{\pi}e^c \log\left(\left|\operatorname{erf}\left(bx\right)\right|\right)}{2\,b}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x),x, algorithm="giac")
```

output
```
1/2*sqrt(pi)*e^c*log(abs(erf(b*x)))/b
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)}\,dx = \frac{\sqrt{\pi}\,\ln\left(\operatorname{erf}(b\,x)\right)\,\mathrm{e}^c}{2\,b}$$

input
```
int(exp(c - b^2*x^2)/erf(b*x),x)
```

output
```
(pi^(1/2)*log(erf(b*x))*exp(c))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c-b^2x^2}}{\mathrm{erf}(bx)}\,dx = \frac{\sqrt{\pi}\,e^c \log(\mathrm{erf}(bx))}{2b}$$

input
```
int(exp(-b^2*x^2+c)/erf(b*x),x)
```

output
```
(sqrt(pi)*e**c*log(erf(b*x)))/(2*b)
```

3.50 $\int \frac{e^{c-b^2x^2}}{\textbf{erf}(bx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^2}\,dx = -\frac{e^c\sqrt{\pi}}{2b\operatorname{erf}(bx)}$$

output
```
-1/2*exp(c)*Pi^(1/2)/b/erf(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^2}\,dx = -\frac{e^c\sqrt{\pi}}{2b\operatorname{erf}(bx)}$$

input
```
Integrate[E^(c - b^2*x^2)/Erf[b*x]^2,x]
```

output
```
-1/2*(E^c*Sqrt[Pi])/(b*Erf[b*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c-b^2x^2}}{\text{erf}(bx)^2}dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{\sqrt{\pi}e^c \int \frac{1}{\text{erf}(bx)^2}d\text{erf}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\pi}e^c}{2b\text{erf}(bx)}$$

input
```
Int[E^(c - b^2*x^2)/Erf[b*x]^2,x]
```

output
```
-1/2*(E^c*Sqrt[Pi])/(b*Erf[b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{e^{c}\sqrt{\pi}}{2b\,\mathrm{erf}(bx)}$	17

input
```
int(exp(-b^2*x^2+c)/erf(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(c)*Pi^(1/2)/b/erf(b*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\mathrm{erf}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2\,b\,\mathrm{erf}\,(bx)}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x)^2,x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*e^c/(b*erf(b*x))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{e^{c-b^2x^2}}{\mathrm{erf}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2b\,\mathrm{erf}\,(bx)}$$

input
```
integrate(exp(-b**2*x**2+c)/erf(b*x)**2,x)
```

output
```
-sqrt(pi)*exp(c)/(2*b*erf(b*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2\,b\operatorname{erf}(bx)}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x)^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(pi)*e^c/(b*erf(b*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2\,b\operatorname{erf}(bx)}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x)^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*e^c/(b*erf(b*x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^2}\,dx = -\frac{\sqrt{\pi}\,e^c}{2\,b\operatorname{erf}(b\,x)}$$

input
```
int(exp(c - b^2*x^2)/erf(b*x)^2,x)
```

output
```
-(pi^(1/2)*exp(c))/(2*b*erf(b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\mathrm{erf}(bx)^2}\,dx = -\frac{\sqrt{\pi}\,e^c}{2\,\mathrm{erf}\,(bx)\,b}$$

input
```
int(exp(-b^2*x^2+c)/erf(b*x)^2,x)
```

output
```
( - sqrt(pi)*e**c)/(2*erf(b*x)*b)
```

3.51 $\int \frac{e^{c-b^2x^2}}{\mathbf{erf}(bx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{e^c\sqrt{\pi}}{4b\operatorname{erf}(bx)^2}$$

output
```
-1/4*exp(c)*Pi^(1/2)/b/erf(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{e^c\sqrt{\pi}}{4b\operatorname{erf}(bx)^2}$$

input
```
Integrate[E^(c - b^2*x^2)/Erf[b*x]^3,x]
```

output
```
-1/4*(E^c*Sqrt[Pi])/(b*Erf[b*x]^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c-b^2x^2}}{\text{erf}(bx)^3} \, dx$$

$$\Big\downarrow \; 6927$$

$$\frac{\sqrt{\pi}e^c \int \frac{1}{\text{erf}(bx)^3}d\text{erf}(bx)}{2b}$$

$$\Big\downarrow \; 15$$

$$-\frac{\sqrt{\pi}e^c}{4b\text{erf}(bx)^2}$$

input
```
Int[E^(c - b^2*x^2)/Erf[b*x]^3,x]
```

output
```
-1/4*(E^c*Sqrt[Pi])/(b*Erf[b*x]^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{e^{c} \sqrt{\pi}}{4 b \operatorname{erf}(b x)^{2}}$	17

input
```
int(exp(-b^2*x^2+c)/erf(b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(c)*Pi^(1/2)/b/erf(b*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{\sqrt{\pi}e^{c}}{4\,b\operatorname{erf}\left(bx\right)^{2}}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x)^3,x, algorithm="fricas")
```

output
```
-1/4*sqrt(pi)*e^c/(b*erf(b*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{\sqrt{\pi}e^{c}}{4b\operatorname{erf}^{2}\left(bx\right)}$$

input
```
integrate(exp(-b**2*x**2+c)/erf(b*x)**3,x)
```

output
```
-sqrt(pi)*exp(c)/(4*b*erf(b*x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{\sqrt{\pi}e^c}{4\,b\operatorname{erf}\left(bx\right)^2}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x)^3,x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*e^c/(b*erf(b*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{\sqrt{\pi}e^c}{4\,b\operatorname{erf}\left(bx\right)^2}$$

input
```
integrate(exp(-b^2*x^2+c)/erf(b*x)^3,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*e^c/(b*erf(b*x)^2)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{\sqrt{\pi}\,\mathrm{e}^c}{4\,b\operatorname{erf}\left(b\,x\right)^2}$$

input
```
int(exp(c - b^2*x^2)/erf(b*x)^3,x)
```

output
```
-(pi^(1/2)*exp(c))/(4*b*erf(b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erf}(bx)^3}\,dx = -\frac{\sqrt{\pi}\,e^c}{4\operatorname{erf}\,(bx)^2\,b}$$

input
```
int(exp(-b^2*x^2+c)/erf(b*x)^3,x)
```

output
```
( - sqrt(pi)*e**c)/(4*erf(b*x)**2*b)
```

3.52 $\int e^{c-b^2x^2}\mathbf{erf}(bx)^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int e^{c-b^2x^2}\text{erf}(bx)^n\,dx=\frac{e^c\sqrt{\pi}\text{erf}(bx)^{1+n}}{2b(1+n)}$$

output
```
1/2*exp(c)*Pi^(1/2)*erf(b*x)^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int e^{c-b^2x^2}\text{erf}(bx)^n\,dx=\frac{e^c\sqrt{\pi}\text{erf}(bx)^{1+n}}{2b(1+n)}$$

input
```
Integrate[E^(c - b^2*x^2)*Erf[b*x]^n,x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^(1 + n))/(2*b*(1 + n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)^n\,dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{\sqrt{\pi}e^c\int \operatorname{erf}(bx)^n d\operatorname{erf}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\pi}e^c\operatorname{erf}(bx)^{n+1}}{2b(n+1)}$$

input
```
Int[E^(c - b^2*x^2)*Erf[b*x]^n,x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^(1 + n))/(2*b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int e^{-b^2x^2+c} \operatorname{erf}(bx)^n \, dx$$

input
```
int(exp(-b^2*x^2+c)*erf(b*x)^n,x)
```

output
```
int(exp(-b^2*x^2+c)*erf(b*x)^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int e^{c-b^2x^2} \operatorname{erf}(bx)^n \, dx = \frac{\sqrt{\pi} \operatorname{erf}(bx)^n \operatorname{erf}(bx) e^c}{2(bn+b)}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x)^n,x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*erf(b*x)^n*erf(b*x)*e^c/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(22) = 44.

Time = 1.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int e^{c-b^2x^2} \operatorname{erf}(bx)^n \, dx = \begin{cases} \tilde{\infty} x e^c & \text{for } b = 0 \wedge n = -1 \\ 0^n x e^c & \text{for } b = 0 \\ \frac{\sqrt{\pi} e^c \log(\operatorname{erf}(bx))}{2b} & \text{for } n = -1 \\ \frac{\sqrt{\pi} e^c \operatorname{erf}(bx) \operatorname{erf}^n(bx)}{2bn+2b} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)*erf(b*x)**n,x)
```

output
```
Piecewise((zoo*x*exp(c), Eq(b, 0) & Eq(n, -1)), (0**n*x*exp(c), Eq(b, 0)),
 (sqrt(pi)*exp(c)*log(erf(b*x))/(2*b), Eq(n, -1)), (sqrt(pi)*exp(c)*erf(b*
x)*erf(b*x)**n/(2*b*n + 2*b), True))
```

Maxima [F]

$$\int e^{c-b^2x^2}\text{erf}(bx)^n\,dx = \int \text{erf}\,(bx)^n\,e^{\left(-b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x)^n,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)^n*e^(-b^2*x^2 + c), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int e^{c-b^2x^2}\text{erf}(bx)^n\,dx = \frac{\sqrt{\pi}\,\text{erf}\,(bx)^{n+1}\,e^c}{2\,b(n+1)}$$

input
```
integrate(exp(-b^2*x^2+c)*erf(b*x)^n,x, algorithm="giac")
```

output
```
1/2*sqrt(pi)*erf(b*x)^(n + 1)*e^c/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int e^{c-b^2x^2}\text{erf}(bx)^n\,dx = \frac{\sqrt{\pi}\,\mathrm{e}^c\,\text{erf}(b\,x)^{n+1}}{2\,b\,\left(n+1\right)}$$

input
```
int(exp(c - b^2*x^2)*erf(b*x)^n,x)
```

output
```
(pi^(1/2)*exp(c)*erf(b*x)^(n + 1))/(2*b*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int e^{c-b^2x^2}\operatorname{erf}(bx)^n\,dx=\frac{\sqrt{\pi}\,e^c\operatorname{erf}(bx)^n\operatorname{erf}(bx)}{2b\,(n+1)}$$

input
```
int(exp(-b^2*x^2+c)*erf(b*x)^n,x)
```

output
```
(sqrt(pi)*e**c*erf(b*x)**n*erf(b*x))/(2*b*(n + 1))
```

3.53 $\int e^{c+dx^2} x^5 \mathbf{erf}(bx)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 285

$$\int e^{c+dx^2} x^5 \mathrm{erf}(bx)\, dx = -\frac{be^{c-(b^2-d)x^2}x}{(b^2-d)\, d^2\sqrt{\pi}} + \frac{3be^{c-(b^2-d)x^2}x}{4\,(b^2-d)^2\, d\sqrt{\pi}} + \frac{be^{c-(b^2-d)x^2}x^3}{2\,(b^2-d)\, d\sqrt{\pi}} + \frac{e^{c+dx^2}\mathrm{erf}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\mathrm{erf}(bx)}{d^2} + \frac{e^{c+dx^2}x^4\mathrm{erf}(bx)}{2d} - \frac{be^c\mathrm{erf}\left(\sqrt{b^2-d}x\right)}{\sqrt{b^2-d}d^3} + \frac{be^c\mathrm{erf}\left(\sqrt{b^2-d}x\right)}{2\,(b^2-d)^{3/2}\, d^2} - \frac{3be^c\mathrm{erf}\left(\sqrt{b^2-d}x\right)}{8\,(b^2-d)^{5/2}\, d}$$

output
```
-b*exp(c-(b^2-d)*x^2)*x/(b^2-d)/d^2/Pi^(1/2)+3/4*b*exp(c-(b^2-d)*x^2)*x/(b
^2-d)^2/d/Pi^(1/2)+1/2*b*exp(c-(b^2-d)*x^2)*x^3/(b^2-d)/d/Pi^(1/2)+exp(d*x
^2+c)*erf(b*x)/d^3-exp(d*x^2+c)*x^2*erf(b*x)/d^2+1/2*exp(d*x^2+c)*x^4*erf(
b*x)/d-b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)^(1/2)/d^3+1/2*b*exp(c)*erf((b
^2-d)^(1/2)*x)/(b^2-d)^(3/2)/d^2-3/8*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)
^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.48

$$\int e^{c+dx^2}x^5\text{erf}(bx)\,dx$$
$$=\frac{e^c\left(\frac{2bde^{\left(-b^2+d\right)x^2}x\left(d\left(7-2dx^2\right)+2b^2\left(-2+dx^2\right)\right)}{\left(b^2-d\right)^2\sqrt{\pi}}+4e^{dx^2}\left(2-2dx^2+d^2x^4\right)\text{erf}(bx)+\frac{b\left(-8b^4+20b^2d-15d^2\right)\text{erfi}\left(\sqrt{-b^2+d}x\right)}{\left(-b^2+d\right)^{5/2}}\right.}{8d^3}$$

input

```
Integrate[E^(c + d*x^2)*x^5*Erf[b*x],x]
```

output

```
(E^c*((2*b*d*E^((-b^2 + d)*x^2)*x*(d*(7 - 2*d*x^2) + 2*b^2*(-2 + d*x^2)))/
((b^2 - d)^2*Sqrt[Pi]) + 4*E^(d*x^2)*(2 - 2*d*x^2 + d^2*x^4)*Erf[b*x] + (b
*(-8*b^4 + 20*b^2*d - 15*d^2)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^(5/2)))/(
8*d^3)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6939, 2641, 2641, 2634, 6939, 2641, 2634, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\text{erf}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{b\int e^{c-\left(b^2-d\right)x^2}x^4dx}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erf}(bx)dx}{d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{b\left(\frac{3\int e^{c-\left(b^2-d\right)x^2}x^2dx}{2\left(b^2-d\right)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erf}(bx)dx}{d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{b\left(\frac{3\left(\frac{\int e^{c-\left(b^2-d\right)x^2}dx}{2\left(b^2-d\right)}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erf}(bx)dx}{d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\int e^{dx^2+c}x^3\text{erf}(bx)dx}{d}-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6939}$$

$$-\frac{2\left(-\frac{b\int e^{c-\left(b^2-d\right)x^2}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(bx)dx}{d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\left(-\frac{b\left(\frac{\int e^{c-\left(b^2-d\right)x^2}dx}{2\left(b^2-d\right)}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(bx)dx}{d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\left(-\frac{\int e^{dx^2+c}x\text{erf}(bx)dx}{d}-\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6936}$$

$$-\frac{2\left(-\frac{\frac{\text{erf}(bx)e^{c+dx^2}}{2d}-\frac{b\int e^{c-\left(b^2-d\right)x^2}dx}{\sqrt{\pi}d}}{d}-\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\left(-\frac{\frac{\text{erf}(bx)e^{c+dx^2}}{2d}-\frac{be^c\text{erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}}{d}-\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erf}(bx)e^{c+dx^2}}{2d}$$

input

```
Int[E^(c + d*x^2)*x^5*Erf[b*x],x]
```

output
```
(E^(c + d*x^2)*x^4*Erf[b*x])/(2*d) - (b*(-1/2*(E^(c - (b^2 - d)*x^2)*x^3)/
(b^2 - d) + (3*(-1/2*(E^(c - (b^2 - d)*x^2)*x)/(b^2 - d) + (E^c*Sqrt[Pi]*E
rf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)^(3/2))))/(2*(b^2 - d))))/(d*Sqrt[Pi]) -
(2*((E^(c + d*x^2)*x^2*Erf[b*x])/(2*d) - ((E^(c + d*x^2)*Erf[b*x])/(2*d) -
 (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d))/d - (b*(-1/2*(E^(c - (b
^2 - d)*x^2)*x)/(b^2 - d) + (E^c*Sqrt[Pi]*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 -
d)^(3/2))))/(d*Sqrt[Pi])))/d
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 5.80 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.09

method	result
default	$\frac{\frac{\operatorname{erf}(bx)\,e^{c}\left(\frac{e^{d x^2} b^6 x^4}{2d}-\frac{2b^2\left(\frac{b^4 x^2 e^{d x^2}}{2d}-\frac{b^4 e^{d x^2}}{2d^2}\right)}{d}\right)}{b^5}-\frac{e^{c}\left(\frac{b^2\left(\frac{b^3 x^3 e^{\left(-1+\frac{d}{b^2}\right)b^2 x^2}}{-2+\frac{2d}{b^2}}-\frac{3\left(\frac{bx\,e^{\left(-1+\frac{d}{b^2}\right)b^2x^2}}{-2+\frac{2d}{b^2}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{1-\frac{d}{b^2}}\,bx\right)}{4\left(-1+\frac{d}{b^2}\right)\sqrt{1-\frac{d}{b^2}}}\right)}{2\left(-1+\frac{d}{b^2}\right)}\right)}{d}+\cdots\right)}{\sqrt{\pi}\,b^5}}{b}$

input
```
int(exp(d*x^2+c)*x^5*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
(erf(b*x)/b^5*exp(c)*(1/2*exp(d*x^2)*b^6*x^4/d-2/d*b^2*(1/2/d*b^4*x^2*exp(
d*x^2)-1/2/d^2*b^4*exp(d*x^2)))-1/Pi^(1/2)/b^5*exp(c)*(1/d*b^2*(1/2/(-1+d/
b^2)*b^3*x^3*exp((-1+d/b^2)*b^2*x^2)-3/2/(-1+d/b^2)*(1/2/(-1+d/b^2)*b*x*ex
p((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2
)^(1/2)*b*x)))+1/d^3*b^6*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)
-2/d^2*b^4*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*Pi^(
1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.91

$$\int e^{c+dx^2} x^5 \operatorname{erf}(bx)\,dx =$$
$$-\frac{\pi(8\,b^5-20\,b^3d+15\,bd^2)\sqrt{b^2-d}\operatorname{erf}\left(\sqrt{b^2-d}\,x\right)e^{c}-4\left(\pi(b^6d^2-3\,b^4d^3+3\,b^2d^4-d^5)x^4-2\,\pi(b^6d-\cdots\right.}{\cdots}$$

input
```
integrate(exp(d*x^2+c)*x^5*erf(b*x),x, algorithm="fricas")
```

output
```
-1/8*(pi*(8*b^5 - 20*b^3*d + 15*b*d^2)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*
e^c - 4*(pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^2*d^4 - d^5)*x^4 - 2*pi*(b^6*d - 3*
b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 + 2*pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3))*e
rf(b*x)*e^(d*x^2 + c) - 2*sqrt(pi)*(2*(b^5*d^2 - 2*b^3*d^3 + b*d^4)*x^3 -
(4*b^5*d - 11*b^3*d^2 + 7*b*d^3)*x)*e^(-b^2*x^2 + d*x^2 + c))/(pi*(b^6*d^3
 - 3*b^4*d^4 + 3*b^2*d^5 - d^6))
```

Sympy [F]

$$\int e^{c+dx^2}x^5\text{erf}(bx)\,dx = e^c\int x^5e^{dx^2}\,\text{erf}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**5*erf(b*x),x)
```

output
```
exp(c)*Integral(x**5*exp(d*x**2)*erf(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x^5\text{erf}(bx)\,dx = \int x^5\,\text{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^5*erf(b*x),x, algorithm="maxima")
```

output
```
1/2*(d^2*x^4*e^c - 2*d*x^2*e^c + 2*e^c)*erf(b*x)*e^(d*x^2)/d^3 - integrate
((b*d^2*x^4*e^c - 2*b*d*x^2*e^c + 2*b*e^c)*e^(-b^2*x^2 + d*x^2), x)/(sqrt(
pi)*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.95

$$\int e^{c+dx^2}x^5\text{erf}(bx)\,dx$$
$$= \frac{1}{2}\left(\frac{c^2e^{(dx^2+c)}}{d^3} - \frac{\left(2\,dx^2 - (dx^2+c)^2 + 2\,(dx^2+c)c - 2\right)e^{(dx^2+c)}}{d^3}\right)\text{erf}\,(bx)$$
$$+ \frac{\sqrt{\pi}bd^2\left(\frac{2\,(2\,b^2x^3-2\,dx^3+3\,x)e^{\left(-b^2x^2+dx^2+c\right)}}{b^4-2\,b^2d+d^2} + \frac{3\,\sqrt{\pi}\,\text{erf}\left(-\sqrt{b^2-d}x\right)e^c}{(b^4-2\,b^2d+d^2)\sqrt{b^2-d}}\right) - 4\,\sqrt{\pi}bd\left(\frac{2\,xe^{\left(-b^2x^2+dx^2+c\right)}}{b^2-d} + \frac{\sqrt{\pi}\,\text{erf}\left(-\sqrt{b^2-d}\right.}{(b^2-d)^{\frac{3}{2}}}\right.}{8\,\pi d^3}$$

input
```
integrate(exp(d*x^2+c)*x^5*erf(b*x),x, algorithm="giac")
```

output
```
1/2*(c^2*e^(d*x^2 + c)/d^3 - (2*d*x^2 - (d*x^2 + c)^2 + 2*(d*x^2 + c)*c -
2)*e^(d*x^2 + c)/d^3)*erf(b*x) + 1/8*(sqrt(pi)*b*d^2*(2*(2*b^2*x^3 - 2*d*x
^3 + 3*x)*e^(-b^2*x^2 + d*x^2 + c)/(b^4 - 2*b^2*d + d^2) + 3*sqrt(pi)*erf(
-sqrt(b^2 - d)*x)*e^c/((b^4 - 2*b^2*d + d^2)*sqrt(b^2 - d))) - 4*sqrt(pi)*
b*d*(2*x*e^(-b^2*x^2 + d*x^2 + c)/(b^2 - d) + sqrt(pi)*erf(-sqrt(b^2 - d)*
x)*e^c/(b^2 - d)^(3/2)) + 8*pi*b*erf(-sqrt(b^2 - d)*x)*e^c/sqrt(b^2 - d))/
(pi*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.86

$$\int e^{c+dx^2}x^5\text{erf}(bx)\,dx = \text{erf}(b\,x)\left(\frac{e^{d\,x^2+c}}{d^3} - \frac{x^2\,e^{d\,x^2+c}}{d^2} + \frac{x^4\,e^{d\,x^2+c}}{2\,d}\right)$$
$$- \frac{b\,e^c\,\text{erf}\left(x\,\sqrt{b^2-d}\right)}{d^3\,\sqrt{b^2-d}} - \frac{b\,e^c\,\text{erfi}\left(x\,\sqrt{d-b^2}\right)}{2\,d^2\,(d-b^2)^{3/2}} + \frac{b\,x\,e^{-b^2\,x^2+d\,x^2+c}}{d^2\,\sqrt{\pi}\,(d-b^2)}$$
$$+ \frac{b\,x^5\,e^c\left(e^{d\,x^2-b^2\,x^2}\left(\frac{3\,\sqrt{-x^2\,(d-b^2)}}{2} + \left(-x^2\,(d-b^2)\right)^{3/2}\right) - \frac{3\,\sqrt{\pi}}{4} + \frac{3\,\sqrt{\pi}\,\text{erfc}\left(\sqrt{-x^2\,(d-b^2)}\right)}{4}\right)}{2\,d\,\sqrt{\pi}\left(-x^2\,(d-b^2)\right)^{5/2}}$$

input
```
int(x^5*exp(c + d*x^2)*erf(b*x),x)
```

output
```
erf(b*x)*(exp(c + d*x^2)/d^3 - (x^2*exp(c + d*x^2))/d^2 + (x^4*exp(c + d*x
^2))/(2*d)) - (b*exp(c)*erf(x*(b^2 - d)^(1/2)))/(d^3*(b^2 - d)^(1/2)) - (b
*exp(c)*erfi(x*(d - b^2)^(1/2)))/(2*d^2*(d - b^2)^(3/2)) + (b*x*exp(c + d*
x^2 - b^2*x^2))/(d^2*pi^(1/2)*(d - b^2)) + (b*x^5*exp(c)*(exp(d*x^2 - b^2*
x^2)*((3*(-x^2*(d - b^2))^(1/2))/2 + (-x^2*(d - b^2))^(3/2)) - (3*pi^(1/2)
)/4 + (3*pi^(1/2)*erfc((-x^2*(d - b^2))^(1/2)))/4))/(2*d*pi^(1/2)*(-x^2*(d
 - b^2))^(5/2))
```

Reduce [F]

$$\int e^{c+dx^2}x^5\operatorname{erf}(bx)\,dx$$
$$= \frac{e^c\left(2e^{b^2x^2+dx^2}\operatorname{erf}(bx)\,b^4d^2\pi\,x^4 - 4e^{b^2x^2+dx^2}\operatorname{erf}(bx)\,b^4d\pi\,x^2 + 4e^{b^2x^2+dx^2}\operatorname{erf}(bx)\,b^4\pi - 4e^{b^2x^2+dx^2}\operatorname{erf}(bx)\,b^2d\right.}{}$$

input
```
int(exp(d*x^2+c)*x^5*erf(b*x),x)
```

output
```
(e**c*(2*e**(b**2*x**2 + d*x**2)*erf(b*x)*b**4*d**2*pi*x**4 - 4*e**(b**2*x
**2 + d*x**2)*erf(b*x)*b**4*d*pi*x**2 + 4*e**(b**2*x**2 + d*x**2)*erf(b*x)
*b**4*pi - 4*e**(b**2*x**2 + d*x**2)*erf(b*x)*b**2*d**3*pi*x**4 + 8*e**(b*
*2*x**2 + d*x**2)*erf(b*x)*b**2*d**2*pi*x**2 - 8*e**(b**2*x**2 + d*x**2)*e
rf(b*x)*b**2*d*pi + 2*e**(b**2*x**2 + d*x**2)*erf(b*x)*d**4*pi*x**4 - 4*e*
*(b**2*x**2 + d*x**2)*erf(b*x)*d**3*pi*x**2 + 4*e**(b**2*x**2 + d*x**2)*er
f(b*x)*d**2*pi - 8*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/e**(b**2*x**2),
x)*b**5 + 20*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/e**(b**2*x**2),x)*b**
3*d - 15*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/e**(b**2*x**2),x)*b*d**2
+ 2*sqrt(pi)*e**(d*x**2)*b**3*d**2*x**3 - 4*sqrt(pi)*e**(d*x**2)*b**3*d*x
- 2*sqrt(pi)*e**(d*x**2)*b*d**3*x**3 + 7*sqrt(pi)*e**(d*x**2)*b*d**2*x))/(
4*e**(b**2*x**2)*d**3*pi*(b**4 - 2*b**2*d + d**2))
```

3.54 $\int e^{c+dx^2}x^3\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int e^{c+dx^2}x^3\text{erf}(bx)\,dx=\frac{be^{c-(b^2-d)x^2}x}{2(b^2-d)\,d\sqrt{\pi}}-\frac{e^{c+dx^2}\text{erf}(bx)}{2d^2}+\frac{e^{c+dx^2}x^2\text{erf}(bx)}{2d}+\frac{be^c\text{erf}\left(\sqrt{b^2-d}\,x\right)}{2\sqrt{b^2-d}\,d^2}-\frac{be^c\text{erf}\left(\sqrt{b^2-d}\,x\right)}{4(b^2-d)^{3/2}\,d}$$

output
```
1/2*b*exp(c-(b^2-d)*x^2)*x/(b^2-d)/d/Pi^(1/2)-1/2*exp(d*x^2+c)*erf(b*x)/d^
2+1/2*exp(d*x^2+c)*x^2*erf(b*x)/d+1/2*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d
)^(1/2)/d^2-1/4*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

$$\int e^{c+dx^2}x^3\text{erf}(bx)\,dx=\frac{e^c\left(\frac{2bde^{(-b^2+d)x^2}x}{(b^2-d)\sqrt{\pi}}+2e^{dx^2}(-1+dx^2)\,\text{erf}(bx)+\frac{b(-2b^2+3d)\text{erfi}\left(\sqrt{-b^2+d}\,x\right)}{(-b^2+d)^{3/2}}\right)}{4d^2}$$

input
```
Integrate[E^(c + d*x^2)*x^3*Erf[b*x],x]
```

output
```
(E^c*((2*b*d*E^((-b^2 + d)*x^2)*x)/((b^2 - d)*Sqrt[Pi]) + 2*E^(d*x^2)*(-1
+ d*x^2)*Erf[b*x] + (b*(-2*b^2 + 3*d)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^(
3/2)))/(4*d^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6939, 2641, 2634, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{erf}(bx) e^{c+dx^2} \, dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{b\int e^{c-(b^2-d)x^2}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(bx)dx}{d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{b\left(\frac{\int e^{c-\left(b^2-d\right)x^2}dx}{2(b^2-d)}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(bx)dx}{d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\int e^{dx^2+c}x\text{erf}(bx)dx}{d}-\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6936}$$

$$-\frac{\frac{\text{erf}(bx)e^{c+dx^2}}{2d}-\frac{b\int e^{c-\left(b^2-d\right)x^2}dx}{\sqrt{\pi}d}}{d}-\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\frac{\text{erf}(bx)e^{c+dx^2}}{2d}-\frac{be^c\text{erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}}{d}-\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{c-x^2(b^2-d)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erf}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^3*Erf[b*x],x]
```

output
```
(E^(c + d*x^2)*x^2*Erf[b*x])/(2*d) - ((E^(c + d*x^2)*Erf[b*x])/(2*d) - (b*
E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d))/d - (b*(-1/2*(E^(c - (b^2 -
 d)*x^2)*x)/(b^2 - d) + (E^c*Sqrt[Pi]*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)^(
3/2))))/(d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\frac{\mathrm{erf}(bx)\,e^{c}\left(\frac{b^{4}x^{2}e^{d\,x^{2}}}{2d}-\frac{b^{4}e^{d\,x^{2}}}{2d^{2}}\right)}{b^{3}}-\frac{e^{c}\left(\frac{b^{2}\left(\frac{bx\,e^{\left(-1+\frac{d}{b^{2}}\right)b^{2}x^{2}}}{-2+\frac{2d}{b^{2}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\,bx\right)}{4\left(-1+\frac{d}{b^{2}}\right)\sqrt{1-\frac{d}{b^{2}}}}\right)}{d}-\frac{b^{4}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\,bx\right)}{2d^{2}\sqrt{1-\frac{d}{b^{2}}}}\right)}{\sqrt{\pi}\,b^{3}}}{b}$	168

input
```
int(exp(d*x^2+c)*x^3*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
(erf(b*x)/b^3*exp(c)*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2))-1/P
i^(1/2)/b^3*exp(c)*(1/d*b^2*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/
4/(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))-1/2/d^2*b^
4*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int e^{c+dx^2}x^3\mathrm{erf}(bx)\,dx$$
$$=\frac{\pi(2\,b^3-3\,bd)\sqrt{b^2-d}\,\mathrm{erf}\left(\sqrt{b^2-d}x\right)e^c+2\,\sqrt{\pi}(b^3d-bd^2)xe^{\left(-b^2x^2+dx^2+c\right)}+2\left(\pi(b^4d-2\,b^2d^2+d^3)x^2-\right.}{4\,\pi(b^4d^2-2\,b^2d^3+d^4)}$$

input
```
integrate(exp(d*x^2+c)*x^3*erf(b*x),x, algorithm="fricas")
```

output
```
1/4*(pi*(2*b^3 - 3*b*d)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e^c + 2*sqrt(pi
)*(b^3*d - b*d^2)*x*e^(-b^2*x^2 + d*x^2 + c) + 2*(pi*(b^4*d - 2*b^2*d^2 +
d^3)*x^2 - pi*(b^4 - 2*b^2*d + d^2))*erf(b*x)*e^(d*x^2 + c))/(pi*(b^4*d^2
- 2*b^2*d^3 + d^4))
```

Sympy [F]

$$\int e^{c+dx^2}x^3\operatorname{erf}(bx)\,dx = e^c\int x^3e^{dx^2}\operatorname{erf}(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**3*erf(b*x),x)
```

output
```
exp(c)*Integral(x**3*exp(d*x**2)*erf(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x^3\operatorname{erf}(bx)\,dx = \int x^3\operatorname{erf}(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erf(b*x),x, algorithm="maxima")
```

output
```
1/2*(d*x^2*e^c - e^c)*erf(b*x)*e^(d*x^2)/d^2 - integrate((b*d*x^2*e^c - b*
e^c)*e^(-b^2*x^2 + d*x^2), x)/(sqrt(pi)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.91

$$\int e^{c+dx^2}x^3\operatorname{erf}(bx)\,dx = \frac{1}{2}\left(\frac{(dx^2+c-1)e^{(dx^2+c)}}{d^2} - \frac{ce^{(dx^2+c)}}{d^2}\right)\operatorname{erf}(bx) + \frac{bd\left(\frac{2\,xe^{\left(-b^2x^2+dx^2+c\right)}}{b^2-d} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b^2-d}x\right)e^c}{(b^2-d)^{\frac{3}{2}}}\right) - \frac{2\sqrt{\pi}b\operatorname{erf}\left(-\sqrt{b^2-d}x\right)e^c}{\sqrt{b^2-d}}}{4\sqrt{\pi}d^2}$$

input
```
integrate(exp(d*x^2+c)*x^3*erf(b*x),x, algorithm="giac")
```

output
```
1/2*((d*x^2 + c - 1)*e^(d*x^2 + c)/d^2 - c*e^(d*x^2 + c)/d^2)*erf(b*x) + 1
/4*(b*d*(2*x*e^(-b^2*x^2 + d*x^2 + c)/(b^2 - d) + sqrt(pi)*erf(-sqrt(b^2 -
 d)*x)*e^c/(b^2 - d)^(3/2)) - 2*sqrt(pi)*b*erf(-sqrt(b^2 - d)*x)*e^c/sqrt(
b^2 - d))/(sqrt(pi)*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int e^{c+dx^2}x^3\text{erf}(bx)\,dx=\frac{b\,x\,e^{-b^2\,x^2+d\,x^2+c}}{2\,\sqrt{\pi}\,\left(b^2\,d-d^2\right)}-\text{erf}(b\,x)\,\left(\frac{e^{d\,x^2+c}}{2\,d^2}-\frac{x^2\,e^{d\,x^2+c}}{2\,d}\right)+\frac{b\,e^c\,\text{erf}\left(x\,\sqrt{b^2-d}\right)}{2\,d^2\,\sqrt{b^2-d}}+\frac{b\,e^c\,\text{erfi}\left(x\,\sqrt{d-b^2}\right)}{4\,d\,{\left(d-b^2\right)}^{3/2}}$$

input
```
int(x^3*exp(c + d*x^2)*erf(b*x),x)
```

output
```
(b*x*exp(c + d*x^2 - b^2*x^2))/(2*pi^(1/2)*(b^2*d - d^2)) - erf(b*x)*(exp(
c + d*x^2)/(2*d^2) - (x^2*exp(c + d*x^2))/(2*d)) + (b*exp(c)*erf(x*(b^2 -
d)^(1/2)))/(2*d^2*(b^2 - d)^(1/2)) + (b*exp(c)*erfi(x*(d - b^2)^(1/2)))/(4
*d*(d - b^2)^(3/2))
```

Reduce [F]

$$\int e^{c+dx^2}x^3\text{erf}(bx)\,dx=\frac{e^c\left(e^{b^2x^2+d\,x^2}\text{erf}(bx)\,b^2d\pi\,x^2-e^{b^2x^2+d\,x^2}\text{erf}(bx)\,b^2\pi-e^{b^2x^2+d\,x^2}\text{erf}(bx)\,d^2\pi\,x^2+e^{b^2x^2+d\,x^2}\text{erf}(bx)\,d\pi+2\sqrt{\pi}\right.}{2e^{b^2x^2}d^2\pi\left(b^2-d\right)}$$

input
```
int(exp(d*x^2+c)*x^3*erf(b*x),x)
```

output
```
(e**c*(e**(b**2*x**2 + d*x**2)*erf(b*x)*b**2*d*pi*x**2 - e**(b**2*x**2 + d
*x**2)*erf(b*x)*b**2*pi - e**(b**2*x**2 + d*x**2)*erf(b*x)*d**2*pi*x**2 +
e**(b**2*x**2 + d*x**2)*erf(b*x)*d*pi + 2*sqrt(pi)*e**(b**2*x**2)*int(e**(
d*x**2)/e**(b**2*x**2),x)*b**3 - 3*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)
/e**(b**2*x**2),x)*b*d + sqrt(pi)*e**(d*x**2)*b*d*x))/(2*e**(b**2*x**2)*d*
*2*pi*(b**2 - d))
```

3.55 $\int e^{c+dx^2} x\,\text{erf}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int e^{c+dx^2} x\text{erf}(bx)\,dx = \frac{e^{c+dx^2}\text{erf}(bx)}{2d} - \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d}$$

output
```
1/2*exp(d*x^2+c)*erf(b*x)/d-1/2*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)^(1/2
)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int e^{c+dx^2} x\text{erf}(bx)\,dx = \frac{e^c\left(e^{dx^2}\text{erf}(bx) - \frac{b\text{erfi}\left(\sqrt{-b^2+d}x\right)}{\sqrt{-b^2+d}}\right)}{2d}$$

input
```
Integrate[E^(c + d*x^2)*x*Erf[b*x],x]
```

output
```
(E^c*(E^(d*x^2)*Erf[b*x] - (b*Erfi[Sqrt[-b^2 + d]*x])/Sqrt[-b^2 + d]))/(2*
d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erf}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{6936}$$

$$\frac{\text{erf}(bx)e^{c+dx^2}}{2d}-\frac{b\int e^{c-(b^2-d)x^2}dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\text{erf}(bx)e^{c+dx^2}}{2d}-\frac{be^c\text{erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}$$

input
```
Int[E^(c + d*x^2)*x*Erf[b*x],x]
```

output
```
(E^(c + d*x^2)*Erf[b*x])/(2*d) - (b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2
- d]*d)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\frac{\operatorname{erf}(bx)b\,\mathrm{e}^{\frac{b^2 d\,x^2+b^2 c}{b^2}}}{2d}-\frac{b\,\mathrm{e}^{c}\operatorname{erf}\left(\sqrt{1-\frac{d}{b^2}}\,bx\right)}{2d\sqrt{1-\frac{d}{b^2}}}}{b}$	67

input
```
int(exp(d*x^2+c)*x*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/2*erf(b*x)*b*exp((b^2*d*x^2+b^2*c)/b^2)/d-1/2*b/d*exp(c)/(1-d/b^2)^(1/2
)*erf((1-d/b^2)^(1/2)*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int e^{c+dx^2}x\mathrm{erf}(bx)\,dx=-\frac{\sqrt{b^2-d}b\operatorname{erf}\left(\sqrt{b^2-d}x\right)e^c-\left(b^2-d\right)\operatorname{erf}\left(bx\right)e^{\left(dx^2+c\right)}}{2\left(b^2d-d^2\right)}$$

input
```
integrate(exp(d*x^2+c)*x*erf(b*x),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b^2 - d)*b*erf(sqrt(b^2 - d)*x)*e^c - (b^2 - d)*erf(b*x)*e^(d*x
^2 + c))/(b^2*d - d^2)
```

Sympy [F]

$$\int e^{c+dx^2}x\mathrm{erf}(bx)\,dx=e^c\int xe^{dx^2}\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(exp(d*x**2+c)*x*erf(b*x),x)
```

output
```
exp(c)*Integral(x*exp(d*x**2)*erf(b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int e^{c+dx^2}x\text{erf}(bx)\,dx = -\frac{b\,\text{erf}\left(\sqrt{b^2-d}x\right)e^c}{2\sqrt{b^2-d}d} + \frac{\text{erf}\left(bx\right)e^{\left(dx^2+c\right)}}{2\,d}$$

input
```
integrate(exp(d*x^2+c)*x*erf(b*x),x, algorithm="maxima")
```

output
```
-1/2*b*erf(sqrt(b^2 - d)*x)*e^c/(sqrt(b^2 - d)*d) + 1/2*erf(b*x)*e^(d*x^2
+ c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int e^{c+dx^2}x\text{erf}(bx)\,dx = \frac{b\,\text{erf}\left(-\sqrt{b^2-d}x\right)e^c}{2\sqrt{b^2-d}d} + \frac{\text{erf}\left(bx\right)e^{\left(dx^2+c\right)}}{2\,d}$$

input
```
integrate(exp(d*x^2+c)*x*erf(b*x),x, algorithm="giac")
```

output
```
1/2*b*erf(-sqrt(b^2 - d)*x)*e^c/(sqrt(b^2 - d)*d) + 1/2*erf(b*x)*e^(d*x^2
+ c)/d
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int e^{c+dx^2}x\text{erf}(bx)\,dx = \frac{e^{d\,x^2}\,e^c\,\text{erf}(b\,x)}{2\,d} - \frac{b\,e^c\,\text{erf}\left(x\,\sqrt{b^2-d}\right)}{2\,d\,\sqrt{b^2-d}}$$

input
```
int(x*exp(c + d*x^2)*erf(b*x),x)
```

output
```
(exp(d*x^2)*exp(c)*erf(b*x))/(2*d) - (b*exp(c)*erf(x*(b^2 - d)^(1/2)))/(2*
d*(b^2 - d)^(1/2))
```

Reduce [F]

$$\int e^{c+dx^2} x \operatorname{erf}(bx)\, dx = e^c \left(\int e^{d\,x^2} \operatorname{erf}(bx)\, x dx \right)$$

input
```
int(exp(d*x^2+c)*x*erf(b*x),x)
```

output
```
e**c*int(e**(d*x**2)*erf(b*x)*x,x)
```

3.56 $\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x}\,dx=\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erf}(bx)}{x},x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erf(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x}\,dx=\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)e^{c+dx^2}}{x} \, dx$$
$$\Big\downarrow \; 6948$$
$$\int \frac{\text{erf}(bx)e^{c+dx^2}}{x} dx$$

input
```
Int[(E^(c + d*x^2)*Erf[b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^{2}+c} \operatorname{erf}\left(b x \right)}{x}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x}\,dx = \int \frac{\mathrm{erf}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(d*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x}\,dx = e^c \int \frac{e^{dx^2}\,\mathrm{erf}\,(bx)}{x}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x)/x,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x}\,dx = \int \frac{\mathrm{erf}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erf}(b\,x)}{x}\,dx$$

input
```
int((exp(c + d*x^2)*erf(b*x))/x,x)
```

output
```
int((exp(c + d*x^2)*erf(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x}\,dx = e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx)}{x}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x,x)
```

output
```
e**c*int((e**(d*x**2)*erf(b*x))/x,x)
```

3.57 $\int \frac{e^{c+dx^2}\mathbf{erf}(bx)}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^3} dx = -\frac{be^{c-(b^2-d)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\operatorname{erf}(bx)}{2x^2} - b\sqrt{b^2-d}e^c\operatorname{erf}\left(\sqrt{b^2-d}x\right) + d\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erf}(bx)}{x}, x\right)$$

output
```
-b*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x-1/2*exp(d*x^2+c)*erf(b*x)/x^2-b*(b^2-d)^(
1/2)*exp(c)*erf((b^2-d)^(1/2)*x)+d*Defer(Int)(exp(d*x^2+c)*erf(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^3} dx = \int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^3} dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^3,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(bx)e^{c+dx^2}}{x^3}dx$$

$$\Big\downarrow \text{6945}$$

$$\frac{b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^2}dx}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\operatorname{erf}(bx)}{x}dx-\frac{\operatorname{erf}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(-2\left(b^2-d\right)\int e^{c-\left(b^2-d\right)x^2}dx-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\operatorname{erf}(bx)}{x}dx-\frac{\operatorname{erf}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$d\int \frac{e^{dx^2+c}\operatorname{erf}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\operatorname{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{6948}$$

$$d\int \frac{e^{dx^2+c}\operatorname{erf}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\operatorname{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\operatorname{erf}(bx)e^{c+dx^2}}{2x^2}$$

input
```
Int[(E^(c + d*x^2)*Erf[b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erf}\,(bx)}{x^3}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^3,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x^3}\,dx = \int \frac{\mathrm{erf}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(d*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^3}\,dx = e^c \int \frac{e^{dx^2}\,\text{erf}\,(bx)}{x^3}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x)/x**3,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^3}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^3}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erf}(b\,x)}{x^3}\,dx$$

input
```
int((exp(c + d*x^2)*erf(b*x))/x^3,x)
```

output
```
int((exp(c + d*x^2)*erf(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x^3}\,dx = e^c\left(\int \frac{e^{d\,x^2}\mathrm{erf}(bx)}{x^3}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^3,x)
```

output
```
e**c*int((e**(d*x**2)*erf(b*x))/x**3,x)
```

3.58 $\int \frac{e^{c+dx^2}\mathbf{erf}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = -\frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} + \frac{b(b^2-d)\,e^{c-(b^2-d)x^2}}{3\sqrt{\pi}x} - \frac{bde^{c-(b^2-d)x^2}}{2\sqrt{\pi}x} - \frac{e^{c+dx^2}\text{erf}(bx)}{4x^4} - \frac{de^{c+dx^2}\text{erf}(bx)}{4x^2} + \frac{1}{3}b\left(b^2-d\right)^{3/2}e^c\text{erf}\left(\sqrt{b^2-d}x\right) - \frac{1}{2}b\sqrt{b^2-d}de^c\text{erf}\left(\sqrt{b^2-d}x\right) + \frac{1}{2}d^2\text{Int}\left(\frac{e^{c+dx^2}\text{erf}(bx)}{x},x\right)$$

output
```
-1/6*b*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x^3+1/3*b*(b^2-d)*exp(c-(b^2-d)*x^2)/Pi
^(1/2)/x-1/2*b*d*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x-1/4*exp(d*x^2+c)*erf(b*x)/x
^4-1/4*d*exp(d*x^2+c)*erf(b*x)/x^2+1/3*b*(b^2-d)^(3/2)*exp(c)*erf((b^2-d)^
(1/2)*x)-1/2*b*(b^2-d)^(1/2)*d*exp(c)*erf((b^2-d)^(1/2)*x)+1/2*d^2*Defer(I
nt)(exp(d*x^2+c)*erf(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^5}\,dx = \int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^5}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^5,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^5, x]
```

Rubi [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erf}(bx)e^{c+dx^2}}{x^5}\,dx$$

$$\Big\downarrow \text{ 6945}$$

$$\frac{b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^4}dx}{2\sqrt{\pi}}+\frac{1}{2}d\int \frac{e^{dx^2+c}\operatorname{erf}(bx)}{x^3}dx-\frac{\operatorname{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^2}dx-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}+\frac{1}{2}d\int \frac{e^{dx^2+c}\operatorname{erf}(bx)}{x^3}dx-\frac{\operatorname{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(-2\left(b^2-d\right)\int e^{c-\left(b^2-d\right)x^2}dx-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}+\frac{1}{2}d\int\frac{e^{dx^2+c}\text{erf}(bx)}{x^3}dx-\frac{\text{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{1}{2}d\int\frac{e^{dx^2+c}\text{erf}(bx)}{x^3}dx+\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{ 6945}$$

$$\frac{1}{2}d\left(\frac{b\int\frac{e^{c-\left(b^2-d\right)x^2}}{x^2}dx}{\sqrt{\pi}}+d\int\frac{e^{dx^2+c}\text{erf}(bx)}{x}dx-\frac{\text{erf}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{2}d\left(\frac{b\left(-2\left(b^2-d\right)\int e^{c-\left(b^2-d\right)x^2}dx-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}+d\int\frac{e^{dx^2+c}\text{erf}(bx)}{x}dx-\frac{\text{erf}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{1}{2}d\left(d\int \frac{e^{dx^2+c}\text{erf}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \textbf{6948}$$

$$\frac{1}{2}d\left(d\int \frac{e^{dx^2+c}\text{erf}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{4x^4}$$

input
```
Int[(E^(c + d*x^2)*Erf[b*x])/x^5,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{e^{d\,x^2+c}\,\text{erf}\,(bx)}{x^5}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^5,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(d*x^2 + c)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 33.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = e^c \int \frac{e^{dx^2}\,\text{erf}\,(bx)}{x^5}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x)/x**5,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erf}(b\,x)}{x^5}\,dx$$

input
```
int((exp(c + d*x^2)*erf(b*x))/x^5,x)
```

output
```
int((exp(c + d*x^2)*erf(b*x))/x^5, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 12.00

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^5}\,dx$$
$$= \frac{e^c\left(-3e^{b^2x^2+d\,x^2}\operatorname{erf}(bx)\,d\pi\,x^2 - 3e^{b^2x^2+d\,x^2}\operatorname{erf}(bx)\,\pi - 4\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{e^{d\,x^2}}{e^{b^2x^2}x^2}dx\right)b^3x^4 + 10\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{e^{d\,x^2}}{e^{b^2x^2}x^2}\right.\right.}{12e^{b^2x^2}\pi\,x^4}$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^5,x)
```

output
```
(e**c*( - 3*e**(b**2*x**2 + d*x**2)*erf(b*x)*d*pi*x**2 - 3*e**(b**2*x**2 +
 d*x**2)*erf(b*x)*pi - 4*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/(e**(b**2
*x**2)*x**2),x)*b**3*x**4 + 10*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/(e*
*(b**2*x**2)*x**2),x)*b*d*x**4 + 6*e**(b**2*x**2)*int((e**(d*x**2)*erf(b*x
))/x,x)*d**2*pi*x**4 - 2*sqrt(pi)*e**(d*x**2)*b*x))/(12*e**(b**2*x**2)*pi*
x**4)
```

3.59 $\int e^{c+dx^2} x^4 \mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{c+dx^2} x^4 \operatorname{erf}(bx)\,dx = -\frac{3be^{c-(b^2-d)x^2}}{4(b^2-d)\,d^2\sqrt{\pi}} + \frac{be^{c-(b^2-d)x^2}}{2(b^2-d)^2\,d\sqrt{\pi}} + \frac{be^{c-(b^2-d)x^2}x^2}{2(b^2-d)\,d\sqrt{\pi}} - \frac{3e^{c+dx^2}x\operatorname{erf}(bx)}{4d^2} + \frac{e^{c+dx^2}x^3\operatorname{erf}(bx)}{2d} + \frac{3\operatorname{Int}\left(e^{c+dx^2}\operatorname{erf}(bx),x\right)}{4d^2}$$

output
```
-3/4*b*exp(c-(b^2-d)*x^2)/(b^2-d)/d^2/Pi^(1/2)+1/2*b*exp(c-(b^2-d)*x^2)/(b
^2-d)^2/d/Pi^(1/2)+1/2*b*exp(c-(b^2-d)*x^2)*x^2/(b^2-d)/d/Pi^(1/2)-3/4*exp
(d*x^2+c)*x*erf(b*x)/d^2+1/2*exp(d*x^2+c)*x^3*erf(b*x)/d+3/4*Defer(Int)(ex
p(d*x^2+c)*erf(b*x),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2} x^4 \operatorname{erf}(bx)\,dx = \int e^{c+dx^2} x^4 \operatorname{erf}(bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^4*Erf[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^4*Erf[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erf}(bx) e^{c+dx^2} \, dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{b\int e^{c-\left(b^2-d\right)x^2}x^3dx}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(bx)dx}{2d}+\frac{x^3\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{b\left(\frac{\int e^{c-\left(b^2-d\right)x^2}xdx}{b^2-d}-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(bx)dx}{2d}+\frac{x^3\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3\int e^{dx^2+c}x^2\text{erf}(bx)dx}{2d}-\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6939}$$

$$-\frac{3\left(-\frac{b\int e^{c-\left(b^2-d\right)x^2}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(bx)dx}{2d}+\frac{x\text{erf}(bx)e^{c+dx^2}}{2d}\right)}{2d}-\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\operatorname{erf}(bx)dx}{2d}+\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d(b^2-d)}+\frac{x\operatorname{erf}(bx)e^{c+dx^2}}{2d}\right)}{2d}-\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\operatorname{erf}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \; 6933$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\operatorname{erf}(bx)dx}{2d}+\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d(b^2-d)}+\frac{x\operatorname{erf}(bx)e^{c+dx^2}}{2d}\right)}{2d}-\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\operatorname{erf}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^4*Erf[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \mathrm{e}^{d\,x^2+c} x^4 \operatorname{erf}(bx)\, dx$$

input
```
int(exp(d*x^2+c)*x^4*erf(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^4*erf(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erf}(bx)\,dx=\int x^4\,\text{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erf(b*x),x, algorithm="fricas")
```

output
```
integral(x^4*erf(b*x)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 46.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^4\text{erf}(bx)\,dx=e^c\int x^4e^{dx^2}\,\text{erf}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**4*erf(b*x),x)
```

output
```
exp(c)*Integral(x**4*exp(d*x**2)*erf(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erf}(bx)\,dx=\int x^4\,\text{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erf(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*erf(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erf}(bx)\,dx=\int x^4\,\text{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erf(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erf(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erf}(bx)\,dx=\int x^4\,\mathrm{e}^{d\,x^2+c}\,\text{erf}(b\,x)\,dx$$

input
```
int(x^4*exp(c + d*x^2)*erf(b*x),x)
```

output
```
int(x^4*exp(c + d*x^2)*erf(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int e^{c+dx^2} x^4 \mathrm{erf}(bx)\, dx = e^c \left(\int e^{d\,x^2} \mathrm{erf}(bx)\, x^4 dx \right)$$

input
```
int(exp(d*x^2+c)*x^4*erf(b*x),x)
```

output
```
e**c*int(e**(d*x**2)*erf(b*x)*x**4,x)
```

3.60 $\int e^{c+dx^2} x^2 \mathbf{erf}(bx)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{c+dx^2} x^2 \operatorname{erf}(bx)\, dx = \frac{b e^{c-(b^2-d)x^2}}{2(b^2-d)\, d\sqrt{\pi}} + \frac{e^{c+dx^2} x \operatorname{erf}(bx)}{2d} - \frac{\operatorname{Int}\left(e^{c+dx^2} \operatorname{erf}(bx), x\right)}{2d}$$

output
```
1/2*b*exp(c-(b^2-d)*x^2)/(b^2-d)/d/Pi^(1/2)+1/2*exp(d*x^2+c)*x*erf(b*x)/d-
1/2*Defer(Int)(exp(d*x^2+c)*erf(b*x),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2} x^2 \operatorname{erf}(bx)\, dx = \int e^{c+dx^2} x^2 \operatorname{erf}(bx)\, dx$$

input
```
Integrate[E^(c + d*x^2)*x^2*Erf[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^2*Erf[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erf}(bx) e^{c+dx^2} \, dx$$

$$\Big\downarrow \ 6939$$

$$-\frac{b \int e^{c-\left(b^2-d\right)x^2} x dx}{\sqrt{\pi} d} - \frac{\int e^{dx^2+c} \text{erf}(bx) dx}{2d} + \frac{x \text{erf}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \ 2638$$

$$-\frac{\int e^{dx^2+c} \text{erf}(bx) dx}{2d} + \frac{b e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi} d \left(b^2-d\right)} + \frac{x \text{erf}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \ 6933$$

$$-\frac{\int e^{dx^2+c} \text{erf}(bx) dx}{2d} + \frac{b e^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi} d \left(b^2-d\right)} + \frac{x \text{erf}(bx) e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^2*Erf[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{d\,x^2+c} x^2 \,\text{erf}\,(bx)\, dx$$

input
```
int(exp(d*x^2+c)*x^2*erf(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^2*erf(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\mathrm{erf}(bx)\,dx = \int x^2\,\mathrm{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erf(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*erf(b*x)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 8.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^2\mathrm{erf}(bx)\,dx = e^c\int x^2e^{dx^2}\,\mathrm{erf}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**2*erf(b*x),x)
```

output
```
exp(c)*Integral(x**2*exp(d*x**2)*erf(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erf}(bx)\,dx = \int x^2\,\text{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erf(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*erf(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erf}(bx)\,dx = \int x^2\,\text{erf}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erf(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erf(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erf}(bx)\,dx = \int x^2\,\mathrm{e}^{d\,x^2+c}\,\text{erf}(b\,x)\,dx$$

input
```
int(x^2*exp(c + d*x^2)*erf(b*x),x)
```

output
```
int(x^2*exp(c + d*x^2)*erf(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int e^{c+dx^2}x^2\text{erf}(bx)\,dx = e^c\left(\int e^{d\,x^2}\text{erf}(bx)\,x^2dx\right)$$

input
```
int(exp(d*x^2+c)*x^2*erf(b*x),x)
```

output
```
e**c*int(e**(d*x**2)*erf(b*x)*x**2,x)
```

3.61 $\int e^{c+dx^2}\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int e^{c+dx^2}\operatorname{erf}(bx)\,dx=\operatorname{Int}\left(e^{c+dx^2}\operatorname{erf}(bx),x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erf(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{c+dx^2}\operatorname{erf}(bx)\,dx=\int e^{c+dx^2}\operatorname{erf}(bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*Erf[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*Erf[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 6933}$$

$$\int \text{erf}(bx)e^{c+dx^2}dx$$

input
```
Int[E^(c + d*x^2)*Erf[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \mathrm{e}^{d\,x^2+c}\,\text{erf}\,(bx)\,dx$$

input
```
int(exp(d*x^2+c)*erf(b*x),x)
```

output
```
int(exp(d*x^2+c)*erf(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erf}(bx)\,dx=\int \operatorname{erf}\left(bx\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x),x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erf}(bx)\,dx=e^c\int e^{dx^2}\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x),x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erf}(bx)\,dx=\int \operatorname{erf}\left(bx\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x),x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\text{erf}(bx)\,dx=\int \text{erf}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x),x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\text{erf}(bx)\,dx=\int \mathrm{e}^{d\,x^2+c}\,\text{erf}(b\,x)\,dx$$

input
```
int(exp(c + d*x^2)*erf(b*x),x)
```

output
```
int(exp(c + d*x^2)*erf(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int e^{c+dx^2}\operatorname{erf}(bx)\,dx = e^c\left(\int e^{d\,x^2}\operatorname{erf}(bx)\,dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x),x)
```

output
```
e**c*int(e**(d*x**2)*erf(b*x),x)
```

3.62 $\int \frac{e^{c+dx^2}\mathbf{erf}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx = -\frac{e^{c+dx^2}\operatorname{erf}(bx)}{x} + \frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}} + 2d\text{Int}\left(e^{c+dx^2}\operatorname{erf}(bx),x\right)$$

output
```
-exp(d*x^2+c)*erf(b*x)/x+b*exp(c)*Ei(-(b^2-d)*x^2)/Pi^(1/2)+2*d*Defer(Int)
(exp(d*x^2+c)*erf(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^2,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)e^{c+dx^2}}{x^2} \, dx$$
$$\Big\downarrow \ 6945$$
$$\frac{2b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}dx}{\sqrt{\pi}} + 2d\int e^{dx^2+c}\text{erf}(bx)dx - \frac{\text{erf}(bx)e^{c+dx^2}}{x}$$
$$\Big\downarrow \ 2639$$
$$2d\int e^{dx^2+c}\text{erf}(bx)dx + \frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)e^{c+dx^2}}{x}$$
$$\Big\downarrow \ 6933$$
$$2d\int e^{dx^2+c}\text{erf}(bx)dx + \frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}} - \frac{\text{erf}(bx)e^{c+dx^2}}{x}$$

input
```
Int[(E^(c + d*x^2)*Erf[b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^{2}+c}\operatorname{erf}\left(bx\right)}{x^{2}}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^2,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx=\int \frac{\operatorname{erf}\left(bx\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(d*x^2 + c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx=e^{c}\int \frac{e^{dx^2}\operatorname{erf}\left(bx\right)}{x^2}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x)/x**2,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^2}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^2}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\operatorname{erf}(b\,x)}{x^2}\,dx$$

input
```
int((exp(c + d*x^2)*erf(b*x))/x^2,x)
```

output
```
int((exp(c + d*x^2)*erf(b*x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^2}\,dx = e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx)}{x^2}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^2,x)
```

output
```
e**c*int((e**(d*x**2)*erf(b*x))/x**2,x)
```

3.63 $\int \frac{e^{c+dx^2}\mathbf{erf}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\mathrm{erf}(bx)}{x^4}\,dx = -\frac{be^{c-(b^2-d)x^2}}{3\sqrt{\pi}x^2} - \frac{e^{c+dx^2}\mathrm{erf}(bx)}{3x^3} - \frac{2de^{c+dx^2}\mathrm{erf}(bx)}{3x} - \frac{b(b^2-d)\,e^c\,\mathrm{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{3\sqrt{\pi}} + \frac{2bde^c\,\mathrm{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{3\sqrt{\pi}} + \frac{4}{3}d^2\mathrm{Int}\left(e^{c+dx^2}\mathrm{erf}(bx),x\right)$$

output
```
-1/3*b*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x^2-1/3*exp(d*x^2+c)*erf(b*x)/x^3-2/3*d
*exp(d*x^2+c)*erf(b*x)/x-1/3*b*(b^2-d)*exp(c)*Ei(-(b^2-d)*x^2)/Pi^(1/2)+2/
3*b*d*exp(c)*Ei(-(b^2-d)*x^2)/Pi^(1/2)+4/3*d^2*Defer(Int)(exp(d*x^2+c)*erf
(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^4}\,dx = \int \frac{e^{c+dx^2}\text{erf}(bx)}{x^4}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^4,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[b*x])/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erf}(bx)e^{c+dx^2}}{x^4}\,dx$$

$$\Big\downarrow \text{6945}$$

$$\frac{2b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^3}dx}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(bx)}{x^2}dx-\frac{\text{erf}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2b\left(-\left(\left(b^2-d\right)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}dx\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(bx)}{x^2}dx-\frac{\text{erf}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(bx)}{x^2}dx+\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \ 6945$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erf}(bx)dx-\frac{\text{erf}(bx)e^{c+dx^2}}{x}\right)+\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \ 2639$$

$$\frac{2}{3}d\left(2d\int e^{dx^2+c}\text{erf}(bx)dx+\frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{x}\right)+\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \ 6933$$

$$\frac{2}{3}d\left(2d\int e^{dx^2+c}\text{erf}(bx)dx+\frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{x}\right)+\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erf}(bx)e^{c+dx^2}}{3x^3}$$

input
```
Int[(E^(c + d*x^2)*Erf[b*x])/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erf}(bx)}{x^4}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^4,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^4,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(d*x^2 + c)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 14.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^4}\,dx = e^c \int \frac{e^{dx^2}\operatorname{erf}(bx)}{x^4}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x)/x**4,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^4}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erf}(bx)}{x^4}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(d*x^2 + c)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erf}(b\,x)}{x^4}\,dx$$

input
```
int((exp(c + d*x^2)*erf(b*x))/x^4,x)
```

output
```
int((exp(c + d*x^2)*erf(b*x))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\operatorname{erf}(bx)}{x^4}\,dx = e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx)}{x^4}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x)/x^4,x)
```

output
```
e**c*int((e**(d*x**2)*erf(b*x))/x**4,x)
```

3.64 $\int e^{c+b^2x^2}x^5\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx = -\frac{2e^cx}{b^5\sqrt{\pi}}+\frac{2e^cx^3}{3b^3\sqrt{\pi}}-\frac{e^cx^5}{5b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erf}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erf}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erf}(bx)}{2b^2}$$

output
```
-2*exp(c)*x/b^5/Pi^(1/2)+2/3*exp(c)*x^3/b^3/Pi^(1/2)-1/5*exp(c)*x^5/b/Pi^(
1/2)+exp(b^2*x^2+c)*erf(b*x)/b^6-exp(b^2*x^2+c)*x^2*erf(b*x)/b^4+1/2*exp(b
^2*x^2+c)*x^4*erf(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx = \frac{e^c\left(-60bx+20b^3x^3-6b^5x^5+15e^{b^2x^2}\sqrt{\pi}(2-2b^2x^2+b^4x^4)\,\text{erf}(bx)\right)}{30b^6\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^5*Erf[b*x],x]
```

output
```
(E^c*(-60*b*x + 20*b^3*x^3 - 6*b^5*x^5 + 15*E^(b^2*x^2)*Sqrt[Pi]*(2 - 2*b^
2*x^2 + b^4*x^4)*Erf[b*x]))/(30*b^6*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6939, 15, 6939, 15, 6936, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{b^2x^2+c}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erf}(bx)dx}{b^2}-\frac{\int e^cx^4dx}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2+c}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erf}(bx)dx}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{6939}$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\text{erf}(bx)dx}{b^2}-\frac{\int e^cx^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\text{erf}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^3}{3\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{6936}$$

$$-\frac{2\left(-\frac{\frac{e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{\int e^cdx}{\sqrt{\pi}b}}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^3}{3\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^4 e^{b^2x^2+c}\operatorname{erf}(bx)}{2b^2} - \frac{2\left(\frac{x^2 e^{b^2x^2+c}\operatorname{erf}(bx)}{2b^2} - \frac{\frac{e^{b^2x^2+c}\operatorname{erf}(bx)}{2b^2} - \frac{e^c x}{\sqrt{\pi}b}}{b^2} - \frac{e^c x^3}{3\sqrt{\pi}b}\right)}{b^2} - \frac{e^c x^5}{5\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^5*Erf[b*x],x]
```

output
```
-1/5*(E^c*x^5)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^4*Erf[b*x])/(2*b^2) - (2*
(-1/3*(E^c*x^3)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^2*Erf[b*x])/(2*b^2) - (-
((E^c*x)/(b*Sqrt[Pi])) + (E^(c + b^2*x^2)*Erf[b*x])/(2*b^2))/b^2))/b^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

method	result
default	$\frac{\frac{\operatorname{erf}(bx)\,\mathrm{e}^{c}\left(\frac{\mathrm{e}^{b^2x^2}b^4x^4}{2}-b^2x^2\mathrm{e}^{b^2x^2}+\mathrm{e}^{b^2x^2}\right)}{b^5}-\frac{\mathrm{e}^{c}\left(\frac{1}{5}b^5x^5-\frac{2}{3}b^3x^3+2bx\right)}{\sqrt{\pi}\,b^5}}{b}$
orering	$\frac{(3b^6x^6+5b^4x^4-10b^2x^2+90)\mathrm{e}^{b^2x^2+c}\operatorname{erf}(bx)}{15b^6}-\frac{(3b^4x^4-10b^2x^2+30)\left(2b^2x^6\mathrm{e}^{b^2x^2+c}\operatorname{erf}(bx)+5\,\mathrm{e}^{b^2x^2+c}x^4\operatorname{erf}(bx)+\frac{2\,\mathrm{e}^{b^2x^2}}{}\right.}{30x^4b^6}$
parallelrisch	$\frac{-6\,\mathrm{e}^{b^2x^2+c}\mathrm{e}^{-b^2x^2}x^5b^5+15\,\mathrm{e}^{b^2x^2+c}x^4\operatorname{erf}(bx)b^4\sqrt{\pi}+20\,\mathrm{e}^{b^2x^2+c}\mathrm{e}^{-b^2x^2}x^3b^3-30\,\mathrm{e}^{b^2x^2+c}x^2\operatorname{erf}(bx)b^2\sqrt{\pi}-60\,\mathrm{e}^{b^2x^2+c}x\,\mathrm{e}^{-b^2x}}{30b^6\sqrt{\pi}}$

input
```
int(exp(b^2*x^2+c)*x^5*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
(erf(b*x)/b^5*exp(c)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2*exp(b^2*x^2)+exp(b^
2*x^2))-1/Pi^(1/2)/b^5*exp(c)*(1/5*b^5*x^5-2/3*b^3*x^3+2*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

$$\int e^{c+b^2x^2}x^5\operatorname{erf}(bx)\,dx = \frac{15\,(2\,\pi+\pi b^4x^4-2\,\pi b^2x^2)\operatorname{erf}(bx)\,e^{(b^2x^2+c)}-2\,\sqrt{\pi}(3\,b^5x^5-10\,b^3x^3+30\,bx)e^c}{30\,\pi b^6}$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erf(b*x),x, algorithm="fricas")
```

output
```
1/30*(15*(2*pi + pi*b^4*x^4 - 2*pi*b^2*x^2)*erf(b*x)*e^(b^2*x^2 + c) - 2*s
qrt(pi)*(3*b^5*x^5 - 10*b^3*x^3 + 30*b*x)*e^c)/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 35.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx$$
$$=\begin{cases} -\frac{x^5e^c}{5\sqrt{\pi}b}+\frac{x^4e^ce^{b^2x^2}\,\text{erf}\,(bx)}{2b^2}+\frac{2x^3e^c}{3\sqrt{\pi}b^3}-\frac{x^2e^ce^{b^2x^2}\,\text{erf}\,(bx)}{b^4}-\frac{2xe^c}{\sqrt{\pi}b^5}+\frac{e^ce^{b^2x^2}\,\text{erf}\,(bx)}{b^6} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x**5*erf(b*x),x)
```

output
```
Piecewise((-x**5*exp(c)/(5*sqrt(pi)*b) + x**4*exp(c)*exp(b**2*x**2)*erf(b*
x)/(2*b**2) + 2*x**3*exp(c)/(3*sqrt(pi)*b**3) - x**2*exp(c)*exp(b**2*x**2)
*erf(b*x)/b**4 - 2*x*exp(c)/(sqrt(pi)*b**5) + exp(c)*exp(b**2*x**2)*erf(b*
x)/b**6, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx =$$
$$-\frac{6\,b^5x^5e^c-20\,b^3x^3e^c-15\left(\sqrt{\pi}b^4x^4e^c-2\,\sqrt{\pi}b^2x^2e^c+2\,\sqrt{\pi}e^c\right)\text{erf}\,(bx)\,e^{\left(b^2x^2\right)}+60\,bxe^c}{30\,\sqrt{\pi}b^6}$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erf(b*x),x, algorithm="maxima")
```

output
```
-1/30*(6*b^5*x^5*e^c - 20*b^3*x^3*e^c - 15*(sqrt(pi)*b^4*x^4*e^c - 2*sqrt(
pi)*b^2*x^2*e^c + 2*sqrt(pi)*e^c)*erf(b*x)*e^(b^2*x^2) + 60*b*x*e^c)/(sqrt
(pi)*b^6)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx = \frac{1}{2}\left(\frac{c^2e^{(b^2x^2+c)}}{b^6} - \frac{\left(2\,b^2x^2-(b^2x^2+c)^2+2\,(b^2x^2+c)c-2\right)e^{(b^2x^2+c)}}{b^6}\right)\text{erf}\,(bx) - \frac{3\,\sqrt{\pi}b^4x^5e^c-10\,\sqrt{\pi}b^2x^3e^c+30\,\sqrt{\pi}xe^c}{15\,\pi b^5}$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erf(b*x),x, algorithm="giac")
```

output
```
1/2*(c^2*e^(b^2*x^2 + c)/b^6 - (2*b^2*x^2 - (b^2*x^2 + c)^2 + 2*(b^2*x^2 +
 c)*c - 2)*e^(b^2*x^2 + c)/b^6)*erf(b*x) - 1/15*(3*sqrt(pi)*b^4*x^5*e^c -
10*sqrt(pi)*b^2*x^3*e^c + 30*sqrt(pi)*x*e^c)/(pi*b^5)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx = \text{erf}(b\,x)\left(\frac{\mathrm{e}^{b^2\,x^2+c}}{b^6}+\frac{x^4\,\mathrm{e}^{b^2\,x^2+c}}{2\,b^2}-\frac{x^2\,\mathrm{e}^{b^2\,x^2+c}}{b^4}\right) - \frac{3\,\mathrm{e}^c\,b^4\,x^5-10\,\mathrm{e}^c\,b^2\,x^3+30\,\mathrm{e}^c\,x}{15\,b^5\,\sqrt{\pi}}$$

input
```
int(x^5*exp(c + b^2*x^2)*erf(b*x),x)
```

output
```
erf(b*x)*(exp(c + b^2*x^2)/b^6 + (x^4*exp(c + b^2*x^2))/(2*b^2) - (x^2*exp
(c + b^2*x^2))/b^4) - (30*x*exp(c) - 10*b^2*x^3*exp(c) + 3*b^4*x^5*exp(c))
/(15*b^5*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int e^{c+b^2x^2}x^5\text{erf}(bx)\,dx$$
$$=\frac{e^c\left(15e^{b^2x^2}\text{erf}(bx)\,b^4\pi\,x^4-30e^{b^2x^2}\text{erf}(bx)\,b^2\pi\,x^2+30e^{b^2x^2}\text{erf}(bx)\,\pi-6\sqrt{\pi}\,b^5x^5+20\sqrt{\pi}\,b^3x^3-60\sqrt{\pi}\,bx\right)}{30b^6\pi}$$

input
```
int(exp(b^2*x^2+c)*x^5*erf(b*x),x)
```

output
```
(e**c*(15*e**(b**2*x**2)*erf(b*x)*b**4*pi*x**4 - 30*e**(b**2*x**2)*erf(b*x
)*b**2*pi*x**2 + 30*e**(b**2*x**2)*erf(b*x)*pi - 6*sqrt(pi)*b**5*x**5 + 20
*sqrt(pi)*b**3*x**3 - 60*sqrt(pi)*b*x))/(30*b**6*pi)
```

3.65 $\int e^{c+b^2x^2}x^3\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx=\frac{e^cx}{b^3\sqrt{\pi}}-\frac{e^cx^3}{3b\sqrt{\pi}}-\frac{e^{c+b^2x^2}\text{erf}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erf}(bx)}{2b^2}$$

output
```
exp(c)*x/b^3/Pi^(1/2)-1/3*exp(c)*x^3/b/Pi^(1/2)-1/2*exp(b^2*x^2+c)*erf(b*x
)/b^4+1/2*exp(b^2*x^2+c)*x^2*erf(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx=\frac{e^c\left(6bx-2b^3x^3+3e^{b^2x^2}\sqrt{\pi}(-1+b^2x^2)\,\text{erf}(bx)\right)}{6b^4\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^3*Erf[b*x],x]
```

output
```
(E^c*(6*b*x - 2*b^3*x^3 + 3*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erf[b*x]))
/(6*b^4*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6939, 15, 6936, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{b^2x^2+c}\text{erf}(bx)\,dx$$

$$\Big\downarrow \textit{6939}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erf}(bx)dx}{b^2}-\frac{\int e^cx^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \textit{15}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erf}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^3}{3\sqrt{\pi}b}$$

$$\Big\downarrow \textit{6936}$$

$$-\frac{\frac{e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{\int e^cdx}{\sqrt{\pi}b}}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^3}{3\sqrt{\pi}b}$$

$$\Big\downarrow \textit{24}$$

$$\frac{x^2e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{\frac{e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx}{\sqrt{\pi}b}}{b^2}-\frac{e^cx^3}{3\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^3*Erf[b*x],x]
```

output
```
-1/3*(E^c*x^3)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^2*Erf[b*x])/(2*b^2) - (-(
(E^c*x)/(b*Sqrt[Pi])) + (E^(c + b^2*x^2)*Erf[b*x])/(2*b^2))/b^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{\operatorname{erf}(bx)\mathrm{e}^{c}\left(\frac{b^{2}x^{2}\mathrm{e}^{b^{2}x^{2}}}{2}-\frac{\mathrm{e}^{b^{2}x^{2}}}{2}\right)}{b^{3}}-\frac{\mathrm{e}^{c}\left(\frac{1}{3}b^{3}x^{3}-bx\right)}{\sqrt{\pi}\,b^{3}}}{b}$	66
parallelrisch	$\frac{-2\,\mathrm{e}^{b^{2}x^{2}+c}\mathrm{e}^{-b^{2}x^{2}}x^{3}b^{3}+3\,\mathrm{e}^{b^{2}x^{2}+c}x^{2}\operatorname{erf}(bx)b^{2}\sqrt{\pi}+6\,\mathrm{e}^{b^{2}x^{2}+c}x\,\mathrm{e}^{-b^{2}x^{2}}b-3\operatorname{erf}(bx)\mathrm{e}^{b^{2}x^{2}+c}\sqrt{\pi}}{6\sqrt{\pi}\,b^{4}}$	104
orering	$\frac{(b^{4}x^{4}-6)\mathrm{e}^{b^{2}x^{2}+c}\operatorname{erf}(bx)}{3b^{4}}-\frac{(b^{2}x^{2}-3)\left(2b^{2}x^{4}\mathrm{e}^{b^{2}x^{2}+c}\operatorname{erf}(bx)+3\,\mathrm{e}^{b^{2}x^{2}+c}x^{2}\operatorname{erf}(bx)+\frac{2\,\mathrm{e}^{b^{2}x^{2}+c}x^{3}\mathrm{e}^{-b^{2}x^{2}}b}{\sqrt{\pi}}\right)}{6x^{2}b^{4}}$	117

input
```
int(exp(b^2*x^2+c)*x^3*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
(erf(b*x)/b^3*exp(c)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp(b^2*x^2))-1/Pi^(1/2
)/b^3*exp(c)*(1/3*b^3*x^3-b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx = -\frac{3\,(\pi-\pi b^2x^2)\,\text{erf}\,(bx)\,e^{(b^2x^2+c)}+2\,\sqrt{\pi}(b^3x^3-3\,bx)e^c}{6\,\pi b^4}$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erf(b*x),x, algorithm="fricas")
```

output
```
-1/6*(3*(pi - pi*b^2*x^2)*erf(b*x)*e^(b^2*x^2 + c) + 2*sqrt(pi)*(b^3*x^3 -
 3*b*x)*e^c)/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx = \begin{cases} -\frac{x^3e^c}{3\sqrt{\pi}b}+\frac{x^2e^ce^{b^2x^2}\,\text{erf}\,(bx)}{2b^2}+\frac{xe^c}{\sqrt{\pi}b^3}-\frac{e^ce^{b^2x^2}\,\text{erf}\,(bx)}{2b^4} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x**3*erf(b*x),x)
```

output
```
Piecewise((-x**3*exp(c)/(3*sqrt(pi)*b) + x**2*exp(c)*exp(b**2*x**2)*erf(b*
x)/(2*b**2) + x*exp(c)/(sqrt(pi)*b**3) - exp(c)*exp(b**2*x**2)*erf(b*x)/(2
*b**4), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx = -\frac{2\,b^3x^3e^c-3\left(\sqrt{\pi}b^2x^2e^c-\sqrt{\pi}e^c\right)\text{erf}\,(bx)\,e^{(b^2x^2)}-6\,bxe^c}{6\,\sqrt{\pi}b^4}$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erf(b*x),x, algorithm="maxima")
```

output
```
-1/6*(2*b^3*x^3*e^c - 3*(sqrt(pi)*b^2*x^2*e^c - sqrt(pi)*e^c)*erf(b*x)*e^(
b^2*x^2) - 6*b*x*e^c)/(sqrt(pi)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx = \frac{1}{2}\left(\frac{(b^2x^2+c-1)e^{(b^2x^2+c)}}{b^4} - \frac{ce^{(b^2x^2+c)}}{b^4}\right)\text{erf}\,(bx) - \frac{b^2x^3e^c - 3\,xe^c}{3\,\sqrt{\pi}b^3}$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erf(b*x),x, algorithm="giac")
```

output
```
1/2*((b^2*x^2 + c - 1)*e^(b^2*x^2 + c)/b^4 - c*e^(b^2*x^2 + c)/b^4)*erf(b*
x) - 1/3*(b^2*x^3*e^c - 3*x*e^c)/(sqrt(pi)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{c+b^2x^2}x^3\text{erf}(bx)\,dx = \frac{3\,x\,\mathrm{e}^c - b^2\,x^3\,\mathrm{e}^c}{3\,b^3\,\sqrt{\pi}} - \text{erf}(b\,x)\left(\frac{\mathrm{e}^{b^2\,x^2+c}}{2\,b^4} - \frac{x^2\,\mathrm{e}^{b^2\,x^2+c}}{2\,b^2}\right)$$

input
```
int(x^3*exp(c + b^2*x^2)*erf(b*x),x)
```

output
```
(3*x*exp(c) - b^2*x^3*exp(c))/(3*b^3*pi^(1/2)) - erf(b*x)*(exp(c + b^2*x^2
)/(2*b^4) - (x^2*exp(c + b^2*x^2))/(2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int e^{c+b^2x^2} x^3 \operatorname{erf}(bx)\, dx = \frac{e^c \left(3e^{b^2x^2} \operatorname{erf}(bx)\, b^2\pi\, x^2 - 3e^{b^2x^2} \operatorname{erf}(bx)\, \pi - 2\sqrt{\pi}\, b^3x^3 + 6\sqrt{\pi}\, bx\right)}{6b^4\pi}$$

input
```
int(exp(b^2*x^2+c)*x^3*erf(b*x),x)
```

output
```
(e**c*(3*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 - 3*e**(b**2*x**2)*erf(b*x)*
pi - 2*sqrt(pi)*b**3*x**3 + 6*sqrt(pi)*b*x))/(6*b**4*pi)
```

3.66 $\int e^{c+b^2x^2}x\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int e^{c+b^2x^2}x\text{erf}(bx)\,dx=-\frac{e^cx}{b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erf}(bx)}{2b^2}$$

output
```
-exp(c)*x/b/Pi^(1/2)+1/2*exp(b^2*x^2+c)*erf(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int e^{c+b^2x^2}x\text{erf}(bx)\,dx=\frac{e^c\left(-\frac{2bx}{\sqrt{\pi}}+e^{b^2x^2}\text{erf}(bx)\right)}{2b^2}$$

input
```
Integrate[E^(c + b^2*x^2)*x*Erf[b*x],x]
```

output
```
(E^c*((-2*b*x)/Sqrt[Pi] + E^(b^2*x^2)*Erf[b*x]))/(2*b^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6936, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{b^2x^2+c}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6936}$$

$$\frac{e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{\int e^c dx}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{24}$$

$$\frac{e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^c x}{\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x*Erf[b*x],x]
```

output
```
-((E^c*x)/(b*Sqrt[Pi])) + (E^(c + b^2*x^2)*Erf[b*x])/(2*b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{-2\,\mathrm{e}^{b^2x^2+c}x\,\mathrm{e}^{-b^2x^2}b+\operatorname{erf}(bx)\mathrm{e}^{b^2x^2+c}\sqrt{\pi}}{2\sqrt{\pi}\,b^2}$	51
parallelrisch	$\frac{-2\,\mathrm{e}^{b^2x^2+c}x\,\mathrm{e}^{-b^2x^2}b+\operatorname{erf}(bx)\mathrm{e}^{b^2x^2+c}\sqrt{\pi}}{2\sqrt{\pi}\,b^2}$	51
orering	$\frac{(b^2x^2+1)\mathrm{e}^{b^2x^2+c}\operatorname{erf}(bx)}{b^2}-\frac{2b^2x^2\mathrm{e}^{b^2x^2+c}\operatorname{erf}(bx)+\mathrm{e}^{b^2x^2+c}\operatorname{erf}(bx)+\frac{2\,\mathrm{e}^{b^2x^2+c}x\,\mathrm{e}^{-b^2x^2}b}{\sqrt{\pi}}}{2b^2}$	98

input
```
int(exp(b^2*x^2+c)*x*erf(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*exp(b^2*x^2+c)*x*exp(-b^2*x^2)*b+erf(b*x)*exp(b^2*x^2+c)*Pi^(1/2))
/Pi^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^{c+b^2x^2}x\mathrm{erf}(bx)\,dx=-\frac{2\,\sqrt{\pi}bxe^c-\pi\,\mathrm{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{2\,\pi b^2}$$

input
```
integrate(exp(b^2*x^2+c)*x*erf(b*x),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(pi)*b*x*e^c - pi*erf(b*x)*e^(b^2*x^2 + c))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int e^{c+b^2x^2} x\text{erf}(bx)\,dx = \begin{cases} -\frac{xe^c}{\sqrt{\pi}b} + \frac{e^c e^{b^2x^2}\text{erf}\,(bx)}{2b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x*erf(b*x),x)
```

output
```
Piecewise((-x*exp(c)/(sqrt(pi)*b) + exp(c)*exp(b**2*x**2)*erf(b*x)/(2*b**2
), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int e^{c+b^2x^2} x\text{erf}(bx)\,dx = -\frac{2\,bxe^c - \sqrt{\pi}\,\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{2\,\sqrt{\pi}b^2}$$

input
```
integrate(exp(b^2*x^2+c)*x*erf(b*x),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*e^c - sqrt(pi)*erf(b*x)*e^(b^2*x^2 + c))/(sqrt(pi)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int e^{c+b^2x^2} x\text{erf}(bx)\,dx = -\frac{xe^c}{\sqrt{\pi}b} + \frac{\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}}{2\,b^2}$$

input
```
integrate(exp(b^2*x^2+c)*x*erf(b*x),x, algorithm="giac")
```

output
```
-x*e^c/(sqrt(pi)*b) + 1/2*erf(b*x)*e^(b^2*x^2 + c)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int e^{c+b^2x^2}x\text{erf}(bx)\,dx=\frac{e^{b^2\,x^2}\,e^c\,\text{erf}(b\,x)}{2\,b^2}-\frac{x\,e^c}{b\,\sqrt{\pi}}$$

input
```
int(x*exp(c + b^2*x^2)*erf(b*x),x)
```

output
```
(exp(b^2*x^2)*exp(c)*erf(b*x))/(2*b^2) - (x*exp(c))/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int e^{c+b^2x^2}x\text{erf}(bx)\,dx=\frac{e^c\left(e^{b^2x^2}\text{erf}(bx)\,\pi-2\sqrt{\pi}\,bx\right)}{2b^2\pi}$$

input
```
int(exp(b^2*x^2+c)*x*erf(b*x),x)
```

output
```
(e**c*(e**(b**2*x**2)*erf(b*x)*pi - 2*sqrt(pi)*b*x))/(2*b**2*pi)
```

3.67 $\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
2*b*exp(c)*x*hypergeom([1/2, 1],[3/2, 3/2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x,x]
```

output
```
(2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}\,dx$$

$$\Big\downarrow \text{ 6942}$$

$$\frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[(E^(c + b^2*x^2)*Erf[b*x])/x,x]
```

output
```
(2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 6942
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b*
E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}\,\text{erf}\left(bx\right)}{x}dx$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x,x)
```

output
```
int(exp(b^2*x^2+c)*erf(b*x)/x,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(b^2*x^2 + c)/x, x)
```

Sympy [A] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x}\,dx = \frac{2bxe^c{}_2F_2\left(\begin{matrix}\frac{1}{2},1\\ \frac{3}{2},\frac{3}{2}\end{matrix}\middle| b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(exp(b**2*x**2+c)*erf(b*x)/x,x)
```

output
```
2*b*x*exp(c)*hyper((1/2, 1), (3/2, 3/2), b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{e^{b^2\,x^2+c}\operatorname{erf}(b\,x)}{x}\,dx$$

input
```
int((exp(c + b^2*x^2)*erf(b*x))/x,x)
```

output
```
int((exp(c + b^2*x^2)*erf(b*x))/x, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x}\,dx = e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x}dx\right)$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x,x)
```

output
```
e**c*int((e**(b**2*x**2)*erf(b*x))/x,x)
```

3.68 $\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^3}\,dx = -\frac{be^c}{\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\text{erf}(bx)}{2x^2} + \frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-b*exp(c)/Pi^(1/2)/x-1/2*exp(b^2*x^2+c)*erf(b*x)/x^2+2*b^3*exp(c)*x*hyperg
eom([1/2, 1],[3/2, 3/2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^3}\,dx = -\frac{2be^c\,{}_2F_2\left(-\frac{1}{2},1;\frac{1}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}x}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^3,x]
```

output
```
(-2*b*E^c*HypergeometricPFQ[{-1/2, 1}, {1/2, 3/2}, b^2*x^2])/(Sqrt[Pi]*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6945, 15, 6942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^3}\,dx$$
$$\Big\downarrow \text{6945}$$
$$b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx+\frac{b\int \frac{e^c}{x^2}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{2x^2}$$
$$\Big\downarrow \text{15}$$
$$b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx-\frac{e^{b^2x^2+c}\text{erf}(bx)}{2x^2}-\frac{be^c}{\sqrt{\pi}x}$$
$$\Big\downarrow \text{6942}$$
$$\frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{2x^2}-\frac{be^c}{\sqrt{\pi}x}$$

input
```
Int[(E^(c + b^2*x^2)*Erf[b*x])/x^3,x]
```

output
```
-((b*E^c)/(Sqrt[Pi]*x)) - (E^(c + b^2*x^2)*Erf[b*x])/(2*x^2) + (2*b^3*E^c*
x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6942
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b*
E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}\operatorname{erf}(bx)}{x^3}dx$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^3,x)
```

output
```
int(exp(b^2*x^2+c)*erf(b*x)/x^3,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Sympy [A] (verification not implemented)

Time = 28.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = -\frac{2be^c{}_2F_2\left(\begin{matrix} -\frac{1}{2},1 \\ \frac{1}{2},\frac{3}{2} \end{matrix}\middle| b^2x^2\right)}{\sqrt{\pi}x}$$

input
```
integrate(exp(b**2*x**2+c)*erf(b*x)/x**3,x)
```

output
```
-2*b*exp(c)*hyper((-1/2, 1), (1/2, 3/2), b**2*x**2)/(sqrt(pi)*x)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\operatorname{erf}(b\,x)}{x^3}\,dx$$

input
```
int((exp(c + b^2*x^2)*erf(b*x))/x^3,x)
```

output
```
int((exp(c + b^2*x^2)*erf(b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x^3}dx\right)$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^3,x)
```

output
```
e**c*int((e**(b**2*x**2)*erf(b*x))/x**3,x)
```

3.69 $\int \frac{e^{c+b^2x^2}\mathrm{erf}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{e^{c+b^2x^2}\mathrm{erf}(bx)}{x^5}\,dx = -\frac{be^c}{6\sqrt{\pi}x^3} - \frac{b^3e^c}{2\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\mathrm{erf}(bx)}{4x^4} - \frac{b^2e^{c+b^2x^2}\mathrm{erf}(bx)}{4x^2} + \frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-1/6*b*exp(c)/Pi^(1/2)/x^3-1/2*b^3*exp(c)/Pi^(1/2)/x-1/4*exp(b^2*x^2+c)*er
f(b*x)/x^4-1/4*b^2*exp(b^2*x^2+c)*erf(b*x)/x^2+b^5*exp(c)*x*hypergeom([1/2
, 1],[3/2, 3/2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

$$\int \frac{e^{c+b^2x^2}\mathrm{erf}(bx)}{x^5}\,dx = -\frac{2be^c\,{}_2F_2\left(-\frac{3}{2},1;-\frac{1}{2},\frac{3}{2};b^2x^2\right)}{3\sqrt{\pi}x^3}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^5,x]
```

output
```
(-2*b*E^c*HypergeometricPFQ[{-3/2, 1}, {-1/2, 3/2}, b^2*x^2])/(3*Sqrt[Pi]*
x^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6945, 15, 6945, 15, 6942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^5}dx$$

$$\Big\downarrow \text{ 6945}$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^3}dx+\frac{b\int \frac{e^c}{x^4}dx}{2\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^3}dx-\frac{e^{b^2x^2+c}\text{erf}(bx)}{4x^4}-\frac{be^c}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{ 6945}$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx+\frac{b\int \frac{e^c}{x^2}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{2x^2}\right)-\frac{e^{b^2x^2+c}\text{erf}(bx)}{4x^4}-\frac{be^c}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx-\frac{e^{b^2x^2+c}\text{erf}(bx)}{2x^2}-\frac{be^c}{\sqrt{\pi}x}\right)-\frac{e^{b^2x^2+c}\text{erf}(bx)}{4x^4}-\frac{be^c}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{ 6942}$$

$$\frac{1}{2}b^2\left(\frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{2x^2}-\frac{be^c}{\sqrt{\pi}x}\right)-\frac{e^{b^2x^2+c}\text{erf}(bx)}{4x^4}-\frac{be^c}{6\sqrt{\pi}x^3}$$

input
```
Int[(E^(c + b^2*x^2)*Erf[b*x])/x^5,x]
```

output
```
-1/6*(b*E^c)/(Sqrt[Pi]*x^3) - (E^(c + b^2*x^2)*Erf[b*x])/(4*x^4) + (b^2*(-
((b*E^c)/(Sqrt[Pi]*x)) - (E^(c + b^2*x^2)*Erf[b*x])/(2*x^2) + (2*b^3*E^c*x
*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6942
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b*
E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
 + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}\,\text{erf}\,(bx)}{x^5}dx$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^5,x)
```

output
```
int(exp(b^2*x^2+c)*erf(b*x)/x^5,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(exp(b**2*x**2+c)*erf(b*x)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^5}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^5}\,dx = \int \frac{e^{b^2x^2+c}\operatorname{erf}(b\,x)}{x^5}\,dx$$

input
```
int((exp(c + b^2*x^2)*erf(b*x))/x^5,x)
```

output
```
int((exp(c + b^2*x^2)*erf(b*x))/x^5, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^5}\,dx = \frac{e^c\left(-3e^{b^2x^2}\operatorname{erf}(bx)\,b^2\pi\,x^2 - 3e^{b^2x^2}\operatorname{erf}(bx)\,\pi - 6\sqrt{\pi}\,b^3x^3 - 2\sqrt{\pi}\,bx + 6\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x}dx\right)b^4\pi\,x^4\right)}{12\pi\,x^4}$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^5,x)
```

output
```
(e**c*( - 3*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 - 3*e**(b**2*x**2)*erf(b*
x)*pi - 6*sqrt(pi)*b**3*x**3 - 2*sqrt(pi)*b*x + 6*int((e**(b**2*x**2)*erf(
b*x))/x,x)*b**4*pi*x**4))/(12*pi*x**4)
```

3.70 $\int e^{c+b^2x^2}x^4\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx=\frac{3e^cx^2}{4b^3\sqrt{\pi}}-\frac{e^cx^4}{4b\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\text{erf}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\text{erf}(bx)}{2b^2}+\frac{3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{4b^3\sqrt{\pi}}$$

output
```
3/4*exp(c)*x^2/b^3/Pi^(1/2)-1/4*exp(c)*x^4/b/Pi^(1/2)-3/4*exp(b^2*x^2+c)*x
*erf(b*x)/b^4+1/2*exp(b^2*x^2+c)*x^3*erf(b*x)/b^2+3/4*exp(c)*x^2*hypergeom
([1, 1],[3/2, 2],b^2*x^2)/b^3/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx=\frac{e^c\left(6b^2x^2-2b^4x^4+2be^{b^2x^2}\sqrt{\pi}x(-3+2b^2x^2)\,\text{erf}(bx)+3\pi\text{erf}(bx)\text{erfi}(bx)-6b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)\right)}{8b^5\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^4*Erf[b*x],x]
```

output
```
(E^c*(6*b^2*x^2 - 2*b^4*x^4 + 2*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*b^2*x^2)*
Erf[b*x] + 3*Pi*Erf[b*x]*Erfi[b*x] - 6*b^2*x^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, -(b^2*x^2)]))/(8*b^5*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6939, 15, 6939, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{b^2x^2+c}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{3\int e^{b^2x^2+c}x^2\text{erf}(bx)dx}{2b^2}-\frac{\int e^cx^3dx}{\sqrt{\pi}b}+\frac{x^3e^{b^2x^2+c}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\int e^{b^2x^2+c}x^2\text{erf}(bx)dx}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6939}$$

$$-\frac{3\left(-\frac{\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}-\frac{\int e^cxdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erf}(bx)}{2b^2}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^2}{2\sqrt{\pi}b}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6930}$$

$$-\frac{3\left(-\frac{e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^2}{2\sqrt{\pi}b}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^cx^4}{4\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^4*Erf[b*x],x]
```

output
```
-1/4*(E^c*x^4)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^3*Erf[b*x])/(2*b^2) - (3*
(-1/2*(E^c*x^2)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erf[b*x])/(2*b^2) - (E^c
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*b*Sqrt[Pi])))/(2*b^2
)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{b^2x^2+c}x^4\,\mathrm{erf}\,(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^4*erf(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^4*erf(b*x),x)
```

Fricas [F]

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx = \int x^4\,\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erf(b*x),x, algorithm="fricas")
```

output
```
integral(x^4*erf(b*x)*e^(b^2*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b**2*x**2+c)*x**4*erf(b*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx = \int x^4\,\text{erf}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erf(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*erf(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx = \int x^4\,\text{erf}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erf(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erf(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx = \int x^4\,\mathrm{e}^{b^2\,x^2+c}\,\text{erf}(b\,x)\,dx$$

input
```
int(x^4*exp(c + b^2*x^2)*erf(b*x),x)
```

output
```
int(x^4*exp(c + b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int e^{c+b^2x^2}x^4\text{erf}(bx)\,dx = e^c\left(\int e^{b^2x^2}\text{erf}(bx)\,x^4dx\right)$$

input
```
int(exp(b^2*x^2+c)*x^4*erf(b*x),x)
```

output
```
e**c*int(e**(b**2*x**2)*erf(b*x)*x**4,x)
```

3.71 $\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx=-\frac{e^cx^2}{2b\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\text{erf}(bx)}{2b^2}-\frac{e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2b\sqrt{\pi}}$$

output
```
-1/2*exp(c)*x^2/b/Pi^(1/2)+1/2*exp(b^2*x^2+c)*x*erf(b*x)/b^2-1/2*exp(c)*x^
2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/b/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx=\frac{e^c\left(-2b^2x^2+\text{erf}(bx)\left(2be^{b^2x^2}\sqrt{\pi}x-\pi\text{erfi}(bx)\right)+2b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)\right)}{4b^3\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^2*Erf[b*x],x]
```

output
```
(E^c*(-2*b^2*x^2 + Erf[b*x]*(2*b*E^(b^2*x^2)*Sqrt[Pi]*x - Pi*Erfi[b*x]) +
2*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]))/(4*b^3*Sqrt[Pi
])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6939, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{b^2x^2+c}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}-\frac{\int e^c x dx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^c x^2}{2\sqrt{\pi}b}$$

$$\Big\downarrow \text{6930}$$

$$-\frac{e^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erf}(bx)}{2b^2}-\frac{e^c x^2}{2\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^2*Erf[b*x],x]
```

output
```
-1/2*(E^c*x^2)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erf[b*x])/(2*b^2) - (E^c*
x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(2*b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int \mathrm{e}^{b^2x^2+c}x^2 \operatorname{erf}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^2*erf(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^2*erf(b*x),x)
```

Fricas [F]

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx = \int x^2 \operatorname{erf}(bx)\, e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erf(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*erf(b*x)*e^(b^2*x^2 + c), x)
```

Sympy [A] (verification not implemented)

Time = 27.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx = \frac{bx^4e^c{}_2F_2\left(\begin{matrix}1,2\\ \frac{3}{2},3\end{matrix}\middle|b^2x^2\right)}{2\sqrt{\pi}}$$

input
```
integrate(exp(b**2*x**2+c)*x**2*erf(b*x),x)
```

output
```
b*x**4*exp(c)*hyper((1, 2), (3/2, 3), b**2*x**2)/(2*sqrt(pi))
```

Maxima [F]

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx = \int x^2\,\text{erf}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erf(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*erf(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx = \int x^2\,\text{erf}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erf(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erf(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx = \int x^2\,\mathrm{e}^{b^2\,x^2+c}\,\text{erf}(b\,x)\,dx$$

input
```
int(x^2*exp(c + b^2*x^2)*erf(b*x),x)
```

output
```
int(x^2*exp(c + b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int e^{c+b^2x^2}x^2\text{erf}(bx)\,dx = e^c\left(\int e^{b^2x^2}\text{erf}(bx)\,x^2dx\right)$$

input
```
int(exp(b^2*x^2+c)*x^2*erf(b*x),x)
```

output
```
e**c*int(e**(b**2*x**2)*erf(b*x)*x**2,x)
```

3.72 $\int e^{c+b^2x^2}\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx=\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

output
```
b*exp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [F]

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx=\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx$$

input
```
Integrate[E^(c + b^2*x^2)*Erf[b*x],x]
```

output
```
Integrate[E^(c + b^2*x^2)*Erf[b*x], x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{b^2x^2+c}\text{erf}(bx)\,dx$$
$$\Big\downarrow \text{6930}$$
$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[E^(c + b^2*x^2)*Erf[b*x],x]
```

output
```
(b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int e^{b^2x^2+c}\,\text{erf}\,(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*erf(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*erf(b*x),x)
```

Fricas [F]

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx = \int \operatorname{erf}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x),x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(b^2*x^2 + c), x)
```

Sympy [A] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx = \frac{bx^2e^c{}_2F_2\left(\begin{matrix}1,1\\ \frac{3}{2},2\end{matrix}\middle|b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(exp(b**2*x**2+c)*erf(b*x),x)
```

output
```
b*x**2*exp(c)*hyper((1, 1), (3/2, 2), b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx = \int \operatorname{erf}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x),x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx = \int \operatorname{erf}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x),x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx = \int \mathrm{e}^{b^2\,x^2+c}\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(exp(c + b^2*x^2)*erf(b*x),x)
```

output
```
int(exp(c + b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int e^{c+b^2x^2}\operatorname{erf}(bx)\,dx = e^c\left(\int e^{b^2x^2}\operatorname{erf}(bx)\,dx\right)$$

input
```
int(exp(b^2*x^2+c)*erf(b*x),x)
```

output
```
e**c*int(e**(b**2*x**2)*erf(b*x),x)
```

3.73 $\int \frac{e^{c+b^2x^2}\mathbf{erf}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = -\frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x} + \frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}} + \frac{2be^c\log(x)}{\sqrt{\pi}}$$

output
```
-exp(b^2*x^2+c)*erf(b*x)/x+2*b^3*exp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*
x^2)/Pi^(1/2)+2*b*exp(c)*ln(x)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \frac{e^c\left(\operatorname{erf}(bx)\left(-e^{b^2x^2}\sqrt{\pi}+b\pi x\operatorname{erfi}(bx)\right)-2b^3x^3\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)+2bx\log(x)\right)}{\sqrt{\pi}x}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^2,x]
```

output
```
(E^c*(Erf[b*x]*(-(E^(b^2*x^2)*Sqrt[Pi]) + b*Pi*x*Erfi[b*x]) - 2*b^3*x^3*Hy
pergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)] + 2*b*x*Log[x]))/(Sqrt[Pi]*x
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6945, 14, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^2}dx$$

$$\Big\downarrow \text{6945}$$

$$2b^2\int e^{b^2x^2+c}\text{erf}(bx)dx+\frac{2b\int \frac{e^c}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}$$

$$\Big\downarrow \text{14}$$

$$2b^2\int e^{b^2x^2+c}\text{erf}(bx)dx-\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}+\frac{2be^c\log(x)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6930}$$

$$\frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}+\frac{2be^c\log(x)}{\sqrt{\pi}}$$

input
```
Int[(E^(c + b^2*x^2)*Erf[b*x])/x^2,x]
```

output
```
-((E^(c + b^2*x^2)*Erf[b*x])/x) + (2*b^3*E^c*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, b^2*x^2])/Sqrt[Pi] + (2*b*E^c*Log[x])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}\,\mathrm{erf}\,(bx)}{x^2}dx$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^2,x)
```

output
```
int(exp(b^2*x^2+c)*erf(b*x)/x^2,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\mathrm{erf}(bx)}{x^2}\,dx = \int \frac{\mathrm{erf}\,(bx)\,e^{(b^2x^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \frac{2b^3x^2e^c{}_2F_2\left(\begin{matrix}1,1\\2,\frac{5}{2}\end{matrix}\middle|b^2x^2\right)}{3\sqrt{\pi}} + \frac{be^c\log(b^2x^2)}{\sqrt{\pi}}$$

input
```
integrate(exp(b**2*x**2+c)*erf(b*x)/x**2,x)
```

output
```
2*b**3*x**2*exp(c)*hyper((1, 1), (2, 5/2), b**2*x**2)/(3*sqrt(pi)) + b*exp
(c)*log(b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\operatorname{erf}(b\,x)}{x^2}\,dx$$

input
```
int((exp(c + b^2*x^2)*erf(b*x))/x^2,x)
```

output
```
int((exp(c + b^2*x^2)*erf(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x^2}dx\right)$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^2,x)
```

output
```
e**c*int((e**(b**2*x**2)*erf(b*x))/x**2,x)
```

3.74 $\int \frac{e^{c+b^2x^2}\textbf{erf}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^4}\,dx = -\frac{be^c}{3\sqrt{\pi}x^2} - \frac{e^{c+b^2x^2}\text{erf}(bx)}{3x^3} - \frac{2b^2e^{c+b^2x^2}\text{erf}(bx)}{3x} + \frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}} + \frac{4b^3e^c\log(x)}{3\sqrt{\pi}}$$

output
```
-1/3*b*exp(c)/Pi^(1/2)/x^2-1/3*exp(b^2*x^2+c)*erf(b*x)/x^3-2/3*b^2*exp(b^2
*x^2+c)*erf(b*x)/x+4/3*b^5*exp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/P
i^(1/2)+4/3*b^3*exp(c)*ln(x)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{e^{c+b^2x^2}\text{erf}(bx)}{x^4}\,dx = -\frac{e^c\left(bx + e^{b^2x^2}\sqrt{\pi}(1+2b^2x^2)\,\text{erf}(bx) - 2b^3\pi x^3\text{erf}(bx)\text{erfi}(bx) + 4b^5x^5\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right) - 4b^3x^3\log\right.}{3\sqrt{\pi}x^3}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erf[b*x])/x^4,x]
```

output
```
-1/3*(E^c*(b*x + E^(b^2*x^2)*Sqrt[Pi]*(1 + 2*b^2*x^2)*Erf[b*x] - 2*b^3*Pi*
x^3*Erf[b*x]*Erfi[b*x] + 4*b^5*x^5*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b
^2*x^2)] - 4*b^3*x^3*Log[x]))/(Sqrt[Pi]*x^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6945, 15, 6945, 14, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^4}\,dx$$

$$\Big\downarrow \text{6945}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^2}dx+\frac{2b\int \frac{e^c}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x^2}dx-\frac{e^{b^2x^2+c}\text{erf}(bx)}{3x^3}-\frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6945}$$

$$\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2+c}\text{erf}(bx)dx+\frac{2b\int \frac{e^c}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}\right)-\frac{e^{b^2x^2+c}\text{erf}(bx)}{3x^3}-\frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2+c}\text{erf}(bx)dx-\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}+\frac{2be^c\log(x)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erf}(bx)}{3x^3}-\frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6930}$$

$$\frac{2}{3}b^2\left(\frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}+\frac{2be^c\log(x)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erf}(bx)}{3x^3}-\frac{be^c}{3\sqrt{\pi}x^2}$$

input
```
Int[(E^(c + b^2*x^2)*Erf[b*x])/x^4,x]
```

output
```
-1/3*(b*E^c)/(Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erf[b*x])/(3*x^3) + (2*b^2*
(-((E^(c + b^2*x^2)*Erf[b*x])/x) + (2*b^3*E^c*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, b^2*x^2])/Sqrt[Pi] + (2*b*E^c*Log[x])/Sqrt[Pi]))/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}\,\mathrm{erf}\left(bx\right)}{x^4}dx$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^4,x)
```

output
```
int(exp(b^2*x^2+c)*erf(b*x)/x^4,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^4,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 81.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \frac{b^3 G_{3,2}^{1,2}\left(\begin{matrix} 2,1 & \frac{5}{2} \\ 2 & 0 \end{matrix} \middle| \frac{e^{-i\pi}}{b^2x^2}\right)e^c}{2}$$

input
```
integrate(exp(b**2*x**2+c)*erf(b*x)/x**4,x)
```

output
```
b**3*meijerg(((2, 1), (5/2,)), ((2,), (0,)), exp_polar(-I*pi)/(b**2*x**2))
*exp(c)/2
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erf(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\operatorname{erf}(b\,x)}{x^4}\,dx$$

input
```
int((exp(c + b^2*x^2)*erf(b*x))/x^4,x)
```

output
```
int((exp(c + b^2*x^2)*erf(b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x^4}dx\right)$$

input
```
int(exp(b^2*x^2+c)*erf(b*x)/x^4,x)
```

output
```
e**c*int((e**(b**2*x**2)*erf(b*x))/x**4,x)
```

3.75 $\int e^{-b^2x^2} x^5 \mathbf{erf}(bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 135

$$\int e^{-b^2x^2} x^5 \operatorname{erf}(bx)\, dx = -\frac{11e^{-2b^2x^2}x}{16b^5\sqrt{\pi}} - \frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{b^6} - \frac{e^{-b^2x^2}x^2\operatorname{erf}(bx)}{b^4} - \frac{e^{-b^2x^2}x^4\operatorname{erf}(bx)}{2b^2} + \frac{43\operatorname{erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}$$

output
```
-11/16*x/b^5/exp(2*b^2*x^2)/Pi^(1/2)-1/4*x^3/b^3/exp(2*b^2*x^2)/Pi^(1/2)-e
rf(b*x)/b^6/exp(b^2*x^2)-x^2*erf(b*x)/b^4/exp(b^2*x^2)-1/2*x^4*erf(b*x)/b^
2/exp(b^2*x^2)+43/64*erf(2^(1/2)*b*x)*2^(1/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int e^{-b^2x^2} x^5 \operatorname{erf}(bx)\, dx = \frac{-\frac{4be^{-2b^2x^2}x(11+4b^2x^2)}{\sqrt{\pi}} - 32e^{-b^2x^2}(2+2b^2x^2+b^4x^4)\operatorname{erf}(bx) + 43\sqrt{2}\operatorname{erf}\left(\sqrt{2}bx\right)}{64b^6}$$

input
```
Integrate[(x^5*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
((-4*b*x*(11 + 4*b^2*x^2))/(E^(2*b^2*x^2)*Sqrt[Pi]) - (32*(2 + 2*b^2*x^2 +
 b^4*x^4)*Erf[b*x])/E^(b^2*x^2) + 43*Sqrt[2]*Erf[Sqrt[2]*b*x])/(64*b^6)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.75, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6939, 2641, 2641, 2634, 6939, 2641, 2634, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{-b^2x^2}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6939}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2}+\frac{\int e^{-2b^2x^2}x^4dx}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2}+\frac{\frac{3\int e^{-2b^2x^2}x^2dx}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2}+\frac{\frac{3\left(\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erf}(bx)dx}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6939}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\int e^{-2b^2x^2}x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2641}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6936}$$

$$\frac{2\left(\frac{\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{x^4e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{2\left(-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}+\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

input
```
Int[(x^5*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
-1/2*(x^4*Erf[b*x])/(b^2*E^(b^2*x^2)) + (-1/4*x^3/(b^2*E^(2*b^2*x^2)) + (3
*(-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3)))/(4*
b^2))/(b*Sqrt[Pi]) + (2*(-1/2*(x^2*Erf[b*x])/(b^2*E^(b^2*x^2)) + (-1/2*Erf
[b*x]/(b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2))/b^2 + (-1/4*x/
(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi]))
)/b^2
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\frac{\operatorname{erf}(bx)\left(-\frac{e^{-b^2x^2}x^4b^4}{2}-e^{-b^2x^2}x^2b^2-e^{-b^2x^2}\right)}{b^5}-\frac{-\frac{43\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{64}+\frac{e^{-2b^2x^2}b^3x^3}{4}+\frac{11\,e^{-2b^2x^2}bx}{16}}{\sqrt{\pi}\,b^5}}{b}$	119

input
```
int(x^5*erf(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
(erf(b*x)/b^5*(-1/2*b^4*x^4/exp(b^2*x^2)-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^
2))-1/Pi^(1/2)/b^5*(-43/64*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)+1/4/exp(b^2*x
^2)^2*b^3*x^3+11/16/exp(b^2*x^2)^2*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.72

$$\int e^{-b^2x^2}x^5\operatorname{erf}(bx)\,dx$$
$$=\frac{43\sqrt{2}\pi\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-32\left(\pi b^5x^4+2\,\pi b^3x^2+2\,\pi b\right)\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}-4\sqrt{\pi}(4\,b^4x^3+11\,b^2x)e^{\left(-2\,b^2x^2\right)}}{64\,\pi b^7}$$

input
```
integrate(x^5*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/64*(43*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 32*(pi*b^5*x^4 +
2*pi*b^3*x^2 + 2*pi*b)*erf(b*x)*e^(-b^2*x^2) - 4*sqrt(pi)*(4*b^4*x^3 + 11*
b^2*x)*e^(-2*b^2*x^2))/(pi*b^7)
```

Sympy [F]

$$\int e^{-b^2x^2}x^5\mathrm{erf}(bx)\,dx = \int x^5e^{-b^2x^2}\,\mathrm{erf}\,(bx)\,dx$$

input
```
integrate(x**5*erf(b*x)/exp(b**2*x**2),x)
```

output
```
Integral(x**5*exp(-b**2*x**2)*erf(b*x), x)
```

Maxima [F]

$$\int e^{-b^2x^2}x^5\mathrm{erf}(bx)\,dx = \int x^5\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^5*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
-1/2*(b^4*x^4 + 2*b^2*x^2 + 2)*erf(b*x)*e^(-b^2*x^2)/b^6 + integrate((b^4*
x^4 + 2*b^2*x^2 + 2)*e^(-2*b^2*x^2), x)/(sqrt(pi)*b^5)
```

Giac [F]

$$\int e^{-b^2x^2}x^5\mathrm{erf}(bx)\,dx = \int x^5\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^5*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*erf(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.42

$$\int e^{-b^2x^2}x^5\mathrm{erf}(bx)\,dx = \frac{\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,x\,\sqrt{b^2}\right)}{2\,b\,{\left(b^2\right)}^{5/2}}-\frac{\mathrm{erfi}\left(x\,\sqrt{-2\,b^2}\right)}{2\,b^3\,{\left(-2\,b^2\right)}^{3/2}}-\frac{x^3\,\mathrm{e}^{-2\,b^2\,x^2}}{4\,b^3\,\sqrt{\pi}} - \mathrm{erf}\left(b\,x\right)\,\left(\frac{\mathrm{e}^{-b^2\,x^2}}{b^6}+\frac{x^4\,\mathrm{e}^{-b^2\,x^2}}{2\,b^2}+\frac{x^2\,\mathrm{e}^{-b^2\,x^2}}{b^4}\right) - \frac{11\,x\,\mathrm{e}^{-2\,b^2\,x^2}}{16\,b^5\,\sqrt{\pi}}+\frac{3\,\sqrt{2}\,x^5}{64\,b\,{\left(b^2\,x^2\right)}^{5/2}}-\frac{3\,\sqrt{2}\,x^5\,\mathrm{erfc}\left(\sqrt{2\,b^2\,x^2}\right)}{64\,b\,{\left(b^2\,x^2\right)}^{5/2}}$$

input
```
int(x^5*exp(-b^2*x^2)*erf(b*x),x)
```

output
```
(2^(1/2)*erf(2^(1/2)*x*(b^2)^(1/2)))/(2*b*(b^2)^(5/2)) - erfi(x*(-2*b^2)^(
1/2))/(2*b^3*(-2*b^2)^(3/2)) - (x^3*exp(-2*b^2*x^2))/(4*b^3*pi^(1/2)) - er
f(b*x)*(exp(-b^2*x^2)/b^6 + (x^4*exp(-b^2*x^2))/(2*b^2) + (x^2*exp(-b^2*x^
2))/b^4) - (11*x*exp(-2*b^2*x^2))/(16*b^5*pi^(1/2)) + (3*2^(1/2)*x^5)/(64*
b*(b^2*x^2)^(5/2)) - (3*2^(1/2)*x^5*erfc((2*b^2*x^2)^(1/2)))/(64*b*(b^2*x^
2)^(5/2))
```

Reduce [F]

$$\int e^{-b^2x^2}x^5\mathrm{erf}(bx)\,dx = \int \frac{\mathrm{erf}(bx)\,x^5}{e^{b^2x^2}}dx$$

input
```
int(x^5*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int((erf(b*x)*x**5)/e**(b**2*x**2),x)
```

3.76 $\int e^{-b^2x^2}x^3\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx=-\frac{e^{-2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erf}(bx)}{2b^2}+\frac{5\text{erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}$$

output
```
-1/4*x/b^3/exp(2*b^2*x^2)/Pi^(1/2)-1/2*erf(b*x)/b^4/exp(b^2*x^2)-1/2*x^2*e
rf(b*x)/b^2/exp(b^2*x^2)+5/16*erf(2^(1/2)*b*x)*2^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx=\frac{-\frac{4be^{-2b^2x^2}x}{\sqrt{\pi}}-8e^{-b^2x^2}(1+b^2x^2)\,\text{erf}(bx)+5\sqrt{2}\text{erf}\left(\sqrt{2}bx\right)}{16b^4}$$

input
```
Integrate[(x^3*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
((-4*b*x)/(E^(2*b^2*x^2)*Sqrt[Pi]) - (8*(1 + b^2*x^2)*Erf[b*x])/E^(b^2*x^2
) + 5*Sqrt[2]*Erf[Sqrt[2]*b*x])/(16*b^4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6939, 2641, 2634, 6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-b^2x^2}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6939}$$

$$\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\int e^{-2b^2x^2}x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}+\frac{\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\int e^{-b^2x^2}x\text{erf}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6936}$$

$$\frac{\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{x^2e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}}{b^2}+\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

input
```
Int[(x^3*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
-1/2*(x^2*Erf[b*x])/(b^2*E^(b^2*x^2)) + (-1/2*Erf[b*x]/(b^2*E^(b^2*x^2)) +
 Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2))/b^2 + (-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sq
rt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\frac{\operatorname{erf}(bx)\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}x^{2}b^{2}}{2}-\frac{\mathrm{e}^{-b^{2}x^{2}}}{2}\right)}{b^{3}}-\frac{-\frac{5\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{16}+\frac{\mathrm{e}^{-2b^{2}x^{2}}bx}{4}}{\sqrt{\pi}\,b^{3}}}{b}$	83

input
```
int(x^3*erf(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
(erf(b*x)/b^3*(-1/2*b^2*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))-1/Pi^(1/2)/b^3*
(-5/16*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)+1/4/exp(b^2*x^2)^2*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int e^{-b^2x^2}x^3\operatorname{erf}(bx)\,dx$$
$$= -\frac{4\sqrt{\pi}b^2xe^{\left(-2\,b^2x^2\right)}-5\sqrt{2}\pi\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)+8\left(\pi b^3x^2+\pi b\right)\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{16\,\pi b^5}$$

input
```
integrate(x^3*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/16*(4*sqrt(pi)*b^2*x*e^(-2*b^2*x^2) - 5*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2
)*sqrt(b^2)*x) + 8*(pi*b^3*x^2 + pi*b)*erf(b*x)*e^(-b^2*x^2))/(pi*b^5)
```

Sympy [F]

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx=\int x^3e^{-b^2x^2}\,\text{erf}\,(bx)\,dx$$

input
```
integrate(x**3*erf(b*x)/exp(b**2*x**2),x)
```

output
```
Integral(x**3*exp(-b**2*x**2)*erf(b*x), x)
```

Maxima [F]

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx=\int x^3\,\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^3*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
-1/2*(b^2*x^2 + 1)*erf(b*x)*e^(-b^2*x^2)/b^4 + integrate((b^2*x^2 + 1)*e^(
-2*b^2*x^2), x)/(sqrt(pi)*b^3)
```

Giac [F]

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx=\int x^3\,\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^3*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*erf(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx = \frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}\,x\,\sqrt{b^2}\right)}{4\,b\,(b^2)^{3/2}} - \frac{\text{erfi}\left(\sqrt{2}\,x\,\sqrt{-b^2}\right)}{4\,b\,(-2\,b^2)^{3/2}} - \text{erf}(b\,x)\left(\frac{e^{-b^2\,x^2}}{2\,b^4} + \frac{x^2\,e^{-b^2\,x^2}}{2\,b^2}\right) - \frac{x\,e^{-2\,b^2\,x^2}}{4\,b^3\,\sqrt{\pi}}$$

input
```
int(x^3*exp(-b^2*x^2)*erf(b*x),x)
```

output
```
(2^(1/2)*erf(2^(1/2)*x*(b^2)^(1/2)))/(4*b*(b^2)^(3/2)) - erfi(2^(1/2)*x*(-
b^2)^(1/2))/(4*b*(-2*b^2)^(3/2)) - erf(b*x)*(exp(-b^2*x^2)/(2*b^4) + (x^2*
exp(-b^2*x^2))/(2*b^2)) - (x*exp(-2*b^2*x^2))/(4*b^3*pi^(1/2))
```

Reduce [F]

$$\int e^{-b^2x^2}x^3\text{erf}(bx)\,dx = \int \frac{\text{erf}(bx)\,x^3}{e^{b^2x^2}}dx$$

input
```
int(x^3*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int((erf(b*x)*x**3)/e**(b**2*x**2),x)
```

3.77 $\int e^{-b^2x^2} x \operatorname{erf}(bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int e^{-b^2x^2} x \operatorname{erf}(bx) \, dx = -\frac{e^{-b^2x^2} \operatorname{erf}(bx)}{2b^2} + \frac{\operatorname{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

output
```
-1/2*erf(b*x)/b^2/exp(b^2*x^2)+1/4*erf(2^(1/2)*b*x)*2^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int e^{-b^2x^2} x \operatorname{erf}(bx) \, dx = \frac{-2e^{-b^2x^2} \operatorname{erf}(bx) + \sqrt{2} \operatorname{erf}\left(\sqrt{2}bx\right)}{4b^2}$$

input
```
Integrate[(x*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
((-2*Erf[b*x])/E^(b^2*x^2) + Sqrt[2]*Erf[Sqrt[2]*b*x])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6936, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-b^2x^2}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{6936}$$

$$\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\text{erf}(\sqrt{2}bx)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

input
```
Int[(x*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
-1/2*Erf[b*x]/(b^2*E^(b^2*x^2)) + Erf[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{-\frac{\operatorname{erf}(bx)\mathrm{e}^{-b^2x^2}}{2b}+\frac{\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{4b}}{b}$	39

input
```
int(x*erf(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*erf(b*x)/b*exp(-b^2*x^2)+1/4/b*2^(1/2)*erf(2^(1/2)*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x\operatorname{erf}(bx)\,dx = -\frac{2\,b\operatorname{erf}(bx)\,e^{(-b^2x^2)}-\sqrt{2}\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,b^3}$$

input
```
integrate(x*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/4*(2*b*erf(b*x)*e^(-b^2*x^2) - sqrt(2)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*
x))/b^3
```

Sympy [F]

$$\int e^{-b^2x^2}x\operatorname{erf}(bx)\,dx = \int xe^{-b^2x^2}\operatorname{erf}(bx)\,dx$$

input
```
integrate(x*erf(b*x)/exp(b**2*x**2),x)
```

output
```
Integral(x*exp(-b**2*x**2)*erf(b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int e^{-b^2x^2}x\text{erf}(bx)\,dx = -\frac{\text{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{2\,b^2}+\frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}bx\right)}{4\,b^2}$$

input
```
integrate(x*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
-1/2*erf(b*x)*e^(-b^2*x^2)/b^2 + 1/4*sqrt(2)*erf(sqrt(2)*b*x)/b^2
```

Giac [F]

$$\int e^{-b^2x^2}x\text{erf}(bx)\,dx = \int x\,\text{erf}\left(bx\right)e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x*erf(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x\text{erf}(bx)\,dx = \frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}\,x\,\sqrt{b^2}\right)}{4\,b\,\sqrt{b^2}}-\frac{{\mathrm{e}}^{-b^2\,x^2}\,\text{erf}(b\,x)}{2\,b^2}$$

input
```
int(x*exp(-b^2*x^2)*erf(b*x),x)
```

output
```
(2^(1/2)*erf(2^(1/2)*x*(b^2)^(1/2)))/(4*b*(b^2)^(1/2)) - (exp(-b^2*x^2)*er
f(b*x))/(2*b^2)
```

Reduce [F]

$$\int e^{-b^2x^2}x\mathrm{erf}(bx)\,dx=\int \frac{\mathrm{erf}(bx)\,x}{e^{b^2x^2}}dx$$

input
```
int(x*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int((erf(b*x)*x)/e**(b**2*x**2),x)
```

3.78 $\int \frac{e^{-b^2x^2}\mathbf{erf}(bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \operatorname{Int}\left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x},x\right)$$

output

```
Defer(Int)(erf(b*x)/exp(b^2*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx$$

input

```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x),x]
```

output

```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx$$
$$\Big\downarrow \text{ 6948}$$
$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}dx$$

input
```
Int[Erf[b*x]/(E^(b^2*x^2)*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx)\,e^{-b^2x^2}}{x}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x,x)
```

output
```
int(erf(b*x)/exp(b^2*x^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(-b^2x^2)}}{x}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(-b^2*x^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx$$

input
```
integrate(erf(b*x)/exp(b**2*x**2)/x,x)
```

output
```
Integral(exp(-b**2*x**2)*erf(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(-b^2x^2)}}{x}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(-b^2x^2)}}{x}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{x}\,dx$$

input
```
int((exp(-b^2*x^2)*erf(b*x))/x,x)
```

output
```
int((exp(-b^2*x^2)*erf(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x,x)
```

output
```
int(erf(b*x)/(e**(b**2*x**2)*x),x)
```

3.79 $\int \frac{e^{-b^2x^2}\mathbf{erf}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = -\frac{be^{-2b^2x^2}}{\sqrt{\pi}x} - \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2x^2} - \sqrt{2}b^2\operatorname{erf}\left(\sqrt{2}bx\right) - b^2\operatorname{Int}\left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x},x\right)$$

output
```
-b/exp(2*b^2*x^2)/Pi^(1/2)/x-1/2*erf(b*x)/exp(b^2*x^2)/x^2-2^(1/2)*b^2*erf
(2^(1/2)*b*x)-b^2*Defer(Int)(erf(b*x)/exp(b^2*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx$$

input
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^3),x]
```

output
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}\,dx$$

$$\Big\downarrow \ 6945$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\int \frac{e^{-2b^2x^2}}{x^2}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}$$

$$\Big\downarrow \ 2643$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\left(-4b^2\int e^{-2b^2x^2}dx-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}$$

$$\Big\downarrow \ 2634$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}$$

$$\Big\downarrow \ 6948$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}$$

input
```
Int[Erf[b*x]/(E^(b^2*x^2)*x^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erf}(bx)\,\mathrm{e}^{-b^2x^2}}{x^3}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^3,x)
```

output
```
int(erf(b*x)/exp(b^2*x^2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(-b^2x^2)}}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^3,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b**2*x**2)/x**3,x)
```

output
```
Integral(exp(-b**2*x**2)*erf(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}\,dx = \int \frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}\,dx = \int \frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\operatorname{erf}(b\,x)}{x^3}\,dx$$

input
```
int((exp(-b^2*x^2)*erf(b*x))/x^3,x)
```

output
```
int((exp(-b^2*x^2)*erf(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^3}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^3,x)
```

output
```
int(erf(b*x)/(e**(b**2*x**2)*x**3),x)
```

3.80 $\int \frac{e^{-b^2x^2}\mathbf{erf}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}\,dx = -\frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3}+\frac{7b^3e^{-2b^2x^2}}{6\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}+\frac{b^2e^{-b^2x^2}\text{erf}(bx)}{4x^2}+\frac{b^4\text{erf}(\sqrt{2}bx)}{\sqrt{2}}+\frac{2}{3}\sqrt{2}b^4\text{erf}\left(\sqrt{2}bx\right)+\frac{1}{2}b^4\text{Int}\left(\frac{e^{-b^2x^2}\text{erf}(bx)}{x},x\right)$$

output
```
-1/6*b/exp(2*b^2*x^2)/Pi^(1/2)/x^3+7/6*b^3/exp(2*b^2*x^2)/Pi^(1/2)/x-1/4*e
rf(b*x)/exp(b^2*x^2)/x^4+1/4*b^2*erf(b*x)/exp(b^2*x^2)/x^2+7/6*b^4*erf(2^(
1/2)*b*x)*2^(1/2)+1/2*b^4*Defer(Int)(erf(b*x)/exp(b^2*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}\,dx$$

input
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^5),x]
```

output
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^5), x]
```

Rubi [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}dx$$

$$\Big\downarrow \text{6945}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}dx+\frac{b\int \frac{e^{-2b^2x^2}}{x^4}dx}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}dx+\frac{b\left(-\frac{4}{3}b^2\int \frac{e^{-2b^2x^2}}{x^2}dx-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}dx+\frac{b\left(-\frac{4}{3}b^2\left(-4b^2\int e^{-2b^2x^2}dx-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}+\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{6945}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\int \frac{e^{-2b^2x^2}}{x^2}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}+\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\left(-4b^2\int e^{-2b^2x^2}dx-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}+\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}+\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{6948}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x}dx\right)+\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{4x^4}+\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}$$

input
```
Int[Erf[b*x]/(E^(b^2*x^2)*x^5),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\text{erf}\,(bx)\,\mathrm{e}^{-b^2x^2}}{x^5}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^5,x)
```

output
```
int(erf(b*x)/exp(b^2*x^2)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{x^5}\,dx = \int \frac{\mathrm{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^5,x, algorithm="fricas")
```

output
```
integral(erf(b*x)*e^(-b^2*x^2)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 16.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-b^2x^2}\mathrm{erf}(bx)}{x^5}\,dx = \int \frac{e^{-b^2x^2}\,\mathrm{erf}\left(bx\right)}{x^5}\,dx$$

input
```
integrate(erf(b*x)/exp(b**2*x**2)/x**5,x)
```

output
```
Integral(exp(-b**2*x**2)*erf(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(-b^2x^2)}}{x^5}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^5,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(-b^2x^2)}}{x^5}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^5}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{x^5}\,dx$$

input
```
int((exp(-b^2*x^2)*erf(b*x))/x^5,x)
```

output
```
int((exp(-b^2*x^2)*erf(b*x))/x^5, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^5}\,dx = \int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^5}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^5,x)
```

output
```
int(erf(b*x)/(e**(b**2*x**2)*x**5),x)
```

3.81 $\int e^{-b^2x^2}x^4\textbf{erf}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx=-\frac{e^{-2b^2x^2}}{2b^5\sqrt{\pi}}-\frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\text{erf}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^3\text{erf}(bx)}{2b^2}+\frac{3\sqrt{\pi}\text{erf}(bx)^2}{16b^5}$$

output
```
-1/2/b^5/exp(2*b^2*x^2)/Pi^(1/2)-1/4*x^2/b^3/exp(2*b^2*x^2)/Pi^(1/2)-3/4*x
*erf(b*x)/b^4/exp(b^2*x^2)-1/2*x^3*erf(b*x)/b^2/exp(b^2*x^2)+3/16*Pi^(1/2)
*erf(b*x)^2/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.76

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx=\frac{e^{-2b^2x^2}\left(-4(2+b^2x^2)-4be^{b^2x^2}\sqrt{\pi}x(3+2b^2x^2)\,\text{erf}(bx)+3e^{2b^2x^2}\pi\text{erf}(bx)^2\right)}{16b^5\sqrt{\pi}}$$

input
```
Integrate[(x^4*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
(-4*(2 + b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x] +
3*E^(2*b^2*x^2)*Pi*Erf[b*x]^2)/(16*b^5*E^(2*b^2*x^2)*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6939, 2641, 2638, 6939, 2638, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-b^2 x^2} \text{erf}(bx) \, dx$$

$$\Big\downarrow \text{6939}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}x^3dx}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2}+\frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2}-\frac{x^2e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{2638}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erf}(bx)dx}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6939}$$

$$\frac{3\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}+\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2638}$$

$$\frac{3\left(\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6927}$$

$$\frac{3\left(\frac{\sqrt{\pi}\int \operatorname{erf}(bx)d\operatorname{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\operatorname{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\operatorname{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \; 15$$

$$-\frac{x^3e^{-b^2x^2}\operatorname{erf}(bx)}{2b^2}+\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}+\frac{3\left(\frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\operatorname{erf}(bx)}{2b^2}-\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}$$

input

```
Int[(x^4*Erf[b*x])/E^(b^2*x^2),x]
```

output

```
(-1/8*1/(b^4*E^(2*b^2*x^2)) - x^2/(4*b^2*E^(2*b^2*x^2)))/(b*Sqrt[Pi]) - (x
^3*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(-1/4*1/(b^3*E^(2*b^2*x^2)*Sqrt[Pi])
 - (x*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (Sqrt[Pi]*Erf[b*x]^2)/(8*b^3)))/(2*b
^2)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^4 \operatorname{erf}(bx)\,\mathrm{e}^{-b^2x^2}dx$$

input
```
int(x^4*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^4*erf(b*x)/exp(b^2*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int e^{-b^2x^2}x^4\operatorname{erf}(bx)\,dx = -\frac{4\,(2\,\pi b^3x^3+3\,\pi bx)\operatorname{erf}(bx)\,e^{(-b^2x^2)}-\sqrt{\pi}\left(3\,\pi\operatorname{erf}(bx)^2-4\,(b^2x^2+2)e^{(-2\,b^2x^2)}\right)}{16\,\pi b^5}$$

input
```
integrate(x^4*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/16*(4*(2*pi*b^3*x^3 + 3*pi*b*x)*erf(b*x)*e^(-b^2*x^2) - sqrt(pi)*(3*pi*
erf(b*x)^2 - 4*(b^2*x^2 + 2)*e^(-2*b^2*x^2)))/(pi*b^5)
```

Sympy [A] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx = \begin{cases} -\frac{x^3e^{-b^2x^2}\text{erf}\,(bx)}{2b^2} - \frac{x^2e^{-2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{3xe^{-b^2x^2}\text{erf}\,(bx)}{4b^4} + \frac{3\sqrt{\pi}\,\text{erf}^2\,(bx)}{16b^5} - \frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*erf(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((-x**3*exp(-b**2*x**2)*erf(b*x)/(2*b**2) - x**2*exp(-2*b**2*x**2
)/(4*sqrt(pi)*b**3) - 3*x*exp(-b**2*x**2)*erf(b*x)/(4*b**4) + 3*sqrt(pi)*e
rf(b*x)**2/(16*b**5) - exp(-2*b**2*x**2)/(2*sqrt(pi)*b**5), Ne(b, 0)), (0,
 True))
```

Maxima [F]

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx = \int x^4\,\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^4*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
1/2*integrate((2*b^2*x^3 + 3*x)*e^(-2*b^2*x^2), x)/(sqrt(pi)*b^3) - 1/16*(
4*(2*b^3*x^3 + 3*b*x)*erf(b*x)*e^(-b^2*x^2) - 3*sqrt(pi)*erf(b*x)^2)/b^5
```

Giac [F]

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx = \int x^4\,\text{erf}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^4*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*erf(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx = -\frac{8\,\text{e}^{-2\,b^2\,x^2} - 3\,\pi\,\text{erf}(b\,x)^2}{16\,b^5\,\sqrt{\pi}} - \frac{x^2\,\text{e}^{-2\,b^2\,x^2}}{4\,b^3\,\sqrt{\pi}} - \frac{3\,x\,\text{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{4\,b^4} - \frac{x^3\,\text{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{2\,b^2}$$

input
```
int(x^4*exp(-b^2*x^2)*erf(b*x),x)
```

output
```
- (8*exp(-2*b^2*x^2) - 3*pi*erf(b*x)^2)/(16*b^5*pi^(1/2)) - (x^2*exp(-2*b^
2*x^2))/(4*b^3*pi^(1/2)) - (3*x*exp(-b^2*x^2)*erf(b*x))/(4*b^4) - (x^3*exp
(-b^2*x^2)*erf(b*x))/(2*b^2)
```

Reduce [F]

$$\int e^{-b^2x^2}x^4\text{erf}(bx)\,dx = \int \frac{\text{erf}(bx)\,x^4}{e^{b^2x^2}}dx$$

input
```
int(x^4*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int((erf(b*x)*x**4)/e**(b**2*x**2),x)
```

3.82 $\int e^{-b^2x^2}x^2\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int e^{-b^2x^2}x^2\operatorname{erf}(bx)\,dx=-\frac{e^{-2b^2x^2}}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x\operatorname{erf}(bx)}{2b^2}+\frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{8b^3}$$

output
```
-1/4/b^3/exp(2*b^2*x^2)/Pi^(1/2)-1/2*x*erf(b*x)/b^2/exp(b^2*x^2)+1/8*Pi^(1
/2)*erf(b*x)^2/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int e^{-b^2x^2}x^2\operatorname{erf}(bx)\,dx=-\frac{\frac{2e^{-2b^2x^2}}{\sqrt{\pi}}+4be^{-b^2x^2}x\operatorname{erf}(bx)-\sqrt{\pi}\operatorname{erf}(bx)^2}{8b^3}$$

input
```
Integrate[(x^2*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
-1/8*(2/(E^(2*b^2*x^2)*Sqrt[Pi]) + (4*b*x*Erf[b*x])/E^(b^2*x^2) - Sqrt[Pi]
*Erf[b*x]^2)/b^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6939, 2638, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-b^2 x^2} \text{erf}(bx) \, dx$$

$$\Big\downarrow \text{ 6939}$$

$$\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2} + \frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b} - \frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{\int e^{-b^2x^2}\text{erf}(bx)dx}{2b^2} - \frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2} - \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

$$\Big\downarrow \text{ 6927}$$

$$\frac{\sqrt{\pi}\int \text{erf}(bx)d\text{erf}(bx)}{4b^3} - \frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2} - \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}\text{erf}(bx)^2}{8b^3} - \frac{xe^{-b^2x^2}\text{erf}(bx)}{2b^2} - \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

input
```
Int[(x^2*Erf[b*x])/E^(b^2*x^2),x]
```

output
```
-1/4*1/(b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erf[b*x])/(2*b^2*E^(b^2*x^2)) + (
Sqrt[Pi]*Erf[b*x]^2)/(8*b^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^2 \operatorname{erf}(bx)\, \mathrm{e}^{-b^2x^2} dx$$

input
```
int(x^2*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^2*erf(b*x)/exp(b^2*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int e^{-b^2x^2}x^2\mathrm{erf}(bx)\,dx = -\frac{4\,\pi bx\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi\,\mathrm{erf}\,(bx)^2 - 2\,e^{\left(-2\,b^2x^2\right)}\right)}{8\,\pi b^3}$$

input
```
integrate(x^2*erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/8*(4*pi*b*x*erf(b*x)*e^(-b^2*x^2) - sqrt(pi)*(pi*erf(b*x)^2 - 2*e^(-2*b
^2*x^2)))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int e^{-b^2x^2}x^2\mathrm{erf}(bx)\,dx = \begin{cases} -\frac{xe^{-b^2x^2}\,\mathrm{erf}\,(bx)}{2b^2} + \frac{\sqrt{\pi}\,\mathrm{erf}^2\,(bx)}{8b^3} - \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*erf(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((-x*exp(-b**2*x**2)*erf(b*x)/(2*b**2) + sqrt(pi)*erf(b*x)**2/(8*
b**3) - exp(-2*b**2*x**2)/(4*sqrt(pi)*b**3), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int e^{-b^2x^2}x^2\mathrm{erf}(bx)\,dx = \int x^2\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^2*erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x*e^(-2*b^2*x^2), x)/(sqrt(pi)*b) - 1/8*(4*b*x*erf(b*x)*e^(-b^2*
x^2) - sqrt(pi)*erf(b*x)^2)/b^3
```

Giac [F]

$$\int e^{-b^2x^2}x^2\mathrm{erf}(bx)\,dx=\int x^2\,\mathrm{erf}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^2*erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*erf(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int e^{-b^2x^2}x^2\mathrm{erf}(bx)\,dx=-\mathrm{erf}(b\,x)\left(\frac{\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{-b^2}\right)}{4\left(-b^2\right)^{3/2}}+\frac{x\,\mathrm{e}^{-b^2\,x^2}}{2\,b^2}\right)-\frac{2\,\mathrm{e}^{-2\,b^2\,x^2}-\pi\,\mathrm{erfi}\left(x\sqrt{-b^2}\right)^2}{8\,b^3\sqrt{\pi}}$$

input
```
int(x^2*exp(-b^2*x^2)*erf(b*x),x)
```

output
```
- erf(b*x)*((pi^(1/2)*erfi(x*(-b^2)^(1/2)))/(4*(-b^2)^(3/2)) + (x*exp(-b^2
*x^2))/(2*b^2)) - (2*exp(-2*b^2*x^2) - pi*erfi(x*(-b^2)^(1/2))^2)/(8*b^3*p
i^(1/2))
```

Reduce [F]

$$\int e^{-b^2x^2}x^2\mathrm{erf}(bx)\,dx=\int\frac{\mathrm{erf}(bx)\,x^2}{e^{b^2x^2}}dx$$

input
```
int(x^2*erf(b*x)/exp(b^2*x^2),x)
```

output
```
int((erf(b*x)*x**2)/e**(b**2*x**2),x)
```

3.83 $\int e^{-b^2x^2}\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{4b}$$

output
```
1/4*Pi^(1/2)*erf(b*x)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx = \frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{4b}$$

input
```
Integrate[Erf[b*x]/E^(b^2*x^2),x]
```

output
```
(Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-b^2x^2}\text{erf}(bx)\,dx$$

$$\Big\downarrow \text{ 6927}$$

$$\frac{\sqrt{\pi}\int \text{erf}(bx)d\text{erf}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}\text{erf}(bx)^2}{4b}$$

input
```
Int[Erf[b*x]/E^(b^2*x^2),x]
```

output
```
(Sqrt[Pi]*Erf[b*x]^2)/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{\pi}\,\mathrm{erf}(bx)^2}{4b}$	15

input
```
int(erf(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*erf(b*x)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int e^{-b^2x^2}\mathrm{erf}(bx)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(bx\right)^2}{4\,b}$$

input
```
integrate(erf(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*erf(b*x)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int e^{-b^2x^2}\mathrm{erf}(bx)\,dx = \begin{cases} \frac{\sqrt{\pi}\,\mathrm{erf}^2\left(bx\right)}{4b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(erf(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((sqrt(pi)*erf(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx=\frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{4\,b}$$

input
```
integrate(erf(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(b*x)^2/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx=\frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{4\,b}$$

input
```
integrate(erf(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(b*x)^2/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx=\frac{\sqrt{\pi}\operatorname{erf}\left(x\sqrt{b^2}\right)\operatorname{erf}(b\,x)}{2\sqrt{b^2}}-\frac{\sqrt{\pi}\operatorname{erf}\left(x\sqrt{b^2}\right)^2}{4\,b}$$

input
```
int(exp(-b^2*x^2)*erf(b*x),x)
```

output
```
(pi^(1/2)*erf(x*(b^2)^(1/2))*erf(b*x))/(2*(b^2)^(1/2)) - (pi^(1/2)*erf(x*(
b^2)^(1/2))^2)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int e^{-b^2x^2}\operatorname{erf}(bx)\,dx=\frac{\sqrt{\pi}\operatorname{erf}(bx)^2}{4b}$$

input
```
int(erf(b*x)/exp(b^2*x^2),x)
```

output
```
(sqrt(pi)*erf(b*x)**2)/(4*b)
```

3.84 $\int \frac{e^{-b^2x^2}\mathbf{erf}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}\,dx=-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}-\frac{1}{2}b\sqrt{\pi}\text{erf}(bx)^2+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-erf(b*x)/exp(b^2*x^2)/x-1/2*b*Pi^(1/2)*erf(b*x)^2+b*Ei(-2*b^2*x^2)/Pi^(1/
2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}\,dx=-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}-\frac{1}{2}b\sqrt{\pi}\text{erf}(bx)^2+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^2),x]
```

output
```
-(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegralEi
[-2*b^2*x^2])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6945, 2639, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx$$

$$\Big\downarrow \text{6945}$$

$$-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx+\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}$$

$$\Big\downarrow \text{2639}$$

$$-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6927}$$

$$-\sqrt{\pi}b\int \text{erf}(bx)d\text{erf}(bx)-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\text{erf}(bx)^2$$

input
```
Int[Erf[b*x]/(E^(b^2*x^2)*x^2),x]
```

output
```
-(Erf[b*x]/(E^(b^2*x^2)*x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegralEi
[-2*b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\text{erf}\left(bx\right)\text{e}^{-b^2x^2}}{x^2}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^2,x)
```

output
```
int(erf(b*x)/exp(b^2*x^2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}\,dx = -\frac{2\,\pi\,\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)} + \sqrt{\pi}\left(\pi bx\,\text{erf}\,(bx)^2 - 2\,bx\text{Ei}(-2\,b^2x^2)\right)}{2\,\pi x}$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*pi*erf(b*x)*e^(-b^2*x^2) + sqrt(pi)*(pi*b*x*erf(b*x)^2 - 2*b*x*Ei(
-2*b^2*x^2)))/(pi*x)
```

Sympy [F]

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}\,dx = \int \frac{e^{-b^2x^2}\,\text{erf}\,(bx)}{x^2}\,dx$$

input
```
integrate(erf(b*x)/exp(b**2*x**2)/x**2,x)
```

output
```
Integral(exp(-b**2*x**2)*erf(b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}\,dx = \int \frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^2, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(-b^2x^2)}}{x^2}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\operatorname{erf}(b\,x)}{x^2}\,dx$$

input
```
int((exp(-b^2*x^2)*erf(b*x))/x^2,x)
```

output
```
int((exp(-b^2*x^2)*erf(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^2}\,dx = \int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^2}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^2,x)
```

output
```
int(erf(b*x)/(e**(b**2*x**2)*x**2),x)
```

3.85 $\int \frac{e^{-b^2x^2}\mathbf{erf}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}\,dx = -\frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2} - \frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3} + \frac{2b^2e^{-b^2x^2}\text{erf}(bx)}{3x} + \frac{1}{3}b^3\sqrt{\pi}\text{erf}(bx)^2 - \frac{4b^3\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{3\sqrt{\pi}}$$

output
```
-1/3*b/exp(2*b^2*x^2)/Pi^(1/2)/x^2-1/3*erf(b*x)/exp(b^2*x^2)/x^3+2/3*b^2*e
rf(b*x)/exp(b^2*x^2)/x+1/3*b^3*Pi^(1/2)*erf(b*x)^2-4/3*b^3*Ei(-2*b^2*x^2)/
Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}\,dx = \frac{1}{3}\left(\frac{e^{-b^2x^2}\left(-1+2b^2x^2\right)\text{erf}(bx)}{x^3} + b^3\sqrt{\pi}\text{erf}(bx)^2 + \frac{b\left(-\frac{e^{-2b^2x^2}}{x^2} - 4b^2\,\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)}{\sqrt{\pi}}\right)$$

input
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^4),x]
```

output
```
(((-1 + 2*b^2*x^2)*Erf[b*x])/(E^(b^2*x^2)*x^3) + b^3*Sqrt[Pi]*Erf[b*x]^2 +
 (b*(-(1/(E^(2*b^2*x^2)*x^2)) - 4*b^2*ExpIntegralEi[-2*b^2*x^2]))/Sqrt[Pi]
)/3
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6945, 2643, 2639, 6945, 2639, 6927, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}dx$$

$$\Big\downarrow \text{6945}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx+\frac{2b\int \frac{e^{-2b^2x^2}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx+\frac{2b\left(-2b^2\int \frac{e^{-2b^2x^2}}{x}dx-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^2}dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6945}$$

$$-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx+\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erf}(bx)dx-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6927}$$

$$-\frac{2}{3}b^2\left(-\sqrt{\pi}b\int \text{erf}(bx)d\text{erf}(bx)-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{3}b^2\left(-\frac{e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}-\frac{1}{2}\sqrt{\pi}b\text{erf}(bx)^2\right)-\frac{e^{-b^2x^2}\text{erf}(bx)}{3x^3}+\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

input
```
Int[Erf[b*x]/(E^(b^2*x^2)*x^4),x]
```

output
```
-1/3*Erf[b*x]/(E^(b^2*x^2)*x^3) + (2*b*(-1/2*1/(E^(2*b^2*x^2)*x^2) - b^2*E
xpIntegralEi[-2*b^2*x^2]))/(3*Sqrt[Pi]) - (2*b^2*(-(Erf[b*x]/(E^(b^2*x^2)*
x)) - (b*Sqrt[Pi]*Erf[b*x]^2)/2 + (b*ExpIntegralEi[-2*b^2*x^2])/Sqrt[Pi]))
/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6945
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m + 1)*E^(c + d*x^2)*(Erf[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(m
+ 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[2*(b/((m +
1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x
]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erf}(bx)\,\mathrm{e}^{-b^2x^2}}{x^4}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^4,x)
```

output
```
int(erf(b*x)/exp(b^2*x^2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = -\frac{\left(\pi - 2\,\pi b^2x^2\right)\operatorname{erf}(bx)\,e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi b^3x^3\operatorname{erf}(bx)^2 - 4\,b^3x^3\operatorname{Ei}\left(-2\,b^2x^2\right) - bxe^{\left(-2\,b^2x^2\right)}\right)}{3\,\pi x^3}$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((pi - 2*pi*b^2*x^2)*erf(b*x)*e^(-b^2*x^2) - sqrt(pi)*(pi*b^3*x^3*erf
(b*x)^2 - 4*b^3*x^3*Ei(-2*b^2*x^2) - b*x*e^(-2*b^2*x^2)))/(pi*x^3)
```

Sympy [F]

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}\,dx = \int \frac{e^{-b^2x^2}\,\text{erf}\,(bx)}{x^4}\,dx$$

input
```
integrate(erf(b*x)/exp(b**2*x**2)/x**4,x)
```

output
```
Integral(exp(-b**2*x**2)*erf(b*x)/x**4, x)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erf}(bx)}{x^4}\,dx = \int \frac{\text{erf}\,(bx)\,e^{(-b^2x^2)}}{x^4}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^4, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\operatorname{erf}(bx)\,e^{(-b^2x^2)}}{x^4}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(erf(b*x)*e^(-b^2*x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{e^{-b^2\,x^2}\,\operatorname{erf}(b\,x)}{x^4}\,dx$$

input
```
int((exp(-b^2*x^2)*erf(b*x))/x^4,x)
```

output
```
int((exp(-b^2*x^2)*erf(b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^4}\,dx = \int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^4}dx$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^4,x)
```

output
```
int(erf(b*x)/(e**(b**2*x**2)*x**4),x)
```

3.86 $\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 342

$$\begin{aligned}\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx=&-\frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2\left(b^2-d\right)^2d\sqrt{\pi}}+\frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2\left(b^2-d\right)d\sqrt{\pi}}\\&-\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d^2}+\frac{e^{c+dx^2}x^2\text{erf}(a+bx)}{2d}\\&+\frac{be^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}d^2}-\frac{a^2b^3e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{5/2}d}\\&-\frac{be^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}d}\end{aligned}$$

output
```
-1/2*a*b^2*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)^2/d/Pi^(1/2)+1/2*b*exp(
-a^2+c-2*a*b*x-(b^2-d)*x^2)*x/(b^2-d)/d/Pi^(1/2)-1/2*exp(d*x^2+c)*erf(b*x+
a)/d^2+1/2*exp(d*x^2+c)*x^2*erf(b*x+a)/d+1/2*b*exp(c+a^2*d/(b^2-d))*erf((a
*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(1/2)/d^2-1/2*a^2*b^3*exp(c+a^2*d/(b^
2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(5/2)/d-1/4*b*exp(c+a^2*d
/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(3/2)/d
```

Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx$$
$$=\frac{e^c\left(2e^{dx^2}(-1+dx^2)\,\text{erf}(a+bx)-\frac{bde^{-a^2-2abx+(-b^2+d)x^2}\left(2(b^2-d)\,(ab+(-b^2+d)x)+\sqrt{b^2-d}((1+2a^2)b^2-d)e^{\frac{(ab+(b^2-d)x)^2}{b^2-d}}\right.}{(b^2-d)^3\sqrt{\pi}}\right.}{4d^2}$$

input
```
Integrate[E^(c + d*x^2)*x^3*Erf[a + b*x],x]
```

output
```
(E^c*(2*E^(d*x^2)*(-1 + d*x^2)*Erf[a + b*x] - (b*d*E^(-a^2 - 2*a*b*x + (-b
^2 + d)*x^2)*(2*(b^2 - d)*(a*b + (-b^2 + d)*x) + Sqrt[b^2 - d]*((1 + 2*a^2
)*b^2 - d)*E^((a*b + (b^2 - d)*x)^2/(b^2 - d))*Sqrt[Pi]*Erf[(a*b + (b^2 -
d)*x)/Sqrt[b^2 - d]]))/((b^2 - d)^3*Sqrt[Pi]) + (2*b*E^((a^2*d)/(b^2 - d))
*Erfi[(-(a*b) + (-b^2 + d)*x)/Sqrt[-b^2 + d]])/Sqrt[-b^2 + d]))/(4*d^2)
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6939, 2671, 2664, 2634, 2670, 2664, 2634, 6936, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{c+dx^2}\text{erf}(a+bx)\,dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{2671}$$

$$-\frac{b\left(\frac{\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{2(b^2-d)}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\downarrow$ 2664

$$-\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{2(b^2-d)}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\downarrow$ 2634

$$-\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\downarrow$ 2670

$$-\frac{b\Bigg(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+}}{2(b^2-d)}}{\sqrt{\pi}d}$$

$$\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\downarrow$ 2664

$$-\frac{b\left(-\frac{ab\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\cdots}}{2(b^2-d\cdots}\right.}{\sqrt{\pi}d}$$

$$\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \; 2634$$

$$-\frac{\int e^{dx^2+c}x\text{erf}(a+bx)dx}{d}-\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\cdots\right.}}{2(b^2-d)}\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \; 6936$$

$$-\frac{\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d}-\frac{b\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{\sqrt{\pi}d}}{d}-\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\cdots\right.}}{2(b^2-d)}\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \; 2664$$

$$-\frac{\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d}-\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{\sqrt{\pi}d}}{d}-$$

$$\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\cdots\right.}}{2(b^2-d)}\cdots\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \ 2634$$

$$-\frac{\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d}-\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}}{d}-$$

$$\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\cdots\right.}}{2(b^2-d)}\cdots\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^3*Erf[a + b*x],x]
```

output
```
(E^(c + d*x^2)*x^2*Erf[a + b*x])/(2*d) - ((E^(c + d*x^2)*Erf[a + b*x])/(2*
d) - (b*E^((b^2*c + a^2*d - c*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b
^2 - d]])/(2*Sqrt[b^2 - d]*d))/d - (b*(-1/2*(E^(-a^2 + c - 2*a*b*x - (b^2
- d)*x^2)*x)/(b^2 - d) + (E^((b^2*c + a^2*d - c*d)/(b^2 - d))*Sqrt[Pi]*Erf
[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(4*(b^2 - d)^(3/2)) - (a*b*(-1/2*E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2)/(b^2 - d) - (a*b*E^((b^2*c + a^2*d - c*
d)/(b^2 - d))*Sqrt[Pi]*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d
)^(3/2))))/(b^2 - d)))/(d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6939
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol] :
> Simp[x^(m - 1)*E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/(2
*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erf[a + b*x], x], x] - Simp[b/(d*Sqrt[Pi]
)   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; FreeQ[
{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{d\,x^2+c}x^3 \operatorname{erf}(bx+a)\,dx$$

input
```
int(exp(d*x^2+c)*x^3*erf(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^3*erf(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.78

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx$$
$$= \frac{\pi(2\,b^5-(2\,a^2+5)b^3d+3\,bd^2)\sqrt{b^2-d}\,\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)}+2\left(\pi(b^6d-3\,b^4d^2+3\,b^2d^3-d^4)x\right.}{4\,\pi(b^6d}$$

input
```
integrate(exp(d*x^2+c)*x^3*erf(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(pi*(2*b^5 - (2*a^2 + 5)*b^3*d + 3*b*d^2)*sqrt(b^2 - d)*erf((a*b + (b^
2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a^2 - c)*d)/(b^2 - d)) + 2*(pi*(b^6*
d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 - pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3
))*erf(b*x + a)*e^(d*x^2 + c) - 2*sqrt(pi)*(a*b^4*d - a*b^2*d^2 - (b^5*d -
 2*b^3*d^2 + b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x + d*x^2 - a^2 + c))/(pi*(b^6*
d^2 - 3*b^4*d^3 + 3*b^2*d^4 - d^5))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**3*erf(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx=\int x^3\,\text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erf(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(d*x^2*e^c - e^c)*erf(b*x + a)*e^(d*x^2)/d^2 - integrate((b*d*x^2*e^c
- b*e^c)*e^(-b^2*x^2 - 2*a*b*x + d*x^2 - a^2), x)/(sqrt(pi)*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.84

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx=\frac{1}{2}\left(\frac{(dx^2+c-1)e^{(dx^2+c)}}{d^2}-\frac{ce^{(dx^2+c)}}{d^2}\right)\text{erf}\,(bx+a)$$
$$-\frac{\frac{2\sqrt{\pi}b\,\text{erf}\left(-\sqrt{b^2-d}\left(\frac{ab}{b^2-d}+x\right)\right)e^{\left(\frac{b^2c+a^2d-cd}{b^2-d}\right)}}{\sqrt{b^2-d}}-\frac{\left(\frac{\sqrt{\pi}\left(2\,a^2b^2+b^2-d\right)\text{erf}\left(-\sqrt{b^2-d}\left(\frac{ab}{b^2-d}+x\right)\right)e^{\left(\frac{b^2c+a^2d-cd}{b^2-d}\right)}}{\sqrt{b^2-d}}+2\left(\left(\frac{ab}{b^2-d}+x\right)b^2-2\,ab-\right.\right.}{b^4-2\,b^2d+d^2}}{4\sqrt{\pi}d^2}$$

input
```
integrate(exp(d*x^2+c)*x^3*erf(b*x+a),x, algorithm="giac")
```

output
```
1/2*((d*x^2 + c - 1)*e^(d*x^2 + c)/d^2 - c*e^(d*x^2 + c)/d^2)*erf(b*x + a)
 - 1/4*(2*sqrt(pi)*b*erf(-sqrt(b^2 - d)*(a*b/(b^2 - d) + x))*e^((b^2*c + a
^2*d - c*d)/(b^2 - d))/sqrt(b^2 - d) - (sqrt(pi)*(2*a^2*b^2 + b^2 - d)*erf
(-sqrt(b^2 - d)*(a*b/(b^2 - d) + x))*e^((b^2*c + a^2*d - c*d)/(b^2 - d))/s
qrt(b^2 - d) + 2*((a*b/(b^2 - d) + x)*b^2 - 2*a*b - (a*b/(b^2 - d) + x)*d)
*e^(-b^2*x^2 - 2*a*b*x + d*x^2 - a^2 + c))*b*d/(b^4 - 2*b^2*d + d^2))/(sqr
t(pi)*d^2)
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.13

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx$$
$$= \frac{\text{erfi}\left(\frac{a\,b-x\,(d-b^2)}{\sqrt{d-b^2}}\right)\,\left(b^3\,e^{\frac{c\,d}{d-b^2}-\frac{a^2\,d}{d-b^2}-\frac{b^2\,c}{d-b^2}}+2\,a^2\,b^3\,e^{\frac{c\,d}{d-b^2}-\frac{a^2\,d}{d-b^2}-\frac{b^2\,c}{d-b^2}}-b\,d\,e^{\frac{c\,d}{d-b^2}-\frac{a^2\,d}{d-b^2}-\frac{b^2\,c}{d-b^2}}\right)}{4\,d\,(d-b^2)^{5/2}}$$
$$-\frac{\frac{a\,b^2\,e^{-a^2-2\,a\,b\,x-b^2\,x^2+d\,x^2+c}}{2\,(d-b^2)^2}+\frac{b\,x\,e^{-a^2-2\,a\,b\,x-b^2\,x^2+d\,x^2+c}}{2\,(d-b^2)}}{d\,\sqrt{\pi}}$$
$$-\text{erf}(a+b\,x)\,\left(\frac{e^{d\,x^2+c}}{2\,d^2}-\frac{x^2\,e^{d\,x^2+c}}{2\,d}\right)+\frac{b\,\text{erf}\left(\frac{a\,b\,1\text{i}-x\,(d-b^2)\,1\text{i}}{\sqrt{d-b^2}}\right)\,e^{c-a^2-\frac{a^2\,b^2}{d-b^2}}\,1\text{i}}{2\,d^2\,\sqrt{d-b^2}}$$

input
```
int(x^3*erf(a + b*x)*exp(c + d*x^2),x)
```

output
```
(erfi((a*b - x*(d - b^2))/(d - b^2)^(1/2))*(b^3*exp((c*d)/(d - b^2) - (a^2
*d)/(d - b^2) - (b^2*c)/(d - b^2)) + 2*a^2*b^3*exp((c*d)/(d - b^2) - (a^2*
d)/(d - b^2) - (b^2*c)/(d - b^2)) - b*d*exp((c*d)/(d - b^2) - (a^2*d)/(d -
 b^2) - (b^2*c)/(d - b^2))))/(4*d*(d - b^2)^(5/2)) - ((a*b^2*exp(c + d*x^2
 - a^2 - b^2*x^2 - 2*a*b*x))/(2*(d - b^2)^2) + (b*x*exp(c + d*x^2 - a^2 -
b^2*x^2 - 2*a*b*x))/(2*(d - b^2)))/(d*pi^(1/2)) - erf(a + b*x)*(exp(c + d*
x^2)/(2*d^2) - (x^2*exp(c + d*x^2))/(2*d)) + (b*erf((a*b*1i - x*(d - b^2)*
1i)/(d - b^2)^(1/2))*exp(c - a^2 - (a^2*b^2)/(d - b^2))*1i)/(2*d^2*(d - b^
2)^(1/2))
```

Reduce [F]

$$\int e^{c+dx^2}x^3\text{erf}(a+bx)\,dx = e^c\left(\int e^{d\,x^2}\text{erf}(bx+a)\,x^3dx\right)$$

input
```
int(exp(d*x^2+c)*x^3*erf(b*x+a),x)
```

output
```
e**c*int(e**(d*x**2)*erf(a + b*x)*x**3,x)
```

3.87 $\int e^{c+dx^2} x \mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int e^{c+dx^2} x\,\mathrm{erf}(a+bx)\,dx = \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{2d} - \frac{b e^{c+\frac{a^2d}{b^2-d}}\mathrm{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}\,d}$$

output
```
1/2*exp(d*x^2+c)*erf(b*x+a)/d-1/2*b*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*
x)/(b^2-d)^(1/2))/(b^2-d)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int e^{c+dx^2} x\,\mathrm{erf}(a+bx)\,dx = \frac{e^c\left(e^{dx^2}\mathrm{erf}(a+bx) - \frac{b e^{\frac{a^2d}{b^2-d}}\mathrm{erfi}\left(\frac{-ab+(-b^2+d)x}{\sqrt{-b^2+d}}\right)}{\sqrt{-b^2+d}}\right)}{2d}$$

input
```
Integrate[E^(c + d*x^2)*x*Erf[a + b*x],x]
```

output
```
(E^c*(E^(d*x^2)*Erf[a + b*x] - (b*E^((a^2*d)/(b^2 - d))*Erfi[(-(a*b) + (-b
^2 + d)*x)/Sqrt[-b^2 + d]])/Sqrt[-b^2 + d]))/(2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6936, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{c+dx^2}\text{erf}(a+bx)\,dx$$

$$\Big\downarrow \text{6936}$$

$$\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d}-\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2664}$$

$$\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d}-\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{e^{c+dx^2}\text{erf}(a+bx)}{2d}-\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}$$

input
```
Int[E^(c + d*x^2)*x*Erf[a + b*x],x]
```

output
```
(E^(c + d*x^2)*Erf[a + b*x])/(2*d) - (b*E^((b^2*c + a^2*d - c*d)/(b^2 - d)
)*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqrt[b^2 - d]*d)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 6936
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Sim
p[E^(c + d*x^2)*(Erf[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(-a^
2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.62

method	result	size
default	$\frac{\frac{\operatorname{erf}(bx+a)b\,\mathrm{e}^{\frac{d\,a^{2}-2da(bx+a)+b^{2}c+d(bx+a)^{2}}{b^{2}}}}{2d}-\frac{b\,\mathrm{e}^{\frac{d\,a^{2}+b^{2}c}{b^{2}}-\frac{a^{2}d^{2}}{b^{4}\left(-1+\frac{d}{b^{2}}\right)}}\operatorname{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\,(bx+a)+\frac{ad}{b^{2}\sqrt{1-\frac{d}{b^{2}}}}\right)}{2d\sqrt{1-\frac{d}{b^{2}}}}}{b}$	139

input
```
int(exp(d*x^2+c)*x*erf(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(1/2*erf(b*x+a)*b*exp((d*a^2-2*d*a*(b*x+a)+b^2*c+d*(b*x+a)^2)/b^2)/d-1/2*b
/d*exp((a^2*d+b^2*c)/b^2-1/b^4*a^2*d^2/(-1+d/b^2))/(1-d/b^2)^(1/2)*erf((1-
d/b^2)^(1/2)*(b*x+a)+1/b^2*a*d/(1-d/b^2)^(1/2)))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int e^{c+dx^2}x\mathrm{erf}(a+bx)\,dx$$
$$=-\frac{\sqrt{b^2-d}b\,\mathrm{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)}-(b^2-d)\,\mathrm{erf}\,(bx+a)\,e^{(dx^2+c)}}{2\,(b^2d-d^2)}$$

input
```
integrate(exp(d*x^2+c)*x*erf(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b^2 - d)*b*erf((a*b + (b^2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (
a^2 - c)*d)/(b^2 - d)) - (b^2 - d)*erf(b*x + a)*e^(d*x^2 + c))/(b^2*d - d^
2)
```

Sympy [F]

$$\int e^{c+dx^2}x\mathrm{erf}(a+bx)\,dx=e^c\int xe^{dx^2}\,\mathrm{erf}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x*erf(b*x+a),x)
```

output
```
exp(c)*Integral(x*exp(d*x**2)*erf(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int e^{c+dx^2}x\mathrm{erf}(a+bx)\,dx=-\frac{b\,\mathrm{erf}\left(\frac{ab}{\sqrt{b^2-d}}+\sqrt{b^2-d}x\right)e^{\left(\frac{a^2b^2}{b^2-d}-a^2+c\right)}}{2\sqrt{b^2-d}d}+\frac{\mathrm{erf}\,(bx+a)\,e^{(dx^2+c)}}{2\,d}$$

input
```
integrate(exp(d*x^2+c)*x*erf(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*b*erf(a*b/sqrt(b^2 - d) + sqrt(b^2 - d)*x)*e^(a^2*b^2/(b^2 - d) - a^2
 + c)/(sqrt(b^2 - d)*d) + 1/2*erf(b*x + a)*e^(d*x^2 + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int e^{c+dx^2}x\text{erf}(a+bx)\,dx=\frac{b\,\text{erf}\left(-\sqrt{b^2-d}\left(\frac{ab}{b^2-d}+x\right)\right)e^{\left(\frac{b^2c+a^2d-cd}{b^2-d}\right)}}{2\sqrt{b^2-d}d}+\frac{\text{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{2\,d}$$

input
```
integrate(exp(d*x^2+c)*x*erf(b*x+a),x, algorithm="giac")
```

output
```
1/2*b*erf(-sqrt(b^2 - d)*(a*b/(b^2 - d) + x))*e^((b^2*c + a^2*d - c*d)/(b^
2 - d))/(sqrt(b^2 - d)*d) + 1/2*erf(b*x + a)*e^(d*x^2 + c)/d
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int e^{c+dx^2}x\text{erf}(a+bx)\,dx=\frac{\text{erf}(a+b\,x)\,e^{d\,x^2+c}}{2\,d}-\frac{b\,\text{erf}\left(\frac{a\,b\,1\text{i}-x\,\left(d-b^2\right)\,1\text{i}}{\sqrt{d-b^2}}\right)\,e^{c-a^2-\frac{a^2\,b^2}{d-b^2}}\,1\text{i}}{2\,d\,\sqrt{d-b^2}}$$

input
```
int(x*erf(a + b*x)*exp(c + d*x^2),x)
```

output
```
(erf(a + b*x)*exp(c + d*x^2))/(2*d) - (b*erf((a*b*1i - x*(d - b^2)*1i)/(d
- b^2)^(1/2))*exp(c - a^2 - (a^2*b^2)/(d - b^2))*1i)/(2*d*(d - b^2)^(1/2))
```

Reduce [F]

$$\int e^{c+dx^2} x\operatorname{erf}(a+bx)\,dx = e^c\left(\int e^{d\,x^2}\operatorname{erf}(bx+a)\,xdx\right)$$

input
```
int(exp(d*x^2+c)*x*erf(b*x+a),x)
```

output
```
e**c*int(e**(d*x**2)*erf(a + b*x)*x,x)
```

3.88 $\int \frac{e^{c+dx^2}\textbf{erf}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx = \text{Int}\left(\frac{e^{c+dx^2}\text{erf}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{6948}$$

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x}dx$$

input
```
Int[(E^(c + d*x^2)*Erf[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erf}\,(bx+a)}{x}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)*e^(d*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 6.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}\,dx = e^c \int \frac{e^{dx^2}\operatorname{erf}(a+bx)}{x}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x+a)/x,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}\,dx = \int \frac{\operatorname{erf}(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}\,dx=\int \frac{\operatorname{erf}(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}\,dx=\int \frac{\operatorname{erf}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}}{x}\,dx$$

input
```
int((erf(a + b*x)*exp(c + d*x^2))/x,x)
```

output
```
int((erf(a + b*x)*exp(c + d*x^2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}\,dx = e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx+a)}{x}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

output
```
e**c*int((e**(d*x**2)*erf(a + b*x))/x,x)
```

3.89 $\int \frac{e^{c+dx^2}\mathbf{erf}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^3}\,dx = &-\frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{2x^2}\\ &-b\sqrt{b^2-d}e^{c+\frac{a^2d}{b^2-d}}\operatorname{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)\\ &-\frac{2ab^2\operatorname{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{\sqrt{\pi}}\\ &+d\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x},x\right)\end{aligned}$$

output

```
-b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/Pi^(1/2)/x-1/2*exp(d*x^2+c)*erf(b*x+a)/
x^2-b*(b^2-d)^(1/2)*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2)
)-2*a*b^2*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d)*x^2)/x,x)/Pi^(1/2)+d*Defe
r(Int)(exp(d*x^2+c)*erf(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^3}\,dx = \int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^3}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^3,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{6945}$$

$$\frac{b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x^2}dx}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\operatorname{erf}(a+bx)}{x}dx-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2672}$$

$$\frac{b\left(-2\left(b^2-d\right)\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+$$
$$d\int \frac{e^{dx^2+c}\operatorname{erf}(a+bx)}{x}dx-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2664}$$

$$\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-2\left(b^2-d\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+$$
$$d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+$$
$$d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2673}$$

$$\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+$$
$$d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{6948}$$

$$\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+$$
$$d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{2x^2}$$

input
```
Int[(E^(c + d*x^2)*Erf[a + b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erf}\left(bx+a\right)}{x^3}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x^3,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x+a)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^3}\,dx = \int \frac{\mathrm{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 25.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^3}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erf}\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x+a)/x**3,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(a + b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^3}\,dx=\int \frac{\text{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^3}\,dx=\int \frac{\text{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{erf}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}}{x^3}\,dx$$

input
```
int((erf(a + b*x)*exp(c + d*x^2))/x^3,x)
```

output
```
int((erf(a + b*x)*exp(c + d*x^2))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^3}\,dx = e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx+a)}{x^3}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x^3,x)
```

output
```
e**c*int((e**(d*x**2)*erf(a + b*x))/x**3,x)
```

3.90 $\int e^{c+dx^2} x^4 \mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int e^{c+dx^2} x^4 \text{erf}(a+bx)\,dx &= -\frac{3be^{-a^2+c-2abx-(b^2-d)x^2}}{4(b^2-d)\,d^2\sqrt{\pi}} + \frac{a^2b^3e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^3\,d\sqrt{\pi}} \\ &\quad + \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2\,d\sqrt{\pi}} - \frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)^2\,d\sqrt{\pi}} \\ &\quad + \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x^2}{2(b^2-d)\,d\sqrt{\pi}} - \frac{3e^{c+dx^2}x\,\text{erf}(a+bx)}{4d^2} \\ &\quad + \frac{e^{c+dx^2}x^3\text{erf}(a+bx)}{2d} - \frac{3ab^2e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}\,d^2} \\ &\quad + \frac{a^3b^4e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{7/2}\,d} \\ &\quad + \frac{3ab^2e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{5/2}\,d} + \frac{3\text{Int}\left(e^{c+dx^2}\text{erf}(a+bx),x\right)}{4d^2}\end{aligned}$$

output
```
-3/4*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)/d^2/Pi^(1/2)+1/2*a^2*b^3*ex
p(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)^3/d/Pi^(1/2)+1/2*b*exp(-a^2+c-2*a*b*
x-(b^2-d)*x^2)/(b^2-d)^2/d/Pi^(1/2)-1/2*a*b^2*exp(-a^2+c-2*a*b*x-(b^2-d)*x
^2)*x/(b^2-d)^2/d/Pi^(1/2)+1/2*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)*x^2/(b^2-
d)/d/Pi^(1/2)-3/4*exp(d*x^2+c)*x*erf(b*x+a)/d^2+1/2*exp(d*x^2+c)*x^3*erf(b
*x+a)/d-3/4*a*b^2*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/
(b^2-d)^(3/2)/d^2+1/2*a^3*b^4*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^
2-d)^(1/2))/(b^2-d)^(7/2)/d+3/4*a*b^2*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d
)*x)/(b^2-d)^(1/2))/(b^2-d)^(5/2)/d+3/4*Defer(Int)(exp(d*x^2+c)*erf(b*x+a)
,x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx=\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^4*Erf[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^4*Erf[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{c+dx^2}\text{erf}(a+bx)\,dx$$

$\big\downarrow$ 6939

$$-\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}x^3dx}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\big\downarrow$ 2671

$$-\frac{b\left(\frac{\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\big\downarrow$ 2670

$$-\frac{b\left(\frac{-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\big\downarrow$ 2664

$$-\frac{b\left(\frac{-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\big\downarrow$ 2634

$$-\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2671}$$

$$-\frac{b\left(-\frac{ab\left(\frac{\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{2\left(b^2-d\right)}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2\cdots\right.}{\sqrt{b^2-\cdots}}\right.}{2\left(b^2-d\right)^{3/2}}}{b^2-\cdots}\cdots\right.}{\sqrt{\pi}d}\cdots$$

$$\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2664}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{2\left(b^2-d\right)}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\cdots}{2\left(b^2-d\right)^{3\cdots}}}{\cdots}\cdots\right.}{\sqrt{\pi}d}\cdots$$

$$\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\cdots}{2\left(b^2-d\right)^{3\cdots}}}{\cdots}\cdots\right.}{\sqrt{\pi}d}\cdots$$

$$\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2670}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+}}{2\left(b^2-d\right)}}{b^2-d}}{\sqrt{\pi}d}$$

$$\frac{3\int e^{dx^2+c}x^2\operatorname{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\operatorname{erf}(a+bx)}{2d}$$

$\downarrow$ 2664

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{-a^2-2abx-x^2}}{2\left(b^2-d\right)}}{b^2-d}}{\sqrt{\pi}d}$$

$$\frac{3\int e^{dx^2+c}x^2\operatorname{erf}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\operatorname{erf}(a+bx)}{2d}$$

$\downarrow$ 2634

$$-\frac{3\int e^{dx^2+c}x^2\text{erf}(a+bx)dx}{2d}-$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2(b^2-d)+c}}{2(b^2-d)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2(b^{2}\cdots}}{2(b^2-d)}\cdots\right)}{b^2-d}\cdots\right)}{\cdots}\cdots\right)}{\sqrt{\pi}d\cdots}$$

$$\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\downarrow$ 6939

$$-\frac{3\left(-\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2(b^2-d)+c}}{2(b^2-d)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2(b^{2}\cdots}}{2(b^2-d)}\cdots\right)}{b^2-d}\cdots\right)}{\cdots}\cdots\right)}{\sqrt{\pi}d\cdots}$$

$$\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\downarrow$ 2670

$$-\frac{3\left(-\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^{2}\right.}}{2\left(b^2-d\right)}\right.}{b^2-d}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{3\left(-\frac{b\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2\cdots\right.}}{2\left(b^2-d\right)}\cdots\right)}{b^2-d}\cdots\right)}{\sqrt{\pi}d\cdots}$$

$$\frac{x^3e^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \; 2634$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\operatorname{erf}(a+bx)dx}{2d}-\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\operatorname{erf}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2 \cdots\right.}}{2\left(b^2-d\right)}\cdots\right.}{b^2-d}\cdots\right.}{\cdots}\cdots\right)}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\operatorname{erf}(a+bx)}{2d}$$

$$\Big\downarrow \; 6933$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\operatorname{erf}(a+bx)dx}{2d}-\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\operatorname{erf}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2 \cdots\right.}}{2\left(b^2-d\right)}\cdots\right)}{b^2-d}\cdots\right)}{\cdots}\cdots\right)}{\sqrt{\pi}d\cdots}$$

$$\frac{x^3e^{c+dx^2}\operatorname{erf}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^4*Erf[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{d\,x^2+c}x^4\operatorname{erf}\left(bx+a\right)dx$$

input
```
int(exp(d*x^2+c)*x^4*erf(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^4*erf(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx=\int x^4\,\text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erf(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^4*erf(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**4*erf(b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx=\int x^4\,\text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erf(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^4*erf(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx=\int x^4\,\text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erf(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^4*erf(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erf}(a+bx)\,dx=\int x^4\,\text{erf}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^4*erf(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^4*erf(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int e^{c+dx^2} x^4 \operatorname{erf}(a+bx)\,dx = e^c \left(\int e^{d\,x^2} \operatorname{erf}(bx+a)\,x^4 dx \right)$$

input
```
int(exp(d*x^2+c)*x^4*erf(b*x+a),x)
```

output
```
e**c*int(e**(d*x**2)*erf(a + b*x)*x**4,x)
```

3.91 $\int e^{c+dx^2} x^2 \mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int e^{c+dx^2} x^2 \text{erf}(a+bx)\,dx = \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)\,d\sqrt{\pi}} + \frac{e^{c+dx^2}x\text{erf}(a+bx)}{2d} + \frac{ab^2e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}\,d} - \frac{\text{Int}\left(e^{c+dx^2}\text{erf}(a+bx),x\right)}{2d}$$

output
```
1/2*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)/d/Pi^(1/2)+1/2*exp(d*x^2+c)*
x*erf(b*x+a)/d+1/2*a*b^2*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^
(1/2))/(b^2-d)^(3/2)/d-1/2*Defer(Int)(exp(d*x^2+c)*erf(b*x+a),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2} x^2 \text{erf}(a+bx)\,dx = \int e^{c+dx^2} x^2 \text{erf}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^2*Erf[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^2*Erf[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.68 (sec), antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{c+dx^2} \text{erf}(a+bx) \, dx$$

$$\Big\downarrow \text{6939}$$

$$-\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-(b^2-d)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2(b^2-d)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{b\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{(ab+(b^2-d)x)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2(b^2-d)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}-\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2(b^2-d)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}$$

$\Big\downarrow$ 6933

$$-\frac{\int e^{dx^2+c}\text{erf}(a+bx)dx}{2d}-\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2(b^2-d)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erf}(a+bx)}{2d}$$

input

```
Int[E^(c + d*x^2)*x^2*Erf[a + b*x],x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{d\,x^2+c}x^2\,\text{erf}\,(bx+a)\,dx$$

input

```
int(exp(d*x^2+c)*x^2*erf(b*x+a),x)
```

output

```
int(exp(d*x^2+c)*x^2*erf(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx=\int x^2\,\text{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erf(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*erf(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 57.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx=e^c\int x^2e^{dx^2}\,\text{erf}\left(a+bx\right)dx$$

input
```
integrate(exp(d*x**2+c)*x**2*erf(b*x+a),x)
```

output
```
exp(c)*Integral(x**2*exp(d*x**2)*erf(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx=\int x^2\,\text{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erf(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*erf(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx=\int x^2\,\text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erf(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*erf(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erf}(a+bx)\,dx=\int x^2\,\text{erf}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^2*erf(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^2*erf(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int e^{c+dx^2} x^2 \operatorname{erf}(a+bx)\, dx = e^c \left(\int e^{d\,x^2} \operatorname{erf}(bx+a)\, x^2 dx \right)$$

input
```
int(exp(d*x^2+c)*x^2*erf(b*x+a),x)
```

output
```
e**c*int(e**(d*x**2)*erf(a + b*x)*x**2,x)
```

3.92 $\int e^{c+dx^2}\mathbf{erf}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int e^{c+dx^2}\operatorname{erf}(a+bx)\,dx=\operatorname{Int}\left(e^{c+dx^2}\operatorname{erf}(a+bx),x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erf(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}\operatorname{erf}(a+bx)\,dx=\int e^{c+dx^2}\operatorname{erf}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*Erf[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*Erf[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx^2}\text{erf}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6933}$$

$$\int e^{c+dx^2}\text{erf}(a+bx)dx$$

input
```
Int[E^(c + d*x^2)*Erf[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \mathrm{e}^{d\,x^2+c}\,\mathrm{erf}\left(bx+a\right)dx$$

input
```
int(exp(d*x^2+c)*erf(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*erf(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erf}(a+bx)\,dx=\int \text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a),x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erf}(a+bx)\,dx=e^c\int e^{dx^2}\,\text{erf}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x+a),x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erf}(a+bx)\,dx=\int \text{erf}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a),x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\operatorname{erf}(a+bx)\,dx=\int \operatorname{erf}(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a),x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\operatorname{erf}(a+bx)\,dx=\int \operatorname{erf}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(erf(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(erf(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{c+dx^2}\operatorname{erf}(a+bx)\,dx = e^c\left(\int e^{d\,x^2}\operatorname{erf}(bx+a)\,dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x+a),x)
```

output
```
e**c*int(e**(d*x**2)*erf(a + b*x),x)
```

3.93 $\int \frac{e^{c+dx^2}\textbf{erf}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^2}\,dx = -\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x} + \frac{2b\text{Int}\left(\frac{e^{-a^2+c-2abx+(-b^2+d)x^2}}{x},x\right)}{\sqrt{\pi}} + 2d\text{Int}\left(e^{c+dx^2}\operatorname{erf}(a+bx),x\right)$$

output
```
-exp(d*x^2+c)*erf(b*x+a)/x+2*b*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d)*x^2)
/x,x)/Pi^(1/2)+2*d*Defer(Int)(exp(d*x^2+c)*erf(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^2}\,dx = \int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^2}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^2,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \ 6945$$

$$\frac{2b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\operatorname{erf}(a+bx)dx-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}$$

$$\Big\downarrow \ 2673$$

$$\frac{2b\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\operatorname{erf}(a+bx)dx-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}$$

$$\Big\downarrow \ 6933$$

$$\frac{2b\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\operatorname{erf}(a+bx)dx-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x}$$

input
```
Int[(E^(c + d*x^2)*Erf[a + b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erf}\left(bx+a\right)}{x^2}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x^2,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^2}\,dx = \int \frac{\mathrm{erf}\left(bx+a\right)e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 8.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^2}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erf}\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x+a)/x**2,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^2}\,dx = \int \frac{\text{erf}\,(bx+a)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^2}\,dx = \int \frac{\text{erf}\,(bx+a)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{erf}(a+b\,x)\;\mathrm{e}^{d\,x^2+c}}{x^2}\,dx$$

input
```
int((erf(a + b*x)*exp(c + d*x^2))/x^2,x)
```

output
```
int((erf(a + b*x)*exp(c + d*x^2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^2}\,dx = e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx+a)}{x^2}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x^2,x)
```

output
```
e**c*int((e**(d*x**2)*erf(a + b*x))/x**2,x)
```

3.94 $\int \frac{e^{c+dx^2}\mathbf{erf}(a+bx)}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{x^4}\,dx =& -\frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x^2}+\frac{2ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x}\\&-\frac{e^{c+dx^2}\operatorname{erf}(a+bx)}{3x^3}-\frac{2de^{c+dx^2}\operatorname{erf}(a+bx)}{3x}\\&+\frac{2}{3}ab^2\sqrt{b^2-d}e^{c+\frac{a^2d}{b^2-d}}\operatorname{erf}\left(\frac{ab+(b^2-d)\,x}{\sqrt{b^2-d}}\right)\\&+\frac{4a^2b^3\text{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}}\\&-\frac{2b(b^2-d)\,\text{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}}\\&+\frac{4bd\text{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}}\\&+\frac{4}{3}d^2\text{Int}\left(e^{c+dx^2}\operatorname{erf}(a+bx),x\right)\end{aligned}$$

output
```
-1/3*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/Pi^(1/2)/x^2+2/3*a*b^2*exp(-a^2+c-2
*a*b*x-(b^2-d)*x^2)/Pi^(1/2)/x-1/3*exp(d*x^2+c)*erf(b*x+a)/x^3-2/3*d*exp(d
*x^2+c)*erf(b*x+a)/x+2/3*a*b^2*(b^2-d)^(1/2)*exp(c+a^2*d/(b^2-d))*erf((a*b
+(b^2-d)*x)/(b^2-d)^(1/2))+4/3*a^2*b^3*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2
+d)*x^2)/x,x)/Pi^(1/2)-2/3*b*(b^2-d)*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d
)*x^2)/x,x)/Pi^(1/2)+4/3*b*d*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d)*x^2)/x
,x)/Pi^(1/2)+4/3*d^2*Defer(Int)(exp(d*x^2+c)*erf(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^4}\,dx=\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^4}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^4,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erf[a + b*x])/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \text{ 6945}$$

$$\frac{2b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x^3}dx}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$\downarrow$ 2672

$$\frac{2b\left(-ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x^2}dx-\left(b^2-d\right)\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}+$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$\downarrow$ 2672

$$\frac{2b\Big(-\Big(\left(b^2-d\right)\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx\Big)-ab\Big(-2\left(b^2-d\right)\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2\cdots}}{x}}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$\downarrow$ 2664

$$\frac{2b\Big(-\Big(\left(b^2-d\right)\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx\Big)-ab\Big(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-2\left(b^2-d\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\cdots\right.}{b\cdots}}}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$\downarrow$ 2634

$$\frac{2b\Big(-ab\Big(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\Big)-\cdots}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$\downarrow$ 2673

$$\frac{2b\Big(-ab\Big(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\Big)-\cdots}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erf}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$\downarrow$ 6945

$$\frac{2b\left(-ab\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)-}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erf}(a+bx)dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{2673}$$

$$\frac{2b\left(-ab\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)-}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erf}(a+bx)dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{6933}$$

$$\frac{2b\left(-ab\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)-}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erf}(a+bx)dx-\frac{e^{c+dx^2}\text{erf}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erf}(a+bx)}{3x^3}$$

input
```
Int[(E^(c + d*x^2)*Erf[a + b*x])/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erf}\left(bx+a\right)}{x^4}dx$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x^4,x)
```

output
```
int(exp(d*x^2+c)*erf(b*x+a)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^4}\,dx = \int \frac{\mathrm{erf}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
integral(erf(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 76.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^4}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erf}\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(exp(d*x**2+c)*erf(b*x+a)/x**4,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erf(a + b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^4}\,dx = \int \frac{\text{erf}\,(bx+a)\,e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erf}(a+bx)}{x^4}\,dx = \int \frac{\text{erf}\,(bx+a)\,e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erf(b*x+a)/x^4,x, algorithm="giac")
```

output
```
integrate(erf(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^4}\,dx = \int \frac{\mathrm{erf}(a+b\,x)\ \mathrm{e}^{d\,x^2+c}}{x^4}\,dx$$

input
```
int((erf(a + b*x)*exp(c + d*x^2))/x^4,x)
```

output
```
int((erf(a + b*x)*exp(c + d*x^2))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\mathrm{erf}(a+bx)}{x^4}\,dx = e^c\left(\int \frac{e^{d\,x^2}\mathrm{erf}(bx+a)}{x^4}dx\right)$$

input
```
int(exp(d*x^2+c)*erf(b*x+a)/x^4,x)
```

output
```
e**c*int((e**(d*x**2)*erf(a + b*x))/x**4,x)
```

3.95 $\int \left(\frac{e^{-b^2x^2}\mathbf{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\mathbf{erf}(bx)}{x}\right)\,dx$

Optimal result

Integrand size = 40, antiderivative size = 62

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)\,dx=-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2x^2}-\sqrt{2}b^2\operatorname{erf}\left(\sqrt{2}bx\right)$$

output
```
-b/exp(2*b^2*x^2)/Pi^(1/2)/x-1/2*erf(b*x)/exp(b^2*x^2)/x^2-2^(1/2)*b^2*erf
(2^(1/2)*b*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)\,dx=-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{2x^2}-\sqrt{2}b^2\operatorname{erf}\left(\sqrt{2}bx\right)$$

input
```
Integrate[Erf[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erf[b*x])/(E^(b^2*x^2)*x),x]
```

output
```
-(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b
^2*Erf[Sqrt[2]*b*x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2e^{-b^2x^2}\text{erf}(bx)}{x}+\frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^{-b^2x^2}\text{erf}(bx)}{2x^2}-\sqrt{2}b^2\text{erf}\left(\sqrt{2}bx\right)-\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

input
```
Int[Erf[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erf[b*x])/(E^(b^2*x^2)*x),x]
```

output
```
-(b/(E^(2*b^2*x^2)*Sqrt[Pi]*x)) - Erf[b*x]/(2*E^(b^2*x^2)*x^2) - Sqrt[2]*b
^2*Erf[Sqrt[2]*b*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{-\frac{\text{erf}(bx)b\,\text{e}^{-b^2x^2}}{2x^2}+\frac{b^3\left(-\frac{\text{e}^{-2b^2x^2}}{bx}-\sqrt{2}\,\sqrt{\pi}\,\text{erf}\left(\sqrt{2}\,bx\right)\right)}{\sqrt{\pi}}}{b}$	67

input
```
int(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x,method=_RETURN
VERBOSE)
```

output
```
(-1/2*erf(b*x)*b/exp(b^2*x^2)/x^2+1/Pi^(1/2)*b^3*(-1/exp(b^2*x^2)^2/b/x-2^
(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)\,dx = -\frac{2\,\sqrt{2}\pi\sqrt{b^2}bx^2\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)+2\,\sqrt{\pi}bxe^{\left(-2\,b^2x^2\right)}+\pi\operatorname{erf}\left(bx\right)e^{\left(-b^2x^2\right)}}{2\,\pi x^2}$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x, algorit
hm="fricas")
```

output
```
-1/2*(2*sqrt(2)*pi*sqrt(b^2)*b*x^2*erf(sqrt(2)*sqrt(b^2)*x) + 2*sqrt(pi)*b
*x*e^(-2*b^2*x^2) + pi*erf(b*x)*e^(-b^2*x^2))/(pi*x^2)
```

Sympy [F]

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)\,dx = \int \frac{(b^2x^2+1)\,e^{-b^2x^2}\operatorname{erf}\left(bx\right)}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b**2*x**2)/x**3+b**2*erf(b*x)/exp(b**2*x**2)/x,x)
```

output
```
Integral((b**2*x**2 + 1)*exp(-b**2*x**2)*erf(b*x)/x**3, x)
```

Maxima [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erf}(bx)}{x}\right)\,dx=\int \frac{b^2\,\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}+\frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x, algorit
hm="maxima")
```

output
```
b*integrate(e^(-2*b^2*x^2)/x^2, x)/sqrt(pi) - 1/2*erf(b*x)*e^(-b^2*x^2)/x^
2
```

Giac [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erf}(bx)}{x}\right)\,dx=\int \frac{b^2\,\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}+\frac{\text{erf}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

input
```
integrate(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x, algorit
hm="giac")
```

output
```
integrate(b^2*erf(b*x)*e^(-b^2*x^2)/x + erf(b*x)*e^(-b^2*x^2)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \left(\frac{e^{-b^2x^2}\text{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erf}(bx)}{x}\right)\,dx=-\frac{\frac{\mathrm{e}^{-b^2\,x^2}\,\text{erf}(b\,x)}{2}+\frac{b\,x\,\mathrm{e}^{-2\,b^2\,x^2}}{\sqrt{\pi}}}{x^2}-\sqrt{2}\,b^2\,\text{erf}\left(\sqrt{2}\,b\,x\right)$$

input
```
int((exp(-b^2*x^2)*erf(b*x))/x^3 + (b^2*exp(-b^2*x^2)*erf(b*x))/x,x)
```

output
```
- ((exp(-b^2*x^2)*erf(b*x))/2 + (b*x*exp(-2*b^2*x^2))/pi^(1/2))/x^2 - 2^(1
/2)*b^2*erf(2^(1/2)*b*x)
```

Reduce [F]

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erf}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\operatorname{erf}(bx)}{x}\right)dx=\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^3}dx+\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x}dx\right)b^2$$

input
```
int(erf(b*x)/exp(b^2*x^2)/x^3+b^2*erf(b*x)/exp(b^2*x^2)/x,x)
```

output
```
int(erf(b*x)/(e**(b**2*x**2)*x**3),x) + int(erf(b*x)/(e**(b**2*x**2)*x),x)
*b**2
```

3.96 $\int \mathbf{erf}(bx) \sin\left(c + ib^2x^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \operatorname{erf}(bx) \sin\left(c + ib^2x^2\right) \, dx = -\frac{ie^{ic}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b} + \frac{ibe^{-ic}x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*I*exp(I*c)*Pi^(1/2)*erf(b*x)^2/b+1/2*I*b*x^2*hypergeom([1, 1],[3/2, 2
],b^2*x^2)/exp(I*c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \operatorname{erf}(bx) \sin\left(c + ib^2x^2\right) \, dx = \frac{(\cos(c) - i\sin(c))\left(4ib^2x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right) + \pi\operatorname{erf}(bx)^2(-i\cos(2c) + \sin(2c))\right)}{8b\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]*Sin[c + I*b^2*x^2],x]
```

output
```
((Cos[c] - I*Sin[c])*((4*I)*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^
2*x^2] + Pi*Erf[b*x]^2*((-I)*Cos[2*c] + Sin[2*c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6958, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx$$

$$\Big\downarrow \ 6958$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erf}(bx)dx-\frac{1}{2}i\int e^{ic-b^2x^2}\text{erf}(bx)dx$$

$$\Big\downarrow \ 6927$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erf}(bx)dx-\frac{i\sqrt{\pi}e^{ic}\int \text{erf}(bx)d\text{erf}(bx)}{4b}$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erf}(bx)dx-\frac{i\sqrt{\pi}e^{ic}\text{erf}(bx)^2}{8b}$$

$$\Big\downarrow \ 6930$$

$$\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{ic}\text{erf}(bx)^2}{8b}$$

input
```
Int[Erf[b*x]*Sin[c + I*b^2*x^2],x]
```

output
```
((-1/8*I)*E^(I*c)*Sqrt[Pi]*Erf[b*x]^2)/b + ((I/2)*b*x^2*HypergeometricPFQ[
{1, 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6958
```
Int[Erf[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[I/2   Int[
E^((-I)*c - I*d*x^2)*Erf[b*x], x], x] - Simp[I/2   Int[E^(I*c + I*d*x^2)*Er
f[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \operatorname{erf}(bx) \sin\left(ib^2x^2 + c\right) dx$$

input
```
int(erf(b*x)*sin(c+I*b^2*x^2),x)
```

output
```
int(erf(b*x)*sin(c+I*b^2*x^2),x)
```

Fricas [F]

$$\int \operatorname{erf}(bx) \sin\left(c + ib^2x^2\right)\,dx = \int \operatorname{erf}(bx) \sin\left(i\,b^2x^2 + c\right)\,dx$$

input
```
integrate(erf(b*x)*sin(c+I*b^2*x^2),x, algorithm="fricas")
```

output
```
integral(1/2*(-I*erf(b*x)*e^(-2*b^2*x^2 + 2*I*c) + I*erf(b*x))*e^(b^2*x^2
- I*c), x)
```

Sympy [F]

$$\int \operatorname{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \sin\left(ib^2x^2+c\right)\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(erf(b*x)*sin(c+I*b**2*x**2),x)
```

output
```
Integral(sin(I*b**2*x**2 + c)*erf(b*x), x)
```

Maxima [F]

$$\int \operatorname{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \operatorname{erf}\left(bx\right)\sin\left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erf(b*x)*sin(c+I*b^2*x^2),x, algorithm="maxima")
```

output
```
-1/8*I*sqrt(pi)*cos(c)*erf(b*x)^2/b + 1/8*sqrt(pi)*erf(b*x)^2*sin(c)/b + 1
/2*I*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erf(b*x)*e^
(b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \operatorname{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \operatorname{erf}\left(bx\right)\sin\left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erf(b*x)*sin(c+I*b^2*x^2),x, algorithm="giac")
```

output
```
integrate(erf(b*x)*sin(I*b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \sin\left(b^2\,x^2\,1\mathrm{i}+c\right)\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(sin(c + b^2*x^2*1i)*erf(b*x),x)
```

output
```
int(sin(c + b^2*x^2*1i)*erf(b*x), x)
```

Reduce [F]

$$\int \operatorname{erf}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \operatorname{erf}(bx)\sin\left(b^2i\,x^2+c\right)dx$$

input
```
int(erf(b*x)*sin(c+I*b^2*x^2),x)
```

output
```
int(erf(b*x)*sin(b**2*i*x**2 + c),x)
```

3.97 $\int \mathrm{erf}(bx) \sin\left(c - ib^2x^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \mathrm{erf}(bx) \sin\left(c - ib^2x^2\right) \, dx = \frac{ie^{-ic}\sqrt{\pi}\mathrm{erf}(bx)^2}{8b} - \frac{ibe^{ic}x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*I*Pi^(1/2)*erf(b*x)^2/b/exp(I*c)-1/2*I*b*exp(I*c)*x^2*hypergeom([1, 1]
,[3/2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \mathrm{erf}(bx) \sin\left(c - ib^2x^2\right) \, dx = \frac{(i\cos(c) + \sin(c))\left(\pi\mathrm{erf}(bx)^2 - 4b^2x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cos(2c) + i\sin(2c))\right)}{8b\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]*Sin[c - I*b^2*x^2],x]
```

output
```
((I*Cos[c] + Sin[c])*(Pi*Erf[b*x]^2 - 4*b^2*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, b^2*x^2]*(Cos[2*c] + I*Sin[2*c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6958, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{6958}$$

$$\frac{1}{2}i\int e^{-b^2x^2-ic}\operatorname{erf}(bx)dx-\frac{1}{2}i\int e^{b^2x^2+ic}\operatorname{erf}(bx)dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{i\sqrt{\pi}e^{-ic}\int \operatorname{erf}(bx)d\operatorname{erf}(bx)}{4b}-\frac{1}{2}i\int e^{b^2x^2+ic}\operatorname{erf}(bx)dx$$

$$\Big\downarrow \text{15}$$

$$\frac{i\sqrt{\pi}e^{-ic}\operatorname{erf}(bx)^2}{8b}-\frac{1}{2}i\int e^{b^2x^2+ic}\operatorname{erf}(bx)dx$$

$$\Big\downarrow \text{6930}$$

$$\frac{i\sqrt{\pi}e^{-ic}\operatorname{erf}(bx)^2}{8b}-\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

input
```
Int[Erf[b*x]*Sin[c - I*b^2*x^2],x]
```

output
```
((I/8)*Sqrt[Pi]*Erf[b*x]^2)/(b*E^(I*c)) - ((I/2)*b*E^(I*c)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6958
```
Int[Erf[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[I/2   Int[
E^((-I)*c - I*d*x^2)*Erf[b*x], x], x] - Simp[I/2   Int[E^(I*c + I*d*x^2)*Er
f[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int -\operatorname{erf}(bx)\sin\left(ib^2x^2-c\right)dx$$

input
```
int(-erf(b*x)*sin(-c+I*b^2*x^2),x)
```

output
```
int(-erf(b*x)*sin(-c+I*b^2*x^2),x)
```

Fricas [F]

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erf}(bx)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erf(b*x)*sin(-c+I*b^2*x^2),x, algorithm="fricas")
```

output
```
integral(1/2*(I*erf(b*x)*e^(-2*b^2*x^2 - 2*I*c) - I*erf(b*x))*e^(b^2*x^2 +
 I*c), x)
```

Sympy [F]

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\int \sin\left(ib^2x^2-c\right)\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(-erf(b*x)*sin(-c+I*b**2*x**2),x)
```

output
```
-Integral(sin(I*b**2*x**2 - c)*erf(b*x), x)
```

Maxima [F]

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erf}\left(bx\right)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erf(b*x)*sin(-c+I*b^2*x^2),x, algorithm="maxima")
```

output
```
1/8*I*sqrt(pi)*cos(c)*erf(b*x)^2/b + 1/8*sqrt(pi)*erf(b*x)^2*sin(c)/b - 1/
2*I*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erf(b*x)*e^(
b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erf}\left(bx\right)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erf(b*x)*sin(-c+I*b^2*x^2),x, algorithm="giac")
```

output
```
integrate(-erf(b*x)*sin(I*b^2*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int \sin\left(c-b^2\,x^2\,1\mathrm{i}\right)\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(sin(c - b^2*x^2*1i)*erf(b*x),x)
```

output
```
int(sin(c - b^2*x^2*1i)*erf(b*x), x)
```

Reduce [F]

$$\int \operatorname{erf}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\left(\int \operatorname{erf}(bx)\sin\left(b^2i\,x^2-c\right)dx\right)$$

input
```
int(-erf(b*x)*sin(-c+I*b^2*x^2),x)
```

output
```
- int(erf(b*x)*sin(b**2*i*x**2 - c),x)
```

3.98 $\int \cos(c + ib^2x^2)\,\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \cos\left(c+ib^2x^2\right)\mathrm{erf}(bx)\,dx=\frac{e^{ic}\sqrt{\pi}\,\mathrm{erf}(bx)^2}{8b}+\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*exp(I*c)*Pi^(1/2)*erf(b*x)^2/b+1/2*b*x^2*hypergeom([1, 1],[3/2, 2],b^2
*x^2)/exp(I*c)/Pi^(1/2)
```

Mathematica [F]

$$\int \cos\left(c+ib^2x^2\right)\mathrm{erf}(bx)\,dx=\int \cos\left(c+ib^2x^2\right)\mathrm{erf}(bx)\,dx$$

input
```
Integrate[Cos[c + I*b^2*x^2]*Erf[b*x],x]
```

output
```
Integrate[Cos[c + I*b^2*x^2]*Erf[b*x], x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6961, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)\cos\left(c+ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{6961}$$

$$\frac{1}{2}\int e^{ic-b^2x^2}\text{erf}(bx)dx+\frac{1}{2}\int e^{b^2x^2-ic}\text{erf}(bx)dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{1}{2}\int e^{b^2x^2-ic}\text{erf}(bx)dx+\frac{\sqrt{\pi}e^{ic}\int \text{erf}(bx)d\text{erf}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2-ic}\text{erf}(bx)dx+\frac{\sqrt{\pi}e^{ic}\text{erf}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{ic}\text{erf}(bx)^2}{8b}$$

input
```
Int[Cos[c + I*b^2*x^2]*Erf[b*x],x]
```

output
```
(E^(I*c)*Sqrt[Pi]*Erf[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, b^2*x^2])/(2*E^(I*c)*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6961
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int[
E^((-I)*c - I*d*x^2)*Erf[b*x], x], x] + Simp[1/2   Int[E^(I*c + I*d*x^2)*Er
f[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \cos \left(i b^{2} x^{2}+c\right) \operatorname{erf}\left(b x\right)dx$$

input
```
int(cos(c+I*b^2*x^2)*erf(b*x),x)
```

output
```
int(cos(c+I*b^2*x^2)*erf(b*x),x)
```

Fricas [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erf}(bx) \, dx=\int { \cos \left(i \, b^{2} x^{2} + c\right) \operatorname{erf}\left(b x\right) \,d x}$$

input
```
integrate(cos(c+I*b^2*x^2)*erf(b*x),x, algorithm="fricas")
```

output
```
integral(1/2*(erf(b*x)*e^(-2*b^2*x^2 + 2*I*c) + erf(b*x))*e^(b^2*x^2 - I*c
), x)
```

Sympy [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cos \left(ib^2x^2+c\right) \operatorname{erf}\left(bx\right)dx$$

input
```
integrate(cos(c+I*b**2*x**2)*erf(b*x),x)
```

output
```
Integral(cos(I*b**2*x**2 + c)*erf(b*x), x)
```

Maxima [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cos \left(i\,b^2x^2+c\right) \operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erf(b*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*cos(c)*erf(b*x)^2/b + 1/8*I*sqrt(pi)*erf(b*x)^2*sin(c)/b + 1/
2*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) - 1/2*I*integrate(erf(b*x)*e^(
b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cos \left(i\,b^2x^2+c\right) \operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erf(b*x),x, algorithm="giac")
```

output
```
integrate(cos(I*b^2*x^2 + c)*erf(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erf}(bx) \, dx = \int \cos \left(b^2\, x^2\, 1\mathrm{i}+c\right)\, \operatorname{erf}(b\, x)\, dx$$

input
```
int(cos(c + b^2*x^2*1i)*erf(b*x),x)
```

output
```
int(cos(c + b^2*x^2*1i)*erf(b*x), x)
```

Reduce [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erf}(bx) \, dx = \int \cos \left(b^2 i\, x^2+c\right) \operatorname{erf}(bx) \, dx$$

input
```
int(cos(c+I*b^2*x^2)*erf(b*x),x)
```

output
```
int(cos(b**2*i*x**2 + c)*erf(b*x),x)
```

3.99 $\int \cos(c - ib^2x^2) \,\mathbf{erf}(bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \cos(c - ib^2x^2) \operatorname{erf}(bx)\, dx = \frac{e^{-ic}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b} + \frac{be^{ic}x^2\, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*Pi^(1/2)*erf(b*x)^2/b/exp(I*c)+1/2*b*exp(I*c)*x^2*hypergeom([1, 1],[3/
2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [F]

$$\int \cos(c - ib^2x^2) \operatorname{erf}(bx)\, dx = \int \cos(c - ib^2x^2) \operatorname{erf}(bx)\, dx$$

input
```
Integrate[Cos[c - I*b^2*x^2]*Erf[b*x],x]
```

output
```
Integrate[Cos[c - I*b^2*x^2]*Erf[b*x], x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6961, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)\cos\left(c-ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{6961}$$

$$\frac{1}{2}\int e^{-b^2x^2-ic}\text{erf}(bx)dx+\frac{1}{2}\int e^{b^2x^2+ic}\text{erf}(bx)dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{1}{2}\int e^{b^2x^2+ic}\text{erf}(bx)dx+\frac{\sqrt{\pi}e^{-ic}\int \text{erf}(bx)d\text{erf}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2+ic}\text{erf}(bx)dx+\frac{\sqrt{\pi}e^{-ic}\text{erf}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-ic}\text{erf}(bx)^2}{8b}$$

input
```
Int[Cos[c - I*b^2*x^2]*Erf[b*x],x]
```

output
```
(Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^(I*c)) + (b*E^(I*c)*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6961
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int[
E^((-I)*c - I*d*x^2)*Erf[b*x], x], x] + Simp[1/2   Int[E^(I*c + I*d*x^2)*Er
f[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \cos \left(i b^2 x^2 - c\right) \operatorname{erf}(bx)\, dx$$

input
```
int(cos(-c+I*b^2*x^2)*erf(b*x),x)
```

output
```
int(cos(-c+I*b^2*x^2)*erf(b*x),x)
```

Fricas [F]

$$\int \cos \left(c - i b^2 x^2\right) \operatorname{erf}(bx)\, dx = \int \cos \left(i\, b^2 x^2 - c\right) \operatorname{erf}(bx)\, dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erf(b*x),x, algorithm="fricas")
```

output
```
integral(1/2*(erf(b*x)*e^(-2*b^2*x^2 - 2*I*c) + erf(b*x))*e^(b^2*x^2 + I*c
), x)
```

Sympy [F]

$$\int \cos\left(c - ib^2x^2\right)\operatorname{erf}(bx)\,dx = \int \cos\left(ib^2x^2 - c\right)\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(cos(-c+I*b**2*x**2)*erf(b*x),x)
```

output
```
Integral(cos(I*b**2*x**2 - c)*erf(b*x), x)
```

Maxima [F]

$$\int \cos\left(c - ib^2x^2\right)\operatorname{erf}(bx)\,dx = \int \cos\left(i\,b^2x^2 - c\right)\operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erf(b*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*cos(c)*erf(b*x)^2/b - 1/8*I*sqrt(pi)*erf(b*x)^2*sin(c)/b + 1/
2*cos(c)*integrate(erf(b*x)*e^(b^2*x^2), x) + 1/2*I*integrate(erf(b*x)*e^(
b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \cos\left(c - ib^2x^2\right)\operatorname{erf}(bx)\,dx = \int \cos\left(i\,b^2x^2 - c\right)\operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erf(b*x),x, algorithm="giac")
```

output
```
integrate(cos(I*b^2*x^2 - c)*erf(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erf}(bx)\, dx = \int \cos\left(c - b^2\, x^2\, 1\mathrm{i}\right)\, \operatorname{erf}(b\, x)\, dx$$

input
```
int(cos(c - b^2*x^2*1i)*erf(b*x),x)
```

output
```
int(cos(c - b^2*x^2*1i)*erf(b*x), x)
```

Reduce [F]

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erf}(bx)\, dx = \int \cos\left(b^2 i\, x^2 - c\right) \operatorname{erf}(bx)\, dx$$

input
```
int(cos(-c+I*b^2*x^2)*erf(b*x),x)
```

output
```
int(cos(b**2*i*x**2 - c)*erf(b*x),x)
```

3.100 $\int \mathbf{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx=-\frac{e^{-c}\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}+\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*Pi^(1/2)*erf(b*x)^2/b/exp(c)+1/2*b*exp(c)*x^2*hypergeom([1, 1],[3/2,
2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx=\frac{\pi\operatorname{erf}(bx)^2(-\cosh(c)+\sinh(c))+4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cosh(c)+\sinh(c))}{8b\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]*Sinh[c + b^2*x^2],x]
```

output
```
(Pi*Erf[b*x]^2*(-Cosh[c] + Sinh[c]) + 4*b^2*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c]))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6964, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

$$\Big\downarrow \text{6964}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erf}(bx)dx-\frac{1}{2}\int e^{-b^2x^2-c}\text{erf}(bx)dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erf}(bx)dx-\frac{\sqrt{\pi}e^{-c}\int \text{erf}(bx)d\text{erf}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erf}(bx)dx-\frac{\sqrt{\pi}e^{-c}\text{erf}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{erf}(bx)^2}{8b}$$

input
```
Int[Erf[b*x]*Sinh[c + b^2*x^2],x]
```

output
```
-1/8*(Sqrt[Pi]*Erf[b*x]^2)/(b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6964
```
Int[Erf[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[1/2   Int
[E^(c + d*x^2)*Erf[b*x], x], x] - Simp[1/2   Int[E^(-c - d*x^2)*Erf[b*x], x
], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \operatorname{erf}(bx)\sinh\left(b^2x^2+c\right)dx$$

input
```
int(erf(b*x)*sinh(b^2*x^2+c),x)
```

output
```
int(erf(b*x)*sinh(b^2*x^2+c),x)
```

Fricas [F]

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \operatorname{erf}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erf(b*x)*sinh(b^2*x^2+c),x, algorithm="fricas")
```

output
```
integral(erf(b*x)*sinh(b^2*x^2 + c), x)
```

Sympy [F]

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \sinh\left(b^2x^2+c\right)\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(erf(b*x)*sinh(b**2*x**2+c),x)
```

output
```
Integral(sinh(b**2*x**2 + c)*erf(b*x), x)
```

Maxima [F]

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \operatorname{erf}\left(bx\right)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erf(b*x)*sinh(b^2*x^2+c),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*erf(b*x)^2*e^(-c)/b + 1/2*integrate(erf(b*x)*e^(b^2*x^2 + c)
, x)
```

Giac [F]

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \operatorname{erf}\left(bx\right)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erf(b*x)*sinh(b^2*x^2+c),x, algorithm="giac")
```

output
```
integrate(erf(b*x)*sinh(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx = \int \sinh\left(b^2\,x^2+c\right)\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(sinh(c + b^2*x^2)*erf(b*x),x)
```

output
```
int(sinh(c + b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int \operatorname{erf}(bx)\sinh\left(c+b^2x^2\right)\,dx = \int \operatorname{erf}(bx)\sinh\left(b^2x^2+c\right)dx$$

input
```
int(erf(b*x)*sinh(b^2*x^2+c),x)
```

output
```
int(erf(b*x)*sinh(b**2*x**2 + c),x)
```

3.101 $\int \mathbf{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx=\frac{e^c\sqrt{\pi}\operatorname{erf}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*exp(c)*Pi^(1/2)*erf(b*x)^2/b-1/2*b*x^2*hypergeom([1, 1],[3/2, 2],b^2*x
^2)/exp(c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx$$
$$=\frac{(\cosh(c)-\sinh(c))\left(-4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)+\pi\operatorname{erf}(bx)^2(\cosh(2c)+\sinh(2c))\right)}{8b\sqrt{\pi}}$$

input
```
Integrate[Erf[b*x]*Sinh[c - b^2*x^2],x]
```

output
```
((Cosh[c] - Sinh[c])*(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x
^2] + Pi*Erf[b*x]^2*(Cosh[2*c] + Sinh[2*c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6964, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6964}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erf}(bx)dx-\frac{1}{2}\int e^{b^2x^2-c}\text{erf}(bx)dx$$

$$\Big\downarrow \text{ 6927}$$

$$\frac{\sqrt{\pi}e^c\int \text{erf}(bx)d\text{erf}(bx)}{4b}-\frac{1}{2}\int e^{b^2x^2-c}\text{erf}(bx)dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}e^c\text{erf}(bx)^2}{8b}-\frac{1}{2}\int e^{b^2x^2-c}\text{erf}(bx)dx$$

$$\Big\downarrow \text{ 6930}$$

$$\frac{\sqrt{\pi}e^c\text{erf}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

input
```
Int[Erf[b*x]*Sinh[c - b^2*x^2],x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^2)/(8*b) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6964
```
Int[Erf[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[1/2   Int
[E^(c + d*x^2)*Erf[b*x], x], x] - Simp[1/2   Int[E^(-c - d*x^2)*Erf[b*x], x
], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int -\operatorname{erf}(bx)\sinh\left(b^2x^2-c\right)dx$$

input
```
int(-erf(b*x)*sinh(b^2*x^2-c),x)
```

output
```
int(-erf(b*x)*sinh(b^2*x^2-c),x)
```

Fricas [F]

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int -\operatorname{erf}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erf(b*x)*sinh(b^2*x^2-c),x, algorithm="fricas")
```

output
```
integral(-erf(b*x)*sinh(b^2*x^2 - c), x)
```

Sympy [F]

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx = -\int \sinh\left(b^2x^2-c\right)\operatorname{erf}\left(bx\right)dx$$

input
```
integrate(-erf(b*x)*sinh(b**2*x**2-c),x)
```

output
```
-Integral(sinh(b**2*x**2 - c)*erf(b*x), x)
```

Maxima [F]

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx = \int -\operatorname{erf}\left(bx\right)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erf(b*x)*sinh(b^2*x^2-c),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*erf(b*x)^2*e^c/b - 1/2*integrate(erf(b*x)*e^(b^2*x^2 - c), x)
```

Giac [F]

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx = \int -\operatorname{erf}\left(bx\right)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erf(b*x)*sinh(b^2*x^2-c),x, algorithm="giac")
```

output
```
integrate(-erf(b*x)*sinh(b^2*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int \sinh\left(c-b^2\,x^2\right)\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(sinh(c - b^2*x^2)*erf(b*x),x)
```

output
```
int(sinh(c - b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int \operatorname{erf}(bx)\sinh\left(c-b^2x^2\right)\,dx=-\left(\int \operatorname{erf}(bx)\sinh\left(b^2x^2-c\right)dx\right)$$

input
```
int(-erf(b*x)*sinh(b^2*x^2-c),x)
```

output
```
- int(erf(b*x)*sinh(b**2*x**2 - c),x)
```

3.102 $\int \cosh\left(c+b^2x^2\right) \mathbf{erf}(bx)\, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \cosh\left(c+b^2x^2\right) \text{erf}(bx)\, dx = \frac{e^{-c}\sqrt{\pi}\text{erf}(bx)^2}{8b} + \frac{be^c x^2\, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*Pi^(1/2)*erf(b*x)^2/b/exp(c)+1/2*b*exp(c)*x^2*hypergeom([1, 1],[3/2, 2
],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \cosh\left(c+b^2x^2\right) \text{erf}(bx)\, dx = \frac{-4b^2x^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)(\cosh(c)+\sinh(c)) + \pi\text{erf}(bx)(\text{erf}(bx)(\cosh(c)-\sinh(c)) + 2\text{erfi}(bx)(\cosh}{8b\sqrt{\pi}}$$

input
```
Integrate[Cosh[c + b^2*x^2]*Erf[b*x],x]
```

output
```
(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]*(Cosh[c] + Sin
h[c]) + Pi*Erf[b*x]*(Erf[b*x]*(Cosh[c] - Sinh[c]) + 2*Erfi[b*x]*(Cosh[c] +
 Sinh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6967, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erf}(bx)\cosh\left(b^2x^2+c\right)\,dx$$

$$\Big\downarrow \ 6967$$

$$\frac{1}{2}\int e^{-b^2x^2-c}\operatorname{erf}(bx)dx+\frac{1}{2}\int e^{b^2x^2+c}\operatorname{erf}(bx)dx$$

$$\Big\downarrow \ 6927$$

$$\frac{1}{2}\int e^{b^2x^2+c}\operatorname{erf}(bx)dx+\frac{\sqrt{\pi}e^{-c}\int \operatorname{erf}(bx)d\operatorname{erf}(bx)}{4b}$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}\int e^{b^2x^2+c}\operatorname{erf}(bx)dx+\frac{\sqrt{\pi}e^{-c}\operatorname{erf}(bx)^2}{8b}$$

$$\Big\downarrow \ 6930$$

$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\operatorname{erf}(bx)^2}{8b}$$

input
```
Int[Cosh[c + b^2*x^2]*Erf[b*x],x]
```

output
```
(Sqrt[Pi]*Erf[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, b^2*x^2])/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6967
```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int
[E^(c + d*x^2)*Erf[b*x], x], x] + Simp[1/2   Int[E^(-c - d*x^2)*Erf[b*x], x
], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \cosh \left(b^2 x^2+c\right) \operatorname{erf}(b x)\, dx$$

input
```
int(cosh(b^2*x^2+c)*erf(b*x),x)
```

output
```
int(cosh(b^2*x^2+c)*erf(b*x),x)
```

Fricas [F]

$$\int \cosh \left(c+b^2 x^2\right) \operatorname{erf}(b x)\, dx=\int \cosh \left(b^2 x^2+c\right) \operatorname{erf}(b x)\, dx$$

input
```
integrate(cosh(b^2*x^2+c)*erf(b*x),x, algorithm="fricas")
```

output
```
integral(cosh(b^2*x^2 + c)*erf(b*x), x)
```

Sympy [F]

$$\int \cosh \left(c+b^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cosh \left(b^2x^2+c\right) \operatorname{erf}\left(bx\right)dx$$

input
```
integrate(cosh(b**2*x**2+c)*erf(b*x),x)
```

output
```
Integral(cosh(b**2*x**2 + c)*erf(b*x), x)
```

Maxima [F]

$$\int \cosh \left(c+b^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cosh \left(b^2x^2+c\right) \operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erf(b*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*erf(b*x)^2*e^(-c)/b + 1/2*integrate(erf(b*x)*e^(b^2*x^2 + c),
 x)
```

Giac [F]

$$\int \cosh \left(c+b^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cosh \left(b^2x^2+c\right) \operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erf(b*x),x, algorithm="giac")
```

output
```
integrate(cosh(b^2*x^2 + c)*erf(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erf}(bx)\,dx = \int \cosh\left(b^2\,x^2+c\right)\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(cosh(c + b^2*x^2)*erf(b*x),x)
```

output
```
int(cosh(c + b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erf}(bx)\,dx = \int \cosh\left(b^2x^2+c\right)\operatorname{erf}(bx)\,dx$$

input
```
int(cosh(b^2*x^2+c)*erf(b*x),x)
```

output
```
int(cosh(b**2*x**2 + c)*erf(b*x),x)
```

3.103 $\int \cosh(c-b^2x^2)\,\mathbf{erf}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int \cosh(c-b^2x^2)\,\mathrm{erf}(bx)\,dx = \frac{e^c\sqrt{\pi}\mathrm{erf}(bx)^2}{8b} + \frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*exp(c)*Pi^(1/2)*erf(b*x)^2/b+1/2*b*x^2*hypergeom([1, 1],[3/2, 2],b^2*x
^2)/exp(c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \cosh(c-b^2x^2)\,\mathrm{erf}(bx)\,dx$$
$$= \frac{4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)(-\cosh(c)+\sinh(c))+\pi\mathrm{erf}(bx)(2\mathrm{erfi}(bx)(\cosh(c)-\sinh(c))+\mathrm{erf}(bx)(\cosh}{8b\sqrt{\pi}}$$

input
```
Integrate[Cosh[c - b^2*x^2]*Erf[b*x],x]
```

output
```
(4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]*(-Cosh[c] + Sin
h[c]) + Pi*Erf[b*x]*(2*Erfi[b*x]*(Cosh[c] - Sinh[c]) + Erf[b*x]*(Cosh[c] +
 Sinh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6967, 6927, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erf}(bx)\cosh\left(c-b^2x^2\right)\,dx$$

$$\Big\downarrow \text{6967}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\operatorname{erf}(bx)dx+\frac{1}{2}\int e^{b^2x^2-c}\operatorname{erf}(bx)dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{1}{2}\int e^{b^2x^2-c}\operatorname{erf}(bx)dx+\frac{\sqrt{\pi}e^c\int \operatorname{erf}(bx)d\operatorname{erf}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2-c}\operatorname{erf}(bx)dx+\frac{\sqrt{\pi}e^c\operatorname{erf}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^c\operatorname{erf}(bx)^2}{8b}$$

input
```
Int[Cosh[c - b^2*x^2]*Erf[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erf[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2
}, b^2*x^2])/(2*E^c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6967
```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erf[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int
[E^(c + d*x^2)*Erf[b*x], x], x] + Simp[1/2   Int[E^(-c - d*x^2)*Erf[b*x], x
], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \cosh\left(b^2x^2-c\right)\operatorname{erf}\left(bx\right)dx$$

input
```
int(cosh(b^2*x^2-c)*erf(b*x),x)
```

output
```
int(cosh(b^2*x^2-c)*erf(b*x),x)
```

Fricas [F]

$$\int \cosh\left(c-b^2x^2\right)\operatorname{erf}(bx)\,dx = \int \cosh\left(b^2x^2-c\right)\operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erf(b*x),x, algorithm="fricas")
```

output
```
integral(cosh(b^2*x^2 - c)*erf(b*x), x)
```

Sympy [F]

$$\int \cosh \left(c-b^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cosh \left(b^2x^2-c\right) \operatorname{erf}\left(bx\right)dx$$

input
```
integrate(cosh(b**2*x**2-c)*erf(b*x),x)
```

output
```
Integral(cosh(b**2*x**2 - c)*erf(b*x), x)
```

Maxima [F]

$$\int \cosh \left(c-b^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cosh \left(b^2x^2-c\right) \operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erf(b*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*erf(b*x)^2*e^c/b + 1/2*integrate(erf(b*x)*e^(b^2*x^2 - c), x)
```

Giac [F]

$$\int \cosh \left(c-b^2x^2\right) \operatorname{erf}(bx)\,dx=\int \cosh \left(b^2x^2-c\right) \operatorname{erf}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erf(b*x),x, algorithm="giac")
```

output
```
integrate(cosh(b^2*x^2 - c)*erf(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(c-b^2x^2\right)\operatorname{erf}(bx)\,dx=\int \cosh\left(c-b^2\,x^2\right)\,\operatorname{erf}(b\,x)\,dx$$

input
```
int(cosh(c - b^2*x^2)*erf(b*x),x)
```

output
```
int(cosh(c - b^2*x^2)*erf(b*x), x)
```

Reduce [F]

$$\int \cosh\left(c-b^2x^2\right)\operatorname{erf}(bx)\,dx=\int \cosh\left(b^2x^2-c\right)\operatorname{erf}(bx)\,dx$$

input
```
int(cosh(b^2*x^2-c)*erf(b*x),x)
```

output
```
int(cosh(b**2*x**2 - c)*erf(b*x),x)
```

3.104 $\int x^5 \mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int x^5 \operatorname{erfc}(bx)\,dx = -\frac{5e^{-b^2x^2}x}{8b^5\sqrt{\pi}} - \frac{5e^{-b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{5\operatorname{erf}(bx)}{16b^6} + \frac{1}{6}x^6\operatorname{erfc}(bx)$$

output
```
-5/8*x/b^5/exp(b^2*x^2)/Pi^(1/2)-5/12*x^3/b^3/exp(b^2*x^2)/Pi^(1/2)-1/6*x^
5/b/exp(b^2*x^2)/Pi^(1/2)+5/16*erf(b*x)/b^6+1/6*x^6*erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int x^5 \operatorname{erfc}(bx)\,dx = \frac{1}{48}\left(-\frac{2e^{-b^2x^2}x(15+10b^2x^2+4b^4x^4)}{b^5\sqrt{\pi}} + \frac{15\operatorname{erf}(bx)}{b^6} + 8x^6\operatorname{erfc}(bx)\right)$$

input
```
Integrate[x^5*Erfc[b*x],x]
```

output
```
((-2*x*(15 + 10*b^2*x^2 + 4*b^4*x^4))/(b^5*E^(b^2*x^2)*Sqrt[Pi]) + (15*Erf
[b*x])/b^6 + 8*x^6*Erfc[b*x])/48
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6916, 2641, 2641, 2641, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6916}$$

$$\frac{b\int e^{-b^2x^2}x^6dx}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{5\int e^{-b^2x^2}x^4dx}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2dx}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}dx}{2b^2}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(\frac{5\left(\frac{3\left(\frac{\sqrt{\pi}\operatorname{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\operatorname{erfc}(bx)$$

input
```
Int[x^5*Erfc[b*x],x]
```

output
```
(b*(-1/2*x^5/(b^2*E^(b^2*x^2)) + (5*(-1/2*x^3/(b^2*E^(b^2*x^2)) + (3*(-1/2
*x/(b^2*E^(b^2*x^2)) + (Sqrt[Pi]*Erf[b*x])/(4*b^3)))/(2*b^2)))/(2*b^2)))/(
3*Sqrt[Pi]) + (x^6*Erfc[b*x])/6
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{8\,\operatorname{erfc}(bx)x^6b^6\sqrt{\pi}-8\,\mathrm{e}^{-b^2x^2}x^5b^5-20\,\mathrm{e}^{-b^2x^2}x^3b^3-30\,\mathrm{e}^{-b^2x^2}bx-15\,\operatorname{erfc}(bx)\sqrt{\pi}}{48b^6\sqrt{\pi}}$	81
derivativedivides	$\frac{\frac{b^6x^6\operatorname{erfc}(bx)}{6}+\frac{-\frac{\mathrm{e}^{-b^2x^2}x^5b^5}{2}-\frac{5\,\mathrm{e}^{-b^2x^2}x^3b^3}{4}-\frac{15\,\mathrm{e}^{-b^2x^2}bx}{8}+\frac{15\,\operatorname{erf}(bx)\sqrt{\pi}}{16}}{3\sqrt{\pi}}}{b^6}$	83
default	$\frac{\frac{b^6x^6\operatorname{erfc}(bx)}{6}+\frac{-\frac{\mathrm{e}^{-b^2x^2}x^5b^5}{2}-\frac{5\,\mathrm{e}^{-b^2x^2}x^3b^3}{4}-\frac{15\,\mathrm{e}^{-b^2x^2}bx}{8}+\frac{15\,\operatorname{erf}(bx)\sqrt{\pi}}{16}}{3\sqrt{\pi}}}{b^6}$	83
parts	$\frac{x^6\operatorname{erfc}(bx)}{6}+\frac{b\left(-\frac{x^5\mathrm{e}^{-b^2x^2}}{2b^2}+\frac{-\frac{5x^3\mathrm{e}^{-b^2x^2}}{4b^2}+\frac{5\left(-\frac{3x\,\mathrm{e}^{-b^2x^2}}{4b^2}+\frac{3\sqrt{\pi}\,\operatorname{erf}(bx)}{8b^3}\right)}{2b^2}}{b^2}\right)}{3\sqrt{\pi}}$	91

input
```
int(x^5*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*erfc(b*x)*x^6*b^6*Pi^(1/2)-8*exp(-b^2*x^2)*x^5*b^5-20*exp(-b^2*x^2
)*x^3*b^3-30*exp(-b^2*x^2)*b*x-15*erfc(b*x)*Pi^(1/2))/b^6/Pi^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

$$\int x^5\operatorname{erfc}(bx)\,dx = \frac{8\,\pi b^6x^6-2\,\sqrt{\pi}(4\,b^5x^5+10\,b^3x^3+15\,bx)e^{(-b^2x^2)}+(15\,\pi-8\,\pi b^6x^6)\operatorname{erf}(bx)}{48\,\pi b^6}$$

input
```
integrate(x^5*erfc(b*x),x, algorithm="fricas")
```

output
```
1/48*(8*pi*b^6*x^6 - 2*sqrt(pi)*(4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*e^(-b^2*
x^2) + (15*pi - 8*pi*b^6*x^6)*erf(b*x))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^5 \operatorname{erfc}(bx)\,dx = \begin{cases} \frac{x^6 \operatorname{erfc}(bx)}{6} - \frac{x^5 e^{-b^2x^2}}{6\sqrt{\pi}b} - \frac{5x^3 e^{-b^2x^2}}{12\sqrt{\pi}b^3} - \frac{5x e^{-b^2x^2}}{8\sqrt{\pi}b^5} - \frac{5 \operatorname{erfc}(bx)}{16b^6} & \text{for } b \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*erfc(b*x),x)
```

output
```
Piecewise((x**6*erfc(b*x)/6 - x**5*exp(-b**2*x**2)/(6*sqrt(pi)*b) - 5*x**3
*exp(-b**2*x**2)/(12*sqrt(pi)*b**3) - 5*x*exp(-b**2*x**2)/(8*sqrt(pi)*b**5
) - 5*erfc(b*x)/(16*b**6), Ne(b, 0)), (x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int x^5 \operatorname{erfc}(bx)\,dx = \frac{1}{6}x^6 \operatorname{erfc}(bx) - \frac{b\left(\frac{2(4b^4x^5+10b^2x^3+15x)e^{(-b^2x^2)}}{b^6} - \frac{15\sqrt{\pi}\operatorname{erf}(bx)}{b^7}\right)}{48\sqrt{\pi}}$$

input
```
integrate(x^5*erfc(b*x),x, algorithm="maxima")
```

output
```
1/6*x^6*erfc(b*x) - 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^(-b^2*x^2)
/b^6 - 15*sqrt(pi)*erf(b*x)/b^7)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int x^5 \operatorname{erfc}(bx)\,dx = -\frac{1}{6}x^6 \operatorname{erf}(bx) + \frac{1}{6}x^6 - \frac{b\left(\frac{2(4b^4x^5+10b^2x^3+15x)e^{(-b^2x^2)}}{b^6} + \frac{15\sqrt{\pi}\operatorname{erf}(-bx)}{b^7}\right)}{48\sqrt{\pi}}$$

input
```
integrate(x^5*erfc(b*x),x, algorithm="giac")
```

output
```
-1/6*x^6*erf(b*x) + 1/6*x^6 - 1/48*b*(2*(4*b^4*x^5 + 10*b^2*x^3 + 15*x)*e^
(-b^2*x^2)/b^6 + 15*sqrt(pi)*erf(-b*x)/b^7)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int x^5\text{erfc}(bx)\,dx=\frac{x^6\,\text{erfc}(b\,x)}{6}-\frac{\frac{5\,\text{erfc}(b\,x)}{16}+\frac{5\,b^3\,x^3\,\mathrm{e}^{-b^2\,x^2}}{12\,\sqrt{\pi}}+\frac{b^5\,x^5\,\mathrm{e}^{-b^2\,x^2}}{6\,\sqrt{\pi}}+\frac{5\,b\,x\,\mathrm{e}^{-b^2\,x^2}}{8\,\sqrt{\pi}}}{b^6}$$

input
```
int(x^5*erfc(b*x),x)
```

output
```
(x^6*erfc(b*x))/6 - ((5*erfc(b*x))/16 + (5*b^3*x^3*exp(-b^2*x^2))/(12*pi^(
1/2)) + (b^5*x^5*exp(-b^2*x^2))/(6*pi^(1/2)) + (5*b*x*exp(-b^2*x^2))/(8*pi
^(1/2)))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int x^5\text{erfc}(bx)\,dx$$
$$=\frac{-8e^{b^2x^2}\text{erf}(bx)\,b^6\pi\,x^6+15e^{b^2x^2}\text{erf}(bx)\,\pi+8e^{b^2x^2}b^6\pi\,x^6-8\sqrt{\pi}\,b^5x^5-20\sqrt{\pi}\,b^3x^3-30\sqrt{\pi}\,bx}{48e^{b^2x^2}b^6\pi}$$

input
```
int(x^5*erfc(b*x),x)
```

output
```
( - 8*e**(b**2*x**2)*erf(b*x)*b**6*pi*x**6 + 15*e**(b**2*x**2)*erf(b*x)*pi
 + 8*e**(b**2*x**2)*b**6*pi*x**6 - 8*sqrt(pi)*b**5*x**5 - 20*sqrt(pi)*b**3
*x**3 - 30*sqrt(pi)*b*x)/(48*e**(b**2*x**2)*b**6*pi)
```

3.105 $\int x^3 \mathrm{erfc}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int x^3\mathrm{erfc}(bx)\,dx=-\frac{3e^{-b^2x^2}x}{8b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^3}{4b\sqrt{\pi}}+\frac{3\mathrm{erf}(bx)}{16b^4}+\frac{1}{4}x^4\mathrm{erfc}(bx)$$

output
```
-3/8*x/b^3/exp(b^2*x^2)/Pi^(1/2)-1/4*x^3/b/exp(b^2*x^2)/Pi^(1/2)+3/16*erf(
b*x)/b^4+1/4*x^4*erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int x^3\mathrm{erfc}(bx)\,dx=\frac{1}{16}\left(-\frac{2e^{-b^2x^2}x(3+2b^2x^2)}{b^3\sqrt{\pi}}+\frac{3\mathrm{erf}(bx)}{b^4}+4x^4\mathrm{erfc}(bx)\right)$$

input
```
Integrate[x^3*Erfc[b*x],x]
```

output
```
((-2*x*(3 + 2*b^2*x^2))/(b^3*E^(b^2*x^2)*Sqrt[Pi]) + (3*Erf[b*x])/b^4 + 4*
x^4*Erfc[b*x])/16
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6916, 2641, 2641, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{erfc}(bx) \, dx$$

$$\Big\downarrow \; 6916$$

$$\frac{b \int e^{-b^2x^2} x^4 dx}{2\sqrt{\pi}} + \frac{1}{4} x^4 \text{erfc}(bx)$$

$$\Big\downarrow \; 2641$$

$$\frac{b \left(\frac{3 \int e^{-b^2x^2} x^2 dx}{2b^2} - \frac{x^3 e^{-b^2x^2}}{2b^2} \right)}{2\sqrt{\pi}} + \frac{1}{4} x^4 \text{erfc}(bx)$$

$$\Big\downarrow \; 2641$$

$$\frac{b \left(\frac{3 \left(\frac{\int e^{-b^2x^2} dx}{2b^2} - \frac{x e^{-b^2x^2}}{2b^2} \right)}{2b^2} - \frac{x^3 e^{-b^2x^2}}{2b^2} \right)}{2\sqrt{\pi}} + \frac{1}{4} x^4 \text{erfc}(bx)$$

$$\Big\downarrow \; 2634$$

$$\frac{b \left(\frac{3 \left(\frac{\sqrt{\pi} \text{erf}(bx)}{4b^3} - \frac{x e^{-b^2x^2}}{2b^2} \right)}{2b^2} - \frac{x^3 e^{-b^2x^2}}{2b^2} \right)}{2\sqrt{\pi}} + \frac{1}{4} x^4 \text{erfc}(bx)$$

input
```
Int[x^3*Erfc[b*x],x]
```

output
```
(b*(-1/2*x^3/(b^2*E^(b^2*x^2)) + (3*(-1/2*x/(b^2*E^(b^2*x^2)) + (Sqrt[Pi]*
Erf[b*x])/(4*b^3)))/(2*b^2)))/(2*Sqrt[Pi]) + (x^4*Erfc[b*x])/4
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{4\,\operatorname{erfc}(bx)x^4\sqrt{\pi}\,b^4-4\,{\rm e}^{-b^2x^2}x^3b^3-6\,{\rm e}^{-b^2x^2}bx-3\,\operatorname{erfc}(bx)\sqrt{\pi}}{16\sqrt{\pi}\,b^4}$	64
derivativedivides	$\frac{\frac{b^4x^4\operatorname{erfc}(bx)}{4}+\frac{-\frac{{\rm e}^{-b^2x^2}x^3b^3}{2}-\frac{3\,{\rm e}^{-b^2x^2}bx}{4}+\frac{3\,\operatorname{erf}(bx)\sqrt{\pi}}{8}}{2\sqrt{\pi}}}{b^4}$	65
default	$\frac{\frac{b^4x^4\operatorname{erfc}(bx)}{4}+\frac{-\frac{{\rm e}^{-b^2x^2}x^3b^3}{2}-\frac{3\,{\rm e}^{-b^2x^2}bx}{4}+\frac{3\,\operatorname{erf}(bx)\sqrt{\pi}}{8}}{2\sqrt{\pi}}}{b^4}$	65
parts	$\frac{x^4\operatorname{erfc}(bx)}{4}+\frac{b\left(-\frac{x^3{\rm e}^{-b^2x^2}}{2b^2}+\frac{-\frac{3x\,{\rm e}^{-b^2x^2}}{4b^2}+\frac{3\sqrt{\pi}\,\operatorname{erf}(bx)}{8b^3}}{b^2}\right)}{2\sqrt{\pi}}$	68

input
```
int(x^3*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*erfc(b*x)*x^4*Pi^(1/2)*b^4-4*exp(-b^2*x^2)*x^3*b^3-6*exp(-b^2*x^2)
*b*x-3*erfc(b*x)*Pi^(1/2))/Pi^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x^3\text{erfc}(bx)\,dx = \frac{4\,\pi b^4x^4 - 2\,\sqrt{\pi}(2\,b^3x^3 + 3\,bx)e^{\left(-b^2x^2\right)} + (3\,\pi - 4\,\pi b^4x^4)\,\text{erf}\,(bx)}{16\,\pi b^4}$$

input
```
integrate(x^3*erfc(b*x),x, algorithm="fricas")
```

output
```
1/16*(4*pi*b^4*x^4 - 2*sqrt(pi)*(2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) + (3*pi -
 4*pi*b^4*x^4)*erf(b*x))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int x^3\text{erfc}(bx)\,dx = \begin{cases} \frac{x^4\,\text{erfc}\,(bx)}{4} - \frac{x^3e^{-b^2x^2}}{4\sqrt{\pi}b} - \frac{3xe^{-b^2x^2}}{8\sqrt{\pi}b^3} - \frac{3\,\text{erfc}\,(bx)}{16b^4} & \text{for}\ b \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erfc(b*x),x)
```

output
```
Piecewise((x**4*erfc(b*x)/4 - x**3*exp(-b**2*x**2)/(4*sqrt(pi)*b) - 3*x*ex
p(-b**2*x**2)/(8*sqrt(pi)*b**3) - 3*erfc(b*x)/(16*b**4), Ne(b, 0)), (x**4/
4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int x^3 \text{erfc}(bx)\,dx = \frac{1}{4}\,x^4\,\text{erfc}\,(bx) - \frac{b\left(\frac{2\,(2\,b^2x^3+3\,x)e^{\left(-b^2x^2\right)}}{b^4} - \frac{3\,\sqrt{\pi}\,\text{erf}(bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

input
```
integrate(x^3*erfc(b*x),x, algorithm="maxima")
```

output
```
1/4*x^4*erfc(b*x) - 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4 - 3*sqrt(
pi)*erf(b*x)/b^5)/sqrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int x^3 \text{erfc}(bx)\,dx = -\frac{1}{4}\,x^4\,\text{erf}\,(bx) + \frac{1}{4}\,x^4 - \frac{b\left(\frac{2\,(2\,b^2x^3+3\,x)e^{\left(-b^2x^2\right)}}{b^4} + \frac{3\,\sqrt{\pi}\,\text{erf}(-bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

input
```
integrate(x^3*erfc(b*x),x, algorithm="giac")
```

output
```
-1/4*x^4*erf(b*x) + 1/4*x^4 - 1/16*b*(2*(2*b^2*x^3 + 3*x)*e^(-b^2*x^2)/b^4
 + 3*sqrt(pi)*erf(-b*x)/b^5)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int x^3 \text{erfc}(bx)\,dx = \frac{x^4\,\text{erfc}(b\,x)}{4} - \frac{\frac{3\,\text{erfc}(b\,x)}{16} + \frac{b^3\,x^3\,\mathrm{e}^{-b^2\,x^2}}{4\,\sqrt{\pi}} + \frac{3\,b\,x\,\mathrm{e}^{-b^2\,x^2}}{8\,\sqrt{\pi}}}{b^4}$$

input
```
int(x^3*erfc(b*x),x)
```

output
```
(x^4*erfc(b*x))/4 - ((3*erfc(b*x))/16 + (b^3*x^3*exp(-b^2*x^2))/(4*pi^(1/2
)) + (3*b*x*exp(-b^2*x^2))/(8*pi^(1/2)))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int x^3 \text{erfc}(bx)\,dx = \frac{-4e^{b^2x^2}\text{erf}(bx)\,b^4\pi\,x^4 + 3e^{b^2x^2}\text{erf}(bx)\,\pi + 4e^{b^2x^2}b^4\pi\,x^4 - 4\sqrt{\pi}\,b^3x^3 - 6\sqrt{\pi}\,bx}{16e^{b^2x^2}b^4\pi}$$

input
```
int(x^3*erfc(b*x),x)
```

output
```
( - 4*e**(b**2*x**2)*erf(b*x)*b**4*pi*x**4 + 3*e**(b**2*x**2)*erf(b*x)*pi
+ 4*e**(b**2*x**2)*b**4*pi*x**4 - 4*sqrt(pi)*b**3*x**3 - 6*sqrt(pi)*b*x)/(
16*e**(b**2*x**2)*b**4*pi)
```

3.106 $\int x\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 46

$$\int x\mathrm{erfc}(bx)\,dx = -\frac{e^{-b^2x^2}x}{2b\sqrt{\pi}} + \frac{\mathrm{erf}(bx)}{4b^2} + \frac{1}{2}x^2\mathrm{erfc}(bx)$$

output
```
-1/2*x/b/exp(b^2*x^2)/Pi^(1/2)+1/4*erf(b*x)/b^2+1/2*x^2*erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int x\mathrm{erfc}(bx)\,dx = \frac{1}{4}\left(\frac{\mathrm{erf}(bx)}{b^2} + 2x\left(-\frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\mathrm{erfc}(bx)\right)\right)$$

input
```
Integrate[x*Erfc[b*x],x]
```

output
```
(Erf[b*x]/b^2 + 2*x*(-(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]))/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6916, 2641, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6916}$$

$$\frac{b\int e^{-b^2x^2}x^2dx}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{\int e^{-b^2x^2}dx}{2b^2}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(\frac{\sqrt{\pi}\text{erf}(bx)}{4b^3}-\frac{xe^{-b^2x^2}}{2b^2}\right)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)$$

input
```
Int[x*Erfc[b*x],x]
```

output
```
(b*(-1/2*x/(b^2*E^(b^2*x^2)) + (Sqrt[Pi]*Erf[b*x])/(4*b^3)))/Sqrt[Pi] + (x
^2*Erfc[b*x])/2
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result	size
parts	$\frac{x^{2} \operatorname{erfc}\left(b x\right)}{2}+\frac{b \left(-\frac{x \,{\mathrm e}^{-b^{2} x^{2}}}{2 b^{2}}+\frac{\sqrt{\pi}\, \operatorname{erf}\left(b x\right)}{4 b^{3}}\right)}{\sqrt{\pi}}$	44
derivativedivides	$\frac{\frac{b^{2} x^{2} \operatorname{erfc}\left(b x\right)}{2}+\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} b x}{2}+\frac{\operatorname{erf}\left(b x\right) \sqrt{\pi}}{4}}{\sqrt{\pi}}}{b^{2}}$	46
default	$\frac{\frac{b^{2} x^{2} \operatorname{erfc}\left(b x\right)}{2}+\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} b x}{2}+\frac{\operatorname{erf}\left(b x\right) \sqrt{\pi}}{4}}{\sqrt{\pi}}}{b^{2}}$	46
parallelrisch	$\frac{2 x^{2} \operatorname{erfc}\left(b x\right) \sqrt{\pi}\, b^{2}-2 \,{\mathrm e}^{-b^{2} x^{2}} b x -\operatorname{erfc}\left(b x\right) \sqrt{\pi}}{4 \sqrt{\pi}\, b^{2}}$	47

input
```
int(x*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*erfc(b*x)+1/Pi^(1/2)*b*(-1/2/b^2*x*exp(-b^2*x^2)+1/4/b^3*Pi^(1/2)*
erf(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x\text{erfc}(bx)\,dx = \frac{2\,\pi b^2x^2 - 2\,\sqrt{\pi}bxe^{\left(-b^2x^2\right)} + \left(\pi - 2\,\pi b^2x^2\right)\text{erf}\left(bx\right)}{4\,\pi b^2}$$

input
```
integrate(x*erfc(b*x),x, algorithm="fricas")
```

output
```
1/4*(2*pi*b^2*x^2 - 2*sqrt(pi)*b*x*e^(-b^2*x^2) + (pi - 2*pi*b^2*x^2)*erf(
b*x))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x\text{erfc}(bx)\,dx = \begin{cases} \frac{x^2\,\text{erfc}\left(bx\right)}{2} - \frac{xe^{-b^2x^2}}{2\sqrt{\pi}b} - \frac{\text{erfc}\left(bx\right)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*erfc(b*x),x)
```

output
```
Piecewise((x**2*erfc(b*x)/2 - x*exp(-b**2*x**2)/(2*sqrt(pi)*b) - erfc(b*x)
/(4*b**2), Ne(b, 0)), (x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x\text{erfc}(bx)\,dx = \frac{1}{2}\,x^2\,\text{erfc}\,(bx) - \frac{b\left(\frac{2\,xe^{\left(-b^2x^2\right)}}{b^2} - \frac{\sqrt{\pi}\,\text{erf}(bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

input
```
integrate(x*erfc(b*x),x, algorithm="maxima")
```

output
```
1/2*x^2*erfc(b*x) - 1/4*b*(2*x*e^(-b^2*x^2)/b^2 - sqrt(pi)*erf(b*x)/b^3)/s
qrt(pi)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x\text{erfc}(bx)\,dx = -\frac{1}{2}\,x^2\,\text{erf}\,(bx) + \frac{1}{2}\,x^2 - \frac{b\left(\frac{2\,xe^{\left(-b^2x^2\right)}}{b^2} + \frac{\sqrt{\pi}\,\text{erf}(-bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

input
```
integrate(x*erfc(b*x),x, algorithm="giac")
```

output
```
-1/2*x^2*erf(b*x) + 1/2*x^2 - 1/4*b*(2*x*e^(-b^2*x^2)/b^2 + sqrt(pi)*erf(-
b*x)/b^3)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int x\text{erfc}(bx)\,dx = \frac{x^2\,\text{erfc}(b\,x)}{2} - \frac{\frac{\text{erfc}(b\,x)}{4} + \frac{b\,x\,\mathrm{e}^{-b^2\,x^2}}{2\,\sqrt{\pi}}}{b^2}$$

input
```
int(x*erfc(b*x),x)
```

output
```
(x^2*erfc(b*x))/2 - (erfc(b*x)/4 + (b*x*exp(-b^2*x^2))/(2*pi^(1/2)))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int x\text{erfc}(bx)\,dx = \frac{-2e^{b^2x^2}\text{erf}(bx)\,b^2\pi\,x^2 + e^{b^2x^2}\text{erf}(bx)\,\pi + 2e^{b^2x^2}b^2\pi\,x^2 - 2\sqrt{\pi}\,bx}{4e^{b^2x^2}b^2\pi}$$

input
```
int(x*erfc(b*x),x)
```

output
```
( - 2*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 + e**(b**2*x**2)*erf(b*x)*pi +
2*e**(b**2*x**2)*b**2*pi*x**2 - 2*sqrt(pi)*b*x)/(4*e**(b**2*x**2)*b**2*pi)
```

3.107 $\int \frac{\text{erfc}(bx)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{\text{erfc}(bx)}{x} \, dx = -\frac{2bx \, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}} + \log(x)$$

output
```
-2*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-b^2*x^2)/Pi^(1/2)+ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{\text{erfc}(bx)}{x} \, dx = -\frac{2bx \, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}} + (\text{erf}(bx) + \text{erfc}(bx))\log(x)$$

input
```
Integrate[Erfc[b*x]/x,x]
```

output
```
(-2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi] +
(Erf[b*x] + Erfc[b*x])*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6913, 6912}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)}{x} \, dx$$

$$\Big\downarrow \; 6913$$

$$\log(x) - \int \frac{\text{erf}(bx)}{x} dx$$

$$\Big\downarrow \; 6912$$

$$\log(x) - \frac{2bx \, {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[Erfc[b*x]/x,x]
```

output
```
(-2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi] +
Log[x]
```

Defintions of rubi rules used

rule 6912
```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

rule 6913
```
Int[Erfc[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[Log[x], x] - Int[Erf[b*x]/x, x
] /; FreeQ[b, x]
```

Maple [F]

$$\int \frac{\operatorname{erfc}(bx)}{x} dx$$

input
```
int(erfc(b*x)/x,x)
```

output
```
int(erfc(b*x)/x,x)
```

Fricas [F]

$$\int \frac{\operatorname{erfc}(bx)}{x} dx = \int \frac{\operatorname{erfc}(bx)}{x} dx$$

input
```
integrate(erfc(b*x)/x,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{erfc}(bx)}{x} dx = -\frac{2bx{}_2F_2\left(\begin{matrix}\frac{1}{2},\frac{1}{2}\\ \frac{3}{2},\frac{3}{2}\end{matrix}\middle| -b^2x^2\right)}{\sqrt{\pi}} + \frac{\log(b^2x^2)}{2}$$

input
```
integrate(erfc(b*x)/x,x)
```

output
```
-2*b*x*hyper((1/2, 1/2), (3/2, 3/2), -b**2*x**2)/sqrt(pi) + log(b**2*x**2)
/2
```

Maxima [F]

$$\int \frac{\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx)}{x}\,dx$$

input
```
integrate(erfc(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)/x, x)
```

Giac [F]

$$\int \frac{\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx)}{x}\,dx$$

input
```
integrate(erfc(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}(b\,x)}{x}\,dx$$

input
```
int(erfc(b*x)/x,x)
```

output
```
int(erfc(b*x)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(bx)}{x}\,dx = -\left(\int \frac{\operatorname{erf}(bx)}{x}dx\right) + \log(x)$$

input
```
int(erfc(b*x)/x,x)
```

output
```
- int(erf(b*x)/x,x) + log(x)
```

3.108 $\int \frac{\mathbf{erfc}(bx)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \frac{\operatorname{erfc}(bx)}{x^3} dx = \frac{be^{-b^2x^2}}{\sqrt{\pi}x} + b^2\operatorname{erf}(bx) - \frac{\operatorname{erfc}(bx)}{2x^2}$$

output
```
b/exp(b^2*x^2)/Pi^(1/2)/x+b^2*erf(b*x)-1/2*erfc(b*x)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)}{x^3} dx = \frac{be^{-b^2x^2}}{\sqrt{\pi}x} + b^2\operatorname{erf}(bx) - \frac{\operatorname{erfc}(bx)}{2x^2}$$

input
```
Integrate[Erfc[b*x]/x^3,x]
```

output
```
b/(E^(b^2*x^2)*Sqrt[Pi]*x) + b^2*Erf[b*x] - Erfc[b*x]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6916, 2643, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)}{x^3} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{b \int \frac{e^{-b^2x^2}}{x^2}dx}{\sqrt{\pi}}-\frac{\text{erfc}(bx)}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-2b^2\int e^{-b^2x^2}dx-\frac{e^{-b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{b\left(\sqrt{\pi}(-b)\text{erf}(bx)-\frac{e^{-b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)}{2x^2}$$

input
```
Int[Erfc[b*x]/x^3,x]
```

output
```
-((b*(-(1/(E^(b^2*x^2)*x)) - b*Sqrt[Pi]*Erf[b*x]))/Sqrt[Pi]) - Erfc[b*x]/(
2*x^2)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
parts	$-\frac{\operatorname{erfc}(bx)}{2x^{2}}-\frac{b\left(-\frac{\mathrm{e}^{-b^{2}x^{2}}}{x}-b\sqrt{\pi}\,\operatorname{erf}(bx)\right)}{\sqrt{\pi}}$	42
parallelrisch	$-\frac{2x^{2}\operatorname{erfc}(bx)\sqrt{\pi}\,b^{2}-2\,\mathrm{e}^{-b^{2}x^{2}}bx+\operatorname{erfc}(bx)\sqrt{\pi}}{2\sqrt{\pi}\,x^{2}}$	46
derivativedivides	$b^{2}\left(-\frac{\operatorname{erfc}(bx)}{2b^{2}x^{2}}-\frac{-\frac{\mathrm{e}^{-b^{2}x^{2}}}{xb}-\operatorname{erf}(bx)\sqrt{\pi}}{\sqrt{\pi}}\right)$	51
default	$b^{2}\left(-\frac{\operatorname{erfc}(bx)}{2b^{2}x^{2}}-\frac{-\frac{\mathrm{e}^{-b^{2}x^{2}}}{xb}-\operatorname{erf}(bx)\sqrt{\pi}}{\sqrt{\pi}}\right)$	51

input
```
int(erfc(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*erfc(b*x)/x^2-1/Pi^(1/2)*b*(-1/x*exp(-b^2*x^2)-b*Pi^(1/2)*erf(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{erfc}(bx)}{x^3} \, dx = -\frac{\pi - 2\sqrt{\pi} bxe^{\left(-b^2x^2\right)} - (\pi + 2\,\pi b^2x^2)\operatorname{erf}(bx)}{2\,\pi x^2}$$

input
```
integrate(erfc(b*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(pi - 2*sqrt(pi)*b*x*e^(-b^2*x^2) - (pi + 2*pi*b^2*x^2)*erf(b*x))/(pi
*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{erfc}(bx)}{x^3} \, dx = -b^2 \operatorname{erfc}(bx) + \frac{be^{-b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)}{2x^2}$$

input
```
integrate(erfc(b*x)/x**3,x)
```

output
```
-b**2*erfc(b*x) + b*exp(-b**2*x**2)/(sqrt(pi)*x) - erfc(b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{erfc}(bx)}{x^3} \, dx = \frac{b^2\sqrt{x^2}\Gamma\left(-\frac{1}{2}, b^2x^2\right)}{2\sqrt{\pi}x} - \frac{\operatorname{erfc}(bx)}{2\,x^2}$$

input
```
integrate(erfc(b*x)/x^3,x, algorithm="maxima")
```

output
```
1/2*b^2*sqrt(x^2)*gamma(-1/2, b^2*x^2)/(sqrt(pi)*x) - 1/2*erfc(b*x)/x^2
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfc}(bx)}{x^3}\,dx$$

input
```
integrate(erfc(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{erfc}(bx)}{x^3}\,dx = -b^2\operatorname{erfc}(b\,x) - \frac{\frac{\operatorname{erfc}(b\,x)}{2} - \frac{b\,x\,e^{-b^2\,x^2}}{\sqrt{\pi}}}{x^2}$$

input
```
int(erfc(b*x)/x^3,x)
```

output
```
- b^2*erfc(b*x) - (erfc(b*x)/2 - (b*x*exp(-b^2*x^2))/pi^(1/2))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{erfc}(bx)}{x^3}\,dx = \frac{2e^{b^2x^2}\operatorname{erf}(bx)\,b^2\pi\,x^2 + e^{b^2x^2}\operatorname{erf}(bx)\,\pi - e^{b^2x^2}\pi + 2\sqrt{\pi}\,bx}{2e^{b^2x^2}\pi\,x^2}$$

input
```
int(erfc(b*x)/x^3,x)
```

output
```
(2*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 + e**(b**2*x**2)*erf(b*x)*pi - e**
(b**2*x**2)*pi + 2*sqrt(pi)*b*x)/(2*e**(b**2*x**2)*pi*x**2)
```

3.109 $\int \frac{\mathbf{erfc}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int \frac{\operatorname{erfc}(bx)}{x^5}\,dx = \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} - \frac{1}{3}b^4\operatorname{erf}(bx) - \frac{\operatorname{erfc}(bx)}{4x^4}$$

output
```
1/6*b/exp(b^2*x^2)/Pi^(1/2)/x^3-1/3*b^3/exp(b^2*x^2)/Pi^(1/2)/x-1/3*b^4*er
f(b*x)-1/4*erfc(b*x)/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{erfc}(bx)}{x^5}\,dx = \frac{1}{12}\left(\frac{2e^{-b^2x^2}(b-2b^3x^2)}{\sqrt{\pi}x^3} - 4b^4\operatorname{erf}(bx) - \frac{3\operatorname{erfc}(bx)}{x^4}\right)$$

input
```
Integrate[Erfc[b*x]/x^5,x]
```

output
```
((2*(b - 2*b^3*x^2))/(E^(b^2*x^2)*Sqrt[Pi]*x^3) - 4*b^4*Erf[b*x] - (3*Erfc
[b*x])/x^4)/12
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6916, 2643, 2643, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)}{x^5}\,dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{b\int \frac{e^{-b^2x^2}}{x^4}dx}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}}{x^2}dx-\frac{e^{-b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}dx-\frac{e^{-b^2x^2}}{x}\right)-\frac{e^{-b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{b\left(-\frac{2}{3}b^2\left(\sqrt{\pi}(-b)\text{erf}(bx)-\frac{e^{-b^2x^2}}{x}\right)-\frac{e^{-b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)}{4x^4}$$

input
```
Int[Erfc[b*x]/x^5,x]
```

output
```
-1/2*(b*(-1/3*1/(E^(b^2*x^2)*x^3) - (2*b^2*(-(1/(E^(b^2*x^2)*x)) - b*Sqrt[
Pi]*Erf[b*x]))/3))/Sqrt[Pi] - Erfc[b*x]/(4*x^4)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result	size
parts	$-\frac{\operatorname{erfc}(bx)}{4x^4}-\frac{b\left(-\frac{{\mathrm e}^{-b^2x^2}}{3x^3}-\frac{2b^2\left(-\frac{{\mathrm e}^{-b^2x^2}}{x}-b\sqrt{\pi}\,\operatorname{erf}(bx)\right)}{3}\right)}{2\sqrt{\pi}}$	62
parallelrisch	$\frac{4\operatorname{erfc}(bx)x^4\sqrt{\pi}\,b^4-4\,{\mathrm e}^{-b^2x^2}x^3b^3+2\,{\mathrm e}^{-b^2x^2}bx-3\operatorname{erfc}(bx)\sqrt{\pi}}{12\sqrt{\pi}\,x^4}$	64
derivativedivides	$b^4\left(-\frac{\operatorname{erfc}(bx)}{4b^4x^4}-\frac{-\frac{{\mathrm e}^{-b^2x^2}}{3b^3x^3}+\frac{2\,{\mathrm e}^{-b^2x^2}}{3xb}+\frac{2\operatorname{erf}(bx)\sqrt{\pi}}{3}}{2\sqrt{\pi}}\right)$	69
default	$b^4\left(-\frac{\operatorname{erfc}(bx)}{4b^4x^4}-\frac{-\frac{{\mathrm e}^{-b^2x^2}}{3b^3x^3}+\frac{2\,{\mathrm e}^{-b^2x^2}}{3xb}+\frac{2\operatorname{erf}(bx)\sqrt{\pi}}{3}}{2\sqrt{\pi}}\right)$	69

input
```
int(erfc(b*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*erfc(b*x)/x^4-1/2/Pi^(1/2)*b*(-1/3/x^3*exp(-b^2*x^2)-2/3*b^2*(-1/x*ex
p(-b^2*x^2)-b*Pi^(1/2)*erf(b*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{\text{erfc}(bx)}{x^5}\,dx = -\frac{3\,\pi + 2\,\sqrt{\pi}(2\,b^3x^3 - bx)e^{\left(-b^2x^2\right)} - (3\,\pi - 4\,\pi b^4x^4)\,\text{erf}\,(bx)}{12\,\pi x^4}$$

input
```
integrate(erfc(b*x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*pi + 2*sqrt(pi)*(2*b^3*x^3 - b*x)*e^(-b^2*x^2) - (3*pi - 4*pi*b^4
*x^4)*erf(b*x))/(pi*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{\text{erfc}(bx)}{x^5}\,dx = \frac{b^4\,\text{erfc}\,(bx)}{3} - \frac{b^3e^{-b^2x^2}}{3\sqrt{\pi}x} + \frac{be^{-b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{erfc}\,(bx)}{4x^4}$$

input
```
integrate(erfc(b*x)/x**5,x)
```

output
```
b**4*erfc(b*x)/3 - b**3*exp(-b**2*x**2)/(3*sqrt(pi)*x) + b*exp(-b**2*x**2)
/(6*sqrt(pi)*x**3) - erfc(b*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{erfc}(bx)}{x^5} \, dx = \frac{b^4 (x^2)^{\frac{3}{2}} \Gamma\left(-\frac{3}{2}, b^2 x^2\right)}{4 \sqrt{\pi} x^3} - \frac{\operatorname{erfc}(bx)}{4 x^4}$$

input
```
integrate(erfc(b*x)/x^5,x, algorithm="maxima")
```

output
```
1/4*b^4*(x^2)^(3/2)*gamma(-3/2, b^2*x^2)/(sqrt(pi)*x^3) - 1/4*erfc(b*x)/x^
4
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^5} \, dx = \int \frac{\operatorname{erfc}(bx)}{x^5} \, dx$$

input
```
integrate(erfc(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)}{x^5} \, dx = -\frac{\frac{\operatorname{erfc}(b\,x)}{4} + \frac{b^3\,x^3\,e^{-b^2\,x^2}}{3\,\sqrt{\pi}} - \frac{b\,x\,e^{-b^2\,x^2}}{6\,\sqrt{\pi}}}{x^4} - \frac{b^5\,\operatorname{erfi}\left(x\,\sqrt{-b^2}\right)}{3\,\sqrt{-b^2}}$$

input
```
int(erfc(b*x)/x^5,x)
```

output
```
- (erfc(b*x)/4 + (b^3*x^3*exp(-b^2*x^2))/(3*pi^(1/2)) - (b*x*exp(-b^2*x^2)
)/(6*pi^(1/2)))/x^4 - (b^5*erfi(x*(-b^2)^(1/2)))/(3*(-b^2)^(1/2))
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^5}\,dx = \frac{3e^{b^2x^2}\operatorname{erf}(bx)\,\pi + 4\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^3x^4 - 3e^{b^2x^2}\pi + 2\sqrt{\pi}\,bx}{12e^{b^2x^2}\pi\,x^4}$$

input
```
int(erfc(b*x)/x^5,x)
```

output
```
(3*e**(b**2*x**2)*erf(b*x)*pi + 4*sqrt(pi)*e**(b**2*x**2)*int(1/(e**(b**2*
x**2)*x**2),x)*b**3*x**4 - 3*e**(b**2*x**2)*pi + 2*sqrt(pi)*b*x)/(12*e**(b
**2*x**2)*pi*x**4)
```

3.110 $\int \frac{\text{erfc}(bx)}{x^7} dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int \frac{\text{erfc}(bx)}{x^7} dx = \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5} - \frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3} + \frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x} + \frac{4}{45}b^6\text{erf}(bx) - \frac{\text{erfc}(bx)}{6x^6}$$

output
```
1/15*b/exp(b^2*x^2)/Pi^(1/2)/x^5-2/45*b^3/exp(b^2*x^2)/Pi^(1/2)/x^3+4/45*b
^5/exp(b^2*x^2)/Pi^(1/2)/x+4/45*b^6*erf(b*x)-1/6*erfc(b*x)/x^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{\text{erfc}(bx)}{x^7} dx = \frac{1}{90}\left(\frac{2be^{-b^2x^2}(3-2b^2x^2+4b^4x^4)}{\sqrt{\pi}x^5} + 8b^6\text{erf}(bx) - \frac{15\text{erfc}(bx)}{x^6}\right)$$

input
```
Integrate[Erfc[b*x]/x^7,x]
```

output
```
((2*b*(3 - 2*b^2*x^2 + 4*b^4*x^4))/(E^(b^2*x^2)*Sqrt[Pi]*x^5) + 8*b^6*Erf[
b*x] - (15*Erfc[b*x])/x^6)/90
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6916, 2643, 2643, 2643, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfc}(bx)}{x^7} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{b \int \frac{e^{-b^2x^2}}{x^6}dx}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-\frac{2}{5}b^2 \int \frac{e^{-b^2x^2}}{x^4}dx-\frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}}{x^2}dx-\frac{e^{-b^2x^2}}{3x^3}\right)-\frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2 \int e^{-b^2x^2}dx-\frac{e^{-b^2x^2}}{x}\right)-\frac{e^{-b^2x^2}}{3x^3}\right)-\frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)}{6x^6}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(\sqrt{\pi}(-b)\operatorname{erf}(bx)-\frac{e^{-b^2x^2}}{x}\right)-\frac{e^{-b^2x^2}}{3x^3}\right)-\frac{e^{-b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)}{6x^6}$$

input
```
Int[Erfc[b*x]/x^7,x]
```

output
```
-1/3*(b*(-1/5*1/(E^(b^2*x^2)*x^5) - (2*b^2*(-1/3*1/(E^(b^2*x^2)*x^3) - (2*
b^2*(-(1/(E^(b^2*x^2)*x)) - b*Sqrt[Pi]*Erf[b*x]))/3))/5))/Sqrt[Pi] - Erfc[
b*x]/(6*x^6)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{8\,\mathrm{erfc}(bx)x^6b^6\sqrt{\pi}-8\,\mathrm{e}^{-b^2x^2}x^5b^5+4\,\mathrm{e}^{-b^2x^2}x^3b^3-6\,\mathrm{e}^{-b^2x^2}bx+15\,\mathrm{erfc}(bx)\sqrt{\pi}}{90\sqrt{\pi}\,x^6}$	81
parts	$-\frac{\mathrm{erfc}(bx)}{6x^6}-\frac{b\left(-\frac{\mathrm{e}^{-b^2x^2}}{5x^5}-\frac{2b^2\left(-\frac{\mathrm{e}^{-b^2x^2}}{3x^3}-\frac{2b^2\left(-\frac{\mathrm{e}^{-b^2x^2}}{x}-b\sqrt{\pi}\,\mathrm{erf}(bx)\right)}{3}\right)}{5}\right)}{3\sqrt{\pi}}$	82
derivativedivides	$b^6\left(-\frac{\mathrm{erfc}(bx)}{6b^6x^6}-\frac{-\frac{\mathrm{e}^{-b^2x^2}}{5b^5x^5}+\frac{2\,\mathrm{e}^{-b^2x^2}}{15b^3x^3}-\frac{4\,\mathrm{e}^{-b^2x^2}}{15xb}-\frac{4\,\mathrm{erf}(bx)\sqrt{\pi}}{15}}{3\sqrt{\pi}}\right)$	87
default	$b^6\left(-\frac{\mathrm{erfc}(bx)}{6b^6x^6}-\frac{-\frac{\mathrm{e}^{-b^2x^2}}{5b^5x^5}+\frac{2\,\mathrm{e}^{-b^2x^2}}{15b^3x^3}-\frac{4\,\mathrm{e}^{-b^2x^2}}{15xb}-\frac{4\,\mathrm{erf}(bx)\sqrt{\pi}}{15}}{3\sqrt{\pi}}\right)$	87

input
```
int(erfc(b*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/90*(8*erfc(b*x)*x^6*b^6*Pi^(1/2)-8*exp(-b^2*x^2)*x^5*b^5+4*exp(-b^2*x^2
)*x^3*b^3-6*exp(-b^2*x^2)*b*x+15*erfc(b*x)*Pi^(1/2))/Pi^(1/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{\mathrm{erfc}(bx)}{x^7}\,dx=-\frac{15\,\pi-2\,\sqrt{\pi}(4\,b^5x^5-2\,b^3x^3+3\,bx)e^{(-b^2x^2)}-(15\,\pi+8\,\pi b^6x^6)\,\mathrm{erf}\,(bx)}{90\,\pi x^6}$$

input
```
integrate(erfc(b*x)/x^7,x, algorithm="fricas")
```

output
```
-1/90*(15*pi - 2*sqrt(pi)*(4*b^5*x^5 - 2*b^3*x^3 + 3*b*x)*e^(-b^2*x^2) - (
15*pi + 8*pi*b^6*x^6)*erf(b*x))/(pi*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{erfc}(bx)}{x^7}\,dx = -\frac{4b^6\operatorname{erfc}(bx)}{45} + \frac{4b^5e^{-b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3e^{-b^2x^2}}{45\sqrt{\pi}x^3} + \frac{be^{-b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\operatorname{erfc}(bx)}{6x^6}$$

input
```
integrate(erfc(b*x)/x**7,x)
```

output
```
-4*b**6*erfc(b*x)/45 + 4*b**5*exp(-b**2*x**2)/(45*sqrt(pi)*x) - 2*b**3*exp
(-b**2*x**2)/(45*sqrt(pi)*x**3) + b*exp(-b**2*x**2)/(15*sqrt(pi)*x**5) - e
rfc(b*x)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int \frac{\operatorname{erfc}(bx)}{x^7}\,dx = \frac{b^6(x^2)^{\frac{5}{2}}\Gamma\left(-\frac{5}{2}, b^2x^2\right)}{6\sqrt{\pi}x^5} - \frac{\operatorname{erfc}(bx)}{6x^6}$$

input
```
integrate(erfc(b*x)/x^7,x, algorithm="maxima")
```

output
```
1/6*b^6*(x^2)^(5/2)*gamma(-5/2, b^2*x^2)/(sqrt(pi)*x^5) - 1/6*erfc(b*x)/x^
6
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^7}\,dx = \int \frac{\operatorname{erfc}(bx)}{x^7}\,dx$$

input
```
integrate(erfc(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{erfc}(bx)}{x^7}\,dx = \int \frac{\text{erfc}(b\,x)}{x^7}\,dx$$

input
```
int(erfc(b*x)/x^7,x)
```

output
```
int(erfc(b*x)/x^7, x)
```

Reduce [F]

$$\int \frac{\text{erfc}(bx)}{x^7}\,dx = \frac{15e^{b^2x^2}\text{erf}(bx)\,\pi - 8\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^5x^6 - 15e^{b^2x^2}\pi - 4\sqrt{\pi}\,b^3x^3 + 6\sqrt{\pi}\,bx}{90e^{b^2x^2}\pi\,x^6}$$

input
```
int(erfc(b*x)/x^7,x)
```

output
```
(15*e**(b**2*x**2)*erf(b*x)*pi - 8*sqrt(pi)*e**(b**2*x**2)*int(1/(e**(b**2
*x**2)*x**2),x)*b**5*x**6 - 15*e**(b**2*x**2)*pi - 4*sqrt(pi)*b**3*x**3 +
6*sqrt(pi)*b*x)/(90*e**(b**2*x**2)*pi*x**6)
```

3.111 $\int x^6 \mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 109

$$\int x^6 \operatorname{erfc}(bx)\,dx = -\frac{6e^{-b^2x^2}}{7b^7\sqrt{\pi}} - \frac{6e^{-b^2x^2}x^2}{7b^5\sqrt{\pi}} - \frac{3e^{-b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\operatorname{erfc}(bx)$$

output
```
-6/7/b^7/exp(b^2*x^2)/Pi^(1/2)-6/7*x^2/b^5/exp(b^2*x^2)/Pi^(1/2)-3/7*x^4/b
^3/exp(b^2*x^2)/Pi^(1/2)-1/7*x^6/b/exp(b^2*x^2)/Pi^(1/2)+1/7*x^7*erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int x^6 \operatorname{erfc}(bx)\,dx = \frac{e^{-b^2x^2}\left(-6 - 6b^2x^2 - 3b^4x^4 - b^6x^6 + b^7e^{b^2x^2}\sqrt{\pi}x^7\operatorname{erfc}(bx)\right)}{7b^7\sqrt{\pi}}$$

input
```
Integrate[x^6*Erfc[b*x],x]
```

output
```
(-6 - 6*b^2*x^2 - 3*b^4*x^4 - b^6*x^6 + b^7*E^(b^2*x^2)*Sqrt[Pi]*x^7*Erfc[
b*x])/(7*b^7*E^(b^2*x^2)*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6916, 2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{erfc}(bx) \, dx$$

$$\Big\downarrow \text{6916}$$

$$\frac{2b \int e^{-b^2x^2}x^7dx}{7\sqrt{\pi}}+\frac{1}{7}x^7\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b\left(\frac{3\int e^{-b^2x^2}x^5dx}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}+\frac{1}{7}x^7\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b\left(\frac{3\left(\frac{2\int e^{-b^2x^2}x^3dx}{b^2}-\frac{x^4e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}+\frac{1}{7}x^7\text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b\left(\frac{3\left(\frac{2\left(\frac{\int e^{-b^2x^2}xdx}{b^2}-\frac{x^2e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}+\frac{1}{7}x^7\text{erfc}(bx)$$

$$\Big\downarrow \text{2638}$$

$$\frac{2b\left(\frac{3\left(\frac{2\left(-\frac{x^2e^{-b^2x^2}}{2b^2}-\frac{e^{-b^2x^2}}{2b^4}\right)}{b^2}-\frac{x^4e^{-b^2x^2}}{2b^2}\right)}{b^2}-\frac{x^6e^{-b^2x^2}}{2b^2}\right)}{7\sqrt{\pi}}+\frac{1}{7}x^7\text{erfc}(bx)$$

input
```
Int[x^6*Erfc[b*x],x]
```

output
```
(2*b*(-1/2*x^6/(b^2*E^(b^2*x^2)) + (3*(-1/2*x^4/(b^2*E^(b^2*x^2)) + (2*(-1
/2*1/(b^4*E^(b^2*x^2)) - x^2/(2*b^2*E^(b^2*x^2))))/b^2))/b^2))/(7*Sqrt[Pi]
) + (x^7*Erfc[b*x])/7
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{\operatorname{erfc}(bx)x^7b^7\sqrt{\pi}-\mathrm{e}^{-b^2x^2}x^6b^6-3\,\mathrm{e}^{-b^2x^2}x^4b^4-6\,\mathrm{e}^{-b^2x^2}x^2b^2-6\,\mathrm{e}^{-b^2x^2}}{7b^7\sqrt{\pi}}$	86
derivativedivides	$\frac{\frac{b^7x^7\operatorname{erfc}(bx)}{7}+\frac{-\frac{\mathrm{e}^{-b^2x^2}x^6b^6}{7}-\frac{3\,\mathrm{e}^{-b^2x^2}x^4b^4}{7}-\frac{6\,\mathrm{e}^{-b^2x^2}x^2b^2}{7}-\frac{6\,\mathrm{e}^{-b^2x^2}}{7}}{\sqrt{\pi}}}{b^7}$	90
default	$\frac{\frac{b^7x^7\operatorname{erfc}(bx)}{7}+\frac{-\frac{\mathrm{e}^{-b^2x^2}x^6b^6}{7}-\frac{3\,\mathrm{e}^{-b^2x^2}x^4b^4}{7}-\frac{6\,\mathrm{e}^{-b^2x^2}x^2b^2}{7}-\frac{6\,\mathrm{e}^{-b^2x^2}}{7}}{\sqrt{\pi}}}{b^7}$	90
parts	$\frac{x^7\operatorname{erfc}(bx)}{7}+\frac{2b\left(-\frac{x^6\mathrm{e}^{-b^2x^2}}{2b^2}+\frac{-\frac{3x^4\mathrm{e}^{-b^2x^2}}{2b^2}+\frac{3\left(-\frac{x^2\mathrm{e}^{-b^2x^2}}{b^2}-\frac{\mathrm{e}^{-b^2x^2}}{b^4}\right)}{b^2}}{b^2}\right)}{7\sqrt{\pi}}$	95

input
```
int(x^6*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*(erfc(b*x)*x^7*b^7*Pi^(1/2)-exp(-b^2*x^2)*x^6*b^6-3*exp(-b^2*x^2)*x^4*
b^4-6*exp(-b^2*x^2)*x^2*b^2-6*exp(-b^2*x^2))/b^7/Pi^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int x^6\operatorname{erfc}(bx)\,dx = -\frac{\pi b^7x^7\operatorname{erf}(bx)-\pi b^7x^7+\sqrt{\pi}(b^6x^6+3\,b^4x^4+6\,b^2x^2+6)e^{(-b^2x^2)}}{7\,\pi b^7}$$

input
```
integrate(x^6*erfc(b*x),x, algorithm="fricas")
```

output
```
-1/7*(pi*b^7*x^7*erf(b*x) - pi*b^7*x^7 + sqrt(pi)*(b^6*x^6 + 3*b^4*x^4 + 6
*b^2*x^2 + 6)*e^(-b^2*x^2))/(pi*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int x^6\mathrm{erfc}(bx)\,dx = \begin{cases} \frac{x^7\,\mathrm{erfc}\,(bx)}{7} - \frac{x^6e^{-b^2x^2}}{7\sqrt{\pi}b} - \frac{3x^4e^{-b^2x^2}}{7\sqrt{\pi}b^3} - \frac{6x^2e^{-b^2x^2}}{7\sqrt{\pi}b^5} - \frac{6e^{-b^2x^2}}{7\sqrt{\pi}b^7} & \text{for } b \neq 0 \\ \frac{x^7}{7} & \text{otherwise} \end{cases}$$

input
```
integrate(x**6*erfc(b*x),x)
```

output
```
Piecewise((x**7*erfc(b*x)/7 - x**6*exp(-b**2*x**2)/(7*sqrt(pi)*b) - 3*x**4
*exp(-b**2*x**2)/(7*sqrt(pi)*b**3) - 6*x**2*exp(-b**2*x**2)/(7*sqrt(pi)*b*
*5) - 6*exp(-b**2*x**2)/(7*sqrt(pi)*b**7), Ne(b, 0)), (x**7/7, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int x^6\mathrm{erfc}(bx)\,dx = \frac{1}{7}\,x^7\,\mathrm{erfc}\,(bx) - \frac{(b^6x^6 + 3\,b^4x^4 + 6\,b^2x^2 + 6)e^{(-b^2x^2)}}{7\,\sqrt{\pi}b^7}$$

input
```
integrate(x^6*erfc(b*x),x, algorithm="maxima")
```

output
```
1/7*x^7*erfc(b*x) - 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^(-b^2*x^2)
/(sqrt(pi)*b^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int x^6\mathrm{erfc}(bx)\,dx = -\frac{1}{7}\,x^7\,\mathrm{erf}\,(bx) + \frac{1}{7}\,x^7 - \frac{(b^6x^6 + 3\,b^4x^4 + 6\,b^2x^2 + 6)e^{(-b^2x^2)}}{7\,\sqrt{\pi}b^7}$$

input
```
integrate(x^6*erfc(b*x),x, algorithm="giac")
```

output
```
-1/7*x^7*erf(b*x) + 1/7*x^7 - 1/7*(b^6*x^6 + 3*b^4*x^4 + 6*b^2*x^2 + 6)*e^
(-b^2*x^2)/(sqrt(pi)*b^7)
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^6\text{erfc}(bx)\,dx = \frac{x^7\,\text{erfc}(b\,x)}{7} - \frac{\frac{6\,\text{e}^{-b^2\,x^2}}{7\,\sqrt{\pi}} + \frac{6\,b^2\,x^2\,\text{e}^{-b^2\,x^2}}{7\,\sqrt{\pi}} + \frac{3\,b^4\,x^4\,\text{e}^{-b^2\,x^2}}{7\,\sqrt{\pi}} + \frac{b^6\,x^6\,\text{e}^{-b^2\,x^2}}{7\,\sqrt{\pi}}}{b^7}$$

input
```
int(x^6*erfc(b*x),x)
```

output
```
(x^7*erfc(b*x))/7 - ((6*exp(-b^2*x^2))/(7*pi^(1/2)) + (6*b^2*x^2*exp(-b^2*
x^2))/(7*pi^(1/2)) + (3*b^4*x^4*exp(-b^2*x^2))/(7*pi^(1/2)) + (b^6*x^6*exp
(-b^2*x^2))/(7*pi^(1/2)))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int x^6\text{erfc}(bx)\,dx = \frac{-e^{b^2x^2}\text{erf}(bx)\,b^7\pi\,x^7 + e^{b^2x^2}b^7\pi\,x^7 - \sqrt{\pi}\,b^6x^6 - 3\sqrt{\pi}\,b^4x^4 - 6\sqrt{\pi}\,b^2x^2 - 6\sqrt{\pi}}{7e^{b^2x^2}b^7\pi}$$

input
```
int(x^6*erfc(b*x),x)
```

output
```
( - e**(b**2*x**2)*erf(b*x)*b**7*pi*x**7 + e**(b**2*x**2)*b**7*pi*x**7 - s
qrt(pi)*b**6*x**6 - 3*sqrt(pi)*b**4*x**4 - 6*sqrt(pi)*b**2*x**2 - 6*sqrt(p
i))/(7*e**(b**2*x**2)*b**7*pi)
```

3.112 $\int x^4 \mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 84

$$\int x^4\text{erfc}(bx)\,dx=-\frac{2e^{-b^2x^2}}{5b^5\sqrt{\pi}}-\frac{2e^{-b^2x^2}x^2}{5b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)$$

output
```
-2/5/b^5/exp(b^2*x^2)/Pi^(1/2)-2/5*x^2/b^3/exp(b^2*x^2)/Pi^(1/2)-1/5*x^4/b
/exp(b^2*x^2)/Pi^(1/2)+1/5*x^5*erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int x^4\text{erfc}(bx)\,dx=e^{-b^2x^2}\left(-\frac{2}{5b^5\sqrt{\pi}}-\frac{2x^2}{5b^3\sqrt{\pi}}-\frac{x^4}{5b\sqrt{\pi}}\right)+\frac{1}{5}x^5\text{erfc}(bx)$$

input
```
Integrate[x^4*Erfc[b*x],x]
```

output
```
(-2/(5*b^5*Sqrt[Pi]) - (2*x^2)/(5*b^3*Sqrt[Pi]) - x^4/(5*b*Sqrt[Pi]))/E^(b
^2*x^2) + (x^5*Erfc[b*x])/5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6916, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erfc}(bx) \, dx$$

$$\Big\downarrow \text{6916}$$

$$\frac{2b \int e^{-b^2x^2} x^5 dx}{5\sqrt{\pi}} + \frac{1}{5} x^5 \text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b \left(\frac{2 \int e^{-b^2x^2} x^3 dx}{b^2} - \frac{x^4 e^{-b^2x^2}}{2b^2}\right)}{5\sqrt{\pi}} + \frac{1}{5} x^5 \text{erfc}(bx)$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b \left(\frac{2 \left(\frac{\int e^{-b^2x^2} x dx}{b^2} - \frac{x^2 e^{-b^2x^2}}{2b^2}\right)}{b^2} - \frac{x^4 e^{-b^2x^2}}{2b^2}\right)}{5\sqrt{\pi}} + \frac{1}{5} x^5 \text{erfc}(bx)$$

$$\Big\downarrow \text{2638}$$

$$\frac{2b \left(\frac{2 \left(-\frac{x^2 e^{-b^2x^2}}{2b^2} - \frac{e^{-b^2x^2}}{2b^4}\right)}{b^2} - \frac{x^4 e^{-b^2x^2}}{2b^2}\right)}{5\sqrt{\pi}} + \frac{1}{5} x^5 \text{erfc}(bx)$$

input
```
Int[x^4*Erfc[b*x],x]
```

output
```
(2*b*(-1/2*x^4/(b^2*E^(b^2*x^2)) + (2*(-1/2*1/(b^4*E^(b^2*x^2)) - x^2/(2*b
^2*E^(b^2*x^2))))/b^2))/(5*Sqrt[Pi]) + (x^5*Erfc[b*x])/5
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{x^{5} \operatorname{erfc}\left(b x\right) b^{5} \sqrt{\pi}-{\mathrm e}^{-b^{2} x^{2}} x^{4} b^{4}-2 \,{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}-2 \,{\mathrm e}^{-b^{2} x^{2}}}{5 b^{5} \sqrt{\pi}}$	69
derivativedivides	$\frac{\frac{b^{5} x^{5} \operatorname{erfc}\left(b x\right)}{5}+\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{4} b^{4}}{5}-\frac{2 \,{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}}{5}-\frac{2 \,{\mathrm e}^{-b^{2} x^{2}}}{5}}{\sqrt{\pi}}}{b^{5}}$	72
default	$\frac{\frac{b^{5} x^{5} \operatorname{erfc}\left(b x\right)}{5}+\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{4} b^{4}}{5}-\frac{2 \,{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}}{5}-\frac{2 \,{\mathrm e}^{-b^{2} x^{2}}}{5}}{\sqrt{\pi}}}{b^{5}}$	72
parts	$\frac{x^{5} \operatorname{erfc}\left(b x\right)}{5}+\frac{2 b \left(-\frac{x^{4} {\mathrm e}^{-b^{2} x^{2}}}{2 b^{2}}+\frac{-\frac{x^{2} {\mathrm e}^{-b^{2} x^{2}}}{b^{2}}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{b^{4}}}{b^{2}}\right)}{5 \sqrt{\pi}}$	72

input
```
int(x^4*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*(x^5*erfc(b*x)*b^5*Pi^(1/2)-exp(-b^2*x^2)*x^4*b^4-2*exp(-b^2*x^2)*x^2*
b^2-2*exp(-b^2*x^2))/b^5/Pi^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int x^4 \mathrm{erfc}(bx)\,dx = -\frac{\pi b^5 x^5 \operatorname{erf}(bx) - \pi b^5 x^5 + \sqrt{\pi}(b^4x^4 + 2\,b^2x^2 + 2)e^{(-b^2x^2)}}{5\,\pi b^5}$$

input
```
integrate(x^4*erfc(b*x),x, algorithm="fricas")
```

output
```
-1/5*(pi*b^5*x^5*erf(b*x) - pi*b^5*x^5 + sqrt(pi)*(b^4*x^4 + 2*b^2*x^2 + 2
)*e^(-b^2*x^2))/(pi*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int x^4 \mathrm{erfc}(bx)\,dx = \begin{cases} \frac{x^5 \operatorname{erfc}(bx)}{5} - \frac{x^4 e^{-b^2x^2}}{5\sqrt{\pi}b} - \frac{2x^2 e^{-b^2x^2}}{5\sqrt{\pi}b^3} - \frac{2e^{-b^2x^2}}{5\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*erfc(b*x),x)
```

output
```
Piecewise((x**5*erfc(b*x)/5 - x**4*exp(-b**2*x**2)/(5*sqrt(pi)*b) - 2*x**2
*exp(-b**2*x**2)/(5*sqrt(pi)*b**3) - 2*exp(-b**2*x**2)/(5*sqrt(pi)*b**5),
Ne(b, 0)), (x**5/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int x^4 \text{erfc}(bx)\,dx = \frac{1}{5}\,x^5\,\text{erfc}\,(bx) - \frac{(b^4x^4 + 2\,b^2x^2 + 2)e^{(-b^2x^2)}}{5\,\sqrt{\pi}b^5}$$

input
```
integrate(x^4*erfc(b*x),x, algorithm="maxima")
```

output
```
1/5*x^5*erfc(b*x) - 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(sqrt(pi)*b
^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int x^4 \text{erfc}(bx)\,dx = -\frac{1}{5}\,x^5\,\text{erf}\,(bx) + \frac{1}{5}\,x^5 - \frac{(b^4x^4 + 2\,b^2x^2 + 2)e^{(-b^2x^2)}}{5\,\sqrt{\pi}b^5}$$

input
```
integrate(x^4*erfc(b*x),x, algorithm="giac")
```

output
```
-1/5*x^5*erf(b*x) + 1/5*x^5 - 1/5*(b^4*x^4 + 2*b^2*x^2 + 2)*e^(-b^2*x^2)/(
sqrt(pi)*b^5)
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int x^4 \text{erfc}(bx)\,dx = \frac{x^5\,\text{erfc}(b\,x)}{5} - \frac{\frac{2\,\mathrm{e}^{-b^2\,x^2}}{5\,\sqrt{\pi}} + \frac{2\,b^2\,x^2\,\mathrm{e}^{-b^2\,x^2}}{5\,\sqrt{\pi}} + \frac{b^4\,x^4\,\mathrm{e}^{-b^2\,x^2}}{5\,\sqrt{\pi}}}{b^5}$$

input
```
int(x^4*erfc(b*x),x)
```

output
```
(x^5*erfc(b*x))/5 - ((2*exp(-b^2*x^2))/(5*pi^(1/2)) + (2*b^2*x^2*exp(-b^2*
x^2))/(5*pi^(1/2)) + (b^4*x^4*exp(-b^2*x^2))/(5*pi^(1/2)))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int x^4 \text{erfc}(bx)\,dx = \frac{-e^{b^2x^2}\text{erf}(bx)\,b^5\pi\,x^5 + e^{b^2x^2}b^5\pi\,x^5 - \sqrt{\pi}\,b^4x^4 - 2\sqrt{\pi}\,b^2x^2 - 2\sqrt{\pi}}{5e^{b^2x^2}b^5\pi}$$

input
```
int(x^4*erfc(b*x),x)
```

output
```
( - e**(b**2*x**2)*erf(b*x)*b**5*pi*x**5 + e**(b**2*x**2)*b**5*pi*x**5 - s
qrt(pi)*b**4*x**4 - 2*sqrt(pi)*b**2*x**2 - 2*sqrt(pi))/(5*e**(b**2*x**2)*b
**5*pi)
```

3.113 $\int x^2 \textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x^2 \mathrm{erfc}(bx)\,dx = -\frac{e^{-b^2x^2}}{3b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^2}{3b\sqrt{\pi}} + \frac{1}{3}x^3\mathrm{erfc}(bx)$$

output
```
-1/3/b^3/exp(b^2*x^2)/Pi^(1/2)-1/3*x^2/b/exp(b^2*x^2)/Pi^(1/2)+1/3*x^3*erf
c(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int x^2 \mathrm{erfc}(bx)\,dx = \frac{1}{3}\left(-\frac{e^{-b^2x^2}(1+b^2x^2)}{b^3\sqrt{\pi}} + x^3\mathrm{erfc}(bx)\right)$$

input
```
Integrate[x^2*Erfc[b*x],x]
```

output
```
(-((1 + b^2*x^2)/(b^3*E^(b^2*x^2)*Sqrt[Pi])) + x^3*Erfc[b*x])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6916, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{ 6916}$$

$$\frac{2b\int e^{-b^2x^2}x^3dx}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{2b\left(\frac{\int e^{-b^2x^2}xdx}{b^2}-\frac{x^2e^{-b^2x^2}}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{2b\left(-\frac{x^2e^{-b^2x^2}}{2b^2}-\frac{e^{-b^2x^2}}{2b^4}\right)}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)$$

input
```
Int[x^2*Erfc[b*x],x]
```

output
```
(2*b*(-1/2*1/(b^4*E^(b^2*x^2)) - x^2/(2*b^2*E^(b^2*x^2))))/(3*Sqrt[Pi]) +
(x^3*Erfc[b*x])/3
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

method	result	size
parts	$\frac{x^{3} \operatorname{erfc}\left(b x \right)}{3}+\frac{2 b \left(-\frac{x^{2} {\mathrm e}^{-b^{2} x^{2}}}{2 b^{2}}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{2 b^{4}}\right)}{3 \sqrt{\pi }}$	49
parallelrisch	$\frac{x^{3} \operatorname{erfc}\left(b x \right) b^{3} \sqrt{\pi }-{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}-{\mathrm e}^{-b^{2} x^{2}}}{3 b^{3} \sqrt{\pi }}$	52
derivativedivides	$\frac{\frac{b^{3} x^{3} \operatorname{erfc}\left(b x \right)}{3}+\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}}{3}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{3}}{\sqrt{\pi }}}{b^{3}}$	54
default	$\frac{\frac{b^{3} x^{3} \operatorname{erfc}\left(b x \right)}{3}+\frac{-\frac{{\mathrm e}^{-b^{2} x^{2}} x^{2} b^{2}}{3}-\frac{{\mathrm e}^{-b^{2} x^{2}}}{3}}{\sqrt{\pi }}}{b^{3}}$	54

input
```
int(x^2*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*erfc(b*x)+2/3/Pi^(1/2)*b*(-1/2/b^2*x^2*exp(-b^2*x^2)-1/2/b^4*exp(-
b^2*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int x^2 \text{erfc}(bx)\,dx = -\frac{\pi b^3 x^3 \,\text{erf}\,(bx) - \pi b^3 x^3 + \sqrt{\pi}(b^2x^2+1)e^{(-b^2x^2)}}{3\,\pi b^3}$$

input
```
integrate(x^2*erfc(b*x),x, algorithm="fricas")
```

output
```
-1/3*(pi*b^3*x^3*erf(b*x) - pi*b^3*x^3 + sqrt(pi)*(b^2*x^2 + 1)*e^(-b^2*x^
2))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^2 \text{erfc}(bx)\,dx = \begin{cases} \frac{x^3\,\text{erfc}\,(bx)}{3} - \frac{x^2 e^{-b^2x^2}}{3\sqrt{\pi}b} - \frac{e^{-b^2x^2}}{3\sqrt{\pi}b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*erfc(b*x),x)
```

output
```
Piecewise((x**3*erfc(b*x)/3 - x**2*exp(-b**2*x**2)/(3*sqrt(pi)*b) - exp(-b
**2*x**2)/(3*sqrt(pi)*b**3), Ne(b, 0)), (x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int x^2 \text{erfc}(bx)\, dx = \frac{1}{3}\, x^3\, \text{erfc}\,(bx) - \frac{(b^2x^2+1)e^{(-b^2x^2)}}{3\,\sqrt{\pi}b^3}$$

input
```
integrate(x^2*erfc(b*x),x, algorithm="maxima")
```

output
```
1/3*x^3*erfc(b*x) - 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int x^2 \text{erfc}(bx)\, dx = -\frac{1}{3}\, x^3\, \text{erf}\,(bx) + \frac{1}{3}\, x^3 - \frac{(b^2x^2+1)e^{(-b^2x^2)}}{3\,\sqrt{\pi}b^3}$$

input
```
integrate(x^2*erfc(b*x),x, algorithm="giac")
```

output
```
-1/3*x^3*erf(b*x) + 1/3*x^3 - 1/3*(b^2*x^2 + 1)*e^(-b^2*x^2)/(sqrt(pi)*b^3
)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x^2 \text{erfc}(bx)\, dx = \frac{x^3\, \text{erfc}(b\, x)}{3} - \frac{\frac{\mathrm{e}^{-b^2\, x^2}}{3\,\sqrt{\pi}} + \frac{b^2\, x^2\, \mathrm{e}^{-b^2\, x^2}}{3\,\sqrt{\pi}}}{b^3}$$

input
```
int(x^2*erfc(b*x),x)
```

output
```
(x^3*erfc(b*x))/3 - (exp(-b^2*x^2)/(3*pi^(1/2)) + (b^2*x^2*exp(-b^2*x^2))/
(3*pi^(1/2)))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x^2 \text{erfc}(bx)\, dx = \frac{-e^{b^2x^2}\text{erf}(bx)\, b^3\pi\, x^3 + e^{b^2x^2}b^3\pi\, x^3 - \sqrt{\pi}\, b^2x^2 - \sqrt{\pi}}{3e^{b^2x^2}b^3\pi}$$

input
```
int(x^2*erfc(b*x),x)
```

output
```
( - e**(b**2*x**2)*erf(b*x)*b**3*pi*x**3 + e**(b**2*x**2)*b**3*pi*x**3 - s
qrt(pi)*b**2*x**2 - sqrt(pi))/(3*e**(b**2*x**2)*b**3*pi)
```

3.114 $\int \mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 27

$$\int \operatorname{erfc}(bx)\,dx = -\frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\operatorname{erfc}(bx)$$

output
```
-1/b/exp(b^2*x^2)/Pi^(1/2)+x*erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \operatorname{erfc}(bx)\,dx = -\frac{e^{-b^2x^2}}{b\sqrt{\pi}} + x\operatorname{erfc}(bx)$$

input
```
Integrate[Erfc[b*x],x]
```

output
```
-(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erfc}(bx)\,dx$$

$$\Big\downarrow \text{ 6904}$$

$$x\operatorname{erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b}$$

input
```
Int[Erfc[b*x],x]
```

output
```
-(1/(b*E^(b^2*x^2)*Sqrt[Pi])) + x*Erfc[b*x]
```

Defintions of rubi rules used

rule 6904
```
Int[Erfc[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erfc[a + b*x]/b)
, x] - Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
parts	$x\,\operatorname{erfc}\left(bx\right)-\frac{{\mathrm e}^{-b^{2} x^{2}}}{\sqrt{\pi}\, b}$	25
derivativedivides	$\frac{bx\,\operatorname{erfc}\left(bx\right)-\frac{{\mathrm e}^{-b^{2} x^{2}}}{\sqrt{\pi}}}{b}$	27
default	$\frac{bx\,\operatorname{erfc}\left(bx\right)-\frac{{\mathrm e}^{-b^{2} x^{2}}}{\sqrt{\pi}}}{b}$	27
parallelrisch	$\frac{x\,\operatorname{erfc}\left(bx\right)\sqrt{\pi}\, b-{\mathrm e}^{-b^{2} x^{2}}}{\sqrt{\pi}\, b}$	30

input
```
int(erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
x*erfc(b*x)-1/Pi^(1/2)/b*exp(-b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \operatorname{erfc}(bx)\,dx=-\frac{\pi bx\operatorname{erf}\left(bx\right)-\pi bx+\sqrt{\pi}e^{\left(-b^{2}x^{2}\right)}}{\pi b}$$

input
```
integrate(erfc(b*x),x, algorithm="fricas")
```

output
```
-(pi*b*x*erf(b*x) - pi*b*x + sqrt(pi)*e^(-b^2*x^2))/(pi*b)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \operatorname{erfc}(bx)\,dx = \begin{cases} x\operatorname{erfc}(bx) - \frac{e^{-b^2x^2}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(erfc(b*x),x)
```

output
```
Piecewise((x*erfc(b*x) - exp(-b**2*x**2)/(sqrt(pi)*b), Ne(b, 0)), (x, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \operatorname{erfc}(bx)\,dx = \frac{bx\operatorname{erfc}(bx) - \frac{e^{\left(-b^2x^2\right)}}{\sqrt{\pi}}}{b}$$

input
```
integrate(erfc(b*x),x, algorithm="maxima")
```

output
```
(b*x*erfc(b*x) - e^(-b^2*x^2)/sqrt(pi))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \operatorname{erfc}(bx)\,dx = -x\operatorname{erf}(bx) + x - \frac{e^{\left(-b^2x^2\right)}}{\sqrt{\pi}b}$$

input
```
integrate(erfc(b*x),x, algorithm="giac")
```

output
```
-x*erf(b*x) + x - e^(-b^2*x^2)/(sqrt(pi)*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \operatorname{erfc}(bx)\,dx = x\operatorname{erfc}(b\,x) - \frac{\mathrm{e}^{-b^2\,x^2}}{b\,\sqrt{\pi}}$$

input
```
int(erfc(b*x),x)
```

output
```
x*erfc(b*x) - exp(-b^2*x^2)/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \operatorname{erfc}(bx)\,dx = \frac{-e^{b^2x^2}\operatorname{erf}(bx)\,b\pi x + e^{b^2x^2}b\pi x - \sqrt{\pi}}{e^{b^2x^2}b\pi}$$

input
```
int(erfc(b*x),x)
```

output
```
( - e**(b**2*x**2)*erf(b*x)*b*pi*x + e**(b**2*x**2)*b*pi*x - sqrt(pi))/(e*
*(b**2*x**2)*b*pi)
```

3.115 $\int \frac{\mathbf{erfc}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\text{erfc}(bx)}{x^2}\,dx = -\frac{\text{erfc}(bx)}{x} - \frac{b\,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-erfc(b*x)/x-b*Ei(-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}(bx)}{x^2}\,dx = -\frac{\text{erfc}(bx)}{x} - \frac{b\,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erfc[b*x]/x^2,x]
```

output
```
-(Erfc[b*x]/x) - (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6916, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)}{x^2} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{2b \int \frac{e^{-b^2x^2}}{x}dx}{\sqrt{\pi}} - \frac{\text{erfc}(bx)}{x}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{b \,\text{ExpIntegralEi}\left(-b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erfc}(bx)}{x}$$

input
```
Int[Erfc[b*x]/x^2,x]
```

output
```
-(Erfc[b*x]/x) - (b*ExpIntegralEi[-(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
parts	$-\frac{\operatorname{erfc}(bx)}{x}+\frac{b\ \mathrm{expIntegral}_1\left(b^2x^2\right)}{\sqrt{\pi}}$	25
derivativedivides	$b\left(-\frac{\operatorname{erfc}(bx)}{bx}+\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{\sqrt{\pi}}\right)$	29
default	$b\left(-\frac{\operatorname{erfc}(bx)}{bx}+\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{\sqrt{\pi}}\right)$	29

input
```
int(erfc(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-erfc(b*x)/x+1/Pi^(1/2)*b*Ei(1,b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{erfc}(bx)}{x^2}\,dx = -\frac{\pi+\sqrt{\pi}bx\mathrm{Ei}(-b^2x^2)-\pi\operatorname{erf}(bx)}{\pi x}$$

input
```
integrate(erfc(b*x)/x^2,x, algorithm="fricas")
```

output
```
-(pi + sqrt(pi)*b*x*Ei(-b^2*x^2) - pi*erf(b*x))/(pi*x)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{erfc}(bx)}{x^2}\,dx = \frac{b\,\mathrm{E}_1\left(b^2x^2\right)}{\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)}{x}$$

input
```
integrate(erfc(b*x)/x**2,x)
```

output
```
b*expint(1, b**2*x**2)/sqrt(pi) - erfc(b*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{erfc}(bx)}{x^2}\,dx = -\frac{b\operatorname{Ei}(-b^2x^2)}{\sqrt{\pi}} - \frac{\operatorname{erfc}(bx)}{x}$$

input
```
integrate(erfc(b*x)/x^2,x, algorithm="maxima")
```

output
```
-b*Ei(-b^2*x^2)/sqrt(pi) - erfc(b*x)/x
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^2}\,dx = \int \frac{\operatorname{erfc}(bx)}{x^2}\,dx$$

input
```
integrate(erfc(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{erfc}(bx)}{x^2}\,dx = -\frac{\operatorname{erfc}(b\,x)}{x} - \frac{b\,\operatorname{ei}(-b^2\,x^2)}{\sqrt{\pi}}$$

input
```
int(erfc(b*x)/x^2,x)
```

output
```
- erfc(b*x)/x - (b*ei(-b^2*x^2))/pi^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{erfc}(bx)}{x^2}\,dx = \frac{-\sqrt{\pi}\,ei(-b^2x^2)\,bx + \operatorname{erf}(bx)\,\pi - \pi}{\pi x}$$

input
```
int(erfc(b*x)/x^2,x)
```

output
```
( - sqrt(pi)*ei( - b**2*x**2)*b*x + erf(b*x)*pi - pi)/(pi*x)
```

3.116 $\int \frac{\text{erfc}(bx)}{x^4} dx$

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{\text{erfc}(bx)}{x^4} dx = \frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{erfc}(bx)}{3x^3} + \frac{b^3 \,\text{ExpIntegralEi}\left(-b^2x^2\right)}{3\sqrt{\pi}}$$

output
```
1/3*b/exp(b^2*x^2)/Pi^(1/2)/x^2-1/3*erfc(b*x)/x^3+1/3*b^3*Ei(-b^2*x^2)/Pi^
(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\text{erfc}(bx)}{x^4} dx = \frac{1}{3}\left(-\frac{\text{erfc}(bx)}{x^3} + \frac{b\left(\frac{e^{-b^2x^2}}{x^2} + b^2 \,\text{ExpIntegralEi}\left(-b^2x^2\right)\right)}{\sqrt{\pi}}\right)$$

input
```
Integrate[Erfc[b*x]/x^4,x]
```

output
```
(-(Erfc[b*x]/x^3) + (b*(1/(E^(b^2*x^2)*x^2) + b^2*ExpIntegralEi[-(b^2*x^2)
]))/Sqrt[Pi])/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6916, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)}{x^4} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{2b\int \frac{e^{-b^2x^2}}{x^3}dx}{3\sqrt{\pi}} - \frac{\text{erfc}(bx)}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{2b\left(b^2\left(-\int \frac{e^{-b^2x^2}}{x}dx\right) - \frac{e^{-b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{\text{erfc}(bx)}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{2b\left(-\frac{1}{2}b^2 \,\text{ExpIntegralEi}\left(-b^2x^2\right) - \frac{e^{-b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{\text{erfc}(bx)}{3x^3}$$

input
```
Int[Erfc[b*x]/x^4,x]
```

output
```
-1/3*Erfc[b*x]/x^3 - (2*b*(-1/2*1/(E^(b^2*x^2)*x^2) - (b^2*ExpIntegralEi[-
(b^2*x^2)])/2))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
parts	$-\frac{\operatorname{erfc}(bx)}{3x^3}-\frac{2b\left(-\frac{\mathrm{e}^{-b^2x^2}}{2x^2}+\frac{b^2\,\mathrm{expIntegral}_1\left(b^2x^2\right)}{2}\right)}{3\sqrt{\pi}}$	46
derivativedivides	$b^3\left(-\frac{\operatorname{erfc}(bx)}{3b^3x^3}-\frac{2\left(-\frac{\mathrm{e}^{-b^2x^2}}{2x^2b^2}+\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{2}\right)}{3\sqrt{\pi}}\right)$	53
default	$b^3\left(-\frac{\operatorname{erfc}(bx)}{3b^3x^3}-\frac{2\left(-\frac{\mathrm{e}^{-b^2x^2}}{2x^2b^2}+\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{2}\right)}{3\sqrt{\pi}}\right)$	53

input
```
int(erfc(b*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*erfc(b*x)/x^3-2/3/Pi^(1/2)*b*(-1/2/x^2*exp(-b^2*x^2)+1/2*b^2*Ei(1,b^2
*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{\text{erfc}(bx)}{x^4}\,dx = -\frac{\pi - \pi\,\text{erf}\,(bx) - \sqrt{\pi}\left(b^3x^3\text{Ei}(-b^2x^2) + bxe^{(-b^2x^2)}\right)}{3\,\pi x^3}$$

input
```
integrate(erfc(b*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(pi - pi*erf(b*x) - sqrt(pi)*(b^3*x^3*Ei(-b^2*x^2) + b*x*e^(-b^2*x^2)
))/(pi*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\text{erfc}(bx)}{x^4}\,dx = -\frac{b^3\,\text{E}_1\left(b^2x^2\right)}{3\sqrt{\pi}} + \frac{be^{-b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{erfc}\,(bx)}{3x^3}$$

input
```
integrate(erfc(b*x)/x**4,x)
```

output
```
-b**3*expint(1, b**2*x**2)/(3*sqrt(pi)) + b*exp(-b**2*x**2)/(3*sqrt(pi)*x*
*2) - erfc(b*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{erfc}(bx)}{x^4}\,dx = \frac{b^3\Gamma(-1,b^2x^2)}{3\sqrt{\pi}} - \frac{\operatorname{erfc}(bx)}{3\,x^3}$$

input
```
integrate(erfc(b*x)/x^4,x, algorithm="maxima")
```

output
```
1/3*b^3*gamma(-1, b^2*x^2)/sqrt(pi) - 1/3*erfc(b*x)/x^3
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}(bx)}{x^4}\,dx$$

input
```
integrate(erfc(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{erfc}(bx)}{x^4}\,dx = \frac{b^3\operatorname{ei}(-b^2x^2)}{3\sqrt{\pi}} - \frac{\frac{\operatorname{erfc}(b\,x)}{3} - \frac{b\,x\,e^{-b^2\,x^2}}{3\sqrt{\pi}}}{x^3}$$

input
```
int(erfc(b*x)/x^4,x)
```

output
```
(b^3*ei(-b^2*x^2))/(3*pi^(1/2)) - (erfc(b*x)/3 - (b*x*exp(-b^2*x^2))/(3*pi
^(1/2)))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{\text{erfc}(bx)}{x^4}\,dx = \frac{\sqrt{\pi}\,e^{b^2x^2}\,ei(-b^2x^2)\,b^3x^3 + e^{b^2x^2}\text{erf}(bx)\,\pi - e^{b^2x^2}\pi + \sqrt{\pi}\,bx}{3e^{b^2x^2}\pi\,x^3}$$

input
```
int(erfc(b*x)/x^4,x)
```

output
```
(sqrt(pi)*e**(b**2*x**2)*ei( - b**2*x**2)*b**3*x**3 + e**(b**2*x**2)*erf(b
*x)*pi - e**(b**2*x**2)*pi + sqrt(pi)*b*x)/(3*e**(b**2*x**2)*pi*x**3)
```

3.117 $\int \frac{\text{erfc}(bx)}{x^6} dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int \frac{\text{erfc}(bx)}{x^6} dx = \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} - \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} - \frac{\text{erfc}(bx)}{5x^5} - \frac{b^5 \text{ExpIntegralEi}(-b^2x^2)}{10\sqrt{\pi}}$$

output
```
1/10*b/exp(b^2*x^2)/Pi^(1/2)/x^4-1/10*b^3/exp(b^2*x^2)/Pi^(1/2)/x^2-1/5*er
fc(b*x)/x^5-1/10*b^5*Ei(-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{\text{erfc}(bx)}{x^6} dx = e^{-b^2x^2}\left(\frac{b}{10\sqrt{\pi}x^4} - \frac{b^3}{10\sqrt{\pi}x^2}\right) - \frac{\text{erfc}(bx)}{5x^5} - \frac{b^5 \text{ExpIntegralEi}(-b^2x^2)}{10\sqrt{\pi}}$$

input
```
Integrate[Erfc[b*x]/x^6,x]
```

output
```
(b/(10*Sqrt[Pi]*x^4) - b^3/(10*Sqrt[Pi]*x^2))/E^(b^2*x^2) - Erfc[b*x]/(5*x
^5) - (b^5*ExpIntegralEi[-(b^2*x^2)])/(10*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6916, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)}{x^6} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{2b \int \frac{e^{-b^2x^2}}{x^5}dx}{5\sqrt{\pi}} - \frac{\text{erfc}(bx)}{5x^5}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{2b\left(-\frac{1}{2}b^2 \int \frac{e^{-b^2x^2}}{x^3}dx - \frac{e^{-b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erfc}(bx)}{5x^5}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{2b\left(-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}}{x}dx\right) - \frac{e^{-b^2x^2}}{2x^2}\right) - \frac{e^{-b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erfc}(bx)}{5x^5}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{2b\left(-\frac{1}{2}b^2\left(-\frac{1}{2}b^2 \,\text{ExpIntegralEi}\left(-b^2x^2\right) - \frac{e^{-b^2x^2}}{2x^2}\right) - \frac{e^{-b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erfc}(bx)}{5x^5}$$

input
```
Int[Erfc[b*x]/x^6,x]
```

output
```
-1/5*Erfc[b*x]/x^5 - (2*b*(-1/4*1/(E^(b^2*x^2)*x^4) - (b^2*(-1/2*1/(E^(b^2
*x^2)*x^2) - (b^2*ExpIntegralEi[-(b^2*x^2)])/2))/2))/(5*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result	size
parts	$-\frac{\operatorname{erfc}(bx)}{5x^5}-\frac{2b\left(-\frac{{\mathrm e}^{-b^2x^2}}{4x^4}-\frac{b^2\left(-\frac{{\mathrm e}^{-b^2x^2}}{2x^2}+\frac{b^2\,\mathrm{expIntegral}_1\left(b^2x^2\right)}{2}\right)}{2}\right)}{5\sqrt{\pi}}$	66
derivativedivides	$b^5\left(-\frac{\operatorname{erfc}(bx)}{5b^5x^5}-\frac{2\left(-\frac{{\mathrm e}^{-b^2x^2}}{4b^4x^4}+\frac{{\mathrm e}^{-b^2x^2}}{4x^2b^2}-\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{4}\right)}{5\sqrt{\pi}}\right)$	71
default	$b^5\left(-\frac{\operatorname{erfc}(bx)}{5b^5x^5}-\frac{2\left(-\frac{{\mathrm e}^{-b^2x^2}}{4b^4x^4}+\frac{{\mathrm e}^{-b^2x^2}}{4x^2b^2}-\frac{\mathrm{expIntegral}_1\left(b^2x^2\right)}{4}\right)}{5\sqrt{\pi}}\right)$	71

input
```
int(erfc(b*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*erfc(b*x)/x^5-2/5/Pi^(1/2)*b*(-1/4/x^4*exp(-b^2*x^2)-1/2*b^2*(-1/2/x^
2*exp(-b^2*x^2)+1/2*b^2*Ei(1,b^2*x^2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx = -\frac{2\,\pi - 2\,\pi\operatorname{erf}(bx) + \sqrt{\pi}\left(b^5x^5\operatorname{Ei}(-b^2x^2) + (b^3x^3 - bx)e^{(-b^2x^2)}\right)}{10\,\pi x^5}$$

input
```
integrate(erfc(b*x)/x^6,x, algorithm="fricas")
```

output
```
-1/10*(2*pi - 2*pi*erf(b*x) + sqrt(pi)*(b^5*x^5*Ei(-b^2*x^2) + (b^3*x^3 -
b*x)*e^(-b^2*x^2)))/(pi*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx = \frac{b^5\operatorname{E}_1(b^2x^2)}{10\sqrt{\pi}} - \frac{b^3e^{-b^2x^2}}{10\sqrt{\pi}x^2} + \frac{be^{-b^2x^2}}{10\sqrt{\pi}x^4} - \frac{\operatorname{erfc}(bx)}{5x^5}$$

input
```
integrate(erfc(b*x)/x**6,x)
```

output
```
b**5*expint(1, b**2*x**2)/(10*sqrt(pi)) - b**3*exp(-b**2*x**2)/(10*sqrt(pi
)*x**2) + b*exp(-b**2*x**2)/(10*sqrt(pi)*x**4) - erfc(b*x)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.33

$$\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx = \frac{b^5\Gamma(-2, b^2x^2)}{5\sqrt{\pi}} - \frac{\operatorname{erfc}(bx)}{5x^5}$$

input
```
integrate(erfc(b*x)/x^6,x, algorithm="maxima")
```

output
```
1/5*b^5*gamma(-2, b^2*x^2)/sqrt(pi) - 1/5*erfc(b*x)/x^5
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx = \int \frac{\operatorname{erfc}(bx)}{x^6}\,dx$$

input
```
integrate(erfc(b*x)/x^6,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx = -\frac{\frac{\operatorname{erfc}(b\,x)}{5} + \frac{b^3\,x^3\,e^{-b^2\,x^2}}{10\sqrt{\pi}} - \frac{b\,x\,e^{-b^2\,x^2}}{10\sqrt{\pi}}}{x^5} - \frac{b^5\operatorname{ei}(-b^2\,x^2)}{10\sqrt{\pi}}$$

input
```
int(erfc(b*x)/x^6,x)
```

output
```
- (erfc(b*x)/5 + (b^3*x^3*exp(-b^2*x^2))/(10*pi^(1/2)) - (b*x*exp(-b^2*x^2
))/(10*pi^(1/2)))/x^5 - (b^5*ei(-b^2*x^2))/(10*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erfc}(bx)}{x^6}\,dx = \frac{-\sqrt{\pi}\,e^{b^2x^2}\,ei(-b^2x^2)\,b^5x^5 + 2e^{b^2x^2}\operatorname{erf}(bx)\,\pi - 2e^{b^2x^2}\pi - \sqrt{\pi}\,b^3x^3 + \sqrt{\pi}\,bx}{10e^{b^2x^2}\pi\,x^5}$$

input
```
int(erfc(b*x)/x^6,x)
```

output
```
( - sqrt(pi)*e**(b**2*x**2)*ei( - b**2*x**2)*b**5*x**5 + 2*e**(b**2*x**2)*
erf(b*x)*pi - 2*e**(b**2*x**2)*pi - sqrt(pi)*b**3*x**3 + sqrt(pi)*b*x)/(10
*e**(b**2*x**2)*pi*x**5)
```

3.118 $\int (c+dx)^3 \textbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 292

$$\begin{aligned}\int (c+dx)^3\text{erfc}(a+bx)\,dx = &-\frac{d^2(bc-ad)e^{-(a+bx)^2}}{b^4\sqrt{\pi}}-\frac{(bc-ad)^3e^{-(a+bx)^2}}{b^4\sqrt{\pi}}\\&-\frac{3d^3e^{-(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}}-\frac{3d(bc-ad)^2e^{-(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}}\\&-\frac{d^2(bc-ad)e^{-(a+bx)^2}(a+bx)^2}{b^4\sqrt{\pi}}-\frac{d^3e^{-(a+bx)^2}(a+bx)^3}{4b^4\sqrt{\pi}}\\&+\frac{3d^3\text{erf}(a+bx)}{16b^4}+\frac{3d(bc-ad)^2\text{erf}(a+bx)}{4b^4}\\&+\frac{(bc-ad)^4\text{erf}(a+bx)}{4b^4d}+\frac{(c+dx)^4\text{erfc}(a+bx)}{4d}\end{aligned}$$

output
```
-d^2*(-a*d+b*c)/b^4/exp((b*x+a)^2)/Pi^(1/2)-(-a*d+b*c)^3/b^4/exp((b*x+a)^2
)/Pi^(1/2)-3/8*d^3*(b*x+a)/b^4/exp((b*x+a)^2)/Pi^(1/2)-3/2*d*(-a*d+b*c)^2*
(b*x+a)/b^4/exp((b*x+a)^2)/Pi^(1/2)-d^2*(-a*d+b*c)*(b*x+a)^2/b^4/exp((b*x+
a)^2)/Pi^(1/2)-1/4*d^3*(b*x+a)^3/b^4/exp((b*x+a)^2)/Pi^(1/2)+3/16*d^3*erf(
b*x+a)/b^4+3/4*d*(-a*d+b*c)^2*erf(b*x+a)/b^4+1/4*(-a*d+b*c)^4*erf(b*x+a)/b
^4/d+1/4*(d*x+c)^4*erfc(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92

$$\int (c+dx)^3\text{erfc}(a+bx)\,dx$$
$$=\frac{e^{-(a+bx)^2}\left(2a(5+2a^2)\,d^3-2bd^2(8(1+a^2)\,c+(3+2a^2)\,dx)+4ab^2d(6c^2+4cdx+d^2x^2)-4b^3(4c^3+6c^2\right.}{}$$

input
```
Integrate[(c + d*x)^3*Erfc[a + b*x],x]
```

output
```
(2*a*(5 + 2*a^2)*d^3 - 2*b*d^2*(8*(1 + a^2)*c + (3 + 2*a^2)*d*x) + 4*a*b^2
*d*(6*c^2 + 4*c*d*x + d^2*x^2) - 4*b^3*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 +
d^3*x^3) + (-16*a^3*b*c*d^2 + 4*a^4*d^3 - 8*a*(2*b^3*c^3 + 3*b*c*d^2) + 12
*a^2*(2*b^2*c^2*d + d^3) + 3*(4*b^2*c^2*d + d^3))*E^(a + b*x)^2*Sqrt[Pi]*E
rf[a + b*x] + 4*b^4*E^(a + b*x)^2*Sqrt[Pi]*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*
x^2 + d^3*x^3)*Erfc[a + b*x])/(16*b^4*E^(a + b*x)^2*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6916, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{erfc}(a+bx)\,dx$$

$$\Big\downarrow \text{6916}$$

$$\frac{b\int e^{-(a+bx)^2}(c+dx)^4dx}{2\sqrt{\pi}d}+\frac{(c+dx)^4\text{erfc}(a+bx)}{4d}$$

$$\Big\downarrow \text{2656}$$

$$\frac{b\int\left(\frac{e^{-(a+bx)^2}(bc-ad)^4}{b^4}+\frac{4de^{-(a+bx)^2}(a+bx)(bc-ad)^3}{b^4}+\frac{6d^2e^{-(a+bx)^2}(a+bx)^2(bc-ad)^2}{b^4}+\frac{4d^3e^{-(a+bx)^2}(a+bx)^3(bc-ad)}{b^4}+\frac{d^4e^{-(a+}}{}\right.}{2\sqrt{\pi}d}$$

$$\frac{(c+dx)^4\text{erfc}(a+bx)}{4d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\left(-\frac{2d^3e^{-(a+bx)^2}(bc-ad)}{b^5}-\frac{2d^3e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{b^5}+\frac{3\sqrt{\pi}d^2(bc-ad)^2\text{erf}(a+bx)}{2b^5}-\frac{3d^2e^{-(a+bx)^2}(a+bx)(bc-ad)^2}{b^5}+\frac{\sqrt{\pi}(bc-ad}{2}\right.}{2\sqrt{\pi}d}$$

$$\frac{(c+dx)^4\text{erfc}(a+bx)}{4d}$$

input
```
Int[(c + d*x)^3*Erfc[a + b*x],x]
```

output
```
(b*((-2*d^3*(b*c - a*d))/(b^5*E^(a + b*x)^2) - (2*d*(b*c - a*d)^3)/(b^5*E^
(a + b*x)^2) - (3*d^4*(a + b*x))/(4*b^5*E^(a + b*x)^2) - (3*d^2*(b*c - a*d
)^2*(a + b*x))/(b^5*E^(a + b*x)^2) - (2*d^3*(b*c - a*d)*(a + b*x)^2)/(b^5*
E^(a + b*x)^2) - (d^4*(a + b*x)^3)/(2*b^5*E^(a + b*x)^2) + (3*d^4*Sqrt[Pi]
*Erf[a + b*x])/(8*b^5) + (3*d^2*(b*c - a*d)^2*Sqrt[Pi]*Erf[a + b*x])/(2*b^
5) + ((b*c - a*d)^4*Sqrt[Pi]*Erf[a + b*x])/(2*b^5)))/(2*d*Sqrt[Pi]) + ((c
+ d*x)^4*Erfc[a + b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.65

method	result
parallelrisch	$\frac{16x\,\mathrm{e}^{-(bx+a)^2}a\,b^2c\,d^2+4d^3\,\mathrm{erfc}(bx+a)x^4\sqrt{\pi}\,b^4+16x\,\mathrm{erfc}(bx+a)c^3\sqrt{\pi}\,b^4+16\sqrt{\pi}\,\mathrm{erfc}(bx+a)a\,b^3c^3-12\sqrt{\pi}\,\mathrm{erfc}(bx+a)}{}$
parts	$\frac{\mathrm{erfc}(bx+a)d^3x^4}{4}+\mathrm{erfc}\,(bx+a)\,d^2c\,x^3+\frac{3\,\mathrm{erfc}(bx+a)d\,c^2x^2}{2}+\mathrm{erfc}\,(bx+a)\,c^3x+\frac{\mathrm{erfc}(bx+a)c^4}{4d}+$
derivativedivides	$\frac{\frac{d^3\,\mathrm{erfc}(bx+a)a^4}{4b^3}-\frac{d^2\,\mathrm{erfc}(bx+a)a^3c}{b^2}-\frac{d^3\,\mathrm{erfc}(bx+a)a^3(bx+a)}{b^3}+\frac{3d\,\mathrm{erfc}(bx+a)a^2c^2}{2b}+\frac{3d^2\,\mathrm{erfc}(bx+a)a^2c(bx+a)}{b^2}+\frac{3d^3\,\mathrm{erfc}(bx+a)a}{2b^3}}{}$
default	$\frac{\frac{d^3\,\mathrm{erfc}(bx+a)a^4}{4b^3}-\frac{d^2\,\mathrm{erfc}(bx+a)a^3c}{b^2}-\frac{d^3\,\mathrm{erfc}(bx+a)a^3(bx+a)}{b^3}+\frac{3d\,\mathrm{erfc}(bx+a)a^2c^2}{2b}+\frac{3d^2\,\mathrm{erfc}(bx+a)a^2c(bx+a)}{b^2}+\frac{3d^3\,\mathrm{erfc}(bx+a)a}{2b^3}}{}$

input
```
int((d*x+c)^3*erfc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*(16*x*exp(-(b*x+a)^2)*a*b^2*c*d^2+4*d^3*erfc(b*x+a)*x^4*Pi^(1/2)*b^4+
16*x*erfc(b*x+a)*c^3*Pi^(1/2)*b^4+16*Pi^(1/2)*erfc(b*x+a)*a*b^3*c^3-12*Pi^
(1/2)*erfc(b*x+a)*b^2*c^2*d+4*exp(-(b*x+a)^2)*a^3*d^3-16*exp(-(b*x+a)^2)*b
^3*c^3+10*exp(-(b*x+a)^2)*a*d^3+16*Pi^(1/2)*erfc(b*x+a)*a^3*b*c*d^2-24*Pi^
(1/2)*erfc(b*x+a)*a^2*b^2*c^2*d+24*Pi^(1/2)*erfc(b*x+a)*a*b*c*d^2+16*d^2*c
*erfc(b*x+a)*x^3*Pi^(1/2)*b^4+24*c^2*d*erfc(b*x+a)*x^2*Pi^(1/2)*b^4-3*Pi^(
1/2)*erfc(b*x+a)*d^3-4*d^3*exp(-(b*x+a)^2)*x^3*b^3-6*x*exp(-(b*x+a)^2)*b*d
^3-16*exp(-(b*x+a)^2)*b*c*d^2-4*x*exp(-(b*x+a)^2)*a^2*b*d^3-24*x*exp(-(b*x
+a)^2)*b^3*c^2*d+4*x^2*exp(-(b*x+a)^2)*a*b^2*d^3-16*x^2*exp(-(b*x+a)^2)*b^
3*c*d^2-16*exp(-(b*x+a)^2)*a^2*b*c*d^2+24*exp(-(b*x+a)^2)*a*b^2*c^2*d-4*Pi
^(1/2)*erfc(b*x+a)*a^4*d^3-12*Pi^(1/2)*erfc(b*x+a)*a^2*d^3)/Pi^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.08

$$\int (c+dx)^3 \text{erfc}(a+bx)\,dx$$
$$= \frac{4\,\pi b^4 d^3 x^4 + 16\,\pi b^4 c d^2 x^3 + 24\,\pi b^4 c^2 d x^2 + 16\,\pi b^4 c^3 x - 2\,\sqrt{\pi}(2\,b^3 d^3 x^3 + 8\,b^3 c^3 - 12\,ab^2 c^2 d + 8\,(a^2+1)bcd}{}$$

input
```
integrate((d*x+c)^3*erfc(b*x+a),x, algorithm="fricas")
```

output
```
1/16*(4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi
*b^4*c^3*x - 2*sqrt(pi)*(2*b^3*d^3*x^3 + 8*b^3*c^3 - 12*a*b^2*c^2*d + 8*(a
^2 + 1)*b*c*d^2 - (2*a^3 + 5*a)*d^3 + 2*(4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (1
2*b^3*c^2*d - 8*a*b^2*c*d^2 + (2*a^2 + 3)*b*d^3)*x)*e^(-b^2*x^2 - 2*a*b*x
- a^2) - (4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 1
6*pi*b^4*c^3*x + pi*(16*a*b^3*c^3 - 12*(2*a^2 + 1)*b^2*c^2*d + 8*(2*a^3 +
3*a)*b*c*d^2 - (4*a^4 + 12*a^2 + 3)*d^3))*erf(b*x + a))/(pi*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(258) = 516.

Time = 1.87 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.55

$$\int (c+dx)^3 \text{erfc}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*erfc(b*x+a),x)
```

output
```
Piecewise((-a**4*d**3*erfc(a + b*x)/(4*b**4) + a**3*c*d**2*erfc(a + b*x)/b
**3 + a**3*d**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b**4)
 - 3*a**2*c**2*d*erfc(a + b*x)/(2*b**2) - a**2*c*d**2*exp(-a**2)*exp(-b**2
*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b**3) - a**2*d**3*x*exp(-a**2)*exp(-b**2*x*
*2)*exp(-2*a*b*x)/(4*sqrt(pi)*b**3) - 3*a**2*d**3*erfc(a + b*x)/(4*b**4) +
 a*c**3*erfc(a + b*x)/b + 3*a*c**2*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b
*x)/(2*sqrt(pi)*b**2) + a*c*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x
)/(sqrt(pi)*b**2) + a*d**3*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(
4*sqrt(pi)*b**2) + 3*a*c*d**2*erfc(a + b*x)/(2*b**3) + 5*a*d**3*exp(-a**2)
*exp(-b**2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b**4) + c**3*x*erfc(a + b*x) +
3*c**2*d*x**2*erfc(a + b*x)/2 + c*d**2*x**3*erfc(a + b*x) + d**3*x**4*erfc
(a + b*x)/4 - c**3*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) -
 3*c**2*d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) - c*d*
*2*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi)*b) - d**3*x**3*
exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(4*sqrt(pi)*b) - 3*c**2*d*erfc(a
+ b*x)/(4*b**2) - c*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sqrt(pi
)*b**3) - 3*d**3*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(8*sqrt(pi)*b*
*3) - 3*d**3*erfc(a + b*x)/(16*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/
2 + c*d**2*x**3 + d**3*x**4/4)*erfc(a), True))
```

Maxima [F]

$$\int (c+dx)^3 \text{erfc}(a+bx)\,dx = \int (dx+c)^3 \,\text{erfc}\,(bx+a)\;dx$$

input
```
integrate((d*x+c)^3*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*erfc(b*x + a), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.49

$$\int (c+dx)^3\text{erfc}(a+bx)\,dx$$
$$=\frac{1}{4}d^3x^4+cd^2x^3+\frac{3}{2}c^2dx^2-\left(x\,\text{erf}\left(bx+a\right)-\frac{\frac{\sqrt{\pi}a\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}\right)c^3$$
$$-\frac{3}{4}\left(2\,x^2\,\text{erf}\left(bx+a\right)+\frac{\frac{\sqrt{\pi}\left(2\,a^2+1\right)\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(b\left(x+\frac{a}{b}\right)-2\,a\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}b}\right)c^2d$$
$$-\frac{1}{2}\left(2\,x^3\,\text{erf}\left(bx+a\right)-\frac{\frac{\sqrt{\pi}\left(2\,a^3+3\,a\right)\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{2\left(b^2\left(x+\frac{a}{b}\right)^2-3\,ab\left(x+\frac{a}{b}\right)+3\,a^2+1\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}b^2}\right)cd^2$$
$$-\frac{1}{16}\left(4\,x^4\,\text{erf}\left(bx+a\right)+\frac{\frac{\sqrt{\pi}\left(4\,a^4+12\,a^2+3\right)\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(2\,b^3\left(x+\frac{a}{b}\right)^3-8\,ab^2\left(x+\frac{a}{b}\right)^2+12\,a^2b\left(x+\frac{a}{b}\right)-8\,a^3+3\,b\left(x+\frac{a}{b}\right)-8\,a\right.}{b}}{\sqrt{\pi}b^3}\right.$$
$$+c^3x$$

input
```
integrate((d*x+c)^3*erfc(b*x+a),x, algorithm="giac")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 - (x*erf(b*x + a) - (sqrt(pi)*a*er
f(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/sqrt(pi))*c^3 - 3/4*(2
*x^2*erf(b*x + a) + (sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x +
a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(sqrt(pi)*b))*c^2*d - 1/2*(2*x
^3*erf(b*x + a) - (sqrt(pi)*(2*a^3 + 3*a)*erf(-b*(x + a/b))/b - 2*(b^2*(x
+ a/b)^2 - 3*a*b*(x + a/b) + 3*a^2 + 1)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(s
qrt(pi)*b^2))*c*d^2 - 1/16*(4*x^4*erf(b*x + a) + (sqrt(pi)*(4*a^4 + 12*a^2
 + 3)*erf(-b*(x + a/b))/b + 2*(2*b^3*(x + a/b)^3 - 8*a*b^2*(x + a/b)^2 + 1
2*a^2*b*(x + a/b) - 8*a^3 + 3*b*(x + a/b) - 8*a)*e^(-b^2*x^2 - 2*a*b*x - a
^2)/b)/(sqrt(pi)*b^3))*d^3 + c^3*x
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.21

$$\int (c+dx)^3\text{erfc}(a+bx)\,dx = \frac{d^3\,x^4\,\text{erfc}(a+b\,x)}{4} - \frac{\text{erfc}(a+b\,x)\,\left(b^2\left(\frac{3\,d\,a^2\,c^2}{2}+\frac{3\,d\,c^2}{4}\right)-b\left(c\,a^3\,d^2+\frac{3\,c\,a\,d^2}{2}\right)+\frac{3\,d^3}{16}+\frac{3\,a^2\,d^3}{4}+\frac{a^4\,d^3}{4}-a\,b^3\,c^3\right)}{b^4} + c^3\,x\,\text{erfc}(a+b\,x) + \frac{e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(2\,a^3\,d^3-8\,a^2\,b\,c\,d^2+12\,a\,b^2\,c^2\,d+5\,a\,d^3-8\,b^3\,c^3-8\,b\,c\,d^2)}{8\,b^4\,\sqrt{\pi}} + \frac{3\,c^2\,d\,x^2\,\text{erfc}(a+b\,x)}{2} + c\,d^2\,x^3\,\text{erfc}(a+b\,x) - \frac{x\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(2\,a^2\,d^3-8\,a\,b\,c\,d^2+12\,b^2\,c^2\,d+3\,d^3)}{8\,b^3\,\sqrt{\pi}} - \frac{d^3\,x^3\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}}{4\,b\,\sqrt{\pi}} + \frac{x^2\,e^{-a^2-2\,a\,b\,x-b^2\,x^2}\,(a\,d^3-4\,b\,c\,d^2)}{4\,b^2\,\sqrt{\pi}}$$

input
```
int(erfc(a + b*x)*(c + d*x)^3,x)
```

output
```
(d^3*x^4*erfc(a + b*x))/4 - (erfc(a + b*x)*(b^2*((3*c^2*d)/4 + (3*a^2*c^2*
d)/2) - b*(a^3*c*d^2 + (3*a*c*d^2)/2) + (3*d^3)/16 + (3*a^2*d^3)/4 + (a^4*
d^3)/4 - a*b^3*c^3))/b^4 + c^3*x*erfc(a + b*x) + (exp(- a^2 - b^2*x^2 - 2*
a*b*x)*(5*a*d^3 + 2*a^3*d^3 - 8*b^3*c^3 - 8*b*c*d^2 + 12*a*b^2*c^2*d - 8*a
^2*b*c*d^2))/(8*b^4*pi^(1/2)) + (3*c^2*d*x^2*erfc(a + b*x))/2 + c*d^2*x^3*
erfc(a + b*x) - (x*exp(- a^2 - b^2*x^2 - 2*a*b*x)*(3*d^3 + 2*a^2*d^3 + 12*
b^2*c^2*d - 8*a*b*c*d^2))/(8*b^3*pi^(1/2)) - (d^3*x^3*exp(- a^2 - b^2*x^2
- 2*a*b*x))/(4*b*pi^(1/2)) + (x^2*exp(- a^2 - b^2*x^2 - 2*a*b*x)*(a*d^3 -
4*b*c*d^2))/(4*b^2*pi^(1/2))
```

Reduce [F]

$$\int (c+dx)^3 \operatorname{erfc}(a+bx)\,dx$$
$$= \frac{-4\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,a\,c^3 - 4\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,b\,c^3x - 4\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int \operatorname{erf}(bx+\right.}{}$$

input
```
int((d*x+c)^3*erfc(b*x+a),x)
```

output
```
( - 4*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c**3 - 4*sqr
t(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c**3*x - 4*sqrt(pi)*e
**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**3,x)*b*d**3 - 12*sqrt(p
i)*e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**2,x)*b*c*d**2 - 12*
sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*c**2*d +
4*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b*c**3*x + 6*sqrt(pi)*e**(a**2
+ 2*a*b*x + b**2*x**2)*b*c**2*d*x**2 + 4*sqrt(pi)*e**(a**2 + 2*a*b*x + b**
2*x**2)*b*c*d**2*x**3 + sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b*d**3*x*
*4 - 4*c**3)/(4*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.119 $\int (c+dx)^2 \mathbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 194

$$\int (c+dx)^2\mathrm{erfc}(a+bx)\,dx = -\frac{d^2e^{-(a+bx)^2}}{3b^3\sqrt{\pi}}-\frac{(bc-ad)^2e^{-(a+bx)^2}}{b^3\sqrt{\pi}} -\frac{d(bc-ad)e^{-(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}} -\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}}+\frac{d(bc-ad)\mathrm{erf}(a+bx)}{2b^3} +\frac{(bc-ad)^3\mathrm{erf}(a+bx)}{3b^3d}+\frac{(c+dx)^3\mathrm{erfc}(a+bx)}{3d}$$

output
```
-1/3*d^2/b^3/exp((b*x+a)^2)/Pi^(1/2)-(-a*d+b*c)^2/b^3/exp((b*x+a)^2)/Pi^(1
/2)-d*(-a*d+b*c)*(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)-1/3*d^2*(b*x+a)^2/b^3
/exp((b*x+a)^2)/Pi^(1/2)+1/2*d*(-a*d+b*c)*erf(b*x+a)/b^3+1/3*(-a*d+b*c)^3*
erf(b*x+a)/b^3/d+1/3*(d*x+c)^3*erfc(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int (c+dx)^2\operatorname{erfc}(a+bx)\,dx$$

$$= \frac{-((-3bcd-6a^2bcd+2a^3d^2+3a(2b^2c^2+d^2))\operatorname{erf}(a+bx)) + \frac{2e^{-(a+bx)^2}\left(-((1+a^2)d^2)+abd(3c+dx)-b^2(3c^2+3cdx+\dots\right.}{\sqrt{\pi}\dots}}{6b^3}$$

input
```
Integrate[(c + d*x)^2*Erfc[a + b*x],x]
```

output
```
(-((-3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + 3*a*(2*b^2*c^2 + d^2))*Erf[a + b*
x]) + (2*(-((1 + a^2)*d^2) + a*b*d*(3*c + d*x) - b^2*(3*c^2 + 3*c*d*x + d^
2*x^2) + b^3*E^(a + b*x)^2*Sqrt[Pi]*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Erfc[a +
 b*x]))/(E^(a + b*x)^2*Sqrt[Pi]))/(6*b^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6916, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\operatorname{erfc}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6916}$$

$$\frac{2b\int e^{-(a+bx)^2}(c+dx)^3dx}{3\sqrt{\pi}d}+\frac{(c+dx)^3\operatorname{erfc}(a+bx)}{3d}$$

$$\Big\downarrow \text{ 2656}$$

$$\frac{2b\int\left(\frac{e^{-(a+bx)^2}(bc-ad)^3}{b^3}+\frac{3de^{-(a+bx)^2}(a+bx)(bc-ad)^2}{b^3}+\frac{3d^2e^{-(a+bx)^2}(a+bx)^2(bc-ad)}{b^3}+\frac{d^3e^{-(a+bx)^2}(a+bx)^3}{b^3}\right)dx}{3\sqrt{\pi}d}+\frac{(c+dx)^3\text{erfc}(a+bx)}{3d}$$

$$\Big\downarrow \; 2009$$

$$\frac{2b\left(\frac{3\sqrt{\pi}d^2(bc-ad)\text{erf}(a+bx)}{4b^4}-\frac{3d^2e^{-(a+bx)^2}(a+bx)(bc-ad)}{2b^4}+\frac{\sqrt{\pi}(bc-ad)^3\text{erf}(a+bx)}{2b^4}-\frac{3de^{-(a+bx)^2}(bc-ad)^2}{2b^4}-\frac{d^3e^{-(a+bx)^2}}{2b^4}-d^3e\cdots\right.}{3\sqrt{\pi}d}$$

$$\frac{(c+dx)^3\text{erfc}(a+bx)}{3d}$$

input
```
Int[(c + d*x)^2*Erfc[a + b*x],x]
```

output
```
(2*b*(-1/2*d^3/(b^4*E^(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(2*b^4*E^(a + b*x
)^2) - (3*d^2*(b*c - a*d)*(a + b*x))/(2*b^4*E^(a + b*x)^2) - (d^3*(a + b*x
)^2)/(2*b^4*E^(a + b*x)^2) + (3*d^2*(b*c - a*d)*Sqrt[Pi]*Erf[a + b*x])/(4*
b^4) + ((b*c - a*d)^3*Sqrt[Pi]*Erf[a + b*x])/(2*b^4)))/(3*d*Sqrt[Pi]) + ((
c + d*x)^3*Erfc[a + b*x])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{2d^2 \operatorname{erfc}(bx+a)x^3\sqrt{\pi}\,b^3+6cd\,x^2 \operatorname{erfc}(bx+a)\sqrt{\pi}\,b^3+6c^2x \operatorname{erfc}(bx+a)\sqrt{\pi}\,b^3+2\sqrt{\pi}\operatorname{erfc}(bx+a)a^3d^2-6\sqrt{\pi}\operatorname{erfc}(bx+a)a^2b\ldots}{}$
parts	$\frac{\operatorname{erfc}(bx+a)d^2x^3}{3}+\operatorname{erfc}(bx+a)\,dc\,x^2+\operatorname{erfc}(bx+a)\,c^2x+\frac{\operatorname{erfc}(bx+a)c^3}{3d}+\frac{2b\left(\frac{c^3\sqrt{\pi}\operatorname{erf}(bx+a)}{2b}+\mathrm{e}^{-\ldots}\right.}{}$
derivativedivides	$\frac{-\frac{d^2\operatorname{erfc}(bx+a)a^3}{3b^2}+\frac{d\operatorname{erfc}(bx+a)a^2c}{b}+\frac{d^2\operatorname{erfc}(bx+a)a^2(bx+a)}{b^2}-\operatorname{erfc}(bx+a)a\,c^2-\frac{2d\operatorname{erfc}(bx+a)ac(bx+a)}{b}-\frac{d^2\operatorname{erfc}(bx+a)a(bx+\ldots}{b^2}}{}$
default	$\frac{-\frac{d^2\operatorname{erfc}(bx+a)a^3}{3b^2}+\frac{d\operatorname{erfc}(bx+a)a^2c}{b}+\frac{d^2\operatorname{erfc}(bx+a)a^2(bx+a)}{b^2}-\operatorname{erfc}(bx+a)a\,c^2-\frac{2d\operatorname{erfc}(bx+a)ac(bx+a)}{b}-\frac{d^2\operatorname{erfc}(bx+a)a(bx+\ldots}{b^2}}{}$

input
```
int((d*x+c)^2*erfc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*d^2*erfc(b*x+a)*x^3*Pi^(1/2)*b^3+6*c*d*x^2*erfc(b*x+a)*Pi^(1/2)*b^3
+6*c^2*x*erfc(b*x+a)*Pi^(1/2)*b^3+2*Pi^(1/2)*erfc(b*x+a)*a^3*d^2-6*Pi^(1/2
)*erfc(b*x+a)*a^2*b*c*d+6*Pi^(1/2)*erfc(b*x+a)*a*b^2*c^2-2*d^2*exp(-(b*x+a
)^2)*x^2*b^2+2*x*exp(-(b*x+a)^2)*a*b*d^2-6*x*exp(-(b*x+a)^2)*b^2*c*d+3*Pi^
(1/2)*erfc(b*x+a)*a*d^2-3*Pi^(1/2)*erfc(b*x+a)*b*c*d-2*exp(-(b*x+a)^2)*a^2
*d^2+6*exp(-(b*x+a)^2)*a*b*c*d-6*exp(-(b*x+a)^2)*b^2*c^2-2*exp(-(b*x+a)^2)
*d^2)/Pi^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\int (c+dx)^2 \operatorname{erfc}(a+bx)\,dx$$
$$= \frac{2\,\pi b^3d^2x^3+6\,\pi b^3cdx^2+6\,\pi b^3c^2x-2\,\sqrt{\pi}(b^2d^2x^2+3\,b^2c^2-3\,abcd+(a^2+1)d^2+(3\,b^2cd-abd^2)x)e^{(-b^2\ldots}}{\ldots}$$

input
```
integrate((d*x+c)^2*erfc(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*pi*b^3*d^2*x^3 + 6*pi*b^3*c*d*x^2 + 6*pi*b^3*c^2*x - 2*sqrt(pi)*(b^
2*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 + 1)*d^2 + (3*b^2*c*d - a*b*d^2)*
x)*e^(-b^2*x^2 - 2*a*b*x - a^2) - (2*pi*b^3*d^2*x^3 + 6*pi*b^3*c*d*x^2 + 6
*pi*b^3*c^2*x + pi*(6*a*b^2*c^2 - 3*(2*a^2 + 1)*b*c*d + (2*a^3 + 3*a)*d^2)
)*erf(b*x + a))/(pi*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(165) = 330$.

Time = 0.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.05

$$\int (c+dx)^2 \operatorname{erfc}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3d^2\operatorname{erfc}(a+bx)}{3b^3} - \frac{a^2cd\operatorname{erfc}(a+bx)}{b^2} - \frac{a^2d^2e^{-a^2}e^{-b^2x^2}e^{-2abx}}{3\sqrt{\pi}b^3} + \frac{ac^2\operatorname{erfc}(a+bx)}{b} + \frac{acde^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b^2} + \frac{ad^2xe^{-a^2}e^{-b^2x^2}e^{-2a}}{3\sqrt{\pi}b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\operatorname{erfc}(a) \end{cases}$$

input
```
integrate((d*x+c)**2*erfc(b*x+a),x)
```

output
```
Piecewise((a**3*d**2*erfc(a + b*x)/(3*b**3) - a**2*c*d*erfc(a + b*x)/b**2
- a**2*d**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(pi)*b**3) + a
*c**2*erfc(a + b*x)/b + a*c*d*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sq
rt(pi)*b**2) + a*d**2*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*sqrt(p
i)*b**2) + a*d**2*erfc(a + b*x)/(2*b**3) + c**2*x*erfc(a + b*x) + c*d*x**2
*erfc(a + b*x) + d**2*x**3*erfc(a + b*x)/3 - c**2*exp(-a**2)*exp(-b**2*x**
2)*exp(-2*a*b*x)/(sqrt(pi)*b) - c*d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*
b*x)/(sqrt(pi)*b) - d**2*x**2*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(3*
sqrt(pi)*b) - c*d*erfc(a + b*x)/(2*b**2) - d**2*exp(-a**2)*exp(-b**2*x**2)
*exp(-2*a*b*x)/(3*sqrt(pi)*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x*
*3/3)*erfc(a), True))
```

Maxima [F]

$$\int (c+dx)^2\text{erfc}(a+bx)\,dx = \int (dx+c)^2\,\text{erfc}\,(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*erfc(b*x + a), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.44

$$\int (c+dx)^2\text{erfc}(a+bx)\,dx$$
$$= \frac{1}{3}\,d^2x^3 + cdx^2 - \left(x\,\text{erf}\,(bx+a) - \frac{\frac{\sqrt{\pi}a\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}\right)c^2 - \frac{1}{2}\left(2\,x^2\,\text{erf}\,(bx+a) + \frac{\frac{\sqrt{\pi}(2\,a^2+1)\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\,\left(b\left(x+\frac{a}{b}\right)-2\,a\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}b}\right)cd - \frac{1}{6}\left(2\,x^3\,\text{erf}\,(bx+a) - \frac{\frac{\sqrt{\pi}(2\,a^3+3\,a)\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{2\,\left(b^2\left(x+\frac{a}{b}\right)^2-3\,ab\left(x+\frac{a}{b}\right)+3\,a^2+1\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}b^2}\right)d^2 + c^2x$$

input
```
integrate((d*x+c)^2*erfc(b*x+a),x, algorithm="giac")
```

output
```
1/3*d^2*x^3 + c*d*x^2 - (x*erf(b*x + a) - (sqrt(pi)*a*erf(-b*(x + a/b))/b
- e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/sqrt(pi))*c^2 - 1/2*(2*x^2*erf(b*x + a)
+ (sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^
2*x^2 - 2*a*b*x - a^2)/b)/(sqrt(pi)*b))*c*d - 1/6*(2*x^3*erf(b*x + a) - (s
qrt(pi)*(2*a^3 + 3*a)*erf(-b*(x + a/b))/b - 2*(b^2*(x + a/b)^2 - 3*a*b*(x
+ a/b) + 3*a^2 + 1)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/(sqrt(pi)*b^2))*d^2 +
c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13

$$\int (c+dx)^2\text{erfc}(a+bx)\,dx = \frac{d^2\,x^3\,\text{erfc}(a+b\,x)}{3} - \frac{\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}\,\left(\frac{b^2\,c^2}{\sqrt{\pi}}-\frac{a\,d\,b\,c}{\sqrt{\pi}}+\frac{a^2\,d^2+d^2}{3\,\sqrt{\pi}}\right)}{b^3} + \frac{\text{erfc}(a+b\,x)\,\left(\frac{a\,d^2}{2}-b\,\left(c\,d\,a^2+\frac{c\,d}{2}\right)+\frac{a^3\,d^2}{3}+a\,b^2\,c^2\right)}{b^3} + c^2\,x\,\text{erfc}(a+b\,x)+c\,d\,x^2\,\text{erfc}(a+b\,x) + \frac{x\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}\,\left(a\,d^2-3\,b\,c\,d\right)}{3\,b^2\,\sqrt{\pi}} - \frac{d^2\,x^2\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}}{3\,b\,\sqrt{\pi}}$$

input
```
int(erfc(a + b*x)*(c + d*x)^2,x)
```

output
```
(d^2*x^3*erfc(a + b*x))/3 - (exp(- a^2 - b^2*x^2 - 2*a*b*x)*((d^2 + a^2*d^
2)/(3*pi^(1/2)) + (b^2*c^2)/pi^(1/2) - (a*b*c*d)/pi^(1/2)))/b^3 + (erfc(a
+ b*x)*((a*d^2)/2 - b*((c*d)/2 + a^2*c*d) + (a^3*d^2)/3 + a*b^2*c^2))/b^3
+ c^2*x*erfc(a + b*x) + c*d*x^2*erfc(a + b*x) + (x*exp(- a^2 - b^2*x^2 - 2
*a*b*x)*(a*d^2 - 3*b*c*d))/(3*b^2*pi^(1/2)) - (d^2*x^2*exp(- a^2 - b^2*x^2
 - 2*a*b*x))/(3*b*pi^(1/2))
```

Reduce [F]

$$\int (c+dx)^2\text{erfc}(a+bx)\,dx = \frac{-3\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,a\,c^2-3\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,b\,c^2x-3\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int \text{erf}(bx+\right.}{}$$

input
```
int((d*x+c)^2*erfc(b*x+a),x)
```

output
```
( - 3*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c**2 - 3*sqr
t(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c**2*x - 3*sqrt(pi)*e
**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**2,x)*b*d**2 - 6*sqrt(pi
)*e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*c*d + 3*sqrt(pi)
*e**(a**2 + 2*a*b*x + b**2*x**2)*b*c**2*x + 3*sqrt(pi)*e**(a**2 + 2*a*b*x
+ b**2*x**2)*b*c*d*x**2 + sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b*d**2*
x**3 - 3*c**2)/(3*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.120 $\int (c+dx)\mathbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 119

$$\int (c+dx)\text{erfc}(a+bx)\,dx=-\frac{(bc-ad)e^{-(a+bx)^2}}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}}+\frac{d\text{erf}(a+bx)}{4b^2}+\frac{(bc-ad)^2\text{erf}(a+bx)}{2b^2d}+\frac{(c+dx)^2\text{erfc}(a+bx)}{2d}$$

output
```
-(-a*d+b*c)/b^2/exp((b*x+a)^2)/Pi^(1/2)-1/2*d*(b*x+a)/b^2/exp((b*x+a)^2)/P
i^(1/2)+1/4*d*erf(b*x+a)/b^2+1/2*(-a*d+b*c)^2*erf(b*x+a)/b^2/d+1/2*(d*x+c)
^2*erfc(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int (c+dx)\text{erfc}(a+bx)\,dx$$
$$=\frac{e^{-(a+bx)^2}\left(-4bc+2ad-2bdx+(-4abc+d+2a^2d)\,e^{(a+bx)^2}\sqrt{\pi}\text{erf}(a+bx)+2b^2e^{(a+bx)^2}\sqrt{\pi}x(2c+dx)\text{er}\right.}{4b^2\sqrt{\pi}}$$

input
```
Integrate[(c + d*x)*Erfc[a + b*x],x]
```

output
```
(-4*b*c + 2*a*d - 2*b*d*x + (-4*a*b*c + d + 2*a^2*d)*E^(a + b*x)^2*Sqrt[Pi
]*Erf[a + b*x] + 2*b^2*E^(a + b*x)^2*Sqrt[Pi]*x*(2*c + d*x)*Erfc[a + b*x])
/(4*b^2*E^(a + b*x)^2*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6916, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{erfc}(a+bx)\,dx$$

$$\Big\downarrow \text{6916}$$

$$\frac{b\int e^{-(a+bx)^2}(c+dx)^2dx}{\sqrt{\pi}d}+\frac{(c+dx)^2\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2656}$$

$$\frac{b\int \left(\frac{e^{-(a+bx)^2}(bc-ad)^2}{b^2}+\frac{2de^{-(a+bx)^2}(a+bx)(bc-ad)}{b^2}+\frac{d^2e^{-(a+bx)^2}(a+bx)^2}{b^2}\right)dx}{\sqrt{\pi}d}+\frac{(c+dx)^2\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{\sqrt{\pi}(bc-ad)^2\text{erf}(a+bx)}{2b^3}-\frac{de^{-(a+bx)^2}(bc-ad)}{b^3}+\frac{\sqrt{\pi}d^2\text{erf}(a+bx)}{4b^3}-\frac{d^2e^{-(a+bx)^2}(a+bx)}{2b^3}\right)}{\sqrt{\pi}d}+\frac{(c+dx)^2\text{erfc}(a+bx)}{2d}$$

input
```
Int[(c + d*x)*Erfc[a + b*x],x]
```

output
```
(b*(-((d*(b*c - a*d))/(b^3*E^(a + b*x)^2)) - (d^2*(a + b*x))/(2*b^3*E^(a +
 b*x)^2) + (d^2*Sqrt[Pi]*Erf[a + b*x])/(4*b^3) + ((b*c - a*d)^2*Sqrt[Pi]*E
rf[a + b*x])/(2*b^3)))/(d*Sqrt[Pi]) + ((c + d*x)^2*Erfc[a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6916
```
Int[Erfc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfc[a + b*x]/(d*(m + 1))), x] + Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)/E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\frac{\operatorname{erfc}(bx+a)da(bx+a)}{b}+\operatorname{erfc}(bx+a)c(bx+a)+\frac{\operatorname{erfc}(bx+a)d(bx+a)^2}{2b}-\frac{-d\left(-\frac{(bx+a)\mathrm{e}^{-(bx+a)^2}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}(bx+a)}{4}\right)+\mathrm{e}^{-(bx+a)^2}b}{\sqrt{\pi}\,b}}{b}$
default	$\frac{-\frac{\operatorname{erfc}(bx+a)da(bx+a)}{b}+\operatorname{erfc}(bx+a)c(bx+a)+\frac{\operatorname{erfc}(bx+a)d(bx+a)^2}{2b}-\frac{-d\left(-\frac{(bx+a)\mathrm{e}^{-(bx+a)^2}}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}(bx+a)}{4}\right)+\mathrm{e}^{-(bx+a)^2}b}{\sqrt{\pi}\,b}}{b}$
parallelrisch	$\frac{2d\,x^2\operatorname{erfc}(bx+a)\sqrt{\pi}\,b^2+4cx\,\operatorname{erfc}(bx+a)\sqrt{\pi}\,b^2-2\sqrt{\pi}\,\operatorname{erfc}(bx+a)a^2d+4\sqrt{\pi}\,\operatorname{erfc}(bx+a)abc-2\,\mathrm{e}^{-(bx+a)^2}bdx-d\,\operatorname{erfc}(bx}{4\sqrt{\pi}\,b^2}$
parts	$\frac{\operatorname{erfc}(bx+a)d\,x^2}{2}+\operatorname{erfc}(bx+a)\,cx+\frac{b\left(\mathrm{e}^{-a^2}d\left(-\frac{x\,\mathrm{e}^{-b^2x^2-2bxa}}{2b^2}-\frac{a\left(-\frac{\mathrm{e}^{-b^2x^2-2bxa}}{2b^2}-\frac{a\sqrt{\pi}\,\mathrm{e}^{a^2}\operatorname{erf}(bx+a)}{2b^2}\right)}{b}+\right.\right.}{\sqrt{\pi}}$

input
```
int((d*x+c)*erfc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*erfc(b*x+a)*d*a*(b*x+a)+erfc(b*x+a)*c*(b*x+a)+1/2/b*erfc(b*x+a)*
d*(b*x+a)^2-1/b/Pi^(1/2)*(-d*(-1/2*(b*x+a)/exp((b*x+a)^2)+1/4*Pi^(1/2)*erf
(b*x+a))+c*b/exp((b*x+a)^2)-d*a/exp((b*x+a)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int (c+dx)\text{erfc}(a+bx)\,dx$$
$$= \frac{2\,\pi b^2dx^2 + 4\,\pi b^2cx - 2\,\sqrt{\pi}(bdx + 2\,bc - ad)e^{(-b^2x^2 - 2\,abx - a^2)} - (2\,\pi b^2dx^2 + 4\,\pi b^2cx + \pi(4\,abc - (2\,a^2 + 1}{4\,\pi b^2}$$

input
```
integrate((d*x+c)*erfc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*pi*b^2*d*x^2 + 4*pi*b^2*c*x - 2*sqrt(pi)*(b*d*x + 2*b*c - a*d)*e^(-
b^2*x^2 - 2*a*b*x - a^2) - (2*pi*b^2*d*x^2 + 4*pi*b^2*c*x + pi*(4*a*b*c -
(2*a^2 + 1)*d))*erf(b*x + a))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int (c+dx)\text{erfc}(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^2d\,\text{erfc}\,(a+bx)}{2b^2} + \frac{ac\,\text{erfc}\,(a+bx)}{b} + \frac{ade^{-a^2}e^{-b^2x^2}e^{-2abx}}{2\sqrt{\pi}b^2} + cx\,\text{erfc}\,(a+bx) + \frac{dx^2\,\text{erfc}\,(a+bx)}{2} - \frac{ce^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b} - \frac{d \\ \left(cx + \frac{dx^2}{2}\right)\text{erfc}\,(a) \end{cases}$$

input
```
integrate((d*x+c)*erfc(b*x+a),x)
```

output
```
Piecewise((-a**2*d*erfc(a + b*x)/(2*b**2) + a*c*erfc(a + b*x)/b + a*d*exp(
-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b**2) + c*x*erfc(a + b*x)
 + d*x**2*erfc(a + b*x)/2 - c*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(sq
rt(pi)*b) - d*x*exp(-a**2)*exp(-b**2*x**2)*exp(-2*a*b*x)/(2*sqrt(pi)*b) -
d*erfc(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*erfc(a), True))
```

Maxima [F]

$$\int (c+dx)\text{erfc}(a+bx)\,dx = \int (dx+c)\,\text{erfc}\,(bx+a)\,dx$$

input
```
integrate((d*x+c)*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*erfc(b*x + a), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.33

$$\int (c+dx)\text{erfc}(a+bx)\,dx$$
$$= \frac{1}{2}\,dx^2 - \left(x\,\text{erf}\,(bx+a) - \frac{\frac{\sqrt{\pi}a\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}\right)c$$
$$- \frac{1}{4}\left(2\,x^2\,\text{erf}\,(bx+a) + \frac{\frac{\sqrt{\pi}(2\,a^2+1)\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\frac{a}{b}\right)-2\,a\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}b}\right)d + cx$$

input
```
integrate((d*x+c)*erfc(b*x+a),x, algorithm="giac")
```

output
```
1/2*d*x^2 - (x*erf(b*x + a) - (sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^
2 - 2*a*b*x - a^2)/b)/sqrt(pi))*c - 1/4*(2*x^2*erf(b*x + a) + (sqrt(pi)*(2
*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*
x - a^2)/b)/(sqrt(pi)*b))*d + c*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int (c+dx)\mathrm{erfc}(a+bx)\,dx = c\,x\,\mathrm{erfc}(a+b\,x) - \mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}\left(\frac{c}{b\,\sqrt{\pi}} - \frac{a\,d}{2\,b^2\,\sqrt{\pi}}\right) - \frac{\mathrm{erfc}(a+b\,x)\,\left(\frac{d\,a^2}{2} - b\,c\,a + \frac{d}{4}\right)}{b^2} + \frac{d\,x^2\,\mathrm{erfc}(a+b\,x)}{2} - \frac{d\,x\,\mathrm{e}^{-a^2-2\,a\,b\,x-b^2\,x^2}}{2\,b\,\sqrt{\pi}}$$

input
```
int(erfc(a + b*x)*(c + d*x),x)
```

output
```
c*x*erfc(a + b*x) - exp(- a^2 - b^2*x^2 - 2*a*b*x)*(c/(b*pi^(1/2)) - (a*d)
/(2*b^2*pi^(1/2))) - (erfc(a + b*x)*(d/4 + (a^2*d)/2 - a*b*c))/b^2 + (d*x^
2*erfc(a + b*x))/2 - (d*x*exp(- a^2 - b^2*x^2 - 2*a*b*x))/(2*b*pi^(1/2))
```

Reduce [F]

$$\int (c+dx)\mathrm{erfc}(a+bx)\,dx = \frac{-2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\mathrm{erf}(bx+a)\,ac - 2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\mathrm{erf}(bx+a)\,bcx - 2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int \mathrm{erf}(bx+a)\right.}{2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}b}$$

input
```
int((d*x+c)*erfc(b*x+a),x)
```

output
```
( - 2*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c - 2*sqrt(p
i)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c*x - 2*sqrt(pi)*e**(a**
2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*d + 2*sqrt(pi)*e**(a**2 +
 2*a*b*x + b**2*x**2)*b*c*x + sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b*d
*x**2 - 2*c)/(2*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.121 $\int \mathbf{erfc}(a + bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \operatorname{erfc}(a+bx)\,dx = -\frac{e^{-(a+bx)^2}}{b\sqrt{\pi}} + \frac{(a+bx)\operatorname{erfc}(a+bx)}{b}$$

output
```
-1/b/exp((b*x+a)^2)/Pi^(1/2)+(b*x+a)*erfc(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \operatorname{erfc}(a+bx)\,dx = -\frac{e^{-(a+bx)^2}}{b\sqrt{\pi}} - \frac{a\operatorname{erf}(a+bx)}{b} + x\operatorname{erfc}(a+bx)$$

input
```
Integrate[Erfc[a + b*x],x]
```

output
```
-(1/(b*E^(a + b*x)^2*Sqrt[Pi])) - (a*Erf[a + b*x])/b + x*Erfc[a + b*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(a+bx)\,dx$$

$$\Big\downarrow \text{6904}$$

$$\frac{(a+bx)\text{erfc}(a+bx)}{b}-\frac{e^{-(a+bx)^2}}{\sqrt{\pi}b}$$

input
```
Int[Erfc[a + b*x],x]
```

output
```
-(1/(b*E^(a + b*x)^2*Sqrt[Pi])) + ((a + b*x)*Erfc[a + b*x])/b
```

Defintions of rubi rules used

rule 6904
```
Int[Erfc[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erfc[a + b*x]/b)
, x] - Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{(bx+a)\,\mathrm{erfc}(bx+a)-\frac{\mathrm{e}^{-(bx+a)^2}}{\sqrt{\pi}}}{b}$	33
default	$\frac{(bx+a)\,\mathrm{erfc}(bx+a)-\frac{\mathrm{e}^{-(bx+a)^2}}{\sqrt{\pi}}}{b}$	33
parallelrisch	$\frac{x\ \mathrm{erfc}(bx+a)\sqrt{\pi}\,b+a\ \mathrm{erfc}(bx+a)\sqrt{\pi}-\mathrm{e}^{-(bx+a)^2}}{\sqrt{\pi}\,b}$	44
parts	$x\ \mathrm{erfc}\,(bx+a)+\frac{2b\left(-\frac{\mathrm{e}^{-b^2x^2-2bxa-a^2}}{2b^2}-\frac{a\sqrt{\pi}\ \mathrm{erf}(bx+a)}{2b^2}\right)}{\sqrt{\pi}}$	57

input
```
int(erfc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*erfc(b*x+a)-1/Pi^(1/2)*exp(-(b*x+a)^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \mathrm{erfc}(a+bx)\,dx=\frac{\pi bx-(\pi bx+\pi a)\,\mathrm{erf}\,(bx+a)-\sqrt{\pi}e^{(-b^2x^2-2\,abx-a^2)}}{\pi b}$$

input
```
integrate(erfc(b*x+a),x, algorithm="fricas")
```

output
```
(pi*b*x - (pi*b*x + pi*a)*erf(b*x + a) - sqrt(pi)*e^(-b^2*x^2 - 2*a*b*x -
a^2))/(pi*b)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \operatorname{erfc}(a+bx)\,dx = \begin{cases} \frac{a\operatorname{erfc}(a+bx)}{b} + x\operatorname{erfc}(a+bx) - \frac{e^{-a^2}e^{-b^2x^2}e^{-2abx}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ x\operatorname{erfc}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(erfc(b*x+a),x)
```

output
```
Piecewise((a*erfc(a + b*x)/b + x*erfc(a + b*x) - exp(-a**2)*exp(-b**2*x**2
)*exp(-2*a*b*x)/(sqrt(pi)*b), Ne(b, 0)), (x*erfc(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \operatorname{erfc}(a+bx)\,dx = \frac{(bx+a)\operatorname{erfc}(bx+a) - \frac{e^{\left(-(bx+a)^2\right)}}{\sqrt{\pi}}}{b}$$

input
```
integrate(erfc(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*erfc(b*x + a) - e^(-(b*x + a)^2)/sqrt(pi))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \operatorname{erfc}(a+bx)\,dx = -x\operatorname{erf}(bx+a) + x + \frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{\sqrt{\pi}}$$

input
```
integrate(erfc(b*x+a),x, algorithm="giac")
```

output
```
-x*erf(b*x + a) + x + (sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*
b*x - a^2)/b)/sqrt(pi)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \operatorname{erfc}(a+bx)\,dx = x\operatorname{erfc}(a+b\,x)+\frac{a\operatorname{erfc}(a+b\,x)}{b}-\frac{\mathrm{e}^{-b^2\,x^2}\,\mathrm{e}^{-a^2}\,\mathrm{e}^{-2\,a\,b\,x}}{b\,\sqrt{\pi}}$$

input
```
int(erfc(a + b*x),x)
```

output
```
x*erfc(a + b*x) + (a*erfc(a + b*x))/b - (exp(-b^2*x^2)*exp(-a^2)*exp(-2*a*
b*x))/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.03

$$\int \operatorname{erfc}(a+bx)\,dx = \frac{-\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,a-\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,bx+\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}bx-1}{\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}b}$$

input
```
int(erfc(b*x+a),x)
```

output
```
( - sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a - sqrt(pi)*e**
(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*x + sqrt(pi)*e**(a**2 + 2*a*b*
x + b**2*x**2)*b*x - 1)/(sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.122 $\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\operatorname{erfc}(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(erfc(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Erfc[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Erfc[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \ 6925$$
$$\int \frac{\text{erfc}(a+bx)}{c+dx}dx$$

input
```
Int[Erfc[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx+a\right)}{dx+c}dx$$

input
```
int(erfc(b*x+a)/(d*x+c),x)
```

output
```
int(erfc(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erfc}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erfc}\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c),x)
```

output
```
Integral(erfc(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erfc}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erfc}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{erfc}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(erfc(a + b*x)/(c + d*x),x)
```

output
```
int(erfc(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{erfc}(a+bx)}{c+dx}\,dx = \frac{-\left(\int \frac{\operatorname{erf}(bx+a)}{dx+c}dx\right)d+\log(dx+c)}{d}$$

input
```
int(erfc(b*x+a)/(d*x+c),x)
```

output
```
( - int(erf(a + b*x)/(c + d*x),x)*d + log(c + d*x))/d
```

3.123 $\int \frac{\text{erfc}(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2} dx = -\frac{\text{erfc}(a+bx)}{d(c+dx)} - \frac{2b\text{Int}\left(\frac{e^{-(a+bx)^2}}{c+dx}, x\right)}{d\sqrt{\pi}}$$

output
```
-erfc(b*x+a)/d/(d*x+c)-2*b*Defer(Int)(1/exp((b*x+a)^2)/(d*x+c),x)/d/Pi^(1/
2)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2} dx = \int \frac{\text{erfc}(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Erfc[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Erfc[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{2b \int \frac{e^{-(a+bx)^2}}{c+dx} dx}{\sqrt{\pi} d} - \frac{\text{erfc}(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2654}$$

$$-\frac{2b \int \frac{e^{-(a+bx)^2}}{c+dx} dx}{\sqrt{\pi} d} - \frac{\text{erfc}(a+bx)}{d(c+dx)}$$

input
```
Int[Erfc[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx+a\right)}{\left(dx+c\right)^2} dx$$

input
```
int(erfc(b*x+a)/(d*x+c)^2,x)
```

output
```
int(erfc(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfc}\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(erfc(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erfc}\,(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erfc}\,(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfc}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erfc}(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(erfc(a + b*x)/(c + d*x)^2,x)
```

output
```
int(erfc(a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 5.50

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^2}\,dx = \frac{-\left(\int \frac{\operatorname{erf}(bx+a)}{d^2x^2+2cdx+c^2}dx\right)c^2-\left(\int \frac{\operatorname{erf}(bx+a)}{d^2x^2+2cdx+c^2}dx\right)cdx+x}{c\,(dx+c)}$$

input
```
int(erfc(b*x+a)/(d*x+c)^2,x)
```

output
```
( - int(erf(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 - int(erf(a + b*
x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + x)/(c*(c + d*x))
```

3.124 $\int \frac{\mathrm{erfc}(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\mathrm{erfc}(a+bx)}{(c+dx)^3}\,dx = -\frac{\mathrm{erfc}(a+bx)}{2d(c+dx)^2} - \frac{b\mathrm{Int}\left(\frac{e^{-(a+bx)^2}}{(c+dx)^2},x\right)}{d\sqrt{\pi}}$$

output
```
-1/2*erfc(b*x+a)/d/(d*x+c)^2-b*Defer(Int)(1/exp((b*x+a)^2)/(d*x+c)^2,x)/d/
Pi^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\mathrm{erfc}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\mathrm{erfc}(a+bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[Erfc[a + b*x]/(c + d*x)^3,x]
```

output
```
Integrate[Erfc[a + b*x]/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{6916}$$

$$-\frac{b\int \frac{e^{-(a+bx)^2}}{(c+dx)^2}dx}{\sqrt{\pi}d}-\frac{\operatorname{erfc}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2650}$$

$$-\frac{b\left(-\frac{2b^2\int e^{-(a+bx)^2}dx}{d^2}+\frac{2b(bc-ad)\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{-(a+bx)^2}}{d(c+dx)}\right)}{\sqrt{\pi}d}-\frac{\operatorname{erfc}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{b\left(\frac{2b(bc-ad)\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{-(a+bx)^2}}{d(c+dx)}-\frac{\sqrt{\pi}b\operatorname{erf}(a+bx)}{d^2}\right)}{\sqrt{\pi}d}-\frac{\operatorname{erfc}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2654}$$

$$-\frac{b\left(\frac{2b(bc-ad)\int \frac{e^{-(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{-(a+bx)^2}}{d(c+dx)}-\frac{\sqrt{\pi}b\operatorname{erf}(a+bx)}{d^2}\right)}{\sqrt{\pi}d}-\frac{\operatorname{erfc}(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Erfc[a + b*x]/(c + d*x)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx+a)}{(dx+c)^3}dx$$

input
```
int(erfc(b*x+a)/(d*x+c)^3,x)
```

output
```
int(erfc(b*x+a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.93

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erfc}(bx+a)}{(dx+c)^3}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [N/A]

Not integrable

Time = 72.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(erfc(a + b*x)/(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)/(d*x + c)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(erfc(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\operatorname{erfc}(a+b\,x)}{(c+d\,x)^3}\,dx$$

input
```
int(erfc(a + b*x)/(c + d*x)^3,x)
```

output
```
int(erfc(a + b*x)/(c + d*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 11.43

$$\int \frac{\operatorname{erfc}(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{-2\left(\int \frac{\operatorname{erf}(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c^2d-4\left(\int \frac{\operatorname{erf}(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^2x-2\left(\int \frac{\operatorname{erf}(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)d^3x}{2d\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int(erfc(b*x+a)/(d*x+c)^3,x)
```

output
```
( - 2*int(erf(a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*
c**2*d - 4*int(erf(a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x)*c*d**2*x - 2*int(erf(a + b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*d**3*x**2 - 1)/(2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.125 $\int x^5\mathbf{erfc}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 178

$$\int x^5\text{erfc}(bx)^2\,dx = \frac{11e^{-2b^2x^2}}{12b^6\pi}+\frac{7e^{-2b^2x^2}x^2}{12b^4\pi}+\frac{e^{-2b^2x^2}x^4}{6b^2\pi}-\frac{5e^{-b^2x^2}x\text{erfc}(bx)}{4b^5\sqrt{\pi}} -\frac{5e^{-b^2x^2}x^3\text{erfc}(bx)}{6b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x^5\text{erfc}(bx)}{3b\sqrt{\pi}}-\frac{5\text{erfc}(bx)^2}{16b^6}+\frac{1}{6}x^6\text{erfc}(bx)^2$$

output
```
11/12/b^6/exp(2*b^2*x^2)/Pi+7/12*x^2/b^4/exp(2*b^2*x^2)/Pi+1/6*x^4/b^2/exp
(2*b^2*x^2)/Pi-5/4*x*erfc(b*x)/b^5/exp(b^2*x^2)/Pi^(1/2)-5/6*x^3*erfc(b*x)
/b^3/exp(b^2*x^2)/Pi^(1/2)-1/3*x^5*erfc(b*x)/b/exp(b^2*x^2)/Pi^(1/2)-5/16*
erfc(b*x)^2/b^6+1/6*x^6*erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.87

$$\int x^5 \text{erfc}(bx)^2 \, dx = \frac{1}{48}\left(8x^6 + 8x^6 \text{erf}(bx)^2 + \frac{e^{-2b^2x^2}\left(44 + 28b^2x^2 + 8b^4x^4 + 4be^{b^2x^2}\sqrt{\pi}x(15 + 10b^2x^2 + 4b^4x^4)\,\text{erf}(bx) - 15e^{2b^2x^2}\pi\text{erf}(bx)^2\right)}{b^6\pi} - 16x^6\left(\text{erf}(bx) + \frac{bx\Gamma\left(\frac{7}{2}, b^2x^2\right)}{\sqrt{\pi}\,(b^2x^2)^{7/2}}\right)\right)$$

input
```
Integrate[x^5*Erfc[b*x]^2,x]
```

output
```
(8*x^6 + 8*x^6*Erf[b*x]^2 + (44 + 28*b^2*x^2 + 8*b^4*x^4 + 4*b*E^(b^2*x^2)
*Sqrt[Pi]*x*(15 + 10*b^2*x^2 + 4*b^4*x^4)*Erf[b*x] - 15*E^(2*b^2*x^2)*Pi*E
rf[b*x]^2)/(b^6*E^(2*b^2*x^2)*Pi) - 16*x^6*(Erf[b*x] + (b*x*Gamma[7/2, b^2
*x^2])/(Sqrt[Pi]*(b^2*x^2)^(7/2))))/48
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.53, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6919, 6940, 2641, 2641, 2638, 6940, 2641, 2638, 6940, 2638, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{erfc}(bx)^2 \, dx$$

$$\Big\downarrow \; 6919$$

$$\frac{2b \int e^{-b^2x^2} x^6 \text{erfc}(bx) dx}{3\sqrt{\pi}} + \frac{1}{6}x^6 \text{erfc}(bx)^2$$

$$\Big\downarrow \; 6940$$

$$\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}x^5dx}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erfc}(bx)dx}{2b^2}-\frac{\frac{\int e^{-2b^2x^2}x^3dx}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erfc}(bx)dx}{2b^2}-\frac{\frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2}-\frac{x^2e^{-2b^2x^2}}{4b^2}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2638}$$

$$\frac{2b\left(\frac{5\int e^{-b^2x^2}x^4\text{erfc}(bx)dx}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \text{6940}$$

$$\frac{2b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}x^3dx}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}+\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{2b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2}-\frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2}-\frac{x^2e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b\cdots}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$$\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2638}$$

$$\frac{2b\left(\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right.}{3\sqrt{\pi}}$$

$$\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \; 6940$$

$$\frac{2b\left(\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\right.}{3\sqrt{\pi}}$$

$$\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \; 2638$$

$$\frac{2b\left(\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e}{}}{}\right.}{3\sqrt{\pi}}$$

$$\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow \; 6928$$

$$\frac{2b\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{\pi}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\right.}{3\sqrt{\pi}}$$

$$\frac{1}{6}x^6\text{erfc}(bx)^2$$

$$\Big\downarrow 15$$

$$\frac{2b\left(-\frac{x^5e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{b^2}-\frac{x^4e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}+\frac{5\left(-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}+\frac{3\left(-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{8b^3}-x\right)}{}\right)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\frac{1}{6}x^6\text{erfc}(bx)^2$$

input
```
Int[x^5*Erfc[b*x]^2,x]
```

output
```
(x^6*Erfc[b*x]^2)/6 + (2*b*(-((-1/4*x^4/(b^2*E^(2*b^2*x^2)) + (-1/8*1/(b^4
*E^(2*b^2*x^2)) - x^2/(4*b^2*E^(2*b^2*x^2)))/b^2)/(b*Sqrt[Pi])) - (x^5*Erf
c[b*x])/(2*b^2*E^(b^2*x^2)) + (5*(-((-1/8*1/(b^4*E^(2*b^2*x^2)) - x^2/(4*b
^2*E^(2*b^2*x^2)))/(b*Sqrt[Pi])) - (x^3*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) + (
3*(1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) -
(Sqrt[Pi]*Erfc[b*x]^2)/(8*b^3)))/(2*b^2)))/(2*b^2)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{8\,\mathrm{erfc}(bx)^{2} x^{6} \pi^{\frac{3}{2}} b^{6}-16\,\mathrm{e}^{-b^{2} x^{2}} x^{5}\,\mathrm{erfc}(bx) b^{5} \pi+8\,\mathrm{e}^{-2 b^{2} x^{2}} x^{4} b^{4} \sqrt{\pi}-40\,\mathrm{e}^{-b^{2} x^{2}} x^{3}\,\mathrm{erfc}(bx) b^{3} \pi+28\,\mathrm{e}^{-2 b^{2} x^{2}} x^{2} b^{2} \sqrt{\pi}-60\,\mathrm{e}^{-b^{2} x^{2}} x}{48 \pi^{\frac{3}{2}} b^{6}}$

input
```
int(x^5*erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*erfc(b*x)^2*x^6*Pi^(3/2)*b^6-16*exp(-b^2*x^2)*x^5*erfc(b*x)*b^5*Pi
+8*exp(-b^2*x^2)^2*x^4*Pi^(1/2)*b^4-40*exp(-b^2*x^2)*x^3*erfc(b*x)*b^3*Pi+
28*exp(-b^2*x^2)^2*x^2*b^2*Pi^(1/2)-60*exp(-b^2*x^2)*x*erfc(b*x)*b*Pi-15*e
rfc(b*x)^2*Pi^(3/2)+44*exp(-b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int x^5 \operatorname{erfc}(bx)^2 \, dx$$
$$= \frac{8\pi b^6x^6 - (15\pi - 8\pi b^6x^6)\operatorname{erf}(bx)^2 - 4\sqrt{\pi}(4b^5x^5 + 10b^3x^3 + 15bx - (4b^5x^5 + 10b^3x^3 + 15bx)\operatorname{erf}(bx}{48\pi b^6}$$

input
```
integrate(x^5*erfc(b*x)^2,x, algorithm="fricas")
```

output
```
1/48*(8*pi*b^6*x^6 - (15*pi - 8*pi*b^6*x^6)*erf(b*x)^2 - 4*sqrt(pi)*(4*b^5
*x^5 + 10*b^3*x^3 + 15*b*x - (4*b^5*x^5 + 10*b^3*x^3 + 15*b*x)*erf(b*x))*e
^(-b^2*x^2) + 2*(15*pi - 8*pi*b^6*x^6)*erf(b*x) + 4*(2*b^4*x^4 + 7*b^2*x^2
 + 11)*e^(-2*b^2*x^2))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

$$\int x^5 \operatorname{erfc}(bx)^2 \, dx$$
$$= \begin{cases} \frac{x^6 \operatorname{erfc}^2(bx)}{6} - \frac{x^5 e^{-b^2x^2} \operatorname{erfc}(bx)}{3\sqrt{\pi}b} + \frac{x^4 e^{-2b^2x^2}}{6\pi b^2} - \frac{5x^3 e^{-b^2x^2} \operatorname{erfc}(bx)}{6\sqrt{\pi}b^3} + \frac{7x^2 e^{-2b^2x^2}}{12\pi b^4} - \frac{5x e^{-b^2x^2} \operatorname{erfc}(bx)}{4\sqrt{\pi}b^5} - \frac{5\operatorname{erfc}^2(bx)}{16b^6} + \frac{11e^{-}}{12} \\ \frac{x^6}{6} \end{cases}$$

input
```
integrate(x**5*erfc(b*x)**2,x)
```

output
```
Piecewise((x**6*erfc(b*x)**2/6 - x**5*exp(-b**2*x**2)*erfc(b*x)/(3*sqrt(pi
)*b) + x**4*exp(-2*b**2*x**2)/(6*pi*b**2) - 5*x**3*exp(-b**2*x**2)*erfc(b*
x)/(6*sqrt(pi)*b**3) + 7*x**2*exp(-2*b**2*x**2)/(12*pi*b**4) - 5*x*exp(-b*
*2*x**2)*erfc(b*x)/(4*sqrt(pi)*b**5) - 5*erfc(b*x)**2/(16*b**6) + 11*exp(-
2*b**2*x**2)/(12*pi*b**6), Ne(b, 0)), (x**6/6, True))
```

Maxima [F]

$$\int x^5 \text{erfc}(bx)^2 \, dx = \int x^5 \, \text{erfc}(bx)^2 \, dx$$

input
```
integrate(x^5*erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*erfc(b*x)^2, x)
```

Giac [F]

$$\int x^5 \text{erfc}(bx)^2 \, dx = \int x^5 \, \text{erfc}(bx)^2 \, dx$$

input
```
integrate(x^5*erfc(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*erfc(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.80

$$\int x^5 \text{erfc}(bx)^2 \, dx = \frac{x^6 \operatorname{erfc}(b\,x)^2}{6} - \frac{\frac{5\,\pi\,\operatorname{erfc}(b\,x)^2}{16} - \frac{11\,e^{-2\,b^2\,x^2}}{12} - \frac{7\,b^2\,x^2\,e^{-2\,b^2\,x^2}}{12} - \frac{b^4\,x^4\,e^{-2\,b^2\,x^2}}{6} + \frac{5\,b^3\,x^3\,\sqrt{\pi}\,e^{-b^2\,x^2}\,\operatorname{erfc}(b\,x)}{6} + \frac{b^5\,x^5\,\sqrt{\pi}\,e^{-b^2\,x^2}\,\operatorname{erfc}(b\,x)}{3} + \frac{5\,b\,x}{}}{b^6\,\pi}$$

input
```
int(x^5*erfc(b*x)^2,x)
```

output
```
(x^6*erfc(b*x)^2)/6 - ((5*pi*erfc(b*x)^2)/16 - (11*exp(-2*b^2*x^2))/12 - (
7*b^2*x^2*exp(-2*b^2*x^2))/12 - (b^4*x^4*exp(-2*b^2*x^2))/6 + (5*b^3*x^3*p
i^(1/2)*exp(-b^2*x^2)*erfc(b*x))/6 + (b^5*x^5*pi^(1/2)*exp(-b^2*x^2)*erfc(
b*x))/3 + (5*b*x*pi^(1/2)*exp(-b^2*x^2)*erfc(b*x))/4)/(b^6*pi)
```

Reduce [F]

$$\int x^5 \text{erfc}(bx)^2 \, dx = \frac{-8e^{b^2x^2} \operatorname{erf}(bx)\, b^6 \pi\, x^6 + 15e^{b^2x^2} \operatorname{erf}(bx)\, \pi + 24e^{b^2x^2} \left(\int \operatorname{erf}(bx)^2\, x^5 dx\right) b^6 \pi + 4e^{b^2x^2} b^6 \pi\, x^6 - 8\sqrt{\pi}\, b^5 x^5 - 20}{24e^{b^2x^2} b^6 \pi}$$

input
```
int(x^5*erfc(b*x)^2,x)
```

output
```
( - 8*e**(b**2*x**2)*erf(b*x)*b**6*pi*x**6 + 15*e**(b**2*x**2)*erf(b*x)*pi
 + 24*e**(b**2*x**2)*int(erf(b*x)**2*x**5,x)*b**6*pi + 4*e**(b**2*x**2)*b*
*6*pi*x**6 - 8*sqrt(pi)*b**5*x**5 - 20*sqrt(pi)*b**3*x**3 - 30*sqrt(pi)*b*
x)/(24*e**(b**2*x**2)*b**6*pi)
```

3.126 $\int x^3 \mathbf{erfc}(bx)^2\, dx$

Optimal result

Integrand size = 10, antiderivative size = 126

$$\int x^3 \operatorname{erfc}(bx)^2\, dx = \frac{e^{-2b^2x^2}}{2b^4\pi} + \frac{e^{-2b^2x^2}x^2}{4b^2\pi} - \frac{3e^{-b^2x^2}x\operatorname{erfc}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{-b^2x^2}x^3\operatorname{erfc}(bx)}{2b\sqrt{\pi}} - \frac{3\operatorname{erfc}(bx)^2}{16b^4} + \frac{1}{4}x^4\operatorname{erfc}(bx)^2$$

output
```
1/2/b^4/exp(2*b^2*x^2)/Pi+1/4*x^2/b^2/exp(2*b^2*x^2)/Pi-3/4*x*erfc(b*x)/b^
3/exp(b^2*x^2)/Pi^(1/2)-1/2*x^3*erfc(b*x)/b/exp(b^2*x^2)/Pi^(1/2)-3/16*erf
c(b*x)^2/b^4+1/4*x^4*erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14

$$\int x^3 \operatorname{erfc}(bx)^2 \, dx = \frac{1}{8}\left(2x^4 - 4x^4 \operatorname{erf}(bx) + 2x^4 \operatorname{erf}(bx)^2 + \frac{e^{-2b^2x^2}\left(8 + 4b^2x^2 + 4be^{b^2x^2}\sqrt{\pi}x(3 + 2b^2x^2)\operatorname{erf}(bx) - 3e^{2b^2x^2}\pi\operatorname{erf}(bx)^2\right)}{2b^4\pi} - \frac{4x\Gamma\left(\frac{5}{2}, b^2x^2\right)}{b^3\sqrt{\pi}\sqrt{b^2x^2}}\right)$$

input
```
Integrate[x^3*Erfc[b*x]^2,x]
```

output
```
(2*x^4 - 4*x^4*Erf[b*x] + 2*x^4*Erf[b*x]^2 + (8 + 4*b^2*x^2 + 4*b*E^(b^2*x
^2)*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x] - 3*E^(2*b^2*x^2)*Pi*Erf[b*x]^2)/(
2*b^4*E^(2*b^2*x^2)*Pi) - (4*x*Gamma[5/2, b^2*x^2])/(b^3*Sqrt[Pi]*Sqrt[b^2
*x^2]))/8
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6919, 6940, 2641, 2638, 6940, 2638, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{erfc}(bx)^2 \, dx$$

$$\Big\downarrow \text{6919}$$

$$\frac{b\int e^{-b^2x^2}x^4 \operatorname{erfc}(bx)dx}{\sqrt{\pi}} + \frac{1}{4}x^4 \operatorname{erfc}(bx)^2$$

$$\Big\downarrow \text{6940}$$

$$\frac{b\left(\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}x^3dx}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2}-\frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2}-\frac{x^2e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2638}$$

$$\frac{b\left(\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

$$\Big\downarrow \text{6940}$$

$$\frac{b\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2638}$$

$$\frac{b\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

$$\Big\downarrow \text{6928}$$

$$\frac{b\left(\frac{3\left(-\frac{\sqrt{\pi}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

$$\Big\downarrow \text{15}$$

$$\frac{b\left(-\frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}+\frac{3\left(-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}\right)}{\sqrt{\pi}}+\frac{1}{4}x^4\text{erfc}(bx)^2$$

input
```
Int[x^3*Erfc[b*x]^2,x]
```

output
```
(x^4*Erfc[b*x]^2)/4 + (b*(-((-1/8*1/(b^4*E^(2*b^2*x^2)) - x^2/(4*b^2*E^(2*
b^2*x^2)))/(b*Sqrt[Pi])) - (x^3*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(1/(4*
b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (Sqrt[Pi
]*Erfc[b*x]^2)/(8*b^3)))/(2*b^2)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.92

method	result	si
parallelrisch	$\frac{4 \operatorname{erfc}(bx)^{2} x^{4} \pi^{\frac{3}{2}} b^{4}-8 \,\mathrm{e}^{-b^{2} x^{2}} x^{3} \operatorname{erfc}(bx) b^{3} \pi+4 \,\mathrm{e}^{-2 b^{2} x^{2}} x^{2} b^{2} \sqrt{\pi}-12 \,\mathrm{e}^{-b^{2} x^{2}} x \operatorname{erfc}(bx) b \pi-3 \operatorname{erfc}(bx)^{2} \pi^{\frac{3}{2}}+8 \,\mathrm{e}^{-2 b^{2} x^{2}} \sqrt{\pi}}{16 \pi^{\frac{3}{2}} b^{4}}$	1

input
```
int(x^3*erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*erfc(b*x)^2*x^4*Pi^(3/2)*b^4-8*exp(-b^2*x^2)*x^3*erfc(b*x)*b^3*Pi+
4*exp(-b^2*x^2)^2*x^2*b^2*Pi^(1/2)-12*exp(-b^2*x^2)*x*erfc(b*x)*b*Pi-3*erf
c(b*x)^2*Pi^(3/2)+8*exp(-b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{erfc}(bx)^2 \, dx = \frac{4 \, \pi b^{4} x^{4} - (3 \, \pi - 4 \, \pi b^{4} x^{4}) \operatorname{erf}\left(bx\right)^{2} - 4 \, \sqrt{\pi}(2 \, b^{3} x^{3} + 3 \, bx - (2 \, b^{3} x^{3} + 3 \, bx) \operatorname{erf}\left(bx\right))e^{\left(-b^{2} x^{2}\right)} + 2 \, (3 \, \pi - 4 \, \pi}{16 \, \pi b^{4}}$$

input
```
integrate(x^3*erfc(b*x)^2,x, algorithm="fricas")
```

output
```
1/16*(4*pi*b^4*x^4 - (3*pi - 4*pi*b^4*x^4)*erf(b*x)^2 - 4*sqrt(pi)*(2*b^3*
x^3 + 3*b*x - (2*b^3*x^3 + 3*b*x)*erf(b*x))*e^(-b^2*x^2) + 2*(3*pi - 4*pi*
b^4*x^4)*erf(b*x) + 4*(b^2*x^2 + 2)*e^(-2*b^2*x^2))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int x^3\text{erfc}(bx)^2\,dx = \begin{cases} \frac{x^4\,\text{erfc}^2\,(bx)}{4} - \frac{x^3e^{-b^2x^2}\,\text{erfc}\,(bx)}{2\sqrt{\pi}b} + \frac{x^2e^{-2b^2x^2}}{4\pi b^2} - \frac{3xe^{-b^2x^2}\,\text{erfc}\,(bx)}{4\sqrt{\pi}b^3} - \frac{3\,\text{erfc}^2\,(bx)}{16b^4} + \frac{e^{-2b^2x^2}}{2\pi b^4} & \text{for } b \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erfc(b*x)**2,x)
```

output
```
Piecewise((x**4*erfc(b*x)**2/4 - x**3*exp(-b**2*x**2)*erfc(b*x)/(2*sqrt(pi
)*b) + x**2*exp(-2*b**2*x**2)/(4*pi*b**2) - 3*x*exp(-b**2*x**2)*erfc(b*x)/
(4*sqrt(pi)*b**3) - 3*erfc(b*x)**2/(16*b**4) + exp(-2*b**2*x**2)/(2*pi*b**
4), Ne(b, 0)), (x**4/4, True))
```

Maxima [F]

$$\int x^3\text{erfc}(bx)^2\,dx = \int x^3\,\text{erfc}\,(bx)^2\,\,dx$$

input
```
integrate(x^3*erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*erfc(b*x)^2, x)
```

Giac [F]

$$\int x^3\text{erfc}(bx)^2\,dx = \int x^3\,\text{erfc}\left(bx\right)^2\,dx$$

input
```
integrate(x^3*erfc(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*erfc(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int x^3\text{erfc}(bx)^2\,dx = \frac{x^4\,\text{erfc}(b\,x)^2}{4} - \frac{\frac{3\,\pi\,\text{erfc}(b\,x)^2}{16} - \frac{e^{-2\,b^2\,x^2}}{2} - \frac{b^2\,x^2\,e^{-2\,b^2\,x^2}}{4} + \frac{b^3\,x^3\,\sqrt{\pi}\,e^{-b^2\,x^2}\,\text{erfc}(b\,x)}{2} + \frac{3\,b\,x\,\sqrt{\pi}\,e^{-b^2\,x^2}\,\text{erfc}(b\,x)}{4}}{b^4\,\pi}$$

input
```
int(x^3*erfc(b*x)^2,x)
```

output
```
(x^4*erfc(b*x)^2)/4 - ((3*pi*erfc(b*x)^2)/16 - exp(-2*b^2*x^2)/2 - (b^2*x^
2*exp(-2*b^2*x^2))/4 + (b^3*x^3*pi^(1/2)*exp(-b^2*x^2)*erfc(b*x))/2 + (3*b
*x*pi^(1/2)*exp(-b^2*x^2)*erfc(b*x))/4)/(b^4*pi)
```

Reduce [F]

$$\int x^3\text{erfc}(bx)^2\,dx = \frac{-4e^{b^2x^2}\text{erf}(bx)\,b^4\pi\,x^4 + 3e^{b^2x^2}\text{erf}(bx)\,\pi + 8e^{b^2x^2}\left(\int \text{erf}(bx)^2\,x^3dx\right)b^4\pi + 2e^{b^2x^2}b^4\pi\,x^4 - 4\sqrt{\pi}\,b^3x^3 - 6\sqrt{\pi}}{8e^{b^2x^2}b^4\pi}$$

input
```
int(x^3*erfc(b*x)^2,x)
```

output
```
( - 4*e**(b**2*x**2)*erf(b*x)*b**4*pi*x**4 + 3*e**(b**2*x**2)*erf(b*x)*pi
+ 8*e**(b**2*x**2)*int(erf(b*x)**2*x**3,x)*b**4*pi + 2*e**(b**2*x**2)*b**4
*pi*x**4 - 4*sqrt(pi)*b**3*x**3 - 6*sqrt(pi)*b*x)/(8*e**(b**2*x**2)*b**4*p
i)
```

3.127 $\int x\text{erfc}(bx)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 72

$$\int x\text{erfc}(bx)^2 \, dx = \frac{e^{-2b^2x^2}}{2b^2\pi} - \frac{e^{-b^2x^2}x\text{erfc}(bx)}{b\sqrt{\pi}} - \frac{\text{erfc}(bx)^2}{4b^2} + \frac{1}{2}x^2\text{erfc}(bx)^2$$

output
```
1/2/b^2/exp(2*b^2*x^2)/Pi-x*erfc(b*x)/b/exp(b^2*x^2)/Pi^(1/2)-1/4*erfc(b*x
)^2/b^2+1/2*x^2*erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int x\text{erfc}(bx)^2 \, dx$$
$$= \frac{2e^{-2b^2x^2}\left(-1+be^{b^2x^2}\sqrt{\pi}x\right)^2 + \left(4be^{-b^2x^2}\sqrt{\pi}x + \pi(2-4b^2x^2)\right)\text{erf}(bx) + \pi(-1+2b^2x^2)\,\text{erf}(bx)^2}{4b^2\pi}$$

input
```
Integrate[x*Erfc[b*x]^2,x]
```

output
```
((2*(-1 + b*E^(b^2*x^2)*Sqrt[Pi]*x)^2)/E^(2*b^2*x^2) + ((4*b*Sqrt[Pi]*x)/E
^(b^2*x^2) + Pi*(2 - 4*b^2*x^2))*Erf[b*x] + Pi*(-1 + 2*b^2*x^2)*Erf[b*x]^2
)/(4*b^2*Pi)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6919, 6940, 2638, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfc}(bx)^2\,dx$$

$$\Big\downarrow \text{6919}$$

$$\frac{2b\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2$$

$$\Big\downarrow \text{6940}$$

$$\frac{2b\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2638}$$

$$\frac{2b\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2$$

$$\Big\downarrow \text{6928}$$

$$\frac{2b\left(-\frac{\sqrt{\pi}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2$$

$$\Big\downarrow \text{15}$$

$$\frac{2b\left(-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(bx)^2$$

input
```
Int[x*Erfc[b*x]^2,x]
```

output
```
(x^2*Erfc[b*x]^2)/2 + (2*b*(1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erfc[b*x
])/(2*b^2*E^(b^2*x^2)) - (Sqrt[Pi]*Erfc[b*x]^2)/(8*b^3)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{2\,\mathrm{erfc}(bx)^2 x^2 \pi^{\frac{3}{2}} b^2 - 4\,\mathrm{e}^{-b^2x^2} x\ \mathrm{erfc}(bx) b\pi - \mathrm{erfc}(bx)^2 \pi^{\frac{3}{2}} + 2\,\mathrm{e}^{-2b^2x^2} \sqrt{\pi}}{4\pi^{\frac{3}{2}} b^2}$	72

input
```
int(x*erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*erfc(b*x)^2*x^2*Pi^(3/2)*b^2-4*exp(-b^2*x^2)*x*erfc(b*x)*b*Pi-erfc(
b*x)^2*Pi^(3/2)+2*exp(-b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int x\mathrm{erfc}(bx)^2\,dx$$
$$= \frac{2\,\pi b^2x^2 - (\pi - 2\,\pi b^2x^2)\,\mathrm{erf}\,(bx)^2 + 4\,\sqrt{\pi}(bx\,\mathrm{erf}\,(bx) - bx)e^{(-b^2x^2)} + 2\,(\pi - 2\,\pi b^2x^2)\,\mathrm{erf}\,(bx) + 2\,e^{(-2\,b^2x^2)}}{4\,\pi b^2}$$

input
```
integrate(x*erfc(b*x)^2,x, algorithm="fricas")
```

output
```
1/4*(2*pi*b^2*x^2 - (pi - 2*pi*b^2*x^2)*erf(b*x)^2 + 4*sqrt(pi)*(b*x*erf(b
*x) - b*x)*e^(-b^2*x^2) + 2*(pi - 2*pi*b^2*x^2)*erf(b*x) + 2*e^(-2*b^2*x^2
))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x\text{erfc}(bx)^2\,dx = \begin{cases} \frac{x^2\,\text{erfc}^2\,(bx)}{2} - \frac{xe^{-b^2x^2}\,\text{erfc}\,(bx)}{\sqrt{\pi}b} - \frac{\text{erfc}^2\,(bx)}{4b^2} + \frac{e^{-2b^2x^2}}{2\pi b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*erfc(b*x)**2,x)
```

output
```
Piecewise((x**2*erfc(b*x)**2/2 - x*exp(-b**2*x**2)*erfc(b*x)/(sqrt(pi)*b)
- erfc(b*x)**2/(4*b**2) + exp(-2*b**2*x**2)/(2*pi*b**2), Ne(b, 0)), (x**2/
2, True))
```

Maxima [F]

$$\int x\text{erfc}(bx)^2\,dx = \int x\,\text{erfc}\,(bx)^2\;dx$$

input
```
integrate(x*erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*erfc(b*x)^2, x)
```

Giac [F]

$$\int x\text{erfc}(bx)^2\,dx = \int x\,\text{erfc}\,(bx)^2\;dx$$

input
```
integrate(x*erfc(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*erfc(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x\mathrm{erfc}(bx)^2\,dx=\frac{\frac{\mathrm{e}^{-2\,b^2\,x^2}}{2}-b\,x\,\sqrt{\pi}\,\mathrm{e}^{-b^2\,x^2}\,\mathrm{erfc}(b\,x)}{b^2\,\pi}-\frac{\frac{\mathrm{erfc}(b\,x)^2}{4}-\frac{b^2\,x^2\,\mathrm{erfc}(b\,x)^2}{2}}{b^2}$$

input
```
int(x*erfc(b*x)^2,x)
```

output
```
(exp(-2*b^2*x^2)/2 - b*x*pi^(1/2)*exp(-b^2*x^2)*erfc(b*x))/(b^2*pi) - (erf
c(b*x)^2/4 - (b^2*x^2*erfc(b*x)^2)/2)/b^2
```

Reduce [F]

$$\int x\mathrm{erfc}(bx)^2\,dx$$
$$=\frac{-2e^{b^2x^2}\mathrm{erf}(bx)\,b^2\pi\,x^2+e^{b^2x^2}\mathrm{erf}(bx)\,\pi+2e^{b^2x^2}\left(\int\mathrm{erf}(bx)^2\,xdx\right)b^2\pi+e^{b^2x^2}b^2\pi\,x^2-2\sqrt{\pi}\,bx}{2e^{b^2x^2}b^2\pi}$$

input
```
int(x*erfc(b*x)^2,x)
```

output
```
( - 2*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 + e**(b**2*x**2)*erf(b*x)*pi +
2*e**(b**2*x**2)*int(erf(b*x)**2*x,x)*b**2*pi + e**(b**2*x**2)*b**2*pi*x**
2 - 2*sqrt(pi)*b*x)/(2*e**(b**2*x**2)*b**2*pi)
```

3.128 $\int \frac{\text{erfc}(bx)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfc}(bx)^2}{x}\,dx = \text{Int}\left(\frac{\text{erfc}(bx)^2}{x},x\right)$$

output
```
Defer(Int)(erfc(b*x)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfc}(bx)^2}{x}\,dx = \int \frac{\text{erfc}(bx)^2}{x}\,dx$$

input
```
Integrate[Erfc[b*x]^2/x,x]
```

output
```
Integrate[Erfc[b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)^2}{x} \, dx$$

$$\Big\downarrow \; 6925$$

$$\int \frac{\text{erfc}(bx)^2}{x}dx$$

input
```
Int[Erfc[b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx\right)^2}{x}dx$$

input
```
int(erfc(b*x)^2/x,x)
```

output
```
int(erfc(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erfc(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfc}^2\left(bx\right)}{x}\,dx$$

input
```
integrate(erfc(b*x)**2/x,x)
```

output
```
Integral(erfc(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erfc(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erfc(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfc}(b\,x)^2}{x}\,dx$$

input
```
int(erfc(b*x)^2/x,x)
```

output
```
int(erfc(b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70

$$\int \frac{\operatorname{erfc}(bx)^2}{x}\,dx = -2\left(\int \frac{\operatorname{erf}(bx)}{x}dx\right) + \int \frac{\operatorname{erf}(bx)^2}{x}dx + \log(x)$$

input
```
int(erfc(b*x)^2/x,x)
```

output
```
- 2*int(erf(b*x)/x,x) + int(erf(b*x)**2/x,x) + log(x)
```

3.129 $\int \frac{\text{erfc}(bx)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{\text{erfc}(bx)^2}{x^3} dx = \frac{2be^{-b^2x^2}\text{erfc}(bx)}{\sqrt{\pi}x} - b^2\text{erfc}(bx)^2 - \frac{\text{erfc}(bx)^2}{2x^2} + \frac{2b^2 \text{ExpIntegralEi}(-2b^2x^2)}{\pi}$$

output
```
2*b*erfc(b*x)/exp(b^2*x^2)/Pi^(1/2)/x-b^2*erfc(b*x)^2-1/2*erfc(b*x)^2/x^2+
2*b^2*Ei(-2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\text{erfc}(bx)^2}{x^3} dx = \frac{2be^{-b^2x^2}\text{erfc}(bx)}{\sqrt{\pi}x} + \left(-b^2 - \frac{1}{2x^2}\right)\text{erfc}(bx)^2 + \frac{2b^2 \text{ExpIntegralEi}(-2b^2x^2)}{\pi}$$

input
```
Integrate[Erfc[b*x]^2/x^3,x]
```

output
```
(2*b*Erfc[b*x])/(E^(b^2*x^2)*Sqrt[Pi]*x) + (-b^2 - 1/(2*x^2))*Erfc[b*x]^2
+ (2*b^2*ExpIntegralEi[-2*b^2*x^2])/Pi
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6919, 6946, 2639, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 6919}$$

$$-\frac{2b\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx}{\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 6946}$$

$$-\frac{2b\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 2639}$$

$$-\frac{2b\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 6928}$$

$$-\frac{2b\left(\sqrt{\pi}b\int \text{erfc}(bx)d\text{erfc}(bx)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2b\left(-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\text{erfc}(bx)^2\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{2x^2}$$

input
```
Int[Erfc[b*x]^2/x^3,x]
```

output
```
-1/2*Erfc[b*x]^2/x^2 - (2*b*(-(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Er
fc[b*x]^2)/2 - (b*ExpIntegralEi[-2*b^2*x^2])/Sqrt[Pi]))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx$$

input
```
int(erfc(b*x)^2/x^3,x)
```

output
```
int(erfc(b*x)^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx = -\frac{\pi - 4\,\pi\sqrt{b^2}bx^2\operatorname{erf}\left(\sqrt{b^2}x\right) - 4\,b^2x^2\operatorname{Ei}(-2\,b^2x^2) + (\pi + 2\,\pi b^2x^2)\operatorname{erf}(bx)^2 + 4\,\sqrt{\pi}(bx\operatorname{erf}(bx) - bx)e^{(-b^2x^2)} - 2\,\pi\operatorname{erf}(bx)}{2\,\pi x^2}$$

input
```
integrate(erfc(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(pi - 4*pi*sqrt(b^2)*b*x^2*erf(sqrt(b^2)*x) - 4*b^2*x^2*Ei(-2*b^2*x^2
) + (pi + 2*pi*b^2*x^2)*erf(b*x)^2 + 4*sqrt(pi)*(b*x*erf(b*x) - b*x)*e^(-b
^2*x^2) - 2*pi*erf(b*x))/(pi*x^2)
```

Sympy [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfc}^2(bx)}{x^3}\,dx$$

input
```
integrate(erfc(b*x)**2/x**3,x)
```

output
```
Integral(erfc(b*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx$$

input
```
integrate(erfc(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x^3, x)
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx$$

input
```
integrate(erfc(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfc}(b\,x)^2}{x^3}\,dx$$

input
```
int(erfc(b*x)^2/x^3,x)
```

output
```
int(erfc(b*x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^3}\,dx$$
$$= \frac{-e^{b^2x^2}\operatorname{erf}(bx)^2\pi + 4e^{b^2x^2}\operatorname{erf}(bx)\,b^2\pi\,x^2 + 2e^{b^2x^2}\operatorname{erf}(bx)\,\pi + 4\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^2}dx\right)b\,x^2 - e^{b^2x^2}\pi + 4\sqrt{\pi}}{2e^{b^2x^2}\pi\,x^2}$$

input
```
int(erfc(b*x)^2/x^3,x)
```

output
```
( - e**(b**2*x**2)*erf(b*x)**2*pi + 4*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2
 + 2*e**(b**2*x**2)*erf(b*x)*pi + 4*sqrt(pi)*e**(b**2*x**2)*int(erf(b*x)/(
e**(b**2*x**2)*x**2),x)*b*x**2 - e**(b**2*x**2)*pi + 4*sqrt(pi)*b*x)/(2*e*
*(b**2*x**2)*pi*x**2)
```

3.130 $\int \frac{\text{erfc}(bx)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 125

$$\int \frac{\text{erfc}(bx)^2}{x^5}\,dx = -\frac{b^2e^{-2b^2x^2}}{3\pi x^2}+\frac{be^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x^3}-\frac{2b^3e^{-b^2x^2}\text{erfc}(bx)}{3\sqrt{\pi}x} +\frac{1}{3}b^4\text{erfc}(bx)^2-\frac{\text{erfc}(bx)^2}{4x^4}-\frac{4b^4\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{3\pi}$$

output
```
-1/3*b^2/exp(2*b^2*x^2)/Pi/x^2+1/3*b*erfc(b*x)/exp(b^2*x^2)/Pi^(1/2)/x^3-2
/3*b^3*erfc(b*x)/exp(b^2*x^2)/Pi^(1/2)/x+1/3*b^4*erfc(b*x)^2-1/4*erfc(b*x)
^2/x^4-4/3*b^4*Ei(-2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{\text{erfc}(bx)^2}{x^5}\,dx = \frac{-\frac{4be^{-b^2x^2}x(-1+2b^2x^2)\text{erfc}(bx)}{\sqrt{\pi}}+\left(-3+4b^4x^4\right)\text{erfc}(bx)^2-\frac{4b^2x^2\left(e^{-2b^2x^2}+4b^2x^2\,\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)}{\pi}}{12x^4}$$

input
```
Integrate[Erfc[b*x]^2/x^5,x]
```

output
```
((-4*b*x*(-1 + 2*b^2*x^2)*Erfc[b*x])/(E^(b^2*x^2)*Sqrt[Pi]) + (-3 + 4*b^4*
x^4)*Erfc[b*x]^2 - (4*b^2*x^2*(E^(-2*b^2*x^2) + 4*b^2*x^2*ExpIntegralEi[-2
*b^2*x^2]))/Pi)/(12*x^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6919, 6946, 2643, 2639, 6946, 2639, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)^2}{x^5} \, dx$$

$$\Big\downarrow \text{6919}$$

$$-\frac{b \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}dx}{\sqrt{\pi}} - \frac{\text{erfc}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{6946}$$

$$-\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx - \frac{2b\int \frac{e^{-2b^2x^2}}{x^3}dx}{3\sqrt{\pi}} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}\right)}{\sqrt{\pi}} - \frac{\text{erfc}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx - \frac{2b\left(-2b^2\int \frac{e^{-2b^2x^2}}{x}dx - \frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}\right)}{\sqrt{\pi}} - \frac{\text{erfc}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{b\left(-\frac{2}{3}b^2 \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx - \frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3} - \frac{2b\left(b^2(-\text{ExpIntegralEi}(-2b^2x^2)) - \frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}} - \frac{\text{erfc}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{6946}$$

$$-\frac{b\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{2b\int\frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2\right.\right.\right.}{3\sqrt{\pi}}}{\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{4x^4}$$

$\Big\downarrow$ 2639

$$-\frac{b\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegra}\right.\right.}{}}{\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{4x^4}$$

$\Big\downarrow$ 6928

$$-\frac{b\left(-\frac{2}{3}b^2\left(\sqrt{\pi}b\int\text{erfc}(bx)d\text{erfc}(bx)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegra}\right.\right.}{}}{\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{4x^4}$$

$\Big\downarrow$ 15

$$-\frac{b\left(-\frac{2}{3}b^2\left(-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\text{erfc}(bx)^2\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2\right.\right.\right.}{3\sqrt{\pi}}}{\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{4x^4}$$

input

```
Int[Erfc[b*x]^2/x^5,x]
```

output

```
-1/4*Erfc[b*x]^2/x^4 - (b*(-1/3*Erfc[b*x]/(E^(b^2*x^2)*x^3) - (2*b*(-1/2*1
/(E^(2*b^2*x^2)*x^2) - b^2*ExpIntegralEi[-2*b^2*x^2]))/(3*Sqrt[Pi]) - (2*b
^2*(-(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpInte
gralEi[-2*b^2*x^2])/Sqrt[Pi]))/3))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5} dx$$

input
```
int(erfc(b*x)^2/x^5,x)
```

output
```
int(erfc(b*x)^2/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5} dx = -\frac{3\pi + 8\pi\sqrt{b^2}b^3x^4 \operatorname{erf}\left(\sqrt{b^2}x\right) + 16b^4x^4\operatorname{Ei}(-2b^2x^2) + 4b^2x^2e^{(-2b^2x^2)} + (3\pi - 4\pi b^4x^4)\operatorname{erf}(bx)^2 + 4\sqrt{}}{12\pi x^4}$$

input
```
integrate(erfc(b*x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*pi + 8*pi*sqrt(b^2)*b^3*x^4*erf(sqrt(b^2)*x) + 16*b^4*x^4*Ei(-2*b
^2*x^2) + 4*b^2*x^2*e^(-2*b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*erf(b*x)^2 + 4*
sqrt(pi)*(2*b^3*x^3 - b*x - (2*b^3*x^3 - b*x)*erf(b*x))*e^(-b^2*x^2) - 6*p
i*erf(b*x))/(pi*x^4)
```

Sympy [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5} dx = \int \frac{\operatorname{erfc}^2(bx)}{x^5} dx$$

input
```
integrate(erfc(b*x)**2/x**5,x)
```

output
```
Integral(erfc(b*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfc}\,(bx)^2}{x^5}\,dx$$

input
```
integrate(erfc(b*x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x^5, x)
```

Giac [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfc}\,(bx)^2}{x^5}\,dx$$

input
```
integrate(erfc(b*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfc}(b\,x)^2}{x^5}\,dx$$

input
```
int(erfc(b*x)^2/x^5,x)
```

output
```
int(erfc(b*x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^5}\,dx$$
$$= \frac{-3e^{b^2x^2}\operatorname{erf}(bx)^2\pi + 6e^{b^2x^2}\operatorname{erf}(bx)\pi + 12\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^4}dx\right)b\,x^4 + 8\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^3x^4 - 3e^{b}}{12e^{b^2x^2}\pi\,x^4}$$

input
```
int(erfc(b*x)^2/x^5,x)
```

output
```
( - 3*e**(b**2*x**2)*erf(b*x)**2*pi + 6*e**(b**2*x**2)*erf(b*x)*pi + 12*sq
rt(pi)*e**(b**2*x**2)*int(erf(b*x)/(e**(b**2*x**2)*x**4),x)*b*x**4 + 8*sqr
t(pi)*e**(b**2*x**2)*int(1/(e**(b**2*x**2)*x**2),x)*b**3*x**4 - 3*e**(b**2
*x**2)*pi + 4*sqrt(pi)*b*x)/(12*e**(b**2*x**2)*pi*x**4)
```

3.131 $\int \frac{\operatorname{erfc}(bx)^2}{x^7} dx$

Optimal result

Integrand size = 10, antiderivative size = 177

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7} dx = -\frac{b^2 e^{-2b^2x^2}}{15\pi x^4} + \frac{2b^4 e^{-2b^2x^2}}{9\pi x^2} + \frac{2be^{-b^2x^2}\operatorname{erfc}(bx)}{15\sqrt{\pi}x^5} - \frac{4b^3 e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x^3} + \frac{8b^5 e^{-b^2x^2}\operatorname{erfc}(bx)}{45\sqrt{\pi}x} - \frac{4}{45}b^6\operatorname{erfc}(bx)^2 - \frac{\operatorname{erfc}(bx)^2}{6x^6} + \frac{28b^6 \operatorname{ExpIntegralEi}(-2b^2x^2)}{45\pi}$$

output

```
-1/15*b^2/exp(2*b^2*x^2)/Pi/x^4+2/9*b^4/exp(2*b^2*x^2)/Pi/x^2+2/15*b*erfc(
b*x)/exp(b^2*x^2)/Pi^(1/2)/x^5-4/45*b^3*erfc(b*x)/exp(b^2*x^2)/Pi^(1/2)/x^
3+8/45*b^5*erfc(b*x)/exp(b^2*x^2)/Pi^(1/2)/x-4/45*b^6*erfc(b*x)^2-1/6*erfc
(b*x)^2/x^6+28/45*b^6*Ei(-2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7}\,dx$$
$$= \frac{e^{-2b^2x^2}\left(-6b^2x^2+20b^4x^4+4be^{b^2x^2}\sqrt{\pi}x(3-2b^2x^2+4b^4x^4)\operatorname{erfc}(bx)-e^{2b^2x^2}\pi(15+8b^6x^6)\operatorname{erfc}(bx)^2+5\right.}{90\pi x^6}$$

input
```
Integrate[Erfc[b*x]^2/x^7,x]
```

output
```
(-6*b^2*x^2 + 20*b^4*x^4 + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3 - 2*b^2*x^2 + 4*b
^4*x^4)*Erfc[b*x] - E^(2*b^2*x^2)*Pi*(15 + 8*b^6*x^6)*Erfc[b*x]^2 + 56*b^6
*E^(2*b^2*x^2)*x^6*ExpIntegralEi[-2*b^2*x^2])/(90*E^(2*b^2*x^2)*Pi*x^6)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6919, 6946, 2643, 2643, 2639, 6946, 2643, 2639, 6946, 2639, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7}\,dx$$
$$\Big\downarrow \text{ 6919}$$
$$-\frac{2b\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^6}dx}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)^2}{6x^6}$$
$$\Big\downarrow \text{ 6946}$$
$$-\frac{2b\left(-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}dx-\frac{2b\int \frac{e^{-2b^2x^2}}{x^5}dx}{5\sqrt{\pi}}-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\operatorname{erfc}(bx)^2}{6x^6}$$
$$\Big\downarrow \text{ 2643}$$

$$-\frac{2b\left(-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}dx-\frac{2b\left(b^2\left(-\int \frac{e^{-2b^2x^2}}{x^3}dx\right)-\frac{e^{-2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2643

$$-\frac{2b\left(-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}dx-\frac{2b\left(-\left(b^2\left(-2b^2\int \frac{e^{-2b^2x^2}}{x}dx-\frac{e^{-2b^2x^2}}{2x^2}\right)\right)-\frac{e^{-2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2639

$$-\frac{2b\left(-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{5x^5}-\frac{2b\left(-\left(b^2\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)\right)-\frac{e^{-2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{6x^6}$$

$\Big\downarrow$ 6946

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx-\frac{2b\int \frac{e^{-2b^2x^2}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{5x^5}-\frac{2b\left(-\left(b^2\left(b^2\left(-\text{ExpIntegral}\cdots\right.\right.\right.\right.}{\cdots}\right.}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2643

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx-\frac{2b\left(-2b^2\int \frac{e^{-2b^2x^2}}{x}dx-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{5x^5}-\frac{2b\left(-\left(b^2\cdots\right.\right.}{\cdots}\right.}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2639

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegralEi}(-2b^2x^2)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{5x^5}-\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 6946$$

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{2b\int\frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpInteg}\right.\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 2639$$

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\right.\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 6928$$

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(\sqrt{\pi}b\int\text{erfc}(bx)d\text{erfc}(bx)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\right.\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 15$$

$$-\frac{2b\left(-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}(-2b^2x^2)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\,\text{erfc}(bx)^2\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpInteg}\right.\right.}{}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfc}(bx)^2}{6x^6}$$

input
```
Int[Erfc[b*x]^2/x^7,x]
```

output
```
-1/6*Erfc[b*x]^2/x^6 - (2*b*(-1/5*Erfc[b*x]/(E^(b^2*x^2)*x^5) - (2*b*(-1/4
*1/(E^(2*b^2*x^2)*x^4) - b^2*(-1/2*1/(E^(2*b^2*x^2)*x^2) - b^2*ExpIntegral
Ei[-2*b^2*x^2])))/(5*Sqrt[Pi]) - (2*b^2*(-1/3*Erfc[b*x]/(E^(b^2*x^2)*x^3)
- (2*b*(-1/2*1/(E^(2*b^2*x^2)*x^2) - b^2*ExpIntegralEi[-2*b^2*x^2]))/(3*Sq
rt[Pi]) - (2*b^2*(-(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/
2 - (b*ExpIntegralEi[-2*b^2*x^2])/Sqrt[Pi]))/3))/5))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7}dx$$

input
```
int(erfc(b*x)^2/x^7,x)
```

output
```
int(erfc(b*x)^2/x^7,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7}\,dx = -\frac{15\,\pi - 16\,\pi\sqrt{b^2}b^5x^6\operatorname{erf}\left(\sqrt{b^2}x\right) - 56\,b^6x^6\operatorname{Ei}(-2\,b^2x^2) + (15\,\pi + 8\,\pi b^6x^6)\operatorname{erf}(bx)^2 - 4\,\sqrt{\pi}(4\,b^5x^5 - 2\,b}{90\,\pi}$$

input
```
integrate(erfc(b*x)^2/x^7,x, algorithm="fricas")
```

output
```
-1/90*(15*pi - 16*pi*sqrt(b^2)*b^5*x^6*erf(sqrt(b^2)*x) - 56*b^6*x^6*Ei(-2
*b^2*x^2) + (15*pi + 8*pi*b^6*x^6)*erf(b*x)^2 - 4*sqrt(pi)*(4*b^5*x^5 - 2*
b^3*x^3 + 3*b*x - (4*b^5*x^5 - 2*b^3*x^3 + 3*b*x)*erf(b*x))*e^(-b^2*x^2) -
 30*pi*erf(b*x) - 2*(10*b^4*x^4 - 3*b^2*x^2)*e^(-2*b^2*x^2))/(pi*x^6)
```

Sympy [F]

$$\int \frac{\text{erfc}(bx)^2}{x^7} \, dx = \int \frac{\text{erfc}^2\,(bx)}{x^7} \, dx$$

input
```
integrate(erfc(b*x)**2/x**7,x)
```

output
```
Integral(erfc(b*x)**2/x**7, x)
```

Maxima [F]

$$\int \frac{\text{erfc}(bx)^2}{x^7} \, dx = \int \frac{\text{erfc}\,(bx)^2}{x^7} \, dx$$

input
```
integrate(erfc(b*x)^2/x^7,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x^7, x)
```

Giac [F]

$$\int \frac{\text{erfc}(bx)^2}{x^7} \, dx = \int \frac{\text{erfc}\,(bx)^2}{x^7} \, dx$$

input
```
integrate(erfc(b*x)^2/x^7,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7} \, dx = \int \frac{\operatorname{erfc}(b\,x)^2}{x^7} \, dx$$

input
```
int(erfc(b*x)^2/x^7,x)
```

output
```
int(erfc(b*x)^2/x^7, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(bx)^2}{x^7} \, dx = \frac{-15e^{b^2x^2}\operatorname{erf}(bx)^2\pi + 30e^{b^2x^2}\operatorname{erf}(bx)\pi + 60\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^6}dx\right)b\,x^6 - 16\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^5x^6 -}{90e^{b^2x^2}\pi\,x^6}$$

input
```
int(erfc(b*x)^2/x^7,x)
```

output
```
( - 15*e**(b**2*x**2)*erf(b*x)**2*pi + 30*e**(b**2*x**2)*erf(b*x)*pi + 60*
sqrt(pi)*e**(b**2*x**2)*int(erf(b*x)/(e**(b**2*x**2)*x**6),x)*b*x**6 - 16*
sqrt(pi)*e**(b**2*x**2)*int(1/(e**(b**2*x**2)*x**2),x)*b**5*x**6 - 15*e**(
b**2*x**2)*pi - 8*sqrt(pi)*b**3*x**3 + 12*sqrt(pi)*b*x)/(90*e**(b**2*x**2)
*pi*x**6)
```

3.132 $\int x^4 \text{erfc}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 165

$$\int x^4\text{erfc}(bx)^2\,dx=\frac{11e^{-2b^2x^2}x}{20b^4\pi}+\frac{e^{-2b^2x^2}x^3}{5b^2\pi}-\frac{43\text{erf}\left(\sqrt{2}bx\right)}{40b^5\sqrt{2\pi}}-\frac{4e^{-b^2x^2}\text{erfc}(bx)}{5b^5\sqrt{\pi}}-\frac{4e^{-b^2x^2}x^2\text{erfc}(bx)}{5b^3\sqrt{\pi}}-\frac{2e^{-b^2x^2}x^4\text{erfc}(bx)}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)^2$$

output
```
11/20*x/b^4/exp(2*b^2*x^2)/Pi+1/5*x^3/b^2/exp(2*b^2*x^2)/Pi-43/80*erf(2^(1
/2)*b*x)/b^5*2^(1/2)/Pi^(1/2)-4/5*erfc(b*x)/b^5/exp(b^2*x^2)/Pi^(1/2)-4/5*
x^2*erfc(b*x)/b^3/exp(b^2*x^2)/Pi^(1/2)-2/5*x^4*erfc(b*x)/b/exp(b^2*x^2)/P
i^(1/2)+1/5*x^5*erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int x^4\text{erfc}(bx)^2\,dx$$

$$=\frac{-43\sqrt{2\pi}\text{erf}\left(\sqrt{2}bx\right)+4\left(be^{-2b^2x^2}x(11+4b^2x^2)-8e^{-b^2x^2}\sqrt{\pi}(2+2b^2x^2+b^4x^4)\,\text{erfc}(bx)+4b^5\pi x^5\text{erfc}(bx\right.}{80b^5\pi}$$

input
```
Integrate[x^4*Erfc[b*x]^2,x]
```

output
```
(-43*Sqrt[2*Pi]*Erf[Sqrt[2]*b*x] + 4*((b*x*(11 + 4*b^2*x^2))/E^(2*b^2*x^2)
 - (8*Sqrt[Pi]*(2 + 2*b^2*x^2 + b^4*x^4)*Erfc[b*x])/E^(b^2*x^2) + 4*b^5*Pi
*x^5*Erfc[b*x]^2))/(80*b^5*Pi)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6919, 6940, 2641, 2641, 2634, 6940, 2641, 2634, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erfc}(bx)^2 \, dx$$

$$\Big\downarrow \text{6919}$$

$$\frac{4b\int e^{-b^2x^2}x^5\text{erfc}(bx)dx}{5\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)^2$$

$$\Big\downarrow \text{6940}$$

$$\frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{\int e^{-2b^2x^2}x^4dx}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{5\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{\frac{3\int e^{-2b^2x^2}x^2dx}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{5\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{\frac{3\left(\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{5\sqrt{\pi}}+\frac{1}{5}x^5\text{erfc}(bx)^2$$

$$\Big\downarrow \text{2634}$$

$$\frac{4b\left(\frac{2\int e^{-b^2x^2}x^3\operatorname{erfc}(bx)dx}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}+\frac{1}{5}x^5\operatorname{erfc}(bx)^2$$

$$\Big\downarrow \; 6940$$

$$\frac{4b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x\operatorname{erfc}(bx)dx}{b^2}-\frac{\int e^{-2b^2x^2}x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}\cdots$$

$$\frac{1}{5}x^5\operatorname{erfc}(bx)^2$$

$$\Big\downarrow \; 2641$$

$$\frac{4b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x\operatorname{erfc}(bx)dx}{b^2}-\frac{\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\cdots}{\sqrt{\pi}b}\cdots}{5\sqrt{\pi}}$$

$$\frac{1}{5}x^5\operatorname{erfc}(bx)^2$$

$$\Big\downarrow \; 2634$$

$$\frac{4b\left(\frac{2\left(\frac{\int e^{-b^2x^2}x\operatorname{erfc}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\cdots}{\sqrt{\pi}b}\cdots}{5\sqrt{\pi}}$$

$$\frac{1}{5}x^5\operatorname{erfc}(bx)^2$$

$$\Big\downarrow \; 6937$$

$$\frac{4b\left(\frac{2\left(\frac{-\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-\cdots}}{4\cdots}\right.}{4b^2}\cdots}{\sqrt{\pi}\cdots}\right.}{5\sqrt{\pi}}$$

$$\frac{1}{5}x^5\text{erfc}(bx)^2$$

$$\Big\downarrow \; 2634$$

$$\frac{4b\left(-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{2\left(\frac{-\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{\cdots}}{\cdots}\right.}{4b^2}\cdots}{\sqrt{\pi}\cdots}\right.}{5\sqrt{\pi}}$$

$$\frac{1}{5}x^5\text{erfc}(bx)^2$$

input

```
Int[x^4*Erfc[b*x]^2,x]
```

output

```
(x^5*Erfc[b*x]^2)/5 + (4*b*(-((-1/4*x^3/(b^2*E^(2*b^2*x^2)) + (3*(-1/4*x/(
b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3)))/(4*b^2))/(b*S
qrt[Pi])) - (x^4*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) + (2*(-((-1/4*x/(b^2*E^(2*
b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi])) - (x^2*Er
fc[b*x])/(2*b^2*E^(b^2*x^2)) + (-1/2*Erf[Sqrt[2]*b*x]/(Sqrt[2]*b^2) - Erfc
[b*x]/(2*b^2*E^(b^2*x^2)))/b^2))/b^2))/(5*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{b^5x^5}{5}-\frac{2\,\mathrm{erf}(bx)b^5x^5}{5}+\frac{-\frac{2\,\mathrm{e}^{-b^2x^2}x^4b^4}{5}-\frac{4\,\mathrm{e}^{-b^2x^2}x^2b^2}{5}-\frac{4\,\mathrm{e}^{-b^2x^2}}{5}}{\sqrt{\pi}}+\frac{\mathrm{erf}(bx)^2b^5x^5}{5}-\frac{4\,\mathrm{erf}(bx)\left(-\frac{\mathrm{e}^{-b^2x^2}x^4b^4}{2}-\mathrm{e}^{-b^2x^2}x^2b^2-\right.}{5\sqrt{\pi}}}{b^5}$
default	$\frac{\frac{b^5x^5}{5}-\frac{2\,\mathrm{erf}(bx)b^5x^5}{5}+\frac{-\frac{2\,\mathrm{e}^{-b^2x^2}x^4b^4}{5}-\frac{4\,\mathrm{e}^{-b^2x^2}x^2b^2}{5}-\frac{4\,\mathrm{e}^{-b^2x^2}}{5}}{\sqrt{\pi}}+\frac{\mathrm{erf}(bx)^2b^5x^5}{5}-\frac{4\,\mathrm{erf}(bx)\left(-\frac{\mathrm{e}^{-b^2x^2}x^4b^4}{2}-\mathrm{e}^{-b^2x^2}x^2b^2-\right.}{5\sqrt{\pi}}}{b^5}$

input
```
int(x^4*erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*b^5*x^5-2/5*erf(b*x)*b^5*x^5+4/5/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^
4*x^4-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))+1/5*erf(b*x)^2*b^5*x^5-4/5*erf(
b*x)/Pi^(1/2)*(-1/2/exp(b^2*x^2)*b^4*x^4-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^
2))+4/5/Pi*(-43/64*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)+11/16/exp(b^2*x^2)^2*
b*x+1/4/exp(b^2*x^2)^2*b^3*x^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int x^4\text{erfc}(bx)^2\,dx$$
$$= \frac{16\,\pi b^6x^5\,\text{erf}\,(bx)^2 - 32\,\pi b^6x^5\,\text{erf}\,(bx) + 16\,\pi b^6x^5 - 43\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\text{erf}\left(\sqrt{2}\sqrt{b^2}x\right) - 32\,\sqrt{\pi}(b^5x^4 + 2\,b^3x^2 -}{80\,\pi b^6}$$

input
```
integrate(x^4*erfc(b*x)^2,x, algorithm="fricas")
```

output
```
1/80*(16*pi*b^6*x^5*erf(b*x)^2 - 32*pi*b^6*x^5*erf(b*x) + 16*pi*b^6*x^5 -
43*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 32*sqrt(pi)*(b^5*
x^4 + 2*b^3*x^2 - (b^5*x^4 + 2*b^3*x^2 + 2*b)*erf(b*x) + 2*b)*e^(-b^2*x^2)
 + 4*(4*b^4*x^3 + 11*b^2*x)*e^(-2*b^2*x^2))/(pi*b^6)
```

Sympy [F]

$$\int x^4\text{erfc}(bx)^2\,dx = \int x^4\,\text{erfc}^2\,(bx)\,dx$$

input
```
integrate(x**4*erfc(b*x)**2,x)
```

output
```
Integral(x**4*erfc(b*x)**2, x)
```

Maxima [F]

$$\int x^4 \text{erfc}(bx)^2 \, dx = \int x^4 \text{erfc}\,(bx)^2 \, dx$$

input
```
integrate(x^4*erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4*erfc(b*x)^2, x)
```

Giac [F]

$$\int x^4 \text{erfc}(bx)^2 \, dx = \int x^4 \text{erfc}\,(bx)^2 \, dx$$

input
```
integrate(x^4*erfc(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*erfc(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{erfc}(bx)^2 \, dx = \int x^4 \,\text{erfc}(b\,x)^2 \, dx$$

input
```
int(x^4*erfc(b*x)^2,x)
```

output
```
int(x^4*erfc(b*x)^2, x)
```

Reduce [F]

$$\int x^4 \mathrm{erfc}(bx)^2 \, dx$$
$$= \frac{-2e^{b^2x^2} \mathrm{erf}(bx) \, b^5 \pi \, x^5 + 5e^{b^2x^2} \left(\int \mathrm{erf}(bx)^2 \, x^4 dx\right) b^5 \pi + e^{b^2x^2} b^5 \pi \, x^5 - 2\sqrt{\pi} \, b^4 x^4 - 4\sqrt{\pi} \, b^2 x^2 - 4\sqrt{\pi}}{5e^{b^2x^2} b^5 \pi}$$

input
```
int(x^4*erfc(b*x)^2,x)
```

output
```
( - 2*e**(b**2*x**2)*erf(b*x)*b**5*pi*x**5 + 5*e**(b**2*x**2)*int(erf(b*x)
**2*x**4,x)*b**5*pi + e**(b**2*x**2)*b**5*pi*x**5 - 2*sqrt(pi)*b**4*x**4 -
 4*sqrt(pi)*b**2*x**2 - 4*sqrt(pi))/(5*e**(b**2*x**2)*b**5*pi)
```

3.133 $\int x^2\text{erfc}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int x^2\text{erfc}(bx)^2\,dx=\frac{e^{-2b^2x^2}x}{3b^2\pi}-\frac{5\text{erf}\left(\sqrt{2}bx\right)}{6b^3\sqrt{2\pi}}-\frac{2e^{-b^2x^2}\text{erfc}(bx)}{3b^3\sqrt{\pi}}-\frac{2e^{-b^2x^2}x^2\text{erfc}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)^2$$

output
```
1/3*x/b^2/exp(2*b^2*x^2)/Pi-5/12*erf(2^(1/2)*b*x)/b^3*2^(1/2)/Pi^(1/2)-2/3
*erfc(b*x)/b^3/exp(b^2*x^2)/Pi^(1/2)-2/3*x^2*erfc(b*x)/b/exp(b^2*x^2)/Pi^(
1/2)+1/3*x^3*erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int x^2\text{erfc}(bx)^2\,dx=\frac{4be^{-2b^2x^2}x-5\sqrt{2\pi}\text{erf}\left(\sqrt{2}bx\right)-8e^{-b^2x^2}\sqrt{\pi}\left(1+b^2x^2\right)\text{erfc}(bx)+4b^3\pi x^3\text{erfc}(bx)^2}{12b^3\pi}$$

input
```
Integrate[x^2*Erfc[b*x]^2,x]
```

output
```
((4*b*x)/E^(2*b^2*x^2) - 5*Sqrt[2*Pi]*Erf[Sqrt[2]*b*x] - (8*Sqrt[Pi]*(1 +
b^2*x^2)*Erfc[b*x])/E^(b^2*x^2) + 4*b^3*Pi*x^3*Erfc[b*x]^2)/(12*b^3*Pi)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6919, 6940, 2641, 2634, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erfc}(bx)^2 \, dx$$

$$\Big\downarrow \text{6919}$$

$$\frac{4b \int e^{-b^2x^2} x^3 \text{erfc}(bx) dx}{3\sqrt{\pi}} + \frac{1}{3} x^3 \text{erfc}(bx)^2$$

$$\Big\downarrow \text{6940}$$

$$\frac{4b \left(\frac{\int e^{-b^2x^2} x \text{erfc}(bx) dx}{b^2} - \frac{\int e^{-2b^2x^2} x^2 dx}{\sqrt{\pi} b} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} \right)}{3\sqrt{\pi}} + \frac{1}{3} x^3 \text{erfc}(bx)^2$$

$$\Big\downarrow \text{2641}$$

$$\frac{4b \left(\frac{\int e^{-b^2x^2} x \text{erfc}(bx) dx}{b^2} - \frac{\frac{\int e^{-2b^2x^2} dx}{4b^2} - \frac{x e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} \right)}{3\sqrt{\pi}} + \frac{1}{3} x^3 \text{erfc}(bx)^2$$

$$\Big\downarrow \text{2634}$$

$$\frac{4b \left(\frac{\int e^{-b^2x^2} x \text{erfc}(bx) dx}{b^2} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} - \frac{\frac{\sqrt{\frac{\pi}{2}} \text{erf}\left(\sqrt{2} bx\right)}{8b^3} - \frac{x e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b} \right)}{3\sqrt{\pi}} + \frac{1}{3} x^3 \text{erfc}(bx)^2$$

$$\Big\downarrow \text{6937}$$

$$\frac{4b \left(\frac{-\frac{\int e^{-2b^2x^2} dx}{\sqrt{\pi} b} - \frac{e^{-b^2x^2} \text{erfc}(bx)}{2b^2}}{b^2} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} - \frac{\frac{\sqrt{\frac{\pi}{2}} \text{erf}\left(\sqrt{2} bx\right)}{8b^3} - \frac{x e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b} \right)}{3\sqrt{\pi}} + \frac{1}{3} x^3 \text{erfc}(bx)^2$$

$$\Big\downarrow \text{2634}$$

$$\frac{4b\left(\frac{-\frac{\text{erf}(\sqrt{2}bx)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(bx)^2$$

input
```
Int[x^2*Erfc[b*x]^2,x]
```

output
```
(x^3*Erfc[b*x]^2)/3 + (4*b*(-((-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Er
f[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi])) - (x^2*Erfc[b*x])/(2*b^2*E^(b^2*x^2
)) + (-1/2*Erf[Sqrt[2]*b*x]/(Sqrt[2]*b^2) - Erfc[b*x]/(2*b^2*E^(b^2*x^2)))
/b^2))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6919
```
Int[Erfc[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfc[b*x]^2
/(m + 1)), x] + Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[(x^(m + 1)*Erfc[b*x])/E
^(b^2*x^2), x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{b^3x^3}{3}-\frac{2\,\mathrm{erf}(bx)b^3x^3}{3}+\frac{-\frac{2\,\mathrm{e}^{-b^2x^2}x^2b^2}{3}-\frac{2\,\mathrm{e}^{-b^2x^2}}{3}}{\sqrt{\pi}}+\frac{\mathrm{erf}(bx)^2b^3x^3}{3}-\frac{4\,\mathrm{erf}(bx)\left(-\frac{\mathrm{e}^{-b^2x^2}x^2b^2}{2}-\frac{\mathrm{e}^{-b^2x^2}}{2}\right)}{3\sqrt{\pi}}+\frac{-\frac{5\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}(\sqrt{2}}{12}}{\pi}}{b^3}$
default	$\frac{\frac{b^3x^3}{3}-\frac{2\,\mathrm{erf}(bx)b^3x^3}{3}+\frac{-\frac{2\,\mathrm{e}^{-b^2x^2}x^2b^2}{3}-\frac{2\,\mathrm{e}^{-b^2x^2}}{3}}{\sqrt{\pi}}+\frac{\mathrm{erf}(bx)^2b^3x^3}{3}-\frac{4\,\mathrm{erf}(bx)\left(-\frac{\mathrm{e}^{-b^2x^2}x^2b^2}{2}-\frac{\mathrm{e}^{-b^2x^2}}{2}\right)}{3\sqrt{\pi}}+\frac{-\frac{5\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}(\sqrt{2}}{12}}{\pi}}{b^3}$

input
```
int(x^2*erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^3*x^3-2/3*erf(b*x)*b^3*x^3+4/3/Pi^(1/2)*(-1/2*b^2*x^2/exp(b^2
*x^2)-1/2/exp(b^2*x^2))+1/3*erf(b*x)^2*b^3*x^3-4/3*erf(b*x)/Pi^(1/2)*(-1/2
*b^2*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))+4/3/Pi*(-5/16*2^(1/2)*Pi^(1/2)*erf
(2^(1/2)*b*x)+1/4/exp(b^2*x^2)^2*b*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int x^2\mathrm{erfc}(bx)^2\,dx$$
$$=\frac{4\,\pi b^4x^3\,\mathrm{erf}\left(bx\right)^2-8\,\pi b^4x^3\,\mathrm{erf}\left(bx\right)+4\,\pi b^4x^3+4\,b^2xe^{\left(-2\,b^2x^2\right)}-5\,\sqrt{2}\sqrt{\pi}\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-8\,\sqrt{\pi}(b^3x^2}{12\,\pi b^4}$$

input
```
integrate(x^2*erfc(b*x)^2,x, algorithm="fricas")
```

output
```
1/12*(4*pi*b^4*x^3*erf(b*x)^2 - 8*pi*b^4*x^3*erf(b*x) + 4*pi*b^4*x^3 + 4*b
^2*x*e^(-2*b^2*x^2) - 5*sqrt(2)*sqrt(pi)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x
) - 8*sqrt(pi)*(b^3*x^2 - (b^3*x^2 + b)*erf(b*x) + b)*e^(-b^2*x^2))/(pi*b^
4)
```

Sympy [F]

$$\int x^2 \text{erfc}(bx)^2 \, dx = \int x^2 \,\text{erfc}^2 \,(bx) \, dx$$

input
```
integrate(x**2*erfc(b*x)**2,x)
```

output
```
Integral(x**2*erfc(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \text{erfc}(bx)^2 \, dx = \int x^2 \,\text{erfc}\,(bx)^2 \, dx$$

input
```
integrate(x^2*erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*erfc(b*x)^2, x)
```

Giac [F]

$$\int x^2 \text{erfc}(bx)^2 \, dx = \int x^2 \,\text{erfc}\,(bx)^2 \, dx$$

input
```
integrate(x^2*erfc(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*erfc(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{erfc}(bx)^2 \, dx = \int x^2 \, \mathrm{erfc}(b\,x)^2 \, dx$$

input
```
int(x^2*erfc(b*x)^2,x)
```

output
```
int(x^2*erfc(b*x)^2, x)
```

Reduce [F]

$$\int x^2 \mathrm{erfc}(bx)^2 \, dx = \frac{-2e^{b^2x^2} \mathrm{erf}(bx)\, b^3\pi\, x^3 + 3e^{b^2x^2} \left(\int \mathrm{erf}(bx)^2 \, x^2 dx\right) b^3\pi + e^{b^2x^2} b^3\pi\, x^3 - 2\sqrt{\pi}\, b^2x^2 - 2\sqrt{\pi}}{3e^{b^2x^2} b^3\pi}$$

input
```
int(x^2*erfc(b*x)^2,x)
```

output
```
( - 2*e**(b**2*x**2)*erf(b*x)*b**3*pi*x**3 + 3*e**(b**2*x**2)*int(erf(b*x)
**2*x**2,x)*b**3*pi + e**(b**2*x**2)*b**3*pi*x**3 - 2*sqrt(pi)*b**2*x**2 -
 2*sqrt(pi))/(3*e**(b**2*x**2)*b**3*pi)
```

3.134 $\int \text{erfc}(bx)^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int \text{erfc}(bx)^2 \, dx = -\frac{\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}bx\right)}{b} - \frac{2e^{-b^2x^2}\text{erfc}(bx)}{b\sqrt{\pi}} + x\text{erfc}(bx)^2$$

output
```
-2^(1/2)/Pi^(1/2)*erf(2^(1/2)*b*x)/b-2*erfc(b*x)/b/exp(b^2*x^2)/Pi^(1/2)+x
*erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \text{erfc}(bx)^2 \, dx = -\frac{\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}bx\right)}{b} - \frac{2e^{-b^2x^2}\text{erfc}(bx)}{b\sqrt{\pi}} + x\text{erfc}(bx)^2$$

input
```
Integrate[Erfc[b*x]^2,x]
```

output
```
-((Sqrt[2/Pi]*Erf[Sqrt[2]*b*x])/b) - (2*Erfc[b*x])/(b*E^(b^2*x^2)*Sqrt[Pi]
) + x*Erfc[b*x]^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6907, 27, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erfc}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6907}$$

$$\frac{4 \int b e^{-b^2 x^2} x \operatorname{erfc}(bx) dx}{\sqrt{\pi}} + x \operatorname{erfc}(bx)^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4b \int e^{-b^2 x^2} x \operatorname{erfc}(bx) dx}{\sqrt{\pi}} + x \operatorname{erfc}(bx)^2$$

$$\Big\downarrow \text{ 6937}$$

$$\frac{4b \left(-\frac{\int e^{-2b^2 x^2} dx}{\sqrt{\pi} b} - \frac{e^{-b^2 x^2} \operatorname{erfc}(bx)}{2b^2}\right)}{\sqrt{\pi}} + x \operatorname{erfc}(bx)^2$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{4b \left(-\frac{\operatorname{erf}\left(\sqrt{2} b x\right)}{2\sqrt{2} b^2} - \frac{e^{-b^2 x^2} \operatorname{erfc}(bx)}{2b^2}\right)}{\sqrt{\pi}} + x \operatorname{erfc}(bx)^2$$

input
```
Int[Erfc[b*x]^2,x]
```

output
```
x*Erfc[b*x]^2 + (4*b*(-1/2*Erf[Sqrt[2]*b*x]/(Sqrt[2]*b^2) - Erfc[b*x]/(2*b
^2*E^(b^2*x^2))))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6907
```
Int[Erfc[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(Erfc[a + b*x]^
2/b), x] + Simp[4/Sqrt[Pi]   Int[(a + b*x)*(Erfc[a + b*x]/E^(a + b*x)^2), x
], x] /; FreeQ[{a, b}, x]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\operatorname{erf}(bx)^2 bx+\frac{2\operatorname{erf}(bx)\mathrm{e}^{-b^2x^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,bx\right)}{\sqrt{\pi}}}{b}$	48
default	$\frac{\operatorname{erf}(bx)^2 bx+\frac{2\operatorname{erf}(bx)\mathrm{e}^{-b^2x^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,bx\right)}{\sqrt{\pi}}}{b}$	48

input
```
int(erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(erf(b*x)^2*b*x+2*erf(b*x)/Pi^(1/2)*exp(-b^2*x^2)-1/Pi^(1/2)*2^(1/2)*e
rf(2^(1/2)*b*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int \operatorname{erfc}(bx)^2 \, dx = \frac{\pi b^2 x \operatorname{erf}(bx)^2 - 2\pi b^2 x \operatorname{erf}(bx) + \pi b^2 x - \sqrt{2}\sqrt{\pi}\sqrt{b^2} \operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right) + 2\sqrt{\pi}(b \operatorname{erf}(bx) - b)e^{(-b^2x^2)}}{\pi b^2}$$

input
```
integrate(erfc(b*x)^2,x, algorithm="fricas")
```

output
```
(pi*b^2*x*erf(b*x)^2 - 2*pi*b^2*x*erf(b*x) + pi*b^2*x - sqrt(2)*sqrt(pi)*s
qrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) + 2*sqrt(pi)*(b*erf(b*x) - b)*e^(-b^2*x^
2))/(pi*b^2)
```

Sympy [F]

$$\int \operatorname{erfc}(bx)^2 \, dx = \int \operatorname{erfc}^2(bx) \, dx$$

input
```
integrate(erfc(b*x)**2,x)
```

output
```
Integral(erfc(b*x)**2, x)
```

Maxima [F]

$$\int \operatorname{erfc}(bx)^2 \, dx = \int \operatorname{erfc}(bx)^2 \, dx$$

input
```
integrate(erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2, x)
```

Giac [F]

$$\int \operatorname{erfc}(bx)^2\,dx = \int \operatorname{erfc}\left(bx\right)^2\,dx$$

input
```
integrate(erfc(b*x)^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(bx)^2\,dx = \int \operatorname{erfc}(b\,x)^2\,dx$$

input
```
int(erfc(b*x)^2,x)
```

output
```
int(erfc(b*x)^2, x)
```

Reduce [F]

$$\int \operatorname{erfc}(bx)^2\,dx = \frac{-2\sqrt{\pi}\,e^{b^2x^2}\operatorname{erf}(bx)\,bx + \sqrt{\pi}\,e^{b^2x^2}\left(\int \operatorname{erf}(bx)^2\,dx\right)b + \sqrt{\pi}\,e^{b^2x^2}bx - 2}{\sqrt{\pi}\,e^{b^2x^2}b}$$

input
```
int(erfc(b*x)^2,x)
```

output
```
( - 2*sqrt(pi)*e**(b**2*x**2)*erf(b*x)*b*x + sqrt(pi)*e**(b**2*x**2)*int(e
rf(b*x)**2,x)*b + sqrt(pi)*e**(b**2*x**2)*b*x - 2)/(sqrt(pi)*e**(b**2*x**2
)*b)
```

3.135 $\int \frac{\text{erfc}(bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfc}(bx)^2}{x^2}\,dx = \text{Int}\left(\frac{\text{erfc}(bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(erfc(b*x)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfc}(bx)^2}{x^2}\,dx = \int \frac{\text{erfc}(bx)^2}{x^2}\,dx$$

input
```
Integrate[Erfc[b*x]^2/x^2,x]
```

output
```
Integrate[Erfc[b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{6925}$$

$$\int \frac{\text{erfc}(bx)^2}{x^2}dx$$

input
```
Int[Erfc[b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx\right)^2}{x^2}dx$$

input
```
int(erfc(b*x)^2/x^2,x)
```

output
```
int(erfc(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx$$

input
```
integrate(erfc(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfc}^2(bx)}{x^2}\,dx$$

input
```
integrate(erfc(b*x)**2/x**2,x)
```

output
```
Integral(erfc(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx$$

input
```
integrate(erfc(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x^2}\,dx$$

input
```
integrate(erfc(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfc}(b\,x)^2}{x^2}\,dx$$

input
```
int(erfc(b*x)^2/x^2,x)
```

output
```
int(erfc(b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 6.90

$$\int \frac{\text{erfc}(bx)^2}{x^2}\,dx = \frac{-2\sqrt{\pi}\,ei(-b^2x^2)\,bx - \text{erf}(bx)^2\,\pi + 2\,\text{erf}(bx)\,\pi + 4\sqrt{\pi}\left(\int \frac{\text{erf}(bx)}{e^{b^2x^2}x}dx\right)bx - \pi}{\pi x}$$

input
```
int(erfc(b*x)^2/x^2,x)
```

output
```
( - 2*sqrt(pi)*ei( - b**2*x**2)*b*x - erf(b*x)**2*pi + 2*erf(b*x)*pi + 4*s
qrt(pi)*int(erf(b*x)/(e**(b**2*x**2)*x),x)*b*x - pi)/(pi*x)
```

3.136 $\int \frac{\text{erfc}(bx)^2}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfc}(bx)^2}{x^4} dx = \text{Int}\left(\frac{\text{erfc}(bx)^2}{x^4}, x\right)$$

output
```
Defer(Int)(erfc(b*x)^2/x^4,x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfc}(bx)^2}{x^4} dx = \int \frac{\text{erfc}(bx)^2}{x^4} dx$$

input
```
Integrate[Erfc[b*x]^2/x^4,x]
```

output
```
Integrate[Erfc[b*x]^2/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 6925}$$

$$\int \frac{\text{erfc}(bx)^2}{x^4}dx$$

input
```
Int[Erfc[b*x]^2/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx\right)^{2}}{x^{4}}dx$$

input
```
int(erfc(b*x)^2/x^4,x)
```

output
```
int(erfc(b*x)^2/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(erfc(b*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfc}^2\left(bx\right)}{x^4}\,dx$$

input
```
integrate(erfc(b*x)**2/x**4,x)
```

output
```
Integral(erfc(b*x)**2/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(erfc(b*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(erfc(b*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfc}(b\,x)^2}{x^4}\,dx$$

input
```
int(erfc(b*x)^2/x^4,x)
```

output
```
int(erfc(b*x)^2/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 13.80

$$\int \frac{\operatorname{erfc}(bx)^2}{x^4}\,dx = \frac{2\sqrt{\pi}\,e^{b^2x^2}\,ei(-b^2x^2)\,b^3x^3 - e^{b^2x^2}\operatorname{erf}(bx)^2\,\pi + 2e^{b^2x^2}\operatorname{erf}(bx)\,\pi + 4\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^3}dx\right)b\,x^3 - e^{b^2x^2}\pi + 2\sqrt{\pi}\,bx}{3e^{b^2x^2}\pi\,x^3}$$

input
```
int(erfc(b*x)^2/x^4,x)
```

output
```
(2*sqrt(pi)*e**(b**2*x**2)*ei( - b**2*x**2)*b**3*x**3 - e**(b**2*x**2)*erf
(b*x)**2*pi + 2*e**(b**2*x**2)*erf(b*x)*pi + 4*sqrt(pi)*e**(b**2*x**2)*int
(erf(b*x)/(e**(b**2*x**2)*x**3),x)*b*x**3 - e**(b**2*x**2)*pi + 2*sqrt(pi)
*b*x)/(3*e**(b**2*x**2)*pi*x**3)
```

3.137 $\int \frac{\text{erfc}(bx)^2}{x^6} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfc}(bx)^2}{x^6} dx = \text{Int}\left(\frac{\text{erfc}(bx)^2}{x^6}, x\right)$$

output
```
Defer(Int)(erfc(b*x)^2/x^6,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfc}(bx)^2}{x^6} dx = \int \frac{\text{erfc}(bx)^2}{x^6} dx$$

input
```
Integrate[Erfc[b*x]^2/x^6,x]
```

output
```
Integrate[Erfc[b*x]^2/x^6, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)^2}{x^6} \, dx$$
$$\Big\downarrow \; 6925$$
$$\int \frac{\text{erfc}(bx)^2}{x^6}dx$$

input
```
Int[Erfc[b*x]^2/x^6,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\,(bx)^2}{x^6}dx$$

input
```
int(erfc(b*x)^2/x^6,x)
```

output
```
int(erfc(b*x)^2/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x^6} dx = \int \frac{\operatorname{erfc}(bx)^2}{x^6} dx$$

input
```
integrate(erfc(b*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral((erf(b*x)^2 - 2*erf(b*x) + 1)/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)^2}{x^6} dx = \int \frac{\operatorname{erfc}^2(bx)}{x^6} dx$$

input
```
integrate(erfc(b*x)**2/x**6,x)
```

output
```
Integral(erfc(b*x)**2/x**6, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfc}(bx)^2}{x^6} dx = \int \frac{\operatorname{erfc}(bx)^2}{x^6} dx$$

input
```
integrate(erfc(b*x)^2/x^6,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2/x^6, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfc}(bx)^2}{x^6}\,dx = \int \frac{\text{erfc}\,(bx)^2}{x^6}\,dx$$

input
```
integrate(erfc(b*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)^2/x^6, x)
```

Mupad [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfc}(bx)^2}{x^6}\,dx = \int \frac{\text{erfc}(b\,x)^2}{x^6}\,dx$$

input
```
int(erfc(b*x)^2/x^6,x)
```

output
```
int(erfc(b*x)^2/x^6, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 14.70

$$\int \frac{\operatorname{erfc}(bx)^2}{x^6}\,dx$$
$$= \frac{-\sqrt{\pi}\,e^{b^2x^2}\,ei(-b^2x^2)\,b^5x^5 - e^{b^2x^2}\operatorname{erf}(bx)^2\,\pi + 2e^{b^2x^2}\operatorname{erf}(bx)\,\pi + 4\sqrt{\pi}\,e^{b^2x^2}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^5}dx\right) b\,x^5 - e^{b^2x^2}\pi - \surd}{5e^{b^2x^2}\pi\,x^5}$$

input
```
int(erfc(b*x)^2/x^6,x)
```

output
```
( - sqrt(pi)*e**(b**2*x**2)*ei( - b**2*x**2)*b**5*x**5 - e**(b**2*x**2)*er
f(b*x)**2*pi + 2*e**(b**2*x**2)*erf(b*x)*pi + 4*sqrt(pi)*e**(b**2*x**2)*in
t(erf(b*x)/(e**(b**2*x**2)*x**5),x)*b*x**5 - e**(b**2*x**2)*pi - sqrt(pi)*
b**3*x**3 + sqrt(pi)*b*x)/(5*e**(b**2*x**2)*pi*x**5)
```

3.138 $\int (c+dx)^2\mathbf{erfc}(a+bx)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 375

$$
\begin{aligned}
\int (c+dx)^2\text{erfc}(a+bx)^2\,dx &= \frac{d(bc-ad)e^{-2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{-2(a+bx)^2}(a+bx)}{3b^3\pi} \\
&\quad -\frac{(bc-ad)^2\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^3} \\
&\quad -\frac{5d^2\text{erf}\left(\sqrt{2}(a+bx)\right)}{6b^3\sqrt{2\pi}}-\frac{2d^2e^{-(a+bx)^2}\text{erfc}(a+bx)}{3b^3\sqrt{\pi}} \\
&\quad -\frac{2(bc-ad)^2e^{-(a+bx)^2}\text{erfc}(a+bx)}{b^3\sqrt{\pi}} \\
&\quad -\frac{2d(bc-ad)e^{-(a+bx)^2}(a+bx)\text{erfc}(a+bx)}{b^3\sqrt{\pi}} \\
&\quad -\frac{2d^2e^{-(a+bx)^2}(a+bx)^2\text{erfc}(a+bx)}{3b^3\sqrt{\pi}} \\
&\quad -\frac{d(bc-ad)\text{erfc}(a+bx)^2}{2b^3} \\
&\quad +\frac{(bc-ad)^2(a+bx)\text{erfc}(a+bx)^2}{b^3} \\
&\quad +\frac{d(bc-ad)(a+bx)^2\text{erfc}(a+bx)^2}{b^3} \\
&\quad +\frac{d^2(a+bx)^3\text{erfc}(a+bx)^2}{3b^3}
\end{aligned}
$$

output
```
d*(-a*d+b*c)/b^3/exp(2*(b*x+a)^2)/Pi+1/3*d^2*(b*x+a)/b^3/exp(2*(b*x+a)^2)/
Pi-(-a*d+b*c)^2*2^(1/2)/Pi^(1/2)*erf(2^(1/2)*(b*x+a))/b^3-5/12*d^2*erf(2^(
1/2)*(b*x+a))/b^3*2^(1/2)/Pi^(1/2)-2/3*d^2*erfc(b*x+a)/b^3/exp((b*x+a)^2)/
Pi^(1/2)-2*(-a*d+b*c)^2*erfc(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)-2*d*(-a*d+
b*c)*(b*x+a)*erfc(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)-2/3*d^2*(b*x+a)^2*erf
c(b*x+a)/b^3/exp((b*x+a)^2)/Pi^(1/2)-1/2*d*(-a*d+b*c)*erfc(b*x+a)^2/b^3+(-
a*d+b*c)^2*(b*x+a)*erfc(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^2*erfc(b*x+a)^2/
b^3+1/3*d^2*(b*x+a)^3*erfc(b*x+a)^2/b^3
```

Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.63

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx$$
$$= \frac{-12b^2\sqrt{\pi}(c+dx)^2\left(\sqrt{2}\text{erf}\left(\sqrt{2}(a+bx)\right)+\text{erfc}(a+bx)\left(2e^{-(a+bx)^2}-\sqrt{\pi}(a+bx)\text{erfc}(a+bx)\right)\right)+6bd(}{}$$

input
```
Integrate[(c + d*x)^2*Erfc[a + b*x]^2,x]
```

output
```
(-12*b^2*Sqrt[Pi]*(c + d*x)^2*(Sqrt[2]*Erf[Sqrt[2]*(a + b*x)] + Erfc[a + b
*x]*(2/E^(a + b*x)^2 - Sqrt[Pi]*(a + b*x)*Erfc[a + b*x])) + 6*b*d*(c + d*x
)*(2/E^(2*(a + b*x)^2) + (4*Sqrt[Pi]*(a + b*x))/E^(a + b*x)^2 - 2*Pi*(a +
b*x)^2 - 2*Pi*Erf[a + b*x] - (4*Sqrt[Pi]*(a + b*x)*Erf[a + b*x])/E^(a + b*
x)^2 + 4*Pi*(a + b*x)^2*Erf[a + b*x] + Pi*Erf[a + b*x]^2 - 2*Pi*(a + b*x)^
2*Erf[a + b*x]^2 + 4*a*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)] + 4*b*Sqrt[2*Pi]*
x*Erf[Sqrt[2]*(a + b*x)] + 2*Pi*(2 + Erfc[-a - b*x]*Erfc[a + b*x]) - 4*Sqr
t[Pi]*(a + b*x)*ExpIntegralE[1/2, (a + b*x)^2]) + d^2*((24*Sqrt[Pi])/E^(a
+ b*x)^2 - 36*b*Pi*x + 12*a^2*b*Pi*x + 12*a*b^2*Pi*x^2 + 4*b^3*Pi*x^3 - (8
*(a + b*x))/E^(2*(a + b*x)^2) - (8*Sqrt[Pi]*(1 + (a + b*x)^2))/E^(a + b*x)
^2 + 12*a*Pi*Erf[a + b*x] + 12*b*Pi*x*Erf[a + b*x] - 8*Pi*(a + b*x)^3*Erf[
a + b*x] + (8*Sqrt[Pi]*(1 + (a + b*x)^2)*Erf[a + b*x])/E^(a + b*x)^2 + 6*P
i*(a + b*x)*Erf[a + b*x]^2 + 4*Pi*(a + b*x)^3*Erf[a + b*x]^2 - 5*Sqrt[2*Pi
]*Erf[Sqrt[2]*(a + b*x)] - 12*a^2*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)] - 12*b
*Sqrt[2*Pi]*x*(2*a + b*x)*Erf[Sqrt[2]*(a + b*x)] - 12*Sqrt[Pi]*ExpIntegral
E[3/2, (a + b*x)^2]))/(12*b^3*Pi)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6922, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6922}$$

$$\frac{\int \left((bc-ad)^2\text{erfc}(a+bx)^2+d^2(a+bx)^2\text{erfc}(a+bx)^2+2d(bc-ad)(a+bx)\text{erfc}(a+bx)^2\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\sqrt{\frac{2}{\pi}}(bc-ad)^2\text{erf}(\sqrt{2}(a+bx))+d(a+bx)^2(bc-ad)\text{erfc}(a+bx)^2+(a+bx)(bc-ad)^2\text{erfc}(a+bx)^2-\frac{2de^{-(a+}}{}}{}$$

input
```
Int[(c + d*x)^2*Erfc[a + b*x]^2,x]
```

output
```
((d*(b*c - a*d))/(E^(2*(a + b*x)^2)*Pi) + (d^2*(a + b*x))/(3*E^(2*(a + b*x
)^2)*Pi) - (b*c - a*d)^2*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)] - (5*d^2*Erf[Sq
rt[2]*(a + b*x)])/(6*Sqrt[2*Pi]) - (2*d^2*Erfc[a + b*x])/(3*E^(a + b*x)^2*
Sqrt[Pi]) - (2*(b*c - a*d)^2*Erfc[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) - (2*
d*(b*c - a*d)*(a + b*x)*Erfc[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) - (2*d^2*(
a + b*x)^2*Erfc[a + b*x])/(3*E^(a + b*x)^2*Sqrt[Pi]) - (d*(b*c - a*d)*Erfc
[a + b*x]^2)/2 + (b*c - a*d)^2*(a + b*x)*Erfc[a + b*x]^2 + d*(b*c - a*d)*(
a + b*x)^2*Erfc[a + b*x]^2 + (d^2*(a + b*x)^3*Erfc[a + b*x]^2)/3)/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6922
```
Int[Erfc[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[1/b^(m + 1)   Subst[Int[ExpandIntegrand[Erfc[x]^2, (b*c - a*d + d*x)^m, x]
, x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)^2 \operatorname{erfc}(bx+a)^2\, dx$$

input
```
int((d*x+c)^2*erfc(b*x+a)^2,x)
```

output
```
int((d*x+c)^2*erfc(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.26

$$\int (c+dx)^2 \operatorname{erfc}(a+bx)^2\, dx = \frac{4\pi b^4 d^2 x^3 + 12\pi b^4 c d x^2 + 12\pi b^4 c^2 x - \sqrt{2}\sqrt{\pi}(12b^2c^2 - 24abcd + (12a^2+5)d^2)\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) -}{}$$

input
```
integrate((d*x+c)^2*erfc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(4*pi*b^4*d^2*x^3 + 12*pi*b^4*c*d*x^2 + 12*pi*b^4*c^2*x - sqrt(2)*sqr
t(pi)*(12*b^2*c^2 - 24*a*b*c*d + (12*a^2 + 5)*d^2)*sqrt(b^2)*erf(sqrt(2)*s
qrt(b^2)*(b*x + a)/b) - 4*pi*(6*a*b^2*c^2 - 3*(2*a^2 + 1)*b*c*d + (2*a^3 +
 3*a)*d^2)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b) + 2*(2*pi*b^4*d^2*x^3 + 6*
pi*b^4*c*d*x^2 + 6*pi*b^4*c^2*x + pi*(6*a*b^3*c^2 - 3*(2*a^2 + 1)*b^2*c*d
+ (2*a^3 + 3*a)*b*d^2))*erf(b*x + a)^2 - 8*sqrt(pi)*(b^3*d^2*x^2 + 3*b^3*c
^2 - 3*a*b^2*c*d + (a^2 + 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x - (b^3*d^2*
x^2 + 3*b^3*c^2 - 3*a*b^2*c*d + (a^2 + 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*
x)*erf(b*x + a))*e^(-b^2*x^2 - 2*a*b*x - a^2) - 8*(pi*b^4*d^2*x^3 + 3*pi*b
^4*c*d*x^2 + 3*pi*b^4*c^2*x)*erf(b*x + a) + 4*(b^2*d^2*x + 3*b^2*c*d - 2*a
*b*d^2)*e^(-2*b^2*x^2 - 4*a*b*x - 2*a^2))/(pi*b^4)
```

Sympy [F]

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx=\int (c+dx)^2\,\text{erfc}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*erfc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*erfc(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx=\int (dx+c)^2\,\text{erfc}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*erfc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*erfc(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx=\int (dx+c)^2\,\text{erfc}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*erfc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*erfc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx=\int \text{erfc}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(erfc(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(erfc(a + b*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\text{erfc}(a+bx)^2\,dx$$
$$=\frac{-6\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,a\,c^2-6\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,b\,c^2x+3\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int \text{erf}(bx+\right.}{}$$

input
```
int((d*x+c)^2*erfc(b*x+a)^2,x)
```

output
```
( - 6*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c**2 - 6*sqr
t(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c**2*x + 3*sqrt(pi)*e
**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)**2,x)*b*c**2 - 6*sqrt(pi)*
e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x**2,x)*b*d**2 - 12*sqrt(
pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*c*d + 3*sqrt(p
i)*e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)**2*x**2,x)*b*d**2 + 6*
sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)**2*x,x)*b*c*d +
3*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b*c**2*x + 3*sqrt(pi)*e**(a**2
+ 2*a*b*x + b**2*x**2)*b*c*d*x**2 + sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**
2)*b*d**2*x**3 - 6*c**2)/(3*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.139 $\int (c+dx)\mathbf{erfc}(a+bx)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 189

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx=\frac{de^{-2(a+bx)^2}}{2b^2\pi}-\frac{(bc-ad)\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b^2}-\frac{2(bc-ad)e^{-(a+bx)^2}\text{erfc}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{-(a+bx)^2}(a+bx)\text{erfc}(a+bx)}{b^2\sqrt{\pi}}-\frac{d\text{erfc}(a+bx)^2}{4b^2}+\frac{(bc-ad)(a+bx)\text{erfc}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{erfc}(a+bx)^2}{2b^2}$$

output
```
1/2*d/b^2/exp(2*(b*x+a)^2)/Pi-(-a*d+b*c)*2^(1/2)/Pi^(1/2)*erf(2^(1/2)*(b*x
+a))/b^2-2*(-a*d+b*c)*erfc(b*x+a)/b^2/exp((b*x+a)^2)/Pi^(1/2)-d*(b*x+a)*er
fc(b*x+a)/b^2/exp((b*x+a)^2)/Pi^(1/2)-1/4*d*erfc(b*x+a)^2/b^2+(-a*d+b*c)*(
b*x+a)*erfc(b*x+a)^2/b^2+1/2*d*(b*x+a)^2*erfc(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.59

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx$$
$$= \frac{4b(c+dx)\left(-\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)+\text{erfc}(a+bx)\left(-\frac{2e^{-(a+bx)^2}}{\sqrt{\pi}}+(a+bx)\text{erfc}(a+bx)\right)\right)+\frac{d\left(2e^{-2(a+bx)^2}+\right.}{}}{}$$

input
```
Integrate[(c + d*x)*Erfc[a + b*x]^2,x]
```

output
```
(4*b*(c + d*x)*(-(Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)]) + Erfc[a + b*x]*(-2/(
E^(a + b*x)^2*Sqrt[Pi]) + (a + b*x)*Erfc[a + b*x])) + (d*(2/E^(2*(a + b*x)
^2) + (4*Sqrt[Pi]*(a + b*x))/E^(a + b*x)^2 - 2*Pi*(a + b*x)^2 - 2*Pi*Erf[a
 + b*x] - (4*Sqrt[Pi]*(a + b*x)*Erf[a + b*x])/E^(a + b*x)^2 + 4*Pi*(a + b*
x)^2*Erf[a + b*x] + Pi*Erf[a + b*x]^2 - 2*Pi*(a + b*x)^2*Erf[a + b*x]^2 +
4*a*Sqrt[2*Pi]*Erf[Sqrt[2]*(a + b*x)] + 4*b*Sqrt[2*Pi]*x*Erf[Sqrt[2]*(a +
b*x)] + 2*Pi*(2 + Erfc[-a - b*x]*Erfc[a + b*x]) - 4*Sqrt[Pi]*(a + b*x)*Exp
IntegralE[1/2, (a + b*x)^2]))/Pi)/(4*b^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6922, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx$$
$$\Big\downarrow \ 6922$$
$$\frac{\int \left((bc-ad)\text{erfc}(a+bx)^2+d(a+bx)\text{erfc}(a+bx)^2\right)d(a+bx)}{b^2}$$
$$\Big\downarrow \ 2009$$

$$\frac{-\sqrt{\frac{2}{\pi}}(bc-ad)\mathrm{erf}\big(\sqrt{2}(a+bx)\big)+(a+bx)(bc-ad)\mathrm{erfc}(a+bx)^2-\frac{2e^{-(a+bx)^2}(bc-ad)\mathrm{erfc}(a+bx)}{\sqrt{\pi}}+\frac{1}{2}d(a+bx)^2\mathrm{erfc}(}{b^2}$$

input
```
Int[(c + d*x)*Erfc[a + b*x]^2,x]
```

output
```
(d/(2*E^(2*(a + b*x)^2)*Pi) - (b*c - a*d)*Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)
] - (2*(b*c - a*d)*Erfc[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) - (d*(a + b*x)*
Erfc[a + b*x])/(E^(a + b*x)^2*Sqrt[Pi]) - (d*Erfc[a + b*x]^2)/4 + (b*c - a
*d)*(a + b*x)*Erfc[a + b*x]^2 + (d*(a + b*x)^2*Erfc[a + b*x]^2)/2)/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6922
```
Int[Erfc[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[1/b^(m + 1)   Subst[Int[ExpandIntegrand[Erfc[x]^2, (b*c - a*d + d*x)^m, x]
, x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)\,\mathrm{erfc}\,(bx+a)^2\,dx$$

input
```
int((d*x+c)*erfc(b*x+a)^2,x)
```

output
```
int((d*x+c)*erfc(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.44

$$\int (c+dx)\mathrm{erfc}(a+bx)^2\,dx$$
$$= \frac{2\pi b^3dx^2 + 4\pi b^3cx - 4\sqrt{2}\sqrt{\pi}\sqrt{b^2}(bc-ad)\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right) - 2\pi(4abc-(2a^2+1)d)\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right. \cdots}{\cdots}$$

input
```
integrate((d*x+c)*erfc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*pi*b^3*d*x^2 + 4*pi*b^3*c*x - 4*sqrt(2)*sqrt(pi)*sqrt(b^2)*(b*c - a
*d)*erf(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 2*pi*(4*a*b*c - (2*a^2 + 1)*d)*sq
rt(b^2)*erf(sqrt(b^2)*(b*x + a)/b) + (2*pi*b^3*d*x^2 + 4*pi*b^3*c*x + pi*(
4*a*b^2*c - (2*a^2 + 1)*b*d))*erf(b*x + a)^2 + 2*b*d*e^(-2*b^2*x^2 - 4*a*b
*x - 2*a^2) - 4*sqrt(pi)*(b^2*d*x + 2*b^2*c - a*b*d - (b^2*d*x + 2*b^2*c -
 a*b*d)*erf(b*x + a))*e^(-b^2*x^2 - 2*a*b*x - a^2) - 4*(pi*b^3*d*x^2 + 2*p
i*b^3*c*x)*erf(b*x + a))/(pi*b^3)
```

Sympy [F]

$$\int (c+dx)\mathrm{erfc}(a+bx)^2\,dx = \int (c+dx)\,\mathrm{erfc}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*erfc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*erfc(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx=\int (dx+c)\,\text{erfc}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*erfc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*erfc(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx=\int (dx+c)\,\text{erfc}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*erfc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*erfc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx=\int \text{erfc}(a+b\,x)^2\,(c+d\,x)\,dx$$

input
```
int(erfc(a + b*x)^2*(c + d*x),x)
```

output
```
int(erfc(a + b*x)^2*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\text{erfc}(a+bx)^2\,dx$$
$$=\frac{-4\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,ac-4\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\text{erf}(bx+a)\,bcx+2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int \text{erf}(bx+a)^2\right.}{}$$

input
```
int((d*x+c)*erfc(b*x+a)^2,x)
```

output
```
( - 4*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a*c - 4*sqrt(p
i)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*c*x + 2*sqrt(pi)*e**(a**
2 + 2*a*b*x + b**2*x**2)*int(erf(a + b*x)**2,x)*b*c - 4*sqrt(pi)*e**(a**2
+ 2*a*b*x + b**2*x**2)*int(erf(a + b*x)*x,x)*b*d + 2*sqrt(pi)*e**(a**2 + 2
*a*b*x + b**2*x**2)*int(erf(a + b*x)**2*x,x)*b*d + 2*sqrt(pi)*e**(a**2 + 2
*a*b*x + b**2*x**2)*b*c*x + sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b*d*x
**2 - 4*c)/(2*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.140 $\int \text{erfc}(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int \text{erfc}(a+bx)^2\,dx=-\frac{\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)}{b}-\frac{2e^{-(a+bx)^2}\text{erfc}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\text{erfc}(a+bx)^2}{b}$$

output
```
-2^(1/2)/Pi^(1/2)*erf(2^(1/2)*(b*x+a))/b-2*erfc(b*x+a)/b/exp((b*x+a)^2)/Pi
^(1/2)+(b*x+a)*erfc(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \text{erfc}(a+bx)^2\,dx=\frac{-\sqrt{\frac{2}{\pi}}\text{erf}\left(\sqrt{2}(a+bx)\right)+\text{erfc}(a+bx)\left(-\frac{2e^{-(a+bx)^2}}{\sqrt{\pi}}+(a+bx)\text{erfc}(a+bx)\right)}{b}$$

input
```
Integrate[Erfc[a + b*x]^2,x]
```

output
```
(-(Sqrt[2/Pi]*Erf[Sqrt[2]*(a + b*x)]) + Erfc[a + b*x]*(-2/(E^(a + b*x)^2*S
qrt[Pi]) + (a + b*x)*Erfc[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6907, 7281, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6907}$$

$$\frac{4\int e^{-(a+bx)^2}(a+bx)\text{erfc}(a+bx)dx}{\sqrt{\pi}}+\frac{(a+bx)\text{erfc}(a+bx)^2}{b}$$

$$\Big\downarrow \text{7281}$$

$$\frac{4\int e^{-(a+bx)^2}(a+bx)\text{erfc}(a+bx)d(a+bx)}{\sqrt{\pi}b}+\frac{(a+bx)\text{erfc}(a+bx)^2}{b}$$

$$\Big\downarrow \text{6937}$$

$$\frac{4\left(-\frac{\int e^{-2(a+bx)^2}d(a+bx)}{\sqrt{\pi}}-\frac{1}{2}e^{-(a+bx)^2}\text{erfc}(a+bx)\right)}{\sqrt{\pi}b}+\frac{(a+bx)\text{erfc}(a+bx)^2}{b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{4\left(-\frac{\text{erf}\left(\sqrt{2}(a+bx)\right)}{2\sqrt{2}}-\frac{1}{2}e^{-(a+bx)^2}\text{erfc}(a+bx)\right)}{\sqrt{\pi}b}+\frac{(a+bx)\text{erfc}(a+bx)^2}{b}$$

input
```
Int[Erfc[a + b*x]^2,x]
```

output
```
((a + b*x)*Erfc[a + b*x]^2)/b + (4*(-1/2*Erf[Sqrt[2]*(a + b*x)]/Sqrt[2] -
Erfc[a + b*x]/(2*E^(a + b*x)^2)))/(b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6907
```
Int[Erfc[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(Erfc[a + b*x]^
2/b), x] + Simp[4/Sqrt[Pi]   Int[(a + b*x)*(Erfc[a + b*x]/E^(a + b*x)^2), x
], x] /; FreeQ[{a, b}, x]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{erf}(bx+a)^2(bx+a)+\frac{2\operatorname{erf}(bx+a)e^{-(bx+a)^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,(bx+a)\right)}{\sqrt{\pi}}}{b}$	59
default	$\frac{\operatorname{erf}(bx+a)^2(bx+a)+\frac{2\operatorname{erf}(bx+a)e^{-(bx+a)^2}}{\sqrt{\pi}}-\frac{\sqrt{2}\operatorname{erf}\left(\sqrt{2}\,(bx+a)\right)}{\sqrt{\pi}}}{b}$	59

input
```
int(erfc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(erf(b*x+a)^2*(b*x+a)+2*erf(b*x+a)/Pi^(1/2)*exp(-(b*x+a)^2)-1/Pi^(1/2)
*2^(1/2)*erf(2^(1/2)*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(63) = 126$.

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.99

$$\int \operatorname{erfc}(a+bx)^2\,dx = -\frac{2\,\pi b^2 x \operatorname{erf}\left(bx+a\right) - \pi b^2 x + 2\,\pi a\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - \left(\pi b^2 x + \pi ab\right)\operatorname{erf}\left(bx+a\right)^2 + \sqrt{2}\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\cdots\right.}{\pi b^2}$$

input
```
integrate(erfc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*pi*b^2*x*erf(b*x + a) - pi*b^2*x + 2*pi*a*sqrt(b^2)*erf(sqrt(b^2)*(b*x
 + a)/b) - (pi*b^2*x + pi*a*b)*erf(b*x + a)^2 + sqrt(2)*sqrt(pi)*sqrt(b^2)
*erf(sqrt(2)*sqrt(b^2)*(b*x + a)/b) - 2*sqrt(pi)*(b*erf(b*x + a) - b)*e^(-
b^2*x^2 - 2*a*b*x - a^2))/(pi*b^2)
```

Sympy [F]

$$\int \operatorname{erfc}(a+bx)^2\,dx = \int \operatorname{erfc}^2\left(a+bx\right)dx$$

input
```
integrate(erfc(b*x+a)**2,x)
```

output
```
Integral(erfc(a + b*x)**2, x)
```

Maxima [F]

$$\int \operatorname{erfc}(a+bx)^2\,dx = \int \operatorname{erfc}\left(bx+a\right)^2\,dx$$

input
```
integrate(erfc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)^2, x)
```

Giac [F]

$$\int \operatorname{erfc}(a+bx)^2\,dx = \int \operatorname{erfc}\left(bx+a\right)^2\,dx$$

input
```
integrate(erfc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(a+bx)^2\,dx = \int \operatorname{erfc}(a+b\,x)^2\,dx$$

input
```
int(erfc(a + b*x)^2,x)
```

output
```
int(erfc(a + b*x)^2, x)
```

Reduce [F]

$$\int \operatorname{erfc}(a+bx)^2\,dx$$
$$= \frac{-2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,a-2\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\operatorname{erf}(bx+a)\,bx+\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}\left(\int\operatorname{erf}(bx+a)^2\,dx\right.}{\sqrt{\pi}\,e^{b^2x^2+2abx+a^2}b}$$

input
```
int(erfc(b*x+a)^2,x)
```

output
```
( - 2*sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*a - 2*sqrt(pi)
*e**(a**2 + 2*a*b*x + b**2*x**2)*erf(a + b*x)*b*x + sqrt(pi)*e**(a**2 + 2*
a*b*x + b**2*x**2)*int(erf(a + b*x)**2,x)*b + sqrt(pi)*e**(a**2 + 2*a*b*x
+ b**2*x**2)*b*x - 2)/(sqrt(pi)*e**(a**2 + 2*a*b*x + b**2*x**2)*b)
```

3.141 $\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx=\text{Int}\left(\frac{\text{erfc}(a+bx)^2}{c+dx},x\right)$$

output
```
Defer(Int)(erfc(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx$$

input
```
Integrate[Erfc[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Erfc[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx$$

$$\Big\downarrow \ 6925$$

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}dx$$

input
```
Int[Erfc[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx+a\right)^{2}}{dx+c}dx$$

input
```
int(erfc(b*x+a)^2/(d*x+c),x)
```

output
```
int(erfc(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{erfc}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erfc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral((erf(b*x + a)^2 - 2*erf(b*x + a) + 1)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{erfc}^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(erfc(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(erfc(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{erfc}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erfc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)^2/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{erfc}\,(bx+a)^2}{dx+c} \, dx$$

input
```
integrate(erfc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{erfc}(a+b\,x)^2}{c+d\,x} \, dx$$

input
```
int(erfc(a + b*x)^2/(c + d*x),x)
```

output
```
int(erfc(a + b*x)^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{erfc}(a+bx)^2}{c+dx}\,dx = \frac{-2\left(\int \frac{\operatorname{erf}(bx+a)}{dx+c}dx\right)d + \left(\int \frac{\operatorname{erf}(bx+a)^2}{dx+c}dx\right)d + \log(dx+c)}{d}$$

input
```
int(erfc(b*x+a)^2/(d*x+c),x)
```

output
```
( - 2*int(erf(a + b*x)/(c + d*x),x)*d + int(erf(a + b*x)**2/(c + d*x),x)*d
 + log(c + d*x))/d
```

3.142 $\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\text{erfc}(a+bx)^2}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(erfc(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx$$

input
```
Integrate[Erfc[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Erfc[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 6925$$

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}dx$$

input
```
Int[Erfc[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(erfc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(erfc(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\operatorname{erfc}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((erf(b*x + a)^2 - 2*erf(b*x + a) + 1)/(d^2*x^2 + 2*c*d*x + c^2),
x)
```

Sympy [N/A]

Not integrable

Time = 11.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{erfc}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfc}^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(erfc(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(erfc(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erfc}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)^2/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\text{erfc}\,(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(erfc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfc}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\text{erfc}(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(erfc(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(erfc(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 8.94

$$\int \frac{\operatorname{erfc}(a+bx)^2}{(c+dx)^2}\,dx$$
$$= \frac{-2\left(\int \frac{\operatorname{erf}(bx+a)}{d^2x^2+2cdx+c^2}dx\right)c^2-2\left(\int \frac{\operatorname{erf}(bx+a)}{d^2x^2+2cdx+c^2}dx\right)cdx+\left(\int \frac{\operatorname{erf}(bx+a)^2}{d^2x^2+2cdx+c^2}dx\right)c^2+\left(\int \frac{\operatorname{erf}(bx+a)^2}{d^2x^2+2cdx+c^2}dx\right)cdx -}{c\,(dx+c)}$$

input
```
int(erfc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
( - 2*int(erf(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 - 2*int(erf(a
+ b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + int(erf(a + b*x)**2/(c**2 +
 2*c*d*x + d**2*x**2),x)*c**2 + int(erf(a + b*x)**2/(c**2 + 2*c*d*x + d**2
*x**2),x)*c*d*x + x)/(c*(c + d*x))
```

3.143 $\int x^2\mathbf{erfc}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int x^2\text{erfc}(d(a+b\log(cx^n)))\,dx = \frac{1}{3}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}x^3(cx^n)^{-3/n}\text{erf}\left(\frac{2abd^2-\frac{3}{n}+2b^2d^2\log(cx^n)}{2bd}\right)+\frac{1}{3}x^3\text{erfc}(d(a+b\log(cx^n)))$$

output
```
1/3*exp(1/4*(-12*a*b*d^2*n+9)/b^2/d^2/n^2)*x^3*erf(1/2*(2*a*b*d^2-3/n+2*b^
2*d^2*ln(c*x^n))/b/d)/((c*x^n)^(3/n))+1/3*x^3*erfc(d*(a+b*ln(c*x^n)))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int x^2\text{erfc}(d(a+b\log(cx^n)))\,dx = \frac{1}{3}\left(e^{\frac{3\left(\frac{\frac{3}{d^2}-4abn}{b^2}-4n\log(cx^n)\right)}{4n^2}}x^3\text{erf}\left(ad-\frac{3}{2bdn}+bd\log(cx^n)\right)+x^3\text{erfc}(d(a+b\log(cx^n)))\right)$$

input
```
Integrate[x^2*Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
(E^((3*((3/d^2 - 4*a*b*n)/b^2 - 4*n*Log[c*x^n]))/(4*n^2))*x^3*Erf[a*d - 3/
(2*b*d*n) + b*d*Log[c*x^n]] + x^3*Erfc[d*(a + b*Log[c*x^n])])/3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6956, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{erfc}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6956}$$

$$\frac{2bdn\int e^{-d^2(a+b\log(cx^n))^2}x^2dx}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2712}$$

$$\frac{2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{2-2abd^2n}dx}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2706}$$

$$\frac{2bdx^3(cx^n)^{-3/n}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2+\frac{(3-2abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2664}$$

$$\frac{2bdx^3(cx^n)^{-3/n}e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\int \exp\left(-\frac{\left(2abd^2+2b^2\log(cx^n)d^2-\frac{3}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{3\sqrt{\pi}}+\frac{1}{3}x^3\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{3}x^3(cx^n)^{-3/n}\,e^{\frac{9-12abd^2n}{4b^2d^2n^2}}\,\text{erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{3}{n}}{2bd}\right)+\frac{1}{3}x^3\text{erfc}(d(a+b\log(cx^n)))$$

input
```
Int[x^2*Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
(E^((9 - 12*a*b*d^2*n)/(4*b^2*d^2*n^2))*x^3*Erf[(2*a*b*d^2 - 3/n + 2*b^2*d
^2*Log[c*x^n])/(2*b*d)])/(3*(c*x^n)^(3/n)) + (x^3*Erfc[d*(a + b*Log[c*x^n]
)])/3
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6956
```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfc[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 + Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^2 \operatorname{erfc}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int(x^2*erfc(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*erfc(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.27

$$\int x^2\operatorname{erfc}(d(a+b\log(cx^n)))\,dx = -\frac{1}{3}\,x^3\operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)+\frac{1}{3}\,x^3 +\frac{1}{3}\,\sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n-3\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{3\left(4\,b^2d^2n\log(c)+4\,abd^2n-3\right)}{4\,b^2d^2n^2}\right)}$$

input
```
integrate(x^2*erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/3*x^3*erf(b*d*log(c*x^n) + a*d) + 1/3*x^3 + 1/3*sqrt(b^2*d^2*n^2)*erf(1
/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 3)*sqrt(b^2*
d^2*n^2)/(b^2*d^2*n^2))*e^(-3/4*(4*b^2*d^2*n*log(c) + 4*a*b*d^2*n - 3)/(b^
2*d^2*n^2))
```

Sympy [F]

$$\int x^2\text{erfc}(d(a+b\log(cx^n)))\,dx=\int x^2\,\text{erfc}\,(ad+bd\log(cx^n))\,dx$$

input
```
integrate(x**2*erfc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*erfc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2\text{erfc}(d(a+b\log(cx^n)))\,dx=\int x^2\,\text{erfc}\,((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*erfc((b*log(c*x^n) + a)*d), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int x^2\text{erfc}(d(a+b\log(cx^n)))\,dx$$
$$=-\frac{1}{3}\,x^3\,\text{erf}\,(bdn\log(x)+bd\log(c)+ad)+\frac{1}{3}\,x^3-\frac{\text{erf}\left(-bdn\log(x)-bd\log(c)-ad+\frac{3}{2\,bdn}\right)e^{\left(-\frac{3\,a}{bn}+\frac{9}{4\,b^2d^2n^2}\right)}}{3\,c^{\frac{3}{n}}}$$

input
```
integrate(x^2*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
-1/3*x^3*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/3*x^3 - 1/3*erf(-b*d*n*l
og(x) - b*d*log(c) - a*d + 3/2/(b*d*n))*e^(-3*a/(b*n) + 9/4/(b^2*d^2*n^2))
/c^(3/n)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{erfc}(d(a+b\log(cx^n)))\,dx = \int x^2\,\text{erfc}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x^2*erfc(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*erfc(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2 \text{erfc}(d(a+b\log(cx^n)))\,dx = -\left(\int \text{erf}(\log(x^n c)\,bd+ad)\,x^2dx\right)+\frac{x^3}{3}$$

input
```
int(x^2*erfc(d*(a+b*log(c*x^n))),x)
```

output
```
( - 3*int(erf(log(x**n*c)*b*d + a*d)*x**2,x) + x**3)/3
```

3.144 $\int x\text{erfc}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x\text{erfc}(d(a+b\log(cx^n)))\,dx=\frac{1}{2}e^{\frac{1-2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{-2/n}\text{erf}\left(\frac{abd^2-\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right)+\frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n)))$$

output
```
1/2*exp((-2*a*b*d^2*n+1)/b^2/d^2/n^2)*x^2*erf((a*b*d^2-1/n+b^2*d^2*ln(c*x^
n))/b/d)/((c*x^n)^(2/n))+1/2*x^2*erfc(d*(a+b*ln(c*x^n)))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int x\text{erfc}(d(a+b\log(cx^n)))\,dx=\frac{1}{2}\left(e^{\frac{\frac{\frac{1}{d^2}-2abn}{b^2}-2n\log(cx^n)}{n^2}}x^2\text{erf}\left(ad-\frac{1}{bdn}+bd\log(cx^n)\right)+x^2\text{erfc}(d(a+b\log(cx^n)))\right)$$

input
```
Integrate[x*Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
(E^(((d^(-2) - 2*a*b*n)/b^2 - 2*n*Log[c*x^n])/n^2)*x^2*Erf[a*d - 1/(b*d*n)
 + b*d*Log[c*x^n]] + x^2*Erfc[d*(a + b*Log[c*x^n])])/2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6956, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfc}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6956}$$

$$\frac{bdn\int e^{-d^2(a+b\log(cx^n))^2}xdx}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2712}$$

$$\frac{bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{1-2abd^2n}dx}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2706}$$

$$\frac{bdx^2(cx^n)^{2\left(abd^2-\frac{1}{n}\right)-2abd^2}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2+\frac{2(1-abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2664}$$

$$\frac{bdx^2e^{\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2\left(abd^2-\frac{1}{n}\right)-2abd^2}\int \exp\left(-\frac{\left(abd^2+b^2\log(cx^n)d^2-\frac{1}{n}\right)^2}{b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}}+\frac{1}{2}x^2\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}x^2e^{\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2\left(abd^2-\frac{1}{n}\right)-2abd^2}\operatorname{erf}\left(\frac{abd^2+b^2d^2\log\left(cx^n\right)-\frac{1}{n}}{bd}\right)+\frac{1}{2}x^2\operatorname{erfc}(d(a+b\log\left(cx^n\right)))$$

input
```
Int[x*Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
(E^((1 - 2*a*b*d^2*n)/(b^2*d^2*n^2))*x^2*(c*x^n)^(-2*a*b*d^2 + 2*(a*b*d^2
- n^(-1)))*Erf[(a*b*d^2 - n^(-1) + b^2*d^2*Log[c*x^n])/(b*d)])/2 + (x^2*Er
fc[d*(a + b*Log[c*x^n])])/2
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6956
```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfc[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 + Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x \operatorname{erfc}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(x*erfc(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*erfc(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int x\mathrm{erfc}(d(a+b\log\left(cx^{n}\right)))\,dx=-\frac{1}{2}\,x^{2}\operatorname{erf}\left(bd\log\left(cx^{n}\right)+ad\right)+\frac{1}{2}\,\sqrt{b^{2}d^{2}n^{2}}\operatorname{erf}\left(\frac{\left(b^{2}d^{2}n^{2}\log\left(x\right)+b^{2}d^{2}n\log\left(c\right)+abd^{2}n-1\right)\sqrt{b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n-1}{b^{2}d^{2}n^{2}}\right)}+\frac{1}{2}\,x^{2}$$

input
```
integrate(x*erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/2*x^2*erf(b*d*log(c*x^n) + a*d) + 1/2*sqrt(b^2*d^2*n^2)*erf((b^2*d^2*n^
2*log(x) + b^2*d^2*n*log(c) + a*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^
2))*e^(-(2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - 1)/(b^2*d^2*n^2)) + 1/2*x^2
```

Sympy [F]

$$\int x\mathrm{erfc}(d(a+b\log{(cx^n)}))\,dx = \int x\,\mathrm{erfc}\,(ad+bd\log{(cx^n)})\,dx$$

input
```
integrate(x*erfc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*erfc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\mathrm{erfc}(d(a+b\log{(cx^n)}))\,dx = \int x\,\mathrm{erfc}\,((b\log{(cx^n)}+a)d)\,dx$$

input
```
integrate(x*erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*erfc((b*log(c*x^n) + a)*d), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int x\mathrm{erfc}(d(a+b\log{(cx^n)}))\,dx\\&= -\frac{1}{2}\,x^2\,\mathrm{erf}\,(bdn\log{(x)}+bd\log{(c)}+ad)+\frac{1}{2}\,x^2\\&\quad-\frac{\mathrm{erf}\,\left(-bdn\log{(x)}-bd\log{(c)}-ad+\frac{1}{bdn}\right)e^{\left(-\frac{2\,a}{bn}+\frac{1}{b^2d^2n^2}\right)}}{2\,c^{\frac{2}{n}}}\end{aligned}$$

input
```
integrate(x*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
-1/2*x^2*erf(b*d*n*log(x) + b*d*log(c) + a*d) + 1/2*x^2 - 1/2*erf(-b*d*n*l
og(x) - b*d*log(c) - a*d + 1/(b*d*n))*e^(-2*a/(b*n) + 1/(b^2*d^2*n^2))/c^(
2/n)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{erfc}(d(a+b\log(cx^n)))\,dx=\int x\,\mathrm{erfc}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x*erfc(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*erfc(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\mathrm{erfc}(d(a+b\log(cx^n)))\,dx=-\left(\int\mathrm{erf}(\log(x^nc)\,bd+ad)\,xdx\right)+\frac{x^2}{2}$$

input
```
int(x*erfc(d*(a+b*log(c*x^n))),x)
```

output
```
( - 2*int(erf(log(x**n*c)*b*d + a*d)*x,x) + x**2)/2
```

3.145 $\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = e^{\frac{1-4abd^2n}{4b^2d^2n^2}}x(cx^n)^{-1/n}\operatorname{erf}\left(\frac{2abd^2-\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right) + x\operatorname{erfc}(d(a+b\log(cx^n)))$$

output
```
exp(1/4*(-4*a*b*d^2*n+1)/b^2/d^2/n^2)*x*erf(1/2*(2*a*b*d^2-1/n+2*b^2*d^2*l
n(c*x^n))/b/d)/((c*x^n)^(1/n))+x*erfc(d*(a+b*ln(c*x^n)))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = e^{\frac{\frac{\frac{1}{d^2}-4abn}{b^2}-4n\log(cx^n)}{4n^2}}x\operatorname{erf}\left(ad-\frac{1}{2bdn}+bd\log(cx^n)\right) + x\operatorname{erfc}(d(a+b\log(cx^n)))$$

input
```
Integrate[Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
E^(((d^(-2) - 4*a*b*n)/b^2 - 4*n*Log[c*x^n])/(4*n^2))*x*Erf[a*d - 1/(2*b*d
*n) + b*d*Log[c*x^n]] + x*Erfc[d*(a + b*Log[c*x^n])]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6952, 2710, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{ 6952}$$

$$\frac{2bdn\int e^{-d^2(a+b\log(cx^n))^2}dx}{\sqrt{\pi}}+x\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{ 2710}$$

$$\frac{2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{-2abd^2n}dx}{\sqrt{\pi}}+x\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{ 2706}$$

$$\frac{2bdx(cx^n)^{-1/n}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2+\frac{(1-2abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}}+ x\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{2bdx(cx^n)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\int \exp\left(-\frac{(2abd^2+2b^2\log(cx^n)d^2-\frac{1}{n})^2}{4b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}}+ x\text{erfc}(d(a+b\log(cx^n)))$$

$$\Big\downarrow \text{ 2634}$$

$$x(cx^n)^{-1/n}e^{\frac{1-4abd^2n}{4b^2d^2n^2}}\text{erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)+x\text{erfc}(d(a+b\log(cx^n)))$$

input
```
Int[Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
(E^((1 - 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*x*Erf[(2*a*b*d^2 - n^(-1) + 2*b^2*d
^2*Log[c*x^n])/(2*b*d)])/(c*x^n)^n^(-1) + x*Erfc[d*(a + b*Log[c*x^n])]
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 6952
```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*
Erfc[d*(a + b*Log[c*x^n])], x] + Simp[2*b*d*(n/Sqrt[Pi])   Int[1/E^(d*(a +
b*Log[c*x^n]))^2, x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Maple [F]

$$\int \operatorname{erfc}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(erfc(d*(a+b*ln(c*x^n))),x)
```

output
```
int(erfc(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = \sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n-1)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{4\,b^2d^2n\log(c)+4\,abd^2n-1}{4\,b^2d^2n^2}\right)} - x\operatorname{erf}\left(bd\log(cx^n)+ad\right)+x$$

input
```
integrate(erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
sqrt(b^2*d^2*n^2)*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a
*b*d^2*n - 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(-1/4*(4*b^2*d^2*n*log(c)
 + 4*a*b*d^2*n - 1)/(b^2*d^2*n^2)) - x*erf(b*d*log(c*x^n) + a*d) + x
```

Sympy [F]

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfc}\left(d(a+b\log(cx^n))\right)dx$$

input
```
integrate(erfc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(erfc(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfc}\left((b\log(cx^n)+a)d\right)\,dx$$

input
```
integrate(erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(erfc((b*log(c*x^n) + a)*d), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = -x\operatorname{erf}\left(bdn\log(x)+bd\log(c)+ad\right)+x - \frac{\operatorname{erf}\left(-bdn\log(x)-bd\log(c)-ad+\frac{1}{2\,bdn}\right)e^{\left(-\frac{a}{bn}+\frac{1}{4\,b^2d^2n^2}\right)}}{c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
-x*erf(b*d*n*log(x) + b*d*log(c) + a*d) + x - erf(-b*d*n*log(x) - b*d*log(
c) - a*d + 1/2/(b*d*n))*e^(-a/(b*n) + 1/4/(b^2*d^2*n^2))/c^(1/n)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfc}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(erfc(d*(a + b*log(c*x^n))),x)
```

output
```
int(erfc(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \operatorname{erfc}(d(a+b\log(cx^n)))\, dx = -\left(\int \operatorname{erf}(\log(x^n c)\, bd + ad)\, dx\right) + x$$

input
```
int(erfc(d*(a+b*log(c*x^n))),x)
```

output
```
- int(erf(log(x**n*c)*b*d + a*d),x) + x
```

3.146 $\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x}\,dx = -\frac{e^{-d^2(a+b\log(cx^n))^2}}{bdn\sqrt{\pi}} + \frac{\text{erfc}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bn}$$

output
```
-1/b/d/exp(d^2*(a+b*ln(c*x^n))^2)/n/Pi^(1/2)+erfc(d*(a+b*ln(c*x^n)))*(a+b*
ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x}\,dx = \frac{-\frac{e^{-d^2\left(a^2+b^2\log^2(cx^n)\right)}(cx^n)^{-2abd^2}}{bd\sqrt{\pi}} - \frac{a\text{erf}(d(a+b\log(cx^n)))}{b} + \text{erfc}(d(a+b\log(cx^n)))\log(cx^n)}{n}$$

input
```
Integrate[Erfc[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-(1/(b*d*E^(d^2*(a^2 + b^2*Log[c*x^n]^2))*Sqrt[Pi]*(c*x^n)^(2*a*b*d^2)))
- (a*Erf[d*(a + b*Log[c*x^n])])/b + Erfc[d*(a + b*Log[c*x^n])]*Log[c*x^n])
/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3039, 7281, 6904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{erfc}(d(a+b\log(cx^n)))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \operatorname{erfc}(ad+b\log(cx^n)\,d)\,d(ad+b\log(cx^n)\,d)}{bdn}$$

$$\Big\downarrow \text{6904}$$

$$\frac{(ad+bd\log(cx^n))\operatorname{erfc}(ad+bd\log(cx^n)) - \frac{e^{-(ad+bd\log(cx^n))^2}}{\sqrt{\pi}}}{bdn}$$

input
```
Int[Erfc[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-(1/(E^(a*d + b*d*Log[c*x^n])^2*Sqrt[Pi])) + Erfc[a*d + b*d*Log[c*x^n]]*(
a*d + b*d*Log[c*x^n]))/(b*d*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 6904
```
Int[Erfc[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erfc[a + b*x]/b)
, x] - Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{(ad+bd\ln(c\,x^{n}))\operatorname{erfc}(ad+bd\ln(c\,x^{n}))-\frac{e^{-(ad+bd\ln(c\,x^{n}))^{2}}}{\sqrt{\pi}}}{nbd}$
default	$\frac{(ad+bd\ln(c\,x^{n}))\operatorname{erfc}(ad+bd\ln(c\,x^{n}))-\frac{e^{-(ad+bd\ln(c\,x^{n}))^{2}}}{\sqrt{\pi}}}{nbd}$
parts	$\ln(x)\operatorname{erfc}(d(a+b\ln(c\,x^{n})))+\frac{2dbn\left(-\frac{e^{-\ln(x)^{2}b^{2}d^{2}n^{2}-2d^{2}(b(\ln(c\,x^{n})-n\ln(x))+a)bn\ln(x)-d^{2}(b(\ln(c\,x^{n})-}}{2b^{2}d^{2}n^{2}}}{}$

input
```
int(erfc(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b/d*((a*d+b*d*ln(c*x^n))*erfc(a*d+b*d*ln(c*x^n))-1/Pi^(1/2)*exp(-(a*d+
b*d*ln(c*x^n))^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{erfc}(d(a + b\log(cx^n)))}{x}\,dx$$
$$= \frac{\pi bdn\log(x) - (\pi bdn\log(x) + \pi bd\log(c) + \pi ad)\operatorname{erf}(bd\log(cx^n) + ad) - \sqrt{\pi}e^{\left(-b^2d^2n^2\log(x)^2 - b^2d^2\log(c)^2 - 2\right.}}{\pi bdn}$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
(pi*b*d*n*log(x) - (pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*erf(b*d*log(
c*x^n) + a*d) - sqrt(pi)*e^(-b^2*d^2*n^2*log(x)^2 - b^2*d^2*log(c)^2 - 2*a
*b*d^2*log(c) - a^2*d^2 - 2*(b^2*d^2*n*log(c) + a*b*d^2*n)*log(x)))/(pi*b*
d*n)
```

Sympy [F]

$$\int \frac{\operatorname{erfc}(d(a + b\log(cx^n)))}{x}\,dx = \int \frac{\operatorname{erfc}(ad + bd\log(cx^n))}{x}\,dx$$

input
```
integrate(erfc(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(erfc(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x}\,dx$$
$$= \frac{(b\log(cx^n)+a)d\operatorname{erfc}((b\log(cx^n)+a)d) - \frac{e^{\left(-(b\log(cx^n)+a)^2d^2\right)}}{\sqrt{\pi}}}{bdn}$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
((b*log(c*x^n) + a)*d*erfc((b*log(c*x^n) + a)*d) - e^(-(b*log(c*x^n) + a)^
2*d^2)/sqrt(pi))/(b*d*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x}\,dx$$
$$= \frac{bdn\log(x)+bd\log(c)+ad-(bdn\log(x)+bd\log(c)+ad)\operatorname{erf}(bdn\log(x)+bd\log(c)+ad) - \frac{e^{\left(-(bdn\,l\right.}}{}}{bdn}$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
(b*d*n*log(x) + b*d*log(c) + a*d - (b*d*n*log(x) + b*d*log(c) + a*d)*erf(b
*d*n*log(x) + b*d*log(c) + a*d) - e^(-(b*d*n*log(x) + b*d*log(c) + a*d)^2)
/sqrt(pi))/(b*d*n)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\operatorname{erfc}(d\,(a+b\,\ln(c\,x^n)))\,\ln(c\,x^n)}{n} + \frac{a\,\operatorname{erfc}(d\,(a+b\,\ln(c\,x^n)))}{b\,n} - \frac{e^{-b^2\,d^2\,\ln(c\,x^n)^2}\,e^{-a^2\,d^2}}{b\,d\,n\,\sqrt{\pi}\,(c\,x^n)^{2\,a\,b\,d^2}}$$

input
```
int(erfc(d*(a + b*log(c*x^n)))/x,x)
```

output
```
(erfc(d*(a + b*log(c*x^n)))*log(c*x^n))/n + (a*erfc(d*(a + b*log(c*x^n))))
/(b*n) - (exp(-b^2*d^2*log(c*x^n)^2)*exp(-a^2*d^2))/(b*d*n*pi^(1/2)*(c*x^n
)^(2*a*b*d^2))
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x}\,dx = -\left(\int \frac{\operatorname{erf}(\log(x^n c)\,bd+ad)}{x}dx\right)+\log(x)$$

input
```
int(erfc(d*(a+b*log(c*x^n)))/x,x)
```

output
```
- int(erf(log(x**n*c)*b*d + a*d)/x,x) + log(x)
```

3.147 $\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{e^{\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\text{erf}\left(\frac{2abd^2+\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)}{x} - \frac{\text{erfc}(d(a+b\log(cx^n)))}{x}$$

output
```
-exp(1/4/b^2/d^2/n^2+a/b/n)*(c*x^n)^(1/n)*erf(1/2*(2*a*b*d^2+1/n+2*b^2*d^2
*ln(c*x^n))/b/d)/x-erfc(d*(a+b*ln(c*x^n)))/x
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{e^{\frac{1+4abd^2n}{4b^2d^2n^2}}(cx^n)^{\frac{1}{n}}\,\text{erf}\left(ad+\frac{1}{2bdn}+bd\log(cx^n)\right)+\text{erfc}(d(a+b\log(cx^n)))}{x}$$

input
```
Integrate[Erfc[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-((E^((1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x^n)^n^(-1)*Erf[a*d + 1/(2*b*d
*n) + b*d*Log[c*x^n]] + Erfc[d*(a + b*Log[c*x^n])])/x)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6956, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x^2}dx$$

$$\Big\downarrow \text{6956}$$

$$-\frac{2bdn\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^2}dx}{\sqrt{\pi}}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2712}$$

$$-\frac{2bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{-2abnd^2-2}dx}{\sqrt{\pi}}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2706}$$

$$-\frac{2bd(cx^n)^{\frac{1}{n}}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2-\frac{(2abnd^2+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}x}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{2bd(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}}\int \exp\left(-\frac{\left(2abd^2+2b^2\log(cx^n)d^2+\frac{1}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}x}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}+\frac{1}{4b^2d^2n^2}}\text{erf}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)}{x}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{x}$$

input
```
Int[Erfc[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-((E^(1/(4*b^2*d^2*n^2) + a/(b*n))*(c*x^n)^n^(-1)*Erf[(2*a*b*d^2 + n^(-1)
+ 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x) - Erfc[d*(a + b*Log[c*x^n])]/x
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6956
```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfc[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 + Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(erfc(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(erfc(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{erfc}(d(a+b\log\left(cx^{n}\right)))}{x^{2}}\,dx = -\frac{\sqrt{b^{2}d^{2}n^{2}}x\operatorname{erf}\left(\frac{\left(2\,b^{2}d^{2}n^{2}\log(x)+2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n+1\right)\sqrt{b^{2}d^{2}n^{2}}}{2\,b^{2}d^{2}n^{2}}\right)e^{\left(\frac{4\,b^{2}d^{2}n\log(c)+4\,abd^{2}n+1}{4\,b^{2}d^{2}n^{2}}\right)}-\operatorname{erf}\left(bd\log\left(cx^{n}\right)+a\right.}{x}$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
-(sqrt(b^2*d^2*n^2)*x*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) +
 2*a*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*n*log
(c) + 4*a*b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d) + 1)/x
```

Sympy [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log\left(cx^{n}\right)))}{x^{2}}\,dx = \int \frac{\operatorname{erfc}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

input
```
integrate(erfc(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(erfc(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erfc}\left((b\log(cx^n)+a)d\right)}{x^2}\,dx$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(erfc((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erfc}\left((b\log(cx^n)+a)d\right)}{x^2}\,dx$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(erfc((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erfc}(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

input
```
int(erfc(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(erfc(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(d(a + b\log(cx^n)))}{x^2}\,dx = \frac{-\left(\int \frac{\operatorname{erf}(\log(x^n c)bd + ad)}{x^2}dx\right)x - 1}{x}$$

input
```
int(erfc(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
( - (int(erf(log(x**n*c)*b*d + a*d)/x**2,x)*x + 1))/x
```

3.148 $\int \frac{\mathrm{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{\mathrm{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\,\mathrm{erf}\left(\frac{1+abd^2n+b^2d^2n\log(cx^n)}{bdn}\right)}{2x^2} - \frac{\mathrm{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

output
```
-1/2*exp((2*a*b*d^2*n+1)/b^2/d^2/n^2)*(c*x^n)^(2/n)*erf((1+a*b*d^2*n+b^2*d
^2*n*ln(c*x^n))/b/d/n)/x^2-1/2*erfc(d*(a+b*ln(c*x^n)))/x^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{\mathrm{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\,\mathrm{erf}\left(ad+\frac{1}{bdn}+bd\log(cx^n)\right)+\mathrm{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Integrate[Erfc[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*(E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(2/n)*Erf[a*d + 1/(b*d*n
) + b*d*Log[c*x^n]] + Erfc[d*(a + b*Log[c*x^n])])/x^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6956, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(d(a+b\log(cx^n)))}{x^3}dx$$

$$\Big\downarrow \text{6956}$$

$$-\frac{bdn\int \frac{e^{-d^2(a+b\log(cx^n))^2}}{x^3}dx}{\sqrt{\pi}}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2712}$$

$$-\frac{bdnx^{2abd^2n}(cx^n)^{-2abd^2}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2}x^{-2abnd^2-3}dx}{\sqrt{\pi}}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2706}$$

$$-\frac{bd(cx^n)^{2\left(abd^2+\frac{1}{n}\right)-2abd^2}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)\,d^2-\frac{2(abnd^2+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}x^2}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{bde^{\frac{2abd^2n+1}{b^2d^2n^2}}(cx^n)^{2\left(abd^2+\frac{1}{n}\right)-2abd^2}\int \exp\left(-\frac{(abnd^2+b^2n\log(cx^n)d^2+1)^2}{b^2d^2n^2}\right)d\log(cx^n)}{\sqrt{\pi}x^2}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{e^{\frac{2abd^2n+1}{b^2d^2n^2}}(cx^n)^{2\left(abd^2+\frac{1}{n}\right)-2abd^2}\text{erf}\left(\frac{abd^2n+b^2d^2n\log(cx^n)+1}{bdn}\right)}{2x^2}-\frac{\text{erfc}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Int[Erfc[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*(E^((1 + 2*a*b*d^2*n)/(b^2*d^2*n^2))*(c*x^n)^(-2*a*b*d^2 + 2*(a*b*d^2
 + n^(-1)))*Erf[(1 + a*b*d^2*n + b^2*d^2*n*Log[c*x^n])/(b*d*n)])/x^2 - Erf
c[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6956
```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfc[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 + Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(erfc(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(erfc(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\sqrt{b^2d^2n^2}x^2\operatorname{erf}\left(\frac{(b^2d^2n^2\log(x)+b^2d^2n\log(c)+abd^2n+1)\sqrt{b^2d^2n^2}}{b^2d^2n^2}\right)e^{\left(\frac{2\,b^2d^2n\log(c)+2\,abd^2n+1}{b^2d^2n^2}\right)}-\operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)}{2\,x^2}$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b^2*d^2*n^2)*x^2*erf((b^2*d^2*n^2*log(x) + b^2*d^2*n*log(c) + a
*b*d^2*n + 1)*sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^((2*b^2*d^2*n*log(c) + 2*
a*b*d^2*n + 1)/(b^2*d^2*n^2)) - erf(b*d*log(c*x^n) + a*d) + 1)/x^2
```

Sympy [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erfc}\left(ad+bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

input
```
integrate(erfc(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(erfc(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erfc}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(erfc((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erfc}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(erfc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(erfc((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erfc}(d\,(a+b\,\ln(c\,x^n)))}{x^3}\,dx$$

input
```
int(erfc(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(erfc(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfc}(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{-2\left(\int \frac{\operatorname{erf}(\log(x^n c)bd+ad)}{x^3}dx\right)x^2-1}{2x^2}$$

input

```
int(erfc(d*(a+b*log(c*x^n)))/x^3,x)
```

output

```
( - 2*int(erf(log(x**n*c)*b*d + a*d)/x**3,x)*x**2 - 1)/(2*x**2)
```

3.149 $\int (ex)^m \mathbf{erfc}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = -\frac{e^{\frac{(1+m)\left(1+m-4abd^2n\right)}{4b^2d^2n^2}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\text{erf}\left(\frac{1+m-2abd^2n-2b^2d^2n\log(cx^n)}{2bdn}\right)}{1+m}+\frac{(ex)^{1+m}\text{erfc}(d(a+b\log(cx^n)))}{e(1+m)}$$

output
```
-exp(1/4*(1+m)*(-4*a*b*d^2*n+m+1)/b^2/d^2/n^2)*x*(e*x)^m*erf(1/2*(1+m-2*a*
b*d^2*n-2*b^2*d^2*n*ln(c*x^n))/b/d/n)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m
)*erfc(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^m\left(e^{\frac{(1+m)\left(1+m-4abd^2n+4b^2d^2n^2\log(x)-4b^2d^2n\log(cx^n)\right)}{4b^2d^2n^2}}x^{-m}\text{erf}\left(ad-\frac{1+m-2b^2d^2n\log(cx^n)}{2bdn}\right)+x\text{erfc}(d(a+b\log(cx\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*((E^(((1 + m)*(1 + m - 4*a*b*d^2*n + 4*b^2*d^2*n^2*Log[x] - 4*b^2
*d^2*n*Log[c*x^n]))/(4*b^2*d^2*n^2))*Erf[a*d - (1 + m - 2*b^2*d^2*n*Log[c*
x^n])/(2*b*d*n)])/x^m + x*Erfc[d*(a + b*Log[c*x^n])]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6956, 2712, 2706, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{ 6956}$$

$$\frac{2bdn\int e^{-d^2(a+b\log(cx^n))^2}(ex)^m dx}{\sqrt{\pi}(m+1)}+\frac{(ex)^{m+1}\text{erfc}(d(a+b\log(cx^n)))}{e(m+1)}$$

$$\Big\downarrow \text{ 2712}$$

$$\frac{2bdn(ex)^m (cx^n)^{-2abd^2} x^{2abd^2n-m}\int e^{-a^2d^2-b^2\log^2(cx^n)d^2} x^{m-2abd^2n}dx}{\sqrt{\pi}(m+1)}+\frac{(ex)^{m+1}\text{erfc}(d(a+b\log(cx^n)))}{e(m+1)}$$

$$\Big\downarrow \text{ 2706}$$

$$\frac{2bdx(ex)^m (cx^n)^{-\frac{-2abd^2n+m+1}{n}-2abd^2}\int \exp\left(-a^2d^2-b^2\log^2(cx^n)d^2+\frac{(-2abnd^2+m+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}(m+1)}+\frac{(ex)^{m+1}\text{erfc}(d(a+b\log(cx^n)))}{e(m+1)}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{2bdx(ex)^m \exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right)(cx^n)^{-\frac{-2abd^2n+m+1}{n}-2abd^2}\int \exp\left(-\frac{(-2abnd^2-2b^2n\log(cx^n)d^2+m+1)^2}{4b^2d^2n^2}\right)d\log(cx}{\sqrt{\pi}(m+1)}$$
$$\frac{(ex)^{m+1}\text{erfc}(d(a+b\log(cx^n)))}{e(m+1)}$$

$\Big\downarrow$ 2634

$$\frac{(ex)^{m+1}\text{erfc}(d(a+b\log(cx^n)))}{e(m+1)} - \frac{x(ex)^m \exp\left(\frac{(m+1)(-4abd^2n+m+1)}{4b^2d^2n^2}\right)(cx^n)^{-\frac{-2abd^2n+m+1}{n}-2abd^2}\text{erf}\left(\frac{-2abd^2n-2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}$$

input
```
Int[(e*x)^m*Erfc[d*(a + b*Log[c*x^n])],x]
```

output
```
-((E^(((1 + m)*(1 + m - 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*x*(e*x)^m*(c*x^n)^(
-2*a*b*d^2 - (1 + m - 2*a*b*d^2*n)/n)*Erf[(1 + m - 2*a*b*d^2*n - 2*b^2*d^2
*n*Log[c*x^n])/(2*b*d*n)])/(1 + m)) + ((e*x)^(1 + m)*Erfc[d*(a + b*Log[c*x
^n])])/(e*(1 + m))
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6956
```
Int[Erfc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfc[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 + Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m/E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m \operatorname{erfc}\left(d(a+b\ln\left(c\,x^n\right))\right) dx$$

input
```
int((e*x)^m*erfc(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*erfc(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.54

$$\int (ex)^m \operatorname{erfc}(d(a+b\log(cx^n)))\,dx =$$

$$-\frac{x \operatorname{erf}\left(bd\log\left(cx^n\right)+ad\right)e^{(m\log(e)+m\log(x))}-\sqrt{b^2d^2n^2}\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n-m-1\right)\sqrt{b^2d^2n^2}}{2\,b^2d^2n^2}\right)}{m+1}$$

input
```
integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-(x*erf(b*d*log(c*x^n) + a*d)*e^(m*log(e) + m*log(x)) - sqrt(b^2*d^2*n^2)*
erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n - m - 1)*
sqrt(b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*m*n^2*log(e) - 4*(b^2*d
^2*m + b^2*d^2)*n*log(c) + m^2 - 4*(a*b*d^2*m + a*b*d^2)*n + 2*m + 1)/(b^2
*d^2*n^2)) - x*e^(m*log(e) + m*log(x)))/(m + 1)
```

Sympy [F]

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{erfc}(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*erfc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*erfc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{erfc}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*erfc((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \int (ex)^m\,\text{erfc}\left((b\log(cx^n)+a)d\right)\,dx$$

input
```
integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*erfc((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \int \text{erfc}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(erfc(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(erfc(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \text{erfc}(d(a+b\log(cx^n)))\,dx = \frac{e^m\left(x^m x-\left(\int x^m \text{erf}(\log(x^n c)\,bd+ad)\,dx\right)m-\left(\int x^m \text{erf}(\log(x^n c)\,bd+ad)\,dx\right)\right)}{m+1}$$

input
```
int((e*x)^m*erfc(d*(a+b*log(c*x^n))),x)
```

output
```
(e**m*(x**m*x - int(x**m*erf(log(x**n*c)*b*d + a*d),x)*m - int(x**m*erf(lo
g(x**n*c)*b*d + a*d),x)))/(m + 1)
```

3.150 $\int e^{c-b^2x^2}\mathbf{erfc}(bx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx=-\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^3}{6b}$$

output
```
-1/6*exp(c)*Pi^(1/2)*erfc(b*x)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx=-\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^3}{6b}$$

input
```
Integrate[E^(c - b^2*x^2)*Erfc[b*x]^2,x]
```

output
```
-1/6*(E^c*Sqrt[Pi]*Erfc[b*x]^3)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-b^2x^2}\text{erfc}(bx)^2\,dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{\sqrt{\pi}e^c\int \text{erfc}(bx)^2d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^3}{6b}$$

input
```
Int[E^(c - b^2*x^2)*Erfc[b*x]^2,x]
```

output
```
-1/6*(E^c*Sqrt[Pi]*Erfc[b*x]^3)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\frac{e^{c}\sqrt{\pi}\,\operatorname{erf}(bx)}{2}-\frac{\sqrt{\pi}\,e^{c}\operatorname{erf}(bx)^{2}}{2}+\frac{\sqrt{\pi}\,e^{c}\operatorname{erf}(bx)^{3}}{6}}{b}$	43

input
```
int(exp(-b^2*x^2+c)*erfc(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*exp(c)*Pi^(1/2)*erf(b*x)-1/2*Pi^(1/2)*exp(c)*erf(b*x)^2+1/6*Pi^(1/2)*
exp(c)*erf(b*x)^3)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx = \frac{\sqrt{\pi}\left(\operatorname{erf}(bx)^3-3\operatorname{erf}(bx)^2+3\operatorname{erf}(bx)\right)e^c}{6\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x)^2,x, algorithm="fricas")
```

output
```
1/6*sqrt(pi)*(erf(b*x)^3 - 3*erf(b*x)^2 + 3*erf(b*x))*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx = \begin{cases} -\frac{\sqrt{\pi}e^c\operatorname{erfc}^3(bx)}{6b} & \text{for } b\neq 0 \\ xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)*erfc(b*x)**2,x)
```

output
```
Piecewise((-sqrt(pi)*exp(c)*erfc(b*x)**3/(6*b), Ne(b, 0)), (x*exp(c), True
))
```

Maxima [F]

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx=\int \operatorname{erfc}\left(bx\right)^2e^{\left(-b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^2*e^(-b^2*x^2 + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx=\frac{\sqrt{\pi}\operatorname{erf}\left(bx\right)^3e^c-3\sqrt{\pi}\operatorname{erf}\left(bx\right)^2e^c+3\sqrt{\pi}\operatorname{erf}\left(bx\right)e^c}{6\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x)^2,x, algorithm="giac")
```

output
```
1/6*(sqrt(pi)*erf(b*x)^3*e^c - 3*sqrt(pi)*erf(b*x)^2*e^c + 3*sqrt(pi)*erf(
b*x)*e^c)/b
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx = -\frac{\sqrt{\pi}\,\mathrm{e}^c\,\operatorname{erfc}(b\,x)^3}{6\,b}$$

input
```
int(exp(c - b^2*x^2)*erfc(b*x)^2,x)
```

output
```
-(pi^(1/2)*exp(c)*erfc(b*x)^3)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^2\,dx = \frac{\sqrt{\pi}\,e^c\operatorname{erf}(bx)\left(\operatorname{erf}(bx)^2-3\operatorname{erf}(bx)+3\right)}{6b}$$

input
```
int(exp(-b^2*x^2+c)*erfc(b*x)^2,x)
```

output
```
(sqrt(pi)*e**c*erf(b*x)*(erf(b*x)**2 - 3*erf(b*x) + 3))/(6*b)
```

3.151 $\int e^{c-b^2x^2}\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx=-\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^2}{4b}$$

output
```
-1/4*exp(c)*Pi^(1/2)*erfc(b*x)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx=-\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^2}{4b}$$

input
```
Integrate[E^(c - b^2*x^2)*Erfc[b*x],x]
```

output
```
-1/4*(E^c*Sqrt[Pi]*Erfc[b*x]^2)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-b^2x^2}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{\sqrt{\pi}e^c\int \text{erfc}(bx)d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^2}{4b}$$

input
```
Int[E^(c - b^2*x^2)*Erfc[b*x],x]
```

output
```
-1/4*(E^c*Sqrt[Pi]*Erfc[b*x]^2)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{\frac{e^{c}\sqrt{\pi}\,\operatorname{erf}(bx)}{2}-\frac{\sqrt{\pi}\,e^{c}\operatorname{erf}(bx)^{2}}{4}}{b}$	30

input
```
int(exp(-b^2*x^2+c)*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/2*exp(c)*Pi^(1/2)*erf(b*x)-1/4*Pi^(1/2)*exp(c)*erf(b*x)^2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx = -\frac{\sqrt{\pi}\left(\operatorname{erf}\left(bx\right)^2-2\operatorname{erf}\left(bx\right)\right)e^c}{4\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

output
```
-1/4*sqrt(pi)*(erf(b*x)^2 - 2*erf(b*x))*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx = \begin{cases} -\frac{\sqrt{\pi}e^c\operatorname{erfc}^2(bx)}{4b} & \text{for } b\neq 0 \\ xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)*erfc(b*x),x)
```

output
```
Piecewise((-sqrt(pi)*exp(c)*erfc(b*x)**2/(4*b), Ne(b, 0)), (x*exp(c), True
))
```

Maxima [F]

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx=\int \operatorname{erfc}(bx)\,e^{(-b^2x^2+c)}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2 + c), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx=-\frac{\sqrt{\pi}\operatorname{erf}(bx)^2\,e^c-2\sqrt{\pi}\operatorname{erf}(bx)\,e^c}{4\,b}$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x),x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*erf(b*x)^2*e^c - 2*sqrt(pi)*erf(b*x)*e^c)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx=-\frac{\sqrt{\pi}\,\mathrm{e}^c\operatorname{erfc}(b\,x)^2}{4\,b}$$

input
```
int(exp(c - b^2*x^2)*erfc(b*x),x)
```

output
```
-(pi^(1/2)*exp(c)*erfc(b*x)^2)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)\,dx=\frac{\sqrt{\pi}\,e^c\operatorname{erf}(bx)\,(-\operatorname{erf}(bx)+2)}{4b}$$

input
```
int(exp(-b^2*x^2+c)*erfc(b*x),x)
```

output
```
(sqrt(pi)*e**c*erf(b*x)*( - erf(b*x) + 2))/(4*b)
```

3.152 $\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)}\,dx = -\frac{e^c\sqrt{\pi}\log(\text{erfc}(bx))}{2b}$$

output
```
-1/2*exp(c)*Pi^(1/2)*ln(erfc(b*x))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)}\,dx = -\frac{e^c\sqrt{\pi}\log(\text{erfc}(bx))}{2b}$$

input
```
Integrate[E^(c - b^2*x^2)/Erfc[b*x],x]
```

output
```
-1/2*(E^c*Sqrt[Pi]*Log[Erfc[b*x]])/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6928, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)}\,dx$$

$$\Big\downarrow \; 6928$$

$$-\frac{\sqrt{\pi}e^c\int \frac{1}{\text{erfc}(bx)}d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \; 14$$

$$-\frac{\sqrt{\pi}e^c\log(\text{erfc}(bx))}{2b}$$

input
```
Int[E^(c - b^2*x^2)/Erfc[b*x],x]
```

output
```
-1/2*(E^c*Sqrt[Pi]*Log[Erfc[b*x]])/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int \frac{e^{-b^{2} x^{2}+c}}{\operatorname{erfc}\left(b x\right)}dx$$

input
```
int(exp(-b^2*x^2+c)/erfc(b*x),x)
```

output
```
int(exp(-b^2*x^2+c)/erfc(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\, dx = -\frac{\sqrt{\pi}e^{c} \log\left(\operatorname{erf}\left(bx\right) - 1\right)}{2\, b}$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*e^c*log(erf(b*x) - 1)/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\, dx = \begin{cases} -\frac{\sqrt{\pi}e^{c} \log\left(\operatorname{erfc}\left(bx\right)\right)}{2b} & \text{for}\: b \neq 0 \\ xe^{c} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)/erfc(b*x),x)
```

output
```
Piecewise((-sqrt(pi)*exp(c)*log(erfc(b*x))/(2*b), Ne(b, 0)), (x*exp(c), Tr
ue))
```

Maxima [F]

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\,dx = \int \frac{e^{\left(-b^2x^2+c\right)}}{\operatorname{erfc}\left(bx\right)}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(e^(-b^2*x^2 + c)/erfc(b*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\,dx = -\frac{\sqrt{\pi}e^c \log\left(\left|\operatorname{erf}\left(bx\right)-1\right|\right)}{2\,b}$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*e^c*log(abs(erf(b*x) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\,dx = -\frac{\sqrt{\pi}\,\ln\left(\operatorname{erfc}(b\,x)\right)\,\mathrm{e}^c}{2\,b}$$

input
```
int(exp(c - b^2*x^2)/erfc(b*x),x)
```

output
```
-(pi^(1/2)*log(erfc(b*x))*exp(c))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)}\,dx = -\frac{\sqrt{\pi}\,e^c \log(\operatorname{erf}(bx)-1)}{2b}$$

input
```
int(exp(-b^2*x^2+c)/erfc(b*x),x)
```

output
```
( - sqrt(pi)*e**c*log(erf(b*x) - 1))/(2*b)
```

3.153 $\int \frac{e^{c-b^2x^2}}{\mathbf{erfc}(bx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)^2}\,dx = \frac{e^c\sqrt{\pi}}{2b\text{erfc}(bx)}$$

output
```
1/2*exp(c)*Pi^(1/2)/b/erfc(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)^2}\,dx = \frac{e^c\sqrt{\pi}}{2b\text{erfc}(bx)}$$

input
```
Integrate[E^(c - b^2*x^2)/Erfc[b*x]^2,x]
```

output
```
(E^c*Sqrt[Pi])/(2*b*Erfc[b*x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)^2} \, dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{\sqrt{\pi}e^c \int \frac{1}{\text{erfc}(bx)^2}d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\pi}e^c}{2b\text{erfc}(bx)}$$

input
```
Int[E^(c - b^2*x^2)/Erfc[b*x]^2,x]
```

output
```
(E^c*Sqrt[Pi])/(2*b*Erfc[b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int \frac{\mathrm{e}^{-b^2x^2+c}}{\operatorname{erfc}(bx)^2}dx$$

input
```
int(exp(-b^2*x^2+c)/erfc(b*x)^2,x)
```

output
```
int(exp(-b^2*x^2+c)/erfc(b*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2\,(b\operatorname{erf}(bx)-b)}$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x)^2,x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*e^c/(b*erf(b*x) - b)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^2}\,dx = \begin{cases} \frac{\sqrt{\pi}e^c}{2b\operatorname{erfc}(bx)} & \text{for } b\neq 0 \\ xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)/erfc(b*x)**2,x)
```

output
```
Piecewise((sqrt(pi)*exp(c)/(2*b*erfc(b*x)), Ne(b, 0)), (x*exp(c), True))
```

Maxima [F]

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^2}\,dx = \int \frac{e^{(-b^2x^2+c)}}{\operatorname{erfc}\left(bx\right)^2}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(e^(-b^2*x^2 + c)/erfc(b*x)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2\,b(\operatorname{erf}\left(bx\right)-1)}$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x)^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*e^c/(b*(erf(b*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{e}^c}{2\,b\,\operatorname{erfc}\left(b\,x\right)}$$

input
```
int(exp(c - b^2*x^2)/erfc(b*x)^2,x)
```

output
```
(pi^(1/2)*exp(c))/(2*b*erfc(b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^2}\,dx = -\frac{\sqrt{\pi}\,e^c\operatorname{erf}(bx)}{2b\left(\operatorname{erf}\left(bx\right)-1\right)}$$

input
```
int(exp(-b^2*x^2+c)/erfc(b*x)^2,x)
```

output
```
( - sqrt(pi)*e**c*erf(b*x))/(2*b*(erf(b*x) - 1))
```

3.154 $\int \frac{e^{c-b^2x^2}}{\mathbf{erfc}(bx)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3} \, dx = \frac{e^c\sqrt{\pi}}{4b\operatorname{erfc}(bx)^2}$$

output
```
1/4*exp(c)*Pi^(1/2)/b/erfc(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3} \, dx = \frac{e^c\sqrt{\pi}}{4b\operatorname{erfc}(bx)^2}$$

input
```
Integrate[E^(c - b^2*x^2)/Erfc[b*x]^3,x]
```

output
```
(E^c*Sqrt[Pi])/(4*b*Erfc[b*x]^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c-b^2x^2}}{\text{erfc}(bx)^3}dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{\sqrt{\pi}e^c\int \frac{1}{\text{erfc}(bx)^3}d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\pi}e^c}{4b\text{erfc}(bx)^2}$$

input

```
Int[E^(c - b^2*x^2)/Erfc[b*x]^3,x]
```

output

```
(E^c*Sqrt[Pi])/(4*b*Erfc[b*x]^2)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6928

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int \frac{\mathrm{e}^{-b^2x^2+c}}{\operatorname{erfc}(bx)^3}dx$$

input
```
int(exp(-b^2*x^2+c)/erfc(b*x)^3,x)
```

output
```
int(exp(-b^2*x^2+c)/erfc(b*x)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3}\,dx = \frac{\sqrt{\pi}e^c}{4\left(b\operatorname{erf}(bx)^2 - 2\,b\operatorname{erf}(bx) + b\right)}$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x)^3,x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*e^c/(b*erf(b*x)^2 - 2*b*erf(b*x) + b)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3}\,dx = \begin{cases} \frac{\sqrt{\pi}e^c}{4b\operatorname{erfc}^2(bx)} & \text{for } b \neq 0 \\ xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)/erfc(b*x)**3,x)
```

output
```
Piecewise((sqrt(pi)*exp(c)/(4*b*erfc(b*x)**2), Ne(b, 0)), (x*exp(c), True)
)
```

Maxima [F]

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3}\,dx = \int \frac{e^{(-b^2x^2+c)}}{\operatorname{erfc}(bx)^3}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x)^3,x, algorithm="maxima")
```

output
```
integrate(e^(-b^2*x^2 + c)/erfc(b*x)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3}\,dx = \frac{\sqrt{\pi}e^c}{4\,b(\operatorname{erf}(bx)-1)^2}$$

input
```
integrate(exp(-b^2*x^2+c)/erfc(b*x)^3,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*e^c/(b*(erf(b*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3}\,dx = \frac{\sqrt{\pi}\,\mathrm{e}^c}{4\,b\operatorname{erfc}(b\,x)^2}$$

input
```
int(exp(c - b^2*x^2)/erfc(b*x)^3,x)
```

output
```
(pi^(1/2)*exp(c))/(4*b*erfc(b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{e^{c-b^2x^2}}{\operatorname{erfc}(bx)^3}\,dx = \frac{\sqrt{\pi}\,e^c}{4b\left(\operatorname{erf}(bx)^2-2\operatorname{erf}(bx)+1\right)}$$

input
```
int(exp(-b^2*x^2+c)/erfc(b*x)^3,x)
```

output
```
(sqrt(pi)*e**c)/(4*b*(erf(b*x)**2 - 2*erf(b*x) + 1))
```

3.155 $\int e^{c-b^2x^2}\mathbf{erfc}(bx)^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int e^{c-b^2x^2}\text{erfc}(bx)^n\,dx=-\frac{e^c\sqrt{\pi}\text{erfc}(bx)^{1+n}}{2b(1+n)}$$

output
```
-1/2*exp(c)*Pi^(1/2)*erfc(b*x)^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int e^{c-b^2x^2}\text{erfc}(bx)^n\,dx=-\frac{e^c\sqrt{\pi}\text{erfc}(bx)^{1+n}}{2b(1+n)}$$

input
```
Integrate[E^(c - b^2*x^2)*Erfc[b*x]^n,x]
```

output
```
-1/2*(E^c*Sqrt[Pi]*Erfc[b*x]^(1 + n))/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-b^2x^2}\text{erfc}(bx)^n\,dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{\sqrt{\pi}e^c\int \text{erfc}(bx)^n d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^{n+1}}{2b(n+1)}$$

input
```
Int[E^(c - b^2*x^2)*Erfc[b*x]^n,x]
```

output
```
-1/2*(E^c*Sqrt[Pi]*Erfc[b*x]^(1 + n))/(b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int e^{-b^2x^2+c} \operatorname{erfc}(bx)^n \, dx$$

input
```
int(exp(-b^2*x^2+c)*erfc(b*x)^n,x)
```

output
```
int(exp(-b^2*x^2+c)*erfc(b*x)^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int e^{c-b^2x^2} \operatorname{erfc}(bx)^n \, dx = \frac{\sqrt{\pi}(-\operatorname{erf}(bx) + 1)^n (\operatorname{erf}(bx) - 1)e^c}{2(bn + b)}$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x)^n,x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*(-erf(b*x) + 1)^n*(erf(b*x) - 1)*e^c/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.

Time = 1.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int e^{c-b^2x^2} \operatorname{erfc}(bx)^n \, dx = \begin{cases} xe^c & \text{for } b = 0 \wedge (b = 0 \vee n = -1) \\ -\frac{\sqrt{\pi}e^c \log(\operatorname{erfc}(bx))}{2b} & \text{for } n = -1 \\ -\frac{\sqrt{\pi}e^c \operatorname{erfc}(bx) \operatorname{erfc}^n(bx)}{2bn+2b} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(-b**2*x**2+c)*erfc(b*x)**n,x)
```

output
```
Piecewise((x*exp(c), Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (-sqrt(pi)*exp(c)
*log(erfc(b*x))/(2*b), Eq(n, -1)), (-sqrt(pi)*exp(c)*erfc(b*x)*erfc(b*x)**
n/(2*b*n + 2*b), True))
```

Maxima [F]

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^n\,dx = \int \operatorname{erfc}(bx)^n\, e^{(-b^2x^2+c)}\,dx$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x)^n,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)^n*e^(-b^2*x^2 + c), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^n\,dx = -\frac{\sqrt{\pi}(-\operatorname{erf}(bx)+1)^{n+1}e^c}{2\,b(n+1)}$$

input
```
integrate(exp(-b^2*x^2+c)*erfc(b*x)^n,x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*(-erf(b*x) + 1)^(n + 1)*e^c/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^n\,dx = -\frac{\sqrt{\pi}\,e^c\operatorname{erfc}(b\,x)^{n+1}}{2\,b\,(n+1)}$$

input
```
int(exp(c - b^2*x^2)*erfc(b*x)^n,x)
```

output
```
-(pi^(1/2)*exp(c)*erfc(b*x)^(n + 1))/(2*b*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int e^{c-b^2x^2}\operatorname{erfc}(bx)^n\,dx=\frac{\sqrt{\pi}\,e^c(-\operatorname{erf}(bx)+1)^n\,(\operatorname{erf}(bx)-1)}{2b\,(n+1)}$$

input
```
int(exp(-b^2*x^2+c)*erfc(b*x)^n,x)
```

output
```
(sqrt(pi)*e**c*( - erf(b*x) + 1)**n*(erf(b*x) - 1))/(2*b*(n + 1))
```

3.156 $\int e^{c+dx^2} x^5 \mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 283

$$\int e^{c+dx^2} x^5 \operatorname{erfc}(bx)\,dx = \frac{be^{c-(b^2-d)x^2}x}{(b^2-d)\,d^2\sqrt{\pi}} - \frac{3be^{c-(b^2-d)x^2}x}{4\,(b^2-d)^2\,d\sqrt{\pi}} - \frac{be^{c-(b^2-d)x^2}x^3}{2\,(b^2-d)\,d\sqrt{\pi}} + \frac{be^c\operatorname{erf}\left(\sqrt{b^2-d}x\right)}{\sqrt{b^2-d}d^3} - \frac{be^c\operatorname{erf}\left(\sqrt{b^2-d}x\right)}{2\,(b^2-d)^{3/2}\,d^2} + \frac{3be^c\operatorname{erf}\left(\sqrt{b^2-d}x\right)}{8\,(b^2-d)^{5/2}\,d} + \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{d^3} - \frac{e^{c+dx^2}x^2\operatorname{erfc}(bx)}{d^2} + \frac{e^{c+dx^2}x^4\operatorname{erfc}(bx)}{2d}$$

output
```
b*exp(c-(b^2-d)*x^2)*x/(b^2-d)/d^2/Pi^(1/2)-3/4*b*exp(c-(b^2-d)*x^2)*x/(b^
2-d)^2/d/Pi^(1/2)-1/2*b*exp(c-(b^2-d)*x^2)*x^3/(b^2-d)/d/Pi^(1/2)+b*exp(c)
*erf((b^2-d)^(1/2)*x)/(b^2-d)^(1/2)/d^3-1/2*b*exp(c)*erf((b^2-d)^(1/2)*x)/
(b^2-d)^(3/2)/d^2+3/8*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)^(5/2)/d+exp(d*
x^2+c)*erfc(b*x)/d^3-exp(d*x^2+c)*x^2*erfc(b*x)/d^2+1/2*exp(d*x^2+c)*x^4*e
rfc(b*x)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.49

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx = \frac{e^c\left(\frac{2bde^{(-b^2+d)x^2}x(b^2(4-2dx^2)+d(-7+2dx^2))}{(b^2-d)^2\sqrt{\pi}}+4e^{dx^2}(2-2dx^2+d^2x^4)\,\text{erfc}(bx)+\frac{b(8b^4-20b^2d+15d^2)\text{erfi}\left(\sqrt{-b^2+d}x\right)}{(-b^2+d)^{5/2}}\right)}{8d^3}$$

input
```
Integrate[E^(c + d*x^2)*x^5*Erfc[b*x],x]
```

output
```
(E^c*((2*b*d*E^((-b^2 + d)*x^2)*x*(b^2*(4 - 2*d*x^2) + d*(-7 + 2*d*x^2)))/
((b^2 - d)^2*Sqrt[Pi]) + 4*E^(d*x^2)*(2 - 2*d*x^2 + d^2*x^4)*Erfc[b*x] + (
b*(8*b^4 - 20*b^2*d + 15*d^2)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^(5/2)))/(
8*d^3)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6940, 2641, 2641, 2634, 6940, 2641, 2634, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\text{erfc}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{6940}$$

$$\frac{b\int e^{c-(b^2-d)x^2}x^4dx}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erfc}(bx)dx}{d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{3\int e^{c-(b^2-d)x^2}x^2dx}{2(b^2-d)}-\frac{x^3e^{c-x^2(b^2-d)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erfc}(bx)dx}{d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{3\left(\frac{\int e^{c-\left(b^2-d\right)x^2}dx}{2\left(b^2-d\right)}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erfc}(bx)dx}{d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\int e^{dx^2+c}x^3\text{erfc}(bx)dx}{d}+\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6940}$$

$$-\frac{2\left(\frac{b\int e^{c-\left(b^2-d\right)x^2}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(bx)dx}{d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{d}+\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\left(\frac{b\left(\frac{\int e^{c-\left(b^2-d\right)x^2}dx}{2\left(b^2-d\right)}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(bx)dx}{d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{d}+\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\left(-\frac{\int e^{dx^2+c}x\text{erfc}(bx)dx}{d}+\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{d}+\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \ 6937$$

$$-\frac{2\left(-\frac{-\frac{b\int e^{c-\left(b^2-d\right)x^2}dx}{\sqrt{\pi}d}+\frac{\text{erfc}(bx)e^{c+dx^2}}{2d}}{d}+\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{d}+\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \ 2634$$

$$-\frac{2\left(-\frac{-\frac{be^c\text{erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}+\frac{\text{erfc}(bx)e^{c+dx^2}}{2d}}{d}+\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{d}+\frac{b\left(\frac{3\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2\left(b^2-d\right)}\right)}{2(b^2-d)}-\frac{x^3e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfc}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^5*Erfc[b*x],x]
```

output
```
(b*(-1/2*(E^(c - (b^2 - d)*x^2)*x^3)/(b^2 - d) + (3*(-1/2*(E^(c - (b^2 - d
)*x^2)*x)/(b^2 - d) + (E^c*Sqrt[Pi]*Erf[Sqrt[b^2 - d]*x])/(4*(b^2 - d)^(3/
2))))/(2*(b^2 - d))))/(d*Sqrt[Pi]) + (E^(c + d*x^2)*x^4*Erfc[b*x])/(2*d) -
 (2*((b*(-1/2*(E^(c - (b^2 - d)*x^2)*x)/(b^2 - d) + (E^c*Sqrt[Pi]*Erf[Sqrt
[b^2 - d]*x])/(4*(b^2 - d)^(3/2))))/(d*Sqrt[Pi]) + (E^(c + d*x^2)*x^2*Erfc
[b*x])/(2*d) - ((b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d) + (E^(c +
 d*x^2)*Erfc[b*x])/(2*d))/d))/d
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 5.48 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{e^{c}\left(\frac{e^{d x^{2}} b^{6} x^{4}}{2 d}-\frac{2 b^{2}\left(\frac{b^{4} x^{2} e^{d x^{2}}}{2 d}-\frac{b^{4} e^{d x^{2}}}{2 d^{2}}\right)}{d}\right)}{b^{5}}-\frac{\operatorname{erf}(b x) e^{c}\left(\frac{e^{d x^{2}} b^{6} x^{4}}{2 d}-\frac{2 b^{2}\left(\frac{b^{4} x^{2} e^{d x^{2}}}{2 d}-\frac{b^{4} e^{d x^{2}}}{2 d^{2}}\right)}{d}\right)}{b^{5}}+\frac{e^{c}\left(\frac{b^{2}\left(\frac{b^{3} x^{3} e^{\left(-1+\frac{d}{b^{2}}\right) b^{2} x^{2}}}{-2+\frac{2 d}{b^{2}}}-\frac{3\left(b \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{b}$

input
```
int(exp(d*x^2+c)*x^5*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/b^5*exp(c)*(1/2*exp(d*x^2)*b^6*x^4/d-2/d*b^2*(1/2/d*b^4*x^2*exp(d*x^2)-
1/2/d^2*b^4*exp(d*x^2)))-erf(b*x)/b^5*exp(c)*(1/2*exp(d*x^2)*b^6*x^4/d-2/d
*b^2*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2)))+1/Pi^(1/2)/b^5*exp
(c)*(1/d*b^2*(1/2/(-1+d/b^2)*b^3*x^3*exp((-1+d/b^2)*b^2*x^2)-3/2/(-1+d/b^2
)*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2)*Pi^(1/2)/(1-d
/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)))+1/d^3*b^6*Pi^(1/2)/(1-d/b^2)^(1/2)*
erf((1-d/b^2)^(1/2)*b*x)-2/d^2*b^4*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*
x^2)-1/4/(-1+d/b^2)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))))/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.26

$$\int e^{c+dx^2}x^5\operatorname{erfc}(bx)\,dx$$
$$=\frac{\pi(8\,b^5-20\,b^3d+15\,bd^2)\sqrt{b^2-d}\operatorname{erf}\left(\sqrt{b^2-d}x\right)e^c-2\sqrt{\pi}(2\,(b^5d^2-2\,b^3d^3+bd^4)x^3-(4\,b^5d-11\,b^3d^2\cdots}{\cdots}$$

input
```
integrate(exp(d*x^2+c)*x^5*erfc(b*x),x, algorithm="fricas")
```

output
```
1/8*(pi*(8*b^5 - 20*b^3*d + 15*b*d^2)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e
^c - 2*sqrt(pi)*(2*(b^5*d^2 - 2*b^3*d^3 + b*d^4)*x^3 - (4*b^5*d - 11*b^3*d
^2 + 7*b*d^3)*x)*e^(-b^2*x^2 + d*x^2 + c) + 4*(pi*(b^6*d^2 - 3*b^4*d^3 + 3
*b^2*d^4 - d^5)*x^4 - 2*pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 + 2*p
i*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3) - (pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^2*d^4
 - d^5)*x^4 - 2*pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 + 2*pi*(b^6 -
 3*b^4*d + 3*b^2*d^2 - d^3))*erf(b*x))*e^(d*x^2 + c))/(pi*(b^6*d^3 - 3*b^4
*d^4 + 3*b^2*d^5 - d^6))
```

Sympy [F]

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx = e^c\int x^5e^{dx^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**5*erfc(b*x),x)
```

output
```
exp(c)*Integral(x**5*exp(d*x**2)*erfc(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx = \int x^5\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^5*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^5*erfc(b*x)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx=\int x^5\,\text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^5*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*erfc(b*x)*e^(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx=\int x^5\,\mathrm{e}^{d\,x^2+c}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x^5*exp(c + d*x^2)*erfc(b*x),x)
```

output
```
int(x^5*exp(c + d*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{c+dx^2}x^5\text{erfc}(bx)\,dx$$
$$=\frac{e^c\left(-4e^{b^2x^2+d\,x^2}\text{erf}(bx)\,b^4\pi-4e^{b^2x^2+d\,x^2}\text{erf}(bx)\,d^2\pi+8\sqrt{\pi}\,e^{b^2x^2}\left(\int\frac{e^{d\,x^2}}{e^{b^2x^2}}dx\right)b^5-8e^{b^2x^2+d\,x^2}b^2d\pi+2e^{b^2x^2}\right.}{}$$

input
```
int(exp(d*x^2+c)*x^5*erfc(b*x),x)
```

output
```
(e**c*( - 2*e**(b**2*x**2 + d*x**2)*erf(b*x)*b**4*d**2*pi*x**4 + 4*e**(b**
2*x**2 + d*x**2)*erf(b*x)*b**4*d*pi*x**2 - 4*e**(b**2*x**2 + d*x**2)*erf(b
*x)*b**4*pi + 4*e**(b**2*x**2 + d*x**2)*erf(b*x)*b**2*d**3*pi*x**4 - 8*e**
(b**2*x**2 + d*x**2)*erf(b*x)*b**2*d**2*pi*x**2 + 8*e**(b**2*x**2 + d*x**2
)*erf(b*x)*b**2*d*pi - 2*e**(b**2*x**2 + d*x**2)*erf(b*x)*d**4*pi*x**4 + 4
*e**(b**2*x**2 + d*x**2)*erf(b*x)*d**3*pi*x**2 - 4*e**(b**2*x**2 + d*x**2)
*erf(b*x)*d**2*pi + 2*e**(b**2*x**2 + d*x**2)*b**4*d**2*pi*x**4 - 4*e**(b*
*2*x**2 + d*x**2)*b**4*d*pi*x**2 + 4*e**(b**2*x**2 + d*x**2)*b**4*pi - 4*e
**(b**2*x**2 + d*x**2)*b**2*d**3*pi*x**4 + 8*e**(b**2*x**2 + d*x**2)*b**2*
d**2*pi*x**2 - 8*e**(b**2*x**2 + d*x**2)*b**2*d*pi + 2*e**(b**2*x**2 + d*x
**2)*d**4*pi*x**4 - 4*e**(b**2*x**2 + d*x**2)*d**3*pi*x**2 + 4*e**(b**2*x*
*2 + d*x**2)*d**2*pi + 8*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/e**(b**2*
x**2),x)*b**5 - 20*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/e**(b**2*x**2),
x)*b**3*d + 15*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)/e**(b**2*x**2),x)*b
*d**2 - 2*sqrt(pi)*e**(d*x**2)*b**3*d**2*x**3 + 4*sqrt(pi)*e**(d*x**2)*b**
3*d*x + 2*sqrt(pi)*e**(d*x**2)*b*d**3*x**3 - 7*sqrt(pi)*e**(d*x**2)*b*d**2
*x))/(4*e**(b**2*x**2)*d**3*pi*(b**4 - 2*b**2*d + d**2))
```

3.157 $\int e^{c+dx^2} x^3 \text{erfc}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int e^{c+dx^2} x^3 \text{erfc}(bx)\,dx = -\frac{be^{c-(b^2-d)x^2}x}{2(b^2-d)\,d\sqrt{\pi}} - \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d^2} + \frac{be^c\text{erf}\left(\sqrt{b^2-d}x\right)}{4(b^2-d)^{3/2}d} - \frac{e^{c+dx^2}\text{erfc}(bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfc}(bx)}{2d}$$

output
```
-1/2*b*exp(c-(b^2-d)*x^2)*x/(b^2-d)/d/Pi^(1/2)-1/2*b*exp(c)*erf((b^2-d)^(1
/2)*x)/(b^2-d)^(1/2)/d^2+1/4*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)^(3/2)/d
-1/2*exp(d*x^2+c)*erfc(b*x)/d^2+1/2*exp(d*x^2+c)*x^2*erfc(b*x)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

$$\int e^{c+dx^2} x^3 \text{erfc}(bx)\,dx = \frac{e^c\left(\frac{2bde^{(-b^2+d)x^2}x}{(-b^2+d)\sqrt{\pi}} + 2e^{dx^2}(-1+dx^2)\,\text{erfc}(bx) + \frac{(2b^3-3bd)\text{erfi}\left(\sqrt{-b^2+d}x\right)}{(-b^2+d)^{3/2}}\right)}{4d^2}$$

input
```
Integrate[E^(c + d*x^2)*x^3*Erfc[b*x],x]
```

output
```
(E^c*((2*b*d*E^((-b^2 + d)*x^2)*x)/((-b^2 + d)*Sqrt[Pi]) + 2*E^(d*x^2)*(-1
 + d*x^2)*Erfc[b*x] + ((2*b^3 - 3*b*d)*Erfi[Sqrt[-b^2 + d]*x])/(-b^2 + d)^
(3/2)))/(4*d^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6940, 2641, 2634, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{erfc}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{6940}$$

$$\frac{b\int e^{c-(b^2-d)x^2}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(bx)dx}{d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$\frac{b\left(\frac{\int e^{c-\left(b^2-d\right)x^2}dx}{2(b^2-d)}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(bx)dx}{d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\int e^{dx^2+c}x\text{erfc}(bx)dx}{d}+\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6937}$$

$$-\frac{\frac{b\int e^{c-\left(b^2-d\right)x^2}dx}{\sqrt{\pi}d}+\frac{\text{erfc}(bx)e^{c+dx^2}}{2d}}{d}+\frac{b\left(\frac{\sqrt{\pi}e^c\text{erf}\left(x\sqrt{b^2-d}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\frac{be^c\operatorname{erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}+\frac{\operatorname{erfc}(bx)e^{c+dx^2}}{2d}}{d}+\frac{b\left(\frac{\sqrt{\pi}e^c\operatorname{erf}\left(x\sqrt{b^2-d}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{x^2\operatorname{erfc}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^3*Erfc[b*x],x]
```

output
```
(b*(-1/2*(E^(c - (b^2 - d)*x^2)*x)/(b^2 - d) + (E^c*Sqrt[Pi]*Erf[Sqrt[b^2
- d]*x])/(4*(b^2 - d)^(3/2))))/(d*Sqrt[Pi]) + (E^(c + d*x^2)*x^2*Erfc[b*x]
)/(2*d) - ((b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d) + (E^(c + d*x^
2)*Erfc[b*x])/(2*d))/d
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{e^{c}\left(\frac{b^{4}x^{2}e^{d\,x^{2}}}{2d}-\frac{b^{4}e^{d\,x^{2}}}{2d^{2}}\right)}{b^{3}}-\frac{\operatorname{erf}(bx)e^{c}\left(\frac{b^{4}x^{2}e^{d\,x^{2}}}{2d}-\frac{b^{4}e^{d\,x^{2}}}{2d^{2}}\right)}{b^{3}}+\frac{e^{c}\left(\frac{b^{2}\left(\frac{bx\,e^{\left(-1+\frac{d}{b^{2}}\right)b^{2}x^{2}}}{-2+\frac{2d}{b^{2}}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\,bx\right)}{4\left(-1+\frac{d}{b^{2}}\right)\sqrt{1-\frac{d}{b^{2}}}}\right)}{d}-\frac{b^{4}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\,bx\right.}{2d^{2}\sqrt{1-\frac{d}{b^{2}}}}\right.}{\sqrt{\pi}\,b^{3}}}{b}$

input
```
int(exp(d*x^2+c)*x^3*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/b^3*exp(c)*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2))-erf(b*x)/b
^3*exp(c)*(1/2/d*b^4*x^2*exp(d*x^2)-1/2/d^2*b^4*exp(d*x^2))+1/Pi^(1/2)/b^3
*exp(c)*(1/d*b^2*(1/2/(-1+d/b^2)*b*x*exp((-1+d/b^2)*b^2*x^2)-1/4/(-1+d/b^2
)*Pi^(1/2)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))-1/2/d^2*b^4*Pi^(1/2)/
(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x)))/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

$$\int e^{c+dx^2}x^3\operatorname{erfc}(bx)\,dx =$$

$$-\frac{\pi(2\,b^3-3\,bd)\sqrt{b^2-d}\,\operatorname{erf}\left(\sqrt{b^2-d}x\right)e^c+2\,\sqrt{\pi}(b^3d-bd^2)xe^{(-b^2x^2+dx^2+c)}-2\left(\pi(b^4d-2\,b^2d^2+d^3)x^2\right.}{4\,\pi(b^4d^2-2\,b^2d^3+}$$

input
```
integrate(exp(d*x^2+c)*x^3*erfc(b*x),x, algorithm="fricas")
```

output
```
-1/4*(pi*(2*b^3 - 3*b*d)*sqrt(b^2 - d)*erf(sqrt(b^2 - d)*x)*e^c + 2*sqrt(p
i)*(b^3*d - b*d^2)*x*e^(-b^2*x^2 + d*x^2 + c) - 2*(pi*(b^4*d - 2*b^2*d^2 +
 d^3)*x^2 - pi*(b^4 - 2*b^2*d + d^2) - (pi*(b^4*d - 2*b^2*d^2 + d^3)*x^2 -
 pi*(b^4 - 2*b^2*d + d^2))*erf(b*x))*e^(d*x^2 + c))/(pi*(b^4*d^2 - 2*b^2*d
^3 + d^4))
```

Sympy [F]

$$\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx=e^c\int x^3e^{dx^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**3*erfc(b*x),x)
```

output
```
exp(c)*Integral(x**3*exp(d*x**2)*erfc(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx=\int x^3\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*erfc(b*x)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx=\int x^3\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*erfc(b*x)*e^(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx = \int x^3\,\mathrm{e}^{d\,x^2+c}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x^3*exp(c + d*x^2)*erfc(b*x),x)
```

output
```
int(x^3*exp(c + d*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{c+dx^2}x^3\text{erfc}(bx)\,dx$$
$$= \frac{e^c\left(-e^{b^2x^2+d\,x^2}\text{erf}(bx)\,b^2d\pi\,x^2 + e^{b^2x^2+d\,x^2}\text{erf}(bx)\,b^2\pi + e^{b^2x^2+d\,x^2}\text{erf}(bx)\,d^2\pi\,x^2 - e^{b^2x^2+d\,x^2}\text{erf}(bx)\,d\pi + e^{b}\right.}{}$$

input
```
int(exp(d*x^2+c)*x^3*erfc(b*x),x)
```

output
```
(e**c*( - e**(b**2*x**2 + d*x**2)*erf(b*x)*b**2*d*pi*x**2 + e**(b**2*x**2
+ d*x**2)*erf(b*x)*b**2*pi + e**(b**2*x**2 + d*x**2)*erf(b*x)*d**2*pi*x**2
 - e**(b**2*x**2 + d*x**2)*erf(b*x)*d*pi + e**(b**2*x**2 + d*x**2)*b**2*d*
pi*x**2 - e**(b**2*x**2 + d*x**2)*b**2*pi - e**(b**2*x**2 + d*x**2)*d**2*p
i*x**2 + e**(b**2*x**2 + d*x**2)*d*pi - 2*sqrt(pi)*e**(b**2*x**2)*int(e**(
d*x**2)/e**(b**2*x**2),x)*b**3 + 3*sqrt(pi)*e**(b**2*x**2)*int(e**(d*x**2)
/e**(b**2*x**2),x)*b*d - sqrt(pi)*e**(d*x**2)*b*d*x))/(2*e**(b**2*x**2)*d*
*2*pi*(b**2 - d))
```

3.158 $\int e^{c+dx^2} x \operatorname{erfc}(bx) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int e^{c+dx^2} x \operatorname{erfc}(bx) \, dx = \frac{b e^c \operatorname{erf}\left(\sqrt{b^2-d}x\right)}{2\sqrt{b^2-d}d} + \frac{e^{c+dx^2} \operatorname{erfc}(bx)}{2d}$$

output
```
1/2*b*exp(c)*erf((b^2-d)^(1/2)*x)/(b^2-d)^(1/2)/d+1/2*exp(d*x^2+c)*erfc(b*
x)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int e^{c+dx^2} x \operatorname{erfc}(bx) \, dx = \frac{e^c \left(e^{dx^2} \operatorname{erfc}(bx) + \frac{b \operatorname{erfi}\left(\sqrt{-b^2+d}x\right)}{\sqrt{-b^2+d}}\right)}{2d}$$

input
```
Integrate[E^(c + d*x^2)*x*Erfc[b*x],x]
```

output
```
(E^c*(E^(d*x^2)*Erfc[b*x] + (b*Erfi[Sqrt[-b^2 + d]*x])/Sqrt[-b^2 + d]))/(2
*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfc}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{6937}$$

$$\frac{b\int e^{c-(b^2-d)x^2}dx}{\sqrt{\pi}d}+\frac{\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{be^c\text{erf}\left(x\sqrt{b^2-d}\right)}{2d\sqrt{b^2-d}}+\frac{\text{erfc}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x*Erfc[b*x],x]
```

output
```
(b*E^c*Erf[Sqrt[b^2 - d]*x])/(2*Sqrt[b^2 - d]*d) + (E^(c + d*x^2)*Erfc[b*x
])/(2*d)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{\frac{b\,{\mathrm e}^{\frac{b^{2} d\,x^{2}+b^{2} c}{b^{2}}}}{2 d}-\frac{\operatorname{erf}\left(b x \right) b\,{\mathrm e}^{\frac{b^{2} d\,x^{2}+b^{2} c}{b^{2}}}}{2 d}+\frac{b\,{\mathrm e}^{c} \operatorname{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\, b x \right)}{2 d \sqrt{1-\frac{d}{b^{2}}}}}{b}$	92

input
```
int(exp(d*x^2+c)*x*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*exp((b^2*d*x^2+b^2*c)/b^2)/d-1/2*erf(b*x)*b*exp((b^2*d*x^2+b^2*c)/b
^2)/d+1/2*b/d*exp(c)/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int e^{c+dx^2} x\operatorname{erfc}(bx)\,dx = \frac{\sqrt{b^2-d}\,b\operatorname{erf}\left(\sqrt{b^2-d}\,x\right)e^c+\left(b^2-\left(b^2-d\right)\operatorname{erf}\left(bx\right)-d\right)e^{\left(dx^2+c\right)}}{2\left(b^2d-d^2\right)}$$

input
```
integrate(exp(d*x^2+c)*x*erfc(b*x),x, algorithm="fricas")
```

output
```
1/2*(sqrt(b^2 - d)*b*erf(sqrt(b^2 - d)*x)*e^c + (b^2 - (b^2 - d)*erf(b*x)
- d)*e^(d*x^2 + c))/(b^2*d - d^2)
```

Sympy [F]

$$\int e^{c+dx^2} x\operatorname{erfc}(bx)\,dx = e^c\int xe^{dx^2}\operatorname{erfc}\left(bx\right)dx$$

input
```
integrate(exp(d*x**2+c)*x*erfc(b*x),x)
```

output
```
exp(c)*Integral(x*exp(d*x**2)*erfc(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x\text{erfc}(bx)\,dx=\int x\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x*erfc(b*x)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x\text{erfc}(bx)\,dx=\int x\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x*erfc(b*x)*e^(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx^2}x\text{erfc}(bx)\,dx=\int x\,\mathrm{e}^{d\,x^2+c}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x*exp(c + d*x^2)*erfc(b*x),x)
```

output
```
int(x*exp(c + d*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{c+dx^2} x \operatorname{erfc}(bx)\, dx = \frac{e^c \left(e^{d\,x^2} - 2\left(\int e^{d\,x^2} \operatorname{erf}(bx)\, x dx\right) d\right)}{2d}$$

input
```
int(exp(d*x^2+c)*x*erfc(b*x),x)
```

output
```
(e**c*(e**(d*x**2) - 2*int(e**(d*x**2)*erf(b*x)*x,x)*d))/(2*d)
```

3.159 $\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx=\text{Int}\left(\frac{e^{c+dx^2}\text{erfc}(bx)}{x},x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfc(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx=\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)e^{c+dx^2}}{x} \, dx$$
$$\Big\downarrow \; 6949$$
$$\int \frac{\text{erfc}(bx)e^{c+dx^2}}{x}dx$$

input
```
Int[(E^(c + d*x^2)*Erfc[b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{{\rm e}^{d\,x^{2}+c} \operatorname{erfc}\left(b x \right)}{x}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx = e^c\int \frac{e^{dx^2}\,\text{erfc}\,(bx)}{x}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x)/x,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx=\int \frac{\text{erfc}\left(bx\right)e^{\left(dx^2+c\right)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x}\,dx=\int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erfc}(b\,x)}{x}\,dx$$

input
```
int((exp(c + d*x^2)*erfc(b*x))/x,x)
```

output
```
int((exp(c + d*x^2)*erfc(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x}\,dx = \frac{e^c\left(ei(d\,x^2) - 2\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx)}{x}dx\right)\right)}{2}$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x,x)
```

output
```
(e**c*(ei(d*x**2) - 2*int((e**(d*x**2)*erf(b*x))/x,x)))/2
```

3.160 $\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx = \frac{be^{c-(b^2-d)x^2}}{\sqrt{\pi}x} + b\sqrt{b^2-d}e^c\text{erf}\left(\sqrt{b^2-d}x\right) - \frac{e^{c+dx^2}\text{erfc}(bx)}{2x^2} + d\text{Int}\left(\frac{e^{c+dx^2}\text{erfc}(bx)}{x},x\right)$$

output
```
b*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x+b*(b^2-d)^(1/2)*exp(c)*erf((b^2-d)^(1/2)*x
)-1/2*exp(d*x^2+c)*erfc(b*x)/x^2+d*Defer(Int)(exp(d*x^2+c)*erfc(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^3,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)e^{c+dx^2}}{x^3}dx$$

$$\Big\downarrow \text{6946}$$

$$-\frac{b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^2}dx}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{b\left(-2\left(b^2-d\right)\int e^{c-\left(b^2-d\right)x^2}dx-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{6949}$$

$$d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}$$

input
```
Int[(E^(c + d*x^2)*Erfc[b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erfc}\left(bx\right)}{x^3}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^3,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 6.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx = e^c\int \frac{e^{dx^2}\,\text{erfc}\,(bx)}{x^3}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x)/x**3,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erfc}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfc}(b\,x)}{x^3}\,dx$$

input
```
int((exp(c + d*x^2)*erfc(b*x))/x^3,x)
```

output
```
int((exp(c + d*x^2)*erfc(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \frac{e^{c+dx^2}\mathrm{erfc}(bx)}{x^3}\,dx = \frac{e^c\left(ei(d\,x^2)\,d\,x^2 - e^{d\,x^2} - 2\left(\int \frac{e^{d\,x^2}\mathrm{erf}(bx)}{x^3}dx\right)x^2\right)}{2x^2}$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^3,x)
```

output
```
(e**c*(ei(d*x**2)*d*x**2 - e**(d*x**2) - 2*int((e**(d*x**2)*erf(b*x))/x**3
,x)*x**2))/(2*x**2)
```

3.161 $\int \frac{e^{c+dx^2}\mathbf{erfc}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^5}\,dx &= \frac{be^{c-(b^2-d)x^2}}{6\sqrt{\pi}x^3} - \frac{b(b^2-d)\,e^{c-(b^2-d)x^2}}{3\sqrt{\pi}x} + \frac{bde^{c-(b^2-d)x^2}}{2\sqrt{\pi}x}\\ &\quad - \frac{1}{3}b\left(b^2-d\right)^{3/2}e^c\operatorname{erf}\left(\sqrt{b^2-d}x\right) + \frac{1}{2}b\sqrt{b^2-d}de^c\operatorname{erf}\left(\sqrt{b^2-d}x\right)\\ &\quad - \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{4x^4} - \frac{de^{c+dx^2}\operatorname{erfc}(bx)}{4x^2} + \frac{1}{2}d^2\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x},x\right)\end{aligned}$$

output
```
1/6*b*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x^3-1/3*b*(b^2-d)*exp(c-(b^2-d)*x^2)/Pi^
(1/2)/x+1/2*b*d*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x-1/3*b*(b^2-d)^(3/2)*exp(c)*e
rf((b^2-d)^(1/2)*x)+1/2*b*(b^2-d)^(1/2)*d*exp(c)*erf((b^2-d)^(1/2)*x)-1/4*
exp(d*x^2+c)*erfc(b*x)/x^4-1/4*d*exp(d*x^2+c)*erfc(b*x)/x^2+1/2*d^2*Defer(
Int)(exp(d*x^2+c)*erfc(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^5}\,dx = \int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^5}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^5,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^5, x]
```

Rubi [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfc}(bx)e^{c+dx^2}}{x^5}\,dx$$

$$\Big\downarrow \; 6946$$

$$-\frac{b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^4}dx}{2\sqrt{\pi}}+\frac{1}{2}d\int \frac{e^{dx^2+c}\operatorname{erfc}(bx)}{x^3}dx-\frac{\operatorname{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 2643$$

$$-\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^2}dx-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}+\frac{1}{2}d\int \frac{e^{dx^2+c}\operatorname{erfc}(bx)}{x^3}dx-\frac{\operatorname{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 2643$$

$$-\frac{b\left(-\frac{2}{3}(b^2-d)\left(-2(b^2-d)\int e^{c-(b^2-d)x^2}dx-\frac{e^{c-x^2(b^2-d)}}{x}\right)-\frac{e^{c-x^2(b^2-d)}}{3x^3}\right)}{2\sqrt{\pi}}+\frac{1}{2}d\int\frac{e^{dx^2+c}\text{erfc}(bx)}{x^3}dx-\frac{\text{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}d\int\frac{e^{dx^2+c}\text{erfc}(bx)}{x^3}dx-\frac{b\left(-\frac{2}{3}(b^2-d)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2(b^2-d)}}{x}\right)-\frac{e^{c-x^2(b^2-d)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{6946}$$

$$\frac{1}{2}d\left(-\frac{b\int\frac{e^{c-(b^2-d)x^2}}{x^2}dx}{\sqrt{\pi}}+d\int\frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}\right)-\frac{b\left(-\frac{2}{3}(b^2-d)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2(b^2-d)}}{x}\right)-\frac{e^{c-x^2(b^2-d)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2}d\left(-\frac{b\left(-2(b^2-d)\int e^{c-(b^2-d)x^2}dx-\frac{e^{c-x^2(b^2-d)}}{x}\right)}{\sqrt{\pi}}+d\int\frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}\right)-\frac{b\left(-\frac{2}{3}(b^2-d)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2(b^2-d)}}{x}\right)-\frac{e^{c-x^2(b^2-d)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}d\left(d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}\right)-\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 6949$$

$$\frac{1}{2}d\left(d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x}dx-\frac{b\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{2x^2}\right)-\frac{b\left(-\frac{2}{3}\left(b^2-d\right)\left(\sqrt{\pi}e^c\left(-\sqrt{b^2-d}\right)\text{erf}\left(x\sqrt{b^2-d}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{x}\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{4x^4}$$

input
```
Int[(E^(c + d*x^2)*Erfc[b*x])/x^5,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erfc}\left(bx\right)}{x^5}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^5,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 31.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^5}\,dx = e^c\int \frac{e^{dx^2}\,\text{erfc}\,(bx)}{x^5}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x)/x**5,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erfc}(b\,x)}{x^5}\,dx$$

input
```
int((exp(c + d*x^2)*erfc(b*x))/x^5,x)
```

output
```
int((exp(c + d*x^2)*erfc(b*x))/x^5, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 15.76

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^5}\,dx$$
$$= \frac{e^c\Big(3e^{b^2x^2}\,ei(d\,x^2)\,d^2\pi\,x^4 + 3e^{b^2x^2+d\,x^2}\operatorname{erf}(bx)\,d\pi\,x^2 + 3e^{b^2x^2+d\,x^2}\operatorname{erf}(bx)\,\pi - 3e^{b^2x^2+d\,x^2}d\pi\,x^2 - 3e^{b^2x^2+d\,x^2}\pi}{1}$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^5,x)
```

output
```
(e**c*(3*e**(b**2*x**2)*ei(d*x**2)*d**2*pi*x**4 + 3*e**(b**2*x**2 + d*x**2
)*erf(b*x)*d*pi*x**2 + 3*e**(b**2*x**2 + d*x**2)*erf(b*x)*pi - 3*e**(b**2*
x**2 + d*x**2)*d*pi*x**2 - 3*e**(b**2*x**2 + d*x**2)*pi + 4*sqrt(pi)*e**(b
**2*x**2)*int(e**(d*x**2)/(e**(b**2*x**2)*x**2),x)*b**3*x**4 - 10*sqrt(pi)
*e**(b**2*x**2)*int(e**(d*x**2)/(e**(b**2*x**2)*x**2),x)*b*d*x**4 - 6*e**(
b**2*x**2)*int((e**(d*x**2)*erf(b*x))/x,x)*d**2*pi*x**4 + 2*sqrt(pi)*e**(d
*x**2)*b*x))/(12*e**(b**2*x**2)*pi*x**4)
```

3.162 $\int e^{c+dx^2} x^4 \textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{c+dx^2} x^4 \text{erfc}(bx)\,dx = \frac{3be^{c-(b^2-d)x^2}}{4(b^2-d)\,d^2\sqrt{\pi}} - \frac{be^{c-(b^2-d)x^2}}{2(b^2-d)^2\,d\sqrt{\pi}} - \frac{be^{c-(b^2-d)x^2}x^2}{2(b^2-d)\,d\sqrt{\pi}} - \frac{3e^{c+dx^2}x\text{erfc}(bx)}{4d^2} + \frac{e^{c+dx^2}x^3\text{erfc}(bx)}{2d} + \frac{3\text{Int}\left(e^{c+dx^2}\text{erfc}(bx),x\right)}{4d^2}$$

output
```
3/4*b*exp(c-(b^2-d)*x^2)/(b^2-d)/d^2/Pi^(1/2)-1/2*b*exp(c-(b^2-d)*x^2)/(b^
2-d)^2/d/Pi^(1/2)-1/2*b*exp(c-(b^2-d)*x^2)*x^2/(b^2-d)/d/Pi^(1/2)-3/4*exp(
d*x^2+c)*x*erfc(b*x)/d^2+1/2*exp(d*x^2+c)*x^3*erfc(b*x)/d+3/4*Defer(Int)(e
xp(d*x^2+c)*erfc(b*x),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx=\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^4*Erfc[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^4*Erfc[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\text{erfc}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \; 6940$$

$$\frac{b\int e^{c-(b^2-d)x^2}x^3dx}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(bx)dx}{2d}+\frac{x^3\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \; 2641$$

$$\frac{b\left(\frac{\int e^{c-(b^2-d)x^2}xdx}{b^2-d}-\frac{x^2e^{c-x^2(b^2-d)}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(bx)dx}{2d}+\frac{x^3\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \; 2638$$

$$-\frac{3\int e^{dx^2+c}x^2\text{erfc}(bx)dx}{2d}+\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6940}$$

$$-\frac{3\left(\frac{b\int e^{c-\left(b^2-d\right)x^2}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfc}(bx)dx}{2d}+\frac{x\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{2d}+\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\text{erfc}(bx)dx}{2d}-\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d(b^2-d)}+\frac{x\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{2d}+\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6934}$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\text{erfc}(bx)dx}{2d}-\frac{be^{c-x^2\left(b^2-d\right)}}{2\sqrt{\pi}d(b^2-d)}+\frac{x\text{erfc}(bx)e^{c+dx^2}}{2d}\right)}{2d}+\frac{b\left(-\frac{x^2e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)}-\frac{e^{c-x^2\left(b^2-d\right)}}{2(b^2-d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfc}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^4*Erfc[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{d\,x^2+c}x^4\,\text{erfc}\,(bx)\,dx$$

input
```
int(exp(d*x^2+c)*x^4*erfc(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^4*erfc(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx=\int x^4\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-(x^4*erf(b*x) - x^4)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 53.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx=e^c\int x^4e^{dx^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**4*erfc(b*x),x)
```

output
```
exp(c)*Integral(x**4*exp(d*x**2)*erfc(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx=\int x^4\,\text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*erfc(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx=\int x^4\,\text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erfc(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx=\int x^4\,\mathrm{e}^{d\,x^2+c}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x^4*exp(c + d*x^2)*erfc(b*x),x)
```

output
```
int(x^4*exp(c + d*x^2)*erfc(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int e^{c+dx^2}x^4\text{erfc}(bx)\,dx = \frac{e^c\left(-3\sqrt{\pi}\,\text{erf}\left(\sqrt{d}\,ix\right)i+4e^{d\,x^2}\sqrt{d}\,d\,x^3-6e^{d\,x^2}\sqrt{d}\,x-8\sqrt{d}\left(\int e^{d\,x^2}\text{erf}(bx)\,x^4dx\right)d^2\right)}{8\sqrt{d}\,d^2}$$

input
```
int(exp(d*x^2+c)*x^4*erfc(b*x),x)
```

output
```
(e**c*( - 3*sqrt(pi)*erf(sqrt(d)*i*x)*i + 4*e**(d*x**2)*sqrt(d)*d*x**3 - 6
*e**(d*x**2)*sqrt(d)*x - 8*sqrt(d)*int(e**(d*x**2)*erf(b*x)*x**4,x)*d**2))
/(8*sqrt(d)*d**2)
```

3.163 $\int e^{c+dx^2} x^2 \text{erfc}(bx) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{c+dx^2} x^2 \text{erfc}(bx) \, dx = -\frac{b e^{c-(b^2-d)x^2}}{2(b^2-d) d\sqrt{\pi}} + \frac{e^{c+dx^2} x \text{erfc}(bx)}{2d} - \frac{\text{Int}\left(e^{c+dx^2} \text{erfc}(bx), x\right)}{2d}$$

output
```
-1/2*b*exp(c-(b^2-d)*x^2)/(b^2-d)/d/Pi^(1/2)+1/2*exp(d*x^2+c)*x*erfc(b*x)/
d-1/2*Defer(Int)(exp(d*x^2+c)*erfc(b*x),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2} x^2 \text{erfc}(bx) \, dx = \int e^{c+dx^2} x^2 \text{erfc}(bx) \, dx$$

input
```
Integrate[E^(c + d*x^2)*x^2*Erfc[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^2*Erfc[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erfc}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 6940}$$

$$\frac{b\int e^{c-(b^2-d)x^2}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfc}(bx)dx}{2d}+\frac{x\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{\int e^{dx^2+c}\text{erfc}(bx)dx}{2d}-\frac{be^{c-x^2(b^2-d)}}{2\sqrt{\pi}d(b^2-d)}+\frac{x\text{erfc}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 6934}$$

$$-\frac{\int e^{dx^2+c}\text{erfc}(bx)dx}{2d}-\frac{be^{c-x^2(b^2-d)}}{2\sqrt{\pi}d(b^2-d)}+\frac{x\text{erfc}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^2*Erfc[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{d\,x^2+c} x^2\,\text{erfc}\left(bx\right)dx$$

input
```
int(exp(d*x^2+c)*x^2*erfc(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^2*erfc(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int e^{c+dx^2}x^2\text{erfc}(bx)\,dx = \int x^2\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-(x^2*erf(b*x) - x^2)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 10.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^2\text{erfc}(bx)\,dx = e^c\int x^2e^{dx^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**2*erfc(b*x),x)
```

output
```
exp(c)*Integral(x**2*exp(d*x**2)*erfc(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erfc}(bx)\,dx = \int x^2\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*erfc(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erfc}(bx)\,dx = \int x^2\,\text{erfc}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erfc(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erfc}(bx)\,dx = \int x^2\,\mathrm{e}^{d\,x^2+c}\,\mathrm{erfc}(b\,x)\,dx$$

input
```
int(x^2*exp(c + d*x^2)*erfc(b*x),x)
```

output
```
int(x^2*exp(c + d*x^2)*erfc(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int e^{c+dx^2}x^2\mathrm{erfc}(bx)\,dx = \frac{e^c\left(\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{d}\,ix\right)i+2e^{d\,x^2}\sqrt{d}\,x-4\sqrt{d}\left(\int e^{d\,x^2}\mathrm{erf}(bx)\,x^2dx\right)d\right)}{4\sqrt{d}\,d}$$

input
```
int(exp(d*x^2+c)*x^2*erfc(b*x),x)
```

output
```
(e**c*(sqrt(pi)*erf(sqrt(d)*i*x)*i + 2*e**(d*x**2)*sqrt(d)*x - 4*sqrt(d)*i
nt(e**(d*x**2)*erf(b*x)*x**2,x)*d))/(4*sqrt(d)*d)
```

3.164 $\int e^{c+dx^2}\text{erfc}(bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int e^{c+dx^2}\text{erfc}(bx)\,dx=\text{Int}\left(e^{c+dx^2}\text{erfc}(bx),x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfc(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{c+dx^2}\text{erfc}(bx)\,dx=\int e^{c+dx^2}\text{erfc}(bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*Erfc[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*Erfc[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erfc}(bx)e^{c+dx^2}\,dx$$
$$\Big\downarrow \text{6934}$$
$$\int \operatorname{erfc}(bx)e^{c+dx^2}dx$$

input
```
Int[E^(c + d*x^2)*Erfc[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \mathrm{e}^{d\,x^2+c}\operatorname{erfc}(bx)\,dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x),x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int e^{c+dx^2}\text{erfc}(bx)\,dx=\int \text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\text{erfc}(bx)\,dx=e^{c}\int e^{dx^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x),x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\text{erfc}(bx)\,dx=\int \text{erfc}\,(bx)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfc}(bx)\,dx=\int \operatorname{erfc}\left(bx\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfc}(bx)\,dx=\int \mathrm{e}^{d\,x^2+c}\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(exp(c + d*x^2)*erfc(b*x),x)
```

output
```
int(exp(c + d*x^2)*erfc(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int e^{c+dx^2}\operatorname{erfc}(bx)\,dx = \frac{e^c\left(-\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{d}\,ix\right)i - 2\sqrt{d}\left(\int e^{d\,x^2}\operatorname{erf}(bx)\,dx\right)\right)}{2\sqrt{d}}$$

input
```
int(exp(d*x^2+c)*erfc(b*x),x)
```

output
```
(e**c*( - sqrt(pi)*erf(sqrt(d)*i*x)*i - 2*sqrt(d)*int(e**(d*x**2)*erf(b*x)
,x)))/(2*sqrt(d))
```

3.165 $\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx=-\frac{e^{c+dx^2}\text{erfc}(bx)}{x}-\frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}}+2d\text{Int}\left(e^{c+dx^2}\text{erfc}(bx),x\right)$$

output
```
-exp(d*x^2+c)*erfc(b*x)/x-b*exp(c)*Ei(-(b^2-d)*x^2)/Pi^(1/2)+2*d*Defer(Int
)(exp(d*x^2+c)*erfc(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx=\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^2,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfc}(bx)e^{c+dx^2}}{x^2} \, dx$$

$$\Big\downarrow \ 6946$$

$$-\frac{2b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}dx}{\sqrt{\pi}} + 2d\int e^{dx^2+c}\text{erfc}(bx)dx - \frac{\text{erfc}(bx)e^{c+dx^2}}{x}$$

$$\Big\downarrow \ 2639$$

$$2d\int e^{dx^2+c}\text{erfc}(bx)dx - \frac{be^c \, \text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}} - \frac{\text{erfc}(bx)e^{c+dx^2}}{x}$$

$$\Big\downarrow \ 6934$$

$$2d\int e^{dx^2+c}\text{erfc}(bx)dx - \frac{be^c \, \text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}} - \frac{\text{erfc}(bx)e^{c+dx^2}}{x}$$

input
```
Int[(E^(c + d*x^2)*Erfc[b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfc}\left(bx\right)}{x^2}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^2,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\mathrm{erfc}(bx)}{x^2}\,dx = \int \frac{\mathrm{erfc}\left(bx\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\mathrm{erfc}(bx)}{x^2}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erfc}\left(bx\right)}{x^2}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x)/x**2,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^2}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^2}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\operatorname{erfc}(b\,x)}{x^2}\,dx$$

input
```
int((exp(c + d*x^2)*erfc(b*x))/x^2,x)
```

output
```
int((exp(c + d*x^2)*erfc(b*x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^2}\,dx = e^c\left(\int \frac{e^{d\,x^2}}{x^2}dx-\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx)}{x^2}dx\right)\right)$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^2,x)
```

output
```
e**c*(int(e**(d*x**2)/x**2,x) - int((e**(d*x**2)*erf(b*x))/x**2,x))
```

3.166 $\int \frac{e^{c+dx^2}\mathbf{erfc}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^4}\,dx &= \frac{be^{c-(b^2-d)x^2}}{3\sqrt{\pi}x^2} - \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{3x^3} - \frac{2de^{c+dx^2}\operatorname{erfc}(bx)}{3x} \\ &\quad + \frac{b(b^2-d)\,e^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{3\sqrt{\pi}} \\ &\quad - \frac{2bde^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{3\sqrt{\pi}} \\ &\quad + \frac{4}{3}d^2\text{Int}\left(e^{c+dx^2}\operatorname{erfc}(bx),x\right)\end{aligned}$$

output
```
1/3*b*exp(c-(b^2-d)*x^2)/Pi^(1/2)/x^2-1/3*exp(d*x^2+c)*erfc(b*x)/x^3-2/3*d
*exp(d*x^2+c)*erfc(b*x)/x+1/3*b*(b^2-d)*exp(c)*Ei(-(b^2-d)*x^2)/Pi^(1/2)-2
/3*b*d*exp(c)*Ei(-(b^2-d)*x^2)/Pi^(1/2)+4/3*d^2*Defer(Int)(exp(d*x^2+c)*er
fc(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^4}\,dx=\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^4}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^4,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[b*x])/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfc}(bx)e^{c+dx^2}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6946}$$

$$-\frac{2b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x^3}dx}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\operatorname{erfc}(bx)}{x^2}dx-\frac{\operatorname{erfc}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{ 2643}$$

$$-\frac{2b\left(-\left(\left(b^2-d\right)\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}dx\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\operatorname{erfc}(bx)}{x^2}dx-\frac{\operatorname{erfc}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(bx)}{x^2}dx-\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{3x^3}$$

$\big\downarrow$ 6946

$$\frac{2}{3}d\left(-\frac{2b\int \frac{e^{c-\left(b^2-d\right)x^2}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfc}(bx)dx-\frac{\text{erfc}(bx)e^{c+dx^2}}{x}\right)-\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{3x^3}$$

$\big\downarrow$ 2639

$$\frac{2}{3}d\left(2d\int e^{dx^2+c}\text{erfc}(bx)dx-\frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{x}\right)-\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{3x^3}$$

$\big\downarrow$ 6934

$$\frac{2}{3}d\left(2d\int e^{dx^2+c}\text{erfc}(bx)dx-\frac{be^c\,\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)}{\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{x}\right)-\frac{2b\left(-\frac{1}{2}e^c\left(b^2-d\right)\text{ExpIntegralEi}\left(-\left(\left(b^2-d\right)x^2\right)\right)-\frac{e^{c-x^2\left(b^2-d\right)}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfc}(bx)e^{c+dx^2}}{3x^3}$$

input
```
Int[(E^(c + d*x^2)*Erfc[b*x])/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erfc}\left(b x\right)}{x^4}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^4,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^4,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(d*x^2 + c)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 14.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^4}\,dx = e^c\int \frac{e^{dx^2}\operatorname{erfc}\left(bx\right)}{x^4}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x)/x**4,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^4}\,dx=\int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfc}(bx)}{x^4}\,dx=\int \frac{\text{erfc}\,(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(d*x^2 + c)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^4}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erfc}(b\,x)}{x^4}\,dx$$

input
```
int((exp(c + d*x^2)*erfc(b*x))/x^4,x)
```

output
```
int((exp(c + d*x^2)*erfc(b*x))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(bx)}{x^4}\,dx = e^c\left(\int \frac{e^{d\,x^2}}{x^4}dx - \left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx)}{x^4}dx\right)\right)$$

input
```
int(exp(d*x^2+c)*erfc(b*x)/x^4,x)
```

output
```
e**c*(int(e**(d*x**2)/x**4,x) - int((e**(d*x**2)*erf(b*x))/x**4,x))
```

3.167 $\int e^{c+b^2x^2}x^5\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx=\frac{2e^cx}{b^5\sqrt{\pi}}-\frac{2e^cx^3}{3b^3\sqrt{\pi}}+\frac{e^cx^5}{5b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfc}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfc}(bx)}{2b^2}$$

output
```
2*exp(c)*x/b^5/Pi^(1/2)-2/3*exp(c)*x^3/b^3/Pi^(1/2)+1/5*exp(c)*x^5/b/Pi^(1
/2)+exp(b^2*x^2+c)*erfc(b*x)/b^6-exp(b^2*x^2+c)*x^2*erfc(b*x)/b^4+1/2*exp(
b^2*x^2+c)*x^4*erfc(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx=\frac{e^c\left(60bx-20b^3x^3+6b^5x^5+15e^{b^2x^2}\sqrt{\pi}(2-2b^2x^2+b^4x^4)\,\text{erfc}(bx)\right)}{30b^6\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^5*Erfc[b*x],x]
```

output
```
(E^c*(60*b*x - 20*b^3*x^3 + 6*b^5*x^5 + 15*E^(b^2*x^2)*Sqrt[Pi]*(2 - 2*b^2
*x^2 + b^4*x^4)*Erfc[b*x]))/(30*b^6*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6940, 15, 6940, 15, 6937, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{b^2x^2+c}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6940}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erfc}(bx)dx}{b^2}+\frac{\int e^cx^4dx}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erfc}(bx)dx}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{6940}$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\text{erfc}(bx)dx}{b^2}+\frac{\int e^cx^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\text{erfc}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^3}{3\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{6937}$$

$$-\frac{2\left(-\frac{\frac{\int e^cdx}{\sqrt{\pi}b}+\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^3}{3\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^4 e^{b^2x^2+c}\text{erfc}(bx)}{2b^2} - \frac{2\left(\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2} - \frac{\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx}{\sqrt{\pi}b}}{b^2} + \frac{e^cx^3}{3\sqrt{\pi}b}\right)}{b^2} + \frac{e^cx^5}{5\sqrt{\pi}b}$$

input

```
Int[E^(c + b^2*x^2)*x^5*Erfc[b*x],x]
```

output

```
(E^c*x^5)/(5*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^4*Erfc[b*x])/(2*b^2) - (2*((
E^c*x^3)/(3*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^2*Erfc[b*x])/(2*b^2) - ((E^c*
x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2))/b^2))/b^2
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6937

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{e^{c}\left(\frac{e^{b^{2}x^{2}}b^{4}x^{4}}{2}-b^{2}x^{2}e^{b^{2}x^{2}}+e^{b^{2}x^{2}}\right)}{b^{5}}-\frac{\operatorname{erf}(bx)e^{c}\left(\frac{e^{b^{2}x^{2}}b^{4}x^{4}}{2}-b^{2}x^{2}e^{b^{2}x^{2}}+e^{b^{2}x^{2}}\right)}{b^{5}}+\frac{e^{c}\left(\frac{1}{5}b^{5}x^{5}-\frac{2}{3}b^{3}x^{3}+2bx\right)}{\sqrt{\pi}\,b^{5}}}{b}$
parallelrisch	$\frac{6\,e^{b^{2}x^{2}+c}e^{-b^{2}x^{2}}x^{5}b^{5}+15\,e^{b^{2}x^{2}+c}x^{4}\operatorname{erfc}(bx)b^{4}\sqrt{\pi}-20\,e^{b^{2}x^{2}+c}e^{-b^{2}x^{2}}x^{3}b^{3}-30\,e^{b^{2}x^{2}+c}x^{2}\operatorname{erfc}(bx)b^{2}\sqrt{\pi}+60\,e^{b^{2}x^{2}+c}x\,e^{-b^{2}x}}{30b^{6}\sqrt{\pi}}$

input
```
int(exp(b^2*x^2+c)*x^5*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/b^5*exp(c)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2*exp(b^2*x^2)+exp(b^2*x^2))
-erf(b*x)/b^5*exp(c)*(1/2*exp(b^2*x^2)*b^4*x^4-b^2*x^2*exp(b^2*x^2)+exp(b^
2*x^2))+1/Pi^(1/2)/b^5*exp(c)*(1/5*b^5*x^5-2/3*b^3*x^3+2*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int e^{c+b^{2}x^{2}}x^{5}\operatorname{erfc}(bx)\,dx$$
$$=\frac{2\sqrt{\pi}(3\,b^{5}x^{5}-10\,b^{3}x^{3}+30\,bx)e^{c}+15\,(2\,\pi+\pi b^{4}x^{4}-2\,\pi b^{2}x^{2}-(2\,\pi+\pi b^{4}x^{4}-2\,\pi b^{2}x^{2})\operatorname{erf}(bx))e^{(b^{2}x^{2}+c}}{30\,\pi b^{6}}$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erfc(b*x),x, algorithm="fricas")
```

output
```
1/30*(2*sqrt(pi)*(3*b^5*x^5 - 10*b^3*x^3 + 30*b*x)*e^c + 15*(2*pi + pi*b^4
*x^4 - 2*pi*b^2*x^2 - (2*pi + pi*b^4*x^4 - 2*pi*b^2*x^2)*erf(b*x))*e^(b^2*
x^2 + c))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 36.56 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx$$
$$=\begin{cases} \frac{x^5e^c}{5\sqrt{\pi}b}+\frac{x^4e^ce^{b^2x^2}\,\text{erfc}\,(bx)}{2b^2}-\frac{2x^3e^c}{3\sqrt{\pi}b^3}-\frac{x^2e^ce^{b^2x^2}\,\text{erfc}\,(bx)}{b^4}+\frac{2xe^c}{\sqrt{\pi}b^5}+\frac{e^ce^{b^2x^2}\,\text{erfc}\,(bx)}{b^6} & \text{for } b\neq 0\\ \frac{x^6e^c}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x**5*erfc(b*x),x)
```

output
```
Piecewise((x**5*exp(c)/(5*sqrt(pi)*b) + x**4*exp(c)*exp(b**2*x**2)*erfc(b*
x)/(2*b**2) - 2*x**3*exp(c)/(3*sqrt(pi)*b**3) - x**2*exp(c)*exp(b**2*x**2)
*erfc(b*x)/b**4 + 2*x*exp(c)/(sqrt(pi)*b**5) + exp(c)*exp(b**2*x**2)*erfc(
b*x)/b**6, Ne(b, 0)), (x**6*exp(c)/6, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx=\int x^5\,\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^5*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx=\int x^5\,\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx$$
$$= \frac{e^c\left(60\,b\,x-20\,b^3\,x^3+6\,b^5\,x^5+30\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfc}(b\,x)-30\,b^2\,x^2\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfc}(b\,x)+15\,b^4\,x^4\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfc}\right.}{30\,b^6\,\sqrt{\pi}}$$

input
```
int(x^5*exp(c + b^2*x^2)*erfc(b*x),x)
```

output
```
(exp(c)*(60*b*x - 20*b^3*x^3 + 6*b^5*x^5 + 30*pi^(1/2)*exp(b^2*x^2)*erfc(b
*x) - 30*b^2*x^2*pi^(1/2)*exp(b^2*x^2)*erfc(b*x) + 15*b^4*x^4*pi^(1/2)*exp
(b^2*x^2)*erfc(b*x)))/(30*b^6*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int e^{c+b^2x^2}x^5\text{erfc}(bx)\,dx$$
$$= \frac{e^c\left(-15e^{b^2x^2}\text{erf}(bx)\,b^4\pi\,x^4+30e^{b^2x^2}\text{erf}(bx)\,b^2\pi\,x^2-30e^{b^2x^2}\text{erf}(bx)\,\pi+15e^{b^2x^2}b^4\pi\,x^4-30e^{b^2x^2}b^2\pi\,x^2+3\right.}{30b^6\pi}$$

input
```
int(exp(b^2*x^2+c)*x^5*erfc(b*x),x)
```

output
```
(e**c*( - 15*e**(b**2*x**2)*erf(b*x)*b**4*pi*x**4 + 30*e**(b**2*x**2)*erf(
b*x)*b**2*pi*x**2 - 30*e**(b**2*x**2)*erf(b*x)*pi + 15*e**(b**2*x**2)*b**4
*pi*x**4 - 30*e**(b**2*x**2)*b**2*pi*x**2 + 30*e**(b**2*x**2)*pi + 6*sqrt(
pi)*b**5*x**5 - 20*sqrt(pi)*b**3*x**3 + 60*sqrt(pi)*b*x))/(30*b**6*pi)
```

3.168 $\int e^{c+b^2x^2}x^3\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx=-\frac{e^cx}{b^3\sqrt{\pi}}+\frac{e^cx^3}{3b\sqrt{\pi}}-\frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erfc}(bx)}{2b^2}$$

output
```
-exp(c)*x/b^3/Pi^(1/2)+1/3*exp(c)*x^3/b/Pi^(1/2)-1/2*exp(b^2*x^2+c)*erfc(b
*x)/b^4+1/2*exp(b^2*x^2+c)*x^2*erfc(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx=\frac{e^c\left(2bx(-3+b^2x^2)+3e^{b^2x^2}\sqrt{\pi}(-1+b^2x^2)\,\text{erfc}(bx)\right)}{6b^4\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^3*Erfc[b*x],x]
```

output
```
(E^c*(2*b*x*(-3 + b^2*x^2) + 3*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erfc[b*
x]))/(6*b^4*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6940, 15, 6937, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{b^2x^2+c}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6940}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erfc}(bx)dx}{b^2}+\frac{\int e^cx^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erfc}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^3}{3\sqrt{\pi}b}$$

$$\Big\downarrow \text{6937}$$

$$-\frac{\frac{\int e^cdx}{\sqrt{\pi}b}+\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^3}{3\sqrt{\pi}b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^2e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}-\frac{\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx}{\sqrt{\pi}b}}{b^2}+\frac{e^cx^3}{3\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^3*Erfc[b*x],x]
```

output
```
(E^c*x^3)/(3*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^2*Erfc[b*x])/(2*b^2) - ((E^c
*x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2))/b^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\frac{e^{c}\left(\frac{b^{2} x^{2} e^{b^{2} x^{2}}}{2}-\frac{e^{b^{2} x^{2}}}{2}\right)}{b^{3}}-\frac{\operatorname{erf}(b x) e^{c}\left(\frac{b^{2} x^{2} e^{b^{2} x^{2}}}{2}-\frac{e^{b^{2} x^{2}}}{2}\right)}{b^{3}}+\frac{e^{c}\left(\frac{1}{3} b^{3} x^{3}-b x\right)}{\sqrt{\pi}\, b^{3}}}{b}$	99
parallelrisch	$\frac{2 e^{b^{2} x^{2}+c} e^{-b^{2} x^{2}} x^{3} b^{3}+3 e^{b^{2} x^{2}+c} x^{2} \operatorname{erfc}(b x) b^{2} \sqrt{\pi}-6 e^{b^{2} x^{2}+c} x\, e^{-b^{2} x^{2}} b-3 \operatorname{erfc}(b x) e^{b^{2} x^{2}+c} \sqrt{\pi}}{6 \sqrt{\pi}\, b^{4}}$	104

input
```
int(exp(b^2*x^2+c)*x^3*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
(1/b^3*exp(c)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp(b^2*x^2))-erf(b*x)/b^3*exp
(c)*(1/2*b^2*x^2*exp(b^2*x^2)-1/2*exp(b^2*x^2))+1/Pi^(1/2)/b^3*exp(c)*(1/3
*b^3*x^3-b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx = \frac{2\sqrt{\pi}(b^3x^3-3\,bx)e^c-3\,(\pi-\pi b^2x^2-(\pi-\pi b^2x^2)\,\text{erf}\,(bx))e^{(b^2x^2+c)}}{6\,\pi b^4}$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erfc(b*x),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(pi)*(b^3*x^3 - 3*b*x)*e^c - 3*(pi - pi*b^2*x^2 - (pi - pi*b^2*
x^2)*erf(b*x))*e^(b^2*x^2 + c))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx = \begin{cases} \frac{x^3e^c}{3\sqrt{\pi}b}+\frac{x^2e^ce^{b^2x^2}\,\text{erfc}\,(bx)}{2b^2}-\frac{xe^c}{\sqrt{\pi}b^3}-\frac{e^ce^{b^2x^2}\,\text{erfc}\,(bx)}{2b^4} & \text{for } b\neq 0 \\ \frac{x^4e^c}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x**3*erfc(b*x),x)
```

output
```
Piecewise((x**3*exp(c)/(3*sqrt(pi)*b) + x**2*exp(c)*exp(b**2*x**2)*erfc(b*
x)/(2*b**2) - x*exp(c)/(sqrt(pi)*b**3) - exp(c)*exp(b**2*x**2)*erfc(b*x)/(
2*b**4), Ne(b, 0)), (x**4*exp(c)/4, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx = \int x^3\,\text{erfc}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx = \int x^3\,\text{erfc}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx = -\frac{e^c\left(6\,b\,x-2\,b^3\,x^3+3\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfc}(b\,x)-3\,b^2\,x^2\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfc}(b\,x)\right)}{6\,b^4\,\sqrt{\pi}}$$

input
```
int(x^3*exp(c + b^2*x^2)*erfc(b*x),x)
```

output
```
-(exp(c)*(6*b*x - 2*b^3*x^3 + 3*pi^(1/2)*exp(b^2*x^2)*erfc(b*x) - 3*b^2*x^
2*pi^(1/2)*exp(b^2*x^2)*erfc(b*x)))/(6*b^4*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int e^{c+b^2x^2}x^3\text{erfc}(bx)\,dx$$
$$=\frac{e^c\left(-3e^{b^2x^2}\text{erf}(bx)\,b^2\pi\,x^2+3e^{b^2x^2}\text{erf}(bx)\,\pi+3e^{b^2x^2}b^2\pi\,x^2-3e^{b^2x^2}\pi+2\sqrt{\pi}\,b^3x^3-6\sqrt{\pi}\,bx\right)}{6b^4\pi}$$

input
```
int(exp(b^2*x^2+c)*x^3*erfc(b*x),x)
```

output
```
(e**c*( - 3*e**(b**2*x**2)*erf(b*x)*b**2*pi*x**2 + 3*e**(b**2*x**2)*erf(b*
x)*pi + 3*e**(b**2*x**2)*b**2*pi*x**2 - 3*e**(b**2*x**2)*pi + 2*sqrt(pi)*b
**3*x**3 - 6*sqrt(pi)*b*x))/(6*b**4*pi)
```

3.169 $\int e^{c+b^2x^2}x\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx=\frac{e^cx}{b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^2}$$

output
```
exp(c)*x/b/Pi^(1/2)+1/2*exp(b^2*x^2+c)*erfc(b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx=\frac{e^cx}{b\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfc}(bx)}{2b^2}$$

input
```
Integrate[E^(c + b^2*x^2)*x*Erfc[b*x],x]
```

output
```
(E^c*x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6937, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{b^2x^2+c}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{ 6937}$$

$$\frac{\int e^c dx}{\sqrt{\pi}b} + \frac{e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2b^2} + \frac{e^c x}{\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x*Erfc[b*x],x]
```

output
```
(E^c*x)/(b*Sqrt[Pi]) + (E^(c + b^2*x^2)*Erfc[b*x])/(2*b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{2\,\mathrm{e}^{b^{2} x^{2}+c} x\,\mathrm{e}^{-b^{2} x^{2}} b+\operatorname{erfc}\left(b x\right) \mathrm{e}^{b^{2} x^{2}+c} \sqrt{\pi}}{2 b^{2} \sqrt{\pi}}$	51
parallelrisch	$\frac{2\,\mathrm{e}^{b^{2} x^{2}+c} x\,\mathrm{e}^{-b^{2} x^{2}} b+\operatorname{erfc}\left(b x\right) \mathrm{e}^{b^{2} x^{2}+c} \sqrt{\pi}}{2 b^{2} \sqrt{\pi}}$	51

input
```
int(exp(b^2*x^2+c)*x*erfc(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*exp(b^2*x^2+c)*x*exp(-b^2*x^2)*b+erfc(b*x)*exp(b^2*x^2+c)*Pi^(1/2))
/b^2/Pi^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int e^{c+b^2x^2}x\operatorname{erfc}(bx)\,dx = \frac{2\,\sqrt{\pi}bxe^{c}+(\pi-\pi\operatorname{erf}(bx))e^{(b^2x^2+c)}}{2\,\pi b^2}$$

input
```
integrate(exp(b^2*x^2+c)*x*erfc(b*x),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(pi)*b*x*e^c + (pi - pi*erf(b*x))*e^(b^2*x^2 + c))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int e^{c+b^2x^2}x\operatorname{erfc}(bx)\,dx = \begin{cases} \frac{xe^c}{\sqrt{\pi}b}+\frac{e^ce^{b^2x^2}\operatorname{erfc}(bx)}{2b^2} & \text{for } b\neq 0\\ \frac{x^2e^c}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x*erfc(b*x),x)
```

output
```
Piecewise((x*exp(c)/(sqrt(pi)*b) + exp(c)*exp(b**2*x**2)*erfc(b*x)/(2*b**2
), Ne(b, 0)), (x**2*exp(c)/2, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx=\int x\,\text{erfc}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx=\int x\,\text{erfc}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int e^{c+b^2x^2}x\text{erfc}(bx)\,dx=\frac{x\,\mathrm{e}^c}{b\,\sqrt{\pi}}+\frac{\mathrm{e}^{b^2\,x^2}\,\mathrm{e}^c\,\mathrm{erfc}(b\,x)}{2\,b^2}$$

input
```
int(x*exp(c + b^2*x^2)*erfc(b*x),x)
```

output
```
(x*exp(c))/(b*pi^(1/2)) + (exp(b^2*x^2)*exp(c)*erfc(b*x))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int e^{c+b^2x^2}x\operatorname{erfc}(bx)\,dx=\frac{e^c\left(-e^{b^2x^2}\operatorname{erf}(bx)\,\pi+e^{b^2x^2}\pi+2\sqrt{\pi}\,bx\right)}{2b^2\pi}$$

input
```
int(exp(b^2*x^2+c)*x*erfc(b*x),x)
```

output
```
(e**c*( - e**(b**2*x**2)*erf(b*x)*pi + e**(b**2*x**2)*pi + 2*sqrt(pi)*b*x)
)/(2*b**2*pi)
```

3.170 $\int \frac{e^{c+b^2x^2}\mathbf{erfc}(bx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x}\,dx = \frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right) - \frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
1/2*exp(c)*Ei(b^2*x^2)-2*b*exp(c)*x*hypergeom([1/2, 1],[3/2, 3/2],b^2*x^2)
/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x}\,dx = \frac{1}{2}e^c\left(\text{ExpIntegralEi}\left(b^2x^2\right) - \frac{4bx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}\right)$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x,x]
```

output
```
(E^c*(ExpIntegralEi[b^2*x^2] - (4*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/
2}, b^2*x^2])/Sqrt[Pi]))/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6943, 2639, 6942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}\,dx$$
$$\Big\downarrow \text{ 6943}$$
$$\int \frac{e^{b^2x^2+c}}{x}dx-\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx$$
$$\Big\downarrow \text{ 2639}$$
$$\frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx$$
$$\Big\downarrow \text{ 6942}$$
$$\frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[(E^(c + b^2*x^2)*Erfc[b*x])/x,x]
```

output
```
(E^c*ExpIntegralEi[b^2*x^2])/2 - (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3
/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6942
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b*
E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

rule 6943
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)])/(x_), x_Symbol] :> Int[E^(c
 + d*x^2)/x, x] - Int[E^(c + d*x^2)*(Erf[b*x]/x), x] /; FreeQ[{b, c, d}, x]
 && EqQ[d, b^2]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}\,\text{erfc}\,(bx)}{x}dx$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x,x)
```

output
```
int(exp(b^2*x^2+c)*erfc(b*x)/x,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x, x)
```

Sympy [A] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx = -\frac{2bxe^c{}_2F_2\left(\begin{matrix}\frac{1}{2},1\\ \frac{3}{2},\frac{3}{2}\end{matrix}\middle|b^2x^2\right)}{\sqrt{\pi}} + \frac{e^c\operatorname{Ei}(b^2x^2)}{2}$$

input
```
integrate(exp(b**2*x**2+c)*erfc(b*x)/x,x)
```

output
```
-2*b*x*exp(c)*hyper((1/2, 1), (3/2, 3/2), b**2*x**2)/sqrt(pi) + exp(c)*Ei(
b**2*x**2)/2
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx = \int \frac{e^{b^2\,x^2+c}\operatorname{erfc}(b\,x)}{x}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfc(b*x))/x,x)
```

output
```
int((exp(c + b^2*x^2)*erfc(b*x))/x, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x}\,dx = \frac{e^c\left(ei(b^2x^2)-2\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x}dx\right)\right)}{2}$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x,x)
```

output
```
(e**c*(ei(b**2*x**2) - 2*int((e**(b**2*x**2)*erf(b*x))/x,x)))/2
```

3.171 $\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx = \frac{be^c}{\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{2x^2} + \frac{1}{2}b^2e^c\,\text{ExpIntegralEi}\left(b^2x^2\right) - \frac{2b^3e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
b*exp(c)/Pi^(1/2)/x-1/2*exp(b^2*x^2+c)*erfc(b*x)/x^2+1/2*b^2*exp(c)*Ei(b^2
*x^2)-2*b^3*exp(c)*x*hypergeom([1/2, 1],[3/2, 3/2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx = -\frac{e^c\left(e^{b^2x^2} - b^2x^2\,\text{ExpIntegralEi}\left(b^2x^2\right) - \frac{4bx\,{}_2F_2\left(-\frac{1}{2},1;\frac{1}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}\right)}{2x^2}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^3,x]
```

output
```
-1/2*(E^c*(E^(b^2*x^2) - b^2*x^2*ExpIntegralEi[b^2*x^2] - (4*b*x*Hypergeom
etricPFQ[{-1/2, 1}, {1/2, 3/2}, b^2*x^2])/Sqrt[Pi]))/x^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6946, 15, 6943, 2639, 6942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^3}dx$$

$$\Big\downarrow \text{ 6946}$$

$$b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}dx-\frac{b\int \frac{e^c}{x^2}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}$$

$$\Big\downarrow \text{ 15}$$

$$b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}dx-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}+\frac{be^c}{\sqrt{\pi}x}$$

$$\Big\downarrow \text{ 6943}$$

$$b^2\left(\int \frac{e^{b^2x^2+c}}{x}dx-\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}+\frac{be^c}{\sqrt{\pi}x}$$

$$\Big\downarrow \text{ 2639}$$

$$b^2\left(\frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}+\frac{be^c}{\sqrt{\pi}x}$$

$$\Big\downarrow \text{ 6942}$$

$$b^2\left(\frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}+\frac{be^c}{\sqrt{\pi}x}$$

input
```
Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^3,x]
```

output
```
(b*E^c)/(Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erfc[b*x])/(2*x^2) + b^2*((E^c*Exp
IntegralEi[b^2*x^2])/2 - (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}
, b^2*x^2])/Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6942
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b*
E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

rule 6943
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)])/(x_), x_Symbol] :> Int[E^(c
 + d*x^2)/x, x] - Int[E^(c + d*x^2)*(Erf[b*x]/x), x] /; FreeQ[{b, c, d}, x]
 && EqQ[d, b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}\operatorname{erfc}(bx)}{x^3}dx$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^3,x)
```

output
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^3,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\mathrm{erfc}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^3, x)
```

Sympy [A] (verification not implemented)

Time = 30.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{e^{c+b^2x^2}\mathrm{erfc}(bx)}{x^3}\,dx = \frac{b^2e^c\operatorname{Ei}(b^2x^2)}{2} + \frac{2be^c{}_2F_2\left(\begin{matrix}-\frac{1}{2},1\\ \frac{1}{2},\frac{3}{2}\end{matrix}\middle|b^2x^2\right)}{\sqrt{\pi}x} - \frac{e^ce^{b^2x^2}}{2x^2}$$

input
```
integrate(exp(b**2*x**2+c)*erfc(b*x)/x**3,x)
```

output
```
b**2*exp(c)*Ei(b**2*x**2)/2 + 2*b*exp(c)*hyper((-1/2, 1), (1/2, 3/2), b**2
*x**2)/(sqrt(pi)*x) - exp(c)*exp(b**2*x**2)/(2*x**2)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\,\text{erfc}(b\,x)}{x^3}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^3,x)
```

output
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^3}\,dx = \frac{e^c\left(ei(b^2x^2)\,b^2x^2 - e^{b^2x^2} - 2\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x^3}dx\right)x^2\right)}{2x^2}$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^3,x)
```

output
```
(e**c*(ei(b**2*x**2)*b**2*x**2 - e**(b**2*x**2) - 2*int((e**(b**2*x**2)*er
f(b*x))/x**3,x)*x**2))/(2*x**2)
```

3.172 $\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^5}\,dx=\frac{be^c}{6\sqrt{\pi}x^3}+\frac{b^3e^c}{2\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\text{erfc}(bx)}{4x^4}-\frac{b^2e^{c+b^2x^2}\text{erfc}(bx)}{4x^2}+\frac{1}{4}b^4e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\frac{b^5e^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
1/6*b*exp(c)/Pi^(1/2)/x^3+1/2*b^3*exp(c)/Pi^(1/2)/x-1/4*exp(b^2*x^2+c)*erf
c(b*x)/x^4-1/4*b^2*exp(b^2*x^2+c)*erfc(b*x)/x^2+1/4*b^4*exp(c)*Ei(b^2*x^2)
-b^5*exp(c)*x*hypergeom([1/2, 1],[3/2, 3/2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^5}\,dx=-\frac{e^c\left(3\sqrt{\pi}\left(e^{b^2x^2}(1+b^2x^2)-b^4x^4\,\text{ExpIntegralEi}\left(b^2x^2\right)\right)-8bx\,{}_2F_2\left(-\frac{3}{2},1;-\frac{1}{2},\frac{3}{2};b^2x^2\right)\right)}{12\sqrt{\pi}x^4}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^5,x]
```

output
```
-1/12*(E^c*(3*Sqrt[Pi]*(E^(b^2*x^2)*(1 + b^2*x^2) - b^4*x^4*ExpIntegralEi[
b^2*x^2]) - 8*b*x*HypergeometricPFQ[{-3/2, 1}, {-1/2, 3/2}, b^2*x^2]))/(Sq
rt[Pi]*x^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6946, 15, 6946, 15, 6943, 2639, 6942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^5}\,dx$$

$$\Big\downarrow \text{6946}$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^3}dx - \frac{b\int \frac{e^c}{x^4}dx}{2\sqrt{\pi}} - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^3}dx - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4} + \frac{be^c}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{6946}$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}dx - \frac{b\int \frac{e^c}{x^2}dx}{\sqrt{\pi}} - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4} + \frac{be^c}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}dx - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2} + \frac{be^c}{\sqrt{\pi}x}\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4} + \frac{be^c}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{6943}$$

$$\frac{1}{2}b^2\left(b^2\left(\int \frac{e^{b^2x^2+c}}{x}dx - \int \frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2} + \frac{be^c}{\sqrt{\pi}x}\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4} + \frac{be^c}{6\sqrt{\pi}x^3}$$

$$\downarrow \text{2639}$$

$$\frac{1}{2}b^2\left(b^2\left(\frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\int\frac{e^{b^2x^2+c}\text{erf}(bx)}{x}dx\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}+\frac{be^c}{\sqrt{\pi}x}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4}+\frac{be^c}{6\sqrt{\pi}x^3}$$

$$\downarrow \text{6942}$$

$$\frac{1}{2}b^2\left(b^2\left(\frac{1}{2}e^c\,\text{ExpIntegralEi}\left(b^2x^2\right)-\frac{2be^cx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{2x^2}+\frac{be^c}{\sqrt{\pi}x}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{4x^4}+\frac{be^c}{6\sqrt{\pi}x^3}$$

input
```
Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^5,x]
```

output
```
(b*E^c)/(6*Sqrt[Pi]*x^3) - (E^(c + b^2*x^2)*Erfc[b*x])/(4*x^4) + (b^2*((b*
E^c)/(Sqrt[Pi]*x) - (E^(c + b^2*x^2)*Erfc[b*x])/(2*x^2) + b^2*((E^c*ExpInt
egralEi[b^2*x^2])/2 - (2*b*E^c*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b
^2*x^2])/Sqrt[Pi])))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6942
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b*
E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, b^2*x^2], x] /; Fr
eeQ[{b, c, d}, x] && EqQ[d, b^2]
```

rule 6943
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)])/(x_), x_Symbol] :> Int[E^(c
 + d*x^2)/x, x] - Int[E^(c + d*x^2)*(Erf[b*x]/x), x] /; FreeQ[{b, c, d}, x]
 && EqQ[d, b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}\,\mathrm{erfc}\,(bx)}{x^5}dx$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^5,x)
```

output
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^5,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\mathrm{erfc}(bx)}{x^5}\,dx = \int \frac{\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(exp(b**2*x**2+c)*erfc(b*x)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{e^{b^2\,x^2+c}\,\text{erfc}(b\,x)}{x^5}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^5,x)
```

output
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^5, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^5}\,dx = \frac{e^c\left(3ei(b^2x^2)\,b^4\pi\,x^4+3e^{b^2x^2}\text{erf}(bx)\,b^2\pi\,x^2+3e^{b^2x^2}\text{erf}(bx)\,\pi-3e^{b^2x^2}b^2\pi\,x^2-3e^{b^2x^2}\pi+6\sqrt{\pi}\,b^3x^3+2\sqrt{\pi}\right.}{12\pi\,x^4}$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^5,x)
```

output
```
(e**c*(3*ei(b**2*x**2)*b**4*pi*x**4 + 3*e**(b**2*x**2)*erf(b*x)*b**2*pi*x*
*2 + 3*e**(b**2*x**2)*erf(b*x)*pi - 3*e**(b**2*x**2)*b**2*pi*x**2 - 3*e**(
b**2*x**2)*pi + 6*sqrt(pi)*b**3*x**3 + 2*sqrt(pi)*b*x - 6*int((e**(b**2*x*
*2)*erf(b*x))/x,x)*b**4*pi*x**4))/(12*pi*x**4)
```

3.173 $\int e^{c+b^2x^2}x^4\textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int e^{c+b^2x^2}x^4\text{erfc}(bx)\,dx=-\frac{3e^cx^2}{4b^3\sqrt{\pi}}+\frac{e^cx^4}{4b\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\text{erfc}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\text{erfc}(bx)}{2b^2}+\frac{3e^c\sqrt{\pi}\text{erfi}(bx)}{8b^5}-\frac{3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{4b^3\sqrt{\pi}}$$

output
```
-3/4*exp(c)*x^2/b^3/Pi^(1/2)+1/4*exp(c)*x^4/b/Pi^(1/2)-3/4*exp(b^2*x^2+c)*
x*erfc(b*x)/b^4+1/2*exp(b^2*x^2+c)*x^3*erfc(b*x)/b^2+3/8*exp(c)*Pi^(1/2)*e
rfi(b*x)/b^5-3/4*exp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/b^3/Pi^(1/2
)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int e^{c+b^2x^2}x^4\text{erfc}(bx)\,dx=-\frac{e^c\left(6be^{b^2x^2}\sqrt{\pi}x+6b^2x^2-4b^3e^{b^2x^2}\sqrt{\pi}x^3-2b^4x^4+2be^{b^2x^2}\sqrt{\pi}x(-3+2b^2x^2)\,\text{erf}(bx)-3\pi\text{erfi}(bx)+3\pi\right.}{8b^5\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^4*Erfc[b*x],x]
```

output
```
-1/8*(E^c*(6*b*E^(b^2*x^2)*Sqrt[Pi]*x + 6*b^2*x^2 - 4*b^3*E^(b^2*x^2)*Sqrt
[Pi]*x^3 - 2*b^4*x^4 + 2*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*b^2*x^2)*Erf[b*x
] - 3*Pi*Erfi[b*x] + 3*Pi*Erf[b*x]*Erfi[b*x] - 6*b^2*x^2*HypergeometricPFQ
[{1, 1}, {3/2, 2}, -(b^2*x^2)]))/(b^5*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6940, 15, 6940, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{b^2x^2+c}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6940}$$

$$-\frac{3\int e^{b^2x^2+c}x^2\text{erfc}(bx)dx}{2b^2}+\frac{\int e^cx^3dx}{\sqrt{\pi}b}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\int e^{b^2x^2+c}x^2\text{erfc}(bx)dx}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6940}$$

$$-\frac{3\left(-\frac{\int e^{b^2x^2+c}\text{erfc}(bx)dx}{2b^2}+\frac{\int e^cxdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{\int e^{b^2x^2+c}\text{erfc}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6931}$$

$$-\frac{3\left(-\frac{\int e^{b^2x^2+c}dx-\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{3\left(-\frac{\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6930}$$

$$-\frac{3\left(-\frac{\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^4}{4\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^4*Erfc[b*x],x]
```

output
```
(E^c*x^4)/(4*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x^3*Erfc[b*x])/(2*b^2) - (3*((
E^c*x^2)/(2*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfc[b*x])/(2*b^2) - ((E^c*Sq
rt[Pi]*Erfi[b*x])/(2*b) - (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b
^2*x^2])/Sqrt[Pi])/(2*b^2)))/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{b^2x^2+c}x^4\,\mathrm{erfc}\,(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^4*erfc(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^4*erfc(b*x),x)
```

Fricas [F]

$$\int e^{c+b^2x^2}x^4\mathrm{erfc}(bx)\,dx=\int x^4\,\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-(x^4*erf(b*x) - x^4)*e^(b^2*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}x^4\mathrm{erfc}(bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b**2*x**2+c)*x**4*erfc(b*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{c+b^2x^2}x^4\mathrm{erfc}(bx)\,dx = \int x^4\,\mathrm{erfc}\left(bx\right)e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^4\mathrm{erfc}(bx)\,dx = \int x^4\,\mathrm{erfc}\left(bx\right)e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}x^4\text{erfc}(bx)\,dx=\int x^4\,\mathrm{e}^{b^2\,x^2+c}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x^4*exp(c + b^2*x^2)*erfc(b*x),x)
```

output
```
int(x^4*exp(c + b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{c+b^2x^2}x^4\text{erfc}(bx)\,dx=\frac{e^c\left(-3\sqrt{\pi}\,\text{erf}(bix)\,i+4e^{b^2x^2}b^3x^3-6e^{b^2x^2}bx-8\left(\int e^{b^2x^2}\text{erf}(bx)\,x^4dx\right)b^5\right)}{8b^5}$$

input
```
int(exp(b^2*x^2+c)*x^4*erfc(b*x),x)
```

output
```
(e**c*( - 3*sqrt(pi)*erf(b*i*x)*i + 4*e**(b**2*x**2)*b**3*x**3 - 6*e**(b**
2*x**2)*b*x - 8*int(e**(b**2*x**2)*erf(b*x)*x**4,x)*b**5))/(8*b**5)
```

3.174 $\int e^{c+b^2x^2}x^2\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int e^{c+b^2x^2}x^2\operatorname{erfc}(bx)\,dx=\frac{e^cx^2}{2b\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\operatorname{erfc}(bx)}{2b^2}-\frac{e^c\sqrt{\pi}\operatorname{erfi}(bx)}{4b^3}+\frac{e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2b\sqrt{\pi}}$$

output
```
1/2*exp(c)*x^2/b/Pi^(1/2)+1/2*exp(b^2*x^2+c)*x*erfc(b*x)/b^2-1/4*exp(c)*Pi
^(1/2)*erfi(b*x)/b^3+1/2*exp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/b/P
i^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int e^{c+b^2x^2}x^2\operatorname{erfc}(bx)\,dx=-\frac{e^c\left(-2be^{b^2x^2}\sqrt{\pi}x-2b^2x^2+\pi\operatorname{erfi}(bx)+\operatorname{erf}(bx)\left(2be^{b^2x^2}\sqrt{\pi}x-\pi\operatorname{erfi}(bx)\right)+2b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x\right.\right.}{4b^3\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^2*Erfc[b*x],x]
```

output
```
-1/4*(E^c*(-2*b*E^(b^2*x^2)*Sqrt[Pi]*x - 2*b^2*x^2 + Pi*Erfi[b*x] + Erf[b*
x]*(2*b*E^(b^2*x^2)*Sqrt[Pi]*x - Pi*Erfi[b*x]) + 2*b^2*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]))/(b^3*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6940, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{b^2x^2+c}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6940}$$

$$-\frac{\int e^{b^2x^2+c}\text{erfc}(bx)dx}{2b^2}+\frac{\int e^cxdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int e^{b^2x^2+c}\text{erfc}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}$$

$$\Big\downarrow \text{6931}$$

$$-\frac{\int e^{b^2x^2+c}dx-\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\int e^{b^2x^2+c}\text{erf}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}$$

$$\Big\downarrow \text{6930}$$

$$-\frac{\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfc}(bx)}{2b^2}+\frac{e^cx^2}{2\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^2*Erfc[b*x],x]
```

output
```
(E^c*x^2)/(2*b*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfc[b*x])/(2*b^2) - ((E^c*S
qrt[Pi]*Erfi[b*x])/(2*b) - (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
b^2*x^2])/Sqrt[Pi])/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{b^2x^2+c}x^2\operatorname{erfc}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^2*erfc(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^2*erfc(b*x),x)
```

Fricas [F]

$$\int e^{c+b^2x^2}x^2\mathrm{erfc}(bx)\,dx = \int x^2\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-(x^2*erf(b*x) - x^2)*e^(b^2*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int e^{c+b^2x^2}x^2\mathrm{erfc}(bx)\,dx = -\frac{bx^4e^c{}_2F_2\left(\begin{matrix}1,2\\ \frac{3}{2},3\end{matrix}\middle|b^2x^2\right)}{2\sqrt{\pi}}+\frac{xe^ce^{b^2x^2}}{2b^2}+\frac{i\sqrt{\pi}e^c\operatorname{erf}(ibx)}{4b^3}$$

input
```
integrate(exp(b**2*x**2+c)*x**2*erfc(b*x),x)
```

output
```
-b*x**4*exp(c)*hyper((1, 2), (3/2, 3), b**2*x**2)/(2*sqrt(pi)) + x*exp(c)*
exp(b**2*x**2)/(2*b**2) + I*sqrt(pi)*exp(c)*erf(I*b*x)/(4*b**3)
```

Maxima [F]

$$\int e^{c+b^2x^2}x^2\text{erfc}(bx)\,dx=\int x^2\,\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^2\text{erfc}(bx)\,dx=\int x^2\,\text{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erfc(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}x^2\text{erfc}(bx)\,dx=\int x^2\,\mathrm{e}^{b^2\,x^2+c}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x^2*exp(c + b^2*x^2)*erfc(b*x),x)
```

output
```
int(x^2*exp(c + b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{c+b^2x^2}x^2\text{erfc}(bx)\,dx = \frac{e^c\left(\sqrt{\pi}\,\text{erf}(bix)\,i + 2e^{b^2x^2}bx - 4\left(\int e^{b^2x^2}\text{erf}(bx)\,x^2dx\right)b^3\right)}{4b^3}$$

input
```
int(exp(b^2*x^2+c)*x^2*erfc(b*x),x)
```

output
```
(e**c*(sqrt(pi)*erf(b*i*x)*i + 2*e**(b**2*x**2)*b*x - 4*int(e**(b**2*x**2)
*erf(b*x)*x**2,x)*b**3))/(4*b**3)
```

3.175 $\int e^{c+b^2x^2}\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int e^{c+b^2x^2}\text{erfc}(bx)\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)}{2b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

output
```
1/2*exp(c)*Pi^(1/2)*erfi(b*x)/b-b*exp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2
*x^2)/Pi^(1/2)
```

Mathematica [F]

$$\int e^{c+b^2x^2}\text{erfc}(bx)\,dx=\int e^{c+b^2x^2}\text{erfc}(bx)\,dx$$

input
```
Integrate[E^(c + b^2*x^2)*Erfc[b*x],x]
```

output
```
Integrate[E^(c + b^2*x^2)*Erfc[b*x], x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{b^2x^2+c}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6931}$$

$$\int e^{b^2x^2+c}dx - \int e^{b^2x^2+c}\text{erf}(bx)dx$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b} - \int e^{b^2x^2+c}\text{erf}(bx)dx$$

$$\Big\downarrow \text{6930}$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b} - \frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[E^(c + b^2*x^2)*Erfc[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) - (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2
, 2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

Maple [F]

$$\int \mathrm{e}^{b^2x^2+c}\operatorname{erfc}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*erfc(b*x),x)
```

Fricas [F]

$$\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx = \int \operatorname{erfc}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx = -\frac{bx^2e^c{}_2F_2\left(\begin{matrix}1,1\\ \frac{3}{2},2\end{matrix}\middle|b^2x^2\right)}{\sqrt{\pi}} - \frac{i\sqrt{\pi}e^c\operatorname{erf}(ibx)}{2b}$$

input
```
integrate(exp(b**2*x**2+c)*erfc(b*x),x)
```

output
```
-b*x**2*exp(c)*hyper((1, 1), (3/2, 2), b**2*x**2)/sqrt(pi) - I*sqrt(pi)*ex
p(c)*erf(I*b*x)/(2*b)
```

Maxima [F]

$$\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx = \int \operatorname{erfc}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx = \int \operatorname{erfc}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx = \int e^{b^2\,x^2+c}\operatorname{erfc}(b\,x)\,dx$$

input
```
int(exp(c + b^2*x^2)*erfc(b*x),x)
```

output
```
int(exp(c + b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{c+b^2x^2}\operatorname{erfc}(bx)\,dx = \frac{e^c\left(-\sqrt{\pi}\operatorname{erf}(bix)\,i-2\left(\int e^{b^2x^2}\operatorname{erf}(bx)\,dx\right)b\right)}{2b}$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x),x)
```

output
```
(e**c*( - sqrt(pi)*erf(b*i*x)*i - 2*int(e**(b**2*x**2)*erf(b*x),x)*b))/(2*
b)
```

3.176 $\int \frac{e^{c+b^2x^2}\mathbf{erfc}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2}\,dx = -\frac{e^{c+b^2x^2}\text{erfc}(bx)}{x} + be^c\sqrt{\pi}\text{erfi}(bx) - \frac{2b^3e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}} - \frac{2be^c\log(x)}{\sqrt{\pi}}$$

output
```
-exp(b^2*x^2+c)*erfc(b*x)/x+b*exp(c)*Pi^(1/2)*erfi(b*x)-2*b^3*exp(c)*x^2*h
ypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)-2*b*exp(c)*ln(x)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2}\,dx = -\frac{e^c\left(e^{b^2x^2}\sqrt{\pi} - b\pi x\text{erfi}(bx) + \text{erf}(bx)\left(-e^{b^2x^2}\sqrt{\pi} + b\pi x\text{erfi}(bx)\right) - 2b^3x^3\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right) + 2bx\log}{\sqrt{\pi}x}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^2,x]
```

output
```
-((E^c*(E^(b^2*x^2)*Sqrt[Pi] - b*Pi*x*Erfi[b*x] + Erf[b*x]*(-(E^(b^2*x^2)*
Sqrt[Pi]) + b*Pi*x*Erfi[b*x]) - 2*b^3*x^3*HypergeometricPFQ[{1, 1}, {3/2,
2}, -(b^2*x^2)] + 2*b*x*Log[x]))/(Sqrt[Pi]*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6946, 14, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^2}dx$$

$$\Big\downarrow \text{6946}$$

$$2b^2\int e^{b^2x^2+c}\text{erfc}(bx)dx-\frac{2b\int \frac{e^c}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}$$

$$\Big\downarrow \text{14}$$

$$2b^2\int e^{b^2x^2+c}\text{erfc}(bx)dx-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}-\frac{2be^c\log(x)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6931}$$

$$2b^2\left(\int e^{b^2x^2+c}dx-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}-\frac{2be^c\log(x)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$2b^2\left(\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}-\frac{2be^c\log(x)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6930}$$

$$2b^2\left(\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}-\frac{2be^c\log(x)}{\sqrt{\pi}}$$

input
```
Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^2,x]
```

output
```
-((E^(c + b^2*x^2)*Erfc[b*x])/x) + 2*b^2*((E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) -
 (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]) - (2*b
*E^c*Log[x])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}\,\mathrm{erfc}\,(bx)}{x^2}dx$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^2,x)
```

output
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^2,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\mathrm{erfc}(bx)}{x^2}\,dx = \int \frac{\mathrm{erfc}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.98 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+b^2x^2}\mathrm{erfc}(bx)}{x^2}\,dx = -\frac{2b^3x^2e^c{}_2F_2\left(\begin{matrix}1,1\\2,\frac{5}{2}\end{matrix}\middle|b^2x^2\right)}{3\sqrt{\pi}} - \frac{be^c\log\left(b^2x^2\right)}{\sqrt{\pi}} - i\sqrt{\pi}be^c\,\mathrm{erf}\,(ibx) - \frac{e^ce^{b^2x^2}}{x}$$

input
```
integrate(exp(b**2*x**2+c)*erfc(b*x)/x**2,x)
```

output
```
-2*b**3*x**2*exp(c)*hyper((1, 1), (2, 5/2), b**2*x**2)/(3*sqrt(pi)) - b*ex
p(c)*log(b**2*x**2)/sqrt(pi) - I*sqrt(pi)*b*exp(c)*erf(I*b*x) - exp(c)*exp
(b**2*x**2)/x
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2} dx = \int \frac{\text{erfc}\,(bx)\,e^{(b^2x^2+c)}}{x^2} dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2} dx = \int \frac{\text{erfc}\,(bx)\,e^{(b^2x^2+c)}}{x^2} dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2} dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\,\text{erfc}(b\,x)}{x^2} dx$$

input
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^2,x)
```

output
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^2}\,dx = e^c\left(\int \frac{e^{b^2x^2}}{x^2}dx - \left(\int \frac{e^{b^2x^2}\text{erf}(bx)}{x^2}dx\right)\right)$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^2,x)
```

output
```
e**c*(int(e**(b**2*x**2)/x**2,x) - int((e**(b**2*x**2)*erf(b*x))/x**2,x))
```

3.177 $\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \frac{be^c}{3\sqrt{\pi}x^2} - \frac{e^{c+b^2x^2}\text{erfc}(bx)}{3x^3} - \frac{2b^2e^{c+b^2x^2}\text{erfc}(bx)}{3x} + \frac{2}{3}b^3e^c\sqrt{\pi}\text{erfi}(bx) - \frac{4b^5e^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{3\sqrt{\pi}} - \frac{4b^3e^c\log(x)}{3\sqrt{\pi}}$$

output
```
1/3*b*exp(c)/Pi^(1/2)/x^2-1/3*exp(b^2*x^2+c)*erfc(b*x)/x^3-2/3*b^2*exp(b^2
*x^2+c)*erfc(b*x)/x+2/3*b^3*exp(c)*Pi^(1/2)*erfi(b*x)-4/3*b^5*exp(c)*x^2*h
ypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)-4/3*b^3*exp(c)*ln(x)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \frac{e^c\left(-e^{b^2x^2}\sqrt{\pi} + bx - 2b^2e^{b^2x^2}\sqrt{\pi}x^2 + e^{b^2x^2}\sqrt{\pi}(1+2b^2x^2)\,\text{erf}(bx) + 2b^3\pi x^3\text{erfi}(bx) - 2b^3\pi x^3\text{erf}(bx)\text{erfi}(b\right.}{3\sqrt{\pi}x^3}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfc[b*x])/x^4,x]
```

output
```
(E^c*(-(E^(b^2*x^2)*Sqrt[Pi]) + b*x - 2*b^2*E^(b^2*x^2)*Sqrt[Pi]*x^2 + E^(
b^2*x^2)*Sqrt[Pi]*(1 + 2*b^2*x^2)*Erf[b*x] + 2*b^3*Pi*x^3*Erfi[b*x] - 2*b^
3*Pi*x^3*Erf[b*x]*Erfi[b*x] + 4*b^5*x^5*HypergeometricPFQ[{1, 1}, {3/2, 2}
, -(b^2*x^2)] - 4*b^3*x^3*Log[x]))/(3*Sqrt[Pi]*x^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6946, 15, 6946, 14, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^4}dx$$

$$\Big\downarrow \text{6946}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^2}dx - \frac{2b\int \frac{e^c}{x^3}dx}{3\sqrt{\pi}} - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x^2}dx - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3} + \frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6946}$$

$$\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2+c}\text{erfc}(bx)dx - \frac{2b\int \frac{e^c}{x}dx}{\sqrt{\pi}} - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3} + \frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2+c}\text{erfc}(bx)dx - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x} - \frac{2be^c\log(x)}{\sqrt{\pi}}\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3} + \frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6931}$$

$$\frac{2}{3}b^2\left(2b^2\left(\int e^{b^2x^2+c}dx - \int e^{b^2x^2+c}\text{erf}(bx)dx\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{x} - \frac{2be^c\log(x)}{\sqrt{\pi}}\right) - \frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3} + \frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{3}b^2\left(2b^2\left(\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}-\frac{2be^c\log(x)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3}+\frac{be^c}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6930}$$

$$\frac{2}{3}b^2\left(2b^2\left(\frac{\sqrt{\pi}e^c\text{erfi}(bx)}{2b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{x}-\frac{2be^c\log(x)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfc}(bx)}{3x^3}+\frac{be^c}{3\sqrt{\pi}x^2}$$

input
```
Int[(E^(c + b^2*x^2)*Erfc[b*x])/x^4,x]
```

output
```
(b*E^c)/(3*Sqrt[Pi]*x^2) - (E^(c + b^2*x^2)*Erfc[b*x])/(3*x^3) + (2*b^2*(-
((E^(c + b^2*x^2)*Erfc[b*x])/x) + 2*b^2*((E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) -
(b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi]) - (2*b*
E^c*Log[x])/Sqrt[Pi]))/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}\operatorname{erfc}(bx)}{x^4}dx$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^4,x)
```

output
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^4,x)
```

Fricas [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^4,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(b^2*x^2 + c)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 82.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^4}\,dx = -\frac{b^3 G_{3,2}^{1,3}\left(\begin{matrix} 2, \frac{5}{2}, 1 & \\ 2 & 0 \end{matrix} \middle| \frac{1}{b^2x^2}\right)e^c}{2\pi}$$

input
```
integrate(exp(b**2*x**2+c)*erfc(b*x)/x**4,x)
```

output
```
-b**3*meijerg(((2, 5/2, 1), ()), ((2,), (0,)), 1/(b**2*x**2))*exp(c)/(2*pi
)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \int \frac{\text{erfc}(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \int \frac{\text{erfc}(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfc(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx = \int \frac{e^{b^2\,x^2+c}\operatorname{erfc}(b\,x)}{x^4}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^4,x)
```

output
```
int((exp(c + b^2*x^2)*erfc(b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx = e^c\left(\int \frac{e^{b^2x^2}}{x^4}dx - \left(\int \frac{e^{b^2x^2}\operatorname{erf}(bx)}{x^4}dx\right)\right)$$

input
```
int(exp(b^2*x^2+c)*erfc(b*x)/x^4,x)
```

output
```
e**c*(int(e**(b**2*x**2)/x**4,x) - int((e**(b**2*x**2)*erf(b*x))/x**4,x))
```

3.178 $\int e^{-b^2x^2}x^5\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 135

$$\int e^{-b^2x^2}x^5\operatorname{erfc}(bx)\,dx=\frac{11e^{-2b^2x^2}x}{16b^5\sqrt{\pi}}+\frac{e^{-2b^2x^2}x^3}{4b^3\sqrt{\pi}}-\frac{43\operatorname{erf}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\operatorname{erfc}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\operatorname{erfc}(bx)}{2b^2}$$

output
```
11/16*x/b^5/exp(2*b^2*x^2)/Pi^(1/2)+1/4*x^3/b^3/exp(2*b^2*x^2)/Pi^(1/2)-43
/64*erf(2^(1/2)*b*x)*2^(1/2)/b^6-erfc(b*x)/b^6/exp(b^2*x^2)-x^2*erfc(b*x)/
b^4/exp(b^2*x^2)-1/2*x^4*erfc(b*x)/b^2/exp(b^2*x^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int e^{-b^2x^2}x^5\operatorname{erfc}(bx)\,dx=\frac{-43\sqrt{2}\operatorname{erf}\left(\sqrt{2}bx\right)+4e^{-2b^2x^2}\left(\frac{bx(11+4b^2x^2)}{\sqrt{\pi}}-8e^{b^2x^2}\left(2+2b^2x^2+b^4x^4\right)\operatorname{erfc}(bx)\right)}{64b^6}$$

input
```
Integrate[(x^5*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
(-43*Sqrt[2]*Erf[Sqrt[2]*b*x] + (4*((b*x*(11 + 4*b^2*x^2))/Sqrt[Pi] - 8*E^
(b^2*x^2)*(2 + 2*b^2*x^2 + b^4*x^4)*Erfc[b*x]))/E^(2*b^2*x^2))/(64*b^6)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.76, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6940, 2641, 2641, 2634, 6940, 2641, 2634, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{-b^2x^2}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{6940}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{\int e^{-2b^2x^2}x^4dx}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{\frac{3\int e^{-2b^2x^2}x^2dx}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{\frac{3\left(\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erfc}(bx)dx}{b^2}-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6940}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\operatorname{erfc}(bx)dx}{b^2}-\frac{\int e^{-2b^2x^2}x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2641}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\operatorname{erfc}(bx)dx}{b^2}-\frac{\frac{\int e^{-2b^2x^2}dx}{4b^2}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\operatorname{erfc}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6937}$$

$$\frac{2\left(\frac{-\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}bx\right)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{x^4e^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{2\left(\frac{-\frac{\text{erf}(\sqrt{2}bx)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfc}(bx)}{2b^2}-\frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}\right)}{b^2}-\frac{\frac{3\left(\frac{\sqrt{\frac{\pi}{2}}\text{erf}(\sqrt{2}bx)}{8b^3}-\frac{xe^{-2b^2x^2}}{4b^2}\right)}{4b^2}-\frac{x^3e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b}$$

input
```
Int[(x^5*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
-((-1/4*x^3/(b^2*E^(2*b^2*x^2)) + (3*(-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[P
i/2]*Erf[Sqrt[2]*b*x])/(8*b^3)))/(4*b^2))/(b*Sqrt[Pi])) - (x^4*Erfc[b*x])/
(2*b^2*E^(b^2*x^2)) + (2*(-((-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[
Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi])) - (x^2*Erfc[b*x])/(2*b^2*E^(b^2*x^2))
 + (-1/2*Erf[Sqrt[2]*b*x]/(Sqrt[2]*b^2) - Erfc[b*x]/(2*b^2*E^(b^2*x^2)))/b
^2))/b^2
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.27

method	result
default	$\frac{\frac{-\frac{e^{-b^2x^2}x^4b^4}{2}-e^{-b^2x^2}x^2b^2-e^{-b^2x^2}}{b^5}-\frac{\operatorname{erf}(bx)\left(-\frac{e^{-b^2x^2}x^4b^4}{2}-e^{-b^2x^2}x^2b^2-e^{-b^2x^2}\right)}{b^5}+\frac{-\frac{43\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{64}+\frac{11\,e^{-2b^2x^2}bx}{16}+\frac{e^{-2b^2x^2}}{4}}{\sqrt{\pi}\,b^5}}{b}$

input
```
int(x^5*erfc(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
(1/b^5*(-1/2/exp(b^2*x^2)*b^4*x^4-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))-erf
(b*x)/b^5*(-1/2/exp(b^2*x^2)*b^4*x^4-b^2*x^2/exp(b^2*x^2)-1/exp(b^2*x^2))+
1/Pi^(1/2)/b^5*(-43/64*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)+1/4/exp(b^2*x^2)^
2*b^3*x^3+11/16/exp(b^2*x^2)^2*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int e^{-b^2x^2}x^5\operatorname{erfc}(bx)\,dx =$$
$$-\frac{43\sqrt{2}\pi\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-4\sqrt{\pi}(4\,b^4x^3+11\,b^2x)e^{(-2\,b^2x^2)}+32\,(\pi b^5x^4+2\,\pi b^3x^2+2\,\pi b-(\pi b^5x^4+}{64\,\pi b^7}$$

input
```
integrate(x^5*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/64*(43*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 4*sqrt(pi)*(4*b^
4*x^3 + 11*b^2*x)*e^(-2*b^2*x^2) + 32*(pi*b^5*x^4 + 2*pi*b^3*x^2 + 2*pi*b
- (pi*b^5*x^4 + 2*pi*b^3*x^2 + 2*pi*b)*erf(b*x))*e^(-b^2*x^2))/(pi*b^7)
```

Sympy [F]

$$\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx = \int x^5e^{-b^2x^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(x**5*erfc(b*x)/exp(b**2*x**2),x)
```

output
```
Integral(x**5*exp(-b**2*x**2)*erfc(b*x), x)
```

Maxima [F]

$$\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx = \int x^5\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^5*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^5*erfc(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx = \int x^5\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^5*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*erfc(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx = \int x^5\,\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x^5*exp(-b^2*x^2)*erfc(b*x),x)
```

output
```
int(x^5*exp(-b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2}x^5\text{erfc}(bx)\,dx = \frac{-2e^{b^2x^2}\left(\int \frac{\text{erf}(bx)x^5}{e^{b^2x^2}}dx\right)b^6 - b^4x^4 - 2b^2x^2 - 2}{2e^{b^2x^2}b^6}$$

input
```
int(x^5*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
( - 2*e**(b**2*x**2)*int((erf(b*x)*x**5)/e**(b**2*x**2),x)*b**6 - b**4*x**
4 - 2*b**2*x**2 - 2)/(2*e**(b**2*x**2)*b**6)
```

3.179 $\int e^{-b^2x^2}x^3\textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\frac{e^{-2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{5\text{erf}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erfc}(bx)}{2b^2}$$

output
```
1/4*x/b^3/exp(2*b^2*x^2)/Pi^(1/2)-5/16*erf(2^(1/2)*b*x)*2^(1/2)/b^4-1/2*er
fc(b*x)/b^4/exp(b^2*x^2)-1/2*x^2*erfc(b*x)/b^2/exp(b^2*x^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\frac{-5\sqrt{2}\text{erf}\left(\sqrt{2}bx\right)+4e^{-2b^2x^2}\left(\frac{bx}{\sqrt{\pi}}-2e^{b^2x^2}(1+b^2x^2)\,\text{erfc}(bx)\right)}{16b^4}$$

input
```
Integrate[(x^3*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
(-5*Sqrt[2]*Erf[Sqrt[2]*b*x] + (4*((b*x)/Sqrt[Pi] - 2*E^(b^2*x^2)*(1 + b^2
*x^2)*Erfc[b*x]))/E^(2*b^2*x^2))/(16*b^4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6940, 2641, 2634, 6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-b^2 x^2} \text{erfc}(bx) \, dx$$

$$\Big\downarrow \text{6940}$$

$$\frac{\int e^{-b^2x^2} x\text{erfc}(bx)dx}{b^2} - \frac{\int e^{-2b^2x^2} x^2 dx}{\sqrt{\pi} b} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$\frac{\int e^{-b^2x^2} x\text{erfc}(bx)dx}{b^2} - \frac{\frac{\int e^{-2b^2x^2}dx}{4b^2} - \frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\int e^{-b^2x^2} x\text{erfc}(bx)dx}{b^2} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} - \frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3} - \frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b}$$

$$\Big\downarrow \text{6937}$$

$$\frac{-\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi} b} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}}{b^2} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} - \frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3} - \frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{-\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}}{b^2} - \frac{x^2 e^{-b^2x^2} \text{erfc}(bx)}{2b^2} - \frac{\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}bx\right)}{8b^3} - \frac{xe^{-2b^2x^2}}{4b^2}}{\sqrt{\pi} b}$$

input
```
Int[(x^3*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
-((-1/4*x/(b^2*E^(2*b^2*x^2)) + (Sqrt[Pi/2]*Erf[Sqrt[2]*b*x])/(8*b^3))/(b*
Sqrt[Pi])) - (x^2*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) + (-1/2*Erf[Sqrt[2]*b*x]/
(Sqrt[2]*b^2) - Erfc[b*x]/(2*b^2*E^(b^2*x^2)))/b^2
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\frac{-\frac{e^{-b^2x^2}x^2b^2}{2}-\frac{e^{-b^2x^2}}{2}}{b^3}-\frac{\mathrm{erf}(bx)\left(-\frac{e^{-b^2x^2}x^2b^2}{2}-\frac{e^{-b^2x^2}}{2}\right)}{b^3}+\frac{-\frac{5\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\,bx\right)}{16}+\frac{e^{-2b^2x^2}bx}{4}}{\sqrt{\pi}\,b^3}}{b}$	118

input
```
int(x^3*erfc(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
(1/b^3*(-1/2*b^2*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))-erf(b*x)/b^3*(-1/2*b^2
*x^2/exp(b^2*x^2)-1/2/exp(b^2*x^2))+1/Pi^(1/2)/b^3*(-5/16*2^(1/2)*Pi^(1/2)
*erf(2^(1/2)*b*x)+1/4/exp(b^2*x^2)^2*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x^3\mathrm{erfc}(bx)\,dx = \frac{4\sqrt{\pi}b^2xe^{(-2b^2x^2)}-5\sqrt{2}\pi\sqrt{b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-8\left(\pi b^3x^2+\pi b-(\pi b^3x^2+\pi b)\,\mathrm{erf}\,(bx)\right)e^{(-b^2x^2)}}{16\,\pi b^5}$$

input
```
integrate(x^3*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/16*(4*sqrt(pi)*b^2*x*e^(-2*b^2*x^2) - 5*sqrt(2)*pi*sqrt(b^2)*erf(sqrt(2)
*sqrt(b^2)*x) - 8*(pi*b^3*x^2 + pi*b - (pi*b^3*x^2 + pi*b)*erf(b*x))*e^(-b
^2*x^2))/(pi*b^5)
```

Sympy [F]

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\int x^3e^{-b^2x^2}\,\text{erfc}\,(bx)\,dx$$

input
```
integrate(x**3*erfc(b*x)/exp(b**2*x**2),x)
```

output
```
Integral(x**3*exp(-b**2*x**2)*erfc(b*x), x)
```

Maxima [F]

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\int x^3\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^3*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^3*erfc(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\int x^3\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^3*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*erfc(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\int x^3\,\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)\;dx$$

input
```
int(x^3*exp(-b^2*x^2)*erfc(b*x),x)
```

output
```
int(x^3*exp(-b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2}x^3\text{erfc}(bx)\,dx=\frac{-2e^{b^2x^2}\left(\int\frac{\text{erf}(bx)x^3}{e^{b^2x^2}}dx\right)b^4-b^2x^2-1}{2e^{b^2x^2}b^4}$$

input
```
int(x^3*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
( - 2*e**(b**2*x**2)*int((erf(b*x)*x**3)/e**(b**2*x**2),x)*b**4 - b**2*x**
2 - 1)/(2*e**(b**2*x**2)*b**4)
```

3.180 $\int e^{-b^2x^2} x\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int e^{-b^2x^2} x\text{erfc}(bx)\,dx = -\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

output
```
-1/4*erf(2^(1/2)*b*x)*2^(1/2)/b^2-1/2*erfc(b*x)/b^2/exp(b^2*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int e^{-b^2x^2} x\text{erfc}(bx)\,dx = -\frac{\sqrt{2}\text{erf}\left(\sqrt{2}bx\right) + 2e^{-b^2x^2}\text{erfc}(bx)}{4b^2}$$

input
```
Integrate[(x*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
-1/4*(Sqrt[2]*Erf[Sqrt[2]*b*x] + (2*Erfc[b*x])/E^(b^2*x^2))/b^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6937, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-b^2x^2}\text{erfc}(bx)\,dx$$
$$\Big\downarrow \text{6937}$$
$$-\frac{\int e^{-2b^2x^2}dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$
$$\Big\downarrow \text{2634}$$
$$-\frac{\text{erf}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

input
```
Int[(x*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
-1/2*Erf[Sqrt[2]*b*x]/(Sqrt[2]*b^2) - Erfc[b*x]/(2*b^2*E^(b^2*x^2))
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{-\frac{e^{-b^2x^2}}{2b}+\frac{\operatorname{erf}(bx)e^{-b^2x^2}}{2b}-\frac{\sqrt{2}\,\operatorname{erf}\left(\sqrt{2}\,bx\right)}{4b}}{b}$	53

input
```
int(x*erfc(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2/b*exp(-b^2*x^2)+1/2*erf(b*x)/b*exp(-b^2*x^2)-1/4/b*2^(1/2)*erf(2^(1/
2)*b*x))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int e^{-b^2x^2}x\text{erfc}(bx)\,dx = -\frac{\sqrt{2}\sqrt{b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{b^2}x\right)-2\,(b\operatorname{erf}(bx)-b)e^{(-b^2x^2)}}{4\,b^3}$$

input
```
integrate(x*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*sqrt(b^2)*erf(sqrt(2)*sqrt(b^2)*x) - 2*(b*erf(b*x) - b)*e^(-
b^2*x^2))/b^3
```

Sympy [F]

$$\int e^{-b^2x^2}x\text{erfc}(bx)\,dx = \int xe^{-b^2x^2}\operatorname{erfc}(bx)\,dx$$

input
```
integrate(x*erfc(b*x)/exp(b**2*x**2),x)
```

output
```
Integral(x*exp(-b**2*x**2)*erfc(b*x), x)
```

Maxima [F]

$$\int e^{-b^2x^2}x\text{erfc}(bx)\,dx = \int x\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x*erfc(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x\text{erfc}(bx)\,dx = \int x\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x*erfc(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x\text{erfc}(bx)\,dx = \int x\,\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)\,dx$$

input
```
int(x*exp(-b^2*x^2)*erfc(b*x),x)
```

output
```
int(x*exp(-b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2} x\text{erfc}(bx)\,dx = \frac{-2e^{b^2x^2}\left(\int \frac{\text{erf}(bx)x}{e^{b^2x^2}}dx\right)b^2 - 1}{2e^{b^2x^2}b^2}$$

input
```
int(x*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
( - 2*e**(b**2*x**2)*int((erf(b*x)*x)/e**(b**2*x**2),x)*b**2 - 1)/(2*e**(b
**2*x**2)*b**2)
```

3.181 $\int \frac{e^{-b^2x^2}\mathbf{erfc}(bx)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x} \, dx = \text{Int}\left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}, x\right)$$

output
```
Defer(Int)(erfc(b*x)/exp(b^2*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x} \, dx = \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x} \, dx$$

input
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x),x]
```

output
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x} \, dx$$
$$\Big\downarrow \text{6949}$$
$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx$$

input
```
Int[Erfc[b*x]/(E^(b^2*x^2)*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfc}\left(bx\right)\text{e}^{-b^2x^2}}{x}dx$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x,x)
```

output
```
int(erfc(b*x)/exp(b^2*x^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x}\,dx = \int \frac{\mathrm{erfc}\,(bx)\,e^{(-b^2x^2)}}{x}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(-b^2*x^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x}\,dx = \int \frac{e^{-b^2x^2}\,\mathrm{erfc}\,(bx)}{x}\,dx$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2)/x,x)
```

output
```
Integral(exp(-b**2*x**2)*erfc(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x}\,dx = \int \frac{\mathrm{erfc}\,(bx)\,e^{(-b^2x^2)}}{x}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)}{x}\,dx$$

input
```
int((exp(-b^2*x^2)*erfc(b*x))/x,x)
```

output
```
int((exp(-b^2*x^2)*erfc(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x}\,dx = \frac{ei(-b^2x^2)}{2} - \left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x}dx\right)$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x,x)
```

output
```
(ei( - b**2*x**2) - 2*int(erf(b*x)/(e**(b**2*x**2)*x),x))/2
```

3.182 $\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \frac{be^{-2b^2x^2}}{\sqrt{\pi}x} + \sqrt{2}b^2\text{erf}\left(\sqrt{2}bx\right) - \frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2} - b^2\text{Int}\left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x},x\right)$$

output
```
b/exp(2*b^2*x^2)/Pi^(1/2)/x+2^(1/2)*b^2*erf(2^(1/2)*b*x)-1/2*erfc(b*x)/exp
(b^2*x^2)/x^2-b^2*Defer(Int)(erfc(b*x)/exp(b^2*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx$$

input
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^3),x]
```

output
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx$$

$$\Big\downarrow \text{6946}$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\int \frac{e^{-2b^2x^2}}{x^2}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\left(-4b^2\int e^{-2b^2x^2}dx-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}$$

$$\Big\downarrow \text{6949}$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}$$

input
```
Int[Erfc[b*x]/(E^(b^2*x^2)*x^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)\, \mathrm{e}^{-b^2x^2}}{x^3} dx$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^3,x)
```

output
```
int(erfc(b*x)/exp(b^2*x^2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^3}\, dx = \int \frac{\operatorname{erfc}(bx)\, e^{(-b^2x^2)}}{x^3}\, dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^3,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(-b^2*x^2)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^3}\, dx = \int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^3}\, dx$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2)/x**3,x)
```

output
```
Integral(exp(-b**2*x**2)*erfc(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3} \, dx = \int \frac{\text{erfc}\,(bx)\, e^{\left(-b^2x^2\right)}}{x^3} \, dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3} \, dx = \int \frac{\text{erfc}\,(bx)\, e^{\left(-b^2x^2\right)}}{x^3} \, dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)}{x^3}\,dx$$

input
```
int((exp(-b^2*x^2)*erfc(b*x))/x^3,x)
```

output
```
int((exp(-b^2*x^2)*erfc(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\,dx = \frac{-e^{b^2x^2}\,ei(-b^2x^2)\,b^2x^2 - 2e^{b^2x^2}\left(\int \frac{\text{erf}(bx)}{e^{b^2x^2}x^3}dx\right)x^2 - 1}{2e^{b^2x^2}x^2}$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^3,x)
```

output
```
( - e**(b**2*x**2)*ei( - b**2*x**2)*b**2*x**2 - 2*e**(b**2*x**2)*int(erf(b
*x)/(e**(b**2*x**2)*x**3),x)*x**2 - 1)/(2*e**(b**2*x**2)*x**2)
```

3.183 $\int \frac{e^{-b^2x^2}\textbf{erfc}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^5}\,dx = \frac{be^{-2b^2x^2}}{6\sqrt{\pi}x^3} - \frac{7b^3e^{-2b^2x^2}}{6\sqrt{\pi}x} - \frac{b^4\text{erf}\left(\sqrt{2}bx\right)}{\sqrt{2}} - \frac{2}{3}\sqrt{2}b^4\text{erf}\left(\sqrt{2}bx\right) - \frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4} + \frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{4x^2} + \frac{1}{2}b^4\text{Int}\left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x},x\right)$$

output
```
1/6*b/exp(2*b^2*x^2)/Pi^(1/2)/x^3-7/6*b^3/exp(2*b^2*x^2)/Pi^(1/2)/x-7/6*b^
4*erf(2^(1/2)*b*x)*2^(1/2)-1/4*erfc(b*x)/exp(b^2*x^2)/x^4+1/4*b^2*erfc(b*x
)/exp(b^2*x^2)/x^2+1/2*b^4*Defer(Int)(erfc(b*x)/exp(b^2*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^5}\,dx$$

input
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^5),x]
```

output
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^5), x]
```

Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^5}\,dx$$

$$\Big\downarrow \text{6946}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}dx-\frac{b\int \frac{e^{-2b^2x^2}}{x^4}dx}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}dx-\frac{b\left(-\frac{4}{3}b^2\int \frac{e^{-2b^2x^2}}{x^2}dx-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}dx-\frac{b\left(-\frac{4}{3}b^2\left(-4b^2\int e^{-2b^2x^2}dx-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}dx-\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{6946}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\int\frac{e^{-2b^2x^2}}{x^2}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}\right)-\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\left(-4b^2\int e^{-2b^2x^2}dx-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}\right)-\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}\right)-\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

$$\Big\downarrow \text{6949}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}dx\right)-\frac{b\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}\right)-\frac{b\left(-\frac{4}{3}b^2\left(\sqrt{2\pi}(-b)\text{erf}(\sqrt{2}bx)-\frac{e^{-2b^2x^2}}{x}\right)-\frac{e^{-2b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{4x^4}$$

input
```
Int[Erfc[b*x]/(E^(b^2*x^2)*x^5),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfc}(bx)\,\mathrm{e}^{-b^2x^2}}{x^5}dx$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^5,x)
```

output
```
int(erfc(b*x)/exp(b^2*x^2)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(-b^2x^2)}}{x^5}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^5,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*e^(-b^2*x^2)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 12.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2)/x**5,x)
```

output
```
Integral(exp(-b**2*x**2)*erfc(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^5,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^5}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\operatorname{erfc}(b\,x)}{x^5}\,dx$$

input
```
int((exp(-b^2*x^2)*erfc(b*x))/x^5,x)
```

output
```
int((exp(-b^2*x^2)*erfc(b*x))/x^5, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.72

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^5}\,dx = \frac{e^{b^2x^2}\,ei(-b^2x^2)\,b^4x^4 - 4e^{b^2x^2}\left(\int \frac{\operatorname{erf}(bx)}{e^{b^2x^2}x^5}dx\right)x^4 + b^2x^2 - 1}{4e^{b^2x^2}x^4}$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^5,x)
```

output
```
(e**(b**2*x**2)*ei( - b**2*x**2)*b**4*x**4 - 4*e**(b**2*x**2)*int(erf(b*x)
/(e**(b**2*x**2)*x**5),x)*x**4 + b**2*x**2 - 1)/(4*e**(b**2*x**2)*x**4)
```

3.184 $\int e^{-b^2x^2}x^4\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = \frac{e^{-2b^2x^2}}{2b^5\sqrt{\pi}}+\frac{e^{-2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{-b^2x^2}x\text{erfc}(bx)}{4b^4} - \frac{e^{-b^2x^2}x^3\text{erfc}(bx)}{2b^2}-\frac{3\sqrt{\pi}\text{erfc}(bx)^2}{16b^5}$$

output
```
1/2/b^5/exp(2*b^2*x^2)/Pi^(1/2)+1/4*x^2/b^3/exp(2*b^2*x^2)/Pi^(1/2)-3/4*x*
erfc(b*x)/b^4/exp(b^2*x^2)-1/2*x^3*erfc(b*x)/b^2/exp(b^2*x^2)-3/16*Pi^(1/2
)*erfc(b*x)^2/b^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = -\frac{-4e^{-2b^2x^2}(2+b^2x^2)+4be^{-b^2x^2}\sqrt{\pi}x(3+2b^2x^2)-6\pi\text{erf}(bx)-4be^{-b^2x^2}\sqrt{\pi}x(3+2b^2x^2)\,\text{erf}(bx)+3\pi\text{er}}{16b^5\sqrt{\pi}}$$

input
```
Integrate[(x^4*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
-1/16*((-4*(2 + b^2*x^2))/E^(2*b^2*x^2) + (4*b*Sqrt[Pi]*x*(3 + 2*b^2*x^2))
/E^(b^2*x^2) - 6*Pi*Erf[b*x] - (4*b*Sqrt[Pi]*x*(3 + 2*b^2*x^2)*Erf[b*x])/E
^(b^2*x^2) + 3*Pi*Erf[b*x]^2)/(b^5*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6940, 2641, 2638, 6940, 2638, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-b^2x^2}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{ 6940}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2} - \frac{\int e^{-2b^2x^2}x^3dx}{\sqrt{\pi}b} - \frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2} - \frac{\frac{\int e^{-2b^2x^2}xdx}{2b^2} - \frac{x^2e^{-2b^2x^2}}{4b^2}}{\sqrt{\pi}b} - \frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erfc}(bx)dx}{2b^2} - \frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2} - \frac{-\frac{x^2e^{-2b^2x^2}}{4b^2} - \frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 6940}$$

$$\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2} - \frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b} - \frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}\right)}{2b^2} - \frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2} - \frac{-\frac{x^2e^{-2b^2x^2}}{4b^2} - \frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2} - \frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2} + \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2} - \frac{x^3e^{-b^2x^2}\text{erfc}(bx)}{2b^2} - \frac{-\frac{x^2e^{-2b^2x^2}}{4b^2} - \frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 6928}$$

$$\frac{3\left(-\frac{\sqrt{\pi}\int \operatorname{erfc}(bx)d\operatorname{erfc}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{x^3e^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}-\frac{-\frac{x^2e^{-2b^2x^2}}{4b^2}-\frac{e^{-2b^2x^2}}{8b^4}}{\sqrt{\pi}b}+\frac{3\left(-\frac{\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\operatorname{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}$$

input
```
Int[(x^4*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
-((-1/8*1/(b^4*E^(2*b^2*x^2)) - x^2/(4*b^2*E^(2*b^2*x^2)))/(b*Sqrt[Pi])) -
 (x^3*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]
) - (x*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (Sqrt[Pi]*Erfc[b*x]^2)/(8*b^3)))/(
2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^4 \operatorname{erfc}(bx)\, \mathrm{e}^{-b^2x^2} dx$$

input
```
int(x^4*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^4*erfc(b*x)/exp(b^2*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int e^{-b^2x^2}x^4\operatorname{erfc}(bx)\,dx = -\frac{4\,(2\,\pi b^3x^3+3\,\pi bx-(2\,\pi b^3x^3+3\,\pi bx)\operatorname{erf}(bx))e^{(-b^2x^2)}+\sqrt{\pi}\Big(3\,\pi\operatorname{erf}(bx)^2-6\,\pi\operatorname{erf}(bx)-4\,(b^2x^2+2}{16\,\pi b^5}$$

input
```
integrate(x^4*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/16*(4*(2*pi*b^3*x^3 + 3*pi*b*x - (2*pi*b^3*x^3 + 3*pi*b*x)*erf(b*x))*e^
(-b^2*x^2) + sqrt(pi)*(3*pi*erf(b*x)^2 - 6*pi*erf(b*x) - 4*(b^2*x^2 + 2)*e
^(-2*b^2*x^2)))/(pi*b^5)
```

Sympy [A] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = \begin{cases} -\frac{x^3e^{-b^2x^2}\,\text{erfc}\,(bx)}{2b^2} + \frac{x^2e^{-2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{3xe^{-b^2x^2}\,\text{erfc}\,(bx)}{4b^4} - \frac{3\sqrt{\pi}\,\text{erfc}^2\,(bx)}{16b^5} + \frac{e^{-2b^2x^2}}{2\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*erfc(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((-x**3*exp(-b**2*x**2)*erfc(b*x)/(2*b**2) + x**2*exp(-2*b**2*x**
2)/(4*sqrt(pi)*b**3) - 3*x*exp(-b**2*x**2)*erfc(b*x)/(4*b**4) - 3*sqrt(pi)
*erfc(b*x)**2/(16*b**5) + exp(-2*b**2*x**2)/(2*sqrt(pi)*b**5), Ne(b, 0)),
(x**5/5, True))
```

Maxima [F]

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = \int x^4\,\text{erfc}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^4*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^4*erfc(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = \int x^4\,\text{erfc}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^4*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*erfc(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = \frac{8\,\mathrm{e}^{-2\,b^2\,x^2}-3\,\pi\,\mathrm{erfc}(b\,x)^2}{16\,b^5\,\sqrt{\pi}}+\frac{x^2\,\mathrm{e}^{-2\,b^2\,x^2}}{4\,b^3\,\sqrt{\pi}} - \frac{3\,x\,\mathrm{e}^{-b^2\,x^2}\,\mathrm{erfc}(b\,x)}{4\,b^4}-\frac{x^3\,\mathrm{e}^{-b^2\,x^2}\,\mathrm{erfc}(b\,x)}{2\,b^2}$$

input
```
int(x^4*exp(-b^2*x^2)*erfc(b*x),x)
```

output
```
(8*exp(-2*b^2*x^2) - 3*pi*erfc(b*x)^2)/(16*b^5*pi^(1/2)) + (x^2*exp(-2*b^2
*x^2))/(4*b^3*pi^(1/2)) - (3*x*exp(-b^2*x^2)*erfc(b*x))/(4*b^4) - (x^3*exp
(-b^2*x^2)*erfc(b*x))/(2*b^2)
```

Reduce [F]

$$\int e^{-b^2x^2}x^4\text{erfc}(bx)\,dx = \frac{3\sqrt{\pi}\,e^{b^2x^2}\text{erf}(bx)-8e^{b^2x^2}\left(\int\frac{\text{erf}(bx)x^4}{e^{b^2x^2}}dx\right)b^5-4b^3x^3-6bx}{8e^{b^2x^2}b^5}$$

input
```
int(x^4*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
(3*sqrt(pi)*e**(b**2*x**2)*erf(b*x) - 8*e**(b**2*x**2)*int((erf(b*x)*x**4)
/e**(b**2*x**2),x)*b**5 - 4*b**3*x**3 - 6*b*x)/(8*e**(b**2*x**2)*b**5)
```

3.185 $\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx=\frac{e^{-2b^2x^2}}{4b^3\sqrt{\pi}}-\frac{e^{-b^2x^2}x\text{erfc}(bx)}{2b^2}-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{8b^3}$$

output
```
1/4/b^3/exp(2*b^2*x^2)/Pi^(1/2)-1/2*x*erfc(b*x)/b^2/exp(b^2*x^2)-1/8*Pi^(1
/2)*erfc(b*x)^2/b^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx$$
$$=\frac{2e^{-2b^2x^2}\left(\frac{1}{\sqrt{\pi}}-2be^{b^2x^2}x\right)+\left(2\sqrt{\pi}+4be^{-b^2x^2}x\right)\text{erf}(bx)-\sqrt{\pi}\text{erf}(bx)^2}{8b^3}$$

input
```
Integrate[(x^2*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
((2*(1/Sqrt[Pi] - 2*b*E^(b^2*x^2)*x))/E^(2*b^2*x^2) + (2*Sqrt[Pi] + (4*b*x
)/E^(b^2*x^2))*Erf[b*x] - Sqrt[Pi]*Erf[b*x]^2)/(8*b^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6940, 2638, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-b^2x^2}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{ 6940}$$

$$\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{\int e^{-2b^2x^2}xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{\int e^{-b^2x^2}\text{erfc}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

$$\Big\downarrow \text{ 6928}$$

$$-\frac{\sqrt{\pi}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{8b^3}-\frac{xe^{-b^2x^2}\text{erfc}(bx)}{2b^2}+\frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3}$$

input
```
Int[(x^2*Erfc[b*x])/E^(b^2*x^2),x]
```

output
```
1/(4*b^3*E^(2*b^2*x^2)*Sqrt[Pi]) - (x*Erfc[b*x])/(2*b^2*E^(b^2*x^2)) - (Sq
rt[Pi]*Erfc[b*x]^2)/(8*b^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^2 \operatorname{erfc}(bx)\, e^{-b^2x^2} dx$$

input
```
int(x^2*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^2*erfc(b*x)/exp(b^2*x^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx = \frac{4\,(\pi bx\,\text{erf}\,(bx) - \pi bx)e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi\,\text{erf}\,(bx)^2 - 2\,\pi\,\text{erf}\,(bx) - 2\,e^{\left(-2\,b^2x^2\right)}\right)}{8\,\pi b^3}$$

input
```
integrate(x^2*erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/8*(4*(pi*b*x*erf(b*x) - pi*b*x)*e^(-b^2*x^2) - sqrt(pi)*(pi*erf(b*x)^2 -
 2*pi*erf(b*x) - 2*e^(-2*b^2*x^2)))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x^2\text{erfc}(bx)\,dx = \begin{cases} -\frac{xe^{-b^2x^2}\,\text{erfc}\,(bx)}{2b^2} - \frac{\sqrt{\pi}\,\text{erfc}^2\,(bx)}{8b^3} + \frac{e^{-2b^2x^2}}{4\sqrt{\pi}b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*erfc(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((-x*exp(-b**2*x**2)*erfc(b*x)/(2*b**2) - sqrt(pi)*erfc(b*x)**2/(
8*b**3) + exp(-2*b**2*x**2)/(4*sqrt(pi)*b**3), Ne(b, 0)), (x**3/3, True))
```

Maxima [F]

$$\int e^{-b^2x^2}x^2\mathrm{erfc}(bx)\,dx = \int x^2\,\mathrm{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^2*erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^2*erfc(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^2\mathrm{erfc}(bx)\,dx = \int x^2\,\mathrm{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^2*erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*erfc(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int e^{-b^2x^2}x^2\mathrm{erfc}(bx)\,dx = \frac{2\,\mathrm{e}^{-2\,b^2\,x^2}-\pi\,\mathrm{erfc}(b\,x)^2}{8\,b^3\,\sqrt{\pi}} - \frac{x\,\mathrm{e}^{-b^2\,x^2}\,\mathrm{erfc}(b\,x)}{2\,b^2}$$

input
```
int(x^2*exp(-b^2*x^2)*erfc(b*x),x)
```

output
```
(2*exp(-2*b^2*x^2) - pi*erfc(b*x)^2)/(8*b^3*pi^(1/2)) - (x*exp(-b^2*x^2)*e
rfc(b*x))/(2*b^2)
```

Reduce [F]

$$\int e^{-b^2x^2}x^2\operatorname{erfc}(bx)\,dx=\frac{\sqrt{\pi}\,e^{b^2x^2}\operatorname{erf}(bx)-4e^{b^2x^2}\left(\int\frac{\operatorname{erf}(bx)x^2}{e^{b^2x^2}}dx\right)b^3-2bx}{4e^{b^2x^2}b^3}$$

input
```
int(x^2*erfc(b*x)/exp(b^2*x^2),x)
```

output
```
(sqrt(pi)*e**(b**2*x**2)*erf(b*x) - 4*e**(b**2*x**2)*int((erf(b*x)*x**2)/e
**(b**2*x**2),x)*b**3 - 2*b*x)/(4*e**(b**2*x**2)*b**3)
```

3.186 $\int e^{-b^2x^2}\textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int e^{-b^2x^2}\text{erfc}(bx)\,dx=-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{4b}$$

output
```
-1/4*Pi^(1/2)*erfc(b*x)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}\text{erfc}(bx)\,dx=-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{4b}$$

input
```
Integrate[Erfc[b*x]/E^(b^2*x^2),x]
```

output
```
-1/4*(Sqrt[Pi]*Erfc[b*x]^2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-b^2x^2}\text{erfc}(bx)\,dx$$

$$\Big\downarrow \text{ 6928}$$

$$-\frac{\sqrt{\pi}\int \text{erfc}(bx)d\text{erfc}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{\pi}\text{erfc}(bx)^2}{4b}$$

input
```
Int[Erfc[b*x]/E^(b^2*x^2),x]
```

output
```
-1/4*(Sqrt[Pi]*Erfc[b*x]^2)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{\pi}\left(-\frac{\operatorname{erf}(bx)^2}{2}+\operatorname{erf}(bx)\right)}{2b}$	22

input
```
int(erfc(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi^(1/2)/b*(-1/2*erf(b*x)^2+erf(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx = -\frac{\sqrt{\pi}\left(\operatorname{erf}\left(bx\right)^2-2\operatorname{erf}\left(bx\right)\right)}{4\,b}$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(pi)*(erf(b*x)^2 - 2*erf(b*x))/b
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx = \begin{cases} -\frac{\sqrt{\pi}\operatorname{erfc}^2(bx)}{4b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((-sqrt(pi)*erfc(b*x)**2/(4*b), Ne(b, 0)), (x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx = -\frac{\sqrt{\pi}\operatorname{erfc}(bx)^2}{4\,b}$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*erfc(b*x)^2/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx = -\frac{\sqrt{\pi}\operatorname{erf}(bx)^2 - 2\sqrt{\pi}\operatorname{erf}(bx)}{4\,b}$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*erf(b*x)^2 - 2*sqrt(pi)*erf(b*x))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int e^{-b^2x^2}\operatorname{erfc}(bx)\,dx = -\frac{\sqrt{\pi}\operatorname{erfc}(b\,x)^2}{4\,b}$$

input
```
int(exp(-b^2*x^2)*erfc(b*x),x)
```

output
```
-(pi^(1/2)*erfc(b*x)^2)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{-b^2x^2}\text{erfc}(bx)\,dx = \frac{\sqrt{\pi}\,\text{erf}(bx)\,(-\text{erf}(bx)+2)}{4b}$$

input
```
int(erfc(b*x)/exp(b^2*x^2),x)
```

output
```
(sqrt(pi)*erf(b*x)*( - erf(b*x) + 2))/(4*b)
```

3.187 $\int \frac{e^{-b^2x^2}\mathbf{erfc}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^2}\,dx=-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x}+\frac{1}{2}b\sqrt{\pi}\operatorname{erfc}(bx)^2-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-erfc(b*x)/exp(b^2*x^2)/x+1/2*b*Pi^(1/2)*erfc(b*x)^2-b*Ei(-2*b^2*x^2)/Pi^(
1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^2}\,dx=-\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x}+\frac{1}{2}b\sqrt{\pi}\operatorname{erfc}(bx)^2-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^2),x]
```

output
```
-(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpIntegral
Ei[-2*b^2*x^2])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6946, 2639, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}\,dx$$

$$\Big\downarrow \text{6946}$$

$$-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}$$

$$\Big\downarrow \text{2639}$$

$$-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6928}$$

$$\sqrt{\pi}b\int \text{erfc}(bx)d\text{erfc}(bx)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\text{erfc}(bx)^2$$

input
```
Int[Erfc[b*x]/(E^(b^2*x^2)*x^2),x]
```

output
```
-(Erfc[b*x]/(E^(b^2*x^2)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpIntegral
Ei[-2*b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}(bx)\, e^{-b^2x^2}}{x^2}dx$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^2,x)
```

output
```
int(erfc(b*x)/exp(b^2*x^2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}\,dx = -\frac{2\,\pi^{\frac{3}{2}}\sqrt{b^2}x\,\text{erf}\left(\sqrt{b^2}x\right) + 2\,(\pi - \pi\,\text{erf}\,(bx))e^{\left(-b^2x^2\right)} - \sqrt{\pi}\left(\pi bx\,\text{erf}\,(bx)^2 - 2\,bx\text{Ei}(-2\,b^2x^2)\right)}{2\,\pi x}$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*pi^(3/2)*sqrt(b^2)*x*erf(sqrt(b^2)*x) + 2*(pi - pi*erf(b*x))*e^(-b
^2*x^2) - sqrt(pi)*(pi*b*x*erf(b*x)^2 - 2*b*x*Ei(-2*b^2*x^2)))/(pi*x)
```

Sympy [F]

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}\,dx = \int \frac{e^{-b^2x^2}\,\text{erfc}\,(bx)}{x^2}\,dx$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2)/x**2,x)
```

output
```
Integral(exp(-b**2*x**2)*erfc(b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^2, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x^2}\,dx = \int \frac{\mathrm{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x^2}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\mathrm{erfc}(b\,x)}{x^2}\,dx$$

input
```
int((exp(-b^2*x^2)*erfc(b*x))/x^2,x)
```

output
```
int((exp(-b^2*x^2)*erfc(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\mathrm{erfc}(bx)}{x^2}\,dx = \frac{-\sqrt{\pi}\,e^{b^2x^2}\mathrm{erf}(bx)\,bx - e^{b^2x^2}\left(\int \frac{\mathrm{erf}(bx)}{e^{b^2x^2}x^2}dx\right)x - 1}{e^{b^2x^2}x}$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^2,x)
```

output
```
( - (sqrt(pi)*e**(b**2*x**2)*erf(b*x)*b*x + e**(b**2*x**2)*int(erf(b*x)/(e
**(b**2*x**2)*x**2),x)*x + 1))/(e**(b**2*x**2)*x)
```

3.188 $\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \frac{be^{-2b^2x^2}}{3\sqrt{\pi}x^2} - \frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3} + \frac{2b^2e^{-b^2x^2}\text{erfc}(bx)}{3x} - \frac{1}{3}b^3\sqrt{\pi}\text{erfc}(bx)^2 + \frac{4b^3\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{3\sqrt{\pi}}$$

output
```
1/3*b/exp(2*b^2*x^2)/Pi^(1/2)/x^2-1/3*erfc(b*x)/exp(b^2*x^2)/x^3+2/3*b^2*e
rfc(b*x)/exp(b^2*x^2)/x-1/3*b^3*Pi^(1/2)*erfc(b*x)^2+4/3*b^3*Ei(-2*b^2*x^2
)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \frac{1}{3}\left(\frac{e^{-b^2x^2}\left(-1+2b^2x^2\right)\text{erfc}(bx)}{x^3} - b^3\sqrt{\pi}\text{erfc}(bx)^2 + \frac{b\left(\frac{e^{-2b^2x^2}}{x^2} + 4b^2\,\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)}{\sqrt{\pi}}\right)$$

input
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^4),x]
```

output
```
(((-1 + 2*b^2*x^2)*Erfc[b*x])/(E^(b^2*x^2)*x^3) - b^3*Sqrt[Pi]*Erfc[b*x]^2
 + (b*(1/(E^(2*b^2*x^2)*x^2) + 4*b^2*ExpIntegralEi[-2*b^2*x^2]))/Sqrt[Pi])
/3
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6946, 2643, 2639, 6946, 2639, 6928, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}dx$$

$$\Big\downarrow \text{6946}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx-\frac{2b\int \frac{e^{-2b^2x^2}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx-\frac{2b\left(-2b^2\int \frac{e^{-2b^2x^2}}{x}dx-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^2}dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2(-\text{ExpIntegralEi}(-2b^2x^2))-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6946}$$

$$-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{2b\int \frac{e^{-2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2(-\text{ExpIntegralEi}(-2b^2x^2))-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfc}(bx)dx-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6928}$$

$$-\frac{2}{3}b^2\left(\sqrt{\pi}b\int \text{erfc}(bx)d\text{erfc}(bx)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{3}b^2\left(-\frac{e^{-b^2x^2}\text{erfc}(bx)}{x}-\frac{b\,\text{ExpIntegralEi}\left(-2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\text{erfc}(bx)^2\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{3x^3}-\frac{2b\left(b^2\left(-\text{ExpIntegralEi}\left(-2b^2x^2\right)\right)-\frac{e^{-2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}$$

input
```
Int[Erfc[b*x]/(E^(b^2*x^2)*x^4),x]
```

output
```
-1/3*Erfc[b*x]/(E^(b^2*x^2)*x^3) - (2*b*(-1/2*1/(E^(2*b^2*x^2)*x^2) - b^2*
ExpIntegralEi[-2*b^2*x^2]))/(3*Sqrt[Pi]) - (2*b^2*(-(Erfc[b*x]/(E^(b^2*x^2
)*x)) + (b*Sqrt[Pi]*Erfc[b*x]^2)/2 - (b*ExpIntegralEi[-2*b^2*x^2])/Sqrt[Pi
]))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6946
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfc}(bx)\,\mathrm{e}^{-b^2x^2}}{x^4}\,dx$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^4,x)
```

output
```
int(erfc(b*x)/exp(b^2*x^2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx = \frac{2\,\pi^{\frac{3}{2}}\sqrt{b^2}b^2x^3\operatorname{erf}\left(\sqrt{b^2}x\right) - (\pi - 2\,\pi b^2x^2 - (\pi - 2\,\pi b^2x^2)\operatorname{erf}(bx))e^{(-b^2x^2)} - \sqrt{\pi}\left(\pi b^3x^3\operatorname{erf}(bx)^2 - 4\,b^3x^3\right.}{3\,\pi x^3}$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(2*pi^(3/2)*sqrt(b^2)*b^2*x^3*erf(sqrt(b^2)*x) - (pi - 2*pi*b^2*x^2 -
(pi - 2*pi*b^2*x^2)*erf(b*x))*e^(-b^2*x^2) - sqrt(pi)*(pi*b^3*x^3*erf(b*x)
^2 - 4*b^3*x^3*Ei(-2*b^2*x^2) - b*x*e^(-2*b^2*x^2)))/(pi*x^3)
```

Sympy [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx = \int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2)/x**4,x)
```

output
```
Integral(exp(-b**2*x**2)*erfc(b*x)/x**4, x)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}(bx)\,e^{(-b^2x^2)}}{x^4}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^4, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \int \frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^4}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*e^(-b^2*x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \int \frac{e^{-b^2\,x^2}\,\text{erfc}(b\,x)}{x^4}\,dx$$

input
```
int((exp(-b^2*x^2)*erfc(b*x))/x^4,x)
```

output
```
int((exp(-b^2*x^2)*erfc(b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\text{erfc}(bx)}{x^4}\,dx = \frac{-3e^{b^2x^2}\left(\int \frac{\text{erf}(bx)}{e^{b^2x^2}x^4}dx\right)x^3 - 2e^{b^2x^2}\left(\int \frac{1}{e^{b^2x^2}x^2}dx\right)b^2x^3 - 1}{3e^{b^2x^2}x^3}$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^4,x)
```

output
```
( - 3*e**(b**2*x**2)*int(erf(b*x)/(e**(b**2*x**2)*x**4),x)*x**3 - 2*e**(b*
*2*x**2)*int(1/(e**(b**2*x**2)*x**2),x)*b**2*x**3 - 1)/(3*e**(b**2*x**2)*x
**3)
```

3.189 $\int e^{c+dx^2} x^3 \mathbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 342

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx=\frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2d\sqrt{\pi}}-\frac{be^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)d\sqrt{\pi}}-\frac{be^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}d^2}+\frac{a^2b^3e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{5/2}d}+\frac{be^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}d}-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d^2}+\frac{e^{c+dx^2}x^2\text{erfc}(a+bx)}{2d}$$

output
```
1/2*a*b^2*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)^2/d/Pi^(1/2)-1/2*b*exp(-
a^2+c-2*a*b*x-(b^2-d)*x^2)*x/(b^2-d)/d/Pi^(1/2)-1/2*b*exp(c+a^2*d/(b^2-d))
*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(1/2)/d^2+1/2*a^2*b^3*exp(c+a^
2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(5/2)/d+1/4*b*exp(
c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(3/2)/d-1/2*ex
p(d*x^2+c)*erfc(b*x+a)/d^2+1/2*exp(d*x^2+c)*x^2*erfc(b*x+a)/d
```

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.75

$$\int e^{c+dx^2}x^3\operatorname{erfc}(a+bx)\,dx =$$

$$-\frac{e^c\left(-2e^{dx^2}(-1+dx^2)+2e^{dx^2}(-1+dx^2)\operatorname{erf}(a+bx)-\frac{bde^{-a^2-2abx+(-b^2+d)x^2}\left(2(b^2-d)(ab+(-b^2+d)x)+\sqrt{b^2-d}\cdots\right.}{(b^2-d)^{\cdots}}\cdots\right)}{4d^2}$$

input
```
Integrate[E^(c + d*x^2)*x^3*Erfc[a + b*x],x]
```

output
```
-1/4*(E^c*(-2*E^(d*x^2)*(-1 + d*x^2) + 2*E^(d*x^2)*(-1 + d*x^2)*Erf[a + b*
x] - (b*d*E^(-a^2 - 2*a*b*x + (-b^2 + d)*x^2)*(2*(b^2 - d)*(a*b + (-b^2 +
d)*x) + Sqrt[b^2 - d]*((1 + 2*a^2)*b^2 - d)*E^((a*b + (b^2 - d)*x)^2/(b^2
- d))*Sqrt[Pi]*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]]))/((b^2 - d)^3*Sqrt[
Pi]) + (2*b*E^((a^2*d)/(b^2 - d))*Erfi[(-(a*b) + (-b^2 + d)*x)/Sqrt[-b^2 +
 d]])/Sqrt[-b^2 + d]))/d^2
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6940, 2671, 2664, 2634, 2670, 2664, 2634, 6937, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{c+dx^2}\operatorname{erfc}(a+bx)\,dx$$

$$\Big\downarrow \text{6940}$$

$$\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\operatorname{erfc}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2671}$$

$$\frac{b\left(\frac{\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{2(b^2-d)}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{2(b^2-d)}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2670}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfc}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$\frac{b\left(-\frac{ab\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2 \cdots\right.}}{2(b^2-d)}\cdots\right.}{\sqrt{\pi}d}$$

$$\frac{\int e^{dx^2+c}x\text{erfc}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{\int e^{dx^2+c}x\text{erfc}(a+bx)dx}{d}+$$

$$\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2 \cdots\right.}}{2(b^2-d)}\cdots\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 6937}$$

$$-\frac{\frac{b\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{\sqrt{\pi}d}+\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d}}{d}+$$

$$\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2 \cdots\right.}}{2(b^2-d)}\cdots\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2664}$$

$$-\frac{-\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{\sqrt{\pi}d}+\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d}}{d}+\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\right.}}{2(b^2-d)}\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\Big\downarrow$ 2634

$$-\frac{-\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}+\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2d}}{d}+\frac{b\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\right.}}{2(b^2-d)}\right.}{\sqrt{\pi}d}$$

$$\frac{x^2e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^3*Erfc[a + b*x],x]
```

output
```
(b*(-1/2*(E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)*x)/(b^2 - d) + (E^((b^2*c
 + a^2*d - c*d)/(b^2 - d))*Sqrt[Pi]*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]]
)/(4*(b^2 - d)^(3/2)) - (a*b*(-1/2*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2)/
(b^2 - d) - (a*b*E^((b^2*c + a^2*d - c*d)/(b^2 - d))*Sqrt[Pi]*Erf[(a*b + (
b^2 - d)*x)/Sqrt[b^2 - d]])/(2*(b^2 - d)^(3/2))))/(b^2 - d)))/(d*Sqrt[Pi])
 + (E^(c + d*x^2)*x^2*Erfc[a + b*x])/(2*d) - ((b*E^((b^2*c + a^2*d - c*d)/
(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 - d]])/(2*Sqrt[b^2 - d]*d) + (
E^(c + d*x^2)*Erfc[a + b*x])/(2*d))/d
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6940
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfc[a + b*x], x], x] + Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(-a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x]) /; Fre
eQ[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{d\,x^2+c} x^3 \,\mathrm{erfc}\left(bx+a\right) dx$$

input
```
int(exp(d*x^2+c)*x^3*erfc(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^3*erfc(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int e^{c+dx^2} x^3 \mathrm{erfc}(a+bx)\,dx = -\frac{\pi(2\,b^5-(2\,a^2+5)b^3d+3\,bd^2)\sqrt{b^2-d}\,\mathrm{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+\left(a^2-c\right)d}{b^2-d}\right)}-2\sqrt{\pi}(ab^4d-ab^2d^2-(b^5d-2}{}$$

input
```
integrate(exp(d*x^2+c)*x^3*erfc(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(pi*(2*b^5 - (2*a^2 + 5)*b^3*d + 3*b*d^2)*sqrt(b^2 - d)*erf((a*b + (b
^2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a^2 - c)*d)/(b^2 - d)) - 2*sqrt(pi)
*(a*b^4*d - a*b^2*d^2 - (b^5*d - 2*b^3*d^2 + b*d^3)*x)*e^(-b^2*x^2 - 2*a*b
*x + d*x^2 - a^2 + c) - 2*(pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3 - d^4)*x^2 -
pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3) - (pi*(b^6*d - 3*b^4*d^2 + 3*b^2*d^3
- d^4)*x^2 - pi*(b^6 - 3*b^4*d + 3*b^2*d^2 - d^3))*erf(b*x + a))*e^(d*x^2
+ c))/(pi*(b^6*d^2 - 3*b^4*d^3 + 3*b^2*d^4 - d^5))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**3*erfc(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx=\int x^3\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^3*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx=\int x^3\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfc(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx=\int x^3\,\text{erfc}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^3*erfc(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^3*erfc(a + b*x)*exp(c + d*x^2), x)
```

Reduce [F]

$$\int e^{c+dx^2}x^3\text{erfc}(a+bx)\,dx=\frac{e^c\left(e^{d\,x^2}d\,x^2-e^{d\,x^2}-2\left(\int e^{d\,x^2}\text{erf}(bx+a)\,x^3dx\right)d^2\right)}{2d^2}$$

input
```
int(exp(d*x^2+c)*x^3*erfc(b*x+a),x)
```

output
```
(e**c*(e**(d*x**2)*d*x**2 - e**(d*x**2) - 2*int(e**(d*x**2)*erf(a + b*x)*x
**3,x)*d**2))/(2*d**2)
```

3.190 $\int e^{c+dx^2} x\mathbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int e^{c+dx^2} x\operatorname{erfc}(a+bx)\,dx = \frac{be^{c+\frac{a^2d}{b^2-d}}\operatorname{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2\sqrt{b^2-d}\,d}+\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

output
```
1/2*b*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(1/2
)/d+1/2*exp(d*x^2+c)*erfc(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int e^{c+dx^2} x\operatorname{erfc}(a+bx)\,dx = \frac{e^c\left(e^{dx^2}\operatorname{erfc}(a+bx)+\frac{be^{\frac{a^2d}{b^2-d}}\operatorname{erfi}\left(\frac{-ab+(-b^2+d)x}{\sqrt{-b^2+d}}\right)}{\sqrt{-b^2+d}}\right)}{2d}$$

input
```
Integrate[E^(c + d*x^2)*x*Erfc[a + b*x],x]
```

output
```
(E^c*(E^(d*x^2)*Erfc[a + b*x] + (b*E^((a^2*d)/(b^2 - d))*Erfi[(-(a*b) + (-
b^2 + d)*x)/Sqrt[-b^2 + d]])/Sqrt[-b^2 + d]))/(2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6937, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{c+dx^2}\operatorname{erfc}(a+bx)\,dx$$

$$\Big\downarrow \text{6937}$$

$$\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}dx}{\sqrt{\pi}d}+\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{\sqrt{\pi}d}+\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{be^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x(b^2-d)}{\sqrt{b^2-d}}\right)}{2d\sqrt{b^2-d}}+\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x*Erfc[a + b*x],x]
```

output
```
(b*E^((b^2*c + a^2*d - c*d)/(b^2 - d))*Erf[(a*b + (b^2 - d)*x)/Sqrt[b^2 -
d]])/(2*Sqrt[b^2 - d]*d) + (E^(c + d*x^2)*Erfc[a + b*x])/(2*d)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 6937
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfc[a + b*x]/(2*d)), x] + Simp[b/(d*Sqrt[Pi])   Int[E^(-
a^2 + c - 2*a*b*x - (b^2 - d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(76) = 152$.

Time = 0.83 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.08

method	result
default	$\frac{\frac{b\,{\mathrm e}^{\frac{d\,a^{2}-2 d a (b x +a)+b^{2} c +d (b x +a)^{2}}{b^{2}}}}{2 d}-\frac{\operatorname{erf}\left(b x +a \right) b\,{\mathrm e}^{\frac{d\,a^{2}-2 d a (b x +a)+b^{2} c +d (b x +a)^{2}}{b^{2}}}}{2 d}+\frac{b\,{\mathrm e}^{\frac{d\,a^{2}+b^{2} c}{b^{2}}-\frac{a^{2} d^{2}}{b^{4} \left(-1+\frac{d}{b^{2}}\right)}}\operatorname{erf}\left(\sqrt{1-\frac{d}{b^{2}}}\,\left(b x +a \right)+\frac{a d}{b^{2} \sqrt{1-\frac{d}{b^{2}}}}\right)}{2 d \sqrt{1-\frac{d}{b^{2}}}}}{b}$

input
```
int(exp(d*x^2+c)*x*erfc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*exp((d*a^2-2*d*a*(b*x+a)+b^2*c+d*(b*x+a)^2)/b^2)/d-1/2*erf(b*x+a)*b
*exp((d*a^2-2*d*a*(b*x+a)+b^2*c+d*(b*x+a)^2)/b^2)/d+1/2*b/d*exp((a^2*d+b^2
*c)/b^2-1/b^4*a^2*d^2/(-1+d/b^2))/(1-d/b^2)^(1/2)*erf((1-d/b^2)^(1/2)*(b*x
+a)+1/b^2*a*d/(1-d/b^2)^(1/2)))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26

$$\int e^{c+dx^2} x \text{erfc}(a+bx)\,dx = \frac{\sqrt{b^2-d}\,b\,\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)e^{\left(\frac{b^2c+(a^2-c)d}{b^2-d}\right)}+(b^2-(b^2-d)\,\text{erf}\,(bx+a)-d)e^{(dx^2+c)}}{2\,(b^2d-d^2)}$$

input
```
integrate(exp(d*x^2+c)*x*erfc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(b^2 - d)*b*erf((a*b + (b^2 - d)*x)/sqrt(b^2 - d))*e^((b^2*c + (a
^2 - c)*d)/(b^2 - d)) + (b^2 - (b^2 - d)*erf(b*x + a) - d)*e^(d*x^2 + c))/
(b^2*d - d^2)
```

Sympy [F]

$$\int e^{c+dx^2} x \text{erfc}(a+bx)\,dx = e^c \int x e^{dx^2}\,\text{erfc}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x*erfc(b*x+a),x)
```

output
```
exp(c)*Integral(x*exp(d*x**2)*erfc(a + b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2} x \text{erfc}(a+bx)\,dx = \int x\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x\text{erfc}(a+bx)\,dx=\int x\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x*erfc(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx^2}x\text{erfc}(a+bx)\,dx=\int x\,\text{erfc}(a+b\,x)\,\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x*erfc(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x*erfc(a + b*x)*exp(c + d*x^2), x)
```

Reduce [F]

$$\int e^{c+dx^2}x\text{erfc}(a+bx)\,dx=\frac{e^c\left(e^{d\,x^2}-2\left(\int e^{d\,x^2}\text{erf}\,(bx+a)\,xdx\right)d\right)}{2d}$$

input
```
int(exp(d*x^2+c)*x*erfc(b*x+a),x)
```

output
```
(e**c*(e**(d*x**2) - 2*int(e**(d*x**2)*erf(a + b*x)*x,x)*d))/(2*d)
```

3.191 $\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx=\text{Int}\left(\frac{e^{c+dx^2}\text{erfc}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx=\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{ 6949}$$

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}dx$$

input
```
Int[(E^(c + d*x^2)*Erfc[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erfc}\left(bx+a\right)}{x}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx=\int \frac{\text{erfc}\,(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 6.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx=e^c\int \frac{e^{dx^2}\,\text{erfc}\,(a+bx)}{x}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x+a)/x,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx=\int \frac{\text{erfc}\,(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx = \int \frac{\text{erfc}\,(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\,dx = \int \frac{\text{erfc}(a+b\,x)\,\,\mathrm{e}^{d\,x^2+c}}{x}\,dx$$

input
```
int((erfc(a + b*x)*exp(c + d*x^2))/x,x)
```

output
```
int((erfc(a + b*x)*exp(c + d*x^2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x}\,dx = \frac{e^c\left(ei(d\,x^2)-2\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx+a)}{x}dx\right)\right)}{2}$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

output
```
(e**c*(ei(d*x**2) - 2*int((e**(d*x**2)*erf(a + b*x))/x,x)))/2
```

3.192 $\int \frac{e^{c+dx^2}\mathbf{erfc}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx = \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{\sqrt{\pi}x}+b\sqrt{b^2-d}e^{c+\frac{a^2d}{b^2-d}}\operatorname{erf}\left(\frac{ab+(b^2-d)\,x}{\sqrt{b^2-d}}\right) - \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{2x^2}+\frac{2ab^2\operatorname{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{\sqrt{\pi}} + d\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x},x\right)$$

output

```
b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/Pi^(1/2)/x+b*(b^2-d)^(1/2)*exp(c+a^2*d/(
b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))-1/2*exp(d*x^2+c)*erfc(b*x+a)/x^
2+2*a*b^2*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d)*x^2)/x,x)/Pi^(1/2)+d*Defe
r(Int)(exp(d*x^2+c)*erfc(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^3}\,dx=\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^3}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^3,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{6946}$$

$$-\frac{b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x^2}dx}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2672}$$

$$-\frac{b\left(-2(b^2-d)\int e^{-a^2-2bxa-(b^2-d)x^2+c}dx-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+$$
$$d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-2\left(b^2-d\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2673}$$

$$-\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{6949}$$

$$-\frac{b\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{2x^2}$$

input
```
Int[(E^(c + d*x^2)*Erfc[a + b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{d\,x^2+c}\operatorname{erfc}\left(bx+a\right)}{x^3}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^3,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx=\int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 23.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx=e^c\int \frac{e^{dx^2}\operatorname{erfc}\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x+a)/x**3,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(a + b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx=\int \frac{\operatorname{erfc}(a+b\,x)\ \mathrm{e}^{d\,x^2+c}}{x^3}\,dx$$

input
```
int((erfc(a + b*x)*exp(c + d*x^2))/x^3,x)
```

output
```
int((erfc(a + b*x)*exp(c + d*x^2))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^3}\,dx=\frac{e^c\left(ei(d\,x^2)\,d\,x^2-e^{d\,x^2}-2\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bx+a)}{x^3}dx\right)x^2\right)}{2x^2}$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^3,x)
```

output
```
(e**c*(ei(d*x**2)*d*x**2 - e**(d*x**2) - 2*int((e**(d*x**2)*erf(a + b*x))/
x**3,x)*x**2))/(2*x**2)
```

3.193 $\int e^{c+dx^2} x^4 \mathbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx &= \frac{3be^{-a^2+c-2abx-(b^2-d)x^2}}{4(b^2-d)\,d^2\sqrt{\pi}} - \frac{a^2b^3e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^3\,d\sqrt{\pi}} \\ &\quad - \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)^2\,d\sqrt{\pi}} + \frac{ab^2e^{-a^2+c-2abx-(b^2-d)x^2}x}{2(b^2-d)^2\,d\sqrt{\pi}} \\ &\quad - \frac{be^{-a^2+c-2abx-(b^2-d)x^2}x^2}{2(b^2-d)\,d\sqrt{\pi}} + \frac{3ab^2e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{3/2}\,d^2} \\ &\quad - \frac{a^3b^4e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{7/2}\,d} \\ &\quad - \frac{3ab^2e^{c+\frac{a^2d}{b^2-d}}\text{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{4(b^2-d)^{5/2}\,d} - \frac{3e^{c+dx^2}x\text{erfc}(a+bx)}{4d^2} \\ &\quad + \frac{e^{c+dx^2}x^3\text{erfc}(a+bx)}{2d} + \frac{3\text{Int}\left(e^{c+dx^2}\text{erfc}(a+bx),x\right)}{4d^2}\end{aligned}$$

output
```
3/4*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)/d^2/Pi^(1/2)-1/2*a^2*b^3*exp
(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)^3/d/Pi^(1/2)-1/2*b*exp(-a^2+c-2*a*b*x
-(b^2-d)*x^2)/(b^2-d)^2/d/Pi^(1/2)+1/2*a*b^2*exp(-a^2+c-2*a*b*x-(b^2-d)*x^
2)*x/(b^2-d)^2/d/Pi^(1/2)-1/2*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)*x^2/(b^2-d
)/d/Pi^(1/2)+3/4*a*b^2*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1
/2))/(b^2-d)^(3/2)/d^2-1/2*a^3*b^4*exp(c+a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x
)/(b^2-d)^(1/2))/(b^2-d)^(7/2)/d-3/4*a*b^2*exp(c+a^2*d/(b^2-d))*erf((a*b+(
b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(5/2)/d-3/4*exp(d*x^2+c)*x*erfc(b*x+a)/d^
2+1/2*exp(d*x^2+c)*x^3*erfc(b*x+a)/d+3/4*Defer(Int)(exp(d*x^2+c)*erfc(b*x+
a),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx=\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^4*Erfc[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^4*Erfc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{c+dx^2}\text{erfc}(a+bx)\,dx$$

$$\Big\downarrow \ 6940$$

$$\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}x^3dx}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \ 2671$$

$$\frac{b\left(\frac{\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \ 2670$$

$$\frac{b\left(\frac{-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \ 2664$$

$$\frac{b\left(\frac{-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \ 2634$$

$$\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}x^2dx}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{x^2e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\downarrow$ 2671

$$\frac{b\left(-\frac{ab\left(\frac{\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{2\left(b^2-d\right)}-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-c\right.}{\sqrt{b^2-d}}\right.}{2\left(b^2-d\right)^{3/2}}}{b^2-d}\right.}{\sqrt{\pi}d}$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\downarrow$ 2664

$$\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{2\left(b^2-d\right)}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\right.}{2\left(b^2-d\right)^{3/2}}}{}\right.}{\sqrt{\pi}d}$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\downarrow$ 2634

$$\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{a}{}\right.}{2\left(b^2-d\right)^{3/2}}}{}\right.}{\sqrt{\pi}d}$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\downarrow$ 2670

$$b\left(-\frac{ab\left(-\frac{ab\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}\right.$$

$$\sqrt{\pi}d$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \; 2664$$

$$b\left(-\frac{ab\left(-\frac{ab\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{b^2-d}+\frac{\sqrt{\pi}e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{4\left(b^2-d\right)^{3/2}}-\frac{xe^{-a^2-2abx-x^2\left(b^2\right.}}{2\left(b^2-d\right)}}{b^2-d}\right.$$

$$\sqrt{\pi}d$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \; 2634$$

$$-\frac{3\int e^{dx^2+c}x^2\text{erfc}(a+bx)dx}{2d}+$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2\right.}}{2\left(b^2-d\right)}\right.}{b^2-d}\right.}{}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\downarrow$ 6940

$$-\frac{3\left(\frac{b\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfc}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erfc}(a+bx)}{2d}\right)}{2d}+$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2\right.}}{2\left(b^2-d\right)}\right.}{b^2-d}\right.}{}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$\downarrow$ 2670

$$-\frac{3\left(\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfc}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erfc}(a+bx)}{2d}\right)}{2d}+$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^{\cdots}\right.}}{2\left(b^2-d\right)}\cdots\right)}{b^2-d}\cdots\right)}{\cdots}\cdots\right)}{\sqrt{\pi}d\cdots}$$

$$\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \ 2664$$

$$-\frac{3\left(\frac{b\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfc}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erfc}(a+bx)}{2d}\right)}{2d}+$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^{2}\ldots\right.}}{2\left(b^2-d\right)}}{b^2-d}\ldots\right.}{\ldots}\ldots\right.}{\sqrt{\pi}d\ldots}$$

$$\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx}{2d}+\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}\right)}{2d}+$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\operatorname{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2\right.}}{2\left(b^2-d\right)}\right.}{b^2-d}\right.}{}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\downarrow \text{6934}$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\text{erfc}(a+bx)dx}{2d}+\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erfc}(a+bx)}{2d}\right)}{2d}+$$

$$\frac{b\left(\frac{-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2\left(b^2-d\right)}}{b^2-d}-\frac{ab\left(-\frac{ab\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2\left(b^2-d\right)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^{2}\right.}}{2\left(b^2-d\right)}\right.}{b^2-d}\right.}{}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^4*Erfc[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{d\,x^2+c}x^4\,\text{erfc}\,(bx+a)\,dx$$

input
```
int(exp(d*x^2+c)*x^4*erfc(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^4*erfc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx=\int x^4\,\text{erfc}\,(bx+a)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfc(b*x+a),x, algorithm="fricas")
```

output
```
integral(-(x^4*erf(b*x + a) - x^4)*e^(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**4*erfc(b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erfc}(a+bx)\,dx=\int x^4\,\text{erfc}\,(bx+a)\,e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^4*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2} x^4 \text{erfc}(a+bx)\,dx = \int x^4 \,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfc(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^4*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2} x^4 \text{erfc}(a+bx)\,dx = \int x^4\,\text{erfc}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^4*erfc(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^4*erfc(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

$$\int e^{c+dx^2} x^4 \operatorname{erfc}(a+bx)\, dx = \frac{e^c \left(-3\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{d}\, ix\right) i + 4e^{d\,x^2} \sqrt{d}\, d\, x^3 - 6e^{d\,x^2} \sqrt{d}\, x - 8\sqrt{d} \left(\int e^{d\,x^2} \operatorname{erf}(bx+a)\, x^4 dx\right) d^2\right)}{8\sqrt{d}\, d^2}$$

input
```
int(exp(d*x^2+c)*x^4*erfc(b*x+a),x)
```

output
```
(e**c*( - 3*sqrt(pi)*erf(sqrt(d)*i*x)*i + 4*e**(d*x**2)*sqrt(d)*d*x**3 - 6
*e**(d*x**2)*sqrt(d)*x - 8*sqrt(d)*int(e**(d*x**2)*erf(a + b*x)*x**4,x)*d*
*2))/(8*sqrt(d)*d**2)
```

3.194 $\int e^{c+dx^2} x^2 \mathbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int e^{c+dx^2} x^2 \operatorname{erfc}(a+bx)\,dx = -\frac{b e^{-a^2+c-2abx-(b^2-d)x^2}}{2(b^2-d)\,d\sqrt{\pi}} - \frac{ab^2 e^{c+\frac{a^2 d}{b^2-d}} \operatorname{erf}\left(\frac{ab+(b^2-d)x}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}\,d} + \frac{e^{c+dx^2} x \operatorname{erfc}(a+bx)}{2d} - \frac{\operatorname{Int}\left(e^{c+dx^2}\operatorname{erfc}(a+bx),x\right)}{2d}$$

output
```
-1/2*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/(b^2-d)/d/Pi^(1/2)-1/2*a*b^2*exp(c+
a^2*d/(b^2-d))*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))/(b^2-d)^(3/2)/d+1/2*exp(
d*x^2+c)*x*erfc(b*x+a)/d-1/2*Defer(Int)(exp(d*x^2+c)*erfc(b*x+a),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2} x^2 \operatorname{erfc}(a+bx)\,dx = \int e^{c+dx^2} x^2 \operatorname{erfc}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^2*Erfc[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^2*Erfc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{c+dx^2} \operatorname{erfc}(a+bx) \, dx$$

$$\Big\downarrow \text{ 6940}$$

$$\frac{b\int e^{-a^2-2bxa-(b^2-d)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2670}$$

$$\frac{b\left(-\frac{ab\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{b\left(-\frac{abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab+\left(b^2-d\right)x\right)^2}{b^2-d}}dx}{b^2-d}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{\int e^{dx^2+c}\text{erfc}(a+bx)dx}{2d}+\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

$$\Big\downarrow \textbf{6934}$$

$$-\frac{\int e^{dx^2+c}\text{erfc}(a+bx)dx}{2d}+\frac{b\left(-\frac{\sqrt{\pi}abe^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)}{2(b^2-d)^{3/2}}-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2(b^2-d)}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erfc}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^2*Erfc[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \mathrm{e}^{d\,x^2+c}x^2\,\text{erfc}\left(bx+a\right)dx$$

input
```
int(exp(d*x^2+c)*x^2*erfc(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^2*erfc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx=\int x^2\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfc(b*x+a),x, algorithm="fricas")
```

output
```
integral(-(x^2*erf(b*x + a) - x^2)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 44.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx=e^c\int x^2e^{dx^2}\,\text{erfc}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**2*erfc(b*x+a),x)
```

output
```
exp(c)*Integral(x**2*exp(d*x**2)*erfc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx=\int x^2\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx = \int x^2\,\text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfc(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*erfc(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx = \int x^2\,\text{erfc}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^2*erfc(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^2*erfc(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int e^{c+dx^2}x^2\text{erfc}(a+bx)\,dx$$
$$=\frac{e^c\left(\sqrt{\pi}\,\text{erf}\left(\sqrt{d}\,ix\right)i+2e^{d\,x^2}\sqrt{d}\,x-4\sqrt{d}\left(\int e^{d\,x^2}\text{erf}(bx+a)\,x^2dx\right)d\right)}{4\sqrt{d}\,d}$$

input
```
int(exp(d*x^2+c)*x^2*erfc(b*x+a),x)
```

output
```
(e**c*(sqrt(pi)*erf(sqrt(d)*i*x)*i + 2*e**(d*x**2)*sqrt(d)*x - 4*sqrt(d)*i
nt(e**(d*x**2)*erf(a + b*x)*x**2,x)*d))/(4*sqrt(d)*d)
```

3.195 $\int e^{c+dx^2}\textbf{erfc}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int e^{c+dx^2}\operatorname{erfc}(a+bx)\,dx=\operatorname{Int}\left(e^{c+dx^2}\operatorname{erfc}(a+bx),x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}\operatorname{erfc}(a+bx)\,dx=\int e^{c+dx^2}\operatorname{erfc}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*Erfc[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*Erfc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx^2}\operatorname{erfc}(a+bx)\,dx$$

$$\Big\downarrow \; 6934$$

$$\int e^{c+dx^2}\operatorname{erfc}(a+bx)dx$$

input
```
Int[E^(c + d*x^2)*Erfc[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \mathrm{e}^{d\,x^2+c}\operatorname{erfc}\left(bx+a\right)dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{c+dx^2}\text{erfc}(a+bx)\,dx=\int \text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a),x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfc}(a+bx)\,dx=e^c\int e^{dx^2}\,\text{erfc}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x+a),x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfc}(a+bx)\,dx=\int \text{erfc}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\mathrm{erfc}(a+bx)\,dx=\int \mathrm{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a),x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\mathrm{erfc}(a+bx)\,dx=\int \mathrm{erfc}(a+b\,x)\;\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(erfc(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(erfc(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int e^{c+dx^2}\operatorname{erfc}(a+bx)\,dx = \frac{e^c\left(-\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{d}\,ix\right)i-2\sqrt{d}\left(\int e^{d\,x^2}\operatorname{erf}(bx+a)\,dx\right)\right)}{2\sqrt{d}}$$

input

```
int(exp(d*x^2+c)*erfc(b*x+a),x)
```

output

```
(e**c*( - sqrt(pi)*erf(sqrt(d)*i*x)*i - 2*sqrt(d)*int(e**(d*x**2)*erf(a +
b*x),x)))/(2*sqrt(d))
```

3.196 $\int \frac{e^{c+dx^2}\mathbf{erfc}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx = -\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x} - \frac{2b\operatorname{Int}\left(\frac{e^{-a^2+c-2abx+(-b^2+d)x^2}}{x},x\right)}{\sqrt{\pi}} + 2d\operatorname{Int}\left(e^{c+dx^2}\operatorname{erfc}(a+bx),x\right)$$

output
```
-exp(d*x^2+c)*erfc(b*x+a)/x-2*b*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d)*x^2
)/x,x)/Pi^(1/2)+2*d*Defer(Int)(exp(d*x^2+c)*erfc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx = \int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^2,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{6946}$$

$$-\frac{2b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx-\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x}$$

$$\Big\downarrow \text{2673}$$

$$-\frac{2b\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx-\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x}$$

$$\Big\downarrow \text{6934}$$

$$-\frac{2b\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\operatorname{erfc}(a+bx)dx-\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x}$$

input
```
Int[(E^(c + d*x^2)*Erfc[a + b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erfc}(bx+a)}{x^2}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^2,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{erfc}(bx+a)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx = e^c\int \frac{e^{dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x+a)/x**2,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^2}\,dx=\int \frac{\text{erfc}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}}{x^2}\,dx$$

input
```
int((erfc(a + b*x)*exp(c + d*x^2))/x^2,x)
```

output
```
int((erfc(a + b*x)*exp(c + d*x^2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^2}\,dx=e^c\left(\int \frac{e^{d\,x^2}}{x^2}dx-\left(\int \frac{e^{d\,x^2}\text{erf}(bx+a)}{x^2}dx\right)\right)$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^2,x)
```

output
```
e**c*(int(e**(d*x**2)/x**2,x) - int((e**(d*x**2)*erf(a + b*x))/x**2,x))
```

3.197 $\int \frac{e^{c+dx^2}\textbf{erfc}(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^4}\,dx &= \frac{be^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x^2}-\frac{2ab^2e^{-a^2+c-2abx-(b^2-d)x^2}}{3\sqrt{\pi}x}\\
&-\frac{2}{3}ab^2\sqrt{b^2-d}e^{c+\frac{a^2d}{b^2-d}}\operatorname{erf}\left(\frac{ab+(b^2-d)\,x}{\sqrt{b^2-d}}\right)\\
&-\frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{3x^3}-\frac{2de^{c+dx^2}\operatorname{erfc}(a+bx)}{3x}\\
&-\frac{4a^2b^3\operatorname{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}}\\
&+\frac{2b(b^2-d)\operatorname{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}}\\
&-\frac{4bd\operatorname{Int}\left(\frac{e^{-a^2+c-2abx+\left(-b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}}\\
&+\frac{4}{3}d^2\operatorname{Int}\left(e^{c+dx^2}\operatorname{erfc}(a+bx),x\right)\end{aligned}$$

output
```
1/3*b*exp(-a^2+c-2*a*b*x-(b^2-d)*x^2)/Pi^(1/2)/x^2-2/3*a*b^2*exp(-a^2+c-2*
a*b*x-(b^2-d)*x^2)/Pi^(1/2)/x-2/3*a*b^2*(b^2-d)^(1/2)*exp(c+a^2*d/(b^2-d))
*erf((a*b+(b^2-d)*x)/(b^2-d)^(1/2))-1/3*exp(d*x^2+c)*erfc(b*x+a)/x^3-2/3*d
*exp(d*x^2+c)*erfc(b*x+a)/x-4/3*a^2*b^3*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^
2+d)*x^2)/x,x)/Pi^(1/2)+2/3*b*(b^2-d)*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+
d)*x^2)/x,x)/Pi^(1/2)-4/3*b*d*Defer(Int)(exp(-a^2+c-2*a*b*x+(-b^2+d)*x^2)/
x,x)/Pi^(1/2)+4/3*d^2*Defer(Int)(exp(d*x^2+c)*erfc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx=\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^4,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfc[a + b*x])/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \text{ 6946}$$

$$-\frac{2b\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x^3}dx}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$$\Big\downarrow \; 2672$$

$$-\frac{2b\left(-ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x^2}dx-\left(b^2-d\right)\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}+$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$$\Big\downarrow \; 2672$$

$$-\frac{2b\left(-\left(\left(b^2-d\right)\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx\right)-ab\left(-2\left(b^2-d\right)\int e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}dx-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)\cdots}}{x}\cdots\right.\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$$\Big\downarrow \; 2664$$

$$-\frac{2b\left(-\left(\left(b^2-d\right)\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx\right)-ab\left(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx-2\left(b^2-d\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\int e^{-\frac{\left(ab-\cdots\right.}{\cdots}}\cdots\right.\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$$\Big\downarrow \; 2634$$

$$-\frac{2b\left(-ab\left(-2ab\int \frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)\cdots\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$$\Big\downarrow \; 2673$$

$$-\frac{2b\left(-ab\left(-2ab\int \frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)\cdots\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfc}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$\Big\downarrow$ 6946

$$-\frac{2b\left(-ab\left(-2ab\int\frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(-\frac{2b\int\frac{e^{-a^2-2bxa-\left(b^2-d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfc}(a+bx)dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$\Big\downarrow$ 2673

$$-\frac{2b\left(-ab\left(-2ab\int\frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(-\frac{2b\int\frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfc}(a+bx)dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

$\Big\downarrow$ 6934

$$-\frac{2b\left(-ab\left(-2ab\int\frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx+\sqrt{\pi}\left(-\sqrt{b^2-d}\right)e^{\frac{a^2d+b^2c-cd}{b^2-d}}\text{erf}\left(\frac{ab+x\left(b^2-d\right)}{\sqrt{b^2-d}}\right)-\frac{e^{-a^2-2abx-x^2\left(b^2-d\right)+c}}{x}\right)}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(-\frac{2b\int\frac{e^{-a^2-2bxa+\left(d-b^2\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfc}(a+bx)dx-\frac{e^{c+dx^2}\text{erfc}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erfc}(a+bx)}{3x^3}$$

input
```
Int[(E^(c + d*x^2)*Erfc[a + b*x])/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erfc}\left(bx+a\right)}{x^4}dx$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^4,x)
```

output
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
integral(-(erf(b*x + a) - 1)*e^(d*x^2 + c)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 66.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx = e^c\int \frac{e^{dx^2}\operatorname{erfc}\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfc(b*x+a)/x**4,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfc(a + b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfc}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{erfc}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfc(b*x+a)/x^4,x, algorithm="giac")
```

output
```
integrate(erfc(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx = \int \frac{\text{erfc}(a+b\,x)\ \mathrm{e}^{d\,x^2+c}}{x^4}\,dx$$

input
```
int((erfc(a + b*x)*exp(c + d*x^2))/x^4,x)
```

output
```
int((erfc(a + b*x)*exp(c + d*x^2))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{e^{c+dx^2}\text{erfc}(a+bx)}{x^4}\,dx = e^c\left(\int \frac{e^{d\,x^2}}{x^4}dx - \left(\int \frac{e^{d\,x^2}\text{erf}(bx+a)}{x^4}dx\right)\right)$$

input
```
int(exp(d*x^2+c)*erfc(b*x+a)/x^4,x)
```

output
```
e**c*(int(e**(d*x**2)/x**4,x) - int((e**(d*x**2)*erf(a + b*x))/x**4,x))
```

3.198 $\int \left(\frac{e^{-b^2x^2}\mathbf{erfc}(bx)}{x^3} + \frac{b^2e^{-b^2x^2}\mathbf{erfc}(bx)}{x} \right) dx$

Optimal result

Integrand size = 40, antiderivative size = 60

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^3} + \frac{b^2e^{-b^2x^2}\operatorname{erfc}(bx)}{x} \right) dx = \frac{be^{-2b^2x^2}}{\sqrt{\pi}x} + \sqrt{2}b^2\operatorname{erf}\left(\sqrt{2}bx\right) - \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{2x^2}$$

output
```
b/exp(2*b^2*x^2)/Pi^(1/2)/x+2^(1/2)*b^2*erf(2^(1/2)*b*x)-1/2*erfc(b*x)/exp
(b^2*x^2)/x^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(\frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{x^3} + \frac{b^2e^{-b^2x^2}\operatorname{erfc}(bx)}{x} \right) dx = \frac{be^{-2b^2x^2}}{\sqrt{\pi}x} + \sqrt{2}b^2\operatorname{erf}\left(\sqrt{2}bx\right) - \frac{e^{-b^2x^2}\operatorname{erfc}(bx)}{2x^2}$$

input
```
Integrate[Erfc[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfc[b*x])/(E^(b^2*x^2)*x),x]
```

output
```
b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2
*E^(b^2*x^2)*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2 e^{-b^2x^2}\text{erfc}(bx)}{x}+\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\sqrt{2}b^2\text{erf}\left(\sqrt{2}bx\right)-\frac{e^{-b^2x^2}\text{erfc}(bx)}{2x^2}+\frac{be^{-2b^2x^2}}{\sqrt{\pi}x}$$

input
```
Int[Erfc[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfc[b*x])/(E^(b^2*x^2)*x),x]
```

output
```
b/(E^(2*b^2*x^2)*Sqrt[Pi]*x) + Sqrt[2]*b^2*Erf[Sqrt[2]*b*x] - Erfc[b*x]/(2
*E^(b^2*x^2)*x^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{-\frac{b\,\mathrm{e}^{-b^2x^2}}{2x^2}+\frac{\mathrm{erf}(bx)b\,\mathrm{e}^{-b^2x^2}}{2x^2}-\frac{b^3\left(-\frac{\mathrm{e}^{-2b^2x^2}}{bx}-\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\,bx\right)\right)}{\sqrt{\pi}}}{b}$	84

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x,method=_RETU
RNVERBOSE)
```

output
```
(-1/2*b/exp(b^2*x^2)/x^2+1/2*erf(b*x)*b/exp(b^2*x^2)/x^2-1/Pi^(1/2)*b^3*(-
1/exp(b^2*x^2)^2/b/x-2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b*x)))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)\,dx$$
$$=\frac{2\sqrt{2}\pi\sqrt{b^2}bx^2\,\text{erf}\left(\sqrt{2}\sqrt{b^2}x\right)+2\sqrt{\pi}bxe^{(-2\,b^2x^2)}-(\pi-\pi\,\text{erf}\,(bx))e^{(-b^2x^2)}}{2\,\pi x^2}$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x, algor
ithm="fricas")
```

output
```
1/2*(2*sqrt(2)*pi*sqrt(b^2)*b*x^2*erf(sqrt(2)*sqrt(b^2)*x) + 2*sqrt(pi)*b*
x*e^(-2*b^2*x^2) - (pi - pi*erf(b*x))*e^(-b^2*x^2))/(pi*x^2)
```

Sympy [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)\,dx=\int \frac{(b^2x^2+1)\,e^{-b^2x^2}\,\text{erfc}\,(bx)}{x^3}\,dx$$

input
```
integrate(erfc(b*x)/exp(b**2*x**2)/x**3+b**2*erfc(b*x)/exp(b**2*x**2)/x,x)
```

output
```
Integral((b**2*x**2 + 1)*exp(-b**2*x**2)*erfc(b*x)/x**3, x)
```

Maxima [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)\,dx$$
$$=\int \frac{b^2\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}+\frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x, algor
ithm="maxima")
```

output
```
integrate(b^2*erfc(b*x)*e^(-b^2*x^2)/x + erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

Giac [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)\,dx$$
$$=\int \frac{b^2\,\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}+\frac{\text{erfc}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^3}\,dx$$

input
```
integrate(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x, algor
ithm="giac")
```

output
```
integrate(b^2*erfc(b*x)*e^(-b^2*x^2)/x + erfc(b*x)*e^(-b^2*x^2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)\,dx=\int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)}{x^3}+\frac{b^2\,\mathrm{e}^{-b^2\,x^2}\,\text{erfc}(b\,x)}{x}\,dx$$

input
```
int((exp(-b^2*x^2)*erfc(b*x))/x^3 + (b^2*exp(-b^2*x^2)*erfc(b*x))/x,x)
```

output
```
int((exp(-b^2*x^2)*erfc(b*x))/x^3 + (b^2*exp(-b^2*x^2)*erfc(b*x))/x, x)
```

Reduce [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erfc}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfc}(bx)}{x}\right)\,dx$$
$$=\frac{-2e^{b^2x^2}\left(\int \frac{\text{erf}(bx)}{e^{b^2x^2}x^3}dx\right)x^2-2e^{b^2x^2}\left(\int \frac{\text{erf}(bx)}{e^{b^2x^2}x}dx\right)b^2x^2-1}{2e^{b^2x^2}x^2}$$

input
```
int(erfc(b*x)/exp(b^2*x^2)/x^3+b^2*erfc(b*x)/exp(b^2*x^2)/x,x)
```

output
```
( - 2*e**(b**2*x**2)*int(erf(b*x)/(e**(b**2*x**2)*x**3),x)*x**2 - 2*e**(b*
*2*x**2)*int(erf(b*x)/(e**(b**2*x**2)*x),x)*b**2*x**2 - 1)/(2*e**(b**2*x**
2)*x**2)
```

3.199 $\int \mathbf{erfc}(bx) \sin(c + ib^2x^2) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \text{erfc}(bx) \sin(c + ib^2x^2) \, dx = \frac{ie^{ic}\sqrt{\pi}\text{erfc}(bx)^2}{8b} + \frac{ie^{-ic}\sqrt{\pi}\text{erfi}(bx)}{4b} - \frac{ibe^{-ic}x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*I*exp(I*c)*Pi^(1/2)*erfc(b*x)^2/b+1/4*I*Pi^(1/2)*erfi(b*x)/b/exp(I*c)-
1/2*I*b*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/exp(I*c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \text{erfc}(bx) \sin(c + ib^2x^2) \, dx$$
$$= \frac{(i\cos(c) + \sin(c)) \left(-4b^2x^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right) + \pi(2\text{erfi}(bx) - 2\text{erf}(bx)(\cos(2c) + i\sin(2c)) + \text{erf}(bx)^2(c\right.}{8b\sqrt{\pi}}$$

input
```
Integrate[Erfc[b*x]*Sin[c + I*b^2*x^2],x]
```

output
```
((I*Cos[c] + Sin[c])*(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x
^2] + Pi*(2*Erfi[b*x] - 2*Erf[b*x]*(Cos[2*c] + I*Sin[2*c]) + Erf[b*x]^2*(C
os[2*c] + I*Sin[2*c]))))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6959, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx$$

$$\Big\downarrow \ 6959$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erfc}(bx)dx-\frac{1}{2}i\int e^{ic-b^2x^2}\text{erfc}(bx)dx$$

$$\Big\downarrow \ 6928$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erfc}(bx)dx+\frac{i\sqrt{\pi}e^{ic}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erfc}(bx)dx+\frac{i\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \ 6931$$

$$\frac{1}{2}i\left(\int e^{b^2x^2-ic}dx-\int e^{b^2x^2-ic}\text{erf}(bx)dx\right)+\frac{i\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \ 2633$$

$$\frac{1}{2}i\left(\frac{\sqrt{\pi}e^{-ic}\text{erfi}(bx)}{2b}-\int e^{b^2x^2-ic}\text{erf}(bx)dx\right)+\frac{i\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \ 6930$$

$$\frac{1}{2}i\left(\frac{\sqrt{\pi}e^{-ic}\text{erfi}(bx)}{2b}-\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)+\frac{i\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

input
```
Int[Erfc[b*x]*Sin[c + I*b^2*x^2],x]
```

output
```
((I/8)*E^(I*c)*Sqrt[Pi]*Erfc[b*x]^2)/b + (I/2)*((Sqrt[Pi]*Erfi[b*x])/(2*b*
E^(I*c)) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*S
qrt[Pi]))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6959
```
Int[Erfc[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[I/2   Int
[E^((-I)*c - I*d*x^2)*Erfc[b*x], x], x] - Simp[I/2   Int[E^(I*c + I*d*x^2)*
Erfc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \operatorname{erfc}(bx)\sin\left(ib^2x^2+c\right)dx$$

input
```
int(erfc(b*x)*sin(c+I*b^2*x^2),x)
```

output
```
int(erfc(b*x)*sin(c+I*b^2*x^2),x)
```

Fricas [F]

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \operatorname{erfc}(bx)\sin\left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erfc(b*x)*sin(c+I*b^2*x^2),x, algorithm="fricas")
```

output
```
integral(1/2*((I*erf(b*x) - I)*e^(-2*b^2*x^2 + 2*I*c) - I*erf(b*x) + I)*e^
(b^2*x^2 - I*c), x)
```

Sympy [F]

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \sin\left(ib^2x^2+c\right)\operatorname{erfc}(bx)\,dx$$

input
```
integrate(erfc(b*x)*sin(c+I*b**2*x**2),x)
```

output
```
Integral(sin(I*b**2*x**2 + c)*erfc(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx = \int \operatorname{erfc}\left(bx\right)\sin\left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erfc(b*x)*sin(c+I*b^2*x^2),x, algorithm="maxima")
```

output
```
1/8*I*sqrt(pi)*cos(c)*erfc(b*x)^2/b - 1/8*sqrt(pi)*erfc(b*x)^2*sin(c)/b +
1/2*I*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erfc(b*x)
*e^(b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx = \int \operatorname{erfc}\left(bx\right)\sin\left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erfc(b*x)*sin(c+I*b^2*x^2),x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*sin(I*b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx = \int \sin\left(b^2\,x^2\,1\mathrm{i}+c\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(sin(c + b^2*x^2*1i)*erfc(b*x),x)
```

output
```
int(sin(c + b^2*x^2*1i)*erfc(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfc}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \sin\left(b^2i\,x^2+c\right)dx-\left(\int \operatorname{erf}(bx)\sin\left(b^2i\,x^2+c\right)dx\right)$$

input
```
int(erfc(b*x)*sin(c+I*b^2*x^2),x)
```

output
```
int(sin(b**2*i*x**2 + c),x) - int(erf(b*x)*sin(b**2*i*x**2 + c),x)
```

3.200 $\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\frac{ie^{-ic}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}-\frac{ie^{ic}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}+\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*I*Pi^(1/2)*erfc(b*x)^2/b/exp(I*c)-1/4*I*exp(I*c)*Pi^(1/2)*erfi(b*x)/b
+1/2*I*b*exp(I*c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=\frac{1}{2}i\left(-\frac{\sqrt{\pi}(-2\operatorname{erf}(bx)(\cos(c)-i\sin(c))+\operatorname{erf}(bx)^2(\cos(c)-i\sin(c))+2\operatorname{erfi}(bx)(\cos(c)+i\sin(c)))}{4b}+\frac{bx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cos(c)+i\sin(c))}{\sqrt{\pi}}\right)$$

input
```
Integrate[Erfc[b*x]*Sin[c - I*b^2*x^2],x]
```

output
```
(I/2)*(-1/4*(Sqrt[Pi]*(-2*Erf[b*x]*(Cos[c] - I*Sin[c]) + Erf[b*x]^2*(Cos[c
] - I*Sin[c]) + 2*Erfi[b*x]*(Cos[c] + I*Sin[c])))/b + (b*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cos[c] + I*Sin[c]))/Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6959, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{6959}$$

$$\frac{1}{2}i\int e^{-b^2x^2-ic}\text{erfc}(bx)dx-\frac{1}{2}i\int e^{b^2x^2+ic}\text{erfc}(bx)dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{1}{2}i\int e^{b^2x^2+ic}\text{erfc}(bx)dx-\frac{i\sqrt{\pi}e^{-ic}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}i\int e^{b^2x^2+ic}\text{erfc}(bx)dx-\frac{i\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6931}$$

$$-\frac{1}{2}i\left(\int e^{b^2x^2+ic}dx-\int e^{b^2x^2+ic}\text{erf}(bx)dx\right)-\frac{i\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{1}{2}i\left(\frac{\sqrt{\pi}e^{ic}\text{erfi}(bx)}{2b}-\int e^{b^2x^2+ic}\text{erf}(bx)dx\right)-\frac{i\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$-\frac{1}{2}i\left(\frac{\sqrt{\pi}e^{ic}\text{erfi}(bx)}{2b}-\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{i\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

input
```
Int[Erfc[b*x]*Sin[c - I*b^2*x^2],x]
```

output
```
((-1/8*I)*Sqrt[Pi]*Erfc[b*x]^2)/(b*E^(I*c)) - (I/2)*((E^(I*c)*Sqrt[Pi]*Erf
i[b*x])/(2*b) - (b*E^(I*c)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2
])/Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6959
```
Int[Erfc[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[I/2   Int
[E^((-I)*c - I*d*x^2)*Erfc[b*x], x], x] - Simp[I/2   Int[E^(I*c + I*d*x^2)*
Erfc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int -\operatorname{erfc}(bx)\sin\left(ib^2x^2-c\right)dx$$

input
```
int(-erfc(b*x)*sin(-c+I*b^2*x^2),x)
```

output
```
int(-erfc(b*x)*sin(-c+I*b^2*x^2),x)
```

Fricas [F]

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erfc}(bx)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erfc(b*x)*sin(-c+I*b^2*x^2),x, algorithm="fricas")
```

output
```
integral(1/2*((-I*erf(b*x) + I)*e^(-2*b^2*x^2 - 2*I*c) + I*erf(b*x) - I)*e
^(b^2*x^2 + I*c), x)
```

Sympy [F]

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\int \sin\left(ib^2x^2-c\right)\operatorname{erfc}(bx)\,dx$$

input
```
integrate(-erfc(b*x)*sin(-c+I*b**2*x**2),x)
```

output
```
-Integral(sin(I*b**2*x**2 - c)*erfc(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int-\operatorname{erfc}\left(bx\right)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erfc(b*x)*sin(-c+I*b^2*x^2),x, algorithm="maxima")
```

output
```
-1/8*I*sqrt(pi)*cos(c)*erfc(b*x)^2/b - 1/8*sqrt(pi)*erfc(b*x)^2*sin(c)/b -
 1/2*I*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) + 1/2*integrate(erfc(b*x
)*e^(b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int-\operatorname{erfc}\left(bx\right)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erfc(b*x)*sin(-c+I*b^2*x^2),x, algorithm="giac")
```

output
```
integrate(-erfc(b*x)*sin(I*b^2*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int\sin\left(c-b^2\,x^2\,1\mathrm{i}\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(sin(c - b^2*x^2*1i)*erfc(b*x),x)
```

output
```
int(sin(c - b^2*x^2*1i)*erfc(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfc}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\left(\int \sin\left(b^2i\,x^2-c\right)dx\right)+\int \operatorname{erf}(bx)\sin\left(b^2i\,x^2-c\right)dx$$

input
```
int(-erfc(b*x)*sin(-c+I*b^2*x^2),x)
```

output
```
- int(sin(b**2*i*x**2 - c),x) + int(erf(b*x)*sin(b**2*i*x**2 - c),x)
```

3.201 $\int \cos(c + ib^2x^2)\,\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \cos(c + ib^2x^2)\,\text{erfc}(bx)\,dx = -\frac{e^{ic}\sqrt{\pi}\text{erfc}(bx)^2}{8b} + \frac{e^{-ic}\sqrt{\pi}\text{erfi}(bx)}{4b} - \frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*exp(I*c)*Pi^(1/2)*erfc(b*x)^2/b+1/4*Pi^(1/2)*erfi(b*x)/b/exp(I*c)-1/2
*b*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/exp(I*c)/Pi^(1/2)
```

Mathematica [F]

$$\int \cos(c + ib^2x^2)\,\text{erfc}(bx)\,dx = \int \cos(c + ib^2x^2)\,\text{erfc}(bx)\,dx$$

input
```
Integrate[Cos[c + I*b^2*x^2]*Erfc[b*x],x]
```

output
```
Integrate[Cos[c + I*b^2*x^2]*Erfc[b*x], x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6962, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\cos\left(c+ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{6962}$$

$$\frac{1}{2}\int e^{ic-b^2x^2}\text{erfc}(bx)dx+\frac{1}{2}\int e^{b^2x^2-ic}\text{erfc}(bx)dx$$

$$\Big\downarrow \text{6928}$$

$$\frac{1}{2}\int e^{b^2x^2-ic}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^{ic}\int\text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2-ic}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6931}$$

$$\frac{1}{2}\left(\int e^{b^2x^2-ic}dx-\int e^{b^2x^2-ic}\text{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{-ic}\text{erfi}(bx)}{2b}-\int e^{b^2x^2-ic}\text{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{-ic}\text{erfi}(bx)}{2b}-\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{\sqrt{\pi}e^{ic}\text{erfc}(bx)^2}{8b}$$

input
```
Int[Cos[c + I*b^2*x^2]*Erfc[b*x],x]
```

output
```
-1/8*(E^(I*c)*Sqrt[Pi]*Erfc[b*x]^2)/b + ((Sqrt[Pi]*Erfi[b*x])/(2*b*E^(I*c)
) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(E^(I*c)*Sqrt[Pi]
))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6962
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int
[E^((-I)*c - I*d*x^2)*Erfc[b*x], x], x] + Simp[1/2   Int[E^(I*c + I*d*x^2)*
Erfc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \cos \left(ib^2x^2+c\right) \operatorname{erfc}\left(bx\right) dx$$

input
```
int(cos(c+I*b^2*x^2)*erfc(b*x),x)
```

output
```
int(cos(c+I*b^2*x^2)*erfc(b*x),x)
```

Fricas [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfc}(bx) \, dx = \int \cos \left(i\, b^2x^2+c\right) \operatorname{erfc}\left(bx\right) \, dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-1/2*((erf(b*x) - 1)*e^(-2*b^2*x^2 + 2*I*c) + erf(b*x) - 1)*e^(b^
2*x^2 - I*c), x)
```

Sympy [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfc}(bx) \, dx = \int \cos \left(ib^2x^2+c\right) \operatorname{erfc}\left(bx\right) dx$$

input
```
integrate(cos(c+I*b**2*x**2)*erfc(b*x),x)
```

output
```
Integral(cos(I*b**2*x**2 + c)*erfc(b*x), x)
```

Maxima [F]

$$\int \cos\left(c+ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(i\,b^2x^2+c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erfc(b*x),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*cos(c)*erfc(b*x)^2/b - 1/8*I*sqrt(pi)*erfc(b*x)^2*sin(c)/b +
 1/2*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) - 1/2*I*integrate(erfc(b*x
)*e^(b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \cos\left(c+ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(i\,b^2x^2+c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(cos(I*b^2*x^2 + c)*erfc(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(c+ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(b^2\,x^2\,1\mathrm{i}+c\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(cos(c + b^2*x^2*1i)*erfc(b*x),x)
```

output
```
int(cos(c + b^2*x^2*1i)*erfc(b*x), x)
```

Reduce [F]

$$\int \cos\left(c+ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(b^2i\,x^2+c\right)dx-\left(\int \cos\left(b^2i\,x^2+c\right)\operatorname{erf}(bx)\,dx\right)$$

input
```
int(cos(c+I*b^2*x^2)*erfc(b*x),x)
```

output
```
int(cos(b**2*i*x**2 + c),x) - int(cos(b**2*i*x**2 + c)*erf(b*x),x)
```

3.202 $\int \cos(c - ib^2x^2)\,\textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \cos(c - ib^2x^2)\operatorname{erfc}(bx)\,dx = -\frac{e^{-ic}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b} + \frac{e^{ic}\sqrt{\pi}\operatorname{erfi}(bx)}{4b} - \frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*Pi^(1/2)*erfc(b*x)^2/b/exp(I*c)+1/4*exp(I*c)*Pi^(1/2)*erfi(b*x)/b-1/2
*b*exp(I*c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [F]

$$\int \cos(c - ib^2x^2)\operatorname{erfc}(bx)\,dx = \int \cos(c - ib^2x^2)\operatorname{erfc}(bx)\,dx$$

input
```
Integrate[Cos[c - I*b^2*x^2]*Erfc[b*x],x]
```

output
```
Integrate[Cos[c - I*b^2*x^2]*Erfc[b*x], x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6962, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\cos\left(c-ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6962}$$

$$\frac{1}{2}\int e^{-b^2x^2-ic}\text{erfc}(bx)dx+\frac{1}{2}\int e^{b^2x^2+ic}\text{erfc}(bx)dx$$

$$\Big\downarrow \text{ 6928}$$

$$\frac{1}{2}\int e^{b^2x^2+ic}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^{-ic}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\int e^{b^2x^2+ic}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6931}$$

$$\frac{1}{2}\left(\int e^{b^2x^2+ic}dx-\int e^{b^2x^2+ic}\text{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{ic}\text{erfi}(bx)}{2b}-\int e^{b^2x^2+ic}\text{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6930}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{ic}\text{erfi}(bx)}{2b}-\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{\sqrt{\pi}e^{-ic}\text{erfc}(bx)^2}{8b}$$

input
```
Int[Cos[c - I*b^2*x^2]*Erfc[b*x],x]
```

output
```
-1/8*(Sqrt[Pi]*Erfc[b*x]^2)/(b*E^(I*c)) + ((E^(I*c)*Sqrt[Pi]*Erfi[b*x])/(2
*b) - (b*E^(I*c)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi
])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6962
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int
[E^((-I)*c - I*d*x^2)*Erfc[b*x], x], x] + Simp[1/2   Int[E^(I*c + I*d*x^2)*
Erfc[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \cos \left(i b^2 x^2-c\right) \operatorname{erfc}(b x) \, dx$$

input
```
int(cos(-c+I*b^2*x^2)*erfc(b*x),x)
```

output
```
int(cos(-c+I*b^2*x^2)*erfc(b*x),x)
```

Fricas [F]

$$\int \cos \left(c-i b^2 x^2\right) \operatorname{erfc}(b x) \, dx=\int \cos \left(i \, b^2 x^2-c\right) \operatorname{erfc}(b x) \, dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-1/2*((erf(b*x) - 1)*e^(-2*b^2*x^2 - 2*I*c) + erf(b*x) - 1)*e^(b^
2*x^2 + I*c), x)
```

Sympy [F]

$$\int \cos \left(c-i b^2 x^2\right) \operatorname{erfc}(b x) \, dx=\int \cos \left(i b^2 x^2-c\right) \operatorname{erfc}(b x) \, dx$$

input
```
integrate(cos(-c+I*b**2*x**2)*erfc(b*x),x)
```

output
```
Integral(cos(I*b**2*x**2 - c)*erfc(b*x), x)
```

Maxima [F]

$$\int \cos\left(c-ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(i\,b^2x^2-c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erfc(b*x),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*cos(c)*erfc(b*x)^2/b + 1/8*I*sqrt(pi)*erfc(b*x)^2*sin(c)/b +
 1/2*cos(c)*integrate(erfc(b*x)*e^(b^2*x^2), x) + 1/2*I*integrate(erfc(b*x
)*e^(b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \cos\left(c-ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(i\,b^2x^2-c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(cos(I*b^2*x^2 - c)*erfc(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(c-ib^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cos\left(c-b^2\,x^2\,1\mathrm{i}\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(cos(c - b^2*x^2*1i)*erfc(b*x),x)
```

output
```
int(cos(c - b^2*x^2*1i)*erfc(b*x), x)
```

Reduce [F]

$$\int \cos \left(c-ib^2x^2\right) \operatorname{erfc}(bx)\, dx = \int \cos \left(b^2 i\, x^2-c\right) dx - \left(\int \cos \left(b^2 i\, x^2-c\right) \operatorname{erf}(bx)\, dx\right)$$

input
```
int(cos(-c+I*b^2*x^2)*erfc(b*x),x)
```

output
```
int(cos(b**2*i*x**2 - c),x) - int(cos(b**2*i*x**2 - c)*erf(b*x),x)
```

3.203 $\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx = \frac{e^{-c}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}+\frac{e^{c}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}-\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*Pi^(1/2)*erfc(b*x)^2/b/exp(c)+1/4*exp(c)*Pi^(1/2)*erfi(b*x)/b-1/2*b*ex
p(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx = \frac{-4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cosh(c)+\sinh(c))+\pi(-2\operatorname{erf}(bx)(\cosh(c)-\sinh(c))+\operatorname{erf}(bx)^2(\cosh(c)-\mathrm{s}}{8b\sqrt{\pi}}$$

input
```
Integrate[Erfc[b*x]*Sinh[c + b^2*x^2],x]
```

output
```
(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c
]) + Pi*(-2*Erf[b*x]*(Cosh[c] - Sinh[c]) + Erf[b*x]^2*(Cosh[c] - Sinh[c])
+ 2*Erfi[b*x]*(Cosh[c] + Sinh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6965, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

$$\Big\downarrow \text{6965}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erfc}(bx)dx-\frac{1}{2}\int e^{-b^2x^2-c}\text{erfc}(bx)dx$$

$$\Big\downarrow \text{6928}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erfc}(bx)dx+\frac{\sqrt{\pi}e^{-c}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erfc}(bx)dx+\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6931}$$

$$\frac{1}{2}\left(\int e^{b^2x^2+c}dx-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)+\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{c}\text{erfi}(bx)}{2b}-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)+\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{c}\text{erfi}(bx)}{2b}-\frac{be^{c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)+\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

input
```
Int[Erfc[b*x]*Sinh[c + b^2*x^2],x]
```

output
```
(Sqrt[Pi]*Erfc[b*x]^2)/(8*b*E^c) + ((E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) - (b*E^
c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6965
```
Int[Erfc[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfc[b*x], x], x] - Simp[1/2   Int[E^(-c - d*x^2)*Erfc[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \operatorname{erfc}(bx)\sinh\left(b^2x^2+c\right)dx$$

input
```
int(erfc(b*x)*sinh(b^2*x^2+c),x)
```

output
```
int(erfc(b*x)*sinh(b^2*x^2+c),x)
```

Fricas [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx = \int \operatorname{erfc}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erfc(b*x)*sinh(b^2*x^2+c),x, algorithm="fricas")
```

output
```
integral(-(erf(b*x) - 1)*sinh(b^2*x^2 + c), x)
```

Sympy [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx = \int \sinh\left(b^2x^2+c\right)\operatorname{erfc}(bx)\,dx$$

input
```
integrate(erfc(b*x)*sinh(b**2*x**2+c),x)
```

output
```
Integral(sinh(b**2*x**2 + c)*erfc(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfc}(bx) \sinh \left(c + b^2x^2\right) \, dx = \int \operatorname{erfc}(bx) \sinh \left(b^2x^2 + c\right) \, dx$$

input
```
integrate(erfc(b*x)*sinh(b^2*x^2+c),x, algorithm="maxima")
```

output
```
integrate(erfc(b*x)*sinh(b^2*x^2 + c), x)
```

Giac [F]

$$\int \operatorname{erfc}(bx) \sinh \left(c + b^2x^2\right) \, dx = \int \operatorname{erfc}(bx) \sinh \left(b^2x^2 + c\right) \, dx$$

input
```
integrate(erfc(b*x)*sinh(b^2*x^2+c),x, algorithm="giac")
```

output
```
integrate(erfc(b*x)*sinh(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(bx) \sinh \left(c + b^2x^2\right) \, dx = \int \sinh \left(b^2\,x^2 + c\right) \, \operatorname{erfc}(b\,x) \, dx$$

input
```
int(sinh(c + b^2*x^2)*erfc(b*x),x)
```

output
```
int(sinh(c + b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c+b^2x^2\right)\,dx = \int \sinh\left(b^2x^2+c\right)dx - \left(\int \operatorname{erf}(bx)\sinh\left(b^2x^2+c\right)dx\right)$$

input
```
int(erfc(b*x)*sinh(b^2*x^2+c),x)
```

output
```
int(sinh(b**2*x**2 + c),x) - int(erf(b*x)*sinh(b**2*x**2 + c),x)
```

3.204 $\int \mathbf{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx=-\frac{e^c\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b}-\frac{e^{-c}\sqrt{\pi}\operatorname{erfi}(bx)}{4b}+\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*exp(c)*Pi^(1/2)*erfc(b*x)^2/b-1/4*Pi^(1/2)*erfi(b*x)/b/exp(c)+1/2*b*x
^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/exp(c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx=-\frac{(\cosh(c)-\sinh(c))\left(-4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)+\pi(2\operatorname{erfi}(bx)-2\operatorname{erf}(bx)(\cosh(2c)+\sinh(2c))+\operatorname{erf}(b\right.}{8b\sqrt{\pi}}$$

input
```
Integrate[Erfc[b*x]*Sinh[c - b^2*x^2],x]
```

output
```
-1/8*((Cosh[c] - Sinh[c])*(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
b^2*x^2] + Pi*(2*Erfi[b*x] - 2*Erf[b*x]*(Cosh[2*c] + Sinh[2*c]) + Erf[b*x]
^2*(Cosh[2*c] + Sinh[2*c]))))/(b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6965, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx$$

$$\Big\downarrow \text{6965}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfc}(bx)dx-\frac{1}{2}\int e^{b^2x^2-c}\text{erfc}(bx)dx$$

$$\Big\downarrow \text{6928}$$

$$-\frac{1}{2}\int e^{b^2x^2-c}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^c\int \text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}\int e^{b^2x^2-c}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6931}$$

$$\frac{1}{2}\left(\int e^{b^2x^2-c}\text{erf}(bx)dx-\int e^{b^2x^2-c}dx\right)-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}\left(\int e^{b^2x^2-c}\text{erf}(bx)dx-\frac{\sqrt{\pi}e^{-c}\text{erfi}(bx)}{2b}\right)-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{1}{2}\left(\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{erfi}(bx)}{2b}\right)-\frac{\sqrt{\pi}e^c\text{erfc}(bx)^2}{8b}$$

input
```
Int[Erfc[b*x]*Sinh[c - b^2*x^2],x]
```

output
```
-1/8*(E^c*Sqrt[Pi]*Erfc[b*x]^2)/b + (-1/2*(Sqrt[Pi]*Erfi[b*x])/(b*E^c) + (
b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(E^c*Sqrt[Pi]))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6965
```
Int[Erfc[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfc[b*x], x], x] - Simp[1/2   Int[E^(-c - d*x^2)*Erfc[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int -\operatorname{erfc}(bx)\sinh\left(b^2x^2-c\right)dx$$

input
```
int(-erfc(b*x)*sinh(b^2*x^2-c),x)
```

output
```
int(-erfc(b*x)*sinh(b^2*x^2-c),x)
```

Fricas [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx = \int -\operatorname{erfc}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erfc(b*x)*sinh(b^2*x^2-c),x, algorithm="fricas")
```

output
```
integral((erf(b*x) - 1)*sinh(b^2*x^2 - c), x)
```

Sympy [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx = -\int \sinh\left(b^2x^2-c\right)\operatorname{erfc}(bx)\,dx$$

input
```
integrate(-erfc(b*x)*sinh(b**2*x**2-c),x)
```

output
```
-Integral(sinh(b**2*x**2 - c)*erfc(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int -\operatorname{erfc}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erfc(b*x)*sinh(b^2*x^2-c),x, algorithm="maxima")
```

output
```
-integrate(erfc(b*x)*sinh(b^2*x^2 - c), x)
```

Giac [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int -\operatorname{erfc}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erfc(b*x)*sinh(b^2*x^2-c),x, algorithm="giac")
```

output
```
integrate(-erfc(b*x)*sinh(b^2*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int \sinh\left(c-b^2\,x^2\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(sinh(c - b^2*x^2)*erfc(b*x),x)
```

output
```
int(sinh(c - b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfc}(bx)\sinh\left(c-b^2x^2\right)\,dx=-\left(\int \sinh\left(b^2x^2-c\right)dx\right)+\int \operatorname{erf}(bx)\sinh\left(b^2x^2-c\right)dx$$

input
```
int(-erfc(b*x)*sinh(b^2*x^2-c),x)
```

output
```
- int(sinh(b**2*x**2 - c),x) + int(erf(b*x)*sinh(b**2*x**2 - c),x)
```

3.205 $\int \cosh(c+b^2x^2)\,\mathbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \cosh(c+b^2x^2)\operatorname{erfc}(bx)\,dx = -\frac{e^{-c}\sqrt{\pi}\operatorname{erfc}(bx)^2}{8b} + \frac{e^{c}\sqrt{\pi}\operatorname{erfi}(bx)}{4b} - \frac{be^{c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*Pi^(1/2)*erfc(b*x)^2/b/exp(c)+1/4*exp(c)*Pi^(1/2)*erfi(b*x)/b-1/2*b*e
xp(c)*x^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \cosh(c+b^2x^2)\operatorname{erfc}(bx)\,dx = \frac{4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)(\cosh(c)+\sinh(c))+\pi(\operatorname{erf}(bx)^2(-\cosh(c)+\sinh(c))+2\operatorname{erfi}(bx)(\cosh(c)+}{8b\sqrt{\pi}}$$

input
```
Integrate[Cosh[c + b^2*x^2]*Erfc[b*x],x]
```

output
```
(4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]*(Cosh[c] + Sinh
[c]) + Pi*(Erf[b*x]^2*(-Cosh[c] + Sinh[c]) + 2*Erfi[b*x]*(Cosh[c] + Sinh[c
]) - 2*Erf[b*x]*(-Cosh[c] + Sinh[c] + Erfi[b*x]*(Cosh[c] + Sinh[c]))))/(8*
b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6968, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfc}(bx)\cosh\left(b^2x^2+c\right)\,dx$$

$$\Big\downarrow \text{6968}$$

$$\frac{1}{2}\int e^{-b^2x^2-c}\text{erfc}(bx)dx+\frac{1}{2}\int e^{b^2x^2+c}\text{erfc}(bx)dx$$

$$\Big\downarrow \text{6928}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^{-c}\int \text{erfc}(bx)d\text{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erfc}(bx)dx-\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6931}$$

$$\frac{1}{2}\left(\int e^{b^2x^2+c}dx-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{c}\text{erfi}(bx)}{2b}-\int e^{b^2x^2+c}\text{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{c}\text{erfi}(bx)}{2b}-\frac{be^{c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{\sqrt{\pi}e^{-c}\text{erfc}(bx)^2}{8b}$$

input
```
Int[Cosh[c + b^2*x^2]*Erfc[b*x],x]
```

output
```
-1/8*(Sqrt[Pi]*Erfc[b*x]^2)/(b*E^c) + ((E^c*Sqrt[Pi]*Erfi[b*x])/(2*b) - (b
*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6968
```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfc[b*x], x], x] + Simp[1/2   Int[E^(-c - d*x^2)*Erfc[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \cosh\left(b^2x^2+c\right)\operatorname{erfc}\left(bx\right)dx$$

input
```
int(cosh(b^2*x^2+c)*erfc(b*x),x)
```

output
```
int(cosh(b^2*x^2+c)*erfc(b*x),x)
```

Fricas [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cosh\left(b^2x^2+c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-cosh(b^2*x^2 + c)*erf(b*x) + cosh(b^2*x^2 + c), x)
```

Sympy [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cosh\left(b^2x^2+c\right)\operatorname{erfc}\left(bx\right)dx$$

input
```
integrate(cosh(b**2*x**2+c)*erfc(b*x),x)
```

output
```
Integral(cosh(b**2*x**2 + c)*erfc(b*x), x)
```

Maxima [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfc}(bx)\,dx = \int \cosh\left(b^2x^2+c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(cosh(b^2*x^2 + c)*erfc(b*x), x)
```

Giac [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfc}(bx)\,dx = \int \cosh\left(b^2x^2+c\right)\operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(cosh(b^2*x^2 + c)*erfc(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfc}(bx)\,dx = \int \cosh\left(b^2\,x^2+c\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(cosh(c + b^2*x^2)*erfc(b*x),x)
```

output
```
int(cosh(c + b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfc}(bx)\,dx=\int \cosh\left(b^2x^2+c\right)dx-\left(\int \cosh\left(b^2x^2+c\right)\operatorname{erf}(bx)\,dx\right)$$

input
```
int(cosh(b^2*x^2+c)*erfc(b*x),x)
```

output
```
int(cosh(b**2*x**2 + c),x) - int(cosh(b**2*x**2 + c)*erf(b*x),x)
```

3.206 $\int \cosh(c-b^2x^2)\,\textbf{erfc}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \cosh(c-b^2x^2)\,\text{erfc}(bx)\,dx = -\frac{e^c\sqrt{\pi}\,\text{erfc}(bx)^2}{8b} + \frac{e^{-c}\sqrt{\pi}\,\text{erfi}(bx)}{4b} - \frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*exp(c)*Pi^(1/2)*erfc(b*x)^2/b+1/4*Pi^(1/2)*erfi(b*x)/b/exp(c)-1/2*b*x
^2*hypergeom([1, 1],[3/2, 2],b^2*x^2)/exp(c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \cosh(c-b^2x^2)\,\text{erfc}(bx)\,dx = \frac{4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)(\cosh(c)-\sinh(c)) - \pi(2\text{erf}(bx)(-\cosh(c)+\text{erfi}(bx)(\cosh(c)-\sinh(c)) - s}{8b\sqrt{\pi}}$$

input
```
Integrate[Cosh[c - b^2*x^2]*Erfc[b*x],x]
```

output
```
(4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)]*(Cosh[c] - Sinh
[c]) - Pi*(2*Erf[b*x]*(-Cosh[c] + Erfi[b*x]*(Cosh[c] - Sinh[c]) - Sinh[c])
 + 2*Erfi[b*x]*(-Cosh[c] + Sinh[c]) + Erf[b*x]^2*(Cosh[c] + Sinh[c])))/(8*
b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6968, 6928, 15, 6931, 2633, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erfc}(bx)\cosh\left(c-b^2x^2\right)\,dx$$

$$\Big\downarrow \text{6968}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\operatorname{erfc}(bx)dx+\frac{1}{2}\int e^{b^2x^2-c}\operatorname{erfc}(bx)dx$$

$$\Big\downarrow \text{6928}$$

$$\frac{1}{2}\int e^{b^2x^2-c}\operatorname{erfc}(bx)dx-\frac{\sqrt{\pi}e^c\int \operatorname{erfc}(bx)d\operatorname{erfc}(bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int e^{b^2x^2-c}\operatorname{erfc}(bx)dx-\frac{\sqrt{\pi}e^c\operatorname{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6931}$$

$$\frac{1}{2}\left(\int e^{b^2x^2-c}dx-\int e^{b^2x^2-c}\operatorname{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^c\operatorname{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{-c}\operatorname{erfi}(bx)}{2b}-\int e^{b^2x^2-c}\operatorname{erf}(bx)dx\right)-\frac{\sqrt{\pi}e^c\operatorname{erfc}(bx)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{1}{2}\left(\frac{\sqrt{\pi}e^{-c}\operatorname{erfi}(bx)}{2b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{\sqrt{\pi}e^c\operatorname{erfc}(bx)^2}{8b}$$

input
```
Int[Cosh[c - b^2*x^2]*Erfc[b*x],x]
```

output
```
-1/8*(E^c*Sqrt[Pi]*Erfc[b*x]^2)/b + ((Sqrt[Pi]*Erfi[b*x])/(2*b*E^c) - (b*x
^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/(E^c*Sqrt[Pi]))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6928
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[(-E^
c)*(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfc[b*x]], x] /; FreeQ[{b, c,
d, n}, x] && EqQ[d, -b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6931
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfc[(b_.)*(x_)], x_Symbol] :> Int[E^(c + d*x^
2), x] - Int[E^(c + d*x^2)*Erf[b*x], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^
2]
```

rule 6968
```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfc[(b_.)*(x_)], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfc[b*x], x], x] + Simp[1/2   Int[E^(-c - d*x^2)*Erfc[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \cosh \left(b^2x^2-c\right) \operatorname{erfc}(bx)\, dx$$

input
```
int(cosh(b^2*x^2-c)*erfc(b*x),x)
```

output
```
int(cosh(b^2*x^2-c)*erfc(b*x),x)
```

Fricas [F]

$$\int \cosh \left(c-b^2x^2\right) \operatorname{erfc}(bx)\, dx=\int \cosh \left(b^2x^2-c\right) \operatorname{erfc}(bx)\, dx$$

input
```
integrate(cosh(b^2*x^2-c)*erfc(b*x),x, algorithm="fricas")
```

output
```
integral(-cosh(b^2*x^2 - c)*erf(b*x) + cosh(b^2*x^2 - c), x)
```

Sympy [F]

$$\int \cosh \left(c-b^2x^2\right) \operatorname{erfc}(bx)\, dx=\int \cosh \left(b^2x^2-c\right) \operatorname{erfc}(bx)\, dx$$

input
```
integrate(cosh(b**2*x**2-c)*erfc(b*x),x)
```

output
```
Integral(cosh(b**2*x**2 - c)*erfc(b*x), x)
```

Maxima [F]

$$\int \cosh\left(c - b^2x^2\right) \operatorname{erfc}(bx)\,dx = \int \cosh\left(b^2x^2 - c\right) \operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erfc(b*x),x, algorithm="maxima")
```

output
```
integrate(cosh(b^2*x^2 - c)*erfc(b*x), x)
```

Giac [F]

$$\int \cosh\left(c - b^2x^2\right) \operatorname{erfc}(bx)\,dx = \int \cosh\left(b^2x^2 - c\right) \operatorname{erfc}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erfc(b*x),x, algorithm="giac")
```

output
```
integrate(cosh(b^2*x^2 - c)*erfc(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(c - b^2x^2\right) \operatorname{erfc}(bx)\,dx = \int \cosh\left(c - b^2\,x^2\right)\,\operatorname{erfc}(b\,x)\,dx$$

input
```
int(cosh(c - b^2*x^2)*erfc(b*x),x)
```

output
```
int(cosh(c - b^2*x^2)*erfc(b*x), x)
```

Reduce [F]

$$\int \cosh\left(c-b^2x^2\right)\operatorname{erfc}(bx)\,dx = \int \cosh\left(b^2x^2-c\right)dx - \left(\int \cosh\left(b^2x^2-c\right)\operatorname{erf}(bx)\,dx\right)$$

input
```
int(cosh(b^2*x^2-c)*erfc(b*x),x)
```

output
```
int(cosh(b**2*x**2 - c),x) - int(cosh(b**2*x**2 - c)*erf(b*x),x)
```

3.207 $\int x^5 \text{erfi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 93

$$\int x^5 \text{erfi}(bx)\,dx = -\frac{5e^{b^2x^2}x}{8b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3}{12b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5}{6b\sqrt{\pi}} + \frac{5\text{erfi}(bx)}{16b^6} + \frac{1}{6}x^6\text{erfi}(bx)$$

output
```
-5/8*exp(b^2*x^2)*x/b^5/Pi^(1/2)+5/12*exp(b^2*x^2)*x^3/b^3/Pi^(1/2)-1/6*ex
p(b^2*x^2)*x^5/b/Pi^(1/2)+5/16*erfi(b*x)/b^6+1/6*x^6*erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.69

$$\int x^5 \text{erfi}(bx)\,dx = \frac{-2be^{b^2x^2}x(15-10b^2x^2+4b^4x^4)+\sqrt{\pi}(15+8b^6x^6)\,\text{erfi}(bx)}{48b^6\sqrt{\pi}}$$

input
```
Integrate[x^5*Erfi[b*x],x]
```

output
```
(-2*b*E^(b^2*x^2)*x*(15 - 10*b^2*x^2 + 4*b^4*x^4) + Sqrt[Pi]*(15 + 8*b^6*x
^6)*Erfi[b*x])/(48*b^6*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6917, 2641, 2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{1}{6}x^6 \text{erfi}(bx)-\frac{b\int e^{b^2x^2}x^6dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6 \text{erfi}(bx)-\frac{b\left(\frac{x^5e^{b^2x^2}}{2b^2}-\frac{5\int e^{b^2x^2}x^4dx}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6 \text{erfi}(bx)-\frac{b\left(\frac{x^5e^{b^2x^2}}{2b^2}-\frac{5\left(\frac{x^3e^{b^2x^2}}{2b^2}-\frac{3\int e^{b^2x^2}x^2dx}{2b^2}\right)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6 \text{erfi}(bx)-\frac{b\left(\frac{x^5e^{b^2x^2}}{2b^2}-\frac{5\left(\frac{x^3e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{xe^{b^2x^2}}{2b^2}-\frac{\int e^{b^2x^2}dx}{2b^2}\right)}{2b^2}\right)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{6}x^6\text{erfi}(bx)-\frac{b\left(\frac{x^5e^{b^2x^2}}{2b^2}-\frac{5\left(\frac{x^3e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{xe^{b^2x^2}}{2b^2}-\frac{\sqrt{\pi}\text{erfi}(bx)}{4b^3}\right)}{2b^2}\right)}{2b^2}\right)}{3\sqrt{\pi}}$$

input
```
Int[x^5*Erfi[b*x],x]
```

output
```
(x^6*Erfi[b*x])/6 - (b*((E^(b^2*x^2)*x^5)/(2*b^2) - (5*((E^(b^2*x^2)*x^3)/
(2*b^2) - (3*((E^(b^2*x^2)*x)/(2*b^2) - (Sqrt[Pi]*Erfi[b*x])/(4*b^3)))/(2*
b^2)))/(2*b^2)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{i\left(\frac{ixb\left(28b^4x^4-70b^2x^2+105\right)e^{b^2x^2}}{84}-\frac{i\left(56b^6x^6+105\right)\operatorname{erfi}(bx)\sqrt{\pi}}{168}\right)}{2b^6\sqrt{\pi}}$	62
derivativedivides	$\frac{\frac{b^6x^6\operatorname{erfi}(bx)}{6}-\frac{\frac{b^5x^5e^{b^2x^2}}{2}-\frac{5b^3x^3e^{b^2x^2}}{4}+\frac{15\,e^{b^2x^2}bx}{8}-\frac{15\operatorname{erfi}(bx)\sqrt{\pi}}{16}}{3\sqrt{\pi}}}{b^6}$	77
default	$\frac{\frac{b^6x^6\operatorname{erfi}(bx)}{6}-\frac{\frac{b^5x^5e^{b^2x^2}}{2}-\frac{5b^3x^3e^{b^2x^2}}{4}+\frac{15\,e^{b^2x^2}bx}{8}-\frac{15\operatorname{erfi}(bx)\sqrt{\pi}}{16}}{3\sqrt{\pi}}}{b^6}$	77
parallelrisch	$\frac{8\operatorname{erfi}(bx)x^6b^6\sqrt{\pi}-8b^5x^5e^{b^2x^2}+20b^3x^3e^{b^2x^2}-30\,e^{b^2x^2}bx+15\operatorname{erfi}(bx)\sqrt{\pi}}{48b^6\sqrt{\pi}}$	78
parts	$\frac{x^6\operatorname{erfi}(bx)}{6}-\frac{b\left(\frac{x^5e^{b^2x^2}}{2b^2}-\frac{5\left(\frac{x^3e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{x\,e^{b^2x^2}}{2b^2}+\frac{i\sqrt{\pi}\operatorname{erf}(ibx)}{4b^3}\right)}{2b^2}\right)}{2b^2}\right)}{3\sqrt{\pi}}$	91

input
```
int(x^5*erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^6/Pi^(1/2)*(1/84*I*x*b*(28*b^4*x^4-70*b^2*x^2+105)*exp(b^2*x^2)-1/
168*I*(56*b^6*x^6+105)*erfi(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int x^5\operatorname{erfi}(bx)\,dx=-\frac{2\sqrt{\pi}(4\,b^5x^5-10\,b^3x^3+15\,bx)e^{(b^2x^2)}-(15\,\pi+8\,\pi b^6x^6)\operatorname{erfi}(bx)}{48\,\pi b^6}$$

input
```
integrate(x^5*erfi(b*x),x, algorithm="fricas")
```

output
```
-1/48*(2*sqrt(pi)*(4*b^5*x^5 - 10*b^3*x^3 + 15*b*x)*e^(b^2*x^2) - (15*pi +
 8*pi*b^6*x^6)*erfi(b*x))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int x^5\mathrm{erfi}(bx)\,dx = \begin{cases} \frac{x^6\,\mathrm{erfi}\,(bx)}{6} - \frac{x^5e^{b^2x^2}}{6\sqrt{\pi}b} + \frac{5x^3e^{b^2x^2}}{12\sqrt{\pi}b^3} - \frac{5xe^{b^2x^2}}{8\sqrt{\pi}b^5} + \frac{5\,\mathrm{erfi}\,(bx)}{16b^6} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*erfi(b*x),x)
```

output
```
Piecewise((x**6*erfi(b*x)/6 - x**5*exp(b**2*x**2)/(6*sqrt(pi)*b) + 5*x**3*
exp(b**2*x**2)/(12*sqrt(pi)*b**3) - 5*x*exp(b**2*x**2)/(8*sqrt(pi)*b**5) +
 5*erfi(b*x)/(16*b**6), Ne(b, 0)), (0, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int x^5\mathrm{erfi}(bx)\,dx = \frac{1}{6}\,x^6\,\mathrm{erfi}\,(bx) - \frac{b\left(\frac{2\,(4\,b^4x^5-10\,b^2x^3+15\,x)e^{\left(b^2x^2\right)}}{b^6} + \frac{15i\,\sqrt{\pi}\,\mathrm{erf}(i\,bx)}{b^7}\right)}{48\,\sqrt{\pi}}$$

input
```
integrate(x^5*erfi(b*x),x, algorithm="maxima")
```

output
```
1/6*x^6*erfi(b*x) - 1/48*b*(2*(4*b^4*x^5 - 10*b^2*x^3 + 15*x)*e^(b^2*x^2)/
b^6 + 15*I*sqrt(pi)*erf(I*b*x)/b^7)/sqrt(pi)
```

Giac [F]

$$\int x^5 \text{erfi}(bx)\,dx = \int x^5\,\text{erfi}\,(bx)\,\,dx$$

input
```
integrate(x^5*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int x^5 \text{erfi}(bx)\,dx = \frac{x^6\,\text{erfi}(b\,x)}{6} - \frac{5\,b\,x^7}{16\,{(-b^2\,x^2)}^{7/2}} - \frac{x^5\,\mathrm{e}^{b^2\,x^2}}{6\,b\,\sqrt{\pi}} + \frac{5\,x^3\,\mathrm{e}^{b^2\,x^2}}{12\,b^3\,\sqrt{\pi}} - \frac{5\,x\,\mathrm{e}^{b^2\,x^2}}{8\,b^5\,\sqrt{\pi}} + \frac{5\,b\,x^7\,\text{erfc}\left(\sqrt{-b^2\,x^2}\right)}{16\,{(-b^2\,x^2)}^{7/2}}$$

input
```
int(x^5*erfi(b*x),x)
```

output
```
(x^6*erfi(b*x))/6 - (5*b*x^7)/(16*(-b^2*x^2)^(7/2)) - (x^5*exp(b^2*x^2))/(
6*b*pi^(1/2)) + (5*x^3*exp(b^2*x^2))/(12*b^3*pi^(1/2)) - (5*x*exp(b^2*x^2)
)/(8*b^5*pi^(1/2)) + (5*b*x^7*erfc((-b^2*x^2)^(1/2)))/(16*(-b^2*x^2)^(7/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int x^5 \text{erfi}(bx)\,dx = \frac{-8\,\text{erf}\,(bix)\,b^6 i\pi\,x^6 - 15\,\text{erf}\,(bix)\,i\pi - 8\sqrt{\pi}\,e^{b^2x^2} b^5x^5 + 20\sqrt{\pi}\,e^{b^2x^2} b^3x^3 - 30\sqrt{\pi}\,e^{b^2x^2} bx}{48b^6\pi}$$

input
```
int(x^5*erfi(b*x),x)
```

output
```
( - 8*erf(b*i*x)*b**6*i*pi*x**6 - 15*erf(b*i*x)*i*pi - 8*sqrt(pi)*e**(b**2
*x**2)*b**5*x**5 + 20*sqrt(pi)*e**(b**2*x**2)*b**3*x**3 - 30*sqrt(pi)*e**(
b**2*x**2)*b*x)/(48*b**6*pi)
```

3.208 $\int x^3 \mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int x^3 \text{erfi}(bx)\,dx = \frac{3e^{b^2x^2}x}{8b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3}{4b\sqrt{\pi}} - \frac{3\text{erfi}(bx)}{16b^4} + \frac{1}{4}x^4\text{erfi}(bx)$$

output
```
3/8*exp(b^2*x^2)*x/b^3/Pi^(1/2)-1/4*exp(b^2*x^2)*x^3/b/Pi^(1/2)-3/16*erfi(
b*x)/b^4+1/4*x^4*erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int x^3 \text{erfi}(bx)\,dx = \frac{-\frac{2be^{b^2x^2}x(-3+2b^2x^2)}{\sqrt{\pi}} + (-3+4b^4x^4)\,\text{erfi}(bx)}{16b^4}$$

input
```
Integrate[x^3*Erfi[b*x],x]
```

output
```
((-2*b*E^(b^2*x^2)*x*(-3 + 2*b^2*x^2))/Sqrt[Pi] + (-3 + 4*b^4*x^4)*Erfi[b*
x])/(16*b^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6917, 2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{erfi}(bx) \, dx$$

$$\Big\downarrow \text{ 6917}$$

$$\frac{1}{4}x^4 \text{erfi}(bx) - \frac{b \int e^{b^2x^2} x^4 dx}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{4}x^4 \text{erfi}(bx) - \frac{b \left(\frac{x^3 e^{b^2x^2}}{2b^2} - \frac{3 \int e^{b^2x^2} x^2 dx}{2b^2} \right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{4}x^4 \text{erfi}(bx) - \frac{b \left(\frac{x^3 e^{b^2x^2}}{2b^2} - \frac{3 \left(\frac{x e^{b^2x^2}}{2b^2} - \frac{\int e^{b^2x^2} dx}{2b^2} \right)}{2b^2} \right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{1}{4}x^4 \text{erfi}(bx) - \frac{b \left(\frac{x^3 e^{b^2x^2}}{2b^2} - \frac{3 \left(\frac{x e^{b^2x^2}}{2b^2} - \frac{\sqrt{\pi} \text{erfi}(bx)}{4b^3} \right)}{2b^2} \right)}{2\sqrt{\pi}}$$

input
```
Int[x^3*Erfi[b*x],x]
```

output
```
(x^4*Erfi[b*x])/4 - (b*((E^(b^2*x^2)*x^3)/(2*b^2) - (3*((E^(b^2*x^2)*x)/(2
*b^2) - (Sqrt[Pi]*Erfi[b*x])/(4*b^3)))/(2*b^2)))/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	size
meijerg	$-\frac{i\left(\frac{i x b\left(-10 b^{2} x^{2}+15\right) {\mathrm e}^{b^{2} x^{2}}}{20}-\frac{i\left(-20 b^{4} x^{4}+15\right) \operatorname{erfi}\left(b x \right) \sqrt{\pi}}{40}\right)}{2 b^{4} \sqrt{\pi}}$	54
derivativedivides	$\frac{\frac{b^{4} x^{4} \operatorname{erfi}\left(b x \right)}{4}-\frac{\frac{b^{3} x^{3} {\mathrm e}^{b^{2} x^{2}}}{2}-\frac{3 \,{\mathrm e}^{b^{2} x^{2}} b x}{4}+\frac{3 \,\operatorname{erfi}\left(b x \right) \sqrt{\pi}}{8}}{2 \sqrt{\pi}}}{b^{4}}$	61
default	$\frac{\frac{b^{4} x^{4} \operatorname{erfi}\left(b x \right)}{4}-\frac{\frac{b^{3} x^{3} {\mathrm e}^{b^{2} x^{2}}}{2}-\frac{3 \,{\mathrm e}^{b^{2} x^{2}} b x}{4}+\frac{3 \,\operatorname{erfi}\left(b x \right) \sqrt{\pi}}{8}}{2 \sqrt{\pi}}}{b^{4}}$	61
parallelrisch	$\frac{4 \,\operatorname{erfi}\left(b x \right) x^{4} \sqrt{\pi}\, b^{4}-4 b^{3} x^{3} {\mathrm e}^{b^{2} x^{2}}+6 \,{\mathrm e}^{b^{2} x^{2}} b x -3 \,\operatorname{erfi}\left(b x \right) \sqrt{\pi}}{16 \sqrt{\pi}\, b^{4}}$	62
parts	$\frac{x^{4} \operatorname{erfi}\left(b x \right)}{4}-\frac{b \left(\frac{x^{3} {\mathrm e}^{b^{2} x^{2}}}{2 b^{2}}-\frac{3 \left(\frac{x \,{\mathrm e}^{b^{2} x^{2}}}{2 b^{2}}+\frac{i \sqrt{\pi}\, \operatorname{erf}\left(i b x \right)}{4 b^{3}}\right)}{2 b^{2}}\right)}{2 \sqrt{\pi}}$	69

input
```
int(x^3*erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^4/Pi^(1/2)*(1/20*I*x*b*(-10*b^2*x^2+15)*exp(b^2*x^2)-1/40*I*(-20*
b^4*x^4+15)*erfi(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int x^3 \text{erfi}(bx)\,dx = -\frac{2\sqrt{\pi}(2\,b^3x^3 - 3\,bx)e^{(b^2x^2)} + (3\,\pi - 4\,\pi b^4x^4)\,\text{erfi}\,(bx)}{16\,\pi b^4}$$

input
```
integrate(x^3*erfi(b*x),x, algorithm="fricas")
```

output
```
-1/16*(2*sqrt(pi)*(2*b^3*x^3 - 3*b*x)*e^(b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*
erfi(b*x))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^3 \text{erfi}(bx)\,dx = \begin{cases} \frac{x^4\,\text{erfi}\,(bx)}{4} - \frac{x^3e^{b^2x^2}}{4\sqrt{\pi}b} + \frac{3xe^{b^2x^2}}{8\sqrt{\pi}b^3} - \frac{3\,\text{erfi}\,(bx)}{16b^4} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erfi(b*x),x)
```

output
```
Piecewise((x**4*erfi(b*x)/4 - x**3*exp(b**2*x**2)/(4*sqrt(pi)*b) + 3*x*exp
(b**2*x**2)/(8*sqrt(pi)*b**3) - 3*erfi(b*x)/(16*b**4), Ne(b, 0)), (0, True
))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int x^3 \text{erfi}(bx)\,dx = \frac{1}{4}\,x^4\,\text{erfi}\,(bx) - \frac{b\left(\frac{2\,(2\,b^2x^3-3\,x)e^{\left(b^2x^2\right)}}{b^4} - \frac{3i\,\sqrt{\pi}\,\text{erf}(i\,bx)}{b^5}\right)}{16\,\sqrt{\pi}}$$

input
```
integrate(x^3*erfi(b*x),x, algorithm="maxima")
```

output
```
1/4*x^4*erfi(b*x) - 1/16*b*(2*(2*b^2*x^3 - 3*x)*e^(b^2*x^2)/b^4 - 3*I*sqrt
(pi)*erf(I*b*x)/b^5)/sqrt(pi)
```

Giac [F]

$$\int x^3 \text{erfi}(bx)\,dx = \int x^3\,\text{erfi}\,(bx)\,\,dx$$

input
```
integrate(x^3*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int x^3 \text{erfi}(bx)\,dx = \frac{x^4\,\text{erfi}(b\,x)}{4} - \frac{3\,b\,x^5}{16\,\left(-b^2\,x^2\right)^{5/2}} - \frac{x^3\,\mathrm{e}^{b^2\,x^2}}{4\,b\,\sqrt{\pi}} + \frac{3\,x\,\mathrm{e}^{b^2\,x^2}}{8\,b^3\,\sqrt{\pi}} + \frac{3\,b\,x^5\,\text{erfc}\left(\sqrt{-b^2\,x^2}\right)}{16\,\left(-b^2\,x^2\right)^{5/2}}$$

input
```
int(x^3*erfi(b*x),x)
```

output
```
(x^4*erfi(b*x))/4 - (3*b*x^5)/(16*(-b^2*x^2)^(5/2)) - (x^3*exp(b^2*x^2))/(
4*b*pi^(1/2)) + (3*x*exp(b^2*x^2))/(8*b^3*pi^(1/2)) + (3*b*x^5*erfc((-b^2*
x^2)^(1/2)))/(16*(-b^2*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int x^3\text{erfi}(bx)\,dx = \frac{-4\,\text{erf}(bix)\,b^4i\pi\,x^4 + 3\,\text{erf}(bix)\,i\pi - 4\sqrt{\pi}\,e^{b^2x^2}b^3x^3 + 6\sqrt{\pi}\,e^{b^2x^2}bx}{16b^4\pi}$$

input
```
int(x^3*erfi(b*x),x)
```

output
```
( - 4*erf(b*i*x)*b**4*i*pi*x**4 + 3*erf(b*i*x)*i*pi - 4*sqrt(pi)*e**(b**2*
x**2)*b**3*x**3 + 6*sqrt(pi)*e**(b**2*x**2)*b*x)/(16*b**4*pi)
```

3.209 $\int x\text{erfi}(bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 45

$$\int x\text{erfi}(bx)\,dx=-\frac{e^{b^2x^2}x}{2b\sqrt{\pi}}+\frac{\text{erfi}(bx)}{4b^2}+\frac{1}{2}x^2\text{erfi}(bx)$$

output
```
-1/2*exp(b^2*x^2)*x/b/Pi^(1/2)+1/4*erfi(b*x)/b^2+1/2*x^2*erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x\text{erfi}(bx)\,dx=\frac{1}{4}\left(-\frac{2e^{b^2x^2}x}{b\sqrt{\pi}}+\left(\frac{1}{b^2}+2x^2\right)\text{erfi}(bx)\right)$$

input
```
Integrate[x*Erfi[b*x],x]
```

output
```
((-2*E^(b^2*x^2)*x)/(b*Sqrt[Pi]) + (b^(-2) + 2*x^2)*Erfi[b*x])/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6917, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{1}{2}x^2\text{erfi}(bx)-\frac{b\int e^{b^2x^2}x^2dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{2}x^2\text{erfi}(bx)-\frac{b\left(\frac{xe^{b^2x^2}}{2b^2}-\frac{\int e^{b^2x^2}dx}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}x^2\text{erfi}(bx)-\frac{b\left(\frac{xe^{b^2x^2}}{2b^2}-\frac{\sqrt{\pi}\text{erfi}(bx)}{4b^3}\right)}{\sqrt{\pi}}$$

input
```
Int[x*Erfi[b*x],x]
```

output
```
(x^2*Erfi[b*x])/2 - (b*((E^(b^2*x^2)*x)/(2*b^2) - (Sqrt[Pi]*Erfi[b*x])/(4*
b^3)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
meijerg	$\frac{i\left(ixb\,{\mathrm e}^{b^{2} x^{2}}-\frac{i\left(6 b^{2} x^{2}+3\right) \operatorname{erfi}\left(b x\right) \sqrt{\pi}}{6}\right)}{2 b^{2} \sqrt{\pi}}$	44
derivativedivides	$\frac{\frac{b^{2} x^{2} \operatorname{erfi}\left(b x\right)}{2}-\frac{\frac{{\mathrm e}^{b^{2} x^{2}} b x}{2}-\frac{\operatorname{erfi}\left(b x\right) \sqrt{\pi}}{4}}{\sqrt{\pi}}}{b^{2}}$	45
default	$\frac{\frac{b^{2} x^{2} \operatorname{erfi}\left(b x\right)}{2}-\frac{\frac{{\mathrm e}^{b^{2} x^{2}} b x}{2}-\frac{\operatorname{erfi}\left(b x\right) \sqrt{\pi}}{4}}{\sqrt{\pi}}}{b^{2}}$	45
parallelrisch	$\frac{2 \operatorname{erfi}\left(b x\right) x^{2} \sqrt{\pi}\, b^{2}-2 \,{\mathrm e}^{b^{2} x^{2}} b x+\operatorname{erfi}\left(b x\right) \sqrt{\pi}}{4 \sqrt{\pi}\, b^{2}}$	45
parts	$\frac{x^{2} \operatorname{erfi}\left(b x\right)}{2}-\frac{b\left(\frac{x\,{\mathrm e}^{b^{2} x^{2}}}{2 b^{2}}+\frac{i \sqrt{\pi}\, \operatorname{erf}\left(i b x\right)}{4 b^{3}}\right)}{\sqrt{\pi}}$	47

input
```
int(x*erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^2/Pi^(1/2)*(I*x*b*exp(b^2*x^2)-1/6*I*(6*b^2*x^2+3)*erfi(b*x)*Pi^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int x\text{erfi}(bx)\,dx = -\frac{2\sqrt{\pi}bxe^{(b^2x^2)} - (\pi + 2\,\pi b^2x^2)\,\text{erfi}\,(bx)}{4\,\pi b^2}$$

input
```
integrate(x*erfi(b*x),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(pi)*b*x*e^(b^2*x^2) - (pi + 2*pi*b^2*x^2)*erfi(b*x))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x\text{erfi}(bx)\,dx = \begin{cases} \frac{x^2\,\text{erfi}\,(bx)}{2} - \frac{xe^{b^2x^2}}{2\sqrt{\pi}b} + \frac{\text{erfi}\,(bx)}{4b^2} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*erfi(b*x),x)
```

output
```
Piecewise((x**2*erfi(b*x)/2 - x*exp(b**2*x**2)/(2*sqrt(pi)*b) + erfi(b*x)/
(4*b**2), Ne(b, 0)), (0, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int x\mathrm{erfi}(bx)\,dx = \frac{1}{2}\,x^2\,\mathrm{erfi}\,(bx) - \frac{b\left(\frac{2\,xe^{\left(b^2x^2\right)}}{b^2} + \frac{i\,\sqrt{\pi}\,\mathrm{erf}(i\,bx)}{b^3}\right)}{4\,\sqrt{\pi}}$$

input
```
integrate(x*erfi(b*x),x, algorithm="maxima")
```

output
```
1/2*x^2*erfi(b*x) - 1/4*b*(2*x*e^(b^2*x^2)/b^2 + I*sqrt(pi)*erf(I*b*x)/b^3
)/sqrt(pi)
```

Giac [F]

$$\int x\mathrm{erfi}(bx)\,dx = \int x\,\mathrm{erfi}\,(bx)\,\,dx$$

input
```
integrate(x*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x*erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int x\mathrm{erfi}(bx)\,dx = \frac{x^2\,\mathrm{erfi}(b\,x)}{2} + \frac{b\,\mathrm{erfi}\left(x\,\sqrt{b^2}\right)}{4\,{(b^2)}^{3/2}} - \frac{x\,\mathrm{e}^{b^2\,x^2}}{2\,b\,\sqrt{\pi}}$$

input
```
int(x*erfi(b*x),x)
```

output
```
(x^2*erfi(b*x))/2 + (b*erfi(x*(b^2)^(1/2)))/(4*(b^2)^(3/2)) - (x*exp(b^2*x
^2))/(2*b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int x\text{erfi}(bx)\,dx = \frac{-2\,\text{erf}(bix)\,b^2 i\pi\,x^2 - \text{erf}(bix)\,i\pi - 2\sqrt{\pi}\,e^{b^2x^2}bx}{4b^2\pi}$$

input
```
int(x*erfi(b*x),x)
```

output
```
( - 2*erf(b*i*x)*b**2*i*pi*x**2 - erf(b*i*x)*i*pi - 2*sqrt(pi)*e**(b**2*x*
*2)*b*x)/(4*b**2*pi)
```

3.210 $\int \frac{\text{erfi}(bx)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{\text{erfi}(bx)}{x} \, dx = \frac{2bx \, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

output
```
2*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}(bx)}{x} \, dx = \frac{2bx \, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erfi[b*x]/x,x]
```

output
```
(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x} \, dx$$

$$\Big\downarrow \text{6914}$$

$$\frac{2bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]/x,x]
```

output
```
(2*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 6914
```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{2bx\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}\right],\left[\frac{3}{2},\frac{3}{2}\right],b^2x^2\right)}{\sqrt{\pi}}$	22

input
```
int(erfi(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
2*b*x*hypergeom([1/2,1/2],[3/2,3/2],b^2*x^2)/Pi^(1/2)
```

Fricas [F]

$$\int \frac{\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)}{x}\,dx$$

input
```
integrate(erfi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\text{erfi}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\begin{matrix}\frac{1}{2},\frac{1}{2}\\ \frac{3}{2},\frac{3}{2}\end{matrix}\middle|b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(erfi(b*x)/x,x)
```

output
```
2*b*x*hyper((1/2, 1/2), (3/2, 3/2), b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int \frac{\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)}{x}\,dx$$

input
```
integrate(erfi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)/x, x)
```

Giac [F]

$$\int \frac{\mathrm{erfi}(bx)}{x}\,dx = \int \frac{\mathrm{erfi}\,(bx)}{x}\,dx$$

input
```
integrate(erfi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{erfi}(bx)}{x}\,dx = \int \frac{\mathrm{erfi}(b\,x)}{x}\,dx$$

input
```
int(erfi(b*x)/x,x)
```

output
```
int(erfi(b*x)/x, x)
```

Reduce [F]

$$\int \frac{\mathrm{erfi}(bx)}{x}\,dx = -\left(\int \frac{\mathrm{erf}(bix)}{x}dx\right)i$$

input
```
int(erfi(b*x)/x,x)
```

output
```
- int(erf(b*i*x)/x,x)*i
```

3.211 $\int \frac{\text{erfi}(bx)}{x^3} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \frac{\text{erfi}(bx)}{x^3} \, dx = -\frac{be^{b^2x^2}}{\sqrt{\pi}x} + b^2\text{erfi}(bx) - \frac{\text{erfi}(bx)}{2x^2}$$

output
```
-b*exp(b^2*x^2)/Pi^(1/2)/x+b^2*erfi(b*x)-1/2*erfi(b*x)/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{\text{erfi}(bx)}{x^3} \, dx = -\frac{be^{b^2x^2}}{\sqrt{\pi}x} + \left(b^2 - \frac{1}{2x^2}\right)\text{erfi}(bx)$$

input
```
Integrate[Erfi[b*x]/x^3,x]
```

output
```
-((b*E^(b^2*x^2))/(Sqrt[Pi]*x)) + (b^2 - 1/(2*x^2))*Erfi[b*x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6917, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x^3} \, dx$$
$$\Big\downarrow \text{6917}$$
$$\frac{b \int \frac{e^{b^2x^2}}{x^2}dx}{\sqrt{\pi}} - \frac{\text{erfi}(bx)}{2x^2}$$
$$\Big\downarrow \text{2643}$$
$$\frac{b\left(2b^2 \int e^{b^2x^2}dx - \frac{e^{b^2x^2}}{x}\right)}{\sqrt{\pi}} - \frac{\text{erfi}(bx)}{2x^2}$$
$$\Big\downarrow \text{2633}$$
$$\frac{b\left(\sqrt{\pi} b\text{erfi}(bx) - \frac{e^{b^2x^2}}{x}\right)}{\sqrt{\pi}} - \frac{\text{erfi}(bx)}{2x^2}$$

input
```
Int[Erfi[b*x]/x^3,x]
```

output
```
-1/2*Erfi[b*x]/x^2 + (b*(-(E^(b^2*x^2)/x) + b*Sqrt[Pi]*Erfi[b*x]))/Sqrt[Pi
]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

method	result	size
parts	$-\frac{\operatorname{erfi}(bx)}{2x^2}+\frac{b\left(-\frac{e^{b^2x^2}}{x}-ib\sqrt{\pi}\,\operatorname{erf}(ibx)\right)}{\sqrt{\pi}}$	43
parallelrisch	$\frac{2\operatorname{erfi}(bx)x^2\sqrt{\pi}\,b^2-2\,e^{b^2x^2}bx-\operatorname{erfi}(bx)\sqrt{\pi}}{2\sqrt{\pi}\,x^2}$	46
derivativedivides	$b^2\left(-\frac{\operatorname{erfi}(bx)}{2b^2x^2}+\frac{-\frac{e^{b^2x^2}}{bx}+\operatorname{erfi}(bx)\sqrt{\pi}}{\sqrt{\pi}}\right)$	47
default	$b^2\left(-\frac{\operatorname{erfi}(bx)}{2b^2x^2}+\frac{-\frac{e^{b^2x^2}}{bx}+\operatorname{erfi}(bx)\sqrt{\pi}}{\sqrt{\pi}}\right)$	47
meijerg	$\frac{ib^2\left(\frac{2i e^{b^2x^2}}{xb}+\frac{i\left(-2b^2x^2+1\right)\operatorname{erfi}(bx)\sqrt{\pi}}{x^2b^2}\right)}{2\sqrt{\pi}}$	54

input
```
int(erfi(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*erfi(b*x)/x^2+1/Pi^(1/2)*b*(-1/x*exp(b^2*x^2)-I*b*Pi^(1/2)*erf(I*b*x)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx = -\frac{2\sqrt{\pi}bxe^{(b^2x^2)} + (\pi - 2\pi b^2x^2)\operatorname{erfi}(bx)}{2\pi x^2}$$

input
```
integrate(erfi(b*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(pi)*b*x*e^(b^2*x^2) + (pi - 2*pi*b^2*x^2)*erfi(b*x))/(pi*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx = b^2\operatorname{erfi}(bx) - \frac{be^{b^2x^2}}{\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)}{2x^2}$$

input
```
integrate(erfi(b*x)/x**3,x)
```

output
```
b**2*erfi(b*x) - b*exp(b**2*x**2)/(sqrt(pi)*x) - erfi(b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx = -\frac{\sqrt{-b^2x^2}\,b\Gamma\left(-\frac{1}{2},-b^2x^2\right)}{2\sqrt{\pi}x} - \frac{\operatorname{erfi}(bx)}{2\,x^2}$$

input
```
integrate(erfi(b*x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*sqrt(-b^2*x^2)*b*gamma(-1/2, -b^2*x^2)/(sqrt(pi)*x) - 1/2*erfi(b*x)/x
^2
```

Giac [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfi}(bx)}{x^3}\,dx$$

input
```
integrate(erfi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx = \frac{b\operatorname{erfc}\left(\sqrt{-b^2\,x^2}\right)\,\sqrt{-b^2\,x^2}}{x} - \frac{b\,\sqrt{-b^2\,x^2}}{x} - \frac{b\,\mathrm{e}^{b^2\,x^2}}{x\,\sqrt{\pi}} - \frac{\operatorname{erfi}(b\,x)}{2\,x^2}$$

input
```
int(erfi(b*x)/x^3,x)
```

output
```
(b*erfc((-b^2*x^2)^(1/2))*(-b^2*x^2)^(1/2))/x - (b*(-b^2*x^2)^(1/2))/x - (
b*exp(b^2*x^2))/(x*pi^(1/2)) - erfi(b*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{erfi}(bx)}{x^3}\,dx = \frac{-2\operatorname{erf}(bix)\,b^2 i\pi\,x^2 + \operatorname{erf}(bix)\,i\pi - 2\sqrt{\pi}\,e^{b^2x^2}bx}{2\pi\,x^2}$$

input
```
int(erfi(b*x)/x^3,x)
```

output
```
( - 2*erf(b*i*x)*b**2*i*pi*x**2 + erf(b*i*x)*i*pi - 2*sqrt(pi)*e**(b**2*x*
*2)*b*x)/(2*pi*x**2)
```

3.212 $\int \frac{\operatorname{erfi}(bx)}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\operatorname{erfi}(bx)}{x^5} dx = -\frac{be^{b^2x^2}}{6\sqrt{\pi}x^3} - \frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x} + \frac{1}{3}b^4\operatorname{erfi}(bx) - \frac{\operatorname{erfi}(bx)}{4x^4}$$

output
```
-1/6*b*exp(b^2*x^2)/Pi^(1/2)/x^3-1/3*b^3*exp(b^2*x^2)/Pi^(1/2)/x+1/3*b^4*e
rfi(b*x)-1/4*erfi(b*x)/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{erfi}(bx)}{x^5} dx = \frac{-\frac{2be^{b^2x^2}x(1+2b^2x^2)}{\sqrt{\pi}} + (-3+4b^4x^4)\operatorname{erfi}(bx)}{12x^4}$$

input
```
Integrate[Erfi[b*x]/x^5,x]
```

output
```
((-2*b*E^(b^2*x^2)*x*(1 + 2*b^2*x^2))/Sqrt[Pi] + (-3 + 4*b^4*x^4)*Erfi[b*x
])/(12*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6917, 2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x^5} \, dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{b \int \frac{e^{b^2x^2}}{x^4}dx}{2\sqrt{\pi}} - \frac{\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{3}b^2 \int \frac{e^{b^2x^2}}{x^2}dx - \frac{e^{b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}} - \frac{\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{3}b^2\left(2b^2 \int e^{b^2x^2}dx - \frac{e^{b^2x^2}}{x}\right) - \frac{e^{b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}} - \frac{\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \text{2633}$$

$$\frac{b\left(\frac{2}{3}b^2\left(\sqrt{\pi}b\text{erfi}(bx) - \frac{e^{b^2x^2}}{x}\right) - \frac{e^{b^2x^2}}{3x^3}\right)}{2\sqrt{\pi}} - \frac{\text{erfi}(bx)}{4x^4}$$

input
```
Int[Erfi[b*x]/x^5,x]
```

output
```
-1/4*Erfi[b*x]/x^4 + (b*(-1/3*E^(b^2*x^2)/x^3 + (2*b^2*(-(E^(b^2*x^2)/x) +
 b*Sqrt[Pi]*Erfi[b*x]))/3))/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{4\,\operatorname{erfi}(bx)x^4\sqrt{\pi}\,b^4-4b^3x^3\mathrm{e}^{b^2x^2}-2\,\mathrm{e}^{b^2x^2}bx-3\,\operatorname{erfi}(bx)\sqrt{\pi}}{12\sqrt{\pi}\,x^4}$	62
parts	$-\frac{\operatorname{erfi}(bx)}{4x^4}+\frac{b\left(-\frac{\mathrm{e}^{b^2x^2}}{3x^3}+\frac{2b^2\left(-\frac{\mathrm{e}^{b^2x^2}}{x}-ib\sqrt{\pi}\,\operatorname{erf}(ibx)\right)}{3}\right)}{2\sqrt{\pi}}$	63
meijerg	$-\frac{ib^4\left(-\frac{4i\left(\frac{b^2x^2}{2}+\frac{1}{4}\right)\mathrm{e}^{b^2x^2}}{3x^3b^3}-\frac{i\left(-4b^4x^4+3\right)\operatorname{erfi}(bx)\sqrt{\pi}}{6x^4b^4}\right)}{2\sqrt{\pi}}$	64
derivativedivides	$b^4\left(-\frac{\operatorname{erfi}(bx)}{4b^4x^4}+\frac{-\frac{\mathrm{e}^{b^2x^2}}{3b^3x^3}-\frac{2\,\mathrm{e}^{b^2x^2}}{3bx}+\frac{2\,\operatorname{erfi}(bx)\sqrt{\pi}}{3}}{2\sqrt{\pi}}\right)$	65
default	$b^4\left(-\frac{\operatorname{erfi}(bx)}{4b^4x^4}+\frac{-\frac{\mathrm{e}^{b^2x^2}}{3b^3x^3}-\frac{2\,\mathrm{e}^{b^2x^2}}{3bx}+\frac{2\,\operatorname{erfi}(bx)\sqrt{\pi}}{3}}{2\sqrt{\pi}}\right)$	65

input
```
int(erfi(b*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*erfi(b*x)*x^4*Pi^(1/2)*b^4-4*b^3*x^3*exp(b^2*x^2)-2*exp(b^2*x^2)*b
*x-3*erfi(b*x)*Pi^(1/2))/Pi^(1/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{\text{erfi}(bx)}{x^5}\,dx = -\frac{2\,\sqrt{\pi}(2\,b^3x^3+bx)e^{(b^2x^2)} + (3\,\pi - 4\,\pi b^4x^4)\,\text{erfi}\,(bx)}{12\,\pi x^4}$$

input
```
integrate(erfi(b*x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(pi)*(2*b^3*x^3 + b*x)*e^(b^2*x^2) + (3*pi - 4*pi*b^4*x^4)*er
fi(b*x))/(pi*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{\text{erfi}(bx)}{x^5}\,dx = \frac{b^4\,\text{erfi}\,(bx)}{3} - \frac{b^3e^{b^2x^2}}{3\sqrt{\pi}x} - \frac{be^{b^2x^2}}{6\sqrt{\pi}x^3} - \frac{\text{erfi}\,(bx)}{4x^4}$$

input
```
integrate(erfi(b*x)/x**5,x)
```

output
```
b**4*erfi(b*x)/3 - b**3*exp(b**2*x**2)/(3*sqrt(pi)*x) - b*exp(b**2*x**2)/(
6*sqrt(pi)*x**3) - erfi(b*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{erfi}(bx)}{x^5}\,dx = -\frac{(-b^2x^2)^{\frac{3}{2}}\,b\Gamma\left(-\frac{3}{2},-b^2x^2\right)}{4\sqrt{\pi}x^3} - \frac{\operatorname{erfi}(bx)}{4x^4}$$

input
```
integrate(erfi(b*x)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(-b^2*x^2)^(3/2)*b*gamma(-3/2, -b^2*x^2)/(sqrt(pi)*x^3) - 1/4*erfi(b*
x)/x^4
```

Giac [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfi}(bx)}{x^5}\,dx$$

input
```
integrate(erfi(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{erfi}(bx)}{x^5}\,dx = \frac{b\,(-b^2\,x^2)^{3/2}}{3\,x^3} - \frac{\operatorname{erfi}(b\,x)}{4\,x^4} - \frac{b^3\,e^{b^2\,x^2}}{3\,x\,\sqrt{\pi}} - \frac{b\,e^{b^2\,x^2}}{6\,x^3\,\sqrt{\pi}} - \frac{b\operatorname{erfc}\left(\sqrt{-b^2\,x^2}\right)\,(-b^2\,x^2)^{3/2}}{3\,x^3}$$

input
```
int(erfi(b*x)/x^5,x)
```

output
```
(b*(-b^2*x^2)^(3/2))/(3*x^3) - erfi(b*x)/(4*x^4) - (b^3*exp(b^2*x^2))/(3*x
*pi^(1/2)) - (b*exp(b^2*x^2))/(6*x^3*pi^(1/2)) - (b*erfc((-b^2*x^2)^(1/2))
*(-b^2*x^2)^(3/2))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{erfi}(bx)}{x^5}\,dx = \frac{-4\operatorname{erf}(bix)\,b^4 i\pi\,x^4 + 3\operatorname{erf}(bix)\,i\pi - 4\sqrt{\pi}\,e^{b^2x^2}b^3x^3 - 2\sqrt{\pi}\,e^{b^2x^2}bx}{12\pi\,x^4}$$

input
```
int(erfi(b*x)/x^5,x)
```

output
```
( - 4*erf(b*i*x)*b**4*i*pi*x**4 + 3*erf(b*i*x)*i*pi - 4*sqrt(pi)*e**(b**2*
x**2)*b**3*x**3 - 2*sqrt(pi)*e**(b**2*x**2)*b*x)/(12*pi*x**4)
```

3.213 $\int \frac{\text{erfi}(bx)}{x^7} dx$

Optimal result

Integrand size = 8, antiderivative size = 93

$$\int \frac{\text{erfi}(bx)}{x^7} dx = -\frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{4b^5e^{b^2x^2}}{45\sqrt{\pi}x} + \frac{4}{45}b^6\text{erfi}(bx) - \frac{\text{erfi}(bx)}{6x^6}$$

output
```
-1/15*b*exp(b^2*x^2)/Pi^(1/2)/x^5-2/45*b^3*exp(b^2*x^2)/Pi^(1/2)/x^3-4/45*
b^5*exp(b^2*x^2)/Pi^(1/2)/x+4/45*b^6*erfi(b*x)-1/6*erfi(b*x)/x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.69

$$\int \frac{\text{erfi}(bx)}{x^7} dx = \frac{-2be^{b^2x^2}x(3 + 2b^2x^2 + 4b^4x^4) + \sqrt{\pi}(-15 + 8b^6x^6)\,\text{erfi}(bx)}{90\sqrt{\pi}x^6}$$

input
```
Integrate[Erfi[b*x]/x^7,x]
```

output
```
(-2*b*E^(b^2*x^2)*x*(3 + 2*b^2*x^2 + 4*b^4*x^4) + Sqrt[Pi]*(-15 + 8*b^6*x^
6)*Erfi[b*x])/(90*Sqrt[Pi]*x^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6917, 2643, 2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x^7}dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{b\int \frac{e^{b^2x^2}}{x^6}dx}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{5}b^2\int \frac{e^{b^2x^2}}{x^4}dx-\frac{e^{b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}}{x^2}dx-\frac{e^{b^2x^2}}{3x^3}\right)-\frac{e^{b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2}dx-\frac{e^{b^2x^2}}{x}\right)-\frac{e^{b^2x^2}}{3x^3}\right)-\frac{e^{b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)}{6x^6}$$

$$\Big\downarrow \text{2633}$$

$$\frac{b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\left(\sqrt{\pi}b\text{erfi}(bx)-\frac{e^{b^2x^2}}{x}\right)-\frac{e^{b^2x^2}}{3x^3}\right)-\frac{e^{b^2x^2}}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)}{6x^6}$$

input
```
Int[Erfi[b*x]/x^7,x]
```

output
```
-1/6*Erfi[b*x]/x^6 + (b*(-1/5*E^(b^2*x^2)/x^5 + (2*b^2*(-1/3*E^(b^2*x^2)/x
^3 + (2*b^2*(-(E^(b^2*x^2)/x) + b*Sqrt[Pi]*Erfi[b*x]))/3))/5))/(3*Sqrt[Pi]
)
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{i b^{6}\left(\frac{4 i \left(\frac{2}{9} b^{4} x^{4}+\frac{1}{9} b^{2} x^{2}+\frac{1}{6}\right) {\mathrm e}^{b^{2} x^{2}}}{5 x^{5} b^{5}}+\frac{i \left(-8 b^{6} x^{6}+15\right) \operatorname{erfi}\left(b x \right) \sqrt{\pi}}{45 x^{6} b^{6}}\right)}{2 \sqrt{\pi}}$	72
parallelrisch	$\frac{8 \,\operatorname{erfi}\left(b x \right) x^{6} b^{6} \sqrt{\pi}-8 b^{5} x^{5} {\mathrm e}^{b^{2} x^{2}}-4 b^{3} x^{3} {\mathrm e}^{b^{2} x^{2}}-6 \,{\mathrm e}^{b^{2} x^{2}} b x -15 \,\operatorname{erfi}\left(b x \right) \sqrt{\pi}}{90 \sqrt{\pi}\, x^{6}}$	78
derivativedivides	$b^{6} \left(-\frac{\operatorname{erfi}\left(b x \right)}{6 b^{6} x^{6}}+\frac{-\frac{{\mathrm e}^{b^{2} x^{2}}}{5 b^{5} x^{5}}-\frac{2 \,{\mathrm e}^{b^{2} x^{2}}}{15 b^{3} x^{3}}-\frac{4 \,{\mathrm e}^{b^{2} x^{2}}}{15 b x}+\frac{4 \,\operatorname{erfi}\left(b x \right) \sqrt{\pi}}{15}}{3 \sqrt{\pi}}\right)$	81
default	$b^{6} \left(-\frac{\operatorname{erfi}\left(b x \right)}{6 b^{6} x^{6}}+\frac{-\frac{{\mathrm e}^{b^{2} x^{2}}}{5 b^{5} x^{5}}-\frac{2 \,{\mathrm e}^{b^{2} x^{2}}}{15 b^{3} x^{3}}-\frac{4 \,{\mathrm e}^{b^{2} x^{2}}}{15 b x}+\frac{4 \,\operatorname{erfi}\left(b x \right) \sqrt{\pi}}{15}}{3 \sqrt{\pi}}\right)$	81
parts	$-\frac{\operatorname{erfi}\left(b x \right)}{6 x^{6}}+\frac{b \left(-\frac{{\mathrm e}^{b^{2} x^{2}}}{5 x^{5}}+\frac{2 b^{2} \left(-\frac{{\mathrm e}^{b^{2} x^{2}}}{3 x^{3}}+\frac{2 b^{2} \left(-\frac{{\mathrm e}^{b^{2} x^{2}}}{x}-i b \sqrt{\pi}\, \operatorname{erf}\left(i b x \right)\right)}{3}\right)}{5}\right)}{3 \sqrt{\pi}}$	82

input
```
int(erfi(b*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/Pi^(1/2)*b^6*(4/5*I/x^5/b^5*(2/9*b^4*x^4+1/9*b^2*x^2+1/6)*exp(b^2*x^
2)+1/45*I/x^6/b^6*(-8*b^6*x^6+15)*erfi(b*x)*Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{erfi}(bx)}{x^7}\,dx = -\frac{2\,\sqrt{\pi}(4\,b^5x^5 + 2\,b^3x^3 + 3\,bx)e^{(b^2x^2)} + (15\,\pi - 8\,\pi b^6x^6)\operatorname{erfi}(bx)}{90\,\pi x^6}$$

input
```
integrate(erfi(b*x)/x^7,x, algorithm="fricas")
```

output
```
-1/90*(2*sqrt(pi)*(4*b^5*x^5 + 2*b^3*x^3 + 3*b*x)*e^(b^2*x^2) + (15*pi - 8
*pi*b^6*x^6)*erfi(b*x))/(pi*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{\text{erfi}(bx)}{x^7}\,dx = \frac{4b^6\,\text{erfi}\,(bx)}{45} - \frac{4b^5e^{b^2x^2}}{45\sqrt{\pi}x} - \frac{2b^3e^{b^2x^2}}{45\sqrt{\pi}x^3} - \frac{be^{b^2x^2}}{15\sqrt{\pi}x^5} - \frac{\text{erfi}\,(bx)}{6x^6}$$

input
```
integrate(erfi(b*x)/x**7,x)
```

output
```
4*b**6*erfi(b*x)/45 - 4*b**5*exp(b**2*x**2)/(45*sqrt(pi)*x) - 2*b**3*exp(b
**2*x**2)/(45*sqrt(pi)*x**3) - b*exp(b**2*x**2)/(15*sqrt(pi)*x**5) - erfi(
b*x)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{\text{erfi}(bx)}{x^7}\,dx = -\frac{(-b^2x^2)^{\frac{5}{2}}\,b\Gamma\left(-\frac{5}{2},-b^2x^2\right)}{6\,\sqrt{\pi}x^5} - \frac{\text{erfi}\,(bx)}{6\,x^6}$$

input
```
integrate(erfi(b*x)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(-b^2*x^2)^(5/2)*b*gamma(-5/2, -b^2*x^2)/(sqrt(pi)*x^5) - 1/6*erfi(b*
x)/x^6
```

Giac [F]

$$\int \frac{\text{erfi}(bx)}{x^7}\,dx = \int \frac{\text{erfi}\,(bx)}{x^7}\,dx$$

input
```
integrate(erfi(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{erfi}(bx)}{x^7}\,dx = -\frac{\operatorname{erfi}(b\,x)}{6\,x^6} - \frac{3\,b\,\mathrm{e}^{b^2\,x^2}+2\,b^3\,x^2\,\mathrm{e}^{b^2\,x^2}+4\,b^5\,x^4\,\mathrm{e}^{b^2\,x^2}+4\,b\,\sqrt{\pi}\,{\left(-b^2\,x^2\right)}^{5/2}-4\,b\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b^2}\,\sqrt{x^2}\right)\,{\left(-b^2\,x^2\right)}^{5/2}}{45\,x^5\,\sqrt{\pi}}$$

input
```
int(erfi(b*x)/x^7,x)
```

output
```
- erfi(b*x)/(6*x^6) - (3*b*exp(b^2*x^2) + 2*b^3*x^2*exp(b^2*x^2) + 4*b^5*x
^4*exp(b^2*x^2) + 4*b*pi^(1/2)*(-b^2*x^2)^(5/2) - 4*b*pi^(1/2)*erfc((-b^2)
^(1/2)*(x^2)^(1/2))*(-b^2*x^2)^(5/2))/(45*x^5*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erfi}(bx)}{x^7}\,dx = \frac{-8\,\mathrm{erf}(bix)\,b^6 i\pi\,x^6+15\,\mathrm{erf}(bix)\,i\pi-8\sqrt{\pi}\,e^{b^2x^2}b^5x^5-4\sqrt{\pi}\,e^{b^2x^2}b^3x^3-6\sqrt{\pi}\,e^{b^2x^2}bx}{90\pi\,x^6}$$

input
```
int(erfi(b*x)/x^7,x)
```

output
```
( - 8*erf(b*i*x)*b**6*i*pi*x**6 + 15*erf(b*i*x)*i*pi - 8*sqrt(pi)*e**(b**2
*x**2)*b**5*x**5 - 4*sqrt(pi)*e**(b**2*x**2)*b**3*x**3 - 6*sqrt(pi)*e**(b*
*2*x**2)*b*x)/(90*pi*x**6)
```

3.214 $\int x^6 \text{erfi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 105

$$\int x^6 \text{erfi}(bx)\,dx = \frac{6e^{b^2x^2}}{7b^7\sqrt{\pi}} - \frac{6e^{b^2x^2}x^2}{7b^5\sqrt{\pi}} + \frac{3e^{b^2x^2}x^4}{7b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^6}{7b\sqrt{\pi}} + \frac{1}{7}x^7\text{erfi}(bx)$$

output
```
6/7*exp(b^2*x^2)/b^7/Pi^(1/2)-6/7*exp(b^2*x^2)*x^2/b^5/Pi^(1/2)+3/7*exp(b^
2*x^2)*x^4/b^3/Pi^(1/2)-1/7*exp(b^2*x^2)*x^6/b/Pi^(1/2)+1/7*x^7*erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int x^6 \text{erfi}(bx)\,dx = \frac{1}{7}\left(\frac{e^{b^2x^2}(6-6b^2x^2+3b^4x^4-b^6x^6)}{b^7\sqrt{\pi}} + x^7\text{erfi}(bx)\right)$$

input
```
Integrate[x^6*Erfi[b*x],x]
```

output
```
((E^(b^2*x^2)*(6 - 6*b^2*x^2 + 3*b^4*x^4 - b^6*x^6))/(b^7*Sqrt[Pi]) + x^7*
Erfi[b*x])/7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6917, 2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{erfi}(bx) \, dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{1}{7}x^7\text{erfi}(bx)-\frac{2b\int e^{b^2x^2}x^7dx}{7\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{7}x^7\text{erfi}(bx)-\frac{2b\left(\frac{x^6e^{b^2x^2}}{2b^2}-\frac{3\int e^{b^2x^2}x^5dx}{b^2}\right)}{7\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{7}x^7\text{erfi}(bx)-\frac{2b\left(\frac{x^6e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{x^4e^{b^2x^2}}{2b^2}-\frac{2\int e^{b^2x^2}x^3dx}{b^2}\right)}{b^2}\right)}{7\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{7}x^7\text{erfi}(bx)-\frac{2b\left(\frac{x^6e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{x^4e^{b^2x^2}}{2b^2}-\frac{2\left(\frac{x^2e^{b^2x^2}}{2b^2}-\frac{\int e^{b^2x^2}xdx}{b^2}\right)}{b^2}\right)}{b^2}\right)}{7\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{7}x^7\operatorname{erfi}(bx)-\frac{2b\left(\frac{x^6e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{x^4e^{b^2x^2}}{2b^2}-\frac{2\left(\frac{x^2e^{b^2x^2}}{2b^2}-\frac{e^{b^2x^2}}{2b^4}\right)}{b^2}\right)}{b^2}\right)}{7\sqrt{\pi}}$$

input
```
Int[x^6*Erfi[b*x],x]
```

output
```
(-2*b*((E^(b^2*x^2)*x^6)/(2*b^2) - (3*((E^(b^2*x^2)*x^4)/(2*b^2) - (2*(-1/
2*E^(b^2*x^2)/b^4 + (E^(b^2*x^2)*x^2)/(2*b^2)))/b^2))/b^2))/(7*Sqrt[Pi]) +
 (x^7*Erfi[b*x])/7
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

method	result	size
meijerg	$\frac{-\frac{12}{7}+\frac{\left(-4b^6x^6+12b^4x^4-24b^2x^2+24\right)e^{b^2x^2}}{14}+\frac{2b^7x^7\sqrt{\pi}\,\mathrm{erfi}(bx)}{7}}{2b^7\sqrt{\pi}}$	62
derivativedivides	$\frac{\frac{b^7x^7\,\mathrm{erfi}(bx)}{7}-\frac{2\left(\frac{b^6x^6e^{b^2x^2}}{2}-\frac{3\,e^{b^2x^2}b^4x^4}{2}+3b^2x^2e^{b^2x^2}-3\,e^{b^2x^2}\right)}{7\sqrt{\pi}}}{b^7}$	82
default	$\frac{\frac{b^7x^7\,\mathrm{erfi}(bx)}{7}-\frac{2\left(\frac{b^6x^6e^{b^2x^2}}{2}-\frac{3\,e^{b^2x^2}b^4x^4}{2}+3b^2x^2e^{b^2x^2}-3\,e^{b^2x^2}\right)}{7\sqrt{\pi}}}{b^7}$	82
parallelrisch	$\frac{b^7x^7\sqrt{\pi}\,\mathrm{erfi}(bx)-b^6x^6e^{b^2x^2}+3\,e^{b^2x^2}b^4x^4-6b^2x^2e^{b^2x^2}+6\,e^{b^2x^2}}{7b^7\sqrt{\pi}}$	82
parts	$\frac{x^7\,\mathrm{erfi}(bx)}{7}-\frac{2b\left(\frac{x^6e^{b^2x^2}}{2b^2}-\frac{3\left(\frac{x^4e^{b^2x^2}}{2b^2}-\frac{2\left(\frac{x^2e^{b^2x^2}}{2b^2}-\frac{e^{b^2x^2}}{2b^4}\right)}{b^2}\right)}{b^2}\right)}{7\sqrt{\pi}}$	91

input
```
int(x^6*erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^7/Pi^(1/2)*(-12/7+1/14*(-4*b^6*x^6+12*b^4*x^4-24*b^2*x^2+24)*exp(b^2
*x^2)+2/7*b^7*x^7*Pi^(1/2)*erfi(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int x^6\mathrm{erfi}(bx)\,dx=\frac{\pi b^7x^7\,\mathrm{erfi}\,(bx)-\sqrt{\pi}(b^6x^6-3\,b^4x^4+6\,b^2x^2-6)e^{(b^2x^2)}}{7\,\pi b^7}$$

input
```
integrate(x^6*erfi(b*x),x, algorithm="fricas")
```

output
```
1/7*(pi*b^7*x^7*erfi(b*x) - sqrt(pi)*(b^6*x^6 - 3*b^4*x^4 + 6*b^2*x^2 - 6)
*e^(b^2*x^2))/(pi*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^6 \operatorname{erfi}(bx)\,dx = \begin{cases} \frac{x^7 \operatorname{erfi}(bx)}{7} - \frac{x^6 e^{b^2x^2}}{7\sqrt{\pi}b} + \frac{3x^4 e^{b^2x^2}}{7\sqrt{\pi}b^3} - \frac{6x^2 e^{b^2x^2}}{7\sqrt{\pi}b^5} + \frac{6e^{b^2x^2}}{7\sqrt{\pi}b^7} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**6*erfi(b*x),x)
```

output
```
Piecewise((x**7*erfi(b*x)/7 - x**6*exp(b**2*x**2)/(7*sqrt(pi)*b) + 3*x**4*
exp(b**2*x**2)/(7*sqrt(pi)*b**3) - 6*x**2*exp(b**2*x**2)/(7*sqrt(pi)*b**5)
 + 6*exp(b**2*x**2)/(7*sqrt(pi)*b**7), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int x^6 \operatorname{erfi}(bx)\,dx = \frac{1}{7}x^7 \operatorname{erfi}(bx) - \frac{(b^6x^6 - 3\,b^4x^4 + 6\,b^2x^2 - 6)e^{(b^2x^2)}}{7\,\sqrt{\pi}b^7}$$

input
```
integrate(x^6*erfi(b*x),x, algorithm="maxima")
```

output
```
1/7*x^7*erfi(b*x) - 1/7*(b^6*x^6 - 3*b^4*x^4 + 6*b^2*x^2 - 6)*e^(b^2*x^2)/
(sqrt(pi)*b^7)
```

Giac [F]

$$\int x^6 \operatorname{erfi}(bx)\,dx = \int x^6 \operatorname{erfi}(bx)\,dx$$

input
```
integrate(x^6*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^6*erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int x^6 \text{erfi}(bx) \, dx = \frac{x^7 \,\text{erfi}(b\,x)}{7} - \frac{\mathrm{e}^{b^2\,x^2}\,\left(b^6\,x^6-3\,b^4\,x^4+6\,b^2\,x^2-6\right)}{7\,b^7\,\sqrt{\pi}}$$

input
```
int(x^6*erfi(b*x),x)
```

output
```
(x^7*erfi(b*x))/7 - (exp(b^2*x^2)*(6*b^2*x^2 - 3*b^4*x^4 + b^6*x^6 - 6))/(
7*b^7*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int x^6 \text{erfi}(bx) \, dx = \frac{-\text{erf}(bix)\, b^7 i\pi\, x^7 - \sqrt{\pi}\, e^{b^2x^2} b^6 x^6 + 3\sqrt{\pi}\, e^{b^2x^2} b^4 x^4 - 6\sqrt{\pi}\, e^{b^2x^2} b^2 x^2 + 6\sqrt{\pi}\, e^{b^2x^2}}{7b^7\pi}$$

input
```
int(x^6*erfi(b*x),x)
```

output
```
( - erf(b*i*x)*b**7*i*pi*x**7 - sqrt(pi)*e**(b**2*x**2)*b**6*x**6 + 3*sqrt
(pi)*e**(b**2*x**2)*b**4*x**4 - 6*sqrt(pi)*e**(b**2*x**2)*b**2*x**2 + 6*sq
rt(pi)*e**(b**2*x**2))/(7*b**7*pi)
```

3.215 $\int x^4\text{erfi}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int x^4\text{erfi}(bx)\,dx = -\frac{2e^{b^2x^2}}{5b^5\sqrt{\pi}}+\frac{2e^{b^2x^2}x^2}{5b^3\sqrt{\pi}}-\frac{e^{b^2x^2}x^4}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfi}(bx)$$

output
```
-2/5*exp(b^2*x^2)/b^5/Pi^(1/2)+2/5*exp(b^2*x^2)*x^2/b^3/Pi^(1/2)-1/5*exp(b
^2*x^2)*x^4/b/Pi^(1/2)+1/5*x^5*erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int x^4\text{erfi}(bx)\,dx = \frac{1}{5}\left(-\frac{e^{b^2x^2}(2-2b^2x^2+b^4x^4)}{b^5\sqrt{\pi}}+x^5\text{erfi}(bx)\right)$$

input
```
Integrate[x^4*Erfi[b*x],x]
```

output
```
(-((E^(b^2*x^2)*(2 - 2*b^2*x^2 + b^4*x^4))/(b^5*Sqrt[Pi])) + x^5*Erfi[b*x]
)/5
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6917, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erfi}(bx) \, dx$$

$$\Big\downarrow \text{ 6917}$$

$$\frac{1}{5} x^5 \text{erfi}(bx) - \frac{2b \int e^{b^2 x^2} x^5 dx}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{5} x^5 \text{erfi}(bx) - \frac{2b \left(\frac{x^4 e^{b^2 x^2}}{2b^2} - \frac{2 \int e^{b^2 x^2} x^3 dx}{b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{5} x^5 \text{erfi}(bx) - \frac{2b \left(\frac{x^4 e^{b^2 x^2}}{2b^2} - \frac{2 \left(\frac{x^2 e^{b^2 x^2}}{2b^2} - \frac{\int e^{b^2 x^2} x dx}{b^2}\right)}{b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{1}{5} x^5 \text{erfi}(bx) - \frac{2b \left(\frac{x^4 e^{b^2 x^2}}{2b^2} - \frac{2 \left(\frac{x^2 e^{b^2 x^2}}{2b^2} - \frac{e^{b^2 x^2}}{2b^4}\right)}{b^2}\right)}{5\sqrt{\pi}}$$

input
```
Int[x^4*Erfi[b*x],x]
```

output
```
(-2*b*((E^(b^2*x^2)*x^4)/(2*b^2) - (2*(-1/2*E^(b^2*x^2)/b^4 + (E^(b^2*x^2)
*x^2)/(2*b^2)))/b^2))/(5*Sqrt[Pi]) + (x^5*Erfi[b*x])/5
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

method	result	size
meijerg	$-\frac{-\frac{4}{5}+\frac{2\left(3 b^{4} x^{4}-6 b^{2} x^{2}+6\right) {\mathrm e}^{b^{2} x^{2}}}{15}-\frac{2 b^{5} x^{5} \sqrt{\pi}\, \operatorname{erfi}\left(b x \right)}{5}}{2 b^{5} \sqrt{\pi}}$	54
derivativedivides	$\frac{\frac{b^{5} x^{5} \operatorname{erfi}\left(b x \right)}{5}-\frac{2 \left(\frac{{\mathrm e}^{b^{2} x^{2}} b^{4} x^{4}}{2}-b^{2} x^{2} {\mathrm e}^{b^{2} x^{2}}+{\mathrm e}^{b^{2} x^{2}}\right)}{5 \sqrt{\pi}}}{b^{5}}$	64
default	$\frac{\frac{b^{5} x^{5} \operatorname{erfi}\left(b x \right)}{5}-\frac{2 \left(\frac{{\mathrm e}^{b^{2} x^{2}} b^{4} x^{4}}{2}-b^{2} x^{2} {\mathrm e}^{b^{2} x^{2}}+{\mathrm e}^{b^{2} x^{2}}\right)}{5 \sqrt{\pi}}}{b^{5}}$	64
parallelrisch	$\frac{b^{5} x^{5} \sqrt{\pi}\, \operatorname{erfi}\left(b x \right)-{\mathrm e}^{b^{2} x^{2}} b^{4} x^{4}+2 b^{2} x^{2} {\mathrm e}^{b^{2} x^{2}}-2 \,{\mathrm e}^{b^{2} x^{2}}}{5 b^{5} \sqrt{\pi}}$	66
parts	$\frac{x^{5} \operatorname{erfi}\left(b x \right)}{5}-\frac{2 b \left(\frac{x^{4} {\mathrm e}^{b^{2} x^{2}}}{2 b^{2}}-\frac{2 \left(\frac{x^{2} {\mathrm e}^{b^{2} x^{2}}}{2 b^{2}}-\frac{{\mathrm e}^{b^{2} x^{2}}}{2 b^{4}}\right)}{b^{2}}\right)}{5 \sqrt{\pi}}$	69

input
```
int(x^4*erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^5/Pi^(1/2)*(-4/5+2/15*(3*b^4*x^4-6*b^2*x^2+6)*exp(b^2*x^2)-2/5*b^5*
x^5*Pi^(1/2)*erfi(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int x^4\text{erfi}(bx)\,dx = \frac{\pi b^5 x^5 \,\text{erfi}\,(bx) - \sqrt{\pi}(b^4x^4 - 2\,b^2x^2 + 2)e^{(b^2x^2)}}{5\,\pi b^5}$$

input
```
integrate(x^4*erfi(b*x),x, algorithm="fricas")
```

output
```
1/5*(pi*b^5*x^5*erfi(b*x) - sqrt(pi)*(b^4*x^4 - 2*b^2*x^2 + 2)*e^(b^2*x^2)
)/(pi*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int x^4\text{erfi}(bx)\,dx = \begin{cases} \frac{x^5\,\text{erfi}\,(bx)}{5} - \frac{x^4e^{b^2x^2}}{5\sqrt{\pi}b} + \frac{2x^2e^{b^2x^2}}{5\sqrt{\pi}b^3} - \frac{2e^{b^2x^2}}{5\sqrt{\pi}b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*erfi(b*x),x)
```

output
```
Piecewise((x**5*erfi(b*x)/5 - x**4*exp(b**2*x**2)/(5*sqrt(pi)*b) + 2*x**2*
exp(b**2*x**2)/(5*sqrt(pi)*b**3) - 2*exp(b**2*x**2)/(5*sqrt(pi)*b**5), Ne(
b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.53

$$\int x^4\text{erfi}(bx)\,dx = \frac{1}{5}\,x^5\,\text{erfi}\,(bx) - \frac{(b^4x^4 - 2\,b^2x^2 + 2)e^{(b^2x^2)}}{5\,\sqrt{\pi}b^5}$$

input
```
integrate(x^4*erfi(b*x),x, algorithm="maxima")
```

output
```
1/5*x^5*erfi(b*x) - 1/5*(b^4*x^4 - 2*b^2*x^2 + 2)*e^(b^2*x^2)/(sqrt(pi)*b^
5)
```

Giac [F]

$$\int x^4\text{erfi}(bx)\,dx = \int x^4\,\text{erfi}\,(bx)\;dx$$

input
```
integrate(x^4*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.53

$$\int x^4\text{erfi}(bx)\,dx = \frac{x^5\,\text{erfi}(b\,x)}{5} - \frac{e^{b^2\,x^2}\,(b^4\,x^4 - 2\,b^2\,x^2 + 2)}{5\,b^5\,\sqrt{\pi}}$$

input
```
int(x^4*erfi(b*x),x)
```

output
```
(x^5*erfi(b*x))/5 - (exp(b^2*x^2)*(b^4*x^4 - 2*b^2*x^2 + 2))/(5*b^5*pi^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int x^4 \text{erfi}(bx)\,dx = \frac{-\text{erf}(bix)\,b^5 i\pi\,x^5 - \sqrt{\pi}\,e^{b^2x^2}b^4x^4 + 2\sqrt{\pi}\,e^{b^2x^2}b^2x^2 - 2\sqrt{\pi}\,e^{b^2x^2}}{5b^5\pi}$$

input
```
int(x^4*erfi(b*x),x)
```

output
```
( - erf(b*i*x)*b**5*i*pi*x**5 - sqrt(pi)*e**(b**2*x**2)*b**4*x**4 + 2*sqrt
(pi)*e**(b**2*x**2)*b**2*x**2 - 2*sqrt(pi)*e**(b**2*x**2))/(5*b**5*pi)
```

3.216 $\int x^2 \mathbf{erfi}(bx)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int x^2 \text{erfi}(bx)\, dx = \frac{e^{b^2x^2}}{3b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^2}{3b\sqrt{\pi}} + \frac{1}{3}x^3\text{erfi}(bx)$$

output
```
1/3*exp(b^2*x^2)/b^3/Pi^(1/2)-1/3*exp(b^2*x^2)*x^2/b/Pi^(1/2)+1/3*x^3*erfi
(b*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int x^2 \text{erfi}(bx)\, dx = \frac{1}{3}\left(\frac{e^{b^2x^2}(1-b^2x^2)}{b^3\sqrt{\pi}} + x^3\text{erfi}(bx)\right)$$

input
```
Integrate[x^2*Erfi[b*x],x]
```

output
```
((E^(b^2*x^2)*(1 - b^2*x^2))/(b^3*Sqrt[Pi]) + x^3*Erfi[b*x])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6917, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{1}{3}x^3\text{erfi}(bx)-\frac{2b\int e^{b^2x^2}x^3dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{3}x^3\text{erfi}(bx)-\frac{2b\left(\frac{x^2e^{b^2x^2}}{2b^2}-\frac{\int e^{b^2x^2}xdx}{b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{3}x^3\text{erfi}(bx)-\frac{2b\left(\frac{x^2e^{b^2x^2}}{2b^2}-\frac{e^{b^2x^2}}{2b^4}\right)}{3\sqrt{\pi}}$$

input
```
Int[x^2*Erfi[b*x],x]
```

output
```
(-2*b*(-1/2*E^(b^2*x^2)/b^4 + (E^(b^2*x^2)*x^2)/(2*b^2)))/(3*Sqrt[Pi]) + (
x^3*Erfi[b*x])/3
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
meijerg	$\frac{-\frac{2}{3}+\frac{\left(-2b^{2}x^{2}+2\right)e^{b^{2}x^{2}}}{3}+\frac{2b^{3}x^{3}\sqrt{\pi}\,\mathrm{erfi}(bx)}{3}}{2b^{3}\sqrt{\pi}}$	46
parts	$\frac{x^{3}\,\mathrm{erfi}(bx)}{3}-\frac{2b\left(\frac{x^{2}e^{b^{2}x^{2}}}{2b^{2}}-\frac{e^{b^{2}x^{2}}}{2b^{4}}\right)}{3\sqrt{\pi}}$	47
parallelrisch	$\frac{b^{3}x^{3}\sqrt{\pi}\,\mathrm{erfi}(bx)-b^{2}x^{2}e^{b^{2}x^{2}}+e^{b^{2}x^{2}}}{3b^{3}\sqrt{\pi}}$	48
derivativedivides	$\frac{\frac{b^{3}x^{3}\,\mathrm{erfi}(bx)}{3}-\frac{2\left(\frac{b^{2}x^{2}e^{b^{2}x^{2}}}{2}-\frac{e^{b^{2}x^{2}}}{2}\right)}{3\sqrt{\pi}}}{b^{3}}$	50
default	$\frac{\frac{b^{3}x^{3}\,\mathrm{erfi}(bx)}{3}-\frac{2\left(\frac{b^{2}x^{2}e^{b^{2}x^{2}}}{2}-\frac{e^{b^{2}x^{2}}}{2}\right)}{3\sqrt{\pi}}}{b^{3}}$	50

input
```
int(x^2*erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3/Pi^(1/2)*(-2/3+1/3*(-2*b^2*x^2+2)*exp(b^2*x^2)+2/3*b^3*x^3*Pi^(1/2
)*erfi(b*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int x^2\text{erfi}(bx)\,dx = \frac{\pi b^3x^3\,\text{erfi}\,(bx) - \sqrt{\pi}(b^2x^2-1)e^{(b^2x^2)}}{3\,\pi b^3}$$

input
```
integrate(x^2*erfi(b*x),x, algorithm="fricas")
```

output
```
1/3*(pi*b^3*x^3*erfi(b*x) - sqrt(pi)*(b^2*x^2 - 1)*e^(b^2*x^2))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x^2\text{erfi}(bx)\,dx = \begin{cases} \frac{x^3\,\text{erfi}\,(bx)}{3} - \frac{x^2e^{b^2x^2}}{3\sqrt{\pi}b} + \frac{e^{b^2x^2}}{3\sqrt{\pi}b^3} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*erfi(b*x),x)
```

output
```
Piecewise((x**3*erfi(b*x)/3 - x**2*exp(b**2*x**2)/(3*sqrt(pi)*b) + exp(b**
2*x**2)/(3*sqrt(pi)*b**3), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x^2\text{erfi}(bx)\,dx = \frac{1}{3}\,x^3\,\text{erfi}\,(bx) - \frac{(b^2x^2-1)e^{(b^2x^2)}}{3\,\sqrt{\pi}b^3}$$

input
```
integrate(x^2*erfi(b*x),x, algorithm="maxima")
```

output
```
1/3*x^3*erfi(b*x) - 1/3*(b^2*x^2 - 1)*e^(b^2*x^2)/(sqrt(pi)*b^3)
```

Giac [F]

$$\int x^2\text{erfi}(bx)\,dx = \int x^2\,\text{erfi}\,(bx)\;dx$$

input
```
integrate(x^2*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x^2\text{erfi}(bx)\,dx = \frac{x^3\,\text{erfi}(b\,x)}{3} - \frac{e^{b^2\,x^2}\,(b^2\,x^2-1)}{3\,b^3\,\sqrt{\pi}}$$

input
```
int(x^2*erfi(b*x),x)
```

output
```
(x^3*erfi(b*x))/3 - (exp(b^2*x^2)*(b^2*x^2 - 1))/(3*b^3*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int x^2 \text{erfi}(bx)\,dx = \frac{-\text{erf}(bix)\,b^3 i\pi\,x^3 - \sqrt{\pi}\,e^{b^2x^2}b^2x^2 + \sqrt{\pi}\,e^{b^2x^2}}{3b^3\pi}$$

input
```
int(x^2*erfi(b*x),x)
```

output
```
( - erf(b*i*x)*b**3*i*pi*x**3 - sqrt(pi)*e**(b**2*x**2)*b**2*x**2 + sqrt(p
i)*e**(b**2*x**2))/(3*b**3*pi)
```

3.217 $\int \text{erfi}(bx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \text{erfi}(bx)\,dx = -\frac{e^{b^2x^2}}{b\sqrt{\pi}} + x\text{erfi}(bx)$$

output
```
-exp(b^2*x^2)/b/Pi^(1/2)+x*erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{erfi}(bx)\,dx = -\frac{e^{b^2x^2}}{b\sqrt{\pi}} + x\text{erfi}(bx)$$

input
```
Integrate[Erfi[b*x],x]
```

output
```
-(E^(b^2*x^2)/(b*Sqrt[Pi])) + x*Erfi[b*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6905}$$

$$x\text{erfi}(bx)-\frac{e^{b^2x^2}}{\sqrt{\pi}b}$$

input
```
Int[Erfi[b*x],x]
```

output
```
-(E^(b^2*x^2)/(b*Sqrt[Pi])) + x*Erfi[b*x]
```

Defintions of rubi rules used

rule 6905
```
Int[Erfi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erfi[a + b*x]/b)
, x] - Simp[E^(a + b*x)^2/(b*Sqrt[Pi]), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
parts	$-\frac{e^{b^2x^2}}{b\sqrt{\pi}}+x\,\mathrm{erfi}\,(bx)$	24
derivativedivides	$\frac{bx\,\mathrm{erfi}(bx)-\frac{e^{b^2x^2}}{\sqrt{\pi}}}{b}$	26
default	$\frac{bx\,\mathrm{erfi}(bx)-\frac{e^{b^2x^2}}{\sqrt{\pi}}}{b}$	26
parallelrisch	$\frac{bx\sqrt{\pi}\,\mathrm{erfi}(bx)-e^{b^2x^2}}{\sqrt{\pi}\,b}$	29
meijerg	$-\frac{-2+2\,e^{b^2x^2}-2bx\sqrt{\pi}\,\mathrm{erfi}(bx)}{2\sqrt{\pi}\,b}$	32

input
```
int(erfi(b*x),x,method=_RETURNVERBOSE)
```

output
```
-exp(b^2*x^2)/b/Pi^(1/2)+x*erfi(b*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \mathrm{erfi}(bx)\,dx=\frac{\pi bx\,\mathrm{erfi}\,(bx)-\sqrt{\pi}e^{\left(b^2x^2\right)}}{\pi b}$$

input
```
integrate(erfi(b*x),x, algorithm="fricas")
```

output
```
(pi*b*x*erfi(b*x) - sqrt(pi)*e^(b^2*x^2))/(pi*b)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \operatorname{erfi}(bx)\,dx = \begin{cases} x\operatorname{erfi}(bx) - \frac{e^{b^2x^2}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(erfi(b*x),x)
```

output
```
Piecewise((x*erfi(b*x) - exp(b**2*x**2)/(sqrt(pi)*b), Ne(b, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \operatorname{erfi}(bx)\,dx = \frac{bx\operatorname{erfi}(bx) - \frac{e^{(b^2x^2)}}{\sqrt{\pi}}}{b}$$

input
```
integrate(erfi(b*x),x, algorithm="maxima")
```

output
```
(b*x*erfi(b*x) - e^(b^2*x^2)/sqrt(pi))/b
```

Giac [F]

$$\int \operatorname{erfi}(bx)\,dx = \int \operatorname{erfi}(bx)\,dx$$

input
```
integrate(erfi(b*x),x, algorithm="giac")
```

output
```
integrate(erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \operatorname{erfi}(bx)\,dx = x\operatorname{erfi}(b\,x) - \frac{e^{b^2\,x^2}}{b\,\sqrt{\pi}}$$

input
```
int(erfi(b*x),x)
```

output
```
x*erfi(b*x) - exp(b^2*x^2)/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \operatorname{erfi}(bx)\,dx = \frac{-\operatorname{erf}(bix)\,bi\pi x - \sqrt{\pi}\,e^{b^2x^2}}{b\pi}$$

input
```
int(erfi(b*x),x)
```

output
```
( - (erf(b*i*x)*b*i*pi*x + sqrt(pi)*e**(b**2*x**2)))/(b*pi)
```

3.218 $\int \frac{\text{erfi}(bx)}{x^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \frac{\text{erfi}(bx)}{x^2} \, dx = -\frac{\text{erfi}(bx)}{x} + \frac{b \, \text{ExpIntegralEi}\left(b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-erfi(b*x)/x+b*Ei(b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}(bx)}{x^2} \, dx = -\frac{\text{erfi}(bx)}{x} + \frac{b \, \text{ExpIntegralEi}\left(b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erfi[b*x]/x^2,x]
```

output
```
-(Erfi[b*x]/x) + (b*ExpIntegralEi[b^2*x^2])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6917, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6917}$$

$$\frac{2b \int \frac{e^{b^2x^2}}{x}dx}{\sqrt{\pi}} - \frac{\text{erfi}(bx)}{x}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{b \,\text{ExpIntegralEi}\left(b^2x^2\right)}{\sqrt{\pi}} - \frac{\text{erfi}(bx)}{x}$$

input
```
Int[Erfi[b*x]/x^2,x]
```

output
```
-(Erfi[b*x]/x) + (b*ExpIntegralEi[b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
parts	$-\frac{\text{erfi}(bx)}{x}-\frac{b\ \text{expIntegral}_1(-b^2x^2)}{\sqrt{\pi}}$	27
derivativedivides	$b\left(-\frac{\text{erfi}(bx)}{bx}-\frac{\text{expIntegral}_1(-b^2x^2)}{\sqrt{\pi}}\right)$	31
default	$b\left(-\frac{\text{erfi}(bx)}{bx}-\frac{\text{expIntegral}_1(-b^2x^2)}{\sqrt{\pi}}\right)$	31
meijerg	$\frac{b\left(4\ln(x)+4\ln(ib)-\frac{2\sqrt{\pi}\ \text{erfi}(bx)}{xb}-2\ln(-b^2x^2)-2\ \text{expIntegral}_1(-b^2x^2)\right)}{2\sqrt{\pi}}$	57

input
```
int(erfi(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-erfi(b*x)/x-1/Pi^(1/2)*b*Ei(1,-b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{\text{erfi}(bx)}{x^2}\,dx=\frac{\sqrt{\pi}bx\text{Ei}(b^2x^2)-\pi\,\text{erfi}\,(bx)}{\pi x}$$

input
```
integrate(erfi(b*x)/x^2,x, algorithm="fricas")
```

output
```
(sqrt(pi)*b*x*Ei(b^2*x^2) - pi*erfi(b*x))/(pi*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{erfi}(bx)}{x^2}\,dx = -\frac{b\,\mathrm{E}_1\left(b^2x^2e^{i\pi}\right)}{\sqrt{\pi}} - \frac{i\operatorname{erfc}\left(ibx\right)}{x} + \frac{i}{x}$$

input
```
integrate(erfi(b*x)/x**2,x)
```

output
```
-b*expint(1, b**2*x**2*exp_polar(I*pi))/sqrt(pi) - I*erfc(I*b*x)/x + I/x
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erfi}(bx)}{x^2}\,dx = \frac{b\mathrm{Ei}(b^2x^2)}{\sqrt{\pi}} - \frac{\operatorname{erfi}\left(bx\right)}{x}$$

input
```
integrate(erfi(b*x)/x^2,x, algorithm="maxima")
```

output
```
b*Ei(b^2*x^2)/sqrt(pi) - erfi(b*x)/x
```

Giac [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^2}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)}{x^2}\,dx$$

input
```
integrate(erfi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{erfi}(bx)}{x^2}\,dx = \frac{b\,\operatorname{ei}(b^2\,x^2)}{\sqrt{\pi}} - \frac{\operatorname{erfi}(b\,x)}{x}$$

input
```
int(erfi(b*x)/x^2,x)
```

output
```
(b*ei(b^2*x^2))/pi^(1/2) - erfi(b*x)/x
```

Reduce [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^2}\,dx = -\left(\int \frac{\operatorname{erf}(bix)}{x^2}dx\right)i$$

input
```
int(erfi(b*x)/x^2,x)
```

output
```
- int(erf(b*i*x)/x**2,x)*i
```

3.219 $\int \frac{\text{erfi}(bx)}{x^4} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int \frac{\text{erfi}(bx)}{x^4} \, dx = -\frac{be^{b^2x^2}}{3\sqrt{\pi}x^2} - \frac{\text{erfi}(bx)}{3x^3} + \frac{b^3 \, \text{ExpIntegralEi}\left(b^2x^2\right)}{3\sqrt{\pi}}$$

output
```
-1/3*b*exp(b^2*x^2)/Pi^(1/2)/x^2-1/3*erfi(b*x)/x^3+1/3*b^3*Ei(b^2*x^2)/Pi^
(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\text{erfi}(bx)}{x^4} \, dx = -\frac{\frac{be^{b^2x^2}x}{\sqrt{\pi}} + \text{erfi}(bx) - \frac{b^3x^3 \, \text{ExpIntegralEi}(b^2x^2)}{\sqrt{\pi}}}{3x^3}$$

input
```
Integrate[Erfi[b*x]/x^4,x]
```

output
```
-1/3*((b*E^(b^2*x^2)*x)/Sqrt[Pi] + Erfi[b*x] - (b^3*x^3*ExpIntegralEi[b^2*
x^2])/Sqrt[Pi])/x^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6917, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x^4} \, dx$$

$$\Big\downarrow \text{ 6917}$$

$$\frac{2b \int \frac{e^{b^2x^2}}{x^3}dx}{3\sqrt{\pi}} - \frac{\text{erfi}(bx)}{3x^3}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2b\left(b^2 \int \frac{e^{b^2x^2}}{x}dx - \frac{e^{b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{\text{erfi}(bx)}{3x^3}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{2b\left(\frac{1}{2}b^2 \text{ExpIntegralEi}\left(b^2x^2\right) - \frac{e^{b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}} - \frac{\text{erfi}(bx)}{3x^3}$$

input
```
Int[Erfi[b*x]/x^4,x]
```

output
```
-1/3*Erfi[b*x]/x^3 + (2*b*(-1/2*E^(b^2*x^2)/x^2 + (b^2*ExpIntegralEi[b^2*x
^2])/2))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{\operatorname{erfi}(bx)}{3x^3}+\frac{2b\left(-\frac{{\rm e}^{b^2x^2}}{2x^2}-\frac{b^2\,\mathrm{expIntegral}_1\left(-b^2x^2\right)}{2}\right)}{3\sqrt{\pi}}$	46
derivativedivides	$b^3\left(-\frac{\operatorname{erfi}(bx)}{3b^3x^3}+\frac{-\frac{{\rm e}^{b^2x^2}}{3x^2b^2}-\frac{\mathrm{expIntegral}_1\left(-b^2x^2\right)}{3}}{\sqrt{\pi}}\right)$	52
default	$b^3\left(-\frac{\operatorname{erfi}(bx)}{3b^3x^3}+\frac{-\frac{{\rm e}^{b^2x^2}}{3x^2b^2}-\frac{\mathrm{expIntegral}_1\left(-b^2x^2\right)}{3}}{\sqrt{\pi}}\right)$	52
meijerg	$-\frac{b^3\left(\frac{2}{b^2x^2}+\frac{10}{9}-\frac{4\ln(x)}{3}-\frac{4\ln(ib)}{3}-\frac{50b^2x^2+90}{45b^2x^2}+\frac{2\,{\rm e}^{b^2x^2}}{3x^2b^2}+\frac{2\sqrt{\pi}\,\operatorname{erfi}(bx)}{3b^3x^3}+\frac{2\ln\left(-b^2x^2\right)}{3}+\frac{2\,\mathrm{expIntegral}_1\left(-b^2x^2\right)}{3}\right)}{2\sqrt{\pi}}$	102

input
```
int(erfi(b*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*erfi(b*x)/x^3+2/3/Pi^(1/2)*b*(-1/2/x^2*exp(b^2*x^2)-1/2*b^2*Ei(1,-b^2
*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx = -\frac{\pi \operatorname{erfi}(bx) - \sqrt{\pi}\left(b^3x^3\operatorname{Ei}(b^2x^2) - bxe^{(b^2x^2)}\right)}{3\,\pi x^3}$$

input
```
integrate(erfi(b*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(pi*erfi(b*x) - sqrt(pi)*(b^3*x^3*Ei(b^2*x^2) - b*x*e^(b^2*x^2)))/(pi
*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx = -\frac{b^3\,\mathrm{E}_1\left(b^2x^2e^{i\pi}\right)}{3\sqrt{\pi}} - \frac{be^{b^2x^2}}{3\sqrt{\pi}x^2} - \frac{i\operatorname{erfc}(ibx)}{3x^3} + \frac{i}{3x^3}$$

input
```
integrate(erfi(b*x)/x**4,x)
```

output
```
-b**3*expint(1, b**2*x**2*exp_polar(I*pi))/(3*sqrt(pi)) - b*exp(b**2*x**2)
/(3*sqrt(pi)*x**2) - I*erfc(I*b*x)/(3*x**3) + I/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx = \frac{b^3\Gamma(-1,-b^2x^2)}{3\sqrt{\pi}} - \frac{\operatorname{erfi}(bx)}{3x^3}$$

input
```
integrate(erfi(b*x)/x^4,x, algorithm="maxima")
```

output
```
1/3*b^3*gamma(-1, -b^2*x^2)/sqrt(pi) - 1/3*erfi(b*x)/x^3
```

Giac [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx = \int \frac{\operatorname{erfi}(bx)}{x^4}\,dx$$

input
```
integrate(erfi(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx = \frac{b^3\operatorname{ei}(b^2x^2)}{3\sqrt{\pi}} - \frac{\operatorname{erfi}(b\,x)}{3\,x^3} - \frac{b\,e^{b^2x^2}}{3\,x^2\sqrt{\pi}}$$

input
```
int(erfi(b*x)/x^4,x)
```

output
```
(b^3*ei(b^2*x^2))/(3*pi^(1/2)) - erfi(b*x)/(3*x^3) - (b*exp(b^2*x^2))/(3*x
^2*pi^(1/2))
```

Reduce [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^4}\,dx = \frac{-\operatorname{erf}(bix)\,b^2 i\pi\,x^2 + \operatorname{erf}(bix)\,i\pi - \sqrt{\pi}\,e^{b^2x^2}bx - \left(\int \frac{\operatorname{erf}(bix)}{x^2}dx\right)b^2 i\pi\,x^3}{3\pi\,x^3}$$

input
```
int(erfi(b*x)/x^4,x)
```

output
```
( - erf(b*i*x)*b**2*i*pi*x**2 + erf(b*i*x)*i*pi - sqrt(pi)*e**(b**2*x**2)*
b*x - int(erf(b*i*x)/x**2,x)*b**2*i*pi*x**3)/(3*pi*x**3)
```

3.220 $\int \frac{\text{erfi}(bx)}{x^6}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 78

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx=-\frac{be^{b^2x^2}}{10\sqrt{\pi}x^4}-\frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2}-\frac{\text{erfi}(bx)}{5x^5}+\frac{b^5\,\text{ExpIntegralEi}\left(b^2x^2\right)}{10\sqrt{\pi}}$$

output
```
-1/10*b*exp(b^2*x^2)/Pi^(1/2)/x^4-1/10*b^3*exp(b^2*x^2)/Pi^(1/2)/x^2-1/5*e
rfi(b*x)/x^5+1/10*b^5*Ei(b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx=\frac{-be^{b^2x^2}x(1+b^2x^2)-2\sqrt{\pi}\text{erfi}(bx)+b^5x^5\,\text{ExpIntegralEi}\left(b^2x^2\right)}{10\sqrt{\pi}x^5}$$

input
```
Integrate[Erfi[b*x]/x^6,x]
```

output
```
(-(b*E^(b^2*x^2)*x*(1 + b^2*x^2)) - 2*Sqrt[Pi]*Erfi[b*x] + b^5*x^5*ExpInte
gralEi[b^2*x^2])/(10*Sqrt[Pi]*x^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6917, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)}{x^6} \, dx$$

$$\Big\downarrow \text{ 6917}$$

$$\frac{2b \int \frac{e^{b^2x^2}}{x^5}dx}{5\sqrt{\pi}} - \frac{\text{erfi}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2b\left(\frac{1}{2}b^2 \int \frac{e^{b^2x^2}}{x^3}dx - \frac{e^{b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erfi}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2b\left(\frac{1}{2}b^2\left(b^2 \int \frac{e^{b^2x^2}}{x}dx - \frac{e^{b^2x^2}}{2x^2}\right) - \frac{e^{b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erfi}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{2b\left(\frac{1}{2}b^2\left(\frac{1}{2}b^2 \text{ExpIntegralEi}\left(b^2x^2\right) - \frac{e^{b^2x^2}}{2x^2}\right) - \frac{e^{b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}} - \frac{\text{erfi}(bx)}{5x^5}$$

input
```
Int[Erfi[b*x]/x^6,x]
```

output
```
-1/5*Erfi[b*x]/x^5 + (2*b*(-1/4*E^(b^2*x^2)/x^4 + (b^2*(-1/2*E^(b^2*x^2)/x
^2 + (b^2*ExpIntegralEi[b^2*x^2])/2))/2))/(5*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
parts	$-\frac{\operatorname{erfi}(bx)}{5x^5}+\frac{2b\left(-\frac{e^{b^2x^2}}{4x^4}+\frac{b^2\left(-\frac{e^{b^2x^2}}{2x^2}-\frac{b^2\,\mathrm{expIntegral}_1\left(-b^2x^2\right)}{2}\right)}{2}\right)}{5\sqrt{\pi}}$
derivativedivides	$b^5\left(-\frac{\operatorname{erfi}(bx)}{5b^5x^5}+\frac{-\frac{e^{b^2x^2}}{10b^4x^4}-\frac{e^{b^2x^2}}{10x^2b^2}-\frac{\mathrm{expIntegral}_1\left(-b^2x^2\right)}{10}}{\sqrt{\pi}}\right)$
default	$b^5\left(-\frac{\operatorname{erfi}(bx)}{5b^5x^5}+\frac{-\frac{e^{b^2x^2}}{10b^4x^4}-\frac{e^{b^2x^2}}{10x^2b^2}-\frac{\mathrm{expIntegral}_1\left(-b^2x^2\right)}{10}}{\sqrt{\pi}}\right)$
meijerg	$\frac{b^5\left(-\frac{1}{b^4x^4}-\frac{2}{3b^2x^2}-\frac{19}{50}+\frac{2\ln(x)}{5}+\frac{2\ln(ib)}{5}+\frac{399b^4x^4+700b^2x^2+1050}{1050b^4x^4}-\frac{\left(21b^2x^2+21\right)e^{b^2x^2}}{105b^4x^4}-\frac{2\sqrt{\pi}\,\operatorname{erfi}(bx)}{5b^5x^5}-\frac{\ln\left(-b^2x^2\right)}{5}-\mathrm{exp}\ldots}{2\sqrt{\pi}}$

input
```
int(erfi(b*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*erfi(b*x)/x^5+2/5/Pi^(1/2)*b*(-1/4/x^4*exp(b^2*x^2)+1/2*b^2*(-1/2/x^2
*exp(b^2*x^2)-1/2*b^2*Ei(1,-b^2*x^2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx = -\frac{2\,\pi\,\text{erfi}\,(bx) - \sqrt{\pi}\left(b^5x^5\text{Ei}(b^2x^2) - (b^3x^3 + bx)e^{(b^2x^2)}\right)}{10\,\pi x^5}$$

input
```
integrate(erfi(b*x)/x^6,x, algorithm="fricas")
```

output
```
-1/10*(2*pi*erfi(b*x) - sqrt(pi)*(b^5*x^5*Ei(b^2*x^2) - (b^3*x^3 + b*x)*e^
(b^2*x^2)))/(pi*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx = -\frac{b^5\,\text{E}_1\,(b^2x^2e^{i\pi})}{10\sqrt{\pi}} - \frac{b^3e^{b^2x^2}}{10\sqrt{\pi}x^2} - \frac{be^{b^2x^2}}{10\sqrt{\pi}x^4} - \frac{i\,\text{erfc}\,(ibx)}{5x^5} + \frac{i}{5x^5}$$

input
```
integrate(erfi(b*x)/x**6,x)
```

output
```
-b**5*expint(1, b**2*x**2*exp_polar(I*pi))/(10*sqrt(pi)) - b**3*exp(b**2*x
**2)/(10*sqrt(pi)*x**2) - b*exp(b**2*x**2)/(10*sqrt(pi)*x**4) - I*erfc(I*b
*x)/(5*x**5) + I/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.36

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx = -\frac{b^5\Gamma(-2,-b^2x^2)}{5\sqrt{\pi}} - \frac{\text{erfi}\,(bx)}{5\,x^5}$$

input
```
integrate(erfi(b*x)/x^6,x, algorithm="maxima")
```

output
```
-1/5*b^5*gamma(-2, -b^2*x^2)/sqrt(pi) - 1/5*erfi(b*x)/x^5
```

Giac [F]

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx = \int \frac{\text{erfi}\,(bx)}{x^6}\,dx$$

input
```
integrate(erfi(b*x)/x^6,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{\text{erfi}(bx)}{x^6}\,dx = \frac{b^5\,\text{ei}(b^2\,x^2)}{10\,\sqrt{\pi}} - \frac{\text{erfi}(b\,x)}{5\,x^5} - \frac{\frac{b\,e^{b^2\,x^2}}{2} + \frac{b^3\,x^2\,e^{b^2\,x^2}}{2}}{5\,x^4\,\sqrt{\pi}}$$

input
```
int(erfi(b*x)/x^6,x)
```

output
```
(b^5*ei(b^2*x^2))/(10*pi^(1/2)) - erfi(b*x)/(5*x^5) - ((b*exp(b^2*x^2))/2
+ (b^3*x^2*exp(b^2*x^2))/2)/(5*x^4*pi^(1/2))
```

Reduce [F]

$$\int \frac{\operatorname{erfi}(bx)}{x^6}\,dx$$
$$= \frac{-\operatorname{erf}(bix)\,b^4 i\pi\,x^4 + 2\operatorname{erf}(bix)\,i\pi - \sqrt{\pi}\,e^{b^2x^2}b^3x^3 - \sqrt{\pi}\,e^{b^2x^2}bx - \left(\int \frac{\operatorname{erf}(bix)}{x^2}dx\right) b^4 i\pi\,x^5}{10\pi\,x^5}$$

input
```
int(erfi(b*x)/x^6,x)
```

output
```
( - erf(b*i*x)*b**4*i*pi*x**4 + 2*erf(b*i*x)*i*pi - sqrt(pi)*e**(b**2*x**2
)*b**3*x**3 - sqrt(pi)*e**(b**2*x**2)*b*x - int(erf(b*i*x)/x**2,x)*b**4*i*
pi*x**5)/(10*pi*x**5)
```

3.221 $\int (c+dx)^3 \mathbf{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 279

$$\begin{aligned}\int (c+dx)^3\text{erfi}(a+bx)\,dx &= \frac{d^2(bc-ad)e^{(a+bx)^2}}{b^4\sqrt{\pi}}-\frac{(bc-ad)^3e^{(a+bx)^2}}{b^4\sqrt{\pi}}\\ &+\frac{3d^3e^{(a+bx)^2}(a+bx)}{8b^4\sqrt{\pi}}-\frac{3d(bc-ad)^2e^{(a+bx)^2}(a+bx)}{2b^4\sqrt{\pi}}\\ &-\frac{d^2(bc-ad)e^{(a+bx)^2}(a+bx)^2}{b^4\sqrt{\pi}}-\frac{d^3e^{(a+bx)^2}(a+bx)^3}{4b^4\sqrt{\pi}}\\ &-\frac{3d^3\text{erfi}(a+bx)}{16b^4}+\frac{3d(bc-ad)^2\text{erfi}(a+bx)}{4b^4}\\ &-\frac{(bc-ad)^4\text{erfi}(a+bx)}{4b^4d}+\frac{(c+dx)^4\text{erfi}(a+bx)}{4d}\end{aligned}$$

output
```
d^2*(-a*d+b*c)*exp((b*x+a)^2)/b^4/Pi^(1/2)-(-a*d+b*c)^3*exp((b*x+a)^2)/b^4
/Pi^(1/2)+3/8*d^3*exp((b*x+a)^2)*(b*x+a)/b^4/Pi^(1/2)-3/2*d*(-a*d+b*c)^2*e
xp((b*x+a)^2)*(b*x+a)/b^4/Pi^(1/2)-d^2*(-a*d+b*c)*exp((b*x+a)^2)*(b*x+a)^2
/b^4/Pi^(1/2)-1/4*d^3*exp((b*x+a)^2)*(b*x+a)^3/b^4/Pi^(1/2)-3/16*d^3*erfi(
b*x+a)/b^4+3/4*d*(-a*d+b*c)^2*erfi(b*x+a)/b^4-1/4*(-a*d+b*c)^4*erfi(b*x+a)
/b^4/d+1/4*(d*x+c)^4*erfi(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.85

$$\int (c+dx)^3\text{erfi}(a+bx)\,dx$$
$$=\frac{-2e^{(a+bx)^2}\left(a(5-2a^2)\,d^3+bd^2(8(-1+a^2)\,c+(-3+2a^2)\,dx)-2ab^2d(6c^2+4cdx+d^2x^2)+2b^3(4c^3+\right.}{}$$

input
```
Integrate[(c + d*x)^3*Erfi[a + b*x],x]
```

output
```
(-2*E^(a + b*x)^2*(a*(5 - 2*a^2)*d^3 + b*d^2*(8*(-1 + a^2)*c + (-3 + 2*a^2
)*d*x) - 2*a*b^2*d*(6*c^2 + 4*c*d*x + d^2*x^2) + 2*b^3*(4*c^3 + 6*c^2*d*x
+ 4*c*d^2*x^2 + d^3*x^3)) + Sqrt[Pi]*(12*b^2*c^2*d + 16*a^3*b*c*d^2 - 3*d^
3 - 4*a^4*d^3 + 12*a^2*d*(-2*b^2*c^2 + d^2) + 8*a*(2*b^3*c^3 - 3*b*c*d^2)
+ 4*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Erfi[a + b*x])/(16*
b^4*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {6917, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{erfi}(a+bx)\,dx$$
$$\Big\downarrow \text{ 6917}$$
$$\frac{(c+dx)^4\text{erfi}(a+bx)}{4d}-\frac{b\int e^{(a+bx)^2}(c+dx)^4dx}{2\sqrt{\pi}d}$$
$$\Big\downarrow \text{ 2656}$$

$$\frac{(c+dx)^4\text{erfi}(a+bx)}{4d}-\frac{b\int\left(\frac{e^{(a+bx)^2}(bc-ad)^4}{b^4}+\frac{4de^{(a+bx)^2}(a+bx)(bc-ad)^3}{b^4}+\frac{6d^2e^{(a+bx)^2}(a+bx)^2(bc-ad)^2}{b^4}+\frac{4d^3e^{(a+bx)^2}(a+bx)^3(bc-ad)}{b^4}+\frac{d^4e^{(a+bx)^2}(a+\ldots}{b^4}\right.}{2\sqrt{\pi}d}$$

$\Big\downarrow$ 2009

$$\frac{(c+dx)^4\text{erfi}(a+bx)}{4d}-\frac{b\left(-\frac{2d^3e^{(a+bx)^2}(bc-ad)}{b^5}+\frac{2d^3e^{(a+bx)^2}(a+bx)^2(bc-ad)}{b^5}-\frac{3\sqrt{\pi}d^2(bc-ad)^2\text{erfi}(a+bx)}{2b^5}+\frac{3d^2e^{(a+bx)^2}(a+bx)(bc-ad)^2}{b^5}+\frac{\sqrt{\pi}(bc-ad)^4\text{er}\ldots}{2b^5}\right.}{2\sqrt{\pi}d}$$

input
```
Int[(c + d*x)^3*Erfi[a + b*x],x]
```

output
```
((c + d*x)^4*Erfi[a + b*x])/(4*d) - (b*((-2*d^3*(b*c - a*d)*E^(a + b*x)^2)
/b^5 + (2*d*(b*c - a*d)^3*E^(a + b*x)^2)/b^5 - (3*d^4*E^(a + b*x)^2*(a + b
*x))/(4*b^5) + (3*d^2*(b*c - a*d)^2*E^(a + b*x)^2*(a + b*x))/b^5 + (2*d^3*
(b*c - a*d)*E^(a + b*x)^2*(a + b*x)^2)/b^5 + (d^4*E^(a + b*x)^2*(a + b*x)^
3)/(2*b^5) + (3*d^4*Sqrt[Pi]*Erfi[a + b*x])/(8*b^5) - (3*d^2*(b*c - a*d)^2
*Sqrt[Pi]*Erfi[a + b*x])/(2*b^5) + ((b*c - a*d)^4*Sqrt[Pi]*Erfi[a + b*x])/
(2*b^5)))/(2*d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.64

method	result
parallelrisch	$\frac{-24\sqrt{\pi}\,\operatorname{erfi}(bx+a)abc\,d^2+16x\,\mathrm{e}^{(bx+a)^2}a\,b^2c\,d^2+16d^2c\,\operatorname{erfi}(bx+a)x^3\sqrt{\pi}\,b^4+24c^2d\,\operatorname{erfi}(bx+a)x^2\sqrt{\pi}\,b^4+16\sqrt{\pi}\,\operatorname{erfi}(bx}{}$
derivativedivides	$\frac{\frac{d^3\operatorname{erfi}(bx+a)a^4}{4b^3}-\frac{d^2\operatorname{erfi}(bx+a)a^3c}{b^2}-\frac{d^3\operatorname{erfi}(bx+a)a^3(bx+a)}{b^3}+\frac{3d\,\operatorname{erfi}(bx+a)a^2c^2}{2b}+\frac{3d^2\operatorname{erfi}(bx+a)a^2c(bx+a)}{b^2}+\frac{3d^3\operatorname{erfi}(bx+a)a^2(}{2b^3}}{}$
default	$\frac{\frac{d^3\operatorname{erfi}(bx+a)a^4}{4b^3}-\frac{d^2\operatorname{erfi}(bx+a)a^3c}{b^2}-\frac{d^3\operatorname{erfi}(bx+a)a^3(bx+a)}{b^3}+\frac{3d\,\operatorname{erfi}(bx+a)a^2c^2}{2b}+\frac{3d^2\operatorname{erfi}(bx+a)a^2c(bx+a)}{b^2}+\frac{3d^3\operatorname{erfi}(bx+a)a^2(}{2b^3}}{b}$
parts	$\frac{\operatorname{erfi}(bx+a)d^3x^4}{4}+\operatorname{erfi}(bx+a)\,d^2c\,x^3+\frac{3\operatorname{erfi}(bx+a)d\,c^2x^2}{2}+\operatorname{erfi}(bx+a)\,c^3x+\frac{\operatorname{erfi}(bx+a)c^4}{4d}-$

input
```
int((d*x+c)^3*erfi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-24*Pi^(1/2)*erfi(b*x+a)*a*b*c*d^2+16*x*exp((b*x+a)^2)*a*b^2*c*d^2+1
6*d^2*c*erfi(b*x+a)*x^3*Pi^(1/2)*b^4+24*c^2*d*erfi(b*x+a)*x^2*Pi^(1/2)*b^4
+16*Pi^(1/2)*erfi(b*x+a)*a^3*b*c*d^2-24*Pi^(1/2)*erfi(b*x+a)*a^2*b^2*c^2*d
-16*exp((b*x+a)^2)*b^3*c^3-10*exp((b*x+a)^2)*a*d^3-3*Pi^(1/2)*erfi(b*x+a)*
d^3-4*x*exp((b*x+a)^2)*a^2*b*d^3-24*x*exp((b*x+a)^2)*b^3*c^2*d+4*x^2*exp((
b*x+a)^2)*a*b^2*d^3-16*x^2*exp((b*x+a)^2)*b^3*c*d^2-16*exp((b*x+a)^2)*a^2*
b*c*d^2+24*exp((b*x+a)^2)*a*b^2*c^2*d+4*d^3*erfi(b*x+a)*x^4*Pi^(1/2)*b^4+1
6*x*erfi(b*x+a)*c^3*Pi^(1/2)*b^4+16*Pi^(1/2)*erfi(b*x+a)*a*b^3*c^3+12*Pi^(
1/2)*erfi(b*x+a)*b^2*c^2*d+4*exp((b*x+a)^2)*a^3*d^3-4*d^3*exp((b*x+a)^2)*x
^3*b^3+6*x*exp((b*x+a)^2)*b*d^3+16*exp((b*x+a)^2)*b*c*d^2-4*Pi^(1/2)*erfi(
b*x+a)*a^4*d^3+12*Pi^(1/2)*erfi(b*x+a)*a^2*d^3)/Pi^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94

$$\int (c+dx)^3\text{erfi}(a+bx)\,dx =$$

$$-\frac{2\sqrt{\pi}(2b^3d^3x^3+8b^3c^3-12ab^2c^2d+8(a^2-1)bcd^2-(2a^3-5a)d^3+2(4b^3cd^2-ab^2d^3)x^2+(12b^3c$$

input
```
integrate((d*x+c)^3*erfi(b*x+a),x, algorithm="fricas")
```

output
```
-1/16*(2*sqrt(pi)*(2*b^3*d^3*x^3 + 8*b^3*c^3 - 12*a*b^2*c^2*d + 8*(a^2 - 1
)*b*c*d^2 - (2*a^3 - 5*a)*d^3 + 2*(4*b^3*c*d^2 - a*b^2*d^3)*x^2 + (12*b^3*
c^2*d - 8*a*b^2*c*d^2 + (2*a^2 - 3)*b*d^3)*x)*e^(b^2*x^2 + 2*a*b*x + a^2)
- (4*pi*b^4*d^3*x^4 + 16*pi*b^4*c*d^2*x^3 + 24*pi*b^4*c^2*d*x^2 + 16*pi*b^
4*c^3*x + pi*(16*a*b^3*c^3 - 12*(2*a^2 - 1)*b^2*c^2*d + 8*(2*a^3 - 3*a)*b*
c*d^2 - (4*a^4 - 12*a^2 + 3)*d^3))*erfi(b*x + a))/(pi*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(258) = 516.

Time = 1.88 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.67

$$\int (c+dx)^3\text{erfi}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*erfi(b*x+a),x)
```

output
```
Piecewise((-a**4*d**3*erfi(a + b*x)/(4*b**4) + a**3*c*d**2*erfi(a + b*x)/b
**3 + a**3*d**3*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(4*sqrt(pi)*b**4) -
3*a**2*c**2*d*erfi(a + b*x)/(2*b**2) - a**2*c*d**2*exp(a**2)*exp(b**2*x**2
)*exp(2*a*b*x)/(sqrt(pi)*b**3) - a**2*d**3*x*exp(a**2)*exp(b**2*x**2)*exp(
2*a*b*x)/(4*sqrt(pi)*b**3) + 3*a**2*d**3*erfi(a + b*x)/(4*b**4) + a*c**3*e
rfi(a + b*x)/b + 3*a*c**2*d*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(
pi)*b**2) + a*c*d**2*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b**
2) + a*d**3*x**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(4*sqrt(pi)*b**2) -
 3*a*c*d**2*erfi(a + b*x)/(2*b**3) - 5*a*d**3*exp(a**2)*exp(b**2*x**2)*exp
(2*a*b*x)/(8*sqrt(pi)*b**4) + c**3*x*erfi(a + b*x) + 3*c**2*d*x**2*erfi(a
+ b*x)/2 + c*d**2*x**3*erfi(a + b*x) + d**3*x**4*erfi(a + b*x)/4 - c**3*ex
p(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b) - 3*c**2*d*x*exp(a**2)*ex
p(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(pi)*b) - c*d**2*x**2*exp(a**2)*exp(b**2*
x**2)*exp(2*a*b*x)/(sqrt(pi)*b) - d**3*x**3*exp(a**2)*exp(b**2*x**2)*exp(2
*a*b*x)/(4*sqrt(pi)*b) + 3*c**2*d*erfi(a + b*x)/(4*b**2) + c*d**2*exp(a**2
)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*b**3) + 3*d**3*x*exp(a**2)*exp(b**
2*x**2)*exp(2*a*b*x)/(8*sqrt(pi)*b**3) - 3*d**3*erfi(a + b*x)/(16*b**4), N
e(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*erfi(a),
 True))
```

Maxima [F]

$$\int (c+dx)^3 \text{erfi}(a+bx)\,dx = \int (dx+c)^3 \,\text{erfi}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)^3*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*erfi(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3 \text{erfi}(a+bx)\,dx = \int (dx+c)^3 \,\text{erfi}\,(bx+a)\;dx$$

input
```
integrate((d*x+c)^3*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*erfi(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.28

$$\int (c+dx)^3 \text{erfi}(a+bx)\,dx = \text{erfi}(a+b\,x)\,\left(c^3\,x+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{d^3\,x^4}{4}\right) - \frac{\frac{e^{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(-2\,a^3\,d^3+8\,a^2\,b\,c\,d^2-12\,a\,b^2\,c^2\,d+5\,a\,d^3+8\,b^3\,c^3-8\,b\,c\,d^2\right)}{4\,b^4}+\frac{d^3\,x^3\,e^{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,b}-\frac{x^2\,e^{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a\,d^3-4\,b\,c\,c\right.}{2\,b^2}}{2\,\sqrt{\pi}} - \frac{\text{erfi}(a+b\,x)\,\left(4\,a^4\,d^3-16\,a^3\,b\,c\,d^2+24\,a^2\,b^2\,c^2\,d-12\,a^2\,d^3-16\,a\,b^3\,c^3+24\,a\,b\,c\,d^2-12\,b^2\,c^2\,d+3\,d^3\right)}{16\,b^4}$$

input
```
int(erfi(a + b*x)*(c + d*x)^3,x)
```

output
```
erfi(a + b*x)*(c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3) - ((exp(
a^2 + b^2*x^2 + 2*a*b*x)*(5*a*d^3 - 2*a^3*d^3 + 8*b^3*c^3 - 8*b*c*d^2 - 12
*a*b^2*c^2*d + 8*a^2*b*c*d^2))/(4*b^4) + (d^3*x^3*exp(a^2 + b^2*x^2 + 2*a*
b*x))/(2*b) - (x^2*exp(a^2 + b^2*x^2 + 2*a*b*x)*(a*d^3 - 4*b*c*d^2))/(2*b^
2) - (x*exp(a^2 + b^2*x^2 + 2*a*b*x)*(b^2*(12*c^2*d - 72*a^2*c^2*d) + b*(4
8*a^3*c*d^2 - 8*a*c*d^2) - 3*d^3 + 20*a^2*d^3 - 12*a^4*d^3))/(b^3*(24*a^2
- 4)))/(2*pi^(1/2)) - (erfi(a + b*x)*(3*d^3 - 12*a^2*d^3 + 4*a^4*d^3 - 16*
a*b^3*c^3 - 12*b^2*c^2*d + 24*a^2*b^2*c^2*d + 24*a*b*c*d^2 - 16*a^3*b*c*d^
2))/(16*b^4)
```

Reduce [F]

$$\int (c+dx)^3 \operatorname{erfi}(a+bx)\,dx$$
$$= \frac{-\sqrt{\pi}\operatorname{erf}(bix+ai)\,a\,c^3 i-\sqrt{\pi}\operatorname{erf}(bix+ai)\,b\,c^3 ix-e^{b^2x^2+2abx+a^2}c^3-\sqrt{\pi}\left(\int \operatorname{erf}(bix+ai)\,x^3dx\right)b\,d^3 i-3}{\sqrt{\pi}\,b}$$

input
```
int((d*x+c)^3*erfi(b*x+a),x)
```

output
```
( - sqrt(pi)*erf(a*i + b*i*x)*a*c**3*i - sqrt(pi)*erf(a*i + b*i*x)*b*c**3*
i*x - e**(a**2 + 2*a*b*x + b**2*x**2)*c**3 - sqrt(pi)*int(erf(a*i + b*i*x)
*x**3,x)*b*d**3*i - 3*sqrt(pi)*int(erf(a*i + b*i*x)*x**2,x)*b*c*d**2*i - 3
*sqrt(pi)*int(erf(a*i + b*i*x)*x,x)*b*c**2*d*i)/(sqrt(pi)*b)
```

3.222 $\int (c+dx)^2 \mathbf{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 186

$$\int (c+dx)^2\text{erfi}(a+bx)\,dx=\frac{d^2e^{(a+bx)^2}}{3b^3\sqrt{\pi}}-\frac{(bc-ad)^2e^{(a+bx)^2}}{b^3\sqrt{\pi}}-\frac{d(bc-ad)e^{(a+bx)^2}(a+bx)}{b^3\sqrt{\pi}}-\frac{d^2e^{(a+bx)^2}(a+bx)^2}{3b^3\sqrt{\pi}}+\frac{d(bc-ad)\text{erfi}(a+bx)}{2b^3}-\frac{(bc-ad)^3\text{erfi}(a+bx)}{3b^3d}+\frac{(c+dx)^3\text{erfi}(a+bx)}{3d}$$

output
```
1/3*d^2*exp((b*x+a)^2)/b^3/Pi^(1/2)-(-a*d+b*c)^2*exp((b*x+a)^2)/b^3/Pi^(1/
2)-d*(-a*d+b*c)*exp((b*x+a)^2)*(b*x+a)/b^3/Pi^(1/2)-1/3*d^2*exp((b*x+a)^2)
*(b*x+a)^2/b^3/Pi^(1/2)+1/2*d*(-a*d+b*c)*erfi(b*x+a)/b^3-1/3*(-a*d+b*c)^3*
erfi(b*x+a)/b^3/d+1/3*(d*x+c)^3*erfi(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int (c+dx)^2\text{erfi}(a+bx)\,dx$$
$$=\frac{-2e^{(a+bx)^2}\left(\left(-1+a^2\right)d^2-abd(3c+dx)+b^2(3c^2+3cdx+d^2x^2)\right)+\sqrt{\pi}(3bcd-6a^2bcd+2a^3d^2+a(6b^2}{6b^3\sqrt{\pi}}$$

input
```
Integrate[(c + d*x)^2*Erfi[a + b*x],x]
```

output
```
(-2*E^(a + b*x)^2*((-1 + a^2)*d^2 - a*b*d*(3*c + d*x) + b^2*(3*c^2 + 3*c*d
*x + d^2*x^2)) + Sqrt[Pi]*(3*b*c*d - 6*a^2*b*c*d + 2*a^3*d^2 + a*(6*b^2*c^
2 - 3*d^2) + 2*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Erfi[a + b*x])/(6*b^3*Sq
rt[Pi])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6917, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \text{erfi}(a+bx)\,dx$$

$$\Big\downarrow \; 6917$$

$$\frac{(c+dx)^3 \text{erfi}(a+bx)}{3d}-\frac{2b\int e^{(a+bx)^2}(c+dx)^3dx}{3\sqrt{\pi}d}$$

$$\Big\downarrow \; 2656$$

$$\frac{(c+dx)^3 \text{erfi}(a+bx)}{3d}-\frac{2b\int \left(\frac{e^{(a+bx)^2}(bc-ad)^3}{b^3}+\frac{3de^{(a+bx)^2}(a+bx)(bc-ad)^2}{b^3}+\frac{3d^2e^{(a+bx)^2}(a+bx)^2(bc-ad)}{b^3}+\frac{d^3e^{(a+bx)^2}(a+bx)^3}{b^3}\right)dx}{3\sqrt{\pi}d}$$

$$\Big\downarrow \; 2009$$

$$\frac{(c+dx)^3 \text{erfi}(a+bx)}{3d}-\frac{2b\left(-\frac{3\sqrt{\pi}d^2(bc-ad)\text{erfi}(a+bx)}{4b^4}+\frac{3d^2e^{(a+bx)^2}(a+bx)(bc-ad)}{2b^4}+\frac{\sqrt{\pi}(bc-ad)^3\text{erfi}(a+bx)}{2b^4}+\frac{3de^{(a+bx)^2}(bc-ad)^2}{2b^4}-\frac{d^3e^{(a+bx)^2}}{2b^4}+\frac{d^3e^{(}}{}\right.}{3\sqrt{\pi}d}$$

input
```
Int[(c + d*x)^2*Erfi[a + b*x],x]
```

output
```
((c + d*x)^3*Erfi[a + b*x])/(3*d) - (2*b*(-1/2*(d^3*E^(a + b*x)^2)/b^4 + (
3*d*(b*c - a*d)^2*E^(a + b*x)^2)/(2*b^4) + (3*d^2*(b*c - a*d)*E^(a + b*x)^
2*(a + b*x))/(2*b^4) + (d^3*E^(a + b*x)^2*(a + b*x)^2)/(2*b^4) - (3*d^2*(b
*c - a*d)*Sqrt[Pi]*Erfi[a + b*x])/(4*b^4) + ((b*c - a*d)^3*Sqrt[Pi]*Erfi[a
 + b*x])/(2*b^4)))/(3*d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.39

method	result
parallelrisch	$\frac{2d^2 \operatorname{erfi}(bx+a)x^3\sqrt{\pi}\,b^3+6cd\,x^2\operatorname{erfi}(bx+a)\sqrt{\pi}\,b^3+6c^2x\operatorname{erfi}(bx+a)\sqrt{\pi}\,b^3+2\sqrt{\pi}\operatorname{erfi}(bx+a)a^3d^2-6\sqrt{\pi}\operatorname{erfi}(bx+a)a^2bc}{}$
derivativedivides	$\frac{-\frac{d^2\operatorname{erfi}(bx+a)a^3}{3b^2}+\frac{d\operatorname{erfi}(bx+a)a^2c}{b}+\frac{d^2\operatorname{erfi}(bx+a)a^2(bx+a)}{b^2}-\operatorname{erfi}(bx+a)a\,c^2-\frac{2d\operatorname{erfi}(bx+a)ac(bx+a)}{b}-\frac{d^2\operatorname{erfi}(bx+a)a(bx+a)}{b^2}}{}$
default	$\frac{-\frac{d^2\operatorname{erfi}(bx+a)a^3}{3b^2}+\frac{d\operatorname{erfi}(bx+a)a^2c}{b}+\frac{d^2\operatorname{erfi}(bx+a)a^2(bx+a)}{b^2}-\operatorname{erfi}(bx+a)a\,c^2-\frac{2d\operatorname{erfi}(bx+a)ac(bx+a)}{b}-\frac{d^2\operatorname{erfi}(bx+a)a(bx+a)}{b^2}}{}$
parts	$\frac{\operatorname{erfi}(bx+a)d^2x^3}{3}+\operatorname{erfi}(bx+a)\,dc\,x^2+\operatorname{erfi}(bx+a)\,c^2x+\frac{\operatorname{erfi}(bx+a)c^3}{3d}-\frac{2b\left(-\frac{ie^{a^2}c^3\sqrt{\pi}\,e^{-a^2}\operatorname{erf}(ib}{2b}\right.}{}$

input
```
int((d*x+c)^2*erfi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*d^2*erfi(b*x+a)*x^3*Pi^(1/2)*b^3+6*c*d*x^2*erfi(b*x+a)*Pi^(1/2)*b^3
+6*c^2*x*erfi(b*x+a)*Pi^(1/2)*b^3+2*Pi^(1/2)*erfi(b*x+a)*a^3*d^2-6*Pi^(1/2
)*erfi(b*x+a)*a^2*b*c*d+6*Pi^(1/2)*erfi(b*x+a)*a*b^2*c^2-2*d^2*exp((b*x+a)
^2)*x^2*b^2+2*x*exp((b*x+a)^2)*a*b*d^2-6*x*exp((b*x+a)^2)*b^2*c*d-3*Pi^(1/
2)*erfi(b*x+a)*a*d^2+3*Pi^(1/2)*erfi(b*x+a)*b*c*d-2*exp((b*x+a)^2)*a^2*d^2
+6*exp((b*x+a)^2)*a*b*c*d-6*exp((b*x+a)^2)*b^2*c^2+2*exp((b*x+a)^2)*d^2)/P
i^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.87

$$\int (c+dx)^2\operatorname{erfi}(a+bx)\,dx =$$
$$-\frac{2\sqrt{\pi}(b^2d^2x^2+3\,b^2c^2-3\,abcd+(a^2-1)d^2+(3\,b^2cd-abd^2)x)e^{(b^2x^2+2\,abx+a^2)}-(2\,\pi b^3d^2x^3+6\,\pi b^3cd}{6\,\pi b^3}$$

input
```
integrate((d*x+c)^2*erfi(b*x+a),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(pi)*(b^2*d^2*x^2 + 3*b^2*c^2 - 3*a*b*c*d + (a^2 - 1)*d^2 + (3
*b^2*c*d - a*b*d^2)*x)*e^(b^2*x^2 + 2*a*b*x + a^2) - (2*pi*b^3*d^2*x^3 + 6
*pi*b^3*c*d*x^2 + 6*pi*b^3*c^2*x + pi*(6*a*b^2*c^2 - 3*(2*a^2 - 1)*b*c*d +
 (2*a^3 - 3*a)*d^2))*erfi(b*x + a))/(pi*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(165) = 330$.

Time = 0.94 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.14

$$\int (c+dx)^2 \operatorname{erfi}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3d^2 \operatorname{erfi}(a+bx)}{3b^3} - \frac{a^2cd \operatorname{erfi}(a+bx)}{b^2} - \frac{a^2d^2e^{a^2}e^{b^2x^2}e^{2abx}}{3\sqrt{\pi}b^3} + \frac{ac^2 \operatorname{erfi}(a+bx)}{b} + \frac{acde^{a^2}e^{b^2x^2}e^{2abx}}{\sqrt{\pi}b^2} + \frac{ad^2xe^{a^2}e^{b^2x^2}e^{2abx}}{3\sqrt{\pi}b^2} - \frac{ad^2 \operatorname{erfi}(}{2b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \operatorname{erfi}(a) \end{cases}$$

input
```
integrate((d*x+c)**2*erfi(b*x+a),x)
```

output
```
Piecewise((a**3*d**2*erfi(a + b*x)/(3*b**3) - a**2*c*d*erfi(a + b*x)/b**2
- a**2*d**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b**3) + a*c*
*2*erfi(a + b*x)/b + a*c*d*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)
*b**2) + a*d**2*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b**2)
- a*d**2*erfi(a + b*x)/(2*b**3) + c**2*x*erfi(a + b*x) + c*d*x**2*erfi(a +
 b*x) + d**2*x**3*erfi(a + b*x)/3 - c**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*
b*x)/(sqrt(pi)*b) - c*d*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)*
b) - d**2*x**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqrt(pi)*b) + c*d*
erfi(a + b*x)/(2*b**2) + d**2*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(3*sqr
t(pi)*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*erfi(a), True))
```

Maxima [F]

$$\int (c+dx)^2\text{erfi}(a+bx)\,dx = \int (dx+c)^2\,\text{erfi}\,(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*erfi(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2\text{erfi}(a+bx)\,dx = \int (dx+c)^2\,\text{erfi}\,(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*erfi(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int (c+dx)^2\text{erfi}(a+bx)\,dx\\&=\frac{\frac{e^{a^2+2\,a\,b\,x+b^2\,x^2}\,(-a^2\,d^2+3\,a\,b\,c\,d-3\,b^2\,c^2+d^2)}{b^3}+\frac{x\,e^{a^2+2\,a\,b\,x+b^2\,x^2}\,(a\,d^2-3\,b\,c\,d)}{b^2}-\frac{d^2\,x^2\,e^{a^2+2\,a\,b\,x+b^2\,x^2}}{b}}{3\,\sqrt{\pi}}\\&\quad+\text{erfi}(a+b\,x)\,\left(c^2\,x+c\,d\,x^2+\frac{d^2\,x^3}{3}\right)\\&\quad+\frac{\text{erfi}(a+b\,x)\,(2\,a^3\,d^2-6\,a^2\,b\,c\,d+6\,a\,b^2\,c^2-3\,a\,d^2+3\,b\,c\,d)}{6\,b^3}\end{aligned}$$

input
```
int(erfi(a + b*x)*(c + d*x)^2,x)
```

output
```
((exp(a^2 + b^2*x^2 + 2*a*b*x)*(d^2 - a^2*d^2 - 3*b^2*c^2 + 3*a*b*c*d))/b^
3 + (x*exp(a^2 + b^2*x^2 + 2*a*b*x)*(a*d^2 - 3*b*c*d))/b^2 - (d^2*x^2*exp(
a^2 + b^2*x^2 + 2*a*b*x))/b)/(3*pi^(1/2)) + erfi(a + b*x)*(c^2*x + (d^2*x^
3)/3 + c*d*x^2) + (erfi(a + b*x)*(2*a^3*d^2 - 3*a*d^2 + 6*a*b^2*c^2 + 3*b*
c*d - 6*a^2*b*c*d))/(6*b^3)
```

Reduce [F]

$$\int (c+dx)^2 \operatorname{erfi}(a+bx)\,dx$$
$$= \frac{-\sqrt{\pi}\,\operatorname{erf}(bix+ai)\,a\,c^2i-\sqrt{\pi}\,\operatorname{erf}(bix+ai)\,b\,c^2ix-e^{b^2x^2+2abx+a^2}c^2-\sqrt{\pi}\left(\int \operatorname{erf}(bix+ai)\,x^2dx\right)b\,d^2i-2}{\sqrt{\pi}\,b}$$

input
```
int((d*x+c)^2*erfi(b*x+a),x)
```

output
```
( - sqrt(pi)*erf(a*i + b*i*x)*a*c**2*i - sqrt(pi)*erf(a*i + b*i*x)*b*c**2*
i*x - e**(a**2 + 2*a*b*x + b**2*x**2)*c**2 - sqrt(pi)*int(erf(a*i + b*i*x)
*x**2,x)*b*d**2*i - 2*sqrt(pi)*int(erf(a*i + b*i*x)*x,x)*b*c*d*i)/(sqrt(pi
)*b)
```

3.223 $\int (c+dx)\text{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int (c+dx)\text{erfi}(a+bx)\,dx = -\frac{(bc-ad)e^{(a+bx)^2}}{b^2\sqrt{\pi}} - \frac{de^{(a+bx)^2}(a+bx)}{2b^2\sqrt{\pi}} + \frac{d\text{erfi}(a+bx)}{4b^2} - \frac{(bc-ad)^2\text{erfi}(a+bx)}{2b^2d} + \frac{(c+dx)^2\text{erfi}(a+bx)}{2d}$$

output
```
-(-a*d+b*c)*exp((b*x+a)^2)/b^2/Pi^(1/2)-1/2*d*exp((b*x+a)^2)*(b*x+a)/b^2/P
i^(1/2)+1/4*d*erfi(b*x+a)/b^2-1/2*(-a*d+b*c)^2*erfi(b*x+a)/b^2/d+1/2*(d*x+
c)^2*erfi(b*x+a)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int (c+dx)\text{erfi}(a+bx)\,dx = \frac{-2e^{(a+bx)^2}(2bc-ad+bdx)+\sqrt{\pi}(4abc+d-2a^2d+4b^2cx+2b^2dx^2)\,\text{erfi}(a+bx)}{4b^2\sqrt{\pi}}$$

input
```
Integrate[(c + d*x)*Erfi[a + b*x],x]
```

output
```
(-2*E^(a + b*x)^2*(2*b*c - a*d + b*d*x) + Sqrt[Pi]*(4*a*b*c + d - 2*a^2*d
+ 4*b^2*c*x + 2*b^2*d*x^2)*Erfi[a + b*x])/(4*b^2*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6917, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{erfi}(a+bx)\,dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{(c+dx)^2\text{erfi}(a+bx)}{2d}-\frac{b\int e^{(a+bx)^2}(c+dx)^2dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2656}$$

$$\frac{(c+dx)^2\text{erfi}(a+bx)}{2d}-\frac{b\int \left(\frac{e^{(a+bx)^2}(bc-ad)^2}{b^2}+\frac{2de^{(a+bx)^2}(a+bx)(bc-ad)}{b^2}+\frac{d^2e^{(a+bx)^2}(a+bx)^2}{b^2}\right)dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^2\text{erfi}(a+bx)}{2d}-\frac{b\left(\frac{\sqrt{\pi}(bc-ad)^2\text{erfi}(a+bx)}{2b^3}+\frac{de^{(a+bx)^2}(bc-ad)}{b^3}-\frac{\sqrt{\pi}d^2\text{erfi}(a+bx)}{4b^3}+\frac{d^2e^{(a+bx)^2}(a+bx)}{2b^3}\right)}{\sqrt{\pi}d}$$

input
```
Int[(c + d*x)*Erfi[a + b*x],x]
```

output
```
((c + d*x)^2*Erfi[a + b*x])/(2*d) - (b*((d*(b*c - a*d)*E^(a + b*x)^2)/b^3
+ (d^2*E^(a + b*x)^2*(a + b*x))/(2*b^3) - (d^2*Sqrt[Pi]*Erfi[a + b*x])/(4*
b^3) + ((b*c - a*d)^2*Sqrt[Pi]*Erfi[a + b*x])/(2*b^3)))/(d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

rule 6917
```
Int[Erfi[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[
(c + d*x)^(m + 1)*(Erfi[a + b*x]/(d*(m + 1))), x] - Simp[2*(b/(Sqrt[Pi]*d*(
m + 1)))   Int[(c + d*x)^(m + 1)*E^(a + b*x)^2, x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{\operatorname{erfi}(bx+a)da(bx+a)}{b}+\operatorname{erfi}(bx+a)c(bx+a)+\frac{\operatorname{erfi}(bx+a)d(bx+a)^2}{2b}+\frac{-d\left(\frac{(bx+a)e^{(bx+a)^2}}{2}-\frac{\sqrt{\pi}\operatorname{erfi}(bx+a)}{4}\right)-e^{(bx+a)^2}bc+da\,e}{b\sqrt{\pi}}}{b}$
default	$\frac{-\frac{\operatorname{erfi}(bx+a)da(bx+a)}{b}+\operatorname{erfi}(bx+a)c(bx+a)+\frac{\operatorname{erfi}(bx+a)d(bx+a)^2}{2b}+\frac{-d\left(\frac{(bx+a)e^{(bx+a)^2}}{2}-\frac{\sqrt{\pi}\operatorname{erfi}(bx+a)}{4}\right)-e^{(bx+a)^2}bc+da\,e}{b\sqrt{\pi}}}{b}$
parallelrisch	$\frac{2d\,x^2\operatorname{erfi}(bx+a)\sqrt{\pi}\,b^2+4cx\operatorname{erfi}(bx+a)\sqrt{\pi}\,b^2-2\sqrt{\pi}\operatorname{erfi}(bx+a)a^2d+4\sqrt{\pi}\operatorname{erfi}(bx+a)abc-2\,e^{(bx+a)^2}bdx+\operatorname{erfi}(bx+a)d}{4\sqrt{\pi}\,b^2}$
parts	$\frac{\operatorname{erfi}(bx+a)d\,x^2}{2}+\operatorname{erfi}(bx+a)\,cx-\frac{b\left(e^{a^2}d\left(\frac{x\,e^{b^2x^2+2bxa}}{2b^2}-\frac{a\left(\frac{e^{b^2x^2+2bxa}}{2b^2}+\frac{ia\sqrt{\pi}\,e^{-a^2}\operatorname{erf}(ibx+ia)}{2b^2}\right)}{b}+\frac{i\sqrt{\pi}\,e^{-}}{}\right.\right.}{\sqrt{\pi}}$

input
```
int((d*x+c)*erfi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*erfi(b*x+a)*d*a*(b*x+a)+erfi(b*x+a)*c*(b*x+a)+1/2/b*erfi(b*x+a)*
d*(b*x+a)^2+1/b/Pi^(1/2)*(-d*(1/2*(b*x+a)*exp((b*x+a)^2)-1/4*Pi^(1/2)*erfi
(b*x+a))-exp((b*x+a)^2)*b*c+d*a*exp((b*x+a)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int (c+dx)\text{erfi}(a+bx)\,dx = -\frac{2\sqrt{\pi}(bdx+2\,bc-ad)e^{\left(b^2x^2+2\,abx+a^2\right)} - \left(2\,\pi b^2dx^2+4\,\pi b^2cx+\pi\left(4\,abc-\left(2\,a^2-1\right)d\right)\right)\text{erfi}\left(bx+a\right)}{4\,\pi b^2}$$

input
```
integrate((d*x+c)*erfi(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(pi)*(b*d*x + 2*b*c - a*d)*e^(b^2*x^2 + 2*a*b*x + a^2) - (2*pi
*b^2*d*x^2 + 4*pi*b^2*c*x + pi*(4*a*b*c - (2*a^2 - 1)*d))*erfi(b*x + a))/(
pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.55

$$\int (c+dx)\text{erfi}(a+bx)\,dx = \begin{cases} -\frac{a^2d\,\text{erfi}\,(a+bx)}{2b^2} + \frac{ac\,\text{erfi}\,(a+bx)}{b} + \frac{ade^{a^2}e^{b^2x^2}e^{2abx}}{2\sqrt{\pi}b^2} + cx\,\text{erfi}\,(a+bx) + \frac{dx^2\,\text{erfi}\,(a+bx)}{2} - \frac{ce^{a^2}e^{b^2x^2}e^{2abx}}{\sqrt{\pi}b} - \frac{dxe^{a^2}e^{b^2x^2}e^{2}}{2\sqrt{\pi}b} \\ \left(cx+\frac{dx^2}{2}\right)\text{erfi}\,(a) \end{cases}$$

input
```
integrate((d*x+c)*erfi(b*x+a),x)
```

output
```
Piecewise((-a**2*d*erfi(a + b*x)/(2*b**2) + a*c*erfi(a + b*x)/b + a*d*exp(
a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(pi)*b**2) + c*x*erfi(a + b*x) +
d*x**2*erfi(a + b*x)/2 - c*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(sqrt(pi)
*b) - d*x*exp(a**2)*exp(b**2*x**2)*exp(2*a*b*x)/(2*sqrt(pi)*b) + d*erfi(a
+ b*x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*erfi(a), True))
```

Maxima [F]

$$\int (c+dx)\text{erfi}(a+bx)\,dx=\int (dx+c)\,\text{erfi}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*erfi(b*x + a), x)
```

Giac [F]

$$\int (c+dx)\text{erfi}(a+bx)\,dx=\int (dx+c)\,\text{erfi}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*erfi(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92

$$\int (c+dx)\text{erfi}(a+bx)\,dx=\frac{\frac{e^{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(\frac{a\,d}{2}-b\,c\right)}{b^2}-\frac{d\,x\,e^{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,b}}{\sqrt{\pi}}+\text{erfi}(a+b\,x)\,\left(\frac{d\,x^2}{2}+c\,x\right)+\frac{\text{erfi}(a+b\,x)\,\left(-2\,d\,a^2\,b+4\,c\,a\,b^2+d\,b\right)}{4\,b^3}$$

input
```
int(erfi(a + b*x)*(c + d*x),x)
```

output
```
((exp(a^2 + b^2*x^2 + 2*a*b*x)*((a*d)/2 - b*c))/b^2 - (d*x*exp(a^2 + b^2*x
^2 + 2*a*b*x))/(2*b))/pi^(1/2) + erfi(a + b*x)*(c*x + (d*x^2)/2) + (erfi(a
 + b*x)*(b*d + 4*a*b^2*c - 2*a^2*b*d))/(4*b^3)
```

Reduce [F]

$$\int (c+dx)\text{erfi}(a+bx)\,dx = \frac{-\sqrt{\pi}\,\text{erf}(bix+ai)\,aci-\sqrt{\pi}\,\text{erf}(bix+ai)\,bcix-e^{b^2x^2+2abx+a^2}c-\sqrt{\pi}\left(\int \text{erf}(bix+ai)\,xdx\right)bdi}{\sqrt{\pi}\,b}$$

input
```
int((d*x+c)*erfi(b*x+a),x)
```

output
```
( - (sqrt(pi)*erf(a*i + b*i*x)*a*c*i + sqrt(pi)*erf(a*i + b*i*x)*b*c*i*x +
 e**(a**2 + 2*a*b*x + b**2*x**2)*c + sqrt(pi)*int(erf(a*i + b*i*x)*x,x)*b*
d*i))/(sqrt(pi)*b)
```

3.224 $\int \text{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \text{erfi}(a+bx)\,dx = -\frac{e^{(a+bx)^2}}{b\sqrt{\pi}} + \frac{(a+bx)\text{erfi}(a+bx)}{b}$$

output
```
-exp((b*x+a)^2)/b/Pi^(1/2)+(b*x+a)*erfi(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \text{erfi}(a+bx)\,dx = \frac{-\frac{e^{(a+bx)^2}}{\sqrt{\pi}} + (a+bx)\text{erfi}(a+bx)}{b}$$

input
```
Integrate[Erfi[a + b*x],x]
```

output
```
(-(E^(a + b*x)^2/Sqrt[Pi]) + (a + b*x)*Erfi[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{6905\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erfi}(a+bx)\,dx$$

$$\Big\downarrow \; 6905$$

$$\frac{(a+bx)\operatorname{erfi}(a+bx)}{b}-\frac{e^{(a+bx)^2}}{\sqrt{\pi}b}$$

input
```
Int[Erfi[a + b*x],x]
```

output
```
-(E^(a + b*x)^2/(b*Sqrt[Pi])) + ((a + b*x)*Erfi[a + b*x])/b
```

Defintions of rubi rules used

rule 6905
```
Int[Erfi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erfi[a + b*x]/b)
, x] - Simp[E^(a + b*x)^2/(b*Sqrt[Pi]), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{erfi}(bx+a)-\frac{e^{(bx+a)^2}}{\sqrt{\pi}}}{b}$	31
default	$\frac{(bx+a)\operatorname{erfi}(bx+a)-\frac{e^{(bx+a)^2}}{\sqrt{\pi}}}{b}$	31
parallelrisch	$\frac{x\ \operatorname{erfi}(bx+a)\sqrt{\pi}\,b+a\ \operatorname{erfi}(bx+a)\sqrt{\pi}-e^{(bx+a)^2}}{\sqrt{\pi}\,b}$	42
parts	$x\ \operatorname{erfi}(bx+a)-\frac{2b\left(\frac{e^{b^2x^2+2bxa+a^2}}{2b^2}+\frac{ia\sqrt{\pi}\ \operatorname{erf}(ibx+ia)}{2b^2}\right)}{\sqrt{\pi}}$	60

input
```
int(erfi(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*erfi(b*x+a)-1/Pi^(1/2)*exp((b*x+a)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \operatorname{erfi}(a+bx)\,dx=\frac{(\pi bx+\pi a)\operatorname{erfi}(bx+a)-\sqrt{\pi}e^{\left(b^2x^2+2\,abx+a^2\right)}}{\pi b}$$

input
```
integrate(erfi(b*x+a),x, algorithm="fricas")
```

output
```
((pi*b*x + pi*a)*erfi(b*x + a) - sqrt(pi)*e^(b^2*x^2 + 2*a*b*x + a^2))/(pi
*b)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \operatorname{erfi}(a+bx)\,dx = \begin{cases} \frac{a\operatorname{erfi}(a+bx)}{b} + x\operatorname{erfi}(a+bx) - \frac{e^{a^2}e^{b^2x^2}e^{2abx}}{\sqrt{\pi}b} & \text{for } b \neq 0 \\ x\operatorname{erfi}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(erfi(b*x+a),x)
```

output
```
Piecewise((a*erfi(a + b*x)/b + x*erfi(a + b*x) - exp(a**2)*exp(b**2*x**2)*
exp(2*a*b*x)/(sqrt(pi)*b), Ne(b, 0)), (x*erfi(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \operatorname{erfi}(a+bx)\,dx = \frac{(bx+a)\operatorname{erfi}(bx+a) - \frac{e^{\left((bx+a)^2\right)}}{\sqrt{\pi}}}{b}$$

input
```
integrate(erfi(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*erfi(b*x + a) - e^((b*x + a)^2)/sqrt(pi))/b
```

Giac [F]

$$\int \operatorname{erfi}(a+bx)\,dx = \int \operatorname{erfi}(bx+a)\,dx$$

input
```
integrate(erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \operatorname{erfi}(a+bx)\,dx = x\operatorname{erfi}(a+b\,x)+\frac{a\operatorname{erfi}(a+b\,x)}{b}-\frac{\mathrm{e}^{a^2}\,\mathrm{e}^{b^2\,x^2}\,\mathrm{e}^{2\,a\,b\,x}}{b\,\sqrt{\pi}}$$

input
```
int(erfi(a + b*x),x)
```

output
```
x*erfi(a + b*x) + (a*erfi(a + b*x))/b - (exp(a^2)*exp(b^2*x^2)*exp(2*a*b*x
))/(b*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.71

$$\int \operatorname{erfi}(a+bx)\,dx = \frac{-\sqrt{\pi}\,\operatorname{erf}(bix+ai)\,ai-\sqrt{\pi}\,\operatorname{erf}(bix+ai)\,bix-e^{b^2x^2+2abx+a^2}}{\sqrt{\pi}\,b}$$

input
```
int(erfi(b*x+a),x)
```

output
```
( - (sqrt(pi)*erf(a*i + b*i*x)*a*i + sqrt(pi)*erf(a*i + b*i*x)*b*i*x + e**
(a**2 + 2*a*b*x + b**2*x**2)))/(sqrt(pi)*b)
```

3.225 $\int \frac{\text{erfi}(a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{erfi}(a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\text{erfi}(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(erfi(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{c+dx} \, dx = \int \frac{\text{erfi}(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Erfi[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Erfi[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \; 6926$$

$$\int \frac{\text{erfi}(a+bx)}{c+dx}dx$$

input
```
Int[Erfi[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}\left(bx+a\right)}{dx+c}dx$$

input
```
int(erfi(b*x+a)/(d*x+c),x)
```

output
```
int(erfi(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfi}\,(bx+a)}{dx+c}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfi}\,(a+bx)}{c+dx}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c),x)
```

output
```
Integral(erfi(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfi}\,(bx+a)}{dx+c}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfi}\,(bx+a)}{dx+c}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = \int \frac{\text{erfi}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(erfi(a + b*x)/(c + d*x),x)
```

output
```
int(erfi(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\text{erfi}(a+bx)}{c+dx}\,dx = -\left(\int \frac{\text{erf}(bix+ai)}{dx+c}dx\right)i$$

input
```
int(erfi(b*x+a)/(d*x+c),x)
```

output
```
- int(erf(a*i + b*i*x)/(c + d*x),x)*i
```

3.226 $\int \frac{\mathbf{erfi}(a+bx)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^2} \, dx = -\frac{\text{erfi}(a+bx)}{d(c+dx)} + \frac{2b\text{Int}\left(\frac{e^{(a+bx)^2}}{c+dx}, x\right)}{d\sqrt{\pi}}$$

output
```
-erfi(b*x+a)/d/(d*x+c)+2*b*Defer(Int)(exp((b*x+a)^2)/(d*x+c),x)/d/Pi^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^2} \, dx = \int \frac{\text{erfi}(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Erfi[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Erfi[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{2b \int \frac{e^{(a+bx)^2}}{c+dx} dx}{\sqrt{\pi} d} - \frac{\text{erfi}(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2654}$$

$$\frac{2b \int \frac{e^{(a+bx)^2}}{c+dx} dx}{\sqrt{\pi} d} - \frac{\text{erfi}(a+bx)}{d(c+dx)}$$

input
```
Int[Erfi[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}\left(bx+a\right)}{\left(dx+c\right)^2} dx$$

input
```
int(erfi(b*x+a)/(d*x+c)^2,x)
```

output
```
int(erfi(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erfi}\,(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{erfi}\,(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(erfi(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfi}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfi}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{erfi}(a+b\,x)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(erfi(a + b*x)/(c + d*x)^2,x)
```

output
```
int(erfi(a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^2}\,dx = -\left(\int \frac{\operatorname{erf}(bix+ai)}{d^2x^2+2cdx+c^2}dx\right)i$$

input
```
int(erfi(b*x+a)/(d*x+c)^2,x)
```

output
```
- int(erf(a*i + b*i*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*i
```

3.227 $\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^3} \, dx = -\frac{\operatorname{erfi}(a+bx)}{2d(c+dx)^2} + \frac{b\operatorname{Int}\left(\frac{e^{(a+bx)^2}}{(c+dx)^2}, x\right)}{d\sqrt{\pi}}$$

output
```
-1/2*erfi(b*x+a)/d/(d*x+c)^2+b*Defer(Int)(exp((b*x+a)^2)/(d*x+c)^2,x)/d/Pi
^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^3} \, dx = \int \frac{\operatorname{erfi}(a+bx)}{(c+dx)^3} \, dx$$

input
```
Integrate[Erfi[a + b*x]/(c + d*x)^3,x]
```

output
```
Integrate[Erfi[a + b*x]/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{6917}$$

$$\frac{b\int \frac{e^{(a+bx)^2}}{(c+dx)^2}dx}{\sqrt{\pi}d}-\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2650}$$

$$\frac{b\left(\frac{2b^2\int e^{(a+bx)^2}dx}{d^2}-\frac{2b(bc-ad)\int \frac{e^{(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{(a+bx)^2}}{d(c+dx)}\right)}{\sqrt{\pi}d}-\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2633}$$

$$\frac{b\left(-\frac{2b(bc-ad)\int \frac{e^{(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{(a+bx)^2}}{d(c+dx)}+\frac{\sqrt{\pi}b\text{erfi}(a+bx)}{d^2}\right)}{\sqrt{\pi}d}-\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2654}$$

$$\frac{b\left(-\frac{2b(bc-ad)\int \frac{e^{(a+bx)^2}}{c+dx}dx}{d^2}-\frac{e^{(a+bx)^2}}{d(c+dx)}+\frac{\sqrt{\pi}b\text{erfi}(a+bx)}{d^2}\right)}{\sqrt{\pi}d}-\frac{\text{erfi}(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Erfi[a + b*x]/(c + d*x)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}(bx + a)}{(dx + c)^3} dx$$

input
```
int(erfi(b*x+a)/(d*x+c)^3,x)
```

output
```
int(erfi(b*x+a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{erfi}(a + bx)}{(c + dx)^3} dx = \int \frac{\operatorname{erfi}(bx + a)}{(dx + c)^3} dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [N/A]

Not integrable

Time = 75.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}(a + bx)}{(c + dx)^3} dx = \int \frac{\operatorname{erfi}(a + bx)}{(c + dx)^3} dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(erfi(a + b*x)/(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^3}\,dx=\int \frac{\text{erfi}\left(bx+a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)/(d*x + c)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^3}\,dx=\int \frac{\text{erfi}\left(bx+a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(erfi(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^3}\,dx = \int \frac{\text{erfi}(a+b\,x)}{(c+d\,x)^3}\,dx$$

input
```
int(erfi(a + b*x)/(c + d*x)^3,x)
```

output
```
int(erfi(a + b*x)/(c + d*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.14

$$\int \frac{\text{erfi}(a+bx)}{(c+dx)^3}\,dx = -\left(\int \frac{\text{erf}(bix+ai)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)i$$

input
```
int(erfi(b*x+a)/(d*x+c)^3,x)
```

output
```
 - int(erf(a*i + b*i*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
*i
```

3.228 $\int x^5 \mathbf{erfi}(bx)^2\, dx$

Optimal result

Integrand size = 10, antiderivative size = 175

$$\int x^5 \text{erfi}(bx)^2\, dx = \frac{11e^{2b^2x^2}}{12b^6\pi} - \frac{7e^{2b^2x^2}x^2}{12b^4\pi} + \frac{e^{2b^2x^2}x^4}{6b^2\pi} - \frac{5e^{b^2x^2}x\text{erfi}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3\text{erfi}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5\text{erfi}(bx)}{3b\sqrt{\pi}} + \frac{5\text{erfi}(bx)^2}{16b^6} + \frac{1}{6}x^6\text{erfi}(bx)^2$$

output
```
11/12*exp(2*b^2*x^2)/b^6/Pi-7/12*exp(2*b^2*x^2)*x^2/b^4/Pi+1/6*exp(2*b^2*x
^2)*x^4/b^2/Pi-5/4*exp(b^2*x^2)*x*erfi(b*x)/b^5/Pi^(1/2)+5/6*exp(b^2*x^2)*
x^3*erfi(b*x)/b^3/Pi^(1/2)-1/3*exp(b^2*x^2)*x^5*erfi(b*x)/b/Pi^(1/2)+5/16*
erfi(b*x)^2/b^6+1/6*x^6*erfi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.57

$$\int x^5 \text{erfi}(bx)^2\, dx = \frac{4e^{2b^2x^2}(11 - 7b^2x^2 + 2b^4x^4) - 4be^{b^2x^2}\sqrt{\pi}x(15 - 10b^2x^2 + 4b^4x^4)\,\text{erfi}(bx) + \pi(15 + 8b^6x^6)\,\text{erfi}(bx)^2}{48b^6\pi}$$

input
```
Integrate[x^5*Erfi[b*x]^2,x]
```

output
```
(4*E^(2*b^2*x^2)*(11 - 7*b^2*x^2 + 2*b^4*x^4) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x
*(15 - 10*b^2*x^2 + 4*b^4*x^4)*Erfi[b*x] + Pi*(15 + 8*b^6*x^6)*Erfi[b*x]^2
)/(48*b^6*Pi)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.54, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6920, 6941, 2641, 2641, 2638, 6941, 2641, 2638, 6941, 2638, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\text{erfi}(bx)^2\,dx$$

$$\Big\downarrow \text{6920}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\int e^{b^2x^2}x^6\text{erfi}(bx)dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6941}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\int e^{b^2x^2}x^4\text{erfi}(bx)dx}{2b^2}-\frac{\int e^{2b^2x^2}x^5dx}{\sqrt{\pi}b}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\int e^{b^2x^2}x^4\text{erfi}(bx)dx}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\int e^{2b^2x^2}x^3dx}{b^2}}{\sqrt{\pi}b}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\int e^{b^2x^2}x^4\text{erfi}(bx)dx}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{\int e^{2b^2x^2}xdx}{2b^2}}{b^2}}{\sqrt{\pi}b}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\int e^{b^2x^2}x^4\text{erfi}(bx)dx}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 6941

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\left(-\frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}-\frac{\int e^{2b^2x^2}x^3dx}{\sqrt{\pi}b}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 2641

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\left(-\frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{\int e^{2b^2x^2}xdx}{2b^2}}{\sqrt{\pi}b}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 2638

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\left(-\frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{b^2}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$\Big\downarrow$ 6941

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\left(-\frac{3\left(-\frac{\int e^{b^2x^2}\text{erfi}(bx)dx}{2b^2}-\frac{\int e^{2b^2x^2}xdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}}{}\right.}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\left(-\frac{3\left(-\frac{\int e^{b^2x^2}\text{erfi}(bx)dx}{2b^2}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{x^2 \cdots}{\cdots}}{\sqrt{\cdots}}\cdots\right.}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6929}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(-\frac{5\left(-\frac{3\left(-\frac{\sqrt{\pi}\int\text{erfi}(bx)d\text{erfi}(bx)}{4b^3}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{2b^2}+\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}\cdots}{\cdots}\cdots\right.}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{6}x^6\text{erfi}(bx)^2-\frac{2b\left(\frac{x^5e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^4e^{2b^2x^2}}{4b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{b^2}}{\sqrt{\pi}b}-\frac{5\left(\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}-\frac{3\left(-\frac{\sqrt{\pi}\text{erfi}(bx)^2}{8b^3}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x \cdots}}{4\sqrt{\pi}b \cdots}\cdots\right.}{2b^2}\cdots\right.}{2b^2}\cdots\right.}{3\sqrt{\pi}}$$

input

```
Int[x^5*Erfi[b*x]^2,x]
```

output
```
(x^6*Erfi[b*x]^2)/6 - (2*b*(-(((E^(2*b^2*x^2)*x^4)/(4*b^2) - (-1/8*E^(2*b^
2*x^2)/b^4 + (E^(2*b^2*x^2)*x^2)/(4*b^2))/b^2)/(b*Sqrt[Pi])) + (E^(b^2*x^2
)*x^5*Erfi[b*x])/(2*b^2) - (5*(-((-1/8*E^(2*b^2*x^2)/b^4 + (E^(2*b^2*x^2)*
x^2)/(4*b^2))/(b*Sqrt[Pi])) + (E^(b^2*x^2)*x^3*Erfi[b*x])/(2*b^2) - (3*(-1
/4*E^(2*b^2*x^2)/(b^3*Sqrt[Pi]) + (E^(b^2*x^2)*x*Erfi[b*x])/(2*b^2) - (Sqr
t[Pi]*Erfi[b*x]^2)/(8*b^3)))/(2*b^2)))/(2*b^2)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{8\,\mathrm{erfi}(bx)^2 x^6 b^6 \pi^{\frac{3}{2}} - 16\,\mathrm{erfi}(bx) e^{b^2x^2} x^5 b^5 \pi + 8\,e^{2b^2x^2} x^4 b^4 \sqrt{\pi} + 40\,\mathrm{erfi}(bx) e^{b^2x^2} x^3 b^3 \pi - 28\,e^{2b^2x^2} x^2 b^2 \sqrt{\pi} - 60\,\mathrm{erfi}(bx) x\, e^{b^2x^2} b}{48 b^6 \pi^{\frac{3}{2}}}$

input
```
int(x^5*erfi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*erfi(b*x)^2*x^6*b^6*Pi^(3/2)-16*erfi(b*x)*exp(b^2*x^2)*x^5*b^5*Pi+
8*exp(b^2*x^2)^2*x^4*b^4*Pi^(1/2)+40*erfi(b*x)*exp(b^2*x^2)*x^3*b^3*Pi-28*
exp(b^2*x^2)^2*x^2*b^2*Pi^(1/2)-60*erfi(b*x)*x*exp(b^2*x^2)*b*Pi+15*erfi(b
*x)^2*Pi^(3/2)+44*exp(b^2*x^2)^2*Pi^(1/2))/b^6/Pi^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.55

$$\int x^5 \mathrm{erfi}(bx)^2\,dx =$$
$$-\frac{4\sqrt{\pi}(4\,b^5x^5 - 10\,b^3x^3 + 15\,bx)\,\mathrm{erfi}\,(bx)\,e^{(b^2x^2)} - (15\,\pi + 8\,\pi b^6x^6)\,\mathrm{erfi}\,(bx)^2 - 4\,(2\,b^4x^4 - 7\,b^2x^2 + 11)e}{48\,\pi b^6}$$

input
```
integrate(x^5*erfi(b*x)^2,x, algorithm="fricas")
```

output
```
-1/48*(4*sqrt(pi)*(4*b^5*x^5 - 10*b^3*x^3 + 15*b*x)*erfi(b*x)*e^(b^2*x^2)
- (15*pi + 8*pi*b^6*x^6)*erfi(b*x)^2 - 4*(2*b^4*x^4 - 7*b^2*x^2 + 11)*e^(2
*b^2*x^2))/(pi*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.96

$$\int x^5 \text{erfi}(bx)^2 \, dx$$
$$= \begin{cases} \frac{x^6 \text{erfi}^2(bx)}{6} - \frac{x^5 e^{b^2x^2} \text{erfi}(bx)}{3\sqrt{\pi}b} + \frac{x^4 e^{2b^2x^2}}{6\pi b^2} + \frac{5x^3 e^{b^2x^2} \text{erfi}(bx)}{6\sqrt{\pi}b^3} - \frac{7x^2 e^{2b^2x^2}}{12\pi b^4} - \frac{5x e^{b^2x^2} \text{erfi}(bx)}{4\sqrt{\pi}b^5} + \frac{11e^{2b^2x^2}}{12\pi b^6} + \frac{5\,\text{erfi}^2(bx)}{16b^6} & \text{fo} \\ 0 & \text{ot} \end{cases}$$

input
```
integrate(x**5*erfi(b*x)**2,x)
```

output
```
Piecewise((x**6*erfi(b*x)**2/6 - x**5*exp(b**2*x**2)*erfi(b*x)/(3*sqrt(pi)
*b) + x**4*exp(2*b**2*x**2)/(6*pi*b**2) + 5*x**3*exp(b**2*x**2)*erfi(b*x)/
(6*sqrt(pi)*b**3) - 7*x**2*exp(2*b**2*x**2)/(12*pi*b**4) - 5*x*exp(b**2*x*
*2)*erfi(b*x)/(4*sqrt(pi)*b**5) + 11*exp(2*b**2*x**2)/(12*pi*b**6) + 5*erf
i(b*x)**2/(16*b**6), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^5 \text{erfi}(bx)^2 \, dx = \int x^5 \,\text{erfi}(bx)^2 \, dx$$

input
```
integrate(x^5*erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*erfi(b*x)^2, x)
```

Giac [F]

$$\int x^5 \text{erfi}(bx)^2 \, dx = \int x^5 \, \text{erfi}\,(bx)^2 \, dx$$

input
```
integrate(x^5*erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*erfi(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int x^5 \text{erfi}(bx)^2 \, dx = \frac{x^6 \,\text{erfi}(b\,x)^2}{6} + \frac{\frac{11\,e^{2\,b^2\,x^2}}{12} + \frac{5\,\pi\,\text{erfi}(b\,x)^2}{16} - \frac{7\,b^2\,x^2\,e^{2\,b^2\,x^2}}{12} + \frac{b^4\,x^4\,e^{2\,b^2\,x^2}}{6} + \frac{5\,b^3\,x^3\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfi}(b\,x)}{6} - \frac{b^5\,x^5\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfi}(b\,x)}{3} - \frac{5\,b\,x\,\sqrt{\pi}\,e^{b^2\,x^{}}}{4}}{b^6\,\pi}$$

input
```
int(x^5*erfi(b*x)^2,x)
```

output
```
(x^6*erfi(b*x)^2)/6 + ((11*exp(2*b^2*x^2))/12 + (5*pi*erfi(b*x)^2)/16 - (7
*b^2*x^2*exp(2*b^2*x^2))/12 + (b^4*x^4*exp(2*b^2*x^2))/6 + (5*b^3*x^3*pi^(
1/2)*exp(b^2*x^2)*erfi(b*x))/6 - (b^5*x^5*pi^(1/2)*exp(b^2*x^2)*erfi(b*x))
/3 - (5*b*x*pi^(1/2)*exp(b^2*x^2)*erfi(b*x))/4)/(b^6*pi)
```

Reduce [F]

$$\int x^5 \text{erfi}(bx)^2 \, dx = -\left(\int \text{erf}\,(bix)^2 \, x^5 dx\right)$$

input
```
int(x^5*erfi(b*x)^2,x)
```

output
```
- int(erf(b*i*x)**2*x**5,x)
```

3.229 $\int x^3 \text{erfi}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 124

$$\int x^3 \text{erfi}(bx)^2 \, dx = -\frac{e^{2b^2x^2}}{2b^4\pi} + \frac{e^{2b^2x^2}x^2}{4b^2\pi} + \frac{3e^{b^2x^2}x\text{erfi}(bx)}{4b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^3\text{erfi}(bx)}{2b\sqrt{\pi}} - \frac{3\text{erfi}(bx)^2}{16b^4} + \frac{1}{4}x^4\text{erfi}(bx)^2$$

output
```
-1/2*exp(2*b^2*x^2)/b^4/Pi+1/4*exp(2*b^2*x^2)*x^2/b^2/Pi+3/4*exp(b^2*x^2)*
x*erfi(b*x)/b^3/Pi^(1/2)-1/2*exp(b^2*x^2)*x^3*erfi(b*x)/b/Pi^(1/2)-3/16*er
fi(b*x)^2/b^4+1/4*x^4*erfi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int x^3 \text{erfi}(bx)^2 \, dx = \frac{4e^{2b^2x^2}(-2+b^2x^2) - 4be^{b^2x^2}\sqrt{\pi}x(-3+2b^2x^2)\,\text{erfi}(bx) + \pi(-3+4b^4x^4)\,\text{erfi}(bx)^2}{16b^4\pi}$$

input
```
Integrate[x^3*Erfi[b*x]^2,x]
```

output
```
(4*E^(2*b^2*x^2)*(-2 + b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2*b^2*x
^2)*Erfi[b*x] + Pi*(-3 + 4*b^4*x^4)*Erfi[b*x]^2)/(16*b^4*Pi)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6920, 6941, 2641, 2638, 6941, 2638, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{erfi}(bx)^2 \, dx$$

$$\Big\downarrow \text{6920}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{b\int e^{b^2x^2}x^4\text{erfi}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6941}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{b\left(-\frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{2b^2} - \frac{\int e^{2b^2x^2}x^3dx}{\sqrt{\pi}b} + \frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{b\left(-\frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{2b^2} - \frac{\frac{x^2e^{2b^2x^2}}{4b^2} - \frac{\int e^{2b^2x^2}xdx}{2b^2}}{\sqrt{\pi}b} + \frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2 - \frac{b\left(-\frac{3\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{2b^2} + \frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\frac{x^2e^{2b^2x^2}}{4b^2} - \frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6941}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2-\frac{b\left(-\frac{3\left(-\frac{\int e^{b^2x^2}\text{erfi}(bx)dx}{2b^2}-\frac{\int e^{2b^2x^2}xdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2-\frac{b\left(-\frac{3\left(-\frac{\int e^{b^2x^2}\text{erfi}(bx)dx}{2b^2}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6929}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2-\frac{b\left(-\frac{3\left(-\frac{\sqrt{\pi}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b^3}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}+\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4\text{erfi}(bx)^2-\frac{b\left(\frac{x^3e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2}}{4b^2}-\frac{e^{2b^2x^2}}{8b^4}}{\sqrt{\pi}b}-\frac{3\left(-\frac{\sqrt{\pi}\text{erfi}(bx)^2}{8b^3}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{2b^2}\right)}{\sqrt{\pi}}$$

input
```
Int[x^3*Erfi[b*x]^2,x]
```

output
```
(x^4*Erfi[b*x]^2)/4 - (b*(-((-1/8*E^(2*b^2*x^2)/b^4 + (E^(2*b^2*x^2)*x^2)/
(4*b^2))/(b*Sqrt[Pi])) + (E^(b^2*x^2)*x^3*Erfi[b*x])/(2*b^2) - (3*(-1/4*E^
(2*b^2*x^2)/(b^3*Sqrt[Pi]) + (E^(b^2*x^2)*x*Erfi[b*x])/(2*b^2) - (Sqrt[Pi]
*Erfi[b*x]^2)/(8*b^3)))/(2*b^2)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{4\,\mathrm{erfi}(bx)^2 x^4 \pi^{\frac{3}{2}} b^4-8\,\mathrm{erfi}(bx)\mathrm{e}^{b^2x^2}x^3b^3\pi+4\,\mathrm{e}^{2b^2x^2}x^2b^2\sqrt{\pi}+12\,\mathrm{erfi}(bx)x\,\mathrm{e}^{b^2x^2}b\pi-3\,\mathrm{erfi}(bx)^2\pi^{\frac{3}{2}}-8\,\mathrm{e}^{2b^2x^2}\sqrt{\pi}}{16\pi^{\frac{3}{2}}b^4}$	112

input
```
int(x^3*erfi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*erfi(b*x)^2*x^4*Pi^(3/2)*b^4-8*erfi(b*x)*exp(b^2*x^2)*x^3*b^3*Pi+4
*exp(b^2*x^2)^2*x^2*b^2*Pi^(1/2)+12*erfi(b*x)*x*exp(b^2*x^2)*b*Pi-3*erfi(b
*x)^2*Pi^(3/2)-8*exp(b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int x^3\mathrm{erfi}(bx)^2\,dx$$
$$= -\frac{4\,\sqrt{\pi}(2\,b^3x^3-3\,bx)\,\mathrm{erfi}\,(bx)\,e^{(b^2x^2)}+(3\,\pi-4\,\pi b^4x^4)\,\mathrm{erfi}\,(bx)^2-4\,(b^2x^2-2)e^{(2\,b^2x^2)}}{16\,\pi b^4}$$

input
```
integrate(x^3*erfi(b*x)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*sqrt(pi)*(2*b^3*x^3 - 3*b*x)*erfi(b*x)*e^(b^2*x^2) + (3*pi - 4*pi
*b^4*x^4)*erfi(b*x)^2 - 4*(b^2*x^2 - 2)*e^(2*b^2*x^2))/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int x^3 \text{erfi}(bx)^2 \, dx = \begin{cases} \frac{x^4 \operatorname{erfi}^2(bx)}{4} - \frac{x^3 e^{b^2x^2} \operatorname{erfi}(bx)}{2\sqrt{\pi}b} + \frac{x^2 e^{2b^2x^2}}{4\pi b^2} + \frac{3xe^{b^2x^2} \operatorname{erfi}(bx)}{4\sqrt{\pi}b^3} - \frac{e^{2b^2x^2}}{2\pi b^4} - \frac{3 \operatorname{erfi}^2(bx)}{16b^4} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erfi(b*x)**2,x)
```

output
```
Piecewise((x**4*erfi(b*x)**2/4 - x**3*exp(b**2*x**2)*erfi(b*x)/(2*sqrt(pi)
*b) + x**2*exp(2*b**2*x**2)/(4*pi*b**2) + 3*x*exp(b**2*x**2)*erfi(b*x)/(4*
sqrt(pi)*b**3) - exp(2*b**2*x**2)/(2*pi*b**4) - 3*erfi(b*x)**2/(16*b**4),
Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^3 \text{erfi}(bx)^2 \, dx = \int x^3 \operatorname{erfi}(bx)^2 \, dx$$

input
```
integrate(x^3*erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*erfi(b*x)^2, x)
```

Giac [F]

$$\int x^3 \text{erfi}(bx)^2 \, dx = \int x^3 \operatorname{erfi}(bx)^2 \, dx$$

input
```
integrate(x^3*erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*erfi(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int x^3 \text{erfi}(bx)^2 \, dx$$
$$= \frac{x^4 \, \text{erfi}(b\,x)^2}{4} - \frac{\frac{e^{2\,b^2\,x^2}}{2} + \frac{3\,\pi\,\text{erfi}(b\,x)^2}{16} - \frac{b^2\,x^2\,e^{2\,b^2\,x^2}}{4} + \frac{b^3\,x^3\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfi}(b\,x)}{2} - \frac{3\,b\,x\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfi}(b\,x)}{4}}{b^4\,\pi}$$

input
```
int(x^3*erfi(b*x)^2,x)
```

output
```
(x^4*erfi(b*x)^2)/4 - (exp(2*b^2*x^2)/2 + (3*pi*erfi(b*x)^2)/16 - (b^2*x^2
*exp(2*b^2*x^2))/4 + (b^3*x^3*pi^(1/2)*exp(b^2*x^2)*erfi(b*x))/2 - (3*b*x*
pi^(1/2)*exp(b^2*x^2)*erfi(b*x))/4)/(b^4*pi)
```

Reduce [F]

$$\int x^3 \text{erfi}(bx)^2 \, dx = -\left(\int \text{erf}(bix)^2 \, x^3 dx\right)$$

input
```
int(x^3*erfi(b*x)^2,x)
```

output
```
- int(erf(b*i*x)**2*x**3,x)
```

3.230 $\int x \text{erfi}(bx)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 71

$$\int x \text{erfi}(bx)^2 \, dx = \frac{e^{2b^2x^2}}{2b^2\pi} - \frac{e^{b^2x^2} x \text{erfi}(bx)}{b\sqrt{\pi}} + \frac{\text{erfi}(bx)^2}{4b^2} + \frac{1}{2}x^2 \text{erfi}(bx)^2$$

output
```
1/2*exp(2*b^2*x^2)/b^2/Pi-exp(b^2*x^2)*x*erfi(b*x)/b/Pi^(1/2)+1/4*erfi(b*x
)^2/b^2+1/2*x^2*erfi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x \text{erfi}(bx)^2 \, dx = \frac{2e^{2b^2x^2} - 4be^{b^2x^2} \sqrt{\pi} x \text{erfi}(bx) + (\pi + 2b^2\pi x^2) \text{erfi}(bx)^2}{4b^2\pi}$$

input
```
Integrate[x*Erfi[b*x]^2,x]
```

output
```
(2*E^(2*b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*Erfi[b*x] + (Pi + 2*b^2*Pi*x
^2)*Erfi[b*x]^2)/(4*b^2*Pi)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6920, 6941, 2638, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfi}(bx)^2\,dx$$

$$\Big\downarrow \ 6920$$

$$\frac{1}{2}x^2\text{erfi}(bx)^2-\frac{2b\int e^{b^2x^2}x^2\text{erfi}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \ 6941$$

$$\frac{1}{2}x^2\text{erfi}(bx)^2-\frac{2b\left(-\frac{\int e^{b^2x^2}\text{erfi}(bx)dx}{2b^2}-\frac{\int e^{2b^2x^2}xdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \ 2638$$

$$\frac{1}{2}x^2\text{erfi}(bx)^2-\frac{2b\left(-\frac{\int e^{b^2x^2}\text{erfi}(bx)dx}{2b^2}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \ 6929$$

$$\frac{1}{2}x^2\text{erfi}(bx)^2-\frac{2b\left(-\frac{\sqrt{\pi}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b^3}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}x^2\text{erfi}(bx)^2-\frac{2b\left(-\frac{\sqrt{\pi}\text{erfi}(bx)^2}{8b^3}+\frac{xe^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2}}{4\sqrt{\pi}b^3}\right)}{\sqrt{\pi}}$$

input
```
Int[x*Erfi[b*x]^2,x]
```

output
```
(x^2*Erfi[b*x]^2)/2 - (2*b*(-1/4*E^(2*b^2*x^2)/(b^3*Sqrt[Pi]) + (E^(b^2*x^
2)*x*Erfi[b*x])/(2*b^2) - (Sqrt[Pi]*Erfi[b*x]^2)/(8*b^3)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{2\,\mathrm{erfi}(bx)^{2} x^{2} \pi^{\frac{3}{2}} b^{2}-4\ \mathrm{erfi}(bx) x\, \mathrm{e}^{b^{2} x^{2}} b \pi+\mathrm{erfi}(bx)^{2} \pi^{\frac{3}{2}}+2\,\mathrm{e}^{2 b^{2} x^{2}} \sqrt{\pi}}{4 \pi^{\frac{3}{2}} b^{2}}$	69

input
```
int(x*erfi(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*erfi(b*x)^2*x^2*Pi^(3/2)*b^2-4*erfi(b*x)*x*exp(b^2*x^2)*b*Pi+erfi(b
*x)^2*Pi^(3/2)+2*exp(b^2*x^2)^2*Pi^(1/2))/Pi^(3/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int x\mathrm{erfi}(bx)^2\,dx = -\frac{4\,\sqrt{\pi}bx\,\mathrm{erfi}\,(bx)\,e^{\left(b^{2}x^{2}\right)} - (\pi + 2\,\pi b^{2}x^{2})\,\mathrm{erfi}\,(bx)^{2} - 2\,e^{\left(2\,b^{2}x^{2}\right)}}{4\,\pi b^{2}}$$

input
```
integrate(x*erfi(b*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(pi)*b*x*erfi(b*x)*e^(b^2*x^2) - (pi + 2*pi*b^2*x^2)*erfi(b*x)
^2 - 2*e^(2*b^2*x^2))/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x\mathrm{erfi}(bx)^2\,dx = \begin{cases} \frac{x^{2}\,\mathrm{erfi}^{2}\,(bx)}{2} - \frac{xe^{b^{2}x^{2}}\,\mathrm{erfi}\,(bx)}{\sqrt{\pi}b} + \frac{e^{2b^{2}x^{2}}}{2\pi b^{2}} + \frac{\mathrm{erfi}^{2}\,(bx)}{4b^{2}} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*erfi(b*x)**2,x)
```

output
```
Piecewise((x**2*erfi(b*x)**2/2 - x*exp(b**2*x**2)*erfi(b*x)/(sqrt(pi)*b) +
 exp(2*b**2*x**2)/(2*pi*b**2) + erfi(b*x)**2/(4*b**2), Ne(b, 0)), (0, True
))
```

Maxima [F]

$$\int x\text{erfi}(bx)^2\,dx = \int x\,\text{erfi}\,(bx)^2\,dx$$

input
```
integrate(x*erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*erfi(b*x)^2, x)
```

Giac [F]

$$\int x\text{erfi}(bx)^2\,dx = \int x\,\text{erfi}\,(bx)^2\,dx$$

input
```
integrate(x*erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*erfi(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x\text{erfi}(bx)^2\,dx = \frac{\frac{b^2\,x^2\,\text{erfi}(b\,x)^2}{2}+\frac{\text{erfi}(b\,x)^2}{4}}{b^2}+\frac{\frac{e^{2\,b^2\,x^2}}{2}-b\,x\,\sqrt{\pi}\,e^{b^2\,x^2}\,\text{erfi}(b\,x)}{b^2\,\pi}$$

input
```
int(x*erfi(b*x)^2,x)
```

output
```
(erfi(b*x)^2/4 + (b^2*x^2*erfi(b*x)^2)/2)/b^2 + (exp(2*b^2*x^2)/2 - b*x*pi
^(1/2)*exp(b^2*x^2)*erfi(b*x))/(b^2*pi)
```

Reduce [F]

$$\int x\text{erfi}(bx)^2\,dx = -\left(\int \text{erf}(bix)^2\,xdx\right)$$

input
```
int(x*erfi(b*x)^2,x)
```

output
```
 - int(erf(b*i*x)**2*x,x)
```

3.231 $\int \frac{\text{erfi}(bx)^2}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfi}(bx)^2}{x} \, dx = \text{Int}\left(\frac{\text{erfi}(bx)^2}{x}, x\right)$$

output
```
Defer(Int)(erfi(b*x)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x} \, dx = \int \frac{\text{erfi}(bx)^2}{x} \, dx$$

input
```
Integrate[Erfi[b*x]^2/x,x]
```

output
```
Integrate[Erfi[b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)^2}{x} \, dx$$

$$\big\downarrow \text{ 6926}$$

$$\int \frac{\text{erfi}(bx)^2}{x}dx$$

input
```
Int[Erfi[b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}\left(bx\right)^2}{x}dx$$

input
```
int(erfi(b*x)^2/x,x)
```

output
```
int(erfi(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erfi(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfi}^2\left(bx\right)}{x}\,dx$$

input
```
integrate(erfi(b*x)**2/x,x)
```

output
```
Integral(erfi(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erfi(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(erfi(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx = \int \frac{\operatorname{erfi}(b\,x)^2}{x}\,dx$$

input
```
int(erfi(b*x)^2/x,x)
```

output
```
int(erfi(b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{erfi}(bx)^2}{x}\,dx = -\left(\int \frac{\operatorname{erf}(bix)^2}{x}dx\right)$$

input
```
int(erfi(b*x)^2/x,x)
```

output
```
 - int(erf(b*i*x)**2/x,x)
```

3.232 $\int \frac{\text{erfi}(bx)^2}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{\text{erfi}(bx)^2}{x^3}\,dx = -\frac{2be^{b^2x^2}\text{erfi}(bx)}{\sqrt{\pi}x} + b^2\text{erfi}(bx)^2 - \frac{\text{erfi}(bx)^2}{2x^2} + \frac{2b^2\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\pi}$$

output

```
-2*b*exp(b^2*x^2)*erfi(b*x)/Pi^(1/2)/x+b^2*erfi(b*x)^2-1/2*erfi(b*x)^2/x^2
+2*b^2*Ei(2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\text{erfi}(bx)^2}{x^3}\,dx = -\frac{2be^{b^2x^2}\text{erfi}(bx)}{\sqrt{\pi}x} + \left(b^2 - \frac{1}{2x^2}\right)\text{erfi}(bx)^2 + \frac{2b^2\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\pi}$$

input

```
Integrate[Erfi[b*x]^2/x^3,x]
```

output

```
(-2*b*E^(b^2*x^2)*Erfi[b*x])/(Sqrt[Pi]*x) + (b^2 - 1/(2*x^2))*Erfi[b*x]^2
+ (2*b^2*ExpIntegralEi[2*b^2*x^2])/Pi
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6920, 6947, 2639, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{6920}$$

$$\frac{2b\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{6947}$$

$$\frac{2b\left(2b^2\int e^{b^2x^2}\text{erfi}(bx)dx+\frac{2b\int \frac{e^{2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2b\left(2b^2\int e^{b^2x^2}\text{erfi}(bx)dx-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{6929}$$

$$\frac{2b\left(\sqrt{\pi}b\int \text{erfi}(bx)d\text{erfi}(bx)-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2b\left(-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\text{erfi}(bx)^2\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{2x^2}$$

input
```
Int[Erfi[b*x]^2/x^3,x]
```

output
```
-1/2*Erfi[b*x]^2/x^2 + (2*b*(-((E^(b^2*x^2)*Erfi[b*x])/x) + (b*Sqrt[Pi]*Er
fi[b*x]^2)/2 + (b*ExpIntegralEi[2*b^2*x^2])/Sqrt[Pi]))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}dx$$

input
```
int(erfi(b*x)^2/x^3,x)
```

output
```
int(erfi(b*x)^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx = \frac{4\,b^2x^2\operatorname{Ei}(2\,b^2x^2) - 4\,\sqrt{\pi}bx\operatorname{erfi}(bx)\,e^{(b^2x^2)} - (\pi - 2\,\pi b^2x^2)\operatorname{erfi}(bx)^2}{2\,\pi x^2}$$

input
```
integrate(erfi(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*b^2*x^2*Ei(2*b^2*x^2) - 4*sqrt(pi)*b*x*erfi(b*x)*e^(b^2*x^2) - (pi
- 2*pi*b^2*x^2)*erfi(b*x)^2)/(pi*x^2)
```

Sympy [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfi}^2(bx)}{x^3}\,dx$$

input
```
integrate(erfi(b*x)**2/x**3,x)
```

output
```
Integral(erfi(b*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^3}\,dx$$

input
```
integrate(erfi(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x^3, x)
```

Giac [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^3}\,dx$$

input
```
integrate(erfi(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3}\,dx = \int \frac{\operatorname{erfi}(b\,x)^2}{x^3}\,dx$$

input
```
int(erfi(b*x)^2/x^3,x)
```

output
```
int(erfi(b*x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^3} \, dx = -\left(\int \frac{\operatorname{erf}(bix)^2}{x^3} dx\right)$$

input
```
int(erfi(b*x)^2/x^3,x)
```

output
```
- int(erf(b*i*x)**2/x**3,x)
```

3.233 $\int \frac{\text{erfi}(bx)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 123

$$\int \frac{\text{erfi}(bx)^2}{x^5}\,dx = -\frac{b^2 e^{2b^2x^2}}{3\pi x^2} - \frac{b e^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x^3} - \frac{2b^3 e^{b^2x^2}\text{erfi}(bx)}{3\sqrt{\pi}x} + \frac{1}{3}b^4\text{erfi}(bx)^2 - \frac{\text{erfi}(bx)^2}{4x^4} + \frac{4b^4\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{3\pi}$$

output
```
-1/3*b^2*exp(2*b^2*x^2)/Pi/x^2-1/3*b*exp(b^2*x^2)*erfi(b*x)/Pi^(1/2)/x^3-2
/3*b^3*exp(b^2*x^2)*erfi(b*x)/Pi^(1/2)/x+1/3*b^4*erfi(b*x)^2-1/4*erfi(b*x)
^2/x^4+4/3*b^4*Ei(2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

$$\int \frac{\text{erfi}(bx)^2}{x^5}\,dx = \frac{-4be^{b^2x^2}\sqrt{\pi}x(1+2b^2x^2)\,\text{erfi}(bx) + \pi(-3+4b^4x^4)\,\text{erfi}(bx)^2 - 4b^2x^2\left(e^{2b^2x^2} - 4b^2x^2\,\text{ExpIntegralEi}\left(2b^2x^2\right)\right)}{12\pi x^4}$$

input
```
Integrate[Erfi[b*x]^2/x^5,x]
```

output
```
(-4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(1 + 2*b^2*x^2)*Erfi[b*x] + Pi*(-3 + 4*b^4*x^
4)*Erfi[b*x]^2 - 4*b^2*x^2*(E^(2*b^2*x^2) - 4*b^2*x^2*ExpIntegralEi[2*b^2*
x^2]))/(12*Pi*x^4)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6920, 6947, 2643, 2639, 6947, 2639, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)^2}{x^5} \, dx$$

$$\Big\downarrow \text{6920}$$

$$\frac{b\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^4}dx}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{6947}$$

$$\frac{b\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx+\frac{2b\int \frac{e^{2b^2x^2}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx+\frac{2b\left(2b^2\int \frac{e^{2b^2x^2}}{x}dx-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{2639}$$

$$\frac{b\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\,\text{ExpIntegralEi}\left(2b^2x^2\right)-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{6947}$$

$$\frac{b\left(\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2}\text{erfi}(bx)dx+\frac{2b\int\frac{e^{2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \ 2639$$

$$\frac{b\left(\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2}\text{erfi}(bx)dx-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\text{ ExpIntegralEi}(2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \ 6929$$

$$\frac{b\left(\frac{2}{3}b^2\left(\sqrt{\pi}b\int\text{erfi}(bx)d\text{erfi}(bx)-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\text{ ExpIntegralEi}(2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

$$\Big\downarrow \ 15$$

$$\frac{b\left(\frac{2}{3}b^2\left(-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\text{ ExpIntegralEi}(2b^2x^2)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\text{erfi}(bx)^2\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{4x^4}$$

input
```
Int[Erfi[b*x]^2/x^5,x]
```

output
```
-1/4*Erfi[b*x]^2/x^4 + (b*(-1/3*(E^(b^2*x^2)*Erfi[b*x])/x^3 + (2*b*(-1/2*E
^(2*b^2*x^2)/x^2 + b^2*ExpIntegralEi[2*b^2*x^2]))/(3*Sqrt[Pi]) + (2*b^2*(-
((E^(b^2*x^2)*Erfi[b*x])/x) + (b*Sqrt[Pi]*Erfi[b*x]^2)/2 + (b*ExpIntegralE
i[2*b^2*x^2])/Sqrt[Pi]))/3))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}dx$$

input
```
int(erfi(b*x)^2/x^5,x)
```

output
```
int(erfi(b*x)^2/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx = \frac{16\,b^4x^4\operatorname{Ei}(2\,b^2x^2) - 4\,b^2x^2e^{(2\,b^2x^2)} - 4\,\sqrt{\pi}(2\,b^3x^3 + bx)\operatorname{erfi}(bx)\,e^{(b^2x^2)} - (3\,\pi - 4\,\pi b^4x^4)\operatorname{erfi}(bx)^2}{12\,\pi x^4}$$

input
```
integrate(erfi(b*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(16*b^4*x^4*Ei(2*b^2*x^2) - 4*b^2*x^2*e^(2*b^2*x^2) - 4*sqrt(pi)*(2*b
^3*x^3 + b*x)*erfi(b*x)*e^(b^2*x^2) - (3*pi - 4*pi*b^4*x^4)*erfi(b*x)^2)/(
pi*x^4)
```

Sympy [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfi}^2(bx)}{x^5}\,dx$$

input
```
integrate(erfi(b*x)**2/x**5,x)
```

output
```
Integral(erfi(b*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx$$

input
```
integrate(erfi(b*x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x^5, x)
```

Giac [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx$$

input
```
integrate(erfi(b*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5}\,dx = \int \frac{\operatorname{erfi}(b\,x)^2}{x^5}\,dx$$

input
```
int(erfi(b*x)^2/x^5,x)
```

output
```
int(erfi(b*x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\text{erfi}(bx)^2}{x^5} \, dx = -\left(\int \frac{\text{erf}(bix)^2}{x^5} dx \right)$$

input
```
int(erfi(b*x)^2/x^5,x)
```

output
```
 - int(erf(b*i*x)**2/x**5,x)
```

3.234 $\int \frac{\text{erfi}(bx)^2}{x^7}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 174

$$\int \frac{\text{erfi}(bx)^2}{x^7}\,dx = -\frac{b^2e^{2b^2x^2}}{15\pi x^4}-\frac{2b^4e^{2b^2x^2}}{9\pi x^2}-\frac{2be^{b^2x^2}\text{erfi}(bx)}{15\sqrt{\pi}x^5}-\frac{4b^3e^{b^2x^2}\text{erfi}(bx)}{45\sqrt{\pi}x^3}-\frac{8b^5e^{b^2x^2}\text{erfi}(bx)}{45\sqrt{\pi}x}+\frac{4}{45}b^6\text{erfi}(bx)^2-\frac{\text{erfi}(bx)^2}{6x^6}+\frac{28b^6\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{45\pi}$$

output

```
-1/15*b^2*exp(2*b^2*x^2)/Pi/x^4-2/9*b^4*exp(2*b^2*x^2)/Pi/x^2-2/15*b*exp(b
^2*x^2)*erfi(b*x)/Pi^(1/2)/x^5-4/45*b^3*exp(b^2*x^2)*erfi(b*x)/Pi^(1/2)/x^
3-8/45*b^5*exp(b^2*x^2)*erfi(b*x)/Pi^(1/2)/x+4/45*b^6*erfi(b*x)^2-1/6*erfi
(b*x)^2/x^6+28/45*b^6*Ei(2*b^2*x^2)/Pi
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int \frac{\text{erfi}(bx)^2}{x^7}\,dx = \frac{-2b^2e^{2b^2x^2}x^2(3+10b^2x^2)-4be^{b^2x^2}\sqrt{\pi}x(3+2b^2x^2+4b^4x^4)\,\text{erfi}(bx)+\pi(-15+8b^6x^6)\,\text{erfi}(bx)^2+56b^6x}{90\pi x^6}$$

input
```
Integrate[Erfi[b*x]^2/x^7,x]
```

output
```
(-2*b^2*E^(2*b^2*x^2)*x^2*(3 + 10*b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(3
 + 2*b^2*x^2 + 4*b^4*x^4)*Erfi[b*x] + Pi*(-15 + 8*b^6*x^6)*Erfi[b*x]^2 + 5
6*b^6*x^6*ExpIntegralEi[2*b^2*x^2])/(90*Pi*x^6)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6920, 6947, 2643, 2643, 2639, 6947, 2643, 2639, 6947, 2639, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)^2}{x^7}\,dx$$

$$\Big\downarrow \; 6920$$

$$\frac{2b\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^6}dx}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 6947$$

$$\frac{2b\left(\frac{2}{5}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^4}dx+\frac{2b\int \frac{e^{2b^2x^2}}{x^5}dx}{5\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 2643$$

$$\frac{2b\left(\frac{2}{5}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^4}dx+\frac{2b\left(b^2\int \frac{e^{2b^2x^2}}{x^3}dx-\frac{e^{2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2643

$$\frac{2b\left(\frac{2}{5}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^4}dx+\frac{2b\left(b^2\left(2b^2\int \frac{e^{2b^2x^2}}{x}dx-\frac{e^{2b^2x^2}}{2x^2}\right)-\frac{e^{2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2639

$$\frac{2b\left(\frac{2}{5}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^4}dx-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}+\frac{2b\left(b^2\left(b^2\,\text{ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)-\frac{e^{2b^2x^2}}{4x^4}\right)}{5\sqrt{\pi}}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)^2}{6x^6}$$

$\Big\downarrow$ 6947

$$2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx+\frac{2b\int \frac{e^{2b^2x^2}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}+\frac{2b\left(b^2\left(b^2\,\text{ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)-\cdots\right.}{5\sqrt{\pi}}\cdots \quad / \quad 3\sqrt{\pi}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2643

$$2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx+\frac{2b\left(2b^2\int \frac{e^{2b^2x^2}}{x}dx-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}+\frac{2b\left(b^2\left(b^2\,\text{ExpIntegralEi}(2b\cdots\right.\right.}{5\sqrt{\cdots}}\cdots \quad / \quad 3\sqrt{\pi}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

$\Big\downarrow$ 2639

$$2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\int \frac{e^{b^2x^2}\text{erfi}(bx)}{x^2}dx-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\,\text{ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right)}{3\sqrt{\pi}}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{5x^5}+\frac{2b\left(b^2\left(b^2\,\text{ExpInte}\cdots\right.\right.}{\cdots}\cdots \quad / \quad 3\sqrt{\pi}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

$$\downarrow 6947$$

$$\frac{2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2}\text{erfi}(bx)dx+\frac{2b\int\frac{e^{2b^2x^2}}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ExpIntegralEi}(2b^2x^2)-\frac{e^{2b^2x^2}}{2x^2}\right.}{3\sqrt{\pi}}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

$$\downarrow 2639$$

$$\frac{2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2}\text{erfi}(bx)dx-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}(2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ExpIntegralEi}(2b^2x^2\right.}{3\sqrt{\pi}}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

$$\downarrow 6929$$

$$\frac{2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\left(\sqrt{\pi}b\int\text{erfi}(bx)d\text{erfi}(bx)-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}(2b^2x^2)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ExpIntegralEi}(2b\right.}{3\sqrt{\pi}}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

$$\downarrow 15$$

$$\frac{2b\left(\frac{2}{5}b^2\left(\frac{2}{3}b^2\left(-\frac{e^{b^2x^2}\text{erfi}(bx)}{x}+\frac{b\,\text{ExpIntegralEi}(2b^2x^2)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}b\,\text{erfi}(bx)^2\right)-\frac{e^{b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2\text{ExpIntegralEi}(2b^2x^2)-\frac{e^{2}}{}\right.}{3\sqrt{\pi}}\right.\right.}{3\sqrt{\pi}}$$

$$\frac{\text{erfi}(bx)^2}{6x^6}$$

input
```
Int[Erfi[b*x]^2/x^7,x]
```

output
```
-1/6*Erfi[b*x]^2/x^6 + (2*b*(-1/5*(E^(b^2*x^2)*Erfi[b*x])/x^5 + (2*b*(-1/4
*E^(2*b^2*x^2)/x^4 + b^2*(-1/2*E^(2*b^2*x^2)/x^2 + b^2*ExpIntegralEi[2*b^2
*x^2])))/(5*Sqrt[Pi]) + (2*b^2*(-1/3*(E^(b^2*x^2)*Erfi[b*x])/x^3 + (2*b*(-
1/2*E^(2*b^2*x^2)/x^2 + b^2*ExpIntegralEi[2*b^2*x^2]))/(3*Sqrt[Pi]) + (2*b
^2*(-((E^(b^2*x^2)*Erfi[b*x])/x) + (b*Sqrt[Pi]*Erfi[b*x]^2)/2 + (b*ExpInte
gralEi[2*b^2*x^2])/Sqrt[Pi]))/3))/5))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)^2}{x^7}dx$$

input
```
int(erfi(b*x)^2/x^7,x)
```

output
```
int(erfi(b*x)^2/x^7,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{erfi}(bx)^2}{x^7}\,dx = \frac{56\,b^6x^6\operatorname{Ei}(2\,b^2x^2) - 4\,\sqrt{\pi}(4\,b^5x^5 + 2\,b^3x^3 + 3\,bx)\operatorname{erfi}(bx)\,e^{(b^2x^2)} - (15\,\pi - 8\,\pi b^6x^6)\operatorname{erfi}(bx)^2 - 2\,(10\,b^4x^4}{90\,\pi x^6}$$

input
```
integrate(erfi(b*x)^2/x^7,x, algorithm="fricas")
```

output
```
1/90*(56*b^6*x^6*Ei(2*b^2*x^2) - 4*sqrt(pi)*(4*b^5*x^5 + 2*b^3*x^3 + 3*b*x
)*erfi(b*x)*e^(b^2*x^2) - (15*pi - 8*pi*b^6*x^6)*erfi(b*x)^2 - 2*(10*b^4*x
^4 + 3*b^2*x^2)*e^(2*b^2*x^2))/(pi*x^6)
```

Sympy [F]

$$\int \frac{\text{erfi}(bx)^2}{x^7} dx = \int \frac{\text{erfi}^2(bx)}{x^7} dx$$

input
```
integrate(erfi(b*x)**2/x**7,x)
```

output
```
Integral(erfi(b*x)**2/x**7, x)
```

Maxima [F]

$$\int \frac{\text{erfi}(bx)^2}{x^7} dx = \int \frac{\text{erfi}(bx)^2}{x^7} dx$$

input
```
integrate(erfi(b*x)^2/x^7,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x^7, x)
```

Giac [F]

$$\int \frac{\text{erfi}(bx)^2}{x^7} dx = \int \frac{\text{erfi}(bx)^2}{x^7} dx$$

input
```
integrate(erfi(b*x)^2/x^7,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{erfi}(bx)^2}{x^7}\,dx = \int \frac{\text{erfi}(b\,x)^2}{x^7}\,dx$$

input
```
int(erfi(b*x)^2/x^7,x)
```

output
```
int(erfi(b*x)^2/x^7, x)
```

Reduce [F]

$$\int \frac{\text{erfi}(bx)^2}{x^7}\,dx = -\left(\int \frac{\text{erf}(bix)^2}{x^7}dx\right)$$

input
```
int(erfi(b*x)^2/x^7,x)
```

output
```
- int(erf(b*i*x)**2/x**7,x)
```

3.235 $\int x^4 \text{erfi}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 162

$$\int x^4 \text{erfi}(bx)^2\,dx = -\frac{11e^{2b^2x^2}x}{20b^4\pi}+\frac{e^{2b^2x^2}x^3}{5b^2\pi}-\frac{4e^{b^2x^2}\text{erfi}(bx)}{5b^5\sqrt{\pi}}+\frac{4e^{b^2x^2}x^2\text{erfi}(bx)}{5b^3\sqrt{\pi}} -\frac{2e^{b^2x^2}x^4\text{erfi}(bx)}{5b\sqrt{\pi}}+\frac{1}{5}x^5\text{erfi}(bx)^2+\frac{43\text{erfi}\left(\sqrt{2}bx\right)}{40b^5\sqrt{2\pi}}$$

output
```
-11/20*exp(2*b^2*x^2)*x/b^4/Pi+1/5*exp(2*b^2*x^2)*x^3/b^2/Pi-4/5*exp(b^2*x
^2)*erfi(b*x)/b^5/Pi^(1/2)+4/5*exp(b^2*x^2)*x^2*erfi(b*x)/b^3/Pi^(1/2)-2/5
*exp(b^2*x^2)*x^4*erfi(b*x)/b/Pi^(1/2)+1/5*x^5*erfi(b*x)^2+43/80*erfi(2^(1
/2)*b*x)/b^5*2^(1/2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int x^4 \text{erfi}(bx)^2\,dx = \frac{4be^{2b^2x^2}x(-11+4b^2x^2)-32e^{b^2x^2}\sqrt{\pi}(2-2b^2x^2+b^4x^4)\,\text{erfi}(bx)+16b^5\pi x^5\text{erfi}(bx)^2+43\sqrt{2\pi}\text{erfi}\left(\sqrt{2}bx\right)}{80b^5\pi}$$

input
```
Integrate[x^4*Erfi[b*x]^2,x]
```

output
```
(4*b*E^(2*b^2*x^2)*x*(-11 + 4*b^2*x^2) - 32*E^(b^2*x^2)*Sqrt[Pi]*(2 - 2*b^
2*x^2 + b^4*x^4)*Erfi[b*x] + 16*b^5*Pi*x^5*Erfi[b*x]^2 + 43*Sqrt[2*Pi]*Erf
i[Sqrt[2]*b*x])/(80*b^5*Pi)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6920, 6941, 2641, 2641, 2633, 6941, 2641, 2633, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\text{erfi}(bx)^2\,dx$$

$$\Big\downarrow \text{6920}$$

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\int e^{b^2x^2}x^5\text{erfi}(bx)dx}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{6941}$$

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\int e^{b^2x^2}x^3\text{erfi}(bx)dx}{b^2}-\frac{\int e^{2b^2x^2}x^4dx}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\int e^{b^2x^2}x^3\text{erfi}(bx)dx}{b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\int e^{2b^2x^2}x^2dx}{4b^2}}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\int e^{b^2x^2}x^3\text{erfi}(bx)dx}{b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\int e^{2b^2x^2}dx}{4b^2}\right)}{4b^2}}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\int e^{b^2x^2}x^3\text{erfi}(bx)dx}{b^2}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 6941

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\left(-\frac{\int e^{b^2x^2}x\text{erfi}(bx)dx}{b^2}-\frac{\int e^{2b^2x^2}x^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{b^2}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 2641

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\left(-\frac{\int e^{b^2x^2}x\text{erfi}(bx)dx}{b^2}-\frac{\frac{xe^{2b^2x^2}}{4b^2}-\frac{\int e^{2b^2x^2}dx}{4b^2}}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{b^2}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 2633

$$\frac{1}{5}x^5\text{erfi}(bx)^2-\frac{4b\left(-\frac{2\left(-\frac{\int e^{b^2x^2}x\text{erfi}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}\right)}{5\sqrt{\pi}}$$

$\downarrow$ 6938

$$\frac{1}{5}x^5\text{erfi}(bx)^2 - \frac{4b\left(-\frac{2\left(-\frac{\frac{e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\int e^{2b^2x^2}dx}{\sqrt{\pi}b}}{b^2}+\frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}(\sqrt{2}bx)}{8b^3}}{\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{e}}{}\right.}{4b^2}}{\sqrt{\pi}b}\right.}{5\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{5}x^5\text{erfi}(bx)^2 - \frac{4b\left(\frac{x^4e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{2\left(\frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\text{erfi}(\sqrt{2}bx)}{2\sqrt{2}b^2}}{b^2}-\frac{\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}(\sqrt{2}bx)}{8b^3}}{\sqrt{\pi}b}\right)}{b^2}-\frac{\frac{x^3e^{2b^2x^2}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}(}{8b^3}\right.}{4b^2}}{\sqrt{\pi}b}\right.}{5\sqrt{\pi}}$$

input
```
Int[x^4*Erfi[b*x]^2,x]
```

output
```
(x^5*Erfi[b*x]^2)/5 - (4*b*((E^(b^2*x^2)*x^4*Erfi[b*x])/(2*b^2) - ((E^(2*b
^2*x^2)*x^3)/(4*b^2) - (3*((E^(2*b^2*x^2)*x)/(4*b^2) - (Sqrt[Pi/2]*Erfi[Sq
rt[2]*b*x])/(8*b^3)))/(4*b^2))/(b*Sqrt[Pi]) - (2*((E^(b^2*x^2)*x^2*Erfi[b*
x])/(2*b^2) - ((E^(b^2*x^2)*Erfi[b*x])/(2*b^2) - Erfi[Sqrt[2]*b*x]/(2*Sqrt
[2]*b^2))/b^2 - ((E^(2*b^2*x^2)*x)/(4*b^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*b*x]
)/(8*b^3))/(b*Sqrt[Pi])))/b^2))/(5*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^4 \operatorname{erfi}(bx)^2 \, dx$$

input
```
int(x^4*erfi(b*x)^2,x)
```

output
```
int(x^4*erfi(b*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.70

$$\int x^4\text{erfi}(bx)^2\,dx$$
$$=\frac{16\,\pi b^6x^5\,\text{erfi}\left(bx\right)^2-32\,\sqrt{\pi}(b^5x^4-2\,b^3x^2+2\,b)\,\text{erfi}\left(bx\right)e^{\left(b^2x^2\right)}-43\,\sqrt{2}\sqrt{\pi}\sqrt{-b^2}\,\text{erf}\left(\sqrt{2}\sqrt{-b^2}x\right)+4\,(4}{80\,\pi b^6}$$

input
```
integrate(x^4*erfi(b*x)^2,x, algorithm="fricas")
```

output
```
1/80*(16*pi*b^6*x^5*erfi(b*x)^2 - 32*sqrt(pi)*(b^5*x^4 - 2*b^3*x^2 + 2*b)*
erfi(b*x)*e^(b^2*x^2) - 43*sqrt(2)*sqrt(pi)*sqrt(-b^2)*erf(sqrt(2)*sqrt(-b
^2)*x) + 4*(4*b^4*x^3 - 11*b^2*x)*e^(2*b^2*x^2))/(pi*b^6)
```

Sympy [F]

$$\int x^4\text{erfi}(bx)^2\,dx=\int x^4\,\text{erfi}^2\left(bx\right)\,dx$$

input
```
integrate(x**4*erfi(b*x)**2,x)
```

output
```
Integral(x**4*erfi(b*x)**2, x)
```

Maxima [F]

$$\int x^4 \text{erfi}(bx)^2 \, dx = \int x^4 \,\text{erfi}\,(bx)^2 \, dx$$

input
```
integrate(x^4*erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4*erfi(b*x)^2, x)
```

Giac [F]

$$\int x^4 \text{erfi}(bx)^2 \, dx = \int x^4 \,\text{erfi}\,(bx)^2 \, dx$$

input
```
integrate(x^4*erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*erfi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{erfi}(bx)^2 \, dx = \int x^4 \,\text{erfi}(b\,x)^2 \, dx$$

input
```
int(x^4*erfi(b*x)^2,x)
```

output
```
int(x^4*erfi(b*x)^2, x)
```

Reduce [F]

$$\int x^4 \text{erfi}(bx)^2 \, dx = -\left(\int \text{erf}(bix)^2 \, x^4 dx\right)$$

input
```
int(x^4*erfi(b*x)^2,x)
```

output
```
- int(erf(b*i*x)**2*x**4,x)
```

3.236 $\int x^2\text{erfi}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int x^2\text{erfi}(bx)^2\,dx=\frac{e^{2b^2x^2}x}{3b^2\pi}+\frac{2e^{b^2x^2}\text{erfi}(bx)}{3b^3\sqrt{\pi}}-\frac{2e^{b^2x^2}x^2\text{erfi}(bx)}{3b\sqrt{\pi}}+\frac{1}{3}x^3\text{erfi}(bx)^2-\frac{5\text{erfi}\left(\sqrt{2}bx\right)}{6b^3\sqrt{2\pi}}$$

output
```
1/3*exp(2*b^2*x^2)*x/b^2/Pi+2/3*exp(b^2*x^2)*erfi(b*x)/b^3/Pi^(1/2)-2/3*ex
p(b^2*x^2)*x^2*erfi(b*x)/b/Pi^(1/2)+1/3*x^3*erfi(b*x)^2-5/12*erfi(2^(1/2)*
b*x)/b^3*2^(1/2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int x^2\text{erfi}(bx)^2\,dx=\frac{4be^{2b^2x^2}x-8e^{b^2x^2}\sqrt{\pi}(-1+b^2x^2)\,\text{erfi}(bx)+4b^3\pi x^3\text{erfi}(bx)^2-5\sqrt{2\pi}\text{erfi}\left(\sqrt{2}bx\right)}{12b^3\pi}$$

input
```
Integrate[x^2*Erfi[b*x]^2,x]
```

output
```
(4*b*E^(2*b^2*x^2)*x - 8*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erfi[b*x] + 4
*b^3*Pi*x^3*Erfi[b*x]^2 - 5*Sqrt[2*Pi]*Erfi[Sqrt[2]*b*x])/(12*b^3*Pi)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6920, 6941, 2641, 2633, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erfi}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6920}$$

$$\frac{1}{3}x^3\text{erfi}(bx)^2 - \frac{4b\int e^{b^2x^2}x^3\text{erfi}(bx)dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 6941}$$

$$\frac{1}{3}x^3\text{erfi}(bx)^2 - \frac{4b\left(-\frac{\int e^{b^2x^2}x\text{erfi}(bx)dx}{b^2} - \frac{\int e^{2b^2x^2}x^2dx}{\sqrt{\pi}b} + \frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{1}{3}x^3\text{erfi}(bx)^2 - \frac{4b\left(-\frac{\int e^{b^2x^2}x\text{erfi}(bx)dx}{b^2} - \frac{\frac{xe^{2b^2x^2}}{4b^2} - \frac{\int e^{2b^2x^2}dx}{4b^2}}{\sqrt{\pi}b} + \frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{1}{3}x^3\text{erfi}(bx)^2 - \frac{4b\left(-\frac{\int e^{b^2x^2}x\text{erfi}(bx)dx}{b^2} + \frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\frac{xe^{2b^2x^2}}{4b^2} - \frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 6938}$$

$$\frac{1}{3}x^3\text{erfi}(bx)^2 - \frac{4b\left(-\frac{\frac{e^{b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\int e^{2b^2x^2}dx}{\sqrt{\pi}b}}{b^2} + \frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\frac{xe^{2b^2x^2}}{4b^2} - \frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{3}x^3\text{erfi}(bx)^2-\frac{4b\left(\frac{x^2e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\frac{e^{b^2x^2}\text{erfi}(bx)}{2b^2}-\frac{\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}}{b^2}-\frac{\frac{xe^{2b^2x^2}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{3\sqrt{\pi}}$$

input
```
Int[x^2*Erfi[b*x]^2,x]
```

output
```
(x^3*Erfi[b*x]^2)/3 - (4*b*((E^(b^2*x^2)*x^2*Erfi[b*x])/(2*b^2) - ((E^(b^2
*x^2)*Erfi[b*x])/(2*b^2) - Erfi[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2))/b^2 - ((E^(2
*b^2*x^2)*x)/(4*b^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi]
)))/(3*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6920
```
Int[Erfi[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Erfi[b*x]^2
/(m + 1)), x] - Simp[4*(b/(Sqrt[Pi]*(m + 1)))   Int[x^(m + 1)*E^(b^2*x^2)*E
rfi[b*x], x], x] /; FreeQ[b, x] && (IGtQ[m, 0] || ILtQ[(m + 1)/2, 0])
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^2\,\mathrm{erfi}\left(bx\right)^2\,dx$$

input
```
int(x^2*erfi(b*x)^2,x)
```

output
```
int(x^2*erfi(b*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int x^2\mathrm{erfi}(bx)^2\,dx = \frac{4\,\pi b^4x^3\,\mathrm{erfi}\left(bx\right)^2+4\,b^2xe^{\left(2\,b^2x^2\right)}-8\,\sqrt{\pi}(b^3x^2-b)\,\mathrm{erfi}\left(bx\right)e^{\left(b^2x^2\right)}+5\,\sqrt{2}\sqrt{\pi}\sqrt{-b^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b^2}x\right)}{12\,\pi b^4}$$

input
```
integrate(x^2*erfi(b*x)^2,x, algorithm="fricas")
```

output
```
1/12*(4*pi*b^4*x^3*erfi(b*x)^2 + 4*b^2*x*e^(2*b^2*x^2) - 8*sqrt(pi)*(b^3*x
^2 - b)*erfi(b*x)*e^(b^2*x^2) + 5*sqrt(2)*sqrt(pi)*sqrt(-b^2)*erf(sqrt(2)*
sqrt(-b^2)*x))/(pi*b^4)
```

Sympy [F]

$$\int x^2 \text{erfi}(bx)^2\,dx = \int x^2\,\text{erfi}^2\,(bx)\,dx$$

input
```
integrate(x**2*erfi(b*x)**2,x)
```

output
```
Integral(x**2*erfi(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \text{erfi}(bx)^2\,dx = \int x^2\,\text{erfi}\,(bx)^2\,dx$$

input
```
integrate(x^2*erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*erfi(b*x)^2, x)
```

Giac [F]

$$\int x^2 \text{erfi}(bx)^2\,dx = \int x^2\,\text{erfi}\,(bx)^2\,dx$$

input
```
integrate(x^2*erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*erfi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{erfi}(bx)^2\,dx = \int x^2\,\text{erfi}(b\,x)^2\,dx$$

input
```
int(x^2*erfi(b*x)^2,x)
```

output
```
int(x^2*erfi(b*x)^2, x)
```

Reduce [F]

$$\int x^2\text{erfi}(bx)^2\,dx = -\left(\int \text{erf}(bix)^2\,x^2dx\right)$$

input
```
int(x^2*erfi(b*x)^2,x)
```

output
```
- int(erf(b*i*x)**2*x**2,x)
```

3.237 $\int \text{erfi}(bx)^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 54

$$\int \text{erfi}(bx)^2 \, dx = -\frac{2e^{b^2x^2}\text{erfi}(bx)}{b\sqrt{\pi}} + x\text{erfi}(bx)^2 + \frac{\sqrt{\frac{2}{\pi}}\text{erfi}\left(\sqrt{2}bx\right)}{b}$$

output
```
-2*exp(b^2*x^2)*erfi(b*x)/b/Pi^(1/2)+x*erfi(b*x)^2+2^(1/2)/Pi^(1/2)*erfi(2
^(1/2)*b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \text{erfi}(bx)^2 \, dx = -\frac{2e^{b^2x^2}\text{erfi}(bx)}{b\sqrt{\pi}} + x\text{erfi}(bx)^2 + \frac{\sqrt{\frac{2}{\pi}}\text{erfi}\left(\sqrt{2}bx\right)}{b}$$

input
```
Integrate[Erfi[b*x]^2,x]
```

output
```
(-2*E^(b^2*x^2)*Erfi[b*x])/(b*Sqrt[Pi]) + x*Erfi[b*x]^2 + (Sqrt[2/Pi]*Erfi
[Sqrt[2]*b*x])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6908, 27, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6908}$$

$$x\text{erfi}(bx)^2 - \frac{4 \int b e^{b^2x^2} x\text{erfi}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 27}$$

$$x\text{erfi}(bx)^2 - \frac{4b \int e^{b^2x^2} x\text{erfi}(bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 6938}$$

$$x\text{erfi}(bx)^2 - \frac{4b\left(\frac{e^{b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\int e^{2b^2x^2}dx}{\sqrt{\pi}b}\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2633}$$

$$x\text{erfi}(bx)^2 - \frac{4b\left(\frac{e^{b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}\right)}{\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]^2,x]
```

output
```
x*Erfi[b*x]^2 - (4*b*((E^(b^2*x^2)*Erfi[b*x])/(2*b^2) - Erfi[Sqrt[2]*b*x]/
(2*Sqrt[2]*b^2)))/Sqrt[Pi]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6908
```
Int[Erfi[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(Erfi[a + b*x]^
2/b), x] - Simp[4/Sqrt[Pi]   Int[(a + b*x)*E^(a + b*x)^2*Erfi[a + b*x], x],
 x] /; FreeQ[{a, b}, x]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int \operatorname{erfi}(bx)^2\,dx$$

input
```
int(erfi(b*x)^2,x)
```

output
```
int(erfi(b*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \operatorname{erfi}(bx)^2\,dx = \frac{\pi b^2 x \operatorname{erfi}(bx)^2 - 2\sqrt{\pi} b \operatorname{erfi}(bx) e^{(b^2x^2)} - \sqrt{2}\sqrt{\pi}\sqrt{-b^2} \operatorname{erf}\left(\sqrt{2}\sqrt{-b^2}x\right)}{\pi b^2}$$

input
```
integrate(erfi(b*x)^2,x, algorithm="fricas")
```

output
```
(pi*b^2*x*erfi(b*x)^2 - 2*sqrt(pi)*b*erfi(b*x)*e^(b^2*x^2) - sqrt(2)*sqrt(
pi)*sqrt(-b^2)*erf(sqrt(2)*sqrt(-b^2)*x))/(pi*b^2)
```

Sympy [F]

$$\int \operatorname{erfi}(bx)^2 \, dx = \int \operatorname{erfi}^2(bx) \, dx$$

input
```
integrate(erfi(b*x)**2,x)
```

output
```
Integral(erfi(b*x)**2, x)
```

Maxima [F]

$$\int \operatorname{erfi}(bx)^2 \, dx = \int \operatorname{erfi}(bx)^2 \, dx$$

input
```
integrate(erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2, x)
```

Giac [F]

$$\int \operatorname{erfi}(bx)^2 \, dx = \int \operatorname{erfi}(bx)^2 \, dx$$

input
```
integrate(erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(bx)^2 \, dx = \int \operatorname{erfi}(b\,x)^2 \, dx$$

input
```
int(erfi(b*x)^2,x)
```

output
```
int(erfi(b*x)^2, x)
```

Reduce [F]

$$\int \operatorname{erfi}(bx)^2 \, dx = -\left(\int \operatorname{erf}(bix)^2 \, dx\right)$$

input
```
int(erfi(b*x)^2,x)
```

output
```
- int(erf(b*i*x)**2,x)
```

3.238 $\int \frac{\text{erfi}(bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfi}(bx)^2}{x^2}\,dx = \text{Int}\left(\frac{\text{erfi}(bx)^2}{x^2},x\right)$$

output
```
Defer(Int)(erfi(b*x)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^2}\,dx = \int \frac{\text{erfi}(bx)^2}{x^2}\,dx$$

input
```
Integrate[Erfi[b*x]^2/x^2,x]
```

output
```
Integrate[Erfi[b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)^2}{x^2} \, dx$$
$$\Big\downarrow \; 6926$$
$$\int \frac{\text{erfi}(bx)^2}{x^2}dx$$

input
```
Int[Erfi[b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}\left(bx\right)^{2}}{x^{2}}dx$$

input
```
int(erfi(b*x)^2/x^2,x)
```

output
```
int(erfi(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^2}\,dx$$

input
```
integrate(erfi(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfi}^2\left(bx\right)}{x^2}\,dx$$

input
```
integrate(erfi(b*x)**2/x**2,x)
```

output
```
Integral(erfi(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^2}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^2}\,dx$$

input
```
integrate(erfi(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)^2}{x^2}\,dx$$

input
```
integrate(erfi(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^2}\,dx = \int \frac{\text{erfi}(b\,x)^2}{x^2}\,dx$$

input
```
int(erfi(b*x)^2/x^2,x)
```

output
```
int(erfi(b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{erfi}(bx)^2}{x^2}\,dx = -\left(\int \frac{\operatorname{erf}(bix)^2}{x^2}dx\right)$$

input
```
int(erfi(b*x)^2/x^2,x)
```

output
```
- int(erf(b*i*x)**2/x**2,x)
```

3.239 $\int \frac{\text{erfi}(bx)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfi}(bx)^2}{x^4}\,dx = \text{Int}\left(\frac{\text{erfi}(bx)^2}{x^4},x\right)$$

output
```
Defer(Int)(erfi(b*x)^2/x^4,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^4}\,dx = \int \frac{\text{erfi}(bx)^2}{x^4}\,dx$$

input
```
Integrate[Erfi[b*x]^2/x^4,x]
```

output
```
Integrate[Erfi[b*x]^2/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)^2}{x^4} \, dx$$
$$\Big\downarrow \text{ 6926}$$
$$\int \frac{\text{erfi}(bx)^2}{x^4}dx$$

input
```
Int[Erfi[b*x]^2/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{erfi}\left(bx\right)^{2}}{x^{4}}dx$$

input
```
int(erfi(b*x)^2/x^4,x)
```

output
```
int(erfi(b*x)^2/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(erfi(b*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)^2/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfi}^2\left(bx\right)}{x^4}\,dx$$

input
```
integrate(erfi(b*x)**2/x**4,x)
```

output
```
Integral(erfi(b*x)**2/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^4}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(erfi(b*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)^2}{x^4}\,dx$$

input
```
integrate(erfi(b*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^4}\,dx = \int \frac{\text{erfi}(b\,x)^2}{x^4}\,dx$$

input
```
int(erfi(b*x)^2/x^4,x)
```

output
```
int(erfi(b*x)^2/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{erfi}(bx)^2}{x^4}\,dx = -\left(\int \frac{\operatorname{erf}(bix)^2}{x^4}dx\right)$$

input
```
int(erfi(b*x)^2/x^4,x)
```

output
```
- int(erf(b*i*x)**2/x**4,x)
```

3.240 $\int \frac{\text{erfi}(bx)^2}{x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{erfi}(bx)^2}{x^6}\,dx = \text{Int}\left(\frac{\text{erfi}(bx)^2}{x^6},x\right)$$

output
```
Defer(Int)(erfi(b*x)^2/x^6,x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{erfi}(bx)^2}{x^6}\,dx = \int \frac{\text{erfi}(bx)^2}{x^6}\,dx$$

input
```
Integrate[Erfi[b*x]^2/x^6,x]
```

output
```
Integrate[Erfi[b*x]^2/x^6, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6} \, dx$$

$$\Big\downarrow \text{ 6926}$$

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}dx$$

input
```
Int[Erfi[b*x]^2/x^6,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}\left(bx\right)^{2}}{x^{6}}dx$$

input
```
int(erfi(b*x)^2/x^6,x)
```

output
```
int(erfi(b*x)^2/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx$$

input
```
integrate(erfi(b*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)^2/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erfi}^2(bx)}{x^6}\,dx$$

input
```
integrate(erfi(b*x)**2/x**6,x)
```

output
```
Integral(erfi(b*x)**2/x**6, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx$$

input
```
integrate(erfi(b*x)^2/x^6,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2/x^6, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erfi}\left(bx\right)^2}{x^6}\,dx$$

input
```
integrate(erfi(b*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2/x^6, x)
```

Mupad [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{erfi}(b\,x)^2}{x^6}\,dx$$

input
```
int(erfi(b*x)^2/x^6,x)
```

output
```
int(erfi(b*x)^2/x^6, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{erfi}(bx)^2}{x^6}\,dx = -\left(\int \frac{\operatorname{erf}(bix)^2}{x^6}dx\right)$$

input
```
int(erfi(b*x)^2/x^6,x)
```

output
```
 - int(erf(b*i*x)**2/x**6,x)
```

3.241 $\int (c+dx)^2 \mathbf{erfi}(a+bx)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 366

$$\begin{aligned}\int (c+dx)^2\text{erfi}(a+bx)^2\,dx &= \frac{d(bc-ad)e^{2(a+bx)^2}}{b^3\pi}+\frac{d^2e^{2(a+bx)^2}(a+bx)}{3b^3\pi} \\ &+\frac{2d^2e^{(a+bx)^2}\text{erfi}(a+bx)}{3b^3\sqrt{\pi}}-\frac{2(bc-ad)^2e^{(a+bx)^2}\text{erfi}(a+bx)}{b^3\sqrt{\pi}} \\ &-\frac{2d(bc-ad)e^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)}{b^3\sqrt{\pi}} \\ &-\frac{2d^2e^{(a+bx)^2}(a+bx)^2\text{erfi}(a+bx)}{3b^3\sqrt{\pi}} \\ &+\frac{d(bc-ad)\text{erfi}(a+bx)^2}{2b^3} \\ &+\frac{(bc-ad)^2(a+bx)\text{erfi}(a+bx)^2}{b^3} \\ &+\frac{d(bc-ad)(a+bx)^2\text{erfi}(a+bx)^2}{b^3} \\ &+\frac{d^2(a+bx)^3\text{erfi}(a+bx)^2}{3b^3} \\ &+\frac{(bc-ad)^2\sqrt{\frac{2}{\pi}}\text{erfi}\left(\sqrt{2}(a+bx)\right)}{b^3}-\frac{5d^2\text{erfi}\left(\sqrt{2}(a+bx)\right)}{6b^3\sqrt{2\pi}}\end{aligned}$$

output
```
d*(-a*d+b*c)*exp(2*(b*x+a)^2)/b^3/Pi+1/3*d^2*exp(2*(b*x+a)^2)*(b*x+a)/b^3/
Pi+2/3*d^2*exp((b*x+a)^2)*erfi(b*x+a)/b^3/Pi^(1/2)-2*(-a*d+b*c)^2*exp((b*x
+a)^2)*erfi(b*x+a)/b^3/Pi^(1/2)-2*d*(-a*d+b*c)*exp((b*x+a)^2)*(b*x+a)*erfi
(b*x+a)/b^3/Pi^(1/2)-2/3*d^2*exp((b*x+a)^2)*(b*x+a)^2*erfi(b*x+a)/b^3/Pi^(
1/2)+1/2*d*(-a*d+b*c)*erfi(b*x+a)^2/b^3+(-a*d+b*c)^2*(b*x+a)*erfi(b*x+a)^2
/b^3+d*(-a*d+b*c)*(b*x+a)^2*erfi(b*x+a)^2/b^3+1/3*d^2*(b*x+a)^3*erfi(b*x+a
)^2/b^3+(-a*d+b*c)^2*2^(1/2)/Pi^(1/2)*erfi(2^(1/2)*(b*x+a))/b^3-5/12*d^2*e
rfi(2^(1/2)*(b*x+a))/b^3*2^(1/2)/Pi^(1/2)
```

Mathematica [F]

$$\int (c+dx)^2 \text{erfi}(a+bx)^2 \, dx = \int (c+dx)^2 \text{erfi}(a+bx)^2 \, dx$$

input
```
Integrate[(c + d*x)^2*Erfi[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^2*Erfi[a + b*x]^2, x]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6923, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \text{erfi}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 6923}$$

$$\frac{\int \left((bc-ad)^2 \text{erfi}(a+bx)^2+d^2 (a+bx)^2 \text{erfi}(a+bx)^2+2d(bc-ad)(a+bx)\text{erfi}(a+bx)^2\right) d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$d(a+bx)^2(bc-ad)\mathrm{erfi}(a+bx)^2+(a+bx)(bc-ad)^2\mathrm{erfi}(a+bx)^2-\frac{2de^{(a+bx)^2}(a+bx)(bc-ad)\mathrm{erfi}(a+bx)}{\sqrt{\pi}}+\frac{1}{2}d(bc-a$$

input
```
Int[(c + d*x)^2*Erfi[a + b*x]^2,x]
```

output
```
((d*(b*c - a*d)*E^(2*(a + b*x)^2))/Pi + (d^2*E^(2*(a + b*x)^2)*(a + b*x))/
(3*Pi) + (2*d^2*E^(a + b*x)^2*Erfi[a + b*x])/(3*Sqrt[Pi]) - (2*(b*c - a*d)
^2*E^(a + b*x)^2*Erfi[a + b*x])/Sqrt[Pi] - (2*d*(b*c - a*d)*E^(a + b*x)^2*
(a + b*x)*Erfi[a + b*x])/Sqrt[Pi] - (2*d^2*E^(a + b*x)^2*(a + b*x)^2*Erfi[
a + b*x])/(3*Sqrt[Pi]) + (d*(b*c - a*d)*Erfi[a + b*x]^2)/2 + (b*c - a*d)^2
*(a + b*x)*Erfi[a + b*x]^2 + d*(b*c - a*d)*(a + b*x)^2*Erfi[a + b*x]^2 + (
d^2*(a + b*x)^3*Erfi[a + b*x]^2)/3 + (b*c - a*d)^2*Sqrt[2/Pi]*Erfi[Sqrt[2]
*(a + b*x)] - (5*d^2*Erfi[Sqrt[2]*(a + b*x)])/(6*Sqrt[2*Pi]))/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6923
```
Int[Erfi[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[1/b^(m + 1)   Subst[Int[ExpandIntegrand[Erfi[x]^2, (b*c - a*d + d*x)^m, x]
, x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)^2 \,\mathrm{erfi}\,(bx+a)^2\, dx$$

input
```
int((d*x+c)^2*erfi(b*x+a)^2,x)
```

output
```
int((d*x+c)^2*erfi(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.77

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}(12\,b^2c^2-24\,abcd+(12\,a^2-5)d^2)\sqrt{-b^2}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{-b^2}(bx+a)}{b}\right)+8\,\sqrt{\pi}(b^3d^2x^2+3\,b^3c^2-3\,ab^2cd\cdots}{\cdots}$$

input
```
integrate((d*x+c)^2*erfi(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/12*(sqrt(2)*sqrt(pi)*(12*b^2*c^2 - 24*a*b*c*d + (12*a^2 - 5)*d^2)*sqrt(
-b^2)*erf(sqrt(2)*sqrt(-b^2)*(b*x + a)/b) + 8*sqrt(pi)*(b^3*d^2*x^2 + 3*b^
3*c^2 - 3*a*b^2*c*d + (a^2 - 1)*b*d^2 + (3*b^3*c*d - a*b^2*d^2)*x)*erfi(b*
x + a)*e^(b^2*x^2 + 2*a*b*x + a^2) - 2*(2*pi*b^4*d^2*x^3 + 6*pi*b^4*c*d*x^
2 + 6*pi*b^4*c^2*x + pi*(6*a*b^3*c^2 - 3*(2*a^2 - 1)*b^2*c*d + (2*a^3 - 3*
a)*b*d^2))*erfi(b*x + a)^2 - 4*(b^2*d^2*x + 3*b^2*c*d - 2*a*b*d^2)*e^(2*b^
2*x^2 + 4*a*b*x + 2*a^2))/(pi*b^4)
```

Sympy [F]

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx = \int (c+dx)^2\,\text{erfi}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*erfi(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*erfi(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx=\int (dx+c)^2\,\text{erfi}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*erfi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*erfi(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx=\int (dx+c)^2\,\text{erfi}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*erfi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*erfi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx=\int \text{erfi}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(erfi(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(erfi(a + b*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\text{erfi}(a+bx)^2\,dx = -\left(\int \text{erf}(bix+ai)^2\,dx\right)c^2 - \left(\int \text{erf}(bix+ai)^2\,x^2dx\right)d^2 - 2\left(\int \text{erf}(bix+ai)^2\,xdx\right)cd$$

input
```
int((d*x+c)^2*erfi(b*x+a)^2,x)
```

output
```
 - int(erf(a*i + b*i*x)**2,x)*c**2 - int(erf(a*i + b*i*x)**2*x**2,x)*d**2
- 2*int(erf(a*i + b*i*x)**2*x,x)*c*d
```

3.242 $\int (c+dx)\text{erfi}(a+bx)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 184

$$\int (c+dx)\text{erfi}(a+bx)^2\,dx=\frac{de^{2(a+bx)^2}}{2b^2\pi}-\frac{2(bc-ad)e^{(a+bx)^2}\text{erfi}(a+bx)}{b^2\sqrt{\pi}}-\frac{de^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)}{b^2\sqrt{\pi}}+\frac{d\text{erfi}(a+bx)^2}{4b^2}+\frac{(bc-ad)(a+bx)\text{erfi}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{erfi}(a+bx)^2}{2b^2}+\frac{(bc-ad)\sqrt{\frac{2}{\pi}}\text{erfi}\left(\sqrt{2}(a+bx)\right)}{b^2}$$

output

```
1/2*d*exp(2*(b*x+a)^2)/b^2/Pi-2*(-a*d+b*c)*exp((b*x+a)^2)*erfi(b*x+a)/b^2/
Pi^(1/2)-d*exp((b*x+a)^2)*(b*x+a)*erfi(b*x+a)/b^2/Pi^(1/2)+1/4*d*erfi(b*x+
a)^2/b^2+(-a*d+b*c)*(b*x+a)*erfi(b*x+a)^2/b^2+1/2*d*(b*x+a)^2*erfi(b*x+a)^
2/b^2+(-a*d+b*c)*2^(1/2)/Pi^(1/2)*erfi(2^(1/2)*(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

$$\int (c+dx)\text{erfi}(a+bx)^2\,dx$$
$$= \frac{2de^{2(a+bx)^2}-4e^{(a+bx)^2}\sqrt{\pi}(2bc-ad+bdx)\text{erfi}(a+bx)+\pi(4abc+d-2a^2d+4b^2cx+2b^2dx^2)\,\text{erfi}(a+b}{4b^2\pi}$$

input
```
Integrate[(c + d*x)*Erfi[a + b*x]^2,x]
```

output
```
(2*d*E^(2*(a + b*x)^2) - 4*E^(a + b*x)^2*Sqrt[Pi]*(2*b*c - a*d + b*d*x)*Er
fi[a + b*x] + Pi*(4*a*b*c + d - 2*a^2*d + 4*b^2*c*x + 2*b^2*d*x^2)*Erfi[a
+ b*x]^2 + 4*(b*c - a*d)*Sqrt[2*Pi]*Erfi[Sqrt[2]*(a + b*x)])/(4*b^2*Pi)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6923, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{erfi}(a+bx)^2\,dx$$
$$\Big\downarrow \; 6923$$
$$\frac{\int \left((bc-ad)\text{erfi}(a+bx)^2+d(a+bx)\text{erfi}(a+bx)^2\right)d(a+bx)}{b^2}$$
$$\Big\downarrow \; 2009$$
$$\frac{(a+bx)(bc-ad)\text{erfi}(a+bx)^2-\frac{2e^{(a+bx)^2}(bc-ad)\text{erfi}(a+bx)}{\sqrt{\pi}}+\sqrt{\frac{2}{\pi}}(bc-ad)\text{erfi}\left(\sqrt{2}(a+bx)\right)+\frac{1}{2}d(a+bx)^2\text{erfi}(a+}{b^2}$$

input
```
Int[(c + d*x)*Erfi[a + b*x]^2,x]
```

output
```
((d*E^(2*(a + b*x)^2))/(2*Pi) - (2*(b*c - a*d)*E^(a + b*x)^2*Erfi[a + b*x]
)/Sqrt[Pi] - (d*E^(a + b*x)^2*(a + b*x)*Erfi[a + b*x])/Sqrt[Pi] + (d*Erfi[
a + b*x]^2)/4 + (b*c - a*d)*(a + b*x)*Erfi[a + b*x]^2 + (d*(a + b*x)^2*Erf
i[a + b*x]^2)/2 + (b*c - a*d)*Sqrt[2/Pi]*Erfi[Sqrt[2]*(a + b*x)])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6923
```
Int[Erfi[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[1/b^(m + 1)   Subst[Int[ExpandIntegrand[Erfi[x]^2, (b*c - a*d + d*x)^m, x]
, x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)\,\mathrm{erfi}\left(bx+a\right)^2\,dx$$

input
```
int((d*x+c)*erfi(b*x+a)^2,x)
```

output
```
int((d*x+c)*erfi(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int (c+dx)\mathrm{erfi}(a+bx)^2\,dx =$$
$$-\frac{4\sqrt{2}\sqrt{\pi}\sqrt{-b^2}(bc-ad)\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{-b^2}(bx+a)}{b}\right)+4\sqrt{\pi}(b^2dx+2\,b^2c-abd)\,\mathrm{erfi}\left(bx+a\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-(}{4\,\pi b^3}$$

input
```
integrate((d*x+c)*erfi(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(2)*sqrt(pi)*sqrt(-b^2)*(b*c - a*d)*erf(sqrt(2)*sqrt(-b^2)*(b*
x + a)/b) + 4*sqrt(pi)*(b^2*d*x + 2*b^2*c - a*b*d)*erfi(b*x + a)*e^(b^2*x^
2 + 2*a*b*x + a^2) - (2*pi*b^3*d*x^2 + 4*pi*b^3*c*x + pi*(4*a*b^2*c - (2*a
^2 - 1)*b*d))*erfi(b*x + a)^2 - 2*b*d*e^(2*b^2*x^2 + 4*a*b*x + 2*a^2))/(pi
*b^3)
```

Sympy [F]

$$\int (c+dx)\text{erfi}(a+bx)^2\,dx=\int (c+dx)\,\text{erfi}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*erfi(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*erfi(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)\text{erfi}(a+bx)^2\,dx=\int (dx+c)\,\text{erfi}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*erfi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*erfi(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)\mathrm{erfi}(a+bx)^2\,dx=\int (dx+c)\,\mathrm{erfi}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*erfi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*erfi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\mathrm{erfi}(a+bx)^2\,dx=\int \mathrm{erfi}(a+b\,x)^2\,(c+d\,x)\,dx$$

input
```
int(erfi(a + b*x)^2*(c + d*x),x)
```

output
```
int(erfi(a + b*x)^2*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\mathrm{erfi}(a+bx)^2\,dx=-\left(\int \mathrm{erf}(bix+ai)^2\,dx\right)c-\left(\int \mathrm{erf}(bix+ai)^2\,xdx\right)d$$

input
```
int((d*x+c)*erfi(b*x+a)^2,x)
```

output
```
- (int(erf(a*i + b*i*x)**2,x)*c + int(erf(a*i + b*i*x)**2*x,x)*d)
```

3.243 $\int \text{erfi}(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 68

$$\int \text{erfi}(a+bx)^2\,dx=-\frac{2e^{(a+bx)^2}\text{erfi}(a+bx)}{b\sqrt{\pi}}+\frac{(a+bx)\text{erfi}(a+bx)^2}{b}+\frac{\sqrt{\frac{2}{\pi}}\text{erfi}\left(\sqrt{2}(a+bx)\right)}{b}$$

output
```
-2*exp((b*x+a)^2)*erfi(b*x+a)/b/Pi^(1/2)+(b*x+a)*erfi(b*x+a)^2/b+2^(1/2)/P
i^(1/2)*erfi(2^(1/2)*(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \text{erfi}(a+bx)^2\,dx =\frac{-2e^{(a+bx)^2}\text{erfi}(a+bx)+\sqrt{\pi}(a+bx)\text{erfi}(a+bx)^2+\sqrt{2}\text{erfi}\left(\sqrt{2}(a+bx)\right)}{b\sqrt{\pi}}$$

input
```
Integrate[Erfi[a + b*x]^2,x]
```

output
```
(-2*E^(a + b*x)^2*Erfi[a + b*x] + Sqrt[Pi]*(a + b*x)*Erfi[a + b*x]^2 + Sqr
t[2]*Erfi[Sqrt[2]*(a + b*x)])/(b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6908, 7281, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 6908}$$

$$\frac{(a+bx)\text{erfi}(a+bx)^2}{b} - \frac{4\int e^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{(a+bx)\text{erfi}(a+bx)^2}{b} - \frac{4\int e^{(a+bx)^2}(a+bx)\text{erfi}(a+bx)d(a+bx)}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 6938}$$

$$\frac{(a+bx)\text{erfi}(a+bx)^2}{b} - \frac{4\left(\frac{1}{2}e^{(a+bx)^2}\text{erfi}(a+bx) - \frac{\int e^{2(a+bx)^2}d(a+bx)}{\sqrt{\pi}}\right)}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{(a+bx)\text{erfi}(a+bx)^2}{b} - \frac{4\left(\frac{1}{2}e^{(a+bx)^2}\text{erfi}(a+bx) - \frac{\text{erfi}\left(\sqrt{2}(a+bx)\right)}{2\sqrt{2}}\right)}{\sqrt{\pi}b}$$

input
```
Int[Erfi[a + b*x]^2,x]
```

output
```
((a + b*x)*Erfi[a + b*x]^2)/b - (4*((E^(a + b*x)^2*Erfi[a + b*x])/2 - Erfi
[Sqrt[2]*(a + b*x)]/(2*Sqrt[2])))/(b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6908
```
Int[Erfi[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(Erfi[a + b*x]^
2/b), x] - Simp[4/Sqrt[Pi]   Int[(a + b*x)*E^(a + b*x)^2*Erfi[a + b*x], x],
 x] /; FreeQ[{a, b}, x]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \operatorname{erfi}(bx+a)^2\,dx$$

input
```
int(erfi(b*x+a)^2,x)
```

output
```
int(erfi(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \operatorname{erfi}(a+bx)^2\,dx = -\frac{2\sqrt{\pi}b\operatorname{erfi}(bx+a)\,e^{(b^2x^2+2\,abx+a^2)} - (\pi b^2x+\pi ab)\operatorname{erfi}(bx+a)^2 + \sqrt{2}\sqrt{\pi}\sqrt{-b^2}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{-b^2}(bx+a)}{b}\right)}{\pi b^2}$$

input
```
integrate(erfi(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*sqrt(pi)*b*erfi(b*x + a)*e^(b^2*x^2 + 2*a*b*x + a^2) - (pi*b^2*x + pi*
a*b)*erfi(b*x + a)^2 + sqrt(2)*sqrt(pi)*sqrt(-b^2)*erf(sqrt(2)*sqrt(-b^2)*
(b*x + a)/b))/(pi*b^2)
```

Sympy [F]

$$\int \operatorname{erfi}(a+bx)^2\,dx=\int \operatorname{erfi}^2(a+bx)\,dx$$

input
```
integrate(erfi(b*x+a)**2,x)
```

output
```
Integral(erfi(a + b*x)**2, x)
```

Maxima [F]

$$\int \operatorname{erfi}(a+bx)^2\,dx=\int \operatorname{erfi}(bx+a)^2\,dx$$

input
```
integrate(erfi(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)^2, x)
```

Giac [F]

$$\int \operatorname{erfi}(a+bx)^2\,dx = \int \operatorname{erfi}\left(bx+a\right)^2\,dx$$

input
```
integrate(erfi(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(a+bx)^2\,dx = \int \operatorname{erfi}(a+b\,x)^2\,dx$$

input
```
int(erfi(a + b*x)^2,x)
```

output
```
int(erfi(a + b*x)^2, x)
```

Reduce [F]

$$\int \operatorname{erfi}(a+bx)^2\,dx = -\left(\int \operatorname{erf}(bix+ai)^2\,dx\right)$$

input
```
int(erfi(b*x+a)^2,x)
```

output
```
- int(erf(a*i + b*i*x)**2,x)
```

3.244 $\int \frac{\text{erfi}(a+bx)^2}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{erfi}(a+bx)^2}{c+dx} \, dx = \text{Int}\left(\frac{\text{erfi}(a+bx)^2}{c+dx},x\right)$$

output
```
Defer(Int)(erfi(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfi}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{erfi}(a+bx)^2}{c+dx} \, dx$$

input
```
Integrate[Erfi[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Erfi[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{erfi}(a+bx)^2}{c+dx} \, dx$$
$$\Big\downarrow \text{ 6926}$$
$$\int \frac{\operatorname{erfi}(a+bx)^2}{c+dx}dx$$

input
```
Int[Erfi[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}\left(bx+a\right)^{2}}{dx+c}dx$$

input
```
int(erfi(b*x+a)^2/(d*x+c),x)
```

output
```
int(erfi(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erfi}(a+bx)^2}{c+dx}\,dx = \int \frac{\operatorname{erfi}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erfi(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{erfi}(a+bx)^2}{c+dx}\,dx = \int \frac{\operatorname{erfi}^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(erfi(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(erfi(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erfi}(a+bx)^2}{c+dx}\,dx = \int \frac{\operatorname{erfi}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erfi(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)^2/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfi}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{erfi}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(erfi(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfi}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{erfi}(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(erfi(a + b*x)^2/(c + d*x),x)
```

output
```
int(erfi(a + b*x)^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{erfi}(a+bx)^2}{c+dx}\,dx = -\left(\int \frac{\operatorname{erf}(bix+ai)^2}{dx+c}dx\right)$$

input
```
int(erfi(b*x+a)^2/(d*x+c),x)
```

output
```
- int(erf(a*i + b*i*x)**2/(c + d*x),x)
```

3.245 $\int \frac{\mathbf{erfi}(a+bx)^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\text{erfi}(a+bx)^2}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(erfi(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx$$

input
```
Integrate[Erfi[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Erfi[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 6926$$

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}dx$$

input
```
Int[Erfi[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{erfi}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(erfi(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(erfi(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{erfi}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfi(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 15.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{erfi}^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(erfi(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(erfi(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{erfi}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfi(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)^2/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erfi}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\operatorname{erfi}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(erfi(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{erfi}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\operatorname{erfi}(a+b\,x)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(erfi(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(erfi(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{\text{erfi}(a+bx)^2}{(c+dx)^2}\,dx = -\left(\int \frac{\text{erf}(bix+ai)^2}{d^2x^2+2cdx+c^2}dx\right)$$

input
```
int(erfi(b*x+a)^2/(d*x+c)^2,x)
```

output
```
- int(erf(a*i + b*i*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.246 $\int x^2 \mathbf{erfi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int x^2\text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{1}{3}x^3\text{erfi}(d(a+b\log(cx^n))) - \frac{1}{3}e^{-\frac{3(3+4abd^2n)}{4b^2d^2n^2}}x^3(cx^n)^{-3/n}\text{erfi}\left(\frac{2abd^2+\frac{3}{n}+2b^2d^2\log(cx^n)}{2bd}\right)$$

output
```
1/3*x^3*erfi(d*(a+b*ln(c*x^n)))-1/3*x^3*erfi(1/2*(2*a*b*d^2+3/n+2*b^2*d^2*
ln(c*x^n))/b/d)/exp(3/4*(4*a*b*d^2*n+3)/b^2/d^2/n^2)/((c*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int x^2\text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{1}{3}\left(x^3\text{erfi}(d(a+b\log(cx^n))) - e^{-\frac{3(3+4abd^2n)}{4b^2d^2n^2}}x^3(cx^n)^{-3/n}\text{erfi}\left(ad+\frac{3}{2bdn}+bd\log(cx^n)\right)\right)$$

input
```
Integrate[x^2*Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*Erfi[d*(a + b*Log[c*x^n])] - (x^3*Erfi[a*d + 3/(2*b*d*n) + b*d*Log[c*
x^n]])/(E^((3*(3 + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(c*x^n)^(3/n)))/3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6957, 2712, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erfi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{ 6957}$$

$$\frac{1}{3}x^3\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdn\int e^{d^2(a+b\log(cx^n))^2}x^2dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2712}$$

$$\frac{1}{3}x^3\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdnx^{-2abd^2n}(cx^n)^{2abd^2}\int e^{a^2d^2+b^2\log^2(cx^n)d^2}x^{2abnd^2+2}dx}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2706}$$

$$\frac{1}{3}x^3\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdx^3(cx^n)^{-3/n}\int \exp\left(a^2d^2+b^2\log^2(cx^n)d^2+\frac{(2abnd^2+3)\log(cx^n)}{n}\right)d\log(cx^n)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{1}{3}x^3\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdx^3(cx^n)^{-3/n}e^{-\frac{3(4abd^2n+3)}{4b^2d^2n^2}}\int \exp\left(\frac{\left(2abd^2+2b^2\log(cx^n)d^2+\frac{3}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{1}{3}x^3\text{erfi}(d(a+b\log(cx^n)))-\frac{1}{3}x^3(cx^n)^{-3/n}e^{-\frac{3\left(4abd^2n+3\right)}{4b^2d^2n^2}}\text{erfi}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{3}{n}}{2bd}\right)$$

input
```
Int[x^2*Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*Erfi[d*(a + b*Log[c*x^n])])/3 - (x^3*Erfi[(2*a*b*d^2 + 3/n + 2*b^2*d^
2*Log[c*x^n])/(2*b*d)])/(3*E^((3*(3 + 4*a*b*d^2*n))/(4*b^2*d^2*n^2))*(c*x^
n)^(3/n))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6957
```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfi[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 - Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^2 \operatorname{erfi}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int(x^2*erfi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*erfi(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.25

$$\int x^2\operatorname{erfi}(d(a+b\log\left(cx^n\right)))\;dx = \frac{1}{3}\,x^3\operatorname{erfi}\left(bd\log\left(cx^n\right)+ad\right) + \frac{1}{3}\,\sqrt{-b^2d^2n^2}\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log\left(x\right)+2\,b^2d^2n\log\left(c\right)+2\,abd^2n+3\right)\sqrt{-b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{3\,\left(4\,b^2d^2n\log(c)+4\,abd^2n+\right.}{4\,b^2d^2n^2}\right.}$$

input
```
integrate(x^2*erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/3*x^3*erfi(b*d*log(c*x^n) + a*d) + 1/3*sqrt(-b^2*d^2*n^2)*erf(1/2*(2*b^2
*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n + 3)*sqrt(-b^2*d^2*n^2)
/(b^2*d^2*n^2))*e^(-3/4*(4*b^2*d^2*n*log(c) + 4*a*b*d^2*n + 3)/(b^2*d^2*n^
2))
```

Sympy [F]

$$\int x^2 \mathrm{erfi}(d(a+b\log(cx^n)))\,dx = \int x^2 \operatorname{erfi}(ad+bd\log(cx^n))\,dx$$

input
```
integrate(x**2*erfi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*erfi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \mathrm{erfi}(d(a+b\log(cx^n)))\,dx = \int x^2 \operatorname{erfi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*erfi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2 \mathrm{erfi}(d(a+b\log(cx^n)))\,dx = \int x^2 \operatorname{erfi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*erfi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \int x^2\operatorname{erfi}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x^2*erfi(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*erfi(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2 \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = -\left(\int \operatorname{erf}(\log(x^n c)\,bdi+adi)\,x^2dx\right)i$$

input
```
int(x^2*erfi(d*(a+b*log(c*x^n))),x)
```

output
```
- int(erf(log(x**n*c)*b*d*i + a*d*i)*x**2,x)*i
```

3.247 $\int x\text{erfi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n)))-\frac{1}{2}e^{-\frac{1+2abd^2n}{b^2d^2n^2}}x^2(cx^n)^{-2/n}\text{erfi}\left(\frac{abd^2+\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right)$$

output
```
1/2*x^2*erfi(d*(a+b*ln(c*x^n)))-1/2*x^2*erfi((a*b*d^2+1/n+b^2*d^2*ln(c*x^n
))/b/d)/exp((2*a*b*d^2*n+1)/b^2/d^2/n^2)/((c*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{1}{2}\left(x^2\text{erfi}(d(a+b\log(cx^n))) - e^{-\frac{\frac{\frac{1}{d^2}+2abn}{b^2}+2n\log(cx^n)}{n^2}}x^2\text{erfi}\left(ad+\frac{1}{bdn}+bd\log(cx^n)\right)\right)$$

input
```
Integrate[x*Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*Erfi[d*(a + b*Log[c*x^n])] - (x^2*Erfi[a*d + 1/(b*d*n) + b*d*Log[c*x^
n]])/E^(((d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n])/n^2))/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6957, 2712, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6957}$$

$$\frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n)))-\frac{bdn\int e^{d^2(a+b\log(cx^n))^2}xdx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2712}$$

$$\frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n)))-\frac{bdnx^{-2abd^2n}(cx^n)^{2abd^2}\int e^{a^2d^2+b^2\log^2(cx^n)d^2}x^{2abnd^2+1}dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2706}$$

$$\frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n)))-\frac{bdx^2(cx^n)^{2abd^2-\frac{2\left(abd^2n+1\right)}{n}}\int \exp\left(a^2d^2+b^2\log^2(cx^n)d^2+\frac{2\left(abnd^2+1\right)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{2}x^2\text{erfi}(d(a+b\log(cx^n)))-\frac{bdx^2e^{-\frac{2abd^2n+1}{b^2d^2n^2}}(cx^n)^{2abd^2-\frac{2\left(abd^2n+1\right)}{n}}\int \exp\left(\frac{\left(abd^2+b^2\log(cx^n)d^2+\frac{1}{n}\right)^2}{b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}x^2\operatorname{erfi}(d(a+b\log(cx^n)))-$$
$$\frac{1}{2}x^2e^{-\frac{2abd^2n+1}{b^2d^2n^2}}(cx^n)^{2abd^2-\frac{2(abd^2n+1)}{n}}\operatorname{erfi}\left(\frac{abd^2+b^2d^2\log(cx^n)+\frac{1}{n}}{bd}\right)$$

input
```
Int[x*Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*Erfi[d*(a + b*Log[c*x^n])])/2 - (x^2*(c*x^n)^(2*a*b*d^2 - (2*(1 + a*b
*d^2*n))/n)*Erfi[(a*b*d^2 + n^(-1) + b^2*d^2*Log[c*x^n])/(b*d)])/(2*E^((1
+ 2*a*b*d^2*n)/(b^2*d^2*n^2)))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6957
```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfi[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 - Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x \,\mathrm{erfi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(x*erfi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*erfi(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

$$\int x\mathrm{erfi}(d(a+b\log\left(cx^{n}\right)))\,dx=\frac{1}{2}\,x^{2}\,\mathrm{erfi}\left(bd\log\left(cx^{n}\right)+ad\right)+\frac{1}{2}\,\sqrt{-b^{2}d^{2}n^{2}}\,\mathrm{erf}\left(\frac{\left(b^{2}d^{2}n^{2}\log\left(x\right)+b^{2}d^{2}n\log\left(c\right)+abd^{2}n+1\right)\sqrt{-b^{2}d^{2}n^{2}}}{b^{2}d^{2}n^{2}}\right)e^{\left(-\frac{2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n+1}{b^{2}d^{2}n^{2}}\right)}$$

input
```
integrate(x*erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/2*x^2*erfi(b*d*log(c*x^n) + a*d) + 1/2*sqrt(-b^2*d^2*n^2)*erf((b^2*d^2*n
^2*log(x) + b^2*d^2*n*log(c) + a*b*d^2*n + 1)*sqrt(-b^2*d^2*n^2)/(b^2*d^2*
n^2))*e^(-(2*b^2*d^2*n*log(c) + 2*a*b*d^2*n + 1)/(b^2*d^2*n^2))
```

Sympy [F]

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx=\int x\,\text{erfi}\,(ad+bd\log(cx^n))\,dx$$

input
```
integrate(x*erfi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*erfi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx=\int x\,\text{erfi}\,((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x*erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*erfi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\text{erfi}(d(a+b\log(cx^n)))\,dx=\int x\,\text{erfi}\,((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x*erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*erfi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{erfi}(d(a+b\log(cx^n)))\,dx=\int x\,\mathrm{erfi}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x*erfi(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*erfi(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\mathrm{erfi}(d(a+b\log(cx^n)))\,dx=-\left(\int \mathrm{erf}(\log(x^nc)\,bdi+adi)\,xdx\right)i$$

input
```
int(x*erfi(d*(a+b*log(c*x^n))),x)
```

output
```
- int(erf(log(x**n*c)*b*d*i + a*d*i)*x,x)*i
```

3.248 $\int \mathrm{erfi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \mathrm{erfi}(d(a+b\log(cx^n)))\,dx = x\mathrm{erfi}(d(a+b\log(cx^n))) - e^{-\frac{1+4abd^2n}{4b^2d^2n^2}}x(cx^n)^{-1/n}\mathrm{erfi}\left(\frac{2abd^2+\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)$$

output
```
x*erfi(d*(a+b*ln(c*x^n)))-x*erfi(1/2*(2*a*b*d^2+1/n+2*b^2*d^2*ln(c*x^n))/b
/d)/exp(1/4*(4*a*b*d^2*n+1)/b^2/d^2/n^2)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \mathrm{erfi}(d(a+b\log(cx^n)))\,dx = x\mathrm{erfi}(d(a+b\log(cx^n))) - e^{-\frac{\frac{\frac{1}{d^2}+4abn}{b^2}+4n\log(cx^n)}{4n^2}}x\mathrm{erfi}\left(ad+\frac{1}{2bdn}+bd\log(cx^n)\right)$$

input
```
Integrate[Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
x*Erfi[d*(a + b*Log[c*x^n])] - (x*Erfi[a*d + 1/(2*b*d*n) + b*d*Log[c*x^n]]
)/E^(((d^(-2) + 4*a*b*n)/b^2 + 4*n*Log[c*x^n])/(4*n^2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6953, 2710, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6953}$$

$$x\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdn\int e^{d^2(a+b\log(cx^n))^2}dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2710}$$

$$x\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdnx^{-2abd^2n}(cx^n)^{2abd^2}\int e^{a^2d^2+b^2\log^2(cx^n)d^2}x^{2abd^2n}dx}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2706}$$

$$x\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdx(cx^n)^{-1/n}\int \exp\left(a^2d^2+b^2\log^2(cx^n)d^2+\frac{(2abnd^2+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2664}$$

$$x\text{erfi}(d(a+b\log(cx^n)))-\frac{2bdx(cx^n)^{-1/n}e^{-\frac{4abd^2n+1}{4b^2d^2n^2}}\int \exp\left(\frac{\left(2abd^2+2b^2\log(cx^n)d^2+\frac{1}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{2633}$$

$$x\text{erfi}(d(a+b\log(cx^n)))-x(cx^n)^{-1/n}e^{-\frac{4abd^2n+1}{4b^2d^2n^2}}\text{erfi}\left(\frac{2abd^2+2b^2d^2\log(cx^n)+\frac{1}{n}}{2bd}\right)$$

input
```
Int[Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
x*Erfi[d*(a + b*Log[c*x^n])] - (x*Erfi[(2*a*b*d^2 + n^(-1) + 2*b^2*d^2*Log
[c*x^n])/(2*b*d)])/(E^((1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 6953
```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*
Erfi[d*(a + b*Log[c*x^n])], x] - Simp[2*b*d*(n/Sqrt[Pi])   Int[E^(d*(a + b*
Log[c*x^n]))^2, x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Maple [F]

$$\int \operatorname{erfi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(erfi(d*(a+b*ln(c*x^n))),x)
```

output
```
int(erfi(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \sqrt{-b^2d^2n^2}\,\operatorname{erf}\left(\frac{\left(2\,b^2d^2n^2\log(x)+2\,b^2d^2n\log(c)+2\,abd^2n+1\right)\sqrt{-b^2d^2n^2}}{2\,b^2d^2n^2}\right)e^{\left(-\frac{4\,b^2d^2n\log(c)+4\,abd^2n+1}{4\,b^2d^2n^2}\right)} + x\operatorname{erfi}\left(bd\log(cx^n)+ad\right)$$

input
```
integrate(erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
sqrt(-b^2*d^2*n^2)*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*
a*b*d^2*n + 1)*sqrt(-b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(-1/4*(4*b^2*d^2*n*log(
c) + 4*a*b*d^2*n + 1)/(b^2*d^2*n^2)) + x*erfi(b*d*log(c*x^n) + a*d)
```

Sympy [F]

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfi}\left(d(a+b\log(cx^n))\right)dx$$

input
```
integrate(erfi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(erfi(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfi}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = \int \operatorname{erfi}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(erfi(d*(a + b*log(c*x^n))),x)
```

output
```
int(erfi(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \operatorname{erfi}(d(a+b\log(cx^n)))\,dx = -\left(\int \operatorname{erf}(\log(x^n c)\,bdi + adi)\,dx\right)i$$

input
```
int(erfi(d*(a+b*log(c*x^n))),x)
```

output
```
- int(erf(log(x**n*c)*b*d*i + a*d*i),x)*i
```

3.249 $\int \frac{\mathbf{erfi}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x}\,dx = -\frac{e^{(ad+bd\log(cx^n))^2}}{bdn\sqrt{\pi}}+\frac{\text{erfi}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bn}$$

output
```
-exp((a*d+b*d*ln(c*x^n))^2)/b/d/n/Pi^(1/2)+erfi(d*(a+b*ln(c*x^n)))*(a+b*ln
(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x}\,dx = \frac{-e^{d^2(a^2+b^2\log^2(cx^n))}(cx^n)^{2abd^2}+d\sqrt{\pi}\text{erfi}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bdn\sqrt{\pi}}$$

input
```
Integrate[Erfi[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-(E^(d^2*(a^2 + b^2*Log[c*x^n]^2))*(c*x^n)^(2*a*b*d^2)) + d*Sqrt[Pi]*Erfi
[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*d*n*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3039, 7281, 6905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \text{erfi}(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int \text{erfi}(ad+b\log(cx^n)d)d(ad+b\log(cx^n)d)}{bdn}$$

$$\Big\downarrow \text{ 6905}$$

$$\frac{(ad+bd\log(cx^n))\text{erfi}(ad+bd\log(cx^n)) - \frac{e^{(ad+bd\log(cx^n))^2}}{\sqrt{\pi}}}{bdn}$$

input
```
Int[Erfi[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(-(E^(a*d + b*d*Log[c*x^n])^2/Sqrt[Pi]) + Erfi[a*d + b*d*Log[c*x^n]]*(a*d
+ b*d*Log[c*x^n]))/(b*d*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 6905
```
Int[Erfi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erfi[a + b*x]/b)
, x] - Simp[E^(a + b*x)^2/(b*Sqrt[Pi]), x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{(ad+bd\ln(c\,x^n))\,\mathrm{erfi}(ad+bd\ln(c\,x^n))-\frac{\mathrm{e}^{(ad+bd\ln(c\,x^n))^2}}{\sqrt{\pi}}}{nbd}$
default	$\frac{(ad+bd\ln(c\,x^n))\,\mathrm{erfi}(ad+bd\ln(c\,x^n))-\frac{\mathrm{e}^{(ad+bd\ln(c\,x^n))^2}}{\sqrt{\pi}}}{nbd}$
parts	$\ln(x)\,\mathrm{erfi}\left(d(a+b\ln(c\,x^n))\right)-\frac{2dbn\left(\frac{\mathrm{e}^{\ln(x)^2 b^2 d^2 n^2+2d^2(b(\ln(c\,x^n)-n\ln(x))+a)bn\ln(x)+d^2(b(\ln(c\,x^n)-n\ln(}}{2b^2d^2n^2}}{}$

input
```
int(erfi(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b/d*((a*d+b*d*ln(c*x^n))*erfi(a*d+b*d*ln(c*x^n))-1/Pi^(1/2)*exp((a*d+b
*d*ln(c*x^n))^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x}\,dx$$
$$= \frac{(\pi bdn\log(x)+\pi bd\log(c)+\pi ad)\operatorname{erfi}(bd\log(cx^n)+ad)-\sqrt{\pi}e^{\left(b^2d^2n^2\log(x)^2+b^2d^2\log(c)^2+2\,abd^2\log(c)+a^2d^2+2\right.}}{\pi bdn}$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*erfi(b*d*log(c*x^n) + a*d) - s
qrt(pi)*e^(b^2*d^2*n^2*log(x)^2 + b^2*d^2*log(c)^2 + 2*a*b*d^2*log(c) + a^
2*d^2 + 2*(b^2*d^2*n*log(c) + a*b*d^2*n)*log(x)))/(pi*b*d*n)
```

Sympy [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\operatorname{erfi}(ad+bd\log(cx^n))}{x}\,dx$$

input
```
integrate(erfi(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(erfi(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x}\,dx$$
$$= \frac{(b\log(cx^n)+a)d\operatorname{erfi}((b\log(cx^n)+a)d)-\frac{e^{\left((b\log(cx^n)+a)^2d^2\right)}}{\sqrt{\pi}}}{bdn}$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
((b*log(c*x^n) + a)*d*erfi((b*log(c*x^n) + a)*d) - e^((b*log(c*x^n) + a)^2
*d^2)/sqrt(pi))/(b*d*n)
```

Giac [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x}\,dx=\int \frac{\operatorname{erfi}\left((b\log(cx^n)+a)d\right)}{x}\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x}\,dx=\frac{\ln(c\,x^n)\ \operatorname{erfi}(a\,d+b\,d\,\ln(c\,x^n))}{n}+\frac{a\,d\operatorname{erfi}\left(a\,\sqrt{d^2}+b\,\ln(c\,x^n)\ \sqrt{d^2}\right)}{b\,n\,\sqrt{d^2}}-\frac{\mathrm{e}^{b^2\,d^2\,\ln(c\,x^n)^2}\,\mathrm{e}^{a^2\,d^2}\,(c\,x^n)^{2\,a\,b\,d^2}}{b\,d\,n\,\sqrt{\pi}}$$

input
```
int(erfi(d*(a + b*log(c*x^n)))/x,x)
```

output
```
(log(c*x^n)*erfi(a*d + b*d*log(c*x^n)))/n + (a*d*erfi(a*(d^2)^(1/2) + b*lo
g(c*x^n)*(d^2)^(1/2)))/(b*n*(d^2)^(1/2)) - (exp(b^2*d^2*log(c*x^n)^2)*exp(
a^2*d^2)*(c*x^n)^(2*a*b*d^2))/(b*d*n*pi^(1/2))
```

Reduce [F]

$$\int \frac{\mathrm{erfi}(d(a+b\log(cx^n)))}{x}\,dx = -\left(\int \frac{\mathrm{erf}(\log(x^n c)\,bdi+adi)}{x}dx\right)i$$

input
```
int(erfi(d*(a+b*log(c*x^n)))/x,x)
```

output
```
- int(erf(log(x**n*c)*b*d*i + a*d*i)/x,x)*i
```

3.250 $\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\text{erfi}(d(a+b\log(cx^n)))}{x} + \frac{e^{-\frac{1}{4b^2d^2n^2}+\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\text{erfi}\left(\frac{2abd^2-\frac{1}{n}+2b^2d^2\log(cx^n)}{2bd}\right)}{x}$$

output
```
-erfi(d*(a+b*ln(c*x^n)))/x+exp(-1/4/b^2/d^2/n^2+a/b/n)*(c*x^n)^(1/n)*erfi(
1/2*(2*a*b*d^2-1/n+2*b^2*d^2*ln(c*x^n))/b/d)/x
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{-\text{erfi}(d(a+b\log(cx^n)))+e^{\frac{-1+4abd^2n}{4b^2d^2n^2}}(cx^n)^{\frac{1}{n}}\,\text{erfi}\left(ad-\frac{1}{2bdn}+bd\log(cx^n)\right)}{x}$$

input
```
Integrate[Erfi[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
(-Erfi[d*(a + b*Log[c*x^n])] + E^((-1 + 4*a*b*d^2*n)/(4*b^2*d^2*n^2))*(c*x
^n)^n^(-1)*Erfi[a*d - 1/(2*b*d*n) + b*d*Log[c*x^n]])/x
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6957, 2712, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx$$

$$\Big\downarrow \text{ 6957}$$

$$\frac{2bdn\int \frac{e^{d^2(a+b\log(cx^n))^2}}{x^2}dx}{\sqrt{\pi}}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{ 2712}$$

$$\frac{2bdnx^{-2abd^2n}(cx^n)^{2abd^2}\int e^{a^2d^2+b^2\log^2(cx^n)d^2}x^{2abd^2n-2}dx}{\sqrt{\pi}}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{ 2706}$$

$$\frac{2bd(cx^n)^{\frac{1}{n}}\int \exp\left(a^2d^2+b^2\log^2(cx^n)\,d^2-\frac{(1-2abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}x}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{2bd(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}-\frac{1}{4b^2d^2n^2}}\int \exp\left(\frac{\left(2abd^2+2b^2\log(cx^n)d^2-\frac{1}{n}\right)^2}{4b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}x}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{(cx^n)^{\frac{1}{n}}e^{\frac{a}{bn}-\frac{1}{4b^2d^2n^2}}\text{erfi}\left(\frac{2abd^2+2b^2d^2\log(cx^n)-\frac{1}{n}}{2bd}\right)}{x}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{x}$$

input
```
Int[Erfi[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-(Erfi[d*(a + b*Log[c*x^n])]/x) + (E^(-1/4*1/(b^2*d^2*n^2) + a/(b*n))*(c*x
^n)^n^(-1)*Erfi[(2*a*b*d^2 - n^(-1) + 2*b^2*d^2*Log[c*x^n])/(2*b*d)])/x
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6957
```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfi[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 - Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(erfi(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(erfi(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{erfi}(d(a+b\log\left(cx^{n}\right)))}{x^{2}}\,dx = -\frac{\sqrt{-b^{2}d^{2}n^{2}}x\operatorname{erf}\left(\frac{\left(2\,b^{2}d^{2}n^{2}\log(x)+2\,b^{2}d^{2}n\log(c)+2\,abd^{2}n-1\right)\sqrt{-b^{2}d^{2}n^{2}}}{2\,b^{2}d^{2}n^{2}}\right)e^{\left(\frac{4\,b^{2}d^{2}n\log(c)+4\,abd^{2}n-1}{4\,b^{2}d^{2}n^{2}}\right)}+\operatorname{erfi}\left(bd\log\left(cx^{n}\right)\right.}{x}$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
-(sqrt(-b^2*d^2*n^2)*x*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c)
+ 2*a*b*d^2*n - 1)*sqrt(-b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*n*l
og(c) + 4*a*b*d^2*n - 1)/(b^2*d^2*n^2)) + erfi(b*d*log(c*x^n) + a*d))/x
```

Sympy [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log\left(cx^{n}\right)))}{x^{2}}\,dx = \int \frac{\operatorname{erfi}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

input
```
integrate(erfi(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(erfi(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erfi}\left((b\log(cx^n)+a)d\right)}{x^2}\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erfi}\left((b\log(cx^n)+a)d\right)}{x^2}\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\operatorname{erfi}(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

input
```
int(erfi(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(erfi(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\mathrm{erfi}(d(a+b\log(cx^n)))}{x^2}\,dx = -\left(\int \frac{\mathrm{erf}(\log(x^n c)\,bdi + adi)}{x^2}dx\right)i$$

input
```
int(erfi(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
- int(erf(log(x**n*c)*b*d*i + a*d*i)/x**2,x)*i
```

3.251 $\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\text{erfi}(d(a+b\log(cx^n)))}{2x^2} + \frac{e^{-\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2/n}\,\text{erfi}\left(\frac{abd^2-\frac{1}{n}+b^2d^2\log(cx^n)}{bd}\right)}{2x^2}$$

output
```
-1/2*erfi(d*(a+b*ln(c*x^n)))/x^2+1/2*(c*x^n)^(2/n)*erfi((a*b*d^2-1/n+b^2*d
^2*ln(c*x^n))/b/d)/exp((-2*a*b*d^2*n+1)/b^2/d^2/n^2)/x^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{-\text{erfi}(d(a+b\log(cx^n))) + e^{\frac{\frac{-\frac{1}{d^2}+2abn}{b^2}+2n\log(cx^n)}{n^2}}\text{erfi}\left(ad-\frac{1}{bdn}+bd\log(cx^n)\right)}{2x^2}$$

input
```
Integrate[Erfi[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(-Erfi[d*(a + b*Log[c*x^n])] + E^(((-d^(-2) + 2*a*b*n)/b^2 + 2*n*Log[c*x^n
])/n^2)*Erfi[a*d - 1/(b*d*n) + b*d*Log[c*x^n]])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6957, 2712, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(d(a+b\log(cx^n)))}{x^3}dx$$

$$\Big\downarrow \text{6957}$$

$$\frac{bdn\int \frac{e^{d^2(a+b\log(cx^n))^2}}{x^3}dx}{\sqrt{\pi}}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2712}$$

$$\frac{bdnx^{-2abd^2n}(cx^n)^{2abd^2}\int e^{a^2d^2+b^2\log^2(cx^n)d^2}x^{2abd^2n-3}dx}{\sqrt{\pi}}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2706}$$

$$\frac{bd(cx^n)^{2abd^2-2\left(abd^2-\frac{1}{n}\right)}\int \exp\left(a^2d^2+b^2\log^2(cx^n)d^2-\frac{2(1-abd^2n)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}x^2}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2664}$$

$$\frac{bde^{-\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2abd^2-2\left(abd^2-\frac{1}{n}\right)}\int \exp\left(\frac{\left(abd^2+b^2\log(cx^n)d^2-\frac{1}{n}\right)^2}{b^2d^2}\right)d\log(cx^n)}{\sqrt{\pi}x^2}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{2633}$$

$$\frac{e^{-\frac{1-2abd^2n}{b^2d^2n^2}}(cx^n)^{2abd^2-2\left(abd^2-\frac{1}{n}\right)}\text{erfi}\left(\frac{abd^2+b^2d^2\log(cx^n)-\frac{1}{n}}{bd}\right)}{2x^2}-\frac{\text{erfi}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Int[Erfi[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/2*Erfi[d*(a + b*Log[c*x^n])]/x^2 + ((c*x^n)^(2*a*b*d^2 - 2*(a*b*d^2 - n
^(-1)))*Erfi[(a*b*d^2 - n^(-1) + b^2*d^2*Log[c*x^n])/(b*d)])/(2*E^((1 - 2*
a*b*d^2*n)/(b^2*d^2*n^2))*x^2)
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6957
```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfi[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 - Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(erfi(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(erfi(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\sqrt{-b^2d^2n^2}x^2\operatorname{erf}\left(\frac{(b^2d^2n^2\log(x)+b^2d^2n\log(c)+abd^2n-1)\sqrt{-b^2d^2n^2}}{b^2d^2n^2}\right)e^{\left(\frac{2\,b^2d^2n\log(c)+2\,abd^2n-1}{b^2d^2n^2}\right)}+\operatorname{erfi}\left(bd\log(cx^n)+c\right.}{2\,x^2}$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-b^2*d^2*n^2)*x^2*erf((b^2*d^2*n^2*log(x) + b^2*d^2*n*log(c) +
a*b*d^2*n - 1)*sqrt(-b^2*d^2*n^2)/(b^2*d^2*n^2))*e^((2*b^2*d^2*n*log(c) +
2*a*b*d^2*n - 1)/(b^2*d^2*n^2)) + erfi(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\operatorname{erfi}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

input
```
integrate(erfi(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(erfi(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\operatorname{erfi}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\operatorname{erfi}((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(erfi(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(erfi((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\operatorname{erfi}(d\,(a+b\,\ln(c\,x^n)))}{x^3}\,dx$$

input
```
int(erfi(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(erfi(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\mathrm{erfi}(d(a+b\log(cx^n)))}{x^3}\,dx = -\left(\int \frac{\mathrm{erf}(\log(x^n c)\,bdi + adi)}{x^3}dx\right)i$$

input
```
int(erfi(d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
- int(erf(log(x**n*c)*b*d*i + a*d*i)/x**3,x)*i
```

3.252 $\int (ex)^m \mathbf{erfi}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int (ex)^m \text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\text{erfi}(d(a+b\log(cx^n)))}{e(1+m)} - \frac{e^{-\frac{(1+m)\left(1+m+4abd^2n\right)}{4b^2d^2n^2}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{1+m+2abd^2n+2b^2d^2n\log(cx^n)}{2bdn}\right)}{1+m}$$

output
```
(e*x)^(1+m)*erfi(d*(a+b*ln(c*x^n)))/e/(1+m)-x*(e*x)^m*erfi(1/2*(1+m+2*a*b*
d^2*n+2*b^2*d^2*n*ln(c*x^n))/b/d/n)/exp(1/4*(1+m)*(4*a*b*d^2*n+m+1)/b^2/d^
2/n^2)/(1+m)/((c*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int (ex)^m \text{erfi}(d(a+b\log(cx^n)))\,dx = \frac{(ex)^m\left(x\text{erfi}(d(a+b\log(cx^n))) - e^{-\frac{(1+m)\left(1+m+4abd^2n-4b^2d^2n^2\log(x)+4b^2d^2n\log(cx^n)\right)}{4b^2d^2n^2}}x^{-m}\text{erfi}\left(\frac{1+m+2abd^2n}{2bdn}+bd\log\right.\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*(x*Erfi[d*(a + b*Log[c*x^n])] - Erfi[(1 + m + 2*a*b*d^2*n)/(2*b*d
*n) + b*d*Log[c*x^n]]/(E^(((1 + m)*(1 + m + 4*a*b*d^2*n - 4*b^2*d^2*n^2*Lo
g[x] + 4*b^2*d^2*n*Log[c*x^n]))/(4*b^2*d^2*n^2))*x^m)))/(1 + m)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6957, 2712, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \text{erfi}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \ 6957$$

$$\frac{(ex)^{m+1}\text{erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdn\int e^{d^2(a+b\log(cx^n))^2}(ex)^m dx}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \ 2712$$

$$\frac{(ex)^{m+1}\text{erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdn(ex)^m(cx^n)^{2abd^2}x^{-2abd^2n-m}\int e^{a^2d^2+b^2\log^2(cx^n)d^2}x^{2abnd^2+m}dx}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \ 2706$$

$$\frac{(ex)^{m+1}\text{erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdx(ex)^m(cx^n)^{2abd^2-\frac{2abd^2n+m+1}{n}}\int \exp\left(a^2d^2+b^2\log^2(cx^n)d^2+\frac{(2abnd^2+m+1)\log(cx^n)}{n}\right)d\log(cx^n)}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \ 2664$$

$$\frac{(ex)^{m+1}\text{erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{2bdx(ex)^m\exp\left(-\frac{(m+1)(4abd^2n+m+1)}{4b^2d^2n^2}\right)(cx^n)^{2abd^2-\frac{2abd^2n+m+1}{n}}\int\exp\left(\frac{(2abnd^2+2b^2n\log(cx^n)d^2+m+1)^2}{4b^2d^2n^2}\right)d\log(cx^n)}{\sqrt{\pi}(m+1)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(ex)^{m+1}\text{erfi}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{x(ex)^m\exp\left(-\frac{(m+1)(4abd^2n+m+1)}{4b^2d^2n^2}\right)(cx^n)^{2abd^2-\frac{2abd^2n+m+1}{n}}\text{erfi}\left(\frac{2abd^2n+2b^2d^2n\log(cx^n)+m+1}{2bdn}\right)}{m+1}$$

input
```
Int[(e*x)^m*Erfi[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*Erfi[d*(a + b*Log[c*x^n])])/(e*(1 + m)) - (x*(e*x)^m*(c*x^n
)^(2*a*b*d^2 - (1 + m + 2*a*b*d^2*n)/n)*Erfi[(1 + m + 2*a*b*d^2*n + 2*b^2*
d^2*n*Log[c*x^n])/(2*b*d*n)])/(E^(((1 + m)*(1 + m + 4*a*b*d^2*n))/(4*b^2*d
^2*n^2))*(1 + m))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 6957
```
Int[Erfi[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(e*x)^(m + 1)*(Erfi[d*(a + b*Log[c*x^n])]/(e*(m + 1))), x]
 - Simp[2*b*d*(n/(Sqrt[Pi]*(m + 1)))   Int[(e*x)^m*E^(d*(a + b*Log[c*x^n]))
^2, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (ex)^m \operatorname{erfi}(d(a + b\ln(c\,x^n)))\,dx$$

input
```
int((e*x)^m*erfi(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*erfi(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.44

$$\int (ex)^m \operatorname{erfi}(d(a + b\log(cx^n)))\,dx$$

$$= \frac{x \operatorname{erfi}(bd\log(cx^n) + ad)\,e^{(m\log(e) + m\log(x))} + \sqrt{-b^2d^2n^2}\operatorname{erf}\left(\frac{(2\,b^2d^2n^2\log(x) + 2\,b^2d^2n\log(c) + 2\,abd^2n + m + 1)\sqrt{-b^2d^2n^2}}{2\,b^2d^2n^2}\right.}{m + 1}$$

input
```
integrate((e*x)^m*erfi(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
(x*erfi(b*d*log(c*x^n) + a*d)*e^(m*log(e) + m*log(x)) + sqrt(-b^2*d^2*n^2)
*erf(1/2*(2*b^2*d^2*n^2*log(x) + 2*b^2*d^2*n*log(c) + 2*a*b*d^2*n + m + 1)
*sqrt(-b^2*d^2*n^2)/(b^2*d^2*n^2))*e^(1/4*(4*b^2*d^2*m*n^2*log(e) - 4*(b^2
*d^2*m + b^2*d^2)*n*log(c) - m^2 - 4*(a*b*d^2*m + a*b*d^2)*n - 2*m - 1)/(b
^2*d^2*n^2)))/(m + 1)
```

Sympy [F]

$$\int (ex)^m \operatorname{erfi}(d(a + b\log{(cx^n)}))\ dx = \int (ex)^m \operatorname{erfi}{(ad + bd\log{(cx^n)})}\, dx$$

input
```
integrate((e*x)**m*erfi(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*erfi(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \operatorname{erfi}(d(a + b\log{(cx^n)}))\ dx = \int (ex)^m \operatorname{erfi}{((b\log{(cx^n)} + a)d)}\ dx$$

input
```
integrate((e*x)^m*erfi(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*erfi((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \text{erfi}(d(a+b\log(cx^n)))\, dx = \int (ex)^m \text{erfi}\left((b\log(cx^n)+a)d\right)\, dx$$

input
```
integrate((e*x)^m*erfi(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*erfi((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \text{erfi}(d(a+b\log(cx^n)))\, dx = \int \text{erfi}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\, dx$$

input
```
int(erfi(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(erfi(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \text{erfi}(d(a+b\log(cx^n)))\, dx = -e^m \left(\int x^m \text{erf}(\log(x^n c)\, bdi + adi)\, dx\right) i$$

input
```
int((e*x)^m*erfi(d*(a+b*log(c*x^n))),x)
```

output
```
- e**m*int(x**m*erf(log(x**n*c)*b*d*i + a*d*i),x)*i
```

3.253 $\int e^{c+b^2x^2}\mathbf{erfi}(bx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int e^{c+b^2x^2}\text{erfi}(bx)^2\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^3}{6b}$$

output
```
1/6*exp(c)*Pi^(1/2)*erfi(b*x)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c+b^2x^2}\text{erfi}(bx)^2\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^3}{6b}$$

input
```
Integrate[E^(c + b^2*x^2)*Erfi[b*x]^2,x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^3)/(6*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{b^2x^2+c}\text{erfi}(bx)^2\,dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)^2d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)^3}{6b}$$

input
```
Int[E^(c + b^2*x^2)*Erfi[b*x]^2,x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^3)/(6*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int e^{b^2x^2+c} \operatorname{erfi}(bx)^2 \, dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)^2,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c+b^2x^2} \operatorname{erfi}(bx)^2 \, dx = \frac{\sqrt{\pi} \operatorname{erfi}(bx)^3 e^c}{6 b}$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)^2,x, algorithm="fricas")
```

output
```
1/6*sqrt(pi)*erfi(b*x)^3*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{c+b^2x^2} \operatorname{erfi}(bx)^2 \, dx = \begin{cases} \frac{\sqrt{\pi} e^c \operatorname{erfi}^3(bx)}{6b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)**2,x)
```

output
```
Piecewise((sqrt(pi)*exp(c)*erfi(b*x)**3/(6*b), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}\text{erfi}(bx)^2\,dx=\int \text{erfi}\,(bx)^2\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^2*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}\text{erfi}(bx)^2\,dx=\int \text{erfi}\,(bx)^2\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^2*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c+b^2x^2}\text{erfi}(bx)^2\,dx=\frac{\sqrt{\pi}\,\mathrm{e}^c\,\text{erfi}(b\,x)^3}{6\,b}$$

input
```
int(exp(c + b^2*x^2)*erfi(b*x)^2,x)
```

output
```
(pi^(1/2)*exp(c)*erfi(b*x)^3)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)^2\,dx = \frac{\sqrt{\pi}\,e^c\operatorname{erf}(bix)^3\,i}{6b}$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)^2,x)
```

output
```
(sqrt(pi)*e**c*erf(b*i*x)**3*i)/(6*b)
```

3.254 $\int e^{c+b^2x^2}\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{4b}$$

output
```
1/4*exp(c)*Pi^(1/2)*erfi(b*x)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{4b}$$

input
```
Integrate[E^(c + b^2*x^2)*Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{b^2x^2+c}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{4b}$$

input
```
Int[E^(c + b^2*x^2)*Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int e^{b^2x^2+c}\,\mathrm{erfi}\,(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c+b^2x^2}\mathrm{erfi}(bx)\,dx=\frac{\sqrt{\pi}\,\mathrm{erfi}\,(bx)^2\,e^c}{4\,b}$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*erfi(b*x)^2*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{c+b^2x^2}\mathrm{erfi}(bx)\,dx=\begin{cases}\frac{\sqrt{\pi}e^c\,\mathrm{erfi}^2\,(bx)}{4b} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x),x)
```

output
```
Piecewise((sqrt(pi)*exp(c)*erfi(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\int \text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\int \text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.33

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{b^2\,x}{\sqrt{b^2}}\right)\,e^c\,\text{erfi}(b\,x)}{2\,\sqrt{b^2}}-\frac{\sqrt{\pi}\,e^c\,\text{erf}\left(x\,\sqrt{-b^2}\right)^2}{4\,b}-\frac{b\,\sqrt{\pi}\,\text{erfi}\left(\frac{b^2\,x}{\sqrt{b^2}}\right)\,e^c\,\text{erf}\left(x\,\sqrt{-b^2}\right)}{2\,\sqrt{b^2}\,\sqrt{-b^2}}$$

input
```
int(exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
(pi^(1/2)*erfi((b^2*x)/(b^2)^(1/2))*exp(c)*erfi(b*x))/(2*(b^2)^(1/2)) - (p
i^(1/2)*exp(c)*erf(x*(-b^2)^(1/2))^2)/(4*b) - (b*pi^(1/2)*erfi((b^2*x)/(b^
2)^(1/2))*exp(c)*erf(x*(-b^2)^(1/2)))/(2*(b^2)^(1/2)*(-b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx = -\frac{\sqrt{\pi}\,e^c\operatorname{erf}(bix)^2}{4b}$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x),x)
```

output
```
( - sqrt(pi)*e**c*erf(b*i*x)**2)/(4*b)
```

3.255 $\int \frac{e^{c+b^2x^2}}{\mathbf{erfi}(bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \frac{e^c\sqrt{\pi}\log(\operatorname{erfi}(bx))}{2b}$$

output
```
1/2*exp(c)*Pi^(1/2)*ln(erfi(b*x))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \frac{e^c\sqrt{\pi}\log(\operatorname{erfi}(bx))}{2b}$$

input
```
Integrate[E^(c + b^2*x^2)/Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Log[Erfi[b*x]])/(2*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6929, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}}{\text{erfi}(bx)}dx$$

$$\Big\downarrow \text{6929}$$

$$\frac{\sqrt{\pi}e^c\int \frac{1}{\text{erfi}(bx)}d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \text{14}$$

$$\frac{\sqrt{\pi}e^c\log(\text{erfi}(bx))}{2b}$$

input
```
Int[E^(c + b^2*x^2)/Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Log[Erfi[b*x]])/(2*b)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}}{\operatorname{erfi}(bx)}dx$$

input
```
int(exp(b^2*x^2+c)/erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)/erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \frac{\sqrt{\pi}e^c\log\left(\operatorname{erfi}\left(bx\right)\right)}{2\,b}$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x),x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*e^c*log(erfi(b*x))/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \begin{cases} \frac{\sqrt{\pi}e^c\log\left(\operatorname{erfi}\left(bx\right)\right)}{2b} & \text{for } b \neq 0 \\ \tilde{\infty}xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)/erfi(b*x),x)
```

output
```
Piecewise((sqrt(pi)*exp(c)*log(erfi(b*x))/(2*b), Ne(b, 0)), (zoo*x*exp(c),
 True))
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \int \frac{e^{\left(b^2x^2+c\right)}}{\operatorname{erfi}\left(bx\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(e^(b^2*x^2 + c)/erfi(b*x), x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \int \frac{e^{\left(b^2x^2+c\right)}}{\operatorname{erfi}\left(bx\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x),x, algorithm="giac")
```

output
```
integrate(e^(b^2*x^2 + c)/erfi(b*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \frac{\sqrt{\pi}\,\ln\left(\operatorname{erfi}(b\,x)\right)\,\mathrm{e}^c}{2\,b}$$

input
```
int(exp(c + b^2*x^2)/erfi(b*x),x)
```

output
```
(pi^(1/2)*log(erfi(b*x))*exp(c))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)}\,dx = \frac{\sqrt{\pi}\,e^c \log(\operatorname{erf}(bix))}{2b}$$

input
```
int(exp(b^2*x^2+c)/erfi(b*x),x)
```

output
```
(sqrt(pi)*e**c*log(erf(b*i*x)))/(2*b)
```

3.256 $\int \frac{e^{c+b^2x^2}}{\mathbf{erfi}(bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{e^{c+b^2x^2}}{\mathrm{erfi}(bx)^2}\,dx = -\frac{e^c\sqrt{\pi}}{2b\,\mathrm{erfi}(bx)}$$

output
```
-1/2*exp(c)*Pi^(1/2)/b/erfi(b*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+b^2x^2}}{\mathrm{erfi}(bx)^2}\,dx = -\frac{e^c\sqrt{\pi}}{2b\,\mathrm{erfi}(bx)}$$

input
```
Integrate[E^(c + b^2*x^2)/Erfi[b*x]^2,x]
```

output
```
-1/2*(E^c*Sqrt[Pi])/(b*Erfi[b*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}}{\text{erfi}(bx)^2} \, dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{\sqrt{\pi}e^c \int \frac{1}{\text{erfi}(bx)^2}d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{\pi}e^c}{2b\text{erfi}(bx)}$$

input
```
Int[E^(c + b^2*x^2)/Erfi[b*x]^2,x]
```

output
```
-1/2*(E^c*Sqrt[Pi])/(b*Erfi[b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}}{\mathrm{erfi}\,(bx)^2}dx$$

input
```
int(exp(b^2*x^2+c)/erfi(b*x)^2,x)
```

output
```
int(exp(b^2*x^2+c)/erfi(b*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c+b^2x^2}}{\mathrm{erfi}(bx)^2}\,dx = -\frac{\sqrt{\pi}e^c}{2\,b\,\mathrm{erfi}\,(bx)}$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x)^2,x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*e^c/(b*erfi(b*x))
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{e^{c+b^2x^2}}{\mathrm{erfi}(bx)^2}\,dx = \begin{cases} -\frac{\sqrt{\pi}e^c}{2b\,\mathrm{erfi}\,(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)/erfi(b*x)**2,x)
```

output
```
Piecewise((-sqrt(pi)*exp(c)/(2*b*erfi(b*x)), Ne(b, 0)), (zoo*x*exp(c), Tru
e))
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^2}\,dx = \int \frac{e^{\left(b^2x^2+c\right)}}{\text{erfi}\left(bx\right)^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(e^(b^2*x^2 + c)/erfi(b*x)^2, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^2}\,dx = \int \frac{e^{\left(b^2x^2+c\right)}}{\text{erfi}\left(bx\right)^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x)^2,x, algorithm="giac")
```

output
```
integrate(e^(b^2*x^2 + c)/erfi(b*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^2}\,dx = -\frac{\sqrt{\pi}\,\mathrm{e}^c}{2\,b\,\text{erfi}\left(b\,x\right)}$$

input
```
int(exp(c + b^2*x^2)/erfi(b*x)^2,x)
```

output
```
-(pi^(1/2)*exp(c))/(2*b*erfi(b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)^2}\,dx = -\frac{\sqrt{\pi}\,e^c i}{2\operatorname{erf}(bix)\,b}$$

input
```
int(exp(b^2*x^2+c)/erfi(b*x)^2,x)
```

output
```
( - sqrt(pi)*e**c*i)/(2*erf(b*i*x)*b)
```

3.257 $\int \frac{e^{c+b^2x^2}}{\mathbf{erfi}(bx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^3}\,dx = -\frac{e^c\sqrt{\pi}}{4b\text{erfi}(bx)^2}$$

output
```
-1/4*exp(c)*Pi^(1/2)/b/erfi(b*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^3}\,dx = -\frac{e^c\sqrt{\pi}}{4b\text{erfi}(bx)^2}$$

input
```
Integrate[E^(c + b^2*x^2)/Erfi[b*x]^3,x]
```

output
```
-1/4*(E^c*Sqrt[Pi])/(b*Erfi[b*x]^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}}{\text{erfi}(bx)^3}\,dx$$

$$\Big\downarrow \; 6929$$

$$\frac{\sqrt{\pi}e^c\int \frac{1}{\text{erfi}(bx)^3}d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \; 15$$

$$-\frac{\sqrt{\pi}e^c}{4b\text{erfi}(bx)^2}$$

input
```
Int[E^(c + b^2*x^2)/Erfi[b*x]^3,x]
```

output
```
-1/4*(E^c*Sqrt[Pi])/(b*Erfi[b*x]^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}}{\text{erfi}\,(bx)^3}dx$$

input
```
int(exp(b^2*x^2+c)/erfi(b*x)^3,x)
```

output
```
int(exp(b^2*x^2+c)/erfi(b*x)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^3}\,dx = -\frac{\sqrt{\pi}e^c}{4\,b\,\text{erfi}\,(bx)^2}$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x)^3,x, algorithm="fricas")
```

output
```
-1/4*sqrt(pi)*e^c/(b*erfi(b*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{e^{c+b^2x^2}}{\text{erfi}(bx)^3}\,dx = \begin{cases} -\frac{\sqrt{\pi}e^c}{4b\,\text{erfi}^2\,(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}xe^c & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)/erfi(b*x)**3,x)
```

output
```
Piecewise((-sqrt(pi)*exp(c)/(4*b*erfi(b*x)**2), Ne(b, 0)), (zoo*x*exp(c),
True))
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)^3}\,dx = \int \frac{e^{(b^2x^2+c)}}{\operatorname{erfi}\,(bx)^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x)^3,x, algorithm="maxima")
```

output
```
integrate(e^(b^2*x^2 + c)/erfi(b*x)^3, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)^3}\,dx = \int \frac{e^{(b^2x^2+c)}}{\operatorname{erfi}\,(bx)^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)/erfi(b*x)^3,x, algorithm="giac")
```

output
```
integrate(e^(b^2*x^2 + c)/erfi(b*x)^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)^3}\,dx = -\frac{\sqrt{\pi}\,\mathrm{e}^c}{4\,b\operatorname{erfi}\,(b\,x)^2}$$

input
```
int(exp(c + b^2*x^2)/erfi(b*x)^3,x)
```

output
```
-(pi^(1/2)*exp(c))/(4*b*erfi(b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{e^{c+b^2x^2}}{\operatorname{erfi}(bx)^3}\,dx = \frac{\sqrt{\pi}\,e^c}{4\operatorname{erf}\left(bix\right)^2 b}$$

input
```
int(exp(b^2*x^2+c)/erfi(b*x)^3,x)
```

output
```
(sqrt(pi)*e**c)/(4*erf(b*i*x)**2*b)
```

3.258 $\int e^{c+b^2x^2}\textbf{erfi}(bx)^n\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int e^{c+b^2x^2}\text{erfi}(bx)^n\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^{1+n}}{2b(1+n)}$$

output

```
1/2*exp(c)*Pi^(1/2)*erfi(b*x)^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int e^{c+b^2x^2}\text{erfi}(bx)^n\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^{1+n}}{2b(1+n)}$$

input

```
Integrate[E^(c + b^2*x^2)*Erfi[b*x]^n,x]
```

output

```
(E^c*Sqrt[Pi]*Erfi[b*x]^(1 + n))/(2*b*(1 + n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{b^2x^2+c}\text{erfi}(bx)^n\,dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)^n d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)^{n+1}}{2b(n+1)}$$

input
```
Int[E^(c + b^2*x^2)*Erfi[b*x]^n,x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^(1 + n))/(2*b*(1 + n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int e^{b^2x^2+c}\operatorname{erfi}(bx)^n\,dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)^n,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)^n\,dx = \frac{\sqrt{\pi}\operatorname{erfi}(bx)^n\operatorname{erfi}(bx)\,e^c}{2\,(bn+b)}$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)^n,x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*erfi(b*x)^n*erfi(b*x)*e^c/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(22) = 44.

Time = 1.72 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)^n\,dx = \begin{cases} \tilde{\infty}xe^c & \text{for } b = 0 \wedge n = -1 \\ 0^n xe^c & \text{for } b = 0 \\ \frac{\sqrt{\pi}e^c\log(\operatorname{erfi}(bx))}{2b} & \text{for } n = -1 \\ \frac{\sqrt{\pi}e^c\operatorname{erfi}(bx)\operatorname{erfi}^n(bx)}{2bn+2b} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)**n,x)
```

output
```
Piecewise((zoo*x*exp(c), Eq(b, 0) & Eq(n, -1)), (0**n*x*exp(c), Eq(b, 0)),
 (sqrt(pi)*exp(c)*log(erfi(b*x))/(2*b), Eq(n, -1)), (sqrt(pi)*exp(c)*erfi(
b*x)*erfi(b*x)**n/(2*b*n + 2*b), True))
```

Maxima [F]

$$\int e^{c+b^2x^2}\text{erfi}(bx)^n\,dx = \int \text{erfi}\,(bx)^n\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)^n,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)^n*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}\text{erfi}(bx)^n\,dx = \int \text{erfi}\,(bx)^n\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)^n,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)^n*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int e^{c+b^2x^2}\text{erfi}(bx)^n\,dx = \frac{\sqrt{\pi}\,\mathrm{e}^c\,\text{erfi}(b\,x)^{n+1}}{2\,b\,(n+1)}$$

input
```
int(exp(c + b^2*x^2)*erfi(b*x)^n,x)
```

output
```
(pi^(1/2)*exp(c)*erfi(b*x)^(n + 1))/(2*b*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)^n\,dx=-\frac{\sqrt{\pi}\,i^n e^c(-1)^n\operatorname{erf}(bix)^n\operatorname{erf}(bix)\,i}{2b\,(n+1)}$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)^n,x)
```

output
```
( - sqrt(pi)*i**n*e**c*( - 1)**n*erf(b*i*x)**n*erf(b*i*x)*i)/(2*b*(n + 1))
```

3.259 $\int e^{c+dx^2} x^5 \mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 257

$$\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx=\frac{3be^{c+(b^2+d)x^2}x}{4d(b^2+d)^2\sqrt{\pi}}+\frac{be^{c+(b^2+d)x^2}x}{d^2(b^2+d)\sqrt{\pi}}-\frac{be^{c+(b^2+d)x^2}x^3}{2d(b^2+d)\sqrt{\pi}}+\frac{e^{c+dx^2}\text{erfi}(bx)}{d^3}-\frac{e^{c+dx^2}x^2\text{erfi}(bx)}{d^2}+\frac{e^{c+dx^2}x^4\text{erfi}(bx)}{2d}-\frac{3be^c\text{erfi}\left(\sqrt{b^2+d}x\right)}{8d(b^2+d)^{5/2}}-\frac{be^c\text{erfi}\left(\sqrt{b^2+d}x\right)}{2d^2(b^2+d)^{3/2}}-\frac{be^c\text{erfi}\left(\sqrt{b^2+d}x\right)}{d^3\sqrt{b^2+d}}$$

output
```
3/4*b*exp(c+(b^2+d)*x^2)*x/d/(b^2+d)^2/Pi^(1/2)+b*exp(c+(b^2+d)*x^2)*x/d^2
/(b^2+d)/Pi^(1/2)-1/2*b*exp(c+(b^2+d)*x^2)*x^3/d/(b^2+d)/Pi^(1/2)+exp(d*x^
2+c)*erfi(b*x)/d^3-exp(d*x^2+c)*x^2*erfi(b*x)/d^2+1/2*exp(d*x^2+c)*x^4*erf
i(b*x)/d-3/8*b*exp(c)*erfi((b^2+d)^(1/2)*x)/d/(b^2+d)^(5/2)-1/2*b*exp(c)*e
rfi((b^2+d)^(1/2)*x)/d^2/(b^2+d)^(3/2)-b*exp(c)*erfi((b^2+d)^(1/2)*x)/d^3/
(b^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx$$
$$=\frac{e^c\left(-\frac{2bde^{(b^2+d)x^2}x(2b^2(-2+dx^2)+d(-7+2dx^2))}{(b^2+d)^2\sqrt{\pi}}+4e^{dx^2}(2-2dx^2+d^2x^4)\,\text{erfi}(bx)-\frac{b(8b^4+20b^2d+15d^2)\text{erfi}\left(\sqrt{b^2+d}x\right)}{(b^2+d)^{5/2}}\right)}{8d^3}$$

input
```
Integrate[E^(c + d*x^2)*x^5*Erfi[b*x],x]
```

output
```
(E^c*((-2*b*d*E^((b^2 + d)*x^2)*x*(2*b^2*(-2 + d*x^2) + d*(-7 + 2*d*x^2)))
/((b^2 + d)^2*Sqrt[Pi]) + 4*E^(d*x^2)*(2 - 2*d*x^2 + d^2*x^4)*Erfi[b*x] -
(b*(8*b^4 + 20*b^2*d + 15*d^2)*Erfi[Sqrt[b^2 + d]*x])/(b^2 + d)^(5/2)))/(8
*d^3)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6941, 2641, 2641, 2633, 6941, 2641, 2633, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\text{erfi}(bx)e^{c+dx^2}\,dx$$
$$\Big\downarrow \text{6941}$$
$$-\frac{b\int e^{(b^2+d)x^2+c}x^4dx}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erfi}(bx)dx}{d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$
$$\Big\downarrow \text{2641}$$
$$-\frac{b\left(\frac{x^3e^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{3\int e^{(b^2+d)x^2+c}x^2dx}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erfi}(bx)dx}{d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\int e^{\left(b^2+d\right)x^2+c}dx}{2\left(b^2+d\right)}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{2\int e^{dx^2+c}x^3\text{erfi}(bx)dx}{d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\int e^{dx^2+c}x^3\text{erfi}(bx)dx}{d}-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6941}$$

$$-\frac{2\left(-\frac{b\int e^{\left(b^2+d\right)x^2+c}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(bx)dx}{d}+\frac{x^2\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\left(-\frac{b\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\int e^{\left(b^2+d\right)x^2+c}dx}{2\left(b^2+d\right)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(bx)dx}{d}+\frac{x^2\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\left(-\frac{\int e^{dx^2+c}x\,\text{erfi}(bx)dx}{d}-\frac{b\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \; 6938$$

$$-\frac{2\left(-\frac{\frac{\text{erfi}(bx)e^{c+dx^2}}{2d}-\frac{b\int e^{\left(b^2+d\right)x^2+c}dx}{\sqrt{\pi}d}}{d}-\frac{b\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \; 2633$$

$$-\frac{2\left(-\frac{\frac{\text{erfi}(bx)e^{c+dx^2}}{2d}-\frac{be^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{2d\sqrt{b^2+d}}}{d}-\frac{b\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{d}-\frac{b\left(\frac{x^3e^{x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{3\left(\frac{xe^{x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4\left(b^2+d\right)^{3/2}}\right)}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^4\text{erfi}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^5*Erfi[b*x],x]
```

output
```
(E^(c + d*x^2)*x^4*Erfi[b*x])/(2*d) - (b*((E^(c + (b^2 + d)*x^2)*x^3)/(2*(
b^2 + d)) - (3*((E^(c + (b^2 + d)*x^2)*x)/(2*(b^2 + d)) - (E^c*Sqrt[Pi]*Er
fi[Sqrt[b^2 + d]*x])/(4*(b^2 + d)^(3/2))))/(2*(b^2 + d))))/(d*Sqrt[Pi]) -
(2*((E^(c + d*x^2)*x^2*Erfi[b*x])/(2*d) - ((E^(c + d*x^2)*Erfi[b*x])/(2*d)
 - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d*Sqrt[b^2 + d]))/d - (b*((E^(c + (b^2
 + d)*x^2)*x)/(2*(b^2 + d)) - (E^c*Sqrt[Pi]*Erfi[Sqrt[b^2 + d]*x])/(4*(b^2
 + d)^(3/2))))/(d*Sqrt[Pi])))/d
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{d\,x^2+c} x^5 \operatorname{erfi}(bx)\,dx$$

input
```
int(exp(d*x^2+c)*x^5*erfi(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^5*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.99

$$\int e^{c+dx^2} x^5 \text{erfi}(bx)\,dx = \frac{\pi(8\,b^5+20\,b^3d+15\,bd^2)\sqrt{-b^2-d}\,\text{erf}\left(\sqrt{-b^2-d}x\right)e^c+4\left(\pi(b^6d^2+3\,b^4d^3+3\,b^2d^4+d^5)x^4-2\,\pi(b^6d+\right.}{}$$

input
```
integrate(exp(d*x^2+c)*x^5*erfi(b*x),x, algorithm="fricas")
```

output
```
1/8*(pi*(8*b^5 + 20*b^3*d + 15*b*d^2)*sqrt(-b^2 - d)*erf(sqrt(-b^2 - d)*x)
*e^c + 4*(pi*(b^6*d^2 + 3*b^4*d^3 + 3*b^2*d^4 + d^5)*x^4 - 2*pi*(b^6*d + 3
*b^4*d^2 + 3*b^2*d^3 + d^4)*x^2 + 2*pi*(b^6 + 3*b^4*d + 3*b^2*d^2 + d^3))*
erfi(b*x)*e^(d*x^2 + c) - 2*sqrt(pi)*(2*(b^5*d^2 + 2*b^3*d^3 + b*d^4)*x^3
- (4*b^5*d + 11*b^3*d^2 + 7*b*d^3)*x)*e^(b^2*x^2 + d*x^2 + c))/(pi*(b^6*d^
3 + 3*b^4*d^4 + 3*b^2*d^5 + d^6))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2} x^5 \text{erfi}(bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**5*erfi(b*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx=\int x^5\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^5*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^5*erfi(b*x)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx=\int x^5\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^5*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*erfi(b*x)*e^(d*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.90

$$\begin{aligned}\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx&=\text{erfi}(b\,x)\left(\frac{e^{d\,x^2+c}}{d^3}-\frac{x^2\,e^{d\,x^2+c}}{d^2}+\frac{x^4\,e^{d\,x^2+c}}{2\,d}\right)\\&-\frac{b\,\text{erfi}\left(x\,\sqrt{b^2+d}\right)\,e^c}{2\,d^2\,\left(b^2+d\right)^{3/2}}-\frac{b\,e^c\,\text{erf}\left(x\,\sqrt{-b^2-d}\right)}{d^3\,\sqrt{-b^2-d}}+\frac{b\,x\,e^{b^2\,x^2+d\,x^2+c}}{d^2\,\sqrt{\pi}\,\left(b^2+d\right)}\\&+\frac{b\,x^5\,e^c\left(\frac{3\,\sqrt{\pi}\,\text{erfc}\left(\sqrt{-x^2\,(b^2+d)}\right)}{4}+e^{b^2\,x^2+d\,x^2}\left(\frac{3\,\sqrt{-x^2\,(b^2+d)}}{2}+\left(-x^2\,\left(b^2+d\right)\right)^{3/2}\right)-\frac{3\,\sqrt{\pi}}{4}\right)}{2\,d\,\sqrt{\pi}\,\left(-x^2\,\left(b^2+d\right)\right)^{5/2}}\end{aligned}$$

input
```
int(x^5*exp(c + d*x^2)*erfi(b*x),x)
```

output
```
erfi(b*x)*(exp(c + d*x^2)/d^3 - (x^2*exp(c + d*x^2))/d^2 + (x^4*exp(c + d*
x^2))/(2*d)) - (b*erfi(x*(d + b^2)^(1/2))*exp(c))/(2*d^2*(d + b^2)^(3/2))
- (b*exp(c)*erf(x*(- d - b^2)^(1/2)))/(d^3*(- d - b^2)^(1/2)) + (b*x*exp(c
 + d*x^2 + b^2*x^2))/(d^2*pi^(1/2)*(d + b^2)) + (b*x^5*exp(c)*((3*pi^(1/2)
*erfc((-x^2*(d + b^2))^(1/2)))/4 + exp(d*x^2 + b^2*x^2)*((3*(-x^2*(d + b^2
))^(1/2))/2 + (-x^2*(d + b^2))^(3/2)) - (3*pi^(1/2))/4))/(2*d*pi^(1/2)*(-x
^2*(d + b^2))^(5/2))
```

Reduce [F]

$$\int e^{c+dx^2}x^5\text{erfi}(bx)\,dx = -e^c\left(\int e^{d\,x^2}\text{erf}(bix)\,x^5dx\right)i$$

input
```
int(exp(d*x^2+c)*x^5*erfi(b*x),x)
```

output
```
 - e**c*int(e**(d*x**2)*erf(b*i*x)*x**5,x)*i
```

3.260 $\int e^{c+dx^2} x^3 \mathbf{erfi}(bx)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 142

$$\int e^{c+dx^2} x^3 \text{erfi}(bx)\, dx = -\frac{be^{c+(b^2+d)x^2}x}{2d(b^2+d)\sqrt{\pi}} - \frac{e^{c+dx^2}\text{erfi}(bx)}{2d^2} + \frac{e^{c+dx^2}x^2\text{erfi}(bx)}{2d} + \frac{be^c\text{erfi}\left(\sqrt{b^2+d}x\right)}{4d(b^2+d)^{3/2}} + \frac{be^c\text{erfi}\left(\sqrt{b^2+d}x\right)}{2d^2\sqrt{b^2+d}}$$

output
```
-1/2*b*exp(c+(b^2+d)*x^2)*x/d/(b^2+d)/Pi^(1/2)-1/2*exp(d*x^2+c)*erfi(b*x)/
d^2+1/2*exp(d*x^2+c)*x^2*erfi(b*x)/d+1/4*b*exp(c)*erfi((b^2+d)^(1/2)*x)/d/
(b^2+d)^(3/2)+1/2*b*exp(c)*erfi((b^2+d)^(1/2)*x)/d^2/(b^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int e^{c+dx^2} x^3 \text{erfi}(bx)\, dx = \frac{e^c\left(-\frac{2bde^{(b^2+d)x^2}x}{(b^2+d)\sqrt{\pi}} + 2e^{dx^2}(-1+dx^2)\,\text{erfi}(bx) + \frac{(2b^3+3bd)\text{erfi}\left(\sqrt{b^2+d}x\right)}{(b^2+d)^{3/2}}\right)}{4d^2}$$

input
```
Integrate[E^(c + d*x^2)*x^3*Erfi[b*x],x]
```

output
```
(E^c*((-2*b*d*E^((b^2 + d)*x^2)*x)/((b^2 + d)*Sqrt[Pi]) + 2*E^(d*x^2)*(-1
+ d*x^2)*Erfi[b*x] + ((2*b^3 + 3*b*d)*Erfi[Sqrt[b^2 + d]*x])/(b^2 + d)^(3/
2)))/(4*d^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6941, 2641, 2633, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{erfi}(bx) e^{c+dx^2} \, dx$$

$$\Big\downarrow \text{6941}$$

$$-\frac{b \int e^{(b^2+d)x^2+c} x^2 dx}{\sqrt{\pi} d} - \frac{\int e^{dx^2+c} x \text{erfi}(bx) dx}{d} + \frac{x^2 \text{erfi}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{b \left(\frac{x e^{x^2(b^2+d)+c}}{2(b^2+d)} - \frac{\int e^{(b^2+d)x^2+c} dx}{2(b^2+d)}\right)}{\sqrt{\pi} d} - \frac{\int e^{dx^2+c} x \text{erfi}(bx) dx}{d} + \frac{x^2 \text{erfi}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\int e^{dx^2+c} x \text{erfi}(bx) dx}{d} - \frac{b \left(\frac{x e^{x^2(b^2+d)+c}}{2(b^2+d)} - \frac{\sqrt{\pi} e^c \text{erfi}\left(x\sqrt{b^2+d}\right)}{4(b^2+d)^{3/2}}\right)}{\sqrt{\pi} d} + \frac{x^2 \text{erfi}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{6938}$$

$$-\frac{\frac{\text{erfi}(bx) e^{c+dx^2}}{2d} - \frac{b \int e^{(b^2+d)x^2+c} dx}{\sqrt{\pi} d}}{d} - \frac{b \left(\frac{x e^{x^2(b^2+d)+c}}{2(b^2+d)} - \frac{\sqrt{\pi} e^c \text{erfi}\left(x\sqrt{b^2+d}\right)}{4(b^2+d)^{3/2}}\right)}{\sqrt{\pi} d} + \frac{x^2 \text{erfi}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\frac{\text{erfi}(bx)e^{c+dx^2}}{2d}-\frac{be^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{2d\sqrt{b^2+d}}}{d}-\frac{b\left(\frac{xe^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{\sqrt{\pi}e^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{4(b^2+d)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{x^2\text{erfi}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^3*Erfi[b*x],x]
```

output
```
(E^(c + d*x^2)*x^2*Erfi[b*x])/(2*d) - ((E^(c + d*x^2)*Erfi[b*x])/(2*d) - (
b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d*Sqrt[b^2 + d]))/d - (b*((E^(c + (b^2 + d
)*x^2)*x)/(2*(b^2 + d)) - (E^c*Sqrt[Pi]*Erfi[Sqrt[b^2 + d]*x])/(4*(b^2 + d
)^(3/2))))/(d*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{d\,x^2+c}x^3\,\text{erfi}\left(bx\right)dx$$

input
```
int(exp(d*x^2+c)*x^3*erfi(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^3*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx = -\frac{\pi(2\,b^3+3\,bd)\sqrt{-b^2-d}\,\text{erf}\left(\sqrt{-b^2-d}x\right)e^c+2\,\sqrt{\pi}(b^3d+bd^2)xe^{(b^2x^2+dx^2+c)}-2\,(\pi(b^4d+2\,b^2d^2+d^3)}{4\,\pi(b^4d^2+2\,b^2d^3+d^4)}$$

input
```
integrate(exp(d*x^2+c)*x^3*erfi(b*x),x, algorithm="fricas")
```

output
```
-1/4*(pi*(2*b^3 + 3*b*d)*sqrt(-b^2 - d)*erf(sqrt(-b^2 - d)*x)*e^c + 2*sqrt
(pi)*(b^3*d + b*d^2)*x*e^(b^2*x^2 + d*x^2 + c) - 2*(pi*(b^4*d + 2*b^2*d^2
+ d^3)*x^2 - pi*(b^4 + 2*b^2*d + d^2))*erfi(b*x)*e^(d*x^2 + c))/(pi*(b^4*d
^2 + 2*b^2*d^3 + d^4))
```

Sympy [F]

$$\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx = e^c\int x^3e^{dx^2}\,\text{erfi}\left(bx\right)dx$$

input
```
integrate(exp(d*x**2+c)*x**3*erfi(b*x),x)
```

output
```
exp(c)*Integral(x**3*exp(d*x**2)*erfi(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx=\int x^3\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*erfi(b*x)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx=\int x^3\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*erfi(b*x)*e^(d*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx=\frac{b\,\text{erfi}\left(x\,\sqrt{b^2+d}\right)\,e^c}{4\,d\,(b^2+d)^{3/2}}-\text{erfi}(b\,x)\left(\frac{e^{d\,x^2+c}}{2\,d^2}-\frac{x^2\,e^{d\,x^2+c}}{2\,d}\right)-\frac{b\,x\,e^{b^2\,x^2+d\,x^2+c}}{2\,\sqrt{\pi}\,\left(b^2\,d+d^2\right)}+\frac{b\,e^c\,\text{erf}\left(x\,\sqrt{-b^2-d}\right)}{2\,d^2\,\sqrt{-b^2-d}}$$

input
```
int(x^3*exp(c + d*x^2)*erfi(b*x),x)
```

output
```
(b*erfi(x*(d + b^2)^(1/2))*exp(c))/(4*d*(d + b^2)^(3/2)) - erfi(b*x)*(exp(
c + d*x^2)/(2*d^2) - (x^2*exp(c + d*x^2))/(2*d)) - (b*x*exp(c + d*x^2 + b^
2*x^2))/(2*pi^(1/2)*(b^2*d + d^2)) + (b*exp(c)*erf(x*(- d - b^2)^(1/2)))/(
2*d^2*(- d - b^2)^(1/2))
```

Reduce [F]

$$\int e^{c+dx^2}x^3\text{erfi}(bx)\,dx = -e^c\left(\int e^{d\,x^2}\text{erf}(bix)\,x^3dx\right)i$$

input
```
int(exp(d*x^2+c)*x^3*erfi(b*x),x)
```

output
```
- e**c*int(e**(d*x**2)*erf(b*i*x)*x**3,x)*i
```

3.261 $\int e^{c+dx^2} x\,\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int e^{c+dx^2} x \operatorname{erfi}(bx)\,dx = \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{2d} - \frac{be^c \operatorname{erfi}\left(\sqrt{b^2+d}\,x\right)}{2d\sqrt{b^2+d}}$$

output
```
1/2*exp(d*x^2+c)*erfi(b*x)/d-1/2*b*exp(c)*erfi((b^2+d)^(1/2)*x)/d/(b^2+d)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int e^{c+dx^2} x \operatorname{erfi}(bx)\,dx = \frac{e^c\left(e^{dx^2}\operatorname{erfi}(bx) - \frac{b\operatorname{erfi}\left(\sqrt{b^2+d}x\right)}{\sqrt{b^2+d}}\right)}{2d}$$

input
```
Integrate[E^(c + d*x^2)*x*Erfi[b*x],x]
```

output
```
(E^c*(E^(d*x^2)*Erfi[b*x] - (b*Erfi[Sqrt[b^2 + d]*x])/Sqrt[b^2 + d]))/(2*d
)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{erfi}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{6938}$$

$$\frac{\text{erfi}(bx)e^{c+dx^2}}{2d}-\frac{b\int e^{(b^2+d)x^2+c}dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\text{erfi}(bx)e^{c+dx^2}}{2d}-\frac{be^c\text{erfi}\left(x\sqrt{b^2+d}\right)}{2d\sqrt{b^2+d}}$$

input
```
Int[E^(c + d*x^2)*x*Erfi[b*x],x]
```

output
```
(E^(c + d*x^2)*Erfi[b*x])/(2*d) - (b*E^c*Erfi[Sqrt[b^2 + d]*x])/(2*d*Sqrt[
b^2 + d])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int e^{d\,x^2+c} x \operatorname{erfi}(bx)\, dx$$

input
```
int(exp(d*x^2+c)*x*erfi(b*x),x)
```

output
```
int(exp(d*x^2+c)*x*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int e^{c+dx^2} x\mathrm{erfi}(bx)\, dx = \frac{\sqrt{-b^2-d}b \operatorname{erf}\left(\sqrt{-b^2-d}x\right) e^c + (b^2+d) \operatorname{erfi}(bx) e^{(dx^2+c)}}{2\,(b^2d+d^2)}$$

input
```
integrate(exp(d*x^2+c)*x*erfi(b*x),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-b^2 - d)*b*erf(sqrt(-b^2 - d)*x)*e^c + (b^2 + d)*erfi(b*x)*e^(d
*x^2 + c))/(b^2*d + d^2)
```

Sympy [F]

$$\int e^{c+dx^2} x\mathrm{erfi}(bx)\, dx = e^c \int xe^{dx^2} \operatorname{erfi}(bx)\, dx$$

input
```
integrate(exp(d*x**2+c)*x*erfi(b*x),x)
```

output
```
exp(c)*Integral(x*exp(d*x**2)*erfi(b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2} x\text{erfi}(bx) \, dx = \int x \,\text{erfi}\,(bx)\, e^{(dx^2+c)} \, dx$$

input
```
integrate(exp(d*x^2+c)*x*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*erfi(b*x)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2} x\text{erfi}(bx) \, dx = \int x \,\text{erfi}\,(bx)\, e^{(dx^2+c)} \, dx$$

input
```
integrate(exp(d*x^2+c)*x*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x*erfi(b*x)*e^(d*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int e^{c+dx^2} x\text{erfi}(bx) \, dx = \frac{e^{d\,x^2}\,e^c\,\text{erfi}(b\,x)}{2\,d} - \frac{b\,e^c\,\text{erf}\left(x\,\sqrt{-b^2-d}\right)}{2\,d\,\sqrt{-b^2-d}}$$

input
```
int(x*exp(c + d*x^2)*erfi(b*x),x)
```

output
```
(exp(d*x^2)*exp(c)*erfi(b*x))/(2*d) - (b*exp(c)*erf(x*(- d - b^2)^(1/2)))/
(2*d*(- d - b^2)^(1/2))
```

Reduce [F]

$$\int e^{c+dx^2} x \operatorname{erfi}(bx)\, dx = -e^c \left(\int e^{d\,x^2} \operatorname{erf}(bix)\, x dx \right) i$$

input
```
int(exp(d*x^2+c)*x*erfi(b*x),x)
```

output
```
- e**c*int(e**(d*x**2)*erf(b*i*x)*x,x)*i
```

3.262 $\int \frac{e^{c+dx^2}\mathbf{erfi}(bx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x}\,dx = \operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x},x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfi(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x}\,dx = \int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)e^{c+dx^2}}{x} \, dx$$

$$\Big\downarrow \; 6950$$

$$\int \frac{\text{erfi}(bx)e^{c+dx^2}}{x}dx$$

input
```
Int[(E^(c + d*x^2)*Erfi[b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{e^{d\,x^2+c}\,\text{erfi}\left(bx\right)}{x}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(d*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx = e^c\int \frac{e^{dx^2}\,\text{erfi}\,(bx)}{x}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x)/x,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x}\,dx=\int \frac{\operatorname{erfi}(bx)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x}\,dx=\int \frac{\mathrm{e}^{d\,x^2+c}\,\operatorname{erfi}(b\,x)}{x}\,dx$$

input
```
int((exp(c + d*x^2)*erfi(b*x))/x,x)
```

output
```
int((exp(c + d*x^2)*erfi(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x}\,dx = -e^c \left(\int \frac{e^{d\,x^2}\operatorname{erf}(bix)}{x}dx\right) i$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(b*i*x))/x,x)*i
```

3.263 $\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx = -\frac{be^{c+(b^2+d)x^2}}{\sqrt{\pi}x}-\frac{e^{c+dx^2}\text{erfi}(bx)}{2x^2}+b\sqrt{b^2+d}e^c\text{erfi}\left(\sqrt{b^2+d}x\right)+d\text{Int}\left(\frac{e^{c+dx^2}\text{erfi}(bx)}{x},x\right)$$

output
```
-b*exp(c+(b^2+d)*x^2)/Pi^(1/2)/x-1/2*exp(d*x^2+c)*erfi(b*x)/x^2+b*(b^2+d)^
(1/2)*exp(c)*erfi((b^2+d)^(1/2)*x)+d*Defer(Int)(exp(d*x^2+c)*erfi(b*x)/x,x
)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^3,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)e^{c+dx^2}}{x^3} \, dx$$

$$\Big\downarrow \text{6947}$$

$$\frac{b\int \frac{e^{\left(b^2+d\right)x^2+c}}{x^2}dx}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(2\left(b^2+d\right)\int e^{\left(b^2+d\right)x^2+c}dx-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{2633}$$

$$d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}$$

$$\Big\downarrow \text{6950}$$

$$d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}$$

input
```
Int[(E^(c + d*x^2)*Erfi[b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{e^{d\,x^2+c}\,\mathrm{erfi}\,(bx)}{x^3}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^3,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^3}\,dx = \int \frac{\mathrm{erfi}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(d*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 6.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx = e^c \int \frac{e^{dx^2}\,\text{erfi}\,(bx)}{x^3}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x)/x**3,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^3}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\operatorname{erfi}(b\,x)}{x^3}\,dx$$

input
```
int((exp(c + d*x^2)*erfi(b*x))/x^3,x)
```

output
```
int((exp(c + d*x^2)*erfi(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^3}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bix)}{x^3}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^3,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(b*i*x))/x**3,x)*i
```

3.264 $\int \frac{e^{c+dx^2}\mathbf{erfi}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^5}\,dx = -\frac{be^{c+(b^2+d)x^2}}{6\sqrt{\pi}x^3} - \frac{bde^{c+(b^2+d)x^2}}{2\sqrt{\pi}x} - \frac{b(b^2+d)\,e^{c+(b^2+d)x^2}}{3\sqrt{\pi}x} - \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{4x^4} - \frac{de^{c+dx^2}\operatorname{erfi}(bx)}{4x^2} + \frac{1}{2}bd\sqrt{b^2+d}e^c\operatorname{erfi}\left(\sqrt{b^2+d}x\right) + \frac{1}{3}b\left(b^2+d\right)^{3/2}e^c\operatorname{erfi}\left(\sqrt{b^2+d}x\right) + \frac{1}{2}d^2\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x},x\right)$$

output
```
-1/6*b*exp(c+(b^2+d)*x^2)/Pi^(1/2)/x^3-1/2*b*d*exp(c+(b^2+d)*x^2)/Pi^(1/2)
/x-1/3*b*(b^2+d)*exp(c+(b^2+d)*x^2)/Pi^(1/2)/x-1/4*exp(d*x^2+c)*erfi(b*x)/
x^4-1/4*d*exp(d*x^2+c)*erfi(b*x)/x^2+1/2*b*d*(b^2+d)^(1/2)*exp(c)*erfi((b^
2+d)^(1/2)*x)+1/3*b*(b^2+d)^(3/2)*exp(c)*erfi((b^2+d)^(1/2)*x)+1/2*d^2*Def
er(Int)(exp(d*x^2+c)*erfi(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^5,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^5, x]
```

Rubi [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)e^{c+dx^2}}{x^5}\,dx$$

$$\Big\downarrow \text{6947}$$

$$\frac{b\int \frac{e^{\left(b^2+d\right)x^2+c}}{x^4}dx}{2\sqrt{\pi}}+\frac{1}{2}d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x^3}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{3}\left(b^2+d\right)\int \frac{e^{\left(b^2+d\right)x^2+c}}{x^2}dx-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}+\frac{1}{2}d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x^3}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{b\left(\frac{2}{3}\left(b^2+d\right)\left(2\left(b^2+d\right)\int e^{\left(b^2+d\right)x^2+c}dx-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}+\frac{1}{2}d\int\frac{e^{dx^2+c}\text{erfi}(bx)}{x^3}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 2633$$

$$\frac{1}{2}d\int\frac{e^{dx^2+c}\text{erfi}(bx)}{x^3}dx+\frac{b\left(\frac{2}{3}\left(b^2+d\right)\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 6947$$

$$\frac{1}{2}d\left(\frac{b\int\frac{e^{\left(b^2+d\right)x^2+c}}{x^2}dx}{\sqrt{\pi}}+d\int\frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(\frac{2}{3}\left(b^2+d\right)\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 2643$$

$$\frac{1}{2}d\left(\frac{b\left(2\left(b^2+d\right)\int e^{\left(b^2+d\right)x^2+c}dx-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int\frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(\frac{2}{3}\left(b^2+d\right)\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 2633$$

$$\frac{1}{2}d\left(d\int\frac{e^{dx^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{2x^2}\right)+\frac{b\left(\frac{2}{3}\left(b^2+d\right)\left(\sqrt{\pi}e^c\sqrt{b^2+d}\,\text{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{4x^4}$$

$$\Big\downarrow \; 6950$$

$$\frac{1}{2}d\left(d\int \frac{e^{dx^2+c}\operatorname{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{\pi}e^c\sqrt{b^2+d}\operatorname{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}-\frac{\operatorname{erfi}(bx)e^{c+dx^2}}{2x^2}\right)+$$
$$\frac{b\left(\frac{2}{3}\left(b^2+d\right)\left(\sqrt{\pi}e^c\sqrt{b^2+d}\operatorname{erfi}\left(x\sqrt{b^2+d}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{x}\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{\operatorname{erfi}(bx)e^{c+dx^2}}{4x^4}$$

input
```
Int[(E^(c + d*x^2)*Erfi[b*x])/x^5,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\operatorname{erfi}\left(bx\right)}{x^5}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^5,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^5}\,dx=\int \frac{\operatorname{erfi}\left(bx\right)e^{\left(dx^2+c\right)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(d*x^2 + c)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 34.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = e^c \int \frac{e^{dx^2}\,\text{erfi}\,(bx)}{x^5}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x)/x**5,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 5.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\text{erfi}(b\,x)}{x^5}\,dx$$

input
```
int((exp(c + d*x^2)*erfi(b*x))/x^5,x)
```

output
```
int((exp(c + d*x^2)*erfi(b*x))/x^5, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^5}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\text{erf}(bix)}{x^5}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^5,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(b*i*x))/x**5,x)*i
```

3.265 $\int e^{c+dx^2} x^4 \textbf{erfi}(bx) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{c+dx^2} x^4 \operatorname{erfi}(bx) \, dx = \frac{b e^{c+(b^2+d)x^2}}{2d\left(b^2+d\right)^2\sqrt{\pi}} + \frac{3b e^{c+(b^2+d)x^2}}{4d^2\left(b^2+d\right)\sqrt{\pi}} - \frac{b e^{c+(b^2+d)x^2} x^2}{2d\left(b^2+d\right)\sqrt{\pi}} - \frac{3e^{c+dx^2} x \operatorname{erfi}(bx)}{4d^2} + \frac{e^{c+dx^2} x^3 \operatorname{erfi}(bx)}{2d} + \frac{3\operatorname{Int}\left(e^{c+dx^2}\operatorname{erfi}(bx), x\right)}{4d^2}$$

output
```
1/2*b*exp(c+(b^2+d)*x^2)/d/(b^2+d)^2/Pi^(1/2)+3/4*b*exp(c+(b^2+d)*x^2)/d^2
/(b^2+d)/Pi^(1/2)-1/2*b*exp(c+(b^2+d)*x^2)*x^2/d/(b^2+d)/Pi^(1/2)-3/4*exp(
d*x^2+c)*x*erfi(b*x)/d^2+1/2*exp(d*x^2+c)*x^3*erfi(b*x)/d+3/4*Defer(Int)(e
xp(d*x^2+c)*erfi(b*x),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2} x^4 \operatorname{erfi}(bx) \, dx = \int e^{c+dx^2} x^4 \operatorname{erfi}(bx) \, dx$$

input
```
Integrate[E^(c + d*x^2)*x^4*Erfi[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^4*Erfi[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{erfi}(bx) e^{c+dx^2} \, dx$$

$$\Big\downarrow \text{ 6941}$$

$$-\frac{b\int e^{(b^2+d)x^2+c}x^3dx}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(bx)dx}{2d}+\frac{x^3\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{b\left(\frac{x^2e^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{\int e^{(b^2+d)x^2+c}xdx}{b^2+d}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(bx)dx}{2d}+\frac{x^3\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{3\int e^{dx^2+c}x^2\text{erfi}(bx)dx}{2d}-\frac{b\left(\frac{x^2e^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{e^{x^2(b^2+d)+c}}{2(b^2+d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 6941}$$

$$-\frac{3\left(-\frac{b\int e^{(b^2+d)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfi}(bx)dx}{2d}+\frac{x\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{2d}-\frac{b\left(\frac{x^2e^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{e^{x^2(b^2+d)+c}}{2(b^2+d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\text{erfi}(bx)dx}{2d}-\frac{be^{x^2(b^2+d)+c}}{2\sqrt{\pi}d(b^2+d)}+\frac{x\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{2d}-\frac{b\left(\frac{x^2e^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{e^{x^2(b^2+d)+c}}{2(b^2+d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfi}(bx)e^{c+dx^2}}{2d}$$

$$\Big\downarrow \; 6935$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\text{erfi}(bx)dx}{2d}-\frac{be^{x^2(b^2+d)+c}}{2\sqrt{\pi}d(b^2+d)}+\frac{x\text{erfi}(bx)e^{c+dx^2}}{2d}\right)}{2d}-\frac{b\left(\frac{x^2e^{x^2(b^2+d)+c}}{2(b^2+d)}-\frac{e^{x^2(b^2+d)+c}}{2(b^2+d)^2}\right)}{\sqrt{\pi}d}+\frac{x^3\text{erfi}(bx)e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^4*Erfi[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{d\,x^2+c}x^4\,\text{erfi}\,(bx)\,dx$$

input
```
int(exp(d*x^2+c)*x^4*erfi(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^4*erfi(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(x^4*erfi(b*x)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 56.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx=e^c\int x^4e^{dx^2}\,\text{erfi}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**4*erfi(b*x),x)
```

output
```
exp(c)*Integral(x**4*exp(d*x**2)*erfi(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*erfi(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erfi(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\mathrm{e}^{d\,x^2+c}\,\text{erfi}(b\,x)\;dx$$

input
```
int(x^4*exp(c + d*x^2)*erfi(b*x),x)
```

output
```
int(x^4*exp(c + d*x^2)*erfi(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 320, normalized size of antiderivative = 18.82

$$\int e^{c+dx^2}x^4\text{erfi}(bx)\,dx$$
$$= \frac{e^c\left(-2e^{d\,x^2}\text{erf}(bix)\,b^4di\pi\,x^3+3e^{d\,x^2}\text{erf}(bix)\,b^4i\pi x-4e^{d\,x^2}\text{erf}(bix)\,b^2d^2i\pi\,x^3+6e^{d\,x^2}\text{erf}(bix)\,b^2di\pi x-2e^{c}\right.}{}$$

input
```
int(exp(d*x^2+c)*x^4*erfi(b*x),x)
```

output
```
(e**c*( - 2*e**(d*x**2)*erf(b*i*x)*b**4*d*i*pi*x**3 + 3*e**(d*x**2)*erf(b*
i*x)*b**4*i*pi*x - 4*e**(d*x**2)*erf(b*i*x)*b**2*d**2*i*pi*x**3 + 6*e**(d*
x**2)*erf(b*i*x)*b**2*d*i*pi*x - 2*e**(d*x**2)*erf(b*i*x)*d**3*i*pi*x**3 +
 3*e**(d*x**2)*erf(b*i*x)*d**2*i*pi*x - 2*sqrt(pi)*e**(b**2*x**2 + d*x**2)
*b**3*d*x**2 + 3*sqrt(pi)*e**(b**2*x**2 + d*x**2)*b**3 - 2*sqrt(pi)*e**(b*
*2*x**2 + d*x**2)*b*d**2*x**2 + 5*sqrt(pi)*e**(b**2*x**2 + d*x**2)*b*d - 3
*int(e**(d*x**2)*erf(b*i*x),x)*b**4*i*pi - 6*int(e**(d*x**2)*erf(b*i*x),x)
*b**2*d*i*pi - 3*int(e**(d*x**2)*erf(b*i*x),x)*d**2*i*pi))/(4*d**2*pi*(b**
4 + 2*b**2*d + d**2))
```

3.266 $\int e^{c+dx^2} x^2 \mathbf{erfi}(bx) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{c+dx^2} x^2 \text{erfi}(bx) \, dx = -\frac{b e^{c+(b^2+d)x^2}}{2d (b^2+d) \sqrt{\pi}} + \frac{e^{c+dx^2} x \text{erfi}(bx)}{2d} - \frac{\text{Int}\left(e^{c+dx^2} \text{erfi}(bx), x\right)}{2d}$$

output
```
-1/2*b*exp(c+(b^2+d)*x^2)/d/(b^2+d)/Pi^(1/2)+1/2*exp(d*x^2+c)*x*erfi(b*x)/
d-1/2*Defer(Int)(exp(d*x^2+c)*erfi(b*x),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2} x^2 \text{erfi}(bx) \, dx = \int e^{c+dx^2} x^2 \text{erfi}(bx) \, dx$$

input
```
Integrate[E^(c + d*x^2)*x^2*Erfi[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^2*Erfi[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{erfi}(bx) e^{c+dx^2} \, dx$$

$$\Big\downarrow \text{ 6941}$$

$$-\frac{b \int e^{(b^2+d)x^2+c} x dx}{\sqrt{\pi} d} - \frac{\int e^{dx^2+c} \text{erfi}(bx) dx}{2d} + \frac{x \text{erfi}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{\int e^{dx^2+c} \text{erfi}(bx) dx}{2d} - \frac{b e^{x^2(b^2+d)+c}}{2\sqrt{\pi} d (b^2+d)} + \frac{x \text{erfi}(bx) e^{c+dx^2}}{2d}$$

$$\Big\downarrow \text{ 6935}$$

$$-\frac{\int e^{dx^2+c} \text{erfi}(bx) dx}{2d} - \frac{b e^{x^2(b^2+d)+c}}{2\sqrt{\pi} d (b^2+d)} + \frac{x \text{erfi}(bx) e^{c+dx^2}}{2d}$$

input
```
Int[E^(c + d*x^2)*x^2*Erfi[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{d\,x^2+c} x^2 \,\text{erfi}\left(bx\right) dx$$

input
```
int(exp(d*x^2+c)*x^2*erfi(b*x),x)
```

output
```
int(exp(d*x^2+c)*x^2*erfi(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}x^2\text{erfi}(bx)\,dx = \int x^2\,\text{erfi}\,(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*erfi(b*x)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 10.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}x^2\text{erfi}(bx)\,dx = e^c\int x^2e^{dx^2}\,\text{erfi}\,(bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**2*erfi(b*x),x)
```

output
```
exp(c)*Integral(x**2*exp(d*x**2)*erfi(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2} x^2 \text{erfi}(bx)\, dx = \int x^2 \,\text{erfi}\,(bx)\, e^{(dx^2+c)}\, dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*erfi(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2} x^2 \text{erfi}(bx)\, dx = \int x^2 \,\text{erfi}\,(bx)\, e^{(dx^2+c)}\, dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erfi(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2} x^2 \text{erfi}(bx)\, dx = \int x^2\, \mathrm{e}^{d\,x^2+c}\, \mathrm{erfi}(b\,x)\, dx$$

input
```
int(x^2*exp(c + d*x^2)*erfi(b*x),x)
```

output
```
int(x^2*exp(c + d*x^2)*erfi(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 6.88

$$\int e^{c+dx^2}x^2\mathrm{erfi}(bx)\,dx$$
$$=\frac{e^c\left(-e^{d\,x^2}\mathrm{erf}(bix)\,b^2i\pi x-e^{d\,x^2}\mathrm{erf}(bix)\,di\pi x-\sqrt{\pi}\,e^{b^2x^2+d\,x^2}b+\left(\int e^{d\,x^2}\mathrm{erf}(bix)\,dx\right)b^2i\pi+\left(\int e^{d\,x^2}\mathrm{erf}(bi\right.\right.}{2d\pi\,(b^2+d)}$$

input
```
int(exp(d*x^2+c)*x^2*erfi(b*x),x)
```

output
```
(e**c*( - e**(d*x**2)*erf(b*i*x)*b**2*i*pi*x - e**(d*x**2)*erf(b*i*x)*d*i*
pi*x - sqrt(pi)*e**(b**2*x**2 + d*x**2)*b + int(e**(d*x**2)*erf(b*i*x),x)*
b**2*i*pi + int(e**(d*x**2)*erf(b*i*x),x)*d*i*pi))/(2*d*pi*(b**2 + d))
```

3.267 $\int e^{c+dx^2}\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int e^{c+dx^2}\text{erfi}(bx)\,dx = \text{Int}\left(e^{c+dx^2}\text{erfi}(bx),x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfi(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{c+dx^2}\text{erfi}(bx)\,dx = \int e^{c+dx^2}\text{erfi}(bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*Erfi[b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*Erfi[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)e^{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 6935}$$

$$\int \text{erfi}(bx)e^{c+dx^2}dx$$

input
```
Int[E^(c + d*x^2)*Erfi[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{d\,x^2+c}\,\text{erfi}\,(bx)\,dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x),x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfi}(bx)\,dx=\int \operatorname{erfi}\left(bx\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfi}(bx)\,dx=e^{c}\int e^{dx^2}\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x),x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfi}(bx)\,dx=\int \operatorname{erfi}\left(bx\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfi}(bx)\,dx=\int \operatorname{erfi}(bx)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int e^{c+dx^2}\operatorname{erfi}(bx)\,dx=\int \mathrm{e}^{d\,x^2+c}\,\operatorname{erfi}(b\,x)\;dx$$

input
```
int(exp(c + d*x^2)*erfi(b*x),x)
```

output
```
int(exp(c + d*x^2)*erfi(b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int e^{c+dx^2}\operatorname{erfi}(bx)\,dx = -e^c\left(\int e^{d\,x^2}\operatorname{erf}(bix)\,dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x),x)
```

output
```
- e**c*int(e**(d*x**2)*erf(b*i*x),x)*i
```

3.268 $\int \frac{e^{c+dx^2}\mathbf{erfi}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^2}\,dx = -\frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x} + \frac{be^c\operatorname{ExpIntegralEi}\left((b^2+d)\,x^2\right)}{\sqrt{\pi}} + 2d\operatorname{Int}\left(e^{c+dx^2}\operatorname{erfi}(bx),x\right)$$

output
```
-exp(d*x^2+c)*erfi(b*x)/x+b*exp(c)*Ei((b^2+d)*x^2)/Pi^(1/2)+2*d*Defer(Int)
(exp(d*x^2+c)*erfi(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^2}\,dx = \int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^2}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^2,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)e^{c+dx^2}}{x^2}dx$$

$$\Big\downarrow \text{6947}$$

$$\frac{2b\int \frac{e^{\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(bx)dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{x}$$

$$\Big\downarrow \text{2639}$$

$$2d\int e^{dx^2+c}\text{erfi}(bx)dx+\frac{be^c\,\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{x}$$

$$\Big\downarrow \text{6935}$$

$$2d\int e^{dx^2+c}\text{erfi}(bx)dx+\frac{be^c\,\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{x}$$

input
```
Int[(E^(c + d*x^2)*Erfi[b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfi}\left(bx\right)}{x^2}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^2,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^2}\,dx = \int \frac{\mathrm{erfi}\left(bx\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(d*x^2 + c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^2}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erfi}\left(bx\right)}{x^2}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x)/x**2,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^2}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\operatorname{erfi}(b\,x)}{x^2}\,dx$$

input
```
int((exp(c + d*x^2)*erfi(b*x))/x^2,x)
```

output
```
int((exp(c + d*x^2)*erfi(b*x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^2}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bix)}{x^2}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^2,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(b*i*x))/x**2,x)*i
```

3.269 $\int \frac{e^{c+dx^2}\mathbf{erfi}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erfi}(bx)}{x^4}\,dx = &-\frac{be^{c+(b^2+d)x^2}}{3\sqrt{\pi}x^2}-\frac{e^{c+dx^2}\operatorname{erfi}(bx)}{3x^3}-\frac{2de^{c+dx^2}\operatorname{erfi}(bx)}{3x}\\&+\frac{2bde^c\,\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)}{3\sqrt{\pi}}\\&+\frac{b(b^2+d)\,e^c\,\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)}{3\sqrt{\pi}}\\&+\frac{4}{3}d^2\text{Int}\left(e^{c+dx^2}\operatorname{erfi}(bx),x\right)\end{aligned}$$

output
```
-1/3*b*exp(c+(b^2+d)*x^2)/Pi^(1/2)/x^2-1/3*exp(d*x^2+c)*erfi(b*x)/x^3-2/3*
d*exp(d*x^2+c)*erfi(b*x)/x+2/3*b*d*exp(c)*Ei((b^2+d)*x^2)/Pi^(1/2)+1/3*b*(
b^2+d)*exp(c)*Ei((b^2+d)*x^2)/Pi^(1/2)+4/3*d^2*Defer(Int)(exp(d*x^2+c)*erf
i(b*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^4}\,dx$$

input

```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^4,x]
```

output

```
Integrate[(E^(c + d*x^2)*Erfi[b*x])/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{erfi}(bx)e^{c+dx^2}}{x^4}dx$$

$$\Big\downarrow \text{6947}$$

$$\frac{2b\int \frac{e^{(b^2+d)x^2+c}}{x^3}dx}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x^2}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2b\left((b^2+d)\int \frac{e^{(b^2+d)x^2+c}}{x}dx-\frac{e^{x^2(b^2+d)+c}}{2x^2}\right)}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x^2}dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(bx)}{x^2}dx+\frac{2b\left(\frac{1}{2}e^c\left(b^2+d\right)\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{6947}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(bx)dx-\frac{\text{erfi}(bx)e^{c+dx^2}}{x}\right)+\frac{2b\left(\frac{1}{2}e^c\left(b^2+d\right)\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2}{3}d\left(2d\int e^{dx^2+c}\text{erfi}(bx)dx+\frac{be^c\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{x}\right)+\frac{2b\left(\frac{1}{2}e^c\left(b^2+d\right)\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{3x^3}$$

$$\Big\downarrow \text{6935}$$

$$\frac{2}{3}d\left(2d\int e^{dx^2+c}\text{erfi}(bx)dx+\frac{be^c\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)}{\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{x}\right)+\frac{2b\left(\frac{1}{2}e^c\left(b^2+d\right)\text{ExpIntegralEi}\left(\left(b^2+d\right)x^2\right)-\frac{e^{x^2\left(b^2+d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{\text{erfi}(bx)e^{c+dx^2}}{3x^3}$$

input
```
Int[(E^(c + d*x^2)*Erfi[b*x])/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfi}\left(bx\right)}{x^4}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^4,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^4}\,dx = \int \frac{\mathrm{erfi}\left(bx\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^4,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(d*x^2 + c)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 14.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^4}\,dx = e^c \int \frac{e^{dx^2}\,\mathrm{erfi}\left(bx\right)}{x^4}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x)/x**4,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(d*x^2 + c)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^4}\,dx = \int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfi}(b\,x)}{x^4}\,dx$$

input
```
int((exp(c + d*x^2)*erfi(b*x))/x^4,x)
```

output
```
int((exp(c + d*x^2)*erfi(b*x))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(bx)}{x^4}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\mathrm{erf}(bix)}{x^4}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x)/x^4,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(b*i*x))/x**4,x)*i
```

3.270 $\int e^{-b^2x^2}x^5\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int e^{-b^2x^2}x^5\text{erfi}(bx)\,dx=\frac{2x}{b^5\sqrt{\pi}}+\frac{2x^3}{3b^3\sqrt{\pi}}+\frac{x^5}{5b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{b^6}-\frac{e^{-b^2x^2}x^2\text{erfi}(bx)}{b^4}-\frac{e^{-b^2x^2}x^4\text{erfi}(bx)}{2b^2}$$

output
```
2*x/b^5/Pi^(1/2)+2/3*x^3/b^3/Pi^(1/2)+1/5*x^5/b/Pi^(1/2)-erfi(b*x)/b^6/exp
(b^2*x^2)-x^2*erfi(b*x)/b^4/exp(b^2*x^2)-1/2*x^4*erfi(b*x)/b^2/exp(b^2*x^2
)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int e^{-b^2x^2}x^5\text{erfi}(bx)\,dx=\frac{\frac{60bx+20b^3x^3+6b^5x^5}{\sqrt{\pi}}-15e^{-b^2x^2}(2+2b^2x^2+b^4x^4)\,\text{erfi}(bx)}{30b^6}$$

input
```
Integrate[(x^5*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
((60*b*x + 20*b^3*x^3 + 6*b^5*x^5)/Sqrt[Pi] - (15*(2 + 2*b^2*x^2 + b^4*x^4
)*Erfi[b*x])/E^(b^2*x^2))/(30*b^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6941, 15, 6941, 15, 6938, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{-b^2x^2}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erfi}(bx)dx}{b^2}+\frac{\int x^4dx}{\sqrt{\pi}b}-\frac{x^4e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\int e^{-b^2x^2}x^3\text{erfi}(bx)dx}{b^2}-\frac{x^4e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{6941}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erfi}(bx)dx}{b^2}+\frac{\int x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\left(\frac{\int e^{-b^2x^2}x\text{erfi}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^3}{3\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{6938}$$

$$\frac{2\left(\frac{\frac{\int 1dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^3}{3\sqrt{\pi}b}\right)}{b^2}-\frac{x^4e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^5}{5\sqrt{\pi}b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{x^4 e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{2\left(-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{\frac{x}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}}{b^2}+\frac{x^3}{3\sqrt{\pi}b}\right)}{b^2}+\frac{x^5}{5\sqrt{\pi}b}$$

input
```
Int[(x^5*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x^5/(5*b*Sqrt[Pi]) - (x^4*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (2*(x^3/(3*b*Sq
rt[Pi]) - (x^2*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x/(b*Sqrt[Pi]) - Erfi[b*x
]/(2*b^2*E^(b^2*x^2)))/b^2))/b^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\left(6b^{5}x^{5}{\mathrm e}^{b^{2}x^{2}}-15\,\mathrm{erfi}\left(b x \right)x^{4}\sqrt{\pi}\,b^{4}+20b^{3}x^{3}{\mathrm e}^{b^{2}x^{2}}-30\,\mathrm{erfi}\left(b x \right)x^{2}\sqrt{\pi}\,b^{2}+60\,{\mathrm e}^{b^{2}x^{2}}b x -30\,\mathrm{erfi}\left(b x \right)\sqrt{\pi}\right){\mathrm e}^{-b^{2}x^{2}}}{30b^{6}\sqrt{\pi}}$	103
parallelrisch	$\frac{\left(6b^{5}x^{5}{\mathrm e}^{b^{2}x^{2}}-15\,\mathrm{erfi}\left(b x \right)x^{4}\sqrt{\pi}\,b^{4}+20b^{3}x^{3}{\mathrm e}^{b^{2}x^{2}}-30\,\mathrm{erfi}\left(b x \right)x^{2}\sqrt{\pi}\,b^{2}+60\,{\mathrm e}^{b^{2}x^{2}}b x -30\,\mathrm{erfi}\left(b x \right)\sqrt{\pi}\right){\mathrm e}^{-b^{2}x^{2}}}{30b^{6}\sqrt{\pi}}$	103

input
```
int(x^5*erfi(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(6*b^5*x^5*exp(b^2*x^2)-15*erfi(b*x)*x^4*Pi^(1/2)*b^4+20*b^3*x^3*exp(
b^2*x^2)-30*erfi(b*x)*x^2*Pi^(1/2)*b^2+60*exp(b^2*x^2)*b*x-30*erfi(b*x)*Pi
^(1/2))/b^6/Pi^(1/2)/exp(b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.74

$$\int e^{-b^2x^2}x^5\mathrm{erfi}(bx)\,dx = \frac{\left(2\,\sqrt{\pi}(3\,b^5x^5+10\,b^3x^3+30\,bx)e^{(b^2x^2)}-15\,(2\,\pi+\pi b^4x^4+2\,\pi b^2x^2)\,\mathrm{erfi}\,(bx)\right)e^{(-b^2x^2)}}{30\,\pi b^6}$$

input
```
integrate(x^5*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/30*(2*sqrt(pi)*(3*b^5*x^5 + 10*b^3*x^3 + 30*b*x)*e^(b^2*x^2) - 15*(2*pi
+ pi*b^4*x^4 + 2*pi*b^2*x^2)*erfi(b*x))*e^(-b^2*x^2)/(pi*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^5\text{erfi}(bx)\,dx=\text{Timed out}$$

input
```
integrate(x**5*erfi(b*x)/exp(b**2*x**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-b^2x^2}x^5\text{erfi}(bx)\,dx=\int x^5\,\text{erfi}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^5*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^5*erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^5\text{erfi}(bx)\,dx=\int x^5\,\text{erfi}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^5*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int e^{-b^2x^2}x^5\mathrm{erfi}(bx)\,dx = \frac{3\,b^4\,x^5+10\,b^2\,x^3+30\,x}{15\,b^5\,\sqrt{\pi}} - \mathrm{erfi}(b\,x)\,\left(\frac{\mathrm{e}^{-b^2\,x^2}}{b^6}+\frac{x^4\,\mathrm{e}^{-b^2\,x^2}}{2\,b^2}+\frac{x^2\,\mathrm{e}^{-b^2\,x^2}}{b^4}\right)$$

input
```
int(x^5*exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
(30*x + 10*b^2*x^3 + 3*b^4*x^5)/(15*b^5*pi^(1/2)) - erfi(b*x)*(exp(-b^2*x^
2)/b^6 + (x^4*exp(-b^2*x^2))/(2*b^2) + (x^2*exp(-b^2*x^2))/b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int e^{-b^2x^2}x^5\mathrm{erfi}(bx)\,dx = \frac{15\,\mathrm{erf}(bix)\,b^4i\pi\,x^4+30\,\mathrm{erf}(bix)\,b^2i\pi\,x^2+30\,\mathrm{erf}(bix)\,i\pi+6\sqrt{\pi}\,e^{b^2x^2}b^5x^5+20\sqrt{\pi}\,e^{b^2x^2}b^3x^3+60\sqrt{\pi}\,e^{b^2x^2}b}{30e^{b^2x^2}b^6\pi}$$

input
```
int(x^5*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
(15*erf(b*i*x)*b**4*i*pi*x**4 + 30*erf(b*i*x)*b**2*i*pi*x**2 + 30*erf(b*i*
x)*i*pi + 6*sqrt(pi)*e**(b**2*x**2)*b**5*x**5 + 20*sqrt(pi)*e**(b**2*x**2)
*b**3*x**3 + 60*sqrt(pi)*e**(b**2*x**2)*b*x)/(30*e**(b**2*x**2)*b**6*pi)
```

3.271 $\int e^{-b^2x^2}x^3\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx=\frac{x}{b^3\sqrt{\pi}}+\frac{x^3}{3b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^4}-\frac{e^{-b^2x^2}x^2\text{erfi}(bx)}{2b^2}$$

output
```
x/b^3/Pi^(1/2)+1/3*x^3/b/Pi^(1/2)-1/2*erfi(b*x)/b^4/exp(b^2*x^2)-1/2*x^2*e
rfi(b*x)/b^2/exp(b^2*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx=\frac{\frac{2bx(3+b^2x^2)}{\sqrt{\pi}}-3e^{-b^2x^2}(1+b^2x^2)\,\text{erfi}(bx)}{6b^4}$$

input
```
Integrate[(x^3*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
((2*b*x*(3 + b^2*x^2))/Sqrt[Pi] - (3*(1 + b^2*x^2)*Erfi[b*x])/E^(b^2*x^2))
/(6*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6941, 15, 6938, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-b^2x^2}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$\frac{\int e^{-b^2x^2}x\text{erfi}(bx)dx}{b^2}+\frac{\int x^2dx}{\sqrt{\pi}b}-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int e^{-b^2x^2}x\text{erfi}(bx)dx}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^3}{3\sqrt{\pi}b}$$

$$\Big\downarrow \text{6938}$$

$$\frac{\frac{\int 1dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}}{b^2}-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^3}{3\sqrt{\pi}b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{x^2e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{\frac{x}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}}{b^2}+\frac{x^3}{3\sqrt{\pi}b}$$

input
```
Int[(x^3*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x^3/(3*b*Sqrt[Pi]) - (x^2*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x/(b*Sqrt[Pi])
 - Erfi[b*x]/(2*b^2*E^(b^2*x^2)))/b^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{\left(2b^{3}x^{3}{\mathrm e}^{b^{2}x^{2}}-3\,\operatorname{erfi}\left(b x \right) x^{2}\sqrt{\pi}\, b^{2}+6\,{\mathrm e}^{b^{2}x^{2}}b x -3\,\operatorname{erfi}\left(b x \right)\sqrt{\pi}\right){\mathrm e}^{-b^{2}x^{2}}}{6\sqrt{\pi}\, b^{4}}$	72
parallelrisch	$\frac{\left(2b^{3}x^{3}{\mathrm e}^{b^{2}x^{2}}-3\,\operatorname{erfi}\left(b x \right) x^{2}\sqrt{\pi}\, b^{2}+6\,{\mathrm e}^{b^{2}x^{2}}b x -3\,\operatorname{erfi}\left(b x \right)\sqrt{\pi}\right){\mathrm e}^{-b^{2}x^{2}}}{6\sqrt{\pi}\, b^{4}}$	72

input
```
int(x^3*erfi(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b^3*x^3*exp(b^2*x^2)-3*erfi(b*x)*x^2*Pi^(1/2)*b^2+6*exp(b^2*x^2)*b*
x-3*erfi(b*x)*Pi^(1/2))/Pi^(1/2)/b^4/exp(b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx = \frac{\left(2\sqrt{\pi}(b^3x^3+3\,bx)e^{(b^2x^2)} - 3\,(\pi+\pi b^2x^2)\,\text{erfi}\,(bx)\right)e^{(-b^2x^2)}}{6\,\pi b^4}$$

input
```
integrate(x^3*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(pi)*(b^3*x^3 + 3*b*x)*e^(b^2*x^2) - 3*(pi + pi*b^2*x^2)*erfi(b
*x))*e^(-b^2*x^2)/(pi*b^4)
```

Sympy [A] (verification not implemented)

Time = 22.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx = \begin{cases} \frac{x^3}{3\sqrt{\pi}b} - \frac{x^2e^{-b^2x^2}\,\text{erfi}\,(bx)}{2b^2} + \frac{x}{\sqrt{\pi}b^3} - \frac{e^{-b^2x^2}\,\text{erfi}\,(bx)}{2b^4} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*erfi(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((x**3/(3*sqrt(pi)*b) - x**2*exp(-b**2*x**2)*erfi(b*x)/(2*b**2) +
 x/(sqrt(pi)*b**3) - exp(-b**2*x**2)*erfi(b*x)/(2*b**4), Ne(b, 0)), (0, Tr
ue))
```

Maxima [F]

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx = \int x^3\,\text{erfi}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^3*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^3*erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx=\int x^3\,\text{erfi}\,(bx)\,e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^3*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx=\frac{\frac{b^2\,x^3}{3}+x}{b^3\,\sqrt{\pi}}-\text{erfi}(b\,x)\,\left(\frac{\mathrm{e}^{-b^2\,x^2}}{2\,b^4}+\frac{x^2\,\mathrm{e}^{-b^2\,x^2}}{2\,b^2}\right)$$

input
```
int(x^3*exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
(x + (b^2*x^3)/3)/(b^3*pi^(1/2)) - erfi(b*x)*(exp(-b^2*x^2)/(2*b^4) + (x^2
*exp(-b^2*x^2))/(2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int e^{-b^2x^2}x^3\text{erfi}(bx)\,dx=\frac{3\,\text{erf}(bix)\,b^2i\pi\,x^2+3\,\text{erf}(bix)\,i\pi+2\sqrt{\pi}\,e^{b^2x^2}b^3x^3+6\sqrt{\pi}\,e^{b^2x^2}bx}{6e^{b^2x^2}b^4\pi}$$

input
```
int(x^3*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
(3*erf(b*i*x)*b**2*i*pi*x**2 + 3*erf(b*i*x)*i*pi + 2*sqrt(pi)*e**(b**2*x**
2)*b**3*x**3 + 6*sqrt(pi)*e**(b**2*x**2)*b*x)/(6*e**(b**2*x**2)*b**4*pi)
```

3.272 $\int e^{-b^2x^2}x\text{erfi}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int e^{-b^2x^2}x\text{erfi}(bx)\,dx=\frac{x}{b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

output
```
x/b/Pi^(1/2)-1/2*erfi(b*x)/b^2/exp(b^2*x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}x\text{erfi}(bx)\,dx=\frac{x}{b\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

input
```
Integrate[(x*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x/(b*Sqrt[Pi]) - Erfi[b*x]/(2*b^2*E^(b^2*x^2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6938, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-b^2x^2}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6938}$$

$$\frac{\int 1dx}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{\sqrt{\pi}b}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

input
```
Int[(x*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x/(b*Sqrt[Pi]) - Erfi[b*x]/(2*b^2*E^(b^2*x^2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\left(2\,\mathrm{e}^{b^{2}x^{2}} b x-\operatorname{erfi}\left(b x\right)\sqrt{\pi}\right)\mathrm{e}^{-b^{2}x^{2}}}{2\sqrt{\pi}\,b^{2}}$	41
parallelrisch	$\frac{\left(2\,\mathrm{e}^{b^{2}x^{2}} b x-\operatorname{erfi}\left(b x\right)\sqrt{\pi}\right)\mathrm{e}^{-b^{2}x^{2}}}{2\sqrt{\pi}\,b^{2}}$	41

input
```
int(x*erfi(b*x)/exp(b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*exp(b^2*x^2)*b*x-erfi(b*x)*Pi^(1/2))/Pi^(1/2)/b^2/exp(b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int e^{-b^2x^2}x\operatorname{erfi}(bx)\,dx = \frac{\left(2\sqrt{\pi}bxe^{\left(b^2x^2\right)} - \pi\operatorname{erfi}\left(bx\right)\right)e^{\left(-b^2x^2\right)}}{2\,\pi b^2}$$

input
```
integrate(x*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(pi)*b*x*e^(b^2*x^2) - pi*erfi(b*x))*e^(-b^2*x^2)/(pi*b^2)
```

Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int e^{-b^2x^2}x\operatorname{erfi}(bx)\,dx = \begin{cases} \frac{x}{\sqrt{\pi}b} - \frac{e^{-b^2x^2}\operatorname{erfi}\left(bx\right)}{2b^2} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*erfi(b*x)/exp(b**2*x**2),x)
```

output
```
Piecewise((x/(sqrt(pi)*b) - exp(-b**2*x**2)*erfi(b*x)/(2*b**2), Ne(b, 0)),
 (0, True))
```

Maxima [F]

$$\int e^{-b^2x^2}x\text{erfi}(bx)\,dx = \int x\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x*erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x\text{erfi}(bx)\,dx = \int x\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x*erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int e^{-b^2x^2}x\text{erfi}(bx)\,dx = \frac{x}{b\,\sqrt{\pi}} - \frac{e^{-b^2\,x^2}\,\text{erfi}(b\,x)}{2\,b^2}$$

input
```
int(x*exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
x/(b*pi^(1/2)) - (exp(-b^2*x^2)*erfi(b*x))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int e^{-b^2x^2}x\text{erfi}(bx)\,dx=\frac{\text{erf}(bix)\,i\pi+2\sqrt{\pi}\,e^{b^2x^2}bx}{2e^{b^2x^2}b^2\pi}$$

input
```
int(x*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
(erf(b*i*x)*i*pi + 2*sqrt(pi)*e**(b**2*x**2)*b*x)/(2*e**(b**2*x**2)*b**2*p
i)
```

3.273 $\int \frac{e^{-b^2x^2}\mathbf{erfi}(bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
2*b*x*hypergeom([1/2, 1],[3/2, 3/2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x}\,dx = \frac{2bx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x),x]
```

output
```
(2*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6944}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx$$
$$\Big\downarrow \text{ 6944}$$
$$\frac{2bx\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x),x]
```

output
```
(2*b*x*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 6944
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b
*E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, (-b^2)*x^2], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int \frac{\text{erfi}\,(bx)\,\mathrm{e}^{-b^2x^2}}{x}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x,x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2)/x,x)
```

Fricas [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2)/x, x)
```

Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = \frac{2bx{}_2F_2\left(\begin{matrix}\frac{1}{2},1\\ \frac{3}{2},\frac{3}{2}\end{matrix}\middle|-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x,x)
```

output
```
2*b*x*hyper((1/2, 1), (3/2, 3/2), -b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erfi}(b\,x)}{x}\,dx$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x,x)
```

output
```
int((exp(-b^2*x^2)*erfi(b*x))/x, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\,dx = -\left(\int \frac{\text{erf}(bix)}{e^{b^2x^2}x}dx\right)i$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x,x)
```

output
```
- int(erf(b*i*x)/(e**(b**2*x**2)*x),x)*i
```

3.274 $\int \frac{e^{-b^2x^2}\mathbf{erfi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = -\frac{b}{\sqrt{\pi}x} - \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{2x^2} - \frac{2b^3x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-b/Pi^(1/2)/x-1/2*erfi(b*x)/exp(b^2*x^2)/x^2-2*b^3*x*hypergeom([1/2, 1],[3
/2, 3/2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = -\frac{2b\,{}_2F_2\left(-\frac{1}{2},1;\frac{1}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}x}$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^3),x]
```

output
```
(-2*b*HypergeometricPFQ[{-1/2, 1}, {1/2, 3/2}, -(b^2*x^2)])/(Sqrt[Pi]*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6947, 15, 6944}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}\,dx$$

$$\Big\downarrow \text{6947}$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}dx\right)+\frac{b\int \frac{1}{x^2}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$b^2\left(-\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}dx\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

$$\Big\downarrow \text{6944}$$

$$-\frac{2b^3x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x^3),x]
```

output
```
-(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2) - (2*b^3*x*Hypergeometri
cPFQ[{1/2, 1}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6944
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b
*E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, (-b^2)*x^2], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)\,\mathrm{e}^{-b^2x^2}}{x^3}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^3,x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2)/x^3,x)
```

Fricas [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^3}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^3,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Sympy [A] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.42

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = -\frac{2b\,{}_2F_2\left(\begin{matrix} -\frac{1}{2}, 1 \\ \frac{1}{2}, \frac{3}{2} \end{matrix}\middle| -b^2x^2\right)}{\sqrt{\pi}x}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x**3,x)
```

output
```
-2*b*hyper((-1/2, 1), (1/2, 3/2), -b**2*x**2)/(sqrt(pi)*x)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^3}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^3}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}\,dx = \int \frac{e^{-b^2\,x^2}\,\text{erfi}(b\,x)}{x^3}\,dx$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x^3,x)
```

output
```
int((exp(-b^2*x^2)*erfi(b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}\,dx = \frac{\text{erf}(bix)\,i\pi - 2\sqrt{\pi}\,e^{b^2x^2}bx + 2e^{b^2x^2}\left(\int \frac{\text{erf}(bix)}{e^{b^2x^2}x}dx\right)b^2i\pi\,x^2}{2e^{b^2x^2}\pi\,x^2}$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^3,x)
```

output
```
(erf(b*i*x)*i*pi - 2*sqrt(pi)*e**(b**2*x**2)*b*x + 2*e**(b**2*x**2)*int(er
f(b*i*x)/(e**(b**2*x**2)*x),x)*b**2*i*pi*x**2)/(2*e**(b**2*x**2)*pi*x**2)
```

3.275 $\int \frac{e^{-b^2x^2}\mathbf{erfi}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = -\frac{b}{6\sqrt{\pi}x^3}+\frac{b^3}{2\sqrt{\pi}x}-\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^4}+\frac{b^2e^{-b^2x^2}\operatorname{erfi}(bx)}{4x^2}+\frac{b^5x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-1/6*b/Pi^(1/2)/x^3+1/2*b^3/Pi^(1/2)/x-1/4*erfi(b*x)/exp(b^2*x^2)/x^4+1/4*
b^2*erfi(b*x)/exp(b^2*x^2)/x^2+b^5*x*hypergeom([1/2, 1],[3/2, 3/2],-b^2*x^
2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = -\frac{2b\,{}_2F_2\left(-\frac{3}{2},1;-\frac{1}{2},\frac{3}{2};-b^2x^2\right)}{3\sqrt{\pi}x^3}$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^5),x]
```

output
```
(-2*b*HypergeometricPFQ[{-3/2, 1}, {-1/2, 3/2}, -(b^2*x^2)])/(3*Sqrt[Pi]*x
^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6947, 15, 6947, 15, 6944}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^5}dx$$

$$\Big\downarrow \text{6947}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}dx+\frac{b\int \frac{1}{x^4}dx}{2\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{4x^4}-\frac{b}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{6947}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}dx\right)+\frac{b\int \frac{1}{x^2}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{4x^4}-\frac{b}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}b^2\left(b^2\left(-\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x}dx\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{4x^4}-\frac{b}{6\sqrt{\pi}x^3}$$

$$\Big\downarrow \text{6944}$$

$$-\frac{1}{2}b^2\left(-\frac{2b^3x\,{}_2F_2\left(\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{4x^4}-\frac{b}{6\sqrt{\pi}x^3}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x^5),x]
```

output
```
-1/6*b/(Sqrt[Pi]*x^3) - Erfi[b*x]/(4*E^(b^2*x^2)*x^4) - (b^2*(-(b/(Sqrt[Pi
]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2) - (2*b^3*x*HypergeometricPFQ[{1/2, 1
}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi]))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6944
```
Int[(E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)])/(x_), x_Symbol] :> Simp[2*b
*E^c*(x/Sqrt[Pi])*HypergeometricPFQ[{1/2, 1}, {3/2, 3/2}, (-b^2)*x^2], x] /
; FreeQ[{b, c, d}, x] && EqQ[d, -b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\text{erfi}\left(bx\right)\text{e}^{-b^2x^2}}{x^5}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^5,x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2)/x^5,x)
```

Fricas [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^5}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^5,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2)/x^5, x)
```

Sympy [A] (verification not implemented)

Time = 63.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = -\frac{2b\,{}_2F_2\left(\begin{matrix} -\frac{3}{2}, 1 \\ -\frac{1}{2}, \frac{3}{2} \end{matrix}\middle| -b^2x^2\right)}{3\sqrt{\pi}x^3}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x**5,x)
```

output
```
-2*b*hyper((-3/2, 1), (-1/2, 3/2), -b**2*x**2)/(3*sqrt(pi)*x**3)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^5}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^5,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^5, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}}{x^5}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\mathrm{e}^{-b^2\,x^2}\,\text{erfi}(b\,x)}{x^5}\,dx$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x^5,x)
```

output
```
int((exp(-b^2*x^2)*erfi(b*x))/x^5, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^5}\,dx = \frac{-3\,\text{erf}(bix)\,b^2i\pi\,x^2+3\,\text{erf}(bix)\,i\pi+6\sqrt{\pi}\,e^{b^2x^2}b^3x^3-2\sqrt{\pi}\,e^{b^2x^2}bx-6e^{b^2x^2}\left(\int\frac{\text{erf}(bix)}{e^{b^2x^2}x}dx\right)b^4i\pi\,x^4}{12e^{b^2x^2}\pi\,x^4}$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^5,x)
```

output
```
( - 3*erf(b*i*x)*b**2*i*pi*x**2 + 3*erf(b*i*x)*i*pi + 6*sqrt(pi)*e**(b**2*
x**2)*b**3*x**3 - 2*sqrt(pi)*e**(b**2*x**2)*b*x - 6*e**(b**2*x**2)*int(erf
(b*i*x)/(e**(b**2*x**2)*x),x)*b**4*i*pi*x**4)/(12*e**(b**2*x**2)*pi*x**4)
```

3.276 $\int e^{-b^2x^2}x^6\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx=\frac{15x^2}{8b^5\sqrt{\pi}}+\frac{5x^4}{8b^3\sqrt{\pi}}+\frac{x^6}{6b\sqrt{\pi}}-\frac{15e^{-b^2x^2}x\text{erfi}(bx)}{8b^6}-\frac{5e^{-b^2x^2}x^3\text{erfi}(bx)}{4b^4}-\frac{e^{-b^2x^2}x^5\text{erfi}(bx)}{2b^2}+\frac{15x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{8b^5\sqrt{\pi}}$$

output
```
15/8*x^2/b^5/Pi^(1/2)+5/8*x^4/b^3/Pi^(1/2)+1/6*x^6/b/Pi^(1/2)-15/8*x*erfi(
b*x)/b^6/exp(b^2*x^2)-5/4*x^3*erfi(b*x)/b^4/exp(b^2*x^2)-1/2*x^5*erfi(b*x)
/b^2/exp(b^2*x^2)+15/8*x^2*hypergeom([1, 1],[3/2, 2],-b^2*x^2)/b^5/Pi^(1/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx=\frac{x^2\left(9+3b^2x^2+4b^4x^4-9\,{}_2F_2\left(1,1;-\frac{3}{2},2;-b^2x^2\right)\right)}{24b^5\sqrt{\pi}}$$

input
```
Integrate[(x^6*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
(x^2*(9 + 3*b^2*x^2 + 4*b^4*x^4 - 9*HypergeometricPFQ[{1, 1}, {-3/2, 2}, -
(b^2*x^2)]))/(24*b^5*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6941, 15, 6941, 15, 6941, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 e^{-b^2x^2}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$\frac{5\int e^{-b^2x^2}x^4\text{erfi}(bx)dx}{2b^2}+\frac{\int x^5dx}{\sqrt{\pi}b}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{5\int e^{-b^2x^2}x^4\text{erfi}(bx)dx}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^6}{6\sqrt{\pi}b}$$

$$\Big\downarrow \text{6941}$$

$$\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}+\frac{\int x^3dx}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^6}{6\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$\frac{5\left(\frac{3\int e^{-b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^6}{6\sqrt{\pi}b}$$

$$\Big\downarrow \text{6941}$$

$$\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfi}(bx)dx}{2b^2}+\frac{\int xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^6}{6\sqrt{\pi}b}$$

$$\Big\downarrow \textbf{15}$$

$$\frac{5\left(\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfi}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^6}{6\sqrt{\pi}b}$$

$$\Big\downarrow \textbf{6932}$$

$$\frac{5\left(\frac{3\left(\frac{x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}\right)}{2b^2}-\frac{x^5e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^6}{6\sqrt{\pi}b}$$

input
```
Int[(x^6*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x^6/(6*b*Sqrt[Pi]) - (x^5*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (5*(x^4/(4*b*Sq
rt[Pi]) - (x^3*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(x^2/(2*b*Sqrt[Pi]) - (
x*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, -(b^2*x^2)])/(2*b*Sqrt[Pi])))/(2*b^2)))/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^6 \operatorname{erfi}(bx) \mathrm{e}^{-b^2x^2} dx$$

input
```
int(x^6*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^6*erfi(b*x)/exp(b^2*x^2),x)
```

Fricas [F]

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx = \int x^6 \operatorname{erfi}(bx)\, e^{(-b^2x^2)}\,dx$$

input
```
integrate(x^6*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
integral(x^6*erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx = \text{Timed out}$$

input
```
integrate(x**6*erfi(b*x)/exp(b**2*x**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx = \int x^6\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^6*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^6*erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx = \int x^6\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^6*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^6*erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx = \int x^6\,\mathrm{e}^{-b^2\,x^2}\,\text{erfi}(b\,x)\,dx$$

input
```
int(x^6*exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
int(x^6*exp(-b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2}x^6\text{erfi}(bx)\,dx$$

$$= \frac{12\,\text{erf}(bix)\,b^4 i\pi\,x^5 + 30\,\text{erf}(bix)\,b^2 i\pi\,x^3 + 45\,\text{erf}(bix)\,i\pi x + 4\sqrt{\pi}\,e^{b^2x^2}b^5x^6 + 15\sqrt{\pi}\,e^{b^2x^2}b^3x^4 + 45\sqrt{\pi}\,e^{b^2x}}{24e^{b^2x^2}b^6\pi}$$

input
```
int(x^6*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
(12*erf(b*i*x)*b**4*i*pi*x**5 + 30*erf(b*i*x)*b**2*i*pi*x**3 + 45*erf(b*i*
x)*i*pi*x + 4*sqrt(pi)*e**(b**2*x**2)*b**5*x**6 + 15*sqrt(pi)*e**(b**2*x**
2)*b**3*x**4 + 45*sqrt(pi)*e**(b**2*x**2)*b*x**2 - 45*e**(b**2*x**2)*int(e
rf(b*i*x)/e**(b**2*x**2),x)*i*pi)/(24*e**(b**2*x**2)*b**6*pi)
```

3.277 $\int e^{-b^2x^2} x^4 \mathbf{erfi}(bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int e^{-b^2x^2} x^4 \operatorname{erfi}(bx)\, dx = \frac{3x^2}{4b^3\sqrt{\pi}} + \frac{x^4}{4b\sqrt{\pi}} - \frac{3e^{-b^2x^2} x \operatorname{erfi}(bx)}{4b^4} - \frac{e^{-b^2x^2} x^3 \operatorname{erfi}(bx)}{2b^2} + \frac{3x^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{4b^3\sqrt{\pi}}$$

output
```
3/4*x^2/b^3/Pi^(1/2)+1/4*x^4/b/Pi^(1/2)-3/4*x*erfi(b*x)/b^4/exp(b^2*x^2)-1
/2*x^3*erfi(b*x)/b^2/exp(b^2*x^2)+3/4*x^2*hypergeom([1, 1],[3/2, 2],-b^2*x
^2)/b^3/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.39

$$\int e^{-b^2x^2} x^4 \operatorname{erfi}(bx)\, dx = \frac{x^2\left(1 + b^2x^2 - {}_2F_2\left(1,1;-\frac{1}{2},2;-b^2x^2\right)\right)}{4b^3\sqrt{\pi}}$$

input
```
Integrate[(x^4*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
(x^2*(1 + b^2*x^2 - HypergeometricPFQ[{1, 1}, {-1/2, 2}, -(b^2*x^2)]))/(4*
b^3*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6941, 15, 6941, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-b^2x^2} \text{erfi}(bx) \, dx$$

$$\Big\downarrow \text{6941}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}+\frac{\int x^3dx}{\sqrt{\pi}b}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\int e^{-b^2x^2}x^2\text{erfi}(bx)dx}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6941}$$

$$\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfi}(bx)dx}{2b^2}+\frac{\int xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int e^{-b^2x^2}\text{erfi}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}$$

$$\Big\downarrow \text{6932}$$

$$\frac{3\left(\frac{x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}\right)}{2b^2}-\frac{x^3e^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^4}{4\sqrt{\pi}b}$$

input
```
Int[(x^4*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x^4/(4*b*Sqrt[Pi]) - (x^3*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (3*(x^2/(2*b*Sq
rt[Pi]) - (x*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, -(b^2*x^2)])/(2*b*Sqrt[Pi])))/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^4 \operatorname{erfi}(bx)\,\mathrm{e}^{-b^2x^2}\,dx$$

input
```
int(x^4*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^4*erfi(b*x)/exp(b^2*x^2),x)
```

Fricas [F]

$$\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx = \int x^4\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^4*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
integral(x^4*erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx = \text{Timed out}$$

input
```
integrate(x**4*erfi(b*x)/exp(b**2*x**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx = \int x^4\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^4*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^4*erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^4*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\mathrm{e}^{-b^2\,x^2}\,\text{erfi}(b\,x)\,dx$$

input
```
int(x^4*exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
int(x^4*exp(-b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2}x^4\text{erfi}(bx)\,dx$$
$$=\frac{2\,\text{erf}\,(bix)\,b^2i\pi\,x^3+3\,\text{erf}\,(bix)\,i\pi x+\sqrt{\pi}\,e^{b^2x^2}b^3x^4+3\sqrt{\pi}\,e^{b^2x^2}b\,x^2-3e^{b^2x^2}\left(\int\frac{\text{erf}(bix)}{e^{b^2x^2}}dx\right)i\pi}{4e^{b^2x^2}b^4\pi}$$

input
```
int(x^4*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
(2*erf(b*i*x)*b**2*i*pi*x**3 + 3*erf(b*i*x)*i*pi*x + sqrt(pi)*e**(b**2*x**
2)*b**3*x**4 + 3*sqrt(pi)*e**(b**2*x**2)*b*x**2 - 3*e**(b**2*x**2)*int(erf
(b*i*x)/e**(b**2*x**2),x)*i*pi)/(4*e**(b**2*x**2)*b**4*pi)
```

3.278 $\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 70

$$\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx=\frac{x^2}{2b\sqrt{\pi}}-\frac{e^{-b^2x^2}x\text{erfi}(bx)}{2b^2}+\frac{x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2b\sqrt{\pi}}$$

output
```
1/2*x^2/b/Pi^(1/2)-1/2*x*erfi(b*x)/b^2/exp(b^2*x^2)+1/2*x^2*hypergeom([1,
1],[3/2, 2],-b^2*x^2)/b/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx=\frac{x^2\left(1-{}_2F_2\left(1,1;\frac{1}{2},2;-b^2x^2\right)\right)}{2b\sqrt{\pi}}$$

input
```
Integrate[(x^2*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
(x^2*(1 - HypergeometricPFQ[{1, 1}, {1/2, 2}, -(b^2*x^2)]))/(2*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6941, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-b^2x^2}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$\frac{\int e^{-b^2x^2}\text{erfi}(bx)dx}{2b^2}+\frac{\int xdx}{\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int e^{-b^2x^2}\text{erfi}(bx)dx}{2b^2}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}$$

$$\Big\downarrow \text{6932}$$

$$\frac{x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}b}-\frac{xe^{-b^2x^2}\text{erfi}(bx)}{2b^2}+\frac{x^2}{2\sqrt{\pi}b}$$

input
```
Int[(x^2*Erfi[b*x])/E^(b^2*x^2),x]
```

output
```
x^2/(2*b*Sqrt[Pi]) - (x*Erfi[b*x])/(2*b^2*E^(b^2*x^2)) + (x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/(2*b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int x^{2} \operatorname{erfi}\left(b x\right) \mathrm{e}^{-b^{2} x^{2}}dx$$

input
```
int(x^2*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
int(x^2*erfi(b*x)/exp(b^2*x^2),x)
```

Fricas [F]

$$\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx = \int x^{2} \operatorname{erfi}\left(b x\right) e^{\left(-b^{2} x^{2}\right)}\,dx$$

input
```
integrate(x^2*erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
integral(x^2*erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 27.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int e^{-b^2x^2}x^2\mathrm{erfi}(bx)\,dx=\frac{bx^4{}_2F_2\left(\begin{matrix}1,2\\ \frac{3}{2},3\end{matrix}\middle|-b^2x^2\right)}{2\sqrt{\pi}}$$

input
```
integrate(x**2*erfi(b*x)/exp(b**2*x**2),x)
```

output
```
b*x**4*hyper((1, 2), (3/2, 3), -b**2*x**2)/(2*sqrt(pi))
```

Maxima [F]

$$\int e^{-b^2x^2}x^2\mathrm{erfi}(bx)\,dx=\int x^2\,\mathrm{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^2*erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(x^2*erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}x^2\mathrm{erfi}(bx)\,dx=\int x^2\,\mathrm{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(x^2*erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx=\int x^2\,\mathrm{e}^{-b^2\,x^2}\,\text{erfi}(b\,x)\,dx$$

input
```
int(x^2*exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
int(x^2*exp(-b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2}x^2\text{erfi}(bx)\,dx=\frac{\text{erf}(bix)\,i\pi x+\sqrt{\pi}\,e^{b^2x^2}b\,x^2-e^{b^2x^2}\left(\int\frac{\text{erf}(bix)}{e^{b^2x^2}}dx\right)i\pi}{2e^{b^2x^2}b^2\pi}$$

input
```
int(x^2*erfi(b*x)/exp(b^2*x^2),x)
```

output
```
(erf(b*i*x)*i*pi*x + sqrt(pi)*e**(b**2*x**2)*b*x**2 - e**(b**2*x**2)*int(e
rf(b*i*x)/e**(b**2*x**2),x)*i*pi)/(2*e**(b**2*x**2)*b**2*pi)
```

3.279 $\int e^{-b^2x^2}\textbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx=\frac{bx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}$$

output
```
b*x^2*hypergeom([1, 1],[3/2, 2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx=\frac{bx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Integrate[Erfi[b*x]/E^(b^2*x^2),x]
```

output
```
(b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{ 6932}$$

$$\frac{bx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]/E^(b^2*x^2),x]
```

output
```
(b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Maple [F]

$$\int \text{erfi}\,(bx)\,\mathrm{e}^{-b^2x^2}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2),x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2),x)
```

Fricas [F]

$$\int e^{-b^2x^2}\operatorname{erfi}(bx)\,dx = \int \operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2),x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{-b^2x^2}\operatorname{erfi}(bx)\,dx = \frac{bx^2\,{}_2F_2\left(\begin{matrix}1,1\\ \frac{3}{2},2\end{matrix}\middle|-b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2),x)
```

output
```
b*x**2*hyper((1, 1), (3/2, 2), -b**2*x**2)/sqrt(pi)
```

Maxima [F]

$$\int e^{-b^2x^2}\operatorname{erfi}(bx)\,dx = \int \operatorname{erfi}(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2), x)
```

Giac [F]

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx = \int \text{erfi}\,(bx)\,e^{\left(-b^2x^2\right)}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2),x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx = \int e^{-b^2\,x^2}\,\text{erfi}(b\,x)\,dx$$

input
```
int(exp(-b^2*x^2)*erfi(b*x),x)
```

output
```
int(exp(-b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int e^{-b^2x^2}\text{erfi}(bx)\,dx = -\left(\int \frac{\text{erf}(bix)}{e^{b^2x^2}}dx\right)i$$

input
```
int(erfi(b*x)/exp(b^2*x^2),x)
```

output
```
- int(erf(b*i*x)/e**(b**2*x**2),x)*i
```

3.280 $\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx = -\frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x} - \frac{2b^3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}} + \frac{2b\log(x)}{\sqrt{\pi}}$$

output
```
-erfi(b*x)/exp(b^2*x^2)/x-2*b^3*x^2*hypergeom([1, 1],[3/2, 2],-b^2*x^2)/Pi
^(1/2)+2*b*ln(x)/Pi^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.43

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx = -\frac{1}{2}bG_{2,3}^{2,1}\left(b^2x^2\middle|\begin{array}{c}0,1\\0,0,-\frac{1}{2}\end{array}\right)$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^2),x]
```

output
```
-1/2*(b*MeijerG[{{0}, {1}}, {{0, 0}, {-1/2}}, b^2*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6947, 14, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}dx$$

$$\Big\downarrow \text{6947}$$

$$-2b^2\int e^{-b^2x^2}\text{erfi}(bx)dx+\frac{2b\int \frac{1}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}$$

$$\Big\downarrow \text{14}$$

$$-2b^2\int e^{-b^2x^2}\text{erfi}(bx)dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6932}$$

$$-\frac{2b^3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x^2),x]
```

output
```
-(Erfi[b*x]/(E^(b^2*x^2)*x)) - (2*b^3*x^2*HypergeometricPFQ[{1, 1}, {3/2,
2}, -(b^2*x^2)])/Sqrt[Pi] + (2*b*Log[x])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)\, \mathrm{e}^{-b^2x^2}}{x^2}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^2,x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2)/x^2,x)
```

Fricas [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\operatorname{erfi}(bx)\, e^{\left(-b^2x^2\right)}}{x^2}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^2,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}\,dx = -\frac{2b^3x^2\,{}_2F_2\left(\begin{matrix} 1,1 \\ 2,\frac{5}{2} \end{matrix}\middle| -b^2x^2\right)}{3\sqrt{\pi}} + \frac{b\log\left(b^2x^2\right)}{\sqrt{\pi}}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x**2,x)
```

output
```
-2*b**3*x**2*hyper((1, 1), (2, 5/2), -b**2*x**2)/(3*sqrt(pi)) + b*log(b**2
*x**2)/sqrt(pi)
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^2}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^2, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^2}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{e^{-b^2\,x^2}\,\text{erfi}(b\,x)}{x^2}\,dx$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x^2,x)
```

output
```
int((exp(-b^2*x^2)*erfi(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^2}\,dx = -\left(\int \frac{\operatorname{erf}(bix)}{e^{b^2x^2}x^2}dx\right)i$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^2,x)
```

output
```
- int(erf(b*i*x)/(e**(b**2*x**2)*x**2),x)*i
```

3.281 $\int \frac{e^{-b^2x^2}\mathbf{erfi}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -\frac{b}{3\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}+\frac{2b^2e^{-b^2x^2}\text{erfi}(bx)}{3x}+\frac{4b^5x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{3\sqrt{\pi}}-\frac{4b^3\log(x)}{3\sqrt{\pi}}$$

output
```
-1/3*b/Pi^(1/2)/x^2-1/3*erfi(b*x)/exp(b^2*x^2)/x^3+2/3*b^2*erfi(b*x)/exp(b
^2*x^2)/x+4/3*b^5*x^2*hypergeom([1, 1],[3/2, 2],-b^2*x^2)/Pi^(1/2)-4/3*b^3
*ln(x)/Pi^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -\frac{bG_{2,3}^{2,1}\left(b^2x^2|\begin{array}{c}0,2\\0,1,-\frac{1}{2}\end{array}\right)}{2x^2}$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^4),x]
```

output
```
-1/2*(b*MeijerG[{{0}, {2}}, {{0, 1}, {-1/2}}, b^2*x^2])/x^2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6947, 15, 6947, 14, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}dx$$

$$\Big\downarrow \text{6947}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}dx+\frac{2b\int \frac{1}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6947}$$

$$-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfi}(bx)dx+\frac{2b\int \frac{1}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfi}(bx)dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}$$

$$\Big\downarrow \text{6932}$$

$$-\frac{2}{3}b^2\left(-\frac{2b^3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x^4),x]
```

output
```
-1/3*b/(Sqrt[Pi]*x^2) - Erfi[b*x]/(3*E^(b^2*x^2)*x^3) - (2*b^2*(-(Erfi[b*x
]/(E^(b^2*x^2)*x)) - (2*b^3*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*
x^2)])/Sqrt[Pi] + (2*b*Log[x])/Sqrt[Pi]))/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)\, e^{-b^2x^2}}{x^4}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^4,x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2)/x^4,x)
```

Fricas [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^4}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^4,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.23

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -\frac{b^3 G_{3,2}^{1,2}\left(\begin{matrix} 2, 1 & \frac{5}{2} \\ 2 & 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{b^2x^2}\right)}{2}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x**4,x)
```

output
```
-b**3*meijerg(((2, 1), (5/2,)), ((2,), (0,)), exp_polar(-2*I*pi)/(b**2*x**
2))/2
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^4}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^4, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^4}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^4}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^4}\,dx = \int \frac{e^{-b^2\,x^2}\,\operatorname{erfi}(b\,x)}{x^4}\,dx$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x^4,x)
```

output
```
int((exp(-b^2*x^2)*erfi(b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^4}\,dx = -\left(\int \frac{\operatorname{erf}(bix)}{e^{b^2x^2}x^4}dx\right)i$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^4,x)
```

output
```
- int(erf(b*i*x)/(e**(b**2*x**2)*x**4),x)*i
```

3.282 $\int \frac{e^{-b^2x^2}\mathbf{erfi}(bx)}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 144

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^6}\,dx = -\frac{b}{10\sqrt{\pi}x^4}+\frac{2b^3}{15\sqrt{\pi}x^2}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}+\frac{2b^2e^{-b^2x^2}\text{erfi}(bx)}{15x^3}-\frac{4b^4e^{-b^2x^2}\text{erfi}(bx)}{15x}-\frac{8b^7x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{15\sqrt{\pi}}+\frac{8b^5\log(x)}{15\sqrt{\pi}}$$

output
```
-1/10*b/Pi^(1/2)/x^4+2/15*b^3/Pi^(1/2)/x^2-1/5*erfi(b*x)/exp(b^2*x^2)/x^5+
2/15*b^2*erfi(b*x)/exp(b^2*x^2)/x^3-4/15*b^4*erfi(b*x)/exp(b^2*x^2)/x-8/15
*b^7*x^2*hypergeom([1, 1],[3/2, 2],-b^2*x^2)/Pi^(1/2)+8/15*b^5*ln(x)/Pi^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.20

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^6}\,dx = -\frac{bG_{2,3}^{2,1}\left(b^2x^2\middle|\begin{array}{c}0,3\\0,2,-\frac{1}{2}\end{array}\right)}{2x^4}$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^6),x]
```

output
```
-1/2*(b*MeijerG[{{0}, {3}}, {{0, 2}, {-1/2}}, b^2*x^2])/x^4
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6947, 15, 6947, 15, 6947, 14, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^6}dx$$

$$\Big\downarrow \text{6947}$$

$$-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}dx+\frac{2b\int \frac{1}{x^5}dx}{5\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{5}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^4}dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}-\frac{b}{10\sqrt{\pi}x^4}$$

$$\Big\downarrow \text{6947}$$

$$-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}dx+\frac{2b\int \frac{1}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}-\frac{b}{10\sqrt{\pi}x^4}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^2}dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}-\frac{b}{10\sqrt{\pi}x^4}$$

$$\Big\downarrow \text{6947}$$

$$-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfi}(bx)dx+\frac{2b\int\frac{1}{x}dx}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}-\frac{b}{10\sqrt{\pi}x^4}$$

$$\Big\downarrow \text{ 14}$$

$$-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-2b^2\int e^{-b^2x^2}\text{erfi}(bx)dx-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}-\frac{b}{10\sqrt{\pi}x^4}$$

$$\Big\downarrow \text{ 6932}$$

$$-\frac{2}{5}b^2\left(-\frac{2}{3}b^2\left(-\frac{2b^3x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{\sqrt{\pi}}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{2b\log(x)}{\sqrt{\pi}}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{3x^3}-\frac{b}{3\sqrt{\pi}x^2}\right)-\frac{e^{-b^2x^2}\text{erfi}(bx)}{5x^5}-\frac{b}{10\sqrt{\pi}x^4}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x^6),x]
```

output
```
-1/10*b/(Sqrt[Pi]*x^4) - Erfi[b*x]/(5*E^(b^2*x^2)*x^5) - (2*b^2*(-1/3*b/(S
qrt[Pi]*x^2) - Erfi[b*x]/(3*E^(b^2*x^2)*x^3) - (2*b^2*(-(Erfi[b*x]/(E^(b^2
*x^2)*x)) - (2*b^3*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sq
rt[Pi] + (2*b*Log[x])/Sqrt[Pi]))/3))/5
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\operatorname{erfi}(bx)\,\mathrm{e}^{-b^2x^2}}{x^6}dx$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^6,x)
```

output
```
int(erfi(b*x)/exp(b^2*x^2)/x^6,x)
```

Fricas [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^6}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(-b^2x^2)}}{x^6}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^6,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(-b^2*x^2)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^6}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^6}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^6,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^6, x)
```

Giac [F]

$$\int \frac{e^{-b^2x^2}\text{erfi}(bx)}{x^6}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^6}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^6,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(-b^2*x^2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^6}\,dx = \int \frac{e^{-b^2\,x^2}\operatorname{erfi}(b\,x)}{x^6}\,dx$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x^6,x)
```

output
```
int((exp(-b^2*x^2)*erfi(b*x))/x^6, x)
```

Reduce [F]

$$\int \frac{e^{-b^2x^2}\operatorname{erfi}(bx)}{x^6}\,dx = -\left(\int \frac{\operatorname{erf}(bix)}{e^{b^2x^2}x^6}dx\right)i$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^6,x)
```

output
```
- int(erf(b*i*x)/(e**(b**2*x**2)*x**6),x)*i
```

3.283 $\int e^{c+b^2x^2}x^5\textbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx=\frac{11e^{c+2b^2x^2}x}{16b^5\sqrt{\pi}}-\frac{e^{c+2b^2x^2}x^3}{4b^3\sqrt{\pi}}+\frac{e^{c+b^2x^2}\text{erfi}(bx)}{b^6}-\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{b^4}+\frac{e^{c+b^2x^2}x^4\text{erfi}(bx)}{2b^2}-\frac{43e^c\text{erfi}\left(\sqrt{2}bx\right)}{32\sqrt{2}b^6}$$

output
```
11/16*exp(2*b^2*x^2+c)*x/b^5/Pi^(1/2)-1/4*exp(2*b^2*x^2+c)*x^3/b^3/Pi^(1/2
)+exp(b^2*x^2+c)*erfi(b*x)/b^6-exp(b^2*x^2+c)*x^2*erfi(b*x)/b^4+1/2*exp(b^
2*x^2+c)*x^4*erfi(b*x)/b^2-43/64*exp(c)*erfi(2^(1/2)*b*x)*2^(1/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx=\frac{e^c\left(-4be^{2b^2x^2}x(-11+4b^2x^2)+32e^{b^2x^2}\sqrt{\pi}(2-2b^2x^2+b^4x^4)\,\text{erfi}(bx)-43\sqrt{2\pi}\text{erfi}\left(\sqrt{2}bx\right)\right)}{64b^6\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^5*Erfi[b*x],x]
```

output
```
(E^c*(-4*b*E^(2*b^2*x^2)*x*(-11 + 4*b^2*x^2) + 32*E^(b^2*x^2)*Sqrt[Pi]*(2
- 2*b^2*x^2 + b^4*x^4)*Erfi[b*x] - 43*Sqrt[2*Pi]*Erfi[Sqrt[2]*b*x]))/(64*b
^6*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.78, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6941, 2641, 2641, 2633, 6941, 2641, 2633, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{b^2x^2+c}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erfi}(bx)dx}{b^2}-\frac{\int e^{2b^2x^2+c}x^4dx}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erfi}(bx)dx}{b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\int e^{2b^2x^2+c}x^2dx}{4b^2}}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erfi}(bx)dx}{b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\int e^{2b^2x^2+c}dx}{4b^2}\right)}{4b^2}}{\sqrt{\pi}b}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\int e^{b^2x^2+c}x^3\text{erfi}(bx)dx}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6941}$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\,\text{erfi}(bx)dx}{b^2}-\frac{\int e^{2b^2x^2+c}x^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \; 2641$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\,\text{erfi}(bx)dx}{b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\int e^{2b^2x^2+c}dx}{4b^2}}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \; 2633$$

$$-\frac{2\left(-\frac{\int e^{b^2x^2+c}x\,\text{erfi}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \; 6938$$

$$-\frac{2\left(-\frac{\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\int e^{2b^2x^2+c}dx}{\sqrt{\pi}b}}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{b^2}+\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}$$

$$\Big\downarrow \; 2633$$

$$\frac{x^4e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{2\left(\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^c\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}}{b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}\right)}{b^2}-\frac{\frac{x^3e^{2b^2x^2+c}}{4b^2}-\frac{3\left(\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}\right)}{4b^2}}{\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^5*Erfi[b*x],x]
```

output
```
(E^(c + b^2*x^2)*x^4*Erfi[b*x])/(2*b^2) - ((E^(c + 2*b^2*x^2)*x^3)/(4*b^2)
 - (3*((E^(c + 2*b^2*x^2)*x)/(4*b^2) - (E^c*Sqrt[Pi/2]*Erfi[Sqrt[2]*b*x])/
(8*b^3)))/(4*b^2))/(b*Sqrt[Pi]) - (2*((E^(c + b^2*x^2)*x^2*Erfi[b*x])/(2*b
^2) - ((E^(c + b^2*x^2)*Erfi[b*x])/(2*b^2) - (E^c*Erfi[Sqrt[2]*b*x])/(2*Sq
rt[2]*b^2))/b^2 - ((E^(c + 2*b^2*x^2)*x)/(4*b^2) - (E^c*Sqrt[Pi/2]*Erfi[Sq
rt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi])))/b^2
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{b^2x^2+c}x^5 \operatorname{erfi}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^5*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^5*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx = \frac{43\sqrt{2}\pi\sqrt{-b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{-b^2}x\right)e^c + 32\left(\pi b^5x^4 - 2\pi b^3x^2 + 2\pi b\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2+c\right)} - 4\sqrt{\pi}(4b^4x^3 - 11b^2x)}{64\pi b^7}$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erfi(b*x),x, algorithm="fricas")
```

output
```
1/64*(43*sqrt(2)*pi*sqrt(-b^2)*erf(sqrt(2)*sqrt(-b^2)*x)*e^c + 32*(pi*b^5*
x^4 - 2*pi*b^3*x^2 + 2*pi*b)*erfi(b*x)*e^(b^2*x^2 + c) - 4*sqrt(pi)*(4*b^4
*x^3 - 11*b^2*x)*e^(2*b^2*x^2 + c))/(pi*b^7)
```

Sympy [F]

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx = e^c\int x^5e^{b^2x^2}\,\text{erfi}\,(bx)\,dx$$

input
```
integrate(exp(b**2*x**2+c)*x**5*erfi(b*x),x)
```

output
```
exp(c)*Integral(x**5*exp(b**2*x**2)*erfi(b*x), x)
```

Maxima [F]

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx = \int x^5\,\text{erfi}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^5*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx = \int x^5\,\text{erfi}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^5*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.43

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx = \text{erfi}(b\,x)\left(\frac{e^{b^2\,x^2+c}}{b^6}+\frac{x^4\,e^{b^2\,x^2+c}}{2\,b^2}-\frac{x^2\,e^{b^2\,x^2+c}}{b^4}\right)-\frac{3\,x^5\,e^c}{8\,b\,(-2\,b^2\,x^2)^{5/2}} + \frac{11\,x\,e^{2\,b^2\,x^2+c}}{16\,b^5\,\sqrt{\pi}}-\frac{x^3\,e^{2\,b^2\,x^2+c}}{4\,b^3\,\sqrt{\pi}}-\frac{\sqrt{2}\,e^c\,\text{erfi}\left(\sqrt{2}\,x\,\sqrt{b^2}\right)}{8\,b^3\,(b^2)^{3/2}} + \frac{3\,x^5\,e^c\,\text{erfc}\left(\sqrt{-2\,b^2\,x^2}\right)}{8\,b\,(-2\,b^2\,x^2)^{5/2}}-\frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}\,x\,\sqrt{-b^2}\right)\,e^c}{2\,b\,(-b^2)^{5/2}}$$

input
```
int(x^5*exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
erfi(b*x)*(exp(c + b^2*x^2)/b^6 + (x^4*exp(c + b^2*x^2))/(2*b^2) - (x^2*ex
p(c + b^2*x^2))/b^4) - (3*x^5*exp(c))/(8*b*(-2*b^2*x^2)^(5/2)) + (11*x*exp
(c + 2*b^2*x^2))/(16*b^5*pi^(1/2)) - (x^3*exp(c + 2*b^2*x^2))/(4*b^3*pi^(1
/2)) - (2^(1/2)*exp(c)*erfi(2^(1/2)*x*(b^2)^(1/2)))/(8*b^3*(b^2)^(3/2)) +
(3*x^5*exp(c)*erfc((-2*b^2*x^2)^(1/2)))/(8*b*(-2*b^2*x^2)^(5/2)) - (2^(1/2
)*erf(2^(1/2)*x*(-b^2)^(1/2))*exp(c))/(2*b*(-b^2)^(5/2))
```

Reduce [F]

$$\int e^{c+b^2x^2}x^5\text{erfi}(bx)\,dx = -e^c\left(\int e^{b^2x^2}\text{erf}(bix)\,x^5dx\right)i$$

input
```
int(exp(b^2*x^2+c)*x^5*erfi(b*x),x)
```

output
```
- e**c*int(e**(b**2*x**2)*erf(b*i*x)*x**5,x)*i
```

3.284 $\int e^{c+b^2x^2}x^3\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx=-\frac{e^{c+2b^2x^2}x}{4b^3\sqrt{\pi}}-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{2b^4}+\frac{e^{c+b^2x^2}x^2\text{erfi}(bx)}{2b^2}+\frac{5e^c\text{erfi}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^4}$$

output
```
-1/4*exp(2*b^2*x^2+c)*x/b^3/Pi^(1/2)-1/2*exp(b^2*x^2+c)*erfi(b*x)/b^4+1/2*
exp(b^2*x^2+c)*x^2*erfi(b*x)/b^2+5/16*exp(c)*erfi(2^(1/2)*b*x)*2^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx=\frac{e^c\left(-4be^{2b^2x^2}x+8e^{b^2x^2}\sqrt{\pi}(-1+b^2x^2)\,\text{erfi}(bx)+5\sqrt{2\pi}\text{erfi}\left(\sqrt{2}bx\right)\right)}{16b^4\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^3*Erfi[b*x],x]
```

output
```
(E^c*(-4*b*E^(2*b^2*x^2)*x + 8*E^(b^2*x^2)*Sqrt[Pi]*(-1 + b^2*x^2)*Erfi[b*
x] + 5*Sqrt[2*Pi]*Erfi[Sqrt[2]*b*x]))/(16*b^4*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6941, 2641, 2633, 6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{b^2x^2+c}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \textbf{6941}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erfi}(bx)dx}{b^2}-\frac{\int e^{2b^2x^2+c}x^2dx}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \textbf{2641}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erfi}(bx)dx}{b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\int e^{2b^2x^2+c}dx}{4b^2}}{\sqrt{\pi}b}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \textbf{2633}$$

$$-\frac{\int e^{b^2x^2+c}x\text{erfi}(bx)dx}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}$$

$$\Big\downarrow \textbf{6938}$$

$$-\frac{\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\int e^{2b^2x^2+c}dx}{\sqrt{\pi}b}}{b^2}+\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}$$

$$\Big\downarrow \textbf{2633}$$

$$\frac{x^2e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^c\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}}{b^2}-\frac{\frac{xe^{2b^2x^2+c}}{4b^2}-\frac{\sqrt{\frac{\pi}{2}}e^c\text{erfi}\left(\sqrt{2}bx\right)}{8b^3}}{\sqrt{\pi}b}$$

input
```
Int[E^(c + b^2*x^2)*x^3*Erfi[b*x],x]
```

output
```
(E^(c + b^2*x^2)*x^2*Erfi[b*x])/(2*b^2) - ((E^(c + b^2*x^2)*Erfi[b*x])/(2*
b^2) - (E^c*Erfi[Sqrt[2]*b*x])/(2*Sqrt[2]*b^2))/b^2 - ((E^(c + 2*b^2*x^2)*
x)/(4*b^2) - (E^c*Sqrt[Pi/2]*Erfi[Sqrt[2]*b*x])/(8*b^3))/(b*Sqrt[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{b^2x^2+c}x^3 \operatorname{erfi}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^3*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^3*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int e^{c+b^2x^2}x^3\mathrm{erfi}(bx)\,dx = -\frac{4\sqrt{\pi}b^2xe^{\left(2\,b^2x^2+c\right)}+5\sqrt{2}\pi\sqrt{-b^2}\operatorname{erf}\left(\sqrt{2}\sqrt{-b^2}x\right)e^c-8\left(\pi b^3x^2-\pi b\right)\operatorname{erfi}(bx)\,e^{\left(b^2x^2+c\right)}}{16\,\pi b^5}$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erfi(b*x),x, algorithm="fricas")
```

output
```
-1/16*(4*sqrt(pi)*b^2*x*e^(2*b^2*x^2 + c) + 5*sqrt(2)*pi*sqrt(-b^2)*erf(sq
rt(2)*sqrt(-b^2)*x)*e^c - 8*(pi*b^3*x^2 - pi*b)*erfi(b*x)*e^(b^2*x^2 + c))
/(pi*b^5)
```

Sympy [F]

$$\int e^{c+b^2x^2}x^3\mathrm{erfi}(bx)\,dx = e^c\int x^3e^{b^2x^2}\operatorname{erfi}(bx)\,dx$$

input
```
integrate(exp(b**2*x**2+c)*x**3*erfi(b*x),x)
```

output
```
exp(c)*Integral(x**3*exp(b**2*x**2)*erfi(b*x), x)
```

Maxima [F]

$$\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx = \int x^3\,\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx = \int x^3\,\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^3*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx = \frac{\sqrt{2}\,\mathrm{e}^c\,\mathrm{erfi}\left(\sqrt{2}\,x\,\sqrt{b^2}\right)}{16\,b\,(b^2)^{3/2}} - \frac{x\,\mathrm{e}^{2\,b^2\,x^2+c}}{4\,b^3\,\sqrt{\pi}} - \mathrm{erfi}(b\,x)\left(\frac{\mathrm{e}^{b^2\,x^2+c}}{2\,b^4} - \frac{x^2\,\mathrm{e}^{b^2\,x^2+c}}{2\,b^2}\right) - \frac{\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,x\,\sqrt{-b^2}\right)\,\mathrm{e}^c}{4\,b\,(-b^2)^{3/2}}$$

input
```
int(x^3*exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
(2^(1/2)*exp(c)*erfi(2^(1/2)*x*(b^2)^(1/2)))/(16*b*(b^2)^(3/2)) - (x*exp(c
 + 2*b^2*x^2))/(4*b^3*pi^(1/2)) - erfi(b*x)*(exp(c + b^2*x^2)/(2*b^4) - (x
^2*exp(c + b^2*x^2))/(2*b^2)) - (2^(1/2)*erf(2^(1/2)*x*(-b^2)^(1/2))*exp(c
))/(4*b*(-b^2)^(3/2))
```

Reduce [F]

$$\int e^{c+b^2x^2}x^3\text{erfi}(bx)\,dx=-e^c\left(\int e^{b^2x^2}\text{erf}(bix)\,x^3dx\right)i$$

input
```
int(exp(b^2*x^2+c)*x^3*erfi(b*x),x)
```

output
```
 - e**c*int(e**(b**2*x**2)*erf(b*i*x)*x**3,x)*i
```

3.285 $\int e^{c+b^2x^2} x\,\text{erfi}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int e^{c+b^2x^2} x\,\text{erfi}(bx)\,dx = \frac{e^{c+b^2x^2}\text{erfi}(bx)}{2b^2} - \frac{e^c\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

output
```
1/2*exp(b^2*x^2+c)*erfi(b*x)/b^2-1/4*exp(c)*erfi(2^(1/2)*b*x)*2^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int e^{c+b^2x^2} x\,\text{erfi}(bx)\,dx = \frac{e^c\left(2e^{b^2x^2}\text{erfi}(bx) - \sqrt{2}\text{erfi}\left(\sqrt{2}bx\right)\right)}{4b^2}$$

input
```
Integrate[E^(c + b^2*x^2)*x*Erfi[b*x],x]
```

output
```
(E^c*(2*E^(b^2*x^2)*Erfi[b*x] - Sqrt[2]*Erfi[Sqrt[2]*b*x]))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6938, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{b^2x^2+c}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6938}$$

$$\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\int e^{2b^2x^2+c}dx}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^c\text{erfi}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2}$$

input
```
Int[E^(c + b^2*x^2)*x*Erfi[b*x],x]
```

output
```
(E^(c + b^2*x^2)*Erfi[b*x])/(2*b^2) - (E^c*Erfi[Sqrt[2]*b*x])/(2*Sqrt[2]*b
^2)
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int e^{b^2x^2+c}x \operatorname{erfi}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{c+b^2x^2}x\text{erfi}(bx)\,dx = \frac{2\,b\,\text{erfi}\left(bx\right)e^{\left(b^2x^2+c\right)} + \sqrt{2}\sqrt{-b^2}\,\text{erf}\left(\sqrt{2}\sqrt{-b^2}x\right)e^c}{4\,b^3}$$

input
```
integrate(exp(b^2*x^2+c)*x*erfi(b*x),x, algorithm="fricas")
```

output
```
1/4*(2*b*erfi(b*x)*e^(b^2*x^2 + c) + sqrt(2)*sqrt(-b^2)*erf(sqrt(2)*sqrt(-
b^2)*x)*e^c)/b^3
```

Sympy [F]

$$\int e^{c+b^2x^2}x\text{erfi}(bx)\,dx = e^c\int xe^{b^2x^2}\,\text{erfi}\left(bx\right)dx$$

input
```
integrate(exp(b**2*x**2+c)*x*erfi(b*x),x)
```

output
```
exp(c)*Integral(x*exp(b**2*x**2)*erfi(b*x), x)
```

Maxima [F]

$$\int e^{c+b^2x^2} x\text{erfi}(bx)\,dx = \int x\,\text{erfi}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2} x\text{erfi}(bx)\,dx = \int x\,\text{erfi}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int e^{c+b^2x^2} x\text{erfi}(bx)\,dx = \frac{e^{b^2\,x^2}\,e^c\,\text{erfi}(b\,x)}{2\,b^2} - \frac{\sqrt{2}\,\text{erf}\left(\sqrt{2}\,x\,\sqrt{-b^2}\right)\,e^c}{4\,b\,\sqrt{-b^2}}$$

input
```
int(x*exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
(exp(b^2*x^2)*exp(c)*erfi(b*x))/(2*b^2) - (2^(1/2)*erf(2^(1/2)*x*(-b^2)^(1
/2))*exp(c))/(4*b*(-b^2)^(1/2))
```

Reduce [F]

$$\int e^{c+b^2x^2}x\operatorname{erfi}(bx)\,dx = -e^c\left(\int e^{b^2x^2}\operatorname{erf}(bix)\,xdx\right)i$$

input
```
int(exp(b^2*x^2+c)*x*erfi(b*x),x)
```

output
```
- e**c*int(e**(b**2*x**2)*erf(b*i*x)*x,x)*i
```

3.286 $\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx=\text{Int}\left(\frac{e^{c+b^2x^2}\text{erfi}(bx)}{x},x\right)$$

output
```
Defer(Int)(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx=\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x,x]
```

output
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x} \, dx$$
$$\Big\downarrow \text{6950}$$
$$\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx$$

input
```
Int[(E^(c + b^2*x^2)*Erfi[b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{b^2x^2+c}\,\text{erfi}\,(bx)}{x}dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(b^2*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx = e^c \int \frac{e^{b^2x^2}\,\text{erfi}\,(bx)}{x}\,dx$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)/x,x)
```

output
```
exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\mathrm{erfi}(bx)}{x}\,dx = \int \frac{\mathrm{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\mathrm{erfi}(bx)}{x}\,dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\,\mathrm{erfi}(b\,x)}{x}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfi(b*x))/x,x)
```

output
```
int((exp(c + b^2*x^2)*erfi(b*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x}\,dx = -e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bix)}{x}dx\right)i$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

output
```
- e**c*int((e**(b**2*x**2)*erf(b*i*x))/x,x)*i
```

3.287 $\int \frac{e^{c+b^2x^2}\mathbf{erfi}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^3}\,dx = -\frac{be^{c+2b^2x^2}}{\sqrt{\pi}x} - \frac{e^{c+b^2x^2}\text{erfi}(bx)}{2x^2} + \sqrt{2}b^2e^c\text{erfi}\left(\sqrt{2}bx\right) + b^2\text{Int}\left(\frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}, x\right)$$

output
```
-b*exp(2*b^2*x^2+c)/Pi^(1/2)/x-1/2*exp(b^2*x^2+c)*erfi(b*x)/x^2+2^(1/2)*b^
2*exp(c)*erfi(2^(1/2)*b*x)+b^2*Defer(Int)(exp(b^2*x^2+c)*erfi(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^3}\,dx = \int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^3}\,dx$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^3,x]
```

output
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^3}\,dx$$

$$\Big\downarrow \text{6947}$$

$$b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\int \frac{e^{2b^2x^2+c}}{x^2}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}$$

$$\Big\downarrow \text{2643}$$

$$b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(4b^2\int e^{2b^2x^2+c}dx-\frac{e^{2b^2x^2+c}}{x}\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}$$

$$\Big\downarrow \text{2633}$$

$$b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}$$

$$\Big\downarrow \text{6950}$$

$$b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}$$

input
```
Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{b^2x^2+c}\,\mathrm{erfi}\,(bx)}{x^3}dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^3,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\mathrm{erfi}(bx)}{x^3}\,dx = \int \frac{\mathrm{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 4.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\mathrm{erfi}(bx)}{x^3}\,dx = e^c\int \frac{e^{b^2x^2}\,\mathrm{erfi}\,(bx)}{x^3}\,dx$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)/x**3,x)
```

output
```
exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\mathrm{erfi}(bx)}{x^3}\,dx = \int \frac{\mathrm{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\mathrm{erfi}(bx)}{x^3}\,dx = \int \frac{\mathrm{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = \int \frac{\mathrm{e}^{b^2\,x^2+c}\operatorname{erfi}(b\,x)}{x^3}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^3,x)
```

output
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^3}\,dx = -e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bix)}{x^3}dx\right)i$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^3,x)
```

output
```
- e**c*int((e**(b**2*x**2)*erf(b*i*x))/x**3,x)*i
```

3.288 $\int \frac{e^{c+b^2x^2}\mathbf{erfi}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx =& -\frac{be^{c+2b^2x^2}}{6\sqrt{\pi}x^3}-\frac{7b^3e^{c+2b^2x^2}}{6\sqrt{\pi}x}-\frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{4x^4}\\&-\frac{b^2e^{c+b^2x^2}\operatorname{erfi}(bx)}{4x^2}+\frac{b^4e^c\operatorname{erfi}\left(\sqrt{2}bx\right)}{\sqrt{2}}\\&+\frac{2}{3}\sqrt{2}b^4e^c\operatorname{erfi}\left(\sqrt{2}bx\right)+\frac{1}{2}b^4\operatorname{Int}\left(\frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x},x\right)\end{aligned}$$

output
```
-1/6*b*exp(2*b^2*x^2+c)/Pi^(1/2)/x^3-7/6*b^3*exp(2*b^2*x^2+c)/Pi^(1/2)/x-1
/4*exp(b^2*x^2+c)*erfi(b*x)/x^4-1/4*b^2*exp(b^2*x^2+c)*erfi(b*x)/x^2+7/6*b
^4*exp(c)*erfi(2^(1/2)*b*x)*2^(1/2)+1/2*b^4*Defer(Int)(exp(b^2*x^2+c)*erfi
(b*x)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^5}\,dx$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^5,x]
```

output
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^5, x]
```

Rubi [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^5}\,dx$$

$$\Big\downarrow \; 6947$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^3}dx+\frac{b\int \frac{e^{2b^2x^2+c}}{x^4}dx}{2\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \; 2643$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^3}dx+\frac{b\left(\frac{4}{3}b^2\int \frac{e^{2b^2x^2+c}}{x^2}dx-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \; 2643$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^3}dx+\frac{b\left(\frac{4}{3}b^2\left(4b^2\int e^{2b^2x^2+c}dx-\frac{e^{2b^2x^2+c}}{x}\right)-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}$$

$$\Big\downarrow \; 2633$$

$$\frac{1}{2}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^3}dx-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}+\frac{b\left(\frac{4}{3}b^2\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \; 6947$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\int \frac{e^{2b^2x^2+c}}{x^2}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}+\frac{b\left(\frac{4}{3}b^2\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \; 2643$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(4b^2\int e^{2b^2x^2+c}dx-\frac{e^{2b^2x^2+c}}{x}\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}+\frac{b\left(\frac{4}{3}b^2\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \; 2633$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}+\frac{b\left(\frac{4}{3}b^2\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}$$

$$\Big\downarrow \; 6950$$

$$\frac{1}{2}b^2\left(b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}dx+\frac{b\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{2x^2}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{4x^4}+\frac{b\left(\frac{4}{3}b^2\left(\sqrt{2\pi}be^c\text{erfi}\left(\sqrt{2}bx\right)-\frac{e^{2b^2x^2+c}}{x}\right)-\frac{e^{2b^2x^2+c}}{3x^3}\right)}{2\sqrt{\pi}}$$

input
```
Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^5,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{b^2x^2+c}\operatorname{erfi}(bx)}{x^5}dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^5,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = \int \frac{\operatorname{erfi}(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 20.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^5}\,dx = e^c\int \frac{e^{b^2x^2}\,\text{erfi}\,(bx)}{x^5}\,dx$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)/x**5,x)
```

output
```
exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^5}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^5}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = \int \frac{e^{b^2\,x^2+c}\,\operatorname{erfi}(b\,x)}{x^5}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^5,x)
```

output
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^5, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{e^{c+b^2x^2}\operatorname{erfi}(bx)}{x^5}\,dx = -e^c\left(\int \frac{e^{b^2x^2}\operatorname{erf}(bix)}{x^5}dx\right)i$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^5,x)
```

output
```
- e**c*int((e**(b**2*x**2)*erf(b*i*x))/x**5,x)*i
```

3.289 $\int e^{c+b^2x^2}x^4\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx=\frac{e^{c+2b^2x^2}}{2b^5\sqrt{\pi}}-\frac{e^{c+2b^2x^2}x^2}{4b^3\sqrt{\pi}}-\frac{3e^{c+b^2x^2}x\text{erfi}(bx)}{4b^4}+\frac{e^{c+b^2x^2}x^3\text{erfi}(bx)}{2b^2}+\frac{3e^c\sqrt{\pi}\text{erfi}(bx)^2}{16b^5}$$

output
```
1/2*exp(2*b^2*x^2+c)/b^5/Pi^(1/2)-1/4*exp(2*b^2*x^2+c)*x^2/b^3/Pi^(1/2)-3/
4*exp(b^2*x^2+c)*x*erfi(b*x)/b^4+1/2*exp(b^2*x^2+c)*x^3*erfi(b*x)/b^2+3/16
*exp(c)*Pi^(1/2)*erfi(b*x)^2/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx=\frac{e^c\left(-4e^{2b^2x^2}(-2+b^2x^2)+4be^{b^2x^2}\sqrt{\pi}x(-3+2b^2x^2)\,\text{erfi}(bx)+3\pi\text{erfi}(bx)^2\right)}{16b^5\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^4*Erfi[b*x],x]
```

output
```
(E^c*(-4*E^(2*b^2*x^2)*(-2 + b^2*x^2) + 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*(-3 + 2
*b^2*x^2)*Erfi[b*x] + 3*Pi*Erfi[b*x]^2))/(16*b^5*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6941, 2641, 2638, 6941, 2638, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{b^2 x^2+c} \text{erfi}(bx) \, dx$$

$$\Big\downarrow \text{6941}$$

$$-\frac{3 \int e^{b^2 x^2+c} x^2 \text{erfi}(bx) dx}{2b^2} - \frac{\int e^{2b^2 x^2+c} x^3 dx}{\sqrt{\pi} b} + \frac{x^3 e^{b^2 x^2+c} \text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \int e^{b^2 x^2+c} x^2 \text{erfi}(bx) dx}{2b^2} - \frac{\frac{x^2 e^{2b^2 x^2+c}}{4b^2} - \frac{\int e^{2b^2 x^2+c} x dx}{2b^2}}{\sqrt{\pi} b} + \frac{x^3 e^{b^2 x^2+c} \text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3 \int e^{b^2 x^2+c} x^2 \text{erfi}(bx) dx}{2b^2} + \frac{x^3 e^{b^2 x^2+c} \text{erfi}(bx)}{2b^2} - \frac{\frac{x^2 e^{2b^2 x^2+c}}{4b^2} - \frac{e^{2b^2 x^2+c}}{8b^4}}{\sqrt{\pi} b}$$

$$\Big\downarrow \text{6941}$$

$$-\frac{3 \left(-\frac{\int e^{b^2 x^2+c} \text{erfi}(bx) dx}{2b^2} - \frac{\int e^{2b^2 x^2+c} x dx}{\sqrt{\pi} b} + \frac{x e^{b^2 x^2+c} \text{erfi}(bx)}{2b^2}\right)}{2b^2} + \frac{x^3 e^{b^2 x^2+c} \text{erfi}(bx)}{2b^2} - \frac{\frac{x^2 e^{2b^2 x^2+c}}{4b^2} - \frac{e^{2b^2 x^2+c}}{8b^4}}{\sqrt{\pi} b}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3\left(-\frac{\int e^{b^2x^2+c}\text{erfi}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2+c}}{4b^2}-\frac{e^{2b^2x^2+c}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{6929}$$

$$-\frac{3\left(-\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)d\text{erfi}(bx)}{4b^3}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}\right)}{2b^2}+\frac{x^3e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2+c}}{4b^2}-\frac{e^{2b^2x^2+c}}{8b^4}}{\sqrt{\pi}b}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^3e^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{\frac{x^2e^{2b^2x^2+c}}{4b^2}-\frac{e^{2b^2x^2+c}}{8b^4}}{\sqrt{\pi}b}-\frac{3\left(-\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{8b^3}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}\right)}{2b^2}$$

input
```
Int[E^(c + b^2*x^2)*x^4*Erfi[b*x],x]
```

output
```
-((-1/8*E^(c + 2*b^2*x^2)/b^4 + (E^(c + 2*b^2*x^2)*x^2)/(4*b^2))/(b*Sqrt[P
i])) + (E^(c + b^2*x^2)*x^3*Erfi[b*x])/(2*b^2) - (3*(-1/4*E^(c + 2*b^2*x^2
)/(b^3*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfi[b*x])/(2*b^2) - (E^c*Sqrt[Pi]*E
rfi[b*x]^2)/(8*b^3)))/(2*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int \mathrm{e}^{b^{2}x^{2}+c}x^{4}\operatorname{erfi}\left(bx\right)dx$$

input
```
int(exp(b^2*x^2+c)*x^4*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^4*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.61

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx = \frac{\left(4\,(2\,\pi b^{3}x^{3}-3\,\pi bx)\operatorname{erfi}\left(bx\right)e^{\left(b^{2}x^{2}\right)}+\sqrt{\pi}\left(3\,\pi\operatorname{erfi}\left(bx\right)^{2}-4\,(b^{2}x^{2}-2)e^{\left(2\,b^{2}x^{2}\right)}\right)\right)e^{c}}{16\,\pi b^{5}}$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erfi(b*x),x, algorithm="fricas")
```

output
```
1/16*(4*(2*pi*b^3*x^3 - 3*pi*b*x)*erfi(b*x)*e^(b^2*x^2) + sqrt(pi)*(3*pi*e
rfi(b*x)^2 - 4*(b^2*x^2 - 2)*e^(2*b^2*x^2)))*e^c/(pi*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx = \begin{cases} \frac{x^3e^ce^{b^2x^2}\,\text{erfi}\,(bx)}{2b^2} - \frac{x^2e^ce^{2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{3xe^ce^{b^2x^2}\,\text{erfi}\,(bx)}{4b^4} + \frac{e^ce^{2b^2x^2}}{2\sqrt{\pi}b^5} + \frac{3\sqrt{\pi}e^c\,\text{erfi}^2\,(bx)}{16b^5} & \text{for}\ b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x**4*erfi(b*x),x)
```

output
```
Piecewise((x**3*exp(c)*exp(b**2*x**2)*erfi(b*x)/(2*b**2) - x**2*exp(c)*exp
(2*b**2*x**2)/(4*sqrt(pi)*b**3) - 3*x*exp(c)*exp(b**2*x**2)*erfi(b*x)/(4*b
**4) + exp(c)*exp(2*b**2*x**2)/(2*sqrt(pi)*b**5) + 3*sqrt(pi)*exp(c)*erfi(
b*x)**2/(16*b**5), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx = \int x^4\,\text{erfi}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx=\int x^4\,\text{erfi}\,(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^4*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx=\text{erfi}(b\,x)\,\left(\frac{x^3\,\mathrm{e}^{b^2\,x^2+c}}{2\,b^2}-\frac{3\,x\,\mathrm{e}^{b^2\,x^2+c}}{4\,b^4}+\frac{3\,\sqrt{\pi}\,\text{erfi}\left(\frac{b^2\,x}{\sqrt{b^2}}\right)\,\mathrm{e}^c}{8\,{\left(b^2\right)}^{5/2}}\right)+\frac{8\,\mathrm{e}^{2\,b^2\,x^2+c}-3\,\pi\,{\text{erfi}\left(\frac{b^2\,x}{\sqrt{b^2}}\right)}^2\,\mathrm{e}^c}{16\,b^5\,\sqrt{\pi}}-\frac{x^2\,\mathrm{e}^{2\,b^2\,x^2+c}}{4\,b^3\,\sqrt{\pi}}$$

input
```
int(x^4*exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
erfi(b*x)*((x^3*exp(c + b^2*x^2))/(2*b^2) - (3*x*exp(c + b^2*x^2))/(4*b^4)
 + (3*pi^(1/2)*erfi((b^2*x)/(b^2)^(1/2))*exp(c))/(8*(b^2)^(5/2))) + (8*exp
(c + 2*b^2*x^2) - 3*pi*erfi((b^2*x)/(b^2)^(1/2))^2*exp(c))/(16*b^5*pi^(1/2
)) - (x^2*exp(c + 2*b^2*x^2))/(4*b^3*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int e^{c+b^2x^2}x^4\text{erfi}(bx)\,dx$$
$$=\frac{e^c\left(-3\sqrt{\pi}\,\text{erf}(bix)^2\,\pi-8e^{b^2x^2}\text{erf}(bix)\,b^3i\pi\,x^3+12e^{b^2x^2}\text{erf}(bix)\,bi\pi x-4\sqrt{\pi}\,e^{2b^2x^2}b^2x^2+8\sqrt{\pi}\,e^{2b^2x^2}\right)}{16b^5\pi}$$

input
```
int(exp(b^2*x^2+c)*x^4*erfi(b*x),x)
```

output
```
(e**c*( - 3*sqrt(pi)*erf(b*i*x)**2*pi - 8*e**(b**2*x**2)*erf(b*i*x)*b**3*i
*pi*x**3 + 12*e**(b**2*x**2)*erf(b*i*x)*b*i*pi*x - 4*sqrt(pi)*e**(2*b**2*x
**2)*b**2*x**2 + 8*sqrt(pi)*e**(2*b**2*x**2)))/(16*b**5*pi)
```

3.290 $\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx=-\frac{e^{c+2b^2x^2}}{4b^3\sqrt{\pi}}+\frac{e^{c+b^2x^2}x\text{erfi}(bx)}{2b^2}-\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{8b^3}$$

output
```
-1/4*exp(2*b^2*x^2+c)/b^3/Pi^(1/2)+1/2*exp(b^2*x^2+c)*x*erfi(b*x)/b^2-1/8*
exp(c)*Pi^(1/2)*erfi(b*x)^2/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx=-\frac{e^c\left(2e^{2b^2x^2}-4be^{b^2x^2}\sqrt{\pi}x\text{erfi}(bx)+\pi\text{erfi}(bx)^2\right)}{8b^3\sqrt{\pi}}$$

input
```
Integrate[E^(c + b^2*x^2)*x^2*Erfi[b*x],x]
```

output
```
-1/8*(E^c*(2*E^(2*b^2*x^2) - 4*b*E^(b^2*x^2)*Sqrt[Pi]*x*Erfi[b*x] + Pi*Erf
i[b*x]^2))/(b^3*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6941, 2638, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{b^2x^2+c}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \ 6941$$

$$-\frac{\int e^{b^2x^2+c}\text{erfi}(bx)dx}{2b^2}-\frac{\int e^{2b^2x^2+c}xdx}{\sqrt{\pi}b}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}$$

$$\Big\downarrow \ 2638$$

$$-\frac{\int e^{b^2x^2+c}\text{erfi}(bx)dx}{2b^2}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

$$\Big\downarrow \ 6929$$

$$-\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)d\text{erfi}(bx)}{4b^3}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

$$\Big\downarrow \ 15$$

$$-\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{8b^3}+\frac{xe^{b^2x^2+c}\text{erfi}(bx)}{2b^2}-\frac{e^{2b^2x^2+c}}{4\sqrt{\pi}b^3}$$

input
```
Int[E^(c + b^2*x^2)*x^2*Erfi[b*x],x]
```

output
```
-1/4*E^(c + 2*b^2*x^2)/(b^3*Sqrt[Pi]) + (E^(c + b^2*x^2)*x*Erfi[b*x])/(2*b
^2) - (E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int e^{b^2x^2+c}x^2 \operatorname{erfi}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*x^2*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*x^2*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx = \frac{\left(4\,\pi bx\,\text{erfi}\,(bx)\,e^{\left(b^2x^2\right)} - \sqrt{\pi}\left(\pi\,\text{erfi}\,(bx)^2 + 2\,e^{\left(2\,b^2x^2\right)}\right)\right)e^c}{8\,\pi b^3}$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erfi(b*x),x, algorithm="fricas")
```

output
```
1/8*(4*pi*b*x*erfi(b*x)*e^(b^2*x^2) - sqrt(pi)*(pi*erfi(b*x)^2 + 2*e^(2*b^
2*x^2)))*e^c/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx = \begin{cases} \frac{xe^ce^{b^2x^2}\,\text{erfi}\,(bx)}{2b^2} - \frac{e^ce^{2b^2x^2}}{4\sqrt{\pi}b^3} - \frac{\sqrt{\pi}e^c\,\text{erfi}^2\,(bx)}{8b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*x**2*erfi(b*x),x)
```

output
```
Piecewise((x*exp(c)*exp(b**2*x**2)*erfi(b*x)/(2*b**2) - exp(c)*exp(2*b**2*
x**2)/(4*sqrt(pi)*b**3) - sqrt(pi)*exp(c)*erfi(b*x)**2/(8*b**3), Ne(b, 0))
, (0, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx = \int x^2\,\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx=\int x^2\,\text{erfi}\,(bx)\,e^{\left(b^2x^2+c\right)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*x^2*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx=\text{erfi}(b\,x)\left(\frac{x\,e^{b^2\,x^2+c}}{2\,b^2}-\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{b^2\,x}{\sqrt{b^2}}\right)\,e^c}{4\,(b^2)^{3/2}}\right)-\frac{2\,e^{2\,b^2\,x^2+c}-\pi\,\text{erfi}\left(\frac{b^2\,x}{\sqrt{b^2}}\right)^2\,e^c}{8\,b^3\,\sqrt{\pi}}$$

input
```
int(x^2*exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
erfi(b*x)*((x*exp(c + b^2*x^2))/(2*b^2) - (pi^(1/2)*erfi((b^2*x)/(b^2)^(1/
2))*exp(c))/(4*(b^2)^(3/2))) - (2*exp(c + 2*b^2*x^2) - pi*erfi((b^2*x)/(b^
2)^(1/2))^2*exp(c))/(8*b^3*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int e^{c+b^2x^2}x^2\text{erfi}(bx)\,dx=\frac{e^c\left(\sqrt{\pi}\,\text{erf}(bix)^2\,\pi-4e^{b^2x^2}\text{erf}(bix)\,bi\pi x-2\sqrt{\pi}\,e^{2b^2x^2}\right)}{8b^3\pi}$$

input
```
int(exp(b^2*x^2+c)*x^2*erfi(b*x),x)
```

output
```
(e**c*(sqrt(pi)*erf(b*i*x)**2*pi - 4*e**(b**2*x**2)*erf(b*i*x)*b*i*pi*x -
2*sqrt(pi)*e**(2*b**2*x**2)))/(8*b**3*pi)
```

3.291 $\int e^{c+b^2x^2}\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{4b}$$

output
```
1/4*exp(c)*Pi^(1/2)*erfi(b*x)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{c+b^2x^2}\text{erfi}(bx)\,dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{4b}$$

input
```
Integrate[E^(c + b^2*x^2)*Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{b^2x^2+c}\text{erfi}(bx)\,dx$$

$$\Big\downarrow \text{6929}$$

$$\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)d\text{erfi}(bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{4b}$$

input
```
Int[E^(c + b^2*x^2)*Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^2)/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

Maple [F]

$$\int e^{b^2x^2+c}\operatorname{erfi}(bx)\,dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x),x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx = \frac{\sqrt{\pi}\operatorname{erfi}(bx)^2 e^c}{4\,b}$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*erfi(b*x)^2*e^c/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx = \begin{cases} \frac{\sqrt{\pi}e^c\operatorname{erfi}^2(bx)}{4b} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x),x)
```

output
```
Piecewise((sqrt(pi)*exp(c)*erfi(b*x)**2/(4*b), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx=\int \operatorname{erfi}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

Giac [F]

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx=\int \operatorname{erfi}(bx)\,e^{(b^2x^2+c)}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.33

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx=\frac{\sqrt{\pi}\,\operatorname{erfi}\left(\frac{b^2x}{\sqrt{b^2}}\right)\,e^c\operatorname{erfi}(b\,x)}{2\sqrt{b^2}}-\frac{\sqrt{\pi}\,e^c\operatorname{erf}\left(x\sqrt{-b^2}\right)^2}{4\,b}-\frac{b\sqrt{\pi}\,\operatorname{erfi}\left(\frac{b^2x}{\sqrt{b^2}}\right)\,e^c\operatorname{erf}\left(x\sqrt{-b^2}\right)}{2\sqrt{b^2}\sqrt{-b^2}}$$

input
```
int(exp(c + b^2*x^2)*erfi(b*x),x)
```

output
```
(pi^(1/2)*erfi((b^2*x)/(b^2)^(1/2))*exp(c)*erfi(b*x))/(2*(b^2)^(1/2)) - (p
i^(1/2)*exp(c)*erf(x*(-b^2)^(1/2))^2)/(4*b) - (b*pi^(1/2)*erfi((b^2*x)/(b^
2)^(1/2))*exp(c)*erf(x*(-b^2)^(1/2)))/(2*(b^2)^(1/2)*(-b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int e^{c+b^2x^2}\operatorname{erfi}(bx)\,dx = -\frac{\sqrt{\pi}\,e^c\operatorname{erf}(bix)^2}{4b}$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x),x)
```

output
```
( - sqrt(pi)*e**c*erf(b*i*x)**2)/(4*b)
```

3.292 $\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx=-\frac{e^{c+b^2x^2}\text{erfi}(bx)}{x}+\frac{1}{2}be^c\sqrt{\pi}\text{erfi}(bx)^2+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}$$

output
```
-exp(b^2*x^2+c)*erfi(b*x)/x+1/2*b*exp(c)*Pi^(1/2)*erfi(b*x)^2+b*exp(c)*Ei(
2*b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx=\frac{1}{2}e^c\left(-\frac{2e^{b^2x^2}\text{erfi}(bx)}{x}+b\sqrt{\pi}\text{erfi}(bx)^2+\frac{2b\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}\right)$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^2,x]
```

output
```
(E^c*((-2*E^(b^2*x^2)*Erfi[b*x])/x + b*Sqrt[Pi]*Erfi[b*x]^2 + (2*b*ExpInte
gralEi[2*b^2*x^2])/Sqrt[Pi]))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6947, 2639, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^2}dx$$

$$\Big\downarrow \text{6947}$$

$$2b^2\int e^{b^2x^2+c}\text{erfi}(bx)dx+\frac{2b\int \frac{e^{2b^2x^2+c}}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}$$

$$\Big\downarrow \text{2639}$$

$$2b^2\int e^{b^2x^2+c}\text{erfi}(bx)dx-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{6929}$$

$$\sqrt{\pi}be^c\int \text{erfi}(bx)d\text{erfi}(bx)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}be^c\text{erfi}(bx)^2$$

input
```
Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^2,x]
```

output
```
-((E^(c + b^2*x^2)*Erfi[b*x])/x) + (b*E^c*Sqrt[Pi]*Erfi[b*x]^2)/2 + (b*E^c
*ExpIntegralEi[2*b^2*x^2])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{\mathrm{e}^{b^2x^2+c}\,\mathrm{erfi}\left(bx\right)}{x^2}dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^2,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx = -\frac{\left(2\,\pi\,\text{erfi}\,(bx)\,e^{(b^2x^2)} - \sqrt{\pi}\left(\pi bx\,\text{erfi}\,(bx)^2 + 2\,bx\text{Ei}(2\,b^2x^2)\right)\right)e^c}{2\,\pi x}$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*pi*erfi(b*x)*e^(b^2*x^2) - sqrt(pi)*(pi*b*x*erfi(b*x)^2 + 2*b*x*Ei
(2*b^2*x^2)))*e^c/(pi*x)
```

Sympy [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx = e^c\int \frac{e^{b^2x^2}\,\text{erfi}\,(bx)}{x^2}\,dx$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)/x**2,x)
```

output
```
exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx = \int \frac{e^{b^2\,x^2+c}\,\text{erfi}(b\,x)}{x^2}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^2,x)
```

output
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^2}\,dx = -e^c\left(\int \frac{e^{b^2x^2}\text{erf}(bix)}{x^2}dx\right)i$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^2,x)
```

output
```
- e**c*int((e**(b**2*x**2)*erf(b*i*x))/x**2,x)*i
```

3.293 $\int \frac{e^{c+b^2x^2}\mathbf{erfi}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -\frac{be^{c+2b^2x^2}}{3\sqrt{\pi}x^2} - \frac{e^{c+b^2x^2}\text{erfi}(bx)}{3x^3} - \frac{2b^2e^{c+b^2x^2}\text{erfi}(bx)}{3x} + \frac{1}{3}b^3e^c\sqrt{\pi}\text{erfi}(bx)^2 + \frac{4b^3e^c\,\text{ExpIntegralEi}\,(2b^2x^2)}{3\sqrt{\pi}}$$

output
```
-1/3*b*exp(2*b^2*x^2+c)/Pi^(1/2)/x^2-1/3*exp(b^2*x^2+c)*erfi(b*x)/x^3-2/3*
b^2*exp(b^2*x^2+c)*erfi(b*x)/x+1/3*b^3*exp(c)*Pi^(1/2)*erfi(b*x)^2+4/3*b^3
*exp(c)*Ei(2*b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -\frac{e^c\left(e^{b^2x^2}\sqrt{\pi}(1+2b^2x^2)\,\text{erfi}(bx) - b^3\pi x^3\text{erfi}(bx)^2 + bx\left(e^{2b^2x^2} - 4b^2x^2\,\text{ExpIntegralEi}\,(2b^2x^2)\right)\right)}{3\sqrt{\pi}x^3}$$

input
```
Integrate[(E^(c + b^2*x^2)*Erfi[b*x])/x^4,x]
```

output
```
-1/3*(E^c*(E^(b^2*x^2)*Sqrt[Pi]*(1 + 2*b^2*x^2)*Erfi[b*x] - b^3*Pi*x^3*Erf
i[b*x]^2 + b*x*(E^(2*b^2*x^2) - 4*b^2*x^2*ExpIntegralEi[2*b^2*x^2])))/(Sqr
t[Pi]*x^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6947, 2643, 2639, 6947, 2639, 6929, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^4}dx$$

$$\Big\downarrow \text{6947}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^2}dx+\frac{2b\int \frac{e^{2b^2x^2+c}}{x^3}dx}{3\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^2}dx+\frac{2b\left(2b^2\int \frac{e^{2b^2x^2+c}}{x}dx-\frac{e^{2b^2x^2+c}}{2x^2}\right)}{3\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2}{3}b^2\int \frac{e^{b^2x^2+c}\text{erfi}(bx)}{x^2}dx-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2e^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)-\frac{e^{2b^2x^2+c}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6947}$$

$$\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2+c}\text{erfi}(bx)dx+\frac{2b\int \frac{e^{2b^2x^2+c}}{x}dx}{\sqrt{\pi}}-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2e^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)-\frac{e^{2b^2x^2+c}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{2}{3}b^2\left(2b^2\int e^{b^2x^2+c}\text{erfi}(bx)dx-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2e^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)-\frac{e^{2b^2x^2+c}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{6929}$$

$$\frac{2}{3}b^2\left(\sqrt{\pi}be^c\int\text{erfi}(bx)d\text{erfi}(bx)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2e^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)-\frac{e^{2b^2x^2+c}}{2x^2}\right)}{3\sqrt{\pi}}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}b^2\left(-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{x}+\frac{be^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)}{\sqrt{\pi}}+\frac{1}{2}\sqrt{\pi}be^c\text{erfi}(bx)^2\right)-\frac{e^{b^2x^2+c}\text{erfi}(bx)}{3x^3}+\frac{2b\left(b^2e^c\,\text{ExpIntegralEi}\left(2b^2x^2\right)-\frac{e^{2b^2x^2+c}}{2x^2}\right)}{3\sqrt{\pi}}$$

input
```
Int[(E^(c + b^2*x^2)*Erfi[b*x])/x^4,x]
```

output
```
-1/3*(E^(c + b^2*x^2)*Erfi[b*x])/x^3 + (2*b*(-1/2*E^(c + 2*b^2*x^2)/x^2 +
b^2*E^c*ExpIntegralEi[2*b^2*x^2]))/(3*Sqrt[Pi]) + (2*b^2*(-((E^(c + b^2*x^
2)*Erfi[b*x])/x) + (b*E^c*Sqrt[Pi]*Erfi[b*x]^2)/2 + (b*E^c*ExpIntegralEi[2
*b^2*x^2])/Sqrt[Pi]))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6947
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m + 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(m + 1)), x] + (-Simp[2*(d/(
m + 1))   Int[x^(m + 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[2*(b/((m
 + 1)*Sqrt[Pi]))   Int[x^(m + 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x],
 x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -1]
```

Maple [F]

$$\int \frac{e^{b^2x^2+c}\operatorname{erfi}(bx)}{x^4}dx$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^4,x)
```

output
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.72

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -\frac{\left((\pi + 2\,\pi b^2x^2)\operatorname{erfi}(bx)\,e^{(b^2x^2)} - \sqrt{\pi}\left(\pi b^3x^3\operatorname{erfi}(bx)^2 + 4\,b^3x^3\operatorname{Ei}(2\,b^2x^2) - bxe^{(2\,b^2x^2)}\right)\right)e^c}{3\,\pi x^3}$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((pi + 2*pi*b^2*x^2)*erfi(b*x)*e^(b^2*x^2) - sqrt(pi)*(pi*b^3*x^3*erf
i(b*x)^2 + 4*b^3*x^3*Ei(2*b^2*x^2) - b*x*e^(2*b^2*x^2)))*e^c/(pi*x^3)
```

Sympy [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = e^c \int \frac{e^{b^2x^2}\,\text{erfi}\,(bx)}{x^4}\,dx$$

input
```
integrate(exp(b**2*x**2+c)*erfi(b*x)/x**4,x)
```

output
```
exp(c)*Integral(exp(b**2*x**2)*erfi(b*x)/x**4, x)
```

Maxima [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Giac [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx)\,e^{(b^2x^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(b^2*x^2+c)*erfi(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*e^(b^2*x^2 + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = \int \frac{e^{b^2\,x^2+c}\,\text{erfi}(b\,x)}{x^4}\,dx$$

input
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^4,x)
```

output
```
int((exp(c + b^2*x^2)*erfi(b*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{c+b^2x^2}\text{erfi}(bx)}{x^4}\,dx = -e^c\left(\int \frac{e^{b^2x^2}\text{erf}(bix)}{x^4}dx\right)i$$

input
```
int(exp(b^2*x^2+c)*erfi(b*x)/x^4,x)
```

output
```
- e**c*int((e**(b**2*x**2)*erf(b*i*x))/x**4,x)*i
```

3.294 $\int e^{c+dx^2} x^3 \mathbf{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 304

$$\int e^{c+dx^2} x^3 \operatorname{erfi}(a+bx)\,dx = \frac{ab^2 e^{a^2+c+2abx+(b^2+d)x^2}}{2d(b^2+d)^2\sqrt{\pi}} - \frac{be^{a^2+c+2abx+(b^2+d)x^2}x}{2d(b^2+d)\sqrt{\pi}} - \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{2d^2} + \frac{e^{c+dx^2}x^2\operatorname{erfi}(a+bx)}{2d} - \frac{a^2b^3e^{c+\frac{a^2d}{b^2+d}}\operatorname{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{2d(b^2+d)^{5/2}} + \frac{be^{c+\frac{a^2d}{b^2+d}}\operatorname{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{4d(b^2+d)^{3/2}} + \frac{be^{c+\frac{a^2d}{b^2+d}}\operatorname{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{2d^2\sqrt{b^2+d}}$$

output
```
1/2*a*b^2*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/d/(b^2+d)^2/Pi^(1/2)-1/2*b*exp(a^
2+c+2*a*b*x+(b^2+d)*x^2)*x/d/(b^2+d)/Pi^(1/2)-1/2*exp(d*x^2+c)*erfi(b*x+a)
/d^2+1/2*exp(d*x^2+c)*x^2*erfi(b*x+a)/d-1/2*a^2*b^3*exp(c+a^2*d/(b^2+d))*e
rfi((a*b+(b^2+d)*x)/(b^2+d)^(1/2))/d/(b^2+d)^(5/2)+1/4*b*exp(c+a^2*d/(b^2+
d))*erfi((a*b+(b^2+d)*x)/(b^2+d)^(1/2))/d/(b^2+d)^(3/2)+1/2*b*exp(c+a^2*d/
(b^2+d))*erfi((a*b+(b^2+d)*x)/(b^2+d)^(1/2))/d^2/(b^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx$$
$$= \frac{e^c\left(2e^{dx^2}(-1+dx^2)\,\text{erfi}(a+bx)+\frac{2be^{\frac{a^2d}{b^2+d}}\text{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{\sqrt{b^2+d}}-\frac{bde^{\frac{a^2d}{b^2+d}}\left(2(b^2+d)e^{\frac{(ab+(b^2+d)x)^2}{b^2+d}}(-ab+(b^2+d)x)+((-\right.}{(b^2+d)^3\sqrt{\pi}}\right.}{4d^2}$$

input
```
Integrate[E^(c + d*x^2)*x^3*Erfi[a + b*x],x]
```

output
```
(E^c*(2*E^(d*x^2)*(-1 + d*x^2)*Erfi[a + b*x] + (2*b*E^((a^2*d)/(b^2 + d))*
Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/Sqrt[b^2 + d] - (b*d*E^((a^2*d)/(
b^2 + d))*(2*(b^2 + d)*E^((a*b + (b^2 + d)*x)^2/(b^2 + d))*(-(a*b) + (b^2
+ d)*x) + ((-1 + 2*a^2)*b^2 - d)*Sqrt[b^2 + d]*Sqrt[Pi]*Erfi[(a*b + (b^2 +
 d)*x)/Sqrt[b^2 + d]]))/((b^2 + d)^3*Sqrt[Pi])))/(4*d^2)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {6941, 2671, 2664, 2633, 2670, 2664, 2633, 6938, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{c+dx^2}\text{erfi}(a+bx)\,dx$$
$$\Big\downarrow \text{ 6941}$$
$$-\frac{b\int e^{a^2+2bxa+(b^2+d)x^2+c}x^2dx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$
$$\Big\downarrow \text{ 2671}$$

$$-\frac{b\left(-\frac{\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{2(b^2+d)}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2664}$$

$$-\frac{b\left(-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}-\frac{e^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx}{2(b^2+d)}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{b\left(-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2670}$$

$$-\frac{b\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{b^2+d}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{ 2664}$$

$$-\frac{b\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{abe^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx}{b^2+d}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\Big\downarrow$ 2633

$$-\frac{\int e^{dx^2+c}x\text{erfi}(a+bx)dx}{d}-\frac{b\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\Big\downarrow$ 6938

$$-\frac{\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{b\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{\sqrt{\pi}d}}{d}-\frac{b\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\Big\downarrow$ 2664

$$-\frac{\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx}{\sqrt{\pi}d}}{d}-\frac{b\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2d\sqrt{b^2+d}}}{d}-\frac{b\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}+\frac{x^2e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^3*Erfi[a + b*x],x]
```

output
```
(E^(c + d*x^2)*x^2*Erfi[a + b*x])/(2*d) - ((E^(c + d*x^2)*Erfi[a + b*x])/(
2*d) - (b*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2 + d)*x)/Sqrt[b^2 + d]
])/(2*d*Sqrt[b^2 + d]))/d - (b*((E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2)*x)/
(2*(b^2 + d)) - (E^(c + (a^2*d)/(b^2 + d))*Sqrt[Pi]*Erfi[(a*b + (b^2 + d)*
x)/Sqrt[b^2 + d]])/(4*(b^2 + d)^(3/2)) - (a*b*(E^(a^2 + c + 2*a*b*x + (b^2
 + d)*x^2)/(2*(b^2 + d)) - (a*b*E^(c + (a^2*d)/(b^2 + d))*Sqrt[Pi]*Erfi[(a
*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*(b^2 + d)^(3/2))))/(b^2 + d)))/(d*Sqr
t[Pi])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6941
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_)^(m_), x_Symbol]
:> Simp[x^(m - 1)*E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] + (-Simp[(m - 1)/
(2*d)   Int[x^(m - 2)*E^(c + d*x^2)*Erfi[a + b*x], x], x] - Simp[b/(d*Sqrt[
Pi])   Int[x^(m - 1)*E^(a^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x]) /; Free
Q[{a, b, c, d}, x] && IGtQ[m, 1]
```

Maple [F]

$$\int \mathrm{e}^{d\,x^{2}+c} x^{3} \operatorname{erfi}\left(b x +a \right)dx$$

input
```
int(exp(d*x^2+c)*x^3*erfi(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^3*erfi(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.86

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx = -\frac{\pi(2\,b^5-(2\,a^2-5)b^3d+3\,bd^2)\sqrt{-b^2-d}\,\text{erf}\left(\frac{(ab+(b^2+d)x)\sqrt{-b^2-d}}{b^2+d}\right)e^{\left(\frac{b^2c+\left(a^2+c\right)d}{b^2+d}\right)}-2\left(\pi(b^6d+3\,b^4d^2+\right.}{}$$

input
```
integrate(exp(d*x^2+c)*x^3*erfi(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(pi*(2*b^5 - (2*a^2 - 5)*b^3*d + 3*b*d^2)*sqrt(-b^2 - d)*erf((a*b + (
b^2 + d)*x)*sqrt(-b^2 - d)/(b^2 + d))*e^((b^2*c + (a^2 + c)*d)/(b^2 + d))
- 2*(pi*(b^6*d + 3*b^4*d^2 + 3*b^2*d^3 + d^4)*x^2 - pi*(b^6 + 3*b^4*d + 3*
b^2*d^2 + d^3))*erfi(b*x + a)*e^(d*x^2 + c) - 2*sqrt(pi)*(a*b^4*d + a*b^2*
d^2 - (b^5*d + 2*b^3*d^2 + b*d^3)*x)*e^(b^2*x^2 + 2*a*b*x + d*x^2 + a^2 +
c))/(pi*(b^6*d^2 + 3*b^4*d^3 + 3*b^2*d^4 + d^5))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**3*erfi(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx = \int x^3\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^3*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx = \int x^3\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^3*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx$$
$$=\frac{\text{erfi}\left(\frac{a\,b+x\,(b^2+d)}{\sqrt{b^2+d}}\right)\left(b^3\,e^{\frac{c\,d}{b^2+d}+\frac{a^2\,d}{b^2+d}+\frac{b^2\,c}{b^2+d}}-2\,a^2\,b^3\,e^{\frac{c\,d}{b^2+d}+\frac{a^2\,d}{b^2+d}+\frac{b^2\,c}{b^2+d}}+b\,d\,e^{\frac{c\,d}{b^2+d}+\frac{a^2\,d}{b^2+d}+\frac{b^2\,c}{b^2+d}}\right)}{4\,d\,(b^2+d)^{5/2}}$$
$$-\frac{\frac{b\,x\,e^{a^2+2\,a\,b\,x+b^2\,x^2+d\,x^2+c}}{2\,(b^2+d)}-\frac{a\,b^2\,e^{a^2+2\,a\,b\,x+b^2\,x^2+d\,x^2+c}}{2\,(b^2+d)^2}}{d\,\sqrt{\pi}}$$
$$-\text{erfi}(a+b\,x)\left(\frac{e^{d\,x^2+c}}{2\,d^2}-\frac{x^2\,e^{d\,x^2+c}}{2\,d}\right)-\frac{b\,e^{c+a^2-\frac{a^2\,b^2}{b^2+d}}\,\text{erf}\left(\frac{a\,b\,1\text{i}+x\,(b^2+d)\,1\text{i}}{\sqrt{b^2+d}}\right)\,1\text{i}}{2\,d^2\,\sqrt{b^2+d}}$$

input
```
int(x^3*erfi(a + b*x)*exp(c + d*x^2),x)
```

output
```
(erfi((a*b + x*(d + b^2))/(d + b^2)^(1/2))*(b^3*exp((c*d)/(d + b^2) + (a^2
*d)/(d + b^2) + (b^2*c)/(d + b^2)) - 2*a^2*b^3*exp((c*d)/(d + b^2) + (a^2*
d)/(d + b^2) + (b^2*c)/(d + b^2)) + b*d*exp((c*d)/(d + b^2) + (a^2*d)/(d +
 b^2) + (b^2*c)/(d + b^2))))/(4*d*(d + b^2)^(5/2)) - ((b*x*exp(c + d*x^2 +
 a^2 + b^2*x^2 + 2*a*b*x))/(2*(d + b^2)) - (a*b^2*exp(c + d*x^2 + a^2 + b^
2*x^2 + 2*a*b*x))/(2*(d + b^2)^2))/(d*pi^(1/2)) - erfi(a + b*x)*(exp(c + d
*x^2)/(2*d^2) - (x^2*exp(c + d*x^2))/(2*d)) - (b*exp(c + a^2 - (a^2*b^2)/(
d + b^2))*erf((a*b*1i + x*(d + b^2)*1i)/(d + b^2)^(1/2))*1i)/(2*d^2*(d + b
^2)^(1/2))
```

Reduce [F]

$$\int e^{c+dx^2}x^3\text{erfi}(a+bx)\,dx=-e^c\left(\int e^{d\,x^2}\text{erf}(bix+ai)\,x^3dx\right)i$$

input
```
int(exp(d*x^2+c)*x^3*erfi(b*x+a),x)
```

output
```
 - e**c*int(e**(d*x**2)*erf(a*i + b*i*x)*x**3,x)*i
```

3.295 $\int e^{c+dx^2} x \,\mathrm{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int e^{c+dx^2} x \,\mathrm{erfi}(a+bx)\,dx=\frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{2d}-\frac{be^{c+\frac{a^2d}{b^2+d}}\mathrm{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{2d\sqrt{b^2+d}}$$

output
```
1/2*exp(d*x^2+c)*erfi(b*x+a)/d-1/2*b*exp(c+a^2*d/(b^2+d))*erfi((a*b+(b^2+d
)*x)/(b^2+d)^(1/2))/d/(b^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int e^{c+dx^2} x \,\mathrm{erfi}(a+bx)\,dx=\frac{e^c\left(e^{dx^2}\mathrm{erfi}(a+bx)-\frac{be^{\frac{a^2d}{b^2+d}}\mathrm{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{\sqrt{b^2+d}}\right)}{2d}$$

input
```
Integrate[E^(c + d*x^2)*x*Erfi[a + b*x],x]
```

output
```
(E^c*(E^(d*x^2)*Erfi[a + b*x] - (b*E^((a^2*d)/(b^2 + d))*Erfi[(a*b + (b^2
+ d)*x)/Sqrt[b^2 + d]])/Sqrt[b^2 + d]))/(2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6938, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{c+dx^2}\text{erfi}(a+bx)\,dx$$

$$\Big\downarrow \text{6938}$$

$$\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{b\int e^{a^2+2bxa+(b^2+d)x^2+c}dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2664}$$

$$\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{(ab+(b^2+d)x)^2}{b^2+d}}dx}{\sqrt{\pi}d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2d}-\frac{be^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{2d\sqrt{b^2+d}}$$

input
```
Int[E^(c + d*x^2)*x*Erfi[a + b*x],x]
```

output
```
(E^(c + d*x^2)*Erfi[a + b*x])/(2*d) - (b*E^(c + (a^2*d)/(b^2 + d))*Erfi[(a
*b + (b^2 + d)*x)/Sqrt[b^2 + d]])/(2*d*Sqrt[b^2 + d])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 6938
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(a_.) + (b_.)*(x_)]*(x_), x_Symbol] :> Si
mp[E^(c + d*x^2)*(Erfi[a + b*x]/(2*d)), x] - Simp[b/(d*Sqrt[Pi])   Int[E^(a
^2 + c + 2*a*b*x + (b^2 + d)*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [F]

$$\int \mathrm{e}^{d\,x^2+c} x\ \mathrm{erfi}\left(bx+a\right)dx$$

input
```
int(exp(d*x^2+c)*x*erfi(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x*erfi(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int e^{c+dx^2} x\mathrm{erfi}(a+bx)\,dx = \frac{\sqrt{-b^2-d}\,b\,\mathrm{erf}\left(\frac{(ab+(b^2+d)x)\sqrt{-b^2-d}}{b^2+d}\right)e^{\left(\frac{b^2c+\left(a^2+c\right)d}{b^2+d}\right)}+\left(b^2+d\right)\mathrm{erfi}\left(bx+a\right)e^{\left(dx^2+c\right)}}{2\left(b^2d+d^2\right)}$$

input
```
integrate(exp(d*x^2+c)*x*erfi(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-b^2 - d)*b*erf((a*b + (b^2 + d)*x)*sqrt(-b^2 - d)/(b^2 + d))*e^
((b^2*c + (a^2 + c)*d)/(b^2 + d)) + (b^2 + d)*erfi(b*x + a)*e^(d*x^2 + c))
/(b^2*d + d^2)
```

Sympy [F]

$$\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx=e^c\int xe^{dx^2}\,\text{erfi}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x*erfi(b*x+a),x)
```

output
```
exp(c)*Integral(x*exp(d*x**2)*erfi(a + b*x), x)
```

Maxima [F]

$$\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx=\int x\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Giac [F]

$$\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx=\int x\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx=\frac{\text{erfi}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}}{2\,d}+\frac{b\,\mathrm{e}^{c+a^2-\frac{a^2\,b^2}{b^2+d}}\,\mathrm{erf}\left(\frac{a\,b\,1\mathrm{i}+x\,\left(b^2+d\right)\,1\mathrm{i}}{\sqrt{b^2+d}}\right)\,1\mathrm{i}}{2\,d\,\sqrt{b^2+d}}$$

input
```
int(x*erfi(a + b*x)*exp(c + d*x^2),x)
```

output
```
(erfi(a + b*x)*exp(c + d*x^2))/(2*d) + (b*exp(c + a^2 - (a^2*b^2)/(d + b^2
))*erf((a*b*1i + x*(d + b^2)*1i)/(d + b^2)^(1/2))*1i)/(2*d*(d + b^2)^(1/2)
)
```

Reduce [F]

$$\int e^{c+dx^2}x\text{erfi}(a+bx)\,dx=-e^c\left(\int e^{d\,x^2}\text{erf}(bix+ai)\,xdx\right)i$$

input
```
int(exp(d*x^2+c)*x*erfi(b*x+a),x)
```

output
```
 - e**c*int(e**(d*x**2)*erf(a*i + b*i*x)*x,x)*i
```

3.296 $\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \text{Int}\left(\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x},x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{ 6950}$$

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x}dx$$

input
```
Int[(E^(c + d*x^2)*Erfi[a + b*x])/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{d\,x^2+c}\operatorname{erfi}\left(bx+a\right)}{x}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)*e^(d*x^2 + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 6.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = e^c\int \frac{e^{dx^2}\,\text{erfi}\,(a+bx)}{x}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x+a)/x,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \int \frac{\text{erfi}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\,dx = \int \frac{\text{erfi}(a+b\,x)\;\mathrm{e}^{d\,x^2+c}}{x}\,dx$$

input
```
int((erfi(a + b*x)*exp(c + d*x^2))/x,x)
```

output
```
int((erfi(a + b*x)*exp(c + d*x^2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bix+ai)}{x}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(a*i + b*i*x))/x,x)*i
```

3.297 $\int \frac{e^{c+dx^2}\mathbf{erfi}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x^3}\,dx =& -\frac{be^{a^2+c+2abx+(b^2+d)x^2}}{\sqrt{\pi}x} - \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{2x^2} \\ &+ b\sqrt{b^2+d}e^{c+\frac{a^2d}{b^2+d}}\operatorname{erfi}\left(\frac{ab+(b^2+d)\,x}{\sqrt{b^2+d}}\right) \\ &+ \frac{2ab^2\operatorname{Int}\left(\frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x},x\right)}{\sqrt{\pi}} \\ &+ d\operatorname{Int}\left(\frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x},x\right)\end{aligned}$$

output
```
-b*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/Pi^(1/2)/x-1/2*exp(d*x^2+c)*erfi(b*x+a)/
x^2+b*(b^2+d)^(1/2)*exp(c+a^2*d/(b^2+d))*erfi((a*b+(b^2+d)*x)/(b^2+d)^(1/2
))+2*a*b^2*Defer(Int)(exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/x,x)/Pi^(1/2)+d*Defer
(Int)(exp(d*x^2+c)*erfi(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x^3}\,dx = \int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x^3}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^3,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 6947}$$

$$\frac{b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x^2}dx}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\operatorname{erfi}(a+bx)}{x}dx-\frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 2672}$$

$$\frac{b\left(2\left(b^2+d\right)\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx+2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\operatorname{erfi}(a+bx)}{x}dx-\frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{b\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+2\left(b^2+d\right)e^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{b\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 2673}$$

$$\frac{b\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 6950}$$

$$\frac{b\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)}{\sqrt{\pi}}+d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{2x^2}$$

input
```
Int[(E^(c + d*x^2)*Erfi[a + b*x])/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfi}\left(bx+a\right)}{x^3}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^3,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{x^3}\,dx = \int \frac{\mathrm{erfi}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 27.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{x^3}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erfi}\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x+a)/x**3,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(a + b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^3}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^3}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x^3}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{erfi}(a+b\,x)\;\mathrm{e}^{d\,x^2+c}}{x^3}\,dx$$

input
```
int((erfi(a + b*x)*exp(c + d*x^2))/x^3,x)
```

output
```
int((erfi(a + b*x)*exp(c + d*x^2))/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{e^{c+dx^2}\operatorname{erfi}(a+bx)}{x^3}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\operatorname{erf}(bix+ai)}{x^3}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^3,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(a*i + b*i*x))/x**3,x)*i
```

3.298 $\int e^{c+dx^2} x^4 \textbf{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int e^{c+dx^2} x^4 \mathrm{erfi}(a+bx)\,dx =& -\frac{a^2b^3e^{a^2+c+2abx+(b^2+d)x^2}}{2d(b^2+d)^3\sqrt{\pi}} + \frac{be^{a^2+c+2abx+(b^2+d)x^2}}{2d(b^2+d)^2\sqrt{\pi}} \\ &+ \frac{3be^{a^2+c+2abx+(b^2+d)x^2}}{4d^2(b^2+d)\sqrt{\pi}} + \frac{ab^2e^{a^2+c+2abx+(b^2+d)x^2}x}{2d(b^2+d)^2\sqrt{\pi}} \\ &- \frac{be^{a^2+c+2abx+(b^2+d)x^2}x^2}{2d(b^2+d)\sqrt{\pi}} - \frac{3e^{c+dx^2}x\,\mathrm{erfi}(a+bx)}{4d^2} \\ &+ \frac{e^{c+dx^2}x^3\mathrm{erfi}(a+bx)}{2d} + \frac{a^3b^4e^{c+\frac{a^2d}{b^2+d}}\mathrm{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{2d(b^2+d)^{7/2}} \\ &- \frac{3ab^2e^{c+\frac{a^2d}{b^2+d}}\mathrm{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{4d(b^2+d)^{5/2}} \\ &- \frac{3ab^2e^{c+\frac{a^2d}{b^2+d}}\mathrm{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{4d^2(b^2+d)^{3/2}} + \frac{3\mathrm{Int}\left(e^{c+dx^2}\mathrm{erfi}(a+bx),x\right)}{4d^2}\end{aligned}$$

output
```
-1/2*a^2*b^3*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/d/(b^2+d)^3/Pi^(1/2)+1/2*b*exp
(a^2+c+2*a*b*x+(b^2+d)*x^2)/d/(b^2+d)^2/Pi^(1/2)+3/4*b*exp(a^2+c+2*a*b*x+(
b^2+d)*x^2)/d^2/(b^2+d)/Pi^(1/2)+1/2*a*b^2*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)*
x/d/(b^2+d)^2/Pi^(1/2)-1/2*b*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)*x^2/d/(b^2+d)/
Pi^(1/2)-3/4*exp(d*x^2+c)*x*erfi(b*x+a)/d^2+1/2*exp(d*x^2+c)*x^3*erfi(b*x+
a)/d+1/2*a^3*b^4*exp(c+a^2*d/(b^2+d))*erfi((a*b+(b^2+d)*x)/(b^2+d)^(1/2))/
d/(b^2+d)^(7/2)-3/4*a*b^2*exp(c+a^2*d/(b^2+d))*erfi((a*b+(b^2+d)*x)/(b^2+d
)^(1/2))/d/(b^2+d)^(5/2)-3/4*a*b^2*exp(c+a^2*d/(b^2+d))*erfi((a*b+(b^2+d)*
x)/(b^2+d)^(1/2))/d^2/(b^2+d)^(3/2)+3/4*Defer(Int)(exp(d*x^2+c)*erfi(b*x+a
),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx=\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^4*Erfi[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^4*Erfi[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{c+dx^2}\text{erfi}(a+bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$-\frac{b\int e^{a^2+2bxa+(b^2+d)x^2+c}x^3dx}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2671}$$

$$-\frac{b\left(-\frac{\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}x^2dx}{b^2+d}+\frac{x^2e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{b^2+d}}{b^2+d}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}x^2dx}{b^2+d}+\frac{x^2e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{abe^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx}{b^2+d}}{b^2+d}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}x^2dx}{b^2+d}+\frac{x^2e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{b\left(-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}x^2dx}{b^2+d}-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}}{b^2+d}+\frac{x^2e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}\right)}{\sqrt{\pi}d}-\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2671}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{2\left(b^2+d\right)}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}\right)}{b^2+d}-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}}\cdots}{\cdots}}{b^2+d}\cdots\right.}{\sqrt{\pi}d\cdots}$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}-\frac{e^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx}{2\left(b^2+d\right)}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}\right)}{b^2+d}-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\cdots}}{\cdots}}{b^2+d}\cdots\right.}{\sqrt{\pi}d\cdots}$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{4\left(b^2+d\right)^{3/2}}+\frac{xe^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}\right)}{b^2+d}-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe\cdots}{\cdots}}{b^2+\cdots}\cdots\right.}{\sqrt{\pi}d\cdots}$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}-\frac{ab\int e^{a^2+2bxa+(b^2+d)x^2+c}dx}{b^2+d}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}\right)}{b^2+d}-\cdots\right)}{\sqrt{\pi}d}\cdots$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\downarrow$ 2664

$$-\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}-\frac{abe^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{(ab+(b^2+d)x)^2}{b^2+d}}dx}{b^2+d}\right)}{b^2+d}-\frac{\sqrt{\pi}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{4(b^2+d)^{3/2}}+\frac{xe^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}\right)}{b^2+d}-\cdots\right)}{\sqrt{\pi}d}\cdots$$

$$\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}+\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\downarrow$ 2633

$$-\frac{3\int e^{dx^2+c}x^2\text{erfi}(a+bx)dx}{2d}-\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}}{b^2+d}-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\right.}{b^2+d}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\downarrow$ 6941

$$-\frac{3\left(-\frac{b\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfi}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erfi}(a+bx)}{2d}\right)}{2d}-\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}}{b^2+d}-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\right.}{b^2+d}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$\downarrow$ 2670

$$-\frac{3\left(-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{b^2+d}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\text{erfi}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\text{erfi}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}}{b^2+d}-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\cdots\right)}{b^2+d}\cdots\right)}{\sqrt{\pi}d\cdots}$$

$$\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\downarrow 2664$$

$$-\frac{3\left(-\frac{b\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}-\frac{abe^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{(ab+(b^2+d)x)^2}{b^2+d}}dx}{b^2+d}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfi}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\operatorname{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{2(b^2+d)^{3/2}}}{b^2+d}-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2(b^2+d)+c}}{2(b^2+d)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\operatorname{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)}{2(b^2+d)^{3/2}}\right)}{b^2+d}-\right.}{b^2+d}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}$$

$$\downarrow 2633$$

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\operatorname{erfi}(a+bx)dx}{2d}-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\operatorname{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}\right)}{2d}-$$

$$\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\operatorname{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}}{b^2+d}-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\operatorname{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\right.}{b^2+d}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}$$

$\downarrow$ 6935

$$-\frac{3\left(-\frac{\int e^{dx^2+c}\text{erfi}(a+bx)dx}{2d}-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erfi}(a+bx)}{2d}\right)}{2d}-\frac{b\left(-\frac{\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}}{b^2+d}-\frac{ab\left(-\frac{ab\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2\left(b^2+d\right)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2\left(b^2+d\right)^{3/2}}\right)}{b^2+d}-\right.}{b^2+d}\right.}{\sqrt{\pi}d}$$

$$\frac{x^3e^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^4*Erfi[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \mathrm{e}^{d\,x^2+c}x^4\,\text{erfi}\left(bx+a\right)dx$$

input
```
int(exp(d*x^2+c)*x^4*erfi(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^4*erfi(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx = \int x^4\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^4*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(d*x**2+c)*x**4*erfi(b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx = \int x^4\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^4*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx=\int x^4\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^4*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^4*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^4\text{erfi}(a+bx)\,dx=\int x^4\,\text{erfi}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^4*erfi(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^4*erfi(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int e^{c+dx^2} x^4 \text{erfi}(a+bx)\, dx = -e^c \left(\int e^{d\,x^2} \text{erf}(bix+ai)\, x^4 dx\right) i$$

input
```
int(exp(d*x^2+c)*x^4*erfi(b*x+a),x)
```

output
```
- e**c*int(e**(d*x**2)*erf(a*i + b*i*x)*x**4,x)*i
```

3.299 $\int e^{c+dx^2}x^2\mathbf{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx=-\frac{be^{a^2+c+2abx+(b^2+d)x^2}}{2d(b^2+d)\sqrt{\pi}}+\frac{e^{c+dx^2}x\text{erfi}(a+bx)}{2d}+\frac{ab^2e^{c+\frac{a^2d}{b^2+d}}\text{erfi}\left(\frac{ab+(b^2+d)x}{\sqrt{b^2+d}}\right)}{2d(b^2+d)^{3/2}}-\frac{\text{Int}\left(e^{c+dx^2}\text{erfi}(a+bx),x\right)}{2d}$$

output
```
-1/2*b*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/d/(b^2+d)/Pi^(1/2)+1/2*exp(d*x^2+c)*
x*erfi(b*x+a)/d+1/2*a*b^2*exp(c+a^2*d/(b^2+d))*erfi((a*b+(b^2+d)*x)/(b^2+d
)^(1/2))/d/(b^2+d)^(3/2)-1/2*Defer(Int)(exp(d*x^2+c)*erfi(b*x+a),x)/d
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx=\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*x^2*Erfi[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*x^2*Erfi[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{c+dx^2} \operatorname{erfi}(a+bx)\,dx$$

$$\Big\downarrow \text{6941}$$

$$-\frac{b\int e^{a^2+2bxa+(b^2+d)x^2+c}xdx}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfi}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{ab\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx}{b^2+d}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfi}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{abe^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx}{b^2+d}\right)}{\sqrt{\pi}d}-\frac{\int e^{dx^2+c}\operatorname{erfi}(a+bx)dx}{2d}+\frac{xe^{c+dx^2}\operatorname{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\int e^{dx^2+c}\text{erfi}(a+bx)dx}{2d}-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2(b^2+d)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

$$\Big\downarrow \textbf{6935}$$

$$-\frac{\int e^{dx^2+c}\text{erfi}(a+bx)dx}{2d}-\frac{b\left(\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2(b^2+d)}-\frac{\sqrt{\pi}abe^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)}{2(b^2+d)^{3/2}}\right)}{\sqrt{\pi}d}+\frac{xe^{c+dx^2}\text{erfi}(a+bx)}{2d}$$

input
```
Int[E^(c + d*x^2)*x^2*Erfi[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \mathrm{e}^{d\,x^2+c}x^2\,\text{erfi}\,(bx+a)\,dx$$

input
```
int(exp(d*x^2+c)*x^2*erfi(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*x^2*erfi(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx=\int x^2\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfi(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 46.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx=e^c\int x^2e^{dx^2}\,\text{erfi}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*x**2*erfi(b*x+a),x)
```

output
```
exp(c)*Integral(x**2*exp(d*x**2)*erfi(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx=\int x^2\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx = \int x^2\,\text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*x^2*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*erfi(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx = \int x^2\,\text{erfi}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(x^2*erfi(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(x^2*erfi(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int e^{c+dx^2}x^2\text{erfi}(a+bx)\,dx=-e^c\left(\int e^{d\,x^2}\text{erf}(bix+ai)\,x^2dx\right)i$$

input
```
int(exp(d*x^2+c)*x^2*erfi(b*x+a),x)
```

output
```
- e**c*int(e**(d*x**2)*erf(a*i + b*i*x)*x**2,x)*i
```

3.300 $\int e^{c+dx^2}\textbf{erfi}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\text{Int}\left(e^{c+dx^2}\text{erfi}(a+bx),x\right)$$

output
```
Defer(Int)(exp(d*x^2+c)*erfi(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\int e^{c+dx^2}\text{erfi}(a+bx)\,dx$$

input
```
Integrate[E^(c + d*x^2)*Erfi[a + b*x],x]
```

output
```
Integrate[E^(c + d*x^2)*Erfi[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6935}$$

$$\int e^{c+dx^2}\text{erfi}(a+bx)dx$$

input
```
Int[E^(c + d*x^2)*Erfi[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \mathrm{e}^{d\,x^2+c}\,\text{erfi}\,(bx+a)\,dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a),x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\int \text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a),x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)*e^(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=e^c\int e^{dx^2}\,\text{erfi}\,(a+bx)\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x+a),x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\int \text{erfi}\,(bx+a)\,e^{(dx^2+c)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\int \text{erfi}\left(bx+a\right)e^{\left(dx^2+c\right)}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a),x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=\int \text{erfi}(a+b\,x)\ \mathrm{e}^{d\,x^2+c}\,dx$$

input
```
int(erfi(a + b*x)*exp(c + d*x^2),x)
```

output
```
int(erfi(a + b*x)*exp(c + d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int e^{c+dx^2}\text{erfi}(a+bx)\,dx=-e^{c}\left(\int e^{d\,x^2}\text{erf}(bix+ai)\,dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a),x)
```

output
```
- e**c*int(e**(d*x**2)*erf(a*i + b*i*x),x)*i
```

3.301 $\int \frac{e^{c+dx^2}\mathbf{erfi}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx = -\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x} + \frac{2b\text{Int}\left(\frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x},x\right)}{\sqrt{\pi}} + 2d\text{Int}\left(e^{c+dx^2}\text{erfi}(a+bx),x\right)$$

output
```
-exp(d*x^2+c)*erfi(b*x+a)/x+2*b*Defer(Int)(exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/
x,x)/Pi^(1/2)+2*d*Defer(Int)(exp(d*x^2+c)*erfi(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx = \int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^2,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{6947}$$

$$\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(a+bx)dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}$$

$$\Big\downarrow \text{2673}$$

$$\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(a+bx)dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}$$

$$\Big\downarrow \text{6935}$$

$$\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(a+bx)dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}$$

input
```
Int[(E^(c + d*x^2)*Erfi[a + b*x])/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{d\,x^2+c}\,\mathrm{erfi}\left(bx+a\right)}{x^2}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^2,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{x^2}\,dx = \int \frac{\mathrm{erfi}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{x^2}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erfi}\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x+a)/x**2,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x^2}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx = \int \frac{\text{erfi}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}}{x^2}\,dx$$

input
```
int((erfi(a + b*x)*exp(c + d*x^2))/x^2,x)
```

output
```
int((erfi(a + b*x)*exp(c + d*x^2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^2}\,dx = -e^c\left(\int \frac{e^{d\,x^2}\text{erf}(bix+ai)}{x^2}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^2,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(a*i + b*i*x))/x**2,x)*i
```

3.302 $\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\begin{aligned}\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx =& -\frac{be^{a^2+c+2abx+(b^2+d)x^2}}{3\sqrt{\pi}x^2}-\frac{2ab^2e^{a^2+c+2abx+(b^2+d)x^2}}{3\sqrt{\pi}x} \\ &-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}-\frac{2de^{c+dx^2}\text{erfi}(a+bx)}{3x} \\ &+\frac{2}{3}ab^2\sqrt{b^2+d}e^{c+\frac{a^2d}{b^2+d}}\text{erfi}\left(\frac{ab+(b^2+d)\,x}{\sqrt{b^2+d}}\right) \\ &+\frac{4a^2b^3\text{Int}\left(\frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}} \\ &+\frac{4bd\text{Int}\left(\frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}} \\ &+\frac{2b(b^2+d)\,\text{Int}\left(\frac{e^{a^2+c+2abx+\left(b^2+d\right)x^2}}{x},x\right)}{3\sqrt{\pi}} \\ &+\frac{4}{3}d^2\text{Int}\left(e^{c+dx^2}\text{erfi}(a+bx),x\right)\end{aligned}$$

output
```
-1/3*b*exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/Pi^(1/2)/x^2-2/3*a*b^2*exp(a^2+c+2*a
*b*x+(b^2+d)*x^2)/Pi^(1/2)/x-1/3*exp(d*x^2+c)*erfi(b*x+a)/x^3-2/3*d*exp(d*
x^2+c)*erfi(b*x+a)/x+2/3*a*b^2*(b^2+d)^(1/2)*exp(c+a^2*d/(b^2+d))*erfi((a*
b+(b^2+d)*x)/(b^2+d)^(1/2))+4/3*a^2*b^3*Defer(Int)(exp(a^2+c+2*a*b*x+(b^2+
d)*x^2)/x,x)/Pi^(1/2)+4/3*b*d*Defer(Int)(exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/x,
x)/Pi^(1/2)+2/3*b*(b^2+d)*Defer(Int)(exp(a^2+c+2*a*b*x+(b^2+d)*x^2)/x,x)/P
i^(1/2)+4/3*d^2*Defer(Int)(exp(d*x^2+c)*erfi(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx=\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx$$

input
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^4,x]
```

output
```
Integrate[(E^(c + d*x^2)*Erfi[a + b*x])/x^4, x]
```

Rubi [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \; 6947$$

$$\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x^3}dx}{3\sqrt{\pi}}+\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 2672

$$\frac{2b\left(ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x^2}dx+\left(b^2+d\right)\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{2x^2}\right)}{3\sqrt{\pi}}+$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 2672

$$\frac{2b\left(\left(b^2+d\right)\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+ab\left(2\left(b^2+d\right)\int e^{a^2+2bxa+\left(b^2+d\right)x^2+c}dx+2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx-\frac{e^{a^2+\cdots}}{\cdots}\right.\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 2664

$$\frac{2b\left(\left(b^2+d\right)\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+ab\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+2\left(b^2+d\right)e^{\frac{a^2d}{b^2+d}+c}\int e^{\frac{\left(ab+\left(b^2+d\right)x\right)^2}{b^2+d}}dx-\frac{e^{\cdots}}{\cdots}\right.\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 2633

$$\frac{2b\left(ab\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)+\left(b^2+d\right)\int \frac{e^{a^2\cdots}}{\cdots}\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 2673

$$\frac{2b\left(ab\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x\left(b^2+d\right)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)+\left(b^2+d\right)\int \frac{e^{a^2\cdots}}{\cdots}\right.}{3\sqrt{\pi}}$$
$$\frac{2}{3}d\int \frac{e^{dx^2+c}\text{erfi}(a+bx)}{x^2}dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 6947

$$\frac{2b\left(ab\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)+\left(b^2+d\right)\int \frac{e^{a^2}}{}\right.}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(a+bx)dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 2673

$$\frac{2b\left(ab\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)+\left(b^2+d\right)\int \frac{e^{a^2}}{}\right.}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(a+bx)dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

$\downarrow$ 6935

$$\frac{2b\left(ab\left(2ab\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx+\sqrt{\pi}\sqrt{b^2+d}e^{\frac{a^2d}{b^2+d}+c}\text{erfi}\left(\frac{ab+x(b^2+d)}{\sqrt{b^2+d}}\right)-\frac{e^{a^2+2abx+x^2\left(b^2+d\right)+c}}{x}\right)+\left(b^2+d\right)\int \frac{e^{a^2}}{}\right.}{3\sqrt{\pi}}$$

$$\frac{2}{3}d\left(\frac{2b\int \frac{e^{a^2+2bxa+\left(b^2+d\right)x^2+c}}{x}dx}{\sqrt{\pi}}+2d\int e^{dx^2+c}\text{erfi}(a+bx)dx-\frac{e^{c+dx^2}\text{erfi}(a+bx)}{x}\right)-$$

$$\frac{e^{c+dx^2}\text{erfi}(a+bx)}{3x^3}$$

input
```
Int[(E^(c + d*x^2)*Erfi[a + b*x])/x^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{d\,x^2+c}\,\mathrm{erfi}\left(bx+a\right)}{x^4}dx$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^4,x)
```

output
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{x^4}\,dx = \int \frac{\mathrm{erfi}\left(bx+a\right)e^{\left(dx^2+c\right)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
integral(erfi(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 62.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\mathrm{erfi}(a+bx)}{x^4}\,dx = e^c\int \frac{e^{dx^2}\,\mathrm{erfi}\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(exp(d*x**2+c)*erfi(b*x+a)/x**4,x)
```

output
```
exp(c)*Integral(exp(d*x**2)*erfi(a + b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx = \int \frac{\text{erfi}\,(bx+a)\,e^{(dx^2+c)}}{x^4}\,dx$$

input
```
integrate(exp(d*x^2+c)*erfi(b*x+a)/x^4,x, algorithm="giac")
```

output
```
integrate(erfi(b*x + a)*e^(d*x^2 + c)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx=\int \frac{\text{erfi}(a+b\,x)\,\mathrm{e}^{d\,x^2+c}}{x^4}\,dx$$

input
```
int((erfi(a + b*x)*exp(c + d*x^2))/x^4,x)
```

output
```
int((erfi(a + b*x)*exp(c + d*x^2))/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{e^{c+dx^2}\text{erfi}(a+bx)}{x^4}\,dx=-e^c\left(\int \frac{e^{d\,x^2}\text{erf}(bix+ai)}{x^4}dx\right)i$$

input
```
int(exp(d*x^2+c)*erfi(b*x+a)/x^4,x)
```

output
```
- e**c*int((e**(d*x**2)*erf(a*i + b*i*x))/x**4,x)*i
```

3.303 $\int \left(\frac{e^{-b^2x^2}\mathbf{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\mathbf{erfi}(bx)}{x}\right)dx$

Optimal result

Integrand size = 40, antiderivative size = 33

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)dx=-\frac{b}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}$$

output
```
-b/Pi^(1/2)/x-1/2*erfi(b*x)/exp(b^2*x^2)/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)dx=-\frac{b}{\sqrt{\pi}x}-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}$$

input
```
Integrate[Erfi[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfi[b*x])/(E^(b^2*x^2)*x),x]
```

output
```
-(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2 e^{-b^2x^2}\text{erfi}(bx)}{x}+\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^{-b^2x^2}\text{erfi}(bx)}{2x^2}-\frac{b}{\sqrt{\pi}x}$$

input
```
Int[Erfi[b*x]/(E^(b^2*x^2)*x^3) + (b^2*Erfi[b*x])/(E^(b^2*x^2)*x),x]
```

output
```
-(b/(Sqrt[Pi]*x)) - Erfi[b*x]/(2*E^(b^2*x^2)*x^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\left(-2\,{\mathrm e}^{b^{2} x^{2}} b x -\operatorname{erfi}\left(b x \right) \sqrt{\pi}\right) {\mathrm e}^{-b^{2} x^{2}}}{2 \sqrt{\pi}\, x^{2}}$	41
parallelrisch	$\frac{\left(-2\,{\mathrm e}^{b^{2} x^{2}} b x -\operatorname{erfi}\left(b x \right) \sqrt{\pi}\right) {\mathrm e}^{-b^{2} x^{2}}}{2 \sqrt{\pi}\, x^{2}}$	41

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x,method=_RETU
RNVERBOSE)
```

output
```
1/2*(-2*exp(b^2*x^2)*b*x-erfi(b*x)*Pi^(1/2))/Pi^(1/2)/x^2/exp(b^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)\,dx=-\frac{\left(2\,\sqrt{\pi}bxe^{\left(b^2x^2\right)}+\pi\,\text{erfi}\left(bx\right)\right)e^{\left(-b^2x^2\right)}}{2\,\pi x^2}$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x, algor
ithm="fricas")
```

output
```
-1/2*(2*sqrt(pi)*b*x*e^(b^2*x^2) + pi*erfi(b*x))*e^(-b^2*x^2)/(pi*x^2)
```

Sympy [A] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)\,dx=\frac{2b^3x{}_2F_2\left(\begin{matrix}\frac{1}{2},1\\\frac{3}{2},\frac{3}{2}\end{matrix}\middle|-b^2x^2\right)}{\sqrt{\pi}}-\frac{2b{}_2F_2\left(\begin{matrix}-\frac{1}{2},1\\\frac{1}{2},\frac{3}{2}\end{matrix}\middle|-b^2x^2\right)}{\sqrt{\pi}x}$$

input
```
integrate(erfi(b*x)/exp(b**2*x**2)/x**3+b**2*erfi(b*x)/exp(b**2*x**2)/x,x)
```

output
```
2*b**3*x*hyper((1/2, 1), (3/2, 3/2), -b**2*x**2)/sqrt(pi) - 2*b*hyper((-1/
2, 1), (1/2, 3/2), -b**2*x**2)/(sqrt(pi)*x)
```

Maxima [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)\,dx=\int \frac{b^2\,\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x}+\frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^3}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x, algor
ithm="maxima")
```

output
```
integrate(b^2*erfi(b*x)*e^(-b^2*x^2)/x + erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Giac [F]

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)\,dx=\int \frac{b^2\,\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x}+\frac{\text{erfi}\,(bx)\,e^{(-b^2x^2)}}{x^3}\,dx$$

input
```
integrate(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x, algor
ithm="giac")
```

output
```
integrate(b^2*erfi(b*x)*e^(-b^2*x^2)/x + erfi(b*x)*e^(-b^2*x^2)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \left(\frac{e^{-b^2x^2}\text{erfi}(bx)}{x^3}+\frac{b^2e^{-b^2x^2}\text{erfi}(bx)}{x}\right)\,dx=-\frac{\mathrm{e}^{-b^2\,x^2}\,\text{erfi}(b\,x)}{2\,x^2}-\frac{b}{x\,\sqrt{\pi}}$$

input
```
int((exp(-b^2*x^2)*erfi(b*x))/x^3 + (b^2*exp(-b^2*x^2)*erfi(b*x))/x,x)
```

output
```
- (exp(-b^2*x^2)*erfi(b*x))/(2*x^2) - b/(x*pi^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \left(\frac{e^{-b^2x^2}\mathrm{erfi}(bx)}{x^3} + \frac{b^2e^{-b^2x^2}\mathrm{erfi}(bx)}{x} \right) dx = \frac{\mathrm{erf}(bix)\, i\pi - 2\sqrt{\pi}\, e^{b^2x^2}bx}{2e^{b^2x^2}\pi\, x^2}$$

input
```
int(erfi(b*x)/exp(b^2*x^2)/x^3+b^2*erfi(b*x)/exp(b^2*x^2)/x,x)
```

output
```
(erf(b*i*x)*i*pi - 2*sqrt(pi)*e**(b**2*x**2)*b*x)/(2*e**(b**2*x**2)*pi*x**
2)
```

3.304 $\int \mathbf{erfi}(bx)\sin(c+ib^2x^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \operatorname{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx=\frac{ie^{-ic}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}-\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*I*Pi^(1/2)*erfi(b*x)^2/b/exp(I*c)-1/2*I*b*exp(I*c)*x^2*hypergeom([1, 1
],[3/2, 2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [F]

$$\int \operatorname{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \operatorname{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx$$

input
```
Integrate[Erfi[b*x]*Sin[c + I*b^2*x^2],x]
```

output
```
Integrate[Erfi[b*x]*Sin[c + I*b^2*x^2], x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6960, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{6960}$$

$$\frac{1}{2}i\int e^{b^2x^2-ic}\text{erfi}(bx)dx-\frac{1}{2}i\int e^{ic-b^2x^2}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{6929}$$

$$\frac{i\sqrt{\pi}e^{-ic}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}-\frac{1}{2}i\int e^{ic-b^2x^2}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{15}$$

$$\frac{i\sqrt{\pi}e^{-ic}\text{erfi}(bx)^2}{8b}-\frac{1}{2}i\int e^{ic-b^2x^2}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{6932}$$

$$\frac{i\sqrt{\pi}e^{-ic}\text{erfi}(bx)^2}{8b}-\frac{ibe^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]*Sin[c + I*b^2*x^2],x]
```

output
```
((I/8)*Sqrt[Pi]*Erfi[b*x]^2)/(b*E^(I*c)) - ((I/2)*b*E^(I*c)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6960
```
Int[Erfi[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[I/2   Int
[E^((-I)*c - I*d*x^2)*Erfi[b*x], x], x] - Simp[I/2   Int[E^(I*c + I*d*x^2)*
Erfi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \operatorname{erfi}(bx)\sin\left(ib^2x^2+c\right)dx$$

input
```
int(erfi(b*x)*sin(c+I*b^2*x^2),x)
```

output
```
int(erfi(b*x)*sin(c+I*b^2*x^2),x)
```

Fricas [F]

$$\int \operatorname{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \operatorname{erfi}(bx)\sin\left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erfi(b*x)*sin(c+I*b^2*x^2),x, algorithm="fricas")
```

output
```
integral(1/2*(-I*erfi(b*x)*e^(-2*b^2*x^2 + 2*I*c) + I*erfi(b*x))*e^(b^2*x^
2 - I*c), x)
```

Sympy [F]

$$\int \operatorname{erfi}(bx)\sin \left(c+ib^2x^2\right)\,dx=\int \sin \left(ib^2x^2+c\right)\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(erfi(b*x)*sin(c+I*b**2*x**2),x)
```

output
```
Integral(sin(I*b**2*x**2 + c)*erfi(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfi}(bx)\sin \left(c+ib^2x^2\right)\,dx=\int \operatorname{erfi}\left(bx\right)\sin \left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erfi(b*x)*sin(c+I*b^2*x^2),x, algorithm="maxima")
```

output
```
1/8*I*sqrt(pi)*cos(c)*erfi(b*x)^2/b + 1/8*sqrt(pi)*erfi(b*x)^2*sin(c)/b -
1/2*I*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) + 1/2*integrate(erfi(b*x
)*e^(-b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \operatorname{erfi}(bx)\sin \left(c+ib^2x^2\right)\,dx=\int \operatorname{erfi}\left(bx\right)\sin \left(i\,b^2x^2+c\right)\,dx$$

input
```
integrate(erfi(b*x)*sin(c+I*b^2*x^2),x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*sin(I*b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx=\int \sin\left(b^2\,x^2\,1\mathrm{i}+c\right)\,\operatorname{erfi}(b\,x)\,dx$$

input
```
int(sin(c + b^2*x^2*1i)*erfi(b*x),x)
```

output
```
int(sin(c + b^2*x^2*1i)*erfi(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfi}(bx)\sin\left(c+ib^2x^2\right)\,dx=-\left(\int \operatorname{erf}(bix)\sin\left(b^2i\,x^2+c\right)dx\right)i$$

input
```
int(erfi(b*x)*sin(c+I*b^2*x^2),x)
```

output
```
- int(erf(b*i*x)*sin(b**2*i*x**2 + c),x)*i
```

3.305 $\int \mathbf{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \text{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\frac{ie^{ic}\sqrt{\pi}\,\text{erfi}(bx)^2}{8b}+\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*I*exp(I*c)*Pi^(1/2)*erfi(b*x)^2/b+1/2*I*b*x^2*hypergeom([1, 1],[3/2,
2],-b^2*x^2)/exp(I*c)/Pi^(1/2)
```

Mathematica [F]

$$\int \text{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int \text{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx$$

input
```
Integrate[Erfi[b*x]*Sin[c - I*b^2*x^2],x]
```

output
```
Integrate[Erfi[b*x]*Sin[c - I*b^2*x^2], x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6960, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6960}$$

$$\frac{1}{2}i\int e^{-b^2x^2-ic}\text{erfi}(bx)dx-\frac{1}{2}i\int e^{b^2x^2+ic}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{1}{2}i\int e^{-b^2x^2-ic}\text{erfi}(bx)dx-\frac{i\sqrt{\pi}e^{ic}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}i\int e^{-b^2x^2-ic}\text{erfi}(bx)dx-\frac{i\sqrt{\pi}e^{ic}\text{erfi}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6932}$$

$$\frac{ibe^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}-\frac{i\sqrt{\pi}e^{ic}\text{erfi}(bx)^2}{8b}$$

input
```
Int[Erfi[b*x]*Sin[c - I*b^2*x^2],x]
```

output
```
((-1/8*I)*E^(I*c)*Sqrt[Pi]*Erfi[b*x]^2)/b + ((I/2)*b*x^2*HypergeometricPFQ
[{1, 1}, {3/2, 2}, -(b^2*x^2)])/(E^(I*c)*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6960
```
Int[Erfi[(b_.)*(x_)]*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[I/2   Int
[E^((-I)*c - I*d*x^2)*Erfi[b*x], x], x] - Simp[I/2   Int[E^(I*c + I*d*x^2)*
Erfi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int -\operatorname{erfi}(bx)\sin\left(ib^2x^2-c\right)dx$$

input
```
int(-erfi(b*x)*sin(-c+I*b^2*x^2),x)
```

output
```
int(-erfi(b*x)*sin(-c+I*b^2*x^2),x)
```

Fricas [F]

$$\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erfi}(bx)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erfi(b*x)*sin(-c+I*b^2*x^2),x, algorithm="fricas")
```

output
```
integral(1/2*(I*erfi(b*x)*e^(-2*b^2*x^2 - 2*I*c) - I*erfi(b*x))*e^(b^2*x^2
 + I*c), x)
```

Sympy [F]

$$\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=-\int \sin\left(ib^2x^2-c\right)\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(-erfi(b*x)*sin(-c+I*b**2*x**2),x)
```

output
```
-Integral(sin(I*b**2*x**2 - c)*erfi(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erfi}\left(bx\right)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erfi(b*x)*sin(-c+I*b^2*x^2),x, algorithm="maxima")
```

output
```
-1/8*I*sqrt(pi)*cos(c)*erfi(b*x)^2/b + 1/8*sqrt(pi)*erfi(b*x)^2*sin(c)/b +
 1/2*I*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) + 1/2*integrate(erfi(b*
x)*e^(-b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int -\operatorname{erfi}\left(bx\right)\sin\left(i\,b^2x^2-c\right)\,dx$$

input
```
integrate(-erfi(b*x)*sin(-c+I*b^2*x^2),x, algorithm="giac")
```

output
```
integrate(-erfi(b*x)*sin(I*b^2*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=\int \sin\left(c-b^2\,x^2\,1\mathrm{i}\right)\,\operatorname{erfi}(b\,x)\,dx$$

input
```
int(sin(c - b^2*x^2*1i)*erfi(b*x),x)
```

output
```
int(sin(c - b^2*x^2*1i)*erfi(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfi}(bx)\sin\left(c-ib^2x^2\right)\,dx=\left(\int \operatorname{erf}(bix)\sin\left(b^2i\,x^2-c\right)dx\right)i$$

input
```
int(-erfi(b*x)*sin(-c+I*b^2*x^2),x)
```

output
```
int(erf(b*i*x)*sin(b**2*i*x**2 - c),x)*i
```

3.306 $\int \cos(c+ib^2x^2)\,\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \cos(c+ib^2x^2)\,\text{erfi}(bx)\,dx = \frac{e^{-ic}\sqrt{\pi}\text{erfi}(bx)^2}{8b} + \frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*Pi^(1/2)*erfi(b*x)^2/b/exp(I*c)+1/2*b*exp(I*c)*x^2*hypergeom([1, 1],[3
/2, 2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [F]

$$\int \cos(c+ib^2x^2)\,\text{erfi}(bx)\,dx = \int \cos(c+ib^2x^2)\,\text{erfi}(bx)\,dx$$

input
```
Integrate[Cos[c + I*b^2*x^2]*Erfi[b*x],x]
```

output
```
Integrate[Cos[c + I*b^2*x^2]*Erfi[b*x], x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6963, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\cos\left(c+ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6963}$$

$$\frac{1}{2}\int e^{ic-b^2x^2}\text{erfi}(bx)dx+\frac{1}{2}\int e^{b^2x^2-ic}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{1}{2}\int e^{ic-b^2x^2}\text{erfi}(bx)dx+\frac{\sqrt{\pi}e^{-ic}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\int e^{ic-b^2x^2}\text{erfi}(bx)dx+\frac{\sqrt{\pi}e^{-ic}\text{erfi}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6932}$$

$$\frac{be^{ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-ic}\text{erfi}(bx)^2}{8b}$$

input
```
Int[Cos[c + I*b^2*x^2]*Erfi[b*x],x]
```

output
```
(Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^(I*c)) + (b*E^(I*c)*x^2*HypergeometricPFQ[{1
, 1}, {3/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6963
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int
[E^((-I)*c - I*d*x^2)*Erfi[b*x], x], x] + Simp[1/2   Int[E^(I*c + I*d*x^2)*
Erfi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \cos \left(i b^2 x^2 + c\right) \operatorname{erfi}\left(b x\right) dx$$

input
```
int(cos(c+I*b^2*x^2)*erfi(b*x),x)
```

output
```
int(cos(c+I*b^2*x^2)*erfi(b*x),x)
```

Fricas [F]

$$\int \cos \left(c + i b^2 x^2\right) \operatorname{erfi}(b x) \, dx = \int \cos \left(i \, b^2 x^2 + c\right) \operatorname{erfi}\left(b x\right) \, dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(1/2*(erfi(b*x)*e^(-2*b^2*x^2 + 2*I*c) + erfi(b*x))*e^(b^2*x^2 - I
*c), x)
```

Sympy [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfi}(bx) \, dx = \int \cos \left(ib^2x^2+c\right) \operatorname{erfi}\left(bx\right) dx$$

input
```
integrate(cos(c+I*b**2*x**2)*erfi(b*x),x)
```

output
```
Integral(cos(I*b**2*x**2 + c)*erfi(b*x), x)
```

Maxima [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfi}(bx) \, dx = \int \cos \left(i\,b^2x^2+c\right) \operatorname{erfi}\left(bx\right) \,dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erfi(b*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*cos(c)*erfi(b*x)^2/b - 1/8*I*sqrt(pi)*erfi(b*x)^2*sin(c)/b +
1/2*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) + 1/2*I*integrate(erfi(b*x
)*e^(-b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfi}(bx) \, dx = \int \cos \left(i\,b^2x^2+c\right) \operatorname{erfi}\left(bx\right) \,dx$$

input
```
integrate(cos(c+I*b^2*x^2)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(cos(I*b^2*x^2 + c)*erfi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfi}(bx)\,dx=\int \cos \left(b^2\,x^2\,1\mathrm{i}+c\right)\,\operatorname{erfi}(b\,x)\,dx$$

input
```
int(cos(c + b^2*x^2*1i)*erfi(b*x),x)
```

output
```
int(cos(c + b^2*x^2*1i)*erfi(b*x), x)
```

Reduce [F]

$$\int \cos \left(c+ib^2x^2\right) \operatorname{erfi}(bx)\,dx=-\left(\int \cos \left(b^2 i\,x^2+c\right) \operatorname{erf}(bix)\,dx\right) i$$

input
```
int(cos(c+I*b^2*x^2)*erfi(b*x),x)
```

output
```
- int(cos(b**2*i*x**2 + c)*erf(b*i*x),x)*i
```

3.307 $\int \cos(c - ib^2x^2)\,\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \cos(c - ib^2x^2)\,\text{erfi}(bx)\,dx = \frac{e^{ic}\sqrt{\pi}\text{erfi}(bx)^2}{8b} + \frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*exp(I*c)*Pi^(1/2)*erfi(b*x)^2/b+1/2*b*x^2*hypergeom([1, 1],[3/2, 2],-b
^2*x^2)/exp(I*c)/Pi^(1/2)
```

Mathematica [F]

$$\int \cos(c - ib^2x^2)\,\text{erfi}(bx)\,dx = \int \cos(c - ib^2x^2)\,\text{erfi}(bx)\,dx$$

input
```
Integrate[Cos[c - I*b^2*x^2]*Erfi[b*x],x]
```

output
```
Integrate[Cos[c - I*b^2*x^2]*Erfi[b*x], x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6963, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erfi}(bx)\cos\left(c-ib^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6963}$$

$$\frac{1}{2}\int e^{-b^2x^2-ic}\operatorname{erfi}(bx)dx+\frac{1}{2}\int e^{b^2x^2+ic}\operatorname{erfi}(bx)dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{1}{2}\int e^{-b^2x^2-ic}\operatorname{erfi}(bx)dx+\frac{\sqrt{\pi}e^{ic}\int \operatorname{erfi}(bx)d\operatorname{erfi}(bx)}{4b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\int e^{-b^2x^2-ic}\operatorname{erfi}(bx)dx+\frac{\sqrt{\pi}e^{ic}\operatorname{erfi}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6932}$$

$$\frac{be^{-ic}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{ic}\operatorname{erfi}(bx)^2}{8b}$$

input
```
Int[Cos[c - I*b^2*x^2]*Erfi[b*x],x]
```

output
```
(E^(I*c)*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -(b^2*x^2)])/(2*E^(I*c)*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6963
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[1/2   Int
[E^((-I)*c - I*d*x^2)*Erfi[b*x], x], x] + Simp[1/2   Int[E^(I*c + I*d*x^2)*
Erfi[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, -b^4]
```

Maple [F]

$$\int \cos\left(i b^2 x^2 - c\right) \operatorname{erfi}\left(bx\right) dx$$

input
```
int(cos(-c+I*b^2*x^2)*erfi(b*x),x)
```

output
```
int(cos(-c+I*b^2*x^2)*erfi(b*x),x)
```

Fricas [F]

$$\int \cos\left(c - i b^2 x^2\right) \operatorname{erfi}(bx)\, dx = \int \cos\left(i\, b^2 x^2 - c\right) \operatorname{erfi}\left(bx\right)\, dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(1/2*(erfi(b*x)*e^(-2*b^2*x^2 - 2*I*c) + erfi(b*x))*e^(b^2*x^2 + I
*c), x)
```

Sympy [F]

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erfi}(bx)\, dx = \int \cos\left(ib^2x^2 - c\right) \operatorname{erfi}\left(bx\right) dx$$

input
```
integrate(cos(-c+I*b**2*x**2)*erfi(b*x),x)
```

output
```
Integral(cos(I*b**2*x**2 - c)*erfi(b*x), x)
```

Maxima [F]

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erfi}(bx)\, dx = \int \cos\left(i\, b^2x^2 - c\right) \operatorname{erfi}\left(bx\right)\, dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erfi(b*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*cos(c)*erfi(b*x)^2/b + 1/8*I*sqrt(pi)*erfi(b*x)^2*sin(c)/b +
1/2*cos(c)*integrate(erfi(b*x)*e^(-b^2*x^2), x) - 1/2*I*integrate(erfi(b*x
)*e^(-b^2*x^2), x)*sin(c)
```

Giac [F]

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erfi}(bx)\, dx = \int \cos\left(i\, b^2x^2 - c\right) \operatorname{erfi}\left(bx\right)\, dx$$

input
```
integrate(cos(-c+I*b^2*x^2)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(cos(I*b^2*x^2 - c)*erfi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erfi}(bx)\, dx = \int \cos\left(c - b^2\, x^2\, 1\mathrm{i}\right)\, \operatorname{erfi}(b\, x)\, dx$$

input
```
int(cos(c - b^2*x^2*1i)*erfi(b*x),x)
```

output
```
int(cos(c - b^2*x^2*1i)*erfi(b*x), x)
```

Reduce [F]

$$\int \cos\left(c - ib^2x^2\right) \operatorname{erfi}(bx)\, dx = -\left(\int \cos\left(b^2 i\, x^2 - c\right) \operatorname{erf}(bix)\, dx\right) i$$

input
```
int(cos(-c+I*b^2*x^2)*erfi(b*x),x)
```

output
```
- int(cos(b**2*i*x**2 - c)*erf(b*i*x),x)*i
```

3.308 $\int \mathbf{erfi}(bx) \sinh \left(c+b^2x^2\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \text{erfi}(bx) \sinh \left(c+b^2x^2\right) \, dx=\frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*exp(c)*Pi^(1/2)*erfi(b*x)^2/b-1/2*b*x^2*hypergeom([1, 1],[3/2, 2],-b^2
*x^2)/exp(c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \text{erfi}(bx) \sinh \left(c+b^2x^2\right) \, dx$$
$$=\frac{4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cosh(c)-\sinh(c))+\pi\text{erfi}(bx)(-2\text{erf}(bx)(\cosh(c)-\sinh(c))+\text{erfi}(bx)(\cosh(c}{8b\sqrt{\pi}}$$

input
```
Integrate[Erfi[b*x]*Sinh[c + b^2*x^2],x]
```

output
```
(4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] - Sinh[c]
) + Pi*Erfi[b*x]*(-2*Erf[b*x]*(Cosh[c] - Sinh[c]) + Erfi[b*x]*(Cosh[c] + S
inh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6966, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

$$\Big\downarrow \; 6966$$

$$\frac{1}{2}\int e^{b^2x^2+c}\text{erfi}(bx)dx-\frac{1}{2}\int e^{-b^2x^2-c}\text{erfi}(bx)dx$$

$$\Big\downarrow \; 6929$$

$$\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}-\frac{1}{2}\int e^{-b^2x^2-c}\text{erfi}(bx)dx$$

$$\Big\downarrow \; 15$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{8b}-\frac{1}{2}\int e^{-b^2x^2-c}\text{erfi}(bx)dx$$

$$\Big\downarrow \; 6932$$

$$\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{8b}-\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

input
```
Int[Erfi[b*x]*Sinh[c + b^2*x^2],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) - (b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
2}, -(b^2*x^2)])/(2*E^c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6966
```
Int[Erfi[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfi[b*x], x], x] - Simp[1/2   Int[E^(-c - d*x^2)*Erfi[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \operatorname{erfi}(bx)\sinh\left(b^2x^2+c\right)dx$$

input
```
int(erfi(b*x)*sinh(b^2*x^2+c),x)
```

output
```
int(erfi(b*x)*sinh(b^2*x^2+c),x)
```

Fricas [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \operatorname{erfi}(bx)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erfi(b*x)*sinh(b^2*x^2+c),x, algorithm="fricas")
```

output
```
integral(erfi(b*x)*sinh(b^2*x^2 + c), x)
```

Sympy [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \sinh\left(b^2x^2+c\right)\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(erfi(b*x)*sinh(b**2*x**2+c),x)
```

output
```
Integral(sinh(b**2*x**2 + c)*erfi(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \operatorname{erfi}\left(bx\right)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erfi(b*x)*sinh(b^2*x^2+c),x, algorithm="maxima")
```

output
```
integrate(erfi(b*x)*sinh(b^2*x^2 + c), x)
```

Giac [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \operatorname{erfi}\left(bx\right)\sinh\left(b^2x^2+c\right)\,dx$$

input
```
integrate(erfi(b*x)*sinh(b^2*x^2+c),x, algorithm="giac")
```

output
```
integrate(erfi(b*x)*sinh(b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx=\int \sinh\left(b^2\,x^2+c\right)\,\operatorname{erfi}(b\,x)\,dx$$

input
```
int(sinh(c + b^2*x^2)*erfi(b*x),x)
```

output
```
int(sinh(c + b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c+b^2x^2\right)\,dx=-\left(\int \operatorname{erf}(bix)\sinh\left(b^2x^2+c\right)dx\right)i$$

input
```
int(erfi(b*x)*sinh(b^2*x^2+c),x)
```

output
```
- int(erf(b*i*x)*sinh(b**2*x**2 + c),x)*i
```

3.309 $\int \mathbf{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=-\frac{e^{-c}\sqrt{\pi}\operatorname{erfi}(bx)^2}{8b}+\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
-1/8*Pi^(1/2)*erfi(b*x)^2/b/exp(c)+1/2*b*exp(c)*x^2*hypergeom([1, 1],[3/2,
 2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx$$
$$=\frac{-4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cosh(c)+\sinh(c))+\pi\operatorname{erfi}(bx)(\operatorname{erfi}(bx)(-\cosh(c)+\sinh(c))+2\operatorname{erf}(bx)(\cosh}{8b\sqrt{\pi}}$$

input
```
Integrate[Erfi[b*x]*Sinh[c - b^2*x^2],x]
```

output
```
(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c
]) + Pi*Erfi[b*x]*(Erfi[b*x]*(-Cosh[c] + Sinh[c]) + 2*Erf[b*x]*(Cosh[c] +
Sinh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6966, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx$$

$$\Big\downarrow \; 6966$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfi}(bx)dx-\frac{1}{2}\int e^{b^2x^2-c}\text{erfi}(bx)dx$$

$$\Big\downarrow \; 6929$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfi}(bx)dx-\frac{\sqrt{\pi}e^{-c}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfi}(bx)dx-\frac{\sqrt{\pi}e^{-c}\text{erfi}(bx)^2}{8b}$$

$$\Big\downarrow \; 6932$$

$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi}e^{-c}\text{erfi}(bx)^2}{8b}$$

input
```
Int[Erfi[b*x]*Sinh[c - b^2*x^2],x]
```

output
```
-1/8*(Sqrt[Pi]*Erfi[b*x]^2)/(b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6966
```
Int[Erfi[(b_.)*(x_)]*Sinh[(c_.) + (d_.)*(x_)^2], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfi[b*x], x], x] - Simp[1/2   Int[E^(-c - d*x^2)*Erfi[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int -\operatorname{erfi}(bx)\sinh\left(b^2x^2-c\right)dx$$

input
```
int(-erfi(b*x)*sinh(b^2*x^2-c),x)
```

output
```
int(-erfi(b*x)*sinh(b^2*x^2-c),x)
```

Fricas [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int -\operatorname{erfi}(bx)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erfi(b*x)*sinh(b^2*x^2-c),x, algorithm="fricas")
```

output
```
integral(-erfi(b*x)*sinh(b^2*x^2 - c), x)
```

Sympy [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=-\int \sinh\left(b^2x^2-c\right)\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(-erfi(b*x)*sinh(b**2*x**2-c),x)
```

output
```
-Integral(sinh(b**2*x**2 - c)*erfi(b*x), x)
```

Maxima [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int -\operatorname{erfi}\left(bx\right)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erfi(b*x)*sinh(b^2*x^2-c),x, algorithm="maxima")
```

output
```
-integrate(erfi(b*x)*sinh(b^2*x^2 - c), x)
```

Giac [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int -\operatorname{erfi}\left(bx\right)\sinh\left(b^2x^2-c\right)\,dx$$

input
```
integrate(-erfi(b*x)*sinh(b^2*x^2-c),x, algorithm="giac")
```

output
```
integrate(-erfi(b*x)*sinh(b^2*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=\int \sinh\left(c-b^2\,x^2\right)\,\operatorname{erfi}(b\,x)\,dx$$

input
```
int(sinh(c - b^2*x^2)*erfi(b*x),x)
```

output
```
int(sinh(c - b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int \operatorname{erfi}(bx)\sinh\left(c-b^2x^2\right)\,dx=\left(\int \operatorname{erf}(bix)\sinh\left(b^2x^2-c\right)dx\right)i$$

input
```
int(-erfi(b*x)*sinh(b^2*x^2-c),x)
```

output
```
int(erf(b*i*x)*sinh(b**2*x**2 - c),x)*i
```

3.310 $\int \cosh\left(c+b^2x^2\right) \mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \cosh\left(c+b^2x^2\right) \text{erfi}(bx)\,dx = \frac{e^c\sqrt{\pi}\text{erfi}(bx)^2}{8b} + \frac{be^{-c}x^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*exp(c)*Pi^(1/2)*erfi(b*x)^2/b+1/2*b*x^2*hypergeom([1, 1],[3/2, 2],-b^2
*x^2)/exp(c)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \cosh\left(c+b^2x^2\right) \text{erfi}(bx)\,dx$$
$$= \frac{4b^2x^2\, {}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)\left(-\cosh(c)+\sinh(c)\right)+\pi\text{erfi}(bx)(2\text{erf}(bx)(\cosh(c)-\sinh(c))+\text{erfi}(bx)(\cosh(\ldots}{8b\sqrt{\pi}}$$

input
```
Integrate[Cosh[c + b^2*x^2]*Erfi[b*x],x]
```

output
```
(4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(-Cosh[c] + Sinh[c
]) + Pi*Erfi[b*x]*(2*Erf[b*x]*(Cosh[c] - Sinh[c]) + Erfi[b*x]*(Cosh[c] + S
inh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6969, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\cosh\left(b^2x^2+c\right)\,dx$$

$$\Big\downarrow \text{ 6969}$$

$$\frac{1}{2}\int e^{-b^2x^2-c}\text{erfi}(bx)dx+\frac{1}{2}\int e^{b^2x^2+c}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{1}{2}\int e^{-b^2x^2-c}\text{erfi}(bx)dx+\frac{\sqrt{\pi}e^c\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\int e^{-b^2x^2-c}\text{erfi}(bx)dx+\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6932}$$

$$\frac{be^{-c}x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^c\text{erfi}(bx)^2}{8b}$$

input
```
Int[Cosh[c + b^2*x^2]*Erfi[b*x],x]
```

output
```
(E^c*Sqrt[Pi]*Erfi[b*x]^2)/(8*b) + (b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
2}, -(b^2*x^2)])/(2*E^c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6969
```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfi[b*x], x], x] + Simp[1/2   Int[E^(-c - d*x^2)*Erfi[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \cosh \left(b^{2} x^{2}+c\right) \operatorname{erfi}\left(b x\right)d x$$

input
```
int(cosh(b^2*x^2+c)*erfi(b*x),x)
```

output
```
int(cosh(b^2*x^2+c)*erfi(b*x),x)
```

Fricas [F]

$$\int \cosh \left(c+b^2x^2\right) \operatorname{erfi}(bx)\,dx=\int \cosh \left(b^{2} x^{2}+c\right) \operatorname{erfi}\left(b x\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(cosh(b^2*x^2 + c)*erfi(b*x), x)
```

Sympy [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfi}(bx)\,dx=\int \cosh\left(b^2x^2+c\right)\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(cosh(b**2*x**2+c)*erfi(b*x),x)
```

output
```
Integral(cosh(b**2*x**2 + c)*erfi(b*x), x)
```

Maxima [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfi}(bx)\,dx=\int \cosh\left(b^2x^2+c\right)\operatorname{erfi}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(cosh(b^2*x^2 + c)*erfi(b*x), x)
```

Giac [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfi}(bx)\,dx=\int \cosh\left(b^2x^2+c\right)\operatorname{erfi}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2+c)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(cosh(b^2*x^2 + c)*erfi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfi}(bx)\,dx=\int \cosh\left(b^2\,x^2+c\right)\,\operatorname{erfi}(b\,x)\,dx$$

input
```
int(cosh(c + b^2*x^2)*erfi(b*x),x)
```

output
```
int(cosh(c + b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int \cosh\left(c+b^2x^2\right)\operatorname{erfi}(bx)\,dx=-\left(\int \cosh\left(b^2x^2+c\right)\operatorname{erf}(bix)\,dx\right)i$$

input
```
int(cosh(b^2*x^2+c)*erfi(b*x),x)
```

output
```
- int(cosh(b**2*x**2 + c)*erf(b*i*x),x)*i
```

3.311 $\int \cosh(c-b^2x^2)\,\mathbf{erfi}(bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \cosh\left(c-b^2x^2\right)\text{erfi}(bx)\,dx=\frac{e^{-c}\sqrt{\pi}\text{erfi}(bx)^2}{8b}+\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}$$

output
```
1/8*Pi^(1/2)*erfi(b*x)^2/b/exp(c)+1/2*b*exp(c)*x^2*hypergeom([1, 1],[3/2,
2],-b^2*x^2)/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \cosh\left(c-b^2x^2\right)\text{erfi}(bx)\,dx$$
$$=\frac{-4b^2x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;b^2x^2\right)(\cosh(c)+\sinh(c))+\pi\text{erfi}(bx)(\text{erfi}(bx)(\cosh(c)-\sinh(c))+2\text{erf}(bx)(\cosh(c}{8b\sqrt{\pi}}$$

input
```
Integrate[Cosh[c - b^2*x^2]*Erfi[b*x],x]
```

output
```
(-4*b^2*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2]*(Cosh[c] + Sinh[c
]) + Pi*Erfi[b*x]*(Erfi[b*x]*(Cosh[c] - Sinh[c]) + 2*Erf[b*x]*(Cosh[c] + S
inh[c])))/(8*b*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6969, 6929, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{erfi}(bx)\cosh\left(c-b^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6969}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfi}(bx)dx+\frac{1}{2}\int e^{b^2x^2-c}\text{erfi}(bx)dx$$

$$\Big\downarrow \text{ 6929}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfi}(bx)dx+\frac{\sqrt{\pi}e^{-c}\int \text{erfi}(bx)d\text{erfi}(bx)}{4b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\int e^{c-b^2x^2}\text{erfi}(bx)dx+\frac{\sqrt{\pi}e^{-c}\text{erfi}(bx)^2}{8b}$$

$$\Big\downarrow \text{ 6932}$$

$$\frac{be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-b^2x^2\right)}{2\sqrt{\pi}}+\frac{\sqrt{\pi}e^{-c}\text{erfi}(bx)^2}{8b}$$

input
```
Int[Cosh[c - b^2*x^2]*Erfi[b*x],x]
```

output
```
(Sqrt[Pi]*Erfi[b*x]^2)/(8*b*E^c) + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -(b^2*x^2)])/(2*Sqrt[Pi])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6969
```
Int[Cosh[(c_.) + (d_.)*(x_)^2]*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[1/2   In
t[E^(c + d*x^2)*Erfi[b*x], x], x] + Simp[1/2   Int[E^(-c - d*x^2)*Erfi[b*x]
, x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, b^4]
```

Maple [F]

$$\int \cosh \left(b^{2} x^{2}-c \right) \operatorname{erfi}\left(b x \right)d x$$

input
```
int(cosh(b^2*x^2-c)*erfi(b*x),x)
```

output
```
int(cosh(b^2*x^2-c)*erfi(b*x),x)
```

Fricas [F]

$$\int \cosh \left(c-b^2x^2\right) \operatorname{erfi}(bx)\,dx=\int \cosh \left(b^{2} x^{2}-c\right) \operatorname{erfi}\left(b x\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erfi(b*x),x, algorithm="fricas")
```

output
```
integral(cosh(b^2*x^2 - c)*erfi(b*x), x)
```

Sympy [F]

$$\int \cosh\left(c - b^2x^2\right)\operatorname{erfi}(bx)\,dx = \int \cosh\left(b^2x^2 - c\right)\operatorname{erfi}\left(bx\right)dx$$

input
```
integrate(cosh(b**2*x**2-c)*erfi(b*x),x)
```

output
```
Integral(cosh(b**2*x**2 - c)*erfi(b*x), x)
```

Maxima [F]

$$\int \cosh\left(c - b^2x^2\right)\operatorname{erfi}(bx)\,dx = \int \cosh\left(b^2x^2 - c\right)\operatorname{erfi}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erfi(b*x),x, algorithm="maxima")
```

output
```
integrate(cosh(b^2*x^2 - c)*erfi(b*x), x)
```

Giac [F]

$$\int \cosh\left(c - b^2x^2\right)\operatorname{erfi}(bx)\,dx = \int \cosh\left(b^2x^2 - c\right)\operatorname{erfi}\left(bx\right)\,dx$$

input
```
integrate(cosh(b^2*x^2-c)*erfi(b*x),x, algorithm="giac")
```

output
```
integrate(cosh(b^2*x^2 - c)*erfi(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(c - b^2x^2\right) \operatorname{erfi}(bx)\, dx = \int \cosh\left(c - b^2\, x^2\right)\, \operatorname{erfi}(b\, x)\, dx$$

input
```
int(cosh(c - b^2*x^2)*erfi(b*x),x)
```

output
```
int(cosh(c - b^2*x^2)*erfi(b*x), x)
```

Reduce [F]

$$\int \cosh\left(c - b^2x^2\right) \operatorname{erfi}(bx)\, dx = -\left(\int \cosh\left(b^2x^2 - c\right) \operatorname{erf}(bix)\, dx\right) i$$

input
```
int(cosh(b^2*x^2-c)*erfi(b*x),x)
```

output
```
- int(cosh(b**2*x**2 - c)*erf(b*i*x),x)*i
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file